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ABSTRACT

This is one of a series that is a collection of translations from the extensive Soviet literature of the past 25 years on research in the psychology of mathematics instruction. It also includes works on methods of teaching mathematics directly influenced by the psychological research. Selected papers and books considered to be of value to the American mathematics educator have been translated from the Russian and appear in this series for the first time in English. The aim of this series is to acquaint mathematics educators and teachers with directions, ideas, and accomplishments in the psychology of mathematical instruction in the Soviet Union. The articles in this volume are concerned with the instruction in problem solving of mentally retarded pupils in the auxiliary schools of the Soviet Union. Both articles in this volume describe research in problem solving and also provide concrete suggestions for improving instruction. The literature reviews contained in these articles provide us with much information on the state of research in the Soviet Union on problem solving in mathematics. (Author/MK)

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SOVIET STUDIES IN THE PSYCHOLOGY OF LEARNING AND TEACHING MATHEMATICS

VOLUME IX

SCHOOL MATHEMATICS STUDY GROUP
STANFORD UNIVERSITY
AND
SURVEY OF RECENT EAST EUROPEAN MATHEMATICAL LITERATURE
THE UNIVERSITY OF CHICAGO

SOVIET STUDIES IN THE PSYCHOLOGY OF LEARNING AND TEACHING MATHEMATICS

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VOLUME IX

PROBLEM-SOLVING PROCESSES OF MENTALLY RETARDED CHILDREN

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PREFACE

The series Soviet Studies in the Psychology of Learning and Teaching Mathematics is a collection of translations from the extensive Soviet literature of the past twenty-five years on research in the psychology of mathematical instruction. It also includes works on methods of teaching mathematics directly influenced by the psychological research. The series is the result of a joint effort by the School Mathematics Study Group at Stanford University, the Department of Mathematics Education at the University of Georgia, and the Survey of Recent East European Mathematical Literature at the University of Chicago. Selected papers and books considered to be of value to the American mathematics educator have been translated from the Russian and appear in this series for the first time in English.

Research achievements in psychology in the United States are outstanding indeed. Educational psychology, however, occupies only a small fraction of the field, and until recently little attention has been given to research in the psychology of learning and teaching particular school subjects.

The situation has been quite different in the Soviet Union. In view of the reigning social and political doctrines, several branches of psychology that are highly developed in the U.S. have scarcely been investigated in the Soviet Union. On the other hand, because of the Soviet emphasis on education and its function in the state, research in educational psychology has been given considerable moral and financial support. Consequently, it has attracted many creative and talented scholars whose contributions have been remarkable.*

Even prior to World War II, the Russians had made great strides in educational psychology. The creation in 1943 of the Academy of Pedagogical Sciences helped to intensify the research efforts and programs in this field. Since then the Academy has become the chief educational research and development center for the Soviet Union. One of the main aims of the Academy is to conduct research and to train research scholars

*A study indicates that 37.5% of all materials in Soviet psychology published in one year was devoted to education and child psychology. See Contemporary Soviet Psychology by Josef Brozek (Chapter 7 of Present-Day Russian Psychology, Pergamon Press, 1966).

in general and specialized education; in educational psychology, and in methods of teaching various school subjects.

The Academy of Pedagogical Sciences of the USSR comprises ten research institutes in Moscow and Leningrad. Many of the studies reported in this series were conducted at the Academy's Institute of General and Polytechnical Education, Institute of Psychology, and Institute of Defectology, the last of which is concerned with the special psychology and educational techniques for handicapped children.

The Academy of Pedagogical Sciences has 31 members and 64 associate members, chosen from among distinguished Soviet scholars, scientists, and educators. Its permanent staff includes more than 650 research associates, who receive advice and cooperation from an additional 1,000 scholars and teachers. The research institutes of the Academy have available 100 "base" or laboratory schools and many other schools in which experiments are conducted. Developments in foreign countries are closely followed by the Bureau for the Study of Foreign Educational Experience and Information.

The Academy has its own publishing house, which issues hundreds of books each year and publishes the collections Izvestiya Akademii Pedagogicheskikh Nauk RSFSR [Proceedings of the Academy of Pedagogical Sciences of the RSFSR], the monthly Sovetskaya Pedagogika [Soviet Pedagogy], and the bimonthly Voprosy Psikhologii [Questions of Psychology]. Since 1963, the Academy has been issuing collection entitled Novye Issledovaniya v Pedagogicheskikh Naukakh [New Research in the Pedagogical Sciences] in order to disseminate information on current research.

A major difference between the Soviet and American conception of educational research is that Russian psychologists often use qualitative rather than quantitative methods of research in instructional psychology in accordance with the prevailing European tradition. American readers may thus find that some of the earlier Russian papers do not comply exactly to U.S. standards of design, analysis, and reporting. By using qualitative methods and by working with small groups, however, the Soviets have been able to penetrate into the child's thoughts and to analyze his mental processes. To this end they have also designed classroom tasks and settings for research and have emphasized long-term, genetic studies of learning.

Russian psychologists have concerned themselves with the dynamics of mental activity and with the aim of arriving at the principles of the learning process itself. They have investigated such areas as: the development of mental operations; the nature and development of thought; the formation of mathematical concepts and the related questions of generalization, abstraction, and concretization; the mental operations of analysis and synthesis; the development of spatial perception; the relation between memory and thought; the development of logical reasoning; the nature of mathematical skills; and the structure and special features of mathematical abilities.

In new approaches to educational research, some Russian psychologists have developed cybernetic and statistical models and techniques, and have made use of algorithms, mathematical logic and information sciences. Much attention has also been given to programmed instruction and to an examination of its psychological problems and its application for greater individualization in learning.

The interrelationship between instruction and child development is a source of sharp disagreement between the Geneva School of psychologists, led by Piaget, and the Soviet psychologists.* The Swiss psychologists ascribe limited significance to the role of instruction in the development of a child. According to them, instruction is subordinate to the specific stages in the development of the child's thinking—stages manifested at certain age levels and relatively independent of the conditions of instruction.

As representatives of the materialistic-evolutionist theory of the mind, Soviet psychologists ascribe a leading role to instruction. They assert that instruction broadens the potential of development, may accelerate it, and may exercise influence not only upon the sequence of the stages of development of the child's thought but even upon the very character of the stages. The Russians study development in the changing conditions of instruction, and by varying these conditions, they demonstrate how the nature of the child's development changes in the process. As a result, they are also investigating tests of giftedness and are using elaborate dynamic, rather than static, indices.

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*See The Problem of Instruction and Development at the 18th International Congress of Psychology by N. A. Menchinskaya and G. G. Saburova, Sovetskaya Pedagogika, 1967, No. 1. (English translation in Soviet Education, July 1967, Vol. 9, No. 9.)

Psychological research has had a considerable effect on the recent Soviet literature on methods of teaching mathematics. Experiments have shown the student's mathematical potential to be greater than had been previously assumed. Consequently, Russian psychologists have advocated the necessity of various changes in the content and methods of mathematical instruction and have participated in designing the new Soviet mathematics curriculum which has been introduced during the 1967-68 academic year.

The aim of this series is to acquaint mathematics educators and teachers with directions, ideas, and accomplishments in the psychology of mathematical instruction in the Soviet Union. This series should assist in opening up avenues of investigation to those who are interested in broadening the foundations of their profession, for it is generally recognized that experiment and research are indispensable for improving content and methods of school mathematics.

We hope that the volumes in this series will be used for study, discussion, and critical analysis in courses or seminars in teacher-training programs or in institutes for in-service teachers at various levels.

At present, materials have been prepared for fifteen volumes. Each book contains one or more articles under a general heading such as The Learning of Mathematical Concepts, The Structure of Mathematical Abilities and Problem Solving in Geometry. The introduction to each volume is intended to provide some background and guidance to its content.

Volumes I to VI were prepared jointly by the School Mathematics Study Group and the Survey of Recent East European Mathematical Literature, both conducted under grants from the National Science Foundation. When the activities of the School Mathematics Study Group ended in August, 1972, the Department of Mathematics Education at the University of Georgia undertook to assist in the editing of the remaining volumes. We express our appreciation to the Foundation and to the many people and organizations who contributed to the establishment and continuation of the series.

Jeremy Kilpatrick
Izaak Wirszup
Edward G. Begle
James W. Wilson

EDITORIAL NOTES

1. Bracketed numerals in the text refer to the numbered references at the end of each paper. Where there are two figures, e.g. [5:123], the second is a page reference. All references are to Russian editions, although titles have been translated and authors' names transliterated.

2. The transliteration scheme used is that of the Library of Congress, with diacritical marks omitted, except that Ю and Я are rendered as "yu" and "ya" instead of "iu" and "ia."

3. Numbered footnotes are those in the original paper, starred footnotes are used for editors' or translator's comments.

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INTRODUCTION

Sandra P. Clarkson

The articles in this volume are concerned with the instruction in problem solving of mentally retarded pupils in the auxiliary schools of the Soviet Union. Both articles in this volume describe research in problem solving and also provide concrete suggestions for improving instruction. The literature reviews contained in these articles provide us with much information of the state of research in the Soviet Union on problem solving in mathematics. The cry is made again and again in the articles for specialized instruction for students with special learning disorders.

Problem solving, resulting in the development of the student's logical thought, is seen as having a therapeutic as well as a pragmatic value for the mentally retarded school pupil. Instruction in problem-solving skills occupies a great deal of time in the auxiliary schools. Mikhal'skii, in the first article, identifies three methods of instruction in problem solving: the method of analogy, or guiding a pupil's solution by giving him easier analogous problems when he encounters difficulty solving the main problem; the method of many problems, where the pupil learns to solve a problem by solving problems of increasing difficulty; and the method of analysis. The latter method is explored in the article.

There are two methods of analysis: analytic and synthetic. In the analytic method of analysis, the pupil decomposes a complex problem into simple problems based on what is asked for in the problem. One asks oneself, "What must I know to solve this problem?" One then sets out to find the information needed. In the synthetic method of analysis, one begins with the data given and asks the question, "If I know this, what else do I know and what can I find out with this information?" One proceeds in this method until the question one asks is the question of the problem. A solution to a problem might reasonably contain both types of analysis.

Mikhal'skii gives a fairly extensive review of the problem-solving studies that used one or both methods of analysis. On the basis of his literature search, he designed and ran a series of investigations to determine the characteristics of auxiliary school pupils' problem-solving processes. The pupils investigated were from grades 3,5, and 7; excellent, average, and poor problem solvers were included in the sample.

The primary goal of the investigator was to determine the pupil's ability to solve problems without the teacher's help. There were four series of problems given. Series one contained ordinary problems of the type used in the classroom as well as problems with too many numbers, too few numbers, and no numbers at all. Series two contained problems in which the pupils had to supply missing numbers. Series three contained problems with data but no question; the question had to be supplied by the pupil. Series four contained typical problems that had to be analyzed before solution.

Mikhal'skii reports in detail the solutions given by individual pupils, but he reaches only a few general conclusions. The third graders solved problems without regard to the question; it did not influence their choice of operations. Most third graders could not identify either the conditions or the question of the problem. Fifth graders likewise neglected to use the question of the problem in determining their solution plan. However, unlike the third graders, they did recognize the conditions and could state the question of the problem. The results for the seventh graders were similar to those for the fifth graders.

With most of the pupils, the method of solution was not influenced by the question of the problem, but instead proceeded because of the conditions and the numerical data. As would be expected, the seventh graders did better than the fifth graders, who did better than the third graders.

The errors made by the pupils in attempting the preliminary analysis of the conditions were analyzed and, based on this analysis, some suggestions were made for problem-solving instruction.

This article provides some helpful suggestions for teaching problem solving, as well as an insight into the processes mentally retarded students use while solving problems.

Kuz'mitskaya, in the other articles of the volume, also attempted to identify the difficulties auxiliary school pupils have in solving problems and to prescribe certain instructional strategies. She assumed that the characteristics of the reproduction of a problem depend on the level of comprehension of the problem.

Problems were presented to 20 fourth grade and 20 sixth grade auxiliary school pupils. The problems chosen were ones they had solved in earlier grades. The same problems were presented to normal second-graders in the public or "mass" school. The pupils were asked to read the problem over several times and then to state the problem orally without reading it. The reproductions were analyzed and possible sources of difficulty identified.

In all phases of problem reproduction, the normal children performed better than the mentally retarded pupils despite the age difference. The auxiliary school pupil's success in reproducing a problem was inversely proportional to its complexity.

The pupil's solution of the problem depended on a correct reproduction of its conditions and question. A further investigation corroborated this observation. When the reproduction was incorrect, the problem was generally solved in accordance with its reproduction, producing an incorrect solution; often the reproduction was distorted while the solution was correct; and sometimes even with a correct reproduction, the problem was solved incorrectly. In general, however, the more correct the reproduction, the more correct the solution.

Several smaller investigations are described, and Kuz'mitskaya identifies the problem types most difficult for the auxiliary school pupils and classifies the most common errors.

A final section on suggestions for improving instruction in arithmetic problem solving is quite refreshing. Kuz'mitskaya suggests field trips to the bakery, the dairy, and other stores to provide experience that would help make arithmetic problems less abstract to pupils. The pupils are then given the opportunity to pretend that they are running a store--selling items, measuring, collecting money, and making change. Verbal problems are gradually introduced that make use of the child's experience. Gradually the mentally retarded pupil's problem-solving ability is developed by providing concrete experiences and then moving to more abstract situations.

Both articles are worthwhile for teachers as well as researchers concerned with the education of mentally retarded children.

THE SOLUTION OF COMPLEX ARITHMETIC PROBLEMS IN THE AUXILIARY SCHOOL

K. A. Mikhal'skii*

Methods of Teaching Problem Solving and the Essence of Methods of Analyzing Them

In preparing the comprehensively developed builders of the communist society, much attention is given to mental training. Among the methods of mental training, one of the best is solving arithmetic problems. In addition to its great practical significance, arithmetical problem solving promotes the development of the pupil's speech and thought, attention, memory and will; it promotes a conscious mastery of mathematical concepts reflecting actual phenomena of the real world. Raising the efficacy of the school's work in the area of problem solving is one of the most important tasks of the methodology of teaching arithmetic.

The published work by J. V. Stalin, Marxism and Questions of Linguistics, marking a new stage in the development of the sciences, allows a novel approach to the determination of the direction and content of the methodology of teaching children. Stalin's indication that language is directly connected with thought and that "language registers the results of mental activity, of the success of man's mental activity, and forms them into words and combinations of words in sentences" [5:22] is an initial proposition, presenting the requirements for the type of work being considered.

Obviously the only insufficiency is that the pupil copes somehow with a specific problem in arithmetic, with the solution of problems and examples. What he learns--the results of his understanding and solving an assignment--should be strengthened in the child's speech,

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*Of the Institute of Psychology, Academy of Pedagogical Sciences of the RSFSR. Published in Proceedings [Izvestiya] of the Academy of Pedagogical Sciences of the RSFSR, 1952, Vol. 41, pp. 13-78. Translated by David A. Henderson.

in the correct verbal formation of what he has understood and done.

Work in developing the child's speech and in enriching his mathematical vocabulary should constitute one of the major pedagogical goals in teaching problem solving to children. This is one of the main methods of developing the pupil's thought, and therefore, one of the methods of developing problem-solving skills, in the broad sense of the word. Obviously the guiding methods of instruction, the methods and types of work on the arithmetic problem, should be those whose primary aim is to develop the student's thinking and those which are the most effective for attaining this goal. Among the problem-solving operations, one method of instruction, as we shall see later, is the preliminary analysis of the conditions. The elaboration of questions connected with methods of preliminary analysis of the conditions of problems, applicable to the auxiliary school, is the main topic of this article.

In the auxiliary school, the pupils' instruction in problem solving acquires, moreover, its own special significance. The auxiliary school is intended for mentally retarded children--the retardation as a result of a disease of the central nervous system. A sharp derangement of intellectual activity--of logical thought--is a characteristic feature of most of the pupils of this school. For this reason, problem-solving instruction is the most difficult part of teaching arithmetic in the auxiliary school. If a teacher says that a pupil is not succeeding in arithmetic, it usually means that the pupil cannot solve problems. At the same time, the solution of arithmetic problems is recognized, in the theory and practice of teaching, as one of the primary ways of promoting the development of the pupils' logical thought. This type of work is aimed directly at minimizing the basic defect in the personality of the mentally retarded; and therefore, in addition to its great practical and educational significance, it is an important correctional and educative method. Therefore, teaching children to solve problems attracts the attention of teachers and defectologists.* "How can children be taught to solve problems, to examine a problem until they come to a

*Specialists in the retarded (Ed.).

conclusion?" This is a primary, and obviously urgent, question, heard most frequently from mathematics educators in auxiliary schools. This work is an attempt to help the teacher answer this question.

The essential step in solving a problem—the preliminary analysis of the conditions—has been isolated here and given the center of attention. For it is at this point of the work on a problem that the child's mental activity is directed toward the deepest comprehension of the conditions of the problem, toward organizing the material, and toward selecting the method of solution; he ascertains the requirement, or question, of the problem; he sorts out known data and establishes their connection and relationship; he chooses arithmetic operations on the basis of the understood conditions. The result of these kinds of mental activity should be the isolation, the "breakdown," the "decomposition" of a complex problem into a series of simple ones, that is, the thorough preparation by a pupil for making a plan and interpreting the written solution.

As can be seen, analysis of the conditions is the basic step: (1) it strengthens the weakest element in the complex problem-solving process of the mentally retarded schoolchildren, since it is directed against a mechanical, insufficiently substantiated performance of arithmetical operations, according to insufficiently recognized conditions; and (2) it promotes the pupil's mental development and, consequently, helps to correct the personality of the mentally retarded child. Remembering this, and taking into account the practical requirements of the school, the author has attempted to elucidate the difficulties that arise during the analysis and solution of complex arithmetic problems by auxiliary school pupils, and to note methodological and pedagogical ways to overcome them.

Concerning the Methods of Teaching Problem Solving

In the theory and practice of teaching children to solve complex arithmetic problems there are various methods[1] and devices used to develop the skill of problem solving. (1) It is possible to teach

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[1]We shall distinguish between the methods of teaching problem solving (the method of analogy, the method of many problems, the method of analysis) and the means of analysis (analytic and synthetic analysis).

children to solve problems by guiding them along the path of solution, using easy analogous statements and problems. When they have a difficulty, the pupils are given an analogous problem that they can solve mentally. An oral solution of an easy problem gives the key to solving a difficult problem. In this method, a breakdown of the conditions of the problems is not given beforehand (the method of analogy). (2) It is also possible to teach children to solve problems through the solution of a great number of problems of gradually increasing difficulty (the method of many problems), without resorting to the methods of analogy or analysis of the conditions. (3) It is also possible to teach children to solve problems through a preliminary analysis of the conditions of the problems (the method of analysis).

When one of the above mentioned methods is emphasized, a unique skill in problem solving is developed in the children, qualitatively different from one developed using a different method. Thus, taught by the method of analogy, the pupil tries to think of an easy problem analogous to the original one that is giving him trouble, often on the basis of external similar and random features (words, statements, situations), without a profound understanding of the sense of the problem proposed to him. He tries to recall, to reproduce in his mind a known solution of an easier problem; in other words, he approaches the solution of the problem, not on the basis of a profound and complete realization of the conditions, but rather by a mechanical transfer of the method of solution of a known easy problem. Therefore, the process of mental reproduction plays a leading role, and narrows the possibilities for creative activity.

In teaching children by the method of many problems, there is danger of their developing habits of solution based on mistaken ideas. We find confirmation of this statement in the psychological investigation of P. A. Shevarev [4]. He writes:

> Mathematics teachers often assume that the independent solution of many examples of a single type in succession promotes the strengthening of the corresponding correct

skill.' But in reality something different occurs—conditions are created under which it is extremely possible that a mistaken connection may arise.[2]

Since the pupil does not dwell long enough on the conscious mastery of the conditions in solving a large number of problems, he becomes habituated to an instinctive, automatized solution of problems without the requisite analysis and understanding of the conditions. Moreover, the solution of a great number of examples and problems of the same type suppresses the active operation of the pupil's thinking. The psychological basis for such a phenomenon was discovered by I. P. Pavlov [3:348].

Qualitatively different from the preceding is the problem-solving skill developed in children by instructing them in the preliminary analysis of the conditions of the problems to be solved. A pupil's mental activity is directed here toward a complete, profound recognition of the conditions and toward the ability to draw a correct conclusion from what is understood--to note the correct method of solution. Such instruction makes it possible to develop problem-solving skill in the broad sense of the word--to master devices for a conscious approach to the solution of problems, devices of creative thought. Recommendations for a definitive analysis of the mutual relationship between the methods of instruction is found in the resolution of the Central Committee of the Communist Party, "on the Elementary and Secondary School," which says "...no one method can be recognized as the basic and universal method of studies...."[3]

Actually, strict application of only one of the investigated methods of teaching problem solving is not only inadvisable, but impossible as well. Thus, for example, the method of a preliminary

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[2]Connections are what Shevarev calls unions of psychic processes, in which the first element of such a union is either recognition of the general characteristics of a specific part of the given conditions, or recognition of the general characteristics of the operation just performed; the second element is the orientation to the performance of a specific kind of operation.

[3]Resolution of the Central Committee of the Communist Party of the Soviet Union of 5 September 1931.

analysis of the conditions requires the solution of a great many problems, as well as the solution of problems analogous to a given problem which facilitate solution of the latter. Also one cannot imagine teaching problem solving treating all the enumerated methods as equivalent--necessary and useful to an identical extent. Obviously, preference should be shown to the most effective method, at the same time recognizing other methods. The selection of the method is to a great extent determined by the problems which face the school.

American bourgeois investigators "prove" in their works the predominance of the many problem method, belittling in every way the method of analyzing the conditions; they try to justify using the many problem method in the American school and to hide its utilitarian nature.

Unlike the narrow practicism of American investigators, who are more concerned about developing the automatized skill of problem solving in schoolchildren, Russian methodologists of the prerevolutionary, and especially the Soviet, period have as their goal the development of the ability to reason, to approach a solution on the basis of profoundly recognized conditions, training "creative devices of thought." To attain these goals, the most expedient method, as we have seen, is that of analyzing the conditions. And actually, when used with others, the method of a preliminary analysis of the conditions is recognized by most of our native authors as the fundamental, most effective method. Thus, our Russian methodology of arithmetic appeared at the very beginning of its development at a more advanced position than contemporary (particularly American) reactionary methodology.

The Essence of the Methods of Analyzing Complex Arithmetic Problems

After becoming familiar with the conditions of the problem, the pupil approaches its analysis. In analyzing the problem he comes to understand the interdependence of the data; he establishes their interdependence; he draws a conclusion on the basis of a series of facts; he asks questions and chooses a means to answer them. Through analysis the complex arithmetic problem is broken down into a series of simple ones. All this work is conducted in class under the teacher's supervision. Gradually understanding the idea of the problem and the ways to solve it through communication with the teacher, the children learn to express what they understand; they learn to formulate and state their ideas. Here, by working on the child's speech, the teacher is working also on the development of his thinking.

An analysis of a problem may be made in two ways—synthetically and analytically. The essence of the synthetic method of analysis of complex arithmetic problems[4] consists in extracting simple problems from a complex problem on the basis of data taken from the conditions, or obtained in simple problems formulated earlier. The question is selected for the data. Extraction of simple problems from a complex one thus goes according to a pattern: knowing specified data, one can find the unknown. The composition of simple problems, together with their possible solutions, is continued until the last simple problem coincides with the question of the complex problem.

The essence of the analytic method of analysis consists in decomposing a complex problem into simple ones, beginning not with the data (as in synthesis) but with the question of the problem. The data necessary to answer the question of the problem are determined in one operation. In this, one and sometimes two given elements are unknown and are the basis for stating the question in composing the next simple problem. The data are selected for the question. Extraction of simple problems from a complex one thus goes according to a pattern: to find the unknown, one must know the specific data. The composition of simple problems by selecting data for the question is continued as long as it is impossible to solve the first simple problem.

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[4] Note. As can be seen, the term "synthetic" method, whose essence consists in decomposing an integer into its parts (a complex problem into a series of simple ones), does not correspond to the scientific (philosophical, psychological) understanding of synthesis as a combination or summation of an integer from its parts. However, the composition of a simple problem by the synthetic method of analysis primarily requires application of a synthetic method of thought (the unification of numerical data for the composition of a simple problem). Moreover, by this method of analysis simple problems are extracted in the order in which they will be needed to compose a pattern and consequent solution. This is why such a method of analysis is called "synthetic." Although we recognize the noncorrespondence of the term with the essence of the synthetic method of analysis, we nevertheless retain it, so as not to bring confusion into this discussion.

Each method of analysis calls more than the analogical method of thought into operation. In the synthetic method of analysis, just as in the analytic method, there occurs a complex mental process, which certainly includes both synthesis and analysis.

F. Engels writes:

> Thought consists as much in analyzing objects of recognition into their elements, as in uniting the interrelated elements. There is no synthesis without analysis [1:40].

In Russian physiology, the works of its founder, I. P. Pavlov, also establish the unity of analysis and synthesis, effected by the large hemispheres of the cortex, a unity of the closed and analyzing apparati. Pavlov says that operation of the large hemispheres is reduced to

> constant analysis and synthesis of the irritations coming from both the external surroundings (this mainly) and within the organism . . . . The essence of the work of the cortex consists in analyzing and synthesizing the entering irritations [3:372].

"Synthesis and analysis of conditioned reflexes (associations)," writes Pavlov, "are essentially those same fundamental processes of our mental activity" [3:238].

However, the unity of analysis and synthesis in the philosophical, psychological and physiological sense of the word cannot be automatically translated into methods of analyzing problems--the synthetic and the analytic methods--predetermining their correlations in the pedagogical and methodological format. The continuous link of analysis and synthesis only helps us in more thoroughly studying the methods of analysis, their peculiarities and the difficulties of each of them.

In the theory and practice of teaching children problem solving, it is known, for example, that different methods of analyzing a problem are not identically assimilated by the children; it is much easier for them to use the synthetic method than the analytic method of analysis. One of the reasons for the relative ease in using the synthetic method is that pupils can deduce on the basis of concrete, contemplated facts, and/or data of the conditions of a problem, while in analysis the pupils' reasoning begins with the unknown, from the question of the problem, for whose answer there is always only one (and sometimes not even one) datum. The child makes a series of deductions (especially the initial ones) without the support of concrete data. For the child it is much more difficult to find premises for deductions than to draw conclusions on the basis of the facts at hand. Thus, the most difficult part of mental operation in solving problems—finding premises, or data, for deductions—is left for the child to do almost entirely in a statement of the question following the analytic method of analysis. The teacher says (analyzing a problem in class): "What data must be found? What must we know to answer the question?" (For a deduction, its premises—the data—must be sought out). In the synthetic method of analysis, however, the teacher asks what may be learned if we know such-and-such data (the premises or data are indicated by the teacher and a deduction must be made from them).

The "train of thought" in the synthetic method of analysis is simpler, since here the whole problem is not being solved, but only a part of it, and this part is often viewed without connecting it with the total content of the problem. In the analytic method of analysis the pupils must evince an ability to isolate simple problems with a definite logical connection and interdependence. In the synthetic method of analysis the simple problems are isolated without common guidelines and the work has no integral character. But in the analytic method the reverse is true--all the work has a more sensible, integral character; a chain of logical deductions is created.

Moreover, the ease of the synthetic method is enhanced by the technique of solving the problem. In the synthetic method a simple problem can be solved immediately upon its extraction from the complex one. Solution of the problem may parallel the analysis. But in the analytic method of analysis, one must extract all simple problems from a complex one, establish the order of their solution (compose a plan) and, only after this, may one approach the solution. In other words, when different methods of analyzing problems are used, mental operations are performed which are somewhat different in scope and nature, a fact which raises the question of the interrelationship of the methods of analysis in teaching children to solve problems. It must be said that there were no experimental data to answer this question. Every methodologist began primarily from his own personal experience and that of his predecessors. Nevertheless, the elaboration of the methodology of problem solving advanced. In our dissertation we follow the development of the topic, beginning with the first statements concerning methods of solving and analyzing problems made by P. S. Gur'ev, V. A. Evtushevskii, and V. A. Latyshev; we examine fervent statements in favor of the analytic method of analysis, voiced by A. I. Gol'denberg and E. Shpital'skii; statements by the outstanding methodologist of the nineteenth and twentieth centuries, F. I. Egorov, who recognized both methods of analysis; the original and unique opinion of S. I. Shokhor-Trotskii, who rejected both methods of analysis; and the further development of the problem by prerevolutionary methodologists V. K. Bellyustin, K. P. Arzhenikov, and F. A. Ern. Finally, we end our survey with an analysis of the positions of contemporary authors of methodological works in arithmetic for the elementary and auxiliary schools.

In discussing the epistemology of Russian methodology of problem solving, we start with recommendations of the founders of Marxism-Leninism concerning the relationship to the historical heritage of the past. Guiding us are the historic decrees by the Central Committee of the Communist Party of the Soviet Union concerning ideological questions which reveal the causes of errors and misinterpretations in ideological work. Holding to the principal positions coming out of the resolutions of the Central Committee, we have attempted to:

a) extract what is progressive and valuable in the methodological compositions on arithmetic by Russian authors regarding questions of teaching the methods of solution of complex arithmetic problems;

b) indicate the positive influences on the formation and subsequent development of original Russian theory and practice of teaching the methods of solution of arithmetic problems.

In their development of questions regarding the solution of complex arithmetic problems, Russian authors have formed a unique methodological notion whose characteristic features are: the use of simple unaffected methods of solution; the ability to begin the solution from a clear, profound and complete understanding of the conditions of a problem; the inculcation of devices of creative thinking; the presence of imaginativeness and independence.

As early as in the works of P. S. Gur'ev, the founder of Russian arithmetic methodology, we find an acute censure of mechanicality and banality in teaching children problem solving, a statement against the memorization of special devices and methods of solution, against use of "particular mechanisms" in solving problems, etc. Gur'ev, opposing the mechanical habit of solving problems, recommends training children in "reasoning ability," and gives in his Handbook "patterns of reasoning" in problem solving. In the author's opinion, a child should not memorize prefabricated methods of solution, but should teach his mind "to test its strength from the early years." Gur'ev stresses the development of creative spontaneous activity and independence in children. These progressive ideas of his, expressing the dogmatism reigning in the school at that time, later had a positive influence on the development of the Russian methodology of arithmetic. Most of the later authors, in elaborating the questions of teaching children problem solving, concentrate on developing the children's ability to reason, or, as Gur'ev liked to say, "to conduct the whole solution by intellect." To attain this goal, the Russian methodologists first posed and successfully resolved a series of problems in teaching problem solving, and pointed out various methodological means and devices.

V. Evtushenskii first pointed out the essential requirement of the problems presented to the child -- their concrete content should correspond to real life situations familiar to the child--and the great importance of diversity in methods of solving problems—the replacement of one device by another, etc.

V. Latyshev opposed mechanicality in teaching children problem solving more substantially and more strongly than did others. He first pointed out the most powerful way to fight the mechanical habit of problem solving—analysis of the conditions of the problem. This is the greatest achievement of Russian methodological thought. The leading method of teaching problem solving, which has been strengthened in the Russian school, is the method of preliminary analysis of the conditions.

After Latyshev had established and emphasized the importance of the analysis of the conditions in developing firm skill in solving problems, the methodologists' aspirations were directed toward the more felicitous use of methods of analysis, toward increasing their efficacy. Among the pedagogs and methodologists there were diverse opinions concerning the methods of analysis. Even when the method of analysis was being originated, there were followers of the predominantly synthetic method (V. Latyshev, K. Arzhenikov) and followers of the predominantly analytic method (A. Gol'denberg, E. Shpital'skii)

However, the farther the development of problem-solving methodology went, the more clearly did the methodological belief in the necessity of using both types--the synthetic and the analytic--prevail. The "equivalence" of these methods was accepted by most authors. We find statements to this effect in the writings of F. Egorov, V. Evtushenskii, V. Bellyustin, F. Ern, et al. Combinations of both methods of analysis were viewed by them as more useful than either one alone for developing the pupils' thinking and for developing firm skills in solving problems. Skill in solving problems was understood by these authors as the inculcation of devices of creative thinking. The analytic method of analysis was shown to be the most powerful means for attaining the goal of developing the mental capability of pupils, but the significance of the synthetic method was not diminished. Children should be taught both analysis and synthesis.

In their consideration of the question, the authors of the Soviet period carried out further development, much better than had been formulated by authors of the prerevolutionary period; the leading method--the method of preliminary analysis of the conditions--was established to be the most expedient method of teaching problem solving, and a more felicitous correlation of the methods of analysis--synthetic and analytic--was established; the ways and time of using the methods of analysis were also ascertained, increasing their efficacy in teaching the children.

In contrast to the foreign bourgeois "theories" of the development of mechanical skill in solving problems, methodologists of the Soviet period supported the idea of the development of skill in the creative solution of questions, on the basis of a profound and complete understanding of the conditions, or as the development of skill on the basis of the comprehensive development of pupils' mental capabilities.

The Soviet methodological notion, at the very beginning of its development, led the relentless struggle with reactionary Anglo-American utilitarianism, narrow practicalism, and narrow-minded attitudes. The positions of Soviet authors, perfect in principle, were the focus of attention in teaching children the preliminary method of analyzing the conditions of problems, that is, in that method of instruction which is ignored or wholly rejected by bourgeois investigators and is considered "excess ornamentation" (Thorndike).

The preliminary analysis of the conditions is considered by modern Russian methodologists the principle step in solving problems. It permits the attainment of the educative and practical goal of arithmetic instruction, permits the promotion of a thorough mastery of mathematical concepts and forms, which reflect the real phenomena of the surrounding world, and thereby helps in the development of the pupils' dialectic thinking. The problem-solving process is regarded by contemporary authors as a complex analytic and synthetic process. Synthesis and analysis appear here as two aspects of one cognitive process.

