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ABSTRACT

Presented is a description of the experimental first-year algebra materials "Algebra Through Applications"; a description of the procedure used in evaluating these materials; and recommendations for changes in the materials, implementation, evaluation, and funding. The evaluation indicates that these materials can be effectively used in a variety of school settings. (MK)

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A REPORT OF THE EVALUATION OF ALGEBRA THROUGH APPLICATIONS

by

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EXECUTIVE SUMMARY

A REPORT OF THE EVALUATION OF ALGEBRA THROUGH APPLICATIONS

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Introduction

During the 1976-77 school year, the National Science Foundation (NSF) sponsored a field evaluation of the experimental first-year algebra materials, Algebra Through Applications. These materials were developed over a two-year period by Dr. Zalman Usiskin of the University of Chicago under a grant from NSF. In them, the usual skills and concepts are developed through applications and models rather than from the field properties. The traditional skills associated with first-year algebra are presented with the exception of factoring of polynomials, fractional expressions and simplification, and artificial word problems. In their place, greater attention is given to operations, linear expressions, sentence solving, and problems arising from real situations. Elementary notions from probability and statistics are integrated into the course. The course is designed for the average student as a substitute for the traditional first-year algebra course. In order to provide for an independent evaluation of these materials, a field evaluation was designed and conducted under the leadership of Dr. Jane Swafford and Dr. Henry Kepner, with the project director serving as a consultant.

Objectives

The major goals for the study were: (1) to evaluate the materials in typical classrooms which would be representative of a broad spectrum of the nation's schools; (2) to evaluate the extent to which students using the materials understand the concepts considered unique to these materials, as well as understand the concepts considered standard in first-year algebra when compared to other first-year algebra students; (3) to evaluate the extent to which student attitudes about the enjoyment and usefulness of mathematics are affected through the use of these materials; (4) to evaluate the extent to which an applications approach helps in solving real-life problems; (5) to evaluate the appropriateness of the reading level of the materials; and (6) to determine the difficulties, if any, of implementing the experimental materials into the school curriculum.

Subjects and Treatments

Twenty (20) schools throughout the United States were selected from volunteer schools on the basis of a geographic and community size distribution. Each school selected to participate was asked to submit the names of two (2) equally capable teachers, each of whom would be willing to teach the experimental materials. One of the two teachers was then selected at random to be the experimental teacher. Each participating school was also asked to provide four comparable first-year algebra classes, of which two (2) would be control classes, and two (2) would be experimental classes. Schools were further asked, insofar as possible, to assign students randomly to these four classes. Students in the two classes assigned to the experimental teacher constituted the experimental population. These were taught using the experimental materials which were provided at no cost to the schools by the project. With the exception of a teacher's guide, Notes to the Teacher, no guidance or in-service was provided to experimental teachers. Each control teacher taught the two control classes using whatever first-year algebra materials were normally used in the school. In all, 2,455 students participated in the study.

In the Fall of 1976, four tests were administered by the teachers to their respective classes. These were the Mathematics Computation Subtest of the Stanford Achievement Test, the ETS Cooperative Mathematics Test: Algebra I, a 25-item Opinion Survey, and a 28-item Consumer Test. In the Spring of 1977, five tests were again administered to all classes. These were the ETS Cooperative Mathematics Test: Algebra I, a project-developed First Year Algebra Test, a modified Opinion Survey, an Algebra I Questionnaire, and a shortened Consumer Test.

In addition to student testing, a site visit was made to each participating school. Textbook Evaluation forms were completed by both control and experimental teachers and end-of-chapter reports and chapter tests were submitted by the experimental teachers. Finally, a reading level comparison of the experimental textbook with two popular commercial first-year algebra textbooks was conducted by Dr. Gerald Kulm, Department of Mathematics, Purdue University.

Results and Conclusions

Due to incomplete data, only seventeen (17) schools were retained in the final achievement analysis. The across-school analysis showed no significant difference between the two treatments on the Stanford Arithmetic Test and ETS Algebra I Test administered in the Fall and again on these two tests in the Spring. School-by-school analysis yielded significant differences in favor of the experimental group in 8 of the 17 schools on the First Year Algebra Test, and in favor of the control group in 8 schools on the ETS Algebra I Test. Overall, in 6 schools in which the experimental group performed significantly better than the control group on the test covering the experimental materials, there were simultaneously no significant differences from the control group on performance on the test covering traditional content. These data indicate that the experimental materials can be used successfully in a variety of school situations, comparing favorably with traditional first-year algebra materials. Item analysis of the two post-tests across schools showed significant differences favoring the experimental group on 13 items on the First Year Algebra Test, 9 of which measured concepts unique to the experimental materials. The performance of the experimental group on topics unique to these materials speaks well of the integration of probability and applications throughout the materials. Significant differences favoring the control group were found for 3 items on the First Year Algebra Test and for 16 items on the ETS Algebra I Test. Analysis of the individual items indicates an apparent weakness in the experimental materials in the area of traditional algebraic skill development.

Attitude data were analyzed by item across 19 schools. In the Fall, there was a significant difference between the responses of the experimental and control groups on only one of 25 items in the survey. From Fall to Spring, there was a decline in attitude in both groups on 10 out of 19 repeated items. In the Spring, there was a significant difference favoring the experimental on 7 items and favoring the control on 1 item. Overall, the experimental group enjoyed word problems and their textbook more than the control group. However, it would seem that the study of algebra, whether through an applications approach or not, does not enhance students' view of the value of mathematics for the real world.

The Consumer Test data were also analyzed by item across 19 schools. Gains from Fall to Spring showed a significant difference in favor of the experimental group on 5 items and in favor of the control group on 2 items. The performance of the experimental group provides evidence that consumer problem-solving skills would be improved with wider attention to real-life application throughout the school mathematics curriculum.

Data from the students and the reading specialist indicate that the reading level of the materials is comparable to other first-year algebra materials. Students found the materials more interesting than most. Perhaps due to the unpolished format of the materials, many teachers perceived that the experimental materials contained more and difficult reading.

Although the mathematics in the experimental materials does not seem to provide an impediment to implementation, their divergence from the traditional first-year algebra syllabus may pose a problem for the use of these materials by traditional teachers without appropriate in-service. The test data indicate, however, that even without adequate preparation and support, the experimental materials can be effectively used in many situations. Additional skill exercises or available supplemental exercises would facilitate their use by the knowledgeable and sympathetic teacher.

Discussion and Recommendations

The Algebra Through Applications materials offer a unique approach to first-year algebra. The field evaluation of the materials indicated that they can be used effectively in a variety of school settings. These materials are responsive to the criticism of school mathematics as irrelevant to the real world. As such they represent a serious departure from the traditional first-year algebra course with its emphasis on skill development. As a prototype of an applications approach to first-year algebra, the materials can be used by those who are familiar with them and share their point of view. They also stand as a source of relevant applications for the traditional first-year algebra course and as a point of departure for the development of a ~~more~~ traditional course with an applications orientation. Support for appropriate dissemination activities is recommended. Consideration of support for subsequent development work or substantial revisions is also recommended.

ACKNOWLEDGMENTS

This is a report of the field evaluation of the experimental first-year algebra materials, Algebra Through Applications. The evaluation took place during the 1976-77 school year and involved 20 schools and 2,455 students from across the country. From planning to completion, the evaluation required 2½ years. A study of this size would be impossible without the help of many people. The project staff would like to express their appreciation to those individuals who contributed to the success of the study.

The evaluation team is particularly indebted to the teachers, students, and principals in the participating schools whose cooperation made the study possible. Thanks must also go to the site visitors who observed in one or more of the participating schools and to the administration and staff of Northern Michigan University for their support during the completion of the study.

Special gratitude is expressed to John Easton of the University of Chicago for his assistance with the development of the study and the distribution of the materials, to Gerald Kulm of Purdue University for conducting the reading evaluation of the materials, and to Joseph Payne of the University of Michigan and James Fey of the University of Maryland for their suggestions concerning the evaluation and the evaluation process.

The evaluator is especially indebted to Sheryl Daniels for her unfailing clerical assistance, to Bill Swafford of Northern Michigan University for his assistance with the organization and

computer management of the data, and to James Wilson of the University of Georgia for his advice and counsel throughout the study.

