DOCUMENT RESUME

ED 182 117 SE 029 682

AUTHOR Higgins, Jon L., Ed.
TITLE Research Reporting Sections, Annual Meeting of the National Council of Teachers of Mathematics (58th, Seattle, Washington, April 16-19, 1980).
INSTITUTION ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, Columbus, Ohio.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
PUB DATE 80
CONTRACT 400-78-0004
NOTE 54p.
AVAILABLE FROM Information Reference Center (ERIC/IRC), The Ohio State University, 1200 Chambers Rd., 3rd Floor, Columbus, Ohio 43212 ($1.75)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Calculus; Educational Games; Elementary Secondary Education; Games; Geometry; Information Processing; *Mathematics Education; *Mathematics Instruction; *Number Concepts; *Problem Solving; *Research; Sex Differences
IDENTIFIERS *National Council of Teachers of Mathematics

ABSTRACT

Presented are abstracts of 14 research reports. Topics covered include: (1) the effects of games on mathematics skills and concepts; (2) the use of problem-solving heuristics in the playing of games involving mathematics; (3) sex differences in electing mathematics; (4) the origins of sex differences in high school mathematics achievement and participation; (5) the long-term consequences of sex differences in high school mathematic achievement; (6) locus of control and mathematics instruction; (7) the interaction of general reasoning ability and Gestalt and analytic strategies of processing spatial tasks with transformational and non-transformational treatments in secondary school geometry; (8) student performances, individual differences, and modes of representation; (9) the development of problem-solving capabilities in primary grade children; (10) story problems; (11) spatial visualization skill and processes used in solving mathematical problems; (12) factors of organization and clarity in mathematics lessons; (13) the psychology of equation solving, an information processing study; and (14) number concepts and the introduction of calculus. (MK)

MATHEMATICS EDUCATION REPORTS

Seattle, Washington
16-19 April 1980

RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

58TH ANNUAL MEETING

Jon L. Higgins, Editor

ERIC Clearinghouse for Science, Mathematics
and Environmental Education
The Ohio State University
College of Education
1200 Chambers Road, Third Floor
Columbus, Ohio 43212

Mathematics Education Reports

Mathematics Education Reports are developed to disseminate information concerning mathematics education documents analyzed at the ERIC Clearinghouse for Science, Mathematics and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific subpopulations of the mathematics education community. Priorities for development of future Mathematics Education Reports are established by the Advisory Board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the Associate Director.

This publication was prepared with funding from the National Institute of Education, U.S. Department of Health, Education, and Welfare under contract no. 400-78-0004. The opinions expressed in this report do not necessarily reflect the positions or policies of NIE or HEW.

FOREWORD

This Mathematics Education Report contains abstracts of all papers presented in the research reporting sessions of the 58th Annual Meeting of the National Council of Teachers of Mathematics. These papers were selected for presentation from a large number of proposals submitted to the NCTM Research Advisory Committee and the program chairman for Research Sections, James M. Rubillo. We wish to thank Dr. Rubillo and the members of the Research Advisory Committee for making them available for this publication.

Jon L. Higgins
Associate Director for
Mathematics Education

TABLE OF CONTENTS

NCTM Research Reporting Sessions
58th Annual Meeting

TABLE OF CONTENTS (Continued)

Reporting Session I

EFFECTS OF GAMES ON MATHEMATICS SKILLS AND CONCEPTS

George W. Bright
Northern Illinois University

John G. Harvey
University of Wisconsin-Madison

Margariete Montague Wheeler
Northern Illinois University

It has been averred that games are important instructional activities (e.g., Dewey, 1928; Bruner, 1960) and that, in mathematics, cognitive effects result from their use (e.g., Biggs and MacLean, 1969, p. 50; Johnson and Rising, 1972, p. 224; Dienes, 1972, p. 64; Sobel and Maletsky, 1975, p. 49). Until recently there had been no systematic investigation of the cognitive effects of games on mathematics learning; as a result until 1976 there was a paucity of empirical research in this area and no substantiated effects (Bright, Harvey, and Wheeler, 1977).

Bright, Harvey, and Wheeler (1977) have proposed a conceptual framework and a research strategy for a systematic investigation of the cognitive effects of games on mathematics learning. Three studies, conducted within this conceptual framework and research strategy, are presented in this research reporting session. One study dealt with the effects of games on logical reasoning; one study explored the relationship between achievement grouping and instructional level; and one study examined the effects when manipulative game constraints were varied.

Study 1: Effects of games on logical reasoning abilities.

Purpose: This study investigated the effects of logical reasoning games at the pre-instructional level. Two games, Mastermind-Regular (Invicta Plastics, 1972) and Number Mastermind (Invicta Plastics, 1976) were used. The subjects were students in seven intact sixth-grade and eight intact eighth-grade classes from two elementary and two middle/junior high schools in northern Illinois and south central Wisconsin; one class in each of the four schools was randomly chosen as a control class.

Procedure: Two pretests and two posttests were administered. One pretest was a formal operations test (Adi, Karplus, and Lawson, in press); the other, a 40-item, investigator-developed test of logical reasoning. One posttest was a parallel form of the 40-item pretest of logical reasoning; the other a 12-item test of game related items generated using photographs of student-completed games. Ranked scores from the formal operations pretest were used to assign students within each of the twelve experimental classrooms to one of the two versions of Mastermind; beginning with ranks one and two, one student from each pair of ranks was randomly assigned to Mastermind-Regular for the entire experimental period, and the other, to Number Mastermind. The games were played for fifteen minutes twice daily for eight weeks during the fall of 1978.

Analysis: To examine the differential effects of the two versions of Mastermind ANCOVA on both posttest scores using the logical reasoning pretest scores as covariate were run for each experimental class; sex was used as a factor in each analysis. Seven of 66 F-values for the ANCOVAs were significant ($p < .05$). Wilcoxon matched-pairs, signed-rank tests indicated that there were no significant differences between the pre- and posttest mean scores on the test of logical reasoning. These results along with other results and the conclusions reached by the investigators will be discussed.

Study 2: Effects of games with different achievement groupings and instructional levels.

Purpose: This study, conducted during the fall of 1978, examined the effects of achievement grouping on the cognitive effects of concept and skill games. Two sets of games were used. One set was the skill game, Order Out (Romberg, Harvey, Moser, and Montgomery, 1974, 1975, 1976), and the other set was eight pairs of concept games similar to the pair Odd or Even Difference and Odd or Even Product (Romberg et al., 1974, 1975, 1976). Subjects were students in eight intact seventh-grade classes in northern Illinois. Four of the eight classes were randomly assigned to play the skill game, and the other four, the concept games.

Procedure: A pre- and posttest was given to each experimental group. The skills pre- and posttests were parallel forms of a 40-item, investigator-developed instrument which measures ability to order common fractions, the content of Order Out. A 28-item, investigator-developed instrument which measures ability to discriminate fair from unfair situations was used as the concepts pre- and posttest. Within each class subjects were separated into four strata on the basis of their pretest scores. In a class of 32 students, using those strata four heterogeneous groups of students were formed in each class by randomly selecting one student from each stratum for each group; the remaining four students in each strata composed a homogeneous group. Students played the games in these groups throughout the treatment period. The skill game was played twice weekly for four weeks; the concept games were played, in pairs, twice weekly for four weeks as well.

Analysis: To examine the differential effects of the two grouping schemes ANCOVA on the posttest scores using the pretest scores as coveriate were run for each experimental class; sex was used as a factor in each analysis. There were no significant differences between the heterogeneous and homogeneous groups. For each of the four treatment groups the pretest and posttest scores were compared by t-tests; in each case there were significant gains ($p < .01$). These data and the conclusions drawn from them will be discussed.

Study 3: Effects of manipulative game constraints.

Purpose: This study investigated differences which occurred when game players (a) manipulated physical objects while playing a game, (b) used pictorial representations of physical objects while playing, or (c) used no physical or pictorial aids while playing. The game was Order Out (Romberg et al., 1974, 1975, 1976). The physical manipulatives were a set of fraction bars; each bar is a unit length which is either undivided or variously divided into halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths and twelfths. The pictorial representation used was a fraction bar page from Developing Mathematical Processes (Romberg et al., 1974, 1975, 1976).

Procedure: The subjects for this study were four intact fifth-grade classes in northern Illinois and eight intact seventh-grade classes in south central Wisconsin. Within each class students were randomly assigned to one of three treatments; they were Order Out with fraction bars, Order Out with the fraction bars page, and Order Out. The game was played for fifteen minutes, twice weekly for five weeks during the spring of 1979.

Four parallel forms of a 40-item, investigator-developed instrument were used as pre- and posttests. Each test item presented two common fractions; students were asked to order them. The 40 items were subdivided into two subtests of 20 items each. The two subtests were randomly ordered and randomly paired with a fraction bar page. Two of the resulting tests were randomly assigned to subjects as pretests; the remaining two tests, to subjects as posttests.

The data have not yet been analyzed for this study as data collection has only recently been completed. To examine differences between the pre- and posttest scores t-tests will be used on the data within each treatment and grade. Analysis of covariance of the posttest scores using the pretest scores as covariate will be used to compare the effects of the three treatments; sex will be used as a factor in the analysis. The results and conclusions will be discussed.

References

Adi, H., Karplus, R., and Lawson, A. E. Conditional reasoning and the four card problem: An exploratory study. Journal for Research in Mathematics Education, in press.

Biggs, E. E. and MacLean, J. R. Freedom to learn: An active learning approach to mathematics. Reading, MA: Addison-Wesley, 1969.

Bright, G. W., Harvey, J. G. and Wheeler, M. M. Cognitive effects of games on mathematics learning. Paper presented at the annual meeting of the National Council of Teachers of Mathematics. Cincinnati, Ohio, April 1977.

Bruner, J. S. Process of education. Cambridge, MA: Harvard University Press, 1960.

Dewey, J. Democracy and education. New York: Macmillan, 1928.

Dienes, Z. P. Some reflections on learning mathematics. In W. E. Lamon (Ed.), Learning and the nature of mathematics. Palo Alto, CA: Science Research Associates, 1972.

Invicta Plastics (USA), Ltd. Mastermind-Regular. New York: The author, 1972.

Invicta Plastics (USA), Ltd. Number Mastermind. New York: The author, 1976.

Johnson, D. A. and Rising, G. R. Guidelines for teaching mathematics. Belmont, CA: Wadsworth, 1972.

Romberg, T. A., Harvey, J. G., Moser, J. M. and Montgomery, M. E. Developing mathematical processes. Chicago: Rand McNally and Co., 1974, 1975, 1976.

Sobel, M. A. and Maletsky, E. M. Teaching mathematics: A sourcebook of aids, activities, and strategies. Englewood Cliffs, NJ: Prentice-Hall, 1975.

