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IDENTIFIERS	*Cambridge Conference on School Mathematics

ABSTRACT

The Cambridge Conference on School Mathematics (CCSM) was an association of prominent mathematicians who had a concern for mathematics education at school level, from kindergarten through grade twelve. These mathematicians organized three main conferences in three areas of mathematics education, and have carried on activities related to the findings of the conferences. The main activity in the area of school education was the Cambridge Conference on School Mathematics held in Cambridge, Massachusetts in 1963. Subsequent activity arising from the original conference led to workshops and a series of feasibility studies. The main activity in the area of teacher education was a conference in 1966. Subsequent activity arising from this conference included some in-service teacher training work in Lexington and Newton, Massachusetts and in Princeton, New Jersey. Pre-service preparation work took place at the University of California, Berkeley; Indiana University; Teachers College, Columbia University; Boston State College and other state colleges in Massachusetts. A third conference was held in 1967 to consider the integration of primary school mathematics and science. The main activity arising from this conference was classroom trials of ideas. The outcome of these trials was a new program entitled Unified Science and Mathematics for Elementary Schools (USMES). This report details activities in the three areas of concern, and includes eleven appendices of relevant materials. (Author/MK)

FINAL REPORT
of
CAMBRIDGE CONFERENCE ON SCHOOL MATHEMATICS

under

NSF Grant #1515 A3

January 1962 - August 1970

Education Development Center
55 Chapel Street
Newton, Massachusetts 02160
August 31, 1970

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Final Report

The Cambridge Conference on School Mathematics (CCSM)*

NSF Grant GE 1515

The Cambridge Conference on School Mathematics (CCSM) was an association of prominent mathematicians who had a concern for mathematics education at school level, from kindergarten through grade twelve. Since 1963 with National Science Foundation support and under the auspices of Education Development Center, these mathematicians have organized three main conferences on mathematics and have carried on activities related to the findings of the conferences. CCSM has been concerned with:

A. School Education
The main activity in this area was the Cambridge Conference on School Mathematics held in Cambridge, Massachusetts in 1963. Subsequent activity arising from the original conference led to workshops held in 1964 and 1965 and a series of feasibility studies carried out in the classroom. A joint conference was held with United Kingdom mathematics curriculum developers at Ditchley Park in Oxfordshire, England.

B. Teacher Education
The main activity was a conference on teacher education held at Pine Manor Junior College in Chestnut Hill, Massachusetts in 1966. Subsequent activity arising from this conference included some in-service teacher training work in Lexington and Newton, Massachusetts and in Princeton, New Jersey. Pre-service preparation work took place at the University of California, Berkeley; Indiana University; Teachers College, Columbia University; Boston State College and other state colleges in Massachusetts.

*List of Steering Committee Members in Appendix A.

C. Integration of Mathematics and Science Education
A third conference was held at Pine Manor in 1967 to consider the integration of primary school mathematics and science. The main activity arising from this conference was classroom trial of ideas in Lexington and Bridgewater, Massachusetts; and Chicago and Urbana, Illinois. The outcome of these trials was a new program entitled Unified Science and Mathematics for Elementary Schools (USMES) which has superseded CCSM.

Mode of Operation

The CCSM did not carry a large permanent staff. The main classroom activities of the program, the feasibility studies and the teacher education development were carried out by consultants. Program continuity was maintained by a small core of key consultants acting as a steering committee and by a small part time administrative staff at EDC. A full time staff member was employed in only one year of the seven years of program activity.

Activities of the program, conferences and classroom operations, were initiated by the steering committee and by other mathematicians receiving steering committee support for classroom trials of recommendations made at the conferences. In some cases the mathematician himself taught the classes concerned, and sometimes the usual class teacher taught under the guidance of the mathematician.

A CCSM Philosophy

It would be wrong to think that initially there existed a strong CCSM philosophy which the program was created to promulgate. The first conference was a meeting of mathematicians and users of mathematics and included many people very active in curriculum development in school mathematics. These people had developed different styles in their curriculum work and laid emphasis in different places. During the discussion at the conference it was possible to identify commonly accepted goals and to reach some agreement on how such goals could be achieved.

The question asked soon after the first conference was "Can the children learn the mathematics proposed?" After experience gained through the feasibility studies in the classrooms it was appreciated that children can deal with quite advanced topics if they are allowed to approach them in a manner suitable to their age. The question now is which of the many topics possible will be most suitable and useful in a school course.

Experience gained from the feasibility studies also reinforced the CCSM feelings about the need for pupil activity and discovery methods in the learning process. There was an increasing awareness that in many cases the mathematical ability of the children is being destroyed by the confinement placed on learning by the teaching methods used. Some people connected with CCSM became strong advocates for the open classroom, for learning by manipulation and the use of materials, for a loosening of curriculum demands and for the encouragement of open-ended learning in which the child is allowed to develop his mathematical thinking to the limits of his desire and immediate ability. In this new mathematics education the teacher role changes to one of advisor, guide, mentor, helper and sometimes teacher.

Lastly, the CCSM became very concerned about pre-service teacher education. The first Goals report intentionally ignored the restrictions teachers' capabilities could put on curriculum improvement and merely stated what were the desired mathematical goals. Classroom experience with the feasibility studies brought the realization that no long-term improvement was possible unless colleges engaged in educating teachers, examined the content and pedagogy of their own mathematics courses. The problem was that new teachers leaving college were not only inadequately prepared to deal with content but also by precept and example in their own learning in college, they had been indoctrinated into a non-active, non-manipulative, non-discovery, non-participatory approach to teaching. The later efforts of the CCSM were aimed at obtaining experience in teacher education and encouraging experimentation in teaching in mathematics education at teachers' colleges. Thus, during the years, as the CCSM emphasis on discovery learning and the use of materials has increased, it has been forced to focus its attention on pre-service teacher education.

The Unified Science and Mathematics for Elementary Schools (USMES) program and the teacher education programs proposed by Professor Springer of Indiana University are examples of the activities reflecting the views of many who worked with CCSM. Others connected with CCSM proposed another similar activity, entitled Cooperative Mathematics Course for Elementary Teachers, which was not funded.

A. School Education

The first conference, which gave the program its title, was held in Cambridge, Massachusetts, in the summer of 1963. Its purpose was to explore curriculum reform needs in mathematics "with a view to a long-range future." The deliberations of the participants, a group of twenty-five mathematicians and users of mathematics, were published by Houghton Mifflin Co., Boston, in a report, entitled Goals for School Mathematics.* In this report the participants, free from the bonds of practical considerations which govern present-

*List of Participants in Appendix B. Summary of Report in Appendix C.

day curriculum reform, were able to outline their exploratory thinking for what, at that time, seemed the distant future. The suggestions for continuing work of mathematics curriculum reform gave rise to considerable comment and discussion in education and mathematics circles. The United States Commissioner of Education at that time, Mr. Francis Keppel, commented in the Foreward to the report:

> "The present report . . . is characterized by a complete impatience with the present capacities of the educational system. It is not only that most teachers will be completely incapable of teaching much of the mathematics set forth in the curricula proposed here; most teachers would be hard up to comprehend it. No brief period of retraining will suffice. Even the first grade curriculum embodies notions with which the average teacher is totally unfamiliar.
>
> "None the less, these are the curricula toward which the schools should be aiming. If teachers cannot achieve them today, they must set their courses so that they may begin to achieve them in ten years, or twenty years, or thirty. If this is what the teacher of the future must know, the schools of education of the present must begin at once to think how to prepare these teachers. There must still be short-term curriculum reforms, they must look upon themselves as constituting a stage toward the larger goals."

An oral description and a printed summary of the findings of the conference were presented at a joint meeting of the American Mathematical Society and the Mathematical Association of America in 1963 and in Boulder, Colorado, and aroused considerable interest and some excitement. Professor Peter Hilton reported on the Conference at the International Congress of Mathematicians in Moscow in 1966.* Following the conference and as a result of the impact of the report on the mathematical world,** there was much activity preparing classroom material based on the ideas of the report and some classroom experimentation with this material. The CCSM itself, in two workshops in 1964 and 1965 and with the assistance of several of the origin

*Hilton Report in Appendix D.
**Sales and Distribution of three Goals Reports in Appendix E.

al conference participants, prepared materials and subsequently carried out some classroom experimentation with these materials. Others, working independently, have used the ideas of the Goals report to develop their own materials and to initiate experimental programs.

The CCSM, however, has never felt that it was principally engaged in the preparation of material for classroom use. The materials developed were not a curriculum, but rather isolated units of mathematical education which were used in the classroom to demonstrate that suggestions in the Goals report were possible and acceptable. After very preliminary tryout to show feasibility, the materials were made available to the mathematical and educational community for more widespread trial and more extensive use by those interested.* Copies of the Feasibility Studies are now available through ERIC Information Analysis Center for Science Education, 1460 West Lane Avenue, Columbus, Ohio 43221.

This first conference of the CCSM focussed the attention of the educational community on the far-reaching changes being envisaged in mathematical education. The conference spoke with authority because the participants included not only many eminent mathematicians but also many who were already deeply involved in developing curriculum materials in mathematics.

The importance of the conference and of its findings is also reflected in the world-wide interest in its report. Requests for conference reports have been received from Europe, India, the Far East, Africa, Australia and countries in South America.

Publications on the first CCSM conference included:

(a) Goals for School Mathematics (Houghton Mifflin, 1963)

(b) "The Cambridge Conference on School Mathematics:" A Report, W. T. Martin, 1965

*List of Feasibility Studies in Appendix F.

(c) The Continuing Work of the Cambridge Conference on School Mathematics (CCSM), Peter Hilton, "The Arithmetic Teacher", February 1966.

Copies of these publications are enclosed separately.

The Ditchley Conference on School Mathematics of Two Countries

The National Science Foundation, the Carnegie Corporation and the Nuffield Foundation provided funds for this conference. For some of the participants the conference was a beginning of a useful exchange of ideas and methods which has proved to be extremely beneficial. It gave added impetus to a movement which has brought new ideas into primary school education in mathematics and which now appear to be spreading into many parts of the United States.

A report on the Ditchley Conference by Earle L. Lomon appeared in the December 1967 issue of the American Mathematical Monthly.* A copy of the report is included with the publications on CCSM activities. A report by Professor Bryan Thwaites, co-chairman of the conference, is also included.

B. Teacher Education

Experience in developing the feasibility studies directed the attention of the CCSM towards the problems of the teachers of the future who will have to teach the mathematics suggested by the original Goals report. It was clear that present day primary teacher preparation procedures, pre-service and in-service, were unlikely to produce the sort of person who would have the mathematical knowledge or the pedagogical philosophy needed by the proposed new curricula in mathematics.

A conference to consider the problem was held in the summer of 1966. Its main objective was to establish guidelines for the mathematical prep-

*Appendix G.

aration of teachers who would be flexible enough to cope with changes in school mathematics curricula.

The outcome of this conference was reported in a second Goals book published by Houghton Mifflin and entitled Goals for the Mathematical Education of Elementary School Teachers. In this report the CCSM made firm recommendations, with alternatives, about the content and pedagogy required in a good teacher education program for mathematics. A summary of the report of the conference* was given in January 1967 at the Annual Meeting of the Mathematical Association of America.

Once again some of the participants in the conference decided that they would like to put into practice some of the ideas discussed at the conference. Teacher training Feasibility Studies were done at Boston State College, and three of the local state colleges in Massachusetts; at Teachers College, Columbia University; at the University of California, Berkeley; and at Indiana University. These studies are now completed and have been sent to ERIC.**

In the meantime the Berkeley materials have been distributed to some colleges in California which have expressed interest in trying out the material. The Boston State College materials are also in continuing use. The Indiana University has developed a much larger teacher education program as a result of its earlier CCSM experiments.

Also, as a result of this conference, a group of interested people developed a proposal for a program which would encourage the initiation of improved courses in mathematics at state colleges which prepared teachers. This proposed project was known as Co-operative Mathematics Course for Elementary Teachers (CMCET).† Unfortunately funding was not available.

*Summary of Report in Appendix H.
**ERIC Information Analysis Center for Science Education, 1460 West Lane Avenue, Columbus, Ohio 43221.
†The materials produced by these efforts are bulky and are therefore attached to this report along with several other published CCSM works.

Other feasibility studies in teacher education were undertaken, including in-service work with teachers in Lexington and Newton, Massachusetts. In this project mathematical consultants, by means of classroom visits and telephone access, made themselves available to teachers trying out new units. The few teachers who made full use of the assistance offered benefitted from the ready contact with advisers. However, in a fully developed scheme of this sort one could expect that teachers would require quite some time to become accustomed to the availability of assistance, before it would be used to its full potential.*

Another in-service teacher effort was undertaken at Princeton achieving some very fine results with children. Unfortunately support of this effort had to be cut off.

A copy of Goals for Mathematical Education of Elementary School Teachers is enclosed with other CCSM publications.

C. Integration of Mathematics and Science Education

The growing concern about the gulf between school mathematics and school science was the reason for the third main CCSM conference which took place in the late summer of 1967. The participants in this conference, including many of the leaders of school curriculum reform in mathematics and science, found that there was considerable amount of agreement on the benefits which mathematical education could derive from properly directed scientific activity, and on the need for mathematics to support science and education. The existing traditional and new curricula in mathematics and science were examined critically. Areas of possible cooperation were identified and areas of curriculum content of questionable value were subject to thorough examination.