Jane O. Swafford
Northern Michigan University
August, 1978

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I. INTRODUCTION

History of the Development Project

The first-year algebra course attracts a large student population. For many students it is their last mathematics course. The First-Year Algebra via Applications Development Project was funded by the National Science Foundation (NSF) in 1974 at a time when no first-year algebra textbook existed with a strong applications orientation. Given the nature and size of this population and the importance of applications of mathematics both to the decision-maker and to the average citizen, such a textbook was desirable. The goals of the Development Project, as stated in the proposal, were to develop an implementable first-year algebra course which (1) offers a picture of the wide range of applications of mathematics from which algebraic symbolism develops naturally, (2) covers the standard skills associated with first-year algebra with only complicated factoring, simplification of complicated radicals, and fractional expressions problems deleted, (3) devotes some time particularly to fundamental ideas from statistics and probability, and (4) is no more difficult than the standard course. The intended student population consisted of average to below-average algebra students (from approximately the 30th to the 85th percentile of ability). The project director was Dr. Zalman Usiskin, Department of Education, University of Chicago.

The first draft of a textbook, Algebra Through Applications, was written during the school year 1974-75 by the project director. It was taught, while being written, by the developer and two other teachers to three average, first-year algebra classes in ~~two~~ Chicago area schools (Addison Trail High School, a public high school in Addison, Illinois; and Unity High School, a parochial, all-girl, inner-city school). As stated in the project renewal proposal, the results from the first year suggested that "the approach was feasible, that numerous small changes were advisable, and that particular attention would have to be given to the development and maintenance of some of the traditional fundamental algebra skills."

A second draft of the textbook was written during the school year 1975-76. Like the first, it was taught by the developer and three other teachers in three Chicago area schools (Addison Trail High School, Proviso West High School in Hillside, Illinois, and Proviso East High School in Maywood, Illinois). In order to give more attention to the development of skills, the preparation of a workbook using a mastery learning approach was begun. Also during the second year, answers to all exercises, notes on each lesson, and suggested assignments and timetables for three ability levels of students were prepared for the teacher's manual, <u>Notes to the Teacher</u>.

In December 1975, a panel of mathematicians, educators, and citizens was asked by NSF to review all Foundation projects and make recommendations. The <u>Algebra Through Applications</u> materials received excellent reviews. The review panel, although pleased with the materials, made the following recommendations:

a) The reading level of the materials should be checked for appropriateness;

b) Preparation of students for later high school mathematics courses and like situations should be tested;

c) The mastery learning aspect should be evaluated;

d) The degree to which student performance meets project objectives should be determined; and

e) An evaluation team independent of the project should be engaged to provide the evaluations recommended in (a), (b), (c), and (d).

These recommendations were in accord with the previous plans of the project. Thus a full-scale national evaluation was planned for 1976-77.

Goals and staff for the field evaluation

A study was proposed by the project director to address the following six questions concerning the materials.

a) To what extent do students who study these materials understand the concepts considered standard in first-year algebra as compared to other first-year algebra students?

b) To what extent do students who study these materials understand the concepts considered unique to these materials?

c) To what extent do these materials help in solving applied problems from real-life situations?

d) Is the reading level of the materials comparable with other first-year algebra materials?

e) To what extent do the mastery learning materials help in improving skills?

f) What are the difficulties, if any, of implementing these materials into the school curriculum?

In order to provide for an independent evaluation, the study was placed under the leadership of two individuals with the project director serving as a consultant. Dr. Jane O. Swafford, Department of Mathematics, Northern Michigan University, was director of the research and responsible for the administration of the study, for the analysis of the data, and for the preparation of the final report and recommendations. Dr. Henry S. Kepner, Jr., the Department of Curriculum and Instruction, University of Wisconsin-Milwaukee, was responsible for the creation and selection of tests and test items. Both evaluators were paid as consultants to the project. Funding for the evaluation was covered in a renewal grant from NSF for the Developmental Project.

The two evaluators, in consultation with the project director, assumed responsibility for the design of the study. The project director participated in most meetings, reviewing tests and test items and suggesting designs and interpretations of the research. Final decisions were made by the two evaluators.

II. DESIGN OF STUDY

Overview

Twenty (20) schools throughout the United States were selected from volunteer schools on the basis of a geographic and community size distribution. Each school selected to participate was asked to submit the names of two (2) equally capable teachers, each of whom would be willing to teach the experimental materials. One of the two teachers was then selected at random to be the experimental teacher. Each participating school was also asked to provide four comparable first-year algebra classes of which two (2) would be control classes, and two (2) would be experimental classes. Schools were further asked, insofar as possible, to assign students randomly to these four classes. Students in the two classes assigned to the experimental teacher constituted the experimental population. These were taught using the experimental materials which were provided at no cost to the schools by the project. With the exception of a teacher's guide, Notes to the Teacher, no guidance or in-service was provided to experimental teachers. Each control teacher taught the two control classes using whatever first-year algebra materials were normally used in the school. In all, 2,455 students participated in the study.

In the Fall of 1976, four tests were administered by the teachers to their respective classes. These were the Mathematics Computation Subtest of the Stanford Achievement Test, the ETS

Cooperative Mathematics Test: Algebra I, a 25-item Opinion Survey, and a 28-item Consumer Test. In the Spring of 1977, five tests were again administered to all classes. These were the ETS Cooperative Mathematics Test: Algebra I, a project-developed First Year Algebra Test, a modified Opinion Survey, an Algebra I Questionnaire, and a shortened Consumer Test.

In addition to student testing, a site visit was made to each participating school. Textbook evaluation forms were completed by both control and experimental teachers, and end-of-chapter reports and chapter tests were submitted by the experimental teachers. Finally, a reading level comparison of the experimental textbook with two popular commercial first-year algebra textbooks was conducted by Dr. Gerald Kulm, Department of Mathematics, Purdue University.

Selection and description of schools, teachers, and students

Schools

Volunteers to participate in a year-long field evaluation of the experimental materials, Algebra Through Applications, were solicited by the project director, Usiskin, during presentations at the 54th Annual Meeting of the National Council of Teachers of Mathematics (NCTM) in Atlanta, Georgia, in April 1976 and at the meeting of the National Council of Supervisors of Mathematics that preceded the NCTM meeting. In addition, a notification of the proposed study was mailed to schools requesting information about the project. Approximately 550 forms were distributed. From these solicitations, indications of interest were received from

88 schools. This list was first reduced to those public schools indicating the availability of at least four classes. The remaining schools were grouped geographically into four categories -- Northeastern, Southeastern, Central, and Western United States. Within each geographic category, schools were further grouped by community size into three subcategories -- urban, suburban, and small-town or rural. The evaluation administrator solicited participants for the empty cells. However, interested schools could not be located for small-town/rural in the Northeast and Central regions. The requirement of two teachers and four classes of first-year algebra automatically excluded many interested small schools. Scheduling constraints and the availability of two willing and equally-capable teachers precluded others from participating. Twenty (20) schools were chosen to participate by selecting at least one school from each non-empty cell and the remainder to balance community size. Of these, six (6) were solicited schools. A distribution of the schools by cell is presented in Table 1. Figure 1 locates the participating schools on a map of the United States. Of the 20 schools selected, 19 completed the study. One school withdrew at mid-year. A list of the 20 schools is given in Appendix A.