AN EXPLORATORY STUDY OF THE USE OF PROBLEM SOLVING HEURISTICS IN THE PLAYING OF GAMES INVOLVING MATHEMATICS

William H. Kraus
Wittenberg University

Purpose

In recent years research on problem solving in mathematics has increased; in particular, there have been an increased number of investigations which have examined the heuristic processes used by subjects while engaged in problem solving. Another area of research, in which there has been only limited activity until recently, is the study of the cognitive effects of games on mathematics learning (Brig t, Harvey, and Wheeler, 1977). At this research reporting session a study will be presented that explored one aspect of the relation between these two areas of research; specifically, the study investigated the use of problem solving heuristics in the playing of games involving mathematics.

Procedure

The two games used in this study were complex, multi-row variants of the game NIM. A computer program was written to play the games. This program was designed to make a specified percentage of good moves, to play better near the end of each game, and to adjust its level of play based on the win/loss record of its opponent.

Two groups of subjects participated in the study. One group (N = 10) consisted of experienced problem solvers, graduate students in mathematics and mathematics education. Data from this group was collected in fall 1978. The second group (N = 20) consisted of novice problem solvers, students randomly selected from 210 eighth-grade students at a south central Wisconsin middle school. Data from this group was collected in spring 1979.

Subjects in both groups were instructed to think aloud while playing the games against the computer. The first game was played a total of eight times by each subject, and the second game, four times. The graduate students averaged about an hour of time spent playing the games, and the eighth grade students averaged about a half-hour. The sessions were recorded on audiotape, and a complete record of game positions and moves was printed out by the computer. Because a wide range of ability levels was represented in the eighth-grade sample, scores on a recent standardized mathematics achievement test were obtained for each subject, and a number sequence test and a digit span test were given. The achievement test and number sequence test were chosen because analysis by Dodson (1972) of data from the National Longitudinal Study of Mathematical Abilities indicated that scores on these two tests discriminated well between ability groups on a test of insightful mathematics problem solving. The digit span test, which has been widely used as a measure of information processing capacity, was chosen because of the large information processing load necessary to "solve" the games.

Analysis

Data collection for both parts of the study has been completed. Initial informal analysis of the data indicates that the games elicited a wide range of problem solving behaviors, including many of the Polya (19') heuristics. Data from the verbal protocols and printed game records are being encoded using an adaptation to a game situation of the coding systems used in the problem solving studies of Kilpatrick (1967) and Lucas (1972). Once intercoder reliability for the coding system has been established, the variety and frequency of heuristics used and errors made by the subjects will be analyzed and related to task performance through correlational techniques, including factor analysis and cluster analysis. In addition, a computer simulation program will be written to play the games using heuristics that subjects in the study used. This program would be used to help describe the subjects' use of heuristics and to help determine the sufficiency of the information derived from the protocols.

Results and conclusions of the study will be discussed. Implications for further research in problem solving in mathematics and in the cognitive effects of games on mathematics learning will also be discussed.

References

Bright, G. W., Harvey, J. G. and Wheeler, M. M. Cognitive effects of games on mathematics learning. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Cincinnati, Ohio, April 1977.

Dodson, J. W. Characteristics of successful insightful problem solvers. In J. W. Wilson and E. G. Begle (Eds.), NLSMA Reports (No. 31), School Mathematics Study Group, 1972.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: An exploratory study (Doctoral dissertation, Stanford University, 1967). Dissertation Abstracts International, 1968, 28, 4380-A. (University Microfilms No. 67-6442)

Lucas, J. F. An exploratory study on the diagnostic teaching of heuristic problem solving strategies in calculus (Doctoral dissertation, University of Wisconsin-Madison, 1972). Dissertation Abstracts International, 1972, 32, 6825-A. (University Microfilms No. 72-15, 368)

Polya, G. How to solve it (2nd ed.). Garden City, NY: Doubleday, 1957

Reporting Session II

DISCRIMINATING FACTORS AND SEX DIFFERENCES IN ELECTING MATHEMATICS - Y POPULATION

Teri Hoch Perl
University of Wisconsin-Madison

Purpose

Women elect fewer mathematics courses than men, thus cutting themselves off from many career options. This difference in 'electing' is too large to be explained by sex differences in ability and achievement. The major objectives of the research reported here are to assess the importance of selected variables on student decisions to continue the study of mathematics, and to identify sex differences in these relationships. In addition this study parallels the author's doctoral research on a similar, though distinct population, thus allowing comparison of results across two large nationally sampled groups.

Procedure

The study to be reported here is based on an analysis of data from one of the two secondary populations of the National Longitudinal Study of Mathematical Abilities (NLSMA), the Y-population. The data base includes over 30,000 observations of seventh to eleventh graders collected over a five-year period (1962-1967). (The study analyzed previously, involved data sampled from a population of college bound 10th-12th graders, whereas the sample analyzed here is one selected from a population of seventh graders, both college and non-college bound.) Included in the data are the coding variable 'electing' versus 'non-electing' mathematics which has been cited in the literature as an important cause of sex differences in mathematics achievement, as well as multiple, and for several variables, longitudinal measures of student ability, achievement and attitude (all in relation to mathematics), sex role identification, socio-economic status, and sex of mathematics teachers. Because of the multivariate character of the data, the major analyses used in this study are factor analysis, discriminant analysis, multiple regression and canonical correlation.

Since many of the same tests were administered to both populations and the scales selected for inclusion in the analyses of both studies are essentially the same, comparison of results across studies should illuminate the findings from each. For example, frequency distributions of students within a four-cell design (girl electors, girl non-electors, boy electors, boy non-electors) show sex differences in electing to be significant in both populations. Proportions, however, differ in each. In the college bound population the eleventh grade distribution of girl electors to non-electors is 53 to 47 percent whereas that for boys, electors to non-electors, is 69 to 31 percent. In the 7th-11th grade population, whereas the proportion of boy electors to non-electors has dropped to that of the girls in the 10th-12th grade, and is now 53 to 47 percent, the figures for girls have dropped even more dramatically, girl electors 39 percent to non-electors, 61 percent.

Results

Preliminary inspection of the 7th-11th grade population data suggests that results of this analysis shall support the findings of the earlier study on factors that affect sex differences in electing patterns. Attitude factors, such as perception of usefulness of mathematics, and variables measuring interest in stereotypically masculine careers, were the major factors to yield significant sex differences in discrimination between electors and non-electors. Other variables, such as ability (as measured by verbal and non-verbal IQ tests) and mathematics achievement, though differentiating most significantly between electors and non-electors, showed sex differences too small to explain the magnitude of sex differences in 'electing.'

Reporting Session II

ORIGINS OF SEX DIFFERENCES IN HIGH SCHOOL MATHEMATICS ACHIEVEMENT AND PARTICIPATION

Lauri Steel
Markita Price
and
Lauress Wise
American Institutes for Research

Purpose

A number of previous studies have found sex differences in mathematics achievement occurring by junior high school and increasing during high school. A variety of explanations of possible determinants have been suggested for such sex-related differences. These factors can be categorized as biological-genetic, spatial aptitude, sex-stereotyping or attitudinal (Sherman, 1977). The present study focused on the last two categories. Numerous studies have also investigated sex-stereotyping and attitudinal factors (Fox, 1976; Haven, 1972; Hilton and Berglund, 1974; Fennema, 1976). Factors which have been identified as possible determinants of differential levels of achievement include sex differences in participation in high school mathematics courses, sex differences in career interests and aspirations and sex differences in attitudes toward mathematics. The present study assessed the relative contribution of these and other factors to the observed sex differences in high school mathematics achievement. The specific objectives were to determine 1) the nature and extent of sex differences in mathematics achievement during high school; 2) the extent to which these differences could be explained by sex differences in participation in math courses during high school; and 3) the extent to which, and the ways in which, specific background, cognitive, social-psychological, or environmental factors contribute to sex differences in participation in high school math and to sex differences in math achievement in high school.

Procedure

Data from the Project TALENT data base was used to investigate these research issues. The TALENT data base contains extensive information on the backgrounds, abilities, interests, plans, and activities of a nationally representative sample of 400,000 students in grades 9 through 12 in 1960, as well as data on their subsequent educational, career, and personal experiences over the next 15 years. Data on characteristics of the schools these students attended was also collected in 1960. A subgroup of 7500 9th-grade students were retested in 1963 as 12th graders. This subgroup was used for most of the analyses of the study.

The large amount of data derived from abilities and information tests, interest inventories, and a student information questionnaire were categorized into six groups of factors which theoretically might affect 12th grade math achievement. These were Family Background, School Characteristics, High School Experiences (e.g., math courses taken), 9th Grade Abilities and Achievement (e.g., general academic aptitude, math achievement, math grades), 12th Grade Characteristics (e.g., educational plans, career plans), and other Social-Psychological Characteristics (e.g., educational plans, career plans, interests, personality traits, activities). The following variables were of particular importance to the results of the study.

Math Achievement. The math achievement measure used in this study was a 54-item test including 16 "word problem" items designed to measure arithmetic reasoning, 24 introductory math items primarily covering elementary algebra and number theory, and 14 advanced math items covering topics generally taught in grades 10 through 12 including plane geometry, analytic geometry, trigonometry, and elementary calculus.

Math Participation. The measure used was the number of semesters of high school mathematics reported in the student information questionnaire.

Math Level of Careers. This variable was determined by the amount of math required by a particular career. There were three levels: 1) math-related careers requiring at least some college level mathematics (e.g., mathematician, physicist, biologist), 2) college non-math careers requiring college preparation and possibly statistics, but not calculus (e.g., lawyer, accountant), and 3) non-math-related careers requiring no college level mathematics (e.g., nurse, sales, unskilled worker).

Results

1. Extent of sex differences in high school math achievement.

Both the TALENT sample and the subgroup of 9th graders retested at grade 12 showed striking sex differences in math achievement by 12th grade. There were no significant differences in the 9th grade; however, by the 12th grade there were significant sex differences with the mean scores of the males averaging one-third of a standard deviation higher than the scores of the females. These differences were sharpest after the 10th grade, when, perhaps not coincidentally, high school math courses became truly elective.

2. Extent to which sex differences in high school math achievement are related to differences in participation in high school math courses.

The two strongest predictors of 12th grade math achievement were 9th grade math achievement ($r = .78$) and the amount of math taken in high school ($r = .73$). To assess the extent to which the sex differences in math participation explained differences in 12th grade math achievement, a two-way ANOVA with sex and amount of math taken as the independent factors was performed. The sex differences were virtually nonexistent after controlling for amount of math taken. There is, however, a small interaction effect with women scoring higher at moderate levels of math participation and men scoring higher at the extremes.