Professor Andrew Gleason of Harvard University writing in "The Bulletin

*Report on the Consultant Access Scheme in Appendix I.

of the National Association of Secondary School Principals" said:

> "There were two major agreements reached at last summer's conference. The first, although extremely important, was easily arrived at. We agreed that education in science and mathematics was not to be thought of in elitist terms. We were not trying to "beef-up" the curriculum in an effort to see how fast we could force-feed our scientifically talented youngsters. Quite the contrary - science and mathematics have become such an integral part of our civilization that it is essential to make them meaningful to every school child.
>
> "Our second major agreement was to think not in terms of coordinating mathematics and science instruction but in terms of integrating them. Instead of having separate mathematics and science classes trying to keep pace with each other, we want a single class devoted to both subjects and emphasizing at any particular time whichever discipline seems appropriate. This is a significant decision and one not to be taken lightly."

The report of the conference on the Integration of Mathematics and Science, was published by Houghton Mifflin under the title Goals for the Correlation of Elementary Science and Mathematics.* A summary of the report was presented at the December 1967/January 1968 meeting of the American Association for the Advancement of Science in New York and at the Annual Meeting of the American Mathematics Society and the Mathematics Association of America. This book has quickly aroused widespread interest and seems likely to have an impact in some ways similar to the first Goals book. At least one university has included it in the required reading list, and there have been very many requests for copies and information from as far afield as the United Kingdom and India.

Local interest, initially aroused at the Conference, has been maintained and increased. Feasibility studies carried out in Massachusetts and

*Summary of the Report in Appendix J; a review of the Report in Appendix K

Illinois have led to the funding of a new program, Unified Science and Mathematics for Elementary Schools (USMES), which will attempt to implement some of the suggestions made at the conference. One pleasing outcome of the conference has been a continuing co-operation between some of the curricula development groups which participated. Some of the CCSM Feasibility Studies will be an integral part of background material for teachers implementing USMES units.

Conclusion

The paragraph which follows was included in the original CCSM proposal presented to the National Science Foundation in November 1962.

> "If for no other reason than that of time-scale, the present proposal should not be considered to affect the continuation of curriculum reform programs now in progress, nor the initiation of others. A new structure will not spring into being overnight. Once created, it may be expected to enter the educational system quite slowly over a long period of time; quite properly the educational system, with its close assocation to the general cultural outlook, possesses great inertia and does not normally accept sudden major change. The most successful process is likely to be one in which revision within the conventional structure is carried on contemporaneously with a revision of the structure itself, and in which the activities interact with one another and over the long term join in effecting a fundamental change."

There is little question that since 1962 there has been a fundamental change in the views of educators and mathematicians about school education and particularly about primary school education. The CCSM has played its part in bringing about this fundamental change. It has helped to change the mathematical expectations of the educators - but also has helped mathematicians to understand and appreciate the role of mathematics in the general education of children. The CCSM has played its part in bringing about major changes in school curriculum, in focusing attention on the need for mathematics and science integration at school level and initiating teacher preparation programs more likely to satisfy the school needs of the present and the future.

APPENDIX A

Cambridge Conference on School Mathematics
Steering Committee

W. T. Martin (Co-Chairman)
Massachusetts Institute of Technology

Peter J. Hilton (Co-Chairman)
Cornell University

James L. Aldrich
Education Development Center

Edward G. Begle
Stanford University

Hugh P. Bradley (Program Director)
Education Development Center

Jerome S. Bruner
Harvard University

Andrew M. Gleason
Harvard University

Earle L. Lomon
Massachusetts Institute of Technology

Edwin E. Moise
Harvard University

Henry O. Pollak
Bell Telephone Laboratories

George Springer
Indiana University

Patrick Suppes
Stanford University

Stephen White
Alfred Sloan Foundation

Jerrold R. Zacharias
Massachusetts Institute of Technology

APPENDIX B

Conference
on
School Mathematics

Cambridge, Massachusetts
Summer 1963

List of Participants

Maurice Auslander, Brandeis University

Edward G. Begle, Stanford University

Jerome S. Bruner, Harvard University

R. Creighton Buck, University of Wisconsin

George Francis Carrier, Harvard University

Julian D. Cole, California Institute of Technology

Robert Davis, Syracuse University

Robert P. Dilworth, California Institute of Technology

Bernard Friedman, University of California

H. L. Frisch, Bell Telephone Laboratories; Yeshiva University

Andrew M. Gleason, Harvard University

Peter J. Hilton, Cornell University

J. L. Hodges, Jr., University of California

Mark Kac, Rockefeller Institute

Seymour H. Koenig, IBM Watson Laboratories; Columbia University

C. C. Lin, Massachusetts Institute of Technology

Earle L. Lomon, Massachusetts Institute of Technology

W. T. Martin, Massachusetts Institute of Technology

Edwin E. Moise, Harvard University

Frederick Mosteller, Harvard University

Henry O. Pollak, Bell Telephone Laboratories

Mina S. Rees, City University of New York

Max M. Schiffer, Stanford University

George Springer, University of Kansas

Patrick Suppes, Stanford University

A. H. Taub, University of Illinois

Stephen White, Educational Services Incorporated

Samuel S. Wilks, Princeton University

Jerrold R. Zacharias, Massachusetts Institute of Technology

Conference on Teacher Education at Pine Manor Junior College, Brookline, Massachusetts June 13 - July 8, 1966

List of Participants

The following, grouped according to time spent at the meeting, participated in the Cambridge Conference on Teacher Training.

Four Weeks

Richard F. Arens, University of California, Los Angeles

Charles W. Curtis, University of Oregon

Morton L. Curtis, Rice University

Donald A. Darling, University of Michigan

F. A. Ficken, New York University

David Gale, Brown University

John W. Green, University of California, Los Angeles (Chairman)

H. J. Greenberg, University of Denver

H. Brian Griffiths, University of Southampton, England

Leon A. Henkin, University of California, Berkeley

William G. Lister, State University of New York at Stonybrook

Mina S. Rees, City University of New York

George Springer, Indiana University

Steven Szabo, University of Illinois Committee on School Mathematics

Shlomo Z. Sternberg, Harvard University

S. James Taylor, University of London, England

Bryan Thwaites, University of Southampton, England

Marion I. Walter, Education Development Center, Inc.

Edwin Weiss, Boston University

Alfred B. Willcox, Amherst College

Stephen S. Willoughby, New York University

Two Weeks

W. T. Martin, Massachusetts Institute of Technology

B. J. Pettis, University of North Carolina

Hassler Whitney, Institute for Advanced Study, Princeton

A Few Days

Edward G. Begle, Stanford University

Robert B. Davis, Webster College

H. L. Frisch, Bell Telephone Laboratories

Andrew M. Gleason, Harvard University

Peter J. Hilton, Cornell University

Samuel Karlin, Stanford University

Burt A. Kaufman, Southern Illinois University

Henry O. Pollak, Bell Telephone Laboratories

Gail S. Young, Jr., Tulane University

From Educational Development Center

Hugh P. Bradley

Jerome S. Bruner, Harvard University

John H. Durston

Edward T. Esty

Ph llis R. Klein

L. Lee Osburn

David A. Page

Messrs. C. W. Curtis, M. L. Curtis, Davis, Durston, Green, Lister, Springer, Szabo, and Willcox composed the writing group for the Report.

Conference
on
Integration of Mathematics and Science Education
at
Pine Manor Junior College, Brookline, Massachusetts
August 21 - September 8, 1967

List of Participants

Professor Andrew M. Gleason, Harvard University, Chairman of the Conference

James L. Aldrich, Education Development Center

Max Beberman, Curriculum Laboratory, University of Illinois

Edward G. Begle, Department of Mathematics, Stanford University

C. B. Bell, Department of Mathematics, Case Institute of Technology

Truman Botts, Department of Mathematics, University of Virginia

Hugh P. Bradley, Education Development Center

Randolph Brown, Education Development Center

Robert B. Davis, Madison Project, Syracuse University

J. A. Easley, Jr., Curriculum Laboratory, University of Illinois

Camilla Fano, Chicago, Illinois

Abraham S. Flexer, Biology Department, Harvard University

Maurice S. Fox, Biology Department, Massachusetts Institute of Technology

Andrew M. Gleason, Mathematics Department, Harvard University

Alan Holden, Chemistry Laboratories, Bell Telephone Laboratories

Robert Karplus, Department of Physics, University of California at Berkeley

John G. King, Department of Physics, Massachusetts Institute of Technology

Edward J. Lofgren, Radiation Laboratory, University of California at Berkeley

Earle L. Lomon, Department of Physics, Massachusetts Institute of Technology

Richard G. Long, Education Development Center

W. T. Martin, Department of Mathematics, Massachusetts Institute of Technology

Paul D. Merrick, Department of Biology, Webster College

Frank J. O'Brien, Staff Engineer, Massachusetts Institute of Technology

Henry O. Pollak, Mathematics Research Center, Bell Telephone Laboratories

Peter B. Shoresman, Elementary School Science Project, University of Illinois

Benson R. Snyder, Medical Department, Massachusetts Institute of Technology

Marion Walter, Graduate School of Education, Harvard University

Ranier Weiss, Department of Physics, Massachusetts Institute of Technology

James H. Werntz, Jr., Department of Physics, University of Minnesota

Constance E. West, Education Development Center

Jerrold R. Zacharias, Department of Physics, Massachusetts Institute of Technology.

Professor David A. Page (EDC) and Mr. Wallace Feurzeig (Bolt Beranek and Newman Inc.) each visited the conference to describe the curriculum units they were developing.

APPENDIX C

GOALS FOR SCHOOL MATHEMATICS

Summary of the Report

of

The Cambridge Conference on School Mathematics

A conference in Cambridge, Massachusetts, sponsored by the National Science Foundation and administered by Educational Services Incorporated,* was held June 18 to July 12, 1963, to discuss the future of mathematics curricula. The main purpose was to reconsider the structure of mathematics education, and to sketch a rough outline of possible new framework for the primary and secondary school. Some twenty-five mathematicians and mathematics users, from university or industry, attended the conference. The fields represented included algebra, geometry, topology, analysis, statistics, applied mathematics, physics, and chemistry.

It was agreed from the outset that, in setting goals for mathematics curricula, the conference would have to defer consideration of the serious and closely related problem of teacher training until its first task was completed. The conference also took account of the possibility that there may be instrinsic limitations on the ability of young children to handle mathematical ideas; however, it felt that the boundaries of these limitations, if they exist, are not well defined, and there is as yet little evidence concerning the degree to which they can be changed by the teaching process. Recognizing then that its work was necessarily of a tentative nature, the conference turned to its main objective, the curriculum from K through 12.

The conference found itself essentially in complete agreement on the mathematical aims of the elementary school.

Through the introduction of the number line, the child would be started immediately on the whole real number system, including negatives. To be sure, at first he would have formal names only for integers and the simplest rational numbers, but all of his work would keep him aware of the existence of other numbers, and the fact that they too have sums, products, etc. By this wedding of arithmetic and geometry at the pre-mathematical level, the intuition of the child would be developed and exploited, and the significance of the arithmetical operations enriched.

Moreover, the child provided with these complementary viewpoints, would have a very good chance to understand the essential nature of mathematics and its relationship to the "real" world.

The order properties of the real number system would be studied from the beginning, and would be used in inequalities, approximation, and order of magnitude estimates.

*Now Education Development Center

The use of Cartesian coordinates ("crossed"number lines) would begin almost as soon as the number line itself. Moreover, we agree with Freudenthal and other pioneers, that an early development of the child's spatial intuition is essential. Study of the standard shapes in two and three dimensions would continue concurrently, and would include discussion of their symmetries.

The notions of functions and set are to be used throughout; of course, set theory and formal logic should not be emphasized as such, but the child should be able to build his early mathematical experience into his habitual language. Informal algebra should be taken up along with the arithmetic operations.

The Conference agreed that reasonable proficiency in arithmetic computation and algebraic manipulations is essential to the student of mathematics. But, this is not an argument in favor of a curriculum devoted primarily to computation with contrived numbers through the whole of grammar school. Long pages of addition and multiplication problems add nothing to a student's understanding of the processes involved; nor do they teach him when to add or multiply. At best, they improve the computational speed of a student who understands how to do the algorithms (an objective that by itself had little appeal to the members of the Conference); at worst, they dissipate or destroy the interest that a good student has in the subject. Entirely adequate practice in computation can be built into problems that, on their own merits, genuinely attract the student's interest.

Because of both its intuitive appeal and its basic importance, there should be an introduction to the elementary ideas of probability and statistical judgment, accompanied by concrete experimentation with random processes.

The concern for motivation, applications, and the interplay between mathematics in the physical world, is a constant theme of the conference report. This is constrained by the limited science experience in the elementary school. However, geometry itself offers a rich area within which the students can explore the relation between physical objects and their idealized mathematical abstractions. As the student's experience deepens, it will be possible to introduce more sophisticated models.

Having studied arithmetic and geometry, mostly informally, in the elementary school, the student will be prepared for a sound treatment of geometry and the algrebra of polynomials, beginning in the seventh grade. The mathematics curriculum for the secondary school can therefore go much further than it commonly does at present. The program of a student who elected mathematics each year will, at the end of the twelfth year, have contained a closely-knit presentation of calculus, linear algebra, and probability, involving a brief introduction to other mathematical topics.