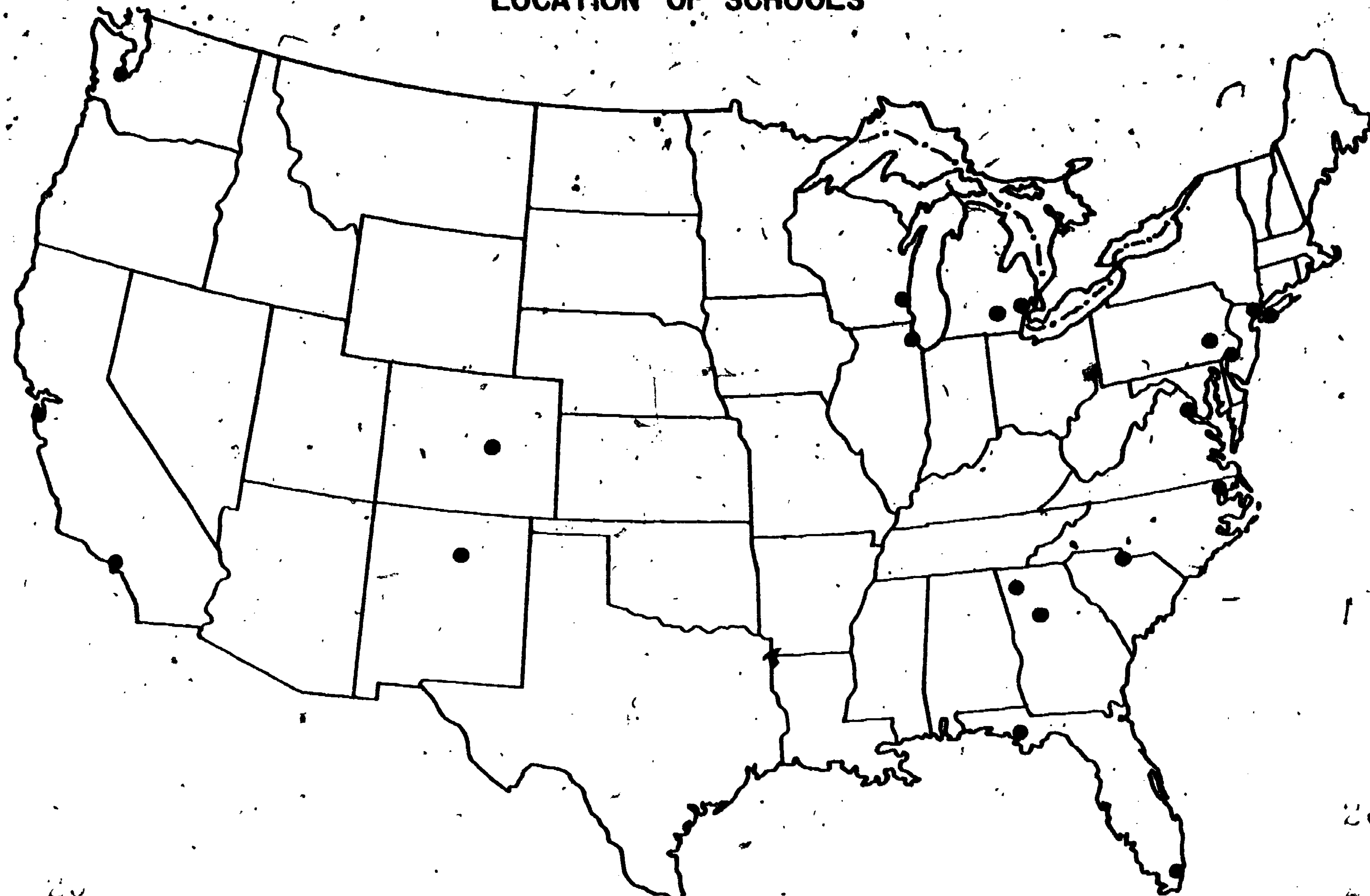
TABLE 1

DISTRIBUTION OF PARTICIPATING SCHOOLS BY GEOGRAPHIC REGION AND COMMUNITY SIZE

	Urban	Suburban	Small-town/rural	Total
Northeast	3	1		4
Southeast	1	2	4	7
Central	2	2		4
West	2	1	2	5
Total	8	6	6	20

Figure 1

LOCATION OF SCHOOLS



The participating schools covered a cross section of American communities and schools. Four were junior high schools (7th-9th grades), 3 were senior high schools (10th-12th grades), and 13 were four-year high schools (9th-12th grades). The schools spread from New York to Miami and from the Atlantic to the Pacific. The larger number of schools selected from the Southeast reflects the location of the NCTM 54th Annual Meeting at which the invitation to participate was made. The schools spanned a range of socio-economic backdrops from the inner city through affluent suburbs to the rural areas, from college and scientific communities to mill towns. The schools also represent racial mixtures from all-white to all-black with clusters of Hispanic, Asian-American, and Native American students.

Teachers

Each school selected was contacted by telephone in the summer of 1976 and asked to provide the names of two "equally capable" teachers, each having at least two classes of first-year algebra in the coming year and willing to teach the experimental materials. One of the two teachers was then selected by the evaluator using a random process. The two classes assigned to this teacher were designated as the experimental group. The other teacher and two classes were designated as the control teacher and control group. A list of the experimental and control teachers by school is contained in Appendix A.

One teacher was assigned to both experimental classes in a school in order to minimize cross-contamination of treatments. It was the decision of the evaluation team that control of the treatment variable would be impossible to maintain if one teacher taught both a class using the experimental materials and a class using the traditional materials. However, this practice introduces variation between treatments due to the differences between teachers. Hence, random assignment of teachers to treatments was made in order to spread the effects of teacher effectiveness evenly across treatments. Although "equally capable" teachers were requested from each school, it is acknowledged that subjective and non-uniform criteria were used in making these judgments. No attempt was made to objectively assess teacher effectiveness.

Both the experimental and control teachers were asked to complete a Teacher Information Form, a copy of which appears in Appendix A. A summary of the information obtained is presented in Table 2. Both sets of teachers are reasonably comparable on the characteristics queried. On the average, both groups were veteran teachers, approximately 70% female with a mean age of 41. Of the experimental teachers, 11 hold master's degrees or better and all but 4 have additional course work beyond their highest degree. Similarly, of the control teachers, 9 hold master's or higher degrees with all but 3 having course work beyond their last degree. One control teacher was a first-year teacher.

TABLE 2

SUMMARY OF TEACHER CHARACTERISTICS

	Experimental	Control
Number of females	14	13
Number of males	5	6
Av. age (age range)	41.9 yrs. (26 to 58)	41.3 yrs. (23 to 59)
Av. yr. teach. experience	12.4 yrs. (2 to 35)	11.9 yrs. (0 to 26)
Av. yr. algebra experience	7.8 yrs. (2 to 22)	9.4 yrs. (0 to 26)
No. with master or above	14	16

Students

In the Fall, 2,455 students were tested. Their distribution by sex and grade level for each of two treatments is presented in Table 3. Of the Fall students, complete pre- and post-achievement data were available for 1,290 students. The distribution by sex and grade level of the 1,290 students is also presented in Table 3. The loss of approximately 48% of the students is due to three factors. In larger schools, students failing the first semester repeat this semester's work in the Spring and hence were not available for post-testing. Other students were available but missed one or more of the four achievement tests. Finally, three schools were not included in the final analysis, accounting for approximately 15% of the participants. Similar but less dramatic losses were experienced with the attitude and consumer data. Complete pre- and post-attitude data were available for 1,621 students and complete consumer test data for 1,490 students. A Chi Square test indicates that significantly more control students were lost than experimental students.

TABLE 3

DISTRIBUTION OF STUDENTS BETWEEN TREATMENTS
BY SEX AND GRADE LEVEL

	Experimental		Control		Total	
	N	(%)	N	(%)	N	(%)
Entire Population N = 2455						
Total	1215	(49.5%)	1240	(50.5%)	2455	(100%)
Female	601	(49.5%)	624	(50.3%)	1225	(49.9%)
Male	614	(50.5%)	616	(49.7%)	1230	(50.1%)
8th	25	(2.1%)	71	(5.7%)	96	(3.9%)
9th	744	(61.2%)	676	(54.5%)	1420	(57.8%)
10th	340	(28.0%)	351	(28.3%)	691	(28.1%)
11th	87	(7.2%)	112	(9.0%)	199	(8.1%)
12th	19	(1.6%)	30	(2.4%)	49	(2.0%)
Restricted* Population N = 1290						
Total	679	(52.6%)**	611	(47.3%)**	1290	(100%)
Female	349	(51.4%)	327	(53.5%)	676	(52.4%)
Male	330	(48.6%)	284	(46.5%)	614	(47.6%)
8th	5	(.7%)	31	(5.1%)	36	(2.8%)
9th	464	(68.3%)	373	(61.0%)	837	(64.9%)
10th	169	(24.9%)	162	(26.5%)	331	(25.7%)
11th	36	(5.3%)	38	(6.2%)	74	(5.7%)
12th	5	(.7%)	7	(1.1%)	12	(.9%)

*Restricted to those students for which complete pre- and post-achievement data are available.

**The number of students lost from the experimental and control groups is significantly different at $\alpha = .05$.

III. DATA COLLECTION

Testing program and description of student instruments

At the beginning of the 1976-77 school year, the following four tests were administered to all classes participating in the study:

a) a 25-item Opinion Survey of Likert-type items;

b) the Mathematics Computation Subtest of the Stanford Achievement Test: Advanced Battery, Form A (1973);

c) the Educational Testing Service (ETS) Cooperative Mathematics Test: Algebra I, Form A (1962); and

d) a 28-item Consumer Test.

