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ABSTRACT

This is one of a series of 20 booklets designed for participants in an in-service course for teachers of elementary mathematics. The course, developed by the University of Illinois Arithmetic Project, is designed to be conducted by local school personnel. In addition to these booklets, a course package includes films showing mathematics being taught to classes of children, extensive discussion notes, and detailed guides for correcting written lessons. This booklet contains exercises on "surrounding" with centimeter blocks, a summary of the problems in the film "Surface Area With Blocks," and the supplement. (MK)

ED187591

THE ARITHMETIC PROJECT COURSE FOR TEACHERS

TOPIC: "Surrounding" With Centimeter Blocks.

FILM: Surface Area With Blocks, Grade 1

SUPPLEMENT: Using Blocks to Introduce Other Bases of Numeration to a Fourth Grade

10

NAME: ....

This booklet is part of a course for teachers produced by The Arithmetic Project in association with Education Development Center. Principal financial support has come from the Carnegie Corporation of New York, the University of Illinois, and the National Science Foundation.

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BOOK TEN

"SURROUNDING" WITH CENTIMETER BLOCKS

By Jack Churchill

"Surrounding" is an activity with centimeter blocks such as those available from several sources.* It offers various types of problems that push students into forming general mathematical solutions and deciding which ones, in various cases, are more "elegant."

The ideas can be pursued in any of the elementary grades. With older children one would move faster and search for general methods more rapidly.

This description is detailed at first, then increasingly sketchy. The first part consists of exercises which should be done by the reader in the order given. Answers to these problems are given for those who wish them. The latter part deals with more general questions and suggestions.

It is important that the reader, as well as his students, have a sufficient number of centimeter blocks at hand when starting this work. Part of the game is to learn how to do the problems without using the blocks, but the blocks should always be available as a last resort.

With a class you will need to give more problems than you will find here. At some point soon after you start, have students write their predictions to each problem before doing the problem with blocks. Encourage students to keep track of results—problem, prediction and actual result—in some kind of table. As suggested above, you will soon find most students confident about the actual result as well as the prediction, all without recourse to the blocks. When this happens, it may be rewarding for you to ask several students to explain how it is that they can be so sure.

The reader will notice that "surrounding" is related to finding the perimeter, but not at all the same thing. The rules for "surrounding" were chosen so as to make a consistent "game" with centimeter blocks. Rules for finding perimeter with blocks could be described, but they would generally be more complicated. At an appropriate stage the relationship of "surrounding" to perimeter can be explored.

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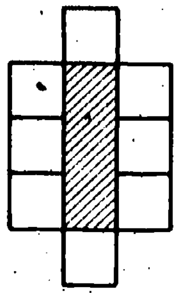
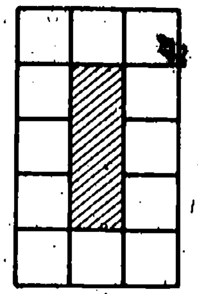
*E. g., Cuisenaire Co. of America, 12 Church St., New Rochelle, N. Y. 10805; or South West Imports, Ltd., P. O. Box 4071, Sta. D., Vancouver, B. C.

How many white blocks does it take to surround a light green block? You can do this in at least two likely ways:



or



For now, we will do it the second way. (Draw it on the board.) Fill in the corners. This is what we will mean by "surrounding" — the rules of the game. Also, for now, surrounding means using white blocks. So, surrounding light green takes 12 blocks.

Problems:

1. Surround ~~red~~: Answer: 10

2. Surround pink: Answer: 14

3. Surround yellow: ____________

4. Without doing it or computing yet, predict how many blocks it will take to surround dark green: ____________
(Check your prediction by doing it with blocks.)

5. Predict how many for blue:* ____________ p. 4

6. Predict how many for orange: ____________ p. 5

---

*Problems left unanswered here are to be worked by the reader as he proceeds. Answers will be found at the bottom of the page indicated at the right of each exercise. For example, the answer to problem 5 is on page 4. By means of this "teaching machine" arrangement, the reader may check the answer to one problem without inadvertently learning the answer to the next.

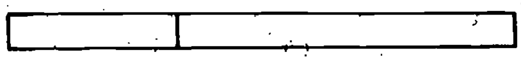
7. Depending on the age and experience of your students, you may soon get some methods for doing these problems. One such method might be to take the length of the block, double it and add 6, which may be abbreviated as $2 \times L + 6$. Give another method.

8. Surround two reds that have been placed end to end: 

_______________ blocks. p. 6

9. Same with two yellows end to end. _______________ p. 7

10. Same with two oranges. _______________ p. 8

11. A yellow and an orange are end to end. 

How many blocks surround them? _______________ p. 9

Separate them. 

Now how many blocks to surround them?* _______________ p. 10

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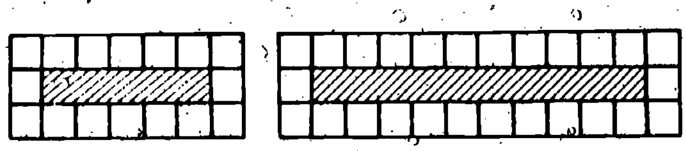
*The blocks are to be surrounded in this manner:



By this time children will usually have invented shortcuts for the laborious process of arranging white blocks. They may decide to use yellows or some other color as measuring blocks. Probably any such system should be encouraged if it gives correct answers in terms of white blocks. The invention of measuring blocks or other fast methods is not only an important idea in itself, but it changes the problem from a simple (if tedious) one of counting white blocks to one of doing such problems as $6 \times 5 + 4 \cdot 12$ or $5 + 5 + 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3$. Such "shortcuts" can be quite hard work for children weak in computation.

page:	4	9	18
answer:	28	36	24

2

12. A brown and a black are end to end. Surround: __________ p. 11

Separate them. Surround: __________ p. 12

13. A blue and a dark green are end to end. Surround: __________ p. 13

Separate them. Surround: __________ p. 14

14. Draw a conclusion:

On the basis of this conclusion, predict what happens in the following. You have surrounded an orange and a blue end to end. You then separate them and surround them. How many more blocks do you need? __________ (Don't write the total number of blocks; write the increase.) p. 15

15. You have surrounded three yellows end to end. Predict how many more blocks you need to surround them separately: __________ If you are in doubt, do it. Were you right? p. 16

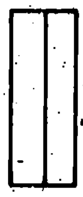
16. You have surrounded 31 black blocks end to end. How many more white blocks needed to surround them separately?

__________ p. 17

17. You have surrounded 931 black blocks end to end. How many more to surround them separately? __________ p. 18

page:	4	9	18
answers:	20	36	26

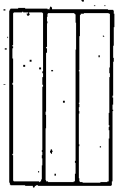
II.