From this dialectic position the authors draw the following methodical conclusions: I. N. Kavun, N. S. Popova, E. S. Berezanskaya recommend the analytic-synthetic device primarily, while not excluding the individual use of both synthetic and analytic methods; N. N. Nikitin, A. S. Pchelko, V. L. Emenov, V. T. Snigirev, and Ya. F. Chekmarev recommend the individual use of both methods. By such instructions in problem solving the authors consider it possible to direct their attention to making first one and then the other method (synthesis or analysis) the major, "stronger," leading method. By this they presume to teach the children to use both synthetic and analytic methods of thinking. In connection with this approach, N. N. Nikitin, with the strongest determination of the Russian authors, supported the use of both methods of analysis in the first year of studying the solution of complex arithmetic problems.

In the auxiliary school the problem of mental development is most closely related to the problem of correcting the personality of the child. The analysis of the conditions as the step most important for the pupils' mental development in solving problems is recognized as the leading method of instruction in the auxiliary school (N. F. Kuz'mina-Syromyatnikova). Both methods of analysis are recommended as necessary for use at the beginning when teaching children to solve complex arithmetic problems; in grades 4-7 preference is given to the analytic method of analysis, which is more important than the synthetic method in the pupils' mental development.

In Table 1 one may see the firmly established methodological line on the correlation and time of application of the methods of analysis. Taking into account data from our investigation of the literature, we constructed an experimental study of the question, and we now begin the presentation of its results.

TABLE I

METHODS OF ANALYSIS AND THEIR USE IN INSTRUCTION

Author	Preferred Method of Analysis				Year of Instruction for Which a Method is Recommended			
	A	S	A,S	A-S S-A	1	2	3	4
Evtushevskii, V. A.			A,S[1]		S	A[2]		
Latyshev, V. A.		S			S	S	S	S
Gol'denberg, A. I.	A				A	A	A	A
Shpital'skii, E.	A				A	A	A	A
Egorov, F. I.			S,A		S → A			
Shokhor-Trotskii, S.I.	Rejected the method of analysis							
Bellyustin, V. K.			S,A		S,A	S,A	S,A	S,A
Arzhenikov, K. P.		S			S	S	S	S
Ern, F. A.			A,S		S	S,A[3]	A,S	A,S
Volkovskii, D. L.			S,A		S	S,A	A,A	S,A
Kavun, I. N. & Popova, N.S.				A-S	A-S	A-S	A-S	A-S
Berezanskaya, E. S.				S-A	S-A	S-A	S-A	S-A
Emenov, V. L.			A,S		S[4]	S,A[5]	S,A	S,A
Nikitin, N. N.			A,S		A-S	A,S	A,S	A,S
Pchelko, A.S.			S,A		S,A[6]	S,A	S,A	S,A
Polyak, G. B.			S,A		S,A	S,A	S,A	S,A
Snigirev, V. T. & Chekmarev, N. F.			S,A		S,A[7]	S,A	S,A	A,S
Kuz'mina-Syromyatnikova, N.F.			S,A		S,A[8]	S,A	A,S	A,S

Key: A = analytic, S = synthetic, A-S = analytic-synthetic,

S-A = synthetic-analytic.

Notes:
1. Evtushevskii calls them "first and second devices of solution."
2. From synthesis, gradual transfer into analysis.
3. Analysis of problems already solved by synthesis.
4. And preparation for analysis.
5. In the beginning of the year, analysis of problems previously solved by synthesis, at the end of the year, analysis may not be preceded by synthesis.
6. Partial, abbreviated analysis and analysis of problems not already solved.
7. Analysis of problems previously solved by synthesis (before the end of the third year of instruction).
8. Instruction in solving complex problems is begun in the second grade of the auxiliary school.

Characteristics of the Analysis and Solution of Arithmetic Problems by Pupils of the Auxiliary School (Summary data of the Experimental Research)

Methods and Results of the Experimental Research

Finding the difficulties encountered by children who have a sharp disorder of intellectual operation, which involve a complex mental process such as that of the solution and analysis of a problem, demands not so much quantitative as qualitative analysis. Children often arrive at identical solutions (correct or incorrect) by various routes. Accenting the qualitative analysis, we are limited, therefore, to a relatively small number of pupils (26) and problems they solved (a total of 521 solutions were obtained).

Following one of the main didactic principles of auxiliary instruction--the principle of the individual approach—we took children with various symptoms for our studies:

a) children with inborn feeblemindedness—oligophrenia, the incomplete development of psychic processes caused by prenatal brain damage or brain damage in the very earliest stages of development;

b) oligophrenic children suffering from additional brain damage (infection, trauma);

c) children with cases of acquired feeblemindedness (8 persons), diagnosed as the result of sustained brain damage (infection, trauma), characterized by a sharp distortion of the psychic processes (predominantly intellectual processes).

Individual lessons were held with pupils who 1) had just begun systematic instruction in solving complex arithmetic problems (grade 3-b), and 2) had already had experience in solving complex arithmetic problems (grades 5-7). To facilitate greater accuracy in comparing results of solution, we took pupils from each grade who were excellent, average, and poor problem solvers. The same problems were given to all the pupils in a single grade. In selecting the various types of problems to be given each pupil, one aim was pursued--to follow, as completely as possible, the analyzing process. For this we considered it necessary to make a preliminary study of the elements comprising analysis (analysis and synthesis) and the processes with which analysis of a problem is most tightly linked (the processes of solving and recognizing a problem).

We made it our primary goal to ascertain how pupils are able to solve problems without interference by the teacher. Does the pupil understand the idea of the problem, or its differences from other problems? With this in mind, we composed the first series from the following four types of problems:

1. ordinary problems, of average difficulty, familiar to the children from instruction,
2. problems with insufficient numerical data,
3. problems with superfluous numerical data, and
4. problems without numbers.

As can be seen, along with ordinary problems that are solved daily in class (problem 1), we proposed problems of a new type (2, 3, 4). The idea is that in solving unstereotyped problems the essence of the solution process and the peculiarities of the child's train of thought are profoundly revealed. We shall see below that the differences in children's solutions, which escaped us during their solution of ordinary problems of average difficulty, come to the surface as soon as the children begin solving problems with an altered form or content. Moreover, the aggregate of problem solutions of series I serves as supplementary material for the study of the fundamental question of the experimental investigation--the synthetic (problem 3) and the analytic (problem 2) methods of analysis.

Before solving the problems of series I, the children were given identical instructions to "solve the problem aloud," but the teacher did not repeat the request if it hindered the pupil in solving a problem. The pupils' solution processes were recorded.

Before approaching the study of the very process of analyzing complex arithmetic problems directly, we learned how pupils do the assignments which are the basis of analysis. With a view to this we composed two more series of assignments, II and III.

In the second series of assignments we gave questions from which it was necessary to select numbers and solve a problem. An aptitude more necessary for the analytic method of analysis was evinced here.

In the third series of assignments we gave only conditions without questions. The pupils were required to state a question fitting the conditions and to solve the problem. Here it was possible to observe one of the major steps of the synthetic method of analysis.

In the fourth series of assignments conditions of problems ordinary (curricular) in their mathematical content and form were actually analyzed. Here it was possible to observe the entire process of analysis directly, as well as its influence on success in solving a problem following the analysis.

The Solution of Ordinary Problems Familiar from Instruction, Problems with Insufficient Numerical Data, Problems with Superfluous Numerical Data, and Problems without Numbers

Third Grade (series I)

The third grade pupils were given the following problems:

1. Forty-five Pioneers went off to a camp along with 23 fewer Octobrists. How many children went to camp?

2. Someone bought 15 albums at 5 rubles apiece and 20 pencils. How much was the total purchase?

3. In a basket there were 75 lemons and 25 oranges. 45 lemons were sold. How many lemons were left in the basket?

4. Pupils were sitting at their desks. One pupil sat at each desk. One half of the desks were not occupied. How many more desks were there than pupils?

Problem 1 was given in usual form, familiar to the children from school instruction. Problems 2, 3, 4 were new for the children; as a rule such problems are not solved in school. Of the four problems of series I, problem 1 was most successfully solved; next came problem 3. Only the best pupils could solve problems 2 and 4.

All the children except Sveta P. and Lena K. answered problem 1 correctly. Despite the identical answer, however, explanations varied for the correct solutions. Lesha S., Yulya S., Nina K., and Vera K. explained their solutions more precisely than the others and asked more logically appropriate questions. Yulii P. and Valya Ch. could not explain their solutions, although they were also correct. Sveta P.

did not explain her wrong solution. What is the reason for this?

We can find a partial answer to this question by analyzing the course followed by the solutions. It appears that the children came to identical results by different routes, by penetrating the idea of the problem in different ways. The pupils who were unable to explain their solutions generally relied, in the solution, not on the concepts of the conditions, the connection and dependence between the data, but on the performance of arithmetical operations in a stereotyped pattern. They hurriedly used one prepared scheme, taken from the solution to problem 1, to solve all the problems. In other words, independently from the conditions of the problem and its question, they performed the same operations with numbers and took them from the conditions in precisely the same order. Their success or failure in the solution depended greatly on the extent to which the selected order of arithmetical operations satisfied the solution of the given problem. In this the pupils held to the stereotyped solution so much that they were unable to leave it, even when there was no numerical data and appropriate story in the conditions for using the chosen stereotype. Then the data were either invented or used twice, and the problem was still solved in its conventional pattern. Such solutions were given by Yulii P., Valya Ch., and Sveta P.

Valya Ch., copying the stereotyped pattern, substituted the main question of the problem for the first question, and the last words of the conditions (those preceding the question of the problem) for the second:

Correct questions (problem 1)	Questions stated by Valya Ch.
1) How many Octobrists went off to camp?	1) "How many children went off to camp?" [Copied from the conditions.]
2) How many children went off to camp all together?	2) "With 23 fewer Octobrists." [Copied from the conditions.]

She also stated such questions in her incorrect solution to problem 3. It is characteristic here that the child's first reasoning was more closely connected with the conditions of the problem. But the more she struggled with the solution, the further she strayed from the conditions and the more firmly fixed she became on the course of the chosen stereotype. In such a case (if the plan used by the pupil diverges from the correct one) the solution does not correspond to the conditions of the problem, the answer obtained does not answer the question of the problem, and the solution is explained incorrectly.

Sveta P., the poorest pupil, used the same stereotyped pattern for the solution of all four problems, more stubbornly and consistently than did Yulii P. She could not explain a single solution correctly, including those solutions coinciding with the stereotype and, hence, accidentally correct (solution of the second question of problem 1 and the second question of problem 3). Sveta P.'s stereotyped character of the solution is obvious when the following four problems, different in content and form, are compared:

Solution to problem 1:
1) 45 Pioneers + 23 Pioneers = 77 Pioneers
1) There were 77 Pioneers
2) 45 Pioneers - 23 Pioneers = 32 Pioneers
2) 32 Pioneers

Solution to problem 2:
1) 20 pencils + 5 pencils = 25 pencils
1) 25 pencils
2) 25 pencils - 15 pencils = 10 pencils
2) 10 pencils

Solution to problem 3:
1) 75 lemons + 25 lemons = 95 lemons
1) 95 lemons
2) 75 lemons - 45 lemons = 12 lemons
2) 12 lemons

Solution to problem 4:
1) 12 desks + 2 desks = 14 desks
1) 14 desks
2) 14 desks - 4 desks = 10 desks
2) 10 desks.

It is not hard to see that, despite the completely different conditions, the solutions to problems 1, 2, 3 and 4 are similar and are performed according to the following stereotyped pattern:

1) From the conditions two numbers are taken and added. The first question in all 4 problems is answered thusly, despite the fact that to answer the first question of problem 1, subtraction, not addition, is needed; for problem 2, multiplication; for problem 3, subtraction; and for problem 4, if it were to be solved with numbers, division.

2) Then the third number, heretofore unused in the problem, is subtracted from the result obtained in 1. If there is no third number (problem 1 has only two numbers), subtraction is performed using the same numbers as in the addition or an invented number is subtracted (problem 4).

Thus to solve a problem according to a stereotyped pattern, the numbers alone are taken from the conditions. Further, the solution is carried out almost independent of the given conditions of the problem or of its main question. But if no numbers are given in the conditions, the conventional solution has no relation to the proposed problem (problem 4). Of course the results of the solution obtained in this manner cannot be explained; actually the subject has not stated a single question in the solution.

Lena S., Yulya S., and Nina P. arrived at a solution by a different route. Even the weakest of them, Nina P., brought her solution into closer, and more precise, agreement with the conditions of the problem. She tried to base the solution upon the relationships of the data and the question of the problem. In accord with her work, she recognized more fully the conditions of the problem and made the questions more precise: "I found out how many... Pioneers... went off all together." She read the conditions, thought a little, then correctly explained the solution: "How many Octobrists and Pioneers went altogether?"

Thus we see qualitatively different beginnings in the pupils' skills in solution being formed:

a) Some pupils quickly learn from instruction a definite model for the solution of all problems, changing it little to conform to the conditions of a problem; and if they do change it, they do so on the basis of the external, formal features of the conditions (Yulii P., Valya Ch., Sveta P.).

b) On the other hand, others try to understand the conditions of a problem and produce a solution as far as they are able. They take fewer unsubstantiated and misunderstood steps and perform fewer operations which they cannot explain (Yulya S., Lesha S., Nina P.). These pupils also evinced a weakness--but a different kind--in the solution. A complete, profound understanding of all the conditions of a problem was still not always accessible to them. The understood separate relationships were not interconnected in the general plan of the solution to a problem. Sometimes (when solving a new and difficult problem) the parts of the conditions which are correctly understood are not enough to solve the whole problem. In such a case the pupil carries out the solution according to the recognized part, without finishing it and without answering the question of the problem. Along with these differences, there is a common feature characteristic of all pupils, a point at which their paths of solution converge:

a) All pupils begin the solution by using the arithmetical operations and numerical data as a guide, and giving little attention to the question of the problem. Some combine the numerical data existing in the conditions according to a prepared stereotyped pattern, while others, guiding themselves by numerical data and trying to understand the relationship between them, construct a plan of solution which is new and original in each problem. In the latter case the recognition of the conditions determines the course and the result of the solution.

b) If the pupils come to understand the conditions and the plan of solution of the problem, it is only as a result of carrying out individual solutions of question after question. Thus the question of the problem is realized <u>post factum</u>--after the solutions possible from the conditions have been carried out. Thus the question of the problem is stated, depending on how it is obtained, after the pupil has blindly solved all possible individual simple problems.

There is a greater possibility in problem 1 of reaching the answer to its question by combining numbers according to a prepared, already-mastered scheme of solution, as well as by a more conscientious solution of consecutive questions. This is because in the first case the prepared, mastered solution pattern coincides with the plan of solution of problem 1, which was given in the usual form, and in the second case both possible solutions eventually lead to the answer of the question of the problem. There are fewer such possibilities in problem 3, even fewer in problem 2, and none at all in problem 4, since in its conditions there are no numerical data which, on being combined, could be fitted into a pattern or could be used to answer the questions blindly.

Primarily for this reason the problems fall into a certain sequence (1, 3, 2, 4) according to the success of the pupils' solution plans. Both individual and general distinctions, particularities of solution characteristic of all third grade pupils, give us a basis for concluding that.

c) Solving the given problems of series I, all the third grade pupils followed the methods of synthesis. The solutions to problems 3, 2 and 4 are illustrative in this respect. The solutions to problem 3 differed very little from each other: average and poor pupils made all possible solutions from the existing numerical data (also utilizing superfluous data) without considering the question of the problem. Strong pupils, too, found themselves on this course at first.

Among the solutions to problem 3 there is a single original solution in which Vera K. used all numerical data, including the superfluous data, but obtained the right answer to the question of the problem:

1) 75 fruits + 25 fruits = 100 fruits.
   How many fruits were there altogether?

2) 100 fruits - 45 fruits = 55 fruits.
   How many fruits were left?

3) 55 fruits - 25 fruits = 30 lemons.
   How many lemons were left?

Instead of the correct and shorter method:

1) How many lemons were left?

   75 lemons - 45 lemons = 30 lemons.

But the method of synthesis, characteristic of most pupils, was observed quite clearly when they were solving problem 2, although they had not enough data to answer the question of the problem. All pupils approached problem 2 in the usual way, answering question after question without considering the question of the problem. In doing so, a) some pupils did not note and did not indicate why the solution was impossible, and incorrectly solved the problem in two steps (Yulii P., Valya Ch., Nina P., Lena K); b) others (the better ones) correctly solved one question from the data of the conditions and did not begin to solve the problem incorrectly, but indicated why the problem was unsolvable (Lesha S., Yulya S., Vera K.).

Incorrect two-step solutions given by the pupils in (a) above are: (1) addition of the number of albums to the cost of one album (solution of the first question), and (2) addition of the obtained result to the other unused number given in the conditions—the number of pencils (solution of the second question).

The pupils indicated why the problem was unsolvable in various ways. Lesha S. and Yulya S. themselves pointed out the reason the problem could not be solved and found the cause in the lack of data. They noted the pattern of the solution: they asked the first question correctly and solved it; they stated the second question correctly, but, not finding the datum needed to answer this question, they said the problem could not be solved.

Here is the solution of Lesha S.:

1) How much do the albums cost?
   15 x 5 = 75 rubles

2) How much do the pencils cost?
   The problem can't be solved—there is no number for the cost of the pencils.

Yulya S. solved the problem similarly:

1) 15 albums x 5 = 75 rubles

2) The problem cannot be solved because we do not know how much a pencil costs.

Vera K. pointed out why the problem was unsolvable without such an analysis of the conditions of the problem, and for a long time could not understand that it was impossible to find the cost of the pencils from the given conditions. She noticed the insufficient data: "There's something here I don't understand. It doesn't say here how much they paid for the pencils." But for a long time she could not understand that this is exactly why the problem could not be solved.

"In the first question we must know how much they paid for the pencils. We can solve it! We take 5, well, maybe 15, no 20. In two stages... I can't solve it. 5 multiplied by 20! No, those are albums. We could probably find out the cost of the pencils, but I don't know how. . . ."

And again she tried to determine how much was paid for the pencils. Finally she was asked "But can you really solve this problem if the cost of the pencils is not given?" She wrote: "This problem cannot be solved; we don't know how much was paid for the pencils."

A strong pupil, Lesha S., solved the problem with insufficient data differently. She independently (not without difficulty and not without trial and error, of course) established why the solution was impossible. She correctly answered the first question of the problem (she indicated the data and the operation for it). Going on to answer the second question, she noticed the missing datum and asked: "How much do pencils cost in this problem?"

It is interesting that this new difficulty caused her to take another tack. She analyzed the second question according to the logical sense of the conditions: "How much do pencils cost?" Then she began searching the conditions for the answer: "30 plus 20? No! 20 taken 4 times? No! 20 taken... taken 5 times? No!" Having failed to select any one of these solutions, she began looking for a different way out: "Well, what is this, huh? What else is it? Where can I find the number? Where is the number telling how much the pencils cost? It's not here at all. Can you really solve it without it? No!" She wrote, "The problem cannot be solved: there is no number designating how much the pencils cost."

The results of the solutions of problem 4 also speak for the synthetic method of solution, which is characteristic of most pupils. Problem 4 (numberless) cannot be solved by relying on numerical data if one has not grasped the idea of the conditions. One cannot solve question after question gradually, blindly, and with no consideration for the question of the problem; numbers cannot be manipulated. It is also impossible to use a mastered stereotype for its solution. The proposed problem may be solved only by guess or by imagination, on the basis of the understood concept; hence it was of almost insurmountable difficulty for all pupils. With some guidance from the teacher, only Yulya S. solved it. All the children, however, tried one way or another to solve the problem; and, in their attempt, they revealed their general approach to the solution. In this respect we can divide the children into two groups:

a) Some of them did nothing understandable or recognizable. They either correctly solved the problem (Yulya S.), gave up the solution (all the strong pupils), or, not having given up, tried to solve it but completed no solution (Nina P.).

b) Others, average and weak pupils, made incorrect solutions (Yulii P., Valya Ch., Vova G., Sveta P.).

Valya Ch., the poorest pupil, without noting that there were no data to solve the problem, began the solution as soon as she had read the conditions (for the second time): "Add 6 to 6! I don't understand this problem. I don't even know why I don't understand it." But this did not stop her from continuing the solution according to the pattern which she used to solve all the problems with data: she added two numbers, then subtracted one of the summands from the sum obtained. She ascribed the question of the problem to the first solution and the end story of the problem to the second solution:

1) 6 pupils + 6 pupils = 12 pupils

2) 12 pupils - 6 pupils = 6 pupils

1) How many times more desks are there than pupils?

2) One-half the desks were occupied.

A similar stereotyped solution was given by Sveta P., but with different numbers and following her own stereotype.

1) 12 desks + 2 desks = 14 desks	1) 14 desks
2) 14 desks - 4 desks = 10 desks	2) 10 desks

Incorrect solutions by average pupils differ from these two solutions.

Vova G. correctly understood one relationship which was most clearly shown in the conditions ("one-half the desks were occupied"). She correctly used it with invented numbers. She thought up the number 100, then subtracted half of it--50.

1) 100 desks - 50 desks = 50 desks.

"I know that they were sitting [pause] at half the desks." But the pupil went no further than one understood relationship, and her subsequent reasoning and solution were wrong. She used the solution to find what she herself had invented--the number of desks:

2) 50 desks + 50 desks = 100 desks

2) How many desks altogether?

In solving the problem Yulii P. noticed the lack of data: "But there aren't any numbers here!" He tried to determine one of the missing data--the number of children. In his attempts he returned (although he did not realize it) to his stereotyped solution of the problem in two stages: "16! Add 4 to 16! 100 : 2! But we must know how many children there were! We have to add 9 to 21, we get 30 pupils, now add 21 to 30 and we have 51." According to pattern he did the solution:

1) 21 desks + 9 desks = 30 desks
2) 21 children + 30 children = 51 children

As we can see, one correct solution was given at first--100 ÷ 2--but it was forgotten and was not cited as a solution; the attempt to solve the problem by stereotype drove it away. It is interesting that when the solution had been done according to the stereotyped pattern the subject tried to make it agree with the conditions of the problem being solved; since the conditions spoke about desks, in the first solution he corrected the word "children" to read "desks," and held to this.

Fifth Grade (series I)

Pupils of the fifth grade were given these four problems:[5]

1. A young Pioneer leader had 150 rubles to buy theater tickets. He bought 37 tickets at 2 rubles 50 kopeks each and 12 tickets at 3 rubles 50 kopeks each. How much money did the leader have left?

2. Ten boxes of apples, each containing 32 kilograms, and 5 boxes of pears were brought into a store. How many kilograms of fruit were brought into the store?

3. Potatoes were being transported in trucks. The first truck carried 3 tons, the second 5 tons, and the third carried half as much as in the second truck. How many tons of potatoes were being carried in the first and second trucks together?

4. There are books in two bookcases, an equal number on each shelf. In the second bookcase there are twice as many shelves, and twice as many books on each shelf. How many times more books are there in the second bookcase than in the first?

The fifth graders most successfully solved problem 1 and then problem 3; problems 2 and 4 were the most difficult for most of the

[5]Problems for the fifth and seventh grades are more complex than the problems given the third graders, according to the requirements of the curriculum.

pupils. The general features of solution characteristic of all the pupils, as well as the features characteristic of individual third graders were present in the pupils of the fifth grade as well. Among the latter there were also:

a) Children, who in learning technique, master the devices for solving identical problems no worse than other children, but stubbornly apply these devices to all other problems, often not taking into account the requirements of the conditions of the given problem. The devices, both feasible and unfeasible, differ little from each other, and the children introduce much from themselves into the problem, especially in solving problems that cannot be solved by ordinary methods.

Such solutions were given by Kolya D., Tonya L., Vita K., Nina Ch., and Nina Ts. In solving problems given in unusual form, these pupils took more unrecognized, incorrect steps (the solutions diverge from the questions stated) than other pupils did. In observing these pupils it may have appeared that they solve problems with conviction, doubting nothing; but in truth, their solutions were reached by a prepared, accustomed pattern. The pupil often cannot abandon the initially accepted, incorrect plan even when he himself notices the error. Thus Nina Ts. incorrectly solved problem 1 in two steps:

1) How much money did the leader spend for 37 tickets?

$$\begin{array}{rl} & \text{2 rubles 50 kopeks} \\ \times & \underline{\text{3 rubles 50 kopeks}} \\ & \underline{\text{6 rubles 250 kopeks}} \\ & \text{8 rubles 50 kopeks} \end{array}$$

2) How much money did he have left?

$$\begin{array}{rl} & \text{150 rubles 100 kopeks} \\ - & \underline{\text{8 rubles 50 kopeks}} \\ & \text{142 rubles 50 kopeks} \end{array}$$

Having gotten on the right track (first question), she could not continue the solution correctly, although she noticed her mistake. Before she finished answering the second question she said, "I didn't do it right... you have to subtract, but I didn't know how many tickets there were altogether. They should be added." But she able to correct the solution.

The solution by Kolya D. is characteristic of this group of children. He thought about the conditions for a long time; reread them several times, circled the letters of the conditions with his pencil, and finally said, "I don't understand... I can make out the words, but I don't understand." But this does not stop him from beginning to solve the problem. Although he worked very slowly he did not stop until he had gotten a solution.

1) How many 2r 50k tickets did she buy?

$$\begin{array}{r} 2r\ 50k \\ +\ \underline{3r\ 50k} \\ 6r \end{array}$$

2) How many 3r 50k tickets did she buy?

$$\begin{array}{r} 6r \\ +\ \underline{3r\ 50k} \\ 9r\ 50k \end{array}$$

3) How many tickets did she buy for both 2r 50k and 3r 50k together?

$$\begin{array}{r} 9r\ 50k \\ +\ \underline{6r\ 50k} \\ 16r \end{array}$$

4) How much money did the leader have left?

$$\begin{array}{r} 150r \\ -\ \underline{16\ r} \\ 134r \end{array}$$

Answer: 134 rubles

The questions stated independently by the pupils differ in the degree of their ambiguity. It is not clear what the pupil wanted to find--the number of tickets or their costs. This was not clear in the solutions.

The formulation of the first, second, and third steps contains an indication of definite numbers of tickets, but there is not a single number in their solutions that is labeled "tickets." Moreover, the number of tickets given in the conditions is not used. All solutions are done using only the cost of the tickets and the amount of money which the leader has.

The stated questions recall the correct plan of solution. The first question mentions tickets priced at 2r 50k, the second mentions tickets at a price of 3r 50k, and the third mentions "both 2r 50k and 3r 50k together"; the fourth question is the question of the problem. The answers to these questions, however, are far from correct. Of the four solutions to the four questions, not one was correct. In solving the problem the pupil tried unsuccessfully to penetrate and understand its conditions; he made an unconscious and unclear plan of the solution.

b) Children who are trying to solve a problem by beginning with its conditions do only what they are able to do from their understanding of the conditions. True, the completed solutions are sometimes not sufficient to answer the question of the problem, but they do not diverge from the concrete data of the conditions. Such solutions were given by Tolya A., Manya V., and Volodya M.

Tolya A. began the solution by determining the number of tickets bought. Having solved this simple but superfluous problem, he stopped and thought. He correctly noted the second and third questions and solved them. But, having still not determined the total amount of money spent, he wanted immediately to answer the question of the problem:

1) How much money did the leader have left?

$$\begin{array}{r} 92r \quad 50k \\ +\ \underline{37r \quad 50k} \\ 130r \end{array}$$

He noticed and corrected his error: "I didn't write that right. I wanted to find out how much he paid for all the tickets. I'll do the question over again. Although everything is correct, I have to subtract in the last question instead of adding."

Tolya A. slowly and indirectly came to the answer to the problem, like Kolya D., Tonya L., and Nina Ch., but took unnecessary and incorrect steps toward an independent analysis of the conditions and toward the solution of the problem. Everything superfluous was discarded, and errors were corrected independently.

He solved problem 2 similarly, with doubts and suppositions. He read the conditions, began to wave his pencil, and was nervous. "But how can I write it here? There aren't any pears, only some kilograms. Are there as many pears as there are apples? We can find out about the apples...." He began to "find out about the apples," solved the first question, wrote "2," and thought. "We can make the same number of pears as apples, also 32 kg. But if they aren't made to be equal, you can't solve it." He assumed that the weight of the boxes of apples and pears was equal, and with this assumption he solved the problem in three steps.

A feature common to the fifth graders as well as to the third graders was their attempt to solve the problem in the usual manner--by synthesis. All the pupils began solving problem 2, and all solved the first question, solvable from the numerical data of the conditions; but they continued the impossible solution differently. Tonya L. and Vova M. wrote the second question of the problem and solved it incorrectly. Manya V., Kolya M., and Nina Ts., after solving the first question and finding no data to solve subsequent questions, gave up the solution.

Unlike the third graders, some of the fifth graders evinced an ability to compose a plan of solving the problem according to the logical sense of the conditions, without support of numerical data. But because numerical data were not given for the solution of the second and third questions of the plan, all pupils gave correct questions but incorrect solutions. (Kolya D., Vitya K., Tolya A., Vova M., and Nina Ch.).

The synthetic method was clearly used in the fifth graders' solutions to problems 3 and 4. Without considering the question of the problem, the children worked out all possible superfluous solutions.

a) Tonya L., Tolya A., Nina Ch., Vova M., and Nina Ts. stated and solved a superfluous question, then, independent of it, they solved the question of the problem:

1) How many tons of potatoes were in the third truck?

5t ÷ 2 = 2t 500 kg*

2) How many tons of potatoes were in the first and second trucks?

3t + 5t = 8t

b) Kolya D. and Manya V., after solving superfluous questions, solved the question of the problem, incorrectly selecting the data (using the results of the superfluous problems).

Kolya M. was an exception. He answered the question of the problem directly, without any superfluous solutions: "But here there are three trucks, but it says only two. Why doesn't it ask about the third truck? There's nothing to solve here! Look! There's 3 tons in the first and 5 tons in the second. Nothing's written about the third one. It's just a first grade problem!"

The simple, numberless problem 4 forced the children to reason, to solve the problem without the opportunity to rely on numerical data. Only two pupils could handle this problem, Vitya K. and Tolya A. Most of the pupils gave up on it (Tonya L., Manya V., Nina Ts., Kolya M.) or gave incorrect solutions (Kolya D., Vova M., Nina Ch.). The absence of numbers in the conditions made it impossible to solve: "You can't solve this problem... it doesn't say here how many books there are... no numbers are given. The problem is awkward. No, it can't be solved anyway" (Nina Ch.).

Incorrect solutions (Vova M., Nina Ch., Kolya D.) represent an attempt to note the questions of the problem by arranging the parts of the conditions in a logical sense, disregarding the existing data and the incorrect solution of the questions thus stated.

1) How many books were on the first shelf?

2 shelves x 2 shelves = 4 shelves

2) How many more books were there in the second bookcase than in the first? (Kolya D.)

*t = metric ton (Ed.)

Seventh Grade (series I)

The seventh grade pupils were given the following four problems:

1. A ship traveled 240 kilometers in 12 hours. In how many hours will it travel 800 kilometers?

2. The floor of a room has to be painted. The room is 7 meters long and 3 meters high. How much does it cost to paint the floor if 1 square meter of paint costs 75 kopeks?

3. A worker earns 18 rubles a day. What is his monthly wage if he works 25 days a month at 6 hours per day?

4. A kolkhoz* sowed a plot with oats and two such plots with wheat. They harvested 4 times as much oats as they sowed and 8 times as much wheat. How much more wheat did they harvest than oats?

Observation of the seventh graders was hindered because they began reasoning aloud only after the plan of solution had almost been completed. We purposely did not interfere with the pupils' solutions, knowing that our request to reason aloud would significantly distort the natural process of solution—speechless analysis, a plan of internal speech—which occurs in children of this age. Therefore the seventh graders' reasoning does not fully reflect their train of thought during the solution, but is more the end result of mental activity on the problem. Children of grades 3 and 5 easily and willingly solve problems by reasoning aloud. The seventh graders preferred to do everything silently and only rarely stated their thoughts (doubt, something not understood, etc.).

Seventh graders, like the third and fifth graders, solved problem 1 most successfully, and then problem 3; problems 2 and 4 were almost impossible for them. The attempt to arrive at a solution on the basis of the numerical data, and not on the basis of understanding the requirements of the question of the problem, was also a characteristic of the seventh grade pupils.

This can be seen even in the solution to problem 1, and even more clearly in the solutions to problems 2, 3, and 4. Solving problem 1, all pupils correctly answered the first question (numerically), but they experienced difficulties, and three pupils even made errors, in

*kolkhoz - a collective (Trans.)

solving the second question of problem 1, applying the numerical data which already existed in the conditions, to the question of the problem (Kolya S., Tamara Sh., Katya B.).

All the pupils solved problem 2 wrong, taking the height of the room for its width. Problem 3, solved in one step, was begun in two steps by all pupils, that is, they made a superfluous solution ("superfluous synthesis"). But during the solution Nina M., Yura B. and Ira G. realized that the problem could be solved in one step. Then they discarded the incorrect solution they had begun and correctly answered the question of the problem.

Unlike the third and fifth grade pupils, the seventh graders, using the characteristic method of synthesis in solution, evinced:

a) an ability to resort to another means of solution—to note a plan of solution following the logical content of the conditions. But their plan often did not agree closely enough with the numerical data in the conditions for the solution to be correct.

Illustrative in this respect are the solutions to problem 4 by Ira G. and Katya B.:

Katya's solution:

1) How much oats was harvested?

4 oats x 4 = 16 oats

2) How much wheat was harvested?

8 oats x 8 = 64 wheat

3) How much larger was the harvest of wheat than that of oats?

64 wheat - 16 oats = 48 wheat Answer: 48 wheat

The questions of the plan were stated by the pupil on the basis of the ideas contained in the conditions rather than from the data itself. But the question of the problem was somewhat changed. As can be seen in the conditions, however, there are no numerical data to solve the questions of the plan composed by the pupil. The pupil erroneously replaced these data with the given relationships of the amount of grain; she did this for a purely formal reason: 4 times more oats were harvested than were sown. From this the pupil knows how much oats was harvested: "40 x 4 = 160." As we can see, the plan composed to solve the problem agreed neither with the conditions of the problem nor with its question.

b) They also showed that they hold less to a stereotyped pattern in the solution; they try to understand the conditions of the problem, and if they cannot, they make incorrect solutions, which are different in each such problem--they are not stereotyped.