At the end of the 1976-77 school year, the following five tests were administered:

e) a 25-item Opinion Survey containing 9 items from the Fall Opinion Survey, together with new or modified items that focused specifically on algebra or the algebra textbook;

f) a 39-item, content-specific Algebra I Questionnaire;

g) a Consumer Test, Form A or B, consisting of 11 and 10 items, respectively, from the Fall Consumer Test;

h) a 33-item First Year Algebra Test; and

i) the ETS Cooperative Mathematics Test: Algebra I, Form A (1962).

Copies of all project-developed tests are included in Appendix B.

The Arithmetic Computation Test was given in the Fall as a check on the equivalence of the control and experimental groups, and to determine the relative entering achievement levels of the students for subsequent analysis. The Stanford Achievement Test, normed in 1972, seemed best suited for this latter task. Permission to reproduce the computation sub-test for our purposes was granted by the publisher, Harcourt Brace Jovanovich, Inc.

The ETS Test was given as a pre- and post-test to measure achievement on objectives common to a broad spectrum of first-year algebra courses. This 40-item, 40-minute test (normed in 1962) is used as a standard end-of-year achievement measure in first-year algebra classes throughout the country. It was considered a valid measure of achievement in the traditional first-year algebra course. A classification of items by objective is also included in Appendix B. Reliabilities for the ETS Test, using a Kuder-Richardson Formula 20, were reported by the publisher as ranging from .84 to .85 for randomly-selected subsamples of the norm groups. Reliabilities for the students involved in the present study were calculated in the Spring using the simpler Kuder-Richardson Formula 21 and were .78 for the experimental group and .82 for the control group.

The First Year Algebra Test was developed to measure achievement on objectives for the traditional first-year algebra course and the experimental materials not measured by the ETS Test. Development of the test by Kepner began with a specification of objectives to be measured using as a point of departure the objectives listed by Usiskin in Notes to the Teacher. Items for each objective were written and trial data obtained from 43 students enrolled in two regular first-year algebra classes and 26 students enrolled in an Algebra Through Applications course. These students were not part of the present study. Final item selection and editing was conducted by the evaluation team. A list of the items by objective appears in Appendix B. A measure of reliability for the First Year Algebra Test was calculated using the Kuder-Richardson Formula 21 yielding .79 for the experimental group and .77 for the control.

The Consumer Test was developed in order to compare improvements on selected consumer problem-solving skills of students presented an applications orientation to first-year algebra with those presented a traditional approach. The items themselves did not, however, require algebraic skills. The Consumer Test was also developed by Kepner, beginning with a list of consumer objectives compiled from the sources listed below:

a) Beckman - Beal Mathematical Competencies Test for Enlightened Citizens. Lincoln, Nebraska. 1973.

b) Edwards, Edgar; Nichols, Eugene and Sharpe, Glyn H. "Mathematical Competencies and Skills Essential for Enlightened Citizens." Mathematics Teacher. November, 1972. pp. 671-677.

c) Henderson, George L. "Essential Mathematics Competencies for Citizenship." Madison, Wisconsin. Written as Mathematics Consultant for Wisconsin Department of Public Instruction.

d) National Assessment of Educational Progress. Mathematics Objectives. Ann Arbor: National Assessment Office. 1970.

e) National Assessment of Educational Progress. Consumer Math: Selécted Results from the First National Assessment of Mathematics. Denver. 1975.

f) National Council of Supervisors of Mathematics. "NCSM Position Paper on Basic Mathematical Skills." Arithmetic Teacher. October, 1977. pp. 19-22.

g) Numerous lists of mathematics and consumer-oriented mathematics developed by state and local groups related to mathematics education.

h) Bell, Max S. "What Does 'Everyman' Really Need from School Mathematics?" Mathematics Teacher. March, 1974. pp. 196-202.

After review and revision by the evaluation team, items were selected or additional items written by Kepner and pilot-tested. Twenty-eight (28) items were selected for the Fall test. These are listed by objective in Appendix B. Due to the concern expressed by some teachers over the number of days required in the Spring for testing, the Fall items were reduced from 28 to 21 items and allocated to two 10-minute tests, each administered in half of the schools. Data from the Fall test were used by the evaluation team in selecting those items to be retained and in distributing the items between Form A and Form B. Item difficulty and objective measured served as criteria for the inclusion and the distribution of items between the two forms. Schools were ranked according to their mean scores on the Fall Consumer Test and alternately assigned to administer Form A or Form B in the Spring. This assignment allowed for the collection of representative data for each item retained.

The Opinion Survey, developed by Swafford, was administered to monitor changes in attitude relative to the enjoyment, usefulness, and nature of mathematics and algebra, as well as to obtain feedback from students on their textbooks. Items were selected from attitude items developed by National Assessment of Educational

Progress (NAEP), International Association for the Evaluation of Educational Achievement (IEA), and Aiken (Journal for Research in Mathematics Education, March 1974) to measure attitudes on the value of mathematics to the individual and to society, on the enjoyment of mathematics, and on the nature of mathematics. These items were reviewed and revised by the evaluation team and additional items written. The final instrument used in the Fall contained 24 Likert-type items and one multiple-choice item. These items are listed by category (enjoyment, usefulness, and nature of mathematics) in Appendix B. It is recognized that attitudes toward mathematics take on dimensions other than those assessed. The chosen dimensions were considered most germane to the present study. Of the original 25 items, 9 were included again in the Spring survey, 7 were modified to read "algebra" instead of "mathematics," and 9 new items were written to explore specific attitudes toward algebra or the algebra textbook. A classification of the items in the Spring survey is also included in Appendix B.

A content-specific attitude survey, the Algebra I Questionnaire, was developed by Swafford for administration in the Spring. This 15-minute, 39-item questionnaire was designed to explore, in more detail, students' attitude relative to the enjoyment, usefulness, and ease of specific topics in first-year algebra rather than attitudes about mathematics or algebra in general. Both the Opinion Survey and the Algebra I Questionnaire were pilot-tested with first-year algebra classes in Marquette, Michigan, before being administered to the students in the study.

The Consumer Test, Opinion Survey, and Algebra I Questionnaire were designed as collections of individual items rather than scales or test batteries. Hence, reliability measures or other total test statistics were not calculated.

All tests were administered by the classroom teachers according to a schedule provided by the evaluators. The order, date, and approximate time for each test are listed in Table 4.

TABLE 4

TESTING SCHEDULE

Test	Day given	Time required
Fall Opinion Survey	1st full day of class	10-15 min.
Stanford Arithmetic Test	2nd or 3rd day	35 min.
ETS Algebra I Test	2nd or 3rd day	40 min.
Fall Consumer Test	6th day (2nd week)	35 min.
Spring Opinion Survey	3 wks from end of sch	10-15 min.
Spring Consumer Test	3 wks from end of sch	10-15 min.
Algebra I Questionnaire	2 wks from end of sch	10-15 min.
First Year Algebra Test	last week of sch	40 min.
ETS Algebra I Test	last week of sch	40 min.

Answers to tests administered in the Fall were recorded on answer sheets or, in the case of the Stanford Arithmetic Test, on the test itself. Answers to the tests administered in the Spring were recorded on prepared computer answer cards, with the exception of the ETS Test, which was not amenable to the card format.

Teacher questionnaires

In order to obtain feedback on the experimental materials and to monitor progress through the materials, each experimental teacher was asked to complete an End-of-Chapter Report form. In addition to the number of days required to cover the chapter, they were asked to identify lessons that went particularly well and those that did not. For the latter, the source(s) of difficulty and suggestions for improvement was solicited. Also, a copy of the chapter test was requested as an informal indicator of the extent to which the experimental approach rather than the traditional approach was being emphasized. A copy of the End-of-Chapter Report form appears in Appendix C.

At the end of the year, both control and experimental teachers were asked to complete an extensive textbook evaluation form. Separate forms were prepared for control and experimental teachers, but some parallel items were included on both for comparison. The forms were developed by Usiskin and Swafford and are also included in Appendix C.

Site visits

In addition to the data collected directly from the students and teachers involved in the study, each participating school was visited during the year. The purpose of the site visit was to verify that the treatment was being implemented, to identify problems, to answer questions concerning the study, and to collect observational data not amenable to pencil and paper reports or tests. During each site visit, the visitor observed in at least one (1) control and one (1) experimental class, talked with each teacher, visited with the department chairman and/or principal, and, in selected cases, interviewed five (5) students in each class. Observations were reported on the forms provided. Copies of all instructions and forms are included in Appendix D.

