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ABSTRACT

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BUDGETARY CONSIDERATION IN SETTING PASSING SCORES

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ABSTRACT

A general model along with four illustrations is presented for the consideration of budgetary constraints in the setting of passing scores in instructional programs involving remedial action for poor test performers. Budgetary constraints normally put an upper limit on any choice of passing score. Given relevant information, this limit may be determined. Alternately, ways to assess the budgetary consequences associated with a given passing score are provided. Such information would be useful in any final decision regarding the passing score.

1. INTRODUCTION

In many instructional programs, such as Individually Prescribed Instruction (Glaser, 1968) or others of a similar nature (Atkinson, 1968; Flanagan, 1967), testing is conducted at the end of every instructional unit to provide feedback to the student and/or teacher in order that appropriate action can be taken. If a student's test score is high, it may be reasonable to grant that student mastery

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of the current unit and to allow him to proceed to a subsequent unit. On the other hand, a low score may indicate that the student might benefit from some remedial action. This is also the case for certification testing such as high school graduation or for minimum competency testing as legislated in several states. Funds are usually allocated for remediation for students whose scores are too low to warrant mastery of the competencies under consideration.

The statistical issues relating to granting or denying mastery status have been approached by several writers, including Huynh (1976, 1977, 1978). Most proposed schemes are by and large quota-free, i.e., the mastery/nonmastery decision process considered by the writers does not take into account the budgetary consequences associated with the denial of mastery status. If funds provided for remediation are limited, then a constraint will have to be imposed on the number of students declared as failures (nonmasters).

The purpose of this paper is to demonstrate how budgetary restrictions may be taken into account in the process of setting passing (mastery) scores or performance standards. Alternately, the presentation provides ways to assess the budgetary consequences associated with an arbitrary passing score. Section 2 describes the overall framework. Illustrations based on the beta-binomial and normal-normal test score models will be provided in subsequent sections.

2. OVERALL FRAMEWORK

It is now assumed that the true ability of a population of subjects may be described by a random variable θ which ranges in the sample space Ω. For the beta-binomial model, θ is the proportion of items that a subject answers correctly in an item pool and Ω is the interval from 0 to 1. For the normal test score model, θ is the traditional true score (Lord & Novick, 1968) and Ω is the entire real line. Let the probability density function (pdf) of θ be $p(\theta)$.

Let x be the score obtained from the administration of an n-item test and let f(x) and $f(x|\theta)$ denote its marginal and conditional probability density functions with respect to θ.

It shall be assumed that all subjects with test scores smaller than a passing (mastery) score c will be denied mastery for the instructional objectives covered by the test and that these subjects will be provided with appropriate remedial learning activities. The remediation is assumed to be so devised that its conclusion will coincide with the mastery status which was previously denied the student. The cost of remediation will be assumed to be a non-increasing function of θ and will be denoted as $\delta(\theta)$. Thus, remediation will cost less for more able students than it will for less able ones.

Consider now a subject with true ability θ. The probability that this person will be declared in need of remediation is given as the sum $\Sigma f(x|\theta)$ or the integral $\int f(x|\theta)dx$, with $x < c$. For the purposes of this section, the summation notation will be used. It follows that the (conditional) expected remediation cost for this subject is

$$\sum_{x<c} f(x|\theta)\delta(\theta) .$$

Hence the (unconditional or marginal) expected remediation cost for a subject drawn randomly from the population is

$$\gamma(c) = \int_\Omega \sum_{x<c} f(x|\theta)\delta(\theta)p(\theta)d\theta . \qquad (1)$$

This function is nondecreasing with respect to its argument c. Its lowest limit is zero (when all subjects are granted mastery status) and its maximum value, $\gamma_{max} = \int_\Omega \delta(\theta)p(\theta)d\theta$, is reached when remediation is provided to all subjects regardless of their test scores.

