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ABSTRACT

Presented are student performance objectives, a student progress chart, and assignment sheets with objective and diagnostic measures for the stated performance objectives in College Algebra II. Topics covered include: differencing and complements; real numbers; factoring; fractions; linear equations; exponents and radicals; complex numbers, relations and functions; quadratics; determinants; factorials, combinations and permutations; binomial theorem; summation motation; and progressions. (MK)

COLLEGE ALGEBRA, II

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INTRODUCTION

During the Summer of 1974 criterion-referenced diagnostic tests were written for college arithmetic and college algebra. The materials included herewith are student performance objectives, student progress chart, assignment sheets/objective and diagnostic measures for each objective.

The materials were prepared for use at specific colleges in the 1974-75 school year and were so named to match the college numbering system.

College Arithmetic is the same as Math 60.

College Arithmetic and Pre-Algebra is the same as Math 50.

College Algebra I is the same as Math A.

College Algebra II is the same as Math D.

OBJECTIVES

MATHEMATICS D

TEACHER OBJECTIVES

1. SETS

1.1 DIFFERENCING AND COMPLEMENTS. Given a universal set and two subsets A and B, the student will find the difference and complements and combinations of these operations.

2. REAL NUMBERS

2.1 FIELD POSTULATES FOR THE REAL NUMBERS. The student will apply a given field axiom to complete a given algebraic equality.

3. FACTORING.

3.1 REVIEW OF FACTORING AND SPECIAL PRODUCTS. The student will factor polynomial expressions of the following types:

(a) expressions containing a common monomial factor,

(b) differences of squares, and

(c) trinomials factorable as a product of binomials.

3.2 SUM OR DIFFERENCE OF TWO CUBES. The student will factor algebraic expressions which are written in the form of a sum or difference of two cubes.

3.3 FACTORING BY GROUPING. The student will factor four-term algebraic expressions by grouping terms and obtaining a common binomial factor.

4. FRACTIONS

4.1 REDUCING, MULTIPLYING, AND DIVIDING FRACTIONS. Given a fraction, the student will reduce it to simplest form. Factoring by grouping and the sum or difference of two cubes will be emphasized. Given two or more fractions, the student will multiply and/or divide them in any combination. Fractions of the form $\frac{x - y}{y - x}$ will be emphasized.

4.2 ADDITION OF FRACTIONS AND SIMPLICATION OF COMPLEX FRACTIONS. Given two or more fractions whose denominators are at most factorable quadratic trinomials, the student will add or subtract in any combination. Given a complex fraction, whose terms may themselves be complex fractions, the student will simplify.

5. LINEAR EQUATIONS

5.1 LINEAR EQUATIONS. Given linear equations, including linear equations containing fractions, the student will find the solution sets. If the variable appears in the denominator, the student will determine which values must be excluded.

5.2 APPLICATIONS OF LINEAR EQUATIONS. The student will translate English statements into linear equations and will determine the solution sets of the equations.

5.3 INEQUALITIES AND ABSOLUTE VALUE. The student will solve first degree inequalities in one variable. He will also transform inequalities involving absolute values to this form and then solve.

6. EXPONENTS AND RADICALS

6.1 EXPONENTS. Given any algebraic expression with rational exponents the student will be able to simplify the expression by applying the laws of exponents. Given an expression containing fractional exponents, the student will change the expression to radical form.

6.2 RADICALS. Given any algebraic expression containing radicals the student will simplify by:

(a) removing factors whose power is greater than the index of the radical,

(b) rationalizing the denominator which is either a monomial or binomial, and

(c) lowering the index of a radical when possible.

Given expressions containing radicals the student will perform the operations of addition, subtraction, multiplication, and division.

6.3 INTRODUCTION TO COMPLEX NUMBERS. Given a complex number the student will recognize it as such and write it in the form a + bi. The student will perform operations with complex numbers.

7. RELATIONS AND FUNCTIONS

7.1 DOMAIN AND RANGE OF A FUNCTION. Given the rule for a function, the student will determine the domain and range.

7.2 LENGTH OF A LINE SEGMENT. Given two points in the plane, the student will determine the length of the line segment joining the two points.

7.3 SLOPE OF A LINE. Given two points in the plane, the student will determine the slope of the line passing through the two points.

7.4 EQUATION OF A LINE IN THE COORDINATE PLANE. Given two points or a point and the slope, the student will write the equation of the line in the form $ax + by + c = 0$. Given the equation of a line, the student will write it in the form $y = mx + b$ and recognize the slope and y-intercept.

8. QUADRATICS

8.1 GENERAL SOLUTION OF QUADRATIC EQUATIONS. Given any quadratic equation with real coefficients, the student will find the solution set. Coefficients for use in the quadratic formula may be binomials, but no more than one binomial per equation. For example, $3x^2 + mx + 3 + k = 0$ but not $3x^2 + mx + x + 3 + k = 0$.

8.2 PROPERTIES OF THE ROOTS OF A QUADRATIC AND OTHER EQUATIONS. Given any quadratic equation the student will use the discriminant to determine conditions for real or complex roots. (Do not use problems like $2x^2 + kx + 7 = 0$ when looking for any but equal roots. (This will lead to a quadratic inequality and this topic is not covered in Math. D now.) Given any quadratic equation, the student will find the sum and product of the roots without solving the equation. The student must have memorized the formulas $r_1 + r_2 = -\frac{b}{a}$ and $r_1 r_2 = \frac{c}{a}$. Given a solution set with as many as 4 roots, the student will find an equation in x which has these roots.

8.3 EQUATIONS WHICH HAVE QUADRATIC FORM. Given any equation, which can be expressed in the form of a quadratic equation, the student will find all real and complex roots.

8.4 EQUATIONS WHICH CONTAIN RADICALS. Given an equation in which the variable occurs under a radical (the radical may have order 2, 3, or 4), the student will find the solution set. This will involve checking by substitution to eliminate extraneous roots.

8.5 CONIC SECTIONS AND THEIR GRAPHS. Given a quadratic equation in two variables, the student will name the shape of the graph of the solution set and plot the graph using the "rapid sketch method". Parabola, circles, ellipses, and hyperbolas are to be included. Given a quadratic equation in two variables the student will determine the real values in the domain and range.

9. DETERMINANTS

9.1 DETERMINANTS. Given a 2×2 or a 3×3 determinant, the student will find the minor and cofactor of any element and expand the determinant by cofactors.