The conference did not reach any substantial agreement as to the order of presentation or the specific content for this program. Indeed the multitude of sound proposals suggest that there is certain to be no unique optimal solution. Two arrangements of the material proposed for the secondary level were developed in some detail.

The conference also arrived at other recommendations which dealt more with methods of presentation than with specific mathematical content. It was felt, for example, that it was desirable to adopt the "spiral" approach, in which every new topic is introduced early under low pressure and is then reconsidered repeatedly, each time with more sophistication, and each time showing more of its interconnections with the rest of the subject. The result should be a sort of guided tour of mathematics. This approach has many important advantages. In the first place, the basic unity of the subject is automatically stressed. Moreover, in the upper grades, this approach implies that the student will be exposed concurrently to a mixture of intuitive "pre-mathematics" and rigorous mathematics. Provided that the distinction is made clear to the student, this will give a much more honest picture of what mathematics is, an organism continuously growing through the interaction of intuition and logical analysis, rather than a static structure walled about by sterile rigor.

A second aspect of the same precept led to the suggestion that topics receive multiple motivation. During the pre-mathematical stage of some topics, it may be wise to give several different informal presentations, each leading up to the desired goal (e.g. the rules for multiplication of negatives), rather than to leave students with the feeling that there is only one correct road. Ideally, this should help to convey to the student the important fact that mathematics is something one <u>does</u>, not something that one absorbs passively. One would hope to strengthen the impression that a mathematical idea appeared first as the solution to some problem by some person. The problems thus become a matter of importance equal to or even greater than that or the textual material itself. It was therefore felt that the design of imaginative problem sequences involving combinations of routine techniques and "discovery" procedures was a matter of the greatest importance in curricular development.

There is much that must be done before the ideas in this proposal can be implemented. Some of the suggestions in the report are already being tried in some of the current educational experiments, either piecemeal or as part of some more extensive program. However, there must be many further experiments to determine just what is possible, and at what age levels. Texts and supplementary materials will have to be written. Unquestionably, the most difficult problem lies in the training of teachers, upon whom the success or failure of curricular reform ultimately rests. Nor is this an isolated problem, for the pressure to advance our mathematical goals is being felt at all levels of the profession, and one facet of the problem cannot be solved in isolation from the others. If the proposals formulated by the conference are to become a reality within the foreseeable future, it is necessary that the entire mathematical community devote considerable attention to the training of teachers at all levels.

The steering committee for this study consisted of E. G. Begle, J. S. Bruner, A. M. Gleason, M. Kac, W. T. Martin (Chairman), E. E. Moise, M. Rees, P. Suppes, S. White, and S. S. Wilks. The conference was organized and administered by Educational Services Incorporated,* Watertown, Massachusetts, under a grant from the National Science Foundation.

The following participated in the conference:

M. Auslander
E. G. Begle
R. C. Buck
G. F. Carrier
J. Cole
R. B. Davis
R. P. Dilworth
B. Friedman
H. L. Frisch
A. M. Gleason
P. J. Hilton
J. L. Hodges
S. Koenig
C. G. Lin
E. L. Lomon
E. E. Moise
F. Mosteller
H. O. Pollak
M. Rees
M. M. Schiffer
G. Springer
P. Suppes
A. H. Taub
S. S. Wilks
J. R. Zacharias

At the close of the conference a draft report was prepared by R. B. Davis, A. M. Gleason. E. L. Lomon, E. E. Moise, and G. Springer. This draft report was reviewed and revised at two meetings held late in August and attended by:

M. Auslander
E. G. Begle
R. B. Davis
H. L. Frisch
A. M. Gleason
P. J. Hilton
S. Koenig
E. L. Lomon
W. T. Martin
E. E. Moise
H. O. Pollak
G. Springer
S. S. Wilks;

by Max Beberman, Director of University of Illinois Committee on School Mathematics, Walter Prenowitz of the African Mathematics Program of Educational Services Incorporated, and Dr. Frank B. Allen, President of the National Council of Teachers of Mathematics; and by John Mays and Richard E. Paulson of the National Science Foundation.

The full report is now being prepared for publication, and will be available in the near future. In its completed form it will run somewhere in the neighborhood of 80 pages.

This summary of the full report was prepared by a sub-committee consisting of R. C. Buck, P. J. Hilton and H. O. Pollak.

September 15, 1963

*Now Education Development Center, Newton, Massachusetts

APPENDIX D

CAMBRIDGE CONFERENCE ON SCHOOL MATHEMATICS

International Congress of Mathematicians, Moscow, 1966

Report

It was decided that a report should be made to the International Congress of Mathematicians, Moscow, 1966, on the activities of the Cambridge Conference on School Mathematics (CCSM). Professor Hilton, co-chairman of CCSM, agreed to make the report which was delivered to Section 15 of the Congress on Friday morning, August 26.

Professor Hilton reviewed the history of curricular reform in mathematics in the United States, drawing heavily on the document written by Dr. Hlavaty for the Ditchley Park CCSM - SMP conference. He then discussed the special role played by CCSM since its inception in the summer of 1963, detailing the activities undertaken and laying particular emphasis on the flexibility of its operations and its attempt to prepare for future - rather than present - needs in mathematical education. A feature of CCSM which aroused particular interest was the involvement of university mathematicians of international reputation in the design of courses at the most elementary levels.

Professor Hilton closed his review by referring to the role CCSM had also tried to play of bringing together representatives of different groups and trends in curricular research to exchange ideas and information about projects in progress or about to be undertaken. He observed that such cooperation should surely extend into the international field and gave as his personal opinion the conviction that an essential requirement was the establishment of an international journal of repute in which descriptive, controversial and critical articles could be read by the scholars and teachers of the world with interests in mathematical education.

It is agreeable to report that the session was remarkably well attended (it should be noted that the 15 sections met simultaneously) and that the discussion was lively and constructive; indeed, it was necessary to allow 45 minutes instead of the usual 20 minutes for the report and discussion. Many requests were received for CCSM literature from European mathematicians, and there was general agreement on the necessity for an international journal. Two points of detail which recurred in the discussion were the following: (a) it was asked whether CCSM has a distinctive philosophy of mathematical education (explicity, does CCSM endorse Polya's view; does CCSM favor the discovery method?), and (b) is it possible for university mathematicians to contribute effectively to curricular work without exposing themselves to a substantial amount of classroom experience? The second question in particular raises issues which are very much in the mind of members of the Steering Committee of CCSM.

Professor Hilton was able to hold many informal discussions during the period of the Congress with mathematicians of various countries (including, of course, the Soviet Union) interested in problems of mathematical education. These contacts should prove of great value in the subsequent development of the work of CCSM.

APPENDIX E

Sales and Distribution

GOALS BOOKS

The three Goals books were published by Houghton Mifflin Company, Boston. Latest information on sales and distribution of each book:

Sales:	21,380	-	Goals for School Mathematics	[1963]
Samples:	4,107			
Sales:	1,848	-	Goals for Mathematical Education of Elementary School Teachers	[1967]
Samples:	3,343			
Sales:	1,688	-	Goals for the Correlation of Elementary Science and Mathematics	[1969]
Samples:	1,597			

FEASIBILITY STUDIES

Prior to sending the Feasibility Studies to ERIC Information Analysis Center for Science Education [1460 West Lane Avenue, Columbus, Ohio 43221] distribution from the EDC office was as follows: (approximate figures)

A total of 650 requests were received and answered.

1. 3,000 assorted studies have been sent in reply to 300 requests.

2. 350 full sets of studies have been sent in reply to the remaining 350 requests.

APPENDIX F

Feasibility Studies

1. A Proposed Syllabus for the Seventh Grade
2. Elementary Modern Mathematics from the Advanced Standpoint
3. Proposed Program for the Tenth Grade
4. Order Structure in Elementary Mathematics
5. A Problem
6. Units
*7. Probability
8. Notes on Desirable Responses at End of Sixth Year
9. Stream of Ideas on Checks, Approximations, and Order of Magnitude Calculations
10. Complex Numbers Leading to Trigonometry
11. Use of Negative Digits in Arithmetic
12. Use of Shift Theorem in Differential Equations
13. Topology in Tenth Grade and After
14. SMSG and the "Gifted Child"
15. What High School Juniors and Seniors Don't Know
16. The Use of Units
17. Exploration
18. The Exponential Function
19. A Proposed Course in Ninth Grade Geometry
20. Multiplication of Negative Numbers
21. Kindergarten
22. Morse School--First Grade (Inequalities Unit)
23. Morse School--Second Grade (Multiplication and The Symmetry of Squares and Triangles)
24. Morse School--Third Grade (Chip Trading & Symmetry Units)
25. Morse School--Third and Sixth Grades (Graphs and Their Applications)
26. Morse School--Third Grade (Vector Geometry)
27. Morse School--Sixth Grade (Elementary Number Theory) Superseded by #35
28. Morse School--Slopes and Limits (Lessons & Commentary)
29. Report of Activities in Cambridge during July and August, 1964 under CCSM
30. Experimental Teaching
31. Palo Alto--Second Grade (Geometry, Logic and Matrices)
*32. Stanford--Eighth Grade (Geometry through Symmetry)
*33. Progress Reports on Estabrook Project, Covering March 1964 through June 1965
34a. Demonstration of Mirror Cards to Estabrook Teachers
*34b. Informal Geometry for Young Children
34c. Symmetry Motions for Elementary School (Parts I and II)
*35. Hosmer School--Sixth Grade 1964-65 (Elementary Number Theory)
36. Report of SMSG/CCSM Conference in March, 1965
*37. Collected Reports of CCSM Writing Conference, Summer, 1965
*38. Inequalities and Real Numbers as a Basis for School Mathematics
*39. Geometry Report
*40. Symmetry Motion Classes
*41. Probability Lessons at Hancock School, Lexington
*42. "Inequality" Lessons at Adams School, Lexington
43a. An Experimental Text in Transformational Geometry - Student Text
43b. An Experimental Text in Transformational Geometry - Teachers' Guide
*44a. Geometry - Teachers' Guide
*44b. Geometry - Children's Worksheets
*45. Averages, Areas and Volumes
46. A Second-Grade Experiment in Mathematics

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*Available from: ERIC Information Analysis Center for Science Education, 1460 West Lane Avenue, Columbus, Ohio 43221

APPENDIX G

The Ditchley Conference

School Mathematics Reform in Two Countries

Earle L. Lomon

The first wave of mathematics curriculm reform has crested in both the United Kingdom and the United States. Those initial efforts were sensitive to the traditional curriculum, school organization and teacher preparation of each country. New curricula had to be separately tested on a small scale before being committed to large-scale use within each country. Until these results were seen, it was unlikely that either party could help the other significantly. Published texts and reports allowed the monitoring of the ideas and progress of the other party. Now that both countries have had experience on a wide scale with the "new math," a second wave of new curricula is forming. At this stage it is appropriate to capitalize on the corrobative, contradictory or complementary experience of the two countries. For such a purpose the Ditchley Conference was called, attended by American and British mathematicians and teachers involved in the development of the new curricula.

Initiative for the Ditchley Conference was taken by Professor Bryan Thwaites and Professor W. T. Martin. The former is director of the School Mathematics Project (S.M.P.), the most extensive project of its type in England. The latter is Chairman of the Cambridge Conference on School Mathematics (C.C.S.M.). S.M.P. undertook to invite the United Kingdom delegation and to obtain the conference facilities. The American delegates were invited by C.C.S.M., whose participation was funded by the Carnegie Corporation and the National Science Foundation. The major British and American curriculum projects were well represented at the conference; this excellent representation indicates the degree of interest aroused in both countries by the topic - a comparative evaluation of American and British curricular innovations in mathematics. The particiants are listed at the end of this article.

The meeting was held September 9 - 11, 1966, in the comfortable setting of Ditchley Park Estate, Oxfordshire. The Ditchley Foundation supports the Estate as an "Anglo-American Conference Centre." This note is intended to relate my personal impressions. A report for distribution in the United Kingdom has been prepared by Professor Thwaites.

Professor Thwaites and Professor Martin, the co-chairmen, skillfully preserved a relaxed atmosphere. The topics of the plenary and group sessions are appended. I shall make no attempt to relate my remarks to particular sessions.

As the two delegations were sufficiently sophisticated to learn as much from differences as from similarities in the two countries' mathematics programs, the conferees were alert to variations of accomplishment and attitutde. A difference affecting all of the secondary school programs is in the degree to which geometry is integrated with algebra or analysis. SMSG and other widely used new American texts largely maintain the traditional separation of these materials. Important exceptions occur in the presentation of such topics as graphing. In the texts of S.M.P. and of the Midlands Mathematical Experiment, these subjects are interlaced in each term, with frequent cross-referencing. A related divergence is the larger degree of systematization and formality in the American courses. Historical reasons for these differences are discernible, but it is more interesting to inquire about the present reactions and future intentions of the two groups.

I digress here to make a point I believe to be essential. There is a wide divergence of attitudes among those active in curriculum development on each side of the Atlantic. The spread of opinion on either side is greater than the difference of the average opinion between the two countries. Any opinion or attitude concerning school mathematics that has substantial support in one country will have important advocates in the other. Present differences of substance are mostly due to the pressure of historical and accidental circumstance. The actual diversity of opinion in the United States is well known to those who have participated. The meeting displayed a similar diversity among the initiators of curriculum reform in the United Kingdom.

