DOCUMENT RESUME

ED 182 105 SE 029 204

AUTHOR Fraser, Barry J.; Koop, Anthony J.
TITLE An Evaluation of Some Motivational Materials Related to the History of Mathematics.
PUB DATE Mar 77
NOTE 85p.; Report of an augmented evaluation study developed by R. Grunseit of Vaucluse Boys High School; Not available in hard copy due to marginal legibility of original document ; Sponsored by the Schools Commission Innovations Program.

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Attitudes; Dramatic Play; *Educational Research; Evaluation; *History; Junior High Schools; Mathematics Education; *Motivation Techniques; Play; *Program Evaluation; Secondary Education; *Secondary School Mathematics; Slides; *Student Attitudes; Surveys; Videotape Recordings

ABSTRACT

This report evaluates motivational materials related to the history of mathematics. Materials produced included plays, written articles, slide-tape presentations, and videotapes. Two of these, plays and written articles, were chosen for intensive study. The study included a survey of teacher opinion, a student questionnaire, and evaluations by mathematics educators, drama critics, and a historian of mathematics of one article and one play. Among the reported findings was that a very favorable opinion of the play, but an appreciably less favorable opinion of the article, was expressed in the teacher survey by a historian of mathematics, and by two mathematics educators. Twelve recommendations are made. (MK)

AN EVALUATION OF SOME MOTIVATIONAL MATERIALS RELATED TO THE HISTORY OF MATHEMATICS

BARRY J. FRASER AND ANTHONY J. KOOP

SCHOOL OF EDUCATION, MACQUARIE UNIVERSITY

A REPORT OF AN AUGMENTED EVALUATION STUDY FUNDED BY A SCHOOLS COMMISSION INNOVATIONS PROGRAM GRANT. THE MATERIALS EVALUATED WERE DEVELOPED BY R. GRUNSEIT OF VAUCLUSE BOYS HIGH SCHOOL UNDER A SEPARATE SCHOOLS COMMISSION INNOVATIONS PROGRAM GRANT (PROJECT NUMBER 74/184/H - DEVELOPMENT OF HIGH INTEREST LEVEL MATHEMATICS MATERIALS).

MARCH, 1977

PREFACE

The materials evaluated in the present study were developed by Rolf Grunseit, Vaucluse Boys High School, Vaucluse, New South Wales, Australia, 2030. Requests for further information about these materials should be sent directly to Mr. Grunseit, while additional details of the research reported herein should be sought from the authors of this report at Macquarie University, North Ryde, New South Wales, Australia, 2113.

ACKNOWLEDGEMENTS

The present study would not have been possible without the co-operation and assistance of the various people mentioned below. Rolf Grunseit kindly provided copies of his materials, spent time in discussion with us, and commented on an earlier draft of this report. The Principal of Vaucluse Boys High School, Mr. D. Lewis, willingly co-operated with us in our visits to his school during the evaluation. John Pottage (Department of History and Philosophy of Science, University of Melbourne), Ken Clements (Faculty of Education, Monash University), and Brian Low, Sandra Alexander and Nel George (all of the School of Education, Macquarie University) provided expert opinion about various aspects of some of the present materials. We are indebted to the 20 experienced mathematics teachers in Sydney suburban schools and the 19 trainee mathematics teachers in their final year of study at Macquarie University for answering two teacher questionnaires. Similarly, we are grateful to the four mathematics teachers who used the mathematical play in their own classrooms, and to their 117 pupils who responded to a student questionnaire on two occasions. Kim Hood provided valuable help as a research assistant and Marilyn Fraser competently typed all material for the study and report. Finally, we thank the Schools Commission Innovations Program for funding the present research.

TABLE OF CONTENTS

[illegible]

LIST OF TABLES

CHAPTER 1

BACKGROUND TO THE STUDY

1.1 Introduction

The purposes of this introductory chapter are threefold. First, a description of the project to be evaluated will be given. Second, in order to allow the significance of the present innovatory project to be seen in a broader context, a discussion of aspects of the background scene in mathematics education will be given. Third, ways in which the present study was narrowed in scope will be described and justified.

1.2 Description of the Project

Some salient descriptive characteristics of the present project have been extracted from the Schools Commission Special Projects file and summarized below.

Project number:	74/184/H
Project director:	Rolf Grunseit, Vaucluse Boys High School
Project title:	Development of High Interest Level Mathematics Materials
Funding:	$4,879 for 2 years commencing December, 1974
Project director's stated aim:	To inculcate in the student a positive attitude to mathematics irrespective of ability
Method of achieving the aim:	By developing a variety of materials related to the history of mathematics for student use
Proposed grade level:	Year 9 approximately

The common thread in the present materials is that each aims to introduce something of the history of mathematics in an interesting way. The materials may be subdivided, however, into one of four main categories according to the different media of presentation used. The four media mainly used for the present materials are plays, written articles, slide-tape presentations and videotapes. Appendix A contains a listing of all materials developed to date as part of the present project, with materials organized into the four subdivisions above where possible. It can be seen from Appendix A that the same historical material has often been used as a basis for a presentation in different media; for example, the article on conics and the slide-tape presentation on conics draw on common historical resources.

The three plays written by the project director form the first set of entries in Appendix A. One of the plays, "Thales of Miletus", is reproduced in full in Appendix B. Although it was envisaged by the project director that the plays would be produced on stage, evidence discussed later in Chapter 3 suggests that the plays could still be useful if simply read aloud by students in class. It should also be pointed out that, at present, these plays remain unpublished and can only be obtained from the project director.

The second set of entries in Appendix A is a list of articles, containing stories on history of mathematics topics, which have appeared in various mathematics teachers' periodicals. It can be seen that so far three such articles have been published covering such areas as the history of conics and the history of maxima. One of these articles, entitled "The Genesis of Conics", has been reproduced fully in Appendix C. The main use envisaged for these articles is that teachers would employ ideas from the articles in preparing lesson introductions.

The third group of things listed in Appendix A consists of slide-tape presentations on topics drawn from the history of mathematics. Each slide-tape presentation originally consists of historical research, slides and a spoken commentary, all provided by the project director. Next, members of the N.S.W. Department of Education Mathematics Equipment Committee modify, edit and expand the materials. The Teachers' Resources Centre of the N.S.W. Department of Education then publishes a set of slides, an audiotape and teachers' notes (cost is approximately $6.50 per kit). So far "The History of Conics" is the only slide-tape kit that has been published although initial stages in the development of five other kits have already been completed.

The fourth set of entries in Appendix A relate to video materials. Students in the project director's own mathematics classes have acted in and assisted in the production of several one-act plays about discoveries in mathematics. As well as some pupils being involved in actually making the videotapes, other pupils are shown these videotapes as introductions to particular mathematical topics. It should be emphasized, however, that the project director feels that the quality of the videotapes is <u>not</u> sufficiently high to warrant their use by other teachers in other schools. Nevertheless, the project director has published an article (see Appendix A) which describes how other teachers can use video equipment themselves in making videotapes of such one-act plays. These possibilities are somewhat limited at present, however, because copies of the scripts of the one-act plays have not been widely disseminated to mathematics teachers.

As well, the project director has used this video equipment at numerous mathematics teachers' in-service and association meetings to illustrate how video might be employed in mathematics teaching.

In addition to the four main classes of materials discussed above (namely, plays, articles, slide-tape presentations and videotapes), other miscellaneous materials were produced; these are listed as the fifth set of entries in Appendix A. These materials include a series of [illegible] biographies of great mathematicians together with drawn likenesses. These are currently being prepared and are under consideration by the N.S.W. Department of Education Mathematics [illegible] Committee.

publication through that department's Teachers Resources Centre. These materials are intended for use by the mathematics teacher in relating a topic being taught to the mathematician who discovered it. Photographs have been taken of the four Platonic solids at the Mining Museum, and it is intended that these will be used with material to be written later to form a unit on the history of geometric solids. Similarly, photographs have been taken of aboriginal petroglyphs as the basis of a unit to be developed on the history of aboriginal mathematics. The remaining materials listed in Appendix A are journal articles which contain discussions of the innovative project as a whole, of suggestions for incorporating history of mathematics into mathematics teaching, and of a number of other miscellaneous areas. These articles, however, can be contrasted with the ones described previously which contain specific stories taken from the history of mathematics.

The above description given of the present project materials will be supplemented with some further discussion in sections 1.4 and 1.5 Beforehand, however, brief consideration will be given to describing some aspects of the contemporary scene in mathematics education and to examining the educational importance of the present materials within that context.

1.3 The Contemporary Scene in Mathematics Education

Mathematics teachers in many countries have had a long-standing concern about the negative attitude towards mathematics held by a sizeable proportion of school students. Furthermore, the study of attitudes to mathematics has developed into a quite active area of educational research in recent years (see reviews of Aiken, 1970, 1976).

Of the many ways which have been suggested to effect improved student attitude to mathematics, use of the history of mathematics in teaching has been suggested by a number of writers (e.g., Ministry of Education, 1958; Francis, 1976.) In fact, the literature of mathematics education has for many decades contained pleas that the historical aspects of mathematics be given a place in mathematics education. For example, the teaching of the history of mathematics was advocated in 1893 by Heppel in Nature, and in 1894 by Collins in Science. More recently, the National Council of Teachers of Mathematics (1969) considered it of sufficient importance to devote a whole yearbook to issues related to history of mathematics in mathematics education. It is of particular interest to note that, despite the many calls for including the history of mathematics in teaching, it has recently been observed that very few teachers actually expose their students to any teaching of the history of mathematics (Green, 1976a) and that very few mathematics textbooks have taken seriously the incorporation of historical material into its mathematics (Green, 1976b).

The brief sketch given of the contemporary scene in mathematics education has indicated that concern for student attitude to mathematics is a real one, that the teaching of history of mathematics has been advocated for various reasons including the promotion of more favourable attitudes, and that mathematics teachers and textbooks largely neglect the history of mathematics. Moreover, Green (1976b) has contended that there is a definite need for historical materials in mathematics which are in a form suitable for use by the teacher in the classroom. **Within this context, then, it can be seen that the present** materials, if successful in their aim of promoting attitudes through the teaching of history of mathematics, could make a worthwhile contribution to mathematics education.

1.4 Deployment of Project Funds and Implications for Evaluation

A financial statement obtained from the project director showed that, from the original grant of $4,879, a sum of approximately $4,000 was spent purchasing video equipment while the remainder was used for the purchase of photographic equipment and for the insurance and repair of equipment. It is vital for a proper understanding of the present report, however, to appreciate that Schools Commission funds were deployed in supporting only some parts of a more comprehensive and ongoing project. Firstly, in terms of timing, Mr. Grunseit had already completed substantial amounts of historical research and the writing of versions of some articles and plays prior to receiving the Schools Commission grant. Moreover, now that the Schools Commission's period of funding is terminated, Mr. Grunseit has every intention of both modifying and extending previous work. Secondly, while Schools Commission funds were spent specifically on the purchase of video and photographic equipment, considerable resources from elsewhere were required to support other aspects of the project. For instance, the Teachers Resources Centre provided support for the production of slide-tape kits, and the vast time commitment and modest expenditure on typing and stationery involved in the historical research and the production of articles and plays were supported either by Vaucluse Boys High School or the project director himself.

1.4.1 Decision to collect formative and summative evaluative information

The situation described above has important implications for the evaluator when deciding whether formative or summative evaluative information, or both, should be collected. Because the funding period was drawing to a close at the time the evaluation was conducted, and because of the potential usefulness of materials in other schools, summative evaluative information was clearly of relevance. However, as the project director has genuine intentions of further modifying materials before disseminating them more widely, formative evaluative information would also be very useful. Consequently, in planning the present study, an attempt was made to ensure the collection of information both about the materials' overall efficacy and usefulness and about specific weaknesses in the materials which could be eliminated in any future rewritten versions.

1.4.2 Comparable importance of different media

The situation described in section 1.4 presents a dilemma for the evaluator. For reasons of accountability, the evaluator may be tempted at first to restrict his attention solely to those aspects of the project (mainly the videotapes) for which most of the Schools Commission funding was deployed. This approach, however, can be considered undesirable for two reasons. Firstly, considerable resources [illegible] were deployed both in conducting the historical research on which the videotapes were based and in developing slide-tape kits, [illegible] articles [illegible]. Secondly, as the key aim of [illegible] [illegible] it could be argued that each different medium in which the material is presented would be of fairly comparable importance in an evaluation study. For the above reasons, materials in all of the four media were included in the initial pool of materials, and later, particular materials were selected for intensive study (see section 1.6).

1.5 Potential Usefulness of Materials in Other Schools

Another important orientation of the present project which is not brought out clearly in the official file relates to the potential usefulness of the materials in other schools. Discussion with the project director made it clear that he perceived that his major goal was to develop materials which could be used by other mathematics teachers in other schools around Australia. That is, although most of the present materials were used by the project director in his own mathematics classes, he perceived that the main purpose of this was to provide feedback information which could be used in further development work. In fact, the project director expressed the concern that his own students could have been used as "guinea pigs" to some extent and could well have been over-exposed to the materials.

From the discussion above, it can be seen that the present project is somewhat atypical of the majority of Schools Commission Innovations Program projects with respect to its wide applicability in other schools. In general, a large number of Schools Commission projects involve some innovatory organization or approach in a single school which, if found successful, is not always easy to replicate at other schools. In contrast, the present project aims at producing curricular products (plays, articles, slide-tape kits) which could be used in an unaltered form in a large range of schools around Australia.

1.5.1 Decision to study a limited range of materials intensively

Because the present materials have the potential of being useful in a wide range of schools, it was considered that examining the applicability of the materials in other schools should form a major focus in the present research. Furthermore, it was thought that the applicability of the materials could be better assessed with available resources by an intensive study of a limited range of materials (involving a relatively large number of mathematics teachers and students) than by a more extensive study of a wider range of offerings. It was therefore decided to select a limited range of materials and to study these fairly intensively.

1.6 Choice of Materials for Intensive Study

In choosing materials for intensive study, considerations of economy and likely applicability of materials in other schools were considered paramount. When videotapes were considered, the fact that the project director felt that existing videotapes were not suitable for use in other schools (see section 1.2) was important. Furthermore, because schools in many Australian states do not have video equipment, the usefulness of the videotapes in other Australian schools must be considered less than that of materials in other media. For these reasons, the videotapes were not included in the present intensive study.

For the slide-tape kit which was completed, namely the one entitled "The History of Conics", costs incurred in reproducing and distributing multiple copies would present [illegible]. However, it was still thought desirable to evaluate these materials because they have high potential applicability in other schools (since the majority of schools do have facilities for projecting slides and playing audio-tapes). An additional problem arose with this kit, however. It was discovered that the Teachers' Resources Centre had made a payment to the project director for his original slides and commentary and then had incorporated modifications and extensions into the presentation prior to publication. Since the copyright of the new version was held by the Teachers' Resources Centre, and since the kits had not come on sale at the time of the present study, the Teachers' Resources Centre expressed some understandable reluctance about making the kit available for the present study. For this reason, the slide-tape kit was not included in the intensive study.

The other two media in which the history of mathematics materials were presented were written plays to be acted or read aloud by students, and written articles. Because of the relatively small expense involved in reproducing and distributing written materials, the costs needed for an intensive study of some of these materials fell within the present budget. Furthermore, because of the relatively small costs of written materials, the articles and plays would have very high potential applicability in a wide range of schools. For the above reasons, it was decided that the present intensive study would focus on materials selected from among the plays and articles.

From the three plays and three articles listed in Appendix A, one play and one article were chosen for intensive study. After carefully scrutinizing all plays and articles, and after consultation with the project director, the play entitled "Thales of Miletus" and article entitled "The Genesis of Conics" were chosen for intensive study. A copy of this play and article have been included in Appendices B and C, respectively.

1.7 Overview of the Present Report

A detailed explication of the design, execution and findings of the present study is, of course, the main purpose of the discussion contained in the remainder of this report. At this stage, however, it may prove useful to provide a brief overview of the contents of the remaining chapters. The purpose of such an overview would be twofold. First, the overview may help to clarify, in a general way, the nature of the present study. Second, such an overview would provide a directory of the location of salient issues in the remainder of the report.

Each of the following four chapters is devoted to different issues. Chapter 2 describes a questionnaire survey of the opinions of teachers, both experienced and in training, about the worth of the Thales play and the conics article. Chapter 3 is devoted to describing a study in which a sample of Year 9 students who actually used the Thales play responded to items measuring affective and cognitive outcomes. Chapter 4 is devoted to an impressionistic evaluation of the Thales play and the conics article, in particular, and the whole set of materials, in general. In particular, reactions to the materials given by a historian of mathematics, mathematics educators and drama critics will be reported. The final chapter, Chapter 5, contains a summary of findings, a discussion of some limitations of the research, a list of tentative conclusions, and suggested recommendations for future development, modifications and dissemination of the project materials.

CHAPTER 2

SURVEY OF TEACHER OPINION ABOUT THE THALES PLAY AND THE CONICS ARTICLE

2.1 Introduction

This chapter describes a questionnaire survey of teacher opinion about the two pieces of material chosen in section 1.6 for intensive study, namely, the Thales play and the conics article. As well as reporting teacher opinion about a wide range of features of the play and article, data analyses will be reported which enable comparison of teachers' opinion of the play and the article, and of experienced and trainee teachers' opinions.

2.2 The Sample

The sample involved in the questionnaire survey consisted of 39 teachers altogether. Of these, 20 were experienced teachers currently teaching mathematics in 12 different private or Government schools in the Sydney metropolitan area. The other 19 were trainee teachers in their final year of training as mathematics teachers at Macquarie University.