9.2. SOLUTION OF SYSTEMS OF EQUATIONS USING CRAMER'S RULE. Given a system of equations in two or three variables the student will find the solution set using Cramer's rule.

9.3 MATRICES. Given suitable matrices, no larger than 3×3, the student will perform the operations of matrix addition and multiplication.

10. FACTORIALS, COMBINATIONS, AND PERMUTATIONS.

10.1 FACTORIALS, COMBINATIONS, AND PERMUTATIONS. The student will compute factorials, combinations, and permutations and apply them to specific problems.

11. BINOMIAL THEOREM.

11.1 BINOMIAL THEOREM. Given a binomial to a positive integral power, the student will expand. The student will find a specified term of a binomial to a positive integral power.

12. SUMMATION NOTATION

12.1 SUMMATION NOTATION. The student will be able to express expanded sums in Σ-notation, and expand sums given in Σ-notation.

13. PROGRESSIONS

13.1 PROGRESSIONS. Given three consecutive entries in a finite arithmetic or geometric progression, the student will find the last term and sum. Given three consecutive terms in an infinite geometric progression ($|r| < 1$), the student will find the sum. Given any two terms in an arithmetic or geometric progression, the student will insert the designated number of means between the two terms.

STUDENT PROGRESS CHART

MATH D

STUDENT PROGRESS CHART

KERN COMMUNITY COLLEGE DISTRICT
vita in aequilibrio
MCMXIII

STUDENT'S NAME			factoring												functions				quadratics										
	1.1 Differencing-Complements	2.1 Field Postulates	3.1 Review-Special Products	3.2 Sum-Difference of Cubes	3.3 Grouping	4.1 Operations	4.2 Complex Fractions	5.1 Linear Equations	5.2 Applications	5.3 Inequalities-Absolute Value	6.1 Exponents	6.2 Radicals	6.3 Complex Numbers	7.1 Domain-Range	7.2 Line Segments	7.3 Slope	7.4 Equation of Line	8.1 Solutions	8.2 Roots	8.3 Quadratic Form	8.4 Radical Form	8.5 Conic Sections	9.1 Determinants	9.2 Systems of Equations	9.3 Matrices	10.1 Combinations-Permutations	11.1 Binomial Theorem	12.1 Summation Notation	13.1 Progressions

ASSIGNMENT SHEET

MATHEMATICS D

ASSIGNMENT SHEET

TOPICS		TEXTS - MATERIALS			
NUMBER	TITLE	RUSSELL & COLLINS	NEWMYER & KLENTOS	TAPE NUMBER	OTHER
	SETS				
1.1	Differencing & Complements	Ex. 1.3, pp.9-11			
	REAL NUMBERS				
2.1	Field Postulates for the Real Numbers	Ex. 1.4, pp.11-14	Unit 3, pp.27-35 problems -11,p. 35	3	
	FACTORING				
3.1	Review of Factoring & Special Products	Ex. 2.2, pp.51-57	Unit 5, pp.45-49	5	
3.2	Sum or Difference of Two Cubes	Ex. 2.3, pp.57-60	Unit 5, p.50, problem set B, p. 51; problem set B, p. 53.	5	
3.3	Factoring by Grouping	Ex. 2.4, pp.60-62	Unit 5, p.50, problem set B, p. 53.	5	
	FRACTIONS				
4.1	Reducing, Multiplying, & Dividing Fractions	Ex. 3.1-3.2, pp. 67-82.	Unit 6 pp.54-59, problem sets B,C, pp.68-69.	6	

NUMBER	TITLE	RUSSELL & COLLINS	NEWMYER & KLENTOS	TAPE NUMBER	OTHER
4.2	Addition of Fractions & Simplification of Complex Fractions	Ex. 3.4-3.5, pp. 84-95.	Unit 6, pp 60-62, problem set B, p. 64.; problem set C, pp.68-69.	6	
	LINEAR EQUATIONS				
5.1	Linear Equations	Ex. 4.1, pp.96-106.	Unit 9, pp.103-107, problem set B, p.110, problem set B, p.114.	9	
5.2	Applications of Linear Equations	Ex. 4.4, pp.126-130.	Unit 9, pp.108-109, problem set D, p. 110, problem set E, p.114.	9	
	EXPONENTS AND RADICALS				
6.1	Exponents	Ex. 5.1-5.2, pp. 131-142.	Unit 7, pp. 71-79, problem set A, p. 81; problem sets A,B, p.84.	7	
6.2	Radicals	Ex. 5.4-5.7, pp. 146-160.	Unit 8, pp.85-94, problem sets A,B, p. 95, problem sets A,B,C, p. 100 -101.	8	
6.3	Introduction to Complex Numbers	Ex. 5.9, pp.163 -166.	Unit 26, pp.325-331, problem sets A,B, p.333, problem sets A,B, p. 335.	26	

NUMBER	TITLE	RUSSELL & COLLINS	NEWMYER & KLENTOS	TAPE NUMBER	OTHER
	RELATIONS & FUNCTIONS				
7.1	Domain & Range of a Function	Ex. 6.1-6.2, pp. 167-176.	Unit 11, pp.133-142.	11	
7.2	Length of a Line Segment	Ex. 6.5, pp.182-183, problem set 6.4, pp.184-185.	Unit 12, p.156.	12	
7.3	Slope of a Line	Ex. 6.6, pp.183--185.	Unit 12, pp.150-151.	12	
7.4	Equation of a line in the Coordinate Plane	Ex. 6.7, pp.185-191.	Unit 12, pp.146-154, problem set C, p.162.	12	
	QUADRATICS				
8.1	General Solution of Quadratic Equations	Ex. 7.2-7.3, pp. 205-217.	Unit 10, pp.115-122, problem sets A-E, p.125.	10	
8.2	Properties of the Roots of Equations	Ex. 7.5-7.6, pp. 220-225.	Unit 10, p.123, problem set F, p.125; problem sets F,G, p.131.	10	
8.3	Equations Which Have Quadratic Form	Ex. 7.7, pp. 226-228.	Unit 21, p. 251.	21	
8.4	Equations Which Contain Radicals	Ex. 7.4, pp.218-220.	Unit 21, pp.249-251.	21	
8.5	Conic Sections & Their Graphs	Ex. 8.7, pp.267-272.	Unit 14, pp.177-186, problem sets A,B,C, p.187, problem sets A,B,C, p.191.	14	

