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ABSTRACT

A remedial mathematics program at the college level is described. The evolution of the course from 1973 until 1979, the program purpose, conceptual framework, procedures, subjects, instruments, and methods are described. Appendices include unit objectives, a chronological development of the program by semester, and data on student achievement. (MK)

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REPORT ON

THE CONTINUOUS SEQUENCE IN BASIC MATHEMATICS

(A COLLEGE REMEDIAL MATH PROGRAM)

A SEVEN YEAR STUDY

AT

CENTRAL MICHIGAN UNIVERSITY

Directors:

Dr. Melvin Poage
Dr. Delano Wegener

PREAMBLE:

Do not follow where the
path may lead
Go, instead, where there
is no path
And Leave a Trail.

Anonymous

No one enjoys teaching remedial mathematics students. It is hard work and there is little reward. The mathematics usually is not very challenging. In addition, we tend to equate the teaching of remedial students with being remedial ourselves. That is, you seldom meet a professor at a conference or colloquium who brags about the fact he is teaching remedial mathematics. We more often are very defensive about this teaching assignment and act as though remedial students are untouchables. This attitude is not limited to college nor to Central Michigan University; it prevails in high schools and junior high schools as well, even in elementary schools. The National Science Foundation recently stated that they had never funded remedial mathematics research at the university level and doubted that they ever will; this, despite the fact that the most critical need at the present time is more information about college remedial mathematics students. We are in a critical period of mathematics education; most of the students entering universities do not have sufficient mathematics backgrounds to meet their needs or complete the degree they choose. We are faced with a long-term situation and we must meet the challenge despite our personal attitudes.

CONTINUOUS SEQUENCE IN BASIC MATHEMATICS

RATIONALE

For the last ten years, the mathematics background of entering freshmen at Central Michgian University has been weaker than it was previously. Many of our students do not have strong enough backgrounds in mathematics to pursue the degree plans they have chosen. In fact, some of our students perform at a level comparable to eighth grade mathematics. This is unfortunate, not only because their preparation is inadequate for their degree plans but also because most of these students are capable of college work. The problem just described is not unique to CMU but exists on a national scale at most universities, colleges, and junior colleges.

In the fall of 1972 it was decided that something would have to be done to improve the Intermediate and College Algebra courses; these courses were not meeting the needs of the students as was evidenced in the fact that the percentage of students successfully completing the course was far too small. The combination of withdrawals and students earning a D or E grade was approximately fifty percent. Students in these courses could be placed into two categories, 1) those who were over-prepared, and 2) those who were under-prepared. The "typical average student" was, for the most part, conspicuously missing. The students who were over-prepared for these courses (those who had more than an adequate prerequisite mathematics background) found the courses boring; these students often failed to attend class, failed to read the text and do assignments and, consequently, many of these students received low or failing grades. Those students who were under-prepared for the courses found that they were confused by the first lecture and each succeeding week they fell further behind and became more confused. Most of these students received low or failing grades. In addition, students who did pass these courses often demonstrated in latter courses very poor retention of the subject matter, this fact caused the inclusion of math review units in latter courses, those offered by other departments as well as those offered by the mathematics department.

Dr. Melvin Poage and Dr. Delano Wegener were asked to research the problem to determine if a program could be developed which would better meet the needs of students who needed to improve mathematical backgrounds at the college remedial level. Armed with some teaching experience in similar situations at other universities and armed with the knowledge that a considerable amount of research and experimenting was being conducted at the remedial level in math and other disciplines, the research directors began experimenting in the Winter Semester of 1973. After seven years of research a remedial mathematics program, Math 105, known as the Continuous Sequence In Basic Mathematics (CSBM) has been developed.

PURPOSE

The goals of the CSBM program at Central Michigan University are as follows:

1. Determine the entry level (based on the mathematics background of students).
2. Provide a flexible program so that students can start at the point commensurate with their backgrounds.
3. Provide a flexible program so that individual sequences for each student can be assigned that will allow students to skip units they already know.
4. Provide instruction in small blocks so that if students need to repeat, the time lost is minimal.
5. Provide materials and instruction that will maximize the students' learning possibilities.
6. Provide varying exit points, determined by the department offering particular degrees.

7. Provide special units of applied mathematics for certain departments.
8. Provide the opportunity for CSBM students to learn and retain mathematics as opposed to memorizing disassociated facts.

CONCEPTUAL FRAMEWORK

The publication of Bloom's taxonomy (1956) stimulated interest in mastery learning. This interest took two basic approaches according to Dr. Bloom (1979), "One is the Keller approach, called 'Personalized System of Instruction (PSI),' in which each student goes at his or her own pace. The other is the group-based approach, in which the teacher teaches a class, then uses feedback as the basis for individualizing the corrective procedures for the students. Both can yield very positive results. ---." A modified version of each of these two approaches was incorporated into the development of CSBM. Most students in the program follow the group based approach now called "Learning for Mastery (LFM);" Bloom (1968), Block (1974), and later Block and Burns, (1977). A few students who first demonstrate they can successfully complete two units and who have demonstrated some self initiative and determination are allowed to use the PSI approach. The need for this restriction is best summarized by Bloom (Brandt, 1979), "The problem with the Keller approach is that while students who finish on time do very well, a sizeable proportion drops out of class. They find they have procrastinated too long."

Some attention was given to developing alternate modes of instruction, particularly, the use of audio-tutorial instruction, the use of math-help labs, and individual tutoring. Guidelines used for the development of audio-tutorial materials were those developed by Postlethwait (1969); modifications of the programs described by Bidwell (1971), Schoen (1974), and

Doggett (1978) were used for the CSBM Math-help labs and the individual tutoring.

The overall design of this study was principally influenced by Bloom (1956), Gagne (1970), Cronback (1967) and Biggs (1969). However, the research conducted in the development of the CSBM program should be classified as applied research as defined by Rising (1978).

PROCEDURE

The general procedure of this study was to identify all variables that had a major affect on learning mathematics by remedial college students. After a variable had been identified, experiments were conducted on the aspect of the program which involved that variable in an attempt to increase the learning and understanding of mathematics. The goal was to improve the conditions that were connected with each variable and maximize the student's learning. In some instances, the logistics of scheduling, space, or record keeping prevented or limited this improvement.

VARIABLES

The list that follows identifies variables by category.

1. Instruction related:

   Attendance, Mode, Memory, Repeating, Content and Sequence, Time, and Objectives.

2. Evaluation related:

   Type, Length, Time, Grades, Feedback, Placement, Subtests, Security, and Mastery Level.

3. Material related:

   Text, Lab, and Audio-Tutorial.

4. Course Management related:

   Computer Management, Number of Students, Instructor Assignment, and Permanent Files.

5. Affective Domain related:

   Student Attitudes, Instructor Attitudes, and Counselor Attitudes.

SUBJECTS

The subjects for this study were the students at CMU enrolled in MTH 105 from the Fall Semester 1972 to the Winter Semester 1979. The number of students by semester is given in Table 1.

Number of Students Enrolled
in Mathematics 105 by Semester

Year	Fall	Winter
1972-73	132	119
1973-74	213	169
1974-75	478	442
1975-76	539	502
1976-77	764	718
1977-78	962	838
1978-79	1582	1279

Table 1

INSTRUMENTS

The directors developed placement tests, called scanners, to measure the entry level of the students' mathematics backgrounds and they developed criterion referenced tests, called criteria tests, for each unit to measure student achievement. The affective domain was measured by an attitude survey called Course Evaluation also developed by the directors.

Three scanners were used to cover the range of content. Questions on each scanner were grouped by units of content. Successful recall level for each unit on a scanner was 52.5%.

Criteria tests were correlated to the set of objectives developed for each unit of the program. Many forms of criteria tests were developed; first to improve the validity and reliability of the tests, and later to accommodate the security needed for large numbers of students. Items were written in a multiple choice format although many were first written in an open-ended format. This procedure enabled directors to improve the construction of the items with respect to clarity and attractiveness of the multiple choice distractors. Mastery level of a criteria test was 87.5%. The criteria tests were designed to measure achievement at four cognitive levels: memory, comprehension, application, and to a very limited degree, higher-level analysis.

Three analyses were used to validate the criteria tests; an objective analysis, an item analysis, and a form comparison analysis. In addition, all instructors were periodically surveyed concerning corrections or suggestions of criteria tests. (See sample analyses Appendix III.)

The Course Evaluation is a modification of attitudinal surveys used by the department and the university for this purpose; the emphasis on instructor evaluation was changed to emphasize evaluation of course and materials.

METHOD (as used the final three years)

PLACEMENT

Each student was given placement examinations upon entry into the sequence to determine the student's mathematics background. The placement tests took 3 to 6 hours to administer, depending upon the particular route the results of tests dictated. Placement tests were usually given for one hour each day and the majority of the students took 4 hours of examinations to be placed. A student started the placement procedure by taking Scanner II which tested his knowledge of the second quarter of the content of the course. The questions on each scanner were grouped by unit and if a student could recall the material tested on Scanner II, the next day he was given Scanner III which tested the third quarter of content; if the student had difficulty recalling the material on Scanner II, he was given Scanner I. Scanners were analyzed by unit and, in most cases, once the correct Scanner had been determined, students were able to recall material in sequence up to a particular unit; beyond that unit, they had difficulty in recalling the material. After the "cut-off point" in the unit sequence had been determined by the Scanners, regular criteria tests for the units below and above this cut-off point were administered to verify the level of mathematics background. In some cases, additional criteria tests were needed to determine the level of a student's mathematics background. If a student did not agree with his placement, additional tests were given until the proper level could be determined. Because of the large number of students and existing policies of the University regarding credit, students were placed, for the most part, in odd-numbered units. The placement results for the Fall Semester 1978 are shown in Table 2.

Unit	Number	Unit	Number
1	52	15	123
3	204	17	89
5	175	19	42
7	49	20	13
9	58	21	11
11	365	23	4
12	214	24	4
13	185	25	1

Table 2

SEQUENCE

Each student was given a unit-sequence to follow in the program. For most students, this sequence consisted of all units between their point of entry and the point of exit as determined by the department offering the degree they were seeking.

Some students demonstrated on the placement tests that they had an area of background weakness below their "cut-off points." These students were assigned unit-sequences beginning with the lowest unit needed, skipping units for which competency had been demonstrated and continuing the unit-sequence to their exit point.

In the above manner, every student in the program was given an individualized unit-sequence to follow. If a student did not agree with his assigned unit-sequence, further tests were administered until the proper sequence was determined.

INSTRUCTION

The content of the program was separated into two-week units. Each unit could be taught in six one-hour lectures; occasionally instructors scheduled an extra lecture in order to complete their presentations. When a student began the program, he was required to attend lectures. (He needed to attend 5 of the 6 lectures to be eligible to take the criteria test.) If the student successfully completed and passed two consecutive units, the attendance requirement was waived and, if he chose, he could study on his own (PSI) and proceed at his own rate. Students studying on their own could not take criteria tests on more than three units in any given two-week period. A student lost the privilege of PSI if he failed a criteria test; he could earn PSI status again by attending lectures and passing two consecutive units.

All students could receive instruction in the math-help labs that were open about 16 hours per week. These labs offer instruction on an individual or small group basis. Occasionally students were required to attend the math help lab for instruction.

Audio-tutorial materials of two types for many units were available in the self-study center in the library; drill skill lessons and tapes, and concept lessons and tapes. Unfortunately, the use of this material by students was quite limited and they were not revised along with the other materials. However, increasing numbers of students in the program may cause the necessity of updating the A-T materials.

Private tutoring was available to students who needed a great deal of help. If a student had failed the same unit two or more times and had demonstrated that he was making an honest effort to learn the material, a tutor was assigned.

ADVANCEMENT AND EVALUATION

To receive credit for a unit and advance to the next unit, a student needed to demonstrate mastery by correctly answering 35 questions on a 40 question multiple choice criteria test. Mastery of each unit at the 87.5% level is required before proceeding to the next unit; 43.12% of the total tests taken in the Winter Semester of 1978 were passed at 87.5% or above.

If test results of a student were below the 87.5% level, the following procedures were followed:

75%-87.5% - subtest on unit

less than 75% and first attempt - repeat unit

less than 75% and second or greater attempt - see counselor

Those student assigned to a subtest attended a special lecture-counsel session. For this session, students had a list of the objectives which were related to the questions they had missed. Instruction for this group of students might be done in small groups or individually during the session, done as an outside study assignment or done as a math-help lab assignment. Students in the subtest group would study concurrently the material related to the objectives they missed as assigned and attend lectures on the next unit. If the student completed the assignment on the missed objectives, he would take a subtest on those objectives. He passed the subtest if he scored 87.5% or better; if he failed the subtest, he repeated the unit. Subtests varied in length according to the number of objectives missed on the original criteria tests. Subtests consisted of all items pertaining to the missed objectives from a randomly selected alternate form criteria test. 89.2% of the subtests taken in the winter semester of 1978 were passed.

If a student scored less than 75% on his first attempt, he was assigned to attend the lectures for this unit a second time. In almost all cases, a different instructor would be teaching the unit.

If a student scored less than 75 and this was his second or greater attempt, he was assigned to a faculty counselor. The counselor had several options: subtest, complete test, working with the student himself, assignment to math-help lab, assign a private tutor, or recommend the student drop the course.

The program was administered in two-week periods. Test rooms were open for criteria tests the last 2½ days of each two-week period. Students could come any time during that time to take a criteria test. Subtests could be taken during the last day of the first week of each period. Special testing times were arranged in case of emergencies. There were no time limits on any CSBM program test. All tests were graded by computer and results were given in terms of objectives. Student test results were available Monday morning following the test.

Students were required to complete course evaluations to measure the affective domain. These evaluations were completed during the last period of each semester. The course evaluation was a 30 item multiple choice questionnaire.

Follow-up studies in a more advanced math course were done to compare students who had been enrolled in the CSBM program with students who did not need the program. In general, the CSBM students completing units required as prerequisites for the more advanced math course earned a higher grade point average than the other students.

CONTENT

The CSBM program was divided into four courses: Course I, Competency Arithmetic - Units 1-7; Course II, Beginning Algebra - Units 8-14; Course III, Intermediate Algebra - Units 15-21; and Course IV, College Algebra - Units 22-28. A list of the objectives for each unit appears in Appendix I of this report.

DEVELOPMENT

A chronological development of the CSBM program appears in Appendix II. While the directors felt, based on their own experience and the research reported in the literature, a program using many of the aspects of PSI and LFM courses would be more effective, they were not able to anticipate all of the problems that would arise and, in many respects, they were too idealistic in their assumptions about remedial students' motivation to learn mathematics. In some ways, Appendix II reflects the struggle between what was practical, what logistics would allow, and idealistic theory.

RESULTS

There were three measures of success of the cognitive domain used, one each for short, medium, and long range results. The short range measure was computed for every two week period; the percent of students who did not fail (those who passed outright plus those who passed by a subtest). The results of this measure is shown in Table 3 and Table 4. Table 3 is for the Fall Semester 1975, approximately that point in the study where the program had been finalized but not all variables maximized. Table 4 contains the data for Winter Semester 1979.

Fall 1975

Period	Tests Given	Passed	Percent
1	486	381	78%
2	473	356	75%
3	453	289	64%
4	403	260	65%
5	395	240	61%
6	345	186	54%
7	243	85	35%
Total	2798	1797	64%

Table 3

Winter 1979

Period	Test Given	Passed	Percent
1	961	677	70%
2	1097	920	83%
3	953	759	79%
4	796	611	76%
5	719	513	71%
6	599	397	66%
7	500	260	51%
Total	5625	4137	73.5%

Table 4.

The medium range was measured by computing the percent of units mastered against units attempted for each semester. Since students could choose not to take math for a given two week period and often did this because of the pressures of other courses, units attempted was used instead of credits enrolled for the base of this measure. This measure shows the percent of students who attempted a particular unit and by some means eventually mastered that unit. Table 5 shows the results for Fall Semester 1975 and the Winter Semester 1979.

Fall 1975			Winter 1979		
Units Attempted	Units Passed	Percent	Units Attempted	Units Passed	Percent
2328	1797	77.2	4816	4137	85.9

Table 5

The long range measure is a measure of the success rate of CSBM students compared to non-CSBM students taking a more advanced math class, MTH 116. The greatest number of CSBM graduates who enroll in more advanced math classes enroll in MTH 116, a pre calculus math for business students. A few enroll in MTH 106, trigonometry, or MTH 130, a pre calculus math for arts and science students. This measure has only been computed for the last three years because the number of students completing the CSBM program and enrolling in MTH 116 was not large enough to be significant. Table 6 contains these results.