2.3 The Questionnaires

The questionnaires used to survey opinions about the Thales play and the conics article are shown in Appendices D and E, respectively. These appendices show that the first 22 items in the questionnaire about the play are identical or parallel to the first 22 items in the questionnaire about the article. Questionnaires were designed in this way so that direct and meaningful comparisons could be made between opinions about the play and opinions about the article.

Appendices D and E also indicate that questionnaire items fall into a number of distinct subgroups. The first eight items on each questionnaire deal with the educational aims likely to be satisfied by the use of the materials. This list of aims, which includes humanizing mathematics, showing that mathematics has practical applications, and providing an awareness of the contribution of mathematics to society, was culled from the literature containing claims about educational aims likely to be satisfied by teaching the history of mathematics (e.g., Kinney and Purdy, 1956; Ministry of Education, 1958). Items 10-17 of each questionnaire are concerned with opinions about which groups of students might be catered for by the materials. Items 18-22 on each questionnaire deal with miscellaneous aspects common to the play and article. Items 23-26 on the questionnaire about the play deal with miscellaneous aspects unique to the play, while Items 23-28 on the questionnaire about the article deal with miscellaneous aspects unique to the article. Item 9 and the last item on each questionnaire are open-ended items, aimed at gathering additional information, which will be discussed in more detail in Chapter 4.

Questionnaires were sent to, and returned from, experienced teachers by mail. Questionnaires were given out to trainee teachers during a regular tutorial time and were returned to a designated location at Macquarie University. Each teacher was provided with a copy of the play, the article and each questionnaire, and was requested to read the play and article and then to respond anonymously to each questionnaire. A follow-up letter reminding teachers to return their completed questionnaires was sent to each teacher one month after questionnaires were originally distributed.

2.4 Comparison of Opinions about the Play and Article

Data from the survey have been organized in Table 1 to permit a comparison of opinions about the play and the article for each of the items common to both questionnaires. This table lists, separately for the play and the article, how many of the 39 subjects selected each alternative response to each item, the mean and the standard deviation for each item, and the results of a t test for dependent samples for differences between opinions about the play and the article.

An overview of the data in Table 1 indicates that opinions expressed about the play were more favourable than those expressed about the article for the large majority of questionnaire items. In fact, it was found that the grand mean of the 21 item means in Table 1 was 2.21 for the play but only 2.02 for the article. Moreover, the last column in Table 1 indicates that opinions about the play were significantly more favourable ($p<.05$) than opinions about the article on 13 of the 21 items.

The first eight items in Table 1, which are concerned with opinions about various aims which could be satisfied by use of the materials, were scored 3, 2 and 1 for the responses Very Useful, Useful and Not Useful, respectively. This table shows that the mean rating of usefulness awarded to the play for satisfying an aim was higher than the mean rating given to the article for all eight aims. Moreover, the mean rating of usefulness given to the play was significantly greater than that given to the article for five of the aims, namely, teaching some history of mathematics, teaching some mathematical concepts, humanizing mathematics, showing that mathematics has practical applications, and providing an awareness of the value of mathematics to society. Table 1 also indicates that the mean score given to the play on the first eight questionnaire items ranged from 1.95 (with eight of the 39 people choosing 'Not Useful') for the aim of providing an awareness of the value of mathematics to society, to 2.36 (with three people choosing 'Not Useful') for the aim of teaching some history of mathematics. For the article, the mean score on the first eight items ranged from 1.57 (with 18 people choosing 'Not Useful') for the aim of providing an awareness of the value of mathematics to society, to 2.08 (with six people choosing 'Not Useful') for the aim of teaching some history of mathematics.

Questionnaire Items 10-17 measure teachers' opinions about what groups of students would be catered for by the materials, and are scored on the same three-point scale as the previous items. For these eight items, significant differences were found in the ratings given to the play and the article in six cases. It was found that opinions about the suitability of materials were significantly more favourable for the play than the article for Year 7 students of low ability, Year 7 students of high ability, Year 9 students of low ability, Year 9 students of high ability and students with a poor attitude to mathematics. This last finding is noteworthy in terms of the project's main aim (of inculcating a positive attitude towards mathematics) because it shows that teachers considered the play more suitable than the article for pupils who already have a poor attitude to mathematics. Opinions about the suitability of materials were more favourable for the article than the play for Year 11 students of high ability. For the play, mean scores for this set of items ranged from 1.34 (with 28 teachers choosing 'Not Useful') for Year 11 students of high ability to 2.25 (with five teachers choosing 'Not Useful') for Year 9 students of high ability. For the article, mean scores ranged from 1.23 (with 29 choosing 'Not Useful') for Year 7 students of low ability to 2.03 (with eight choosing 'Not Useful') for students with a good attitude towards mathematics.

Data in Table 1 for Items 10-15 provide useful information about the grade level at which the present sample of teachers thought that the materials would best be employed. For the play, mean scores indicate that the play was thought most useful for Year 7 students of high ability and Year 9 students. On the other hand, mean scores indicate that the article was considered most suitable for use by Grade 9 students of high ability and Year 11 students.

Items 18-22 in Table 1 cover various miscellaneous aspects common to the play and article, and were scored by allotting 4, 3, 2, 1, respectively, for the responses Strongly Agree, Agree, Disagree and Strongly Disagree. With this scoring scheme, then, a neutral attitude would correspond to a score of 2.5. Results for each of these five items are discussed below.

The mean score for Item 18 indicates that reactions to the statement that most students would find the material interesting were virtually neutral for the article and only slightly positive for the play. This finding is consistent with responses to Items 10-17 which indicated that the materials were considered differentially suitable for different students rather than being suitable for all students.

Results for Item 19 reflect teacher opinion about the amount of time that would be needed if the materials were used with students. The mean score of 2.93 for the article reflects reasonable agreement that the time required to use the article would <u>not</u> be excessive. In contrast to this, however, the mean score of 2.36 for the play indicates that teachers held a slightly negative attitude to the statement that the time required to use the play would <u>not</u> be excessive. Furthermore, differences in opinion about the play and the article were significant for this item. This finding is an interesting one because it shows that, despite the relatively high opinions expressed about the play on most other questionnaire items, teachers were concerned that the amount of time involved in using a mathematical play in their teaching could be excessive.

Table 1 shows that, like the previous item, opinions about Item 20 were significantly more negative for the play than for the article. It was found that teachers held only a slightly positive attitude to the

TABLE 1. Pattern of Responses given by the Total Sample of Teachers to each Questionnaire Item common to the Play and the Article, and Significance Tests for Differences in Opinion about the Play and Article

Questionnaire Item	Play/ Article	No. Selecting each Alternative: Very useful	Useful	Not useful	Mean	Stan Dev	t
Aims Satisfied by Play/Article							
1. Arousing student interest before teaching the topic of similar triangles/conics[a]	Play	13	18	8	2.13	0.73	1.50
	Article	12	12	14	1.92	0.83	
2. Teaching some history of mathematics	Play	17	19	3	2.36	0.63	2.72**
	Article	9	24	6	2.08	0.63	
3. Teaching some important concepts related to similar triangles/conics[a]	Play	11	20	8	2.08	0.71	3.79***
	Article	4	16	1[illegible]	1.64	0.67	
4. Humanizing mathematics	Play	13	23	3	2.26	0.60	2.[illegible]2*
	Article	8	22	9	1.98	0.67	
5. Showing that mathematics has practical applications	Play	15	21	3	2.31	0.62	4.98***
	Article	6	19	14	1.80	0.70	
6. Providing an awareness of the value of mathematics to society	Play	6	25	8	1.95	0.62	4.42***
	Article	1	19	1[illegible]	1.57	0.55	
7. Providing an appreciation of ancient civilization	Play	7	24	8	1.98	0.63	0.22
	Article	8	21	10	1.95	0.69	
8. Promoting a better attitude towards mathematics	Play	6	27	6	2.00	0.56	1.96
	Article	4	20	14	1.75	0.64	
Students Catered for by Play/Article							
10. Year [illegible] students of low ability	[illegible]	[illegible]	11	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	
11. Year [illegible] students of high ability	[illegible]	[illegible]	[illegible]		[illegible]	[illegible]	[illegible]
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	
12. Year 9 students of low ability	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	
	[illegible]				[illegible]	[illegible]	
13. Year 9 students of high ability	[illegible]				[illegible]	[illegible]	
	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	

cont'd.

TABLE 1. (Cont'd.)

Questionnaire Item	Play/ Article	No. Selecting each Alternative: Very useful	Useful	Not useful		Mean	Stan Dev	t
Students Catered for by Play/Article (Cont'd.)								
14. Year 11 students of low ability	Play Article	3 8	15 16	19 15		1.59 1.82	0.64 0.75	-1.39
15. Year 11 students of high ability	Play Article	2 9	6 18	28 11		1.34 1.95	0.57 0.76	-4.54***
16. Students with a good attitude towards mathematics	Play Article	12 9	20 21	7 8		2.13 2.03	0.70 0.73	0.66
17. Students with a poor attitude towards mathematics	Play Article	12 4	17 15	9 16		2.08 1.54	0.74 0.67	3.68***
Miscellaneous (Play and Article)		SA	A	D	SD			
18. Most students would find the material interesting	Play Article	6 1	19 21	13 16	1 1	2.77 2.57	0.74 0.60	1.60
19. The amount of time taken up if the materials were used in the classroom would not be excessive	Play Article	0 5	20 27	14 6	4 1	2.36 2.93	0.68 0.63	-3.74***
20. The use of the materials requires only knowledge and skills which mathematics teachers already have	Play Article	5 7	18 22	13 10	2 0	2.68 2.93	0.77 0.67	-2.04*
21. Materials like this are not readily available elsewhere	Play Article	11 7	25 25	3 6	0 1	3.21 2.98	0.57 0.67	1.86
22. The average mathematics teacher would not be able to write material like this himself	Play Article	10 7	19 24	9 8	1 0	2.98 2.98	0.73 0.63	0.00

* $p < .05$, ** $p < .01$, *** $p < .001$

\- Mean for the article was larger than mean for the play

a 'Similar triangles' and 'conics' are contained in the questionnaires about the play and article, respectively.

statement that the use of the play would require only skills which the teacher would already have, but that more positive attitudes were held to the same statement applied to the article. This finding is also an interesting one as it suggests lack of confidence on the teachers' part about using plays in their mathematics teaching.

In responding to Items 19 and 20, it is likely that many teachers gave their opinions in terms of the time and skill needed to produce the play on stage. However, evidence discussed later in Chapter 3 suggests that the present play can be used quite successfully by teachers in a relatively short time in their ordinary mathematics classes. It is suggested that more favourable responses could have been obtained for Items 19 and 20 if teachers had been asked to answer specifically in terms of the use of the play in the classroom.

Responses to the last two items in Table 1 were quite similar and indicate reasonably positive opinions about the statements that material like the play and article are <u>not</u> readily available elsewhere and could <u>not</u> readily be written by the average mathematics teacher. These responses, then, highlight a major educational merit of the present materials, namely, that teachers perceive them as unique, not readily available elsewhere and not able to be written by the average teacher.[1]

2.5 Responses to Items Unique to the Play or Article

It can be seen from the two questionnaires shown in the appendices that the last four items in Appendix D apply only to the play while the last six items in Appendix E apply solely to the article. Because it is not possible to make the same comparisons for these 10 items that were made for the first 21 items, Table 2 has been constructed to show the pattern of responses for items unique to either the play or the article. In scoring each of the items shown in Table 2, the responses, Strongly Agree, Agree, Disagree and Strongly Disagree were allotted respectively, scores of 4, 3, 2 and 1 as for some of the previous items.

The mean response to each of the four items unique to the play in Table 2 indicates a positive opinion about each item. The mean of 2.80 for Item 23 indicates a slightly positive opinion to the statement that the play could be used profitably in the classroom without the need for a stage or costumes. For Item 24, the mean of 3.31 indicates a very positive reaction to the statement that the play is very suitable for use at the end of the school year. Similarly, the mean of 3.21 for Item 25 indicates a very positive opinion about the statement that the play would be useful for integrating Mathematics with other subjects

---

[1] Responses to Item 21 can be considered a little surprising because, while mathematical plays are indeed scarce, there exists an abundance of books and articles dealing with the history of mathematics (e.g., see bibliographies in Read, 1966; National Council of Teachers of Mathematics, 1969; Dittrich, 1973). This observation, however, does not undermine the usefulness of the present article since the fact that teachers perceive a dearth of other materials makes the present article all the more valuable to them.

TABLE 2. Pattern of Responses given by the Total Sample of Teachers to each Questionnaire Item Unique to the Play or Article

Questionnaire Item	Play/ Article	No. Selecting each Alternative				Mean	Stan Dev
		SA	A	D	SD		
<u>Miscellaneous</u> (<u>Play only</u>)							
23. The play could be used profitably in the classroom without the need for a stage, costumes, etc.	Play	6	21	10	2	2.80	0.77
24. The play would be very suitable for use near the end of the school year.	Play	14	23	2	0	3.31	0.57
25. The play would be useful for integrating Mathematics with other subjects like English or Ancient History.	Play	11	25	3	0	3.21	0.57
26. I would use a play like this in my own mathematics classes.	Play	6	21	7	4	2.76	1.34
<u>Miscellaneous</u> (<u>Article Only</u>)							
23. The number of different characters and ideas is <u>not</u> too large for an article of this size.	Article	1	29	7	2	2.75	0.60
24. An article like this does <u>not</u> need a list of references.	Article	2	18	14	5	2.44	0.79
25. The story related in this article is clearly relevant to the topic of conics.	Article	0	16	19	4	2.31	0.66
26. The mathematics on pages 32-36 is clearly relevant to the rest of the article.	Article	2	15	18	3	2.43	0.71
27. This article would be suitable for direct student use in mathematics projects.	Article	2	25	9	3	2.6[illegible]	0.70
28. I would use an article like this in planning my own mathematics lessons.	Article	2	20	14	3	2.54	0.72

like English or Ancient History. This finding is consistent with claims in the literature that teaching of the history of mathematics is a useful way of integrating mathematics with other subjects like English and History (e.g., Ministry of Education, 1958; Curcic, 1976). Despite the high means for Items 24 and 25, however, Table 2 indicates that only a moderately positive opinion, with a mean of 2.76, was expressed for the statement about actually using the play in the teachers' own classrooms. That is, despite the fact that teachers expressed laudatory opinions about numerous features of the play, only 27 of the 39 teachers agreed that they would actually use such a play in their own classrooms. This pattern of responses, however, is consistent with data shown for Items 19 and 20 in Table 1 which suggest the existence of a feeling among teachers that the play could absorb excessive amounts of class time and could require some skills not possessed by the average mathematics teacher.

The data in Table 2 indicate that a fairly neutral opinion was held of each of the six questionnaire items applicable to the article only, with mean item scores covering a small range from 2.31 for Item 25 to 2.75 for Item 23. Although strongly negative opinions were not expressed about any of these items related to the article, the lack of any strongly positive opinions can be contrasted with the findings for Items 24 and 25 related to the play. The consistent pattern of fairly neutral attitudes to Items 23 to 26 is noteworthy as they provide important feedback information about teachers' neutral reactions to four specific aspects of the writing style of the present article. It would be speculated, then, that teachers' attitude to the present article might improve if it were rewritten so that it incorporated less characters and ideas, contained a list of references, showed more clearly the connection between the story and the topic of conics, and better related the mathematics on pages 32-36 of the article to the rest of the article.

2.6 Differences in Opinion between Experienced and Trainee Teachers

Whereas the previous section was devoted to a discussion of the opinions held of the play and the article by the total sample of teachers, attention in this section will turn to comparing and contrasting the opinions held by the 20 experienced teachers with those held by the 19 trainee teachers. Data on differences in opinion between experienced and trainee teachers are shown in Table 3 for the play and in Table 4 for the article. These tables show, for each questionnaire item, the experienced teachers' mean, the trainee teachers' mean, and the results of t tests for independent samples for differences between experienced teachers' and trainee teachers' opinions.

The mean scores recorded for the play in Table 3 indicate that, with the exception of only a few items, trainee teachers expressed a more favourable opinion than did experienced teachers. This pattern can be further illustrated by examining the grand mean of all 25 item means in Table 3, which was 2.52 for trainee teachers but only 2.18 for experienced teachers. Furthermore, the t tests revealed that these differences in opinion between experienced and trainee teachers were significant for the following eight items: trainee teachers' ratings of the usefulness of the play were significantly higher than those of experienced teachers for four aims (arousing student interest in the topic of similar triangles before teaching it, teaching some important topics related to similar triangles, humanizing mathematics, and promoting a better attitude towards mathematics); trainee teachers' ratings of the suitability of the play for Year 9 students of low ability and for students with a poor attitude towards mathematics were significantly more favourable than the ratings given by experienced teachers; trainee teachers' ratings of the usefulness of the play for integrating Mathematics with other subjects like English or Ancient History were significantly higher than the ratings given by experienced teachers; and trainee teachers' attitudes to actually using such a play in their own classrooms were significantly more favourable than experienced teachers' attitudes. The results for this last item are particularly interesting and reveal that the willingness to actually use a mathematical play in one's teaching was considerably greater among trainee teachers than amongst experienced teachers.

Data like those reported for the play in Table 3 are shown for the article in Table 4. The data in Table 4 indicate high overall similarity in the opinions of the article held by experienced and trainee teachers. This similarity is further illustrated by the grand mean of the item means which was 2.14 for experienced teachers and 2.13 for trainee teachers. Of the 27 items in the questionnaire about the play, significant differences in the opinions of experienced and trainee teachers occurred for only two items, namely Items 24 and 25. The interpretation of these two findings was that experienced teachers expressed more favourable opinions than trainee teachers to the statement that such an article does <u>not</u> need a list of references and to the statement that the story related in the article is clearly relevant to the topic of conics.

