DOCUMENT RESUME

ED 187 540 SE 030 768

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TITLE The Relationship between Volume Conservation and the Learning of a Volume Algorithm for a Cuboid.
PUB DATE Apr 80
NOTE 26p.: Paper presented at the Biennial Southeastern Conference on Human Development (6th, Alexandria, VA, April 17-19, 1980).

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Algorithms; *Cognitive Development; *Conservation (Concept); *Educational Research; Elementary Education; *Elementary School Mathematics; Grade 6; Mathematics Curriculum; *Mathematics Instruction; Measurement; *Research
IDENTIFIERS *Volume (Mathematics)

ABSTRACT

This study investigated the relationship between the level of conservation of displaced volume and the degree to which sixth graders learn the volume algorithm of a cuboid, i.e., volume = length x width x height (v = l x w x h). The problem is a consequence of an apparent discrepancy between the present school programs and the theory of Piaget concerning the time to introduce the volume algorithm. Data showed that sixth graders could apply the algorithm to computation and comprehension questions regardless of their volume conservation level. There was also an improvement of students' conservation levels regardless of their volume achievement scores or their treatments. (Author/MK)

THE RELATIONSHIP BETWEEN VOLUME CONSERVATION AND THE LEARNING OF A VOLUME ALGORITHM FOR A CUBOID*

by

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* Paper presented at the Sixth Biennial Southeastern Conference on Human Development in Alexandria, Virginia, on April 17-19, 1980.

** Financial support for the project was provided through a grant from the Educational Institute of British Columbia.

*** This paper is based on the author's doctoral dissertation, The University of British Columbia, 1979, under the direction of Douglas T. Owens.

ABSTRACT

This study investigated the relationship between the level of conservation of displaced volume and the degree to which sixth graders learn the volume algorithm of a cuboid i.e. "Volume = length × width × height ($V = \ell \times w \times h$)." The problem is a consequence of an apparent discrepancy between the present school programs and the theory of Piaget concerning the time to introduce the volume algorithm. Data showed that sixth graders could apply the algorithm to computation and comprehension questions regardless of their volume conservation level. There was also an improvement of students' conservation levels regardless of their volume achievement scores or their treatments.

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INTRODUCTION

This study was designed to investigate the relationship between the level of conservation of displaced volume and the degree to which sixth graders learn the volume algorithm of a rectangular parallelepiped (cuboid) i.e. "Volume = length × width × height ($V = \ell \times w \times h$)" at the computation and comprehension levels (Begle and Wilson, 1970, p. 373). The problem is a consequence of an apparent discrepancy between present school programs and Piaget's theory concerning the time to introduce the volume algorithm of a cuboid.

Two widely used textbook series in British Columbia introduce the algorithm "$V = \ell \times w \times h$" in grade 5 (age 10) (Dilley et al., 1974 and Eicholz et al., 1974). Another series introduces the algorithm formally in grade 4 (age 9) (Elliot et al., 1974) and uses it informally in grade 3 (age 8).

Most proponents of Piaget's theory would disagree with such early introduction of the algorithm and claim that most children do not develop the necessary cognitive abilities for learning it before grade 6 (age 11). Piaget (1960) himself, for example, holds that "it is not until stage IV (formal operational) that children understand how they can arrive at an area or volume simply by multiplying boundary edges" (p. 408).

Piaget (1960) considers the concept of conservation to be necessary for any meaningful computation in both area and volume:

> ... Children attain a certain kind of conservation of area [and volume], based on the primitive conception of area (and volume) as that which is bounded by lines (or faces). That understanding comes long before the ability to calculate areas and volumes by mathematical multiplication, involving relations between units of different powers... (Piaget, 1960, p. 355)

On the other hand, many educators believe that "acquisition of formal scientific reasoning may be far more dependent on specific instructional experiences and far less dependent on general maturation than hypothesized by Inhelder and Piaget (1960)" (Siegler and Atlas, 1976, p. 368). Graves (1972, p. 223), for example, considered education and experience to be necessary for volume conservation. Lovell (1971, p. 179) went further to suggest that even seven and eight year olds (grades 2 and 3) can learn how to use the algorithm "$V = \ell \times w \times h$" in order to calculate the volume.