The existence of reform curricula in both countries has effected a separation from the historical requirements. This allows the succeeding reforms in each country to progressively approach a common result. In my opinion the major circumstance causing the present difference of emphasis in the two countries is the relative brevity of experience in the United Kingdom with new mathematics in the elementary school. The first large-scale elementary school project was introduced only last September, by the Nuffield Mathematics Project. This project is sophisticated, and well in advance of the first such American attempts. However, its impact has not yet been felt. For most of the British curriculum developers, this leaves the early secondary school mathematics as the beginning of the "spiral" of mathematics ideas and applications. An intuitive approach at this level is thus prescribed. The dominantly intuitive development of many topics is now expected by Americans to take place in elementary school. It follows naturally that a larger degree of systematization and deductive reasoning is planned for secondary school courses. I would expect a similar evolution in the United Kingdom to take place between the coming revision and the one succeeding it.

On the other hand, the impact of the first reform has made the American schools more flexible with respect to the organization of topics. The United States projects will probably take the next opportunity to use several specific effective examples, developed in the United Kingdom and here, of combining geometric and algebraic material.

An important example of complementary curriculum investigations exists in the experience of each country with motion (or transformation) geometry. The S.M.P. program uses symmetry as the organizing principle for much of its geometry. We can here observe on a large scale the efficacy of the approach in the context of a related program. In the United States the experiments have been directed towards finding an effective intuitional development of symmetry motions in elementary school, developing through the junior high school into a systematic basis for geometry and modern algebra in high school. These experiments have been on a small scale, and not in the context of a related mathematics course. The United Kingdom experience can be taken as strong encouragement for the large-scale development of the American materials.

One of the claims most often made for the "new math" is that it teaches unifying concepts which help the student generalize and transfer from one problem to another. The introduction of "sets" at an early primary grade is usually justified in that way. It is thus salutary that delegates from both countries felt that we are only coming to grips with a unifying language and imagery. The role of "set theory" or "functions" in concisely stating the structure of different branches of mathematics was said to require more experience and sophistication in mathematics than the student would have through his school years. A corollary is that the introduction of such topics as sets and functions must avoid being pretentious or formal. There was more hope and intention among Americans than among Britons that the improvement of the curriculum would eventually permit some revelation of structurally unifying signficance of concepts.

Both groups stressed the importance of developing and continually presenting good applications of the mathematical ideas and tools. The best applications use the appropriate mathematics to study situations of importance. They elucidate the modeling process giving the student the power to devise his own applications. Unfortunately, mathematical statements are usually illustrated by an artificial question whose result is of little relevance. Often the mathematics to be practiced is not the most relevant to the problem. In both countries there have been collections made of good applications.

Good applications tend to go deeply into the field of the application, so that more than the mathematics curriculum is involved. In this country we are beginning explorations of the correlation of the mathematics with the science curricula. The absence of science material in the United Kingdom elementary schools has left the matter of correlation less urgent.

In the light of the need for good applications, the teaching of some probability and a little statistics is very appealing. All of arithmetic is applied in obtaining distributions from data, and in calculating theoretical distributions. The mathematics of probability is in turn applied to problems in almost every scientific and sociological endeavour. Experimental probability can be introduced in the earliest years, and the theory developed as facility with sets, algebra and analysis is successively developed. The subject shows what so few laymen realize, that mathematics can deal with imprecise information and indeterminate models. I expect that probability and perhaps statistics will soon have a major role in the curricula of both countries.

In present practice the British give more emphasis to cardinal numbers and discrete algebraic systems, and in the United States there has been a tendency to develop the concept of real numbers at an early stage, and later treat fields and analysis. Present opinions in both countries are widely divergent.

The program of the conference directed us to discuss matters of teaching style and method, as well as content. It is perhaps surprising that the United Kingdom emphasis on informality in the presentation of mathematics is not correlated with strong emphasis on an "open-ended discovery" classroom approach. We viewed a film in which a deeply involved Madison Project class was purposefully non-directed by Professor Robert Davis. Most, if not all, of the British present felt that the students could not extract useful results from the emotional discussion, or they questioned whether the usual teacher could handle the situation. Lest one is still tempted to oversimplify the contrast of United States and United Kingdom attitutdes, it should be noted that Professor Davis has an interest in the teaching of axiomatic structures and some level of logic to the very young!

Consideration was given to the results of behavioural science and learning studies of psychologists. In both countries there have been only minor attempts at devising curricula oriented about approaches suggested by these studies. Their results on perception and mental skills at different ages have been interpreted as indicating important but not insurmountable obstacles to the goals of the mathematics curriculum. The major programs in both countries take the findings of psychologists into account by, for instance, a careful development of spatial perception at the age they think it relevant. A more direct reaction to those findings would be to ignore specific types of perception until they develop "naturally." Very little curriculum development has been based on the latter reaction.

Professor A. Gleason suggested a direct use of the educational process that is perhaps best understood - the conditioned reflex. There are many good reasons to concentrate on more interesting mathematics in elementary school than the traditional arithmetic drill. Professor E. Begle had reported on a very satisfying SMSG experiment in which seventh grade children behind grade level in computation were relieved of all computation for a year. At the end of the year they had, on the average gained two and a half years in arithmetic reasoning and also one and a half years in computation! But there will be children who are poor computers at the end of several grades of stimulating and mathematical ideas, just as there are at the end of several grades of boring drill. Professor Gleason pointed out that the skill of the behavioural scientist at inculcating responses may provide a comparatively painless and rapid way of remedying the situation for these children.

The effect of technical revolutions on the teaching of mathematics received much attention. The British delegates were impressed by the appearance of computer consoles in many American secondary schools. Although there was little enthusiasm for courses in programming as such, integrated use of a computer in the mathematical course had wide appeal. Many advantages to the availability of a computer were cited. In problem solving additional insight would be given into the procedure of solution, even for analytically soluble problems. Application could be introduced, for which the comparison of computed numbers with measured numbers would not have otherwise been possible. The programming itself was regarded as a training in careful, precise thought and in logic. On another level the use of a computer provided motivation to the student, confidence through checking and a release from boring computation.

The reaction was very different when attention was directed towards computers as teaching machines; that is, programmed to teach the student rather than programmed by the student. The expectation was expressed that the programming of sufficiently flexible and subtle teaching would be a formidable task. Very costly investments would be required in the highly competitive computer industry. It was feared that the resultant industry pressure may lead to early adoption preceding the proper development and testing of teaching programs. This would leave schools with the burden of expensive and perhaps harmful computer-based teaching machines.

The recommendation was that the mathematical community should devote serious study to computer-based instruction with the object of producing an acceptable product or at least being an effective critic. The British felt that there was little immediate danger of funds being available in their country for an over-commitment.

The need for large-scale testing was a matter of concern to the British. This is needed in the relative evaluation of curricula (as in the National Longitudinal Study of Mathematical Abilities of SMSG) if not for the grading of increasing numbers of students. Professor R. P. Dilworth presented impressive evidence of the reliability of carefully prepared objective machine-scored tests. It would appear that they are more reliable predictors (of something!) than the conventional "essay type" responses that require careful reading. As Professor Dilworth pointed out, an advantage of large-scale machine-scored testing is the possibility of statistically analyzing pre-tests. The modified test prepared on this basis contains a higher percentage of meaningful questions. The British expressed the desire for the help of American testing experts in devising tests of their own.

The growing exchange of information between the resulting curriculum development in both countries will tend to bring them closer together. However, differences in organization and attitude, noted at the conference, will condition the relative direction of progress for some time to come. There is an existing difference in the standards for teachers (masters) and in the content of their training. Standards are more uniform in the United Kingdom and on the high side of the United States levels. The smaller percentage of college bound students in the United Kingdom will be a factor in determining the content of secondary school instruction. The presence in the U.S.S.R. of schools specialized to mathematics or science was discussed and the few American parallels noted. This has impact on only a small part of the student population. In the United Kingdom some specialization takes place in secondary schools prior to college entrance, as it does in differing forms in the United States.

I detected a difference of attitude that may lead to long-term differences in the curricula of the two countries. The dominant American attitude (remember the individual diversity!) is that some ability to use the subtler mathematical ideas associated with symmetry, continuity, probability, and modeling should be widely diffused through the population. In the United Kingdom the desire is, it seems to me, to present these more esoteric topics to the general population only at the conceptual level, reserving any formal command for the more "practical" skills. Calculus, for example, is considered relevant only for students going on to be scientists, engineers or mathematicians. There is clearly a division on this issue in this country, and this is not the place to present the arguments for either approach. But the next "wave" here is likely to present a program designed to enable the bulk of the population to make some headway in modeling and analyzing real situations involving, for instance, probability. In the United Kingdom the more formal material may be designed for the smaller population that is specially motivated and able.

The direction of the reform movements may also be affected by the relatively smaller participation of university mathematicians in the United Kingdom. In the United States of America, university mathematicians direct curriculum development groups, are on the textbook writing teams, and are occasionally in the experimental classrooms. It was noted that some internationally known Soviet mathematicians teach classes of young children on a regular basis. In the United Kingdom the university participation has mostly been on the level of general formulation and advice. The writing, directing and participation in experimental work is by schoolmasters, some of whom have been on University faculties. There is a handful of research mathematicians participating actively in the curriculum reform.

Most of the American delegates arrived at the conference knowing little of the scope of United Kingdom accomplishments and intentions in mathematics curriculum reform. It is my impression that on the whole the United Kingdom contingent was better informed of American accomplishments, but as poorly informed of the present direction of our thinking and effort as we were of theirs. Only a few delegates from either country had previously had sufficient contacts, such as in the African Program, to reveal their orientation to each other.

On leaving we felt that we had only scratched the surface. Some of us, previously ignorant like myself, had learned of the important scale on which new and very enjoyable courses have been introduced in the United Kingdom. We had become familiar with the organizations and some of the key people involved. Their special interests and their publications are now largely known to us. This is a critical first step in being able to efficiently gather further information by correspondence and in looking forward to working contacts. Effective liaison will undoubtedly expand rapidly from this beginning. There was some talk of long-term visits by members of one project to a project in the other country. I look forward to another meeting of these two groups in which more detailed curriculum planning may be accomplished.

The Ditchley Mathematical Conference

Participants

Co-Chairmen
Professor W. T. Martin, Massachusetts Institute of Technology
Professor B. Thwaites, Southampton University

GREAT BRITAIN
Professor W. H. Cockcroft, Hull University
Dr. H. M. Cundy, Sherborne School
Professor H. B. Griffiths, Southampton University
Dr. J. M. Hammersley, Oxford University
Mr. C. S. Hope, Worcester Training College
Dr. A. G. Howson, Southampton University
Dr. G. Matthews, Nuffield Foundation
Dr. H. R. Pitt, Reading University
Mr. D. A. Quadling, Marlborough College
Mr. A. R. Tammadge, Abingdon School
Dr. A. J. Weir, University of Sussex

UNITED STATES
Mr. J. L. Aldrich, Education Development Center
Professor E. G. Begle, Stanford University
Mr. H. P. Bradley, Education Development Center
Professor R. B. Davis, Webster College
Professor R. P. Dilworth, California Institute of Technology
Professor A. Gleason, Massachusetts Institute of Technology
Professor P. J. Hilton, Cornell University
Dr. J. H. Hlavaty, New Rochelle, New York
Professor E. L. Lomon, Massachusetts Institute of Technology
Mr. R. S. Pieters, Phillips Andover Academy
Dr. H. Pollak, Bell Telephone Laboratories

GUESTS ATTENDING ON SUNDAY EVENING
Professor M. F. Atiyah
Dr. M. J. Lighthill
Professor C. A. Coulson
Mr. R. Lyness
Mr. R. W. Morris
Mr. B. M. W. Young

The Ditchley Mathematical Conference

Professor Bryan Thwaites
Westfield College, Hampstead, England

1. Original Aims

The idea of the conference grew out of three emergent circumstances. First, there is a feeling growing on both sides of the Atlantic that the first wave of syllabus reform in mathematics is nearly spent, that the work of the initial set of major curriculum study groups will soon be completed. Second, the emphasis on the similarities between American and British projects which in the past has been felt to be necessary for the moral of the mathematical evangelists has recently been giving way to critical interest in the substantial differences between the approaches of the two countries. Third, personal contacts between individual members of projects in the two countries have strengthened in the last two or three years to the point of warm regard and friendship, to the point indeed at which some formal dialogue was becoming feasible.

In August 1965, therefore, Professor W. T. Martin of the Massachusetts Institute of Technology and chairman of the African Mathematics Program and the Cambridge Conference on School Mathematics (CCSM), reform programs of a non-profit organization, Educational Services Incorporated,* and Professor Bryan Thwaites of the University of Southampton and director of the British School Mathematics Project decided to investigate the possibility of a small Anglo-American conference on School mathematics whose deliberations would help them to see more clearly the way ahead and, in particular, the nature of the second wave of curricular reform which they felt was soon to gather strength. They thought that a gathering of some twelve of the most distinguished reformers from each of the two countries would be enormously profitable for both sides.

