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ABSTRACT

Subjects for this study were female participants in a mathematics anxiety treatment program at a midwestern university. A series of test instruments were administered. Statistical analysis indicates that for this female sample, the domain of mathematics anxiety is best described as primarily test anxiety and only secondarily as anxiety associated with mathematics course activities. The discussion section lists six assertions related to mathematics anxiety suggested by this and other research. (MK)

Factor Structure of the Mathematics Anxiety Rating Scale

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Abstract

The measurement of mathematics anxiety has become increasingly important for the identification of math-anxious students and the evaluation of treatment programs for mathematics anxiety. The purpose of the present study was to examine the dimensionality and domain of one such measure of mathematics anxiety, the Mathematics Anxiety Rating Scale (MARS). The responses to 94 MARS items were obtained for 350 female participants in a mathematics anxiety program. The items were intercorrelated and the correlation matrix factored by a principal axes technique, using squared multiple correlations as communality estimates, with rotation to a direct oblimin and varimax criterion. Two factors were identified and labeled as Mathematics Test Anxiety and Numerical Anxiety. Factor-derived scales were developed and correlated with five specific anxiety scales and an arithmetic test. Results from this analysis supported the factor interpretations and the expected discriminant and convergent relationships. The concept, measures, and treatment of mathematics anxiety are discussed.

Recently, a considerable amount of attention has been paid to mathematics anxiety as an explanatory variable for the sex-related differences in mathematics performance and enrollment in mathematics curricula. Numerous newspaper and magazine articles (Math Mystique: Fear of Figuring, 1977; Stent, 1977; Tobias, 1976, 1978; Zanca, 1978) have been written calling attention to the concept and effects of mathematics anxiety. Programs for the alleviation of mathematics anxiety have been designed and implemented at several colleges and universities (e.g., Wellesley College, University of Minnesota, Wesleyan University, Iowa State University, Mills College). Several reviews (Aiken, 1970, 1976; Fennema, 1977; Fox, 1977) of the influence of affective variables on mathematics learning have suggested that mathematics anxiety may contribute to mathematics avoidance and poor mathematics performance. Researchers have developed measures o mathematics anxiety (Fennema & Sherman, 1976; Richardson & Suinn, 19 , and a recent study (Betz, 1978) has attempted to document the prevalence of mathematics anxiety. Various interventions strategies for the reduction of mathematics anxiety have been investigated (Addleman, 1972; Brown, 1971; Crouch, 1970; Hendel & Davis, 1978; Hyman, 1974; Natkin, 1967; Nash, 1970; Richardson & Suinn, 1972; Suinn, Edie & Spinelli, 1970; Suinn & Richardson, 1971).

Before psychologists can understand the effects of mathematics anxiety, it is imperative that considerably more empirical research is conducted examining the construct, especially the instrumentation of the construct. Many of the ambiguities of the mathematics anxiety construct may be directly traced to a lack of agreement among researchers on the conceptualization of mathematics anxiety. Several researchers have provided definitions of mathematics anxiety; however, none of these definitions have included a discussion of the domain of mathematics anxiety. The present study addresses the issue of the dimensionality and domain of mathematics anxiety.

Dreger and Aiken (1957), and more recently, Richardson and Suinn (1972) and Fennema and Sherman (1976) have constructed scales and instruments to measure what alternately has been referred to as mathematics anxiety, number anxiety, and mathemaphobia. Dreger and Aiken (1957), relying on an earlier definition of mathemaphobia by Gough (1954), constructed a three item scale of number anxiety. Number anxiety was defined as the "presence of a syndrome of emotional reactions to arithmetic and mathematics" (Dreger & Aiken, 1957, p. 344). Correlations between the Number Anxiety scale and a modified Taylor Manifest Anxiety Scale and mathematics grades were .33 and -.44, respectively, for 704 female and male college students. However, the interpretation of these correlations is somewhat questionable, because one of the three Number Anxiety items seems to be a measure of self-report estimation of mathematics skill ("I was never as good in math as in other subjects").

Fennema and Sherman (1976) have developed an instrument, named the Fennema-Sherman Mathematics Attitude Scales, to measure each of nine affective variables hypothesized as factors affecting mathematics achievement. One such affective variable is mathematics anxiety. The Fennema-Sherman Mathematics Anxiety Scale is a 12-item scale emphasizing feelings of anxiety associated with mathematics classes, courses, problems, and tests. The Mathematics Anxiety Scale is intended to assess "feelings of anxiety, dread, nervousness and associated bodily symptoms related to doing mathematics . . . scale is not intended to measure confidence in or enjoyment of mathematics" (Fennema & Sherman, 1976, p.4). However, after reporting that the correlation between the Confidence In Learning Scale and the Mathematics Anxiety Scale was .89 (Fennema & Sherman, 1976), Fennema (1977) has suggested that confidence and anxiety should be conceptualized as a single dimension.