The results of a second two-way analysis of variance (sex by math taken) where 9th grade math achievement is controlled as a covariate showed that females with math participation and ability equal to males had scores averaging .1 standard deviation lower than male scores. While this difference is statistically significant, it is not large. Math taken and 9th grade achievement can account for 71.1 percent of the variation in 12th grade achievement. Sex can account for 1.6 percent of the variation in 12th grade achievement when other factors are not controlled, but only .2 percent after controls are introduced. Thus, roughly seven-eighths of the relationship between sex and 12th grade math achievement can be attributed to math taken and initial math achievement.

3. Other factors influencing math participation and achievement.

Several exploratory regression analyses were performed on other background, 9th grade characteristics and school environment variables to identify additional predictors of math participation and achievement. With regard to participation in high school math, expected level of education, intention to take a college-preparatory curriculum (closely related to educational expectations), interest in math itself, and interest in math-related careers versus interest in office work were found to be predictive of math participation above and beyond the influence of 9th grade math achievement. Sex differences in the interest variables account for most of the sex differences in participation.

Regression analysis regarding math achievement indicated that differences in amount of math taken are by far the most significant component of achievement differences. Interest in math and math-related careers relative to interest in office work also plays a very significant role. Participation in extracurricular activities also played a modest role in influencing sex differences in math achievement.

4. Summary model of relationship of sex to high school math achievement.

To determine the process by which sex and other independent variables relate to high school math participation and achievement, structural models incorporating those variables found to have the strongest relationships to participation and achievement were developed and tested using the LISREL IV programs. The model determined that the primary effects of sex on math achievement were through high school math participation. The relationship between sex and math participation is somewhat more complex, with the sex effect being primarily attributable to women's lesser interest in math ($b = -.23$), interest in math-related careers ($b = -.65$), and to the lower math level of women's planned careers ($b = -.24$). The combined indirect effect of sex through the moderator variables was $b = -.13$ which exceeds the direct effect of sex ($b = -.09$).

Conclusions

The goal of this study was to identify the origins of observed sex differences in high school mathematics achievement. At the beginning of high school, boys and girls did not differ significantly in their levels of math achievement, but by 12th grade the boys had gained over twice as much as the girls. The findings from this study reveal that virtually all the sex differences in math achievement can be almost entirely explained by sex differences in participation in high school math.

The number of math courses taken during high school was predicted most strongly by math achievement and educational expectations in 9th grade, both measures on which only minimal sex differences were found. Among students of equal ability and educational aspirations, sex differences in math participation appeared to be due to differences in interest in mathematics and in math-related careers. These sex differences in interests and career plans were already evident by the 9th grade.

The findings of the study indicate that efforts to redress the sex differences in math achievement should focus on the processes by which females (and males) develop career-related interests and expectations during elementary and intermediate school. In particular, interventions aimed at reducing sex differences in math participation and achievement should seek to 1) increase young women's interest in math and math-related careers and in other alternatives to office work; and 2) increase their educational expectations, so that they are more appropriate to their abilities. If women are to develop their math abilities to the same extent as men and to maintain their options to pursue science and other math-related careers, it is imperative that factors resulting in sex-typing of career interests and expectations of males and females be identified and eliminated.

References

Fennema, E. Influences of selected cognitive, affective and educational variables (on sex-related differences) in mathematics learning and studying. Madison, Wisconsin: University of Wisconsin, 1976.

Fox, L. H. The effects of sex role socialization on mathematics participation and achievement. Baltimore: The Johns Hopkins University, 1976.

Haven, Z. W. Factors associated with the selection of advance mathematics courses by girls in high school. Research Bulletin 72-12. Princeton, New Jersey: Educational Testing Service, 1972

Hilton, T. L. and Berglund, G. W. Sex differences in mathematics achievement - A longitudinal study. The Journal of Educational Research, 1974, 67, 231-7.

Sherman, J. Effects of biological factors on sex-related differences in mathematics achievement. Madison, Wisconsin: Women's Research Institute of Wisconsin, Inc., 1977.

Reporting Session II

LONG TERM CONSEQUENCES OF SEX DIFFERENCES IN HIGH SCHOOL MATHEMATICS ACHIEVEMENT

Lauress Wise
American Institutes for Research

Purpose

Sells (1973) pointed out that the most heavily male-dominated professions are those requiring college level mathematics. The present study was designed to explore, in detail, the hypothesis that the development of mathematical skills at the high school level is a necessary prerequisite for the completion of the college curriculum required for entrance into careers in mathematics, engineering, and the physical sciences. The study takes advantage of the wealth of data available in the Project TALENT data base on a national sample of roughly 400,000 students who were first tested and surveyed in high school and then followed up with additional surveys one, five, and eleven years after their high school graduation. The most recent survey, completed in 1976, reached the participants at age 29 by which time nearly all of them had completed their formal education and embarked upon a life occupation.

The purpose of this paper is to report the findings from this NIE-sponsored study on the critical importance of decisions regarding elective mathematics in high school for maintaining a wide range of professional career options. These findings should be of interest and importance to mathematics educators who counsel students regarding mathematics curriculum at the high school level.

Procedure

The analyses reported in this paper were based on a subsample of the original TALENT participants consisting of 12,759 students who were tested as 12th graders and who responded to all three follow-up surveys. Case weights correcting for differential sampling ratios and attrition (Abeles and Wise, in press) were used to obtain nationally representative norms.

Regression analyses and analyses of covariance were used to assess the relationship between the development of mathematics skills in high school and persistence in math-related career plans at different points controlling for a wide range of background and high school characteristics. The primary variables used in this study were:

Main independent variables

- mathematics achievement in 12th grade measured by a 54-item test covering arithmetic reasoning, elementary algebra and number theory, plane geometry, analytic geometry, trigonometry and elementary calculus

- mathematics participation defined as the number of semesters of high school mathematics

Outcome variables

- math-level of college major(s) planned 1 year after high school and completed 5 years after high school defined on a 6-level scale measuring the amount of college mathematics generally required for completion of the major

- math-level of career plans at ages 19 and 23 and actual occupation at age 29 defined on the same 6-point scale

Primary control variables

- socioeconomic status of the participant's family at the time of high school

- educational aspirations in high school

- career interests in high school measured with a 205-item battery producing scores on 17 scales

- actual career plans at time of high school graduation

- sex

Results

The results of these analyses indicated that high school mathematics achievement was, indeed, critical to the development of math-related careers. These careers were highly selective in that only a small percentage of those planning such careers (18 percent of the males and 8 percent of the females) actually persisted in those plans through age 29, and only 3 percent of the males and virtually none of the females who were not already planning such careers in high school were later successful in switching into such careers. Those planning math-related careers had average high school achievement scores at the 75th percentile, but the average scores among the persisters was at the 90th percentile.

High school mathematics achievement was found to be significantly related to persistence in math-related career plans at each stage from high school to age 29, even after controlling for college major and the amount of education completed at each stage. In addition to its direct effect on career plan persistence, high school mathematics achievement had an even stronger indirect effect on persistence through its effect on persistence in and completion of a math-related college major. Mathematics achievement and participation also had an indirect effect on career plan persistence through their effect on overall educational attainment.

Sex differences in entrance into math-related careers was explained primarily by sex differences in high school career plans and also by sex differences in mathematics achievement that emerged during the high school years.

Conclusions

The results of this study indicate the importance of early planning for math-related careers and the necessity of taking elective high school mathematics courses. Because of the need for taking calculus during the first year of college in order to complete math-related majors, those students who fell behind in high school had very little chance of ever catching up. Thus, without even realizing it, many of them closed out for good any chance of pursuing a math-related career when they failed to take elective mathematics during high school.

The conclusions of this study are particularly important for high school girls who generally had less encouragement to pursue elective mathematics than the boys, even though they had just as much potential when they entered high school.

References

Abeles, R. and Wise, L. Coping with attrition in longitudinal studies. Journal of Economic Behavior, in press.

Sells, L. High school mathematics as the critical filter in the job market in developing opportunities for minorities in graduate education. Proceedings of the Conference on Minority Graduate Education at the University of California, Berkeley. May, 1973.

Reporting Session III

LOCUS OF CONTROL AND MATHEMATICS INSTRUCTION: THREE EXPLORATORY STUDIES

Douglas B. McLeod
and
Verna M. Adams
San Diego State University

Purpose

In recent years student personality characteristics, as well as cognitive aptitudes, have begun to play a prominent role in research on learning. The search for interactions between student characteristics (aptitudes) and instructional strategies (treatments), which concentrated originally on cognitive measures, has now broadened its scope to include more research on personality dimensions. This change in emphasis is reflected in recent reviews of aptitude-treatment-interaction (ATI) research (Cronbach and Snow, 1977).

One personality dimension that has recently received some attention in ATI research in Rotter's locus-of-control variable (Rotter, 1966). According to Rotter's theory, individuals with an internal locus of control perceive the outcomes of their actions as being due to their own behavior; those who are classified as external, however, tend to attribute the consequences of their actions to chance or to fate. Recent research on locus of control has been reviewed by Lefcourt (1976).

Most ATI studies using locus of control have focused on the relationship of this personality dimension to the organization of instruction. Daniels and Stevens (1976), for example, found an interaction between locus of control and treatments that differed in whether or not they used student-teacher contracts as a means of organizing instruction. In another study, there was an interaction between locus of control and treatments that differed in whether or not they used student-teacher contracts as a means of organizing instruction. In another study, there was an interaction between locus of control and instruction in computer programming where treatments varied in the level of structure provided to the students (Parent, Forward, Canter, and Mohling, 1975). Robin (1976) discusses several studies where locus of control appeared to interact with "behavioral instruction" (such as Keller's Personalized System of Instruction) as opposed to more traditional lecture-discussion classes. All of these interactions were in the direction predicted by the theory; students with an internal locus of control ~~tended~~ to learn more in the treatment that gave them more responsibility for their own learning, but students with an external locus of control seemed to do better in a more traditional class where the teacher took responsibility for student learning.

The purpose of the three present studies was to extend these earlier results on the organization of instruction to treatments that differed in various dimensions of discovery learning in mathematics. The dimensions included level of guidance of instruction; use of inductive, as opposed to deductive, sequences of instruction; and the use of small groups, rather than individual work. Treatments using a low level of guidance, inductive sequences, or small groups were designed to encourage student discovery. The conjecture was that students with an internal locus of control would do best in treatments that required that they discover concepts independently, and students with an external orientation would learn most in an expository setting.

Procedure

Three ATI studies were conducted to search for interactions between locus of control and three different dimensions of discovery learning. In Experiment 1, students in five classes were randomly assigned to either a low-guidance or high-guidance treatment using the topic of networks, including equivalence of networks, traversability and its applications, and Euler's formula. In the high-guidance treatment, students had to complete tables and draw some fairly obvious conclusions. In the low-guidance treatment, students were expected to gather relevant data, organize it, and test hypotheses about networks. A total of 150 minutes (one week) of class time was used for the treatments and a 15-minute posttest.