It is interesting to contrast the present findings for the article with the previous findings for the play. Whereas trainee teachers held clearly more favourable opinions of the play than did experienced teachers, opinions of the article were fairly comparable for experienced and trainee teachers. While there were significant differences for eight items on the questionnaire about the play (with trainee teachers expressing more favourable attitudes in all cases), there were significant differences for only two items on the questionnaire about the article (with experienced teachers expressing more favourable attitudes in both these cases).

2.7 Summary

The present chapter described a survey of the opinions held by 20 experienced teachers and 19 trainee teachers towards various features of the Thales play and the conics article. As well as providing valuable data on teacher opinion about numerous specific aspects of the materials, two more general patterns emerged. Firstly, teachers' opinions of the play were generally more favourable than their opinions of the article. Secondly, while experienced and trainee teachers were found to hold fairly comparable opinions about the article, it was found that trainee teachers' opinions of the play were appreciably more favourable than those held by experienced teachers.

TABLE 3. Means of, and Significance Tests for Differences between, Experienced Teachers and Trainee Teachers for each Item of the Questionnaire about the Play

Questionnaire Item	Teachers' Mean Experienced	Teachers' Mean Trainee	*t*
Aims Satisfied by the Play			
1. Arousing student interest before teaching the topic of similar triangles.	1.80	2.48	3.22**
2. Teaching some history of mathematics.	2.30	2.42	0.59
3. Teaching some important concepts related to similar triangles.	1.80	2.37	2.75**
4. Humanizing mathematics.	1.95	2.58	3.87***
5. Showing that mathematics has practical applications.	2.10	2.53	0.82
6. Providing an awareness of the value of mathematics to society.	1.90	2.00	0.51
7. Providing an appreciation of ancient civilization.	2.05	1.90	-0.77
8. Promoting a better attitude towards mathematics.	1.75	2.27	3.19**
Students Catered for by the Play			
10. Year 7 students of low ability	1.60	1.95	1.34
11. Year 7 students of high ability	2.10	2.37	1.27
12. Year 9 students of low ability	1.55	2.58	2.04*
13. Year 9 students of high ability	2.10	2.37	0.45
14. Year 11 students of low ability	1.45	1.74	1.41
15. Year 11 students of high ability	1.25	1.42	0.92
16. Students with a good attitude towards mathematics	1.95	2.32	1.68
17. Students with a poor attitude towards mathematics	1.70	2.4[illegible]	3.82***
Miscellaneous			
18. Most students would find the material interesting.	2.[illegible]	3.21	1.[illegible]
19. The amount of time taken up if the materials were used in the classroom would [illegible] excessive.	2.[illegible]	2.4[illegible]	0.[illegible]

Cont'd.

TABLE 3. (Cont'd.)

Questionnaire Item	Teachers' Mean		t
	Experienced	Trainee	
Miscellaneous (Cont'd.)			
20. The use of the materials requires only knowledge and skills which mathematics teachers already have.	2.65	2.70	0.07
21. Material like this is not readily available elsewhere.	3.25	3.16	-0.49
22. The average mathematics teacher would not be able to write material like this himself.	3.20	2.74	-1.94
23. The play could be used profitably in the classroom without the need for a stage, costumes, etc.	2.80	2.79	-0.04
24. The play would be very suitable for use near the end of the school year.	3.15	3.48	1.84
25. The play would be useful for integrating Mathematics with other subjects like English or Ancient History.	3.00	3.42	2.45*
26. I would use a play like this in my own mathematics classes.	2.33	3.21	3.83**

* p .05, ** p .01, *** p .001

- Experienced teachers' mean was larger than trainee teachers' mean

TABLE 4. Means of, and Significance Tests for Differences between, Experienced Teachers and Trainee Teachers for each Item of the Questionnaire about the Article

Questionnaire Item	Teachers' Mean		*t*
	Experienced	Trainee	
Aims Satisfied by the Article			
1. Arousing student interest before teaching the topic of conics.	1.85	2.06	0.77
2. Teaching some history of mathematics.	2.15	2.00	-0.75
3. Teaching some important concepts related to conics.	1.60	1.69	0.12
4. Humanizing mathematics.	1.90	2.06	0.72
5. Showing that mathematics has practical applications.	1.65	1.95	1.34
6. Providing an awareness of the value of mathematics to society.	1.60	1.53	-0.42
7. Providing an appreciation of ancient civilization.	2.05	1.84	-0.95
8. Promoting a better attitude towards mathematics.	1.75	1.74	-0.07
Students Catered for by the Article			
10. Year 7 students of low ability	1.20	1.32	0.81
11. Year 7 students of high ability	1.50	1.48	-0.13
12. Year 9 students of low ability	1.45	1.37	-0.50
13. Year 9 students of high ability	1.75	1.95	0.81
14. Year 11 students of low ability	1.70	1.95	1.01
15. Year 11 students of high ability	1.75	2.16	1.82
16. Students with a good attitude towards mathematics	[illegible]	1.95	-0.72
17. Students with a poor attitude towards mathematics	[illegible]	1.79	0.90

TABLE 4. (Cont'd.)

Questionnaire Item	Teachers' Mean Experienced	Teachers' Mean Trainee	t
Miscellaneous			
18. Most students would find the material interesting.	2.45	2.69	1.24
19. The amount of time taken up if the materials were used in the classroom would not be excessive.	2.85	3.00	0.75
20. The use of the materials requires only knowledge and skills which mathematics teachers already have.	2.85	3.00	0.70
21. Material like this is not readily available elsewhere.	2.95	3.00	0.23
22. The average mathematics teachers would not be able to write material like this himself.	3.05	2.90	-0.77
23. The number of different characters and ideas is not too large for an article of this size.	2.80	2.69	-0.60
24. An article like this does not need a list of references.	2.70	2.16	-2.25*
25. The story related in this article is clearly relevant to the topic of conics.	2.55	2.06	-2.52*
26. The mathematics on pages 32-36 is clearly relevant to the rest of the article.	2.58	2.27	-1.38
2[illegible]. This article would be suitable for direct student use in mathematics projects.	2.[illegible]	2.48	-1.71
[illegible] I would use an article like this in planning my own mathematics lessons.	2.[illegible]	2.42	-1.00

[illegible] ** p .01, *** p .001

[illegible]ienced teachers' mean was larger than trainee teachers' me[illegible]

CHAPTER 3

Changes in Affective and Cognitive Outcomes Among Pupils using the Thales Play

3.1 Introduction

Whereas the previous chapter described a questionnaire survey of teacher opinion about certain materials, this chapter describes an investigation of affective and cognitive outcomes among students actually using some materials. The conics article was excluded from this investigation because, as the articles are intended mainly for use by teachers in preparing introductions to lessons, it would be extremely difficult to ascertain whether any observed changes in pupils should be attributed to the article *per se* or to the teacher's skill in preparing and teaching. On the other hand, the Thales play was considered suitable for the present investigation because it had been written for direct use by students. The main purpose of this chapter is to describe the design, execution and results of an investigation of changes experienced by a sample of pupils during their use of the Thales play.

3.2 The Sample

The sample of students involved in the present part of the study consisted of 117 Year 9 students in four different classes, each in a different school. Of these four schools, two were coeducational high schools in the Sydney metropolitan area, one was a Catholic girls' school in the Sydney metropolitan area, and the other was a Catholic boys' school in a country town close to Sydney. The number of boys in the sample was 55 while the number of girls was 62.

It should be appreciated that, for a variety of reasons, the sample employed in the present study was relatively small and not randomly chosen. Instead, the sample was chosen for reasons of convenience and in such a way as to obtain a reasonable spread of geographic areas, types of schools, and pupil general ability, attitude to mathematics and sex. For these reasons, caution should be exercised in generalizing any findings from the present research to a wider population of interest. In particular, as the teachers involved volunteered to use the play, it is possible that these teachers could be atypical of the population of mathematics teachers in some respects.

3.3 Use of the Play

The Thales play was used in each of the four classes in the present sample during the last month of the 1976 school year. In each case, the play was not produced on the stage but used in normal mathematics periods, with different students reading aloud the dialogue for various characters and with students acting out parts of the play where appropriate. Although the project director intended his plays to be actually produced on stage, the Thales play was simply used in regular mathematics classrooms because it was thought that the play would ultimately be used by more mathematics teachers in this way than as a more elaborate stage production. The four teachers involved in the study devoted two or three forty-minute class periods to the use of the play.

Prior to using the play in the classroom, each teacher administered the test of affective and cognitive outcomes described in the next section. Teachers readministered the same test as a posttest soon after completing use of the play. Moreover, teachers were instructed not to teach the topic of similar triangles as a follow-up to the play until after the posttesting was finished. This simple precaution was taken so that, if any changes were observed in pupil understanding of the topic of similar triangles, these changes could be attributed to the experience of doing the play rather than to some teaching on the specific topic occurring between pretest and posttest.

3.4 Student Questionnaire

The questionnaire which was answered by the students, together with scoring procedures, is shown fully in Appendix F. Appendix F indicates that the first page of the questionnaire contains 14 Likert-type attitude items whilst the second page contains five multiple-choice items measuring cognitive achievement.

The attitude items in Appendix F fall into two distinct groups. The first group, namely Items 2, 7, 10 and 13, measure four specific pupil attitudes which could change during the time of use of the play. The four specific attitudes are attitude to learning about the lives of ancient mathematicians (Item 2), attitude to learning about the history of mathematics (Item 7), attitude to the practical applications of mathematics in daily life (Item 10), and attitude to including mathematical plays in mathematics lessons (Item 13). Scores on these four attitude items were not combined to form a meaningful total score but, instead, were used as four distinct attitudinal criteria.

The second group of attitude items in Appendix F comprise a 10-item attitude to mathematics scale developed by Keeves (1974). A student's total score is obtained by summing scores obtained for Items 1, 3, 4, 5, 6, 8, 9, 11, 12 and 14. While Keeves (1974) reported an α reliability estimate of 0.83 when this scale was used with a sample of 140 Year 7 pupils in Canberra, the α coefficient for the present sample of 117 Year 9 students was found to be 0.85 at pretesting and 0.88 at posttesting.

The reason for using Keeves' attitude scale was not that it was thought that pupils would undergo appreciable changes in general attitude to mathematics during the use of a single play. Rather, it was used to furnish data which would permit an investigation of the differential effectiveness of the play for pupils of differing attitudes towards mathematics. This whole issue of the differential effectiveness of the play for different students will form the basis of discussion in the next section. It should also be pointed out that, because of the purposes to which the attitude to mathematics data were put, a single measure of this attitude at [illegible] instance in time would have sufficed. The attitude to mathematics items, however, were administered at pretesting and posttesting for two reasons. Firstly, it proved convenient because it reduced the costs involved in printing two separate questionnaires and because it simplified teachers' instructions for administration by having identical questionnaires on the two occasions. Secondly, the intention of the four items measuring specific attitudes relevant to the play was made a little less obvious to pupils by interspersing these items with the 10 items measuring general attitude to mathematics.

The five multiple-choice items in Appendix F measure areas of cognitive achievement which could be expected to improve due to using the play. While Item 1 measures knowledge of the approximate time when Thales lived, Item 2 measures knowledge of Thales' birthplace, and Items 3-5 each measure application of the concept of similar triangles.

3.5 Choice of Mediating Variables

It is quite possible that a given set of curriculum materials may not be equally effective for all pupils but, instead, be differentially effective for different pupils. For this reason, various writers have advocated the inclusion of certain variables, termed mediating variables, in curriculum evaluation research in order to permit the investigation of differential curricular effectiveness for different sorts of students (e.g., McKeachie, 1963; Cronbach, 1967).

In the present study, three such mediating variables were included. First, pupil general ability scores, obtained from school records by participating teachers, were included. General ability was considered a particularly important variable to include because the project director's stated aim was to inculcate favourable attitudes irrespective of student ability (see section 1.2). Second, pupil general attitude to mathematics was measured with Keeves' (1974) scale described in the previous section. Third, student sex was included as a mediating variable because of the weight of evidence in the literature indicating that boys can vary appreciably from girls in mathematics achievement and attitudes towards mathematics (Turner, 1971; Keeves, 1973; Fennema, 1974). The purpose of including these three variables, then, was to permit an examination of the differential effectiveness of the present play in promoting desirable changes on the present affective and cognitive criteria for pupils of different general abilities, different attitudes towards mathematics and different sexes.

6 Results for Attitudinal Outcomes

Table 5 shows, for each of the four items measuring specific attitudes related to the play, pretest means, posttest means, and results of t tests for dependent samples for changes occurring between pretest and posttest. These data are shown in Table 5 for the whole sample of 117 students and separately for the sample stratified according to student general ability, general mathematics attitude and sex. Each pupil was classified as having either a high or a low level of general ability and either a high or a low level of general mathematics attitude according to whether his or her scores were above or below the total sample's median for general ability and general mathematics attitude, respectively. Using this procedure, it was found that 52 pupils formed the high general ability group while 65 formed the low general ability group, and that 56 pupils formed the high mathematics attitude group while 61 formed the low mathematics attitude group.

The data for Item 2 in Table 5 indicate that a negligible change occurred during the use of the play in attitude to the statement that it would be a waste of time learning about the lives of ancient mathematicians. It was found that this small and nonsignificant change for Item 2 occurred for the sample as a whole and for the sample stratified according to each of the three variables of general ability, mathematics attitude and sex.

For Item 7, it was found that the total sample experienced a significant improvement ($p<.05$) between pretest and posttest in attitude to the statement that learning about the history of mathematics would be interesting. It was also found that a significant improvement on Item 7 occurred for both the high and the low general ability group, and for both the high and the low general mathematics attitude group. When the sample was stratified according to sex, however, it was found that the changes in attitude undergone on Item 7 differed markedly for boys and girls. Whereas boys' changes in attitude on this item were relatively small and nonsignificant, girls' changes were larger and significant. This finding, then, suggests that the play was more effective for girls than boys in promoting a favourable attitude to learning some history of mathematics. This latter finding is particularly noteworthy because, as will be discussed in more detail in Chapter 4, the Thales play contains only a small number of very minor parts for girls.

Results for Item 10 in Table 5 indicate that a quite small and nonsignificant change occurred for the whole sample in attitude to the statement that mathematics has few practical applications to daily life. This finding of a small and nonsignificant difference also occurred for the sample stratified according to general ability, mathematics attitude and sex.

Data for the last item in Table 5 indicate that the sample as a whole experienced a significant improvement between pretest and posttest in attitude towards using mathematical plays in mathematics lessons. Moreover, when the sample was stratified according to general mathematics attitude, it was found that the play proved differentially effective for pupils of different general mathematics attitude in promoting improved attitudes towards mathematical plays. Whereas pupils with a higher level of general attitude towards mathematics experienced a significant improvement in their attitude to mathematical plays, students of lower general mathematics attitude experienced a smaller and nonsignificant change in their attitude to mathematical plays. This finding, then, suggests that pupils who already have a more favourable attitude to learning mathematics are more likely to enjoy using mathematical plays than pupils with less favourable attitudes.

In summary, it can be seen from Table 5 that the total sample experienced a significant improvement on two of the four specific attitudes measured. This pattern of results can be interpreted quite

TABLE 5. **Significance Tests for Differences between Pretest and Posttest on each Attitude Item for the Whole Sample and for Subsamples Stratified according to General Ability, Mathematics Attitude and Sex**

Item	Sample	Means		t
		Pre	Post	
2. It would be a waste of time learning about the lives of mathematicians who lived thousands of years ago.[a]	Whole sample	2.98	3.05	0.60
	High general ability	3.23	3.44	1.35
	Low general ability	2.78	2.72	-0.38
	High math. attitude	3.32	3.54	1.57
	Low math. attitude	2.67	2.61	-0.37
	Boys	2.62	2.66	0.22
	Girls	3.31	3.39	0.52
7. Learning about the history of mathematics would be interesting.	Whole sample	2.61	3.00	3.93***
	High general ability	2.88	3.33	2.92**
	Low general ability	2.38	2.74	2.65**
	High math. attitude	3.04	3.48	3.38**
	Low math. attitude	2.21	2.56	2.30*
	Boys	2.22	2.49	1.97
	Girls	2.95	3.45	3.49**
10. Mathematics has few practical applications to daily life.[a]	Whole sample	3.57	3.69	1.47
	High general ability	3.67	3.77	0.84
	Low general ability	3.49	3.63	1.20
	High math. attitude	3.87	4.02	1.21
	Low math. attitude	3.30	3.39	0.56
	Boys	3.82	3.95	1.07
	Girls	3.35	3.47	1.00
13. I would enjoy mathematics lessons more if they included some plays related to mathematics.	Whole sample	3.12	3.38	2.22*
	High general ability	3.27	3.52	1.83
	Low general ability	3.12	3.28	1.32
	High math. attitude	3.25	3.52	2.27*
	Low math. attitude	3.13	3.26	1.00
	Boys	3.07	3.27	1.59
	Girls	3.29	3.4[illegible]	1.54

* p<.05, ** p<.01, *** p<.001

\- Post test scores were lower than pretest scores

a Scoring on this item has been reversed so that a higher score corresponds to a more positive attitude favourably especially when it is remembered that the play only occupied about two or three normal teaching periods. Moreover, it is of interest to compare the magnitude of the changes occurring in the mean score for these two items with the magnitude of those occurring for the general attitude to mathematics scale. It was found that the mean score on the whole 10-item general attitude to mathematics scale changed only about 0.14 of a raw score point (or 0.014 points per item) between pretest and posttest. Of course, this result is to be anticipated as a play occupying two or three class periods could not be expected to have an appreciable impact on pupils' general attitude to mathematics. These very small changes in means for items measuring general attitude to mathematics, however, can be contrasted with the changes in item means of 0.39 and 0.26 which occurred for the two specific attitudes, namely those measured by Items 7 and 13 for which significant improvements occurred between pretest and posttest.