There seems to be a discrepancy between the present school programs and the theory of Piaget. Some of the present programs introduce the volume algorithm of a cuboid as early as grade 3. This position seems to be backed by some educators who claim that scientific reasoning is more dependent on training and instruction than maturation. Piaget and his proponents seem to argue that conservation of volume is a prerequisite for any meaningful calculation of volume. However, one ought not necessarily delay the introduction of the volume algorithm "$V = \ell \times w \times h$" until all students conserve volume. Studies have indicated that the majority of adults do not conserve volume (Elkind, 1962; Towler and Wheatley, 1971; Graves, 1972). In such a predicament there seems to be a need for research in order to justify our present school curriculum or suggest it's modification. This need has been acknowledged by educators such as DeVault who advocates that "it seems reasonable ... to assert that the studies most likely to produce useful results for curriculum work would be experimental studies (DeVault, 1966, p. 639)."

It was an intention of the investigator to provide necessary data of the relationship between volume conservation and learning of a volume algorithm. Such data was used for making recommendations related to the justification for teaching the volume algorithm prior to grade 6.

METHOD

The original sample consisted of 171 sixth graders of three suburban schools in British Columbia. However, after eliminating subjects who missed any test or treatment session the sample was reduced to 105 students. Subjects were classified as nonconservers (N=57), partial conservers (N=16) and conservers (N=32) using a nonverbal multiple choice test of volume conservation. The criteria used in developing the items were based on Piaget's (1960) testing procedures of volume conservation. The subjects were then divided into two experimental groups and one control group, by randomizing each conservation group across three treatments. One of the two experimental groups (N=36) was taught the volume algorithm using an approach (volume treatment) that resembles those of school programs used in North America. Such resemblance was necessary in order to determine the effectiveness of those programs. On the other hand, this treatment was considered an improvement over school approaches because it was more comprehensive, more intense and required more students' active involvement than these approaches do. Activities of this treatment included finding the volume of cuboids by counting cubes and later by using the algorithm "$V = \ell \times w \times h$." The other experimental group (N=39) was taught the algorithm using a method that emphasized multiplication skills (multiplication treatment). This treatment included training on compensating factors with respect to variations in others. For example, given that $36 = 2 \times 3 \times 6$, the students were trained to complete statements such as $36 = 4 \times \square \times \square$. This task was supplemented by a brief discussion of the volume algorithm "$V = \ell \times w \times h$." The control group (N=30) was taught a unit on numeration systems. Three teachers taught each of the treatments on each of the four days of instruction.

The four different tests used were: Volume Conservation (11 items), Volume Achievement (27 items), Multiplication Achievement (20 items) and the Computation section (45 items) of the Stanford Achievement Test (SAT). The pretests were: Volume Conservation, Volume Achievement and Computation. The posttests and retention tests were: Volume Conservation, Volume Achievement and Multiplication Achievement. The Hoyt estimates of reliabilities for the Volume Achievement Pretest, Posttest and Retention test were 0.94, 0.95 and 0.94 respectively; the reliabilities for the Volume Conservation Pretest, Posttest and Retention Test were 0.78, 0.85 and 0.82 respectively; the reliabilities for the multiplication Achievement Posttest and Retention Test were 0.86 and 0.79 respectively. Data of the posttests and retention tests were analyzed separately by using 3×3 fully crossed two-way analysis of covariance. One of the main factors was made up of conservation levels, while the other was treatments.

FINDINGS

The major aims of this study and the findings related to each of these aims are summarized below. Additional findings of the Post Hoc Qualitative Analyses are also reported in this section.

Aim 1. To determine the various degrees to which conservers, partial conservers, and nonconservers of volume learn the volume algorithm of a cuboid "$V = \ell \times w \times h$."

<u>Findings</u>. The results of the posttest showed a significant ($p \leq 0.05$) superiority of the conservers group over the partial conservers group in the Volume Achievement Posttest (see Table 1). No significant difference was found between any other conservation groups at the 0.05 level. There was no significant difference found in Volume Achievement Retention Test scores between conservation groups at the 0.05 level (see Table 2).