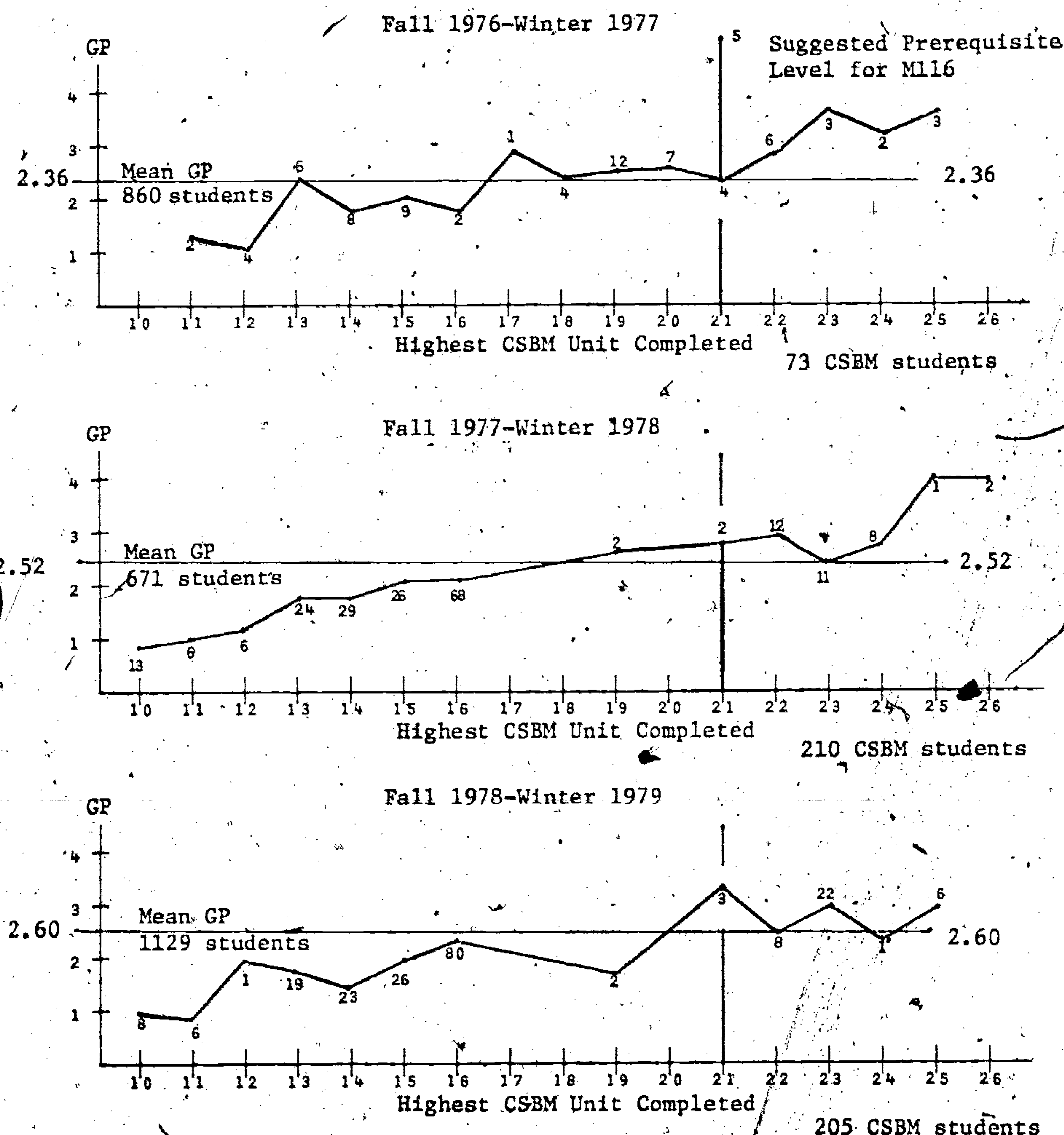

Table 6

Long-range evaluations are critical because we do not always teach what we think we teach. That is, we think we teach so that our students learn math; however, evaluations of students (consequently, an evaluation of teaching success) measured one or more years after students complete a course have indicated that most remedial students have little understanding of previously learned mathematics. Results from our early attempts to measure long term effects agreed with the research literature which indicated that traditional methods and materials were not effective for the long term. Early long-term measures in CSBM were obtained by comparing the exit point with re-entry placement of students who left the program for one or more semesters. With improvement in materials which put less stress on memory and more emphasis on understanding, the difference between exit and re-entry was reduced to 1 unit or less for most students.

On the attitudinal part of the Course Evaluation, students had the choice of four responses to each question: strongly agree, agree, disagree, and strongly disagree. The responses strongly agree and agree were considered favorable; all other responses, including no response to an item was considered unfavorable. Table 7 lists the percent of favorable responses by item and by semester.

In general the percent of favorable responses declined as the number of students in the program increased. There are two main factors that influenced this decline: as the student population increased the program became more pressed to treat students as individuals (the program had to rely more on student initiative to come forward when they needed help), and as the student population increased the program needed more instructors (the program had to rely on an increasing number of instructors that were not as favorable to the program and its purposes). The questions in the

Table 7

Question	F'74	W'75	F'75	W'76	F'76	W'77	F'77	W'78	F'78	W'79
1. I prefer this integrated lecture-independent study approach to the system with lecture only.	84	82	85	83	86	83	75	78	75	70
2. I learned a great deal in this course.	86	85	85	86	85	81	71	72	70	61
3. I would recommend this course to my friends.	78	78	79	85	81	75	66	64	58	38
4. My attitude toward mathematics has improved.	75	75	70	71	74	66	59	60	53	45
5. I was able to achieve the objectives in most of the units.	86	86	91	92	93	93	85	84	84	78
6. The placement test correctly determined the proper beginning point for me.	76	85	68	75	66	71	64	72	60	58
7. I found the lectures useful and helpful.	95	89	81	79	82	77	68	77	73	66
8. The lecture instructors were well prepared.	85	81	79	80	78	78	70	76	69	64
9. The text materials were a great help to me.	79	77	89	87	86	84	82	84	83	74
10. The text was clearly written.*	69	62	79	75	71	73	75	70	73	61
11. I found the lab useful.*	81	81	76	85	80	77	63	64	63	70
12. It was always possible to get into the lab, even during peak hours.*	80	78	70	78	63	70	38	51	45	42
13. The lab instructors were always free to help me when I needed help.*	64	71	66	74	64	61	43	50	49	48
14. The lab instructors were very helpful.*	81	85	84	89	81	76	69	71	67	69
15. The lab instructors were well prepared.*	71	75	77	78	67	64	70	66	66	65
16. The tutors were helpful.*	87	88	92	91	91	80	77	73	71	72
Mean Favorable percent on questions 1-16.	80	80	79	82	78	76	67	70	66	61

*Indicates that only those student who actually used the procedure in question were considered.

Course Evaluation can be separated into three groups: those that had a favorable-percent decrease more than the mean decrease (questions 2, 3, 4, 7, and 12), those that decreased about the same as the mean (questions 6, 8, and 13), and those that had a favorable-percent decrease less than the mean decrease (questions 5, 9, 10, 11, 14, 15, and 16). Improvement in text materials and a better program for training lab instructors and tutors probably accounts for the more favorable result in the last group of questions.

ANALYSIS

The test results of the CSBM program generated a large amount of data each two week period and since most units were taught each period, the data collected in a period was, in many respects, equivalent to that collected in a semester in other programs. Three basic types of analyses were used to pinpoint problems regarding students mastery of concepts: a question response analysis, an objective by test analysis, and a total objective analysis. Early in the development of the program a fourth analysis was used, a question difficulty analysis. Samples of each analysis is shown in Appendix III.

The question response analysis lists the percent of responses to each distractor and correct answer for each question. In general, any question that did not have a correct response between 65% and 92% was reviewed. Questions that had a correct response greater than 92% were reviewed to see if the plausibility of the distractors could be improved or if a different question might be a better test of that concept. Questions that had a correct response less than 65% were reviewed for correctness, clarity, and the possibility of restating the question. If the question was acceptable, the low correct response was attributed to poor text material and the text was modified accordingly.

The objective by test analysis lists the percent of correct responses to each objective. Most objectives were tested by two questions, a few by three or more questions, and a few by only one question. Objectives that did not have a correct response greater than 72% were reviewed. For those objectives that were tested by only one question this analysis did not add any new information, but for most objectives it provided comparative data on questions of the same test concerning that objective as well as sets of questions on different tests concerning that objective. Criteria tests were prepared in pairs, that is, forms A1 and A2 would contain the same questions but the order of the questions would be permuted. Tests were given an alphanumeric form code to identify author and pair. Many units had 16 or more test forms of which 3 to 5 would be in use at any given time. Information gained from this analysis helped improve test questions and pinpoint weak areas in the text material. Early in the development of the program information from this analysis helped to improve the objectives for each unit. Sometimes it became necessary to rewrite an objective that encompassed too much material as two or more simpler objectives or it became necessary to rewrite an objective in a more testable format.

Information from the total objective analysis was used to reorder the sequence of units, primarily placing more difficult units later in the sequence. The total objective analysis listed the percent of correct responses to an objective using the data from all test forms. If the total correct response to an objective was 72% or greater and if the questions concerning that objective were satisfactory according to other criteria, then that objective was considered acceptable. In Units 1-24 there were a total of 549 objectives, of these 441 (80.3%) were acceptable as of Fall semester 1978. Table 8 summarizes this data.

Fall Semester 1978

Unit	Correct Response Less than 72%	Correct Response Greater than 72%	Total Objectives	Unit	Correct Response Less than 72%	Correct Response Greater than 72%	Total Objectives
1	3	30	33	13	7	19	26
2	8	10	18	14	1	19	20
3	4	19	23	15	4	20	24
4	3	24	27	16	1	19	20
5	10	13	23	17	3	17	20
6	2	21	23	18	2	23	25
7	8	12	20	19	5	17	22
8	4	21	25	20	5	14	19
9	7	17	24	21	6	16	22
10	1	23	24	22	4	28	32
11	7	15	22	23	4	19	23
12	2	18	20	24	6	18	24

Table 8

Information from this analysis was used in planning the sequences of units and later in altering that sequence. The final sequence of the units in the CSBM program does not agree with traditional organization of material in all respects (see Appendix I). This difference is most apparent in the sequence for Units 15-21.

There are three Units that had a percent of acceptable objectives considerably less than the average of 80.3%: Unit 2 (55.6%), Unit 5 (56.5%), and Unit 7 (60%). Most students whowere placed in Unit 1 (Operations with

Whole Numbers) or Unit 2 (Operations with Decimals) probably have mathematical backgrounds so deficient that it is unlikely they will be able to complete a university degree. Often these students have weaknesses in other content areas, particularly English, as well as mathematics. The sections of Unit 2 that cause the most trouble are those that are concerned with the algorithms for multiplying and dividing decimals. Unit 5 (Operations Involving Percents) repeatedly causes problems in most curriculums. As many math instructors can testify, students who do well in mathematics frequently have some trouble with operations and problems involving percents. The sections of Unit 5 that particularly cause trouble are those concerning "mixed notation," such as 66 2/3% and .25%, and solving word problems involving percent. Unit 7 (Metric Measurement) seems to cause trouble because most students are unfamiliar with metric units of measurement. The sections of Unit 7 that cause the most problems are those concerning area and volume (capacity).

DISCUSSION AND CONCLUSIONS PART I

In the development of the CSBM program, every attempt was made to identify all variables that had an affect on the learning of remedial mathematics. Some of these variables were labeled critical as they had a great affect on that learning and they were variables that could be manipulated and controlled by instructors. There were four critical variables identified by the research connected with the development of the CSBM program: memory, math reading, short content units and objective organization.

Part I of this section will concern itself with an in depth discussion of the four critical variables. This will be followed in Part II by shorter discussions of the remaining variables.

ROLE OF MEMORY

There is a long list of items that should be and usually are memorized in mathematics. This list begins with the basic facts and includes such items as algorithms for operating with real numbers, Pythagorean Theorem, etc. However, the total of math cannot be memorized. While the list of items to be memorized varies for different math classes and for different students, at some point in each math class an understanding of the topic must occur if a student is to "learn" that topic. We have found that in a typical remedial program too much of each topic is memorized and too little is understood.

Traditionally, the "modus operanda" for teaching remedial as well as most mathematics is to present a new topic by rules and examples which is supposed to convey meaning and understanding to the student; then make an assignment where the exercises can be successfully completed by blindly applying the procedure exhibited in the examples (Laing 1973). We have found that rules and examples by themselves seldom result in understanding a true knowledge of the topic. When students say, "Don't explain it, just show me how to do it," they are expressing that pattern of teaching fostered by the traditional approach. By making this and similar statements, students indicate that they know that math need not be understood but only memorized. Remedial students tend to view math as a great collection of disjointed facts and algorithms that must be memorized.

We have found that remedial students have non-effective study habits, almost always limiting math study and homework to "the night before a test." Such cramming for a math test is often considered as the best course of action, based on the student's empahsis on the role of memory, if one wishes to pass that test. Further, in the mind of these students, passing a math test by sheer memory is equivalent to understanding the topic.

At this point one has to become subjective to determine what aspects of instruction improve the understanding of a topic. Certainly teaching for understanding does not prevent memory loss, perhaps it may retard the loss rate, but a fact or process that is not used frequently is lost from immediate recall whether or not it is understood. Our experience has led us to believe that the difference between students who learn through understanding and those students who memorize is not memory loss but reconstruction ability. Evidence of lack of reconstruction ability is illustrated in the following examples. Recently the math department decided it was necessary to have tests available for the purpose of advising freshmen and transfer students. Some of the questions selected for these tests were taken from the question bank of the CSBM program. One question of the type, "What is the area of a 3 by 7 rectangle? (a) 10 sq. units, (b) 20 sq. units, (c) 21 sq. units, (d) can't remember the formula, (3) none of these," became interesting in the context of reconstruction ability. When this question was used in a CSBM criteria test, distractor "(d)" was never picked by the students. Because of this fact, we were surprised that when this question was used in the advisement test, distractor "(d)" was picked by 10% to 15% of the students. Students who understood the concept of area would, if necessary, reconstruct this concept by making a sketch of a 3 by 7 rectangle and then computing its area by counting the squares. In contrast, those students who thought of area as a set of memorized formulas chose distractor "(d)" if they could not remember the formula.

The second example concerns students who occasionally come in for help in chemistry. In chemistry powers of ten are used in scientific notation and metric measurement. Usually a student in this situation, given the problem $834.64 \cdot 10^{-5}$ to change to scientific notation, will state, "I know you move the decimal point 2 places, but I don't know whether $8.3464 \cdot 10^{-7}$ or $8.3464 \cdot 10^{-3}$ is the answer." Traditionally the teaching of rules such as, "move decimal point right (left) when ..." are favorite methods for many instructors and students. The memory loss of these rules, together with the lack of understanding about operations involving powers of ten has contributed to reducing this student's ability to reconstruct. Our experience indicates that teaching of rules and formulas in cookbook fashion prevents students from understanding related concepts.

Research has indicated that the human brain has the ability to memorize non-trivial mathematical facts and algorithms without understanding the related concepts. Unfortunately, such memorized facts and algorithms are short-term and virtually not reconstructable; as a consequence, remedial students taught by traditional methods have to be retaught every two to four years. While memory is an important ingredient in the total learning process, it cannot be the only ingredient if utilization of math by the student is a goal. Remedial students can understand and need explanations of concepts. These explanations must, however, be given in their language and must relate to and extend previously learned concepts whenever possible. Memory should be preceded by concept explanation and rules should be developed as a consequence of this explanation.

In conclusion, our experience shows that one of the major instruction goals should be to increase a student's ability to reconstruct.

READING AND VOCABULARY

In the preceding section, mention was made that the language used for explanations should be compatible with the language understood by remedial students. Too often, because this difference between math language (vocabulary) and student language (vocabulary) is so great, explanations are completely left out of existing remedial text material. The following, a conversation between a man and his son, illustrates the mathematics reading problem. Son, "May I go out and play ball now? I worked all the problems." Father, "No, I think you should read and study your math text so that you will understand the topic better." Son, "Aw-Dad, nobody ever reads a math text." Unfortunately, we would agree with the son. Historically, the area of math reading and vocabulary has been one of low priority. In fact, we have created for remedial math students a constitutional right, the right of no responsibility for reading math texts. Research indicates that the percent of mathematical terms and symbols understood by students, in particular, remedial students, is very small, Klun (1976). Further, student initiated text reading as part of doing homework is seldom done. We have been able to verify these research results through experiences and student surveys in our own program.

Our student surveys indicate that remedial students, if they study math on a regular basis, use the following approach:

1. Attempt to do all homework problems assigned.
2. If there is difficulty in doing a problem or group of problems on the homework,
    a. A small percent of student attempt to read and gain understanding from their lecture notes. This group is limited because many do not take notes and others who do cannot read their notes.
    b. A large percent call on a friend for help. (This procedures has been a long-standing excuse for high-school or college students to make telephone calls or meet with friends.)
3. If all else fails - ask instructor the next day in class for help.

Further, our surveys indicate that students' choice of textbooks are those written without explanations, proofs, and other expository material.

This unfortunate student preference often places instructors in a situation where they must attempt to do the learning for the student.

In the story above, when the father asked his son to read and study the text material, he was making a logical request within his own frame of reference. As a successful business man, he knows from his own experience that when a topic needs to be studied and understood, reading must be part of that process. In terms already used in this report, the father was stating that not only is reading a necessary part of understanding but that there is a direct relationship between reading and a student's ability to reconstruct. That is, if you have successfully used reading to help understand a concept, you create confidence in your ability to use reading to reconstruct that knowledge again at a later date.

In our quest to seek information about the teaching of reading and vocabulary in mathematics, we were extremely disappointed that we were not able to find (outside of several individual instructors) any program--math or reading--in our K-college education system that considered reading math important enough to investigate methods or develop materials to increase this skill for students. As a result we decided to prepare our own text and other materials.

In preparing our own materials, a number of decisions had to be made. We found out that it was not possible to get a group of instructors to agree on a precise curriculum (set of objectives) for remedial math. We needed a precisely stated set of objectives to carry out the creation and preparation of materials in line with other decisions we had made. Some of the other decisions about materials were:

1) Limited Vocabulary - Level of Language

2) Inclusion of Special Features to Teach Reading

3) Limited Topics

4) Organized in Two-Week Blocks

5) Organized by Objectives

A limited mathematics vocabulary was used to lessen the differences between math language and student language. Since few if any of our remedial students become mathematicians we felt that mathematical rigor and elegance as provided in its langauge was not a top priority. Instead we used terms from high school math and common English with which students would be more familiar. However, not wanting to create a new language, math terms were introduced and used when necessary, and the creation of new quasi-math terms was limited. We also felt that the level of language should increase as the student progressed through the program. As a result, a non-mathematical term accepted and used in an early unit would be replaced with its mathematical counterpart in later units. An example of this is "opposite" which is used in high school and "negative" used in college texts. Support of this approach has come recently from Watkins (1979) who states that her research suggests that texts be written in a style closer to ordinary English than in a style using mathematical language; Fey (1979) states that it is ironic that mathematics teachers so frequently voice concern over the limited reading skills of their students when reading is the main focus of elementary schooling. Perhaps skills for reading mathematics need to be specifically taught; Rising (1979) stated that we must make every effort to express mathematics so that it can be understood by the non-mathematical student; and finally Phillips (1979) stated that the vocabulary used in math needed special attention because it contained many double or multiple meaning words:

Research on teaching reading of mathematics is almost non-existent, Kulm (1976), Earle (1976), and West (1977) were the only resources found. To augment this knowledge, discussions were held with reading teachers and reading consultants in elementary and secondary schools. Most of the suggestions made concerning the teaching of reading required a substantial amount of time and we felt that such amounts of increased time to teach math reading could not be justified. For example, one such suggestion was to have students take turns reading the text aloud and discuss what was read. However, based on the information and suggestions we collected, we decided to include two features to aid in the specific teaching of the reading of math. These two features, called Reading Guides and Key Words, were first put in a separate Study Guide for the students; however, we found that they were more effective if they were incorporated into the text. One feature used, Key Words, had been used previously in mathematics text books, a vocabulary list at the end of each unit. The other feature, Reading Guides, had not been used previously and seemed to have some potential, to ask students questions about what they had read. The list of questions could be in written form and the style could be "fill in the blank" or "complete the sentence." Such a list is usually compiled by choosing important sentences from the textual material and deleting one or more words. Our investigations have shown that the instructors' use of Reading Guides and Key Words does improve reading and hence improves a student's ability to reconstruct.