In Experiment 2, students in three classes were randomly assigned to either an inductive treatment or a deductive treatment using the topics of precision of measurement, significant digits, and their effects on calculations with approximate data. In the inductive treatment, students worked examples (using calculators) and then were encouraged to generate a rule suggested by the examples. In the deductive treatment, students were given the rules, and then asked to apply the rules to simple problems constructed so they did not need calculators. Students were given 75 minutes to work on the materials and took a posttest two days later.

In Experiment 3, students in five classes were randomly assigned to either small groups or individual work. These two treatment groups used exactly the same printed materials on the topic of networks. The printed materials were written in an inductive mode, where students were encouraged to generalize a rule from a number of examples. In small-group instruction, students were assigned to work in groups of four and were told to ask the teacher for help if the group could not solve the problems. In the individual treatment, students were told to work by themselves and to direct their questions to the teacher. One week of class time was used for the treatments and a 15-minute posttest.

Students who participated in the three studies were enrolled in mathematics classes for prospective elementary school teachers. In each experiment, students were assessed on a measure of locus of control. After instruction, students were tested for immediate achievement and then for retention several weeks later. SAT scores were used as a measure of general ability.

Locus of control was assessed using the Mathematics Achievement Questionnaire (MAQ), an instrument based on earlier work by Crandall, Katkovsky, and Crandall (1965). The MAQ was developed to assess locus of control in the specific environment of the mathematics classroom as suggested by Lefcourt (1976) and Rotter (1975).

Results

In each case, data were analyzed using multiple regression techniques. In Experiments 1 and 2, there were no significant interactions with either MAQ or SAT scores. There were, however, significant treatment effects in favor of the high-guidance treatment in Experiment 1 on both the posttest $F(1,25) = 13.0$, $p < .002$, and the retention test, $F(1,25) = 13.2$, p .002, and in favor of the deductive group in Experiment 2 on the posttest, $F(1,27) = 5.47$, $p = < .027$.

In Experiment 3, there was a significant disordinal interaction in the predicted direction between locus of control and treatment on the retention test, $F(1,49) = 4.73$, $p = .034$. Students with an internal locus of control were better off in small-group instruction, and students with an external locus of control learned more in individual instruction where they received help from the teacher. The interaction between general ability and treatment was not significant, but the joint contribution of the two interaction vectors was significant, $F(2,48) = 3.20$, $p = .050$.

References

Crandall, V. C., Katkovsky, W., and Crandall, V. J. Children's beliefs in their own control of reinforcements in intellectual-academic achievement situations. Child Development, 1965, 36, 91-109.

Cronbach, L. J. and Snow, R. E. Aptitudes and instructional methods. New York: Irvington, 1977.

Daniels, R. L. and Stevens, J. P. The interaction between the internal-external locus of control and two methods of college instruction. American Educational Research Journal, 1976, 13, 103-113.

Lefcourt, H. M. Locus of control: Current trends in theory and research. Hillsdale, New Jersey: Erlbaum, 1976.

Parent, J., Forward, J., Canter, R. and Mohling, J. Interactive effects of teaching strategy and personal locus of control on student performance and satisfaction. Journal of Educational Psychology, 1975, 67, 764-769.

Robin, A. L. Behavioral instruction in the college classroom. Review of Educational Research, 1976, 46, 313-354.

Rotter, J. B. Generalized expectancies for internal versus external control of reinforcement. Psychological Monographs, 1966, 80 (1, Whole No. 609).

Rotter, J. B. Some problems and misconceptions related to the construct of internal versus external control of reinforcement. Journal of Consulting and Clinical Psychology, 1975, 43, 56-67.

INTERACTION OF GENERAL REASONING ABILITY AND GESTALT AND ANALYTIC STRATEGIES OF PROCESSING SPATIAL TASKS WITH TRANSFORMATIONAL AND NON-TRANSFORMATIONAL TREATMENTS IN SECONDARY SCHOOL GEOMETRY

V. J. DuRapau
Xavier University of Louisiana

Purpose

The purpose of this study was to investigate the relationship between selected cognitive abilities and two instructional treatments of topics from secondary school geometry. More specifically, it was an experiment to test for interaction between instructional treatments in point and line symmetry in Euclidean two-space using a verbal-gestalt-figural approach and a verbal-analytic-figural approach and two aptitude variables, cognition of figural transformations and cognition of figural relations, in their effect on mathematics achievement. A secondary purpose was to investigate the interaction between general reasoning ability and the treatment variables in their effect on achievement in mathematics.

Studies by Carry (1968), Webb (1971), Eastman (1972), and Salhab (1973) provided evidence of the existence of aptitude-treatment interaction (ATI) using aptitude variables of spatial visualization and general reasoning and treatment variables characteristic of graphical and analytical approaches. The exact nature of the interaction, however, remained unclear.

Guilford's (1967) Structure-of-the-Intellect (SI) model and Guay, McDaniel, and Angelo's (1978) study of 31 widely-used spatial tests suggested that treatments could be designed which facilitate the use of the preferred strategy of subjects for processing spatial tasks. Guilford's content dimension in the SI model suggested that content of the treatment would be an important factor in any ATI found.

Cattell's (1971) hierarchical model of human intelligence suggested that the part played by more general abilities, such as general reasoning, in any ATI might be significant. The researcher's interpretation of Wittrock's (1974) theory of generative processing, together with the position taken by Cronbach and Snow (1977), suggested that a treatment that left much of the burden of organization and interpretation to the individual would show a steeper regression slope than a treatment that gave explicit rules and procedures. The consistent role of the Necessary Arithmetic Operations test in the studies of Carry (1968), Webb (1971), Salhab (1973), and McLeod and Briggs (1977) supported this conjecture.

Procedure

An experiment designed to clarify the nature of the relationship between certain aptitudes and treatments in their effect on achievement in mathematics was undertaken in January, 1979. General reasoning ability and gestalt and analytic strategies for processing spatial tasks were used as aptitudes. Two differing linearly programmed instructional treatments in point and line symmetry in the plane were designed for use in the experiment. The transformational (T) treatment utilized the isometries of the plane and the non-transformational (N) treatment utilized the concepts of distance and perpendicularity in a static sense to teach the concept of symmetry. The N treatment tended to be more explicit and rule-oriented than the T treatment.

The experiment was conducted during a 5-day period. One hundred thirty-two (132) secondary school geometry students were administered a battery of three aptitude tests on the first two days. The students then were randomly assigned to one of the two treatments for two days. On the fifth day a 34-item achievement test was administered to measure learning and transfer. Complete data were obtained on 111 students.

Results

Multiple linear regression techniques were used to analyze the data. The T and N treatments were not substantially different in producing learning, but the T treatment was a significantly ($p < .01$) better facilitator of transfer. Three significant ($p < .02$) ATI were found using the transfer subtest of the achievement test as the criterion measure. The interactions involving general reasoning and the treatment variables and gestalt processing of spatial tasks and the treatment variables were in the direction predicted by the theory. The interaction between analytic processing of spatial tasks and the treatment variables was in a direction opposite that predicted. (The construct validity of the measure of analytic processing is suspect.)

Conclusions

Both a transformational and a non-transformational treatment are suitable approaches for teaching the concept of symmetry. A high school geometry teacher who has specific knowledge about a student's spatial visualization and general reasoning abilities can use this information in teaching the student the concept of symmetry. An understanding by the teacher of the nature of the interactions is of more practical value at this writing than any attempt to use for placement purposes the ATI found in this investigation.

On a more fundamental level, the synthesis of Wittrock's (1974) theory of generative processing and Cronbach and Snow's (1977) thorough review of ATI literature sheds new light on the nature of ATI. Treatments can be designed which effectively teach the same content but which differ in extent of elaboration. A treatment is fully elaborated if it gives explicit, detailed information in a rule-oriented fashion. Such a treatment does much of the organization and structuring of information for the learner. A second treatment may leave much of the organization and structuring to the learner. The former treatment (fully elaborated) has relatively little reliance on general reasoning ability; the latter, much dependence on this ability. Wittrock (1974) said, "Learning with understanding is a generative process" [p. 190]. Students with high general reasoning scores profit more from a treatment in which the organization and structuring essential to generative processing is left to the individual. Students with low general reasoning scores profit more from a fully elaborated treatment, one that explicitly provides the organizational structure that relates new information to previous experience.

References

Carry, L. R. Interaction of visualization and general reasoning abilities with instructional treatment in algebra. Unpublished doctoral dissertation, Stanford University, 1968.

Cattell, R. B. Abilities: Their structure, growth, and action. Boston: Houghton Mifflin, 1971.

Cronbach, L. J. and Snow, R. E. Aptitudes and instructional methods. New York: Irvington Publishers, 1977.

Eastman, P. M. The interaction of spatial visualization and general reasoning abilities with instructional treatment in quadratic inequalities: A follow-up study. Unpublished doctoral dissertation, The University of Texas at Austin, 1972.

Guay, R. B., McDaniel, E. and Angelo, S. Analytic factor confounding spatial ability measurement. Paper proposal for the Annual Convention of the American Psychological Association, Toronto, 1978.

Guilford, J. P. The nature of human intelligence. New York: McGraw-Hill, 1967.

McLeod, D. B. and Briggs, J. T. The interaction of field independence and general reasoning ability with inductive instruction in mathematics. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, Cincinnati, 1977.

Salhab, M. T. The interaction between selected cognitive abilities and instructional treatments on absolute value equations. Unpublished doctoral dissertation, The University of Texas at Austin, 1973.

Webb, L. F. Interaction effects between selected cognitive abilities and instructional treatment in algebra. Unpublished doctoral dissertation, The University of Texas at Austin, 1971.

Wittrock, M. C. A generative model of mathematics learning. Journal for Research in Mathematics Education, 1974, 5, 181-196.

STUDENT PERFORMANCES, INDIVIDUAL DIFFERENCES, AND MODES OF REPRESENTATION

Helen Adi
and
Marilyn Behr
Northern Illinois University

Purpose

Student performances on mathematical tasks in manipulative, pictorial, and symbolic representational modes are of interest to mathematics educators. Theoretical models of instruction (Lesh, 1979), and of memory strategies and learning outcomes (Gagné and White, 1978) suggest that the "translation" or the interplay between the different representational modes enhances meaningful mathematical learning. Observations of student performances on mathematical tasks suggest that such an interplay is extremely difficult; students seem to proceed entirely in one mode or the other, and then, if requested they solve the problem in the other mode (Behr, 1975). In other words, the alternation between the different representational modes is difficult to many students.