Insert Tables 1 & 2 about here

<u>Aim 2</u>. To determine the degree of effectiveness for each of the two teaching methods on learning the volume algorithm for a cuboid.

<u>Findings</u>. The results showed a significant superiority of volume treatment group over multiplication treatment group ($p \leq 0.01$) and over control treatment group ($p \leq 0.01$) on the Volume Achievement Posttest (see Table 1). No significant difference was found between multiplication treatment group and control treatment group at the 0.05 level. Similarly, the results of the retention test showed a significant superiority of volume treatment group over multiplication treatment group ($p \leq 0.01$) and over control treatment group ($p \leq 0.01$) (see Table 2): no significant difference was found between multiplication treatment group and control treatment group at the 0.05 level.

<u>Aim 3</u>. To determine the effect of learning the volume algorithm of a cuboid on the transition from one volume conservation level to another.

<u>Findings</u>. The transition from a lower to a higher level of conservation between the pretest and the posttest was found to be independent of volume achievement scores at the 0.05 level (see Table 3). Similarly, the transition from a lower to a higher level of conservation between the pretest and the retention test was found to be independent of volume achievement scores at the

0.05 level (see Table 3). Likewise, the transition from a higher to a lower level of conservation was found to be independent of volume achievement scores between the pretest and the posttest and between the pretest and the retention test at the 0.05 level (see Table 3).

Even though the transition from one conservation level to another was found to be independent of volume achievement scores there was a general improvement in the subjects' conservation levels.

Insert Table 3 about here

Aim 4. To determine the relationship between sex and the levels of conservation of volume.

Findings. The Volume Conservation Pretest revealed that the initial level of conservation of males was found to be significantly ($p \leq 0.01$) better than that of the females (see Table 4). Out of a total 150 students, 76 were males and 74 were females. The nonconservers were 33 males and 45 females, the partial conservers were 13 males and 9 females and the conservers were 30 males and 20 females.

Insert Table 4 about here

Aim 5. To determine the relationship between sex and the degree of learning the volume algorithm for a cuboid.

Findings. The Volume Achievement Posttest scores were not found to be significantly ($p \leq 0.05$) correlated to sex (see Table 5). Similarly, the Volume Achievement Retention Test scores were not found significantly correlated with sex at the 0.05 level (see Table 5).

Insert Table 5 about here

Aim 6. To determine the relationship between mathematics achievement and the levels of conservation of volume.

Findings. The initial level of conservation was found to be independent of the mathematics achievement scores of the pretest as measured by the computation section of SAT.

Aim 7. To determine the relationship between mathematics achievement and the degree of learning the volume algorithm for a cuboid.

Findings. The Volume Achievement Posttest scores were found to be significantly ($r = 0.35$, $p \leq 0.001$) correlated to the mathematics achievement pretest scores measured by SAT (see Table 6). Similarly, Volume Achievement Retention Test scores were found to be significantly ($r = 0.37$, $p \leq 0.001$) correlated to the pretest of SAT scores (see Table 6).

Insert Table 6 about here

POST HOC QUALITATIVE ANALYSES

Transition between Conservation Levels

The findings related to Aim 3 revealed that the transition between conservation levels from pretest to posttest, pretest to retention test and posttest to retention test was independent of treatments at the 0.05 level. The following, however, are observations based on the detailed contingency tables of transition among conservation levels. These tables are not included in this summary paper but are a part of the complete report.

1. The regression of conservers to nonconservers occurred very rarely. Only two of 32 (6.25%) conservers regressed to nonconservers between the pretest and the posttest. None of the 32 conservers regressed to nonconservers between the pretest and retention test. Similarly, none of the 44 conservers regressed to nonconservers between the posttest and the retention test.

2. There does not seem to be any observable difference in the progress of nonconservers and partial conservers to higher levels of conservation. For example, between the pretest and the posttest 19 of 57 (33%) nonconservers and 4 of 14 (29%) partial conservers progressed to a higher level of conservation.