3.7 Results for the Cognitive Outcomes

Whereas Table 5 summarized analyses for the four specific attitudinal outcomes, Table 6 summarizes data from analogous analyses performed for the five cognitive outcomes (which are measured by items appearing on the second page of the questionnaire in Appendix F). It should be pointed out, however, that the *t* tests used previously with data for the attitude items, which were scored on a five-point scale, are not applicable to the dichotomously scored multiple-choice cognitive items. Instead, the statistical technique chosen for testing the significance of differences between pretest and posttest performance on the dichotomously scored cognitive items was a *z* test for correlated proportions. The original development of this *z* test has been described by McNemar (1947), while its application in curriculum evaluation studies has been discussed in more detail in Fraser (1973).

Table 6 shows, for each of the five cognitive criteria, the number of students correct on the pretest, the number correct on the posttest, and the results of the *z* tests for differences between pretest and posttest performance. Furthermore, this table displays these data for the sample as a whole and separately for the sample stratified according to pupil general ability, general attitude to mathematics and sex.

The results in Table 6 for the first cognitive item, which measures knowledge of the approximate time when Thales lived, indicate that a significant improvement in performance on this item did occur for the sample as a whole. Moreover, it was found that improvement on this particular item did vary with pupil general ability, mathematics attitude and sex. While the high general ability group, the high mathematics attitude group and boys experienced a significant increase in performance on Item 1 between pretest and posttest, changes were nonsignificant for the low general ability group, the low mathematics attitude group and girls. This finding suggests that the play was more successful in bringing about improved knowledge of the time when Thales lived for pupils of high general ability, for pupils of high general attitude to mathematics and for boys.

The aim of the second item in Table 6 is to measure knowledge of Thales' birthplace. The data shown for this item indicate that a significant improvement in performance between pretest and posttest did occur for the sample as a whole and for each subsample formed by stratifying according to general ability, mathematics attitude and sex. The present pattern of findings, therefore, suggest that the play was successful in promoting an improvement in knowledge of Thales' birthplace and that this improvement was independent of student general ability, mathematics attitude and sex.

The pattern of results shown in Table 6 is very similar for Items 3, 4 and 5, which each measure application of the concept of similar triangles. It was found that differences between pretest and posttest performance were nonsignificant both for the sample as a whole and for each subsample formed by stratifying according to general ability, mathematics attitude and sex. These results suggest that the use of the play was ineffective in promoting the aim of applying the concept of similar triangles. This finding that pupils actually experienced little gain in their learning about similar triangles can be contrasted with the data from the teacher survey (Table 1) which showed that teachers tended to feel that the play would be effective in teaching concepts related to similar triangles. It should also be emphasized that, because many students in the present sample of Year 9 students already had a reasonable understanding of the topic of similar triangles prior to using the play, the number getting Items 3 and 4 correct at pretesting was high and left little scope for improvement. However, this was not the case for Item 5, and there was plenty of scope for an improvement in performance on this item if the play were effective.

It is interesting to contrast the results for the first two cognitive items measuring simple historical knowledge with the last three items measuring application of mathematical concepts. It was found (Table 6) that the sample as a whole experienced a significant improvement on both of the items measuring simple historical knowledge but underwent smaller and nonsignificant changes on all three items measuring application of mathematical concepts. It would appear, then, that the present use of the Thales play was more effective in promoting improvement in historical knowledge than in promoting improvement in mathematical achievement.

3.8 Discussion

This chapter has described an investigation of the changes in affective and cognitive outcomes occurring among a sample of Year 9 students who used the Thales play. Although a number of interesting findings emerged, results should be considered tentative for two important reasons outlined below. Firstly, it has already been pointed out in section 3.2 that the present sample, although representing a wide cross-section of students along several dimensions, was still fairly small, nonrandomly chosen and comprised of students whose teachers had volunteered to use the play. Secondly, the method of exploring differential curricular effectiveness in Table 5 by using t tests on subsamples does not provide as rigorous a test as some analysis of covariance techniques would. The latter technique, however, was considered unnecessarily sophisticated for the nature of the present data and, at any rate, no technique analogous to analysis of covariance is available for the dichotomously-scored data of Table 6.

TABLE 6. **Significance Tests for Differences between Pretest and Posttest on each Achievement Item for the Whole Sample and for Subsamples Stratified According to General Ability, Mathematics Attitude and Sex**

Aim Measured by Item	Sample	Sample Size	No. Correct Pre	No. Correct Post	$\underline{z}$
1. Knowledge of the approximate time when Thales lived.	Whole sample	117	39	58	2.90**
	High general ability	52	19	35	3.58***
	Low general ability	65	20	23	0.63
	High math. att.	56	17	29	2.56*
	Low math. att.	61	22	29	1.53
	Boys	55	19	30	2.29*
	Girls	62	20	28	1.79
2. Knowledge of Thales' birthplace.	Whole sample	117	49	107	7.49***
	High general ability	52	24	50	5.10***
	Low general ability	65	25	57	5.49***
	High math. att.	56	25	51	4.91***
	Low math. att.	61	24	56	5.66***
	Boys	55	20	51	5.57***
	Girls	62	29	56	5.01***
3. Application of the concept of similar triangles.	Whole sample	117	111	106	-1.51
	High general ability	52	52	49	1.73
	Low general ability	65	59	57	0.71
	High math. att.	56	53	53	0.00
	Low math. att.	61	58	53	-1.89
	Boys	55	54	52	-1.41
	Girls	62	57	54	1.00

(Cont'd. next page)

TABLE 6. (Cont'd.)

Aim Measured by Item	Sample	Sample Size	No. Correct Pre	No. Correct Post	z
4. Application of the concept of similar triangles.	Whole sample	117	107	108	0.45
	High general ability	52	52	51	-1.00
	Low general ability	65	55	57	1.00
	High math. att.	56	49	52	1.73
	Low math. att.	61	58	56	-1.41
	Boys	55	55	53	-1.41
	Girls	62	52	55	1.73
5. Application of the concept of similar triangles.	Whole sample	117	69	72	0.69
	High general ability	52	40	38	0.63
	Low general ability	65	29	34	1.66
	High math. att.	56	34	35	0.38
	Low math. att.	61	35	37	0.58
	Boys	55	43	45	0.58
	Girls	62	26	27	0.38

* $p<.05$, ** $p<.01$, *** $p<.001$

\- Number of students correct on the posttest was smaller than the number correct on the pretest.

The analyses reported in this chapter revealed that the present sample underwent significant changes on both affective and cognitive outcomes during the time of using the Thales play. Of the four specific attitudes measured, significant improvements were found to occur between pretest and posttest on two, namely attitude to learning about the history of mathematics and attitude to using historical plays. Significant changes were also found to occur on the two cognitive items measuring knowledge of simple historical facts, namely the time when Thales lived and Thales' birthplace. However, differences between pretest and posttest performance were nonsignificant for each of the three cognitive items measuring application of the concept of similar triangles. The present finding of significant differences on four out of nine outcomes considered, which is about nine times the number expected by chance, suggests that the Thales play did have an appreciable and favourable impact on students.

This finding of several changes in students is particularly noteworthy because an average of only two or three class periods was devoted to the use of the play and because the study was conducted during the last month of the school year when student morale could be expected to have been relatively low.

Some evidence was also found to suggest that the Thales play was differentially effective for different students. Girls, but not boys, experienced a significant improvement in attitude to learning about the history of mathematics. While students of higher general attitude towards mathematics experienced a significant improvement in attitude to using mathematical plays, students of lower general attitude experienced a small and nonsignificant change. Improvement in pupil knowledge of the time when Thales lived was significant for pupils of higher general ability, higher general mathematics attitude and boys, but was nonsignificant for pupils of lower general ability, lower general mathematics attitude and girls.

CHAPTER 4

IMPRESSIONISTIC EVALUATION

4.1 Introduction

The previous two chapters have described parts of the present evaluative study which involved a fairly structured examination of a limited range of materials. In fact, Chapter 2 described a survey of teacher opinion about the Thales play and the conics article while Chapter 3 described changes in cognitive and affective outcomes experienced by pupils using the Thales play. In contrast to previous chapters, this chapter is devoted to a more impressionistic evaluation of a broader variety of aspects and range of materials.

One source of impressionistic evaluative information was a group of five experts - a historian of mathematics, two mathematics educators and two drama critics - who provided their views about the Thales play, the conics article, or both. Each of the experts was left free to comment on any aspect of the material they wished rather than being requested to respond to specific questions. Nevertheless, each expert was specifically requested to provide opinions about the overall efficacy and usefulness of the materials (summative evaluation) and to identify any specific weaknesses which might be improved in any future rewritten versions of the materials (formative evaluation).

4.2 Impressions of a Historian of Mathematics

Appendix G contains the impressions of the Thales play and the conics article provided by a historian of mathematics, namely Dr John Pottage of the Department of History and Philosophy of Science at the University of Melbourne. While the reader is encouraged to read the illuminating comments in Appendix G in full, some salient points are listed below.

The following overall comments were made about the play and article.

- The play is considered valuable and should be published for school use after modification.
- The conics article is not considered valuable because it is historically misleading and because of the confusing juxtaposition of mathematics and Greek myths.

The above comments are particularly interesting because they are consistent with the finding from the survey of teacher opinion (Chapter 2) which indicated that teachers tended to view the Thales play more favourably than the conics article.

The following specific criticisms were made of the Thales play.

- Applications of the method of similar triangles can be found in historical records which pre-date the time of Thales by centuries.

- The assertion that Thales was exactly 60 years old in 640 B.C. is more precise than present historical knowledge permits.
- Thales' method of measuring the height of a pyramid would not work if Thales' shadow were a half or a third of his height because, then, the pyramid would throw no shadow at all owing to its shape.
- Measuring a ship's distance out to sea using a five foot staff may not be any more accurate than an experienced look-out's estimate.
- Greeks of this period did not measure angles in "degrees" as suggested in the play.

In addition to modifications suggested by the above comments, other positive suggestions were to

- include a few historical notes (disclaimers) with the play so that the reader wouldn't take everything too literally.
- add suggestions about possible discussion and field work which the teacher could conduct after the play had been used.

4.3 Impressions of Two Mathematics Educators

Appendix H contains a statement of impressions of the Thales play and the conics article provided by two mathematics educators responsible for the training of mathematics teachers in two different universities. The first opinion in Appendix H was obtained from Mr Ken Clements of the Faculty of Education at Monash University while the second opinion came from Mr Brian Low of the School of Education at Macquarie University. Some of the salient comments from Appendix H are summarized below.

- Both mathematics educators contended that the Thales play was more worthwhile than the conics article. (This is consistent with discussion in section 4.2)
- A booklet containing numerous mathematical plays would be valuable.
- The idea of bringing the history of mathematics into the classroom is laudatory.
- The conics article is suitable mainly for students who already have a favourable attitude to mathematics.
- The Thales play may be more suitable for use as a follow-up to work on similar triangles than as an introduction to the topic.
- In-service education would be needed before teachers could put the materials to optimum use.

4.4 Impressions of Two Drama Critics

Appendix I contains the impressions of the Thales play solicited from two drama critics from the School of Education at Macquarie University. The first statement in Appendix I was made by Ms Sandra Alexander, who is involved in media education, and the second statement was made by Mrs Nel George, who is responsible for the training of English teachers. Some of the comments contained in Appendix I are summarized below.

- Both drama critics considered that the Thales play had a number of positive dramatic qualities including its dialogue, settings and movement.
- The play is useful for integrating subject areas.
- The cast is large enough to involve a sizeable proportion of a class.
- The play would be relatively easy to produce.
- There is a lack of parts for girls.

4.5 Teachers' Impressions

The main source of teachers' impressions of the Thales play and conics article were responses to some open-ended questions contained in the questionnaires used in the survey described in Chapter 2. It can be seen from Appendices D and E that Item 9 and the last item in each questionnaire are open-ended questions designed to solicit teachers' impressions. The following comments made by teachers on the open-ended questions were virtually identical to experts' comments discussed in sections 4.2 - 4.4.

- It is impossible to distinguish history and myth in the conics article.
- There are too few femal parts in the Thales play.

Some of the other interesting comments made by teachers on the open-ended items are listed below.

- The play should be modified to make it suitable for use in the classroom rather than on stage.
- The play requires some explicit directions to actors for demonstrating the method of measuring distances of ships out to sea.
- The play would be useful for motivating students but not for teaching some mathematics.
- The play would give students of low mathematical ability a chance to really participate in some mathematics lessons.
- There is a need for this sort of article, but the conics article is not very well written.
- The article is too condensed and contains too many ideas and characters.
- The article doesn't make it really clear how conics were used to solve the Delian problem.

The four teachers who actually used the Thales play in their classrooms for the part of the evaluation study described in Chapter 3 were able to provide insightful reactions to the play. In general, these four teachers were in agreement that the play

- was favourably received by students.
- could be used effectively in the classroom, without the need for a time-consuming stage production.

4.6 Project Director's Impressions

In any innovative project, the role played by the project director is crucial in determining the nature and success of the project. Also, in the evaluation of an innovative project, the impressions of the project director himself constitute a most valuable source of evaluative information. For this reason, the project director was asked to provide a statement of his impressions. Some salient points from the project director's statement, which is fully reproduced in Appendix J, are summarized below.

- Some real interest has been shown in the project, as evidenced by numerous invitations to the project director to talk at in-service or association meetings of mathematics teachers, and an invitation to contribute to a Victorian magazine, Pursuit.
- The video equipment purchased with Schools Commission funds provided considerable motivation to the project director in his total endeavour, and provided a useful medium of dissemination at mathematics teachers' meetings.
- The project director is seriously disappointed at his lack of opportunity to see his plays produced on stage. (Evidence reported in section 4.5, however, suggests that these plays could be profitably used in a less elaborate way in mathematics classes.)
- The project director is aware that his materials may contain some historically inaccurate ideas. (This is consistent with the historian of mathematics' comments in section 4.2.)
- The project director is aware that the level of presentation and editing of some of his materials needs improving. (This is consistent with previous evidence, particularly section 4.5.)
- The project director feels a serious sense of isolation and experiences a real desire for feedback and constructive criticism.

This last comment, which reflects the project director's desire for feedback, emphasizes one of the major merits of an augmented evaluation like the present one. In addition to providing evaluative information for other teachers likely to use the materials, a major purpose satisfied during the present evaluation has been to provide contact with and encouragement to the project director and to collect [illegible] the materials.

4.7 Evaluation of Dissemination

It was pointed out in section 1.5 that the present materials have the potential of being useful in a wide range of schools around Australia. Consequently, the dissemination of the materials is both an important aspect of the project and an issue which warrants mention in this evaluation report.

So far, three avenues of dissemination have been employed. Firstly, various articles have been published in state (but not national) teachers' journals (see Appendix A). Secondly, the project director has spoken at numerous teacher in-service and mathematics associations meetings. Thirdly, the Teachers' Resources Centre has published some materials (and will publish more in the future) based on the project director's work.

While the above description clearly indicates that a substantial amount of dissemination has already taken place, it is likely that even wider dissemination might have been achieved. For example, the use of national instead of state journals would have enabled more teachers to be reached. This issue of dissemination, which is clearly an important one, will be reconsidered in the last chapter where certain recommendations about future dissemination activities will be made.

4.8 Other Evaluative Information

While the previous sections have dealt specifically with the impressions of a historian of mathematics, mathematics educators, drama critics, teachers and the project director, the points below summarize any other evaluative information which was uncovered in the present study but has not yet been described in the report.

- The Thales play was produced on stage at St. Ives High School, with high ability Year 9 pupils acting the parts before a large audience of students from various classes in the school. Several salient comments were made by the mathematics teachers involved: first, the students performing the parts had a rewarding experience; second, pupils in the audience were generally disinterested; third, the large amount of time and effort involved in producing the play on stage was considered excessive for the benefits achieved.
- The editor of <u>The Australian Mathematics Teacher</u> saw one of the present articles and considered that it was limited in usefulness because of the absence of specific suggestions about ways in which the teacher could use the article in mathematics classes.
- The chairman of the Mathematics Equipment Committee of the N.S.W. Department of Education reported that the slide-tape presentation "The History of Conics" has been exhibited at numerous in-service courses and has met with considerable approval from teachers.
- The video equipment purchased with School [illegible] funds [illegible] very useful for recording activities at Vaucluse Boys' High School's annual mathematics ski camp at Jindabyne National Fitness Centre.

- Other staff members teaching various subjects at Vaucluse Boys High School have sought the project director's advice on how to make effective use of video equipment in their teaching.

4.9 Summary

This chapter has been devoted mainly to reporting the impressions held by various people towards the present innovative project. In particular, discussion focussed on opinions expressed by a historian of mathematics, mathematics educators, drama critics, various mathematics teachers and the project director himself. In contrast to the present chapter, Chapter 2 described a survey of teacher opinion about some of the materials while Chapter 3 described an investigation of changes in affective and cognitive outcomes experienced by students during the use of the Thales play. In the following chapter, the evaluative information described in Chapters 2-4 will be briefly summarized to form the basis for drawing conclusions and making recommendations.