While it is observed that many students find it difficult to solve problems requiring an interplay between representational modes (mixed mode tasks), it is also true that there exists considerable individual variability in performance on such tasks. Several sources for this variability are of interest.

The present investigation analyzed the performances of preservice elementary school teachers to study whether the individual difference variables of field dependence/independence, and spatial visualization, along with the performance outcomes on tasks in only the pictorial mode and tasks in only the symbolic mode, would account for the variability in performance on tasks requiring an interplay between the pictorial and the symbolic representational modes.

The theory associated with the cognitive style of field dependence/independence provides a measure of individual differences which may account for some of the variability in performance on tasks requiring an interplay between representational modes. Field independence refers to a preference to experience the environment in analytic terms, while field dependence refers to a preference to experience the environment in a global undifferentiated fashion. Thus, if a task requires an interplay between the pictorial and the symbolic modes for problem solution, the field independent student is expected to dissociate the component aspects of the two representations more readily than the field dependent student. Further theoretical analyses suggest that given a problem requiring an interplay between two solution strategies, the field dependent student will approach the problem with the more familiar strategy first, and then, if requested, with the other strategy; whereas, the field independent students will drop such an approach to construct a rather continuous interplay between the two solution strategies (Pascual-Leone, 1969; Witkin and Goodenough, 1977).

Investigations on brain hemispheric specialization provide another individual difference variable which may account for some of the variability in performance on tasks requiring an interplay between the pictorial and the symbolic modes. The left hemisphere is the predominant processing component of symbolic activity, and the right hemisphere is the predominant processing component for spatial relationships (Cohen, 1972; Galin and Ornstein, 1972, 1974). Individual differences were identified in hemispheric brain dominance (Brown and Jaffee, 1975; Galin and Ornstein, 1974; Tomlinson-Keasey, Kelly and Brown, 1978). Spatial visualization ability was one of the identified factors that correlated highly with right hemispheric dominance. Thus, given a task requiring an interplay between the symbolic and the pictorial modes, normal subjects would alternate the processing of information between the left and the right hemispheres (Galin and Ornstein, 1972). Unless the subjects have about equally dominant capabilities in both hemispheres, both time and information could be lost during the transference of information across the corpus callosum from one hemisphere to the other (Cohen, 1972).

Gagné's (1970) learning theory further suggests that in order for a student to solve a problem requiring an interplay between the symbolic and the pictorial modes, prerequisite capabilities of solving the problem in its entirety in the symbolic, and in the pictorial mode are necessary.

Procedure

Two intact classes of 94 preservice elementary school teachers enrolled in a methods course participated in the study. Paper and pencil tests were administered to all students to measure field dependence/independence, and spatial visualization abilities. A one-week instructional treatment (the same for all subjects) on whole number addition algorithms based on the popsicle stick manipulative aid followed. For example, problems like $12 + 13 = \square$, and $14 + 18 = \square$ were taught. Manipulative experiences, pictorial representations, and their corresponding symbolizations were emphasized during instruction. Immediately following instruction a learning test was administered. Three weeks later the same test was administered to measure retention. Only retention test scores were later considered for analysis.

The Gottschaldt Hidden Figures Test (Crutchfield, 1975) was administered to measure field dependence/independence (Cronbach alpha = 0.87). The Purdue Spatial Visualization Test was administered to measure spatial visualization abilities (Cronbach alpha = 0.87). This test was reported to correlate highly with right hemispheric processing of information.

The retention test consisted of three parts. In part one, students were asked to solve six addition algorithms, using popsicle sticks, in the pictorial mode only; in part two, the same problems were presented in the symbolic mode; and in part three, the same problems were also used but an interplay between pictorial and symbolic representations was required for problem solutions. All three parts were paper-and-pencil, multiple choice, group-administered tests. Administration time for all three parts was one hour. The Cronbach alpha measures for the three parts were 0.56, 0.77 and 0.40, and for the total retention test it was 0.87.

Results

The data have been collected and partially analyzed. A complete report of the results will be available at the Seattle meeting.

Performance on the mixed mode retention test (max = 6.00, mean = 4.89, SD = 1.20) was the dependent measure, and performances on

(1) the pictorial mode retention test (max = 6.00, mean = 3.79, SD = 1.62),

(2) the symbolic mode retention test (max = 5.12, mean = 4.89, SD = 1.42),

(3) the Hidden Figures Test (max = 20.00, mean = 7.63, SD = 4.87), and

(4) the Purdue Spatial Visualization Test (max = 80, mean = 52.61, SD = 10.22),

were the independent measures in a stepwise regression analysis. Table 1 presents the correlation coefficients between the variable measures. The predictor variables were entered into the regression equation according to the magnitudes of their partial correlations with the dependent measure. Symbolic performance was the best predictor of mixed mode performance ($\underline{F}$ = 82.91). The next best predictor was pictorial performance ($\underline{F}$ = 2.21). Spatial visualization entered third into the regression equation, however, it accounted for only 0.23 percent of the variance of the mixed mode performance. Field dependence/independence was not a significant predictor in the regression equation.

Students of extreme individual difference measures (high and low) will be identified and their performances on all three retention subtests will be carefully analyzed and compared by ANOVA.

Table 1

Multiple Correlation Matrix

Variables	1.	2.	3.	4.	5.
1. Mixed mode	1.00	0.66	0.84	0.31	0.31
2. Pictorial mode	—	1.00	0.71	0.25	0.36
3. Symbolic mode	—	—	1.00	0.31	0.30
4. Spatial visualization	—	—	—	1.00	0.51
5. Field dependence/ independence	—	—	—	—	1.00

Reporting Session IV

THE DEVELOPMENT OF PROBLEM SOLVING CAPABILITIES IN PRIMARY GRADE CHILDREN

C. Mauritz Lindvall
and
Cheryl Gibbons Ibarra
University of Pittsburgh

Purpose

This study investigated a simple theoretical structure which attempts to describe the problem solving strategies employed by first grade students in solving simple arithmetic story problems. Specifically, the study examined the relationships between (1) the ability to generate a model for a story and the ability to solve the story, (2) the ability to model a story and the ability to write a number sentence for the story, and (3) the ability to solve a story problem and the ability to write a number sentence for it.

Studies on the processes that children use in solving story problems suggest that the more successful students analyze a story in terms of what is given and what is to be found and then build some general representation or "model," either mental or physical, to represent the essential elements and manipulations described in the story (Wilson, 1967). That is, they approach these stories as a true problem solving activity. Heller and Greeno (1978) have presented a strong case for the point that even apparently simple arithmetic problems present most of the characteristics of a real problem situation. A structure for describing what children do in solving arithmetic story problems, derived from general problem solving theory (Newell and Simon, 1972; Simon and Simon, 1978; Larkin, 1977), may be outlined in the following three stages.

Stage 1. The substantive problem is identified and comprehended.

Stage 2. The essentials of the problem are abstracted and the problem is represented in the form of some simplified general conceptual model.

Stage 3. The general model is reformulated as a mathematical model that can be used in the problem solution.

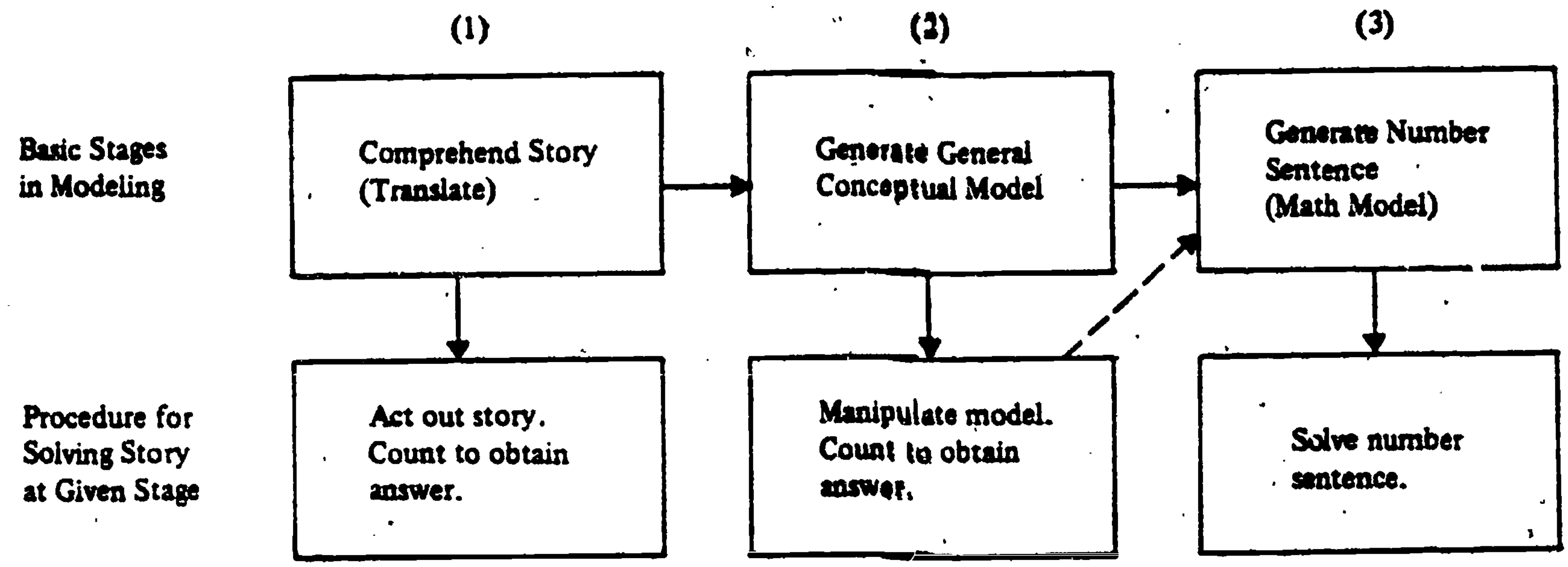

Figure 1. Suggested Stages Used by Students in Developing Appropriate Number Sentence (or Mathematical Model) for a Story Problem Showing the Problem Solving Procedure that could be Used at Each Stage.

The diagram in Figure 1 is intended to show the relationship of these three stages and also to indicate that with simple arithmetic story problems children may solve the story by using procedures appropriate to any one of the three stages of understanding or modeling. Since previous studies by the writers (Ibarra and Lindvall, 1979) had shown that kindergarten pupils who were unable to solve a story problem under other conditions were able to solve it when they were permitted to act it out or had it acted out for them (i.e., were able to solve it using Stage 1 procedures), the present study focused on Stages 2 and 3.