3. Even though there was a general improvement of conservation levels among all treatment groups, the control group seemed to have undergone a steady progress with respect to conservation levels. That is, subjects in all groups progressed and regressed but those in the control group did not seem to regress as much as the subjects in the other two treatment groups. The two of 30 subjects (6.66%) in the control group who regressed between the pretest and posttest, progressed back to the original level in the retention test. Two other subjects in the control group regressed between the posttest and the retention test. In the volume and multiplication treatments there were four in each who regressed from pretest to retention test.

The high stability of the conservation level of conservers throughout the experiment is not surprising. Some research reviewed in the doctoral thesis indicated that natural conservers showed stability of their conservation level and even resisted misleading cues. However, it was curious to note that the conservation level of subjects in the control group did not seem to regress as much as the level of subjects in the other treatment groups. The instability of the other two groups could be explained by the influence of experience in volume activities on the partial conservers since most of those who regressed were partial conservers in the volume and multiplication groups. The experience in volume activities could have disturbed the partial conservers so that they incorrectly applied knowledge acquired in the treatments to volume conservation tasks. Those who were in the control group could have used their intuitive understanding of volume conservation.

Students' Reasons for their Responses on a Volume Conservation Question

Question 11 of the Volume Conservation Test concerned two identical plasticine balls, one of which was later deformed. In this question the students were asked to give reasons for their judgments in comparing the volumes of the final shapes. The reasons were intended to provide validity information for the classification scheme. The reason given by each student for the response on Question 11 was first classified as consistent, inconsistent, or unclassifiable. A reason was classified as consistent if it did not contradict the response. A reason was coded as unclassifiable if it was not possible to understand the reason given by the student.

Most of those who answered Question 11 correctly gave a reason related to size. However, a considerable number of subjects, including nine conservers in each of the pretest and retention test, who answered Question 11 correctly gave a reason related to weight. If those nine classified as conservers are only weight conservers and not volume conservers there is doubt about the validity of the conservation test used in this study. However, an earlier part of the conservation test (Items 4, 5 and 6) was designed to detect those who were only weight conservers. These weight conservers were classified as nonconservers of volume. Language factors could have prevented some of those subjects from expressing their reason more appropriately. The reasons given by students for their responses to Question 11 do not seem to provide sufficient information for conclusive evidence about the validity of the classification scheme used in the Volume Conservation Test.

Consistency between Levels of Conservation and Responses to Question 12 of the Volume Conservation Test

Question 12 of the Volume Conservation Test concerned two identical tile boxes, one of which was closed and the other open. This question involved two unequal volumes and was not a part of the conservation classification scheme since it was not typical of the usual Piagetian questioning protocol. It was included because it was believed that students might be able to give reasons for inequality more easily than for equality.

It appears that between the pretest and posttest there was a general improvement among all conservation groups in answering Question 12 correctly. The conservers seem to have improved the most (15% improvement) followed by partial conservers (12%) followed by the nonconservers (5%). On the other hand the number of nonconservers who answered Question 12 correctly seems to have increased steadily in the pretest, posttest and retention test. The percents of conservers and partial conservers who answered it correctly seem to have increased in the posttest and then decreased in the retention test. The increased percents of correct responses of these two groups followed by decreased percents could possibly be attributed to the tendency of regression toward the mean in successive observations.

Students' Reasons for their Responses on Question 12

All the reasons given by the students for their judgment on Question 12 were first classified as consistent, inconsistent, unclassifiable, or no response. There were very few cases of no response. There were none in the pretest, one in the posttest and one in the retention test. There were only two unclassifiable responses in the pretest, none in the posttest and three in the retention test.

Most students who responded correctly to Question 12 could give an explicit reason which was related to the fact that one of the containers was open (closed) and water would (would not) go inside it. Of those who answered Question 12 correctly 77%, 79% and 77% on the pretest, posttest and retention test respectively gave a clear reason related to the fact mentioned above. On the other hand, it was previously noted that most of those who responded correctly to Question 11 gave an imprecise reason that was related to size. It appears that it was easier for students who responded correctly to give more explicit reasons about inequality of volumes in Question 12 than about equality in Question 11.