The site visits were conducted by Swafford (5 schools), by Kepner (3 schools), and by 9 other qualified educators, each visiting one or two schools in his area (11 schools). The developer of the experimental materials, Usiskin, did not visit any school in the study during the year. A list of observers, schools visited, and dates is included in Appendix D.

During the classroom observations, the observer was asked to note the size and composition of the class, the name of the control textbook, the day's activities, and give general impressions about the class, the teacher, and how the algebra was going. In particular, the observer was asked to report any notable differences between the control and experimental classes.

In addition, some observers were asked to select five students from each class, using a random process, and to interview them informally during the last part of the class. Five questions were suggested that explored students' enjoyment of algebra, their perceived usefulness of algebra, and the enjoyment and readability of the textbook. By posing open-ended questions, feedback not obtainable with a Likert-type survey was sought.

Each observer interviewed both the control and experimental teachers, asking a series of specific questions regarding the comparability of the two classes in the study; the testing program; and, for the experimental teacher, questions concerning aspects of the experimental materials that had been brought into question. The observer also solicited other concerns or comments from the teachers.

Finally, each observer was asked to call on the principal and/or department chairman. During this interview, the observer tried to ascertain if there were any problems with the experimental materials or the study itself, specifically the testing program, from the supervisor's perspective.

Following the visit and the receipt of the site visit forms, the evaluator reviewed the visit by telephone or in person with the observer to discuss their report and perceptions.

Reading level evaluation

The experimental materials place a heavy emphasis on reading mathematics. Real-life problems are described and incorporated routinely into the exposition and problem sets rather than relegated to a separate chapter or the end of the exercises. A set of exercises entitled "Questions Concerning the Reading" follows each exposition to query understanding of the facts and concepts presented. Because of this emphasis on reading, an evaluation of the readability of the text was undertaken. This endeavor was given further impetus by concerns expressed by some of the teachers involved. Dr. Gerald Kulm of Purdue University was contacted and asked to do a reading-level evaluation of the experimental materials, and, in particular, to compare the reading level of <u>Algebra Through Applications</u> with the level of two other first-year algebra textbooks, <u>Holt Algebra I</u> and Houghton-Mifflin's <u>Algebra Structure and Method</u>, Book 1.

IV. ANALYSIS OF DATA AND FINDINGS

Achievement data

Overall achievement

Four achievement tests were administered, two in the Fall and two in the Spring. An Arithmetic Test and the ETS Test were given in the Fall. In the Spring, the ETS Test was repeated, together with a project-developed First Year Algebra Test (FYAT). For the analysis, only data for those students who took all four tests were retained. Furthermore, insufficient data were received from two of the 19 schools completing the study. In each of the 17 schools included in the analysis, the two classes in each treatment were combined into one unit. In all, there were 1,290 students retained in the achievement analysis; 679 in the experimental group; and 611 in the control. Mean scores on the achievement tests were analyzed by treatment and by school. Analysis of achievement on individual items was also conducted by treatment across schools. In all analyses, the level of significance was set at $\alpha = .05$.

Observational reports and preliminary examination of the data suggested that each of the 17 schools in the study represented a unique situation. Each was unique with ~~respect to~~ organization, clientele, and learning environment of the school as well as the confounding effect of the community environment surrounding it. For example, one school experienced a three-week teacher's strike;

in another, attendance was influenced by a local transit strike; in two schools, students were subjected to a change of teacher at mid-year; absentee rates varied markedly from school to school. In some schools it became administratively unfeasible to maintain intact groups throughout the year.

Table 5 presents the school means and treatment means for each of the four achievement tests and the sum of the means on the two Spring Achievement tests. The variability among the school means is apparent. A one-way analysis of variance by school (ignoring treatment) was conducted for each test to determine whether the means of the 17 schools were equal. Table 6 summarizes these analyses. Significant differences were obtained for each test.

The differences noted above suggested the existence of 17 separate experimental settings. To examine the treatment effect across all schools, 17 matched pairs (experimental group matched with control group in each of the 17 schools) were formed. For each of the four achievement tests a matched pair t-statistic was computed. This analysis is summarized in Table 7. Using this analysis, no significant difference was found between treatments either in the Fall or in the Spring. The lack of significant differences in the Fall attests to the comparability of the two treatment groups of the onset of the experiment. To clarify the lack of significant differences in the Spring, further exploration of between-school differences was undertaken.

A two-way treatment x school analysis of variance was conducted for each of the four achievement tests and the sum of the means on the two Spring Tests. In each of these, the unit of analysis was the students score. Means for each treatment within schools are

TABLE 5

SCHOOL AND TREATMENT MEANS ON ACHIEVEMENT TESTS

	N	Fall Arith	Fall ETS	Spring ETS	Spring FYAT*	Spring ETS+FYAT
All students	1290	32.32	12.57	20.79	13.04	33.83
All exp	679	32.43	12.25	19.82	13.84	33.66
All control	611	32.19	12.92	21.87	12.15	34.01
School 1	77	30.09	12.25	17.56	11.04	28.60
Exp	39	30.28	12.38	17.54	13.49	31.03
Cont	38	29.89	12.11	17.58	8.52	26.11
School 2	99	29.49	10.64	13.97	7.68	21.65
Exp	49	29.59	10.89	13.90	8.43	27.33
Cont	50	29.38	10.38	14.04	6.94	20.98
School 3	85	33.67	13.00	21.31	14.18	35.48
Exp	42	36.31	13.17	21.41	17.10	38.50
Cont	43	31.09	12.84	21.21	11.33	32.53
School 4	93	32.72	11.79	20.43	12.59	33.02
Exp	47	32.98	11.55	18.72	12.60	31.32
Cont	46	32.46	12.02	22.17	12.59	34.76
School 5	57	30.13	10.60	17.35	11.56	28.91
Exp	36	31.00	11.06	18.17	13.17	31.33
Cont	21	28.62	9.81	15.95	8.81	24.76
School 6	44	24.66	8.91	11.36	6.86	18.23
Exp	27	24.48	8.93	10.22	6.30	16.52
Cont	17	24.94	8.88	13.18	7.76	20.94
School 7	67	31.26	10.91	18.37	10.31	28.69
Exp	44	31.02	10.55	17.02	10.02	27.05
Cont	23	31.70	11.61	20.96	10.87	31.83
School 8	52	31.37	12.23	19.90	12.10	32.00
Exp	31	31.71	11.45	20.97	12.03	33.00
Cont	21	30.86	13.38	18.33	12.19	30.52
School 9	84	35.25	14.26	23.39	15.20	38.60
Exp	45	34.89	14.18	27.73	16.33	40.07
Cont	39	35.67	14.36	23.00	13.90	36.90
School 10	76	33.15	14.21	19.22	11.71	30.96
Exp	34	31.38	13.68	15.76	12.92	28.68
Cont	42	34.57	14.64	22.02	10.74	32.76

*First Year Algebra Test

TABLE 5 -- Continued

	N	Fall		Spring		
		Arith	ETS	ETS	FYAT	ETS+FYAT
School 11	70	28.96	11.50	22.24	13.40	35.64
Exp	36	30.11	10.81	21.28	14.28	35.56
Cont	34	27.74	12.24	23.26	12.47	35.74
School 12	67	29.60	11.58	19.73	10.61	30.34
Exp	29	29.66	11.97	19.21	11.10	30.31
Cont	38	29.55	11.29	20.13	10.24	30.37
School 13	71	35.52	14.68	25.80	18.00	43.80
Exp	35	36.00	14.14	24.14	20.14	44.29
Cont	36	35.06	15.19	27.42	15.92	43.33
School 14	69	33.21	12.76	21.29	13.84	35.13
Exp	37	32.97	12.65	20.84	16.32	37.16
Cont	32	33.47	12.88	21.81	10.97	32.78
School 15	122	38.33	16.45	29.04	20.20	49.25
Exp	64	37.98	16.08	27.20	20.38	47.58
Cont	58	38.71	16.86	31.05	20.02	51.09
School 16	87	35.53	13.15	24.70	14.23	38.93
Exp	45	35.44	12.49	22.67	14.24	36.91
Cont	42	35.62	13.86	26.88	14.21	41.10
School 17	70	27.87	10.30	18.84	11.41	30.26
Exp	39	28.23	9.13	17.46	10.87	28.33
Cont	31	27.42	11.77	20.58	12.10	32.68

TABLE 6

ANALYSIS OF VARIANCE FOR THE ACHIEVEMENT TESTS BY SCHOOL

Arithmetic Test

Source	df	ss	ms	F	Sig of F
Schools	16	13856.52	866.03	24.29	<.001
Error	1273	45392.45	35.66		
Total	1289				