Little or no research has been reported examining the validity and reliability of Dreger and Aiken's Numerical Anxiety scale and Fennema and Sherman's Mathematics Anxiety Scale--research results based on modifications of the Fennema-Sherman Mathematics Anxiety Scale (e.g., Betz, 1978) confound the problem. The lack of reliability and validity data seriously impedes the use of these measures for research purposes and lends little credibility to results based on these instruments. However, such data is not lacking for the Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972), a 98-Likert item measure of mathematics anxiety.

In two articles detailing the development of the Mathematics Anxiety Rating Scale (MARS), Richardson and Suinn (1972) and Suinn, Edie, Nicoletti, and Spinelli (1972) presented psychometric data including normative, reliability, and validity data for the MARS. Richardson and Suinn (1972) have defined mathematics anxiety as ". . . involving feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems . . ." (p. 551). Evidence for the reliability of the MARS consisted of a two and seven week test-retest reliability coefficients of .78 (N=119) and .85 (N=35), respectively, and an internal consistency alpha coefficient of .97 (N=397). Evidence for the construct validity was provided by three studies in which MARS scores decreased after behavior therapy and two studies in which MARS scores correlated negatively (r= -.64, N=30; r= -.35, N=44) with scores on a highly speeded (10 minutes completion time) version of the Differential Aptitude Test. More recently, Hendel and Rounds (Note 1) found a smaller correlation (r= -.58, N=124) than expected between the MARS and the Fennema-Sherman Mathematics Anxiety Scale. This result suggests that these two instruments may be measuring different components of mathematics anxiety and research conclusions based on one measure may not be applicable to the other measure.

Underlying these validity studies (Richardson and Suinn, 1972; Suinn, Edie, Nicoletti, and Spinelli, 1972) is the assumption that mathematics anxiety, as measured by the MARS, is an unidimensional construct. Unfortunately, important validity data pertaining to the unidimensionality of the MARS were not adequately presented.

Richardson and Suinn (1972) did state that the high alpha coefficient "shows that the average intercorrelation of the items in the test is high. It confirms that the test is highly reliable and indicates that the test items are heavily dominated by a single homogeneous factor . . ." (p. 553). However, as demonstrated by [illegible]en, Lissitz, and Mulaik (1977), the average intercorrelation and coeffici[illegible] are poor indices of item homogeneity. Homogeneous items (tests) usually [illegible]er to the case where these items measure a single common factor (Lord & Novick, 1968, p. 95). Green, Lissitz, and Mulaik (1977) offer "numerical counter examples to show how coefficient alpha and the item-total score correlations can be high when the component items are not homogeneous" (p. 827). It seems, therefore, that the unidimensionality of the MARS is yet to be demonstrated.

The issue of the unidimensionality of the MARS is especially crucial for the interpretation of the pretherapy-to-post therapy validity. Post therapy decreases in MARS scores as reported (Hendel & Davis, 1978; Hyman, 1974;

and Richardson & Suinn, 1972) could be accounted for by as few as 20% of the 98 MARS items. Furthermore, several of these studies may have utilized the MARS items or very similar items in a desensitization hierarchy, a practice advocated by Richardson & Suinn (1972) and Hendel and Davis (1978). In turn, this practice could also confound the results of these pretherapy-to-post therapy studies. If the MARS is multi-dimensional, pretherapy-to post therapy decreases in MARS scores could presumably be accounted for by dimension(s) that may or may not be mathematics anxiety.

A likely dimension to account for these pretherapy-to-post therapy reductions in MARS scores is test anxiety. Hendel (in press) correlated the MARS and the Suinn Test Anxiety Behavior Scale (Suinn, 1969), and a measure of test anxiety. Hendel found a correlation of .65 for 69 females enrolled in a math anxiety treatment program which almost approaches the test-retest reliability of the MARS. Results from the studies (Richardson & Suinn, 1972; Suinn, Edie, Nicoletti, & Spinelli, 1972) in which the MARS and the Differential Aptitude Test were correlated also may support the idea that the MARS predominantly measures test anxiety. Assuming that test anxiety is increased by a speeded ability test, a test anxiety component of the MARS could account for the high to moderate relationships found between the speeded Differential Aptitude Test and the MARS. Finally, one of the authors, while hand-scoring the MARS, noticed that participants enrolled in a mathematics anxiety treatment program usually reported high anxiety for items referring to math tests and low anxiety for items referring to number manipulation.

One approach to establishing the homogeneity of the MARS item pool would be through factor analysis. Factor analysis would also contribute to further understanding of the mathematics anxiety domain as represented by MARS items. The present study was a factor analysis of the Mathematics Anxiety Rating Scale items. The purposes of the present study were as follows:

(a) to identify the independent dimension(s) underlying the MARS item pool; (b) to develop factor scale(s) to measure these dimension(s); (c) to explore the relationship between the MARS factor scale(s) and other specific anxiety scales.