SUMMARY OF FINDINGS, CONCLUSIONS AND DISCUSSION

Importance of Conservation Levels

This study was an attempt to determine the relationship between the level of conservation of volume and the degree to which sixth grade students learn the volume algorithm. The only significant result in this connection was that the conservers scored higher than the partial conservers on the Volume Achievement Posttest. The conservers did not score significantly higher than the nonconservers on the posttest; partial conservers scored lower, though not significantly lower, than the nonconservers on the posttest. On the retention test, the scores of the conservation groups on the Volume Achievement Test did not differ significantly. Futhermore, students of the volume treatment group scored 65% on each of the Volume Achievement Posttest and Volume Achievement Retention Test. Children in grade 6 seem to be able to apply the volume algorithm for a cuboid, at the computation and comprehension levels, regardless of their conservation level.

So far as students who have reached the 6th grade level are concerned, it appears, that conservation level is not an important factor in learning the volume algorithm at the computation and comprehension levels. It is possible, although this study has no data to support it, that conservation level might likewise be relatively unimportant as a factor that influences successful learning of the volume algorithm by students in, say, grades 4 and 5. If this be so, then the present school programs which do present the volume algorithm in those grades may not be unreasonable. So long as the criterion for reasonability is learnability, the present study does not support the idea that the introduction of the volume algorithm should not take place before the learners have become conservers of volume. However, there may be factors other than volume conservation which would also influence the learnability of the volume algorithm. The need for further research, regarding the time to introduce the algorithm, will be discussed later in this report.

Effect of Treatments

Subjects who were in the volume treatment did significantly better, on the Volume Achievement Posttest and Volume Achievement Retention Test, than those subjects who were in each of the other treatments; the subjects in the other two treatments did not differ significantly in volume performance. The subjects who were in the multiplication treatment did significantly better, on the Multiplication Achievement Posttest, than those who were in the volume treatment and the control treatment; those who were in the volume treatment and those who were in the control treatment did not differ significantly. This indicated that at the posttest level the multiplication treatment was successful. That is, the students had learned the multiplication material which was taught.

In short, it may be concluded that, at the posttest level, the subjects who were in the multiplication treatment learned the multiplication material but the subjects who were in the volume treatment did significantly better than those who were in the multiplication treatment, on the Volume Achievement Posttest.

The conclusions mentioned above seem to suggest that the volume treatment is better than the multiplication treatment in teaching sixth graders the volume algorithm of a cuboid. The volume treatment included activities for determining the volume of cuboids by building them with cubes and counting the number of cubes; this method later used the algorithm "$V = \ell \times w \times h$" for computing the volume of a cuboid. The multiplication treatment consisted mainly of studying the effect of varying factors on their product and varying factors when their product is constant; this task was supplemented by a brief application to the volume algorithm "$V = \ell \times w \times h$."

The results of this study do not, therefore, support the conjecture that students who are proficient in varying factors of a fixed product can rapidly predict and determine the volumes or dimensions of cuboids. On the contrary, the results seem to support Piaget's claim that "it is one thing to multiply two numbers together and quite another to multiply two lengths or three lengths and understand that their product is an area or a volume ... (Piaget et al., 1960, p. 408)."

Transition between Conservation Levels

Results of the study revealed that there was, generally, an improvement of the students' conservation levels regardless of their volume achievement scores or their treatments. The transition from a lower to a higher level of conservation between the pretest and each of the posttest and retention test was found independent of volume achievement scores and of treatments.

It appears that the improvement of the subjects' conservation level was influenced by some factor(s) other than treatments and volume achievement scores. Possible factors could have been growth, peer influence, test influence (sensitization) and 'Hawthorne effect'. Growth is suspected to have been a factor because the experiment lasted about two months during which subjects in grade 6, especially those who were "on the doorstep" of conservation, could have developed from one stage of cognitive development to the next. Uncontrollable students' discussions (peer influence) of the Volume Conservation Test outside the classroom could have influenced the results of the posttest and the retention test since these tests were identical to the pretest. The test itself could have influenced some subjects to think seriously about conservation tasks and to correct their own errors in later tests. Finally, the development of the students from one conservation level to the next could have been partially attributed to the fact that they were chosen for the experiment (Hawthorne effect) and consequently to the influence of feeling special and worthy.