Let us suppose, furthermore, that testing is to be conducted for a total of m subjects and the total cost of possible remediation cannot exceed the value B. If the passing score c is selected, then the total expected remediation cost will be $m\gamma(c)$. Hence any choice for c must satisfy the budgetary constraint $m\gamma(c) \leq B$. If $\gamma_{max} \leq B$, any cutoff score will be acceptable. However, if $B < \gamma_{max}$, then the passing score c must be less than or equal to c_1, where c_1 is the highest score satisfying the inequality

$$\gamma(c_1) \leq B/m. \tag{2}$$

For discrete test scores, such as those of the binomial error model, Inequation (2) may be solved by computing the values of $\gamma(a)$ one by one, starting with c as the smallest test score, and stopping when the value c_1 is reached. For continuous test data, numerical procedures for solving the nonlinear equation $\gamma(c_1) = B/m$ might be needed.

3. THE BETA-BINOMIAL MODEL WITH CONSTANT COSTS

Consider now the beta-binomial model as defined by the following pdf's:

$$f(x|\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}, \; x = 0,1,\ldots,n$$

and

$$p(\theta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}, \; 0 < \theta < 1.$$

The two parameters α and β may be estimated from sample data via one of several estimation techniques such as the moment procedure or the maximum likelihood procedure. Let $\bar{x}$ and s be the sample test score mean and standard deviation. In addition, let $\hat{\alpha}_{21}$ be the KR21 reliability coefficient as defined by

$$\hat{\alpha}_{21} = \frac{n}{n-1}\left(1 - \frac{\bar{x}(n-\bar{x})}{ns^2}\right). \tag{3}$$

(In the case of a negative $\hat{\alpha}_{21}$, simply replace the value computed from Equation (3) by any positive reliability estimate.) The moment estimates for α and β are given as

$$\hat{\alpha} = (-1 + 1/\hat{\alpha}_{21})\bar{x} \tag{4}$$

and

$$\hat{\beta} = -\hat{\alpha} + n/\hat{\alpha}_{21} - n. \tag{5}$$

We will now focus on the simple case where a single true passing score (or criterion level) θ_o, separating true masters from true nonmasters, has been specified. Let the remediation cost be constant and equal to γ_o for a true nonmaster and γ_1 for a true master. Thus the cost function is of the form

$$\delta(\theta) = \begin{cases} \gamma_o & \text{if } \theta < \theta_o \\ \gamma_1 & \text{if } \theta \geq \theta_o . \end{cases}$$

The nonincreasing nature of $\delta(\theta)$ is satisfied whenever $\gamma_o > \gamma_1$.

The expected remediation cost per student as shown by Equation (1) is now given as

$$\gamma(c) = \frac{1}{B(\alpha,\beta)} \sum_{x=o}^{c-1} \binom{n}{x} \left[\gamma_1 \int_{\theta_o}^{1} \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x-1} d\theta + \gamma_o \int_o^{\theta_o} \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x-1} d\theta \right]$$

or

$$\gamma(c) = \frac{1}{B(\alpha,\beta)} \sum_{x=o}^{c-1} \binom{n}{x} \left[\gamma_1 B(\alpha+x, n+\beta-x) + (\gamma_o-\gamma_1) \int_o^{\theta_o} \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x-1} d\theta \right].$$

It may be noted that the marginal beta-binomial pdf of x is given as

$$f(x) = \binom{n}{x} B(\alpha+x, n+\beta-x)/B(\alpha,\beta) \tag{6}$$

and that the incomplete beta function $I(\alpha+x, n+\beta-x; \theta_o)$ is defined as

$$I(\alpha+x, n+\beta-x; \theta_o) = \int_o^{\theta_o} \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x-1} d\theta / B(\alpha+x, n+\beta-x).$$

It follows that

$$\gamma(c) = \sum_{x=o}^{c-1} f(x)\left(\gamma_1 + (\gamma_o-\gamma_1) I(\alpha+x, n+\beta-x; \theta_o)\right). \tag{7}$$