NUMBER	TITLE	RUSSELL & COLLINS	NEWMYER & KLENTOS	TAPE NUMBER	OTHER
	DETERMINANTS				
9.1	Determinants	Ex. 8.6, pp.261-263.	Unit 22, pp.275-276.	22	
9.2	Solutions of Systems of Equations Using Cramer's Rule	Ex. 8.6, pp.262-267.	Unit 22, pp.276-279, problem set D, p.280.	22	
9.3	Matrices				
10.1	FACTORIALS, COMBINATIONS, & PERMUTATIONS		Unit 27, pp.337-346. Unit 28, pp.347-355.	27 28	
11.1	THE BINOMIAL THEOREM	Ex. 13.1-13.2, pp.368-372.	Unit 24, pp.301-310.	24	
12.1	SUMMATION NOTATION		Unit 25, p.312.	25	
13.1	PROGRESSIONS	Ex. 12.1-12.3, pp.353-363.	Unit 25, pp.311-324.	25	

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PRETESTS

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MATHEMATICS D

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UNIT TESTS

TOPIC D-1.1

DIFFERENCING AND COMPLEMENTS

OBJECTIVES:

- Given two sets, A and B, to find the difference.
- Given a Universal set, U, and a subset A, to find the complement of A.

A. Given $U = \{1,2,3,\ldots,10\}$, $A = \{1,2,3,4\}$, $B = \{3,5,7\}$

Find:

1. $A - B$
2. $B - A$
3. A'
4. $A - (A \cap B)'$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Write the definition of difference, A - B, in set builder notation.

2. Is the operation differencing commutative?

3. Prove your answer to 2.

4. What is A - A?

C. 1. Write the definition of complement, A', in set builder notation.

2. What is (A')'?

3. What is U'?

4. What is (U-A)'?

TOPIC D-2.1

FIELD POSTULATES FOR THE REAL NUMBERS

OBJECTIVE:

- To apply a given field axiom to a particular algebraic expression.

A. Copy and complete the missing part of the following equalities using the field axiom listed to its right.

1.	$-cd + cd =$ ________	1. additive inverse axiom
2.	$8(xy + a) =$ ________	2. distributive axiom
3.	$a(x + y) =$ ________	3. commutative axiom for multiplication
4.	$(3a)\,x =$ ________	4. associative axiom for multiplication
5.	$(x + y)\ (\underline{\ ?\ }) = 1$	5. multiplicative inverse axiom
6.	$(a + b) + y =$ ________	6. commutative axiom for addition
7.	If $a \in R$ and $b \in R$, then _____ $\in R$, where R is the set of real numbers.	7. closure axiom for multiplication

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and answer as true or false:

1. $3 + 2 = 2 + 3$ is an example of the commutative axiom of addition. 1. ______

2. $5 \cdot 7 = 7 \cdot 5$ is an example of the associative axiom of multiplication. 2. ______

3. $y + 0 = y$ is an example of the additive inverse axiom. 3. ______

4. $5 + (-5) = 0$ is an example of the additive inverse axiom. 4. ______

5. $3(a + b) = 3a + 3b$ is an example of the distributive property. 5. ______

6. $3a \frac{1}{3a} = 1$ is an example of the multiplicative identity axiom. 6. ______

C. Copy and check (✓) the words that are used in the field axiom for real numbers.

1. absolute value 1. ____
2. associative 2. ____
3. additive inverse 3. ____
4. coefficient 4. ____
5. exponent 5. ____
6. distributive 6. ____
7. prime 7. ____
8. identity for addition 8. ____

TOPIC D-3.1

REVIEW OF FACTORING AND SPECIAL PRODUCTS

OBJECTIVE:

- To factor polynomials of the following type:

  a. $a^2 - b^2$

  b. $ac + bc$

  c. $ax^2 + bx + c$ (where factorable into two binomials)

A. Take diagnostic tests A-7.1, 7.2, 7.3

TOPIC D-3.2

SUM OR DIFFERENCE OF TWO CUBES

OBJECTIVE:

- To factor the sum or difference of two cubes.

  Example: $125x^3 - 27y^3 = (5x - 3y)(25x^2 + 15xy + 9y^2)$

A. Copy and factor.

1. $27a^3 + b^3$
2. $\frac{1}{8} - x^3y^3$
3. $16s^3 - 54r^3$
4. $x^6y^3 - 64a^3$
5. $(a + b)^3 + 8$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy, factor, or complete.

1. $a^3 + b^3$

2. $x^3 - y^3$

3. $x^3 + 8$

4. $t^3 - 27 = (t - 3)$ (___?___)

5. $64 + x^3 =$ (___?___) $(16 - 4x + x^2)$

TOPIC D-3.3

FACTORING BY GROUPING

OBJECTIVE:

- To factor algebraic expressions by grouping.

Examples: (1) $x^2 + 2x + 1 - y^2 = (x + y + 1)(x - y + 1)$

(2) $bx + 2b + cx + 2c = (x + 2)(b + c)$

A. Copy and factor.

1. $6ay - 9ax + 10by - 15bx$
2. $x^2 + 4x + 4 - y^2$
3. $4x^2 - 9y^2 + 2x - 3y$
4. $a^2 - y^2 - 6y - 9$
5. $2x^2 - 18y^2 - x + 3y$
6. $x^3 + 8y^3 + x + 2y$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. If unsuccessful see Topic A-7.4.

TOPIC D-4.1

REDUCING, MULTIPLYING, AND DIVIDING FRACTIONS

OBJECTIVES:

- To reduce a fraction to simplest form.
- To multiply two or more fractions.
- To divide two fractions

Example: $\frac{x+y}{x} \div \frac{3x^2}{x^2-y^2} \div \frac{x}{y} \div \frac{x-y}{x+y} = \frac{3y(x+y)}{x(x-y)^2}$

A. Copy and work the following:

1. Reduce to lowest terms:

$$\frac{c^3+d^3}{ac+ad-c^2-cd}$$

In 2, 3 and 4 perform the indicated operations and simplify:

2. $\frac{m^2-n^2}{6m^3} \cdot \frac{20m}{4n-4m}$

3. $\frac{4a^2+3ab-10b^2}{6a^2+13ab-8b^2} \div \frac{3ab+6b^2}{6a^2+16ab}$

4. $\frac{x^2-2xy+y^2}{x^2-y^2} \div \frac{x+y}{x^2+y^2} \cdot \frac{y}{y-x}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Reduce: $\frac{x + 16}{12}$

2. Reduce: $\frac{a - b}{b - a}$

Perform the indicated operations and simplify:

3. $\frac{a}{b} \div \frac{b}{a} \cdot \frac{a}{b}$

4. $\frac{a}{b} \div \frac{b}{a} \div \frac{a}{b}$

C. Factor into prime factors.

1. $8x^3 - 27$

2. $ax - ay + xy - y^2$

3. $20a^2 - 3ab - 9b^2$

4. $3a^4 - 27a^2y^2$

TOPIC D-4.2

ADDITION OF FRACTIONS AND SIMPLIFICATION OF COMPLEX FRACTIONS

OBJECTIVES:

- To add two or more fractions and simplify.