Procedure

Approximately twenty first and second grade pupils from a school in each of four different types of communities were interviewed to obtain data on pupil capabilities. Each pupil was tested on six different arithmetic story problems, each of which could be represented by an addition or subtraction sentence with a specific term missing. With each problem the story was read to the student and he or she was asked: (1) to give the answer, (2) to write a number sentence that could be used to represent the story and to find the answer, and (3) to use counting cubes to explain the story. For each of these three tasks pupil performance was judged as "pass" or "fail."

Analysis

To investigate the relationships among the three measures used in this study the pass-fail results for each possible pairing of the three variables were examined through the use of simple two-by-two contingency tables.

Table 1. Contingency Tables Showing Relationships Among Ability to Build a General Conceptual Model of a Story, to Write an Appropriate Number Sentence, and to Solve the Story (for one type of story; $a - b = \square$)

(a.)

Build Model	Solve Story F	Solve Story P
P	2	50
F	24	5

$r_{phi} = .81$

(b.)

Write Number Sentence	Build Model F	Build Model P
P	0	46
F	29	6

$r_{phi} = .86$

(c.)

Write Number Sentence	Solve Story F	Solve Story P
P	2	44
F	24	11

$r_{phi} = .68$

Results

Table 1 presents data concerning the three relationships that were of interest in this study. While this table presents data on only one of the six types of stories used in this study, the results are somewhat representative of all story types. The data (Table 1(a)) indicate that there is a strong positive relationship ($r_{phi} = .81$) between the ability to solve the story problem and the ability to build a model for the problem.

Since the data in Table 1(b) show that no persons were able to write a correct number sentence if they were unable to build a model of the story, this supports the prerequisite relationship proposed in Figure 1. The data in Table 1(c) indicate that at this early stage in their introduction to simple story problems and the associated number sentences, children probably solve the story and then write the sentence rather than use the sentence as a means of solving the story. This supports an earlier finding of Steffe, Spikes, and Hirstein (1976). Note that the data also show that it is not necessary to be able to write a number sentence to be able to solve the story.

Conclusions

The results of the study indicate that the students who are successful in solving arithmetic stories of the type used in this study are able to generate some type of general conceptual model. This is taken to mean that successful problem solvers abstract the general and essential elements of a story and then proceed to organize and manipulate these elements, either physically or mentally. The study also supports the idea that pupils who can write a number sentence for a story are also able to develop a general conceptual model and to solve the story using such a model. That is, the students appear to be able to write the number sentence for a story only when they are already able to solve it by using a general conceptual model. The number sentence, then, is only a way of modeling a solution process that has already been completed.

In general, the results of this study imply that teaching pupils how to solve arithmetic story problems should always involve the intermediate step of making certain that the pupils can translate the story into some type of meaningful general conceptual model before they attempt to write a number sentence for it. Placing an emphasis on this intermediate step should mean that students would not only become more proficient in solving these arithmetic story problems but should also increase the likelihood that they would acquire a general problem solving strategy that appears to be characteristic of effective adult problem solvers.

Reporting Session IV

TWO STUDIES ON STORY PROBLEMS: DRAWING VS VERBAL FORMATS A PROPOSAL FOR A REPORTING SESSION, NCTM ANNUAL MEETING, 1980

Judith A. Threadgill
University of Calgary
and
Larry Sowder
Northern Illinois University

Purpose

These studies explored, for "story" problems presented with only words and numerals (verbal) or with drawings and fewer words (drawing).

(a) whether there are differences in achievement (number correct, use of correct processes) on the two formats,

(b) how selected measures--e.g., field dependence/independence, spatial visualization, general reasoning--are related to achievement on the two formats, and

(c) whether aptitude-format relations are similar at different age levels.

Paivio has contended that "the most general assumption is that verbal and nonverbal information are represented and processed (internally) in distinct but interconnected symbolic systems" (1974, p. 8). The two formats of problems used in the studies could lead to different representing and processing by students differing in levels of cognitive factors or in age.

Presentations by diagrams or pictures on tests have often seemed to give better performances than words alone (Neil, 1969; Portis, 1973), except when a picture accompanied a verbal statement (O'Flaherty, 1971). With older students in group-testing situations, an accurate drawing or diagram accompanying a verbal problem gave better performance than the verbal presentation alone (Sherrill, 1973; Webb and Sherrill, 1974). In one study, presentations involving drawings did not give the best performance (Kulm, et al., 1974); this study did, however, use an unusual presentation mode.

The above studies did not involve instruction. Drawings for story problems as part of an instructional treatment seem not to have been used as a variable, although diagrams have, with mixed results (Nelson, 1975; Nickel, 1971; Shoecraft, 1972). (Note that important within-format variations, such as those studied by Jerman and his students and by Campbell, are not examined here.)

The field-independent student theoretically is better able to isolate parts of a context than a field-dependent student is. The additional salience of the essential information allowed by a drawing of a story problem could negate any handicap a field-dependent student might have with the problem in word form.

As suggested earlier, students with differing abilities at visual imagery might process words-only story problems with different degrees of effectiveness and consequently solve problems at different levels But how does one measure visual imagery ability? Cronbach and Snow note the uncertain relevance of common aptitude measures to learning from drawings (1977, p. 273). Our studies utilized "strong" measures —i.e., measures involving visual imagery as well as the mental manipulation of a figure. (Study 2 also uses a self-report measure of visual imagery.)

In Study 1, NLSMA's Arithmetic Reasoning Test was used for two reasons: it loads on a general reasoning factor, and its format (words only) reflected the verbal format of the problems in Study 1.

The purposes described above motivated both Study 1 and Study 2. The studies are not replications at different age levels, however. Different versions of aptitude measures and different problems were used since the subjects in the two studies were of different ages.

Procedures

Study 1: Fifth Graders

The 234 students participating in all phases of Study 1 were from ten fifth grade classes located in a large public school system. Classes were selected on the basis of teacher interest in the study and similarity in students' mathematical backgrounds.

The Hidden Figures Test (a measure of field independence/dependence), Punched Holes Test (a measure of spatial visualization), and Arithmetic Reasoning Test (a measure of general reasoning) were selected from the NLSMA X-population test battery and were administered to the students. At the time of this testing, the teachers were given multiple copies of 32 problems, arranged in four problem sets, to be worked by their students during the following month. The problems were similar to those found in most fifth grade mathematics texts, and included whole number operations and money problems. Five of the teachers, selected at random, were given these problems in verbal format, while the other five teachers were given the same problems in drawing format. Thus, approximately half of the students had experience with the drawing format prior to posttesting, while the other half did not.

Each student received a 16-problem posttest a month later. The test had two versions, which were randomly distributed to the students within each classroom. On one of the versions, eight problems in verbal format alternated with eight problems in drawing format. On the other version, each of the verbal problems from the first version was presented via a drawing, while each of the drawn problems from the first version was presented in a verbal format. It is therefore possible to obtain information on each problem in both formats. Scoring of the problems in each format and for each version will be recorded separately, with the scores reflecting both procedures and number correct.

Study 2: Mathematically Naive College Freshmen

The subjects were 67 students in a college mathematics class for mathematically unsophisticated, but willing, learners. A Hidden Figures Test adapted from NLSMA gave a measure of field dependence/independence; Punched Holes (from the French Kit) gave a measure of spatial visualization (and perhaps visual imagery); and a self-report test (Sheehan, 1967) gave a measure of visual imagery.

All these were administered well before a test made up of six story problems was given. Three contexts were used: two $d = rt$ situations; two simple interest contexts; two situations in which the mean of three numbers and two of the numbers were given. Each student received one problem from each context in each format (verbal/drawing). Across the whole class, each context was represented in each format, and two orders of problems were used, to provide some control for context and order-within-test. The four different forms were passed out randomly to the students. The problems were scored right/wrong, as well as for correctness of procedure.

One week later, after instruction on the principles and after homework problems in both formats, the students were given a quiz made up of similar problems (in a fixed order). These were also scored on both right/wrong and correct-procedures bases.

Analysis

The data analyses for the two studies are just beginning. Expectations are that students scoring low on the aptitude measures will profit more from the drawing presentations of problems, whereas students scoring high will be less affected by mode of presentation. Younger students are expected to find the drawing versions easier than the verbal versions, especially if they have received prior exposure to the drawing format.

SPATIAL VISUALIZATION SKILL AND PROCESSES USED IN SOLVING MATHEMATICAL PROBLEMS

Mary Ann Konsin
Carthage College

Purpose

Mathematical problem solving has been an area of concern for mathematics educators and researchers in the 1970s. Documentation of this concern can be found in the Journal for Research in Mathematics Education (1973, 1976, 1978) and Suydam's (1974) Categorized Listing of Research on Mathematics Education K-12): 1964-1973. One of the cognitive variables most frequently identified with mathematical problem solving is the spatial or visual factor. Spatial representations are found at various levels of teaching mathematics, from the Piagetian conservation tasks of preschool programs to the determination of volumes of solids of revolution in the calculus class. Elementary teachers use number lines when working with representations of whole numbers; high school teachers investigate conic sections by slicing a cone with a plane; college teachers sketch quadric surfaces in three dimensions. Each of these use spatial visualization skill in that the student is required to "mentally" manipulate a given object.

Very little information has been presented in terms of how spatial visualization is used in solving mathematical problems. Investigations have focused on identifying high achievers in mathematics and then describing their spatial skills. Guay and McDaniel (1977) found that among elementary school children high mathematics achievers have high spatial ability (i.e., three-dimensional) and low mathematics achievers have low spatial ability (i.e., two-dimensional). Schonberger (1976) found a positive relationship between using a diagram to solve a problem and the subject's spatial ability. Moses (1977) found that subjects with high spatial ability do not necessarily use visual solution processes while solving a mathematical problem. Is the representation of the relationships in a mathematical problem in terms of "visual images" on a paper related to spatial visualization? Is the process variable "draws a figure" identified in the work of Kilpatrick (1967), Zalewski (1974), and Webb (1975) representing the spatial visualization skill of the subject who uses it while solving a mathematical problem? Werdelin (1961) very early concluded that if one could attack a problem either verbally or visually, one would be more apt to solve it. If in some way spatial visualization skill can aid in solving mathematical problems it seems important to try to identify the connecting link between that skill and the procedures a subject uses to solve mathematical problems.

The investigation presented here follows the chain of inquiry presented by the work of Kilpatrick (1967), Kantowski (1974), Zalewski (1974), Webb (1975), and Krutetskii (1976). The specific purpose was to investigate the relationship between spatial visualization skill and processes used by subjects in solving mathematical problems. The following questions were the foci of the investigation:

1. Can subjects identified with diverse levels of spatial visualization skill be identified by the procedures they use to solve mathematical problems?

2. Do types of problem solving sub-sequences and methods of solutions distinguish between subjects with diverse levels of spatial visualization skill?

3. Can subjects identified with diverse levels of spatial visualization and verbal skills be identified by the procedures they use to solve mathematical p· blems?