Selection of Missing Numerical Data for the Conditions and the Question

Third Grade (series II)

The pupils of the third grade were given the following five problems to solve, for which it was necessary to select numerical data in advance:

1. Books and albums were bought.
   How much money was paid for the whole purchase?

2. Four books and one album were bought.
   How much was paid for the whole purchase?

3. Four books, at 3 rubles each, and one album was bought. How much was paid for the whole purchase?

4. Four books and one album were bought. Money was given to the cashier. How much change was received?

5. Books and an album were bought. 50 rubles was given to the cashier. How much change was received?

In the conditions of problem 1, besides the question, it is indicated that "books and albums were bought." As we see, in order to answer the question the cost of the albums and of the books is needed. The total cost of all may be shown, but the price of one book or of one album and the number of each bought is enough to determine the cost.

In problem 2 one numerical datum is given: "4 books and one album were bought." In selecting numerical data for this problem, the person solving it is shown the necessity of giving the price of one book (since the quantity of books is already given), of giving the price of an album and of solving the problem in two operations.

In problem 3 two numerical data are given: "4 books at 3 rubles each and one album were bought." Having given the price of the album, a pupil was able to solve the problem in two operations.

After solving three problems, a pupil was asked to solve two others—4 and 5. The content of the first problem ("books and albums were bought") is included in the conditions of these two problems, and a supplementary condition is given ("money was given to the cashier" in problem 4, and "50 rubles was given to the cashier" in problem 5). Finally, a question is stated for each problem: "How much change was received?" To solve this series of problems the pupils were required to choose numbers suitable to the existing questions and conditions. This type of exercise is recommended by methodologists as exercise preparatory to the analytic method of analysis of complex arithmetic problems. Let us see how our pupils handled these tasks.

Success in solving these problems was in direct proportion to the success of the pupils. Excellent pupils selected data and solved most of the problems correctly. Average pupils selected numbers, but they could not solve all problems. Poor pupils did not select numbers for all the problems, and they solved only two (problems 1 and 2) of the possible 5.

a) In selecting numerical data, the excellent pupils tried to imagine the concrete situation, the concrete object, and, determining its cost, to show it in the conditions of the problem.

The pupils selected numbers for the questions by imagining the concrete situation in life: "How many books do I get? How much does the Young Guard cost? Well, say 20 rubles" (Vera K., problem 5).

"There are books at 2 rubles each and some kind of album there for pictures; it costs 6 rubles" (Lesha S., problem 4).

"Don't you know how much an album costs in the store?" (Lena K., problem 1).

In this, the presented situation again is easily translated into a plan of arithmetical operations. Having thought up the numbers, the pupil returns to the conditions of the problem and solves it correctly.

b) Average pupils tried to understand the sense, the content of the conditions and did not try to imagine the concrete situation.

c) Poor pupils could do neither—they could not imagine a concrete situation, and could not understand and use the meaning contained in the problem. They selected the data according to conditions they made up, which were far from the conditions of the given problem.

<u>Sveta P.</u>'s solutions are characteristic in this respect. With help she selected numbers for problem 4 alone. She unsuccessfully tried to select numbers for problem 5. In choosing the numbers, the pupil worked very little on the basis of the conditions of the problem and made incorrect deductions. Reading problem 1, she stated a question in which data completely irrelevant to the problem were present: "Mama had 34 rubles. She spent 34 rubles. How much did she have left?" The pupil was asked to answer the question "What was bought?" This evoked new reasoning in her, and she thought up new conditions, but with the old question:

> 34 books and one album for 23 rubles... not so many books, say 24 books. How much money did she have left?

Here she gave the answer: "15 rubles.... Can the problem be solved?"

She began the solution with an incorrect question:

1) 34 books - 24 books = 10 books
   How many books did she have left?

2) 10 books + 20 books = 30 books
   For the entire purchase she paid 30 books.

The divergence of her reasoning from the conditions given is even more noticeable in her solution to problem 2. Having read the problem, she began building a totally different problem instead of selecting the missing numbers: "Mama bought 24 apples. . . ."

Sveta was reminded that it was not necessary to think up a new problem, but to choose numbers for this problem; but she continued what she had begun: "Mama had 13 rubles, she bought 10 apples."

She is asked to read the question of the problem. She reads it. What does it say in the conditions about what was bought?

"Mama bought 24 materials."

She is asked to read the conditions aloud. We repeat the question: "Mama bought 24 books and an album for 30 rubles."

Without indicating the price of the books, she began her incorrect solution. Books are added with rubles, and books are obtained:

1) 24 books + 30 rubles = 54 books

She explained her solution, "They paid 54 books for the album." She considered this result the answer to the question of the problem.

She approached the solution to problem 3 similarly. Having read the conditions, she distorted them, naming a different datum in place of the existing one ("4 books"): "Mama bought 50 books."

Experimenter: Read here; how many books were bought?

She reads the conditions aloud: 4 books and an album were bought.

Exp.: How many books were bought?

Pupil: 50 books and an album.

Exp.: No, that's wrong, read the conditions again.

Then she "corrects" herself: instead of "Mama bought 50 books," she says "Papa bought 50 books."

Again Sveta was asked not to make up a new problem, but to make up numbers for this given problem.

Exp.: What numbers are missing here?

Pupil: ..and one album.

Exp.: What were they buying, according to the conditions here?

Pupil: 4 books and 1 album.

Exp.: And does it say here how much they paid for each book?

Pupil: 4 books!

Exp.: 4 books were bought, and how much did they pay for a book?

Pupil: 3 rubles each.

Exp.: And how much was paid for the album? Is it given in the problem?

Pupil: No.

Exp.: Think up a number for how much they paid for the album.

Pupil: 3 albums. Now should I solve the problem?

Exp.: And now do you have all the numbers? What did they buy?

Pupil: 4 books at 3 rubles and 1 album at 3 albums! 3 albums at 3 rubles.

In solving problem 3 she gave the only correct answer to the first question, following the data in the conditions, but she erred in terminology: 1) "4 books x 3 = 12 books. How much was paid for the books?"

The number of books obtained ("12 books") in the second question was then added to the number of albums: 2) "12 books + 3 albums = 15 albums. How many albums were left?"

The solution to problem 4 also began with the reasoning "Papa bought 50 books.... The pupil bought 50 books and 13 albums." She added the invented numbers and thought she had found the number of books the pupil had bought: 1) "50 books + 13 books = 63 books. The pupil bought 63 books."

The problem ended with this solution. After this Sveta was asked to imagine the concrete situation. She was asked if she had been in a bookstore and bought anything.

"I was. I bought some notebooks at 20 kopeks each and an album for 3 rubles. I paid the cashier 4 rubles and got 40 kopeks in change."

Her action is compared with the purchase in the conditions of the given problem. The problem is elucidated: take a price, put the price here at which they bought books and albums above, and how many they bought, and then solve the problem. After this she selected all the data correctly. Nevertheless, she still could not solve the problem: she added the price of the books and the album and subtracted the result from the money given the cashier (she was doing the solution according to her own model).

She also selected the numerical data for problem 5 by recalling a situation similar to that in the problem, but not quite identical to the one described in the given problem:

"A teacher bought 80 books and an album." She writes: "80 books." "And paid 30 rubles for the album and 5 rubles for the books." According to these figures, the purchase will amount to 430 rubles.

5 rubles x 80 = 400 rubles
400 rubles + 30 rubles = 430 rubles.

But in the conditions it says that only 50 rubles were paid. This noncorrespondence is not noticed by the pupil. The solution greatly diverges from the conditions:

1) 80 books - 50 books = 30 books The teacher gave 30 books.
2) 30 rubles + 30 rubles = 60 rubles She gave 60 rubles.

Thus the pupil did many useless solutions irrespective of the question of the problem, and even of the conditions in general; the solutions *per se* are incorrect. She correctly solved only the first question of problem 3, using two data given in the conditions.

What is the cause of such a great many unrecognized and unsubstantiated conclusions during the solution of a problem? Weakness of memory is one reason. Premises for forming a conclusion are not retained in the memory even for the time necessary to form a conclusion. In its turn, the conclusion thus drawn is also forgotten, without having been related to the solution of other questions. What was said is forgotten. What has been stated is repeated with difficulty and inaccuracy, and the meaning is gradually distorted. For example, she stated the data for problem 1: "34 books and an album were bought for 23 rubles." As she began writing, she forgot what she had just said and instead of these numbers she wrote "20 books" and "24 rubles." She correctly stated the first question in problem 3: "How much did they pay for the books?" She began to write it down: "How much did they pay for... I forgot, what were they paying for?" She incorrectly added, "for the total purchase."

In solving all the problems the pupil worked without interruption, hurrying somewhat; she could not stop to concentrate on anything, and she answered absentmindedly and irrelevantly. She answered questions hurriedly and, as a rule, she did not think over the question, often saying what she had been thinking before a question had been asked, and failing to answer the question. Her thinking ran its course with little influence from external considerations (questions, the conditions of the problem).

Average (and some weak) pupils selected numbers for the conditions, but could not answer the question of the problem. Moreover, the question of the problem was sometimes not even the last question in the solution plan. On this basis we may assume that in selecting numerical data the pupils began at some point other than the question of the problem; when selecting the data for its conditions, they, the pupils, relied on individual parts or words of the conditions. Having selected numerical data, the pupils tried to make all possible solutions from the conditions of problems thus obtained, and did not try to answer the question of the problem (Lena K., Yulii P. and others).

In solving the problems of series II, the third grade pupils showed characteristic traits which did not depend upon the pupils' success:

1) Some pupils were almost able to draw correct oral conclusions from the conditions, but on beginning the written solution, stated questions incorrectly and gave incorrect solutions; that is, the pupils based their written solutions not on their stated thoughts, but on the stereotyped plan which they had prepared for the solution of all problems (Yulii P., Vova G); they correctly selected the numerical data, but they were unable to solve the problem.

Yulii P. read problem 1 and correctly named the lacking data: "For the album they paid 3 rubles, and 5 rubles more than that for the book." From such numerical data it would have been possible to answer the question of problem 1 (3r + 5r = 8r ; 8r + 3r = 11r). The pupil tried orally, as soon as he had named the data, to note such a solution: "How much money did they pay for the total purchase? We must add 3 to 5." As soon as he began writing the solution, however, he did not do what he had said nor did he use the numbers he had named:

1) 5 albums + 4 albums = 9 albums
2) 9 albums ÷ 4 = 3 books

Answer: 3 books. (problem 1)

Yulii P. did the same thing in solving problems 2 and 3. In answering the question of these two problems ("How much was paid for the entire purchase?"), he wrote:

1) 10 books + 4 books = 14
2) 14 books ÷ 4 = 3 books (2) albums

Answer: 3 books (2) albums (problem 2)

1) 6 rubles + 3 rubles = 9 rubles
9 rubles ÷ 4 rubles = 3 rubles

Answer: 3 rubles (problem 3)

In solving problem 4, the pupil immediately stated the scheme of solution he had given earlier, without first naming the missing data: "Here there are no numbers. We must add 7 to 10 books, that'll be 17 books. Then divide 17 by 3...."

After this the pupil was asked to first select the numbers, then solve the problem, which he did: he selected the cost of the books

("12 rubles"), of the albums ("3 rubles"), and the amount of money given the cashier ("25 rubles"). But he performed the solution incorrectly, following his prepared scheme.

1) 25 rubles + 12 rubles = 37 rubles
   37 rubles ÷ 3 = 17 rubles
   Answer: 17 rubles

Yulii P. also selected the missing data to solve problem 5, but he solved it incorrectly. This pupil solved all problems according to one stereotyped plan: he added two numbers and divided the result by one of the summands (problems 1 and 2) or by another number given in the conditions (problems 3, 4, 5).

Vova G. selected numerical data for the conditions with more precision than Yulii P., but he too failed to solve a single problem. Vova G. solved all problems following a stereotyped plan, but differently than Yulii P.; two numbers were added (he found the value of the album and the books). Then he again added the first summand to the result. Unlike Yulii P., Vova G. carried out operations only on numbers given in the conditions, or selected and written down at first. He did not think up other numbers during the solution.

2) In selecting numbers for the questions, Lesha S., Yulya S., and Nina P. satisfied the requirements of the conditions and questions more precisely than did the other pupils. They felt the fine distinctions of the given conditions more acutely and tried to use them for the solution. They noticed both the general similarity of the idea and the fine distinctions of the problems. In solving the problems of this series, Vera K., Yulii P., and Valya Ch. noticed the similarity in the general idea of the problems, but did not notice their finer distinctions.

Fifth Grade (series II)

The fifth grade pupils were given the following three problems whose solutions require numbers to be selected first:

1. Oats were sent in trucks and carts to a grain collecting station by a kolkhoz. How many tons of oats altogether did the kolkhoz send?

2. A kolkhoz sent oats to a grain collecting station in carts and in two trucks carrying 3 tons each. How many tons of oats altogether did the kolkhoz send?

3. A kolkhoz sent oats to a grain collecting station in 8 carts and in two trucks carrying 3 tons each. How many tons of oats altogether did the kolkhoz send?

As can be seen, the same question is applied to all three problems. For the answer to the question in problem 1 there are conditions only--no numerical data: "Oats were sent in trucks and carts to a grain collecting station by a kolkhoz."

In problem 2, two data are introduced: "A kolkhoz sent oats to a grain collecting station in carts and in two trucks carrying 3 tons each."

In problem 3 the number of carts is further indicated: "A kolkhoz sent oats to a grain collecting station in 8 carts and in two trucks carrying 3 tons each."

All fifth grade pupils selected the numerical data for the questions correctly, but the problems could still not be solved by Kolya D. (problems 2 and 3), Vitya K. (1 and 3), and Nina Ts. (2 and 3). The difference in the pupils' completion of the assignment lies not so much in the end results as in the ways of achieving them: the children differed in the method of selection of the correct numerical data and in their mistakes in the solution.

All pupils handled problem 1 more successfully than other problems. It was easier to select numerical data for problem 1, since its conditions permit a greater selection of arbitrary data; and the problem is solved more easily than the others. There is therefore no significant difference in the pupils' solution of problem 1.

The pupils used different methods to solve problems 2 and 3. A narrower selection of data is required for problems 2 and 3, since their conditions somewhat limit an arbitrary choice of numbers; their solutions, moreover, require a more profound understanding of the existing conditions than that of problem 1, since the supplementary numerical data have made the solution more complex.

a) Excellent pupils correctly selected numerical data and solved the problems with the data. The children selected the data differently, however, and went about solving the problems in different ways.

Tonya L. had little trouble transferring the understood solution of problem 1 to the solutions of the subsequent problem despite the diversity of the conditions:

60 tons + 50 tons = 80 tons (problem 1)

40 tons + 50 tons = 90 tons (problem 2)

60 tons + 40 tons = 100 tons (problem 3)

Tolya A. went about selecting the numbers and solving the problems very differently. He selected the numerical data in strict accordance with the conditions, taking account of the changes of the conditions of each problem, and used all numerical data correctly in solving the problem. Having correctly selected the data and solved problem 1, he chose other data for problem 2; these data were closer to the conditions of this problem. Here he no longer indicated the total amount of oats in the trucks, but used the data given in the conditions. He indicated the amount of oats in the carts in conformity with that in the trucks "2t" in the carts and "3t in the trucks") and solved the problem correctly.

In solving problem 3, he again wrote down the missing data immediately, but now silently this time. The problem seemed too easy for him; he wanted to make it more complex by numbers: "Can I add another kilogram? Well, 50 kg more--2 t 50 kg ?" He solved the problem correctly. in 3 stages. Thus Tolya A. did not repeat a single similar solution in solving the problems. He correctly used all existing data and correctly selected the missing numbers.

b) Average pupils also approached solution of these problems in different ways.

Vova M. outlined a plan of solution and selected the numbers at the same time. As the conditions of the problems changed, the solutions too were changed; appropriate data wereselected, and those data in the conditions were used correspondingly.

Kolya D. did not use the numerical data--the number of trucks--in solving problems 2 and 3, and solved these problems wrong. Kolya M. did not take the number of carts in problem 3 into account and therefore also solved this problem incorrectly.

c) Poor pupils gave diverse solutions. Manya V. used all numerical data given in the conditions and selected only the numbers which were needed to solve the given problems. Vitya K. could not determine which numerical data were missing in problem 1--the number of carts or the amount of oats. For problem 2 he selected a number that was given there. He began solving problem 3 without first selecting the missing data. Nina Ts. solved only the first problem. Neither the selection of numerical data nor the solutions of problems 2 and 3 agreed precisely enough with the conditions.

From the fifth grade pupils' solutions of problems in series II it is evident that:

1) Some pupils selected numerical data without understanding the meaning of the conditions of the problems; they selected numbers for individual parts of the conditions. Then they began to solve a problem, again without accounting for its question, and either approached the solution to its question blindly or left it unsolved, not even mentioning it in the solution (Kolya D. did this).

2) Other pupils selected the numerical data in close connection with the conditions and the question of the problem, and even had a plan of solution for the problem-to-be. Vova M. outlined a plan of solution at the same time he selected numerical data; Manya V. began to solve problem 2 immediately after solving the question of the problem.

Seventh Grade (series II)

The seventh grade pupils were given the following three problems whose solution required that missing numbers be supplied:

1. Shirts of one size were sewn from silk and from satin. A meter of silk is more expensive than a meter of satin. By how many rubles is a silk shirt more expensive than a satin one?

2. Shirts of one size were sewn from silk and from satin. 3m of material went into each shirt. A meter of silk is more expensive than a meter of satin. By how many rubles is a silk shirt more expensive than a satin one?

3. Shirts of one size were sewn from silk and from satin. 3m of material went into each shirt. A meter of silk is 40 rubles, and a meter of satin is cheaper. By how many rubles is a silk shirt more expensive than a satin one?

The same question is asked in each problem, but the problems differ in their amount of numerical data. Unlike the problems given to the third- and fifth graders, all three problems of the seventh grade have two possibilities for selecting numerical data. Here, selection of numbers for the conditions without the requisite orientation toward the question of the problem complicates the conditions so much that the solution of the problem becomes almost impossible. Thus, if the numbers are selected according to the existing data, then the first condition--"Shirts of one size were sewn from silk and from satin"--lacks an indication of the number of meters of silk and satin, which is not needed to answer the question of the problem, but, on the contrary, would serve as the basis of superfluous solutions which would tend to distract one from the correct path.

Sometimes, however, the children begin selecting numbers not from the beginning, but from the end of the conditions, often because this part is more obvious for the solution--in this part the requirement for arithmetical operations, for direct solution, is more noticeable than in other parts of the conditions; for example:

a) In the conditions "They brought 5t in the first truck, 3t in the second and in the third they brought twice as much as in the second," despite the question of the problem, "How many tons of oats did they bring in the first and second trucks?" the pupils first find out how much oats was brought in the third truck (cf. series I);

b) In the conditions of problem 1, "Shirts of one size were sewn from silk and from satin. A meter of silk is more expensive than one of satin," numerical data are first selected for the second part, "A meter of silk...." Thus, having begun the selection of numerical data from the end of the conditions, one may state the price of a meter of silk and a meter of satin, and then the number of meters going into each shirt; that is, one may select all data needed to solve the proposed questions irrespective of the central question, beginning not from the question of the problem but from its conditions. However, the likelihood of such a selection is less than that described in the first case.

Let us ascertain when and how numerical data are selected for the conditions, irrespective of the question of the problem, and when numerical data are selected by beginning from the question of the problem than from the conditions.

Seventh grade pupils' solutions of problems are more individualized than the solutions made in the third and fifth grades. Over the period of instruction the children gradually form their own particular approach to solving arithmetic problems. What was observed in embryonic form in solutions by pupils of lower grades grows over years of instruction into a firm individual quality that manifests itself in a unique approach to the solution of problems and is peculiar and characteristic of each pupil. Nevertheless, despite this, seventh graders have something in common, something powerful, that unites them in the following two methods of solution of the problems of series II:

First Method

After becoming acquainted with the conditions, the pupil does not outline a plan of solving the problem. Numerical data are selected (correctly or incorrectly) for the logical bits and pieces of the conditions (often correctly), or for individual words according to a formal feature and not according to content (often incorrectly). Next the solution is done according to the individual parts of the conditions, almost unrelated to the questions. At the end of such a solution, also without any connection, the pupil states (copies from the conditions) the question of the problem (doing this by habit acquired over the years of studying problem solving) and solves it, usually incorrectly (Katya B., Kolya S., Olya T.). Or, having selected data for the conditions, the pupil gives up on the solution altogether (Ira G.).

Sometimes there are necessary solutions among solutions of superfluous questions, but they are usually lost amidst the incorrect ones; they are not isolated and used to solve the question of the problem (Ira G. problems 1, 2, 3). The solution of the isolated part itself is obtained as follows:

a) the question is stated according to the sense of the part of the conditions used for the solution, and numerical data are selected (correctly or incorrectly) for its solution;

b) or the question is stated and solved by two numbers taken at random from different parts of the conditions.

Katya B., having read problem 1 once, began to whisper something, continually moving her pencil along the lines of text, pretending she was thinking studiously, but for a long time did not move. Finally, she selected numbers for the first part of the conditions: "There are 10m of silk and 5m of satin. 5m more expensive than satin...." From this data she incorrectly determined the numerical data for the second half of the conditions ("A meter of silk is more expensive than a meter of satin"): "10 minus 5 is 5. 5m more expensive." Having made up only two numbers, both unnecessary for solving the problem, the pupil began solving the problem.

In trying to guide her to a correct selection of numbers, we asked her what must be known to determine how much more expensive a silk shirt is than a satin one. Without thinking, she replied, "By 2 rubles. 10 divided by 5 is 2 rubles!" She also incorrectly answered the question whether she had thought up all the numbers necessary to know how much more expensive a silk shirt is than a satin one: "All. Can I solve it? Should I write down the questions?"

Beginning the solution of problem 1, Katya stated various questions (correctly following from the conditions) but not those needed to answer the stated question of the problem. She copied the question of the problem as the last question. None of the three stages in the plan of solution were interrelated in either the logical sense or in their utilization of the results of the solutions. Incorrect solutions were given all three questions and differed only in the arithmetical operations on the numbers 10 and 5:

1) How many shirts of one size did they sew?

   10 + 5m = 15m

This question was corrected after the solution of the problem had been completed:

1) How much silk and satin was there in all?

   10 + 5m = 15m

2) How much more expensive is a meter of silk than a meter of satin?

   10m − 5 = 5m

3) How many rubles more expensive is a silk shirt than a satin one?

   10m : 5m = 2 rubles. Answer: 2 rubles.

For problem 2, she also selected the number of meters of silk ("20m") and satin ("15m") and immediately noted the questions for the solution: "The first is how much silk and satin there was—20 and 15 is 35m. For each shirt we must divide 35 by 3, how many shirts they sewed. Then, how much more expensive silk is than satin—20 minus 15 is 5. The fourth question is how much more expensive a silk shirt is than a satin one . . . ."

In the conditions of problem 2 there was one datum--the amount of material used in one shirt (3m); because of this Katya solved still another superfluous question (relative to problem 1):

2) How many shirts were sewn?

35m ÷ 3m = 11.6 shirts.

This is the only feature distinguishing the solution of problem 1 from that of problem 2.

For problem 3, in which only the price of silk was omitted, she selected this datum ("30 rubles"). But even though there was then enough data to solve the problem, she selected (as in the first two problems) extra numbers: "How much silk and satin were used? 3m + 8m."

From the data she had then, the pupil solved 4 superfluous questions: the first two as in the preceding problems and the last two from a new combination of data; and what is remarkable is that all four questions were solved correctly from the given conditions. But not one of them advanced the solution of the question of the problem. After solving the four superfluous questions, she wrote the question of the problem as the fifth one and solved it incorrectly:

Problem No. 3.

1) How much silk and satin was there?

4m + 8m = 12m

2) How many shirts will they sew?

12m : 3m = 4 shirts

3) How much does 4m of silk cost

40 rubles x 4 = 160 rubles

4) How much does 8m of satin cost?

30 rubles x 8 = 240 rubles

5) How many more rubles does a silk shirt cost than a satin one?

240 rubles - 160 = 80 rubles

Answer: 80 rubles more expensive than the satin.

As can be seen, none of these questions and solutions are related; they have no general direction toward the goal of answering the question of the problem. This is the result of a "blind" solution with no regard for the question of the problem. The question of the problem is given as the last question, and is not logically connected with the existing solutions of the four questions. For solving the fifth question—the question of the problem--the pupil used only the numerical data--the results of the solution of the third and fourth questions--and they are used according to a formal feature: from the cost of 4m of silk she wanted to subtract the cost of 8m of satin and thus answer the question of how much more expensive a silk shirt is than a satin one. But it appeared that even this could not be done, since the cost of 4m of silk (160 rubles) is less than the cost of 8m of satin (240 rubles). Then the pupil simply stopped at this trouble spot, saying, "You can't take 240 from 160! But what if I turn it around: take 160 from 240? If I mixed up the numbers...." She stopped here; she subtracted 160 from 240.

Second Method

After becoming acquainted with the conditions, the pupil outlines a general plan of solving the problem, then selects the numerical data. In this method: a) the numerical data are selected for the questions of the plan outlined, and the solution is obtained without performing any superfluous solutions (Nina M., Yura B.); b) the plan of solution is outlined (with some assistance, however), and stated orally, but is not realized in the written solution and does not determine the selection of numerical data (Tamara Sh.). In cases similar to Tamara's solution, written solutions are incorrect and are, rather, solutions of the conditions of the problem, as described in the first way (see above).

For clarity let us cite the solutions given by Nina M. and Tamara Sh.

Nina M. logically outlined a plan of solving the problem and began to select numbers for the orally stated question of the plan: "We must find out how much one silk shirt costs and how much one satin shirt costs. This means we must divide. [Pause.] To know how much one shirt costs, we must know how much the material costs: 1m of silk and 1m of satin, and then how many meters went into making a shirt [pause], say 3m [writes in the conditions "3m in each"]. A meter of silk costs 40 rubles, and a meter of satin, 18--no! I say 21 rubles [writes in conditions]."

She solved the problem easily and correctly in 3 stages. She solved problems 2 and 3 just as easily, understanding that they were solved like problem 1: "This is the same one. You made a mistake. You gave me this problem before. I just have to do the same thing over again. The same numbers....". Nina M. even understood the system of these problems, how they differed: "Are we going to solve some more? Probably the same kind again, but here the price of satin is given!"

Tamara Sh. selected numbers and solved problems very differently. She read the conditions of problem 1, shook her shoulders, sat silently and did not begin the solution. She correctly answered the question of what must be found in the problem: "How many rubles more expensive is a silk shirt than a satin one? We need to know how much a silk shirt costs and... how much a satin one costs." But she could not apply or concretize this correct general plan.

Thus only two pupils--Nina M. and Yura B.--independently outlined the plan of solution and then selected numerical data for its question. Most of the pupils began selecting data for the conditions and solving the problem irrespective of its question; even work such as selection of numbers for solving a stated question, which requires an analytic means of analyzing problems, was done by the pupils with all possible variations of the synthetic method of exposition and solution of the conditions.

The first method of solution is closer to the synthetic method, and the second is closer to the analytic method of decomposing and solving complex arithmetic problems. However, neither was completed here. The first way--the most characteristic of the pupils--was not used as a rule after "blind" solutions of individual parts of the conditions, after "superfluous synthesis," to answer the question of the problem. In the second way the general plan of solution is not realized before the end, according to the concrete conditions of the problem.

While we attempted to guide the pupils (which we usually did when they were solving problem 2) toward a correct plan of solution and selection of the data by shortened analysis, they correctly stated the general plan of solution but followed their own habitual method when writing the solution. In their method of solving individual parts of the conditions by synthesis, they forgot about the orally composed plan. We had more success in directing the pupils along the correct path of solution by a preliminary oral solution of a simple, analogous, concrete problem.

<u>Selection of a Question for Numerical Data of the Conditions</u>

<u>Third Grade</u> (series III)

The third grade pupils were given the following two conditions (A and B) and asked to state a question corresponding to them and to solve the problem thus obtained:

<u>Condition A</u>. For May Day a school bought 62m of red satin. 32m was used for posters, 24m for banners and the rest for flags.

<u>Condition B</u>. 4 birdhouses and one little table were made from 17 boards. 2 boards were used for each birdhouse and the rest for the table.

The question following immediately from condition A is, "How many meters of satin were used for the flags?" From condition B it is, "How many boards were used for the table?"

Condition B is somewhat harder to solve than A because:

1) The mathematical content of B is more complicated than that of A (the first question is solved by multiplication, the second by subtraction, while condition A requires either addition and subtraction or only subtraction).

2) The disposition of the numerical data in B is also different: it hinders the selection of data for the solution (data for solving the first question are placed in two semantically distinct parts of the conditions).

3) The question follows more easily from A and has logical sense: bought--used for; (spent)--remained--how much remained.

In B there is no such clear sequence in the placement of the parts of the conditions.

Several methods may be used in stating the questions for conditions A and B:

First Method

The question for the conditions is stated in the logical sense of the whole condition, without solving the problem from its numerical data; only then is the solution for the stages of the plan indicated (numerical data, arithmetical operations). In this case the solution of the problem may be correct; but a mistake may appear when the logically correct plan of solution is not realized precisely enough in terms of the existing concrete data of the conditions.

Second Method

One may begin the solution from the existing numerical data, and, in solving the problem, may answer the last question ending the solution. The last result may be written as the answer to the problem. The solution to the problem may be correct, if the subject composes a correct plan and the final question coincides with the possible question of the problem; or it may be incorrect, if the questions solved blindly are not interrelated but instead are superfluous or incorrect.

Third Method

The question may be stated with respect to a feature of the words at the end of the conditions: "...and the rest for flags" (ending of condition A) means one may ask, "How much was used for the flags?" "...and the rest for the table" (ending of condition B) means, "How much was used for the table?"

Such a solution to the problem is often wrong. Let us examine how our pupils selected questions for the conditions, and how they then solved their own problems. The third graders selected the questions and then solved the problems by various methods; the difference was more noticeable in average and poor pupils:

1) Some pupils selected questions from the logical sense of the conditions, stating in passing the solutions to the questions of the plan (Yuliya S.); others began the solution without having selected the questions for the conditions. However, these solutions were

accompanied by the questions, and the last question for the last solution is taken for the question of the whole conditions (Lesha S.). In both cases, however, the questions of the plan and their solutions were closely interrelated, the difference being which of them was stated first--the question or its solution.

Yuliya S. and Lesha S. stated the questions and performed the solutions simultaneously, without disconnecting them. Yuliya S. stated a question and immediately corroborated it with its solution. Lesha S. explained a solution with the correct question. If, however, the pupils did not succeed in stating the correct questions of the plan, the solution suffered--they then gave incorrect solutions to incorrect problems (Nina P.).

Yuliya S., having read condition B, correctly selected the question for it and began outlining questions of the plan, reciting their solutions: "Here we need how many boards were used for the table... 17 boards...2 boards taken 4 times is 8....8 boards for birdhouses [writes and correctly solves the first question]. In the second question we find out...take 8 from 17...how many boards were used for the table" [she also solves the second problem correctly].

Nina P. tried first to outline the questions of the plan: "How much satin they bought for flags...for posters and flags...how much was used for... for...?" Thus, not having explained the solution and not understanding what she learned by this solution, she began to carry it out; then she added "62m + 32m = 95m." She stated the wrong question for this--not the one she had outlined first, but the last one: "How much was used for posters?"

Nina P. incorrectly stated questions for conditions A and B, not because of the solutions, but because she did not completely understand the general sense of the plan; she did not imagine the concrete data and relationships given in the conditions. The solutions were given according to one pattern: the first two data from the conditions are added, and the last datum is subtracted from this result. Furthermore, the solutions obtained were not always explained; the questions orally stated were not used in the written solution.

2) Other pupils, primarily the poor ones, immediately stated questions for all the conditions of the problem according to a formal

feature--the words at the end of the conditions--or, not having stated a question, they began manipulating the numerical data, giving no attention to their own question selected for the entire problem. The question of the problem remained unsolved (Vova P., Sveta P., Yulii P.).

Yulii P. began performing arithmetical operations on numbers without being able to explain them, without even stating a question to the problem. Having read the conditions, he began solving it; "We need to add 32 to 24... it is 56... then take 62 from 56..." When he was asked, "What do you learn by subtracting 62 from 56?" he answered, "No! I made a mistake... first you have to take 32 from 62 [to find] how much material was left."

He did not write the specific question, "How many meters of material were left?" He began to solve the problem following the pattern we have seen before. He subtracted the second datum from the first, then added the last one from the numerical data in the conditions to the result:

1) 62m - 32m = 30m

2) 30m + 24m = 54m

Answer: 54m.

He could not correctly explain his solutions. The pupil thought that in the first solution he was finding out "that 30m of satin went into flags," and in the second "how much satin was left."

Thus the question was fitted to the conditions, not according to the logical semantics of the conditions and not as a result of the solution of correct questions, but by a false solution, based on a serious misunderstanding of the semantics of the conditions. The conditions stated that satin "went into" a flag. There is a question implied in the conditions, "How many meters of material were left?" Statement of the question of the problem preceded a series of attempts to solve the conditions without a question, "We have to take 17 twice; we must add 4 to 17; we must take 4 from 17," and so on.

Having written the question for condition B, the pupil gave this solution:

1) 17 boards - 4 boards = 13 boards went into making the birdhouse

13 boards : 2 = 12 (1)* 12 boards were left.

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*(1) indicates a remainder of 1 (Ed.).

Characteristic traits of these children (Vova G., Sveta P., Yuliya P.) are their insufficient understanding of the sense of the conditions and the fact that they brought something from themselves into the problems, resulting in a situation different from the one proposed in the problem. Vova G., having stated the correct question to the conditions, gave an incorrect solution: "27 ÷ 4 = ___" and explained it, disregarding the conditions. "How much went into making boards? How much went into making a chair? How much went into making a fence?" Nothing is said in the conditions about a chair or a fence. Sveta P., at the same time she read the problem, stated something that was not there: "2 boards went into... each board, 2 boards, 2 logs went into each..." while there was no mention of logs in the problem.