The values of f(x) may be computed via the following inductive formulae:

$$f(0) = \prod_{i=1}^{n} \frac{n+\beta-i}{n+\alpha+\beta-i} \tag{8}$$

and

$$f(x+1) = f(x) \cdot \frac{(n-x)(\alpha+x)}{(x+1)(n+\beta-x-1)}, \quad x = 0,1,\ldots,n-1. \tag{9}$$

The following recurrence formula, on the other hand, will quicken the evaluation of the incomplete beta functions:

$$I(\alpha+x+1, n+\beta-x-1; \theta_o) = -\frac{\theta^{\alpha+x}(1-\theta)^{n+\beta-x-1}}{(\alpha+x)B(\alpha+x, n+\beta-x)} + I(\alpha+x, n+\beta-x; \theta_o). \quad (10)$$

Finally, as in Section 2, let B be the maximum funds allocated for possible remediation involving a group of m subjects. Then the passing score cannot exceed the highest integer c_1 at which $\gamma(c_1) \leq B/m$.

Numerical Example 1

A maximum sum of B = \$4000 has been allocated for remediation in an instructional program with m = 100 students. Thus B/m = \$40. For the program under study, assume that $\theta_o = .60$ and the remediation costs are γ_o = \$150 for each student with true ability $\theta < .60$ and γ_1 = \$50 for students with $\theta \geq .60$. Now suppose a 5-item test is administered and the test scores yield the estimates $\hat{\alpha} = 3$ and $\hat{\beta} = 2$. At the passing scores c = 1, 2, 3, 4, and 5, the expected remediation costs $\gamma(c)$ are \$7.02, \$19.06, \$31.83, \$41.25, and \$47.19, respectively. Since $\gamma(c_1) \leq \$40$, it follows that $c_1 = 3$. The budget constraint imposes an upper limit of 3 on the passing score. If 3 is used, the expected cost of remediation amounts to \$3183. If the next higher passing score, 4, were used, the expected remediation cost would be \$4125, over the maximum budgeted sum of \$4000.

4. THE BETA-BINOMIAL MODEL WITH LINEAR COSTS

Let us suppose now that the cost function may be written as

$$\delta(\theta) = (\gamma_o - \gamma_1)(1-\theta) + \gamma_1, \quad (11)$$

in which $\gamma_1 < \gamma_o$. Thus the cost is a linear function of θ. It is equal to γ_o when $\theta = 0$ and γ_1 when $\theta = 1$.

Under the beta-binomial model as described in the first paragraph of Section 3, the expected cost per student is given as

$$\gamma(c) = \frac{1}{B(\alpha,\beta)} \sum_{x=0}^{c-1} \binom{n}{x} \left[(\gamma_0-\gamma_1) \int_0^1 \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x+1-1} d\theta + \gamma_1 \int_0^1 \theta^{\alpha+x-1}(1-\theta)^{n+\beta-x-1} d\theta \right]$$

$$= \frac{1}{B(\alpha,\beta)} \sum_{x=0}^{c-1} \binom{n}{x} \left[(\gamma_0-\gamma_1) B(\alpha+x, n+\beta-x+1) + \gamma_1 B(\alpha+x, n+\beta-x) \right].$$

By noting that

$$B(\alpha+x, n+\beta-x+1) = \frac{n+\beta-x}{n+\alpha+\beta} B(\alpha+x, n+\beta-x)$$

it may be deduced that

$$\gamma(c) = \sum_{x=0}^{c-1} f(x) \frac{(\gamma_0-\gamma_1)(n+\beta-x)}{n+\alpha+\beta} + \gamma_1$$

$$= \sum_{x=0}^{c-1} f(x) \frac{\alpha_0(n+\beta-x) + \gamma_1(\alpha+x)}{n+\alpha+\beta}. \qquad (12)$$

As in the previous section, the values of $f(x)$ may be computed inductively via Equations (8) and (9).