Fall ETS

Source	df	ss	ms	F	Sig of F
Schools	16	4588.54	286.78	18.09	<.001
Error	1273	20186.05	15.86		
Total	1289				

Spring ETS

Source	df	ss	ms	F	Sig of F
Schools	16	23141.55	1446.35	49.45	<.001
Error	1273	60376.41	29.25		
Total	1289				

First Year Algebra Test

Source	df	ss	ms	F	Sig of F
Schools	16	14925.28	932.83	41.77	<.001
Error	1273	28428.82	22.33		
Total	1289				

TABLE 7

MEAN CORRECT RESPONSES BY TREATMENT FOR ACHIEVEMENT TESTS
N = 17 MATCHED PAIRS

	Experimental	Control	mean diff (exp-control)	s_d	t-value	sig p<
Arith. Test (48 items)	32.00	31.59	0.418	1.78	0.97	.346
Fall ETS (40 items)	12.07	12.97	-0.900	2.26	-1.64	.120
Spring ETS (40 items)	19.67	19.58	0.094	7.30	0.05	.958
FYAT (33 items)	13.51	12.24	1.271	3.45	1.52	.148

also reported in Table 5. A summary of the analysis of variances is presented in Table 8.

As noted in Table 8, a significant interaction of treatment with school was obtained for all but the Fall ETS Test. That test showed a significant main effects F-value ($p = .019$) in favor of the control group. Efforts to identify the source of the interaction obtained on the other tests by logical groupings of schools met with no success.

Because of the significant interaction of treatment with schools, the achievement data were subjected to a separate analysis for each school. While an analysis based on the use of multiple t-tests should be used with caution, this approach was utilized because of the varied school settings. Significant differences on the Arithmetic Test were found in two schools in the Fall, one in favor of the experimental group and one in favor of the control. Significant differences in favor of the control group on the Fall ETS Test were observed at one school. At the .05 level of significance, differences should be expected in one out of twenty schools by chance. Hence, this further analysis confirms the overall comparability of the two treatment groups in the Fall.

On the Spring First Year Algebra Test, significant differences were found in eight schools, each favoring the experimental group. On the Spring ETS Test, significant differences were found in eight schools, each favoring the control. On the combined algebra achievement score, obtained by summing the two Spring achievement scores, significant t-values were obtained for nine schools, four favoring the experimental group and five favoring the control group. In all cases the significance on the combined score merely reflects

TABLE 8

TREATMENT BY SCHOOL ANALYSIS OF VARIANCE

FALL

Two-way Analysis of Variance for Arithmetic Test

Source	df	ss	ms	F	Sig of F
Main Effects	17	13906.90	818.05	23.16	<.001
Treatment	1	50.37	50.37	1.43	<.233
Schools	16	13889.07	868.07	24.57	<.001
Interaction	16	982.04	61.38	1.73	<.035
Residual	1256	44360.04	35.32		
Total	1289				

Two-way Analysis of Variance for Fall ETS

Source	df	ss	ms	F	Sig of F
Main Effects	17	4675.86	275.05	17.43	<.001
Treatment	1	87.33	87.33	5.53	<.019
Schools	16	4533.70	283.36	17.95	<.001
Interaction	16	274.80	17.18	1.09	<.361
Residual	1256	19823.92	15.78		
Total	1289				

SPRING

Two-way Analysis of Variance for Spring ETS

Source	df	ss	ms	F	Sig of F
Main Effects	17	24268.28	1427.55	52.07	<.001
Treatment	1	1126.73	1126.73	41.10	<.001
Schools	16	22922.71	1432.67	52.26	<.001
Interaction	16	1673.60	104.60	3.82	<.001
Residual	1256	34434.54	27.42		
Total	1289				

TABLE 8 -- Continued

Two-way Analysis of Variance for First Year Algebra Test

Source	df	ss	ms	F	Sig of F
Main Effects	17	15975.52	939.74	45.77	<.001
Treatment	1	1050.24	1050.24	51.16	<.001
Schools	16	15059.61	941.23	45.85	<.001
Interaction	16	1592.16	99.51	4.85	<.001
Residual	1256	25786.42			
Total	1289				

Two-way Analysis of Variance for ETS+FYAT*

Source	df	ss	ms	F	Sig of F
Main Effects	17	74052.69	4356.04	56.44	<.001
Treatment	1	1.34	1.34	0.02	<.895
Schools	16	74011.50	4625.72	59.93	<.001
Interaction	16	4689.86	293.12	3.80	<.001
Residual	1256	96944.44	77.19		
Total	1289				

*First Year Algebra Test

the magnitude and direction of the difference on one of the two Spring achievement tests.

Because of the significant difference obtained in three schools on one of the Fall pre-tests, analysis of covariance using the Arithmetic Test and Fall ETS Test as covariants were also conducted, recognizing that the assumptions for such an analysis are tenuous. This analysis confirmed all of the significant differences resulting from the previous t-tests and, additionally, identified two more significant differences. One of these showed a significant difference in favor of the experimental group on the First Year Algebra Test, while the other showed a significant difference in favor of the control group on the Spring ETS Test. In the second case, the initial superiority of the experimental group on the Arithmetic Test in the Fall was apparently compensated for with the use of that test as a covariate. Significant differences had been found on Fall pre-tests in two other cases. The use of these tests as covariates in these cases did not contribute to the analysis. On the sum of the two Spring tests, the use of covariates confirmed the results of the previous t-tests with three additional refinements. At two schools, the use of covariates reduced the difference between the experimental and control groups, while at a third, the use of covariates accentuated the difference. In all three cases, the differences were in favor of the experimental treatment. A summary of the findings of the 17 separate t-tests and analyses of covariance (ANCOVA) are presented in Table 9.

TABLE 9

SIGNIFICANT DIFFERENCES BY SCHOOL
(Summary of t-tests by treatment)

School	Fall: Arith	Fall: ETS	Spring: ETS	Spring: FYAT	Spring: ETS+FYAT
1				Exp	Exp
2				Exp	
3	Exp		Control*	Exp	Exp**
4			Control		Control
5				Exp	Exp**
6			Control		Control
7			Control		Control
8					
9				Exp	Exp*
10	Control		Control	Exp	
11				Exp*	
12					
13			Control	Exp	
14				Exp	Exp
15			Control		Control
16			Control		Control
17		Control	Control		

* significant difference only with Fall tests as covariates

** no significant difference with Fall tests as covariates

Item analysis

Items on the ETS Test and the First Year Algebra Test measure specific objectives of first-year algebra programs. The First Year Algebra Test was developed from a list of objectives not tested by the ETS Test but covered either by the experimental or traditional materials or both. Twelve of the 33 items on that test reflect the unique objectives of the experimental materials. Items of the ETS Test are categorized by its publisher by objectives and, generally, reflect objectives common to traditional first-year algebra programs. Inasmuch as total achievement test scores are only meaningful when the total test reflects the objectives and emphases of the course, further analysis of the achievement data was undertaken to explore the difference between the experimental and control groups by objectives as measured by items.

Items on the Spring ETS Test and First Year Algebra Test were analyzed by treatment across schools. For each item, the Chi Square statistic was used to test the dependence of the treatment and the number of correct and incorrect responses. The number and percent of correct responses for each item on the Spring ETS Test and on the First Year Algebra Test are presented in Tables 10 and 11, respectively.

On the Spring ETS Test, significant differences were found on 16 of the 40 items, all in favor of the control group. On the First Year Algebra Test significant differences were found on 16 of the 33 items, 13 in favor of the experimental group. A list of items by objective may be found in Appendix B.