4. Do types of problem solving sub-sequences and methods of solutions distinguish between subjects with diverse levels of spatial visualization and verbal skills?

Procedure

From a population of 242 middle school students, sixty-one seventh and eighth grade students were assigned to two different samples based on their scores on Space Relations of the Differential Aptitude Test (Bennett, Seashore and Wesman, 1973) and Vocabulary Test of the Cognitive Abilities Test: Verbal Battery (Thorndike and Hagen, 1975). The first sample was composed of thirty-two subjects identified relative to their spatial skill; the second sample consisted of twenty-nine subjects categorized with respect to their spatial/verbal skills. Each subject was asked to solve ten problems on the Problem Solving Test using the thinking aloud technique and any procedures they wished. The problems on the Problem Solving Test had arithmetic and geometry as their content and had been selected from Krutetskii (1976) and Schonberger (1976). One of the major criteria for selection of the problems was that they could be solved using "visual images" as well as analytical procedures. The investigator conducted individual audio-taped interviews of the sixty-one subjects over a ...-week period of time. The subjects were interviewed away from the school setting and were paid for their participation in the study.

Twenty-two variables were selected a priori by the investigator as being important for solving the problems on the Problem Solving Test. The variables were placed in four categories; processes, process outcomes, error and comments, and types of sub-sequences. The investigator coded the transcribed interviews using an adaptation of Kilpatrick's (1967) coding form.

Analysis

T-tests were used to test the differences between the frequency means with respect to processes, process outcomes, problem solving subsequences, and methods of solution for the various groups. A series of Cluster Analyses of Cases were performed in order to identify subjects who used similar procedures when solving the problems on the Problem Solving Test. The groups of interest to the investigator were those where differences on Space Relations were statistically significant.

Results

Although the analyses have not been completed at this time some preliminary results have been observed. Comparing frequencies of use of the variables identified for investigation have not distinguished between subjects with diverse spatial skills. When cluster analyses are performed using specific combinations of processes groups of subjects have been identified which differ significantly on their Space Relations scores. These groups in turn can be described by their use of combinations of processes when solving the problems on the Problem Solving Test. Similar results have been obtained when specific combinations of processes and process outcomes are used in the cluster analyses. The analyses will be completed and the results will be given at the reporting session.

References

Bennett, G. K., Seashore H. G. and Wesman, A. G. Differential aptitude tests: Administrator's handbook. New York: The Psychological Corporation, 197[illegible].

Guay, R. and McDaniel, E. D. The relationship between mathematics achievement and spatial abilities among elementary school children. Journal for Research in Mathematics Education, 1977, 8, 211-215.

Kantowski, M. G. Processes involved in mathematical problem solving. Unpublished doctoral dissertation. University of Georgia, 1974.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: An exploratory study. Unpublished doctoral dissertation. Stanford University, 1967.

Krutetskii, V. A. The psychology of mathematical abilities in schoolchildren. (J. Kilpatrick and I. Wirszup, Eds. and J. Teller, Trans.) Chicago: University of Chicago Press, 1976.

Moses, B. The nature of spatial ability and its relationship to mathematical problem solving. Unpublished doctoral dissertation. Indiana University, 1977.

Schonberger, A. K. The interrelationship of sex, visual spatial abilities and mathematical problem solving in grade 7. Unpublished doctoral dissertation. University of Wisconsin, 1976.

Suydam, M. N. A categorized Listing of Research on Mathematics Education (K-12) 1964-1973. Columbus: ERIC Information Analysis Center for Science, Mathematics and Environmental Education, 1974.

Suydam, M. N. and Weaver, J. F. Research on Mathematics Education (K-12) Reported in 1972. Journal for Research in Mathematics Education, 1973, 4, 205-242.

Suydam, M. N. and Weaver, J. F. Research on Mathematics Education (K-12) Reported in 1975. Journal for Research in Mathematics Education, 1976, 4, 193-257.

Suydam, M. N. and Weaver, J. F. Research on Mathematics Education Reported in 1977. Journal for Research in Mathematics Education, 1978, 4, 244-310.

Thorndike, R. L. and Hagen, E. Cognitive Abilities Test, Form 1 (Levels A-H). Boston: Houghton Mifflin, 1975.

Webb, N. L. An exploration of mathematical problem-solving processes. Unpublished doctoral dissertation. Stanford University, 1975.

Werdelin, I. Geometrical ability and the space factor in boys and girls. Lund, Sweden: C. W. K. Gleerup, 1961.

Zalewski, D. L. An exploratory study to compare two performance measures: An interview-coding scheme of mathematical problem solving and a written test. Unpublished doctoral dissertation. University of Wisconsin, 1974.

Reporting Session V

FACTORS OF ORGANIZATION AND CLARITY IN MATHEMATICS LESSONS

Nira Hativa
and
Richard E. Snow
Stanford University

Purpose

Good organization of oral messages has been found to improve listeners' understanding and retention by researchers in education, psychology and communication. Lesson organization has been found to have special importance in teaching mathematics and sciences. This study attempted to investigate factors in mathematics teachers' messages that make a lesson organized and clear, to measure a lesson's organization and a teacher's ability to organize knowledge and decode it into a clear lecture, to determine which of teachers' strategies of organization are the most discriminating between teachers rated high or low on lesson organization, and to explore relationships between a teacher's motivation, experience and general scholastic ability and lesson organization ability.

Organization, a major dimension of teacher clarity of presentation is viewed in the present study as a network of non-content communication messages provided by the teacher in order to increase message intelligibility and retainability. 'Non-content' means that these messages do not contain new content or material, but rather that they highlight aspects of the lesson and give it structure.

Some strategies of lesson organization are:

1. Three-division lesson structuring: introduction, body, conclusion.

2. Topic sequencing: from familiar to unfamiliar, from easy to more difficult, from concrete to abstract, in a smooth flow, without skipping intermediate steps or taking-off to irrelevant materials.

3. Using transitions appropriately for smooth continuity and for clarification of the relationships between and relative emphasis of the statements and topics.

4. Framing: use of devices for efficient search in memory to identify pre-existing materials in a learner cognitive structure which are relevant to the new material being studied, and of devices for organizing the newly learned material in a structure that can be efficiently stored in

memory to facilitate its retrieval. Some of the framing strategies are:

(a) Previewing in the introduction: reviews, overviews, advance organizers, behavioral objectives.

(b) Explicit outlining: stating the outline in the introduction, referring to each main point while presenting the material, reviewing the outline in the conclusion.

(c) Using summaries, comparison/contrast tables, and summary statements: a lesson summary in the conclusion, summaries of topics and of subtopics either in advance or after the presentations of these topics.

(d) Indicating interrelations: between the new concepts and the previously learned ones.

(e) Using references: indicating connections between new and previously mentioned materials.

(f) Naming: assigning titles to procedures to serve as a direct access to the procedure storage in memory (in order to call the procedure from memory).

(g) Providing mnemonics: for helping students recall certain formulas, theorems and procedures.

(h) Using visual devices to provide imagery background for clear perception of new concepts.

5. Emphasizing: motivating the students to pay special attention to what is being said and to help them identify and remember the most important points of the message. Some procedures for emphasizing are: writing important points on the board, repetitions of ideas (redundancy), and using different kinds of 'markers'—sentences that motivate to remember topics for special qualifications—such as: markers of importance, of difficulty, of simplicity, of interest, of usefulness, etc.

6. Rationalizing steps: teachers' explanations of what they are doing, why they are doing it, and why in this method and not in another way.

Procedure

In a pilot study 20 mathematics lessons in high schools and a junior college were tape-recorded and analyzed to identify teachers' strategies of lesson organization.

In the main study, 38 graduate mathematics students at Stanford University who taught mathematics in all 50 undergraduate courses of a calculus sequence during the academic year 1978-9 were rated by their students on their lesson organization and clarity. About 16 teachers who were rated high and low were chosen to form two groups of subjects. Each subject was tape recorded twice while teaching in class and also while performing two laboratory tasks of decoding different sources of information into a lecture. Subjects' GRE scores were recorded. A follow-up study of student ratings on participating teachers will take place throughout the next academic year. Two experienced mathematics teachers will be trained to score each transcribed lesson on a rating sheet which breaks down a lesson into components—low inference factors—of organization.

Analysis

Analysis for this study will include analyzing the transcribed lessons to identify strategies of organization, determining which of these strategies are the most discriminating between the two groups of teachers, devising a lesson score on organization and a teacher score on lesson organization, constructing a theoretical model that represents teacher lesson organization as a cognitive construction, and looking for relationships among three different measures of teachers' lesson organization and their GRE scores, motivation, and experience.

At present, only the stage of data collection has been completed. The analysis is planned to be completed in December 1979, and only preliminary results may be reported at this time. There do not seem to exist any definite relationships between teachers' scholastic ability as measured by their GRE scores and their lesson organization as measured by students' ratings.

Direct relationships between teacher experience and his lesson organization as measured by students' ratings, has been found to exist only for teachers with low experience: of teaching from zero through three courses in the mathematics department. After teaching more than three quarters, the correlation is about zero. There seems to be some stability of experienced teachers' ratings by different groups of students in various courses in different quarters. There exists high correlation between teachers' ratings by students as taken at midquarter as part of this study and as taken at the end of the quarter on the same dimension on a Stanford University form.

This study may contribute to knowledge about factors of organization and clarity in mathematics lessons, about meaningful learning, and about teacher encoding knowledge from different sources and communicating adequately his knowledge structure to the learner. Thus, the study may help improve mathematics instruction and learning. It may also form a basis for research in several areas in education, cognitive psychology, and speech communication. Results may be generalizable to written messages, other subject matters—especially sciences—and teaching in undergraduate courses in universities and in other school levels and settings.

Reporting Session V

THE PSYCHOLOGY OF EQUATION SOLVING: AN INFORMATION PROCESSING STUDY[1]

John E. Bernard
Clayton Lewis
and
L. Ray Carry
The University of Texas at Austin

Purpose

This study aims at an understanding of the equation solving methods and strategies available to college students. Both successful and unsuccessful performances are being investigated. The intent is to develop a coherent description of effective and efficient equation solving and of those factors that interfere with such performance. The results should have implications for the learning and teaching of elementary algebra.

The mathematics teaching community can benefit directly from research if the theoretical models offered by the research describe both fluent and weak performance in a given domain. The present research is an attempt to develop such a model for equation solving in elementary algebra.

The framework of the model is adapted from Bundy (no date). A solver is assumed to possess, first, a repertoire of operations for transforming equations and expressions and, second, a method of selecting operations to be used to transform a given equation to a suitable solved form. Accordingly a fluent solver has an adequate set of operations from which to choose and uses them to attain the solution in a minimal number of steps. Fluent solvers might have single operations which can do the work of common sequences of more basic transformations.