Method

Instruments

The Mathematics Anxiety Rating Scale (MARS; Richardson & Suinn, 1972) was used to measure mathematics anxiety. The MARS used in this study consists of 94 items in a Likert format[1]. For each item, individuals are requested to indicate how much they are "frightened by it nowdays" on a five point scale (ranging from 1 = "not at all" to 5 = "very much"). The MARS items are summed, resulting in a single scale score, ranging from a low of 94 to a high of 470, which reflects the intensity of an individual's mathematics anxiety. As discussed by the present authors (see the introduction), normative, reliability, and validity data for the MARS have been provided by Richardson & Suinn (1972) and Suinn, Edie, Nicoletti, and Spinelli (1972).

Five other specific anxiety scales which measure anxiety dimensions that are hypothesized to relate to mathematics anxiety were included in the study: the Suinn Test Anxiety Behavior Scale (Suinn, 1969), the Achievement Anxiety Test (Alpert & Haber, 1960), Fear of Negative Evaluation Scale (Watson & Friend, (1969), and the Fennema-Sherman Math Anxiety Scale (Fennema & Sherman, 1976).

The Suinn Test Anxiety Behavior Scale (STABS) is a 50 item Likert scale composed of behavioral situations which are expected to arouse different levels of test anxiety. A total test anxiety score is calculated by assigning a value of 1 ("not at all" anxious) to 5 ("very much" anxious) corresponding to the level of anxiety checked for each item and then, summing the item scores. Total test anxiety scores range from 50 to 250 with high scores reflecting high levels of test-taking anxiety. As reported by Suinn (1969), test-retest reliability coefficients for the STABS were .78 (N = 158) and .74 (N = 75) for a four and six week interval, respectively. The correlation between the STABS and the Test Anxiety Scale (Sarason, 1957) was .60 (N = 158) and .59 (N = 75).

The Achievement Anxiety Test (AAT) consists of two subscales; a 9-item Facilitating Anxiety Scale (hereafter referred to as AAT+) and a 10-item Debilitating Anxiety Scale (AAT-) which measure the facilitating and debilitating effects of anxiety on achievement performance. For both scales item responses on a 5-point Likert format are separately scored and summed with high scores indicating high levels of anxiety.

The Fear of Negative Evaluation (FNE) scale, a measure of social-evaluative anxiety, is composed of 17 true and 13 false keyed items with total scores ranging from 0 to 30. Watson and Friend (1969) define the construct of fear of negative evaluation "as apprehension about others' evaluations, distress over their negative evaluations, avoidance of evaluative situations, and the expectations that others would evaluate oneself negatively" (p. 449). Watson and Friend (1969) report KR-20 coefficients for the FNE items of .94 (N = 205) and .96 (N = 154) and 1-month FNE scale test-retest correlations of .78 (N = 154) and .94 (N = 29).

The Fennema-Sherman Math Anxiety Scale is a 12-item measure of mathematics anxiety. Instructions for the Math Anxiety Scale request individuals "to indicate the extent to which (they) agree or disagree with the ideas expressed" on a five-point Likert scale (ranging from 1 = "strongly disagree" to 5 = "strongly agree"). Since six of the items are positively worded and six items negatively worded, the scoring of the six negatively worded items was reversed so that low scores indicate high math anxiety.

The final instrument used in the present study was the arithmetic placement test which is a 68-item multiple choice test composed of 13 different types of items (e.g., multiplication of whole numbers, meaning of decimals, problem solving, properties of numbers). This instrument had been designed and was currently being used by mathematics instructors for purposes of identifying those students who should enroll in either an Arithmetic and Elementary Algebra course or more advanced Algebra courses. No data are available concerning the extent to which the arithmetic placement test correlates with other more standardized measures of mathematics achievement.

Data Collection and Sample

Subjects for the present study were female participants in a mathematics anxiety treatment program at a large midwestern university. This program, generally offered each quarter of the academic year through a continuing education division of the university, is designed for individuals who are anxious about mathematics and/or concerned about their performance in mathematics courses. The instruments were administered by counseling psychologists during an initial 3-hour diagnostic session designed to provide an assessment of the participant's mathematics anxiety and mathematics skills. The data were collected at four diagnostic sessions conducted during Spring and Fall 1976, Fall 1977 and Winter 1978.

The MARS was administered to each of 350 female participants. In addition, 67 of the participants (hereafter referred to as Sample 1) completed the STABS, AAT, FNE, and the arithmetic placement test during Fall

1976 and 111 of the participants (Sample 2) completed the Math Anxiety Scale and the arithmetic placement test during Fall 1977.