Fifth Grade (series III)

The fifth grade pupils were given the following conditions and asked to state the question for them and then solve the problem: "For their work on a new building the plasterers were paid 2,890 rubles and the painters were paid one tenth as much as this."

The following questions may be applied to these conditions:

1. How much money was paid to the painters?
2. How much money was paid to the painters and the plasterers?
3. How many more rubles were paid the plasterers than the painters?

There are several approaches to fulfilling the assignment:

a) Either state question 1 from the two data of the conditions, solve it, and then from the amount of money paid the painters and plasterers, state question 2 or 3; or, do not state questions 2 and 3, but stop with the solution of question 1, which is a possible solution.

b) Or, without solving question 1, directly select question 2 or 3, following the logical content of the conditions, write down the selected question, and solve the problem.

The problem except for the novelty of its form—conditions without a question--is not difficult.

The fifth graders' solutions of the problems differ not so much in the results as in the ways they are obtained. The fifth graders fulfilled the assignment similarly to the third graders:

a) Some pupils, reading the conditions, stated the appropriate question and then began to solve the problem (Tolya A., Vova M., Nina Ts., Vitya K.);

b) Others, without stating a question for the entire condition, began the solution (Tonya L., Kolya D., Kolya M., Manya V.).

Of the pupils in group a, Vitya K. was unsuccessful in correctly stating and solving the question of the problem. He read the conditions and noticed the absence of the question of the problem: "But what is the question here...? There isn't any! Oh, it must be how much they plastered... how much the plasterers were paid for 3 rooms...."

He stated and solved this incorrect question, exceeding the bounds of the given conditions:

1) How much did they pay the plasterers for 3 rooms?
 2890 rubles ÷ 3 = 963 (1) rubles

When requested to state and solve another question, Vitya gave another question and solved it, now not even using the numerical data from the conditions but making it up himself: 2) "How much did they pay the painters for 1 room?" He began seeking data, wrote down "10," changed the question from "1 room" to "2 rooms" (obviously so he would be able to divide 10 by 2), and gave this solution: "10 ÷ 2 = 5 times. Answer: 5 times." He himself was amazed at his inept solution: "What's this? It comes out 5 times!". He thought a moment and agreed with himself, took this solution as correct, since "it doesn't come out any more...."

All pupils in group b, who did the solution according to their habitual synthetic method, solving question after question, completed the assignment correctly. However, having solved the first question, they did not continue solving the problem unless reminded of this.

Seventh Grade (series III)

The seventh ~~grade~~ pupils were given the following conditions and were asked to select a question and solve the problem obtained: "Two bicyclists each traveled a distance of 52.8 km. The first was on the road for 6 hours, and the second for 4 hours."

For these conditions there are two questions for whose answer all the given data must be used:

1. How many kilometers per hour did each bicyclist travel?

2. How many kilometers per hour faster did the second bicyclist go than the first?

To solve question 2 one must first solve two independent questions: How many kilometers per hour did the first bicyclist travel? How many kilometers per hour did the second bicyclist travel?

The problem is characterized by the difficulty of establishing the relationship between its data (distance-speed-time of travel).

Only Nina M. could handle this task successfully. No other pupil could cope with such a task, although they made various attempts to complete it. Each pupil tried to do the task in his own unique manner; as a consequence there are no identical answers among the obtained results. The solutions differ in the questions selected for each condition, and in the methods of solution of the problems obtained.

Among the questions selected by the pupils there are none alike. The pupils selected questions such as:

"How much earlier did the second bicyclist return than the first?" (Kolya S.)

"How many kilometers did they both travel together?" (Tamara Sh.)

"How much did the first bicyclist travel?" (Katya B.)

"How many kilometers did each bicyclist travel in 1 hour?" (Nina M.)

"How many kilometers did both bicyclists travel together in 1 hour?" (Nina M.)

"How many more kilometers in 1 hour did the second bicyclist travel than the first?" (Nina M.)

"How much did they travel altogether?" (hours?) (Olya T.)

"How many kilometers did each travel?" (Yura B.)

"In how many hours will they meet?" (Ira S.)

Such diverse questions convinced us again of the exclusively individual approach to the solution of problems, characteristic of pupils in the seventh grade. Identical conditions were understood and solved differently by each pupil. In doing this: (1) some pupils carried all the possible solutions and made mistakes, but did not alter the conditions or introduce something from themselves into the conditions; they did not distort the original sense; (2) others, however, with little regard for the given conditions, imagined a problem similar to, but not quite the same as, the given problem and solved it, distorting its conditions.

The pupils in group (1) (Nina M., Olya T.) carried out the solution according to the given conditions, without introducing anything from themselves or distorting the sense. Either they understood all the conditions and carried out all the solutions possible from them (Nina M.), or they used only a part of the conditions for the solution (Olya T.). In both cases the stated questions and solutions did not deviate from the given conditions.

The pupils in group (2) (Kolya S., Tamara Sh., Katya B., Yura B., Ira S., and Ira G.), misunderstood the conditions or, having changed them at their own discretion, stated questions which deviated from the conditions, and, therefore, could not be answered correctly. At the time the pupil was reading the conditions, he was trying to understand the given problem, and he was also answering the teacher's questions; that is, he was solving a concrete problem. Therefore, the questions which the pupil stated and solved sometimes combine two plans of solution for two problems--the given concrete problem and the problem imagined by the pupil. Because of this fact, the questions are distinguished by their extreme vagueness. In such cases the solutions almost always proceed independent of the questions; they digress from the stated questions, and usually reflect the conditions of the given problem, rather than the imagined one.

Yura B. quickly stated an incorrect question to the conditions: "How many kilometers did each bicyclist travel?" He assumed that the bicyclists were traveling toward each other. He made it his aim to find:

1) How many kilometers did the first bicyclist travel in 6 hours?
2) How many kilometers did the second bicyclist travel in four hours?

For such an assumption, these questions cannot be answered from the given data. In his solutions to the stated questions, Yura determined, not what he had indicated in the questions, but the speed of the first and second bicyclists in 1 hour.

Katya B.'s solution is even more characteristic. She assumed that the bicyclists traveled the 52.8km going toward each other, that each one traveled half the route (which is also not given in the conditions), and that each bicyclist was en route for half the time indicated in the conditions. From these suppositions, but without utilizing the relationship between the data, which is given in the conditions, she solved four incorrect problems:

1) How far did... What was the distance between the two bicyclists?

   52.8km ÷ 2 = 26.4km

2) How long did the first bicyclist travel?

   6 hours ÷ 2 = 3 hours

3) How long did the second bicyclist travel?

   4 hours ÷ 2 = 2 hours

4) How long did 1 bicyclist travel?

   3 hours - 1 hour = 1 hour

   Answer: He traveled for 1 hour.

As we see, Katya carried out the solution, not on the basis of the conditions given her, but rather on the basis of the problem she had made up herself, which she imagined as she read the given conditions.

In conclusion we present an original solution of the problem by Kolya S. He was able to choose numerical data with only one terminology: he stated questions either on finding the time of travel or on finding the distance. Kolya could not establish the relationship between the numerical data (route--time--speed). Judging the time of travel, he thought only about that; judging the distance, he imagined it, too, outside any connection with other data (time--speed). He tried to solve the problem, stating a series of incorrect questions which diverged from the stated conditions. These thoughts and questions, stated in passing about the conditions, do not correctly reflect a single relationship between the data and the conditions: "By how many hours...in how many hours did the first bicyclist travel...how much longer... I don't know, I'll break my head over this...."

He stopped stating questions on the numerical data expressing time (hours) and began stating questions on the data expressing distance (the route in kilometers); "What distance more did the second bicyclist travel than the first... and how many more kilometers did the first bicyclist travel than the second...?"

The pupil was asked to write the questions he had stated and to solve them, "Please, you write, then they'll swear... I have to take a look... I don't understand." And again he gave incorrect questions: "What distance did the bicyclists travel toward each other?" This question is not lacking all meaning if it is assumed that the bicyclists traveled the 52.8 km going toward each other; but such a supposition deviates from the data of the conditions.

At this point Kolya S. stopped stating questions, holding to the conditions, and began recalling a similar problem he had once solved: "We solved a problem once... how much earlier did the second bicyclist return than the first? But I don't know how to solve it... take 4 hours from 6?"

Finally he wrote one single incorrect and unclearly formulated question:

1) How much earlier did the second bicyclist return than the first?

6 hours - 4 hours = 2 hours

Answer: 2 hours.

Preliminary Analysis of the Conditions, Composition of the Oral Plan, and Written Solution of the Problem

Third Grade (series IV)

Preliminary analysis of the conditions by the analytic method

The third grade pupils were given a composite arithmetic problem solvable in two operations (multiplication and subtraction): "From 17m of cloth 4 coats and 1 smock were sewn. 3m of cloth were used for each coat. How many meters of cloth were used for the smock?"

Each pupil was asked to read the conditions of the problem aloud and, with the solved problem before him, to answer the teacher's questions. The teacher's questions and the pupil's answers aimed toward examining the problem analytically.

1. What was sewn from the cloth?

1a. And what else? (If the pupil does not give a complete answer.)

2. What must be found out in the problem?

3. What else was the cloth used for besides the smock?

4. What must be known in order to determine how much cloth was used in one smock?

5. How many meters of cloth were there altogether?

6. Is it known in the problem how much cloth was used in all the coats?

7. How can we find out how many meters of cloth were used in all the coats?

8. By what arithmetical operation can we find this? (What do we have to do for this?)

9. How do we find out how many meters of cloth were used for the smock, if we know how many meters of cloth were used in the coats?

After answering these questions, the pupil orally composed a plan and began the written solution of the problem.

The aim of such an individual lesson was to ascertain: a) which questions the pupils find hard to answer, and why; and where and what are the greatest difficulties encountered by the children in the analytic method of examining the given concrete problem; and b) how much this analysis of the problem will help the pupil solve the problem;

in other words, what is the relationship between the ability to answer questions in analyzing a problem, and the ability to compose a plan of solution and solve the problem independently. We should examine the latter, since in the preceding series the pupils showed a divergence between their oral arguments and their subsequent written solution of the problem.

In order to answer the questions posed in (a) and (b), let us examine the results of the pupils' solutions. Analytic analysis of the problem was not made identically by the pupils,[6] and showed a varying influence on their subsequent solution. There were differences between good, average, and weak pupils. At the same time, there was a common factor, characteristic of all pupils, in their analysis and solution of problems:

1. A common trait observed in all the pupils is their inability to extract the question of the problem from the conditions and state it independently. In reading the conditions, no child could immediately tell what was to be found in the problem. Instead of indicating the question of the problem, they began to read the conditions, to try to solve the problem; they repeated the isolated parts of the conditions which first came to their attention.

2. In elucidating the question, with the teacher's assistance, all the children had trouble taking the next step in analytic analysis; they could not indicate both data needed for the solution to the question of the problem. Instead of the two data, the pupils:

a) named only one datum, and always an unknown: "We must determine how many meters went for the coats"; that is, they stated the first question in the plan of solving the problem (Lesha S., Yulya S., and Vera K.).

b) named no data at all for solving the question of the problem, but solved the first question of the plan (correctly or incorrectly); that is, in answer to the question of what must be known in order to

[6] As might be expected, weak pupils, and some average ones, experienced the greatest difficulties in analyzing and solving problems. It is the solutions of these pupils which we shall principally consider here.

answer the question of the problem, the pupils indicated arithmetical operations on data in the conditions, either correct ones needed to begin the solution ("We have to multiply 4 x 3," Lena K., Vova G.), or incorrect operations on the first two numbers from the conditions ("We must have 17 - 4... 17 - 3... 17 + 4... 17 ÷ 4," Yulii P., Nina M.).

After analyzing the conditions, the pupils composed an oral plan of solution and then began the written solution of the problem. In composing the oral plan, none of the pupils indicated the questions of the plan of solution, but stated the solution at first (arithmetical operation and the data), and only then stated questions for the solutions. Written solutions were also begun by solving questions of the plan, and only then was the solution somehow explained.

It is remarkable that the analysis of the conditions influenced the pupils' composition of the oral plan and solution of the problem in different ways.

a) Yulii P. and Sveta P., in their analysis of the conditions, answered questions both correctly and incorrectly. Beginning composition of the oral plan, they forgot and could not be guided by the conclusions (correct in the end) which they had stated in the analysis. In composing the oral plan mentally, their first, incorrect answers were used primarily; the later answers, which they obtained with the teacher's assistance, were forgotten;

b) Like Yulii P. and Sveta P., Nina P., in analyzing the problem, first gave incorrect and then correct answers. But unlike them, she did not repeat a single incorrect question or solution in composing the oral plan. She did her solution on the basis of her correct, but not immediately isolated, answers.

c) Vova G. correctly answered all the teacher's questions, but erred in composing the plan; he did not immediately outline the correct questions of the plan.

Preliminary analysis of the conditions by the synthetic method (third grade)

The pupils of the third grade were given an arithmetic problem of the same difficulty as the problem given for solution by the analytic method of preliminary analysis. "From 18 sheets of iron 8 pans and one trough were made. Two sheets of iron were used for each pan. How many sheets of iron were used for the trough?"

After reading the conditions, the pupils answered the following questions, which directed their thought toward the synthetic method of analyzing the problem:

1. What was made from the iron?
2. How many troughs were made?
3. How many pans were made?
4. How many sheets of iron were used for each pan?
5. What can we find out if it is known that 8 pans were made and that 2 sheets of iron were used for each pan?
6. How can we learn this?
7. What do we obtain if we multiply 8 by 2?
8. What was made from the rest of the iron?
9. How many sheets of iron were there altogether?
10. How many sheets of iron were used for the pans?
11. What can we find out if it is known that there were 18 sheets of iron altogether and we know how many were used for pans?
12. How can we learn this?
13. What do we find out if we subtract 16 sheets from 18 sheets?

After synthetically analyzing the conditions, the pupil composed an oral plan and began the written solution.

Like the analytic analysis, the pupils synthetically analyzed the conditions with varying degrees of success as they encountered their own particular difficulties; the analysis had a varying influence on the subsequent written solution of the problem.

In their synthetic analysis of the conditions, the pupils experienced the following difficulties:

1) For most of the pupils, it was difficult to respond to the questions with the prepared answers existing in the conditions (that is, they could not isolate, and extract from the conditions, data

necessary for the answer). Lena K., Sveta P., and Nina P. could not even answer the first question, i.e., "What was made from the iron?"

Vera K., Yulii P., Lesha S., Yuliya S., Nina P. and Vova G. could not state the number of troughs made--a datum given in the conditions (in answer to question 2). One of the reasons for this is purely formal--the number of troughs ("1") was given not as a numeral, but as a word ("one trough").

2) For all pupils, however, the main difficulty was with question 5, in which a question is stated using two numerical data. Only one pupil, Vera K., stated the question and then indicated its solution, The majority of pupils, however, gave the solution (numerical data and arithmetical operations) without stating the question:

a) correctly: Yulii P., Valya Ch., Lesha S., Yuliya S., Nina P., Vova G.

b) incorrectly: Lena K., Sveta P.

No pupils who indicated a correct solution of the first question could explain it correctly (the result obtained by multiplying 2 sheets by 8). The children first stated an incorrect question to the correct solution they gave, and some of them could not correctly explain this solution (Valya C., Nina P., Lena K.--second solution).

After a synthetic analysis of the conditions, the pupils composed an oral plan and then began the written solution of the problem.

Identical analyses of the conditions were not given by the pupils:

1) Yulii P. answered incorrectly and with difficulty; he did not listen to questions attentively and did not realize the logical direction of the questions he was asked, a realization which could have helped him solve the problem. It was felt that he approached the solution in his own special manner, shifting with difficulty to the teacher's questions. However, despite the incorrect oral answers given during his analysis of the problem, he correctly solved the problem in writing.

Valya Ch. answered the questions proposed and orally composed a correct plan of solution; in the written solution, however, she did not state the first question of the plan which she gave orally; in other words, the oral argument of both Valya Ch. and Yulii P. differed from the written solutions.

2) A different quality is observed in Lesha S., Yuliya S., and Nina P. These children employed the orally stated ideas more directly in the solution. They answered the teacher's questions correctly and then correctly solved the problem (Lesha S., Yuliya S.); they erred in their argument, however, and in the solution (stating the question for the solutions—Nina P.).

3) Sveta P. answered only a few questions, requiring great effort and the teacher's assistance. She even incorrectly selected the data given in the conditions for her answer. Incorrect results were given independent of the incorrect answers made during the analysis of the conditions.

Fifth Grade (series IV)

Preliminary analysis of the conditions by the analytic method

The fifth grade pupils were given a compound arithmetic problem solvable in three operations (subtraction and two questions of addition): "A harvest of rye and oats was collected on a kolkhoz. From one field of rye they collected 132t 4c * and from another, 9t 2c less. 124t 7c of oats were collected from the first field, and 17t 3c more than this from the second. How many tons of grain were harvested from all the fields of the kolkhoz?"

There are two ways to solve the problem.

The first way leads more quickly to an answer for the question of the problem:

1) How much rye was harvested from the second field?

2) How much oats was harvested from the second field?

3) How many tons of grain did the kolkhoz harvest from all its fields?

The second way is longer, in a plan of five questions, and contains two superfluous questions (2 and 4):

1) How much rye was harvested from the second field?

2) How much rye in all was harvested from the two fields?

3) How much oats was harvested from the second field?

4) How much oats in all was harvested from the two fields?

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*c = centner = 100kg (Trans.)

5) How many tons of grain did the kolkhoz harvest from all its fields?

After he read the conditions, each pupil was given questions from the analytic method of analysis:

1) What must we find out in the problem?

2) What does "how many tons of grain in all" mean? Is this how many tons of rye or of oats?

3) How many fields were planted with rye?

4) With oats?

5) What do we have to know to determine how many tons of grain the kolkhoz harvested from all its fields?

6) From which fields do we know how many tons of grain were harvested?

7) And from which fields is it not known how many tons of grain were harvested, a quantity which we must determine.

8) How do we find out how many tons of rye were harvested from the second field?

9) How do we find out how many tons of oats were harvested from the second field?

10) How do we then find out how many tons of rye and oats altogether were harvested from all the fields by the kolkhoz?

After the analytic analysis, the pupil is asked to compose a plan ("Tell me how you are going to solve this problem"), after which he begins an independent solution of the problem.

The excellent fifth grade pupils were relatively more successful in analytically analyzing the problem than were the third graders. The poor pupils (and Kolya D., an average pupil), however, encountered the same difficulties which had been observed in the third graders (see above). Kolya D. could not say what was to be found out in the problem. Only at the teacher's request did he read the question of the problem. When asked question 5, what must be known in order to answer the question of the problem, he named only one unknown datum. This is the cause of the error in Kolya's solution; he did not select the known data given in the problem (the quantity of grain from the first fields) in order to solve the third question.

The poor pupils (Manya V., Nina Ts., Vitya K.) did not recognize the question of the problem. When asked what must be found out in the problem, they stated the first question of the plan of solution: "How much rye was harvested from the second field?" Just like the third graders, the poor fifth graders had trouble answering question 5 (what must be known to answer the question of the problem?). Instead of the answer--"We must know how much grain the kolkhoz harvested from each field"--the pupils stated an arithmetical operation (addition):

"We must add..." (Vitya K.).

"We must add... from the first and from the second" (Manya V.).

"We must join oats and rye together" (Nina Ts.).

Besides the general difficulties common to third and fifth graders, individual fifth graders encountered special troubles. For a long time, for instance, Vitya K. could not understand what was already known from the conditions (from what fields rye was harvested) and what was unknown. Manya V. gave three incorrect answers to the question, "From how many fields was rye harvested?"

Each pupil composed his own oral plan and solved the problem in writing. <u>Vitya K.</u> orally outlined three correct questions of the plan and named the correct data for solving the third question (the question of the problem); but in the written solution he did not use all the data for solving the third question, and hence did not answer the question of the problem. <u>Manya V.</u>, despite mistakes in analysis, outlined a detailed and time consuming way of solving the problem in five questions, and obtained the precise answer to the question of the problem (she alone of all the poor pupils did this). <u>Nina Ts.</u>, composing an oral plan, stated necessary and unnecessary questions and finally became completely confused. However, as soon as she was asked, "What is the first question?" she stated the correct first question and was even able to further orally outline the plan of solution in three stages. But she erred again in the written solution. She did not indicate all data for the solution of the third question (the question of the problem), but added only the quantity of oats and rye harvested from the second fields.

Why do the poor pupils (including a good pupil, Tolya A., and an average pupil, Kolya D.), when asked to state the question of the problem state the first question of the plan of solution? It appears, that as soon as the pupil has read the conditions he begins the solution of the question—an attempt is made to state the question from the given data (synthesis). He states this solution as the answer to the teacher's question. Moreover, the question for directing the subject's thinking ("What must be found out in the problem?"), when he has already found out something in the problem, is perceived by him as a confirmation of his [illegible]. This is why the pupil answers without even noticing his mista[illegible] he states the first question of the plan of solution. As can be seen, the basis for this tendency is the habit of solving a problem without the requisite understanding of the requirement of the question of the problem, the habit of approaching analysis of a problem in the synthetic manner.

Another fault characteristic of most pupils of the fifth grade, and of all pupils of the third grade, in answering the fundamental questions of an analytic character (our question 5), is the "half-and-half answer"—the indication of only one datum necessary for solving the question of the problem and, as we have seen, an unknown datum. What is the cause of this characteristic peculiarity? Besides the flaw in instruction, in which the children are usually not required to state two data, the pupils understand the question, "What must be known in order to answer the question of the problem?" as the simpler question, "What must be found in the problem, which unknowns are to be determined, in order to answer the question of the problem?" Therefore the pupils, almost without noticing their error, state only the unknown datum for solving the questions of the problem, losing sight of the known data given in the conditions. For this reason, as we saw above, the question of the problem was incorrectly solved by several pupils (Kolya D., Vitya K., Nina Ts.).

The good and average fifth grade pupils, unlike the third grade pupils, were able to isolate and state the questions of the problem. Moreover, they show (qualitatively) increased abilities in plan composition and in problem solving. The pupils outlined the oral plan not by indicating the arithmetical operations and the data, but by first stating the questions; they rarely stated the solutions to them.

Written solutions were also always begun with a statement of the questions; then the solutions were given. The questions of the oral plan and the written solution did not differ.

Preliminary analysis of the conditions by the synthetic method (fifth grade).

The fifth grade pupils were given a compound arithmetic problem with the same mathematical content as the problem given for solution using preliminary analysis by the analytic method: "Some fruits and vegetables were bought. 24r 40k were paid for pears and 4r 70k less than this for apples. 7r 40k were paid for potatoes and 14r 60k more than this for onions. How much money was paid for the entire purchase?"

There are two ways to solve the problem:

The first way (the shorter one) uses a plan with three questions:

1) How much money was paid for the apples?

2) How much money was paid for the onions?

3) How much money was paid for the entire purchase?

The second way is longer, requiring the solution of these three questions and two others.

After he has read the conditions, the pupil is given questions from the synthetic method of analyzing the problem:

1. What fruits and vegetables were bought?

2. How much was paid for potatoes? For onions? For pears? For apples?

2a. For which items do you know how much was paid, and for which do you not know?

3. How much was paid for the pears? For the apples? What may be found out from these numbers?

4. How can you find out how much was paid for the apples?

5. What else was bought?

6. How much was paid for the potatoes? For the onions? What may be found out from these numbers?

7. How can you find out how much was paid for the onions?

8. When you have found how much was paid for the apples and the onions, and the problem indicates how much was paid for the pears and the potatoes, what will you then be able to find out?

9. How will you be able to find out how much was paid for the entire purchase?

After the synthetic analysis, the pupils began oral composition of the plan; they then solved the problem in writing.

The fifth graders handled this type of assignment with more success than the third graders.

The results of the analysis and subsequent solution of the problem are somewhat different in children of different degrees of aptitude; this distinction is most noticeable in children of weaker aptitude and less noticeable in the good and average pupils.

a) All the good and average pupils correctly answered the questions in the analysis, composed oral plans and correctly solved the problems. One average pupil, Kolya D., erred in solving the third question (in selecting the data).

b) The poorer pupils had trouble answering, and gave incorrect answers to several questions in the problem analysis; they could not orally outline a correct plan, and they all solved the last question of the plan incorrectly (they erred in selecting the data).

The greatest difficulty for third and fifth graders alike was presented by questions 3 and 6. To answer both these questions it is necessary to state and solve a question using two data. Even though they had already answered question 3 correctly, some children (Kolya D., Vitya K., Manya V.) had trouble with question 6 (requiring the same ability to state a question using two numbers).

Why is this so? Why did the children draw a correct conclusion from the existing data in the first case and err in the second?

It appears that the success and correctness of the answer depends greatly on whether the teacher's question coincides with the pupil's train of thought as he goes through the solution. It is not hard to see that in the first case the pupils answered the teacher's question correctly because it coincided with their already fixed solution of the question, a coincidence which was not found in the second question.

Seventh Grade (series IV)

Preliminary analysis of the conditions by the analytic method

The seventh grade pupils were given this compound arithmetic problem: "A kolkhoz harvested 120 centners of oats and 3 times as much rye. Three-fifths of the grain was given to the government and 75% of the grain remaining in the kolkhoz was distributed to the kolkhozniks. How many centners of grain were given to the kolkhozniks?"

We conducted an analytic analysis with the pupils after they read the conditions; we asked:

1. What is to be found out in the problem?
2. Can you immediately answer the question of the problem?
3. What must we know in order to determine how many centners of grain were distributed to the kolkhozniks?
4. What must we know in order to determine how much grain the kolkhoz harvested altogether?
5. How can we determine how many centners of grain the kolkhoz collected?
6. How can we determine how many centners of grain the kolkhoz gave the government?
7. What must we know in order to determine how many centners of grain remained in the kolkhoz?
8. How then can we find out how many centners of grain were distributed to the kolkhozniks?

After the analysis the pupils were first asked to tell how they were going to solve the problem, and were then allowed to begin the solution. The seventh grade pupils' answers to the question (the analysis of the conditions), composition of the plan, and solution of the problems differed in their greater individuality. Some pupils grasped the general idea of the problem, outlined a general plan from it, but still did not imagine that they might use this plan and the existing conditions to solve the problem. They stated the plan, but appeared incapable of carrying it out, using the existing conditions of the concrete problem (Kolya S., Tamara Sh., Ira S., Yura B.). Others outlined incorrect questions in a general plan, but gave correct solutions.

Characteristic is the solution by Ira G. The pupil's questions of the plan reflect her rather general and imprecise understanding of the idea of the problem. But the solutions to these questions (arithmetical operations and numerical data) are correct:

1) How many centners of rye were there?

$120 \times 3 = 360c$

2) How many centners of grain were given to the government?

$120c + 360c = 480c$

3) How many centners of grain remained from the government?

$3/5$ of $480c = 288c$

4) How many centners of grain were left?

$480c - 288c = 192c$

5) How many centners of grain were distributed to the kolkhozniks?

75% of $192c = 144c$

Answer: 144c.

Still other seventh graders outlined a correct oral plan, and correctly used it to solve the problem (Nina M.).

Preliminary analysis of the conditions by the synthetic method (seventh grade)

The seventh grade pupils were given a problem identical in difficulty and mathematical content to the problem given for solution using a preliminary analysis by the analytic method: "A shoe factory made 240 pairs of shoes and 3 times as many slippers. Two-fifths of all the footwear was sent to a store and 25% of the remainder was sent to a booth. How many pairs of footwear were sent to the booth?"

After reading the conditions, the pupils were given these questions for a synthetic analysis of the problem:

1. How many pairs of footwear were made at the factory?

2. How many times more slippers were made than shoes?

3. What can you learn if you are told that 240 pairs of shoes were made and 3 times as many slippers were made?

4. How can you find out how many pairs of slippers were made?

5. What can you find out if you are told that 240 pairs of shoes were made, and you learn how many pairs of slippers were made?

6. How can you find out how many pairs of footwear were made altogether?

7. If you know how many pairs of footwear were made altogether, and you know that 2/5 of these went to the store, what can you find out?

8. When it is known how many pairs of footwear there were in all, and how many pairs were sent to the store, what will you be able to find out?

9. How can you find out how many pairs of footwear were left?

10. If you knew how many pairs of footwear were left after some had been sent to the store, and it is given in the problem that 25% of the remaining footwear was sent to the booth, what could you then find out?

After the analysis, the pupils were asked to compose an oral plan, and then begin the written solution.

The pupils' answers to the questions in the analysis and their solutions differed greatly (and this difference depended very little on the aptitude of the pupils): a) some pupils correctly answered the questions, composed a correct plan, and solved the problem (Nina M., Olya T., Yura B., Ira S.); b) others answered the individual questions incorrectly and made various errors in the solution: for a long time Ira S. did not understand how to answer question 5, and she erred in the written solution of questions 5 and 6. Kolya S. and Tamara Sh. could not immediately answer questions 5 and 6 (where it was necessary to state a question from the data), but, nevertheless, correctly composed (orally) the plan of solution, although they erred in the written solution (Kolya S. selected incorrect data for solving correct questions--the fourth and fifth--of the plan, and Tamara Sh. did not state and did not solve the fourth question of the plan); Katya B. answered all questions correctly, but she could not orally compose a precise and correct solution plan (among the correct questions she gave several incorrect ones), and she stated a third question incorrectly in the written solution; despite all this, the solutions were correct and led her to the exact answer to the question of the problem.

Comparison of the Analytic and Synthetic Methods of Analyzing Complex Arithmetic Problems (from experimental data)

In our conclusions for each grade (third, fifth, and seventh) we considered, as far as the factual material allowed, the general and individual difficulties characteristic of the pupils in employing the analytic and synthetic methods of analysis. Now let us make a comparative evaluation, on the basis of the data obtained from this series, of the synthetic and analytic methods of analysis, using as criteria their comprehensibility and efficacy.[7]

From the results obtained let us answer these interrelated, mutually conditioning questions:

1) In analyzing a concrete problem, which of the methods of analysis was the easiest and most comprehensible for the children?

2) With which method of analysis did the children have most success in the oral composition of a plan and the independent written solution of the problem?

Having answered these two questions, we can consider the most difficult question, which requires special study and profound pedagogical and psychological analysis:

3) What is the role and significance of analysis, the influence of one or the other method of analysis on the subsequent process of solving the problem? In other words, during which of the methods of analysis--the analytic or the synthetic one--is the child's mind best attuned to the correct way of solving a problem; with which method does the child best grasp the logical system of questions which leads to establishing relationships among the data and to composing a plan of solution?

The difficulty in answering the third question is that the solution itself does not permit us to judge the role of analysis with thoroughness. There are two reasons for this: first, in this and other series we observed a discrepancy between our pupils' arguments and their solutions; that is, the results which the child obtained in an analysis of the conditions were not always used in the solution; second, since to a certain extent the solution is conducted independently of the arguments in the analysis, the pupil can understand in the solution process something he had not understood in the analysis. It cannot always be established whether the pupil arrived at the correct solution during the oral analysis, or during the written solution. An intermediate, transitional stage aids somewhat in understanding the dependence of the solution upon the analysis. The stage is the composition of the oral plan, after the analysis of the conditions, and

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[7] Here one must remember that, in the school where the material was gathered, the basic method of analysis in teaching children problem solving during the first years of instruction was the synthetic method.

before the written solution of the problem.

Let us now answer the three questions posed above.

1. Which of the methods of analysis was the most comprehensible to the pupils in analyzing a problem?

Among pedagogs and methodologists there is a rather widespread opinion that the analytic method is more difficult, and is therefore recommended only in the final years of instruction in the elementary school. The results of series IV of our experiments do not uphold this opinion. In analyzing problem conditions, for 68% of our children, if synthesis[8] was strong, analysis[9] was also strong; if the pupil correctly answered questions during synthetic analysis, he answered questions equally well during the analytic method of analysis. The 68% was divided in the following way: 44% had trouble with both the synthetic and the analytic methods of analysis; 24% correctly answered questions in both types of analysis. The other 32% were pupils for whom the analytic and synthetic methods presented varying difficulty. Of these 20% gave more incorrect answers during the synthetic analysis than during the analytic analysis, and 12% answered more questions incorrectly using the synthetic. The third grade subjects gave more incorrect answers during the synthetic method and the fifth and seventh grade subjects gave more incorrect answers during the analytic method. Thus, no significant advantages in the ease and comprehensibility of either the analytic or the synthetic methods of analysis were observed for our pupils.

2. After which method of analysis did the pupils more successfully compose an oral plan and give a written solution to the problem?

a. The pupils' composition of the oral plan of solution after each of the methods of analysis of the conditions.

The quantitative correlation of the qualitatively different types of oral plan composition among the pupils changed in relation to the methods of analysis. We saw above that, after one or the other method of analysis, the subjects approached composition of the oral plan in different ways: a) some of them outlined the questions of the plan, b) some also showed the solutions to the questions they stated (that is,

[8,9] With regard to the pupils' answers to questions during the appropriate method of analysis.

arithmetical operations, numerical data), and some composed incorrect plans by not stating the questions, but indicating solutions only, or else by first stating the solutions and only then posing questions to fit solutions.

The dependence of these qualitatively different types of plan composition upon the method of analysis are presented in Table 2.

The results shown in Table 2 indicate that after the analytic method of analysis the number of cases in which the oral plan of solution was composed by stating the questions increased (as compared to the synthetic method) and the number of cases in which it was composed by indicating the solutions (operations, data) decreased; this shows the influence of the analytic method of analysis (which facilitates the composition of the plan, and makes it more logical than the synthetic method) upon the subjects of the third and fifth grades. The differences in composing a plan seem to depend less upon the different methods of analysis among the seventh grade subjects.

After using the synthetic method, not one of the third grade subjects outlined the plan by stating questions, while after the analytic method, 8% first stated the questions, and half of these also indicated the solution to the questions. Cases of composing a plan by indicating solutions (and not questions) decreased to 16% in third grade pupils after the analytic method of analysis (from 34.6% during the synthetic method of analysis).

The fifth grade behaved similarly. All subjects of the fifth grade, after the analytic method of analysis, stated the questions of the solution plan of the problem, while, after the synthetic method, 7.7% had outlined a plan by indicating solutions, after which they stated the questions.