The experimental group did significantly better than the control group on all items dealing with relative frequency and proba-

TABLE 10

NUMBER AND PERCENT OF CORRECT RESPONSES BY ITEM FOR SPRING ETS TEST

Item	Experimental N = 679 No. correct	(% correct)	Control N = 611 No. correct	(% correct)	χ^2	sig
1	646	(95.1)	586	(96.4)	1.03	
2	508	(74.8)	511	(83.6)	16.10	p<.0001
3	407	(59.9)	413	(67.6)	8.61	p<.0033
4	409	(60.2)	364	(59.6)	.00	
5	588	(86.6)	564	(92.3)	12.35	p<.0004
6	526	(77.5)	495	(81.0)	2.90	
7	161	(23.7)	134	(21.9)	.40	
8	582	(85.7)	542	(88.7)	3.21	
9	417	(71.4)	382	(62.5)	.24	
10	295	(43.4)	235	(38.5)	2.79	
11	386	(56.8)	359	(58.8)	.58	
12	492	(72.5)	480	(78.6)	7.06	p<.0079
13	545	(80.3)	479	(78.4)	.29	
14	417	(61.4)	371	(60.1)	.00	
15	395	(58.2)	497	(73.2)	32.80	p<.0001
16	160	(23.6)	170	(27.8)	3.06	
17	566	(83.4)	515	(84.3)	.39	
18	366	(53.9)	457	(74.8)	61.98	p<.0001
19	128	(18.9)	230	(37.6)	56.70	p<.0001
20	315	(46.4)	327	(53.5)	6.80	p<.0091
21	291	(42.9)	318	(52.0)	11.19	p<.0008
22	359	(52.9)	391	(64.0)	16.90	p<.0001
23	105	(15.5)	112	(18.3)	1.82	
24	273	(40.2)	332	(54.3)	26.23	p<.0001
25	201	(29.6)	256	(41.9)	21.45	p<.0001
26	254	(37.4)	232	(38.0)	.06	
27	349	(51.4)	330	(54.0)	.99	
28	259	(38.1)	323	(52.9)	28.57	p<.0001
29	281	(41.4)	285	(46.6)	3.76	
30	354	(52.1)	336	(55.0)	1.18	

TABLE 10 -- Continued

Item	Experimental N = 679 No. correct	(% correct)	Control N = 611 No. correct	(% correct)	χ^2	sig
31	442	(65.0)	408	(66.8)	.53	
32	168	(24.7)	159	(26.0)	.28	
33	233	(34.3)	249	(40.8)	5.82	p<.0159
34	190	(38.0)	222	(36.3)	10.41	p<.0013
35	473	(69.5)	392	(64.1)	3.55	
36	317	(46.7)	272	(44.5)	.39	
37	71	(10.4)	149	(24.4)	43.78	p<.0001
38	190	(28.0)	171	(28.0)	.00	
39	118	(17.4)	105	(17.2)	.00	
40	261	(38.4)	227	(37.1)	.11	

TABLE 11

NUMBER AND PERCENT OF CORRECT RESPONSES BY ITEM FOR FIRST YEAR ALGEBRA TEST

Item	Experimental N = 679 No. correct	(% correct)	Control N = 611 No. correct	(% correct)	χ^2	sig
1	467	(68.8)	436	(71.4)	1.24	
2	170	(25.0)	144	(23.6)	.24	
3	438	(64.5)	326	(53.4)	15.04	p<.0001
4	473	(69.7)	252	(41.2)	101.69	p<.0001
5	192	(28.3)	242	(39.6)	18.64	p<.0001
6	306	(45.1)	168	(27.5)	40.88	p<.0001
7	326	(48.0)	283	(46.3)	.20	
8	418	(61.6)	247	(40.4)	54.93	p<.0001
9	354	(52.1)	165	(27.0)	81.75	p<.0001
10	301	(44.3)	319	(52.2)	8.27	p<.0040
11	315	(46.4)	210	(34.4)	17.98	p<.0001
12	182	(26.8)	148	(24.2)	.87	
13	265	(39.0)	188	(30.8)	8.78	p<.0030
14	259	(38.1)	203	(33.2)	2.89	
15	220	(32.4)	152	(24.9)	8.11	p<.0044
16	356	(52.4)	356	(58.3)	.00	
17	263	(38.7)	178	(29.1)	12.19	p<.0005
18	324	(47.7)	300	(49.1)	.30	
19	77	(11.3)	33	(5.4)	13.55	p<.0002
20	169	(24.9)	149	(24.4)	.01	
21	287	(42.3)	50	(8.2)	190.01	p<.0001
22	333	(49.0)	288	(47.1)	.27	
23	316	(46.5)	309	(50.6)	2.24	
24	105	(15.5)	209	(34.2)	61.28	p<.0001
25	176	(25.9)	161	(26.4)	.03	
26	472	(69.5)	409	(66.9)	.60	
27	260	(38.3)	234	(38.3)	.00	
28	191	(28.1)	167	(27.3)	.04	
29	300	(44.2)	204	(33.4)	14.61	p<.0001
30	202	(29.7)	198	(32.4)	1.09	
31	300	(44.2)	258	(42.2)	.31	
32	340	(50.1)	231	(37.8)	18.27	p<.0000
33	215	(31.7)	192	(31.4)	.00	

bility (FYAT: 3, 8, 11, 19). It should be noted that probability is not included in the traditional algebra course. However, information from the teachers indicated that 11 of 15 experimental teachers reporting omitted Chapter 13 which emphasizes probability. Since the majority of students in the experimental group did not receive the intended instruction in probability, the significant differences noted may be due to the informal introduction of probability concepts throughout the applications format.

Other topics unique to the experimental materials on which the experimental group did significantly better than the control are metric (FYAT 6), models for multiplication (FYAT 4), percentage decrease (FYAT 15), compound interest (FYAT 21), and rate of change as slope (FYAT 29). Some topics, while not unique to the experimental materials, were more heavily emphasized than in the standard course. Of these, no significant difference was found between the two groups on the use of subscripted variables (FYAT 7), use of the rate model of division in a proportion (FYAT 23), and conversion within the metric system (FYAT 31).

The control group did significantly better on many items dealing with elementary algebraic manipulations. Some of these (*e.g.*, division of rational expressions (ETS 28), division of polynomials (ETS 25), and trinomial factorization (ETS 24)) were explicitly omitted from the experimental materials and the superiority of the control group is not unexpected.

Other of these elementary algebraic skills were included in the experimental textbook, but perhaps not emphasized to the extent as in some standard textbooks. These include integer arithmetic (ETS 3), evaluating exponential expressions (ETS 22), multiplying using the distributive property (ETS 15), using the FOIL technique (ETS 21 and FYAT 24), monomial factoring (ETS 18), and simplifying radicals using the property $\sqrt{xy} = \sqrt{x}\sqrt{y}$ (FYAT 10).

The control group did significantly better on the ETS Test on items dealing with factoring (ETS 18 and 24) and quadratics (ETS 31 and 34). However, there was no significant difference between the two groups on the First Year Algebra Test on items involving solving quadratics using the formula (FYAT 14), and solving quadratics in factored form (FYAT 22).

The two groups were more comparable on items requiring the translation from verbal to algebraic expression. The control group did better on 2 items, the experimental on 4, with no differences on 6. Each group did better on those items explicitly taught in their materials. The two groups were relatively comparable on solving linear equations and inequalities, substituting into algebraic expressions and equations, and combining terms. However, the experimental group performed better on items involving slope.

Achievement of average students

The intended population of the experimental materials is the average to below-average algebra student, defined by Usiskin as students from approximately the 30th to the 85th percentile of ability. National norms for the Arithmetic Test indicate boundaries of the 30th to 85th percentile for all beginning 9th grade students as raw scores of 23 and 38. On would expect better scores from algebra students. Accordingly, for students in the study, the 30th to 85th percentile on the Arithmetic Test is bounded by raw scores of 29 and 39. For this study these scores were used to identify "average students." On the national norms these scores encompass the 50th to 85th percentile at the beginning of grade nine, and the 40th to 82nd percentile at the end of grade nine.

Means for the "average student" group are reported in Table 12. Since "average students" were not taught as an intact group, and since the proportion of "average students" varied from school to school, data for "average students" was analyzed by treatment across schools. The t-values and their significance are also reported in Table 12. These results parallel the findings for the total population.

TABLE 12

ACHIEVEMENT TEST MEANS FOR "AVERAGE STUDENTS"

	Fall		Spring		
	Arith.	ETS	ETS	FYAT	ETS+FYAT
Total population	32.32	12.25	20.79	13.04	33.83
			Average students		
Average exp	34.38	12.59	20.41	14.17	34.58
Average control	34.13	13.26	22.53	12.16	34.69
t-value (E-C)	1.06	-2.40	-4.97	5.36	-0.16
significance	p<.292	p<.017	p<.001	p<.001	p<.876
			All students		
All exp	32.43	12.25	19.82	13.84	33.66
All control	32.19	12.92	21.87	12.15	34.01
t-value (E-C)	0.62	-2.73	-5.42	5.27	-0.55
significance	p<.534	p<.006	p<.001	p<.001	p<.583

Attitude data

Opinion Survey

The Opinion Surveys, given in the Fall and Spring (See Appendix B), were composed of 25 items designed to assess student attitudes on the enjoyment, value, and nature of mathematics, algebra, and their algebra textbook. Although attitude scales were not developed, clusters of items probing similar or closely-related attitudes were given. Due to their purpose and construction, data from the Opinion Surveys were analyzed by item across schools and by treatment. Only those 1,521 students who completed both the Fall and Spring Opinion Surveys were retained for this analysis. For each item, the Chi Square statistic was used to test whether the distribution of responses was independent of the treatment. For the Likert-type items, responses were scores, and t-tests were also used to test for differences between the two groups.