Biographical information was available for 311 of the 350 participants. These 311 female participants' ages ranged from 18 to 65; the mean age was 35.6 (SD = 11.3). Although participants varied in their educational backgrounds, the majority either had completed some college (34%) or had a four-year college degree (21%). An average of 16.5 years had elapsed since the participants had received formal instruction in mathematics.

Analyses

Two sets of analyses were performed. First, a factor analysis was performed to examine the dimensionality of the MARS items. The 94 MARS items were intercorrelated and the correlation matrix factored by the principal axes technique, using squared multiple correlations as communality estimates. Since the MARS items are an internally consistent domain of items, correlations among the factors were expected. Therefore, the factors were rotated to an oblique simple structure using the normalized direct ~~quartimin~~ oblimin procedure (Jennrich & Sampson, 1966), with $\gamma = 0$. An orthogonal solution may result from the more general oblique solution (Harman, 1976). Hence, the factors were also rotated to approximate orthogonal simple structure using the normalized varimax algorithm (Kaiser, 1958).

In selecting the best factor solution, the following multiple criteria were used: (a) Cattell's (Cattell, 1966) "scree" test of residual eigenvalues, (b) factor interpretability, and (c) extraction until a residual factor emerges (Cattell, 1958; Rummel, 1970). A residual factor was defined for this study as one for which fewer than four factor loadings could meet the joint condition of being $\geq .30$ and the only factor loading for an item $\geq .30$. These criteria were applied to each of the one through four factor solutions. Items with loadings of $\geq .30$ were then used to define and interpret the factors which met the above criteria.

The second set of analyses was performed to develop scales representative of the MARS factors, thereby facilitating interpretation of the MARS factors, and to test for certain expected discriminant or convergent relationships between the factor-derived measures of mathematics anxiety and the other specific anxiety scales and the measure of mathematics performance. A sequential strategy of item selection was used with the total female sample to create the 15-to-20 item MARS factor-derived scales. For each factor, items with loadings of $\geq .40$ were initially selected for further examination. The female sample ($\underline{N}$ = 350) was scored on the preliminary factor-derived scales. Then, for each of the items selected for each factor-derived scale, product moment correlations of the item with its intended scale and with all other irrelevant scales were calculated. Those items with low item-total score correlations with intended scale and high correlations with the irrelevant scales were sequentially deleted until 15 to 20 items remained per scale. Hypotheses specifying the expected relationships between these MARS factor-derived scales and the specific anxiety scales and arithmetic placement test were developed. These MARS factor-derived scales were subsequently correlated with each of the specific anxiety scales and the arithmetic placement test for Sample 1 and Sample 2.

Results

Factor Analysis

The eigenvalues for the first through the sixth factor were 29.12, 7.68, 3.59, 3.33, 2.76, and 2.45, respectively. The number of factors to be rotated was initially estimated by Cattell's scree test as two. Trial rotations with the direct oblimin and varimax criteria of two-to-four factors indicated that a two-factor solution provided the most interpretable structure. In these trial rotations, a residual factor was found for both the three-factor oblique and orthogonal solutions. Across solutions for the two through four factors, the loadings on the first two factors remained stable; however, the composition of the third factor varied between solutions. There is clear evidence of the existence of at least two patterns of factors. The two factor solution rather than the three factor solution was selected as fitting the data best since it provided the most interpretable structure.

As previously noted, the two factor solution was rotated to both a direct oblimin and a varimax criterion. Somewhat unexpectedly, the results from the oblique and orthogonal rotations were very similar. Salient factor loadings were virtually the same for the two solutions. Coefficients of congruence (Mulaik, 1972; Tucker, 1951) computed between the direct oblimin pattern matrix and the varimax matrix were .98 and .94 for the first and second factors, respectively, indicating that the interpretation of the factors would be very similar for either solution. The direct oblimin solution provides a more parsimonious representation of these results than the varimax solution does; therefore, it was chosen for presentation. (The direct oblimin and varimax factor loading matrices, and the other factor solutions previously discussed may be obtained from the second author upon request.)

Factor I and Factor II were defined by 42 and 44 MARS items, respectively, with factor patterns loadings of $\geq .30$ and loadings of $\leq .30$ on the irrelevant factor. Three MARS items had factor loadings of $\geq .30$ on both factors. Five MARS items did not have factor loadings of $\geq .30$ on either factor.

For Factor I, thirteen (31%) of the 42 salient ($\geq .30$) MARS items reflect apprehension about anticipating, taking, and receiving the results of mathematics tests. Of the 42 salient items, these 13 items had the highest factor pattern loadings, ranging from .52 to .83 with a mean factor pattern loading of .70. Illustrative items included (factor pattern loadings in parenthesis): Thinking about an upcoming math test one day before (.83); Taking an examination (quiz) in a math course (.67); and, Receiving your final math grade in the mail (.70). The other 29 (69%) salient MARS items refer (with one exception) to activities that are directly associated with mathematics courses and classes. Illustrative items included: Listening to a lecture in a math class (.66); Solving a square root problem (.50); Buying a math textbook (.48); Listening to another student explain a math formula (.60); and, Opening a math or stat book and seeing a page full of problems (.77). Overall, 31 (74%) of the 42 salient items have the word "math" embedded in the item statement. Only one salient item--Reading a formula in chemistry (.40)--referred to something other than activities associated with mathematics courses. Factor I appears to be a measure of mathematics test anxiety or mathematics course anxiety (hereafter Factor I is labeled Mathematics Test Anxiety).