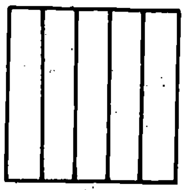
1. Surround 2 yellows side by side:  ________ p. 19

2. Surround 2 dark greens side by side: ________ p. 20

3. Surround 2 blues side by side: ________ p. 21

4. Surround 2 oranges side by side: ________ p. 2

5. Surround 3 yellows side by side:  ________ p. 3

6. Surround 5 yellows side by side:  ________ p. 5

7. Surround 12 pinks side by side: ________ p. 6

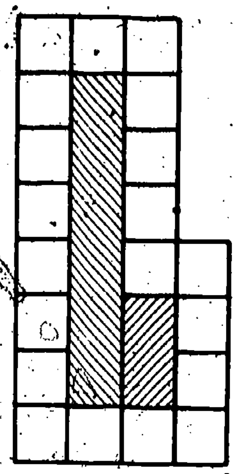
Recall the general methods of predicting answers when surrounding single blocks of various lengths. Some, such as $2 \times L + 6$, are not useful for solving the last few problems, but may suggest new rules that are. Give two possibilities:

p. 7

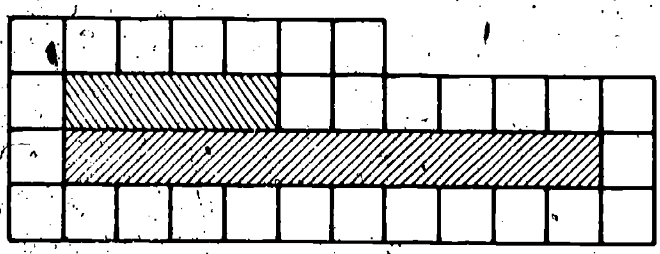
8. Use one of your methods to find how many blocks would be needed to surround 47 blues side by side. ________ p. 8

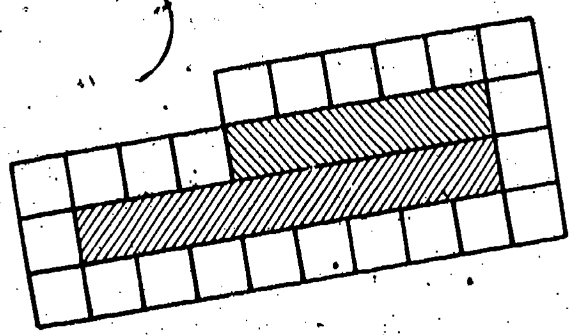
page:	1	9	18
answer:	24	36	28

When two blocks of different lengths are side by side, we shall surround them this way:



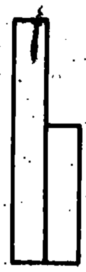
Note that every corner as well as every side is completely covered, but that no white blocks which do not cover a side or touch* a corner are used. Thus the following are <u>not</u> correctly surrounded:





*At least, nearly.

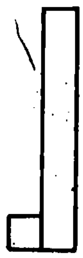
pa	1	4	10	18
an	26	24	28	30

1. Surround black and pink side by side:  ____________ p. 9

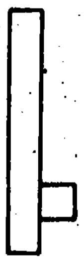
2. Without doing it, predict how many blocks will be needed to surround a black and yellow, side by side. Prediction: ____________

3. Actual answer for black and yellow side by side: ____________ p. 10

4. Surround 2 blacks side by side: ____________ p. 11

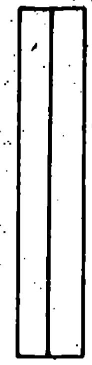
5. Surround a black and a white:  ____________ p. 12

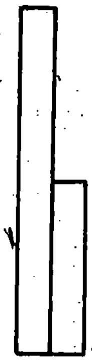
(Be careful to count only the whites used in surrounding, not the one being surrounded.)

6. Surround a black and a white:  ____________ p. 13

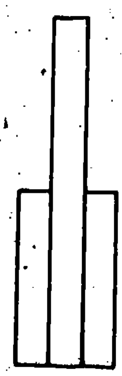
(The white block is one centimeter up from the corner of the black block.)

page:	2	4	10	18
answers:	14	36	28	28

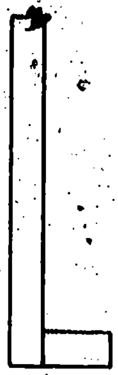
7. Surround 2 oranges:  ____________ p. 14

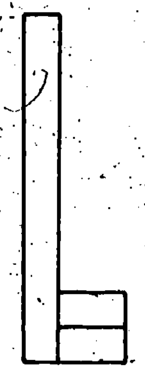
8. Surround an orange and a yellow:  ____________ p. 15

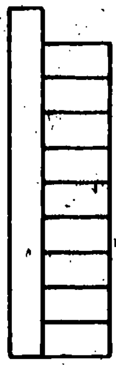
9. Surround an orange and 2 yellows:  ____________ p. 16

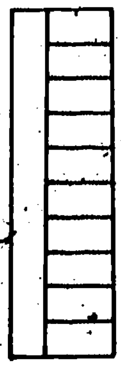
10. Surround an orange and 2 yellows:  ____________ p. 17

page:	2	4	10	18
answer:	26	Some possibilities: $2 \times L + 6 + 2 \times (n - 1)$ $2 \times (L + n + 2)$ $2 \times (L + n) + 4$ (n is the number of blocks)	28	28

11. Surround an orange and a red:  ________ p. 18

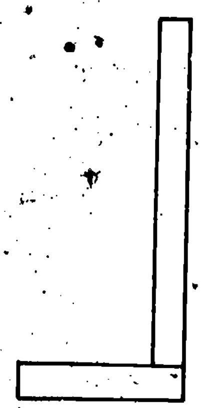
12. Surround an orange and 2 reds:  ________ p. 19

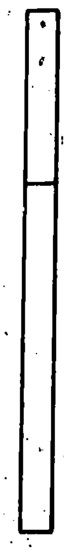
13. Surround an orange and 9 reds:  ________ p. 20

14. Surround an orange and 10 reds:  ________ p. 21

page:	2	4	10	19
answer:	46	116	28	28

15. Surround an orange and a yellow:  ____________ p. 2

16. Surround an orange and a yellow:  ____________ p. 3

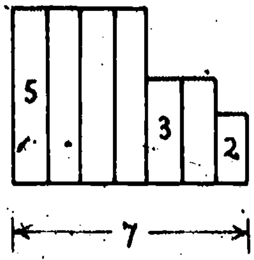


17. Surround an orange and a yellow: ____________ p. 4

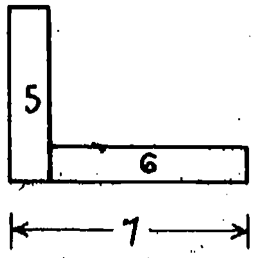
page:	2	6	10	19
answer:	36	22	28	28

IV.

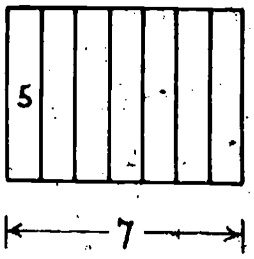
The notions developed in the preceding section suggest that one way of surrounding



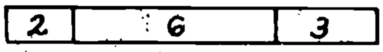
and



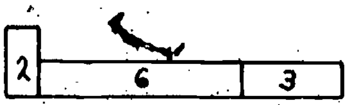
is to look at them as though they were



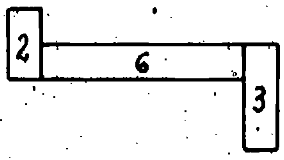
Now consider the following:

1. Surround 

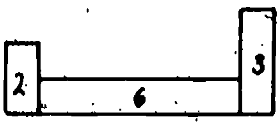
______ p. 5

2. Surround 

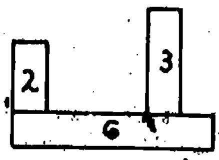
______ p. 6

3. Surround 

______ p. 7

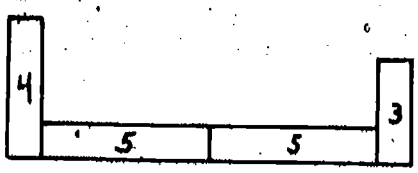
4. Surround 

______ p. 8

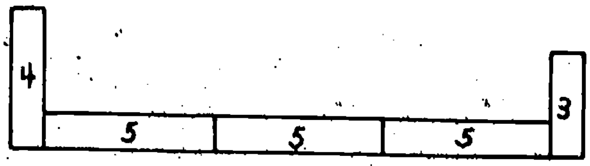
5. Surround 

______ p. 9

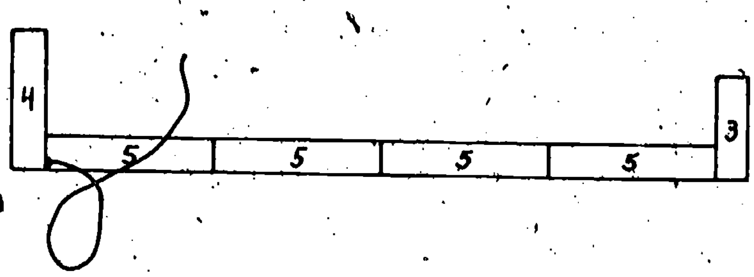
page:	2	6	11	20
answer:	42	22	40	72 and 44

6. 