Numerical Example 2

Consider the basic data of the first numerical example, namely B/m = \$40, γ_0 = \$150, γ_1 = \$50, $\hat{\alpha} = 3$, $\hat{\beta} = 2$, and $n = 5$ items. At the passing scores of 1, 2, 3, 4, and 5, the expected remediation costs $\gamma(c)$ are \$5.71, \$18.81, \$37.86, \$59.29, and \$78.33. Hence the passing score cannot exceed 3, where the maximum value of the expected cost of remediation would amount to \$3786. Had a score of 4 been selected, the expected cost would have amounted to as much as \$5929.

To close this section, it should be mentioned that simple expressions for $\gamma(c)$ such as the one of Equation (12) may be worked out for all cost functions $\delta(\theta)$ which can be represented as integral polynomials of θ.

5. THE BIVARIATE NORMAL MODEL WITH CONSTANT COSTS

Now consider the case where the true score θ and the observed score x are jointly distributed according to a bivariate normal distribution. Without any loss of generality, it may be assumed that x is in its standardized form with zero mean and unit variance. Let ρ be the reliability of the test for the normal population of subjects under consideration. The true score θ has a mean of zero, a standard deviation of $\sqrt{\rho}$, and a correlation of $\sqrt{\rho}$ with the test score x. The joint pdf of x and θ is

$$f(x,\theta) = \frac{1}{2\pi\sqrt{\rho(1-\rho)}} \exp\left[-\frac{1}{2(1-\rho)}\left(x^2 - 2x\theta + \frac{\theta^2}{\rho}\right)\right]. \qquad (13)$$

As in Section 3, it will be assumed that the cost function $\delta(\theta)$ is constant, taking the values of γ_o for $\theta < \theta_o$ and the value of γ_1 for $\theta \geq \theta_o$. It follows from Equation (1) that at any passing score c, the remediation cost for a subject drawn randomly from the population is expected to be

$$\gamma(c) = \gamma_o \int_{-\infty}^{c} \int_{-\infty}^{\theta_o} f(x,\theta)d\theta dx + \gamma_1 \int_{-\infty}^{c} \int_{\theta_o}^{\infty} f(x,\theta)d\theta dx$$

$$= \gamma_1 Pr(x \leq c) + (\gamma_o-\gamma_1) \int_{-\infty}^{c} \int_{-\infty}^{\theta_o} f(x,\theta)d\theta dx. \qquad (14)$$

The maximum passing score c_1 satisfies the equation $\gamma(c_1) = B/m$. This value of c_1 exists as long as $B < \gamma_{max}$ where

$$\gamma_{max} = \gamma_o Pr(\theta < \theta_o) + \gamma_1 Pr(\theta \geq \theta_o).$$

Solutions may be found via numerical procedures such as the Newton iterative solution for nonlinear equations. To apply this technique, it may be noted that the derivative of $\gamma(c)$ with respect to c is

$$\gamma'(c) = \gamma_1 f_N(c) + (\gamma_o-\gamma_1) \int_{-\infty}^{\theta_o} f(c,\theta)d\theta$$

where $f_N(.)$ denotes the pdf of x (the unit normal variable). In other words,

$$f_N(c) = \frac{1}{\sqrt{2\pi}} e^{-c^2/2}.$$

It may also be noted that

$$\int_{-\infty}^{\theta_o} f(c,\theta)d\theta = f_N(c) \cdot F_N\left(\frac{\theta_o - \rho c}{\sqrt{\rho-\rho^2}}\right)$$

where $F_N(.)$ is the (cumulative) distribution function of the unit normal variable.

In summary,

$$\gamma'(c) = f_N(c)\left(\gamma_1 + (\gamma_o - \gamma_1)F_N\left(\frac{\theta_o - \rho c}{\sqrt{\rho-\rho^2}}\right)\right). \qquad (15)$$

Both $\gamma(c)$ and $\gamma'(c)$ may be evaluated via computer programs such as those described in the IMSL (1977). They may also be obtained by use of appropriate tables for the univariate and bivariate normal distributions.