CHAPTER 5

SUMMARY AND IMPLICATIONS

5.1 Introduction

This last chapter covers three areas. Firstly, salient aspects from previous chapters will be summarized. Secondly, problems encountered in attempting to interpret findings of the study will be discussed. Thirdly, suggestions will be made for future modification, development and dissemination of project materials.

5.2 Summary of Previous Chapters

It is not possible to adequately summarize a report of this size in a few paragraphs and, therefore, the reader is urged to return to relevant parts of the report for clarification of the brief statements below. Chapter 1 provided a description of the innovative project, emphasized that the project was atypical of many Schools Commission Innovations Program projects in that a major orientation was to produce products that would be useful in a wide range of schools, and provided a justification for deciding to study a limited range of materials (namely the Thales play and the conics article) intensively. Chapter 2 described a survey of teacher opinion about the Thales play and the conics article; in particular a comparison was made between opinions about the play and opinions about the article, and between experienced teachers' opinions and trainee teachers' opinions. Chapter 3 described the use of the Thales play with Year 9 students, a study of changes in affective and cognitive outcomes experienced by pupils using the play, and an investigation of the effectiveness of the play for pupils of varying general ability, general attitude to mathematics and sex. Chapter 4 was devoted to impressionistic evaluation and described the opinions held about the materials by various people including a historian of mathematics, two mathematics educators, two drama critics, teachers and the project director himself.

Salient findings from the previous chapters are summarized below.

- A very favourable opinion of the play, but an appreciably less favourable opinion of the article, was expressed in the teacher survey (section 2.4), by a historian of mathematics (section 4.2) and by two mathematics educators (section 4.3).

- Trainee teachers' opinions of the play were more favourable than experienced teachers' opinions, but opinions about the article were fairly comparable for trainee and experienced teachers (section 2.6).

- The play was considered to have some positive dramatic qualities (section 4.4).

- Pupils who used the Thales play experienced a significant improvement in attitude to learning history of mathematics.

attitude to using historical plays, knowledge of the approximate time when Thales lived, and knowledge of Thales' birthplace (sections 3.6 and 3.7).

- Pupils experienced small, nonsignificant changes on three items measuring application of the concept of similar triangles (section 3.7). A possible explanation for this is that the exposition of similar triangles in the play may not be clear (section 4.2).

- Evidence was found to suggest that the play was differentially effective for different students: girls experienced a greater improvement in attitude to learning history of mathematics; pupils of more favourable general attitude to mathematics experienced a greater improvement in attitude to using mathematical plays; and improvement in pupil knowledge of the time when Thales lived was greater for pupils of higher general ability, pupils of higher general attitude to mathematics, and girls.

- A major educational merit of the play and article is that teachers perceived them as neither readily available elsewhere nor able to be written by the average teacher (section 2.4).

- The play was considered to be useful for integrating mathematics with other subjects like Ancient History and English (sections 2.5 and 4.4).

- Teachers tended to be worried that the use of the play would require excessive time and capabilities not possessed by mathematics teachers (section 2.4). It is suggested, however, that these worries could be largely overcome if the play were used in normal mathematics classes rather than in a full-scale stage production.

- Specific faults in the article were perceived to be that it involved too many characters and ideas, lacked a list of references and was not clearly related to the topic of conics (sections 2.5 and 4.5).

- The juxtaposition of myth and mathematics in the article was considered of dubious value (sections 4.2 and 4.5).

- It was thought that the amount of mathematics contained in the play and article was relatively small (section 4.5).

- A historian of mathematics found a number of historical inaccuracies in the play and article. While historical accuracy is certainly desirable, Dittrich has contended that materials containing historical inaccuracies may still have educational merit:

> Teachers with only amateur interest in this aspect of mathematics - philosophy and history - will jumble facts, repeat gossip, and miss or misrepresent trends; nevertheless, they can generate first-rate enthusiasm and excite students to [illegible] firsthand research, even if

> only to correct the teachers' misconceptions (Dittrich, 1973, p. 35).

- There is a lack of female parts in the play (sections 4.4 and 4.5).

- There is a need for plays and articles to be accompanied by suggestions for the way teachers can use the materials and provide follow-up activities (sections 4.2 and 4.8).

5.3 Problems in Interpreting Findings

While the previous section included a summary of the actual findings from the present research, discussion in this section will be devoted to various problems involved in attempting a straightforward interpretation of these findings. In discussing curriculum evaluation research, Tawney has commented that

> (n)ewcomers to the area are led to expect objective techniques which will lead to unquestioned judgements ... (Tawney, 1976, p. 34).

In the present study, although a lot of valuable feedback on strengths and weaknesses of the materials has been gathered, a simple unquestioned judgement about the materials certainly cannot be made. Also, in interpreting findings from the present study, it should be emphasized that the range of evaluative aspects considered was only a small sample of the whole domain of relevant aspects. The importance of interpreting particular evaluations against a backdrop consisting of all relevant evaluative aspects has been pointed out by Forehand:

> A study that purports to evaluate a curriculum ... is doomed to be judged inadequate by comparison with all relevant studies that might have been done ... (E)valuation efforts can be made more effective by ... a comparison of what it is doing with what it might be doing (Forehand, 1971, pp. 577-578).

Some caution in generalizing findings is also needed because the present samples of teachers, students and schools were relatively small and nonrandomly chosen. Ideally, the findings from the present investigation should be checked in further replication studies before more confidence is placed in the results. Nevertheless, the present sample included a wide range of teachers and students and, in order to make the present study useful to educators, it is recommended that tentative conclusions be drawn while awaiting replication studies.

5.4 Recommendations

It was pointed out previously that, although the period of Schools Commission funding has already terminated for the present project, the project director continues to be actively engaged in modifying and extending his materials. Some suggestions, derived from findings of the present evaluation study, which may assist the project director in his future efforts are listed below.

- **More effort in the immediate future should be devoted to modifying and consolidating past work than to embarking on new ventures.**

- **Where possible, opinions on preliminary versions of materials be sought from people with expertise in editing, history of mathematics and drama before completing the final versions.**

- **A greater proportion of the time spent on the project be devoted to plays, both in modifying existing ones and developing new ones.**

- Plays should be adapted so that they are suitable for use in the classroom as well as on the stage.

- More female parts should be included in the plays.

- Future articles should have an approach which is different from past articles in that there is less justaposition of myth and mathematics, that the number of ideas and characters in a single article be limited, and that a list of source references be provided.

- Some guidance be given to teachers about possible ways of incorporating articles and plays into their teaching, and about possible follow-up discussions and activities for the plays and articles.

The other recommendations to be made here involve the dissemination of materials. Evidence accrued in this evaluation has indicated the potential usefulness of some of the present project materials in other schools around Australia. An important future concern of the project director, then, should be with the dissemination of his materials. Some suggestions for dissemination are given below.

- The project director should continue his association with the N.S.W. Department of Education Mathematics Equipment Committee as this provides both a production outlet (Teachers' Resources Centre) and access to other teachers on the committee and to editorial and technical assistance.

- An attempt should be made to publish important articles in national instead of state mathematics teachers' journals. (This, however, would require an improved quality of presentation.)

- The project director should continue to speak at in-service and teachers' association meetings where possible. However, in order to make these of maximum usefulness, the project director should discuss with various people how he could best present his materials.

- Some avenue for making the scripts of plays (including the one act plays on videotape) available widely.

should be sought. Suggestions include attempting to publish a small booklet of plays, or approaching the editor of a national journal for mathematics teachers.

- The project director might find it profitable to solicit the help of other interested persons so that developing, modifying and editing materials could become a team effort.

5.5 Conclusion

The main purpose of this chapter has been to summarize salient findings from the present evaluation study and, based on these findings, make recommendations for future modification, extension and dissemination of the project materials. Some of the problems involved in interpreting findings from the evaluation study were also considered, and it was stressed that the present findings should be considered somewhat tentative until more evidence about the effectiveness of the present materials has accrued.

BIBLIOGRAPHY

Aiken, L. R., Jr. Attitudes toward mathematics. Review of Educational Research, 1970, 40, 551-596.

Aiken, L. R., Jr. Update on attitudes and other affective variables in learning mathematics. Review of Educational Research, 1976, 46, 293-311.

Cronbach, L. J. How can instruction be adapted to individual differences? In R. M. Gagné (Ed.), Learning and individual differences. Columbus, Ohio: Merrill, 1967.

Curcic, B. Mathematical quotations for all occasions. Mathematics Teacher, 1976, 69, 40-44.

Dittrich, A. B. An experiment in teaching the history of mathematics. Mathematics Teacher, 1973, 66, 35-38

Fennema, E. Mathematics learning and the sexes: A review. Journal for Research in Mathematics Education, 1974, 5, 126-139.

Forehand, G. A. An evaluation system for curriculum innovation. Teachers College Record, 1971, 72, 577-591.

Francis, R. L. History of mathematics in the training program for elementary teachers. The Arithmetic Teacher, 1976, 23, 248-250.

Fraser, B. J. Curriculum evaluation Part II: An illustration using ASEP trial data. Australian Science Teachers Journal, 1973, 19(2), 42-53.

Green, D. R. History in mathematics teaching: Before the advent of modern mathematics. Mathematics in School, 1976a, 5(3), 15-17.

Green, D. R. History in mathematics teaching: Modern times. Mathematics in School, 1976b, 5(4), 5-9.

Keeves, J. P. Differences between the sexes in mathematics and science courses. International Review of Education, 1973, 19, 47-63.

Keeves, J. P. Some attitude scales for educational research purposes. Melbourne: Australian Council for Educational Research, 1974.

Kinney, L. B. & Purdy, C. R. Teaching mathematics in the secondary school, (3rd edition). New York: Rinehart, 1956.

McKeachie, W. J. Problems and perils in controlled research in teaching. In E. R. Steinberg (Ed.), Needed research in the teaching of English. Washington: U.S. Office of Education, 1963.

McNemar, Q. Note on the sampling error of the difference between correlated proportions or percentages. Psychometrika, 1947, 12, 153-157.

Ministry of Education. Teaching mathematics in secondary schools, London: HMSO, 1958.

National Council of Teachers of Mathematics. Historical topics for the mathematics classroom, Thirty-first Yearbook, Washington, D.C.: The Council, 1968.

Read, C. B. The history of mathematics - A bibliography of articles in English appearing in seven periodicals. School Science and Mathematics, 1966, 66.

Tawney, D. Evaluation - Information for decision-makers. In D. Tawney (Ed.), Curriculum evaluation today: Trends and implications, London: Macmillan, 1976.

Turner, N. D. Can the girls be creeping up on the boys? International Journal of Mathematical Education in Science and Technology, 1971, 2, 337-340.

APPENDIX A

LIST OF PROJECT MATERIALS

PLAYS:

1. Pythagoras
2. Thales of Miletus
3. Archimedes of Syracuse

ARTICLES:

1. The history of conics. Pivot, 3(1), 1976; The genesis of conics. A Mathematics Bulletin, No. 25, 1976.
2. Dido and the founding of Carthage - A history of the quadratic equation and maxima. A Mathematics Bulletin, No. 25, 1976.
3. Eratosthenes of Cyrene. A Mathematics Bulletin. (in press)

SLIDE-TAPE PRESENTATIONS:

1. The History of Conics (published, together with teachers' notes, by the Teachers' Resources Centre).
2. Other slide-tape presentations on Eratosthenes, Thales, Pythagoras, Archimedes and Fibonacci are in various stages of preparation.

VIDEOTAPES:

1. Students in the project director's mathematics classes have made videotapes of one-act plays about discoveries in mathematics.
2. The use of video equipment in recording one-act plays has been described in the following article written by the project director: Filming and the use of motivational video film. Vinculum, No. 5, 1975; A Mathematics Bulletin, No. 26, 1976.

OTHER MATERIALS:

1. A portrait and biography for each of 30 mathematicians are in preparation and under consideration by the Mathematics Equipment Committee for recommendation for publication through the Teachers' Resources Centre.

2. Photographs have been taken of the four Platonic solids at the Mining Museum and of Australian aboriginal pertroglyphs. It is intended that these photographs be used in conjunction with materials to be written later on the history of geometric solids and aboriginal mathematics, respectively.

3. A series of journal articles have been written which describe the innovative project in toto and various suggestions for incorporating history of mathematics into teaching. These journal articles are listed below.

   . Encouragement and innovations, Vinculum, September, 1975.

   . With a little encouragement, Teacher Feedback, September, 1976.

   . The classroom as a teaching aid, Vinculum, October, 1975; A Mathematics Bulletin, No. 24, 1975.

   . High interest level mathematics activities camps, A Mathematics Bulletin, No. 24, 1975.

   . Around the world mathematical tours, Vinculum, May, 1976; A Mathematics Bulletin, No. 26, 1976.

   . The history of mathematics in the classroom - Teaching strategies, Vinculum, September, 1976.

   . Estimating time, Pivot, 3(3), 1976.

APPENDIX B

THE 'THALES OF MILETUS' PLAY

ACTORS IN THE PLAY

Thales **(In his maturity)**

Anaximander **(His pupil and companion)**

Merchant **(Eager to trade, frightened of war, yet still eager for business)**

First Egyptian Priest: **Nuiserra**

Second Egyptian Priest: **Manetho**

Third Egyptian Priest: **Zoser**

Narrator

Soldier 1

Soldier 2

Pirate Leader

Servant

Egyptians

Dancing Girls, **Wrestlers, Acrobats**

Guide

Guards, Crowds

ENTER NARRATOR DRESSED AS AN ANCIENT GREEK

NARRATOR: This is the story of Thales of Miletus. A simple man, and a gentleman, he was given the title of "One of the seven wise men of Greece". The year in which this play occurs is 540 B.C. The places, Miletus and Egypt. Come with me now and look at the civilised world in which Thales lived. Civilised? - Perhaps! It was much the same as today - peopled by men and women of destiny, whose sole ambition was to become great at the expense of others. There were wars, revolutions and innocent people dying, but there was progress also because some people were progressive and contributed to the quality of life.

Thales, the Founder of Modern Mathematics, was such a man.

Think back to the golden days of Miletus. Here we are at the busy port where merchants are busily selling their goods for profit, and clever men trying to buy these goods for next to nothing with other peoples' money, so that everyone can make a profit - that's right - that's business.

I have said enough. Curtains. Let the play begin.

- SCENE 1 -

THE SCENE TAKES PLACE AT THE DOCKS AT A PORT AT MILETUS AROUND 540 B.C. IN THE BACKGROUND THERE ARE FISHING NETS, OARS AND SAIL. THERE IS A CONSTANT MOVEMENT OF PEOPLE, MERCHANTS INSPECTING MERCHANDISE, AND HAGGLING OVER PRICES; SAILORS REPAIRING NETS, FISHERMEN WITH THEIR CATCHES, AND THE OCCASIONAL SOLDIER PATROLING THE WATERFRONT.

ON THE FRONT STAGE THERE ARE SOME CRATES; ONE IS OPENED AND ITS CONTENTS ARE BEING EXAMINED BY TWO MEN, THALES AND A MERCHANT. THALES IS AN ALERT AND ACTIVE MAN OF SIXTY. THE MERCHANT IS PORTLY AND DIGNIFIED AND KEEN TO SUCCEED IN HIS BUSINESS TRANSACTIONS WITH THALES. THALES IS EXAMINING AN OIL PRESS.

THALES: These oil presses look solid enough.

MERCHANT: (Slaps the press as he makes his point) Solid? Iron hard! There's years of life in these presses. They'll crush all the olives picked here in Miletus, ... but you only want to rent them from me for one season?

THALES: That's right, one season. These all you've got?

MERCHANT: All ... In fact all in Miletus.

THALES: (Smiles) Good ... I'll rent the lot for one season, provided of course that the price is fair.

MERCHANT: You know, it took me a long time to find all these presses.

THALES: Hm ... yes, I'm sure it did.

MERCHANT: To gather and carry all of them took my sons considerable effort, and then there are my expenses, hm ... let's say thirty five gold pieces. (Watches Thales for a moment) It's a fair price, I could not do it for less.

THALES: (Quiet for a moment as he contemplates) Hm ...

MERCHANT: It's the best I can do.

THALES: Well then, I'll take them.

MERCHANT: (He smiles and holds out his arm for the traditional arm clasp, binding an agreement) It's a deal then.

THALES: A deal.

MERCHANT: I can't help wondering ...

THALES: Wondering? About what?

MERCHANT: What are you going to do with them all?

THALES: What do you think I am going to do with many oil presses? Make money, what else?

MERCHANT: Well, I've been trying to make money for a long time, but with [illegible] great success. (Displays his clothes) Look at me!

THALES: It's all in the mind ... it's your attitude.

MERCHANT: Are you very rich?

THALES: Not really.

MERCHANT: Well, what happened to your attitude?

THALES: Nothing, I haven't tried it yet. It's my first time in business. There's more to life than money of course, why there's ... (Continues talking softly with noise, shouts and alarms, outside.)

ANAXIMANDER, THALES' PUPIL, RUSHES IN.

ANAXIMANDER: May the gods help us! The port is being attacked from the sea. [illegible] we [illegible] run, - this way.

MERCHANT: [illegible] the deal, the deal sir, we haven't signed anything yet. If [illegible] here sir, just along the ...

[illegible] what are we to do?

[illegible] quickly, run for your life!

[illegible] to Egypt?

[illegible] on the waters [illegible] for our lives.

[illegible] If you pay me now I'm sure we [illegible] gold pieces. [illegible] two [illegible]

[illegible] while you a note promising you the money.

[illegible] word. [illegible]

[illegible]

SOLDIERS RUSH IN.