The seventh grade pupils, on the other hand, produced more correctly composed plans after the synthetic method than after the analytic method.

b. The pupils' completion of the written solution after each of the methods of analysis.

After being introduced to both the synthetic and analytic methods of analysis and composition of the oral plan, the pupils solved the problems independently. We did not observe as great an influence from the method used in the preliminary analysis of the conditions in the written solution as we noticed in the composition of the oral plan of solution.

TABLE 2

COMPOSITION OF AN ORAL PLAN AFTER ANALYZING THE CONDITIONS

Method of Preliminary Analysis	Questions of the Plan			Questions of the Plan & Solutions			Plan Composed by Indicating Questions	Plan Composed Incorrectly		
	Grade III	Grade V	Grade VII	Grade III	Grade V	Grade VII		Grade III	Grade V	Grade VII
Synthetic	0	26.9	23.1	0	0	7.7	57.7	57.1	22.3	37.5
Analytic	4.0	36.0	20.0	4.0	0	8.0	72.0	28.5	11.2	62.5

Note--Entries in the first seven columns appear to be percents of all students using a given method of preliminary analysis; entries in the last three columns appear to be percents of all students in a given grade using a given method of preliminary analysis (Ed.).

For convenience of comparison we have put this in Table 3.

TABLE 3

INCORRECT QUESTIONS AND INCORRECT WRITTEN SOLUTIONS OF THE PROBLEM AFTER ANALYSIS OF THE CONDITIONS AND COMPOSITION OF THE PLAN

Method of Preliminary Analysis	Incorrect Questions*			Incorrect Solutions		
	Grade III	Grade V	Grade VII	Grade III	Grade V	Grade VII
Synthetic	6.2	3.7	8.6	21.3	18.5	17.2
Analytic	7.2	0	11.4	21.1	11.5	17.1

*8.6% of the students did not state questions.

NOTE--Entries are represented as percents of all students in a given grade who solved the problem (Ed.).

From Table 3 it can be seen that in fifth grade pupils, after the analytic method of analysis, there is a slight decrease (compared to the synthetic method) in incorrectly stated questions and solutions. In the seventh grade pupils, however, there is a slight increase in mistakes. The written solutions of the third graders after the analytic analysis were almost the same as the written solutions after synthetic analysis.

3. What influence does the preliminary analysis have on the process of the solution of the problem?

On the basis of the data (given in answer to questions 1 and 2 above) of this series, let us now discuss how the solution process depends upon the preliminary method of analysis. From what has been presented it is not hard to see some very essential facts.

a) The composition of the oral plan is determined by the preliminary method of analysis. After he analytically analyzes the conditions, the subject is more successful in outlining the solution plan; this dependence of the plan composition upon the method of analysis decreases gradually over the years of instruction. Thus, in the third grade, the number of incorrectly composed oral plans was 28.6% less after the analytic method of analysis than after the synthetic method; in the fifth grade the decrease was 11.1%. In the seventh grade, however, there were 25% more correctly composed plans after the synthetic method than after the analytic method (see Table 2).

b) However, the success of the written solution depends little upon which one of the methods was used to make the analysis; it depends more on how the analysis of the conditions was carried out, how the children answered the stated questions. After the analytic method of analysis the number of incorrect questions decreased by 3.7% in fifth grade pupils. In seventh grade pupils, however, it decreased by 2.8% after the synthetic method was used to analyze the incorrect written plans; this is in accordance with the success of the preliminary analysis (see Table 3).

It follows from this that the increase of correct reasoning after the analytic method, observed in third and fifth grade pupils during oral composition of the plan, was not realized fully in the written solution. In other words, when beginning the written solution of the problem, the children were influenced little by their oral reasoning.

The results of the aforementioned qualitative analysis of the pupils' performance of assignments in other series helps us to explain this fact. In particular, we saw (see series II, seventh grade) that one of the characteristic features of the pupils' solution of a problem was the insufficient use of conclusions drawn while mastering the conditions of the problem. When returning to the conditions, the pupil tries to recall, not what he understood and indicated during the analysis, but how he had once solved a similar, familiar problem; the success of the solution is determined more by earlier mastered devices of problem solving, sometimes simply by a mastered stereotyped pattern of solution (see series I, third grade).

In the results of series II we noted that the upper-grade pupils give evidence of their own individual approaches to the solution of problems, with all the weaknesses acquired over years of instruction. Hence the analytic method of analysis and the subsequent composition of the plan, as well as the written solution of the problem, gives way in the seventh grade pupils to the habitual synthetic method of analysis and solution.

Thus the qualitative feature of the pupils' approach to solving the problem explains the observed discrepancies between the oral composition of the plan and its written realization. The results of this series once again confirm the existence of this peculiarity, characteristic of our children's approach to the solution of problems. The pupils carry out more correct arguments and conclusions with the analytic method than with the synthetic, but use them less in the written solution. This can be explained by the peculiarities of the types of approach as well as by the need for switching from oral reasoning in an analytic plan to the problem's written solution, which is always done according to a synthetic plan. Obviously, the pupils have insufficient ability to make such a transition, which demands flexibility and quick-wittedness. Since the transition is not reproduced by the pupils, it escapes the teacher's attention while she is teaching them to solve problems. The methods of analyzing problems bypass this goal. Thus we see in the results of this series that the analytic method may be used as successfully as the synthetic method in the problem analysis of pupils of the lower grades.

The seventh graders' ability to apply the analytic method of analysis is weaker than their ability to apply the synthetic method. These pupils hold more successfully and stubbornly to the synthetic mode which predominates in school practice, than do pupils in the lower grades, who have been less influenced by instruction. The pupils of the lower grades analyze the conditions of a problem by the analytic method just as successfully as by the synthetic method.

Thus, on the basis of the experimental data we draw this conclusion, confirming our hypothesis based on a search of the literature: the analytic method of analysis should be used in the later years of study, thereby making it possible to consolidate the ability to use the synthetic method, leading to the development of less conscious solutions of problems by the isolation of the solutions of separate parts of the conditions.

If we consider, furthermore, that the auxiliary school pupils hold more to what they have learned (often not consciously enough) in the form of mechanical devices, schemes and stereotypes of the solution of a problem, and that they try to operate with them, but not with conscious reasoning according to the given conditions, then for the auxiliary school our conclusion is the first step in correcting the mentally retarded child's personality; problem solving can help attain this goal.

Only when both the synthetic and the analytic methods are used simultaneously in problem-solving instruction is it possible to further overcome the insufficiency characteristic of auxiliary school children's approach to problem solving—the attempt not to reason, but just to operate; not to explain or substantiate one's actions, one's steps along the road to solution, but to "solve the problem"; to outline a barren, incoherent course of solution. How can I solve it, what should I do? Our pupils concentrated primarily on such a question and were little interested in the more essential questions—what had to be solved, and why must this and not that operation be used? Why must such-and-such a question be stated, why must this and not that operation be selected? Why should precisely these and not those data be used? And so on.

Application of the analytic method of analysis from the first years of instruction would be directed against the main defect, the weakest link in the development of the skill of solving problems—against the yearning to arrive at the answer to the question of the problem by a barren, insufficiently considered and unsubstantiated solution.

Some Features of Recognition, of the Solution of Problems, and of the Preliminary Analysis of the Conditions

The function of analysis is not limited merely to the direct breakdown of a complex arithmetic problem into a series of simple ones. In our opinion this is the end result of that mental activity which is active during the analysis, before it, and after it, which begins as soon as the pupil has read (or heard) the conditions of the problem. The analysis of the conditions of the problem is inseparable, from the psychological standpoint, from the process of recognition and solution. On the contrary, the process of analysis promotes recognition of the conditions, complete understanding, and solution of the problems.

Therefore, before dwelling on the features of the analysis of problems, and those difficulties observed in our pupils during such analysis, let us point out several features of recognition and of the solution of problems.

Some Features of Problem Recognition Characteristic of Auxiliary School Pupils

We can point out only certain general and individual peculiarities of the pupils' recognition of problems. The pupils' recognition of a problem changes depending on instructional experience. Qualitatively, however, this change is insignificant:

a) The third graders are best aware of the problem as conditions without a question. The question of the problem remains outside the pupil's cognition and does not determine or guide the solution--it does not influence the selection of data or of operation (see the solutions in series I). The third grade pupils, when requested, could not even extract the question of the problem from the conditions (see series IV). Moreover, they recognized the conditions of the problem as something nondifferentiable, an indivisible unit. The children had trouble not only in extracting the question of the problem, but in stating the separate numerical data, words, etc. Instead of the one required datum they named it along with a series of associated words, and sometimes took into consideration the entire conditions; here the recognition of the conditions, even without a question, occurs only when the problem appears in a form familiar to the children from school instruction. If its content is changed, for instance, by omitting the numerical data but leaving the story of the problem and the question (series II), only the strong pupils will recognize the problem. Average pupils no longer will recognize the conditions as a whole even without the question; they will understand only separate words, only parts of the conditions. Some poor pupils perceive the stated problem with its meaning distorted and find things in it that are not there; they compose a different problem of their own making.

b) In the fifth graders the question of the problem also did not determine the solutions. Recognition of separate parts of the conditions was somewhat better, however. The fifth grade pupils carried out little related solutions, using separate parts of the conditions more freely, and they could already state the question of the problem, which was usually copied down last in the solution.

c) The seventh graders freely produced all possible solutions of the separate parts of noncomplex conditions (series II), but the question of the problem was also often insufficiently recognized. At best the strong and average pupils became aware of the question as a result of the individual solutions. Some of the weak pupils also showed insufficient recognition of the conditions, altering the conditions at their own desire, or imagining and solving a problem different from the given one (series III). Strong and average pupils could extract the question of the problem. The poor seventh graders, when requested, could not immediately point out what was required to be found in the problem (series IV).

The ways in which our pupils realize the condition of the problem and understand the question of the problem generally coincide with the results obtained by N. A. Menchinskaya [2] in her experiments with pupils of the mass school. The process of problem recognition in our pupils, however, is qualitatively different. While the pupils of the mass school (first grade) operated in their solutions upon the partial extracted conditions, and were aware of their semantic aspect, our pupils, even those of older school age (of the third, and sometimes even the fifth grade), could not take into consideration the semantic aspect of the given conditions.

Such were the general characteristics of most of the pupils. However, there were more subtle distinctions in individual features of recognizing the conditions. Some pupils did not remark on the similarity of semantically identical problems differing only in insignificant ways; others remarked on the general similarity and the finer concrete requirements of the conditions of the problem (series II, III).

Other pupils, in place of an unknown situation, part of a condition, or a relationship between data of a problem, named made-up situations, numerical data, etc., distorting the original problem. Still others stopped when they came to something they did not understand, made no incorrect calculations, and did not distort the given conditions.

With some pupils the recognition of the conditions did not determine the success of the solution. We saw that individual pupils stated correct judgments, by which we could infer that the conditions were understood. But as soon as the subjects began the written solution, as soon as they pondered the question of how to solve the problem; they forgot the conditions which they understood and produced in their solutions something quite different from what they had stated, what they had decided to do (series IV).

Some Features of Problem Solving Characteristic of Auxiliary School Pupils

The thought process in solving problems is a complex, intellectual process, whose nature is still not clear even in a man whose intellectual activity is not disturbed. One can therefore imagine all the complexity and difficulty of studying the problem-solving process in children with various disturbances of psychological, mainly intellectual, activity. It is hardly possible to imagine two persons, even without brain damage, who think identically when solving one and the same problem. It is all the more difficult to find this generality in children who have sustained various types of brain damage--various in time, depth, and strength. Therefore, however hard we may try, we can discover only certain general characteristics of problem solving common to all children.

The abovementioned peculiarity of problem recognition, like that of conditions without a question, is more or less characteristic of all pupils; the possible method of solution is conditioned accordingly in this case--a solution on the basis of separate parts, words, and numerical data of the conditions irrespective of the question of the problem (the synthetic method).

Actually we have seen above that almost all our pupils, despite the qualitatively distinct individual peculiarities in the solution, went through the solution carrying out all possible operations, based on the conditions of the given problem, irrespective of its question. In doing this, a) the third graders did not perform all the operations and did not clearly recognize the conditions of the problem. Individual solutions were not linked with each other, they disagreed not only with the question of the problem, but with each other; b) the fifth graders produced more possible solutions in terms of the existing, understood conditions of the problem, but they also selected them poorly for the answer to the question of the problem, and related them poorly to each other; c) the seventh graders produced possible solutions from the given conditions of the problem and often finally came to understand the problem and the answer to the question of the problem. But this occurred only when the problem was given the children in a form familiar from instruction. We had only to present the children a problem in a form new to them (for example, without giving numerical data, as in series II), and they produced all the possible (both necessary and unnecessary) solutions to the question of the problem. Some pupils, however, in solving the last question, could not isolate the solutions necessary for it from the total number of existing ones; they became confused and did not obtain a correct answer.

We cannot overlook mentioning something qualitatively new which was observed in the seventh graders' solutions when they were given a problem in a new form; i.e., the ability to compose a general plan of solution, to state the questions of the plan, beginning from the question of the problem. The questions of the plan, however, were often composed without enough awareness of the concrete data of the proposed conditions. In such cases we observed, on the one hand, an abstraction of the analytically composed oral plan of solution from the concrete data of the conditions, and the absence of direction in the written plan on the other. Despite such a general path of solution, there were diverse individual qualitative peculiarities in various children. There are children, who when working on a solution, strive to apply a particular device learned in instruction, a particular sequence of arithmetical operations used with insufficient understanding of the meaning of the question and the meaning of the conditions of the problem presented. In each grade this peculiarity appeared differently during various assignments.

In their approach to solving problems of various types, some third grade pupils employ, almost exclusively, a single stereotyped pattern which they have mastered in instruction. The child's first oral argument, made on the basis of the understood individual parts of the conditions, is much more correct and precise than later conclusions, especially when the child begins thinking of the written solution of the problem, of how to solve the problem; that is, when he begins to recall and apply the mastered stereotype. Thus in these children, written completion of the solution to the problem deviates from oral argument.

Some fifth graders already possess a large store and a limited variety of learned solution stereotypes. But one of them, once adopted for solving a problem, is used no less stubbornly than was done by the third grade pupils. The fifth graders could not drop an adopted plan even when they themselves noticed that the selected plan of solution was incorrect; they deliberately continued along the incorrect path (series I, II, III). In solving a problem they did not consider the finer requirements of the conditions (series I); problems different in form and content were solved identically (series I). In solving problems of unusual form (in which it was harder to use the stereotype), even greater unrecognized and unsubstantiated steps were taken; the pupil implies that he knows and understands all, and that he has little trouble in a solution. He does not ponder the correctness of his solution, and he solves a problem "with certainty"; but in fact he slides impetuously over the conditions of the problem; hurrying to effect the solution according to a prepared plan, a stereotype, he combines the possible and the impossible in the solution.

Some seventh graders attempted to solve problems without relying on a stereotype already mastered. This was more successfully accomplished in oral reasoning; the pupils could now outline a plan of solving the problem, beginning from the meaning (and not from the numerical data) of the conditions, and the better pupils could form it from the question of the problem. But the composed plan is related too vaguely to the concrete conditions of the problem. The solutions (numerical data, arithmetical operations), however, take their own route independent of the stated (sometimes incorrect) questions. In the seventh grade pupils, in particular, there is instilled a strong desire to get along without stereotypes in the oral solution. In the written solution, however, "the logic of operations takes over the logic of the question."

As we saw above, there are pupils in whom the "logic of operations" also determine, to a large extent, the entire course of the solution; but such logic gives way to the "logic of the question" under the influence of instruction more quickly in the children just indicated in group I.

Some third grade pupils are no longer characterized by stubbornness in using one mastered order of arithmetical operations; the more the child works on the solution and analyzes the conditions, the more accurately he recognizes it; he makes subsequent deductions more correctly than others. What is stated orally is put in the written solution; if he errs in the oral solution, he errs in the written solution; if he states correct ideas, he produces them in the written solution. No discrepancy between the written and the oral solutions of these children was observed; the pupils satisfied the finer requirements of the conditions more precisely (series II); their solutions, moreover, were closer to the concrete conditions (series I). They gave more correct solutions and questions and took fewer incorrect, unsubstantiated steps (series III); they took up and completed more often what was within their capabilities, what they recognized. The children of this group, however, display their own weakness in solution; the solutions they made, which were within their capabilities, were often not enough to answer the question of the problem (series I); individual questions were not always combined in a general plan of the solution (series I).

The fifth grade pupils of this group also evinced greater flexibility in that they imagined their solutions within the concrete framework of the problem; they satisfied the conditions and requirements of the assignments in series II and III precisely, which the pupils with the characteristics of solution mentioned earlier could not do. Solutions of the various problems were different, and were obtained from more than one plan (series I, II, III); in these pupils, we could notice mental operations on the conditions--they doubted the correctness of their solutions, they wavered; they tried to find in the conditions a confirmation of their action (series I). Pupils of this group, in other words, are distinguished more by a critical approach and an independence in seeking ways of solution.

All seventh graders completed solutions of which they were capable, according to the existing conditions of the problem; they did not alter or rework the problem, and they did not introduce anything personal into the problem.

Thus the pupils of identical grades displayed various weaknesses in their solutions; in turn these solutions changed from year to year under the influence of instruction.

Difficulties Encountered by Pupils during Preliminary Analysis of the Conditions of the Problem

The main characteristics of an understanding of the conditions and the solution of problems which we were able to notice in pupils in their individual pursuits were significantly related to the analysis of problems, and therefore explain some some difficulties which the pupils encountered when directly analyzing problems. Of these difficulties we were able to observe the following:

a) Some had difficulty (during the analytic method of analysis) in extracting the question of the problem from the conditions in the form in which it is given (all third graders and weak fifth graders);

b) Some had difficulty in stating both data necessary to solve the question of the problem (most of the pupils of all grades). Instead of two data, the third graders and weak fifth graders stated only one datum;

c) Some had difficulty in stating the question from two data (in the synthetic method of analysis);

d) Some had difficulty in attuning themselves to, or remaining upon, the definite track, designated by the teacher, of breaking down a complex problem into simple problems (in both methods of analysis).

After becoming acquainted with the conditions, the pupil set about solving the problem in his own way and answered the teacher's questions (in both analytic and synthetic methods of analysis) with the thoughts which he had while trying to solve the problem independently.

Although the pupil was able to give correct answers to our questions (after incorrect ones) during analysis of a problem, the pupil could not always, as he began the independent written solution, distinguish correct answers from incorrect ones; correct answers did not serve, moreover, as the basis for a correct solution;

e) Some pupils did not use everything which they had obtained and recognized from the analysis in their solutions; in such cases the written solution differed from the oral argument;

f) Some had difficulty (following the analytic method) in changing over to the independent written solution of the problem, since the questions of the written plan were in reverse order from the questions obtained in the analytic method of analysis.

This difficulty with the analytic method is the main correctional advantage, compared with the synthetic method, because the necessity of shifting from "one's own" method to the analytic method of decomposition, and then to composing a plan of the solution (in a reverse order to that obtained through the analytic method) is directed against the basic disorder of the mind, the slow progress of the pupil's thinking.

Pedagogical and Methodological Ways of Teaching Problem Solving Concerning the Methods of Teaching Problem Solving

The aforementioned peculiarities of the processes of the recognition, analysis, and solution of a problem make it possible to answer a series of questions connected with the methodology of teaching mentally retarded children how to solve complex arithmetic problems.

We did not aim to select the most rational method of teaching problem solving (the method of analyzing the conditions, the method of many problems, the method of analogy) and we did not do comparative research in this direction. There can be no universal method of instruction. Nevertheless, considering the peculiarities of problem solution noticed in our pupils, one may speak of the advantage of one or another method of teaching problem solving.

We saw above that children, through instruction, have a tendency to master a single stereotype and to apply it for solution of various problems; use of the mastered convention in problem solving often occurs almost irrespective of the conditions of the proposed concrete problem. In such a case it seems that the mentally retarded child is not solving the problem given him. No matter how many problems are given the child, they are all "solved" by him according to one stereotype, in the same way he solved the preceding problem. At best only numerical data from the conditions are used in carrying out such a solution. Obviously the method of instruction involving solution of a large number of homogeneous problems cannot be recommended as the basic method of teaching auxiliary school pupils.

In applying the method of analogy, without which it is difficult to proceed in teaching problem solving, there are, however, not enough possibilities for overcoming mechanical and unimaginative ways of solving problems. We saw above that pupils recalled a problem which they had once solved not by understanding the meaning of the entire given problem, but on the basis of its individual words. This led to their recalling a problem which was not analogous—a different, almost dissimilar problem. Instead of helping the child, it diverted him from the correct path of the solution, from his attempt to delve into the conditions of the given problem and understand them. Such mental labor--i.e., recalling or thinking up an easy problem analogous to the given one—was not within our pupils' powers. Moreover, even when the child was able to solve an analogous problem, orally, he was unable to solve it in writing. However, there are no experimental data on the transfer of the problem-solving skill of mentally retarded schoolchildren; this question has not been studied yet. Therefore we cannot decisively reject use of the method of analogy.

The method of teaching through preliminary analysis of the conditions of problems is notable for its comparative advantages. During the analysis of the conditions of a problem, the child's thinking is directed toward a more profound recognition of the conditions of a given problem, toward deductions and solutions on the basis of what he has understood. The method of solution here is "revealed" during the intellectual activity of recognizing the conditions and the question of the problem, rather than recalling a known method of solution (the latter occurs with the method of analogy). In other words, there is developed the skill of beginning a solution from a thorough recognition of the conditions. Preliminary analysis of the conditions of a problem is directed toward overcoming one of the major defects in the thinking of the mentally retarded child--"reproductive thought"--and attempts to eliminate the appearance in the solutions of mechanical and unimaginative methods which arise when the conditions are not quite recognized. Undoubtedly in this we see a correctional advantage of analysis of the conditions over other methods of teaching mentally retarded children problem solving.

These are the possibilities of the method of preliminary analysis of the conditions, whose realization, however, depends on many things. Among them the most important are the use of existing methods (analytic and synthetic) of analysis in their correct relationship, taking into account the peculiarities displayed by pupils in their solution of problems.

We mentioned the mutual relationships of methods of analysis above, when we were comparing both methods (see conclusions for series IV) on the basis of the experimental data we obtained. Let us note here that elementary school methodologists, such as N. N. Nikitin and A. S. Pchelko, and auxiliary school methodologists, such as N. F. Kuz'mina-Syromyatnikova, point out the need for using the analytic method at the same time as the synthetic method, that is, in the early years of instruction. N. A. Menchinskaya came to a similar conclusion in her experimental psychological investigation conducted in the first grade of the elementary school [2].

Applying the Methods of Analysis

Let us examine the possibilities of using the methods of analysis. Work in the school on eliminating the existing flaws, the weak links in the complex problem-solving process which are observed in auxiliary school pupils, can be successfully carried on during the preliminary analysis of the conditions.

Above all it is necessary, during analysis of the conditions of a problem, to improve the pupils' differentiated understanding of the conditions, primarily the extraction from the conditions and recognition of the question of the problem. This goal of analysis should remain as the children are being taught to solve problems, since insufficient recognition of the question of the problem was observed in children of all grades. In the later years of instruction, the pupils' attention, during analysis should be especially directed toward establishing the interconnections between the separate solutions and their relationship to the question of the problem. To surmount this basic flaw, characteristic of all pupils, there are obviously greater possibilities in the analytic method of analysis, which begins with finding out and solving the question of the problem.

The following devices can be recommended to attain this goal.

a) The pupils retell the conditions of the problem without numbers (tell the "picture of the story").[10] This device, verified in practice, facilitates recognition of the conditions of the problem, dividing the material into logical, semantic, and numerical material; the child's thought is directed toward the differentiated recognition of the parts of the conditions, mainly toward recognizing the meaning of the conditions and the question of the problem. This makes it much easier to conduct the subsequent analysis of the problem by the analytic method;

b) Notation (on the blackboard) of the main unknown question of the problem in an abbreviated formulation;

c) Extraction from the conditions and notation (on the blackboard) of the unknowns (in words) and, apart from them, the known data; study of numerical data (value, units, connection and relationship), and, only after this, analysis of the conditions.

These devices greatly facilitate the analysis of the conditions and make it more comprehensible to the pupils; they help strengthen the weak links in problem solving characteristic of mentally retarded schoolchildren.

Changing the very process of analysis, its course or sequence, can scarcely be justified, considering the peculiarities of mentally retarded children. To the contrary, the analysis of the problem by a specific method in strict sequence is highly desirable in teaching problem solving to auxiliary school pupils; it is necessary to attain precision of formulation and a complete statement of the pupil's answer. For example, the pupil must be required to name both given data needed to solve the problem, not just one of them (the children often name an unknown). This need for work on the child's oral formulation derives from the necessity of attaining the primary goal in solving problems--the goal of developing thought, which is directly connected with language.

Here it must not be forgotten that, although the separation of the synthetic and analytic methods is conditional in terms of psychology, each of the methods has its faults and its advantages in terms of

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[10] This device is widely and successfully used by D. V. Tsvetkov in the Kostromskaya Auxiliary School; whose children solve problems well.

methodology. If we consider, moreover, that auxiliary school pupils have trouble mastering what they are given in instruction and blindly retain what they have learned, it becomes a necessity to <u>teach the children both methods of analysis</u> as soon as possible, alternating and transferring from one to the other. For example, they may solve a problem using preliminary analysis of the conditions by one method, and then, after several days, use the other method of analysis.[11]

In analyzing a problem and trying to take account of the pupils' individual differences in its solution, it is necessary to remember the insufficient flexibility of the psychic process which appears in pupils' solutions through the concentration of all their attention and all their mental powers on one object: a) either the pupil tries to establish a simple, elementary relationship among the numerical data (which he often does on the basis of a purely external, formal, similar feature) and, once the simple connection is established, is unable to proceed beyond it to an understanding of the other conditions and a general plan of solution; b) or he strives to understand, noting the general plan of solution without attempting to establish a relationship between the concrete data of the conditions; the plan outlined appears to be too general and the possibility of solving the problem with the existing data is not considered.

In a problem analysis one must strive to join these two mental operations (indicated in points (a) and (b) above). After doing one operation, the child has great difficulty transferring to the other. Therefore it is not enough merely to help the pupil when he is mentally struggling with a difficulty (only within the limits of one of the tendencies indicated in (a) and (b)); the best aid for him would obviously be to direct his mental efforts toward another object of which he has not yet become aware.

When analyzing the conditions one must take into account individual differences in the results of the pupils' mental activity when they are solving problems:

a) Pupils who are able to <u>better</u> the results of their mental

[11] This device is recommended by N. F. Kuz'mina-Syromyatnikova.

activity by moving from stage to stage in the solution of a problem do not require the teacher's undivided attention. They are more or less capable of independently solving separate questions—sometimes all of them. For them the initial moment of the solution is not of decisive significance; its imprecision, unsuitability, and incorrect first steps are corrected later. Here the pupils can successfully use both the analytic and the synthetic methods of analysis.

b) Pupils whose solutions display no improvement in reasoning and who cannot abandon a plan adopted at the very beginning, need the teacher's help. How the pupil will end a solution depends greatly on how he began it; therefore all the teacher's attention should be concentrated here on the initial solution of a problem. The child's thought should be prevented in all possible ways from starting off on an incorrect route in the solution. While the child's consciousness is not yet entirely occupied by the solution of individual questions, it is necessary to help him understand the relationship of all the conditions. The latter is most conveniently realized with the analytic method of analysis.

c) Pupils whose mental activity gives increasingly worse results as they work on a problem require the teacher's exclusive attention during the course of the entire solution. Here it is highly important to determine the moment at which, after a correct operation, the pupil begins to stray from the correct path (which most often happens when he comes to a difficult place in the solution). In such cases it is best to interrupt the solution and review the supplementary analysis of the problem with the pupil, stressing the difficult spot.

Teaching the pupils the analytic and synthetic methods of analysis of complex arithmetic problems should be begun with systematic <u>preparatory exercises</u>,[12] in each (especially the analytic) method of analysis. Here one must remember that the sequence in a system of preparatory exercises for the analytic method designed for the auxiliary school pupils should be different from that recommended for the pupils of the mass school. For our pupils it was easier (see series II) to select

[12]The types of preparatory exercises have been shown by V. A. Latyshev and, in even more detail, by F. A. Ern.

all the data for the question, and harder to select the missing second datum, when only one datum was given. Such are the possibilities for overcoming the weak links in the pupils' problem solving process during the preliminary analysis of the conditions. This type of work, however, must not be considered the only kind, completely sufficient, deciding all methodological questions of teaching children problem solving.

Some practitioners think that, by daily classroom review of work in solving a certain type of problem familiar to the children, they can further develop a firm skill in problem solving. In fact, however, with such instruction the pupils master only a specific device for solving one type of problem, and apply it later, relying little on the conditions of the problem being solved. Anything new in the problem (in form or content) leaves the children in a blind alley; they disregard it in the solution, a tendency characteristic of not only the poor pupils, but also of pupils who can solve the familiar, typical types of problems.

To attain skill in problem solving, not only must homogeneous, habitual work on problems be conducted daily, but there also must be review of the diversity of forms of mental activity which demand the pupils' systematic independence. In other words, work on the arithmetic problem in the auxiliary school should be constructed on the principle of more variation than is presently found in practice.

There are two ways to realize the principle of variation:

1. In a system of problems given in instruction (diversity of form and of mathematical content).

2. In types of work on arithmetic problems. Among these, besides the analysis of the conditions, the following are recommended:

a) Composition of two simple problems from numerical data—one after the other, with subsequent composition of a third problem from the two simple ones. For example, composition and solution of these problems:

Problem 1	Problem 2	Problem 3
4kg of groats at 5 rubles per kilogram.	2kg of sugar at 13 rubles per kilogram.	(composed from problems 1 and 2)

b) Composition of two simple problems from questions and subsequent composition of a third problem from the two simple ones obtained. For example, composition and solution of these problems:

Problem 1	Problem 2	Problem 3
How much does one book cost?	How much does one notebook cost?	How much more expensive is a book than a notebook?

c) Selection of numbers and a question for a situation (in one, two, or three questions). For example, select numbers, state the question, and solve the problems:

1. Two boys were gathering mushrooms. One boy collected more than the other.
2. Three boys were planting trees. Sasha planted fewer than Vanya, and Kolya planted more than Vanya.

d) Composition and solution of a problem from a given arithmetic operation and question. For example, composition and solution of these problems:

Problem 1	Problem 2	Problem 3
+ (on addition)	– (on subtraction)	+/– (on subtraction and addition)
How much money did Vasya have?	How much money did Misha have?	How much money did Vasya and Misha have together?

e) Composition and solution of arithmetic problems from an assigned arithmetical operation. For example, composition of any problem on addition, subtraction, multiplication, or division; composition of any problem requiring that a number be increased, decreased, taken several times; etc.).

f) Composition of arithmetic problems from numbers and operations. For example, composition and solution of these problems:

Problem 1	Problem 2
370 – 150 = 220	160 : 4 = 40

g) Solution of all simple problems from the conditions of a complex problem without a question, then with the added question. For example, solution of any questions:

A hat costs 80 rubles, a suit is 5 times as expensive, and a coat costs 800 rubles.

After the pupils solve all possible questions, they are asked to solve this same problem, first with one, then with another question:

1. How many times more expense is a coat than a suit?
2. How much was the entire purchase?

Work of this nature helps the pupils to understand the relationship between the solutions and the main question of the problem.

h) Composition and solution of a problem in a specified number of questions. For example, composition and solution of any problem in one, two, three, or four questions.

i) Composition of problems from data obtained from the pupils' practical activity (measuring, weighing in class, buying and selling, etc.).

j) Composition and solution of problems on a given topic. For example, composition of a problem on mushrooms, trees, candy, etc.

k) Composition and solution of any problems.

Activities of this type[13] show that the pupils handle the given assignments with difficulty and that the teacher's assistance is necessary at first; later the pupils become increasingly better. Lesson by lesson the pupils approach the solution of problems, the selection of numerical data, and the statement of the question more boldly and more consciously; they relate to their solutions more critically; they check them. Interest is awakened in the children, flashes of independent mental activity occur which are phenomena observed even when they are solving ordinary types of problems; they come to have questions about the possibility and impossibility of a solution, of different methods of solution, etc.

The teacher, however, must be warned against excessively increasing the types of work in problem solving which are offered. Experience has shown that it is best to do this type of work once a week as a special lesson with an obligatory connection with the curriculum material and requirements, enriching instruction in solving curriculum problems in only one way—the method of preliminary analysis of the condition.

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[13] They were conducted in the 1949-50 and 1950-51 school years by teachers of Moscow Auxiliary School No. 111 with the author's participation.

There can be no doubt that work in this direction, as well as the experimentation of teachers, will help to increase the effectiveness of teaching children problem solving in the environs of the auxiliary school.

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BASIC DIFFICULTIES ENCOUNTERED BY AUXILIARY SCHOOL PUPILS IN SOLVING ARITHMETIC PROBLEMS

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This investigation was based upon pedagogical practice, and its conclusions are directed to the improvement of the content and method of teaching problem solving to auxiliary school students.

The analysis and processing of written work in arithmetic by auxiliary school students of the Russian Federation was conducted by the Ministry of Education and the Scientific Research Institute of Defectology for use in studying the students' progress over many years. This study has permitted us to develop a method of investigating the difficulties arising in teaching mentally retarded children how to solve arithmetic problems.

We set the following goals for our investigation:

1) to reveal the fundamental difficulties experienced by auxiliary school students (in solving problems).

2) to reveal the causes of peculiarities in problem solving by mentally retarded schoolchildren.

3) to plot the proper direction of work on improving the content and method of teaching problem solving in the auxiliary school, on the basis of observations, experiments, and the past experience of the best teachers.

To determine the causes of the basic difficulties in solving arithmetic problems encountered by mentally retarded schoolchildren, we conducted two series of experiments with the children.

In one series, problem solving followed oral reproduction of the problem's condition. In the other series, a problem was solved

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without a preliminary statement of the conditions. In these experiments the children solved all the basic kinds of simple problems. At the same time, in order to study the peculiarities of their use of nomenclature, certain problems requiring several operations were employed.