Their idea commended itself to others, and, as a result, CCSM and S.M.P. agreed to sponsor, jointly, a three day meeting of twelve-a-side, a kind of confrontation at which distinctions of approach would be discussed constructively in the light of the differing educational systems of two countries. The topics in which distinctions were originally thought to hold special interest were listed in the leaflet which served as an agenda:

1. Axiomatisation, and its role in mathematical education;
2. Logical processes, and the need for formal treatment;
3. Unifying concepts;
4. Relative emphasis on discrete and continuous systems;
5. Aims and content for geometry, with special reference to its study through transformations;

*Now Education Development Center

6. The role of calculus and the lead up to it;
7. The impact of computers on mathematics;
8. Statistics;
9. Linear spaces.

In the event, these were rather changed.

2. Membership

While such a small gathering could not hope for complete coverage, great care was taken over the choice of membership and the following who accepted invitations together held a breadth of knowledge and experience which, perhaps, has not been assembled before.*

3. The Conference Procedure

On Thursday 8th September the two co-chairmen with four other members met to plan the agenda in detail and their first conclusion was that the topics listed above did not all do justice to the total experience available at the conference. They also decided that certain subjects could be discussed profitably at once in plenary session with only a short introduction prepared in advance by one speaker, whereas other subjects needed examination in small groups before presentation at and discussion by a plenary session.

The programme as finally carried out therefore consisted of the following:

Plenary Discussions

1. "Axiomatisation and its role in mathematical education; at what age and for what people?" introduced by Professor H.B. Griffiths.

2. "Relative emphasis on discrete and continuous systems" introduced by Professor P.J. Hilton.

3. "Special treatment for highly gifted mathematical children" introduced by Dr. J.H. Hlavaty.

4. "Examining procedures" introduced by Professor R.P. Dilworth.

*List of participants on page G-viii.

Group Discussions

5. "Teaching and learning methods, computers in teaching, and adaptation to cognitive styles" led by Professor R.B. Davis.

6. "Aims and content for geometry, with special reference to its study through transformations" led by Dr. A.J. Weir.

7. "The impact of computers on mathematics and on the curriculum" led by Dr. H. Pollak.

8. "Statistics" led by Dr. J.M. Hammersley.

9. "Unifying concepts; sets, functions, etc." led by Mr. D.A. Quadling.

10. "Teacher training and the shortage of teachers" led by Mr. C.S. Hope.

11. "Acquisition of computational skill; arithmetical operations" led by Professor A. Gleason.

12. "Communication with the sciences at school level" led by Professor E. L. Lomon.

It was clear from the start that time was far too short for agreed conclusions to be reached. Group and plenary meetings were therefore aimed primarily at exchanges of views between the two countries, the leaders taking the responsibility for producing working summaries rather than firm recommendations or agreements.

Three types of reports are to be made of the conference. The first will be a detailed account, almost in the form of minutes, which will include the substance of all the papers produced during the meeting together with full reports of the discussions of these papers; this report will be available to the members of the conference only. The second will be an account drawn up by CCSM for the purposes of the various ESI programs. And the third is this report which is issued as an S.M.P. document.

It must be emphasised that this present document does not necessarily carry the agreement of all the members of the conference, either American or British. Furthermore, its following paragraphs do not necessarily correspond to any of the topics listed earlier. It is simply an account written by the British co-chairman and issued on his sole responsibility. Nevertheless, every effort has been made to ensure that all members have had the opportunity of criticising the first draft of this report, and it is hoped that it fairly represents such consensus as was reached on occasions.

The reader must bear in mind constantly what only slowly was borne in upon the conference members, namely that in many, if not in most, matters there is no such thing as an American view of a British practice, an American method or a British philosophy. In a country as vast as the United States, there is room for every kind of experiment. Perhaps, however, in our much smaller country we should not be too complacent: too many university mathematicians seem still woefully ignorant of the movements in school curricula and it is hoped that this Report will help to spread information in schools and universities.

Finally, it is stressed again that the paragraphs which follow are barely more than notes about the matters which struke the reporter as being of the greatest interest and significance. They are in no sense at all a record of the full transactions of the meeting.

4. Curriculum

The trouble with talking about the curriculum is that it is difficult to know where to begin, but once begun it is impossible to know where to stop. Nothing new can be said within a mere three days, but there were four points which seem worthy of record.

I. Geometry

Over the last few years the differences in the approach to geometry between the two countries have been marked. Broadly speaking the American projects have centred on improving the Euclidean axioms, whereas British projects have tended to break away from the Euclidean development of geometry from a motion or vector-space point of view. A much more recent tendency in the U.S. is to build up the intuitive background for geometry over several grades before a large block of deductive geometry is attempted and to experiment with a variety of approaches to the deductive experience. In the future, therefore, it is likely that intuitive mathematical experience including a variety of ideas in geometry will be emphasised in Grades 7, 8, 9, and that the more formal work based on this experience will come later in Grades 10, 11, 12. In contrast, most recent British development suggests that more work on the deductive aspect of geometry will be introduced in the next few years at late pre-O-level stages. For example, one would expect that Book 5 of the 11+ S.M.P. course would contain far more deductive work on geometry than is contained in the 13+ T and T4 books. The tendency to treat geometry in smaller chunks is part of a general trend to unify the entire mathematical experience and break down the large chunks of isolated material; but the U.S. system of year-long units in any one topic is extremely difficult to change because of the fierce mobility of the population.

II. Computers

In the U.S.A. several schools are already equipped with at least one console connected to a central time-sharing computer - a natural corollary to the fast-approaching situation where each scientist in a research laboratory will be supplied with immediate computer access in his own room. The development of this facility seems a matter of historical inevitability and is likely to proceed ever more rapidly as computer technology reduces the size and cost of computing devices. Britain appears to be lagging most seriously behind both the U.S.A. and the U.S.S.R. in this and it may be useful to mention some of the great advantages of school pupils having immediate access in their classroom to a computer. First, there is the undeniable motivation which the actual control of the computer gives to pupils and which is not given by a computer to which programs have to be posted for return a few days later. There is then the possibility of mathematical experiments; for example, pupils can investigate hypotheses in the simple theory of numbers which would be quite beyond their capacity without a computer, or they can investigate the effects of a parameter in a formula. For example, it is both a very interesting and a mathematically valuable exercise to investigate the effect of the value of the parameter k in the following generalisation of Newton's iterative formula for finding the square root of a positive number A

$$(1 + k)x_{n+1} = kx_n + \frac{A}{x_n}$$

Another advantage is that in statistics significant work can be undertaken since there is no practical limit to the amount of data which can be fed in and indeed use of a computer should overcome that fear of numbers, large or small, which so many pupils have. Obviously, the use by the teacher and the pupils of a console in a classroom would have a profound effect upon teaching methods and possibly also on the whole development of the mathematical curriculum; it does not seem, however, to be in line with present thinking that there should be separate courses about computers or their programming at the secondary level. It should perhaps be finally mentioned that the use of computers offers an entirely new way of beginning arithmetic at the grade 1 level; some teachers are already thinking of postpoing the development of algorithms in arithmetic until a much later stage, say grade 5 or 6, and introducing the operations of arithmetic in a purely functional sense in the earlier grades. Attention was also drawn to the existence in America of very small computers which would accept stored programs through touch-button input which cost only about £ 3,000 and are the size of a portable typewriter.

III. Statistics

It seems worth recording a feeling which seemed to be quite widespread that statistics and probability should not be treated as subjects distinct from the main mathematics course and that much more effort should be put into drawing statistical ideas from the general development of mathematical curriculum. The age of 11 is certainly not too young for the ideas of statistics to be introduced and it was also felt to be unrealistic to rely on any substantial knowledge of the calculus when designing a sixth-form course of statistics. This latter point may lead to less emphasis being placed upon continuous didstributions than has been the custom.

IV. Honest Applications

An interesting point arose when the typical British approach to applied mathematics was contrasted to the virtual absence of physical applications in many of the American programs. The British approach as characterised, for example, by some of the applied questions in the 1966 S.M.P. A-level examination, was criticised as being too abstract or unrealistic and perhaps everyone who talked about this at Ditchley agreed that a mathematical curriculum should abound in all kinds of examples and applications drawn from the real world but that these should display an honesty or a reality which is rarely found at the moment.

5. Unifying Concepts

In view of the oft repeated claim of new curricula projects that they present a treatment which makes use of the unifying concepts of sets, functions and so on, one of the groups spent some time in examining the meangfulness of such claims. This group seemed to conclude that the notion of a function, for example, possessed no inherent power of unification nor, indeed, had any special value as a pivotal point in a syllabus. The emphasis perhaps should lie more on the use of these ideas as a component of the normal mathematical vocabulary and the main value of many of the concepts which tend to be labelled "modern" is that they extend the word power and the area from which examples can be drawn. In this discussion it was suggested, too, that children should be brought up to be very flexible in their use of notation and should be allowed great freedom in the way that they are allowed to set out mathematical work. So long as written work can be clearly understood and communicates the ideas in it well, then there is no need to impose standard styles of presentation.

6. Evaluation

Two points are worthy of record here. First of all, researchers in the U.S. are not attempting to make comparative evaluations between different curricula projects; even if this were thought desirable (which it is not) far more information than is at present available would be needed to serve as criteria for such comparison. Second, the English team was fascinated to hear about the national longitudinal study of mathematical abilities headed by Professor Begle. This has been running for four years already and has been planned on what seems to us a massive scale. The basic intention is to try to understand the interaction between pupils understanding and knowledge and all the other influences which bear upon him, namely the teacher, the parent, his own contemporaries, his environment, and so on. To this end the N.L.S.M.A. drew up a very long list consisting of practically every question that anyone could think of asking about the effects of a mathematical curriculum and answers to these questions have been sought over the years through a multitude of specially designed tests covering many thousands of children. It is expected that some of the first provisional results from this enquiry will be available next year By contrast, the number of people involved in evaluation in England is very small indeed and the results of such researches as are going on are very little-known. Although one must not underestimate the amount of evaluation which is inherent in a programme such as the S.M.P.'s in which material is written over and over again on the basis of classroom experience, it should nevertheless be admitted that this procedure is basically a subjective one.

7. Computerised Education

One of the surprises of the conference was the time which was spent in discussing the possibilities for automatic education which are now just around the corner. These go far beyond the ordinary type of teaching aid with which we are more or less familiar in England; the flexibility and total content of a programmed learning routine is enhanced by several orders of magnitude if the details of the routine are stored in a large computer. In the ultimate state of development which can be envisaged at the moment each pupil will have his own individual values of a series of parameters which describe the characteristics of his learning processes and current knowledge - a suggested phrase for this set of parameters could be "cognitive style." A pupil would therefore go into a classroom, seat himself at a console which would, of course, be remote from the central computer, feed into the console his "cognitive style" and also, of course, his name and other information which will inform the computer where his knowledge stands at the moment and from then on the computer would do the rest. It would set the pupil an example to do or a passage of text to read, would demand some sort of response from the pupil and then, in its turn, make the appropriate response to the pupil's performance. In this way, the pupil would receive infinitely more detailed care than any teacher in front of the class could give. And lest such a system be dubbed as too impersonal it should be recorded that such experiments as have already been made along these lines in America and also in Russia suggest that the pupil is very highly motivated by the response behavior of the machine. On the other hand the teacher himself is released by the machine for much more highly individual and deeper work of his own with his pupils. It was clear that the American members of the conference were very anxious about the situation which is developing in the United States in which many textbook publishers are being bought up by computer firms who may be eager for quick returns on their investment; there is therefore a serious danger that computer learning programs will be issued after very hasty compilation and without the deep research which such a revolutionary development in teacher methods deserves. It may be that the British members of the conference took the view that "it cannot happen here", but it seems that we sould be wise in this country to anticipate this development by setting up an official unit to conduct research into this method of teaching.

8. Examinations

Another prediction for the future is that the number of candidates in all kinds of examinations will be so large that even for no other reason examinations will have to be machine scorable but there is a natural reluctance on the part of those who have cherished their skill at setting and marking examinations of a traditional style to admit that machine methods can be as discerning.

It is commonly argued that essay-type examinations are preferable since such questions enable the examiner to test deeper aspects of mathematical understanding and to follow the student's detailed line of reasoning. However, experts in the development of multiple choice questions have devised techniques which test these aspects by means of questions in the multiple choice format. On the other hand, the constructive of effective multiple choice examinations is a very dificult undertaking and is not a job for amateurs. The individual questions must be pre-tested and a full statistical analysis made. On this basis, the contribution of the question to the examination as a whole can be determined. Comparisons of the predictive ability (in terms of future performance in mathematics courses) of well-prepared multiple choice tests and comparable essay-type tests have shown that the multiple choice tests are at least as effective as the essay-type tests. Research in England on machine testing on the basis of multiple choice questions should be pressed forward vigorously, preferably with the full-time help of a consultant American expert.