  Example: $\frac{a + b}{a} + \frac{2a}{b + a} = \frac{3a^2 + 2ab + b^2}{a(a + b)}$

- To simplify a complex fraction.

  Example: $\frac{\frac{1}{x} - x}{1 + \frac{1}{x}} = 1 - x$

A. Copy and write as a simplified fraction.

1. $\frac{3}{a - b} + \frac{2}{b - a} + \frac{5}{a^2 - b^2}$

2. $\frac{4}{a^2 + 6a + 8} - \frac{2}{a^2 + 7a + 12} - \frac{a}{a^2 + 5a + 6}$

3. $\frac{\frac{1}{2} - \frac{1}{k}}{\frac{1}{k^2} - \frac{1}{4}}$

4. $\frac{\frac{1}{x - 1} + 1}{1 + \frac{1}{x^2 - 1}}$

5. $\frac{m^2 + \frac{1}{m}}{1 - \frac{m}{1 - \frac{m}{m - 1}}}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Give an equivalent fraction with the indicated denominator.

1. $\frac{6}{b - 2y} = \frac{?}{2y - b}$

2. $\frac{a - 3}{a + 4} = \frac{?}{3a^2 + 11a - 4}$

3. Combine into a single term.

$$1 - \frac{x - 2}{x^2 - 1}$$

4. Give the complex fraction which is part of this larger complex fraction.

$$\frac{m^2 + \frac{1}{m}}{1 - \frac{m}{1 - \frac{m}{m - 1}}}$$

C. Find the L.C.D. for each set of fractions.

1. $\left\{\frac{2}{3x^2}, \frac{5}{3x^2 - 9x}\right\}$

2. $\left\{\frac{5}{x - 1}, \frac{3}{1 - x}\right\}$

Find the L.C.M. for the following sets of polynomials.

3. $\{2a^2 - 5ab - 3b^2, \ a^2 - 2ab - 3b^2\}$

4. $\{x, \ x - 1, \ x^2 - x, \ x^2 - 1\}$

TOPIC D-5.1

LINEAR EQUATIONS

OBJECTIVE:

- To find the solution set of a linear equation in one variable.

  Example: $3x + 2 = 7$. Solution set is $\left\{\frac{5}{3}\right\}$.

A. Copy and find the solution set for each of the following.

1. $5[2 + 3(x - 2)] + 20 = 0$

2. $\frac{3}{4}x - 1 = 2x + 9$

3. $\frac{2x}{3} - \frac{2x + 5}{6} = \frac{1}{2}$

4. $\frac{x}{x - 2} = \frac{-2}{x - 2} + 7$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Explain the difference between the two symbols {0} and { }.

2. Explain the procedure for clearing an equation of fractions.

3. Remove the grouping symbols: $a - (2b + c)$

4. Why is 2 not the solution to $\frac{1}{x + 2} + \frac{2}{x - 2} = \frac{4}{x^2 - 4}$?

C. Find the solution set.

1. $2x - 3 = 13$

2. $-\frac{3}{5}x = \frac{1}{6}$

3. $5m - 6 = -8m + 4$

4. $5 - x = 13 - x$

TOPIC D-5.2

APPLICATIONS OF LINEAR EQUATIONS

OBJECTIVE:

- To translate a statement into an algebraic equation and find the solution.

Example: Find 3 consecutive even integers whose sum is 138.

$n + (n + 2) + (n + 4) = 138$. The numbers are 44, 46, 48.

A. Solve the following and show all work on additional paper.

1. A man has to exert 40 pounds of pressure on a 10 foot bar to raise a stone from the ground. If the fulcrum is 6 inches from the end of the bar, how heavy is the stone?

2. Bob is two-thirds as old as Jim is now. Six years ago Bob was half as old as Jim was then. How old is Bob now?

3. A boy walked into the country at $2\frac{1}{2}$ miles per hour and rode back over the same route at 20 miles per hour. If the total time was $4\frac{1}{2}$ hours, how far did he walk?

4. How much water must be added to 10 gallons of a 50% solution of acid to dilute to a 20% solution of acid?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. If Joe is x years old today, how old was he:

a. 3 years ago

b. 5 years ago

c. 4 years from now

2. Rate x time = distance, i.e. rt = d. Solve for:

a. r

b. t

TOPIC D-5.3

INEQUALITIES AND ABSOLUTE VALUE

OBJECTIVE:

- To solve inequalities in one variable, including absolute values, and graph the solution on a real number line.

  Example: $2x + 3 > 1$

  Solution: $\{x \mid x > -1\}$

  Example: $|3x - 1| < 2$

  Solution: $\{x \mid -\frac{1}{3} < x < 1\}$

A. Copy and find the solution set and graph on a real number line.

1. $-1 < \frac{1}{3} - 2x < 5$
2. $\frac{1 - 3x}{5} < \frac{x}{3} + 1$
3. $|2 - 3x| > 4$
4. $\left|\frac{-3x - 2}{5}\right| < 1$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and find the solution set and graph on a real number line.

1. $2x \ngtr -3$

2. $5 - x < 1$

3. $|3x| < 2$

4. $|x - 1| > 2$

C. Graph the following on a real number line.

1. $x < 5$

2. $x \geq 0$

3. $-2 < x < 3$

4. $\{x|x < -2\} \cup \{x|x > 1\}$

TOPIC D-6.1

EXPONENTS

OBJECTIVES:

- To simplify expressions containing exponents.

  Example: $\frac{x^3y^{-2}}{x^{-3}y^3} = \frac{x^6}{y^5}$

- To apply the laws of exponents.

  Example: $(a^k)^{k+1} = a^{k^2+k}$

- To write expressions in equivalent exponential and radical notation.