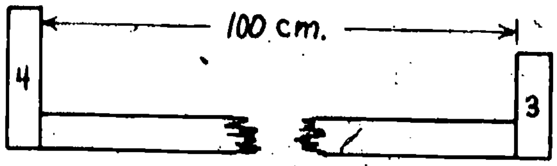
________ p. 10

7. 

________ p. 12

8. 

________ p. 13

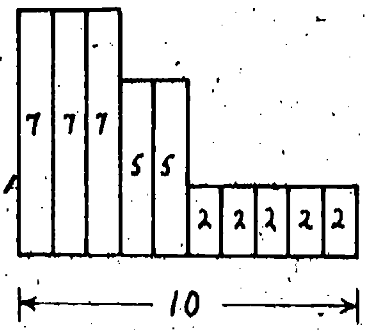
9. 

________ p. 14

page:	3	6
answer:	36	22

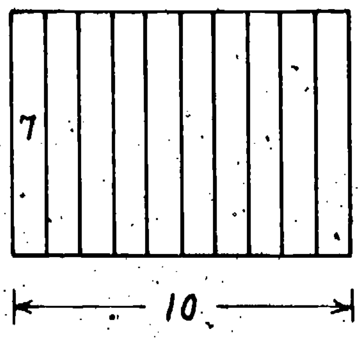
V.

In the foregoing you may have been somewhat surprised to observe that



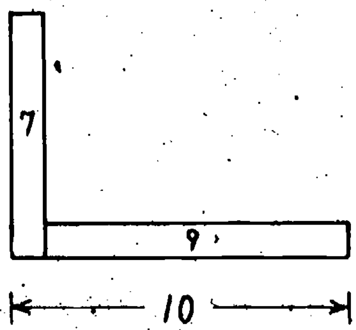
(a)

requires the same number of whites for surrounding as



(b)

and that both are the same for surrounding as



(c)

page:	3	6	11
answer:	42	22	50

As we saw, problems like (a) and (b), above, suggested such formulas for surrounding as this:

$$S = 2 \times L_{max} + 2 \times n + 4$$

where L_{max} is the length of the longest block, n is the number of blocks to be surrounded, and S is the number of white blocks required for surrounding. (The 4 added at the end represents the four corners.) In (a), $L_{max} = 7$ and $n = 10$. Thus the number of white blocks needed is

$$2 \times 7 + 2 \times 10 + 4 = 38$$

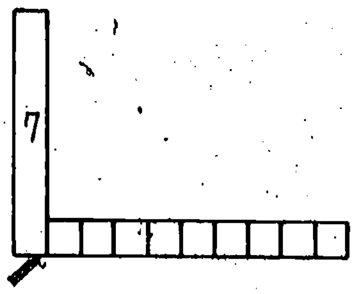
But (c), which gave the same result for surrounding, suggested an elegant, different formula, namely,

$$S = 2 \times L_1 + 2 \times L_2 + 6,$$

where L_1 is the length of one block, and L_2 is the length of the other. (The 6 accounts for the whites needed for the ends of the train.) In (c), $L_1 = 7$ and $L_2 = 9$, so that

$$S = 2 \times 7 + 2 \times 9 + 6 = 38$$

Two observations come to mind, bearing on the pleasant fact that the results of the two formulas agree. One is that they have to give the same answer, since they deal with the same problem. A nice exercise for a fifth grader is to show this. How can the formulas always give the same result when one has a 4 added at the end and the other a 6? Observe that if (c) is considered as

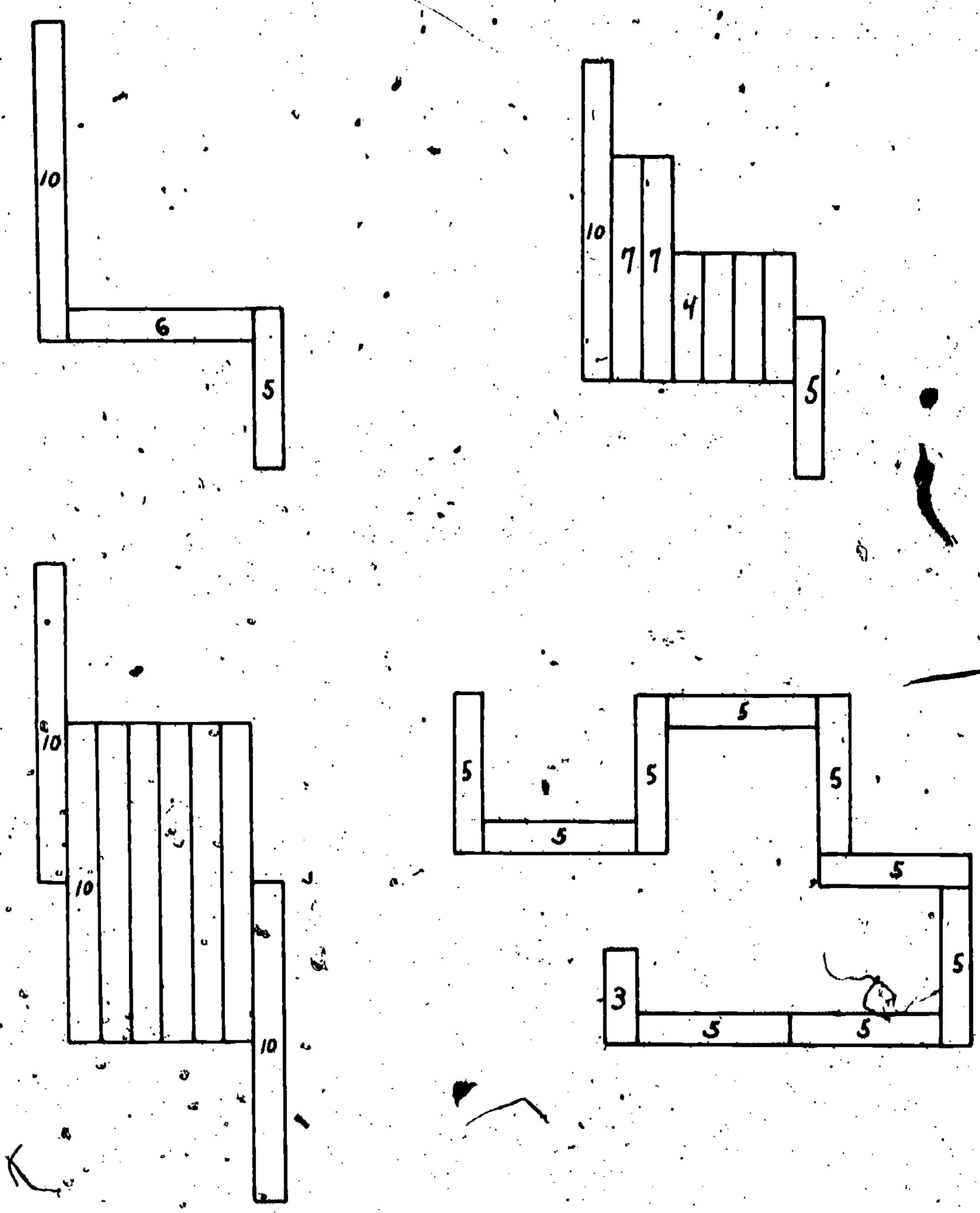


it is amenable to treatment by the formula for (a) and (b). (It seems to be inherent in that formula that all blocks "run the same way"!)