9. Provision for Specially Gifted Children

Very interesting discussions were centred upon what is now known to be the Russian custom of giving special treatment from about the age of fifteen to children with special mathematical gifts (and indeed other subjects too), and it may be helpful to describe even in a few sentences the Russian method. The selection of specially gifted children takes place annually by means of the system of Olympiad examinations which are organised at the city, region, Republic and Union levels. According to the number of places available at each higher stage of the educational ladder so the first so many successful candidates arranged in order of merit of marks, pass on upwards. The teaching which a boy or girl gets in his school to prepare him for these Olympiad examinations is supplemented if he is good enough by the work which he will do in his weekly mathematical circle, which is a kind of mathematical club. The distinctive feature of these circles is that the tutors and teachers at them are usually drawn from the post-graduate students or the lecturers and professors of the nearest university or institute of higher education and it seems clear that tremendous enthusiasm is engendered in these circles both for teaching and for learning. For those pupils who pass into the special mathematical schools of which there are some hundreds all over the U.S.S.R. (over and above the four famous boarding special schools for mathematics) there is a basic curriculum of six periods a week mathematics and a further six periods which are devoted to learning how to use computers (as digital or logical machines). Thus the training of the most able mathematical students in the U.S.S.R. is heavily oriented towards computers. In the U.S.A. the nearest resemblance to this system of strict selection by merit in a single subject are the advanced placement programs now so prevalent. In the United Kingdom it could be said that the system of specialisation in the sixth-form is more or less equivalent to the Russian system. What is therefore of interest is that there is a tendency in the United Kingdom to reduce the number of periods spent in specialised study at the sixth-form level whereas in Russia and in the States there is a tendency to increase it. The increase on the one hand and the decrease on the other indicate a "coming together" of attitutdes on the need for such specialisation. It seems certain that the discussions which we shall be having at the National level in Britain over the next year or two about the nature of the sixth-form curriculum should draw heavily on comparisons with experiences in the U.S.A. and U.S.S.R.

10. Teacher Trainining

One point only is here reported on the discussion about the very difficult problem of teacher training and especially of the up-dating and in-service training of teachers. A most interesting feature of the American reform scene in the last few years has been the production of large numbers of films which are aimed at helping the teacher to learn how to present the new math. Many Englishmen have now had the opportunity to see many of these films which, as a consequence, have come in for some heavy criticism. We would therefore like to stress here, while the English film effort is still in its earliest stages, that to be effective a film has to have a most precise objective which must itself be explained fully and in detail during the course of the film itself.

11. Participation of University Mathematicians in Curriculum Reform

One of the outstanding differences between the American and the English methods of curriculum reform lies in the degree of participation of university mathematicians and in looking to the future two points seem to be worth making in connection with the English situation. First, the English university mathematician has hardly been involved at all in the reform movements and yet within the next few years is going to be hit by the products of the first wave coming up from the schools into the universities. It may be that many university departments have not yet fully realised the influence that this first wave must necessarily have on the style of first year teaching and the nature of the university curriculum especially in the first year. Secondly, although the first wave of reform in England has been on the whole most successfully carried out by the efforts of school masters there must be some doubt as to whether the depth of mathematical knowledge and experience in the schools can be sufficient for a proper development of the second wave of reform. It seems quite clear that the consensus of opinion both in the U.S.A. and again in the U.S.S.R. is that university mathematicians must continue to be deeply involved in reform if it is to be successfully carried on. Finally, it was the feeling among many members of the conference that our two countries have much to learn from Russia - and we make no apology for once again referring to the U.S.S.R. - and observing the degree of commitment that some of Russia's top research mathematicians have in school-teaching; as one example, Professor Kolmogorov devotes about one half of his total teaching time to school-mastering and perhaps at the least in our two countries we should honestly answer the question as to whether we are not losing something of essential value by having rather different customs.

12. Some Concluding Remarks

This conference was really a deck-clearing operation; no one quite knew whether all the bric-a-brac of experience they brought with them was going to be usefully piled together or was going to be swept away for action of another kind. In the event, much less time than as expected was spent on details of mathematical syllabus and for two main reasons: first, there was a mutual recognition of the huge influence which certain developments were likely to have on the whole structure of mathematical education, and second, there seemed a tacit acknowledgement that the first wave is indeed nearly over and that the job of the leaders in the two countries is now to plan a controlled build-up of the second wave. In all this, many members of the conference felt that continued working contact between American and English researchers would bring substantial benefits and a suggestion was heard that, as a small start, a year's exchange should be made between two authors of the S.M.S.G. and S.M.P. teams. Certainly, the English team left Ditchley profoundly impressed by the depth and professionalism of the latest American work in the field of mathematical curricular research, and we from England must conclude this report by thanking, once again, the American side for having put aside the time to come.

APPENDIX H

Summary

of the

Cambridge Conference
on
Teacher Education

Introduction

In the summer of 1963, a group of mathematicians gathered in Cambridge, Massachusetts, at the invitation of Educational Services Incorporated to discuss the current state and possible future of mathematics in the elementary schools. The group adopted the name Cambridge Conference on School Mathematics (CCSM) and issued a formal report Goals for School Mathematics (herein referred to as Goals) which was published by Houghton Mifflin Company, Boston

The CCSM presented proposals for mathematics curricula for the schools of two of three decades hence which were considered by many to be radical and ambitious, but it agreed to defer consideration of the important problem of teacher training. While the outstanding work of the Committee on the Undergraduate Program in Mathematics (CUPM) has given promise of alleviating the shortage of teachers adequately trained to handle University of Illinois Committee on School Mathematics (UICSM), School Mathematics Study Group (SMSG) and other material recently developed, it was clear that further steps had to be taken in the near future if the Goals projection of the mathematics curriculum of a couple of decades hence is the accurate one, or even approximately the right one. Accordingly, during the summer of 1966, again under a grant from the National Science Foundation, the Cambridge Conference on Teacher Training (CCTT) was held to consider the crucial question of preparing teachers to teach a Goals-like curriculum in the elementary schools. The group of 33 was composed in the main of persons holding university positions in pure or applied mathematics or statistics. Several American school teachers and British mathematicians involved in curriculum improvement attended the sessions.

The many divergent views expressed by the participants did finally polarize around two fairly distinct points of view and produced the outlines of two curricula, which they believed would adequately prepare teachers to cope with any elementary school mathematical material that may be developed to meet a Goals-like proposal. It is interesting, and perhaps significant, that although the two outlines are based on quite different premises, they turned out to have many common points and to differ less than might be expected of two programs developed independently of each other. A brief summary of these two proposals is presented below.

The CCTT also discussed several other problems which affect the training of elementary teachers; such topics as the general environment of the elementary teacher, the uses of mathematics specialists, technological teaching aids, and the participation of mathematicians in elementary education. In addition to the curricular proposals, specific recommendations were made for the preparation of materials for use in training teachers, for the dissemination of information about curriculum developments and experiments, and for the continuing education of teachers. These are also summarized below.

Curriculum Proposal 1

Both proposals started from the assumption that students would enter the program with a background approximately equivalent to tenth grade SMSG mathematics. The group working on this proposal first elaborated the Goals outline for K-6 to gain more insight into the needs of elementary school teachers. (This more detailed outline for K-6 appears in Part III of the complete report.) What the group sought were topics in mathematics that would be quite new to the prospective teachers, would demand of them only a rudimentary knowledge of mathematics, would deepen their insight into mathematics, would offer good prospects of enjoyment, and would reveal similarities of mathematical structure with the topics in the K-6 curriculum. Courses A (one year,)B (one-half year,) and C (one-half year) were the result.

The main topics of K-6, of course, are arithmetic and geometry. The group looked for a topic that would use elementary arithmetic and apply to geometry -this led to the choice of matrices for Course A. Since the early stages of the study of matrices involve only the most elementary knowledge of arithmetic, the prospective teachers, while developing an algebra with properties different from those of the ordinary number system, would at the same time have an opportunity to review the rules of arithmetic in a context in which the basic laws have greater interest. As the course progresses, matrices are related to transformations of the plane and thus used to study geometry in the context of motions. Further, the notion of angle is studied in connection with rotation matrices, offering a natural introduction to a certain amount of trigonometry.

- Analytic geometry (lines, circles, triangles, inequalities, convexity).
- Matrices and their operations (up to 3x3).
- Group properties of matrix multiplication (emphasis on 2x2).
- Systems of linear equations.
- Study of certain linear and affine transformations of the plane, using matrices (enlargements, translations, reflections, shear,...).

- Sensed angles, sine and cosine, measurement of angles.
- Elements of vectors (via Cartesian coordinates).
- Composition of transformations and the associated matrix multiplication.
- Transformations of equations, with application to the development of ellipses and their equations from distortions of circles.

Course B undertakes to illustrate the close ties between abstract algebra and the arithmetic of numbers. The algebraic structure emerges from a study of polynomial functions, which also allows an introduction to the formal differential calculus of polynomials. It is then brought into the open by a discussion of the algebra of integral domains, and finally driven home with an introduction to the theory of numbers, during which frequent references are made to parallels with the earlier polynomial algebra.

- Functions (concept, examples, representation, operations with, inverses).
- Linear functions (graph and slope).
- Quadratic functions (examples and motivation; graphs, zeros).
- Quadratic formula.
- Polynomial functions and polynomial algebra.
- Integral domains.
- Elementary number theory.

Course C is devoted to applications of mathematics to science and society and to certain mathematical topics which seem particularly amenable to an approach through real problems. In addition to as many examples as possible, and some exposition of the nature and use of computers in the applications of mathematics, it contains the following mathematical topics:

- Systems of linear equations and optimization problems.
- Intuitive calculus.
- Probability and statistics.
- Logic (if time permits).

Curriculum Proposal 2

The second group began with the premise that the prospective elementary teacher may well have acquired attitudes of uncertainty, fear, and hostility toward mathematics, and therefore first priority should go toward choosing courses which will rekindle the student's interest and build confidence by developing his power to cope with mathematical ideas. In particular, this precept should take precedence over the notion that all materials in the K-6 curriculum must be covered. Nevertheless, it was felt that four semester-length courses will be needed.

The courses are divided into short, relatively independent units which the second group felt had several advantages, primarily flexibility and the repeated fresh starts for students who fail to understand certain topics.

The inclusion of calculus is justified by the fact that the real number system is important in mathematics less for intrinsic interest in it, than for the many complex and useful structures of higher mathematics which it supports. The second group felt that some introduction to the powerful ideas which evolved in the 18th and 19th centuries from the real number system (vector geometry, calculus, probability theory) should be an essential part of the elementary teacher's training.

This group devised two curricula, the second (Alternate) program being considered to be somewhat more difficult. The topics to be covered and approximate number of class hours are listed below.

Sample program

Course I: Number theory (16)
Vectors in the line and plane (8)
Transformations and functions (12)

Course II: The real number system (25)
Combinations and probability (12)

Course III: Intuitive differential calculus (22)
Linear transformations and matrices (15)

Course IV: Isometries and symmetry groups (12)
Quadratic forms and conics (10)
Intuitive integral calculus (15)

Alternate sample program

Course I': Circular functions and complex numbers (10)
The real number system (16)
Counting problems and induction (8)
Functions (6)

Course II': Vectors in the line, plane and space (20)

Course III': Rings and unique factorization (20)
Computational matrix theory and applications (20)

Course IV': Intuitive integral calculus (15)
Probability (15)
Statistics (10)
or Impossibility of angle trisection (10)

General Considerations

1. From their beginnings such groups as the University of Illinois Committee on School Mathematics and the School Mathematics Study Group have been constructively concerned with teacher training. Future activity must build upon their work and continue to advance the general line of the commendable and immediately applicable CUPM recommendations for elementary teachers. The CCTT proposals are aimed (hopefully) at a time when the CUPM recommendations will have been strongly implemented.

The elementary teacher has a short vocational life expectancy. Within three or four years a large percentage are no longer teaching, although some return to the classroom two to twenty years later when family demands have diminished. Thus for many teachers, the preservice training is their only exposure to courses affecting their short teaching careers, emphasizing the need for courses that adequately prepare them.

2. The Conference believes that mathematics specialists can make important contributions to elementary education by teaching mathematics to the students (either independently, or in cooperation with the classroom teacher), by conferences with classroom teachers, by curriculum and test planning, etc. Indeed, some seventeen possible uses of specialists are included in the report. To train these mathematics specialists, the report suggests certain courses selected from the regular college curriculum for mathematics majors.

3. The Conference members were shown some forty mathematics education films and some individual comments about these films and films in general are included in the report. There was considerable discussion of computer-assisted instruction, with the general conclusion that although there are grave dangers involved in this process, the future potential is promising.

4. There are two very direct ways in which mathematicians may contribute to teacher training. The first is by cooperation with schools or departments of education in their universities. Specific actions include cooperation in curriculum planning and teaching courses in the education school. Another, less direct, means of aiding in teacher training is for the mathematician to teach some classes of elementary students, i.e., the mathematician performs one of the functions of the elementary mathematics specialist.

Recommendations

I. The Conference produced guidelines for the construction of a program for training future elementary school teachers. The next step is to take off from the outlines given in the Report and produce material to be used in experimental classroom situations, and finally to revise this material for general distribution. To accomplish this, the Conference recommends that three (or some such number) mathematicians be commissioned to spend a summer individually developing material. In the following year each would try his material on an experimental class of prospective elementary school teachers at his university. The writers would get together the following summer to compare notes on their courses, revise the material, and bring out a version that could be tested on a large sample in many colleges. After this larger test and subsequent revision, a final version would be published for general use.

II. Even after text materials for the proposed courses are available, their acceptance and use will depend on effective, widespread and systematic communication of the aims of the report to the faculties of teacher training institutions and school administrators. To this end it is recommended that regional meetings, involving mathematicians and professors teaching mathematics courses for elementary teachers be held on a regular basis, modeled on those of CUPM. It is also recommended that professional mathematical organizations be urged to include in their meetings sessions relating to elementary mathematics education.