SHORT CONTENT UNITS

In previewing existing textbooks as part of developing the program, we felt that many of the topics included in these texts were not necessary for remedial students. Instead, it would be better to concentrate on core topics for mastery; remedial students often become overwhelmed by the amount of content they are supposed to learn in math (not their favorite subject). Closely associated with this problem is that of organizing text material into two-week blocks. We felt that in order to have a successful program which allowed for multiple entry and exit points and provided an individualized unit sequence for each student, high flexibility would be needed. Such high flexibility would not take place within the framework of the traditional semester based course. In addition, students who failed or could not master particular units should not have to wait a full semester before having a second chance. Such needs cannot be met unless text materials exist in short units of equal teaching length. Because such materials did not exist, another reason for creating new materials for the CSBM program presented itself. The directors found that mathematics does not lend itself easily to being packaged in short units of equal teaching length. A great amount of rewriting was necessary in order to modify and adjust content into these short units; a two week period (six-lectures) was finally selected as the most practical time for the short units. In the final version of the CSBM materials there are some differences in the amount of time needed to teach various units. While most units can be taught in five lectures allowing a lecture for review and summary, a few units require six lectures for complete presentation and occasionally, for a particular group of students, an instructor may need an extra lecture. The trade-off of the problems caused by attempting to package math content into two week units with the high flexibility gained by doing this, is well worth the effort.

OBJECTIVE ORGANIZATION

At the beginning of each section of each unit, a coded list of performance objectives for that section is given. A restatement of these objectives then serve as titles for the appropriate subsections. The objective alpha-numeric code is placed in the margin opposite related text material, reading guides, exercises, and self-test questions. With this alpha-numeric code, students and instructors can quickly and easily find all materials related to a particular objective. In this sense, the text materials have been organized by performance objectives. Because the text material was organized by objectives, it was relatively easy to write computer programs to analyze the effectiveness of the text material for each subsection (objective). After materials were taught by several instructors, student test results were analyzed, since each subsection of the text is tested by two or three questions; any subsection that did not have correct student response in excess of 72% was revised and rewritten until the desired results were obtained. Occasionally, algorithms that traditionally appear in math texts had to be replaced because remedial students could not understand and master the traditional algorithm at a high enough level. We found that the proposition that states, "the algorithms and schemes used in learning mathematics for mathematically inclined individuals, such as instructors, will also be effective in teaching remedial students" is, by no means, a provable statement. When we accomplished the goal of correct student responses exceeding 72%, we felt we had created a text that is readable by most students and is teachable by most instructors.

The problems of the role of memory, reading mathematics, short content units, and objective organization are all interrelated and yet individually each is a problem of many facets. Instructors in other programs have found our results or materials useful in their own programs (even though their program may be substantially different from the CSBM program) in coping with these four critical variables.

DISCUSSION AND CONCLUSIONS PART II

INSTRUCTIONAL RELATED VARIABLES

In Part I we discussed some of the critical variables related to instruction: memory, reading vocabulary, length of units, and objectives. In Part II we will briefly discuss the other variables related to instruction: attendance, mode, repeating and sequence.

Attendance

We have found that mathematics is seldom a favorite course for remedial students, in fact, many state openly their dislike of math. The consequence of this student attitude is that math, if studied at all, is the last subject to be studied and often does not receive its share of the remedial student's time and effort. This situation, in turn, requires that some aspect of remedial programs must concern themselves with applying pressure on students for attempting and completing math assignments. One possible way of successfully applying such pressure is through attendance. In the CSBM program students were required to attend 5 out of 6 lectures before they could take a criteria test on that unit. If a student passed (got a score equal or greater than 87.5%) on two consecutive units, the attendance requirement was waived. At any given time about 28% of the students in the program had the required attendance waived. If a student did not pass a unit, the attendance requirement was reimposed. Since the CSBM program was computer managed, attendance lists or decks of cards were easy to generate. There are several alternate ways to apply pressure on students to do mathematics and perhaps one of these would be more effective. Table 9 contains the CSBM attendance policy.

CSBM Attendance Policy
(Fall 1978)

1. All students (unless they have special assignment by director) must attend classes 5 or more times to be able to take tests. (Green Cards)
2. Students passing their tests with Procedures 1 or 2, for two consecutive periods earn a yellow attendance card and full options. (Self-Study)
3. Students lose yellow cards if they fail a unit.
4. Students will use signed attendance cards to signify they are able to take tests.
5. Students who drop out for 2 consecutive periods will be removed from active class enrollment (No Card)
6. Students on inactive enrollment can activate their status by seeing the CSBM Secretary or the CSBM Director.
7. Instructors return cards to CSBM office, these will be filed and used for test permission.

Table 9

Mode

We experimented with four modes of instruction; lecture, math-help lab, audio-tutorial, and private tutor. Most students seem to prefer the lecture mode. At first, this surprised us since most of the literature available at that time inferred that students would choose alternate modes of instruction if they were available; according to the literature, students would do this because most alternate modes involved a greater degree of student participation and this resulted in improved achievement. However, Smith (1978) states: "The findings of this study fail to support the conclusions made in prior studies that the degree of student participation significantly influenced achievement. One possible reason for this result is that many studies concerning pupil participation have involved elementary school or high school students rather than college students. It may be that most college students have become accustomed to playing a passive role during lesson presentations and thus do not improve significantly, at least at first, if they are allowed to take a more active part in the learning. A second possible reason for this finding is that there is no guarantee that students are really 'passive' learners when they are not encouraged to work problems during a lesson. Likewise, students are not necessarily 'active' learners when they do try to work problems during a lesson. Each student may process information in his own way and the format for reinforcement of this process may be of secondary importance, particularly for college students." Earlier Hoetker and Ahlbrand (1969) stated that if recitation (lecture) is a poor pedagogical approach, why does it persist; maybe most students are used to this method and it is, therefore the most practical approach.

Less than 20% of the students enrolled in CSBM during the first years of development used the audio-tutorial mode as a significant source of instruction; as a result, as time and other pressures increased, the audio-tutorial materials were not up-dated or revised. During the same period of time approximately 30% of the students used the lab as a significant source of instruction; the lab materials which were initially separate were up-dated and integrated into the text materials.

In general, our surveys indicated that the students most likely to use alternate modes of instruction successfully were the older students returning to school after a period of several years of employment or military service. In contrast, the students least likely to use alternate modes successfully were freshmen.

Private tutoring is discussed in the next section.

Repeating

In developing the CSBM program there were certain areas, such as repeating, about which the directors had strong feelings. In these areas experimenting was limited. Our feelings concerning repeating were:

1) student should be allowed to repeat,
2) the period of time to be repeated should be minimal, and
3) repeating should not be unlimited.

Since most remedial students do not have studying math as a high priority, it is only realistic to expect a certain amount of repeating in a remedial program. Further, the repeating time should not be a semester but be as short as possible and still be practical; we decided that the time period should be two weeks. After discussions with other program directors we decided against unlimited repeating. We felt that unlimited repeating would encourage some students to try and "test out" of a unit without making any effort to learn the material. As a result of some investigation we agreed upon the following policy: If a student had made an honest effort to study a unit and he was unable to pass the criteria test on that unit for the second (or more) time, then he would be assigned to a counselor (professor) who would evaluate his attempt, diagnose his weaknesses, and determine a prescriptive course of action. The counselor is given the responsibility of determining whether or not the student has made an honest effort to learn the unit. There are several options that a counselor may use, ranging from suggesting the student drop the course or repeat the unit making an effort to learn the materials, to working with the student or assigning a private tutor. The number of students assigned to counselors increased as the semester progressed, but was always less than 10%. In turn, the number of students assigned to private tutors was less than 1%. Private tutors were math majors and secondary math education majors who applied for these positions. Potential tutors were screened and those accepted were given some instruction on tutoring remedial students. Tutors were paid through a program sponsored by the federal government.

Content and Sequence

As mentioned earlier in this report, it is difficult to get an agreement on the content and sequence for remedial mathematics program. When it became necessary for us to write the CSBM materials, we started with a content that was a composite of existing texts written for this level; to this program we deleted or added as suggested by reports from MAA - CUPM for Basic Mathematics, NCTM and others. Two content areas emerged as being particularly difficult for remedial students to master; the area involving concepts using fractions, rational numbers or rational expressions, and the area involving concepts using exponents or exponential expressions. After considerable experimenting with the amount and placement of content, the first area was taught in the following units:

Unit 4 Operations with Fractions,

Unit 5 Operations Involving Percents,

Unit 10 Rational Numbers

Unit 19 Rational Expressions, and

Unit 20 Solving Different Types of Equations.
(Sections A and B - Equations Involving Rational Expressions.)

The second was taught in the following units:

Unit 1 Whole Numbers
(Section C8 - Squares and Square Roots),

Unit 12 Integral Exponents,

Unit 15 Radicals

Unit 18 Rational Exponents, and

Unit 20 Solving Different Types of Equations.
(Sections C and D - Equations that Involve Exponentials with Rational Exponents.)

These presentations represent a significant departure from traditional treatments which consist of at most two review style chapters on each content area. Review style materials do not correct real and major mathematics weaknesses in remedial students' backgrounds. Because of the amount of data generated for each two-week period, it was relatively easy to spot weaknesses and measure the effect of modification in content in the CSBM text material.

EVALUATION RELATED VARIABLES

Type

Because of the problems of grading, analyzing, and providing feedback to students and instructors, it was imperative that test grading and test results be in a style easily computerized. This in turn meant we must use multiple-choice tests. At first we had apprehensions about using only multiple-choice tests; however, as we experimented with tests of this type, most of those apprehensions were alleviated. Suydam (1974) states: "In general, scores on multiple-choice tests were comparable to those that would be obtained from free-response tests, for the same level of content." Further Suydam states that multiple-choice tests have a high reliability, can be controlled as to the number of questions per objective, and can provide a more adequate measure of many objectives than other objective type tests. Multiple-choice tests can be used to measure a student's ability to: recognize facts or relationships, discriminate, interpret, analyze, make inferences, and solve problems.

To avoid problems using multiple-choice type tests, effort must be made to generate good tests. Some procedures which we found helpful for doing this are stated below. Whenever possible new tests started as a free-response type; commonly recurring student errors were then used to create distractors for a multiple-choice test. Commonly recurring errors noted in the classroom, lab, or by counselors were also used for this purpose. Each of the five distractor positions (foils) were used as the correct response equally often on each test. We occasionally used "all of these" or "none of these" as distractors to discourage a student from starting with the answers and working backwards to try and reconcile their own attempts. In certain situations we also used distractors "Sometimes, Always, Never" in place of "True, False." The use of "Sometimes, Always, Never" allowed us to measure a student's understanding of the critical role of a clause like $n \neq 0$ in a statement like "$\frac{n}{n} = 1$."

We made every effort to ensure that tests did not contain "trick" questions, that is, the answers to all questions or the procedures for solving all problems are given in the text material. Instructors were requested to discuss tests and test procedures with students before the first test each semester. Particularly, we wanted students to understand our use of special distractors like "all of these," "none of these," or "Sometimes, Always, Never."

Some experimenting was done with the set of questions used for a particular test in an attempt to create a bi-modal distribution as shown in Figure 1.

UNIT NO. A FORM NO. N

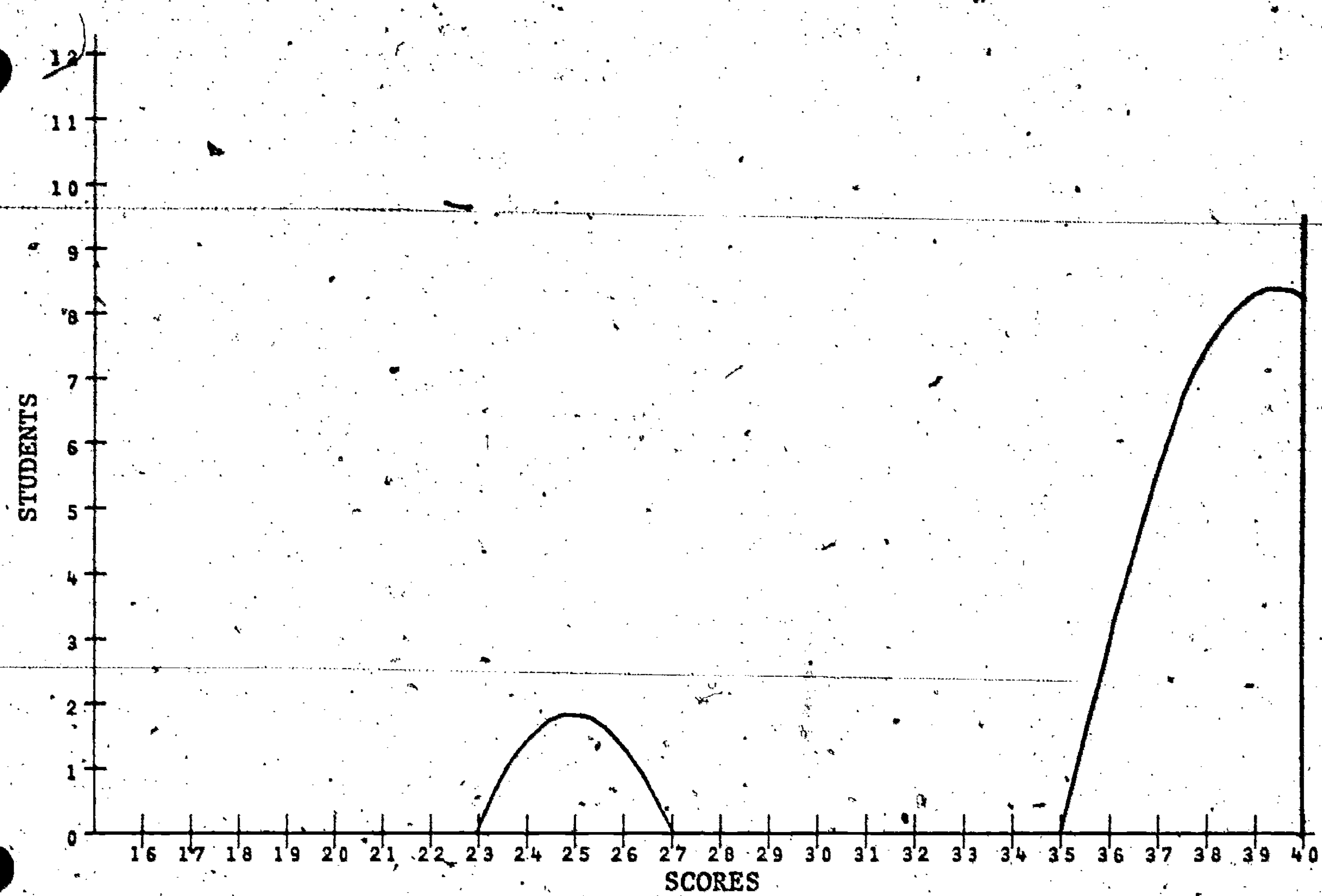

Figure 1

Contrary to research, Byrnes (1978), we found that student success on the same set of questions could often be influenced by whether or not the first few questions of a test were easy or were difficult to answer. Specifically, two forms containing the same questions in different order were constructed; on one form the 6 questions with lowest degree of difficulty were placed first and on the other form the 6 questions with the highest degree of difficulty were placed first. The disagreement with research was probably due to the following factors: populations used in research were not remedial, in most cases subject content was not mathematics, and the degree of difficulty in many cases was determined subjectively. The difference in student success on the two forms seems to be related to remedial students' study and learning patterns; since remedial students often attempt to learn math by memorizing, the anxiety level of these students can be altered by these two forms. Because the success difference was more pronounced at the lower end of the unit sequence, it is doubtful that students who have some understanding of the material tested are affected by the two test forms.

We also found that a mix of 1/3 of the questions based on information covered by the Reading Guides together with 2/3 on information covered by the Exercises produced good results. A typical distribution for one of our criteria tests is shown in Figure 2.