A second question is, if the formulas are the same, which of them is likely to be the more powerful or useful? To find out, one might consider

page:	3	6	11
answer:	36	22	60

which approach works with least modification for such problems as these:



VI.

1. Surround a brown. ________ p. 15

2. Surround two blacks side by side. ________ p. 16

3. How many dark greens side by side can be surrounded by 22 whites?

________ p. 17

page:	3	7	11
answer:	42	28	220

4. What other groups containing blocks all of one color side by side can be surrounded by 22 whites?

At this point a student observed: "You just add. One brown is one eight, and you add 1 + 8. The others are 2 + 7, 3 + 6, 4 + 5, and so on till you get 8 + 1 which is eight white blocks. They all add to the same thing, and they all surround with the same number. That means if you had twelve blues you could add 12 + 9 to get 21, and the answer would be the same as the answer for 20 + 1 or surrounding two oranges end to end."

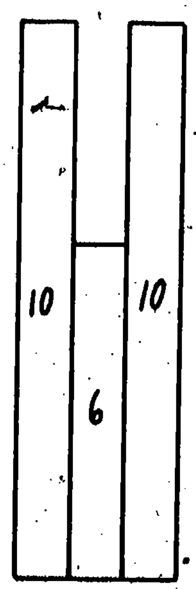
Another student might have observed: "And that would also be true for a blue together with eleven of any other smaller block side by side."

Based on this idea, here is another formula for surrounding:

$$2 \times (L_{max.} + n - 1) + 6$$

VII.

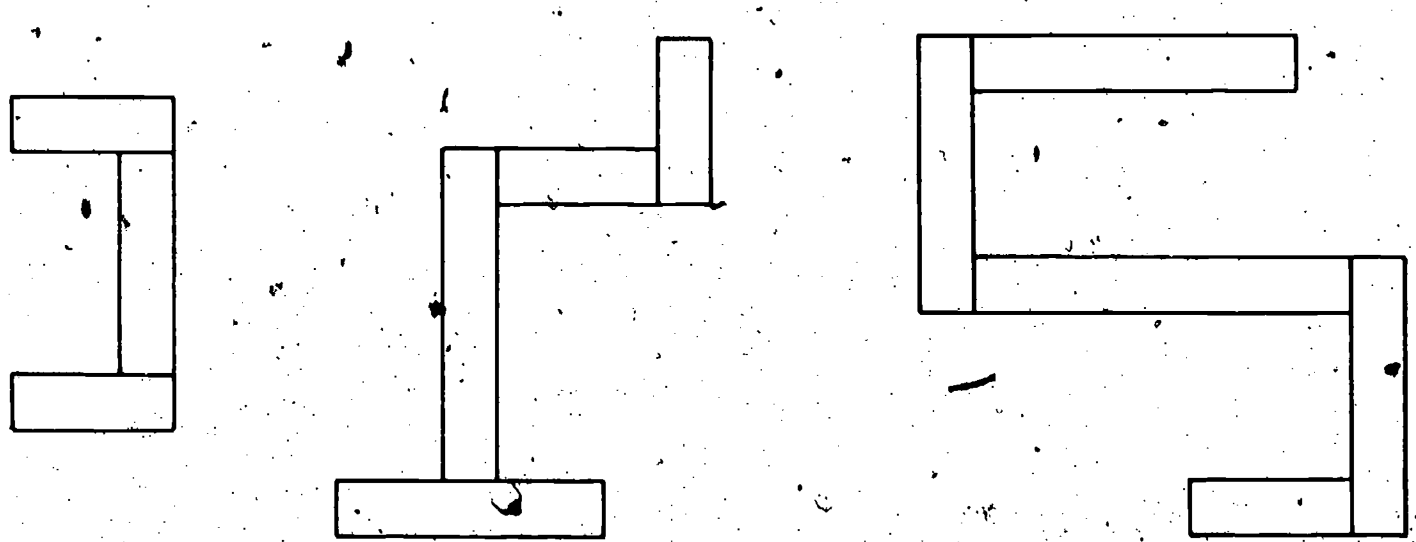
The reader will have noted that one can get into trouble if one allows what might be called "hollow places one block wide" such as this:



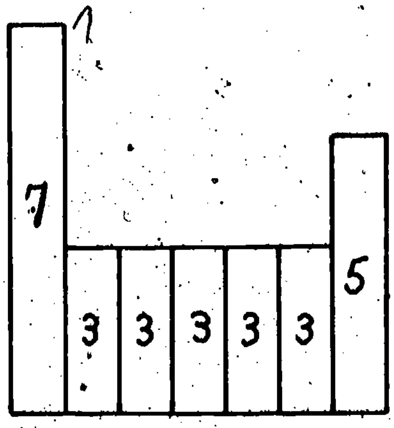
page:	3	7	14
answer:	6	28	22

It may be wise to announce early that such problems will be ruled out altogether, at least temporarily. Many of the otherwise powerful early generalizations will fail with such figures because they cannot be "properly" surrounded. "Fixing up" the generalizations may be undertaken later if desirable; the teacher might have an interested student try to make the necessary modifications or merely try to formulate precisely what has to be avoided.

Other kinds of hollows, however, may be dealt with. Some of the problems in section IV suggest that under some circumstances surrounding without modifying general rules is possible even when portions of a figure are hollows. Each of the three figures below may be surrounded according to the second formula in section V (for example, $S = 2 \times L_1 + 2 \times L_2 + 2 \times L_3 + 6$):

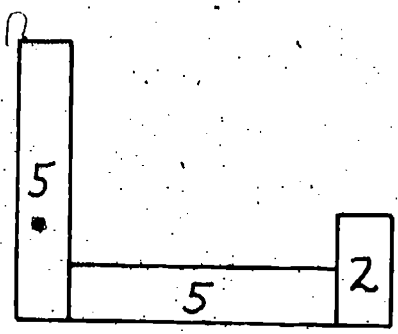


None of the formulas given so far, however, will work for this figure:

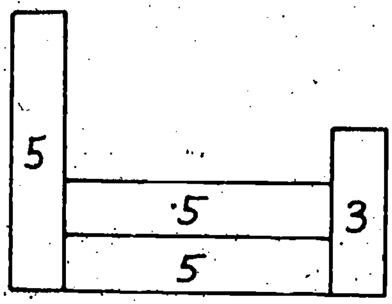


page:	3	7	14
answer:	12	30	22

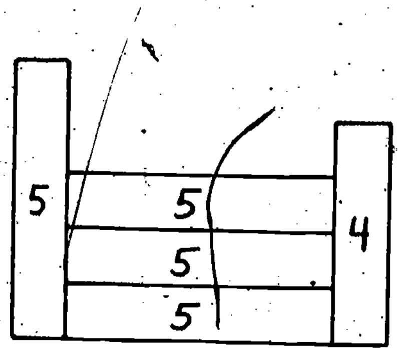
Let us now see if we can find a rule for dealing with hollows wherever they may be found, providing they are "legal" — that is, more than 1 centimeter across, so that they can in fact be properly surrounded. Such a rule is suggested by the following sequences of problems:

1. 

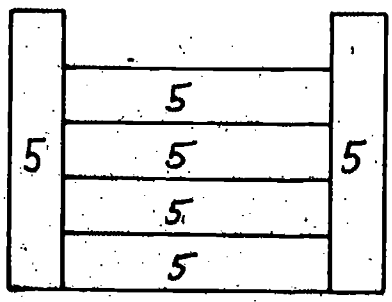
__________ p. 21

2. 