The Conference also proposes the establishment of a quarterly journal consisting of reports on experimental work in pre-college mathematical education. These brief reports will serve to keep both the curriculum experimenters and the mathematical community informed of the current activities of other groups.

III. Continuing education of teachers is becoming increasingly important as the change of mathematics curricula becomes a steady-state situation. Summer institutes are valuable and should be continued, but the Conference feels that these are insufficient. The typical schedule of a teacher gives her little or no time for class preparation during school hours. She has little or nothing to say about the curriculum taught or the texts used. The Conference recommends that elementary teachers be freed from some classroom time so that they may prepare classes and take in-service courses. The Conference also recommends that a sabbatical system be established in the schools to enable teachers to return to colleges for more intensive and prolonged study.

Cambridge Conferences and Teacher Education

Earle L. Lomon

During the summer of 1963, a group of twenty-five professional mathematicians and scientists met in Cambridge, Massachusetts to review school mathematics and to suggest goals for elementary and secondary school mathematics programs. The report of this conference, Goals for School Mathematics,[1] has provoked widespread discussion among mathematics educators.

In that report, a large range of new topics were suggested for inclusion in the elementary school. Some of these topics, to my knowledge, were not even used in experimental classrooms at that time, although most have been tried in small scale experimentation by now. Among the more relevant topics with implications for teacher education was emphasis on the real number system in the early grades. Children were introduced to inequalities and general laws applicable to all real numbers at one time, rather than to integers, then rationals, then real numbers as the child progressed in school. In the third and fourth grades, symmetry, transformation, vectors, and elementary Diophantine problems were recommended. Before the end of the elementary grades, mathematical and experimental probability was introduced. Intuitive calculus was suggested as early as the sixth or seventh grades, although this was not explicity built into the curriculum.

Educating teachers to adequately teach content such as that outlined above was the focus of a second Cambridge Conference, held during the summer of 1966.[2] A crucial question of that conference, and of this one is, "How can teachers be prepared so they adequately handle rapidly evolving mathematics programs with some depth, range, and adventuresomeness?"

Teaching the content suggested in the 1963 conference requires significantly greater background in mathematics than is typical of elementary teachers today. Developing in elementary teachers this adequate understanding of mathematics was viewed in the 1966 teacher training conference as the most important task that can readily be accomplished. A large portion of the report discusses college curricula and approaches by which schools of education and mathematics departments might better prepare teachers.

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[1]"Goals for School Mathematics:" The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Co., Boston, 1963.

[2]The report of this conference, just published, is "Goals for Mathematical Education of Elementary School Teachers:" A Report of the Cambridge Conference on Teacher Training, Houghton Mifflin, Co., Boston, 1967.

The increased amount of mathematics envisaged in the 1963 report is equally challenging for the pupil as for the teacher. How can they be expected to learn ten times more mathematics in approximately the same time? The 1963 conference suggested several teaching strategy approaches that might facilitate pupil development. Three of these are important for us.

The first was the spiral approach in teaching. The first treatment of a topic was to be on an intuitive or concrete level. Then another topic was developed, then the first topic considered again on a slightly more formal level, using some of the ideas from the intervening topic. In this way a combination of the effective use of intuition, concrete experience, and carefully developed content would greatly accelerate a pupil's rate of learning. Properly combining topics should improve the efficiency of the learning process through cross-referencing.

The second suggested change that was hoped would accelerate learning was in the selection of topics. The stimulation of exciting topics and compelling open-ended applications may challenge many children to spend a greater proportion of their time in mathematics.

Third, it was hypothesized that more germane thinking may result through the discovery and dialogue techniques than through a normal expository presentation. When challenged, the child must establish the correctness of his views. He can not rely on the authority of the teacher as a crutch. Discussions among pupils in his class becomes more of a goad to him than a squelch. Experience with these approaches convinces us that they are effective in accelerating the elementary mathematics program. Hopefully, too, the same kind of acceleration can be accomplished in training prospective teachers.

Conference on Teacher Education

The 1966 Cambridge Conference on Teacher Training relied upon the experiences of the University of Illinois Committee on School Mathematics (UICSM) and the School Mathematics Study Group (SMSG). Both had been involved in workshops, summer institutes, and courses on teacher training directed toward new materials. Relevant also was the previous formulation of goals for pre-service mathematics for elementary school teachers by the Committee on Undergraduate Program in Mathematics (CUPM). Experience in implementing CUPM goals provided data on the effectiveness of these kinds of materials. The problem that CUPM worked with was of a different magnitude, however, than one that would provide sufficient background to teach the content outlined in the 1963 conference. If preparation time for teachers remains at the same level as suggested by CUPM, more efficient time utilization is a major concern.

Two alternate four-semester college math courses are suggested by the 1966 report for elementary school generalists. One alternative stresses contact with the K-6 curricula, as written and developed in the 1963 report.[3]

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[3]"Goals for School Mathematics:" The Report of the Cambridge Conference on School Mathematics, Houghton Mifflin Co., Boston, 1963.

In fact, they rewrote parts of that curriculum so it was more explicit for their purposes. They wanted to insure that college students were taught ideas that are germane so that which would be asked of them by such a curriculum. At the same time they wanted to present the college student with fresh and exciting material rather than some sort of rehash or previously studied concepts. To accomplish these two objectives, this group suggested that the content be built around matrix algebra and transfomrations. They felt that this was a new topic for the college students, that presented new algebraic properties, that showed them that not everything in mathematics behaved like numbers, and at the same time had many applications and contacts in the K-6 curriculum proposed by the 1963 report.

In the second alternative program, the group was more wary about the flexibility and depth that would be achieved by the college student. They felt that a solution for their problem was one that involved shorter units which were self-contained and which allowed students to influence the course to some extent. In their proposal, students could select from several units those that were intrinsically interesting and exciting.

Another topic discussed at the conference was the role of the mathematics specialist. Seventeen different activities for the mathematics specialist are suggested.[4] He might teach a class or special courses, be involved mostly with testing, concern himself with curriculum planning, and experimentation, and consulting with teachers, parents or students. One of their primary tasks is simply to remain current with changes occurring in the mathematics curriculum.

The conference stressed that the existence and importance of specialists was not predicated upon elimination of the homeroom environment with its close child-teacher relationships. There are many intermediate systems possible in which the specialist is used in various capacities, and children spend most if not all of their time, with one teacher.

The conference also advocated that adequate opportunity be given to the in-service teacher for continuing his education. The related questions of salary schedules, time devoted to classroom instruction, and sabbatical leaves were considered.

More specifically, the conference recommended that the Education Development Center should sponsor small writing and testing teams to develop materials for pre-service education. Two professors from an education department and two from a mathematics department jointly would write appropriate materials. The materials could then be tried provisionally, rewritten, and tried again as polishing progressed. One such effort has been initiated at Indiana University by Dr. George Springer. This summer several faculty are writing materials, with experimentation slated for the 1967-68 school year. In addition, Drs. E. Weiss and S.Z. Sternberg are working with faculty at Boston State College to implement a similar plan

[4] "Goals for Mathematical Education of Elementary School Teachers", op. cit. p. 30

in which students work for a concentrated period each trimester rather than in a regular course.

One intriguing suggestion included in the appendix of the report is a proposal for a model-city project. Obviously planning and implementing such a project would require the combined efforts of many people from several disciplines. In such a city, teachers would teach twenty pupils three hours per day. Parenthetically, I would suggest two teachers in some classrooms, one working with the group while the other concentrates on individuals with learning or discipline problems. The cost of such an educational program in plant and faculty is about three times that usually spent.

Such a cost is not prohibitive. Most of the public is convinced that education is important, but they must be convinced that spending three times as much money will do at least twice as much good. The model system would be an opportunity to demonstrate that it can.

Implications and Extensions

The recommendations of the Cambridge Conference on Teacher Training are important and undoubtedly should be part of a teacher preparation curriculum. But it is inadequate to meet certain needs of the elementary school teacher who accepts the challenge of this type of curriculum. In particular, the teacher needs to feel comfortable in teaching a rapidly changing curriculum. Obviously this can not be accomplished in preservice education. To adequately handle difficult and changing topics, in-service education must be continual.

Training and confidence in using open-ended material and a discovery format is also essential. Prospective teachers must not only know relevant subject matter, but also effective teaching strategies. So often, when faced with unfamiliar subject matter to teach, the instructor reverts to a teaching style with which he is familiar and comfortable. Many are insecure in teaching new content with an open format or question-answer approach. They mistakenly feel they must be prepared to answer almost all questions students raise about the topic. Such a requirement certainly limits the boundaries of class exploration.

Further, teachers often want well-defined objectives for each class session. With such a procedure the goal must be defined so as to be achieved in a class period. This is clearly incompatible with an open ended or discovery approach. In my opinion it is undesirable to package content in neat little bundles. It is impossible to do so if the teacher honestly follows the argumentation and reasoning of the children of the class. Rather than unifying mathematics content, it segments it. When considering any one topic, several others become evident. If possible it is better to treat them at that time.

In some of our experimental efforts, we have worked with teachers on a one-to-one basis. An interested teacher cooperates in teaching a new unit. We work closely together, conferring perhaps once a week, anwering technical questions, observing her class, teaching the class, then discussing her observations. On that basis, the teacher usually does a marvelous job. She gains the spirit of a modern approach to mathematics and this spills over into her other work. While the quality of such in-service education is good, the quantity is ridiculous. The problems in today's schools could never be solved with such an approach.

Could this same technique be used on a many-to-one basis? We selected two schools in greater Boston to work with. The in-service program was centered around symmetry motions from geometry and related topics for the elementary school. These same units had been used previously. General discussions were held every week or two, sometimes with university faculty present, sometimes in teacher group discussion and planning. Teachers were encouraged to telephone the university people at any time for immediate assistance. While two or three teachers readily caught the spirit of this mode of teaching, the general results were less than we expected. Most did not get their feet wet enough for the process to bear fruit.

Robert Davis of the Madison Project has organized summer workshops in large cities such as Philadelphia, San Diego, New York, and Chicago to extend teachers' understanding of this process of teaching. Direct involvement of participants is an integral part of the workshops. Elementary pupil material is taught to them while they pretend they are children. This, of course, raises some difficulties as one's pretend level fluctuates. The participants then practice teaching the material to each other. Their direct experience in the process gave them confidence that this teaching strategy would work, and that they could handle it.

The Madison Project does not extend far enough. To insure saturation of the discovery approach to teaching, this should be a major teaching technique throughout a teacher's training. Further, the content should be extended to include the ideas incorporated in the 1963 Cambridge Conference; not limited to isolated lessons as presently handled. The in-service experience should be extended to include frequent contacts with new material workshops during the school year.

What are the implications of these experiences for pre-service training of teachers? The discovery approach should be part of the teaching technique used with these prospective teachers. The 1966 Conference advocated a correlation between the methods and content courses in college. This does not go far enough. This relation must be more than that, there are two types of courses. Some, perhaps all, of these courses should be so integrated that content is taught in the same way as advocated for teaching children in elementary schools. While this approach illustrates effective teaching strategies, it also is the most effective way for the college student to learn. They too should benefit from having to think, reason, and answer questions for themselves.

In combination with this, the prospective teacher should spend several hours each week in schools working with children all during their program. Opportunities to try out techniques used with them, and to test their effectiveness, would build up confidence. To wait until near the end of their college undergraduate program is too late. Motivation and discrimination in a student's professional coursework is adversely affected by lack of immediate and continual contact with children.

Correlation of Science and Mathematics

In ten days, a third Cambridge Conference will begin. This conference will solve no problems for you, and will likely create a few. For three weeks, a group of mathematicians and scientists will study the correlation of mathematics and science. These fields are so closely related that more effective learning might result from careful interrelation. Minnemast and the American Association for the Advancement of Science (AAAS) projects have developed some curriculum programs using mathematics as a tool in science. While many recognize the importance of this approach, the range of possibilities is great. A conference stressing the goals of such a combination curriculum is vital at this time. One potential approach in integrating mathematics and science might simply be to roughly correlate the content through reciprocal agreement, with each discipline identifying that content needed from the other, and the time it was needed. At the other extreme, the two subjects would be treated as one. Every lesson would involve mathematics and science together, each dependent on the other. To learn science one must be able to handle mathematical models; and to maintain the momentum and intuition in mathematics one must have challenging problems from science.

Not only will the generalist in the elementary school need to be adequately trained for teaching mathematics and for teaching science; but more than that he must be able to treat both together. The problem is not nearly so crucial in the elementary school with its tradition of generalists as in the secondary school where teachers are specialists. The elementary school presents a great opportunity to experiment with this approach to handling science and mathematics.

APPENDIX I

SELECTIONS

FROM
WINTER - SPRING 1967 PROGRESS REPORTS
OF
FEASIBILITY STUDIES (CCSM)

Consultant Access Scheme

Underwood School, Newton (January 18 - April 7, 1967)

January 20, Marion Walter gave deomonstration classes on informal geometry to second grade and fifth grade classes.