SOLDIER 1: Arm yourselves. We're being attacked by Samian pirates.

MERCHANT: (Inches over to the crates) I've some new arms in the crates here. Swords, shields, helmets. Here, grab them.

THALES: And will I have to pay for their use afterwards?

ANAXIMANDER: Please Thales, run. Don't joke about this.

MERCHANT: We'll discuss the price after ... if we're still alive.

THALES: It will be a pleasure. But why was the garrison not alerted before this?

ANAXIMANDER: I've no idea. Let's run.

SOLDIER 2:(Rushes in herding a group of people before him) To the citadel! Leave everything and run for your lives!

THALES: But surely you could see them coming?

MERCHANT: There are lots of ships passing our port every day. We can't have a garrison rushing out at full alert every time they see a ship. Even if we could, there's no telling how far out to sea they are. There is no way by which we can tell the distance of a ship out to sea. If we could gauge the distance, then ships coming too close could be watched, and if its intention were hostile ample warning could be given.

SOLDIER 1: Come on sir, pick up your arms and join the men defending the square.

ANAXIMANDER: He's a visitor to the city.

SOLDIER 1: I don't think the Samians will be interested in introductions, sir.

THALES: But Samos is a beautiful island, surely not worthy of pirates.

SOLDIER 1: These Samians are renegades, or act so. Their booty probably supplies the treasury though ... Fight back to back, it's the safest way.

ANAXIMANDER: Thank you but I told you we're not fighting. (Shouting while running away with Thales) We're visitors.

PIRATE LEADER: (Rushing on) So are we! (Fighting follows)

END OF SCENE

- SCENE 2 -

[illegible]. IN THE BACKGROUND ARE THE PYRAMIDS. A TABLE IS IN THE FOREGROUND WITH BRIGHTLY COVERED CHAIRS. SLAVES STAND AROUND.

ENTER [illegible] CARRIED ON CHAIRS

[illegible]: [illegible] of the priests) Just help me to get out of this [illegible] instrument. (The slaves rush over to pull him out and walk him over [illegible] chair where they sit him down) Let me have some water. [illegible] hand. A pewter cup of water is put in it) My back is poisoned [illegible] that [illegible] do not wish to poison my system, too. Do your work, taster. [illegible] away, I can't bear to look at them a moment longer.

[illegible]

[illegible] are, [illegible]

[illegible] At my [illegible], the waste of time annoys [illegible] it' the [illegible] possession I have other [illegible] wisely. Now where is this wonderous [illegible]

[illegible]

ENTER PRIEST 3, CARRIED ON A CHAIR.

ZOSER: Sorry I'm late - have I missed anything? ... Ah! There ...

ENTER THALES, ANAXIMANDER, AND A GUIDE.

GUIDE: (Bowing low) Your Holiness, the Miletian Thales and his man.

THALES: (Bowing) And his student, Anaximander. I am deeply honoured that you can spare me your precious time.

NUISSERA: Your reputation preceeded you. Stories of your wisdom have come to our ears. We are curious.

MANETHO: Our land is old. The pyramids there are over two and a half thousand years old.

ZOSER: And our history and knowledge is recorded in writing. We have been doing that for over three thousand years.

MANETHO: Do you have anything to add to this great store of knowledge?

THALES: I come from a little city with hardly any history at all, and certainly not recorded like yours. I am curious and want to learn, but where I come from there are very few written scrolls or people of knowledge to read them. So I must look at the world around me and the heavens above and observe.

ZOSER: You must feel very deprived.

MANETHO: And this is why you've come to us. To learn. How very nice of you.

THALES: Thank you, but I've also come to do some business. I sell oil.

NUISERRA: A merchant? How is it that one of the seven wise men must earn money like a common merchant?

ANAXIMANDER: With respect, Your Holiness, Thales is a merchant, but not a common one.

NUISERRA: I did not mean to be rude, young man. When we are born, the gods decide the number of days in our lives. Each day is like a precious jewel and not to be thrown away. Unfortunately we who are old and count our remaining days know this only too well. What I meant about Thales was for a man of his genius, the quest for wealth is a useless exercise in greed, unworthy and most surprising.

THALES: It is true that our days are numbered and I try to enjoy every one. I like to meet people and understand their life styles. I love my physical luxuries. I love to travel, see wondrous things and meet wise men, such as yourselves. But how can a poor Miletian afford all of this? In Miletus there are few opportunities to better oneself except by selling one's services to someone of great wealth. I did so; I sold my services to King Croesus of Lyddia, and these he used for war. No thank you, I prefer to be self-employed.

ZOSER: And how are you self-employed?

THALES: As an oil merchant. Last year I observed that Miletus was going to have a record olive crop, so I rented all the available oil presses. The farmers who wanted to press their olives for oil had to come to me for presses. I rented the presses to the farmers for a higher price and made my money. I have also bought much of their oil and am reselling it now throughout the civilised world.

MANETHO: It sounds very simple.

NUISERRA: All good ideas sound simple, Manetho. [illegible] I have [illegible] to meet you, Thales, and not your legend. [illegible] appearance of [illegible] and simplicity belies your true genius. [illegible] expression of your true self that I have come to see.

[illegible] I'm afraid I'll disappoint you. [illegible] as "old" as yours.

[illegible] will disappoint you, Nuiserra. [illegible]

MANETHO: [illegible]

ANAXIMANDER: [illegible]

THALES: Please, Anaximander ...

NUISERRA: Look behind you, Thales, look at our pyramids. They were built over two and a half thousand years ago by our ancestors ... two and a half thousand years ago. How high do you think they are?

THALES: About the same length as one hundred and sixty paces.

NUISERRA: A good guess ... better still, how would you measure them exactly?

ANAXIMANDER: He's trying to discredit you.

THALES: Are you asking me for a simple method of measuring the height, or do you also want me to measure the height here and now?

MANETHO: I suppose you want men and equipment to measure the height.

THALES: It's a simple method you want, isn't it?

NUISERRA: Show us how you would do it, for no man has ever measured the height since they were built.

THALES: I need only a man and some wooden pegs.

ZOSER: (To Manetho) Do you really think he can measure the height?

MANETHO: Not a chance. No one has ever answered the problem. The physical effort to measure the pyramid is just too great. The arrogant and conceited always meet defeat with this question. Let's watch him squirm.

THALES: Now you shall see how I would do it. Hm ... (Zoser and Manetho smile at each other and settle in their chairs, comfortably as if expecting a long wait. Anaximander looks expectantly at Thales)

THALES: Sorry ... sorry for the delay. I'll just make myself comfortable on the ground, like this. (Stretches himself out) Anaximander ...

ANAXIMANDER: (Puzzled but eager to do whatever Thales asks him) Yes, Thales, right here. (Whispers) Are you all right?

THALES: Yes, of course. Just do what I ask you. Mark out the length of my body. Put one peg here at my head and the other at my feet. Have you done it? That's right, thank you. Now help me up.

NUISERRA: Will you explain what you are doing, Thales?

THALES: Certainly. My student just marked out the length of my body on the ground. See between these two pegs. Now if I stand here, right where my head was, and the sun casts a shadow of my body the same length on the ground, then the shadow of the pyramid is the same length as its height ... See?

ZOSER: What's that you're saying?

THALES: I'll explain again. It's really very simple ... I lie down in the sand thus ... and my body makes an impression equal to my length. Now I stand up ... For goodness sake, Anaximander, help me stand up ... Now I stand up near where my head was, here. When my shadow, caused by the sun, is the same length as my body impression in the sand (Points to sun and points to sand as he says the words), then the shadow of the pyramid is the same length as its height - right? No - don't answer. So all we have to do is to wait for my shadow to be the same length as my body impression and, at the same time, measure the pyramid's shadow taken from the centre of its side. Right?

ANAXIMANDER: (Big smile) Right!

[illegible] angled triangle - two sides [illegible] about the right angle. [illegible] triangle there is.

[illegible] ise!

[illegible] appointed, Thales.

[illegible] simple.

[illegible] wait. [illegible] the sand. [illegible] the pyramid's shadow [illegible] measured [illegible]

[illegible]

ANAXIMANDER: Providing you have time to wait.

NUISERRA: Time, time yes ... Thales, I'll get some of my servants to measure the height of the pyramid. Let us go back to the house and you join me in a cool glass of wine.

THALES: Thank you, we'll join you.

2 PRIESTS STAND, WALK TO THEIR CHAIRS AND ARE CARRIED AWAY

THALES: Oh, have you a guide? Anaximander would like to stay and see the sights for a longer time.

ZOSER: I will show him personally if he would like. The pupil of such a clever man.

END OF SCENE

- SCENE 3 -

HOME OF NUISERRA. SPLENDID, WELL-LIT COURTYARD, WITH MURALS, SCULPTURE, PAPYRUS IN GREAT CLUSTERS AROUND A FOUNTAIN. IN THE CENTRE ARE TWO LOW CHAIRS COVERED IN RICH FABRIC, WITH A VERY LOW TABLE BETWEEN THEM.

IN COMES A CHAIR BEARING NUISERRA, FOLLOWED BY ONE BEARING THALES

THALES: You have rather an efficient transport service, don't you? In Miletus we have to walk everywhere.

NUISERRA: Yes, but these bearers are new. The last team I had were Nubian and very experienced. This lot gives rather a bumpy ride.

THALES: (Rubbing his back) Yes, a bit bumpy. But surely all these riches ... Aren't they the possessions of the greedy and unworthy?

NUISERRA: I suppose I was rather unjust with that statement and I am sorry. To show my apology I must beg you to stay tonight.

THALES: But what about Anaximander?

NUISERRA: I can send a servant to your lodgings and have him brought here also.

THALES: Then, in that case, I would be greatly honoured to stay.

NUISERRA: Good. Now, how about that glass of wine I promised you? (Claps his hands - enter slave) Some refreshments for my guest and myself. (Slave bows low and leaves) They should not be long.

THALES: Ah! Good. I have had a long morning without refreshment.

NUISERRA: By the way, my personal congratulations as to the solving of our riddle. The measuring of the pyramids is a problem that some of the greatest minds of our civilised world have been unable to solve.

THALES: Thank you. By the way, is there somewhere I may go to wash?

NUISERRA: I should have thought of it earlier. (Claps his hands - slave enters) Water for my guest to wash. (Slave returns, Thales washes, then the wine is laid out with fruit, biscuits, nuts and sweetmeats)

THALES: This is not refreshment, this is a banquet!

NUISERRA: You are impressed.

THALES: I'm too dazzled to be impressed. This wine is [illegible].

NUISERRA: What do you dine on in Miletus, Thales?

THALES: I would eat a [illegible] meal to this at night; [illegible] salad or some cakes, wine and olives. I have said before we [illegible] people in Miletus.

NUISERRA: Oh ... [illegible], let us have [illegible]. (Claps his hands)

[illegible] FORM

[illegible]

- SCENE 4 -

THE WATERFRONT AT MILETUS. COLOUR AND ACTIVITY - ON THE LEFT FRONT STAGE SOLDIERS ARE ASSEMBLING A CATAPULT. IN THE FOREGROUND, THALES IS TALKING TO ANAXIMANDER. THEY ARE LOOKING DOWN AT THE HARBOUR AND SHIPS.

ANAXIMANDER: How time passes quickly! Why it was only two years since we were in Egypt. You have grown very wealthy since then. You are surely almost as rich as King Croeseus.

THALES: Perhaps I am. But I have been away from Miletus for far too long. It is surely worthwhile to visit other cultures, but I have almost forgotten my business and the problems of Miletus.

ANAXIMANDER: Since you have become one of the biggest ship owners and traders in our civilised world you certainly cannot afford to forget the problems of Miletus. Sir, the pirates still attack and plunder our ports. Our people will not go to sea, they are too afraid. Surely, sir, you had forgotten this when you ordered your men to buy all the ships they could manage! Sir, why do you buy all these ships?

THALES: Because the owners are selling cheaply, of course! What better time to buy? Surely, Anaximander, you would have learned that by now?

ANAXIMANDER: Perhaps - but is that the only reason you have bought those ships?

THALES: No, of course not. The point is that as long as my business is expanding and I have to rely on shipping more and more, then I may as well own my fleet. What better time is there to buy ships?

ANAXIMANDER: But what of the pirates? How are you going to survive their attacks?

THALES: I've thought about it. Just look down there. Can you see what the soldiers are doing?

ANAXIMANDER: They're positioning great catapults and other defences on the wharves.

THALES: That's right. They're confident that they can get them to work before the pirates arrive. I've shown the garrison how to be able to tell how far a ship is out to sea. The garrison will be able to tell if a ship is sailing too close to the shore, and if they look suspicious, man the catapults and knock holes in the ships, sinking them.

ANAXIMANDER: I expect you are thinking that I'll ask you how you did it?

THALES: Oh no! I'm sure you will come to the same simple solution to the problem as I did.

ANAXIMANDER: (Smiles sheepishly) You know I can't. Tell me how you did it. I know you can't solve such a hard problem ... simply.

THALES: Believe me it is simple. About four days ago I stood on the beach over there watching a merchant-man sail past. I had my walking staff in my hand, [illegible] now and again for walking ... I am not as young as I used to [illegible] ... [illegible] the staff stuck in the sand, and watching that ship sail by, [illegible] the top of my staff, like this. (Demonstrates) On the [illegible] there was another stick (looks at the ground and finds [illegible]) [illegible] sighted the ship again over the staff and lined it up with [illegible]. I now had an imaginary right angled triangle, [illegible] the staff, the [illegible] side made by the stick being [illegible] distance from the base of the staff to the ship is the [illegible].

[illegible] measure the [illegible] the distance [illegible].

[illegible]

right angled triangle. The distance from the base to the ship is then measured when I turn around and face the whole triangle away from the sea to the beach. I can easily measure the distance on the beach by just pacing it out. Now what do you think?

ANAXIMANDER: Simple. (Smiles)

ALARMS - SOLDIERS RUSH ACROSS THE STAGE

THALES: Our pirate friends again. I think they will get a surprise very soon when they find an armed guard from the city waiting for the pirates swimming ashore from their sunken ships.

MORE SOLDIERS MARCH ACROSS THE STAGE

THALES: Naturally I hired a carpenter and had him make hinged staves of my own design with an angle measuring device. I also hired two scribes who made up the distance tables for every degree. These are now being sold to all who need them: the ships' captains, garrisons, generals, and so on.

ANAXIMANDER: Just like that! (Wanders away while Thales gets warmed to his subject)

THALES: I've been thinking. I make one triangle out to sea and then turn it around to land. If I draw this (demonstrates on a board), I can see that two triangles are exactly alike ... If they have one side and two angles respectively equal ... and see the whole triangle is isosceles. Look, two equal sides. Do you know, I think I've just discovered something else, that the base angles of the isosceles triangle are equal. Now, what do you think of that?

ANAXIMANDER: (Makes a face) Exciting!

THE SOUND OF CHEERING AND MARCHING. ENTER SOLDIERS DRAGGING PIRATES AFTER THEM. THEY STOP FOR A MOMENT.

PIRATE: (To Anaximander) Hail, we meet you again. Remember the last time?

ANAXIMANDER: Of course, we were both visitors then ...

THALES: And this time you can be our guest for as long as we like. We insist.

GUARD: Yes, we insist that you be our guest ... (Prods with his sword) Come on. Let's entertain you.

THALES: Come, Anaximander: we have more work to do also.

CURTAIN

APPENDIX C

'The Genesis of Conics' Article

THE GENESIS OF CONICS

ROLF GRUNSEIT, Vaucluse Boys High School

INTRODUCTION

Students often ask their mathematics teachers how a certain concept was discovered. More often than not the teacher cannot answer the question, does not see the relevance of it, and so ignores one of the most interesting and human aspects of mathematics.

In some cases a most unlikely beginning develops into a mathematical challenge, the solution of which leads to the discovery of new concepts.

The duplication or doubling of the cube required a method to be found for determining the $\sqrt[3]{2}$. Several methods were forthcoming with the most exciting and the most elegant from Menaechmus, a contemporary of Plato. The only instruments allowed were the compass and straight edge.

Menaechmus invented conics to solve the problem. I have pieced together the two stories of the duplication of the cube, both from mythology and from history. They certainly make fascinating telling.

When a small boy chased a mouse across a palace courtyard around 1500 B.C., this single act set in motion a series of events, one of which led to the discovery of conics, that is the mathematics of the cone.

The palace courtyard belonged to the King of Knossos, Minos, on the island of Crete. Knossos was a wealthy and very powerful kingdom, with beautiful buildings, painted walls and magnificent sculptures. Minos, the King, encouraged gifted artists, musicians and poets to come and entertain and to enrich it. In this way Knossos became a great centre of learning and culture and today those times are remembered and recorded as the Minoan civilization or Culture after King Minos.

Knossos also attracted Daedalus, the legendary sculptor and greatest craftsman and inventor of those times. Daedalus was born in Athens where he became its greatest sculptor. So lifelike were his figures that people believed he was endowed with magical powers. He spent all his time perfecting his talents and enjoyed the glory that comes with fame.

His sister had a son, Calos, a most gifted artist. Calos had been taught by the finest teachers in Athens, and now he was ready to be taught those finishing touches by the master himself, his Uncle Daedalus.

Daedalus willingly undertook the teaching of Calos, and very quickly came to realize that here was a more gifted artist than himself. He realized that soon he would be second best to Calos, a fact that he couldn't face. In a fit of blind jealousy, he murdered Calos.

Horrified by his crime, Daedalus hid in the outskirts of Athens. In the meantime the *Areopagus* or council, sentenced him to death for murder, and Daedalus, on the run, disguised himself and fled to Knossos.