__________ p. 20

3. 

__________ p. 19

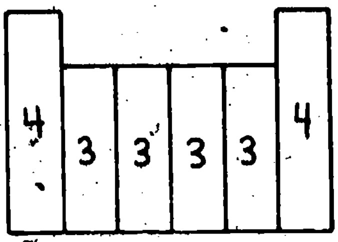
4. 

__________ p. 18

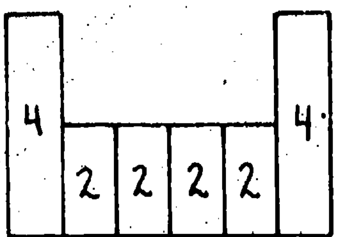
page:	3	7	14
answers:	180	30	3

5. 

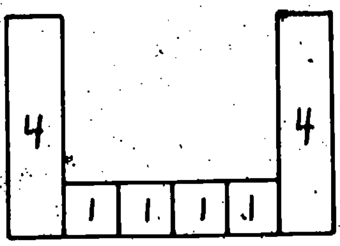
__________ p. 2

6. 

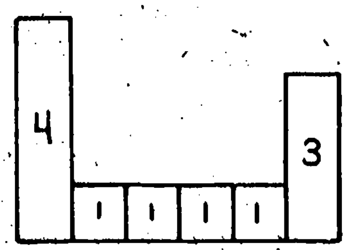
__________ p. 3

7. 

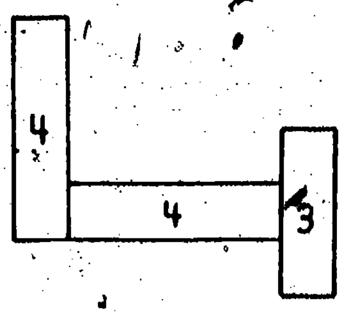
__________ p. 4

8. 

__________ p. 5

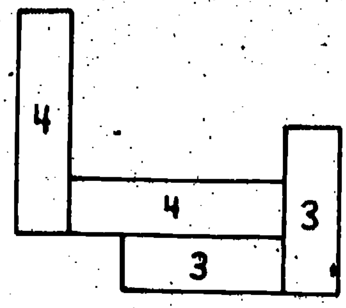
9. 

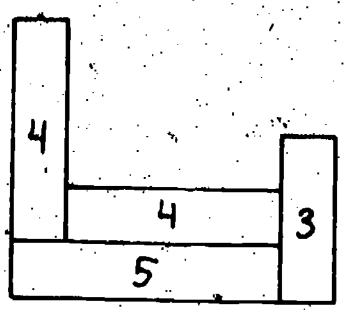
__________ p. 6

10. 

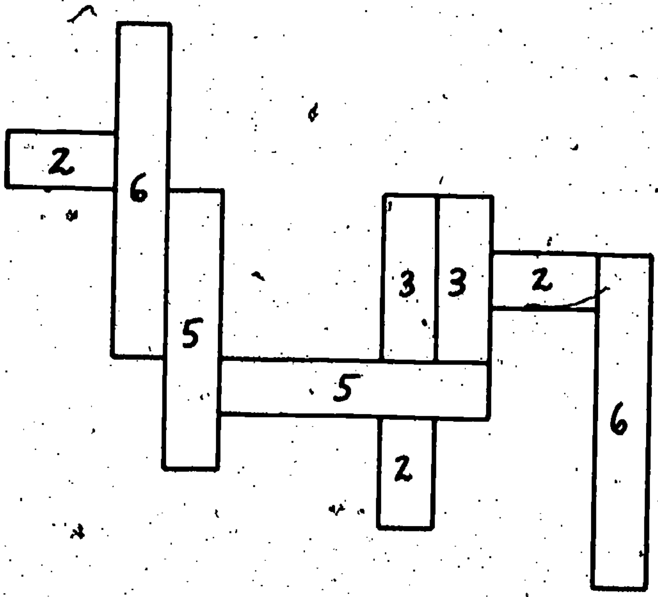
__________ p. 7

page:	3	8	17
answers:	5580	30	30

11.  ______________ p. 8

12.  ______________ p. 9

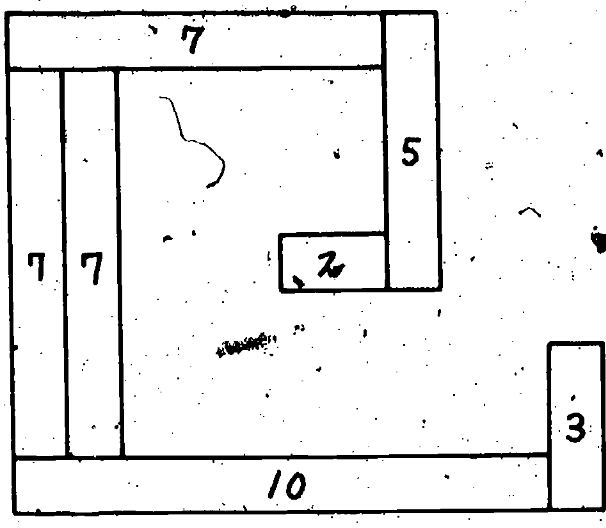
The fact that hollows can be looked for and their effects computed allows solutions to rather complex figures such as this one:

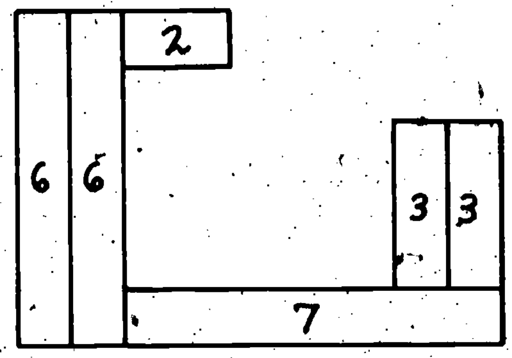


This figure has overall dimensions of 12×10, and it has three hollows, with depths of 3cm., 1cm., and 4cm. The reader may verify that surrounding it would take $2 \times 12 + 2 \times 10 + 4 + 2(3 + 1 + 4)$, or 64 whites. Such a rule

page:	4	8	17
answer:	18	30	30

can be extended to deal with more complex hollows such as these:

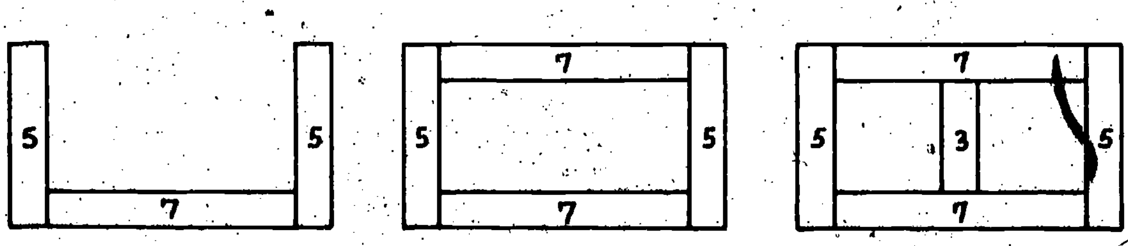




p. 10

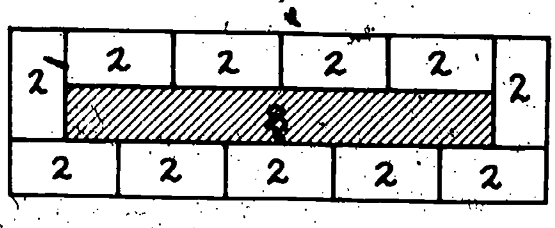
The range of problems one can tackle with "surrounding" allows many teachers to go quite far with a class into some of the ideas of mathematics. In kindergarten and first grade children have the problem of counting correctly and so invent such techniques as measuring blocks and multiplication. Uncovering numerical relationships as a way of predicting simple cases is a special bonus. Sixth graders, in their turn, can puzzle over the equivalence of apparently quite different formulas — and so find themselves concerned with problems of algebraic manipulation.