Factor II has high loading items that refer to everyday, concrete situations requiring some form of number manipulation (such as addition and multiplication). None of the 44 salient items referred to number manipulation associated with mathematics courses or classes. Ten (23%) of the 44 salient items referred to the use of elementary arithmetic skills without a context for the application of these skills. Illustrative items included: Adding up 976 + 777 on paper (.56); Having someone watch you as you total up a column of figures (.57); and, Dividing a five digit number by a two digit number in private with pencil and paper (.56). The majority (52%) of the salient items refer to the use of practical arithmetic skills necessary for making monetary decisions. Illustrative items included: Totaling up a dinner bill that you think overcharged you (.66); Figuring the sales tax on a purchase that costs more than $1.00 (.63); and, Being responsible for collecting dues for an organization and keeping track of the amount (.63). Finally, eleven salient items (25%) did not seem susceptible to the above categorization. These item statements refer to a wide variety of practical situations involving numbers. Illustrative items were: Determining the grade point average for your last term (.49); Deciding which courses to take in order to come out with the proper number of credit hours for full time enrollment (.51); and, Studying for a driver's license test and memorizing the figures involved, such as the distances it takes to stop a car going at different speeds (.44). The interpretation of these salient items indicates that Factor II could be labeled as Numerical Anxiety.

Factor Scales and Correlates

Two scales were constructed to measure the Mathematics Test Anxiety and Numerical Anxiety Factors. The resulting scales each included 15 items. Table 1 and Table 2 show the MARS item composition of these scales along with the item means and standard deviations for the total female sample (N=350). For the total sample, the mean Mathematics Test Anxiety scale score was 52.76 (SD=13.51) and the mean Numerical Anxiety scale score was 27.49 (SD=9.36). With the exception of one item (Hearing friends make bets on a game as they quote the odds), the mean item scores for the numerical anxiety scale were lower than the mean item scores for the Mathematics Test Anxiety scale. In general, the item score standard deviations for the Numerical Anxiety scale were smaller than the item score standard deviations for the Mathematics Test Anxiety scale. Comparison of the mean scale scores, mean item scores, and item score standard devia   of the Mathematics Test Anxiety scale and the Numerical Anxiety scale indicated that this female sample reported considerably more apprehension about mathematics tests and activities associated with mathematics courses than about everyday, practical numerical manipulation. In fact, on the average the female participants in the math anxiety program were reporting "not at all" to "a little" apprehension to the Numerical Anxiety items and "a fair amount" to "much" apprehension to the Mathematics Test Anxiety items.

Insert Table 1 and Table 2 about here

The internal consistency reliability of these scales was examined by calculating coefficient alpha (Nunnally, 1967). Coefficient alpha was .93 for the Mathematics Test Anxiety scale and .87 for the Numerical Anxiety scale. These coefficients compare favorably with the .97 ($N=397$) coefficient alpha, as reported by Richardson and Suinn (1972) for the 98-item MARS.

The attempt to construct two independent mathematics anxiety scales was partially successful. The Pearson product-moment coefficient between the Mathematics Test Anxiety score and the Numerical Anxiety score was .34 ($N=350$, $p \leq .01$). Although some common variance exists between these two scales, the small amount (12%) of common variance compares very favorably to other reported attempts (see Alpert & Haber, 1960; Watson & Friend, 1969) to develop discriminant relationships between specific anxiety scales.

Certain discriminant and convergent relationships were expected between the Mathematics Test Anxiety and Numerical Anxiety scales and the specific anxiety scales. These expected relationships were based on the previous content interpretations of the factors and the subsequent factor scales.

First, it was expected that a higher correlation would be found between the Mathematics Test Anxiety scale and the STABS, a measure of test anxiety, than between the Numerical Anxiety scale and the STABS.

Second, the ATT+ and ATT-, constructed to measure the facilitative and debilitative effects of anxiety on test performance in an academic testing situation, were also expected to correlate more highly with the Mathematics Test Anxiety scale than with the Numerical Anxiety scale. Third, very similar correlations were expected between the arithmetic placement test and the Mathematics Test Anxiety and the Numerical Anxiety scales, as the arithmetic test measures very simple, practical everyday arithmetic skills. No expectations were developed concerning the relationship between the factor scales and the FNE scale and the Fennema-Sherman Math Anxiety Scale. All of the correlations were expected to be of moderate size, with the exception of the large correlation expected between the Mathematics Test Anxiety scale and the STABS.