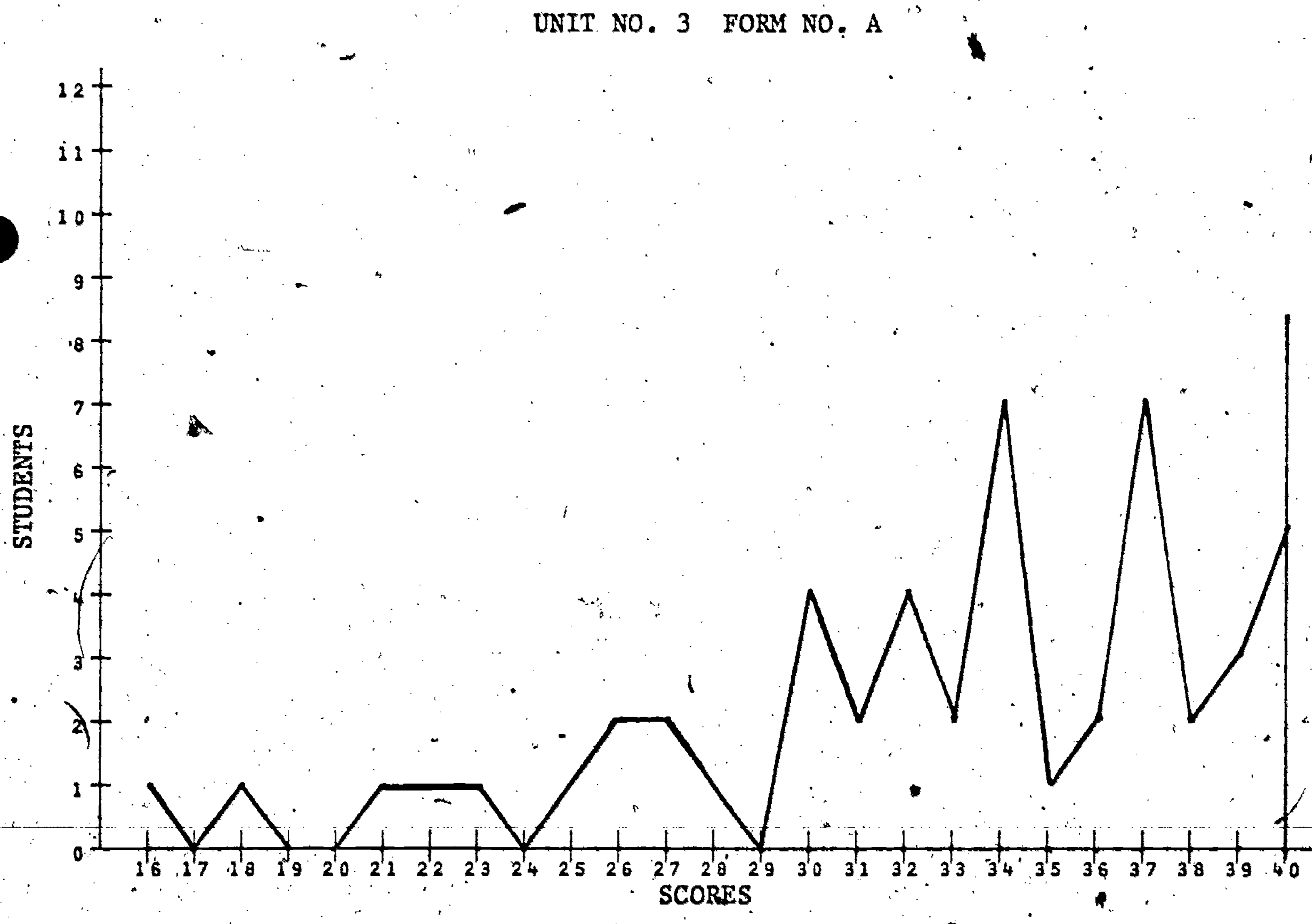

Figure 2

Length

Because the average number of objectives per unit is 20 and because we wanted to test each objective with 2 or 3 questions, it was natural to think of tests that would contain 40 to 50 questions. However, computer printout doubled if we exceeded 40 questions and exceeding this limitation also increased some aspects of the CSBM work for the staff. Because of this, except for scanners, all tests contained 40 questions. We did not find this limitation a severe handicap.

Time

Remedial students often express anxiety in taking mathematics tests. This anxiety is often based on a student's previous experiences of not being able to complete math tests within a given time limit. There were no time limits on CSBM tests. Test rooms were provided during the last three days of each two-week period and when students felt prepared they could take the tests using as much time as needed. The result of this policy was that, with very few exceptions, most students completed tests in less time than we would have allowed under a more traditional setting. The average test time was 35 to 40 minutes. We recommended to those students studying more than one unit that they take only one test a day.

Grades (Credit)

After lengthy discussions within the department it was decided that no grades would be offered, only 1/2 hour credit for each unit in a student's sequence that was successfully completed, and no-credit otherwise. During the Winter Semester 1979 the department recommended that credit given for Units 1-14 not be counted toward graduation. This recommendation has been approved by the University Senate. An argument in opposition to this policy is that since the university had accepted students who had mathematical weaknesses, since these weaknesses were real, since the effort required by most of these students to remove these weaknesses was not minimal, and since at most universities there exist credit courses that require less student effort; any credit earned by a student in the CSBM program should count toward graduation as long as the student has met all other requirements for that particular degree.

The maximum credit earned by a student in one semester was 7 hours (14 units passed), the minimum was, of course, zero. Most students enroll for 3 hours (6 units) and are successful in earning this credit.

Feedback

As stated by most studies, feedback of test results is critical. In the CSBM program all criteria tests were taken during the last 3 days of the second week and sub-tests during the last day of the first week of each two-week period. Students marked opscan answer sheets and these were batch processed on Saturday and Sunday of each week by the computer center. On Monday morning following the tests, students could pick up their results at the CSBM office. The criteria test feedback instrument was called "Star Card." A student's Star Card contained the following data: student name, student number, date, test, procedure information, credit earn status along with an array that identified by objective the test items missed by the student. The 8 by 8 array contained asterisks to show objectives of the missed test items; a sample Star Card is shown in Figure 3.

```
                                14 OCT 78          MATH 105 - CSBM
(STUDENT NAME)                  UNIT 13              CREDIT 1.0
(STUDENT NUMBER)                FORM A1           NO CREDIT 2.0

>YOU MUST ATTEND SUBTEST LECTURE FOR UNIT 13 AT 1:00 IN PE 128 TODAY.  ON
FRIDAY OCT 20 YOU MUST TAKE THE SUBTEST BETWEEN 10-2.  THIS SUBTEST CONSISTS
OF QUESTIONS RELATED TO OBJECTIVES: A1 A6 A7 B1 C3 D2 D3 E2.  YOU MAY ALSO
START UNIT 14.

THE NUMBER OF QUESTIONS YOU MISSED, LISTED BY OBJECTIVE
```

A	B	C	D	E	F	G	H
1**	1*	1	1	1	1	1	1
2	2	2	2*	2*	2	2	2
3	3	3*	3*	3	3	3	3
4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5
6*	6	6	6	6	6	6	6
7*	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8

Figure 3

The credit information on this Star Card indicates that the student enrolled for 3 hours, has successfully mastered 2 units (1 hour credit) and has 2 hours of credit still to be earned.

A sample of the sub-test feedback instrument is shown in Figure 4.

```
(STUDENT NAME)                RESULTS OF SUBTEST - PROCEDURE 2
(STUDENT NUMBER)              21 OCT 78 - CSBM

TEST UNIT 16
FORM A1

       C4

       D3
     2-D4            PASSED

       E1
     2-E2

         7 QUESTIONS CORRECT, OUT OF 7
```

Figure 4

For each student taking a sub-test the computer selected at random an alternate test form and generated a card listing the questions on that form which the student must do for the sub-test. Normally students are not allowed to see copies of criteria tests other than when they take the tests; this has not been a problem since the objective listing gives a student all the information he needs for studying.

Placement

The placement variable has, for the most part, already been discussed; however, it is worthwhile to mention again that students should have a right to disagree with placement results and some procedure should be provided to obtain additional data to help make correct decisions.

Sub-Tests

The need for sub-tests grew from our concern for those students who did well on a criteria test but did not pass. We experimented with various options and multiple options but decided that the use of written sub-tests as the best solution. This concern was motivated by our belief in the mastery learning theory. Bloom (1974): "Virtually all students could learn excellently, mastery learning contended, if instruction were approached systematically, if students were helped when and where they had learning difficulties,---." Block (1979): "---mastery learning provides the classroom teacher with one framework for orchestrating and executing fine generic teaching behaviors--diagnosis, prescription, orientation, feedback, and correction." Anderson (1979): "LFM programs attempt to accommodate students by identifying errors and misunderstandings as they occur and then providing supplementary instruction to help students correct those errors and misunderstandings." Our research has verified that Learning For Mastery (LFM) type courses emphasizing diagnostic and prescriptive procedures increases the number of students who learn mathematics.

Further, that the sub-test procedure is effective in accomplishing this diagnosis and prescription.

Security

When large numbers of students are taking tests on various units on a walk-in basis it is very difficult even under the best circumstances to maintain security regarding tests. Although students taking copies of tests from the test room was not common, it occasionally did happen. The best deterrent against such happenings was a large number of forms of the test for each unit. For many units we had 12 different forms, for some units we had 16. Creating, typing, proof reading, and producing new test forms takes a great deal of time and effort. We hope to have the hardware and software available soon for computer generated tests.

Mastery Level

We have already discussed the mastery levels (87.5%) used in the CSBM program. The mastery levels are dependent on the goals of the course, the objectives of each unit, and the tests used. We feel our tests are not difficult in the sense of containing hard questions but they are complete in the sense that every objective is tested. If we have erred in choosing the mastery level, we have erred by using a mastery level that is too low. We have verified that, if tests and mastery levels are reasonable, students will raise (lower) their efforts to meet expectations.

MATERIAL RELATED VARIABLES

The text, lab, and audio-tutorial materials were independent and separately bound at first, and, as already discussed, were later integrated into a single text. The integrated materials had several advantages, the greatest advantage was that students could more easily keep track of one item than several separate ones. In the process of integrating the materials one type of laboratory exercise which was an effective learning tool was deleted. This type of laboratory exercise was based on the Piagetan concept of reversibility. After a student had demonstrated his ability to work a particular set of exercises he was then asked to make up his own set of exercises for another student or the lab instructor to work. In addition these students were asked to have the correct answers to their exercises and be able to explain how to work them to other students. The higher cognitive levels required to make up and explain how to do a set of exercises gave these students better understandings of mathematical concepts.

COURSE MANAGEMENT RELATED VARIABLES

Computer Management

In a remedial program involving a large number of students in which tests must be graded and evaluated, student feedback of results generated, analysis made, lists in various categories determined and printed, and records kept; computer management must be used. We developed management programs which gave us a high degree of flexibility thus allowing us to maximize and individualize learning by our students.

Number of Students

The CSBM program began with 132 students in the Fall Semester 1978. Certainly as the number of students increases it places an ever increasing strain on the ability of the program to maintain a quality education for all the students; particularly the individualized aspects of the program are more difficult to maintain. Because of this problem the department decided to limit CSBM enrollment to 1000 students a semester beginning with the Winter Semester 1979. Unfortunately this limitation on enrollment has already created a backlog of students wishing to enroll in the program; this backlog is of sufficient size that the limitation on enrollment will have to be removed. It is very possible that the enrollment in CSBM will exceed 2000 students a semester within the next few years.

When developing a program for remedial mathematics, it is natural to have in mind the relative size of that program; we had invisioned the relative size of this program to be 600 students a semester. As student enrollments reached 800-1000 students, we began to experience problems with some of our procedures. We became totally computer dependent and super sensitive to computer slow downs and breakdowns. It became obvious that we must be able to exercise more direct control over the computer management programs and the department purchased a micro-computer system to achieve the desired control.

Instructor Assignment

The staff for the program was composed of five different types of personnel; undergraduates, graduate assistants, instructors on part-time assignment, instructors on full-time assignment, and instructors assigned as counselors. We used two different groups of undergraduates in the CSBM program. Students enrolled in Secondary Mathematics Teaching Methods course, MTH 461, were required as part of that course to work one hour a week in the CSBM math-help lab. In this situation they learned how to manage a math lab and how to instruct remedial students individually or in small groups. Students who have completed MTH 461 may enroll in MTH 562, this is a practicum course where students attend seminars, observe professors teaching units in the program, and teach units under the direction of the professor assigned to MTH 562. Frequently CSBM students have commented that they feel these undergraduate instructors are the "best."

There are a few problems connected with the assignment of instructors; in addition to the usual types of problems related to personal preferences of the level of unit to be taught, there was one unexpected statistic. In general, if an instructor was assigned to teach the same unit consecutively, he did significantly poorer as determined by the percent of students who passed that unit.

Permanent Files

Which records to create and how to store them is mostly dependent on the particular situation and the facilities available. However, students do return to programs after missing several semesters, and it is very helpful to be able to look up previous sequences and the success of these students. At the present time we are fortunate to have the flexibility to have all of our records in both printed form and computer storage.

AFFECTIVE DOMAIN RELATED VARIABLES

Student Attitudes

There is an adage that states: "You can lead a horse to water, but you cannot make him drink." A paraphrase of this statement concerning remedial students and learning mathematics is certainly true. In the CSBM program where we have made an effort to individualize the program, to require a reasonable level of mastery, and to provide several avenues for student grievances; we feel that beyond this point if a college student does not make an honest effort to learn, he is wasting both of our times and money. We have asked the program counselors to be forthright in advising such students to drop the program. Most of the students in the program have or adopt the attitude that this is their last chance and their weaknesses in math are standing in the way of future success in their chosen fields. Because of this attitude they make an honest effort to learn mathematics albeit sometimes the lowest priority when compared to other subjects.

One interesting facet of student attitude developed because the CSBM was a computer managed program. This attitude allowed instructors to play the game "us against the computer." That is, since all tests are computer graded and evaluated and since student feedback is a computer generated card, students felt free to enlist the help of instructors to "beat the computer." This team attitude was refreshing in light of the fact that remedial students usually adopt the attitude "us against the instructor."

Instructor Attitudes

Most directors of programs agree that there is little doubt that the most influential variable concerning student success is the instructors. The best program with poor instructors will not accomplish as much as the poorest program with good instructors. The CSBM program was fortunate in having a large number of excellent instructors; however, we also had a few instructors, for whatever the reason, who preferred not to teach remedial courses and told this to their students. Because students had a different instructor for each two-week unit, the rapport between students and directors or counselors was stronger than that between students and instructors. As a result, if an instructor made a derogatory remark to students, directors or counselors were so advised by justifiably upset students. It is difficult to understand why some instructors do not realize the tremendous effect their personal attitude toward their students has upon the success of those students learning the material they are teaching. In general, the math department and the CSBM directors feel that we have an ethical obligation to teach all students who are admitted to the university and further we have the responsibility to provide a program that will enhance the learning opportunities of these students in mathematics. The cognitive domain is inseparable from the affective domain; students are most successful in a congenial environment.

Counselor's Attitudes

Since the main function of the counselors in the program was to listen to student grievances and advise students having special difficulties with mathematics, attitudes of counselors toward remedial students was critical. It must be noted here that all of the counselors in the program deserve a standing ovation for their resolution of the trials and tribulations they experienced as a counselor in the program.

Concluding Remark

Spock (1979), a friendly adversary of current educational practices, stated: "American schools are failing to educate because they put too much emphasis on memory, do not stress mastery, and are over responding to the 'Back-to-Basic' movement." Our seven year developmental study results indicate that while emphasizing memory pays off in the short-term, it leaves us bankrupt in the long-term.

REFERENCES

1. Anderson, Loren W. "Adaptive Education." Educational Leadership 37 (November 1979): 140-143.

2. Bidwell, James K. "A Tutoring Program for Prospective Secondary Mathematics Teachers." The Mathematics Teacher 64 (December 1971): 721-724.

3. Biggs, Edith E. and James R. MacLean. Freedom to Learn M. Poage, editor. Don Mills, Ontario: Addison-Wesley (Canada) Ltd., 1969.

4. Block, J. "A Description and Comparison of Bloom's Learning for Mastery Strategy and Keller's Personalized System of Instruction." In: J. Block, editor. Schools, Society and Mastery Learning. New York: Holt, Rinehart and Winston, 1974.

5. Block, James H. "Mastery Learning: The Current State of the Craft." Educational Leadership 37 (November 1979): 114-117.

6. Block, J., and R. Burns. "Mastery Learning." In: L. Shulman, editor. Review of Research in Education, Vol. 4. Itasca, Illinois: F.E. Peacock, 1977.

7. Bloom, B.S. (editor), M.D. Engelhart, E.J. Furst, W.H. Hill, and D.R. Krathwohl. Taxonomy of Educational Objectives. Handbook I: Cognitive Domain. New York: McKay, 1956.

8. Bloom, B.S. "Learning for Mastery." UCLA-CSEIP Evaluation Comment 1: 1968.

9. Bloom, B.S. "An Introduction to Mastery Learning." In: J.H. Block, editor. Schools, Society and Mastery Learning. New York: Holt, Rinehart and Winston, 1974.

10. Brandt, Ron. "A Conversation with Benjamin Bloom." Educational Leadership 37 (November 1979): 157-161.

11. Byrnes, Barbara B. A Review of the Literature on Item-Arrangement in Achievement Testing. Masters Paper, Central Michigan University, 1975.

12. Cronback, Lee J. "How Can Instruction be Adapted to Individual Differences." In: R. Gagne, editor, Learning and Individual Differences. Columbus, Ohio: Charles E. Merril, 1967.

13. Doggett, Maran. "Aiding the Seriously Deficient Learner in Computation." The Mathematics Teacher 71 (September 1978): 488-493.

14. Earle, Richard A. Teaching Reading and Mathematics. IRA reading aid series. Newark, Delaware: International Reading Association, 1976.

15. Fey, James T. "Mathematics Teaching Today: Perspectives From Three National Surveys." The Mathematics Teacher 72 (October 1979): 490-504.

16. Gagne, Robert. Conditions of Learning (2nd Ed.). New York: Holt, Rinehart and Winston, 1970.

17. Hoetker, J. and W. Ahlbrand. "The Persistence of the Recitation." American Educational Research Journal 7 (1969): 163.

18. Keller, F. "Goodbye, Teacher . . . ." Journal of Applied Behavior Analysis 1 (1968): 79-89.

19. Kulm, Gerald. "What Makes Math Books Hard to Read?" Paper presented at the MCTM Conference, October 9, 1976, Lansing, Michigan.

20. Laing, Robert A. and Peterson, John C. "Assignments: Yesterday, Today, and Tomorrow--Today." Mathematics Teacher 66 (October 1973): 508-518.

21. Phillips, Jo. "Reading Mathematical Content." Paper presented at the NCTM Name-of-Site Conference, October 12, 1979, Cleveland, Ohio.