  Example: $y^{5/6} = (\sqrt[6]{y})^5$

A. Copy and simplify.

1. $\frac{x^ky^kx^3}{x^{k-1}y}$

2. $\left(\frac{3^2a^{-2}b^3}{6a^4c}\right)^{-2}$

3. $\left(a^k + a^{k+1}\right)^2$

4. $(mx)^{2/3}$

5. $\left(\frac{x^{4n}y^{2n-1}}{x^{3n}y^{2n}}\right)^{1/2}$

6. $\frac{2x^{-1} - y^{-2}}{(2x)^{-2}}$

7. Evaluate $64^{-2/3}$

8. $y^{1/2} + y^{-1/2} = y^{-1/2}(? + ?)$

9. $(x^{-1} + y^{-1})^{-1}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Write in radical form.

a. $a^{1/3}$

b. $a^{1/3} b^{2/3}$

2. Express each of the following with fractional exponents.

a. $\sqrt[4]{m}$

b. $\sqrt[5]{ky^2}$

c. $\sqrt[6]{5} \cdot \sqrt[4]{3}$

If further difficulty is encountered see test A-14.1.

TOPIC D-6.2

RADICALS

OBJECTIVES:

- To simplify expressions containing radicals.

  Example: $4\sqrt{8x^3} = 8x\sqrt{2x}$

- To perform the fundamental operations with radicals.

  Example: $3(\sqrt{6} + \sqrt{5}) = 3\sqrt{6} + 3\sqrt{5}$

A. Copy and perform the indicated operations and simplify.

1. $\sqrt{200x^7y^6z^5}$
2. $4\sqrt{28} - 3\sqrt{63}$
3. $\sqrt[4]{\dfrac{40a}{3b^2}}$
4. $\dfrac{3\sqrt{2} + \sqrt{3}}{2 - \sqrt{3}}$
5. $(2\sqrt{5} - 3)^2$
6. $12x\sqrt[3]{54x^4} - 3\sqrt[3]{16x^7}$
7. $\sqrt[3]{9}(2\sqrt[3]{9} - 4\sqrt[3]{8})$
8. $\sqrt{m + \dfrac{1}{m}}$
9. $\sqrt[6]{8}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. If difficulty is encountered see tests A-14.2 and A-14.3.

TOPIC D-6.3

INTRODUCTION TO COMPLEX NUMBERS

OBJECTIVES:

- To express complex numbers in the form a + bi and to identify pure imaginaries.
- To perform fundamental operations with complex numbers and to simplify.

Example: $(4 + 2i) - (3 - 2i) = 1 + 4i$

A. Copy and express each of the following as a complex number in the form of a + bi and simplify. Which of the following are pure imaginaires?

1. $\sqrt{-16}$
2. $-3\sqrt{-8}$
3. $(1 + 2i)^2$
4. $\dfrac{1 - \sqrt{-2}}{1 + \sqrt{-2}}$
5. $\dfrac{16 + \sqrt{-48}}{12}$
6. $3(1 - \sqrt{-3}^2 + 2(1 - \sqrt{-3}) + 5$
7. i^{17}

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and perform the indicated operations and simplify.

1. i^2

2. $\sqrt{-25}$

3. $-\sqrt{-24}$

4. $(a + bi)^2$

5. $(1 + i)(1 - i)$

6. i^5

C. The symbol "i" is used to represent which of the following?

1. 1
2. -1
3. $\sqrt{-1}$
4. $-\sqrt{1}$

TOPIC D-7.1

DOMAIN AND RANGE OF A FUNCTION

OBJECTIVE:

- To find the domain and range of a function.

Example: $\{(x,y) \mid y = \sqrt{x}\}$, Domain $= \{x \mid x \geq 0\}$, Range $= \{y \mid y \geq 0\}$

A. Copy and give the domain and range of the following (x and y are real).

1. $f = \{(x,y) \mid y = 2x - 3\}$
2. $g = \{(x,y) \mid y = x^2\}$
3. $h = \{(x,y) \mid y = \sqrt{x^2 - 1}\}$
4. $f = \{(x,y) \mid y = \frac{x^2 - 1}{x - 1}\}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Define domain and range of a function.

2. Graph and give domain and range.

a. $f = \{(x,y) | y = x + 1\}$

b. $f = \{(x,y) | y = 2\}$

c. $f = \{(x,y) | y = |x|\}$

TOPIC D-7.2

LENGTH OF LINE SEGMENT

OBJECTIVE:

- To find the length of a line segment.

Example: Find the length of the line segment joining the points (1,2) and (4,6).

Solution: 5

A. Copy and find the length of the line segment joining the two given points.

1. (3,2); (8,6)
2. (-2,7); (3,0)
3. (-1,2); (-7,-3)
4. ($\frac{1}{2}$,3); (-2,$\frac{2}{3}$)

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and find the difference between the two numbers and square that difference.

1. 3,7

2. -5,1

3. 7,-3

4. -6,-1

C. Choose the correct formula for distance.

1. $d = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$

2. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. $d = \sqrt{(x_2 - y_2)^2 + (x_1 - y_1)^2}$

4. $d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

D. Copy and simplify the following:

1. $\sqrt{18}$

2. $\sqrt{36}$

3. $\sqrt{\frac{2}{3}}$

4. $\sqrt{\frac{25}{2}}$

TOPIC D-7.3

SLOPE OF A LINE

OBJECTIVE:

- To find the slope of a line passing through two points.

Example: Find the slope of the line passing through (1,2) and (4,6).

Solution: Slope is $\frac{4}{3}$.

A. Find the slope of the line passing through the given points.

1. (3,1); (4,2)
2. (-7,1); (2,-3)
3. (-2,-4); (-1,5)
4. $(\frac{7}{2}, -\frac{1}{2})$; $(4, -\frac{3}{2})$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Define slope.

2. Find $\frac{(3) - (-2)}{(-4) - (1)}$

3. Find $\frac{(-5) - (2)}{(3) - (-1)}$

TOPIC D-7.4

EQUATION OF A LINE IN THE COORDINATE PLANE

OBJECTIVES:

- Given two points or a point and the slope, to find the equation of a line.

  Example: Write the equation of a line passing through (-1,2), (3,5)

  Solution: $3x - 4y - 10 = 0$

- To express a line in general form, $ax + by + c = 0$, and slope intercept form, $y = mx + b$.

- Given a line in general form, to find the slope and y-intercept.