To plan course work based on the demonstration classes, Walter and Lomon met with nine teachers (the teachers were Miss Byron, Miss Davies, Miss Flanagan, Mrs. Hauser, Mr. Hunt, Miss Kuhl, Mrs. Nadeau [assistant principal], Miss Scully and Mrs. Stewart, and the principal, Mr. Atkins), on January 24. Reactions to the January 20 demonstration were expressed. The teachers said that they found the presentation most important in understanding our intentions. They were given the written guide for the informal geometry unit. This guide is oriented toward the sixth grade level. Added to it was a report of past first and third grade classes with suggestions for further work with young children. Most of the concern was not with questions of mathematical content. In the first meetings the teachers had been most concerned about obtaining fuller explanations of the mathematics and where it was leading. This time the only mathematically related questions appropriately probed the differences between reflections, rotations and translations.

The teachers asserted that they could begin classes on the basis of Miss Walter's notes and the demonstration classes. They were urged to phone Miss Walter or Prof. Lomon with questions arising between class sessions. Four classes promised to carry on with the material. It seemed likely that other classes would try at least a part of the material.

During March there were telephone conversations with two of the teachers. Miss Walter also had discussions in the course of visits to the school for other purposes.

Walter and Lomon find that two aspects of the activity at Underwood are not meeting expectations. First, the number of class sessions per month being devoted to this material is less than the desirable once-a-week. Secondly, they are not taking advantage of telephone inquiries to the extent that seems needed. They do not seem to have severe difficulty in teaching the material. Several of them (not all) claim that their schedule allows them insufficient time to plan the lessons and prepare material. They are uncomfortable with the development of a skill and its related concept over a span of several lessons, being used to a self-contained single-lesson plan.

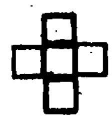
We think it may be advisable to encourage telephone interviews by some administrative means. Perhaps an initial schedule of class would "break the ice."

Lexington Schools (January 18 - April 7, 1967)

In Lexington the sessions on inequalities and probability have proceeded on a regular basis. The "Scope and Sequence" chart of the school system allows the teacher to choose the actual material to be used in reaching the over-all goals. Thus little difficulty with "getting the regular program done" arises.

Several meetings were held with Miss Lyn McLane, Mrs. B. Fitzgerald and Miss K. Dillmore. In addition, telephone contacts with those teachers have averaged about once a week and have proved very useful.

Underwood School, Newton (April 7 - June 3, 1967)

On May 4, Mr. Atkins reported by phone that Miss Flanagan was nearly finished with Marion Walter's "Informal Geometry" and had started to work with "Symmetry Motions" of an equilateral triangle. Mrs. Stewart's class was working on the symmetries of  and its compounded motions.

She reported that her SMSG work (grade 4, part 2) was proceeding quickly and credited that fact to the class' work with informal geomety. Mr. Hunt was also finished with much of the infomal goemetry. Miss Flanagan phoned Lomon to describe and discuss her start with symmetry motions of a triangle.

Lexington Schools (April 7 - June 3, 1967)

Between April 7 and June 3, Miss Lyn McLane conducted seven sessions on probability with Miss Kubasiewicz's sixth grade class at Hancock School. They found open sentences that matched the values of their range and peak results for nickel spinning. They used these sentences to make predictions, some of which they checked. They then began discussion of and experimentation with the combined outcome of several (presumably independent) events. The class has accumulated strong evidence for several assemblies of indivdual events on the appropriate counting of separate elements of the sample set and of the relevency of multiplying elementary probabilities.

Lomon observed and assisted at the lesson of May 4. McLane and Lomon met three times and communicated by phone four times in the preparation of these lessons. Tapes were made and notes prepared by Miss McLane.

APPENDIX J

Toward Integrated Science and Mathematics in the Schools

Last summer the Cambridge Conference on School Mathematics held a conference to discuss the relation between science and mathematics in the public schools.* Participants included about twenty-five mathematicians, physicists, biologists and chemists. Among these were representatives of several well-known curriculum reform projects. Most of the group had previously attended seminars held in various parts of the country during the year 1966-67. A full report of the considerations of the Conference is in preparation and will be published in the coming year. The present selected outline is intended only as an introduction to the AAAS Symposium of 29 December 1967.

During the conference, the topic was considered from many points of view, from broad questions of educational philosophy to concrete problems of developing specific experiments. While no one would claim that such a large topic was exhausted, the majority of those who were present would like to make several recommendations which, while not entirely novel, would have far reaching consequences when carried out.

It was generally agreed that the advantages to both subject areas of emphasizing their organic connections would far outweigh any disadvantages, especially in elementary school. An integrated math-science curriculum will be difficult to implement, and perhaps the greatest difficulty will be the problem of training teachers to handle the material. Nevertheless it appears that an integrated curriculum designed to bring out these connections is necessary in view of two facts; quantitative thinking is the essence of the power of the scientific method; many pupils are unable to grasp the connections between the mathematics they are taught and the real world. This curriculum would comprise a variety of units and activities which could be variously described as:

- Math for math
- Math for science
- Math and science
- Science for math
- Science for science

Of these five categories, only the first has any long history in the schools, while science in any sort of quantitative form has only recently been introduced below the high school level. In the third category, where science and mathematics play synergistic roles in explaining the world about us, we have very little classroom material, indeed. Yet it is in this category that we must put our greatest effort if we are to succeed in conveying the symbiotic relationship between mathematics and science that characterizes the modern, basically scientific world.

Units for a combined curriculum can and should be open-ended, always leaving the opportunity for the talented student to go deeper. Such units have the potential of being more "real" and "honest" than is possible in the individual subject areas as they are now divided. There should be a "bank" of units which present at many levels all the important facts, theoretical

*See Appendix B, page vi, for list of participants.

concepts, and mathematical structures. With such a bank, the teacher could individualize instruction by introducing units dictated by the student's reactions and needs. The teacher will find that the combination of science and mathematics imposes a natural mixed style of exploratory observation and discussion followed by some organization of facts and theories. This style of instruction imposes a great burden on the teacher, and to facilitate his work it will be important to have a teacher's super-manual which catalogues and describes the available materials and suggests possible sequences of units relevant to the pupils' recent experience.

Clearly, it will be some time before such a system can be put into operation. Beginnings in this direction have been made by some of the materials developed by SMSG, AAAS, and, in particular, by MINNE-MAST. Much effort at the conference was devoted to changes that could be adopted very soon. Most of these explicit changes are discussed within the framework of mathematical categories, probably because the mathematics at this level is more structured and because there is so much more experience with mathematics teaching than with science teaching.

Some of the "new math" programs have virtually outlawed the use of numbers to which units are intrinsically attached, such as "8 inches". These are known classically as denominate numbers. Lengths, time intervals and weights have a reality that is in some respects greater than the reality of "pure numbers", and the relationships among these quantities are governed by the physical reality experienced by children. These entities may all be regarded as elements of an appropriate mathematical system that is, admittedly, more complicated than the system of pure numbers. The algebraic structure of these entities is often avoided by resorting to the concept of measure, as in "the measure in inches of a one foot unit of length is twelve". In spite of some formal complications, a system that gives meaning to the more convenient statement "one foot = twelve inches" is more desirable. This implies a great deal more attention, starting in the earliest grades, to the problems of measurement.

Another aspect of measurement is its connection to rational numbers. The relation can be developed by teaching that measurement leads fundamentally to an interval: "The length of this stick is between 5 and 6 inches". Successive refinement of the intervals used implies the need for rational numbers. The idea of rounded measurement (e.g. to the nearest inch) is valuable, to be sure, but should be taught only after the idea of interval is thoroughly understood. This change would be very helpful both in science and in more advanced mathematics.

In school the topic of "ratios and proportions" has traditionally taken an undue share of time and trouble. It was concluded that this should instead arise as a special case of functions, a topic of far wider interest and power. Functions are the most frequently used models of reality and are thus central to the math-science correlation. The concept of functions should therefore be developed from very early grades. Proportions are simply linear functions, and ratios and the numerical coefficients associared with these functions. The linear function, and hence, ratios, can be treated using graphs from grade one.

to examine functions starting from their graphs is made, the periodic functions obtrude. They too can become familiar objects long before they are associated with trigonometry.

The Conference once again questioned the very long time now devoted to mastery of specific algorithms for long multiplication and long division. Most of the conferees thought that the time could be better spent on learning estimation and to use the slide rule or desk calculator. In the meantime, the algorithmic method can be explored by the students in response to the challenging questions.

Our world is not two-dimensional. Many children and most adults have great difficulty in visualizing three-dimensional objects. An enjoyable experimental approach was proposed involving the study of polyhedra, spirals and snail-shells, and highway cloverleafs. We even built a kaleidoscope big enough to get into. (Children love it.)

Although the Conference was unable to discuss computers in any detail, it was recognized that the computer has many contributions to make in mathematics and science education: it can serve as a teaching machine; it can do large and otherwise impossible calculations that would make some experiments impractical. A student who learns to program is obliged to organize his ideas carefully. Even the simplest programs are more demanding in this respect than the hardest algebra problems considered in high school, yet junior high school students are eager and able to accept the challenge from the machine.

Probability and statistics are topics of central importance for a combined math-science curriculum and are clearly important for experimental science. At the same time they are best developed in an experimental context with dice, coins and urns and in tests of hypotheses concerning more realistic systems. Probability by itself is too abstract for children, but inferential statistics makes the subject come alive. A tentative outline, involving both experiments and theory, was prepared for grades one to twelve. This outline suggests that the use of rank ordering and similar non-parametric methods can lead to effective statistical inferences with a minimum of analytical tools.

The success of the general approach proposed here requires the development of a series of individual units which excite and involve the children, develop their curiosity and abilities and honestly represent the challenges of life's problems. The three week meeting permitted only the barest beginnings in these directions, yet activities were started involving microbe cultures, product testing, random walk phenomena, sine function generators and the instruction of various three dimensional figures. We advocate continuing efforts of this kind with the collaboration of mathematicians and scientists - engineers and artists.

APPENDIX K

A REVIEW:

"Goals for the Correlation of Elementary Science and Mathematics:"
The Report of the Cambridge Conference on the Correlation of Science and Mathematics in the Schools

John R. Mayor
American Association
for the
Advancement of Science

(From "The Arithmetic Teacher", March 1970, pp. 271-272)

This report deserves wide attention. It could become one of the most important educational pronouncements of the decade. The report forthrightly identifies issues that will become critical in the next decade and suggests ways in which solutions may be found. It is presented as a majority report, along with a range of opinions of the conferees. It seems certain that few readers will agree with all of its implications. On the other hand, it is equally certain that all readers will be stimulated, and some may even redirect their thinking about elementary school mathematics and science.

The conference was organized by the Cambridge Conference on School Mathematics, a part of the Education Development Center, Inc. The National Science Foundation provided the support. Andrew M. Gleason was chairman of the conference. Among the thirty participants, seven can be identified as university mathematicians, six as members of university physics departments, one from chemistry, and three from biology. Engineering and medicine were also represented. No elementary school teacher participated.

The report consists of an introduction, five chapters, and twenty-five appendices. The chapters review the broad goals of education for the modern world, the implications of these goals for the mathematics-science curriculum, topics in the curriculum, teacher training, and recommendations for immediate implementation. The appendices describe class exercises, a number in sufficient detail to be tried out by the teacher of mathematics or science in the elementary school. Not all of these are new, but many of them have probably never been tried in an elementary school. Among interesting titles of the appendices are: "The Kaleidoscope," "A Highway Cloverleaf Theorem," "Sampling and Tracer Techniques with Houseflies," "A Fine-Grained Normal Distribution."

The value of dialogues on important issues of our time, including educational issues, is emphasized in the statement of purposes of the conference and report. A call is made for dialogues on correlating elementary school science and mathematics among professionals who know and use the subject matter (mathematicians, scientists, engineers, and also lawyers, physicians, and architects), school administrators and curriculum experts, personnel of the publishing industry, professionals from schools of education, classroom teachers, and representatives from foundations. The report is addressed to all of these groups with the hope that dialogues will begin.

The proposed integrated curriculum "seeks to increase the relevance of mathematics and science to each other and to the daily experiences of the student." While the greater importance of this report is long-range, that is, correlation of mathematics and science in another decade, the recommendations for immediate implementation are also important. Something can be done about these now. No long-range goal will be achieved without a beginning. This beginning depends first on the needed dialogues, but it will also depend upon successful implementation of many of the recommendations of chapter V. Samples of these recommendations are:

a) Each mathematics curriculum project now functioning should add to its staff a specialist in contemporary science curricula and, the other way around; each science curriculum staff should include a specialist in contemporary mathematics curricula.

b) Methods courses for teachers should be coordinated with strong, rigorous content courses offered by academic departments. (This is not a new idea, but it is one that is rarely implemented.)

c) Colleges and universities should begin to develop courses designed to train mathematics-science specialists, and to organize summer institutes for teachers functioning as mathematics or science specialists.

d) School systems should move immediately towards unification, at administrative levels, of all curriculum planning in elementary school science and mathematics.

e) Each teacher should add to his classroom program at least one mathematics-science unit each school year.

f) The teacher should try not only the available new science and mathematics materials, but also some of the activities presented in this report.

This report should be required reading for many of the persons to whom it is addressed. It develops a provocative topic for discussion groups at professional meetings. It is an essential reference to all curriculum planning groups in science and mathematics.

This reviewer is clearly biased in favor of the point of view that is so succinctly set forth in the report. Furthermore, one who has had extensive experience in curriculum development in both mathematics and science, the report makes crystal clear how far we have yet to go, and it supports an inference that the mathematics curriculum groups have further to go than the science curriculum groups. The challenge of the report is properly directed to mathematicians and to mathematics teachers.