22. Postlethwait, S.N., Novak, J., and Murray, Jr. H.T. The Audio-Tutorial Approach to Learning (2nd Ed.). Minneapolis, Minn.: Burgess, 1969.

23. Rising, Gerald R. "How Pure Should We Be?" Journal for Research in Mathematics Education 9 (November 1978): 379-383.

24. Rising, Gerald. "Should We Integrate High School Mathematics." Paper presented at NCTM Name-of-Site Conference, October 1[illegible], 1979, Cleveland, Ohio.

25. Schoen, Harold L. "A Plan to Combine Individualized Instruction with the Lecture Method." The Mathematics Teacher 67 (November 1974): 647-649.

26. Smith, Lyle R. "Degrees of Teacher Vagueness and Pupil Participation as They Relate to Student Learning of Mathematical Concepts and Generalizations." In: J.L. Higgins, editor, Research Reporting Sections National Council of Teachers of Mathematics 56th Annual Meeting. Columbus, Ohio: ERIC Center for Science, Mathematics and Environmental Education, April 1978.

27. Spock, B. "American Schools are Failing to Educate." Paper presented to American Principals Association, July 3, 1979, Chicago, Illinois.

28. Suydam, Marilyn M. Evaluation in the Mathematics Classroom. Columbus, Ohio: ERIC Science, Mathematics and Environmental Education Clearinghouse, January 1974.

29. Watkins, Ann E. "The Symbols and Grammatical Structures of Mathematical English and the Reading Comprehension of College Students." Journal for Research in Mathematical Education 10 (May 1979): 216-218.

30. West, Tommie A. "Rx for Verbal Problems: A Diagnostic-Prescriptive Approach." Arithmetic Teacher 25 (November 1977): 57-58.

APPENDIX I

COURSE: I. COMPETENCY ARITHMETIC

UNIT 1: OPERATIONS WITH WHOLE NUMBERS

ADDITION

OBJECTIVES

When you have satisfactorily completed this unit, you should be able to:

A1. Label the parts of an addition problem.
A2. Add two whole numbers using a number line.
A3. Identify and use the basic properties of addition.
A4. Identify and use the special property of zero.
A5. Complete a table of basic addition facts.
A6. Add two or more numbers written in expanded notation.
A7. Add two or more whole numbers.

SUBTRACTION

B1. Label the parts of a subtraction problem.
B2. Relate subtraction problems to addition problems.
B3. Complete a table of basic subtraction facts.
B4. Determine the difference of two whole numbers.
B5. Determine the difference of two whole numbers involving borrowing (regrouping).

MULTIPLICATION OF WHOLE NUMBERS

C1. Label the parts of a multiplication problem.
C2. Interpret multiplication as repeated addition or as a Cartesian product.
C3. Identify and use the basic properties of multiplication.
C4. Identify and use the special property of 1.
C5. Complete the table of basic multiplication facts.
C6. Multiply using expanded notation.
C7. Multiply whole numbers.
C8. Identify Squares and Square Roots.

DIVISION OF WHOLE NUMBERS

D1. Recognize the three notations used for division.
D2. Label the parts of a division problem.
D3. Relate a division problem to a multiplication problem.
D4. Recognize that division by zero is not defined.
D5. Complete the table of the basic division facts.
D6. Identify and use the procedure for division.
D7. Determine the quotient of two whole numbers.

SOLVING WORD PROBLEMS

E1. Translate mathematical statements to English statements.
E2. Select an appropriate variable to represent the unknown part of an English statement.
E3. Select the appropriate mathematical operation for an English statement.
E4. Select the appropriate mathematical relation for an English statement.
E5. Translate English statements to mathematical statements.

UNIT 2. OPERATIONS WITH DECIMALS

INTRODUCTION TO DECIMALS

OBJECTIVES

When you have satisfactorily completed this unit, you should be able to:

A1. Identify the place value of a digit in a decimal fraction.
A2. Compare and order two decimal fractions.
A3. Round or chop a given decimal fraction to a given place value.
A4. Identify or list a decimal fraction between two given decimal fractions.

ADDITION AND SUBTRACTION OF DECIMALS

B1. Identify "like" terms in decimals.
B2. Represent addition of decimals on a number line.
B3. State and use the procedure for adding two decimals.
B4. State and use the procedure for subtracting two decimals.
B5. Round a sum or difference.

MULTIPLICATION OF DECIMALS

C1. Represent a multiplication problem by an array.
C2. State and use the procedure for multiplying two decimals.
C3. Multiply a number by a power of 10.
C4. Represent a decimal in expanded notation.
C5. Round a product.

DIVISION OF DECIMALS

D1. Divide a decimal by a natural number.
D2. Identify equivalent division problems.
D3. State and identify the procedure for dividing two decimals.
D4. Determine the quotient when the division involves a non zero remainder.

UNIT 3: PRIMES, FACTORIZATIONS, LEAST COMMON MULTIPLES

FACTORS, DIVISORS, MULTIPLES, AND FACTORIZATIONS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify and state the definition of a factor.
A2. Identify and state the definition of a divisor.
A3. Identify and state the definition of a multiple.
A4. Use the divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10.
A5. Determine all the factors of a number.
A6. State the definition for or identify a factorization.

PRIMES, COMPOSITES, AND SPECIAL NUMBERS

B1. Identify and state the definition of a prime number.
B2. Identify and state the definition of a composite number.
B3. Identify and state the definition of a special number.
B4. Use the sieve of Eratosthenes to determine prime numbers.

FACTORIZATIONS

C1. Determine the prime factorization of a number.
C2. Determine all factorizations of a number.
C3. Identify pairs of factorizations that are essentially the same.
C4. Identify pairs of factorizations that are essentially different.

THE FUNDAMENTAL THEOREM OF ARITHMETIC

D1. State and identify the Fundamental Theorem of Arithmetic.
D2. Identify the two characteristics of the Fundamental Theorem of Arithmetic.
D3. Determine if two numbers are equal by examining their prime factorizations.
D4. Determine the product of two numbers given their prime factorizations.
D5. Determine the quotient of two numbers given their prime factorizations.

LEAST COMMON MULTIPLES

E1. Determine the set of common multiples of two numbers.
E2. Identify and state the definition of the least common multiple of two numbers.
E3. Identify the two characteristics of the least common multiple of two numbers.
E4. Determine the least common multiple of two numbers.

UNIT 4. OPERATIONS WITH FRACTIONS

INTRODUCTION TO FRACTIONS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify the number, the division problem and the ratio represented by a given fraction.
A2. Identify the numerator and denominator of a given fraction.
A3. State the reason a denominator of a fraction cannot be zero.
A4. State the definition of a fraction.
A5. Shade in a fractional part of a geometric figure.
A6. Represent whole numbers as fractions.

MULTIPLICATION AND DIVISION OF FRACTIONS

B1. Shade a geometric figure to illustrate the product of two fractions.
B2. Multiply two fractions.
B3. Divide two fractions.

EQUIVALENT FRACTIONS

C1. Use a fraction to represent 1.
C2. State the definition of equivalent fractions.
C3. Expand a fraction to a fraction with a given denominator.
C4. Reduce a fraction to its simplest form.
C5. Determine whether or not two fractions are equivalent.

MULTIPLICATION AND DIVISION OF FRACTIONS INVOLVING CANCELLATION

D1. Change division problems to multiplication problems.
D2. Determine the reciprocal of a given fraction.
D3. Use cancellation in computations.
D4. Identify the operations in word problems.
D5. Simplify complex fractions.

ADDITION AND SUBTRACTION OF FRACTIONS

E1. Add two fractions with common denominators.
E2. Add fractions with different denominators.
E3. Determine the lowest common denominator (LCD) of two or more fractions.
E4. Subtract fractions.
E5. Change mixed numerals to common fractions.

COMPARING FRACTIONS

F1. Compare two fractions with common denominators.
F2. Compare two fractions with different denominators.

UNIT 5. OPERATIONS INVOLVING PERCENTS

INTRODUCTION TO PERCENT

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Represent a percent by a geometric figure.
A2. Change fractions to decimals.
A3. Change decimals to fractions.
A4. Change fractions to percents.
A5. Change percents to fractions.
A6. Change decimals to percents.
A7. Change percents to decimals.

MIXED NOTATION

B1. Change decimals and fractions into percents that involve decimals.
B2. Change percents that involve decimals to fractions and decimals.
B3. Change decimals and fractions to percents that involve fractions.
B4. Change percents that involve fractions to fractions and decimals.
B5. Determine equivalent notations for families of percents.
B6. Determine equivalent notations for percents less than 1%.
B7. Determine equivalent notations for percents greater than 100%.

OPERATIONS INVOLVING PERCENTS

C1. Determine the sum (difference) of two percents.
C2. Determine the product of a number and a percent.
C3. Determine the quotient of a number divided by a percent.
C4. Relate division and multiplication problems involving percents.

SOLVING PROBLEMS INVOLVING PERCENTS

D1. Identify a rate.
D2. Solve rate problems.
D3. Determine the percentage in a basic rate problem.
D4. Determine the rate in a basic rate problem.
D5. Determine the base in a basic rate problem.

UNIT 6. GEOMETRY

POINT, LINE AND PLANE

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify and use point and line.

A2. Identify and use line segment and its measure.
A3. Identify and use a ray.
A4. Identify and use plane and space.

ANGLES AND LINES

B1. Identify and use angles.
B2. Identify the measure of an angle.
B3. Use angle measurement to classify right, acute, obtuse, complementary and supplementary angles.
B4. Identify and use the terms intersecting lines, vertical angles, linear pairs of angles, and perpendicular lines.
B5. Identify and use the terms parallel lines, transversal, and alternate interior angles.

TRIANGLES

C1. Identify and classify curves.
C2. Identify a triangle and its parts.
C3. Classify triangles by their angle measures.
C4. Classify triangles by the measures of their sides.
C5. Identify and use the Pythagorean Theorem.

POLYGONS AND CIRCLES

D1. Identify quadrilaterals, parallelograms, rectangles and squares.
D2. Identify polygons and their diagonals and altitudes.
D3. Identify a regular polygon.
D4. Identify circles and their diameters and radii.

CONGRUENT AND SIMILAR TRIANGLES

E1. Identify congruent line segments and congruent angles.
E2. Identify congruent triangles.
E3. Identify and use SSS, SAS, and ASA.
E4. Identify congruent polygons.
E5. Identify similar triangles and use AAA and proportional sides.

UNIT 7. METRIC MEASUREMENT

MEASUREMENT PROCESS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Understand and classify standard and non-standard units.
A2. Understand and use the metric system of measuring units.
A3. Understand the use of estimation in measurement.

A4. Understand and use the terms accuracy and precision.
A5. Understand the measurement process.

LENGTH

B1. Identify and use the metric units of length.
B2. Determine the perimeter of geometric polygons.
B3. Determine the radius, diameter and circumference of circles.

AREA

C1. Identify and use the metric units for area.
C2. Determine the area of parallelograms.
C3. Determine the area of triangles and other polygons.
C4. Determine the area of circles.

VOLUME

D1. Use and understand metric volume measures.
D2. Determine the volumes of right prisms.
D3. Determine the volumes of right pyramids and cones.
D4. Determine the volumes of spheres.

CAPACITY, TEMPERATURE, AND WEIGHT

E1. Identify and use metric units of capacity.
E2. Convert capacity units to volume and vice-versa.
E3. Identify and use metric units of temperature.
E4. Identify and use the units of metric weight.

COURSE II. BEGINNING ALGEBRA

UNIT 8. INTEGERS AND OPERATIONS WITH INTEGERS

INTRODUCTION TO INTEGERS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify or state the definition of a negative integer.
A2. Construct an integer number line.
A3. Determine the opposite of any integer.
A4. Determine the correct order relationship between two integers.

ADDITION OF INTEGERS

B1. Illustrate addition of integers on a number line.
B2. Determine the sum of a number and its opposite.
B3. Identify the special property of zero.
B4. Determine the sum of two integers.

SUBTRACTION OF INTEGERS

C1. Identify the parts of a subtraction problem.
C2. Change a subtraction problem to an equivalent addition problem.
C3. Determine the difference of two integers.
C4. Determine the difference of two opposites.
C5. Determine the sum of more than two integers.

MULTIPLICATION OF INTEGERS

D1. Demonstrate a knowledge of multiplication as repeated addition.
D2. Determine the product of a natural number times a negative integer.
D3. Represent the opposite of any integer as negative 1 times that integer.
D4. Determine the product of two negative integers.
D5. Represent the opposite of any expression as negative 1 times that expression.

DIVISION OF INTEGERS

E1. Change a division problem to a related multiplication problem.
E2. Determine the correct sign for the quotient of two integers.
E3. Determine the correct quotient of two integers.

ABSOLUTE VALUE

F1. Determine the absolute value of a given number.
F2. Understand and use the definition of distance.

UNIT 9. BASIC NUMBER THEORY

PRIMES AND COMPOSITES

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Demonstrate an understanding of the terms: factor, factorization, and divisor.
A2. Determine all factors of an integer.
A3. Identify prime integers.
A4. Identify composite integers.

FUNDAMENTAL THEOREM OF ARITHMETIC

B1. Identify prime factorizations.
B2. Identify pairs of factorizations that are essentially the same.
B3. Identify pairs of factorizations that are essentially different.
B4. State the Fundamental Theorem of Arithmetic.
B5. Demonstrate an understanding of the Fundamental Theorem of Arithmetic.
B6. Determine the prime factorization of any integer.

EXPONENTIAL NOTATION

C1. Write repeated factors in expanded notation.
C2. Identify squares and cubes in expanded notation.
C3. Use exponential notation to write the prime factorization of an integer.

COMPARING INTEGERS

D1. Determine if two integers are equal by examining their prime factorizations.
D2. Determine the product of two integers when given their prime factorizations.
D3. Determine the quotient of two integers when given their prime factorizations.

LEAST COMMON MULTIPLE

E1. Determine the set of multiples of an integer.
E2. Determine the set of common multiples of two or more integers.
E3. State two main characteristics of the LCM of a set of integers.
E4. Find the LCM of two integers.
E5. Find the LCM of three or more integers.

UNIT 10. RATIONAL NUMBERS

INTRODUCTION TO RATIONAL NUMBERS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify an integer.
A2. Mark and identify integers on a number line.
A3. Mark and identify rational numbers on a number line.
A4. Identify a rational number.

MULTIPLYING RATIONAL NUMBERS

B1. State and use the procedure for multiplying two rational numbers.

MULTIPLYING RATIONAL NUMBERS

B2. Determine the correct sign for the product of two integers.
B3. Expand any fraction to a fraction with a given denominator.
B4. Reduce a fraction to simplest terms.
B5. Use cancellation to determine the product of two or more rational numbers.

DIVISION OF RATIONAL NUMBERS

C1. Determine the reciprocal of any given fraction.
C2. Determine the sign of the quotient of two integers.
C3. Determine the quotient of two rational numbers by changing a division problem to a multiplication problem.
C4. Simplify complex fractions.

THE THREE SIGNS OF A FRACTION

D1. Determine the opposite of an integer and the opposite of a fraction.
D2. Identify the three signs of a fraction.
D3. Use the three signs of a fraction to write four equivalent forms of a fraction.
D4. Use the three signs of a fraction to write four equivalent forms of the opposite of a fraction.

ADDITION AND SUBTRACTION OF RATIONAL NUMBERS

E1. Determine the sum of two rational numbers with the same denominator.
E2. Determine the LCD of two or more fractions.
E3. Determine the sum of two or more rational numbers.
E4. Determine the difference of two rational numbers by changing a subtraction problem to an addition problem.
E5. Determine the correct order relationship between two rational numbers.

UNIT 11. INTRODUCTION TO ALGEBRA

EVALUATION OF ALGEBRAIC STATEMENTS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Use the convention for the order of operations.
A2. Use the conventions for grouping symbols.
A3. Evaluate algebraic statements and formulas.

MODELS IN MATHEMATICS

B1. State the purpose of a model.
B2. Relate algebra to a game model.

B3. Relate algebra to a language model.
B4. State the undefined terms for algebra.
B5. Define subtraction, division, less than, and greater than.

THE BASIC AXIOMS OF ALGEBRA

C1. State and use the substitution axiom.
C2. State and use the uniqueness of sums axioms.
C3. State and use the uniqueness of products axiom.
C4. State and use the commutative axioms of addition and multiplication.
C5. State and use the associative axioms of addition and multiplication.

THE SOMETIMES, ALWAYS, NEVER IN ALGEBRA

D1. Understand and use always and never in algebra.
D2. Develop a counterexample.
D3. Understand and use sometimes in algebra.
D4. Change a sometimes statement to an always or never statement.
D5. Identify a well formed formula.

UNIT 12. INTEGRAL EXPONENTS

POSITIVE EXPONENTS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Write a product of the form $x \cdot x \cdot x \cdot \cdots \cdot x$ in exponential form and write an exponential expression as a product of the form $x \cdot x \cdot x \cdot \cdots \cdot x$.
A2. Identify the parts of an exponential expression.
A3. Evaluate exponential expressions of the form $+n^a$.
A4. Evaluate exponential expressions of the forms $(-n)^a$ and $-n^a$.
A5. Use exponential notation to write the prime factorization of an integer.

ZERO AND NEGATIVE EXPONENTS

B1. Evaluate any number or expression raised to the zero power as 1.
B2. Write a product of the form $\frac{y}{x} \cdot \frac{y}{x} \cdot \cdots \cdot \frac{y}{x}$ in exponential form with negative exponents and write an exponential with a negative exponent as a product of the form $\frac{y}{x} \cdot \frac{y}{x} \cdot \cdots \cdot \frac{y}{x}$.
B3. Change an exponential with a negative exponent to an expression with a positive exponent.
B4. Evaluate exponential expressions of the form $+n^{-a}$.
B5. Evaluate exponential expressions of the forms $(-n)^{-a}$ and $-n^{-a}$.