  Example: Find the slope and y-intercept of the line $2x + 3y + 1 = 0$

  Solution: $m = -\frac{2}{8}$, $b = -\frac{1}{3}$

A. Copy and find the equation of the lines described in problems 1 - 4 and express in general form::

1. Passes through (-2,3), (3,-7)

2. Passes through (-2,3) with slope $-\frac{2}{3}$

3. Passes through $(\frac{1}{2}, 1)$, $(-\frac{3}{4}, \frac{5}{3})$

4. Has x-and y-intercepts of 3 and 4, respectively.

5. Find the slope-intercept form of the equation of a line that passes through (-3,2) and (0,0) and give the slope and y-intercept.

Find the slope and y-intercept:

6. $5x + 2y - 7 = 0$

7. $3x + 4y + 5 = 0$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Find the slope of line through

a. (1,3) and (4,5)

b. (x,y) and (-2,1)

2. Rewrite in the form ax + by + c = 0:

$$\frac{y - 1}{x + 2} = -\frac{2}{5}$$

TOPIC D-8.1

GENERAL SOLUTION OF QUADRATIC EQUATIONS

OBJECTIVE:

- To find the solution set for a quadratic equation in one variable.

  Example: $\{x|3x^2 - 7x + 8 = 0\}$

  Solution: Solution set is $\left\{\frac{7 \pm i\sqrt{47}}{6}\right\}$

A. Copy and find the solution sets for the following:

1. $\{x|2x^2 + 5x + 9 = 0\}$
2. $\{x|3x^2 + px - p = 0\}$
3. Solve by factoring: $\{x|90x^2 - 75x + 10 = 0\}$
4. $\{y|ay^2 - ay - a - 1 = 0\}$
5. $\{x|\frac{x^2}{2} - 3x - 5 = 0\}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Which of the following is the quadratic formula used to solve the equation $ax^2 + bx + c = 0$?

1. $x = \dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. $x = \dfrac{b \pm \sqrt{b^2 + 4ac}}{2a}$

4. $x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

C. For each equation identify a, b, and c for use in the quadratic formula. (x is the variable.)

1. $2x^2 - yx + 5 = 0$

2. $kx^2 + x^2 + 2x - k = 0$

3. $bx^2 + 6ax + a + 1 = 0$

4. $5x^2 + 2x + kx + 3 + \pi = 0$

D. Express as complex numbers in the form a + bi.

1. $\sqrt{-9}$

2. $4 \pm \sqrt{-18}$

3. $\dfrac{6 \pm \sqrt{-24}}{18}$

4. $\dfrac{10 \pm i\sqrt{p^2 - 4p^2q}}{5}$

TOPIC D-8.2

PROPERTIES OF THE ROOTS OF EQUATIONS

OBJECTIVES:

- To determine the numerical coefficients of a quadratic equation having specified types of roots.

  Example: For which values of k will $kx^2 + 2x + 3 = 0$ have real roots?

  Solution: $k \leq \frac{1}{3}$

- To find the sum and product of the roots of a quadratic equation.

  Example: Find the sum and the product of the roots of $7y^2 + 3y - 8 = 0$.

  Solution: Sum of the roots is $-\frac{3}{7}$

  Product of the roots is $-\frac{8}{7}$

- To find an equation when roots are given.

  Example: Find an equation in x whose solution set is $\{2, 3i, -3i\}$.

  Solution: $x^3 - 2x^2 + 9x - 18 = 0$

A.

1. For which values of k will $kx^2 - 2x + 5 = 0$ have equal roots?
2. For which values of k will $3x^2 - 3x + k = 0$ have complex roots?
3. Find the sum and the product of the roots of the equation $5x^2 - 3x + 2 = 0$. (Do not solve the equation.)
4. Find an equation in x whose solution set is $\{0, 1 \pm \sqrt{3}\}$.

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Give the discriminant of the quadratic equation $ax^2 + bx + c = 0$.

2. If the discriminant is less than zero then the roots are __?__.

3. Equal roots occur when the discriminant is __?__.

4. What is the discriminant of $5y^2 + 3y - 2 = 0$?

5. Under what conditions will a quadratic equation have rational roots?

C. 1. If r_1 and r_2 are the roots of $ax^2 + bx + c = 0$,

a. What is $r_1 + r_2$?

b. What is $r_1 r_2$?

2. Given the solution set $\{3,2\}$, what is the degree of the equation?

3. Given the solution set $\{-1,2,\pm 5\}$, what is the degree of the equation?

4. Write a quadratic equation with the binomial factors $(x - 2)$ and $(x - 5)$.

D. 1. Given $p = -3$, $q = -1$, evaluate $-p-2q$.

2. Multiply $(x - 3 + \sqrt{2})(x - 3 - \sqrt{2})$

TOPIC D-8.3

EQUATIONS WHICH HAVE QUADRATIC FORM

OBJECTIVE:

- To solve an equation which can be expressed as a quadratic equation.

  Example: Find the solution set for $x^4 - 3x^2 + 2 = 0$.

  Solution: Solution set is $\{\pm\sqrt{2}, \pm 1\}$

A. Copy and find all roots of the following equations.

1. $x^4 - 5x^2 + 4 = 0$
2. $x^6 - 7x^3 - 8 = 0$
3. $(x^2 + 3x)^2 - 2x(x + 3) - 8 = 0$
4. $x - 5x^{1/2} - 6 = 0$
5. $x^{2/3} - x^{1/3} - 2 = 0$
6. $3\left(\frac{1}{x}\right)^2 + 2\frac{1}{x} - 1 = 0$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Write each equation as a quadratic equation in w ($aw^2 + bw + c = 0$) by substituting w for the appropriate expression. Show the expression which w replaces.

1. $x^4 - 8x^2 - 9 = 0$

2. $6x^{2/3} - 5x^{1/3} - 1 = 0$

3. $(k + 1)^2 - 3(k + 1) + 2 = 0$

4. $3(4x^2 - 2)^2 - 7(4x^2 - 2) + 2 = 0$

5. $\frac{9}{x^2} + \frac{3}{x} - 1 = 0$

C. 1. If $w = a^{1/3}$ and $w = 3$, find a

2. If $w = \frac{1}{x}$ and $w = \pm\frac{3}{5}$, find x.

3. If $w = x^2 + 5x$ and $w = -4$, find x.

4. If $w = x$ and $w = 20$, find x.

5. If $w = x$ and $w = 27$, find x.

TOPIC D-8.4

EQUATIONS WHICH CONTAIN RADICALS

OBJECTIVE:

- To find the solution set for equations which contain radicals.