Table 3 shows the correlations of the factor scales with the STABS, ATT+ scale, ATT- scale, FNE scale, and the arithmetic placement test for Sample 1 and with the Fennema-Sherman Math Anxiety Scale and the arithmetic placement test for Sample 2. The mean Mathematics Test Anxiety score was 51.60 (SD=13.65) for Sample 1 and 51.35 (SD=12.85) for Sample 2. The mean Numerical Anxiety score was 27.13 (SD=9.50) for Sample 1 and 27.06 (SD=9.16) for Sample 2. As shown in Table 3, most of the hypothesized discriminant and convergent relationships were confirmed--with several exceptions. One such exception was the higher than expected correlation (.41, $p < .01$) between the Numerical Anxiety scale and the STABS. However, a significant difference (t (64)=3.64, $p < .01$) between the correlations of Mathematics Test Anxiety and Numerical Anxiety scales with the STABS was found in the expected direction. Another exception was that the Numerical

Anxiety scale correlated more highly with the arithmetic placement test than did the Mathematics Test Anxiety scale. The differences between the correlations were statistically significant for Sample 1 (t (66)=2.05, $p < .05$) but not for Sample 2 (t (110)=.84, $p > .05$). Finally, the Fennema-Sherman Math Anxiety Scale was highly correlated (-.65, $p \leq .01$) with the Mathematics Test Anxiety scale and only moderately correlated (-.27, $p \leq .01$) with the Numerical Anxiety scale. Seemingly, the Fennema-Sherman Math Anxiety Scale is predominantly a measure of mathematics test anxiety.

Insert Table 3 about here

Discussion

Recently, Fox (1977) has claimed that "The construct of a specific anxiety about numbers has been validated" (p. 30). Treatment programs and intervention strategies for the alleviation of mathematics anxiety have been implemented and studies investigating the prevalence of mathematics anxiety have been conducted. However, few studies have examined the measures of mathematics anxiety. In this study, the homogeneity of the MARS was investigated. The results indicated that contrary to previous statements (Richardson & Suinn, 1972), mathematics anxiety as measured by the MARS is not an unidimensional construct. It seems that the correlations among the MARS items can be represented by two factors.

These two factors were identified and labeled as Mathematics Test Anxiety and Numerical Anxiety. Factor-derived scales were developed to measure these two factors. Correlation of the Mathematics Test Anxiety scale and the Numerical Anxiety scale with specific anxiety scales and an arithmetic test provided results that further confirmed the factor analytic interpretations.

Factor I, which we have named Mathematics Test Anxiety, might also have been labeled mathematics course anxiety. Many of the salient items of Factor I denote activities associated with academic mathematics courses. However, the most salient items for Factor I involved anticipation, completion, and receiving the results of mathematics tests. The factor-derived Mathematics Anxiety scale correlated highly ($r=.75$, $p \leq .01$) with the STABS, a measure of general test anxiety. The mean STABS score ($M=149.72$) for Sample 1 participants shows that the average female participant is at the 79th percentile of Suinn's college student norms (Suinn, 1969; see Table 3) and/or reports a level of test anxiety similar to students who voluntarily seek assistance for math anxiety (Suinn & Richardson, 1971; see Table 1). The high mean Mathematics Test Anxiety score when compared to the mean STABS score (the STABS uses the same item format and test instructions as the MARS) indicates that Sample 1 participants are somewhat more anxious about mathematics tests than about tests in general. In addition, the large mean score difference between the ATT- ($M=29.87$) and the ATT+ ($M=20.90$) also supports the interpretation that the female participants (Sample 1) were highly anxious about test taking in general.

Factor II was labeled Numerical Anxiety. Salient items for Factor II refer to everyday, concrete situations requiring some form of number manipulation. As expected, the Numerical Anxiety scale correlated lower than the Mathematics Test Anxiety scale with the three measures of test anxiety. Unexpectedly, the Numerical Anxiety scale correlated higher with the arithmetic test than did the Mathematics Test Anxiety scale. However, this finding is somewhat less perplexing upon examination of the item composition of the arithmetic test. Most of the items measure arithmetic skills which are like those skills needed for everyday, practical numerical manipulations. The mean Numerical Anxiety scale score is considerably lower than the mean

Mathematics Test Anxiety scale score for the total sample. Comparison of the distributions of Numerical Anxiety scores and Mathematics Test Anxiety scores shows that 72% of the Mathematics Test Anxiety scores and 6% of the Numerical Anxiety scores are above a score of 44 (scores above 44 indicate "a fair amount" or more of anxiety). These data indicate that the female participants are not only much more anxious about mathematics tests than they are about practical numerical tasks, but also that they are relatively unconcerned about numerical manipulation in the context of daily activities.