EXPONENT LAWS FOR MULTIPLYING AND DIVIDING

C1. Use the law $x^m x^n = x^{m+n}$ in appropriate multiplication problems.
C2. Form reciprocals of exponentials by changing exponents.
C3. Change division problems to equivalent multiplication problems by changing exponents.
C4. Use the law $x^m \div x^n = x^{m-n}$ in appropriate division problems.
C5. Change an exponential with rational base and negative exponent to an exponential with a positive exponent.

EXPONENT LAWS FOR RAISING TO A POWER

D1. Correctly use the law $(x^m)^n = x^{mn}$.
D2. Correctly use the law $(xy)^m = x^m y^m$.
D3. Correctly use the law $(x^m y^n)^k = x^{mk} y^{nk}$.
D4. Correctly use the law $(\frac{x}{y})^m = \frac{x^m}{y^m}$.
D5. Correctly use the law $(\frac{x^m}{y^n})^k = \frac{x^{mk}}{y^{nk}}$.

UNIT 13. POLYNOMIALS

INTRODUCTION TO POLYNOMIALS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify a term.
A2. Identify the numerical coefficient of a term.
A3. Recognize a polynomial.
A4. Recognize a monomial, binomial or trinomial.
A5. Write a polynomial in descending order.
A6. Identify the degree of a polynomial.
A7. Evaluate a polynomial.

ADDITION OF POLYNOMIALS

B1. Identify like terms.
B2. Determine the sum of two polynomials in horizontal form and in vertical form.

SUBTRACTION OF POLYNOMIALS

C1. Identify the notation for the opposite of a polynomial.
C2. Compute the opposite of a polynomial.
C3. Compute the difference of two polynomials.

MULTIPLICATION OF POLYNOMIALS

D1. Determine the product of two monomials.
D2. Determine the product of a monomial and a polynomial.
D3. Determine the product of two polynomials in horizontal form.
D4. Determine the product of two polynomials in vertical form.

DIVISION OF POLYNOMIALS

E1. Determine the quotient of two monomials.
E2. Determine if one monomial is divisible by another.
E3. Determine the quotient of a polynomial divided by a monomial.
E4. Determine the quotient of two polynomials.
E5. Determine if one polynomial is divisible by another.
E6. Determine the quotient of two polynomials with more than three terms.

COMMON MONOMIAL FACTORS

F1. Identify a common monomial factor of a polynomial.
F2. Identify the greatest common monomial factor of a polynomial.
F3. Write the common monomial factor and remaining polynomial in factored form.

UNIT 14. LINEAR EQUATIONS AND INEQUALITIES

MEANINGFUL LINEAR STATEMENTS AND SOLUTION SETS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify meaningful (well-formed) statements.
A2. State the definition of and identify open statements.
A3. State the definition of solution set.
A4. Identify simplest statements.
A5. State three conditions necessary for a statement to be linear.
A6. Identify linear statements.

EQUIVALENT STATEMENTS

B1. State the definition of equivalent statements.
B2. State and use two ways of generating equivalent equations.
B3. State and use three ways of generating equivalent inequalities.

PROCEDURE FOR SOLVING LINEAR EQUATIONS

C1. Solve equations of the form $x + b = d$.
C2. Identify the numerical coefficient of a term.
C3. Solve equations of the form $ax + b = d$.
C4. Solve equations of the form $ax + b = cx + d$.

PROCEDURE FOR SOLVING LINEAR INEQUALITIES

D1. Solve inequalities of the form $ax < d$ (a is positive).
D2. Solve inequalities of the form $-ax < d$ ($-a$ is negative).
D3. Solve inequalities of the forms $ax + b < d$ and $ax + b < cx + d$.
D4. Sketch the graph of solution sets of linear statements.

SOLVING LITERAL EQUATIONS

E1. Evaluate variables in literal equations or formulas.
E2. Solve for a given variable in a literal equation.
E3. Solve for a given variable in a literal equation when that variable is contained in more than one term.

RATIO AND PROPORTION

F1. Identify a ratio.
F2. Identify a proportion.
F3. Solve proportions.
F4. Solve proportions involving integers.

COURSE III. INTERMEDIATE ALGEBRA

UNIT 15. RADICALS

INTRODUCTION TO RADICALS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify the four types of decimals.
A2. Write and use the definition of square root.
A3. Use a table to determine and write a rational approximation to an irrational number.
A4. Recognize and use the terminology and notation for irrational roots.
A5. Recognize and use negative square roots.
A6. Understand the existence of irrational numbers as determined (historically) by line segments.

A7. Determine the square root of an exponential expression.
A8. Recognize that square roots of negative numbers are not defined in the real number system.

MULTIPLICATION OF RADICALS

B1. Determine the product of two radicals.
B2. Determine the quotient of two radicals.
B3. Identify and simplify those radicals that can be simplified.
B4. Determine the product or quotient of two or more radicals and simplify the results.

ADDITION OF RADICALS

C1. Identify like, or common radicals and determine their sum (difference).
C2. Identify and write opposites of radical expressions.
C3. Determine the sum and difference of two or more radicals.
C4. Simplify and then determine the sum or difference of two or more radicals.
C5. Identify and use the radical sign as a grouping symbol.

FRACTIONS INVOLVING RADICALS

D1. Rationalize denominators containing one term.
D2. Determine the binomial needed to rationalize a given binomial.
D3. Rationalize denominators containing two terms.
D4. Rationalize numerators containing one term.
D5. Rationalize numerators containing two terms.

EQUATIONS THAT INVOLVE RADICALS

E1. Remove radicals by squaring and solve the resulting linear equations.
E2. Identify a quadratic equation.
E3. Solve equations of the form $x^2 = a^2$.

UNIT 16. FACTORIZATION OF QUADRATIC POLYNOMIALS

PRIME AND COMPOSITE POLYNOMIALS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify and label polynomial factors.
A2. Identify divisors and factorizations of polynomials.
A3. Determine if two polynomial factorizations are essentially the same.
A4. Write the definition of prime polynomials.
A5. Determine the prime factorization of a first-degree polynomial.

A6. Determine the prime factorization of a polynomial containing a common monomial factor.
A7. Identify a, b, and c in the general quadratic polynomial $ax^2 + bx + c$.

SPECIAL PRODUCTS

B1. Identify and write the prime factorization of the difference of two squares.
B2. Identify the sum of two squares as a prime polynomial.
B3. Identify and write the prime factorization of trinomials that are binomial squares.

FACTORING TRINOMIALS WITH LEADING COEFFICIENT EQUAL TO 1

C1. Determine the signs and the terms of the binomial factors of a trinomial.
C2. Determine and write the prime factorization of a trinomial with a positive last term.
C3. Determine and write the prime factorization of a trinomial with a negative last term.
C4. Recognize that a factorable trinomial has exactly two binomial factors.

FACTORING TRINOMIALS WITH LEADING COEFFICIENT GREATER THAN 1

D1. Determine the signs in the binomial factors of a general quadratic trinomial of the form $ax^2 + bx + c$.
D2. Determine the cross products of the binomial factors of a trinomial.
D3. Determine the prime factorization of a general quadratic trinomial.

COMBINATIONS OF FACTORING TECHNIQUES

E1. Determine and write the prime factorization of the difference of two squares that contain a common factor.
E2. Determine and write the prime factorization of a general quadratic polynomial which contains a common factor.
E3. Determine and write the prime factorization of polynomials that are products of sums and differences of two squares.
E4. Determine and write the prime factorization of polynomials that contain both monomial and binomial common factors.

UNIT 17. SOLVING QUADRATIC EQUATIONS

FACTORS AND ZEROS OF POLYNOMIALS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify the zeros of a quadratic trinomial.

A2. ~~State the theorem concerning a zero product.~~
A3. State the relationship between factors and zeros of a polynomial.
A4. Write the two linear equations associated with a factorable quadratic trinomial.
A5. Solve a quadratic equation that contains a factorable trinomial.

COMPLETING THE SQUARE

B1. Determine if a given trinomial is a binomial square.
B2. Solve a binomial square quadratic equation by extracting square roots.
B3. Use completing the square method to solve quadratic equations with coefficient of x^2 equal to 1.
B4. Use completing the square method to solve quadratic equations with coefficient of x^2 greater than 1.

A FORMULA FOR SOLVING QUADRATIC EQUATIONS

C1. Fill in missing steps in the development of the quadratic formula.
C2. Use the quadratic formula to solve quadratic equations.
C3. Determine factors of a trinomial by using the quadratic formula.
C4. Use the quardratic formula to determine a factorization of a trinomial.
C5. Determine the discriminant of a given quadratic trinomial.
C6. Use the discriminant to determine if a quadratic equation has two, one, or no real solutions.

ANOTHER LOOK AT PRIME TRINOMIALS

D1. Recognize the need for restrictions when discussing prime numbers or prime polynomials.
D2. Identify a prime trinomial when the restrictions require the coefficients of the factors to be integers.
D3. Identify a prime trinomial when the restrictions require the coefficients of the factors to be rational numbers.
D4. Identify a prime trinomial when the restrictions require the coefficients of the factors to be real numbers.
D5. State and use the most commonly used definition of prime trinomial.
D6. Use the discriminant of a trinomial to determine if it is prime or composite under the most commonly used definition of prime trinomial.

UNIT 18. RATIONAL EXPONENTS

INTEGRAL EXPONENTS REVISTED

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Use the definitions of integral exponents to change an exponential with a negative exponent to an exponential with a positive exponent.

A2. Evaluate exponential expressions of the form ~~$+a^{+n}$ and $(+a)^{+n}$~~.
A3. Use the laws of exponents to compute exponential multiplication and division problems.

COMPUTING EXPONENTIAL PROBLEMS

B1. Correctly use the law $(x^m y^n)^k = x^{mk} y^{nk}$.
B2. Correctly use the law $\left(\frac{x^m}{x^n}\right)^k = \frac{x^{mk}}{y^{nk}}$.
B3. Simplify exponentials of the form $\left(\frac{x^a y^b z^c}{x^d y^e z^f}\right)^k$.
B4. Add and subtract certain exponential expressions.

RATIONAL EXPONENTS

C1. Define a rational exponent.
C2. Change exponentials to radicals and change radicals to exponentials.
C3. Correctly use the laws of exponents in problems involving rational exponents.
C4. Determine quotients of exponentials with rational exponents.

OPERATIONS INVOLVING EXPONENTIALS WITH RATIONAL EXPONENTS

D1. Multiply a sum of exponentials by an exponential.
D2. Multiply a sum of exponentials by a sum of exponentials.
D3. Factor an exponential with a rational exponent from a sum of exponentials.

UNIT 19. RATIONAL EXPRESSIONS

INTRODUCTION TO RATIONAL EXPRESSIONS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Identify the domain of a meaningful rational expression.
A2. Identify rational expressions that equal 1.
A3. Multiply two rational expressions.
A4. Expand a rational expression.
A5. Reduce a rational expression.
A6. Identify two equivalent rational expressions.

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

B1. Multiply two rational expressions involving canceling.

B2. Determine the reciprocal of a rational expression.
B3. Divide two rational expressions.
B4. Simplify a complex fraction involving rational expressions.

THE THREE SIGNS OF A RATIONAL EXPRESSION

C1. Use three signs of a fraction to write four equivalent forms of a rational expression.
C2. Identify equivalent expressions involving three signs of a fraction.
C3. Write a rational expression in standard form.
C4. Determine the product of multiplication problems involving the three signs of a fraction.

ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS

D1. Add two rational expressions with the same denominators.
D2. Write four forms of the opposite of a rational expression.
D3. Change subtraction problems to equivalent addition problems.
D4. Determine the LCD of two rational expressions.
D5. Add or subtract two rational expressions with different denominators.

UNIT 20. SOLVING DIFFERENT TYPES OF EQUATIONS

THREE SIGNS OF A QUOTIENT

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Use three signs of a fraction to write four equivalent forms of a rational expression and four equivalent forms of the opposite of a rational expression.
A2. Use three signs of a product to write four equivalent forms of a product of integers and four equivalent forms of the opposite of a product of integers.
A3. Use three signs of a product to write four equivalent forms of a product of binomials and four equivalent forms of the opposite of a product of binomials.
A4. Use many signs of a product or quotient to write equivalent forms of that product or quotient.
A5. Use many signs of product or quotient to reduce algebraic expressions and simplify equations.

EQUATIONS INVOLVING RATIONAL EXPRESSIONS

B1. Generate a linear equation by multiplying both sides of the original equation by the LCD.

B2. Determine the solution of an equation involving rational numbers.
B3. Determine the solution of an equation involving rational expressions.
B4. Determine the solution of an equation involving rational expressions when the generated equation is quadratic.

SOLVING EQUATIONS THAT INVOLVE EXPONENTIALS WITH RATIONAL EXPONENTS

C1. Remove radicals or exponentials by squaring and solving the resulting equation.
C2. Make an appropriate substitution in an equation that is quadratic in form and then solve.

MORE EQUATIONS INVOLVING RATIONAL EXPONENTS

D1. Use factoring to solve equations which involve expressions with fractional or negative exponents.

UNIT 22. COMPLEX NUMBERS

INTRODUCTION TO COMPLEX NUMBERS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Recognize reasons for extending number systems.
A2. Write the definition of the complex number i.
A3. Recognize a + bi as the general form of a complex number.
A4. Recognize real and complex components of a complex number.
A5. State and use the definition of equality for complex numbers.

GRAPHING, ADDING, AND MULTIPLYING

B1. Recognize that it is impossible to define an order relation for the complex numbers.
B2. Associate complex numbers with points in the plane.
B3. Add and subtract complex numbers.
B4. Multiply complex numbers.
B5. Recognize that the closure, commutative, associative, and distributive axioms are true for addition and multiplication of complex numbers.

CONJUGATES, NORMS, AND DISTANCES

C1. Compute the conjugate of any complex number.
C2. Compute the norm of any complex number.
C3. State the distance formula.
C4. Compute the distance between any two points.
C5. Associate the norm of a complex number with its distance from the origin.

DIVISION AND FACTORING

D1. Divide a complex number by a real number.
D2. Compute the reciprocal of a complex number.
D3. Convert a division problem to an equivalent multiplication problem.
D4. Compute the quotient of two complex numbers.
D5. Factor the sum of two squares.

EQUATIONS INVOLVING COMPLEX NUMBERS

E1. Solve quadratic equations with real coefficients and complex solutions.
E2. Recognize that every quadratic equation with real coefficients may be solved with the quadratic formula.
E3. Solve equations of the form x + a = b where a and b are complex numbers.
E4. Solve equations of the form ax + c = b where a, b, and c are complex numbers.
E5. Solve equations of the form ax + b = cx + d where a, b, c, and d are complex numbers.

UNIT 23: FUNCTIONS AND LINEAR FUNCTIONS

INTRODUCTION TO FUNCTIONS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. State the definition of function.
A2. Realize that every domain element is used and each domain element is associated with exactly one range element.
A3. State the domain and range of RECP, OPP, CONJ, ID, SUB, DIV, ADD, and SQRT.
A4. State both the verbal statement and the equation form of the rule for RECP, OPP, CONJ, ID, SUB, DIV, ADD, and SQRT.
A5. State the three important parts of functional notation.
A6. When given the rule, compute range elements associated with a specific domain element.
A7. State the definition of a constant function and recognize constant functions when given the rule.

ZEROS OF FUNCTIONS--RECTANGULAR COORDINATE SYSTEM

B1. Determine the domain of a function when given its defining equation.
B2. State the definition of zero of a function.
B3. Determine the equation to be solved in order to find the zeros of a given function.
B4. Find all zeros of a given function.
B5. Plot points corresponding to given coordinates.
B6. Find coordinates of plotted points.

GRAPHING FUNCTIONS

C1. Plot points on the graph of a function.
C2. Translate statements of the form "(a,b) is on the graph of f" to statements of the form "f(a) = b" and conversely.
C3. Sketch the graph of ID, OPP, RECP, $CONST_0$, and $CONST_k$.
C4. Use vertical line test to determine if a given relation is a function.
C5. Determine domain element and corresponding range element when given a point on the graph of a function.

LINEAR FUNCTIONS

D1. Know the definition of slope of a line and compute slope when two points are given.
D2. Identify linear functions.
D3. Identify the slope, y-intercept, and x-intercept of a linear function.
D4. Sketch the graph of any linear function.
D5. Write the rule of a linear function with a given slope and y-intercept.
D6. Write the rule of a linear function passing through a given point with a given slope.
D7. Write the rule of a linear function passing through two given points.

UNIT 24. QUADRATIC FUNCTIONS

INTRODUCTION TO QUADRATIC FUNCTIONS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Recognize the defining rule of a quadratic function.
A2. Find the y-intercept of a quadratic function.
A3. Find the x-intercepts of a quadratic function.
A4. Find the zeros of a quadratic function.
A5. State and use the relation between zeros and x-intercepts of quadratic functions.
A6. State and use the relation between zeros and factors of quadratic functions.

GRAPHING QUADRATIC FUNCTIONS

B1. Recognize the graph of a quadratic function as a parabola.
B2. Determine from the leading coefficient whether the graph opens up or down.
B3. Write the equation for the line of symmetry.
B4. Determine the vertex.
B5. Rapidly sketch the graph of any quadratic function.

CONSTRUCTING QUADRATIC FUNCTIONS WHICH SATISFY GIVEN CONDITIONS

C1. Determine the quadratic function which has two given y-intercept.
C2. Determine the quadratic function which has two given x-intercepts and passes through a given third point.
C3. Write the discriminant of any quadratic function.
C4. Use the discriminant to determine the nature of the zeros and hence the number of x-intercepts of a quadratic function.
C5. Determine the quadratic function which has only one given x-intercept and a given y-intercept.
C6. Determine the quadratic function whose graph passes through three given points.

INTERSECTIONS OF GRAPHS

D1. Determine the number of points of intersection of two linear functions by examining their rules.
D2. Determine all points of intersection of the graph of two linear functions.
D3. Determine all points of intersection of the graphs of one linear function and one quadratic function.
D4. Determine all points of intersection of the graphs of two quadratic functions.
D5. Determine the associated "intersection finder" function for any two given linear or quadratic functions.
D6. State and use the relation between the zeros of the "intersection finder" function and points of intersection of two functions.