  Example: $\sqrt{x} - \sqrt{x+2} = 3$

  Solution: Solution set is $\emptyset$

A. Copy and find solution set.

1. $3 + \sqrt{8y - 15} = 2y$
2. $(x - 4)^{1/2} - (2x - 1)^{1/2} = (3x + 1)^{1/2}$
3. $\sqrt{x^2 - \sqrt{x + 7}} = x - 1$
4. $\sqrt[3]{1 - y} = 2$
5. $\sqrt{2x - 4} - \sqrt{3x + 4} + 2 = 0$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and square each expression.

1. $\sqrt{x} - 3$

2. $\sqrt{x + 2} + \sqrt{x - 3}$

3. $5\sqrt{2x - 3}$

4. $2\sqrt{x - 1} - \sqrt{3x - 2}$

C. 1. Is -1 a root of $\sqrt{x + 5} = x - 1$?

2. Is $\left\{-\frac{1}{3}, \frac{2}{3}\right\}$ the solution set of $\sqrt{3 - 3x} - \sqrt{3x + 2} = 3$?

3. Is $\left\{\frac{8}{5}, 3\right\}$ the solution set of $\sqrt{5x + 1} + 5x = 11$?

4. Is $\{-5, 2\}$ the solution set of $\sqrt{x}\sqrt{x + 3} = \sqrt{10}$?

TOPIC 8.5.

CONIC SECTIONS AND THEIR GRAPHS

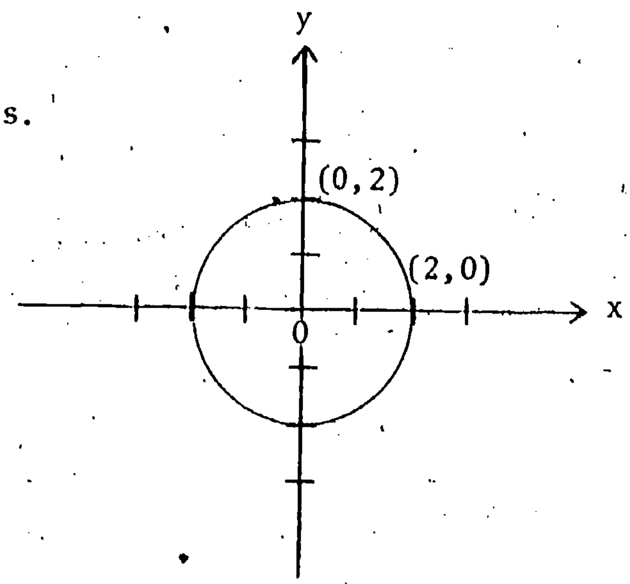
OBJECTIVES:

- To rapid sketch the graphs of the conic sections.
- To determine the domain and range of the curve.

Example: $x^2 + y^2 = 4$

Solution: Domain: $\{x | -2 \leq x \leq 2\}$

Range: $\{y | -2 \leq y \leq 2\}$



A. Copy (1 - 5), graph on the real plane the following and indicate the conic section it represents.

1. $3x^2 + 3y^2 = 15$
2. $6x^2 + y^2 = 36$
3. $xy = 4$
4. $x = \sqrt{5y^2 + 20}$
5. $y - x^2 + 3x - 2 = 0$
6. Restrict the domain and range of $\{(x,y) | x^2 + 4y^2 = 16\}$ so that the equation expresses y as a real-valued function of x.
7. Restrict the domain and range of $\{(x,y) | y^2 = x^2 - 8\}$ so that the equation expresses y as a real-valued function of x.

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Identify the type of conic section which is the graph of each set.

1. $\{(x,y) \mid y - x = x^2\}$

2. $\{(x,y) \mid 3x^2 + y^2 = 10\}$

3. $\{(x,y) \mid y = \frac{1}{x}\}$

4. $\{(x,y) \mid 8x^2 + 8y^2 = 1\}$

5. $\{(x,y) \mid 2x^2 = 3y^2 + 6\}$

C. Find the real-valued domain and range for each of the following relations: (problems 1 and 2).

1. $\{(x,y) \mid y = -x^2 + 3\}$

2. $\{(x,y) \mid 2x^2 + 4y^2 = 12\}$

Choose the word "horizontal" or "vertical" which makes the following statements true.

3. If a curve represents a function, then any ___?___ line passed through the curve will intersect the curve only once.

4. If any ___?___ line is passed through the graph of a function and intersects that curve only once, then we know that the curve represents a one to one function.

D. For what values of x will the following represent real numbers?

1. $\sqrt{x + 3}$

2. $\sqrt{x^2 - 16}$

3. $\sqrt{x^2 + 10}$

4. $\sqrt{\frac{4}{x}}$

TOPIC D-9.1

DETERMINANTS

OBJECTIVE:

- To expand 2 x 2 and 3 x 3 determinants by cofactors.

Example: $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 10$

A. Copy.

1. What is the cofactor of the element in row 3, column 2?

$$\begin{vmatrix} 2 & 1 & 3 \\ -1 & 0 & 4 \\ -5 & -2 & 1 \end{vmatrix}$$

2. Evaluate $\begin{vmatrix} -3 & 1 \\ -2 & 0 \end{vmatrix}$

3. Evaluate $\begin{vmatrix} 0 & 1 & 0 \\ 3 & -2 & 4 \\ 5 & -1 & 8 \end{vmatrix}$

4. Evaluate the determinant in problem (1).

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Define the <u>minor</u> of an element.

2. Indicate in the circles the "sign of position."

$$\begin{vmatrix} \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \end{vmatrix}$$

3. Evaluate (a) $5(-2) - (-3)(-4)$

(b) $3(10 - 4) - (-2)(-5 + 7) + 6(0 - 3)$

TOPIC D-9.2

SOLUTION OF SYSTEMS OF EQUATIONS USING CRAMER'S RULE

OBJECTIVE:

- To solve systems of equations using determinants (Cramer's Rule).

Example: $2x - 3y = 9$

$3x + 5y = 4$

Solution: $x = \dfrac{\begin{vmatrix} 9 & -3 \\ 4 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 3 & 5 \end{vmatrix}} = 3$ and $y = -1$

Solution set is $\{(3, -1)\}$.