The results of this study suggest that, for this female sample, the domain of mathematics anxiety as measured by the MARS is best described as primarily test anxiety and only secondarily as anxiety associated with mathematics course activities. However, the fact that mathematics is a very broad field makes this and other definitions of the mathematics anxiety domain problematic. Fennema (1977) has noted that a serious problem in mathematics attitude research is the use of global definitions of mathematics. Fennema (1977) has further noted that "Mathematics is a complex discipline involving many kinds of related but diverse subject matters and skills. To assume that a person feels the same towards each part of mathematics is not reasonable." (p. 103-104). The usefulness of present instrumentation of mathematics anxiety is hampered by the lack of an adequate definition of mathematics.

Richardson and Suinn (1972) specified the mathematics anxiety domain as involving "the manipulation of numbers and the solving of mathematical problems" (p. 551). If the "solving of mathematical problems" is considered within the context of mathematics tests, the two MARS factors identified are a good fit to that definition of mathematics anxiety domain. Results also indicate that mathematics anxiety as conceptualized by Richardson and Suinn (1972) can be measured with the factor-derived scales. The factor-derived scales are highly internally consistent and show expected convergent and divergent relationships with other specific anxiety scales. Compared to the

98-item MARS, these scales have several advantages. An obvious advantage of the two 15-item factor-derived scales is that these scales will take less time to complete than the MARS. As noted by Peterson (1965), shorter self-report instruments are superior to more lengthy and cumbersome instruments when cost is taken into account. Since the assessment of a client's status with multifactorial rather than unifactorial measures may mask therapeutic changes, Bergin and Lambert (1978) have recommended the development of indices that are more situation specific than the presently used global improvement indices. The MARS factor-derived scales meet the necessary criteria of internal consistency and homogeneity.

The present study has several implications for the treatment of mathematics anxiety. Administration of a measure of test anxiety to mathematics anxiety program participants should be implemented, especially in those programs that serve "returning women" students. In a study of the effects of comparative treatments of mathematics anxiety, Suinn and Richardson (1971) unexpectedly discovered "that the treated subjects of both groups also show STABS scores significantly higher than the nonanxious control sample" (p. 506). Betz (1978) found a moderately high relationship between the modified Fennema-Sherman Math Anxiety Scale and Spielberger's Test Anxiety Inventory for 182 students enrolled in mathematics courses at Ohio State University. Results from the present study showed that the participants were almost as apprehensive about tests in general as tests in particular (mathematics).

Although most treatment studies of mathematics anxiety have used the items from the pretest-post test measure in the math anxiety hierarchies, thereby raising questions about the generalizability of treatment effects, the cognitive and self-control therapies (see Mahoney & Arnkoff, 1978; for a review) as presented by Suinn and Richardson (1971) and Hendel and Davis, (1978) seem more effective in reducing mathematics anxiety than do insight-oriented therapy and systematic desensitization. Previous research (Crouch, 1971; Nash, 1970) has shown that insight-oriented therapy has not been effective for reducing mathematics anxiety. Mixed results (Addleman, 1972; Crouch, 1971; Nash, 1970; Richardson & Suinn, 1972) have been found with systematic desensitization in the treatment of mathematics anxiety. The cognitive therapies seem to be more amenable not only to tailoring the treatment to the individual, but--more importantly in terms of mathematics anxiety--to treating multiple specific anxieties and to preparing the client to cope with future anxieties and problems (Mahoney & Arnkoff, 1978).

Recently, Casserly (Note 2), writing about how to increase enrollment in mathematics courses for women, has stated that mathematics anxiety is "often used to conveniently lump together all sorts of phenomena associated with learning mathematics or not learning mathematics" (p. 7). Mathematics anxiety has been postulated--especially for women--to affect enrollment in mathematics courses, learning of mathematics, and mathematics performance, thereby affecting a student's educational and career goals. Little or no evidence has been presented to support these assertions. Research results suggest that (a) mathematics anxiety is not limited to females but is a phenomenon which is prevalent among students who are poorly prepared in mathematics (Betz, 1978; Casserly, Note 2; Dreger & Aiken, 1957; Hendel, Note 3; Hendel, in press; Hendel & Rounds, Note 1), (b) the large disparity between female and male enrollment in mathematics courses can best be explained by other more established and parsimonious constructs than by mathematics anxiety (Aiken, 1975; Casserly, 1975; Fennema, 1977; Fox, 1977), (c) mathematics anxiety measures (in their relationships with mathematics ability and achievement tests) have not yet demonstrated incremental validity beyond that of trait and state (test) anxiety measures, measures of mathematics attitudes, self-estimates of abilities, and previous mathematics preparation and performance (Aiken, 1970, 1976; Callahan & Glennon, 1975; Crosswhite, 1972; Hendel, Note 3; Fennema &

Sherman, 1977; Sherman & Fennema, 1977), (d) reduction in mathematics anxiety was not related to mathematics grades (Hyman, 1974) and mathematics anxiety adds little to the prediction of mathematics grades (Betz, Note 4), and (e) moderate to high relationships exist between mathematics anxiety measures and measures of test anxiety and mathematics attitudes (Betz, 1978; Hendel, in press)--in some cases almost as high as the relationship between alternative measures of mathematics anxiety (Hendel & Rounds, Note 1). These results and results from the present study suggest that the concept and/or measures of mathematics anxiety may need revision to be of sufficient value to enrich our understanding of mathematics learning and performance for women or men.