Schoolchildren from all of the auxiliary school grades participated in experiments to determine their characteristic ways of solving simple problems. In our study on the reproduction of problems we tested students mainly from the upper classes. To gain a deeper insight into the peculiarities of the perceptual activities of the auxiliary school students, we also conducted experimental studies with students from the mass school.

All the experimental material underwent quantitative and qualitative examination and was compared with what we observed in the lessons, as well as with the teachers' systematic evaluation of the children's knowledge. In the concluding chapter, which contains several proposals for improving the methods of work in problem solving in the auxiliary school, we summarized the experience of some of our best teachers; such experience confirmed the basic conclusions of this special investigation.

The Reproduction of Simple Arithmetic Problems by Mentally Retarded Pupils

Study of the reproduction of the conditions of a problem in order to learn the characteristic way in which the pupils comprehend the problem.

We know from pedagogical and psychological investigations that features of students' thinking are revealed in their solution of arithmetic problems. Investigations into the methods used by normal children to solve problems have enriched our knowledge of the psychology of the children's thought processes, and have contributed to the improvement of methods of teaching arithmetic.

Observations of mentally retarded schoolchildren engaged in problem solving show that they are unable to formulate their ideas in detail, and that their statements sometimes lack enough consistency to lead to a solution; therefore, a study of their reasoning does not always reveal the course of a problem's solution. The thought processes of mentally retarded children solving problems are therefore usually judged by the nature of the mistakes that arise in the course of their solution, as well as by the solution itself.

There is, however, one other method for determining how mentally retarded children comprehend the contents of a problem. We refer to the study of the characteristic features of the students' reproduction of the conditions of a problem. We proceed from the assumption that the character of the reproduction of the material reflects the level of comprehension of that material. Investigations by Soviet psychologists devoted to the problem of reproduction showed that correct reproduction depends greatly upon comprehension of the material. It was also shown that reproduction depends upon the nature of the material offered the children, upon the conditions under which the material was memorized, and upon the goal towards which memorization and reproduction are directed. The data used by Soviet psychology provides a basis for utilizing the characteristics of the students' reproduction of the conditions of a problem, in order to study their comprehension of its contents.

We conducted individual experiments with 20 fourth grade and with 20 sixth grade students in the auxiliary school; for comparison, we conducted analogous experiments with 20 second grade students in the elementary school. The problems posed were selected to correspond to the curriculum for the first grade in the mass school, which corresponds to the curriculum for the third grade in the auxiliary school. Problems were offered to the children from sections of the curriculum that had already been studied. By the time the experiments were conducted, the children had, in their lessons, already solved problems which were more complicated in structure; thus, the problems which we offered should not have presented any special difficulties for these schoolchildren.

Each pupil was given a card on which the text of the problem was printed and which the children read over two or three times. After this the pupil orally reproduced the conditions of the problem. We wrote down their reproduction verbatim; the material gathered was then processed correspondingly.

After the first experiments, it became clear that the pupils from the auxiliary school experience great difficulties in repeating the conditions of a problem. Before analyzing the causes of this difficulty, let us examine the ways in which the reproduction of some other sort of oral material.

The reproduction of a problem--i.e., repeating its conditions--differs, for example, from recounting a literary, historical, or geographical text. When reproducing such texts one is not forbidden to recount it in one's own words; indeed, one is encouraged to do so. In reproducing such texts an alteration or even a violation of sequence is sometimes permissible, so long as the sequence of events, etc., does not suffer as a result.

Other demands must be met in reproducing the conditions of a problem. The text of a problem is generally not very lengthy. The students must closely approximate the text; only insignificant variations in their reproduction of the contents of the problem are permissible, and a completely accurate reproduction of numbers and of the question in the problem is required. The reproduction of a problem, however, cannot be identified with learning prose or poetry by heart, since in that case it is mandatory that the text be reproduced literally. Furthermore, the memorization of poetry is facilitated by the presence of rhyme and rhymed verse, which do not exist in the text of a problem.

Returning to the topic of correct reproduction of the conditions of a problem, we should note that for this purpose it is necessary to carry out a specific logical analysis of the text, having as its aim a full determination of all of its parts; since, in reproducing a problem, all of its contents must be given, preserving without fail the relationship among the contents. The significance of these features in the reproduction of oral material has been noted in a series of psychological investigations. "A precise understanding of the internal relationships among the parts of a text is essential, so that these parts do not stand out in the student's mind as separate units, but as unified members of an interconnected whole" [21: 75].

These statements are particularly significant in connection with the reproduction of a problem, since perfect memorization and reproduction of the problem are impossible without comprehending each of its links and actually showing the relationships between these links.

Psychological investigations show that in the process of reproduction, essential changes are introduced into the material which has already been perceived. "When we mentally reproduce something, we not so much recall it as deduce it from our previous experience" [17:148].

It can be supposed that mentally retarded children will make more significant alterations in material they have perceived than will normal children, and that these alterations will have a somewhat different character. Mentally retarded children do indeed have difficulty in reproducing previous acquired knowledge, as well as in ascertaining the relationships between sentences and groups of sentences in a text. These peculiarities in the way auxiliary school children reproduce an oral text clearly show that the difficulties they experience in establishing the relationship between parts of the text lead to distortion and cause them to substitute other relationships for the original relationships. These peculiarities in the memory of mentally retarded children should be apparent in their reproduction of the conditions of a problem too.

The formation and preservation of the new temporary relationships which are the physiological bases of memory are closely related to the characteristics of the highest nervous activities in man (strength, even temper, and flexibility in the processes of stimulation and inhibition).

In the investigations of N. I. Krasnogorskii and his coworkers certain facts are revealed concerning the highest nervous activities of mentally retarded children. In normal children conditioned reactions are the most delicate and specific of all reactions. Their high specificity, quick formation, and slow disintegration, their stability with time--such are the characteristic traits of conditioned reflexes. On the other hand, imbeciles and debilitated children have difficulty forming conditioned reflexes [8].

Parallel to "sequential stimulation," Krasnogorskii examines "sequential inhibitation." He notes that these mechanisms appear to be more complex, and arise later, than those mechanisms whose activities are not connected with the signal system set aside at the time. Normally they are already formed by the age of three, whereas in cases of irregular development, they are formed only with extreme difficulty. Thus, the "traces" of past experience in auxiliary school students do not bear the same characteristics as the "traces" in mass school students.

The statement of I. P. Pavlov about the relationship between memorization and understanding has great significance as a revelation of the physiological bases of memory: "When a new relationship is formed, that is, what we call 'association,' it is, without a doubt, knowledge about definite relationships in the external world; when it is employed the next time it is called understanding --i.e., the use of the knowledge that has been acquired about relationships. This is understanding" [13:579]. The physiological mechanism of comprehension is regarded here as the ability to make use of previously acquired relationships, the ability to put them into practice. This statement implies that comprehension rests upon retaining traces; it is detrimental in this case for previously acquired temporary relationships not to be retained and consolidated, to be changed or erased. These statements by Pavlov are essential in determining the characteristic way in which auxiliary school students reproduce the conditions of a problem.

The parts of a problem always have a definite interrelationship. Whether this relationship is manifest depends upon the process of interpreting the problem. Unless the problem is properly understood, as we will show below, the conditions of the problem cannot be fully reproduced. Comprehension of the problem, therefore, appears to be an essential requirement in reproducing the problem correctly. Errors made by mentally retarded schoolchildren in repeating the text of the problem are usually the symptoms of difficulties in interpreting and comprehending the problem. We believe that an analysis of the way in which mentally retarded schoolchildren reproduce the text of problems may also ascertain the causes of the difficulties they encounter in solving arithmetic problems.

A child's work in orally formulating a problem begins when he becomes acquainted with it. An arithmetic problem can be presented in many ways; usually, the teacher reads the problem aloud, or the pupil himself reads the problem. Sometimes the contents of the problem are illustrated by visual material as well; this is not our consideration here. The questions that concern us are related to the interpretation of a problem read by the pupil independently. Practicing teachers repeatedly observe that auxiliary school students react to their first encounter with problems in a very distinctive way and that they have great difficulty in interpreting a problem.

These difficulties may result from the students' insufficient acquaintance with the objects or situations referred to in the problem, from their inability to picture the situation on the basis of an oral reading of the text; they may be caused by the students' lack of understanding of the relationship between the parts of the problem, that is, by their inability to interpret the structure of the problem. In reproducing a problem the student should visualize the situation set forth in the problem. In other words, the reproduction of the problem is made on the basis of what has been recreated in the imagination. On the basis of the description given in the problem one must imagine the situation which reflects the conditions of the problem. This situation should include both the basic facts of the problem, and also the alterations that will be brought about according to the conditions of the problem.

Consequently, it is necessary to picture the outcome of the events as predetermined by the contents of the question. Take, for example, the problem, "There were previously so many books standing on the shelf. Then such-and-such a quantity of books were taken away. How many books remain on the shelf?" It is a serious assignment for mentally retarded schoolchildren to picture the outcome of these events. It is not enough that the auxiliary school student perceive the conditions of a problem by reading or hearing it; in order for him to understand it, it is also necessary that the images he visualizes will so embody the material of the problem that they will ensure its reproduction.

Such a re-creation by the imagination is possible only when there is a normal interaction of both signal systems; this is precisely what investigators show to be the disturbance in auxiliary school students. That is why their words and expressions are often not related to substantive objects. Although they may be used in speech, they are really incorrectly understood (in too broad or narrow a sense). The disturbance in the interaction of the two signal systems in auxiliary school students is the source of many of their difficulties in solving arithmetic problems. It is necessary to take this into consideration when analyzing the reproduction of problems.

Characteristics of the reproduction of the conditions of simple problems by pupils of grades 4 and 6 in the auxiliary school.

In order to discover the causes that hamper mentally retarded schoolchildren in solving problems, we analyzed their reproductions of the conditions of the problems. A series of arithmetic problems differing in structural complexity but having only one operation was presented to the children. These problems had been solved by the pupils in the mass school by the end of their first year of schooling, while in the auxiliary school they had been solved in the 2nd and 3rd grades. The numerical data in these problems could be handled by second year students in the auxiliary school.

We attempted to ascertain the difficulties experienced by mentally retarded schoolchildren in reproducing different types of simple problems. Two problems in addition and two in subtraction were presented to them. Below are the texts of the problems:

1. In one cage there are 17 rabbits, and in another, 13 rabbits. Altogether, how many rabbits are in the two cages? (There are 21 elements in the problem.)[1]

2. There were 19 plain* pencils, and 6 more colored than plain pencils. How many colored pencils were there? (There are 18 elements in the problem.)

3. There were 25 notebooks in the cupboard; 13 notebooks were distributed to the students. How many notebooks remained in the cupboard? (There are 21 elements in the problem.)

4. A pupil spent 15 kopeks; he had 20 kopeks left. How much money had the pupil previously? (There are 17 elements in the problem.)

The difference between the number of elements in the conditions of the problem is not great. There are between 17 and 21 elements in the texts of the four problems. The students were quite familiar with

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[1] Each word and each number is taken as a single element.

*black-lead (Ed.)

the objects introduced in the problems.

The situations in the problems, that is, the relationships into which the objects entered as a result of the conditions, were not the same. On the basis of the features of these relationships, the above problems can be divided into two groups, in accordance with the varying complexity of their texts. In problems 1 and 3 the relationships between the components of the problem are given openly in the conditions themselves. In problems 2 and 4 the internal relationships between the components of the problem are introduced indirectly.

The problems in the first group (problems 1 and 3) are characterized by the fact that the "unknown" is closely related to the "given data," and the question asked is frankly joined to it. In the second group of problems (problems 2 and 4) the "unknown" and the "given data" are not connected by such direct, clear relationships. Indeed, although the purchase made by the pupil is mentioned in the text of problem 4, it is necessary, in solving the problem, to find out what the situation was before this purchase--that is, how much money the pupil had before he made the purchase. It is known from the conditions of problem 2 that there were colored pencils, but the question in the problem demands that the quantity of colored pencils be determined by their relationships to the black pencils. It is apparent that the relationship between the "given" and the "unknown" in problems 2 and 4 is different than in problems 1 and 3; consequently, it is more difficult to solve them than to solve those problems where the "unknown" follows directly from the "given."

The two groups of problems indicated above were presented to 20 fourth grade and 20 sixth grade auxiliary school pupils. The method used in our investigation was similar to usual teaching conditions. Although the problems corresponded to the third grade curriculum in the auxiliary school, their solution gave rise to serious difficulties not only for the fourth grade pupils, but for the sixth grade pupils as well. This forced us to pay special attention to the solution of problems by the sixth grade pupils.

Let us first examine how individual schoolchildren accomplished the reproduction of those four problems.

First we will look into the reproduction of the two problems (1 and 3) in which the conditions provided a direct relationship between the "given" and the "unknown" (the first group of problems). These problems were solved by two pupils from the fourth grade in the auxiliary school, Grisha K. and Kolya V.

Grisha K. reproduced the problems of the first group in the following way: "In one cage there are 17, and in another, 13. There will be 30" (Problem 1). Thus, the conditions of the problem were not fully reproduced: the nomenclature and the question were absent. The reproduction of the conditions and the solution are not separate from one another. The meaning of the problem, however, was caught rightly; the solution was also correct.

Problem 3 - "There were 25 notebooks in the cupboard. Thirteen notebooks were distributed. How many notebooks remained?" - the pupil reproduced it correctly and then solved it correctly as well.

Let us examine the peculiarities of reproducing the second group of problems.

Problem 2 was reproduced thus: "There were 20 plain pencils and 10 fewer colored ones. How many colored pencils were there?" The structure of the problem was given, but the numbers and the relationship between its components were incorrectly reproduced. The problem indicated that there were more colored pencils than plain ones. The reproduction said that there were fewer. The problem had the numbers "19" and "6," while in the reproduction the numbers "20" and "10" were named. Having read problem 4, the pupil was silent for a few minutes, and then refused to reproduce it.

Kolya V., a student in the same ~~school and~~ grade, reproduced the problems presented to him in the following way: "In one cage there were 13 rabbits; in another, 17 rabbits. How many rabbits were there in all?" (Problem 1). Here the numbers were given in his conditions, although the concrete words "in two cages" were lacking in his question; but the meaning of the problem was correctly given.

In reproducing problem 3-- "There were 25 notebooks in the cupboard; 13 were returned. How many were left?"-- the pupil substituted one word: instead of "distributed," he said "returned." The very precise words "in the cupboard" were lacking in the question, but the meaning of the problem as a whole was reproduced correctly.

In reproducing problem 2-- "There were 16 plain ones and 6 fewer colored ones. How many were there in all?" -- the pupil omitted the names of the objects and altered the relationship between the colored and the plain pencils, changing the first number as well. The question in the problem relating to the quantity of colored pencils was replaced by the more general question, "How many were there in all?"

Problem 4 was reproduced thus: "The pupil bought a pencil for 15 kopeks, and he had 30 kopeks left." The pupil repeated only the beginning of the problem correctly; he altered the second number given in the conditions, and he did not reproduce the question in the problem at all.

In examining the reproduction of the conditions of all the problems, it can be noted that not only the two pupils mentioned, but also the remaining sixth grade students, often omit the precise conditions governing the operation and that they often drop the names of the objects when reproducing problems. Often the numerical data of the problem is not retained. Sometimes the relationships between the numbers are changed by mistake; for example, instead of "more" the children said "fewer." The question is either completely omitted, giving way to a spontaneous solution, or it is altered. In cases where the question was altered, the specific question asked in the problem becomes the extremely general question, "How many were there in all?" which is not found in the given problem.

In a general survey of the reproduction of all of the problems, the children's answers can be grouped in this way:

1. A reproduction which is fundamentally correct and sufficiently complete: the structure of the problem is retained and the numbers are correctly reproduced, although the question sometimes contains an insignificant error. There are partial changes which do not essentially affect the problem.

2. A reproduction in which the structure of the problem is fundamentally retained, although individual words and their sequence are altered, and the numbers are incorrectly reproduced.

3. A reproduction which contains the individual components of the conditions of the problem, but does not retain the structure of the problem as a whole or the numbers.

4. Refusal to reproduce the problem: "I forgot; I cannot repeat it."

The results of the analysis of twenty sixth grade students' reproductions of the problems are presented in Table 1 (the numbers indicate the number of pupils who reproduced the problems in one form or another).

TABLE 1

FORMS OF REPRODUCTIONS OF PROBLEMS

Group	Problem Number	Correct Repro-duction	Structure of Problem Retained, but Altered	Parts of Problem Retained	Refusal
I	1	2	6	10	2
	3	2	12	4	2
II	2	0	6	14	0
	4	0	2	14	4

It is apparent from the table that only problems No. 1 and No. 3 were correctly reproduced, although in an extremely limited number of cases. None of the students were able to reproduce problems No. 2 and No. 4 correctly. The differences which we attributed to the reproduction of problems in group I and group II are thus confirmed.

It can be surmised that the problems in group I turned out to be easy to memorize because the relationships between the parts of each problem were reflected in the structure of the problems, in the way in which their conditions were formulated, and in the terms used.

In memorizing the problems in Group II the children themselves should have established the relationships between the components of the problem, since they had not received them in a ready form (problems that had an "indirect form" were introduced in this group). Consequently, despite the fact that these problems were solvable by one operation, they were more complex and less comprehendible, because of the peculiarities of their oral formulation. This explains why the problems in group I were reproduced more accurately.

In addition, it indicates that when the internal relationships between the components of a problem are not openly displayed, reproduction of the problem is hampered. It is imperative that the students comprehend these relationships in order to remember and to reproduce the conditions of the problem.

We know from investigations of mentally retarded schoolchildren that they reproduce these texts better when the relationships between consecutive events are set forth using corresponding terms, for example, with the help of causal connections [22]. If the text of the problems has no such connecting terms, then this probably also hampers reproduction.

Furthermore, it is apparent from Table I that the students had the least trouble reproducing problem 3, in which it was necessary to find a remainder; 14 students reproduced its structure and contents fundamentally correctly, while in the remaining cases the numerical data were altered. Eight students reproduced problem 1 satisfactorily. Problem 4 was the most difficult to reproduce--18 students did this unsatisfactorily. Fourteen students reproduced problem 2 unsatisfactorily.

If we combine problems 1 and 3 into one group we can see that the mentally retarded schoolchildren in the sixth grade reproduced even these simple types of problems satisfactorily in only 55% of the cases. The more complicated problems, 2 and 4, could be reproduced satisfactorily in only 20% of the cases.

Naturally, the fourth grade students made an even poorer showing when they reproduced the problems. Thus, the simpler problems, 1 and 3, were reproduced satisfactorily in only 35% of the cases, and problems 2 and 4, in only 10% of the cases.

Thus, the investigation showed that the reproduction of simple problems containing almost the same quantity of elements varied, even when performed by the same auxiliary school students. The reproduction depended upon the type of problem, that is, upon the character of the relationships contained in it. As is apparent from the data we have introduced, half the students reproduced the texts of problems 1 and 3 somewhat accurately. The auxiliary school students reproduced problems 2 and 4 considerably less accurately, although some students attempted to retell them using even more words than were used in the text. These attempts, however, did not lead to a correct reproduction of the text; on the contrary, we observed a complete distortion of the meaning of the problem in a series of cases.

Having established the fact that reproduction of simple problems varied considerably, it was important to examine what was retained and exactly what was dropped or altered in the reproduction of the different types of simple problems.

The students, in most of the cases, reproduced the beginning of problems 1 and 3 correctly; the first number given in the conditions of the problem was also remembered correctly. In cases of incomplete reproduction the second part of the conditions was dropped most often and the question in the problem was somewhat distorted. In the reproduction of these problems we did not encounter any rearrangement or substitution of words which would have grossly distorted their meaning.

Problem 3 reflected a set of circumstances customary in schools; notebooks kept in a cupboard are distributed, some being left in the cupboard. The words of the text and the way in which they are combined directly reflected the situation. This problem was reproduced more accurately than the others. Problem 1, whose question concerns the number of rabbits left in two cages, was reproduced significantly less accurately (here it was necessary to carry out the addition mentally, since the problem does not mention all of the rabbits being together in one cage). The question, "How many are there in all?" used in this problem was generalized; it was not as closely related to the contents of the problem as in problem 3, "How many remained?" which concerned notebooks which actually remained in the cupboard. Therefore, problem 1 was reproduced less accurately than problem 3, and the question in the problem was often dropped.

A somewhat more complex relationship between the text and the reality being reflected was set forth in problems 2 and 4.

We have already noted that the number of words in the conditions of all of the problems differs little.

Let us recall the contents of problem 2. It referred to both plain and colored pencils. In the conditions the quantity of plain pencils was indicated, while the quantity of colored pencils was to be determined from their relationship to the plain pencils; this relationship was given in the text of the problem in the form of a number which contrasted the quantities of colored and plain pencils. There was no direct information in the problem about colored pencils.

The character of the reproduction of this problem is unique, as we see from the difference in the reproduction of the two parts: the given data about the plain pencils was usually reproduced with comparative accuracy; the information regarding the colored pencils was poorly reproduced--the correct data often replaced by indefinite words, or by blanks left in the conditions of the problem. Let us introduce a few examples of such reproduction.

Olya E.: "There were 19 plain pencils, and how many other pencils were there?"

Manya O.: "There were 19 plain pencils by 6 times less. How many pencils were there?"

Senya B.: "There are 19 plain pencils in one, and there are 6 fewer colored ones. How many pencils were there?"

Katya E.: "There were 18 plain pencils and 10 colored ones. How many colored pencils were there?"

In reproducing this problem, pupils usually retained the first group of data; the second group of data was entirely lost or distorted. The question was reproduced correctly in rare cases; of the four reproductions given above, it was retained in one case; in the remaining cases the question was changed and lost its specific character. In these cases the question asked was not about <u>colored</u> pencils, as the problem requires, but about pencils in general. The relationships between the plain and colored pencils were not logical; even the colored pencils themselves were dropped from the problem. If the children comprehended these relationships to some degree, the objects (the colored pencils) were left in the conditions of the problem.

Let us examine another case of reproduction.

Kolya I.: "How many pencils lay... 17 pencils and fewer." The child named the objects--pencils--and formulated the question of the problem in concrete form: "How many pencils lay?" The reproduction of the question, in the very beginning of the problem, and later, of only certain of the problem's data, and, in particular, of the term "fewer," show that the child did not perceive the concrete relationships included in the problem and therefore could not reproduce it.

Another case: "There were 19 plain pencils by 6 times less." Here the first component of the conditions of the problem was retained; the second was distorted, incomplete and lacking in content. In addition, the schoolboy reproduced certain expressions characteristic of arithmetic problems. But in this case they are "free" from definite content, and lacking in meaning.

The relationships present in problem 4 are distorted even more than those examined above in problem 2; although, at the same time, individual terms, and sometimes even whole parts of the text, are retained. The question of the problem suffers even greater damage than in the remaining cases. Let us introduce a few examples:

Misha Z.: "A boy bought a pencil for 15 kopeks. He spent 20 kopeks. How much did he have left? Take 15 from 20."

Katya B.: "A boy had 15 pencils. The pupil had 17 pencils. How many kopeks did the pupil have left?". (Answer: 15)."

In reproducing this problem the same persons and objects are mentioned as in the conditions, but the person's action upon the objects is not retained. This leads us to the conclusion that the relationships in the conditions of the problem are often misunderstood by the schoolchildren, and, thus, are not fully reproduced or are reproduced incorrectly. Certain expressions, however, which are characteristic of arithmetic problems or which are used to communicate their specific relationships, are included in the problem when it is reproduced, although they have no organic relationship to the problem. By retaining arithmetical terminology the pupil sees a well known way out of the difficulties which have arisen.

This singular discrepancy between the mathematical expressions and the objects contained in an arithmetic problem is characteristic of the auxiliary school students' reproduction of the conditions of a problem. In many of the examples the emasculated verbal shells are retained in the reproduction while the objects contained in the problem are lost. In these cases certain arithmetical terms are often replaced by others, which often have an opposite meaning; for example, in the repetition of the text of the problem, the word "more" is replaced by the word "less," and so forth.

The question in the problem underwent the most significant change when problem 2 was reproduced. It was most often substituted by another, more general, question.

Thus, for example, in problem 2, instead of posing the question: "How many colored pencils were there?" the pupil asked: "How many pencils were there in all?" In problem No. 4 instead of the question: "How much money did the pupil have to begin with?" the pupil posed the question: "How much money was left?" In this case the question was simplified.

The character of the change made in the question is important in understanding how the children go about solving the problem, since the question in the problem suggests the choice of the operation used.

Obviously, in the second group, special difficulties in the reproduction of the question arise in connection with the problem's structure, and, in particular, with the relationship of the question to the conditions. That is, the structure of the problems, the peculiarities of their verbal formulation, and their text appear to be the primary source of difficulties in understanding and reproducing them.

We have suggested that reproducing the problems will hamper mentally retarded schoolchildren in their oral formulation of the text of the problems, but that they will be able to remember the objects in the problems, since the problems always contain very familiar objects and since they are required to reproduce only one operation with these objects.

The results of the investigation showed, however, that the defects in reproduction included not only inferior oral formulation of the problems, but also loss of the objects contained in the problems. This hampered the selection of the necessary arithmetical operation.

In reproducing the problems the pupils restated individual words and their combinations, but neither communicated the true relationships contained in the problem nor expressed them in meaningful, grammatically formulated chains of words. It was not the peculiarities of the words and the terms themselves that hampered reproduction--the pupils remembered them and used them. The fact is that these terms were not always used appropriately. The reproduction of the question distinctly revealed this tendency, through the generality of the questions. "How much will it be?", "How much was obtained?" was often substituted for the specific question in the problem. Questions which were general and non-specialized clearly predominated over more specialized questions.

Consequently, the more complicated the indirect relationships between the "given" and the "unknown" in the conditions of the problem, the more difficult it is for the students to reproduce the problem. The children more easily reproduce problems in which the "given" and the "unknown" are directly connected with the conditions of the problem than those in which this connection is not direct. It is important to note, however, that mentally retarded children reproduced even the simplest problems poorly. This fact demands a special explanation.

In general, the conditions of a simple, one operation problem are formed in the following way: first, certain data regarding objects are conveyed, for example: "12 notebooks lay in the cupboard," or "There were 9 rabbits in the cage." Quantities which must be retained are included in this part of the problem. Next new data appear in the conditions which, although they refer to the same objects, now begin to bear certain relationships to the previously named data. For example 12 notebooks lay in the cupboard (I), 6 of them were distributed to the children (II); in one cage there were 8 rabbits (I), in another, 3 rabbits (II). The children should understand these relationships.

The study of auxiliary school students' reproductions of the simplest problems has shown that they have difficulty establishing these relationships. Consequently, it is necessary to teach them to establish these relationships using visually presented material so that they will achieve clear oral formulations of the conditions.

Only if they practice at length, using the visual material to establish such a connection between the components of the problem, will they be able to orally reproduce the conditions of the problems. In using visual educational material, the children must learn to designate their operations with the material verbally; i.e., it is imperative that they achieve a unity of activity between both signal systems (visual and verbal); only then will the words be associated with their concrete substance [18].

One of the causes of the students' inability to reproduce the texts of problems is the overly rapid changeover from work with visual material in auxiliary schools, to the solution of orally formulated problems. Such a rapid changeover is not justified when instructing mentally retarded children; it further disconnects the activity of the two signal systems.

The auxiliary school students experienced special difficulties in reproducing the last part of the problem—the question.

The formulation of the question qualifies the relationships in the text. The question expresses in concrete form the changes which take place in a concrete situation described in the conditions of a problem. It is the question of the problem which suffers particular damage when it is reproduced by mentally retarded children.

Our attention is drawn to the circumstance that, when incorrectly reproduced, the question loses its specific quality, and is reduced to the extremely general question of the type, "How much was it?"

It is necessary to note that during the reproduction of problems the actual substantive content is lost. Only the remainder of the verbal formula and the individual words are retained, which shows the dissociation of the activities of mentally retarded schoolchildren's signal systems.

The students' inability to communicate what has happened to the objects, what was done with them, what changes took place involving them, as well as their retention of emasculated verbal formulas, are undoubtedly symptoms of "verbalism" in the instruction of mentally retarded schoolchildren. Instructors must struggle with these "verbalisms" in teaching mentally retarded children.

<u>Characteristics of the reproduction of conditions of simple arithmetic problems by second grade students in the mass school.</u>

We compared the data concerning auxiliary school students with the results of the reproduction and solution of arithmetic problems by students in the mass school. To that end we conducted corresponding studies with 60 second grade students. The results of their reproduction of the problems are presented in Table 2 (the numbers indicate how many pupils reproduced problems in one form or another).

TABLE II

HOW THE PROBLEMS WERE REPRODUCED

Group	Problem Number	Correct Reproduction	Problem's Structure Retained with Certain Elements Distorted	Parts of Problem Retained	Refusal to Reproduce
I	1	24	36	0	0
	3	24	30	6	0
II	2	42	12	6	0
	4	18	18	18	6

Note: Values here are the number of students reproducing problems in each manner.

It is apparent from the table that reproducing problems 1, 2 and 3 was not difficult for the majority of the students; only the reproduction of problem 4 presented certain difficulties for them.

It is useful to examine the mistakes made by the mass school students in reproducing these problems. The mistakes in reproducing problems 1 and 3 most often consisted of changes in numbers. These changes occurred because the reproduction of the problem is so closely related to its solution that the numbers given in the conditions were replaced: instead of 25 notebooks, 19 notebooks were designated; instead of 13 rabbits, 15 rabbits. The structure of the problem was retained in the reproduction.

In their repetition of the text the children replaced certain words of the problem with their own words, although they did not distort its essential arithmetical meaning; for example: "Twenty-five notebooks lay in the cupboard. They took 13 notebooks (in the text: They gave them out to the pupils). How many notebooks remained?" or, "There were 25 notebooks in the cupboard. Thirteen notebooks were distributed. How many remained in the cupboard?" We also observed how the pupils reproduced "in their own words" the text of problem 2, for example: "There were 19 plain pencils, and 6 more varicolored than plain ones. How many varicolored pencils were there?" The problem was communicated correctly, but the pencils were called varicolored rather than colored.

Converting a given problem into another type of problem is encountered only as an exception among the mass school pupils. For example, in one case a student altered the problem so as to change its arithmetical meaning; having said that there were so many more colored than plain pencils, he formulated the question of the problem thus: "How many fewer colored pencils than plain pencils were there?"

In the reproduction of problem 4 we observed that there were changes in the form of "ad libs." Thus, one pupil who rarely diverged from the original in his reproduction said, "The boy bought 5 notebooks for 17 kopeks each. He had 20 kopeks left. How much money had he had?" Despite such distortion of the conditions of the problem he nevertheless reproduced its structure and question (spent the money, had money left, had how much money).

Sometimes the students forgot the second half of the problem, but remembered the question. In several cases in which the children clearly understood the problem but hurried in solving it, we observed that they abbreviated the text in reproducing it. For example, the problem may have been reproduced thus, "The boy spent 15 kopeks. Twenty kopeks were left. How much had he had?" Such an abbreviation of the problem is not only related to, but is probably because of, complete understanding of the problem.

In reproducing a problem the students in the mass school usually remembered the problem; almost always the question was correctly reproduced. The children failed to reproduce the question only in those few cases in which the solution of the problem followed directly from the reproduction of its conditions; for example, "The pupil bought a pencil for 15 kopeks and he had 20 kopeks left. Fifteen added to 20 would be 35 kopeks." or, "There were 25 notebooks in the cupboard; 13 notebooks were distributed. Thirteen taken from 25 would be 12." Such cases in which the question is absent were also noted by N. A. Menchinskaya [10].

Thus, the reproduction of all four problems by the second grade students in the mass school did not involve great difficulties. It must be pointed out, in comparing fourth graders of the auxiliary school with second graders of the mass school, that the latter reproduced all types of problems better.

However, not only the auxiliary school students but also the mass school students had difficulty with the problem having the most complex structure (No. 4).

We can conclude, in comparing the two groups of children, that the mass school students have less difficulty in reproducing the problems in which the "unknown" and the "given" are directly related, that is, in which the internal relationships are less complex. Moreover, the mass school students solved difficult problems more accurately than did the auxiliary school students.

Conclusions. 1. The reproduction of the text of an arithmetic problem appears to be a complex thought process which depends upon comprehension of the situation described in the conditions of the problem, and the relationships between its components. The depth and character of the understanding of the problem are manifested in the quality of the reproduction.

2. The recollection and reproduction of the problem demand the correct use of previously acquired knowledge, which is the basis for attaining new knowledge. In order for a problem to be reproduced successfully, the knowledge that has been acquired previously must be firmly retained in the memory; and sufficiently differentiated systems for new knowledge, formed in the process of mastering specific types of problems, must be developed.

3. Simple arithmetic problems (problems having one operation) which differ in logical structure are not reproduced equally well by auxiliary school students even when they contain an equal number of words. This investigation showed that when problems have different verbal formulas they create unidentical conditions for understanding their situation, the relationship between the components of their conditions, and their question.

4. Success in reproducing and recollecting a problem depends

upon understanding the relationship between the data of the problem and comprehending the relationship between the "given" and the "unknown." Problems in which the "given" and the "unknown" data are directly related enable one to visualize the situation easily and directly, and are, therefore, satisfactorily reproduced by the auxiliary school students (such a problem is 3, which involves finding a remainder, and problem 1, which involves finding the sum). The simple relationship between the "given" and the "unknown" in the text of the problem is easily represented by visual images familiar to the students, since such images have been consolidated by their past experience. Such texts provide the most favorable conditions for the understanding of problems and for their rigorous reproduction.

5. Problems in which the relationships between the "given" and the "unknown" are not, by words or their combination, directly expressed in concrete form create less favorable conditions for reproducing problems.

But, since these problems demand "indirect" comprehension of the text and, by the same token, contribute to the development of the children's thinking, special attention should be given to exercises in solving such problems.