A. Copy and find the solution set using determinants.

1. $x - 2y = 7$
   $2x + 3y = 0$

2. $3x + 2y = 2a$
   $x + 3y = 5b$

3. $x + 2y + z = 4$
   $x - 3y + 2z = 6$
   $2x + y - z = 2$

4. $x - y - z = 0$
   $3x + 4y + 2z = -8$
   $2x + 3y - z = -4$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. 1. Express the solution for x as a ratio of determinants for the following system. Do not evaluate.

$$\begin{aligned} 2x + 3y &= 7 \\ 5x - y &= 9 \end{aligned}$$

2. Express the solution for y as a ratio of determinants for the following system. Do not evaluate.

$$\begin{aligned} x + 2y - z &= 1 \\ x - y + 2z &= 1 \\ 2x + y - z &= 1 \end{aligned}$$

3. If unable to evaluate a determinant return to Topic D-9.1.

TOPIC D-9.3

MATRICES

OBJECTIVE:

- To add and multiply matrices.

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 17 & -11 \end{bmatrix}$

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$

A. Copy and perform the indicated operation.

1. $\begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 5 & -1 \\ 2 & 4 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 5 & -2 \\ 4 & 1 & -3 \end{bmatrix}$

3. $\begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

4. $\begin{bmatrix} 3 & -1 & -3 \\ 1 & 0 & -1 \\ 2 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 1 & 5 \\ -3 & -2 \end{bmatrix}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

TOPIC D-10.1

FACTORIALS, COMBINATIONS AND PERMUTATIONS

OBJECTIVES:

- To compute combinations and permutations
- To apply combinations and permutations to specific problems.

Examples: 1. $C_{5,2} = 10$

2. $P_{6,2} = 30$

A. Copy and answer.

1. $\frac{5!}{2!3!}$

2. $C_{6,3}$

3. $P_{7,2}$

4. $P_{3,3}$

5. $4!(C_{4,1})(C_{3,1})$

6. $\frac{n!}{(n-1)!}$

7. How many committees of 2 men and 3 women can be formed from a group of 4 men and 5 women?

8. In how many orders may 6 books be arranged on a shelf?

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and circle the correct answer.

1. $6! = ?$ 6^6; $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$; 6.1

2. $C_{7,2} = ?$ $\frac{7!}{5!2!}$; $\frac{7!}{2!}$; $(7!)\ (2!)$

3. $P_{8,5} = ?$ $\frac{8!}{5!}$; $(8!)(5!)$; $\frac{8!}{3!}$

4. $\frac{5!}{4!} = ?$ 5; $\frac{5}{4}$; $(5 - 4)!$

C. Simplify.

1. $\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

2. $\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

3. $5!$

4. $\frac{4!}{0!}$

TOPIC D-11.1

THE BINOMIAL THEOREM

OBJECTIVES:

- To expand a binomial to a positive integral power.

  Example: Expand $(x + y)^4$

  Solution: $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$.

- To find a specified term of a binomial raised to a positive integral power.

  Example: Find the 4th term in the expansion of $(a + b)^8$

  Solution: The answer is $56\ a^5b^3$.

A. Copy and answer.

1. Expand.

   a. $(p - 2q)^6$ b. $(x + \frac{1}{2})^4$ c. $(a^2 - x^3)^5$

2. Find the specified term and simplify.

   a. The 5th term of $(y - m)^8$

   b. The 4th term of $(2x - 3y)^6$

   c. The term involving y^4 in the expansion of $(x + y)^8$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and answer.

1. The number of terms in the expansion of $(x + y)^4$ is ___?___

2. Find 5!

3. Simplify $\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{(7 - 1)!}$

4. Simplify $(2x^3)^4$

5. Simplify $\left(-\frac{1}{4} y^4\right)^6$

6. Given $(x + y)^n$

   a. What is the formula for the n^{th} term in its expansion?

   b. What term involves y^3?

TOPIC D-12.1

SUMMATION NOTATION

OBJECTIVES:

- Given a Σ-notation, to express in expanded form.

  Example: $\sum_{k=1}^{6} k = 1 + 2 + 3 + 4 + 5 + 6$

- Given a polynomial, to express the sum in Σ-notation.

  Example: $a^1 + a^2 + a^3 + a^4 = \sum_{k=1}^{4} a^k$

A. Copy and answer.

1. Express the following in expanded form.

   a. $\sum_{k=1}^{6} k$

   b. $\sum_{k=2}^{7} \frac{k - 1}{k + 1}$

   c. $\sum_{i=1}^{4} \frac{x^i}{3}$

   d. $\sum_{j=1}^{3} (x_j + 2)$

2. Express the following in Σ-notation.

   a. $2 + 4 + 6 + \ldots + 16$

   b. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n + 1)$

   c. $\underbrace{4 + 4 + \ldots + 4}_{n \text{ times}}$

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B. Copy and answer.

1. Given $\sum_{i=1}^{4} i$

   a. How many terms are in its sum?

   b. What are the different values of i?

   c. What is the 4th term in the sum?

   d. Write out the expansion.

2. $\sum_{j=1}^{5} (x_j - 1)$

   a. What is meant by the symbol Σ?

   b. What is the first term of the expansion?

   c. What is the last term of the expansion?

   d. Write out the expansion.

TOPIC D-13.1

PROGRESSIONS

OBJECTIVES:

- To find the last term, sum, and means of finite arithmetic and geometric progressions.

  Example: Find the last term and sum of the arithmetic progression 1,3,5,...,to 6 terms.

  Solution: L = 11, S = 36

  Example: Insert three means in the arithmetic progression: 2, ___, ___, ___, 14.

  Solution: 5, 8, 11

- To find the sum of an infinite geometric progression, $|n| < 1$.

  Example: Find the sum of the infinite geometric progression 1, ½, ¼, ...

  Solution: 2

A. Copy and find the sum and last term.

1. 11, 8, 5, ... to 10 terms
2. 2, $\frac{2}{3}$, $\frac{2}{9}$, ... to 5 terms

Copy and insert the means.

3. Five means in the arithmetic progression between 2 and 11.
4. In a geometric progression the 3rd and 6th terms are 5 and $\frac{40}{27}$. Find the first term.
5. Find the sum of the infinite geometric progression 1, $\frac{3}{5}$, $\frac{9}{25}$, ...

HAVE YOUR WORK CHECKED BEFORE PROCEEDING!

B 1. Find the common difference in the arithmetic progression 1, 3, 5, ...

2. What is the 5th term in problem B(1)?

3. What is the common ratio in the geometric progression 1, $\frac{1}{2}$, $\frac{1}{4}$, ...?

4. Insert 3 means in the arithmetic progression 2, ___, ___, ___, 10.

5. Insert 2 means in the geometric progression 2, ___, ___, 16.