As was mentioned earlier, what surrounding usually does is closely related to finding perimeter. In fact, a few exercises will reveal that for most of the problems in this paper the perimeter is just 4 less than S. Is this true for all problems? For example, predict before working it out whether there is a difference of 4 between perimeter and S in these cases:

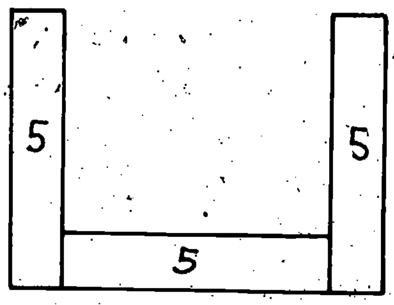


page:	4	8	17
answers:	20	30	30

At some point, particularly after children have started to use measuring blocks, you might consider surrounding by blocks other than the whites. Now the number of blocks needed is not the only question. One also wonders if a figure can in fact be completely surrounded — that is, with no gaps, just as though it were surrounded by whites. Here is a brown surrounded by reds:



Questions: Using reds, is it possible to surround a blue? Three blues side by side? This figure:



Make some predictions about surrounding with light greens. Finally, what happens when you surround in three dimensions? (Again one should avoid hollows with a width of 1 centimeter.) How can all the foregoing rules and generalizations be consistently extended?

As might be expected, the range of ideas that can be covered by a given teacher with his class depends only on his interests and those of his students.

page:	4	8	17
answers:	26	30	30

Summary of Problems in the Film
"Surface Area With Blocks"

1st Grade, James Russell Lowell School, Watertown, Massachusetts
Teacher: Phyllis R. Klein

I know that you have worked with these blocks before. How many white blocks does it take to build this yellow block? (5)

How many white blocks to build this red block? (2)
Does it take more or less than 10 white blocks to build this blue one? (Less)

Now, you have to think of a big pile of "postage stamps". (I have some right here.) Each postage stamp is as big as one side of this white block. They are all the same size. How many postage stamps would it take to cover the whole outside of this white block? (6)

Now you want to cover the outside of the red block.
(Wrong answer: 12).
(10)

What happened to the other two?
(The two white blocks have to be apart to make 12 because if you put them together two of the ends are together and you can't count those.)

How many postage stamps will it take to cover all of this orange block?
(Wrong answers of 100, 52)
(42)

How did you get 42 ?
("Each one is 10. I counted 10, 20, 30, 40, and then the two ends, you count one for each of them, so it would be 42.")

We are going to start covering this dark green block with postage stamps, and we'll keep counting them until we have covered the whole thing.

(Whispered answers: 26, 26, 26, 46)

No, not 46.

Now you have to think of a block that's very big; it takes 100 white blocks to build this block. I want to cover this very tall block with postage stamps. How many stamps will I need? (402)

Now think of a very tiny block that's even smaller than the white block. It's called a half block.

How many stamps will I need to cover half? ($\frac{1}{2}$ of a stamp)

Here's half the stamp and I will show you a block that's cut in half. There is the half block that you have to cover. And here is a stamp. Will you need more than one to cover the whole thing?

(You would have to cut some stamps in half.)

How many stamps would you like me to cut in half?

(2)

We now have all the sides covered and we used 2 stamps for that.

Now what? (Put a whole one on top.)

And now you've covered everything? (Except the bottom)

What will you need for the bottom? (One whole)

One whole, and how many stamps will it take to cover the whole thing?

(4)

Can you think of a block that's even smaller than the $\frac{1}{2}$ block?

(A $\frac{1}{4}$ block)

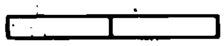
The same question: here is a $\frac{1}{4}$ block and you want to cover the whole block with these stamps. (Wrong answer of 6)

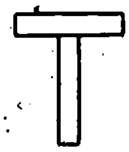
Yes, you would need 6 parts, but I don't think that it would be 6 whole stamps. (3)

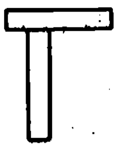
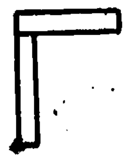
How did you get that? (I just counted the sides right here and that makes one whole and then I counted the bottom and the top and that's three.)

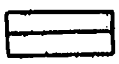
How many stamps to cover this yellow block? (22)

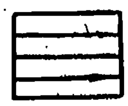
Here are two yellows out in the air separated. How many stamps do I need for both of them? (44)

Now take some glue and I will glue them together. How many stamps does it take now?  (42)

Now we are working on this one:  How many stamps to cover that? (Wrong answer: 43) (42)

Now what about this one?  And this one? 

And how about this:  (34)

How many yellow blocks do I have here?  (4)

How many stamps will it take to cover these blocks? (58)

Here is another stack of blocks. There are four orange blocks here. (108)

A problem to think about that we won't do now: 8 yellow blocks all suspended in air. How many stamps would it take to cover it?

Supplement

Using Blocks to Introduce Other Bases of Numeration to a Fourth Grade

Teacher: David A. Page

(First Day)

Several white blocks are lined up.

Teacher: How many blocks?

Student: 8

Teacher: If we couldn't use the word "eight", what other ways ~~could~~ you say it?

Scott: 4 + 4

Teacher: If you don't use any ordinary words for numbers?

Joyce: 5 + 3

Teacher: All these words, such as one, two, three, and so on—you can't use them. When you couldn't count, how would you say how many blocks?

Ronnie: With fingers!

Teacher: Yes, can you think of other ways?

Edward: H blocks, because H is the 8th letter in the alphabet.

Teacher: Good.

Scott: You could say "some" blocks, but that's not exact.

Diane: "Do" blocks, like in music.

Mark: Make a number line on top of the blocks.

Teacher: How about I, II, III . . .

Class: Roman Numerals.

Laura: You'd want VIII.

Teacher: Or X – II.

Andy: IX is eleven.

Teacher: No, XI is eleven; IX is nine. Maybe IIX could be eight. What we're going to do today is to find out about a different way to write numbers.

The teacher puts out 13 white blocks.

Teacher: How many blocks?

Jane: Thirteen.

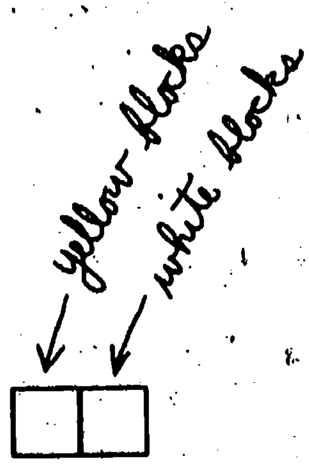
The teacher writes 13 on the board.

Teacher: When you write 13, the "1" tells you what?

Billy: Ten.

Teacher: That's one big block. (He puts 1 orange in place of 10 white blocks.) When we write in yellow chalk from now on, we'll not measure with the orange block, but instead we'll measure with these yellows. So now we're asking how many of <u>these</u> (yellow) and how much more? To measure these (holds up eight white ones), how many yellows do I need, and how much more?

Writes on board:



Diane: A yellow is five.

Teacher: But let's not say that word. How many yellows do I need?

Diane: One yellow and three whites.