6. Not all parts of the text of a problem are reproduced with equal success. Those parts which describe a present situation (there was so much) are usually reproduced better than those parts in which changes in the present situation are involved. Numerical data are also reproduced inaccurately, especially when they involve changes in the present situation. Specific arithmetical terms are usually retained in the reproduction of the text of a problem. However, they are frequently subject to distortion (instead of "greater," the word "fewer" is used; the phrase of "so much more" is reproduced as "by so much more," etc.). Notably, the arithmetical terms that become part of the reproduction are not always related to the text of the problem; sometimes they are not even related to the numbers which are referred to in the conditions of the problem (for example: "There were 16 pencils in the box... fewer pencils... How many pencils are there in all?"). It must be noted that the arithmetical terms play the role of directors, guiding the

comprehension of the problem; they are unique, no matter how they are interpreted. The problem may say that the pupil spent a certain quantity of money and that he had so much left, the initial sum remaining to be determined. Directing their attention to the arithmetical terms "spent" and "had left" and their customary combination, the students reproduce this problem as though it were independent of its context. They treat it as a problem involving finding the remainder, and solve it by the operation of subtraction, without taking the remaining conditions into consideration.

7. The problem's question is frequently omitted; it is transformed from a specific into an extremely general, non-specific question which differs in content from the text of the problem. The more difficult the situation in the problem is to understand, the more the mentally retarded children will distort the wording of the question.

8. The peculiar discrepancy between the arithmetical terminology and the text appears to be the source of difficulties in reproducing and solving problems in arithmetic. As a consequence, there also appears to be a great deal of discrepancy between the oral formulation of the problem and its subjective content ("verbalism" in the wording).

9. In reproducing a problem the children frequently "alter" it from one type to another. This phenomenon, which is indicative of the insufficiency of lasting knowledge and of the insufficient differentiation in mentally retarded schoolchildren, should attract the serious attention of methodologists. At the same time, it is not enough to take them into account, since as yet few pedagogical measures have been devised to counteract such a phenomenon.

10. Our investigations permit us to conclude that single and even dual perception of the conditions of a problem (reading by the teacher or by a pupil) frequently leads to unsatisfactory reproduction of the problem even by sixth grade auxiliary school students; reproduction of the problem by fourth grade students appears even less satisfactory under these conditions. Consequently, a special method should be devised for working on the assimilation and reproduction of the conditions of a problem.

Mistakes Made by Auxiliary School Students in Solving Simple Arithmetic Problems

The effect which reproducing the conditions of a problem has upon its solution.

In the preceding chapter we examined the common features of reproducing problems, and the dependence of the reproduction upon the structure of the problem. Let us now examine how the correct reproduction of a problem affects its solution, when such a solution directly follows the reproduction. This solution is based upon the students' remembering the conditions of the problem, since they did not have the text before them while they were in the process of solving it. In Table 3 the correlation between the reproduction and solution of a problem by sixth grade students is represented. The left column of the table represents data pertinent to reproduction, whereby the fundamental conditions of the problem are retained. The right column has data connected with the correct solution to the problems (the numbers indicate the percentage of students who correctly reproduced or solved a problem).

On the basis of the data we may conclude that if the pupils' repetition of the text of the problem was fundamentally correct, the choice of the operation was also correct in most cases, and the problem was usually solved correctly. This conclusion pertains only to the problems given to the children--and we do not extend it.

To corroborate the idea that, in the fourth and sixth grades in the auxiliary school, the solution to problems is dependent upon their reproduction, we also conducted a special investigation which corroborated the facts we obtained earlier (Table 4).

TABLE 3

REPRODUCTIONS AND CORRECT SOLUTIONS

Group	Problem Number	Reproduction	Correct Solution
I	1	40	50
	3	70	50
II	2	30	30
	4	10	30

Note: Numbers are percents of all students in each group.

TABLE 4

REPRODUCTIONS AND CORRECT SOLUTIONS BY GRADE LEVEL

Group	Problem Number	Grade 4 Correct Reproduction	Grade 4 Correct Solution	Grade 6 Correct Reproduction	Grade 6 Correct Solution
I	1	40	30	60	40
	3	60	60	100	100
II	2	20	10	20	40
	4	0	0	0	0

Note: Numbers are given as percents of all students in each group for each grade level.

The table clearly shows that the solution to a problem depends upon its reproduction, although there may be certain deviations. It may be observed that the sixth grade students often solved problem 2 while familiarizing themselves with its text. This apparently explains the predominance of correct solutions over correct reproductions in that grade.

We also observed in the investigation that the solution to a problem depended upon the character of the recollection of its text.

1. In problems 1 and 3, if the first part of the conditions was reproduced without the second part or the question, the problem was not solved.

2. When the repetition of the conditions of the problems was abridged at the cost of forgetting only the question, the solution was often correct. This can be explained by the fact that the question was "inferred" by the students, for example: "In one cage there were 17, and in another, 13; there will be 30."

Consequently, in order to solve a problem correctly, the children must recognize the presence of at least two quantities in the problem, and understand the relationships that exist between them. We can guess that whenever the "unknown" is closely connected with the "given" data of a problem the solution can be implemented correctly, even without reproducing the question.

3. When the question was reproduced incorrectly, and the conditions satisfactorily, the solution was sometimes correct. Let us take as an example the reproduction of problem 2 by sixth grade pupil N. S.: "In one there were 19 black pencils, and there were 6 fewer colored pencils. How many pencils were there in all?" The solution, which immediately followed the reproduction, was essentially correct and answered the question, "How many colored pencils were there?" ("6 taken from 19 is 13 colored pencils"). Thus, the solution corresponded to the requirement of the problem, and was not related to the character of the reproduction of the question.

In reproducing problem 3 the pupil K. M. reproduced the question, "How many notebooks were in the cupboard in all?" instead of the question, "How many notebooks remained in the cupboard?" However, he solved the problem correctly. Consequently, the relationship between the conditions of the problem and the actual question was understood, and thus led to the solution of the problem. When reproduced, the question did not have the function of guiding the solution. There is a certain discrepancy between the reproduction of, and the solution to a problem, one which apparently reflects the discrepancy between recollection and reproduction.

4. When the conditions of a problem were distorted the solution proceeded otherwise. An incorrect solution was usually related to the character of the distortion. The following reproduction of the text of the problem made by the pupil K. may serve as an example: "There were 19 plain pencils, by 6 times more. How many pencils were there?" The problem was solved by multiplying 19 x 6. Consequently, whenever there are definite arithmetical relationships in the distorted reproduction of the conditions of problems, the students solve the problem in correspondence with the conditions they have reproduced. In terms of these conditions, their solution is correct; in relation to the proposed problem the solution is erroneous.

5. Finally, there were cases in which, although the problem was reproduced correctly, its solution was wrong. This usually occurred in the solution to problem 4. One may believe that in these cases the children misunderstood it: they simplified, as we described above, the contents of the problem, attributed the wrong meaning to its separate parts, and acted in correspondence with their understanding.

Thus, the correspondence between the peculiarities of the reproduction of a problem and the character of its solution is a highly complex one. We have shown only a few of the relationships which arise between the reproduction of a problem and its solution. The data obtained allow us to draw certain conclusions.

1. In those cases in which the conditions of a problem were reproduced erroneously, it was usually solved in accordance with the reproduction.

2. In several cases the reproduction of the conditions of a problem was incomplete or distorted, but its solution was correct. It can be supposed that there was a discrepancy between the subjective content of the problem in its printed form and the oral formulation of this subjective content.

3. Finally, even when the reproduction was satisfactory, the solution to the problem was sometimes incorrect. The cause of the erroneous solution to a problem may be a misunderstanding of the question of the problem, which is often, due to certain characteristics, wrongly identified with another question, similarly formulated, but not identical to it. This error in comprehension is important. We shall analyze this later.

Despite all the complexity of the relationships which exist between the reproduction of the problem and its solution, the material which we have obtained indicates that the application of one solution or another depends to a certain degree upon the peculiarities of the reproduction of the problem as a whole, and of its separate parts.

We have examined only the peculiarities of the first reproduction of a problem, which, in most cases, turned out to be far from complete. From the experience of the best teachers, we know that if children are systematically taught to state the problem correctly, the quality of the reproduction is improved and, in addition, the quality of the correct solutions is raised.

It is particularly important to note the complications which the sixth grade students introduced in their reproduction and solution of simple problems. These can be explained by the fact that the auxiliary school students begin to solve problems involving several operations in the higher grades, and solve few problems involving one operation. The new method of solving problems, which is being developed while solving problems involving two or three operations, is consolidated by daily exercises. Since it is not sufficiently differentiated from the old method of solving simple problems involving one operation, the pupil applies it to the solution of all simple problems. In such cases a problem involving one operation is solved as though it were a problem involving two operations; something resembling the phenomenon which Pavlov called "generalization by virtue of insufficient differentiation of similar stimuli" takes place.

This explanation of the situation is confirmed by the way in which fourth grade pupils, who have already solved problems having two operations, solve problems. Since their ability to solve these problems was not yet significantly consolidated, it had a reorganizing influence on the previously attained methods. In the period preceding the experiment the pupils solved many problems involving one operation, and relatively few problems involving two operations. Therefore, they did not "convert" the simple problems into complex problems containing two operations, as did the sixth grade pupils.

Moreover, we observed that the problems which the fourth grade pupils reproduced and solved were influenced by the problems which they had solved earlier in the same lesson. This influence was manifested by the similarities of the problems. Thus, one pupil reproduced problem 1 in the following way: "There were 17 rabbits in one cage, and 13 rabbits in another. How many were there in all?" This reproduction corresponded to the original. But he altered problem 2 by likening it to the first problem: "There were 19 pencils in one box and 13 pencils in the second." The same thing happened with problem 3: "There were 17 notebooks in one cupboard; in the other, 13 notebooks." Thus, the second and third problems were likened to the first problem and were solved by addition.

These facts have already been noted in literature on the subject, but here they once more show that in mentally retarded children previously attained knowledge is insufficiently differentiated and is difficult to consolidate. This may explain the students' occasional desire to reread the text of the problem several times before reproducing it.

The second grade students in the mass school relate the reproduction of a problem to its solution more closely than do mentally retarded children. When they reproduce a problem perfectly it was usually accompanied by a correct solution (Table 5). The table shows that the correct reproduction of problems usually corresponded with a correct solution. Consequently, perfect reproduction of the text of a problem is very important to its comprehension and solution.

TABLE 5

CORRECT REPRODUCTION AND CORRECT SOLUTION

Group	Problem Number	Correct Reproduction	Correct Solution.
I	1	80	90
	3	90	90
II	2	90	80
	4	60	60

Note: Values are the number of students.

It has been determined that the oral reproduction of a problem is often an insufficient basis for building comprehension of even the simplest arithmetic problems by auxiliary school students. To this end other methods and systems are necessary; these still need to be developed.

Characteristics of the solution of simple problems by third and fourth grade auxiliary school students.

Above we examined the typical errors which arise in solving arithmetic problems. To uncover the causes of the errors and to show how these errors depended upon the character of the problem's structure, special experiments were conducted.

Observations of the solution of arithmetic problems in school and data about the level of the students' knowledge, showed the most characteristic and constant errors involved the choice of operations and the nomenclature. In order to define the conditions under which errors in choosing operations arise, we observed the course of the solution of a series of selected problems.

A. S. Pchelko believes that the fundamental prerequisites of successful solution to all arithmetic problems are evolved in children during instruction in solving simple problems--i.e., problems involving one operation [14]. In fact, when a pupil is solving such problems, he first discovers what the problem represents and what its elements are (conditions, numerical data, question). When solving simple problems, the children learn to understand the relationship between quantities and to apply the arithmetical operations correctly. The simple problems provide a preliminary acquaintance with the arithmetical operations and develop a lasting ability to understand these operations. Although uniform in the number of their operations, these problems differ in their arithmetical and logical structure, and, consequently, they also vary in the difficulty of their solution.

Simple problems can be classified according to the type of arithmetical operation used: thus, four basic groups of problems arise, those involving addition, subtraction, multiplication, and division. However, methodologists are not satisfied with the isolation of these basic groups; they have developed narrower classifications of simple problems. Several such classifications have been suggested.

G. B. Polyak distinguishes 15 types of problems [15], A. S. Pchelko divides all simple problems into 12 types [14], and L. N. Skatkin divides them into 24 types [16].

In our investigations we utilized the classifications that many methodologists (N. N. Nikitin and others [12]) have applied. This classification was suggested as a basis for the program for arithmetic in the auxiliary school. According to such a classification simple problems fall into 11 types, grouped in relation to the arithmetical operation and its use:

addition - 2 types of problems;
subtraction - 3 types of problems;
multiplication - 2 types of problems;
division - 4 types of problems.

All eleven types are included in the arithmetic program for the mass and auxiliary schools. In the auxiliary school they are studied from the first through the fifth grades [3], and in the mass school, from the first through the third grades [2].

Experimental studies were conducted with 182 students from the third, fourth, fifth, sixth, and seventh grades in two auxiliary schools in Moscow and the Moscow area. Similar studies were conducted with 147 students from the second grades of two mass schools.

All the students were given identical problems to solve. The choice and system of the arrangement of the problems corresponded to the system for arranging them in the programs of the mass and auxiliary schools. The students were given only those types of problems which they had studied in school. Operations involving computation presented no difficulties for the students, since the problems required operations within the limits of one hundred only.

The third and fourth grade students in the auxiliary school and the second grade students in the mass school were given only four types of simple problems; the auxiliary school students in the higher grades solved all types of problems. We present the texts of these eleven problems.

1. (Finding the sum). In one cage there were 6 rabbits, and in another, 4 rabbits. Altogether, how many rabbits were there in the two cages?

2. (Finding the remainder). There were 6 notebooks in the cupboard and 5 notebooks were passed out. How many notebooks remained?

3. (Addition of the type "so many more"). Fifteen apples were in the basket, and there were 10 more pears than apples. How many pears are in the basket?

4. (Subtraction of the type "so many fewer"). There were 17 plain pencils, and 6 fewer colored pencils. How many colored pencils were there?

5. (Division into parts). Twelve notebooks were divided among 6 pupils equally. How many notebooks did each pupil receive?

6. (Reduction by a multiple). A ruler costs 32 kopeks, while a pencil is 1/4 as much. How much does a pencil cost?

7. (Finding the product). Five chandeliers were hanging in the hall. There were 3 light bulbs in each chandelier. Altogether, how many light bulbs were hanging in the hall?

8. (Division into subgroups). Thirty-five tomatoes were planted, with 7 tomatoes in a row. How many rows were obtained?

9. (Increasing by a multiple). Petya caught 4 fish, while Vanya caught 3 times as many. How many fish did Vanya catch?

10. (Comparison by determining the difference). An automobile factory used to produce 50 cars a day, but now it produces 75. How many more cars has the factory begun to produce daily?

11. (Comparison by determining the ratio). One pupil has 24 notebooks, while another has 6 notebooks. How many times as many notebooks as the second pupil does the first pupil have?

The experimental lessons were conducted in the third quarter under ordinary classroom conditions. The problems were printed on separate sheets of paper which were given to each pupil.

In the lower grades the teacher first read the conditions of the problem aloud, and then the children received the sheets of paper with the problem and read them to themselves. In the upper grades the pupils read each problem to themselves. The students did not copy down the conditions of the problem, and the time for solving the problem was not limited.

The third and fourth grade pupils in the auxiliary school solved four types of simple problems (problems 1-4). It must be said that by the time the experiments were being conducted the students in these grades were already solving uncomplicated problems involving two operations.

The solutions to the problems were considered to be correct only when the correct operation was chosen and the calculation was correct. In Table 6 the results of the solutions to each problem are presented.

TABLE 6

PERCENT OF STUDENTS WHO SOLVED THE PROBLEM CORRECTLY

Type of School	Grade	Problem 1	Problem 2	Problem 3	Problem 4
Mass school	II	97.0	96.0	91.0	93.0
Auxiliary school	III	33.0	31.5	30.5	29.0
Auxiliary school	IV	52.0	51.0	46.0	43.0

It is apparent from the table that considerably fewer auxiliary school pupils solved the problems correctly than did the mass school pupils. Thus, the percentage of the second grade mass school pupils' correct solutions to the four problems varied between 91% and 97%; the third grade auxiliary school pupils' correct solutions varied between 29% and 33%, while the fourth grade pupils' scores varied between 43% and 52%.

The table shows that the ability of the auxiliary school pupils to solve problems grows with the extent of their education. Thus, in the third grade there were 31% correct solutions, while in the fourth grade the quantity of correct solutions rose to 49%. Since the percentage of correct solutions in the second grade of the mass school is 94%, it is easy to see that the proportion of correct solutions of the four problems in the third grade at the auxiliary school is one-third that of the second grade at the mass school, and in the fourth grade, almost half.

We have analyzed the students' errors in solving the four problems into the following groups:

1. Errors in calculation
2. Errors in applying operations
   a. Incorrect choice of operations
   b. Applying too many operations
   c. Arithmetical manipulations with the numbers.

A few words must be said about the second group of errors. All of the subgroups were selected by teachers, and are reflected in pedagogical literature. We nevertheless wanted to illuminate our understanding of these errors, and to define their contents more precisely. In the first subgroup we referred to cases in which the wrong arithmetical operation was used, due to the influence of a similar problem. This influence arose because the given problem was mistaken for another problem superficially similar to it. Comprehension of the new problem depended upon the similarity in the verbal expression of the two problems--i.e., the newly solved problem and the previously solved one. An analysis of the children's work shows that erroneous solutions usually occur whenever the conditions of the proposed problem are taken to be the conditions of another type of problem.

In the second subgroup we referred to erroneous arithmetical operations which do not stem from the incorrect interpretation of the problem, but are caused by the direct influence of the preceding problem. We refer here to those cases in which addition is applied instead of subtraction because, having solved the first problem by addition, the mentally retarded child solved the three subsequent problems according to this pattern.

We present the following table illustrating the characteristic errors made by the students in the mass and auxiliary schools (Table 7). In examining the table one should remember that the errors were made in only 6% of the cases by the second grade students in the mass school, but in 69% of the cases by third grade students and in 49% of the cases by fourth grade students in the auxiliary school. The errors of a given type made by students in a given class are expressed as a percent of the total number of errors made by those students.

TABLE 7

ERRORS IN SOLUTIONS

Type of School	Grade	Errors in Calculation	Errors in Applying the Operation: Incongruous Choice of Numbers and Operations	Errors in Applying the Operation: Incorrect Choice of Operations	Errors in Applying the Operation: Too many Operations	Total
Mass school	II	0	2	38	60	100
Auxiliary school	III	5	25	49	21	100
	IV	3	20	35	42	100

Note: Numbers are percents of all errors committed.

The second grade students in the mass school made no errors in calculation at all. Cases in which the wrong operation was chosen were encountered rather often (38%), but errors involving the application of too many operations were predominant (60%). The latter is explained by the fact that while the experiments were being conducted the children were doing many exercises in problem solving involving two operations.

It is evident from the table that, even for auxiliary school pupils, errors in calculation are rare. More characteristic of the third grade auxiliary school students are errors involving the incorrect choice of operations, that is, incorrect identification of the type of problem being solved, in which the pupil mistakes it for a similar problem. Analogously the auxiliary school pupils very often (12 times as often as the mass school students) replaced the solution to the problem by arithmetical manipulations with numbers. This was probably a consequence of their low level of comprehension of the problem. Since the third grade had not yet done problems involving two operations often enough to gain confidence in their ability to solve them, cases involving too many operations were rare. The fourth grade students mistook a simple problem for another type of problem somewhat less frequently than did the third grade students. They less frequently solved all the problems according to one pattern. The number of times they solved a simple problem as though it involved two operations, however, was twice that of the third grade pupils. In the fourth grade, problems involving two operations occupy more time than they do in the third grade; this circumstance engenders errors involving the application of too many operations.

The distinguishing features in the way auxiliary school students solve problems are even more sharply revealed by the percentage of pupils who solve all the proposed problems: 80% of the second grade pupils in the mass school solved all of them, compared with 6% of the third graders and 25% of the fourth graders in the auxiliary school.

This fact testifies to the serious discrepancies between the programmed requirements and the factual knowledge of the auxiliary school students. In the second grade of the mass school 80% of the students answered the questions correctly (it must be noted that we worked in a mass school with many poor students).

One might ask how much "differentiated and lasting knowledge" fourth grade auxiliary school students can possess if only 25% of the students can solve four types of simple problems correctly. Nevertheless, the auxiliary school curriculum stipulates that the children should solve complex problems involving two operations beginning with the second grade, and begin, in the fourth grade, complex problems involving three operations.

Characteristics of the method auxiliary school students in the upper grades use in solving simple problems.

In the auxiliary school the process of acquainting the students with simple problems ends with the fifth grade. According to the existing program, the students in that grade should be able to solve simple problems involving comparison by determining the difference, comparison by determining the ratio, division into subgroups, and computing time. In addition, they should be able to solve various complex problems involving two or three operations. To solve these problems conscientiously the students should master the lasting and differentiated skills needed to solve simple problems. To what extent is this achieved in practice?

All eleven types of simple problems whose texts were introduced above were presented to the pupils in the fifth through seventh grades at the auxiliary school.

Seventy-eight third grade students in two Moscow mass schools and 133 fifth, sixth, and seventh grade pupils in two Moscow auxiliary schools participated in the work.

The majority of the third grade pupils in the mass school solved all eleven types of simple problems correctly. Of the whole number of students only 14% committed errors in solving them. Consequently, the level of the knowledge and ability of the third grade students in the mass school corresponded in general to the requirements set forth in the program. The situation was quite the contrary when these problems were solved by the upper grade students in the auxiliary school (Table 8).

TABLE 8

PERCENT OF STUDENTS WHO SOLVED ALL TYPES OF SIMPLE PROBLEMS CORRECTLY

Type of School	Grade	Percent of Students
Mass school	III	86
Auxiliary School	V	18
Auxiliary School	VI	25
Auxiliary School	VII	48

Note: Percentages of the total number of students in a given grade.

It is evident from the table that the fifth grade students in the auxiliary school solved the problems poorly: only 18% of the pupils were able to solve the proposed problems correctly. However, 86% of the third grade pupils in the mass school solved these problems. The number of seventh grade pupils in the auxiliary school who were able to cope with the problems was three times that of the fifth grade. However, at the end of the course, only half the students had solved the proposed problems, and there were twice as many of them as there were third graders at the mass school.

We analyzed the errors made in solving the eleven types of simple problems. We conducted our analysis of the errors committed by the upper grade students in solving the simple problems using the same method employed in our analysis of the errors made by the lower grades.

The results of processing the collected material are reflected in Table 9 (the proportion of the various errors experienced by each group is reflected in the percentage of the entire number of errors committed by the students in a given grade).

By comparing the data contained in Table 9 with the data presented in Table 7, we can see the difference between the solutions to simple problems by the upper and lower grades.

In the upper grades the manipulation of digits, which composed 25% of the errors of the third grade students and 20% of the errors of the fourth grade students in the auxiliary school, did not occur. The attempts to use too many operations disappeared: this provides an opportunity to compare the way the pupils in the upper grades distinguished the various types of problems, to the way in which the lower grades compared them. There were refusals to solve problems, a fact which testifies to the growth of the children's critical attitude toward activities involving thought.

The fundamental category of errors made by pupils in all grades of the auxiliary school involved the incorrect choice of operations. The same is true for third grade pupils in the mass school.

TABLE 9

ERRORS IN SOLUTION

Type of School	Refusal to Solve the Problems	Errors in Calculation	Errors in Applying the Operation: Manipulation of Numbers	Incorrect Choice of Operations	Too many Operations	Total
Mass School Grade III	0	0	0	100	0	100
Auxiliary School Grade V	8	0	0	92	0	100
Auxiliary School Grade VI	23	7	0	70	0	100
Auxiliary School Grade VII	0	0	0	92	8	100

Note: Numbers are in percents of all errors committed in each school and grade level.

Let us explain the character of the most widespread errors, and determine which problems, under what conditions, engendered the erroneous choice of operations. In most cases the errors arose in solving the following problems:

1. The problem in comparison by determining the difference: "An automobile factory used to produce 50 cars a day, but now it produces 75. How many more cars has the factory begun to produce daily?" In the erroneous solution to this problem, instead of the required subtraction, the operation of addition was usually performed; this is a typical error. This solution accounted for 80% of the errors committed.

In the remaining 20% of the cases the problem was erroneously solved by multiplication. The root of this error is concealed by the external similarity between the wording of this problem and problems involving addition. This similarity to problems involving addition, apart from everything else, is amplified by the presence in the text of the term "more." The same problem is sometimes even taken to resemble problems solved by multiplication. This occurs less often, however, apparently because the difference between the numbers in the proposed problem is not sufficiently great to allow for a solution by multiplication.

2. Another problem whose solution gave rise to many errors, even in the upper grades, is the problem involving division into subgroups: "Thirty-five tomatoes were planted, with 7 tomatoes in a row. How many rows were obtained?" A typical error made in solving this problem was the use of multiplication instead of division. The conditions of the problem bear a strong resemblance to those in a problem involving multiplication, primarily because of the wording: "So many tomatoes, so many in each row"; besides, there is a great difference between the two quantities (35 and 7), and this is a characteristic of multiplication problems. Thus, this problem is erroneously understood to be a multiplication problem of the type: "Thirty-five tomatoes were planted in each row. There were 7 rows. How many tomatoes were planted?"

3. The most difficult problem for the children involved comparison by determining the ratio: "One pupil has 24 notebooks, while another has 6 notebooks. How many times as many notebooks as the second pupil does the first pupil have?" The errors committed in solving this problem were more diverse than those in the above cases; 48% of the erroneous solutions were caused by use of addition, 41% of the errors involved the use of multiplication and 13% of the errors came about through the mistaken application of subtraction.

Once again the basis of these errors appeared to be a certain similarity between the problem being solved and other types of problems, along with the students' inability to differentiate them. It is interesting that the proposed problem was taken to resemble three different types of problems, although the typical errors appeared only in applying multiplication and addition.

The extent to which the correct interpretation and solution of simple problems depends upon the type of problem is shown by the graph (Figure 1). The eleven types of problems are numbered along the horizontal line (the order corresponds to the numbering of the problems as indicated on pp. 132-133 The percentages of correct solutions to these problems by students in grades 5 through 7 are indicated along the vertical line in Figure 1.

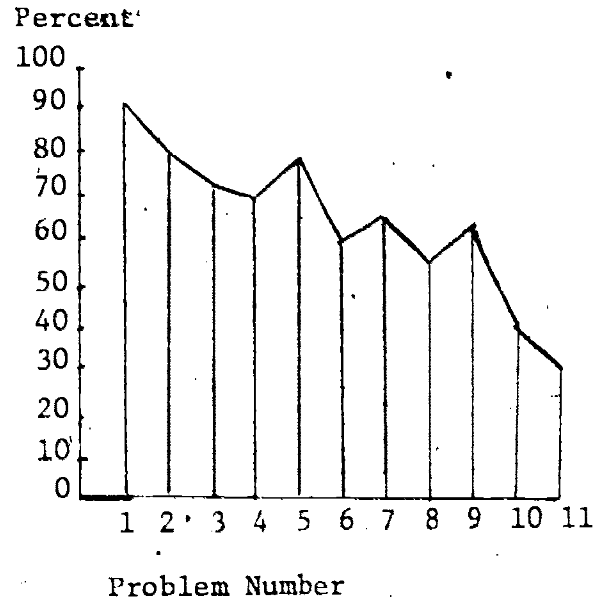

Figure 1. Percent of Correct Solutions of Simple Problems.

All the simple problems were not solved equally well by the students in the upper grades at the auxiliary school. At the same time certain problems appeared to be simpler for them than we expected (for example, the problems involving division into parts). On the other hand, other problems required more attention and more careful working methods.

We believe that a more careful analysis of problem solving permits us to divide the problems into groups according to their complexity. (Table 10).

The first group includes problems in which the operation required to solve the problem is indicated directly in the conditions of the problem by means of the terms added, subtracted, divided. In other words, this group is made up of the so-called direct operation problems which, as we know, are the simplest for the children.

In the second group are problems whose wording describing the relationships between the numbers must be interpreted and expressed arithmetically before the problem can be solved correctly. To choose the operations one must understand terms like "so many more" and "so many fewer."

The third group of simple problems is the most difficult for mentally retarded students. These problems involve the most complex relationships--comparison by determining the difference, comparison by determining the ratio between two quantities, and division into subgroups. The solution of these problems is especially important in the development of the students' thinking. These problems were not mastered satisfactorily. They are difficult; and not only is insufficient time spent on solving them in school, but also no method of teaching them has been worked out.

TABLE 10

PERCENT OF CORRECT SOLUTIONS FOR EACH PROBLEM TYPE

Group I	% of Correct Solutions	Group II	% of Correct Solutions	Group III	% of Correct Solutions
1. Finding the sum	94	1. Increasing by several units	73	1. Division into subgroups	53
2. Finding the remainder	82	2. Multiplication by several units	71	2. Comparison by determining the difference	42
3. Division into parts	81	3. Finding the product	64	3. Comparison by determining the ratio	31
		4. Reduction by a multiple	64		
		5. Increasing by a multiple	61		
Average Percent of Correct Solutions	86		66		42

These were the data we obtained by studying the auxiliary school students' solutions to the eleven problems newly presented. From our point of view they are especially significant for the development of a method for teaching arithmetic.

We were also especially interested in how the auxiliary school students in the upper grades solved the problems which some authors (Skatkin, Nikitin) call indirect problems, and other authors (Menchinskaya) call inverse problems.

To verify the role of the problem's structure in the solution process, we conducted supplementary experiments with the sixth grade in the auxiliary school. Two simple problems involving addition were presented to the children, one directly, the other indirectly.

First Problem	Second Problem
Misha had 30 kopeks; Mama gave him 20 kopeks. How much money did Misha then have?	Petya spent 20 kopeks and had 30 left. How much money did Petya start out with?

Twelve of 13 pupils solved the first problem correctly. The second problem was solved by 4 of the 13 pupils. The remaining pupils solved the problems incorrectly: six children were influenced by the terms spent and had left and used subtraction (30 kopeks - 20 kopeks= 10 kopeks); three pupils transformed it into a problem involving two operations.

Teachers must pay special attention to teaching the solution of "indirect" problems in the auxiliary school.

After completing the analysis in order to verify the data obtained, we again conducted analogous exercises with 138 fifth , sixth, and seventh grade students in three Moscow auxiliary schools. They were given only seven problems from the eleven, beginning with the fifth problem (see pp. 132-133). One should note that the exercises in arithmetic in these schools were conducted by experienced teachers.

The results of the solutions of the proposed problems are represented in the graph (Figure 2).

In this connection the similarity in the configuration of the curves draws our attention. The highest points are observed in the solutions to problems 5 and 9; these problems are consistently the most successfully solved. Finally, we must not fail to note that the point corresponding to the last, the eleventh, problem, occupies the lowest position; i.e., its solution appears to be the most difficult.

A qualitative analysis of the errors confirmed previously obtained results; the incorrect use of operations predominated, because the children mistook the proposed problems for another type of problem more familiar to them.

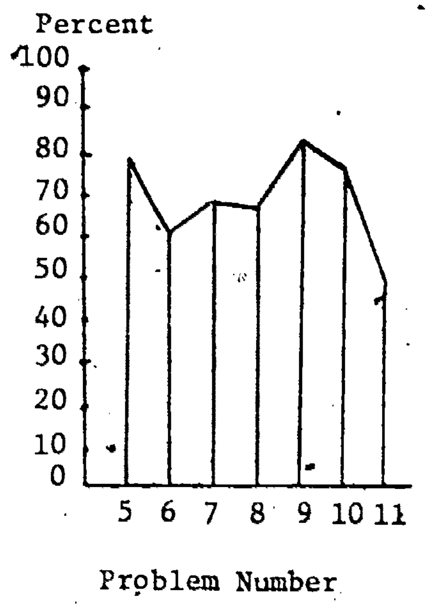

Figure 2. Percent of Correct Solutions of Proposed Problems.

Thus, the solutions to various types of problems involving one operation present different degrees of difficulty for mentally retarded students; the degree of difficulty depends upon the problem's <u>type of structure</u>.

The number of correct solutions is increased in the fifth and sixth grades, but this increase occurs because the problems are more simple. The ability to solve problems, which requires the perception of the relationships between the numbers, remains entirely unsatisfactory. The most difficult problems are those involving comparison by determining the difference and comparison by determining the ratio. The most widespread errors in the solutions to these problems involve the incorrect choice of operation. An analysis of these errors shows that in most cases the students failed to recognize the problem. Relying on nonessential external signs and taking them out of context, the children incorrectly determined what sort of problems they were, and thus solved them unsatisfactorily by applying operations corresponding to different types of problems.

These errors, which persist until the upper grades, testify to the great difficulty experienced by auxiliary school pupils in consolidating a clearly differentiated and tenacious system of elementary arithmetical knowledge.

Errors made by auxiliary school students in writing the nomenclature when solving problems.

An analysis of the typical errors made by auxiliary school students in solving problems would be incomplete if we did not illuminate the errors they make in writing nomenclature. These errors are very common, and they accompany correct solutions as well as erroneous solutions.

In mentally retarded schoolchildren, correct understanding of the subjective content of a problem is directly related to writing the nomenclature in the process of solving the problem. The correct designation of the nomenclature in solving arithmetic problems indicates the pupil's conscious consideration of the conditions of the problem, and of the arithmetical operation he chooses. Thus, many elementary school methodologists have, for a long time, justifiably emphasized the significance of writing the nomenclature in performing arithmetical operations. "The descriptive notation of the solution of a problem serves as a fulcrum for the work of the pupil's imagination and memory, and by the same token facilitates his thinking...." "When solving a problem in elementary school, the written operation must include the name of the units both in the given data and in the result; exceptions, of course, are the multiplier in multiplication, the divisor in division into a given number of parts, and the quotient in division into subgroups" [6].

Other methodologists emphasize that, by writing the nomenclature, the children's thinking becomes directed into the proper channel. "The statement of the nomenclature gives the notation of the problem a more descriptive character; without the nomenclature the notation appears to be abstract and only faintly reveals the character of the students' thinking. Writing the nomenclature disciplines the students' thinking and induces the pupil to look more deeply into the components of the arithmetic problem" [14]. A similar thought is expressed by A. I. Volodina, a teacher: "The statement of the nomenclature indicates the conscious attitude of the pupil toward his work and gives us the opportunity to control this conscious activity" [20].