Numerical Example 3

Let the parameters defining the problem be $\rho = .64$, $\theta_o = 1$, $\gamma_o = \$150$, $\gamma_1 = \$50$, and $B/m = \$40$. Numerical procedure yields the maximum standardized passing score $c_1 = -.475$. If the test scores have a mean of 50 and a standard deviation of 20, then the passing score cannot exceed 40.5.

6. THE BIVARIATE NORMAL MODEL WITH NORMAL-OGIVE COST

Now consider the case where the cost function $\delta(\theta)$ is of the form

$$\delta(\theta) = (\gamma_o - \gamma_1)\left(1 - F_N\left(\frac{\theta - \theta_o}{\sigma}\right)\right) + \gamma_1 \qquad (16)$$

where, as before, $F_N(.)$ represents the distribution function of the unit normal variable. In the context of decision theory, expressions similar to those of Equation (16) have been proposed as utility functions (e.g., Lindley, 1976, and Novick and Lindley, 1978). As in the case of the beta-binomial model with linear costs, γ_o and γ_1 represent the remediation costs associated with the least able ($\theta = -\infty$) and the most able ($\theta = +\infty$) subjects. On the other hand, the parameter θ_o is the location at which the cost is

$(\gamma_o+\gamma_1)/2$ and $1/\sigma$ indicates the extent to which $\delta(\theta)$ decreases at this location.

The expected remediation cost $\gamma(c)$ may now be written as

$$\gamma(c) = \int_{-\infty}^{c} \int_{-\infty}^{+\infty} f(x,\theta)\,\delta(\theta)\,d\theta\,dx$$

$$= \gamma_o \Pr(x \le c) - (\gamma_o - \gamma_1) \int_{-\infty}^{c} \phi(x) f_N(x)\,dx \qquad (17)$$

where

$$\phi(x) = \int_{-\infty}^{+\infty} f(\theta|x) F_N\left(\frac{\theta-\theta_o}{\sigma}\right) d\theta .$$

The conditional pdf $f(\theta|x)$ is given as

$$f(\theta|x) = \frac{1}{\sqrt{2\pi\rho(1-\rho)}} \exp\left(-\frac{(\theta-\rho x)^2}{2\rho(1-\rho)}\right).$$

It follows that

$$\phi(x) = \frac{1}{2\pi\sigma\sqrt{\rho(1-\rho)}} \int_{-\infty}^{+\infty} \{\exp\left(-\frac{(\theta-\rho x)^2}{2\rho(1-\rho)}\right) \int_{-\infty}^{\theta} \exp\left(-\frac{(t-\theta_o)^2}{2\sigma^2}\right) dt\}\,d\theta .$$

It should be noted that the expression

$$\frac{1}{2\pi\sigma\sqrt{\rho(1-\rho)}} \exp\left(-\frac{(\theta-\rho x)^2}{2\rho(1-\rho)} - \frac{(t-\theta_o)^2}{2\sigma^2}\right)$$

acts as the joint pdf of two <u>independent</u> normal random variables θ and t with means ρx and θ_o, and with variances $\rho(1-\rho)$ and σ^2.

Now let us introduce the new random variable $u = \theta - t$ for which the mean is $\rho x - \theta_o$ and the variance is $\rho - \rho^2 + \sigma^2$. Since the condition $t < \theta$ is equivalent to $u > 0$, it follows that $\phi(x)$ may be expressed simply as

$$\phi(x) = \int_{o}^{\infty} \int_{-\infty}^{\infty} g_{\theta u}(\theta,u)\,d\theta\,du ,$$

where $g_{\theta u}(\theta,u)$ is the bivariate normal pdf of θ and u. Hence

$$\phi(x) = \Pr(u \ge 0) = 1 - \Pr(u < 0)$$

or

$$\phi(x) = 1 - F_N\left[\frac{\theta_o - \rho x}{\sqrt{\rho-\rho^2+\sigma^2}}\right]. \qquad (18)$$