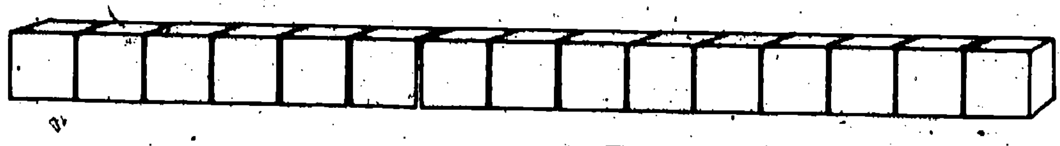
Teacher: We'll write that | 1 | 3 |

The teacher now puts out 12 white blocks.

Teacher: The question is, how many yellows and how many more?

Student: Two yellows and two whites: [2][2]

Teacher: Now how many?



Student: Three yellows and no whites: [3][0]

Teacher: I won't always write the boxes. I may just write, with yellow chalk, 13, 22, or 30. The yellow chalk tells you I'm talking about yellow blocks and white blocks. We'll pronounce it "one-three", "two-two", and "three-oh". Now let's count in new language.

He begins to write on board: 1, 2, 3, 4, ?

Student: 10

Teacher: Next?

Karen: 6

Teacher: But in yellows and whites?

Karen: 11

Teacher: Next? Everybody.

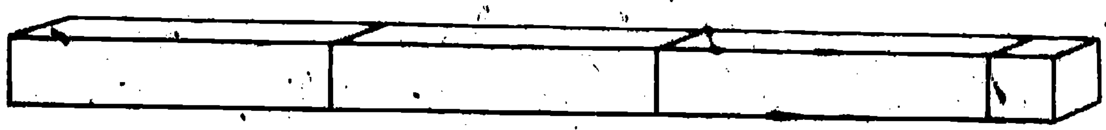
Everyone: 12, 13, 14.

Teacher: What comes next?

Student: 20, 21, 22, 23, 24, 30.

Gayle gets stuck on the next one.

Teacher builds it with yellows and whites:



Gayle:	31
Teacher:	Good. Who can translate that back to old language?
Joyce:	16
Teacher:	Yes. In new language it's 3 yellows and 1 white. Now who can count quite a way in a hurry?
Steve:	32, 33, 34, 35—oh! 40, 41, 42, 43.
Teacher:	Who can translate 43 into old language?
Gayle:	23
Teacher:	Soon we'll have a problem.
Diane:	43, 44, 50.
Teacher:	Now we're in trouble. Our new language doesn't have any fives. You were thinking of this. (He builds 44 + 1 .)
Karen:	"Instead of yellow, get a bigger block so it won't add up to five" for a while.
Teacher:	Imagine an odometer, the thing in your car that tells how far you drive.

0	0	0	0	9

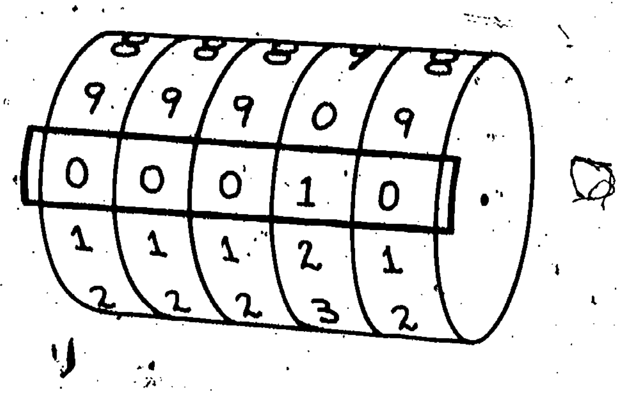
Now, when you drive another mile, what happens?

Mark:	It changes to 10.
Teacher:	This changes to 0,

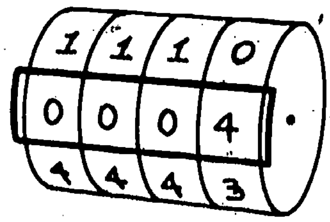
0	0	0	1	0

and when it does that, it moves this wheel so that 1 shows.

Each of these is a little wheel with numbers on it:



Now imagine an odometer that has only 0, 1, 2, 3, and 4 on each wheel:



Imagine the same thing again. You drive one more mile. The right hand wheel moves one unit so that it shows zero, and the wheel next to it moves one unit. What shows now?

Student: 10

Teacher: Yes. And it goes on just the way we've been counting until you get

0	0	4	4

Now what will it do when you go another mile?

Ronnie: One hundred — 100.

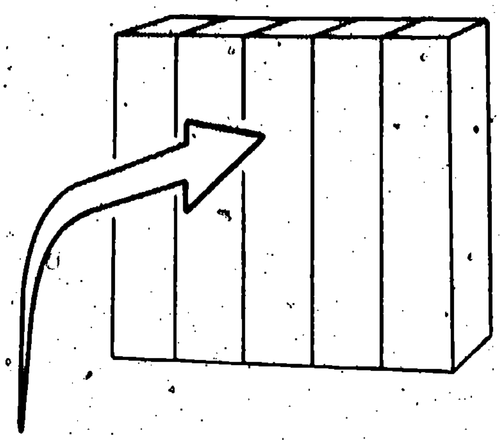
Teacher: Translate 100 back to old language.

Mark: 50?

Teacher: $44_{\text{new language}}$ $24_{\text{old language}}$

Mark: 25

The teacher holds up five yellows side by side:



Teacher: What's this face—what's it called?

Various answers: wall, block, square.

Teacher: So 100 (one-oh-oh) is like a square. Now we'll do arithmetic in new language.

Teacher: Answer these in new language.*

(Writes the following problems in yellow chalk.)

4 + 3 = ☐ p. 31

4 + 4 + 4 + 1 = ☐ (Builds this in white blocks.) p. 32

Teacher: To answer this problem (above), how many yellows do I need to trade for as many whites as I can?

(Trades with Jimmy: two yellows for ten whites.)

More problems:

2 + 2 + 2 + 2 + 2 = ☐ p. 33

12 + 13 = ☐ p. 34

Teacher: Who can translate this whole problem (above) into old language?

Peter: 7 + 8 = 15

---

* Problems left unanswered here are to be worked by the reader as he proceeds. Answers will be found at the bottom of the page indicated at the right of each exercise. For example, the answer to the first problem is on page 31. By means of this "teaching machine" arrangement, the reader may check the answer to one problem without inadvertently learning the answer to the next.

Teacher: Now I'm mixing. You keep track. The box tells what language the answer should be in. If the box is yellow, the answer should be in new language.*

8_{old}	+	8_{old}	= $\square_{new}$	p. 35
4_{old}	×	5_{old}	= $\square_{new}$	p. 36
5_{old}	×	5_{old}	= $\square_{new}$	p. 37
12_{old}	+	12_{new}	= $\square_{old}$	p. 38
12_{old}	+	12_{new}	= $\square_{new}$	p. 39

Mark says 14 for an answer to the last problem. The teacher builds 14 in new language, then builds old 12 and new 12.

Teacher: How many yellows, if we trade these (old 12 and new 12)?

Donna: Three.

Teacher: And how many whites left over?

Mark: Four.

Teacher: Let's see. Is 10_{old} an even number or an odd number?

Andy: Even.

Teacher: Does anyone know whether this is even or odd? (Builds 33_{new} using 3 yellows and 3 whites.)

Steve: Even. Even in new language and odd in old.

Teacher: Is the number of white blocks here an even number or odd number?

---

* In the class, the distinction between "old" and "new" language was made by the color of chalk: yellow for "new" and white for "old". The subscripts "old" and "new" are used here to avoid two-color printing.

Page	30	33	34	35	38	39
Answer	12	75	15	2,000	4,444	122

Steve: Even.

Teacher: Yes. 33_{new} looks like an odd number but it's really even. Suppose I build 33_{new} a different way? (Builds it as 2 yellows and 8 whites.)

Andy: That's still 33_{new}. Still even.