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Footnotes

Requests for reprints should be sent to Darwin D. Hendel, Measurement Services Center, University of Minnesota, 9 Clarence Avenue SE, Minneapolis, Minnesota, 55414.

[1] Due to an error in printing by the Rocky Mountain Behavioral Science Institute, a 94-item version of the MARS was used instead of the standard 98-item version. Therefore, items numbered 95, 96, 97, and 98 were not included in this analysis.

Table 1

Means and Standard Deviations of the

MARS Items Composing the Mathematics

Text Anxiety Scale

MARS Item	M	SD
26. Signing up for a math course.	3.20	1.35
28. Walking into a math class.	2.92	1.40
36. Walking on campus and thinking about a math course.	2.80	1.30
39. Sitting in a math class and waiting for the instructor to arrive.	2.38	1.31
45. Raising your hand in a math class to ask a question.	3.29	1.46
54. Taking an examination (final) in a math course.	4.36	1.01
73. Thinking about an upcoming math test one week before.	3.49	1.24
74. Thinking about an upcoming math test one day before.	4.00	1.16
75. Thinking about an upcoming math test one hour before.	4.13	1.12
76. Thinking about an upcoming math test five minutes before.	4.13	1.18
78. Waiting to get a math test returned in which you expected to do well.	3.32	1.26
79. Waiting to get a math test returned in which you expected to do poorly.	3.96	1.18
81. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.	3.65	1.34
85. Receiving your final math grade in the mail.	3.48	1.33
91. Being given a "pop" quiz in a math class.	3.65	1.27

Note n = 350

Table 2

Means and Standard Deviations of the

MARS Items Composing the Numerical

Anxiety Scale

MARS Item	M	SD
1. Determining the amount of change you should get back from a purchase involving several items.	1.82	1.00
5. Dividing a five digit number by a two digit number in private with pencil and paper.	1.48	.92
7. Listening to a salesman show you how you would save money by buying his higher priced product because it reduces long term expenses.	2.21	1.22
8. Listening to a person explain how he figured out your share of expenses on a trip, including meals, transportation, housing, etc.	2.08	1.10
14. Adding up 976 + 777 on paper.	1.23	.57
33. Reading your W-2 form (or other statement showing your annual earning and taxes).	2.06	1.34
47. Reading a cash register receipt after your purchase.	1.33	.67
48. Figuring the sales tax on a purchase that costs more than $1.00.	1.75	1.04
50. Figuring out which of two summer job offers is the most lucrative: where one involves a lower salary, room and board, and travel, while the other one involves a higher salary but no other benefits.	1.81	1.01
59. Hearing friends make bets on a game as they quote the odds.	2.56	1.32
63. Juggling class times around at registration to determine the best schedule.	1.96	1.10
64. Deciding which courses to take in order to come out with the proper number of credit hours for full time enrollment.	1.86	1.09
65. Working a concrete, <u>everyday application</u> of mathematics that has meaning to <u>me, e.g., figuring out</u> how much I can spend on recreational purposes after paying other bills.	1.81	1.09
67. Being given a set of numerical problems involving addition to solve on paper.	1.67	.94
90. Figuring out your monthly budget.	1.86	1.13

Note n = 350

Table 3

Means, Standard Deviations, and Correlations of the Anxiety Scales and Arithmetic Placement Test with the MARS Factor Scales, by Sample

Sample 1

Scale	Math Test	Numerical	t[a]	n	M	SD
Test Anxiety (STABS)	.75**	.41**	3.64**	65	149.72	39.24
Facilitating Anxiety	-.29**	-.21*	.56	61	21.90	6.13
Debilitating Anxiety	.49**	.32**	1.32	61	29.87	7.94
Fear of Negative Evaluation	.44**	.36**	.61	62	15.97	6.82
Arithmetic Placement Test	-.21*	-.47**	2.05*	67	46.01	14.13

Sample 2[b]

Scale	Math Test	Numerical	t[a]	M	SD
Fennema-Sherman Mathematics Anxiety	-.65**	-.27**	4.49**	23.51	8.47
Arithmetic Placement Test	-.19*	-.28**	.84	25.42	5.73

[a]Two-tailed t-test of the difference between correlations.

[b]n = 111

* $p \leq .05$

**$p \leq .01$