Effect of Mathematics Achievement

Results of the study revealed that volume achievement scores on the posttest and the retention test were correlated with mathematics achievement scores measured by the computation section of SAT. The initial level of conservation of volume was found to be independent of the mathematics achievement scores measured by the computation section of SAT.

The above-mentioned results seem to suggest that a competency in mathematics computation may indicate a competency in volume achievement or vice versa. Furthermore, the mathematics achievement score and volume conservation level seem to be independent.

Effect of Sex. The effect of sex on the degree of learning the volume algorithm of a cuboid and on the initial level of conservation of volume was examined. The degree of learning the volume algorithm of a cuboid, at the posttest and retention test levels was not found to be related to sex. On the other hand, the males were found to have a significantly ($p \leq 0.01$) higher initial level of conservation than the females.

The above-mentioned superiority of males over females in volume conservation has been also reported by other researchers such as Graves (1972), Elkind (1961 and 1962) and Towler and Wheatley (1971). The superiority of the males to the females in the initial level of conservation could be attributed to the more active participation of males in practical experiences involving manipulative skills. (Price-Williams et al., 1969 and Graves, 1972).

IMPLICATIONS FOR EDUCATIONAL PRACTICE

Implication 1.

The students of the volume treatment had an adjusted mean score of 65% on both the Volume Achievement Posttest and Volume Achievement Retention Test. This seems to indicate that students in grade 6 are capable of learning the volume algorithm for a cuboid "$V = \ell \times w \times h$." The volume achievement scores of such students do not seem to be affected by their levels of conservation of volume.

Since the conservation level did not seem to be an important factor in learning the volume algorithm, using conservation as a criterion, the present school programs that introduce the algorithm prior to grade 6 are not proven unreasonable. This study does not, therefore, suggest the delay of introducing the volume algorithm for a cuboid "$V = \ell \times w \times h$." This is not to say that the prevalent school practices are justified with respect to the theory of Piaget. The section on future research outlines possible ways for pursuing the matter of justification of the school programs.

Implication 2.

The results and conclusions of this study indicate that the activity-oriented volume treatment was successful. This treatment was based on determining the volume of cuboids by building them with cubes and counting the number of cubes; later the algorithm "$V = \ell \times w \times h$" was used for computing the volume of a cuboid. It seems appropriate, therefore, to teach the volume algorithm of a cuboid using an activity-oriented method.

Implication 3.

The mathematics computation scores of SAT were found to be positively correlated with volume achievement scores. This seems to suggest that a competency in mathematics computation may indicate a competency in volume achievement or vice versa.

Implication 4.

Females seem to be as capable as males in learning the volume algorithm for a cuboid "$V = \ell \times w \times h$." However, the superiority of the males to the females in the initial level of conservation could be attributed to the more active participation of males in practical experiences involving manipulative skills. Activity-oriented programs in the teaching of volume concepts may be beneficial to the acquisition of volume conservation by females.

RECOMMENDATIONS FOR FUTURE RESEARCH

The purpose of this study was to determine the relationship between the level of conservation of volume and the degree to which students learn the volume algorithm for a cuboid "$V = \ell \times w \times h$." On the basis of the findings, conclusions and implications of the study, further research is needed on this topic.

It is recommended that the experiment be replicated on a larger sample. The sample of this study consisted of 105 students only 16 of which were partial conservers. A larger sample might influence the results found in this study.

It is also recommended that the experiment be replicated on subjects in a grade lower than six. The independence of conservation level and volume achievement could have been influenced by the high grade level chosen. Students in grade 6 could have developed learning habits that were very effective, or those students could have been "on the doorstep" of volume conservation and became conservers early in the experiment. The choice of a lower grade and consequently a lower level of development might reveal the importance of volume conservation level more clearly.

A further recommendation is to conduct a study, in which volume learning at higher cognitive levels than computation and comprehension is investigated with respect to a correlation with volume conservation.