Teacher: How about this? (Writes 34_{new}.) Odd or even?

Jane: Odd.

Teacher: Even or odd? 12_{new}

Diane: Odd.

Teacher: Yes. Old language?

Diane: 7

Teacher: Odd or even?

Diane: Odd.

Teacher: 100, new language:

Mark: Odd.

Teacher: 24_{new}

Everybody: Odd.

Teacher: Fooled you. Here's the 2. (Holds up 2 yellow blocks.)

Tommy: 25 is 100, so 24 is even.

Teacher: You're right, but two-four in new language is much smaller than twenty-four in old language.

What is $121_{new} = \square_{old}$ p. 40

Diane: 20

Andy: 36

Page	30	33	34	36	39
Answer	23	10[illegible]	30	444	37

Teacher: Yes. Even or odd?

Andy: Even, because 1 = 1 yellow square = 25_{old}

2 = 2 yellows = 10_{old}

1 = 1 white block = $\frac{1}{36_{old}}$

Teacher: $300_{new} = \square_{old}$ p. 31

$400_{new} = \square_{old}$ p. 32

$444_{new} = \square_{old}$ p. 34

Ronnie: 114

Teacher: Close. You did 424.

David: 124

Teacher: Yes. Now go one bigger than 444_{new}. What do we write?

Cheryl: 1000

Teacher: What's that in old language?

Cheryl: 125

Teacher: Problem for tomorrow:

Even or odd: 121_{new} p. 35

400_{new} p. 36

And how can you tell fast?

Page	30	34	36	38	39
Answer	20	$30_{new} = 15_{old}$	1,000	10,000	yes

(Second day)

Teacher: $4_{old} + 4_{old} = \square_{old}$ p. 36

$4_{new} + 4_{new} = \square_{old}$ p. 37

Mark: 40

Teacher: Let's measure with yellows and whites.

Donna: 1 yellow and 3 whites.

Teacher: $4_{new} + 2_{new} = \square_{new}$ p. 38

David: 1 yellow and 1 white.

Teacher: $4_{new} + 4_{new} + 4_{new} + 4_{new} = \square_{new}$ p. 39

Tommy: 3 yellows and 1 white.

Teacher: $4_{new} \times 4_{new} = \square_{new}$ p. 40

Cheryl: 3 and 1?

Scott: 31_{new} ("Three-one")

Teacher: $10_{old} + 10_{new} = \square_{old}$ p. 31

$10_{old} + 10_{new} = \square_{new}$ p. 32

$13_{new} + 12_{new} = \square_{new} = \square_{old}$ p. 33

$22_{new} + 23_{new} = \square_{new}$ p. 35

Dennis: 8 yellows and 3 whites.

Paula: 8 yellows and 3 whites.

Jane: 9 yellows.

Page	30	33	36	39	40
Answer	30	124	2,000	13	42

The teacher builds the problem: builds 22_{new} and starts to build the 23_{new}. Hands are in the air. Jimmy answers: 45.

Teacher: But you can't say 5, so what is it?

Jimmy: 100 (One-oh-oh).

Teacher: $22_{new} + 23_{new} = \square_{new} = \square_{old}$ p. 36

$3_{new} \times 3_{new} = \square_{old}$ p. 37

$3_{new} \times 3_{new} = \square_{new}$ p. 38

$231_{new} = \square_{old}$ p. 39

Laura: 66. Every time a number is 100, it's 25. So it's 50; and 3 is 15; and 15 + 50 is 65, plus 1 is 66.*

Teacher: $444_{new} = \square_{old}$ p. 40

Joyce: 444

Teacher: Now translate it back to regular language.

Tommy: 124

Teacher: $250_{old} = \square_{new}$ p. 31

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*Laura's remarks are reproduced here without subscripts because a child will frequently omit them in verbal discourse. The reader (and teacher) is encouraged not to interrupt the explanation, which is essentially correct, but to supply the details later if desired.

Page	31	33	34	36	39	40
Answer	31	even	100	4,010	10	22

Karen: 888

Teacher: There aren't any 8's in our new language.

Karen: 1000

Teacher: Warm, but don't say "one thousand."

Ronnie: One-oh-oh.

Cheryl: One-oh-oh-oh.

Teacher: $124_{old} = \square_{new}$ p. 32

Cheryl: 444

Teacher: $125_{old} = \square_{new}$ p. 33

Cheryl: 500

Teacher: No fives in new language.

Mark: 100

Teacher: Better to read the digits.

Scott: One-oh-oh-oh.

Teacher: Now, Cheryl: $250_{old} = \square_{new}$ p. 34

Cheryl: Two-oh-oh-oh.

Teacher: $505_{old} = \square_{new}$ p. 35

Andy: One-oh-oh-oh-oh.

Teacher: You're ahead of us, Andy.

Page	31	33	34	35	37	39	40
Answer	40	even	8	$100_{new} = 25_{old}$	4,044	11	132

Steve: 4000

Teacher: Steve, you did this one: $500_{old} = \boxed{4000}_{new}$

So now do this: $505_{old} = \boxed{}_{new}$

Steve: 4010

Teacher $524_{old} = \boxed{}_{new}$ p. 36

Richard: 4040?

Teacher: That says take 4 of these (the yellow cubes) and 4 of these (yellows), so it would be $4040_{new} = 520_{old}$

Richard: 4044

Teacher: $550_{old} = \boxed{}_{new}$ p. 38

$525_{old} = \boxed{}_{new}$ p. 39

Scott: 4070

Denis: 4060

Mark: 10000

Teacher: You know that $500_{old} = 4000_{new}$ and that $25_{old} = 100_{new}$

So what's $525_{old} = \boxed{}_{new}$?

Student: 4100

Page	31	34	35	39	40
Answer	100	8	9	31	42

Teacher: And $550_{old} = \square_{new}$

Linda: 4200

Teacher: $600_{old} = \square_{new}$ p. 40

David: 4400

Teacher: $624_{old} = \square_{new}$ p. 31

Karen A: 4440

Teacher: Figure out how much this is: $500_{old} + 100_{old} + 20_{old} = \square_{old}$

Now, how much should we add to get 624_{old} ?

Student: Oh! I see. $\square_{new}$

Teacher: What's my next question?

David: $625_{old} = \square_{new}$ p. 33

Teacher: What's the answer? $\square_{new}$

Page	31	34	35	37	39	40
Answer	19	11	14	4,200	12	42

Teacher: Now some quick questions in yellow chalk (new language):

4 + 4 =	☐	p. 34
3 + 2 =	☐	p. 35
3 × 2 =	☐	p. 36
4 × 4 =	☐	p. 37
3 + 4 =	☐	p. 38
2 + 2 =	☐	p. 40

Teacher: Do this problem first in new language, then translate it and work it in old language. Do the answers agree?

$$
\begin{array}{r}
44_{new} \\
+\ 23_{new} \\
\hline
\square_{new}
\end{array}
$$

p. 31

$$
\begin{array}{r}
24_{old} \\
+\ 13_{old} \\
\hline
\square_{old}
\end{array}
$$

p. 32

Agree? ____________ p. 33

Page	31	34	35	37
Answer	34	31	66	4,100

Teacher: Do the same for this:

$$14_{new} + 23_{new} = \square_{new}$$ p. 34

$$9_{old} + 13_{old} = \square_{old}$$ p. 35

Teacher: Last problem: (In each equation, put the same number in both boxes.)

$$\square_{new} + \square_{new} + 3 = 322_{new}$$ p. 36

Hint: $\square_{old} + \square_{old} + 3 = 87_{old}$ p. 37

A later hint: $\square_{old} + \square_{old} = 84_{old}$ p. 38

Page	32	34	35	38	39
Answer	36	31	124	4,400	4