King Minos welcomed Daedalus to his court, and encouraged him to create great works of art. The more he worked, the happier he became, and the greater were his fame and rewards. He soon became a familiar figure at court where a great many of his creations were to be seen. It seemed at last that his troubles were over, and that he could live a happy life.

Unfortunately for Daedalus, Queen Pasiphae, wife of Minos, observing a magnificent white bull in the King's pastures, desired to be with this beautiful animal, yet each time she approached it, it kept on grazing and moved away. Wishing to be with the animal, Pasiphae enlisted the aid of Daedalus, who with his incredible skill made a hollow wooden cow for the Queen to slip into. In this way she was able to be with her beloved animal.

Time went by and one day she gave birth to a monster, the Minotaur, half man and half bull. Not wishing to anger the gods by killing it, the King decided to keep the monster in a special compound. He forced Daedalus to design and construct the labyrinth, a great maze where the monster was reared. The top of the labyrinth was covered with huge paving stones and became part of the palace grounds.

Years passed and the King and his wife had other children. One was a bright young boy named Glaucus. He was an inquisitive child, who loved to explore all the mysterious dark areas around the palace and in this way, whilst playing over the labyrinth chasing a mouse that he had noticed, he had an accident. One of the huge paving stones had slipped, leaving a gap large enough for a small boy to slip through. Curious young Glaucus peered in and to his joy noticed that just below were great earthenware jars used by the kitchen to store honey. He reached towards the nearest jar to dip in his hand, when suddenly the large paving stone on which he rested tilted, sending him hurtling headlong into the stored honey. The paving stone swung back into place covering the gap through which young Glaucus fell.

Later that evening when the family assembled for the evening meal, Pasiphae noticed that Glaucus was missing; a search was organized but there was no trace of the boy. The search lengthened into days and still no trace of Glaucus.

In desperation Minos and Pasiphae sent a delegation to Delphi, to ask the oracle there for guidance. The Pythoness, as the oracle was called, answered the query in the form of a riddle. She informed them that whoever could most aptly describe an amazing event which had just occurred in Crete would find who had been lost. Minos immediately made inquiries and learned that a heifer calf had recently been born, and it had the amazing ability to change its colouring from white to red and from red to black everyday. Minos on hearing this, summoned his soothsayers and posed the question to them.

"Think of an apt description, a simile, to describe the calf's changing colour," he demanded. They all tried but it was Polyeidus, the Argive, who gave the best answer. He described the calf as resembling a ripe mulberry, white at blossoming, ripening to red and then turning black.

Minos knew that the correct answer had been given, and he therefore ordered Polyeidus to search and find Glaucus. Polyeidus began to search the Labyrinthian Palace without success. Almost despairing he came upon an owl, sitting at the entrance of a cellar frightening away a swarm of bees with the flapping of its wings. Polyeidus took this as an omen. Below in the cellar stood the great jars of honey, over which rested the loose paving stone. Entering the cellar and searching the jars of honey, he found Glaucus drowned in one of them, head downwards.

Grief stricken by the discovery Minos consulted with the priests of Curetes. Their advice to him was to make Polyeidus restore Glaucus to life. Horrified by the royal command Polyeidus replied that though he was skilled in medicine, he was incapable of restoring life to a dead body such as the great Asklepius could!

"Then I will lock you up together with Glaucus in the tomb, and there you shall remain forever unless you restore him back to life."

The King then ordered a magnificent cubical tomb to be built. When it was completed he inspected it and was infuriated with what he saw. It was far too small, certainly not fit for a King's son to be buried in and he commanded that a new tomb be built.

"I want the new tomb to be twice the size of this one."

The Royal architects then sat down to calculate the dimensions of the new tomb.

It seemed an easy task at first, and a new tomb was built. It was larger [illegible] not double the size of the original one.

The problem of how to make one cube double in volume to another remained. Glaucus was then interred with all the ceremony due to a King's son. A ceremonial sword was placed by his side [illegible] before the tomb was sealed. Polyeidus was sealed in as well.

Inside the tomb in utter despair Polyeidus slumped against the wall and there waited for the inevitable end. [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The wise men councilled and decided to send a delegation to the island of Delos, where an oracle gave her pronouncements from the temple of Apollo. Delos was one of the legendary birthplaces of Apollo, son of Zeus. Apollo was worshipped as being the preventer and curer of all physical illness, evil and moral decadence. No one was allowed to stay overnight on the island, since a possible death would defile the holy sanctuary. The temple of Apollo contained a larger than life statue of the god with an altar the shape of a cube, standing in front of it. The Athenians sought the oracle's help and her pronouncement was to pose the problem that the architects of King Minos had failed to solve. [illegible] build a cube double the volume of a given cube. The oracle told the Athenians that they had to build a cubical altar double the volume of Apollo's present altar.

The Athenians went back with what they thought was a simple task. When the altar was constructed its volume was not double the volume of Apollo's altar at Delos. Meantime the plague continued. Some misguided architects believed that by doubling the dimensions of each side of the altar the volume would also be doubled, but what it actually did was to increase the volume eightfold.

Years passed and the plague ran its full course leaving behind in its wake the dead, the crippled and the sorrowing. The problem of how to double a cube remained. It was now given a name, the "Delian problem".

Eventually several mathematicians solved the problem but one Menaechmus who lived around [illegible] produced two solutions. Menaechmus was a pupil of both Plato and Eudoxus. To produce these solutions he invented the conic sections.

When a cone is cut by a plane the shape of the section produced is called a section of a cone or conic section or simply a conic. There are four such conic sections possible: the circle, the ellipse, the parabola and the hyperbola.

Menaechmus thus became the first man known to have considered the conic from [illegible] of view. He used three types of cones to produce his conic sections. Each cone had a different [illegible] angle: an acute angle, a right angle and an obtuse angle. The cones were then cut by a plane [illegible] to one edge forming an ellipse, a parabola and a hyperbola respectively.

Many mathematicians followed Menaechmus in the study of conics [illegible] Aristaeus [illegible] Euclid [illegible] (287-212 B.C.) and Apollonius [illegible]

[illegible]

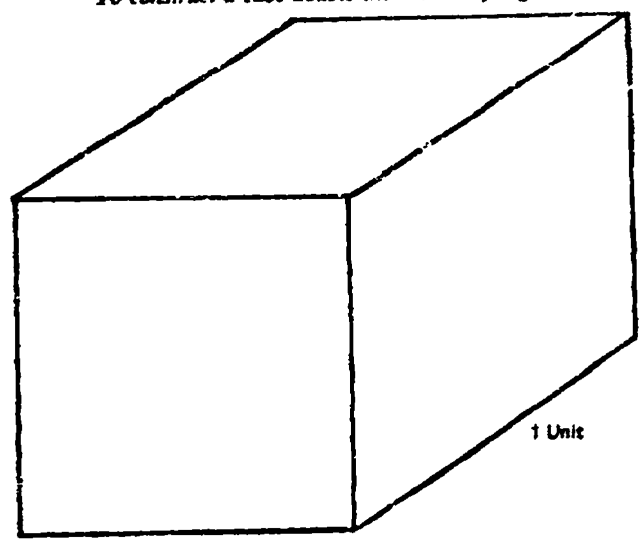
The introduction of gunpowder, the compass [illegible] printing brought back to Europe [illegible]

[illegible]

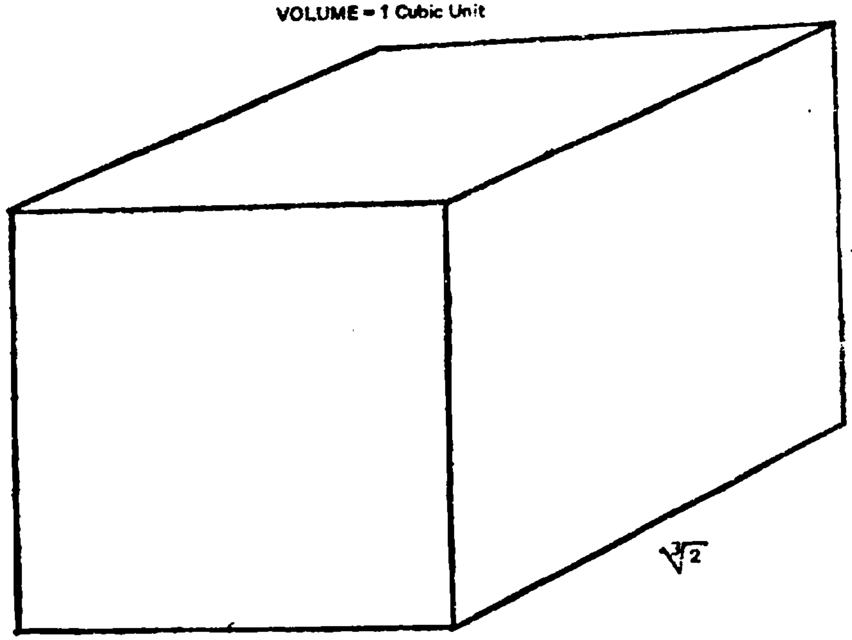
59

THE DELIAN PROBLEM

"To construct a cube double the volume of a given cu "



VOLUME = 1 Cubic Unit



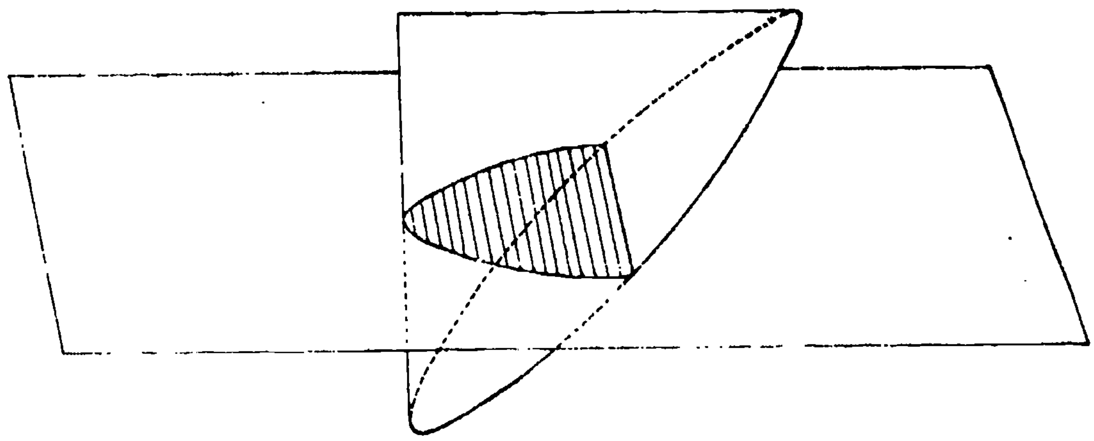
VOLUME = 2 Cubic Units

THE METHOD USED TO SOLVE IT

The Triad of Menaechmus Discovered and used to solve the Delian Problem. The Ellipse. The cone has an apical angle less than 90

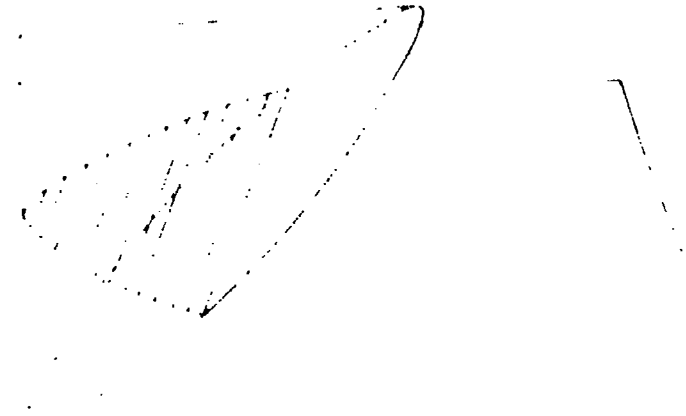
G 4691- 3

The Parabola. The apical angle of the cone equals 90°.

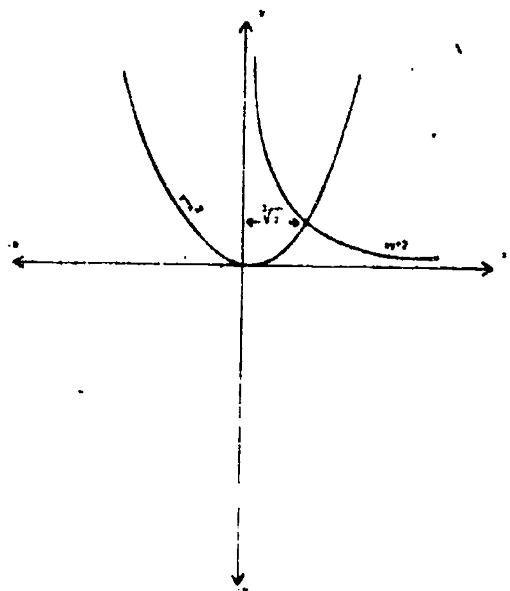


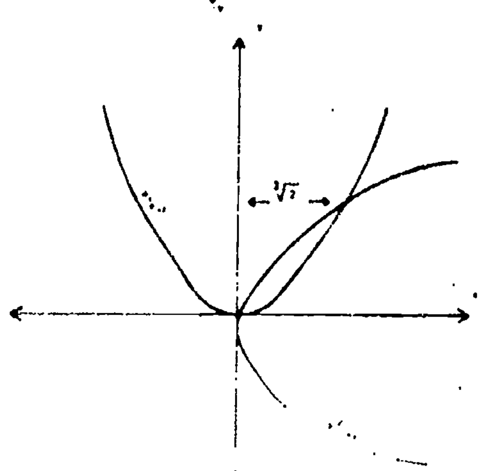
34

[illegible] Hyperbola. The apical angle is greater than 90



The two solutions by Menaechmus using his newly discovered conic sections.





APPENDIX D

QUESTIONNAIRE ABOUT THE PLAY 'THALES OR MILETUS'

School ....

Please complete this questionnaire and return it to B. Fraser and A. Koop at Macquarie University by November 19.

AIMS SATISFIED BY THE PLAY

Indicate with a TICK whether you think that this play would be VERY USEFUL, USEFUL or NOT USEFUL in satisfying each of the following aims.

	Very useful	Useful	Not useful
1. Arousing student interest before teaching the topic of similar triangles			
2. Teaching some history of mathematics			
3. Teaching some important concepts related to similar triangles			
4. Humanizing mathematics			
5. Showing that mathematics has practical applications			
6. Providing an awareness of the value of mathematics to society			
7. Providing an appreciation of ancient civilization			
8. Promoting a better attitude towards mathematics			

9. List any other aims which you think might be satisfied by this play.

____

____

____

____

____

____

STUDENTS CATERED FOR BY THE PLAY

Indicate with a TICK whether you think the play would be VERY USEFUL, USEFUL or NOT USEFUL for the following groups of students.

	Very useful	Useful	Not useful
10. Year [illegible] students of low ability			
11. Year [illegible] students of high ability			
12. Year 9 students of low ability			
13. Year 9 students of high ability			
14. Year 11 students of low ability			
15. Year 11 students of high ability			
16. Students with a good attitude towards mathematics			
17. Students with a [illegible] attitude towards mathematics			

MISCELLANEOUS

Indicate with a TICK whether you STRONGLY AGREE, AGREE, DISAGREE or STRONGLY DISAGREE with each of the following statements about the play.

	Strongly agree	Agree	Disagree	Strongly disagree
18. Most students would find the material interesting.				
19. The amount of time taken up if the materials were used in the classroom would not be excessive.				
20. The use of the materials requires only knowledge and skills which mathematics teachers already have.				
21. Material like this is not readily available elsewhere.				
22. The average mathematics teacher would not be able to write material like this himself.				
23. The play could be used profitably in the classroom without the need for a stage, costumes, etc.				
24. The play would be very suitable for use near the end of the school year.				
25. The play would be useful for integrating Mathematics with other subjects like English or Ancient History.				
26. I would use a play like this in my own mathematics classes.				

27. Please provide any further comments, reactions or criticisms about this play.

APPENDIX E

QUESTIONNAIRE ABOUT THE ARTICLE 'THE GENESIS OF CONICS'

School ....

Please complete this questionnaire and return it to B. Fraser and A. Koop at Macquarie University by November 19.

AIMS SATISFIED BY THE ARTICLE

Indicate with a TICK whether you think that this article would be VERY USEFUL, USEFUL or NOT USEFUL in satisfying each of the following aims.

	Very useful	Useful	Not useful
1. Arousing student interest before teaching the topic of conics			
2. Teaching some history of mathematics			
3. Teaching some important concepts related to conics			
4. Humanizing mathematics			
5. Showing that mathematics has practical applications			
6. Providing an awareness of the value of mathematics to society			
7. Providing an appreciation of ancient civilization			
8. Promoting a better attitude towards mathematics			

9. List any other aims which you think might be satisfied by this article.

____

____

____

____

____

____

[illegible] CATERED FOR BY THE ARTICLE

Indicate with a TICK whether you think the article would be VERY USEFUL, USEFUL or [illegible] USEFUL for the following groups of students.

	Very useful	Useful	Not useful
10. Year [illegible] students of low ability			
11. Year [illegible] students of high ability			
12. Year [illegible] students of low ability			
13. Year [illegible] students of high ability			
14. Year 11 students of low ability			
15. Year 11 students of high ability			
16. [illegible] with [illegible] attitude towards mathematics			
17. [illegible] with [illegible] attitude towards mathematics			

MISCELLANEOUS

Indicate with a TICK whether you **STRONGLY AGREE, AGREE, DISAGREE or STRONGLY DISAGREE** with each of the following statements about the article.