It is also recommended that a volume conservation posttest and a volume conservation retention test be developed which are parallel forms of, but not identical to, the volume conservation pretest. The identical conservation tests given in the pretest, posttest and retention test could have allowed a greater peer influence or sensitization effect than if they were not identical.

It is also recommended that the following observations which were made in the Post Hoc Qualitative Analyses be investigated further.

1. It was noted that there was a general progress in the conservation levels of subjects in all groups. However, in cases of regression, the partial conservers who were given experiences in volume activities seem to have regressed the most. Further investigation, which includes interviewing of subjects, is needed for cases where regression occurs. Such investigation may reveal the effect of volume activities on the stability of the conservation level of subjects, particularly the partial conservers.

2. Students were asked, in Question 11 of the Volume Conservation Test, to write reasons for their responses. Those responses did not seem to provide sufficient validity information for the final judgment about the test used. A considerable number of students gave an incorrect response and a reason which might support the correct response while other students gave a correct response and a reason related to weight. Further investigation is needed to validate the assessing of volume conservation. It is recommended that methodological studies be undertaken to determine the relationship between nonverbal and interrogation methods of assessing conservation of volume.

3. Question 12 of the Volume Conservation Test concerned two unequal volumes where students were asked to write reasons for their responses. It was noted that the number of partial conservers and conservers, who answered Question 12 correctly, increased in the posttest and decreased in the retention test. The number of nonconservers who answered Question 12 correctly seemed to have improved steadily in the pretest, posttest and retention test. On the other hand, most correct responses to Question 12 were supported by explicit reasons. Further research is needed for Question 12 in particular and cases of conservation of inequality of volume in general.

Finally, in this study the relationship between volume conservation and the degree of learning a volume algorithm involving multiplication skills was investigated. It is recommended that similar research be conducted to determine the relationship between various conservation tasks and the degree of learning other algorithms involving multiplication skills in the elementary school. For example, research may be designed to investigate the relationship between area conservation and learning the algorithm for the area of a rectangular region, "$A = \ell \times w$", that is, area equals length times width.

In summary, further research is needed before the prevalent school practice of introducing the volume algorithm for a cuboid ($V = \ell \times w \times h$) in grades earlier than grade 6 can be justified on the basis of the cognitive theory of Piaget.

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Table 1
Analysis of Covariance of Volume Achievement Posttest Scores

Source	df	MS	F
Conservation Level	2	88.88	3.20*
Treatment	2	339.56	12.24**
Conservation Level × Treatment	4	31.50	1.14
Error	95	27.74	

* $p < 0.05$ ** $p < 0.001$

Table 2
Analysis of Covariance of Volume Achievement Retention Scores

Source	df	MS	F
Conservation Level	2	74.16	2.67
Treatment	2	217.01	7.81*
Conservation Level × Treatment	4	23.35	0.84
Error	95	27.79	

* $p < 0.001$

Table 3
Biserial Correlation Coefficients Between Volume Achievement Scores on the Posttest, Retention Test and Transition to a Higher or Lower Level of Conservation

	Transition Higher	Transition Lower
Posttest-Pretest	0.13 (0.18)[1]	0.03 (0.79)
Retention Test-Pretest	0.09 (0.34)	0.03 (0.77)

[1] Number in ( ) indicates the significance level

Table 4
Contingency Table: Pretest Conservation Level Versus Sex

Conservation Level	Males	Females	Total
Nonconservers	33	45	78
Partial Conservers	13	9	22
Conservers	30	20	50
Total	76	74	150

Table 5
Point-Biserial Correlation Coefficients and Significance Levels Between Volume Achievement Scores and Sex on the Posttest and the Retention Test

	Sex	Significance
Volume Achievement Posttest	0.14	0.13
Volume Achievement Retention Test	0.12	0.23

Table 6
Pearson's Product Moment Correlation Coefficients and Significance Levels Between Volume Achievement Scores and SAT Scores on the Posttest and the Retention Test

	SAT	Significance
Volume Achievement Posttest	0.35	0.00026
Volume Achievement Retention Test	0.37	0.00011