With this new expression for $\phi(x)$, the expected remediation cost as defined in Equation (17) may be written as

$$\gamma(c) = \gamma_1 Pr(x < c) + (\gamma_o - \gamma_1) \int_{-\infty}^{c} F_N\left[\frac{\theta_o - \rho x}{\sqrt{\rho-\rho^2+\sigma^2}}\right] f_N(x)dx. \qquad (19)$$

The integral found in Equation (19) may be written as

$$Z(c) = \int_{-\infty}^{c} \int_{-\infty}^{h(x)} f_N(w) f_N(x) dwdx,$$

where $h(x) = (-\rho x + \theta_o)/\sqrt{\rho-\rho^2+\sigma^2}$, and $f_N(\cdot)$ is again the pdf of a unit normal variable. Let

$$v = w - h(x) = w + (\rho x - \theta_o)/\sqrt{\rho-\rho^2+\sigma^2}.$$

Then x and v follow a joint bivariate normal pdf, $g_{xv}(x,v)$, with means, variances, and correlation given, respectively, as

$$\mu_x = 0,$$

$$\mu_v = -\theta_o/\sqrt{\rho-\rho^2+\sigma^2},$$

$$\sigma_x^2 = 1, \qquad (20)$$

$$\sigma_v^2 = (\rho+\sigma^2)/(\rho-\rho^2+\sigma^2),$$

and

$$\rho_{xv} = \rho/\sqrt{\rho+\sigma^2}.$$

Hence the integral Z(c) takes a simpler form given as

$$Z(c) = \int_{-\infty}^{c} \int_{-\infty}^{o} g_{xv}(x,v)dvdx,$$

and the expected remediation cost $\gamma(c)$ may be written as

$$\gamma(c) = \gamma_1 Pr(x < c) + (\gamma_o - \gamma_1) \int_{-\infty}^{c} \int_{-\infty}^{o} g_{xv}(x,v)dvdx. \qquad (21)$$

The numerical values of $\gamma(c)$ may be computed via tables or computer programs dealing with the univariate and bivariate normal distributions.

Numerical procedures such as the Newton iteration process may be used to solve the equation $\gamma(c) = B/m$. The derivative of $\gamma(c)$ with respect to c, from Equation (19), is found to be

$$\gamma'(c) = f_N(c)\left\{\gamma_1 + (\gamma_o - \gamma_1)F_N\left(\frac{\theta_o - \rho c}{\sqrt{\rho - \rho^2 + \sigma^2}}\right)\right\} . \tag{22}$$

It may be noted that by taking $\sigma^2 = 0$, Equations (19) and (22) of this section will reduce to Equations (14) and (15) of Section 5. This is expected since the normal-ogive cost function $\delta(\theta)$ as defined in (16) will degenerate into the constant cost function of Section 5 when σ^2 tends to zero. Finally, the maximum expected remediation cost (per random subject) may be deduced from Equation (21) by letting $c = +\infty$. It is

$$\gamma_m = \gamma_1 + (\gamma_o - \gamma_1)F_N\left(\frac{\theta_o}{\sqrt{\rho + \sigma^2}}\right). \tag{23}$$

Numerical Example 4

Let the parameters of the problem be $\rho = .64$, $\theta_o = 1$, $\sigma = 2$, $\gamma_o = \$150$, $\gamma_1 = \$50$, and $B/m = \$40$. The Newton iteration procedure for solving the equation $\gamma(c_1) = B/m$ yields the solution $c_1 = -.362$. If the test scores have a mean of 50 and a standard deviation of 20, then the test passing score cannot exceed 42.76.

7. SOME CONCLUDING REMARKS

In this paper a general model along with four separate illustrations is provided for the consideration of budgetary constraints in the setting of passing scores in instructional programs involving remediation for subjects with poor test performance. The illustrations are not meant to be exhaustive. Budgetary constraints normally impose a limit on the number of students allowed to take remedial learning activities and, hence, restrict the range in which a choice for the passing score is to be made. The paper also provides ways to assess the budgetary requirement associated with each passing score. This information would be a factor in decisions regarding passing scores and budgets for remediation.

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