	Strongly agree	Agree	Disagree	Strongly disagree
18. Most students would find the material interesting.				
19. The amount of time taken up if the materials were used in the classroom would not be excessive.				
20. The use of the materials requires only knowledge and skills which mathematics teachers already have.				
21. Material like this is not readily available elsewhere.				
22. The average mathematics teacher would not be able to write material like this himself.				
23. The number of different characters and ideas is not too large for an article of this size.				
24. An article like this does not need a list of references.				
25. The story related in this article is clearly relevant to the topic of conics.				
26. The mathematics on pages 32-36 is clearly relevant to the rest of the article.				
27. This article would be suitable for direct student use in mathematics projects.				
28. I would use an article like this in planning my own mathematics lessons.				

29. Please provide any further comments, reactions or criticisms to this article.

APPENDIX F

STUDENT QUESTIONNAIRE ABOUT THE THALES PLAY

Scoring Procedure for the Student Questionnaire

Attitude items: The 14 attitude items on the first page of the questionnaire are scored on a five-point scale. For Items 1, 3, 6, 7, 9, 11 and 13, responses SA, A, N, D, SD are scored 5, 4, 3, 2, 1, respectively. For Items 2, 4, 5, 8, 10, 12 and 14, responses SA, A, N, D, SD are scored 1, 2, 3, 4, 5, respectively. Omitted or invalidly answered items are scored 3. Items 2, 7, 10, 13 measure specific attitudes related to the play, and scores on these items cannot be summed to obtain a meaningful total. Scores on Items 1, 3, 4, 5, 6, 8, 9, 11, 12 and 14 are summed to form a general attitude to mathematics score.

Cognitive items: The five cognitive items on the second page of the student questionnaire are multiple-choice items which are scored 1 for the keyed response and 0 for all others. The keyed responses for these five items, in order, are C, D, B, B, A. Scores on these items cannot be added to form a meaningful total.

STUDENT QUESTIONNAIRE

Name: ____________________

School: ____________________

Boy/Girl: ____________

G	
M	

Directions

Give your opinion about each statement on this page by drawing a circle around

- SA if you STRONGLY AGREE with the statement.
- A if you AGREE with the statement.
- N if you are NOT SURE.
- D if you DISAGREE with the statement.
- SD if you STRONGLY DISAGREE with the statement.

Be sure to give an answer for all statements. If you change your mind about an answer, just cross it out and circle another one.

1.	I like mathematics lessons more than any other lessons.	SA	A	N	D	SD
2.	It would be a waste of time learning about the lives of mathematicians who lived thousands of years ago.	SA	A	N	D	SD
3.	I like to try to solve mathematical puzzles and problems.	SA	A	N	D	SD
4.	I think mathematics is a dull and uninteresting subject.	SA	A	N	D	SD
5.	Outside the classroom, I don't like to think about mathematics.	SA	A	N	D	SD
6.	I like to make up sums and problems and do them at home.	SA	A	N	D	SD
7.	Learning about the history of mathematics would be interesting.	SA	A	N	D	SD
8.	I don't like mathematics because the examples are too hard and make me think.	SA	A	N	D	SD
9.	Mathematics is one of the most interesting subjects, and I want to do as much mathematics as possible.	SA	A	N	D	SD
10.	Mathematics has few practical applications to daily life.	SA	A	N	D	SD
11.	Mathematics is a most important subject and more time should be given to it at school.	SA	A	N	D	SD
12.	I dislike mathematics and will do as little mathematics as possible at school.	SA	A	N	D	SD
13.	I would enjoy mathematics lessons more if they included some plays related to mathematics.	SA	A	N	D	SD
14.	Mathematics is a difficult subject which I don't enjoy.	SA	A	N	D	SD

Directions

For each question on this page, draw a circle around the letter (A, B, C, D or E) corresponding to the one correct answer. If you change your mind about an answer, cross it out and circle another letter.

Show any working you do in the space provided.

Working space

1. The mathematician Thales lived about
   - A 100 years ago.
   - B 1,000 years ago.
   - C 2,000 years ago.
   - D 3,000 years ago.
   - E 4,000 years ago.

2. Thales was born in
   - A Egypt.
   - B Samos.
   - C Minos.
   - D Miletus.

3. A flag pole makes a shadow 100 centimetres long. At the same time, a man who is 170 centimetres tall makes a shadow 170 centimetres long. The height of the flag pole is
   - A 85 centimetres.
   - B 100 centimetres.
   - C 135 centimetres.
   - D 170 centimetres.
   - E 340 centimetres.

4. A man of height 180 centimetres makes a shadow 90 centimetres long. At the same time, a tree of height 16 metres would make a shadow
   - A 4 metres long.
   - B [illegible] metres long.
   - C 16 metres long.
   - D 90 metres long.
   - E 1[illegible]0 metres long.

5. A tower is [illegible]0 metres high and makes a shadow 100 metres long. How high is a building which would make a shadow 40 metres long at the same time?
   - A 20 metres
   - B 40 metres
   - C 50 metres
   - D [illegible]0 metres
   - E 100 metres

APPENDIX G

IMPRESSIONS OF A HISTORIAN OF MATHEMATICS

Comments of Dr John Pottage, Department of History and Philosophy of Science, University of Melbourne

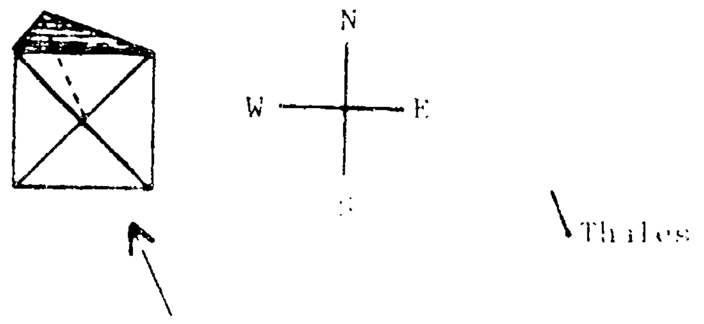
Thales Play

Perhaps the first thing to be mentioned is the dubious nature of the source material. Neugebauer went so far as to write that

> "the traditional stories of discoveries made by Thales or Pythagoras must be regarded as totally unhistorical. Thales, for example, is credited with having discovered that the area of a circle is divided into two equal parts by a diameter. This story clearly reflects the attitude of a much more advanced period when it had become clear that facts of this type require a proof before they can be utilized for subsequent theorems ..." etc. etc. (Exact Sciences in Antiquity, 2nd ed. p. 148)

The latest short account of Thales is in the fine Dictionary of Scientific Biography, of which about 13 volumes have so far appeared. Here the dates of Thales are briefly discussed and the best estimates are given as 625 (?) and 547 (?). So the assertion that Thales was 60 in 540 B.C. is too precise (page 1 of play). For Dating of the Pyramids, see I. E. S. Edwards, The Pyramids of Egypt; for dating in general, see Sir Alan Gardiner, Egypt of the Pharaohs.

Most importantly, some reconsideration of the two applications of similar triangles is in order. If Thales' shadow were a half or a third as long as Thales is high, the pyramid would cast no shadow owing to its shape. Even when the shadow is equal to the height (except perhaps twice in a year) the situation would be as indicated below in plan, so that the passage from measurement to deduced height would be much more complicated than is suggested in the play. As for the distance of the ships being equal to a measurable distance along the beach (need to keep both base lines horizontal), a five foot staff of uncertain perpendicularity would be a quite unsuitable basis for any more reliable data than any experienced look-out might reasonably be expected to supply by looking out to sea. At least Thales could stress how important it is to keep the staff vertical; and perhaps his "angle measuring device" instead of being on sale "to all who need them" could have taken the form of a look-out tower 40 feet (say) above the water, then the angle of depression might not unreasonably be used as an indicator of the distance out to sea. Of course, the turning through a right-angle to sight out a measurable distance along the beach would only be necessary in connection with the graduation of the instrument.



No Greek ever used "degrees" for measuring angles, so far as any record survives, until well into the Hellenistic Period. Archimedes and Apollonius still used fractions of a right angle to specify even very small angles. And, of course, the suggestion that any scale of angles would need to enter as intermediaries is quite unnecessary (and historically misleading). The natural thing would be mark the scale directly in distances. Needless to say, no precise information survives, as to what methods were used; but the application of similar or of congruent triangles pre-dates even the most primitive "trigonometry" by many centuries. Even some of the problems in the Rhind Papyrus show an application of what amounts to the method of similar triangles - and in connection with heights of pyramids too, though not in connection with shadows. But that is another story.

If Mr. Grunseit is prepared to work a little more on his script it should be worth publishing for school use. Possibly he could include a few historical notes (disclaimers!) so that his readers won't take him too literally. Possibly also some questions for discussion concerning the practicality of the applications of similar triangles, or some suggestions for field work by students.

The Conics Article

You ask for a comment on the value of this as something "which teachers could use as a basis for preparing lessons in mathematics".
Answer: I'm afraid it has no value as it stands. The slight mathematical content is just too misleading. If "the only instruments allowed were the compass and the straight edge" what is the poor reader to make of the method of Menaechmus, which one presumes is being presented here as a brilliant method (as it was)? The final page with modern graphs, even to the cartesian equations (!), is disturbing to anyone interested in the history of the subject. The generator of a cone should not be referred to as an "edge" (p. 31). For Apollonius (not Appolonius), the Dictionary of Scientific Biography, should be consulted. The dates as given on p. 31 are sheer guess work on someone's part.

The Greek myths may well have a place in secondary (as well as primary and indeed tertiary education) - they might have a greater value than mathematics - but the combination, I should say, juxtaposition, of myth and mathematics, as in this contribution helps nobody.

APPENDIX H

IMPRESSIONS OF TWO MATHEMATICS EDUCATORS

Comments of Mr Ken Clements, Faculty of Education, Monash University

My wife and I acted out the Thales play tonight. Actually my gut reaction is that Grunseit's work is very creative, and could add interest and variety to secondary mathematics classrooms. There would need to be a fair bit of associated in-service education, but this would probably be easily arranged.

The Thales play is more interesting and would be more likely to succeed in the classroom than the "Discovery of Conics" story.

I applaud the idea of bringing history into mathematics - the kids would be richer for having it. In my opinion Grunseit's efforts deserve every encouragement.

What I would like to see is a small booklet (perhaps 80 pages) with quite a few little plays such as the one on Thales.

Comments of Mr Brian Low, School of Education, Macquarie University

My thoughts on the Thales play are recorded below.

1. The play appears to be interesting in a purely dramatic sense.

2. I would see it as very useful in arousing interest in the history of mathematics, and the applications of mathematics in earlier civilizations.

3. I don't think it is very useful as an advance organizer for similar triangles work - rather as an interesting follow-up to the work. I hold this opinion because some of the concepts introduced by the play are a little obscure for Year 8 or Year 9 students who have had no experience of similar triangles.

4. Provided the medium of plays is not overdone (i.e., placed in a large number of content areas), I think this play would promote a much better attitude to mathematics.

5. It is an interesting way of showing some simple practical applications of mathematics.

6. Overall, I think the play is very useful. I could not recommend any improvements.

My thoughts on the conics article are recorded below.

1. I am not impressed as much by this article as by the play. I am not sure that the majority of children would find it fascinating or terribly interesting.

2. Perhaps it could be used to arouse student interest before teaching conics, but I do not see it as applying to the majority of students - probably only the better ones or the 'kinky' ones.

3. For those students interested in the article I think it would promote a better attitude to mathematics.

4. This article has more import for the teaching of history of mathematics than has the play. If it is important to teach the history of mathematics then this probably goes some way towards achieving the objective.

5. Overall, I am not as impressed by this article as by the play. Perhaps it is the medium more than the content which disappoints me. I think it is most applicable to better students who already have a better attitude to mathematics.

APPENDIX I

IMPRESSIONS OF TWO DRAMA CRITICS

<u>Comments of Ms Sandra Alexander, School of Education, Macquarie University</u>

The Thales play is a creditable attempt to present to students the discovery of some basic mathematical concepts in a historical and social setting. The discoveries are presented as a response to the needs of the societies among which the mathematician Thales moved.

I make no comment on the mathematical or historical accuracy of the play, but only upon its suitability as drama for lower secondary school students.

To recommend it, the play has the virtue of incorporating material from a number of different subject areas, and thus could be included in programs of study emphasizing integration of subject areas. It has quite a large cast, and provision for numerous extras, so that all students in a class could be actively involved in the performance and rehearsals. It contains a fair measure of action and movement, difficult to accomplish in a play designed to convey abstract concepts. The settings are colourful and exotic, but reasonably easy to accomplish within the usual constraints of school drama production - the odd sheet, potted palm and decorated chair are readily available, although a backdrop of the pyramids might prove expensive or difficult in some circumstances. A production would otherwise present few problems of stage mechanics.

My first criticism is that the characters and their inter-relationships are very sketchily developed. In particular, the relationship between Thales and Anaximander does not evolve as it surely would have done given the changes of time and circumstances between the second and third scenes. The addition of some humour would also make the play more attractive.

The play provides no major parts for girls, indeed, no speaking parts at all. Their presence is acknowledged only by the suggestion that 'dancing girls' might perform at one point in the action. The history and archeology of Egypt attest to the fact that some women held positions of considerable power and influence, and the inclusion of some female speaking parts is surely not an outrageous demand in a play apparently intended for production in coeducational schools.

A further, more specific concern I have is that the information explaining the mathematics concepts is presented in somewhat indigestible lumps. This could be overcome by having another character (Anaximander?) elicit the explanations from Thales in question and answer dialogue. This would be more easily comprehensible to children watching the play than are the present long speeches.

All my criticisms could be answered without major changes to the structure of the play, and I would suggest that the author consult an experienced drama editor to give a final form to a potentially valuable educational innovation.

Comments of Mrs Nel George, School of Education, Macquarie University

The Thales play has a number of positive dramatic qualities:-

1. With the exception of the very long conversation on page 4 (which could possibly be enlivened with some action to match the very individual characters of the people on stage), the movement of the play flows well.

2. There s a development from the outlining of the problem in Scene 1 to its successful solution in Scene 4.

3. There is humour and irony both of situation and dialogue.

4. The dialogue moves well and is in character. There is only one change of idiom (and this may have been used to gain an archaic effect: "How time passes quickly" instead of "How quickly time passes").

5. The characters are consistent throughout, and all the qualities of the main characters develop in a credible fashion.

6. There has been an effort at creating an appropriate setting for each scene, and the details are woven effectively into the fabric of the play.

Maybe the role of the narrator could be eliminated. He merely sets the initial scene and the play can stand by itself without this introduction.

APPENDIX J

PROJECT DIRECTOR'S IMPRESSIONS

Comments of Mr Rolf Grunseit, Vaucluse Boys High School

The articles

I enjoy the research as well as writing about my findings. I also find that this material is more useful in making my lessons more interesting.

Another result has been invitations to address in-service groups as well as being the guest speaker at the various Mathematics Association Meetings. In this way I experience some feedback and some correspondence. Several magazines have asked me to contribute to them (e.g., "Pursuit" in Victoria).

I am not always happy with the editing of my articles. I would like to rewrite quite a few of them.

I am rewriting several of the articles for inclusion in student magazines. I am also in the process of writing several more as a result of my research.

Quite often my stories are reconstructions from many sources, some of which are historically doubtful (e.g., the life of Pythagoras from the age of 18 to 52, or again Zeus' predeliction for sex being the prime inspiration for the discovery of conics). Though some of the stories are apocryphal, they make superb motivational material.

Video and photographic equipment

The video equipment and the camera gear purchased has helped to stimulate me. It has enabled me to express myself visually and to explore completely new ways of conveying mathematical concepts.

I am constantly using this medium whether at in-service conferences, Mathematical Associations or in the classroom.

[illegible] and that is the amount of material that I must carry.

[illegible]

[illegible] inability to see any of my plays produced on [illegible] hindered me in rewriting my existing scripts, and developing [illegible] which make it possible to convey mathematics integrated [illegible] and drama in the classroom or stage. [illegible]less I can see [illegible] the writing of more [illegible] a waste of time.

In my haste to have them printed I neglect the necessary rewriting which my articles need. The editing by the magazines also leaves a lot to be desired.

I can see the time when I will rewrite these stories and make them readable enough to please the students from the 12 to 15 age group.

Negative points

I receive little or no feedback from teachers.
I have spoken at numerous inservice meetings, and I am getting bored at the sound of my own voice.
Being self critical, I find it hard to work in isolation without anyone to discuss my work with or to provide constructive criticism.

ABOUT THE AUTHORS

BARRY J. FRASER is a Lecturer in Education at Macquarie University and teaches courses in educational evaluation and research methods. He holds a B.Sc. from Melbourne University, and a B.Ed. and a Ph.D. (in the area of curriculum evaluation) from Monash University. Previously, he has been a teacher of science and mathematics in Victorian secondary schools, a research officer in the evaluation section of the Australian Science Education Project, and a staff member of the Faculty of Education at Monash University.

ANTHONY J. KOOP is a Lecturer in Education at Macquarie University. He holds a B.A. from Sydney University and an M.A. from Macquarie University. He currently teaches introductory courses in Education and conducts workshops in mathematics curriculum development in a special program for teachers and lecturers from developing countries. He has coordinated the development of a primary mathematics curriculum in Western Samoa and worked on mathematics curriculum projects for Tonga and the Gilbert and Ellice Islands. Previously, he has been a teacher of mathematics in New South Wales secondary schools and a member of the staff of the Teacher Education Programme at Macquarie University.