UNIT 25. INTERPRETING GRAPHS AND SOLVING INEQUALITIES

INTERPRETING GRAPHS

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. Understand the relation between the sign of the second coordinate of a point and its position relative to the x-axis.
A2. State the definition of "graph of a function."
A3. Understand the relation between the sign of f(x) and the position of the corresponding point on the graph of f, relative to the x-axis.
A4. State the definition of "zero of a function."
A5. Use the graph of f to determine which domain elements cause f(x) = 0.
A6. Use the graph of f to determine which domain elements cause f(x) > 0.
A7. Use the graph of f to determine which domain elements cause f(x) < 0.

INTERNAL NOTATION

B1. Correctly read and write the notation [a, b].

B2. Correctly read and write the notation (a, b).
B3. ~~Write the definition of open and closed intervals.~~
B4. Correctly read and write set-builder notation to describe intervals and rays.
B5. Correctly read and write the notation (a, b] and [a, b).
B6. Correctly read and write "interval" notation to describe rays.

LINEAR AND QUADRATIC INEQUALITIES

C1. Use the graphical method to solve linear inequalities of the forms $ax + b < 0$ and $ax + b > 0$.
C2. Use the graphical method to solve quadratic inequalities of the forms $ax^2 + bx + c < 0$ and $ax^2 + bx + c > 0$.
C3. Relate inequalities of the type $f(x) < g(x)$ with appropriate inequalities involving the intersection finder function.
C4. Use the graphical method to solve inequalities of the form $f(x) < g(x)$ where f and g are linear functions.
C5. Use the graphical method to solve inequalities of the form $f(x) < g(x)$ where f and g are quadratic functions.
C6. Use the graphical method to solve inequalities of the form $f(x) < g(x)$ where one of f and g is a linear function and the other is a quadratic function.

INEQUALITIES INVOLVING PRODUCTS AND QUOTIENTS OF LINEAR AND QUADRATIC FUNCTIONS

D1. Divide the real number line into rays and intervals as determined by the real zeros of a product of linear and quadratic functions.
D2. Solve inequalities involving the product of linear and quadratic functions.
D3. Divide the real number line into rays and intervals as determined by the real zeros and points where a quotient is undefined.
D4. Solve inequalities involving a quotient of linear and quadratic functions.
D5. Solve inequalities involving a quotient where both the numerator and denominator are products of linear and quadratic functions.

UNIT 26. THREE SPECIAL FUNCTIONS (ABS, SQRT, & DIST)

THE ABSOLUTE VALUE FUNCTION

OBJECTIVES

When you have satisfactorily completed this unit you should be able to:

A1. State the definition of ABS.
A2. For ABS, compute the range element corresponding to a given domain element.
A3. Sketch the graph of ABS.
A4. State and use ABS(xy) = ABS(x)ABS(y).

A5. Explain the proofs of $ABS(xy) = ABS(x)ABS(y)$ and $ABS(\frac{1}{y}) = \frac{1}{ABS(y)}$.

A6. State, use, and explain the proof of $ABS(\frac{x}{y}) = \frac{ABS(x)}{ABS(y)}$.

A7. Explain the proof of $ABS(x) = ABS(-x)$.

GRAPHING |F|

B1. Compute ABS(f(x)) when f is a known function.
B2. Read and write effectively with the || notation for ABS.
B3. Sketch the graph of $|mx + b|$.
B4. Sketch the graph of $|ax^2 + bx + c|$.
B5. Sketch the graph of $|f|$ when the graph of f is known.

SOLVING EQUATIONS INVOLVING ABSOLUTE VALUE OF LINEAR FUNCTIONS

C1. Solve equations of the form $|ax + b| = c$ and geometrically interpret the solution.
C2. Solve equations of the form $|ax + b| = cx + d$ and geometrically interpret the solution.
C3. Solve equations of the form $|ax + b| = |cx + d|$ and geometrically interpret the solution.

THE SQUARE ROOT FUNCTION

D1. State the definition of SQRT.
D2. For SQRT, compute the range element corresponding to a given domain element.
D3. Sketch the graph of SQRT.
D4. Use SQRT to define ABS.

APPENDIX II

CHRONOLOGICAL DEVELOPMENT OF CSBM (BY SEMESTER)

The evolution of the program was, as might be expected, faster paced in the first years and concentrated more on refinement in the last years. The major changes are outlined by semester in this appendix.

Fall 1972: A decision was made to develop a college remedial program in mathematics. Professor Melvin Poage and Professor Delano Wegener were asked to be the directors for this research and development. A review of the literature and existing instructional materials was conducted.

Winter 1973: Math-help labs were established and some students were individually tutored. After completing math-help lab sessions, students were allowed to retake any chapter test to improve their grades.

Fall 1973: A totally new program was proposed to replace the existing MTH 102, Beginning Algebra. The new program, Continuous Sequence in Basic Mathematics (CSBM), was to be run on an experimental basis for the Fall Semester. The CSBM program consisted of a series of two-week units; for several reasons, existing materials would not work in the proposed two-week instruction periods and new materials were written and tested.

The initial sequence consisted of 10 units and students were given placement tests to determine which unit was the most appropriate place to begin. Students received 6 hours of lecture and spent 2 hours in the math-help lab. To demonstrate mastery, students had to score 80% on a criteria test. The program offered the following options to students.

1. Additonal instruction was available by audio-tutorial or private tutoring.
2. Students not passing a unit could repeat that unit in the following two-week period.
3. Students could take more than one unit (by studying on their own) during the same two-week period.
4. If a student felt he knew the material in the next unit, he could take the criteria test on that unit and if he passed, he could skip that unit.

Winter 1974: It was decided to continue the CSBM program and extend the program to include MTH 104, Intermediate Algebra, for the Fall 1974 semester. Material for six additional units was written and tested on students enrolled in MTH 104. Items on the criteria tests for existing units were analyzed and the criteria tests revised. Some changes in the supplementary audio-tutorial materials were made to make them conform with present units.

Summer 1974: In the summer of 1974 the great amount of data collected throughout the first year of operation of CSBM was analyzed. In this respect, the CSBM program is very fortunate; since all units are taught each two-week period, the data collected in a two-week period from the CSBM program is equivalent to a semester's data from other programs. Hence, data from the 16 periods of CSBM for the school year 1973-74 was very complete. In addition, the directors felt they needed some help with developing individualized instruction and, consequently, enrolled in a PSI workshop at the University of Chicago. Because of the data analysis and the workshop, most of the CSBM materials were revised for use in the Fall, 1974. The main thrust of this revision was:

1. to more clearly define the learning objectives of each unit.
2. to create a study guide to help students read and comprehend. (It was also felt that the study guide would help the more capable students to study independently, thus giving them more control over their progress.)
3. to improve the validity of placement and criteria tests.
4. to reorganize and improve text presentation.

Finally, the procedures for test grading, test reporting, attendance, and all others needed for record keeping were revised, updated, and computerized.

Fall 1974: The following changes in options conerning criteria tests were implemented:

1. mastery level was raised from 80% to 87.5%.
2. students receiving a score between 62.5% and 87.5% had their tests analyzed by objectives--detailed diagnosis and prescriptions were provided.

The individual self-study mode of instruction was changed to a PSI structure. Material was written and tested for five new units covering material for college arithmetic to meet the needs of students at this level.

Winter 1975: Investigation was conducted to improve the use of study guides; as the study progressed, it became apparent that if understanding was a major goal, then reading and comprehension were key factors. Because the study guides were a separate entity, most students and some instructors thought them to be supplementary and, as such, did not use them or used them sparingly. Consequently it was decided to incorporate the study guides into the texts to make them a more integral part of the learning process.

Summer 1975: Materials for 12 units were rewritten: some revision and re-organization of content or presentation was done, but integrating the study guides into the text was the main purpose.

Fall 1975 and Winter 1976: The materials for the remaining units were re-written to integrate the study guides. Two addition units on functions were written and tested, and outlines for 5 more units at the college algebra level were planned.

Fall 1976 and Winter 1977: With text materials in a workable state, attention for the next three years focused on the options available to students in respect to their test results. At first, the trend was to create more possible options and provide more detailed prescriptions but as the number of students in the program grew, this trend was reversed. Options available for 1976-77 school year were as follows:

1. pass (87.5%)
2. Oral test (80%) test given by one of the four counselors.
3. Subtest (70%)
4. Repeat (less than 70% and first attempt)
5. See counselor (less than 70% and second or greater attempt).

Fall 1977 and Winter 1978: Options available to students in respect to their test results for the 1977-78 school year were:

1. Pass (87.5%)
2. Oral Test (82.5%)
3. Subtest (77.5%)

4. Lab (67.5%), student was given prescription and sent to math-help lab for instruction and concurrently he could take next unit.
5. Repeat (less than 67.5% and first attempt)
6. See counselor (less than 67.5% and second attempt)
7. See Director (less than 67.5% and third or greater attempt)
or
8. Special class (less than 67.5% and third or greater attempt).

Summer 1978: Final revision of materials--two different units were rewritten as three units. The unit on Sets was removed from the main sequence of topics. Finally, some reorganization so that students would not have to take units they did not need took place.

Fall 1978 and Winter 1979: Options available to students in respect to their test results were:

1. Pass (87.5%)
2. Subtest (80%)
3. Repeat (less than 80% and first attempt--on second attempt, only options 1 and 4 were available to these students)
4. See counselor (less than 80% and second or greater attempt).

Summer 1979: Research and program development completed. A permanent program director was hired to run the CSBM program.

APPENDIX III

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM A1

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	0	69*	1	1	1	0	95.83
2	0	0	4	66*	2	0	91.67
3	0	69*	1	0	2	0	95.83
4	0	10	62*	0	0	0	86.11
5	0	10	10	45*	3	4	62.50
6	0	1	0	3	0	68*	94.44
7	0	12	2	1	57*	0	79.17
8	0	1	65*	0	1	5	90.28
9	0	54*	3	11	4	0	75.00
10	0	0	2	2	68*	0	94.44
11	0	6	1	0	62*	3	86.11
12	0	2	4	8	12	46*	63.89
13	0	45*	6	21	0	0	62.50
14	0	9	59*	0	4	0	81.94
15	0	3	3	66*	0	0	91.67
16	0	1	0	70*	0	1	97.22
17	1	3	2	0	59*	7	81.94
18	0	3	0	0	67*	2	93.06
19	0	1	0	70*	0	1	97.22
20	0	4	3	2	2	61*	84.72
21	0	4	1	0	65*	2	90.28
22	0	1	1	1	66*	3	91.67
23	1	5	51*	6	4	5	70.83
24	0	9	63*	0	0	0	87.50
25	0	67*	4	1	0	0	93.06
26	0	52*	6	5	7	2	72.22
27	0	43*	15	6	8	0	59.72
28	0	2	3	0	66*	1	91.67
29	1	25	6	40*	0	0	55.56
30	1	3	52*	0	3	13	72.22
31	0	1	66*	1	3	1	91.67
32	0	6	2	0	9	55*	76.39
33	0	53*	2	1	16	0	73.61
34	0	8	3	1	1	59*	81.94
35	0	63*	5	0	0	4	87.50
36	0	13	53*	5	1	0	73.61
37	0	2	3	0	2	65*	90.28
38	1	8	1	1	4	57*	79.17
39	0	2	1	59*	3	7	81.94
40	2	4	2	55*	3	6	76.39

72 Total Tests 82.57

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM A3

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	3	260*	35	0	0	23	81.00
2	0	233*	49	16	17	6	72.59
3	0	5	265*	12	14	25	82.55
4	0	301*	7	3	1	9	93.77
5	1	1	15	283*	6	15	88.16
6	0	23	9	249*	31	9	77.57
7	0	14	0	6	15	286*	89.10
8	0	1	7	313*	0	0	97.51
9	1	1	11	1	246*	61	76.64
10	1	86	234*	0	0	0	72.90
11	1	1	3	312*	3	1	97.20
12	4	33	3	1	7	273*	85.05
13	0	33	12	37	232*	7	72.27
14	0	15	300*	6	0	0	93.46
15	0	1	307*	1	11	1	95.64
16	1	50	247*	23	0	0	76.95
17	0	16	284*	0	19	2	88.47
18	1	18	19	0	16	267*	83.18
19	0	128	30	162*	0	1	50.47
20	1	7	14	47	5	247*	76.95
21	2	10	309*	[illegible]	0	0	96.26
22	1	213*	29	41	21	16	66.36
23	0	12	5	303*	1	0	94.39
24	0	282*	39	0	0	0	87.8[illegible]
25	0	320*	0	0	1	0	99.6[illegible]
26	0	36	25	1	1	258*	80.37
27	0	24	0	0	292*	5	90.97
28	0	35	276*	0	9	1	85.98
29	0	0	7	5	301*	8	93.77
30	0	20	11	257*	7	26	80.06
31	1	1	9	2	304*	4	94.70
32	0	1	12	53	253*	2	78.82
33	0	6	15	23	19	258*	80.37
34	0	54	213*	52	1	1	66.36
35	0	20	3	266*	9	23	82.87
36	0	266*	13	24	12	6	82.87
37	1	2	1	5	0	312*	97.20
38	0	15	0	218*	1	87	67.91
39	0	257*	16	36	9	3	80.06
40	4	10	52	6	28	221*	68.85

321 Total Tests 83.24

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM B1

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	0	1	8*	1	0	1	72.73
2	0	9*	1	1	0	0	81.82
3	1	8*	2	0	0	0	72.73
4	0	0	1	1	0	9*	81.82
5	0	0	10*	1	0	0	90.91
6	0	0	0	0	11*	0	100.00
7	0	8*	1	1	1	0	72.73
8	0	0	11*	0	0	0	100.00
9	0	2	0	1	8*	0	72.73
10	0	9*	0	0	0	2	81.82
11	0	2	0	0	9*	0	81.82
12	0	2	0	0	9*	0	81.82
13	0	2	0	0	0	9*	81.82
14	0	8*	1	2	0	0	72.73
15	0	0	1	0	9*	1	81.82
16	0	0	1	0	0	10*	90.91
17	0	4	7*	0	0	0	63.64
18	0	1	0	0	0	10*	90.91
19	0	2	0	0	2	7*	63.64
20	0	1	0	10*	0	0	90.91
21	0	0	0	10*	1	0	90.91
22	0	1	6*	0	1	3	54.55
23	0	0	1	8*	1	1	72.73
24	0	1	0	10*	0	0	90.91
25	0	0	0	0	10*	1	90.91
26	0	0	1	1	9*	0	81.82
27	0	7*	2	1	1	0	63.64
28	0	0	0	10*	1	0	90.91
29	1	0	0	0	9*	1	81.82
30	0	0	1	1	1	8*	72.73
31	0	1	9*	0	0	1	81.82
32	0	0	1	4	1	5*	45.45
33	0	9*	0	1	1	0	81.82
34	0	11*	0	0	0	0	100.00
35	0	2	1	8*	0	0	72.73
36	0	1	10*	0	0	0	90.91
37	0	0	10*	0	0	1	90.91
38	0	1	0	10*	0	0	90.91
39	0	1	0	0	10*	0	90.91
40	0	9*	0	0	1	1	81.82
11 Total Tests							81.14

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM B3

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	4	20	0	0	244*	2	90.37
2	2	10	1	257*	0	0	95.19
3	2	80*	188	0	0	0	29.63
4	3	0	10	46	210*	1	77.78
5	2	2	11	3	248*	4	91.85
6	2	64	152*	51	1	0	56.30
7	2	9	245*	1	11	2	90.74
8	2	233*	7	17	9	2	86.30
9	2	216*	25	12	8	7	80.00
10	2	1	6	1	251*	9	92.96
11	2	37	225*	1	2	3	83.33
12	3	223*	28	2	0	14	82.59
13	3	2	235*	7	13	10	87.04
14	3	23	2	5	9	228*	84.44
15	4	34	214*	18	0	0	79.26
16	2	4	8	236*	2	18	87.41
17	2	2	251*	5	10	0	92.96
18	2	1	3	257*	4	3	95.19
19	2	30	22	1	1	214*	79.26
20	3	0	24	14	21	208*	77.04
21	2	265*	0	2	1	0	98.15
22	3	0	10	0	215*	42	79.63
23	2	10	253*	4	1	0	93.70
24	2	213*	8	30	14	3	78.89
25	3	8	258*	1	0	0	95.56
26	3	248*	18	1	0	0	91.85
27	2	99	30	137*	2	0	50.74
28	2	17	14	38	198*	1	73.33
29	2	188*	25	15	21	19	69.63
30	2	21	5	202*	28	12	74.81
31	2	11	15	209*	7	26	77.41
32	3	2	3	259*	1	2	95.93
33	3	10	3	4	27	223*	82.59
34	2	5	0	10	0	253*	93.70
35	2	242*	17	3	0	6	89.63
36	4	4	16	60	2	184*	68.15
37	3	17	5	230*	2	13	85.19
38	2	16	14	4	22	212*	78.52
39	3	12	6	189*	1	59	70.00
40	4	6	44	13	24	179*	66.30