The model points to a variety of difficulties which poor solvers may have. First, necessary operations may be missing from the repertoire. Second, the repertoire may contain operations that are not mathematically correct. Third, even correct operations may lead to errors if they are applied incorrectly, for example when the solver uses an incorrect interpretation of the grouping imposed by parentheses. Finally, the selection mechanism may have defects, so that even if only correct operations are used the solution is never found or is found only after some false starts and backtracking.

[1]This research is supported by grant SED78-22293 under the program for Research on Cognitive Processes in Science and Mathematics of the National Institute of Education and the National Science Foundation.

The validity of these suggestions about the organization of the solution process, and about the differences between fluent and weak solvers, can be assessed by examining students' written solutions, and their explanations of their work.

Procedure

Written solutions and spoken comments were collected from college students, using work sheets and audio and video recordings. During the first of two sessions subjects solved fourteen equations, designed to exercise a variety of elementary solving skills. Half of the problems were solved under "thinking aloud" instructions (Newell and Simon, 1972) and half under "explain" instructions in which the subjects were asked to explain their solutions as if to a beginning algebra student. In the second session subjects were asked to comment on various aspects of their solutions from the first session, and to solve and comment on additional problems designed to raise issues of interest.

Two groups of subjects participated in the study. One group consisted of 19 volunteers from a freshman-level psychology course. This group contains mostly weak solvers. The fluent solvers who participated were mostly junior-level engineering students, who scored high on a screening test of speed and accuracy in equation solving. These subjects were paid.

Results

A number of aspects of the subjects' written solutions and spoken comments provide evidence of the appropriateness or inappropriateness of the operation-selection model. These include errors, wasted steps, judgments of problem difficulty, expressions of uncertainty, and checking behavior, as well as the specific steps taken in working out solutions. These aspects of performance are being catalogued for each subject and for each problem. Comparisons can then be made between fluent and weak solvers.

This analysis is not complete, but it appears that some of the suggestions of the model about the basis of weakness and fluency are supported. However, some of the features of the data suggest that this model nevertheless offers only a partial account of solving behavior, especially for weak solvers, and so needs to be extended.

Subjects' spoken comments and the pattern of occurrence of some errors across problems are consistent with the use of specific incorrect operations, for example in cancellation, or are consistent with the lack of certain operators, especially in the handling of reciprocals. Difficulties in selection of operations show up when subjects perform a sequence of legal operations that do not lead to a solution. In some cases it is possible to suggest what particular feature of the problem causes inappropriate operations to be selected. The suggestions of the model about fluent solvers seem to be less explicitly supported, perhaps because the data from these subjects have been less thoroughly examined thus far. There is little indication that fluent solvers use a different repertoire of operations than the weak solvers, in general. Their operation sequences are more economical, however.

The major aspect of the data that requires modification of the operation-selection model as it stands is the uncertainty that seems to dominate the solving behavior of many weak solvers. Weak solvers are unsure of the correctness of the operations they carry out, and their behavior reflects their response to this uncertainty in various ways. They monitor their progress towards solution. They sometimes attempt to validate operations by comparing a problem to previously solved problems, to simpler but analogous problems they construct for this purpose, or by comparing the results of different methods of solving the same problem. None of this validation activity has a place in the operation-selection model as it stands.

A second feature of the behavior of weak solvers that lies outside the scope of the operation-selection model is the nature of the process of applying operations. In performing an operation subjects seem often not to be simply applying a well-defined procedure, as called for in the operation-selection model, but rather to be devising a procedure that will be appropriate. Here they seem to be influenced by general ideas about the character of valid operations in algebra (Matz, 1979) as well as by fragmentary recall of a similar correct operation.

Thus, contrary to the suggestions of the operation model a weaker solver is not just a fluent solver with deficient operation repertoire and selection mechanism. Rather, it appears that these deficiencies force the weak solver to engage in behaviors that make him differ in other ways from a fluent solver.

This evaluation of the operation-selection model can contribute to an understanding of learning and teaching in algebra. Many ideas are suggested by the errors and comments that point to incorrect or missing operators and shaky selection. A more general point is that the monitoring activities and attempts to devise appropriate operations, just discussed, are constructive efforts by students to deal with their own difficulties. Further investigation of the success and failures of these efforts should result in important information for teachers.

References

Bundy, A. Analyzing mathematical proofs. D.A.I. Research Report No. 2 Department of Artificial Intelligence, University of Edinburgh, no date.

Matz, M. Toward a theory of high school algebra errors. Paper presented at AERA convention, San Francisco, 1979.

Newell, A. and Simon, H. A. Human problem solving. Englewood Cliffs, New Jersey: Prentice-Hall, 1972.

Reporting Session V

NUMBER CONCEPTS AND THE INTRODUCTION OF CALCULUS

Jere Confrey
Cornell University

Purpose

A disturbing number of very capable students experience difficulty during the transition from precalculus mathematics courses into calculus. As a result, many students abandon mathematics entirely after a first course in calculus. Although this is frequently attributed to the multitude of course offerings at a university, I think that the difficulty in transition to calculus is the result of a clash between the student's concept of number and the concept of number embedded in calculus.

This research investigates the following claim: the undertaking of the study of calculus requires a change in a student's concept of number from a discrete concept of number which is embedded in the precalculus courses to a continuous concept of number embedded in calculus. The research includes an analysis of the history of calculus for the development of the concept of a real number, and an empirical study with precalculus students to determine their concepts of number.

The analysis of the development of calculus historically reveals that different number concepts were used by different mathematical cultures and individuals, and that the fruitfulness and limitations of a particular number concept influence the mathematics which can or cannot be explained. For example, the Greek's concept of number, based on geometric magnitude, was a ratio, either the ratio of two line segments, two areas or two volumes. Therefore, to conceive of a "number zero" made no sense to the Greeks for no ratio can give zero. For the same reason, they were also restricted in their concept of infinity, and irrationals or incommensurables were not considered numbers, only as magnitudes. Five different concepts of number are presented, and the implications of each for calculus are discussed. Briefly, the five number concepts are: (1) a "ratio" concept of number, (2) a "set" concept of number, (3) a "non-terminating decimal" concept of number, (4) a "point-number correspondence" concept of a number, and (5) a "variable" concept of a number.

A contrast of the implications of these different concepts of numbers reveal two general classes of number concepts exist, here referred to as a discrete concept of number and a continuous concept of number. A student when exhibiting a discrete concept of number believes that numbers are essentially separate, independent entities, each one completed and fixed. The image which conveys the discrete concept of number is an infinite deck of cards. Between any two cards, another card can always be inserted, and the cards themselves remain distinct. Both density and infinity can be accounted for in such a model, while discreteness is preserved. A student when exhibiting a continuous concept of number believes that numbers are not necessarily discrete entities, and will complete numbers whose construction requires an infinite process, allowing them to blend together. A continuous concept of number is illustrated by a continuous quantity, the number line, space or time, for example.

The relationship between the two general concepts of number, discrete and continuous is neither dichotomous or class inclusive. A student does not replace a discrete concept of number, but modifies and extends it to account for mathematics in which numbers are associated with continuous quantities, and this is called a continuous concept of number. The following criteria differentiate between when a student is exhibiting a discrete concept of number and when a student is exhibiting a continuous concept of number:

1. A student exhibiting a continuous concept of number will complete an infinite extension of a number, allowing for example, .9 to equal 1. A student exhibiting a discrete concept of number cannot mentally complete such a process.

2. A student exhibiting a continuous concept of number will believe that numbers correspond with points, but unlike the student exhibiting the discrete concept of number, those points are neither definite nor determinable.

3. A student exhibiting a continuous concept of number will reject any claim of "nextness" in describing numbers, because an atomistic view of numbers is incompatible with continuity. A student with a discrete concept of number will accept such a claim.

4. A student with a continuous concept of number rejects the claim that a continuous whole can be accounted for precisely by an infinite set of parts. That is, time is not a succession of instants; nor is motion a succession of positions. In continuous quantities, which are those lacking a natural unit, the whole is logically prior to the parts.

In order to understand calculus, a student must consider the result of attaching numbers to continuous quantities, and the difficulties inherent in such a process. The discrete concept of number which is embedded in the high school curriculum inhibits understanding such a process and consequently, the need for limits, a definition of continuity and delta-epsilon proofs becomes inexplicable. In schools, the number system is built up from the integers which are intrinsically discrete. The addition of the rational numbers in definitional form, p/q where p and q are integers and p/q is in lowest terms, poses no threat to the student's discrete number concept, only the concept of density is added. Since the irrational numbers are normally introduced as non-repeating, non-terminating decimals, and not as limits of infinite series, a student does not see them as sufficiently different to challenge his or her discrete concept of number. The concepts of calculus then appear to the student to be non-intuitive, unnecessarily obtuse, and contrived. If the student were led to understand the difficulties inherent in attaching numbers to continuous quantities, and thus extend his or her discrete number concept to a continuous concept of number, then the concepts of calculus would seem more natural and intuitive.

Procedure

In order to determine a student's concept of number, a series of problems were designed to conflict with a discrete concept of number and to provoke a discussion of numbers. Ten students enrolled in a pre-calculus course at Cornell University were asked to solve the problems and discuss their answers. The problems included a form of Zeno's paradox, an infinite repeating decimal, a derivative problem and a discussion of continuity. The sessions were audio-recorded and the transcripts were analyzed to determine how the students conceived of numbers and how they resolved the inherent conflicts. Using the four criteria to distinguish when students exhibit discrete and continuous number concepts, the transcripts were analyzed to describe each student's concept of number, and the changes and modifications in it over time.

Results

The result of the analysis was that a discrete concept of number is prevalent in students entering calculus; however, students display different degrees of commitment to this concept. Depending on the student's conception of mathematics, the discrete concept of number is variably easy to displace. If a student is insistent on consistency among his or her responses, then the continuous concept of number is more likely to displace the discrete concept of number. In most cases, it is very apparent that the underlying number concept does exert considerable influence on the mathematics which a student does.

The significance of the research to the classroom teacher lies in the understanding of discrete and continuous number concepts and their relationship to other mathematics. From this understanding the following specific outcomes are possible:

1. Teachers of calculus can facilitate the transition to calculus by discussing number concepts early into calculus. The result may be greater ease for students in comprehending the concepts of calculus.

2. The high school curriculum can be examined in order to determine if a continuous concept of number can be included earlier to make the transition to calculus less abrupt.

3. The discrete and continuous number concept will prove insightful in helping elementary school teachers to introduce numbers to children. To introduce numbers to children exclusively through sets is to inhibit the development of a continuous concept of number. Using length, volume or time to introduce numbers as well as sets will provide students with alternative concepts of number.

AUTHOR INDEX