The statements of the methodologists in the mass school are also important for developing methods of teaching arithmetic in the auxiliary school. Indeed, in order to do an arithmetic problem, the pupils must picture, in concrete form, the objects upon which one or another operation is being performed, using definite quantities stated in the conditions of the problem.

Knowledge of the object mentioned in the conditions of the problem is directly connected with its name; knowledge of the quantities, with their names. Understanding the conditions of the problem is closely related to understanding coherent speech, that is, knowledge of the language. Children should know the names of the objects and the quantities mentioned in the problem and should understand the significance of the arithmetical terms signifying the operations to be performed in the problem. They should fully understand how words agree, and what governs them; they should understand the meaning of combinations of words in the sentences constituting the conditions of the problems, and finally, they should know the interrogative form of sentences in order to understand the question in the problem.

Children become acquainted with the contents of problems by reading them. The texts of arithmetic problems, by virtue of their compression, are difficult to understand. One must be able to read the text fluently and intelligently in order to understand the problem. Schoolchildren in the elementary grades at the auxiliary school are often poorly prepared for this task.

For a pupil to understand the text of a problem, the words and sentences must seem a part of reality, and must summon its reflection to his mind. Only under these circumstances is it possible to choose the right arithmetical operation for the required problem. Formulation of the nomenclature, which tells whether or not the contents of the problem have been understood, presents no difficulties for students under these circumstances.

It is customarily the procedure in arithmetic to distinguish two types of nomenclature: nomenclature involving operations with so-called concrete or objective numbers and nomenclature of numbers which are obtained by measurement, called denominate numbers.

We consider the mistakes which the auxiliary school students make in formulating the nomenclature while solving problems with concrete (objective) numbers to be the first stage of work with them. This stage in teaching the formulation of nomenclature appears to be necessary for a mastery of the ability to work with concrete numbers. Acquainting the students with the simplest measurements and enabling them to form the necessary skills in that sphere appear to be the most important problems in teaching arithmetic at the auxiliary school so that students will be prepared to do practical work. But, as we know, operations involving concrete numbers, especially composite numbers, are extremely difficult for auxiliary school students; therefore, some teachers decline this work and do not equip the students with the measuring skills which are so necessary to a person at each step of his practical activities. Examining this question as a whole, we note that proper study of the nomenclature of concrete numbers would certainly have prepared the students to use the system of nomenclature, and, by the same token, would have facilitated their achievement of skills in measuring.

The correct formulation of nomenclature in problem solving is not given the necessary attention; some auxiliary school teachers are quite satisfied with the solution to a problem if the nomenclature is missing, as long as the operation is correctly performed.

We must note that there is no one set of procedures for teaching children to write nomenclature. Certain elementary mistakes in writing nomenclature arise just because of this lack of an established method of teaching such notation, or as a result of improper training. Teachers and methodologists do not agree on this question of method.

Some teachers do not instruct the children to write the nomenclature; their pupils write down the solution without the nomenclature. The students hear from other teachers that it is necessary only to put down the nomenclature in the results. We even came across a case where all the students in one class diligently copied out the nomenclature separately from the numbers, on another line.

Thus, for example, in the notebook of one pupil in that class we saw this notation for the solution to the problem, "5 pencils lay on the table, and 2 more pencils were laid there. How many pencils were there altogether?"

$$5 + 2 = 7$$

pencil + pencil = pencils.

An analysis of the pupils' written work showed that the teachers often permit them to drop the nomenclature in solving problems; this appears to be one of the causes for the children's forgetting the concept of the objective contents of the problem.

During the first stages of instruction, when the child does not yet distinguish the numbers from the concrete totality of the objects, he operates not with numbers, but with objects. Certain elements of abstraction and generalization are involved in the transition from operating with objects to counting on fingers. In the process of becoming acquainted with numbers, the child, on the one hand, abstracts himself from the objects and turns to abstract calculation, and, on the other hand, begins to relate numbers to the specific objective totality. This occurs during the period when the child is studying problem solving. Under an improper method this instruction becomes a mere illustration of one or another arithmetical operation. The child attempts to compute as quickly as possible the total data given in the conditions of the problem. If he does this correctly he receives the teacher's approval. The entire activity of the students when they solve a problem in this manner amounts to performing operations with numbers. The process of solving arithmetic problems in such cases is replaced by solving examples. In this connection the objective content of the problems disappears from the child's mind.

Lack of attention to the notation of the nomenclature in solving arithmetic problems stems from the tendency of many auxiliary school teachers to reduce all instruction in arithmetic to calculation.

In our analysis of the children's work we established that the quantity of errors in the use of nomenclature fluctuated between 15% in some problems, and 80% in others, which indicates the relationship between these errors and the type of problem.

In problems involving the application of the operations of addition and subtraction, one may note three types of errors:

1. The nomenclature is not written completely; these errors are encountered very frequently.

2. The nomenclature is written for only one component of the problem (in the first term or in the minuend), for example: "2 dresses + 5 = 7"; "8 apples - 2 = 6"; these mistakes are not encountered

frequently;

3. The nomenclature is written only in the result—in the sum or the difference, for example: "2 + 5 = 7 (dresses);" "8 - 2 = 6 (apples);" similar notations of the nomenclature appear to be very widespread.

We must distinguish in particular those cases in which the students, in writing the results of the solution to problems involving addition, had to substitute the specific nomenclature with the generic, for example, when it is necessary to add a quantity of girls and a quantity of boys in order to find out how many children there are in the class, or to add apples and pears and determine the amount of fruit. Preliminary work in which the students are taught the transition from specific to generic concepts is meaningful in that it explains these difficult cases to the students. In order to avoid errors, special exercises are necessary whereby the children gain the ability to generalize, to bring the specific concepts under the generic, and to put down the nomenclature correctly.

There is another kind of difficulty which arises in the formulation of nomenclature for the solution of problems in multiplication and division. The children very often commit errors in multiplication which indicate how complex the process of solving the problem is for them and what great demands it makes upon the child's thinking. Thus, the problem, "One pen costs 3 kopeks; how much will 3 pens cost?" is solved in the following way: "3 kopeks x 3 pens = 9 kopeks"; answer: "3 pens cost 9 kopeks." It is apparent that the child retained the objective contents of the problem; he computed both the kopeks and the pens. However, he did not achieve the necessary level of generalization; the child did not understand the logical meaning of the operation of multiplication which does not permit kopeks to be multiplied by pens; therefore, his notation appears to be erroneous. This same type of difficulty is encountered in writing the nomenclature in division into parts. For example, this problem was given: "2 dresses took 10 meters of material. How much material does one dress take?" A typical error is observed in the notation, "10 meters : 2 dresses = 5 meters"; answer: "One dress took 5 meters of material."

It is clear that these difficulties are increased in cases where it is necessary to apply two operations -- division and multiplication.

To study more deeply the nature of the difficulties involved in stating the nomenclature, we conducted a special investigation of the process of solving problems involving two operations. Ninety-six pupils from six grades in five Moscow auxiliary schools participated in the experiments. There were no failing students in these classes.

As a means of comparison, these same problems were solved by 86 second grade pupils in the mass school. They were given problems solvable by finding the unit measure; these problems required multiplication and division. Here is one of the problems: "20 dresses were sewn in the workshop out of 80 meters of material. How many meters of material did 50 of these dresses take?"

Below is a table representing data on the number of mass and auxiliary school students (in percentages of the total number) who refused to solve this problem, committed various errors in solving it, or who solved the problem correctly (Table II).

TABLE II

ERRORS BY MASS AND AUXILIARY SCHOOL PUPILS

Grades	Refusal to Solve	Errors in Solution: Incorrect Choice of Operation	Errors in Calculation	Errors in Nomenclature*	Correct Solution
Grade II in the Mass School	0	0	2	34	64
Grade IV in the Auxiliary School	6	24	10	56	4

*Errors in stating the nomenclature were also present in the first and second group of errors.

Note: Entries are percentages of the total number of students in each category.

It is apparent from the table that the pupils' refusals and their incorrect choice of operations were present only in the work of the auxiliary school students. But errors in nomenclature were extremely widespread for students in both schools. If we take into account the fact that errors in nomenclature accompanied the errors in choosing the operation and in calculations, we see that such errors are encountered in 90% of the auxiliary school students whose work we have analyzed. This is almost three times the number of mass school pupils who committed these errors. Let us examine typical errors in stating the nomenclature. There were frequent errors in the solution to the first question of the problem, which reflected weakness in the ability of mentally retarded pupils to generalize, but which showed at the same time that they understood the concrete contents of the problem correctly. These cases were characteristically written thus: "80 meters ÷ 20 dresses = 4 meters." There are definite relationships in the problem between the objects in the operation (meters of material and dresses) and the quantities of these objects. In solving the first problem the children expressed the relationships between the quantities correctly. The relationships between the objects were represented truly, but were not generalized. This is a sort of "intelligent" error which shows that in solving the problem the pupil is not carrying out division into parts independent of what these parts are made of--dresses or pieces of material, etc.

To teach children not to use the nomenclature when working with division into parts, it is necessary to show them that, in practice, the material should first be divided into parts, and only then can it be used in the preparation of dresses. Not only is it necessary to demonstrate this several times, but it is also necessary to give each pupil the opportunity to carry out division into parts using various objects. It is especially necessary to show the children that these various parts of the material can be used in different ways (to sew a dress, a robe, a skirt), but that in order to do this it is first necessary to divide the material into parts. In other words, the necessity of generalization in a given case must be taught specifically.

The origin of another group of errors in which one of the objects disappears from the written nomenclature appears to be more complex.

The operation is performed correctly, but all of the numbers have identical nomenclature, for example: "80 meters ÷ 20 meters = 4 meters." This notation is evidence of a certain distortion of the contents of the problem, since the connection between the number and the definite object characterized by that number is severed, and only the relation between the given numbers remains. A formulation wherein all the numbers have identical nomenclature just as they do in addition and subtraction, shows that the complex relationships in the problem have been simplified and transformed into more elementary ones. It should be considered that due to the loss of one of the objects in the conditions of the problem the orientators for the solution of the second question in the problem are weakened. This type of error borders closely on the type in which the second object (in this case, the material) disappears, thus simplifying the problem sharply, although the division into parts, required by the conditions of the problem, is retained; e.g., "80 dresses ÷ 20 = 4 dresses."

In these cases, which account for 30% of all errors, it is distinctly apparent that the relationships come under various classifications--relationships between the objects (nomenclature), relationships between the objects and their quantity, and, finally, relationships among the quantities (the numbers)--and this disturbance of the relationship between the object and the quantity leads to the disturbance of the relationship between the objects, that is, to the disturbance of the relationships among all of the classifications. A problem involving two operations is, in fact, transformed into a less complex problem involving one operation, wherein only the correct relationships between the numbers are retained. These sorts of errors in solving problems have been described in patients having brain damage [1].

There is another type of error whereby the nomenclature is retained in the ~~divisor and~~ the dividend, but is missing in the quotient, for example: "80 meters ÷ 20 meters = 4." Here simplification of the relationships between the objects accompanies changing the character of the operation. The problem, which requires division into parts and the determination of what these parts are quantitatively, is transformed into a problem in division into subgroups, which leads the pupil away from the given conditions and hampers further solution.

Finally, where the nomenclature is missing in the divisor and the dividend, but appears in the quotient "80 : 20 = 4 meters," a complex problem is turned into an example; the students' attention is directed toward the arithmetical operation as in solving an example, while the nomenclature is used only in the final answer as an illustration.

The errors encountered in the performance of the second operation in the problem, that is, in multiplication, are analogous and similar to the errors in division described above.

Our examination of all the errors in the notation of the nomenclature allows us to group them into two general categories. First there are the errors caused by the difficulty of abstracting from a concrete situation. This concerns the statement of the nomenclature in the multiplier and the divisor (in division into parts). In utilizing concrete nomenclature in division or multiplication the transition from concrete material to certain generalizations appears to be complex. The presence of these errors should be considered a manifestation that is fully anticipated, since the procedural means used at present do not ensure to a sufficient degree the formulation of that type of generalization.

In other words, there are cases in which the erroneous notation of the nomenclature testifies to the loss of the concrete contents of the problem. This is a double error. In the first place, this is a case in which the problem is transformed into an example, whereby the nomenclature is included only in the result of the operation. In the second place, these are cases in which, in performing the operations of division or multiplication, the nomenclature is placed as though it were in addition or subtraction. In these cases the choice of the operation appears to be correct, and the correct relationships between the numbers are retained, but the relationships between the quantities and the objects are distorted; this is evident in cases in which the notation of the nomenclature is clearly nonsensical, for example: "4 dresses x 50 dresses = 200 dresses." Here there is a peculiar disassociation of the quantitative notations from the designated objects to which these quantities are related. This disassociation is a signal that the objective contents of the problem are either misunderstood or lost.

Thus, an analysis of the errors made in stating the nomenclature showed that the numbers in word problems are often perceived by auxiliary school students as though they were significant apart from the objects; that is, the numbers are not related to the objects they determine. A mentally retarded schoolchild's interpretation of a problem is often limited only by his definition of the relation between the numbers as given in its conditions. The students do not always succeed in penetrating the present and past relations between the objects which compose the real contents of the problem. Because of such a comprehension of the problem the children, naturally, either drop the nomenclature or distribute it incorrectly.

The nomenclature for the quantity obtained as a result of solving an arithmetic problem can be correctly determined when the actual objective contents of the problem are understood. It is important that the teacher understand that in solving a problem the choice of operation may be brought about even without the student's comprehending the real contents of the problem; the correctness of the choice of operation is not always a sign of the correct interpretation of the problem. The correct formulation of the nomenclature serves as a more convincing argument here.

It follows from what has been said that the lack of a suitable method for teaching formulation of nomenclature aggravates the situation even when the numerical data and the objective contents of the problem are not disassociated, as we observed to be the case when the auxiliary school students were in the initial stages of problem solving.

Conclusions. 1. After four or five years of instruction in the auxiliary schools, the students have usually mastered firm skills in performing operations in calculating within the limits of one hundred and even of one thousand. Nevertheless, they do not solve satisfactorily the types of problems that are indicated in the program for the third, fourth and fifth grades. Most pupils in the sixth and seventh grades continue to solve these problems unsatisfactorily.

2. By reproducing the complex relationships in the conditions of the problem, the interpretation and solution of the problem are manifested. When the second grade mass school children have correctly reproduced a problem, their solution is usually correct; after an incorrect reproduction of the problem there is very seldom a correct

solution. Such a connection between the reproduction and solution of problems, although observed in most cases, does not appear to be always true for mentally retarded children, who sometimes, notwithstanding an incomplete and distorted reproduction of a problem, solved it correctly and, conversely, a correct reproduction notwithstanding, sometimes solved it incorrectly.

3. Even when the problem is reproduced correctly it does not always follow that the relationships contained in it have been understood. The cases in which this occurs are characterized by a peculiar disassociation of the words from their concrete contents. The emasculation of the verbal reproduction of the problem is shown by the fact that it not only fails to guide the solution, but, apparently, also fails to give rise to the images behind these words. Mentally retarded children sometimes memorize a short text verbatim without sufficiently understanding it. This appears to be a symptom of the insufficient unity of thought and speech, a disassociation of the activities of the two signal systems.

This is evidenced by those cases in which the problem is reproduced fragmentarily, incompletely, even erroneously, but nevertheless solved correctly. In these cases, valid operations are accompanied by quite unsatisfactory wording.

4. Whenever one type of problem is changed into another type in the reproduction, the problem is usually solved as though it were of the type indicated in the reproduction. The fact that the problem was changed is evidence of insufficient differentiation of the system for learning in mentally retarded schoolchildren. This shows that they perceive the terms and expressions contained in the conditions of the problem in an extremely general, diffuse form. Consequently, mentally retarded children express the situation in the problem incorrectly.

5. Changing a problem from one type into another type occurs not only when a problem has been reproduced and subsequently solved, but also when the solution took place without a preliminary reproduction. Such a change is caused by the special "guiding" role played by the terms expressing specific arithmetical relationships in the text of the problem.

Again, this testifies to the complexity of the interaction of the signal systems in oligophrenics.

6. The peculiarities in the way auxiliary school students learn the language are revealed by the exaggerated role which they give to arithmetical terms and expressions, and their mistaken interpretation of these terms and expressions. These peculiarities consist not only in an oversimplified interpretation of the terms, comparing them to each other because of a certain external resemblance, but also in an insufficient understanding of their relationship to the other words in the sentence.

7. The peculiarities in the way mentally retarded children learn the language are even more distinctly revealed when they solve simple problems given in an indirect form (in other words, problems in which the operation is reversed). Psychological research shows that the texts of such problems are difficult to understand not only for auxiliary school students but for normal schoolchildren as well [11]. The difficulties experienced by auxiliary school students in solving indirect problems are magnified by their misinterpretation of arithmetical expressions and terms. It is particularly easy for them to lose the objective contents in such problems, so that the solution is directed along an erroneous path.

8. The predominant error in solving simple problems appears to be choosing the wrong operation. We must note that this error is not due solely to the reasons we indicated in the previous paragraphs, but also to the conditions under which the solution takes place, in particular, to the character of the problem solved immediately before the new one. Mentally retarded children display a tendency to liken proposed problems to what, in their experience with problem solving, is the "freshest" in their memory.

9. Certain errors in solving one or another simple problem were peculiar to that type of problem. The number of erroneous solutions, and the character of the errors, permitted us to divide the eleven problems presented to the children into three groups, in accordance with the degree of difficulty they posed for the students.

Problems with direct operations enter into the first group with the simplest problems--the addition of two quantities, finding the remainder, and division into parts. In the second group we add problems in which terms like "so many more, fewer" enter into the conditions, problems in increasing and decreasing "by so many times," and problems in finding the product. The children make more mistakes in solving these problems than they do in solving those belonging to the first group. And, finally, the most difficult problems, involving comparison by determining the difference and the ratio and problems in division into subgroups, constituted the third group.

10. An analysis of the errors made by mentally retarded children in writing nomenclature in the solution of problems showed that they often perceived the numbers in the text of the problems out of the context of the objects to which the numbers refer. Mentally retarded schoolchildren often fail to grasp the entire system of relationships within a problem and are limited to isolating the relations between the numbers. Since they do not grasp the relations between the objects and the relations between the numbers and the objects, the children either drop the nomenclature or place it incorrectly. The arrangement of the nomenclature serves as a convincing proof of the character of the auxiliary school students' comprehension of the problem.

11. Errors in the notation of nomenclature in problem solving can be caused, in the first place, by difficulty in abstracting from the graphic situation contained in the problem, and, in the second place, by the students' changing the objective contents of the problem and disassociating these contents.

12. The errors that rise through changing the objective contents of the problem or disassociating them show that the students' comprehension of the real contents of the problem is insufficient, although their choice of the operation in the solution may be essentially correct. In teaching children how to solve arithmetic problems it is necessary to pay serious attention to stating the nomenclature correctly, and to present a single system of requirements for the way that it should be written, since using the nomenclature helps the children to retain the objective contents of the problem in their minds.

13. The characteristic way in which auxiliary school students solve arithmetic problems testifies to the profoundly unique quality of their perceptual activities in comparison with students in the mass school.

There is an urgent need for the auxiliary school teachers to subtly and precisely consider these peculiarities in developing methods of pedagogical influence upon mentally retarded children. This is of primary concern in the teaching program. There can be no doubt that in this way a significant increase in the level of the auxiliary school students' knowledge and in their skills will be achieved.

Some Proposals for Improving Instruction in Arithmetic Problem Solving in the Auxiliary School

Having investigated the characteristics of the way auxiliary school students reproduce and solve arithmetic problems, we are convinced that the level of their knowledge and ability in this sphere is insufficient, but that the possibilities for developing their knowledge and abilities have not only not been exhausted, but to a significant degree have not even been utilized.

The successes and failures in the schoolchildren's mastery of the fundamentals of the sciences are always tied to the concrete organization of the pedagogical work, which provides the conditions for the achievement of the children's mental development. As we observed over a period of several years how mentally retarded schoolchildren master arithmetical knowledge, we became convinced that we should not consider the characteristics of their perceptual activities unchangeable; these peculiarities are closely related to a definite system of teaching the children, and depend upon the character of that system.

Improvement in the curriculum, the textbooks, and the method of teaching in the auxiliary school does not, of course, completely remove all the peculiarities in the thinking of mentally retarded children, which are due to brain defects, but it does give the children a chance to attain the ability to solve the practical problems of life more successfully. The search for a right way to teach mentally retarded children becomes more successful the more we know about the characteristics of the way they attain knowledge.

The chief distinguishing feature of the way mentally retarded children interpret and solve arithmetic problems is the peculiar disassociation of the wording from its concrete contents, and, by the same token, the neglect of the objective contents of the problem. We note that in these children the second signal system is isolated from the first. Thus, in teaching the children how to solve arithmetic problems, it is necessary that they perceive the correct correspondence between their practical experience and the verbal expression of this experience.

The materials in our investigation and the experience of our best teachers allow us to make several proposals for a method of teaching problem solving in arithmetic in the auxiliary school. Above all, we must see that the students' practical experience in calculation and measurement is enriched. It was noted in a series of investigations that schoolchildren of age 8 or 9, upon entering the first grade in the auxiliary school, do not have the necessary practical experience and do not possess the knowledge which normal preschoolers already have at the age of 4 or 5 years [19].

The auxiliary school arithmetic syllabus and a series of methodological aids [7] recommend preparatory studies in the first grade which should enrich the children's spatial and quantitative concepts and should prepare the children to understand arithmetic problems [9].

In studying pedagogical experience we succeeded in establishing that the best teachers attempt to pass on to the students their own practical experience; such experience should be consolidated and generalized by using arithmetical terms and expressions in speaking to them. They direct the child to analyze the reality surrounding him. To give the children knowledge about large and small, narrow and wide, high and low and so forth, good teachers help them to see these properties in the objects around them. Operating with didactic material can only complete the child's perceptual activity and ensure generalization and consolidation of his knowledge in dealing with real objects. Only after sufficiently acquainting the children with real objects do experienced teachers turn to working with didactic material and consolidating the children's knowledge by using pictures.

It is well known that mentally retarded children have difficulty in applying knowledge obtained under particular conditions when solving practical problems. Pedagogical experience tells us that if didactic material is used chiefly in teaching these children, they will not always be able to transfer the knowledge and skills they have acquired to an objective situation. The principle of the visual methods in instruction will be formally observed, but its result will be insignificant. Didactic material must be regarded as an auxiliary method which is significant only when the children already have a definite, sufficiently high level of orientation in the surrounding reality; it helps give the children an ability to abstract from concrete reality and to generalize about this reality. If the child's practical experience is sufficiently rich, working with didactic material elevates the child to a new, higher level; if the children's practical experience is lacking, working with didactic material does not replace it, and is therefore useless.

Before teaching mentally retarded children to solve problems it is necessary to help them build and comprehend everyday experience in that sphere of reality and existence in which calculation and measurement are used the most. When he comes to the auxiliary school, the pupil has a vague conception of where milk, sugar, meat, and bread are sold; he does not know what methods are used to measure these products in selling them, although he may have been in a store and seen this process many times. He usually does not know the monetary symbols, and does not understand the very process of buying and selling.

The students must be given a systematic education in order for them to develop more precise, systematic concepts in this sphere, to enrich their knowledge and vocabulary, and to become acquainted with the terms-- this develops their thinking and speech. Acquainting the children with this sphere of life through pictures and books has been shown by pedagogical experience to be ineffective.

The best teachers of mentally retarded pupils prefer to extend the children's experience by giving them a practical acquaintance with the surrounding reality. Under these conditions the words, terms, and expressions are closely related to the objective world.

To illustrate these statements let us describe a series of pedagogical methods used by teachers:

1. excursions accompanied by demonstration and practical execution of the actions and operations connected with the purchase of products,

2. organization of the game "to the store" in class and after school.

To acquaint the children with what is sold in one store or another, several excursions must be made. One excursion was organized around a special assignment--to see what kinds of stores, booths, and stalls there are on this street--"Here is a bakery. What do they sell inside?"; "Here is a dairy store. What do they deal with?"; "And what is sold in a pharmacy?"

The following series of excursions to a store were made with the aim of observing the process of buying and selling. The children discovered that there are piece goods and goods that are measured or weighed. Here with the help of the teacher, they bought something, and also became acquainted with the corresponding terms. Thus, the children became acquainted with various stores. They discovered what is measured in meters, liters, what is weighed in kilograms, and so forth.

When second graders visit stores, they must learn that it is possible to count objects by twos, fives, and tens, and that certain goods are sold in packs (matches, cigarettes), boxes (candy, pastry), on spools, in balls, in skeins, packages, etc. The results of each excursion were discussed and consolidated in special lesson-games (the game "in the bakery," "at the dairy store," "at the pharmacy," and so forth), in which coins are used freely--first one kopek, then two and three kopeks. Each pupil glued together a box for himself to be used as a coin box and kept his coins there.

A strict system and progression should be observed in conducting these lessons. It is impossible to convey a great quantity of new information to the children at once on a single excursion; it is necessary to secure a certain amount of information, and the corresponding terms, on each excursion. The lesson-dramatizations which we mentioned above (for example, the game, "to the store") have a special significance. Each such lesson should have a definite goal, for example, to give the children in the third grade information about the relationship between the quantity of goods and the amount of money necessary to purchase them. The children should apply this information independently in their practical activity, for example, in buying lunch in the school cafeteria. Measuring is a necessary part of the work in teaching elementary arithmetic in the auxiliary school. It is an essential stage in the studies which prepare mentally retarded children for solving arithmetic problems.

According to the contemporary methods for teaching arithmetic, questions involving instruction in measurement are introduced rather late, namely, in the section on the study of metric measure and of compound concrete numbers. In accordance with the program for arithmetic in the auxiliary school, the children in the second grade are acquainted with measures only in the measuring of segments and liquids, and in the third grade, using the measures indicated, they are given practice in measurement. We are convinced that children should learn to measure in the first grade when they are first introduced to numbers. They should acquire skill in measuring long before they become acquainted with the metric system of measure.

The mentally retarded child, during his initial instruction in calculation, forms the concept that a quantity is an aggregate of objects. This concept is created as a result of enumerating groups of objects, but it can also be created as a result of measuring. This is one of the most important sources for the formation of number concepts, as the foremost prerevolutionary methodologists (Gur'ev [5], Galanin [4], and others) pointed out repeatedly.

Pedagogical experience shows that if, during the very first stages of instruction, the child, in practice, carries out measurements using conventional, everyday units of measurement (a spoon of sugar, two spoons of sugar, three glasses of tea, two bowls of soup, one lump of sugar, two steps, and so forth), he will also, in this way, obtain a concept of number. Children can measure liquids in glasses, or pour jars of sand into a pail in order to strew it on the garden path afterwards; in lessons involving manual work they can measure the length of ribbons, string, and so forth, using colored rulers having a definite length. In this way the child forms an elementary concept of how many times the ruler is laid down according to the length of the tape, and so forth.

Measurement in the first stages of instruction is usually performed with conventional units: steps, rope, rulers, pieces of paper having a specific length, glasses, jars, etc. At first the children learn to do manual work with a "measuring technique"; e.g., they weave mats out of colored paper, thereby gaining a sound knowledge of the significance of the terms "long--short" and learning to measure by placing short strips of one color onto long strips of another color. A great many such lessons involving work with strips must be conducted in order to consolidate skills in measuring. These skills can also be consolidated in lessons involving manual work wherein the length of strips of paper, the length of laces and ribbons, pieces of material, plasticine, etc., are measured.

Such a series of lessons in the first grade in arithmetic and manipulative work should be organically connected with lessons in the study of numbers and with lessons in improving skills in calculating. The fundamental reason for such lessons is that mentally retarded children obtain knowledge of arithmetic on the basis of practical activity, implemented not only by hearing and sight, but also by touch and muscular sensations. When the children possess a knowledge of elementary arithmetic based on practical activity, it is undoubtedly easier for them to approach the solution of problems.

The methods we have set forth are directed toward giving the pupil an opportunity to accumulate practical experience to use in future problem solving.

The teacher should pay special attention to extending the children's practical experience, so that they may comprehend the functional relations which form part of the conditions of the problem (time and distance, speed and time, and so forth). In this connection it is important to note that the teacher should, in the process of teaching, utilize the children's practical experience, not only in the initial, but also in the subsequent stages in which the students learn to thoughtfully examine the conditions of a problem.

Many psychological investigations and methodologies emphasize the meaning of comparison organized and conducted in different ways as a means which contributes to a better mastery of the material in a particular educational discipline. We believe that, even in the initial process of comprehending various arithmetical relations, it is useful to take advantage of comparison; e.g., "There were some candies on the table. I put another piece of candy on the table. Were there more or fewer candies then?"; "Mama went to the store and bought some bread. Did she have more or less money afterwards?"; "Vasya bought one piece of candy, and Misha, five pieces of the same type of candy. Who paid more money?"

These problems give the students practice in ascertaining various mathematical relations. Although such an analysis of the conditions of the problem appears to be generalized, it nevertheless serves as a stepping stone to a higher level of analysis. In solving problems in addition and subtraction, the children should always focus their attention on the fact that in addition the number obtained becomes greater and, conversely, that in subtraction the number obtained will be less than the first number. Many exercises should also be done to consolidate the concept of "as many as." The pupil should name the quantities which are greater than and less than the given quantity.

All these exercises should be performed on concrete material under the conditions of a well-defined activity. This can be effected in handicraft lessons, in which there are various segments of ribbon, braid, and paper. The students can perform the measurements with the segments, although they do not know the conventional measures. Experience has shown that comprehension of the essentials of one or another arithmetical operation is most effectively achieved by the independent activities of the child. Experienced teachers understand that if two piles of books or notebooks are lying on the table and the pupil is given the assignment, "Add one pile to the other," "Pile the books together," "Shift all the books together from the table to the cupboard," "Add one pile of notebooks to the other and give them to me," or "Bring the chairs from the other room and put them all together," while performing the practical operations he is learning to add at the same time. These are practical problems to illustrate the operation of subtraction: certain objects must be taken away, removed, cut off, or carried away from the total quantity of objects.

After the children learn to follow such instructions it is possible to combine these operations with calculation. For example, before piling the groups together, the child should count the number of books in the first pile and in the second. These practical operations should prepare the children to picture in reality the conditions of the problems, and to become accustomed to taking into account the sequence of the changes in the quantities contained in the problem.

The teacher's serious attitude to such exercises helps the children accumulate the necessary experience with practical operations and wording. They will use this experience as a basis in the future, for solving word problems, and for situations in which they have to imagine a situation on the basis of the conditions of the problem. Thus, before passing on to the solution of arithmetic problems in a text, it is necessary to conduct a great deal of preparatory work in developing and enriching the children's concepts and imaginations.

We believe that at this stage, when the students are doing exercises in solving problems with real objects, it is already necessary to teach them to verbalize the operation they have performed. The child should say, for example: "I piled the books together," "I gathered the notebooks together," "I added," "I took away," and so forth. In the next stage of the exercises these verbalizations should be related to the other pupils: "Petya piled the books together," "Vasya added...," "Lenya carried away, took away...," and so forth. Then the pupils should be taught to compose several consistent related sentences describing the operations they are performing, for example: "Lenya put two notebooks on the table. Petya put down five more and piled them together." These verbalizations should accompany the operation or should follow its performance.

The verbalization of the operations helps the child generalize and consolidate them in his mind. The arithmetical language mastered by the child plays a decisive role in the development of his arithmetic thought, and in the transition from visual conceptions of quantities to generalize numerical concepts. Special work must be conducted in order to teach the student to verbalize the operations which they have performed. Such exercises serve as a good preparation for understanding word problems and have great importance for developing the correct interaction between the two signal systems in mentally retarded students.

Studying the existing practice of arithmetic instruction in the auxiliary school, we noted the extraordinary speed with which the teachers shift the students from solving problems involving real objects to solving verbally formulated problems. This unwarranted speed leads to the eventuality that the students do not see either the concrete contents or the essence of the mathematical expressions in the problem.

To prepare the mentally retarded child for solving arithmetic problems, to prevent his disassociating the words from the objective contents, it is necessary to enrich and generalize his practical experience, to systematize and verbalize it, and not to give the child word problems too quickly. Moreover, the pupils should, whenever necessary, return to actual operations with real objects.

Our material permits one more conclusion of serious pedagogical significance; it is necessary to consolidate carefully the ability to solve problems of a particular type. Pavlov's experience in the differentiation of conventional factors shows that the frequent repetition of one or another conventional factor does not make it a more specialized stimulus. Specialization is achieved by contrasting it with closely related factors. Our observations of the reproduction and solution of simple problems showed that the frequent repetition of word problems of a certain type, having similar components and similar wordings, really did not specialize the recognition of this type of problem and its correct solution. It can be supposed that in these cases the comparison of one type of problem with another would have furthered the pupils' formation of the ability to distinguish and recognize them correctly.

An examination of the methodological literature dealing with problem solving, an analysis of the auxiliary school students' achievement level, and special experimental investigations devoted to the detection of characteristic ways that they reproduce and solve problems, have shown that one of the main sources of unsatisfactory results in teaching mentally retarded children to solve problems is that special teaching methods are altogether inadequately evolved and utilized, considering the profound uniqueness of the mentally retarded child's cognitive activities.

The material which we have collected dealing with the level of the auxiliary school students' knowledge in arithmetic, the data about their reproduction and solution of problems, and also an analysis of the experience of the best teachers testifies to the necessity of giving serious attention to the improvement of the curriculum and textbooks, and to the perfection of the methodology for teaching problem solving in the auxiliary school. The data we obtained indicates the necessity of long-term instruction preparatory to solving problems.

In compiling textbooks, the inclusion of each type of simple problem asked of the children should be justified, and serious attention should be given to the order in which such problems are organized. Methods of teaching the solution of each type of simple problem must be evolved. Special attention should be devoted to the transition from solving one type of problem to solving another type of problem with a view to ensuring that the pupils will recognize and solve them correctly.

Methodological devices of education should be specialized, in order to better take into account the uniqueness of the mentally retarded schoolchild.

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