270 Total Tests 81.33

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM C1

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	1	0	0	6	6	30*	69.77
2	0	9	1	3	30*	0	69.77
3	0	1	0	1	0	41*	95.35
4	0	8	35*	0	0	0	81.40
5	0	0	0	2	0	41*	95.35
6	0	3	2	36*	1	1	83.72
7	0	39*	0	2	2	0	90.70
8	0	11	26*	6	0	0	60.47
9	0	2	4	36*	0	1	83.72
10	0	5	7	27*	2	2	62.79
11	0	0	0	2	39*	2	90.70
12	0	37*	0	2	3	1	86.05
13	0	5	16	22*	0	0	51.16
14	0	4	35*	3	1	0	81.40
15	0	5	1	36*	0	1	83.72
16	0	2	31*	3	0	7	72.09
17	0	42*	0	1	0	0	97.67
18	0	41*	1	1	0	0	95.35
19	0	4	0	0	39*	0	90.70
20	0	0	1	0	42*	0	97.67
21	0	0	0	0	43*	0	100.00
22	0	0	0	0	33*	10	76.74
23	0	1	42*	0	0	0	97.67
24	0	3	3	37*	0	0	86.05
25	0	1	5	25*	5	7	58.14
26	0	3	0	0	6	34*	79.07
27	0	4	39*	0	0	0	90.70
28	0	41*	0	1	1	0	95.35
29	0	3	0	0	0	40*	93.02
30	0	39*	1	0	0	3	90.70
31	0	0	35*	2	1	5	81.40
32	0	0	1	0	42*	0	97.67
33	0	0	7	11	1	24*	55.81
34	0	0	1	38*	3	1	88.37
35	0	42*	0	0	1	0	97.67
36	0	3	3	0	35*	2	81.40
37	0	0	36*	4	3	0	83.72
38	0	22*	21	0	0	0	51.16
39	0	1	37*	1	4	0	86.05
40	0	0	3	39*	0	1	90.70

43 Total Tests 83.02

ACCUMULATIVE FALL 1978 (7 PERIODS)

ITEM ANALYSIS
ASTERISK (*) DENOTES CORRECT RESPONSE
UNIT 14 FORM D1

Number Question	None	Responses 1	2	3	4	5	Percent Correct
1	0	12	8	80*	0	6	75.47
2	0	94*	3	7	1	1	88.68
3	0	17	3	4	82*	0	77.36
4	0	8	17	48*	8	25	45.28
5	0	0	91*	1	14	0	85.85
6	0	51*	52	1	0	2	48.11
7	0	3	76*	21	4	2	71.70
8	0	2	2	1	90*	11	84.91
9	0	3	102*	0	0	1	96.23
10	0	8	1	2	87*	8	82.08
11	1	8	64*	18	6	9	60.38
12	0	7	7	91*	1	0	85.85
13	0	5	11	75*	4	11	70.75
14	0	0	0	0	104*	2	98.11
15	0	91*	1	5	6	3	85.85
16	0	97*	2	5	1	1	91.51
17	0	15	7	84*	0	0	79.25
18	1	91*	3	5	5	1	85.85
19	0	0	4	89*	12	1	83.96
20	0	2	8	15	17	64*	60.38
21	0	16	5	65*	12	8	61.32
22	0	2	1	22	0	81*	76.42
23	0	22	2	3	79*	0	74.53
24	1	5	15	23	7	55*	51.89
25	0	95*	0	2	2	7	89.62
26	0	1	3	0	101*	1	95.28
27	0	1	3	1	96*	5	90.57
28	0	0	90*	0	1	15	84.91
29	0	3	103*	0	0	0	97.17
30	0	12	29	65*	0	0	61.32
31	0	92*	5	5	3	1	86.79
32	0	1	0	1	0	104*	98.11
33	0	18	84*	4	0	0	79.25
34	1	99*	2	0	3	1	93.40
35	0	15	0	0	0	91*	85.85
36	0	3	6	89*	7	1	83.96
37	0	28	67*	11	0	0	63.21
38	1	7	11	0	81*	6	76.42
39	0	7	0	0	23	76*	71.70
40	1	15	73*	6	5	6	68.87

106 Total Tests 78.70

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM A1

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	144	124	86.11	2	4 11
A2	72	45	62.50	1	13
A3	144	110	76.39	2	16 29
A4	144	128	88.89	2	21 24
A5	72	66	91.67	1	28
A6	144	118	81.94	2	33 37
Section A Totals	720	591	82.08		
B1	144	123	85.42	2	7 15
B2	144	95	65.97	2	27 30
B3	144	118	81.94	2	34 39
Section B Totals	432	336	77.78		
C1	144	137	95.14	2	1 6
C2	216	197	91.20	3	10 14 19
C3	216	199	92.13	3	22 25 31
C4	144	120	83.33	2	35 38
Section C Totals	720	653	90.69		
D1	72	66	91.67	1	2
D2	216	145	67.13	3	5 9 12
D3	216	180	83.33	3	18 20 26
D4	144	108	75.00	2	32 36
Section D Totals	648	499	77.01		
E1	72	69	95.83	1	3
E2	144	124	86.11	2	8 17
E3	144	106	73.61	2	23 40
Section E Totals	360	299	83.06		
Form A1 Totals	2880	2378	82.57		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM A3

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	642	527	82.09	2	1 18
A2	321	300	93.46	1	14
A3	321	312	97.20	1	11
A4	642	510	79.44	2	27 38
A5	321	304	94.70	1	31
A6	642	499	77.73	2	7 22
Section A Totals	2889	2452	84.87		
B1	642	579	90.19	2	23 28
B2	642	512	79.75	2	9 36
B3	642	504	78.50	2	5 40
Section B Totals	1926	1595	82.81		
C1	642	621	96.73	2	21 37
C2	963	857	88.99	3	17 25 32
C3	963	921	95.64	3	8 15 29
C4	642	574	89.41	2	4 12
Section C Totals	3210	2973	92.62		
D1	642	513	79.91	2	16 35
D2	642	465	72.43	2	2 13
D3	963	750	77.88	3	6 10 26
D4	963	632	65.63	3	19 34 39
Section D Totals	3210	2360	73.52		
E1	321	282	87.85	1	24
E2	642	505	78.66	2	20 33
E3	642	522	81.31	2	3 30
Section E Totals	1605	1309	81.56		
Form A3 Totals	12840	10689	83.24		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM B1

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	22	19	86.36	2	29 37
A2	11	8	72.73	1	7
A3	22	18	81.82	2	21 35
A4	22	19	86.36	2	5 11
A5	11	10	90.91	1	25
A6	22	19	86.36	2	16 33
Section A Totals	110	93	84.55		
B1	22	19	86.36	2	12 38
B2	22	14	63.64	2	3 22
B3	22	20	90.91	2	18 28
Section B Totals	66	53	80.30		
C1	22	17	77.27	2	30 40
C2	33	26	78.79	3	17 20 26
C3	33	30	90.91	3	2 6 36
C4	22	18	81.82	2	10 13
Section C Totals	110	91	82.73		
D1	11	8	72.73	1	23
D2	33	20	60.60	3	9 27 32
D3	33	27	82.82	3	4 14 39
D4	22	15	68.18	2	1 19
Section D Totals	99	70	70.70		
E1	11	11	100.00	1	34
E2	22	18	81.82	2	15 31
E3	22	21	95.45	2	8 24
Section E Totals	55	50	90.91		
Form B1 Totals	440	357	81.14		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM B3

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	540	435	80.56	2	12 38
A2	270	253	93.70	1	23
A3	270	257	95.19	1	18
A4	540	433	80.19	2	1 39
A5	270	248	91.85	1	5
A6	540	411	76.11	2	29 33
Section A Totals	2430	2037	83.83		
B1	540	482	89.26	2	2 11
B2	540	448	82.96	2	8 22
B3	540	415	76.85	2	16 40
Section B Totals	1620	1345	83.02		
C1	540	511	94.63	2	25 34
C2	810	720	88.89	3	4 7 21
C3	810	761	93.95	3	10 17 32
C4	540	470	87.04	2	14 35
Section C Totals	2700	2462	91.19		
D1	540	444	82.22	2	15 37
D2	540	414	76.67	2	9 28
D3	810	496	61.23	3	3 19 30
D4	810	502	61.98	3	6 24 27
Section D Totals	2700	1856	68.74		
E1	270	248	91.85	1	26
E2	540	392	72.59	2	20 36
E3	540	444	82.22	2	13 31
Section E Totals	1350	1084	80.30		
Form B3 Totals	10800	8784	81.33		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM C1

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	86	77	89.53	2	29 39
A2	43	39	90.70	1	7
A3	86	48	55.81	2	8 13
A4	86	72	83.72	2	24 36
A5	43	42	97.67	1	20
A6	86	72	83.72	2	3 16
Section A Totals	430	350	81.40		
B1	86	81	94.19	2	27 35
B2	86	72	83.72	2	22 30
B3	86	76	88.37	2	12 19
Section B Totals	258	229	88.76		
C1	86	63	73.26	2	5 38
C2	129	107	82.95	3	26 31 34
C3	129	119	92.25	3	15 18 23
C4	86	66	76.74	2	2 9
Section C Totals	430	355	82.56		
D1	43	30	69.77	1	1
D2	129	100	77.52	3	25 37 40
D3	129	119	92.25	3	14 17 32
D4	86	74	86.05	2	4 11
Section D Totals	387	323	83.46		
E1	43	43	100.00	1	21
E2	86	65	75.58	2	28 33
E3	86	63	73.26	2	6 10
Section E Totals	215	171	79.53		
Form C1 Totals	1720	1428	83.02		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
ANALYSIS BY TEST
UNIT 14 FORM D1

Objective	Number of Response	Number Correct	Percent Correct	Number of Questions	Question Numbers
A1	212	182	85.85	2	5 35
A2	106	91	85.85	1	15
A3	212	132	62.26	2	30 37
A4	212	165	77.83	2	17 38
A5	106	96	90.57	1	27
A6	212	145	68.40	2	11 22
Section A Totals	1060	811	76.51		
B1	212	202	95.28	2	29 34
B2	212	185	87.26	2	8 25
B3	212	173	81.60	2	3 18
Section B Totals	636	560	88.05		
C1	212	155	73.11	2	6 32
C2	318	255	80.19	3	19 28 39
C3	318	279	87.74	3	1 9 16
C4	212	170	80.19	2	12 23
Section C Totals	1060	859	81.04		
D1	106	64	60.38	1	20
D2	318	199	62.58	3	4 7 13
D3	318	266	83.65	3	26 31 40
D4	212	171	80.66	2	10 33
Section D Totals	954	700	73.38		
E1	106	104	98.11	1	14
E2	212	149	70.28	2	2 24
E3	212	154	72.64	2	21 36
Section E Totals	530	407	76.79		
Form D1 Totals	4240	3337	78.70		

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 8

Objective	Number of Response	Number Correct	Percent Correct
A1	110	74	67.27
A2	110	107	97.27
A3	179	178	99.44
A4	289	274	94.81
Section A Totals	688	633	92.01
B1	220	212	96.36
B2	110	106	96.36
B3	110	105	95.45
B4	330	275	83.33
Section B Totals	770	698	90.65
C1	110	68	61.82
C2	289	270	93.43
C3	330	291	88.18
C4	110	72	65.45
C5	220	190	86.36
Section C Totals	1059	891	84.14
D1	110	108	98.18
D2	110	84	76.36
D3	289	254	87.89
D4	330	307	93.03
D5	220	181	82.27
Section D Totals	1059	934	88.20
E1	220	213	96.82
E2	110	106	96.36
E3	330	317	96.06
Section E Totals	660	636	96.36
F1	41	34	82.93
F2	41	25	60.98
F3	41	38	92.68
F4	41	34	82.93
Section F Totals	164	131	79.88
Unit 8 Totals	4400	3923	89.16

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 9

Objective	Number of Response	Number Correct	Percent Correct
A1	124	91	73.39
A2	124	118	95.16
A3	335	284	84.78
A4	248	209	84.27
A5	161	113	70.19
A6	124	94	75.81
Section A Totals	1116	909	81.45
B1	248	154	62.10
B2	124	115	92.74
B3	422	397	94.08
B4	74	58	78.38
Section B Totals	868	724	83.41
C1	248	233	93.95
C2	124	102	82.26
C3	124	64	51.61
C4	124	96	77.42
C5	248	192	77.42
C6	248	229	92.34
Section C Totals	1116	916	82.08
D1	248	186	75.00
D2	248	213	85.89
D3	248	206	83.06
Section D Totals	744	605	81.32
E1	161	91	56.52
E2	248	190	43.95
E3	248	162	65.32
E4	248	193	77.82
E5	211	140	66.35
Section E Totals	1116	695	62.28
Unit 9 Totals	4960	3849	77.60

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 10

Objective	Number of Response	Number Correct	Percent Correct
A1	696	549	78.88
A2	1098	1064	96.90
A3	696	635	91.24
A4	1392	922	66.24
Section A Totals	3882	3170	81.66
B1	1098	981	89.34
B2	696	626	89.94
B3	1392	1059	76.08
B4	1683	1205	71.60
B5	696	649	93.25
Section B Totals	5565	4520	81.22
C1	1098	1017	92.62
C2	696	622	89.37
C3	1686	1472	87.31
C4	1392	1094	78.59
Section C Totals	4872	4205	86.31
D1	1686	1421	84.28
D2	699	567	81.12
D3	1392	1253	90.01
D4	1095	809	73.88
Section D Totals	4872	4050	83.13
E1	696	674	96.84
E2	1392	1231	88.43
E3	1392	1266	90.95
E4	1689	1404	83.13
Section E Totals	5169	4575	88.51
F1	696	596	85.63
F2	1392	1122	80.60
F3	1392	1015	72.92
Section F Totals	3480	2733	78.53
Unit 10 Totals	27840	23253	83.52

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 11

Objective	Number of Response	Number Correct	Percent Correct
A1	340	317	93.24
A2	510	443	86.86
A3	457	331	72.43
Section A Totals	1307	1091	83.47
B1	170	156	91.76
B2	340	315	92.62
B3	340	282	82.94
B4	340	202	59.41
B5	340	231	67.94
Section B Totals	1530	1186	77.52
C1	223	175	78.48
C2	170	130	76.47
C3	170	122	71.76
C4	340	216	63.53
C5	340	297	87.35
C6	340	238	70.00
Section C Totals	1583	1178	74.42
D1	340	233	68.53
D2	340	246	72.35
D3	170	64	37.65
D4	170	122	71.76
D5	340	250	73.53
Section D Totals	1360	915	67.28
E1	340	246	72.35
E2	340	182	53.53
E3	340	267	78.53
Section E Totals	1020	695	68.14
Unit 11 Totals	6800	5065	74.49

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 12

Objective	Number of Response	Number Correct	Percent Correct
A1	2022	1922	95.05
A2	3033	2731	90.04
A3	1011	855	84.57
A4	2022	1644	81.31
A5	1011	770	76.16
Section A Totals	9099	7922	87.06
B1	2022	1633	80.76
B2	2022	1585	78.39
B3	2022	1856	91.79
B4	2022	1488	73.59
B5	3033	2189	72.17
Section B Totals	11121	8751	78.69
C1	2022	1612	79.72
C2	2022	1098	54.30
C3	2022	1474	72.90
C4	3033	2066	68.12
C5	1011	900	89.02
Section C Totals	10110	7150	70.72
D1	2022	1804	89.22
D2	2022	1644	81.31
D3	2022	1695	83.83
D4	2022	1774	87.73
D5	2022	1477	73.05
Section D Totals	10110	8394	83.03
Unit 12 Totals	40440	32217	79.67

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 13

Objective	Number of Response	Number Correct	Percent Correct
A1	1646	1288	78.25
A2	823	537	65.25
A3	1143	863	75.50
A4	1646	1339	81.35
A5	823	611	74.24
A6	823	648	78.74
A7	823	617	74.97
Section A Totals	7727	5903	76.39
B1	1326	997	75.19
B2	2469	1997	80.88
B3	640	528	82.50
Section B Ttoals	4435	3522	79.41
C1	1966	1679	85.40
C2	1326	1093	82.43
C3	1646	1287	78.19
Section C Totals	4938	4059	82.20
D1	1646	1309	79.53
D2	1646	1381	83.90
D3	1646	1300	78.98
D4	823	572	69.50
Section D Totals	5761	4562	79.19
E1	1646	1242	75.46
E2	823	527	64.03
E3	1646	1082	65.74
E4	823	594	72.17
E5	503	267	53.08
E6	503	341	67.79
Section E Totals	5944	4053	68.19
F1	823	688	83.60
F2	1646	1359	82.56
F3	1646	1119	67.98
Section F Totals	4115	3166	76.94
Unit 13 Totals	32920	25265	76.75

ACCUMULATIVE FALL 1978 (7 PERIODS)
ANALYSIS OF SUCCESS ON EACH OBJECTIVE
COMBINED LISTING
UNIT 14

Objective	Number of Response	Number Correct	Percent Correct
A1	1646	1364	82.87
A2	823	736	89.43
A3	1055	877	83.13
A4	1646	1327	80.62
A5	823	766	93.07
A6	1646	1264	76.79
Section A Totals	7639	6339	82.92
B1	1646	1486	90.28
B2	1646	1326	80.56
B3	1646	1306	79.34
Section B Totals	4938	4118	83.39
C1	1646	1504	91.37
C2	2469	2162	87.57
C3	2469	2309	93.52
C4	1646	1418	86.15
Section C Totals	8230	7393	89.83
D1	1414	1125	79.56
D2	1878	1343	71.51
D3	2469	1838	74.44
D4	2237	1502	67.14
Section D Totals	7998	1508	72.62
E1	823	757	91.98
E2	1646	1253	76.12
E3	1646	1310	79.59
Section E Totals	4115	3320	80.68
Unit 14 Totals	32920	26973	81.93

ACCUMULATIVE FALL 1973 (7 PERIODS)
DIFFICULTY ANALYSIS
TEST 3 FORM A
N=52 STUDENTS.

Question	Indices of Discrimination	& Difficulty
1	.230	.884
2	.307	.769
3	.307	.769
4	.692	.653
5	.153	.923
6	.538	.653
7	.230	.884
8	.307	.846
9	.230	.730
10	.461	.692
11	.307	.846
12	.076	.884
13	.076	.884
14	.384	.807
15	.000	.923
16	.461	.769
17	.153	.923
18	.846	.500
19	.615	.692
20	.230	.884
21	.769	.615
22	.153	.846
23	.153	.923
24	.230	.884
25	.461	.769
26	.307	.846
27	.769	.538
28	.461	.769
29	.769	.615
30	.307	.846
31	.230	.807
32	.230	.884
33	.384	.807
34	.384	.807
35	.307	.846
36	.076	.884
37	.538	.653
38	.461	.769
39	.153	.923
40	.615	.692