On the Fall Opinion Survey, a significant Chi Square value was obtained on only one item. In 25 items one difference should be expected by chance at a .05 level of significance. On this item, "I plan to take another mathematics course after this one," 65.0% of the control group responded affirmatively in the Fall as compared to only 58.4% of the experimental group; while 34.4% of the experimental as opposed to only 28.7% of the control was undecided at this point. In the Spring, however, there was no significant difference between the two groups in their responses to this item. In both groups 79% indicate that they planned to take another mathematics course.

Of the 25 items in the Fall Opinion Survey, 24 were Likert-type items. Responses to these items were assigned scores from 1 to 5 with 5 representing the more favorable response. Mean scores by group were then calculated for each item. Differences between these means were analyzed with t-tests. A significant difference was obtained on 1 item, Item 20. Data for this item are presented below. In the Fall the control group had a more favorable view of the need of mathematics in jobs outside of science and engineering. Again at a .05 level of significance, difference on 1 out of 24 items should be expected by chance. This particular item was not repeated in the Spring. The overall lack of significant differences in the Fall on the Opinion Survey further attests to the comparability of the two treatment groups at the onset of the evaluation.

TABLE 13

RESPONSES TO ITEM 20: FALL OPINION SURVEY

"Outside of science and engineering there is little need for mathematics in jobs."

	SA	A	U	D	SD	Mean	t-value
Exp	25.1%	51.8%	9.9%	10.6%	2.6%	3.86	-2.14
Control	27.5%	53.9%	8.7%	8.0%	2.0%	2.97	

On the Spring Opinion Survey, significant Chi Squares were obtained on 7 of the 25 items. The distribution of responses for these 7 items is presented in Table 14. Five of these items surveyed opinions about the algebra textbook. In all cases, a larger proportion of the experimental group found their textbook

TABLE 14

DISTRIBUTION OF RESPONSES TO SELECT ITEMS FROM THE SPRING OPINION SURVEY

Item 1: Algebra is an interesting subject.

	SD	D	U	A	SA	mean	t-value
Exp	8.9%	18.1%	23.7%	39.2%	10.1%	3.24	-2.78**
Control	5.6%	17.9%	20.4%	43.4%	12.6%	3.39	

$\chi^2 = 11.06^*$

Item 2: Algebra is not important in everyday life.

	SD	D	U	A	SA	mean	t-value
Exp	19.1%	39.9%	22.2%	13.3%	5.6%	3.54	2.04*
Control	15.4%	40.1%	21.8%	16.7%	6.0%	3.42	

$\chi^2 = 6.25$

Item 5: Explanations in my algebra book helped me to understand algebra.

	SD	D	U	A	SA	mean	t-value
Exp	13.6%	21.8%	19.0%	37.3%	8.2%	3.05	2.71**
Control	15.9%	28.0%	16.3%	32.0%	7.8%	2.88	

$\chi^2 = 11.97^*$

Item 10: I enjoy working word problems.

	SD	D	U	A	SA	mean	t-value
Exp	24.0%	33.4%	21.2%	16.1%	5.3%	2.45	5.25**
Control	38.2%	29.0%	15.4%	15.4%	2.1%	2.14	

$\chi^2 = 44.93^{**}$

Item 12: Explanations in my algebra book were of no help in doing the problems.

	SD	D	U	A	SA	mean	t-value
Exp	14.8%	43.6%	19.4%	13.0%	9.2%	3.42	3.09**
Control	11.7%	39.8%	18.9%	18.9%	10.6%	3.23	

$\chi^2 = 13.12^*$

Item 20: The math book we used this year was ____________ than most math books.

	more interesting	less interesting	neither
Exp	38.4%	18.1%	42.9%
Control	21.6%	16.0%	61.8%

$\chi^2 = 63.24^{**}$

TABLE 14 -- Continued

Item 23: I read the explanations in my math book.

	almost never	some of the time	half of the time	most of the time	almost always
Exp	9.5%	21.2%	17.8%	27.9%	23.5%
Control	14.0%	26.4%	15.6%	27.2%	16.9%

$\chi^2 = 19.91$**

Item 24: The math book we used this year was ____________ to read and understand.

	very easy	moderately easy	neither	moderately difficult	very difficult
Exp	6.5%	30.1%	26.5%	23.3%	13.6%
Control	3.2%	18.9%	26.9%	34.5%	16.4%

$\chi^2 = 45.90$**

* significant at $\alpha = .05$

** significant at $\alpha = .01$

more useful, interesting, or readable compared to students using standard algebra textbooks.

Of the 25 items of the Spring Opinion Survey, 19 were Likert-type items. Responses of these items were scored and t-tests were used to further analyze the difference between the two groups on the mean scores on each item. Significant differences were found on 5 of the 19 items. The t-test confirmed the differences in response patterns found by the Chi Square tests on Items 1, 5, 10, and 12, and further identified a significant difference on Item 2. All differences favored the experimental group except for Item 1, "Algebra is an interesting subject." All t-values, except for Item 2, were significant at $\alpha = .01$. Data for these 6 items are also presented in Table 14.

Of the 25 items given in the Fall, 15 Likert-type and 1 multiple-choice items were repeated again in the Spring. Of the 15, 7 were modified by changing "mathematics" to "algebra." For the 15 Likert-type repeated items, changes in attitude from Fall to Spring were analyzed by means of t-tests on paired data, Fall scores paired with Spring scores for each student. For both the experimental and control group, a decline in attitude from Fall to Spring was observed. For the experimental group, attitude declined significantly on 9 of the 15 repeated items, and for the control group on 7 of the 15 repeated items. Comparison of the change of attitude between the two groups from Fall to Spring revealed only one significant difference. The significant decline in attitude on Item 1 for the experimental group is reflected in this significant difference in the two groups from Fall to Spring and in the significant difference between the two groups in the Spring. Both groups registered a decline on Item 6, but the experimental group registered both a significant decline and a significantly greater decline than the control. Data for the 15 repeated items are presented in Table 15.

Content specific attitudes

The Algebra I Questionnaire was designed to assess students' attitudes about the ease, enjoyment and usefulness of specific topics in first-year algebra. Thirteen topics common to first-year algebra courses were queried. Students were asked to respond to the following three dimensions on each topic:

a) easy to learn - - hard to learn

b) like to do - - dislike to do

c) useful after high school - - useless after high school

Positive responses were assigned a value of 5; negative responses a value of 1; neutral, a value of 3; and "did not study," a value of 0. Mean scores for each item were computed by treatment. Differences between treatments across schools were analyzed with t-tests. Table 16 presents the means and t-values for each item. Significant differences favoring the experimental group were found for solving word problems (easy, like), solving inequalities (like), and using the quadratic formula (easy, like). Significant differences favoring the control group were found for solving linear equations (easy, like), factoring trinomials (easy, like), determining slope, y-intercept, or graph of linear equations (easy, like), working with functions (easy, like, useful), working with positive and negative numbers (like), and translating words into algebraic expressions (like, useful). Some of these differ-

TABLE 15

REPEATED ITEMS FROM FALL OPINION SURVEY

		N	Fall mean	Spring mean	Mean difference
Item 1	Exp	787	3.43	3.23	-.198*
	Control	730	3.45	3.39	-.058
Item 2[1]	Exp	788	4.23	3.54	-.699*
	Control	728	4.14	3.42	-.717*
Item 3	Exp	784	3.54	3.48	-.060
	Control	728	3.51	3.50	-.010
Item 6[1]	Exp	781	3.44	3.22	-.224*
	Control	724	3.39	3.25	-.145*
Item 7[1]	Exp	762	2.95	2.67	-.282*
	Control	698	2.97	2.71	-.256*
Item 8	Exp	787	4.05	3.78	-.267*
	Control	724	3.98	3.68	-.303*
Item 9	Exp	768	4.00	3.96	-.037
	Control	703	3.95	4.00	+.048
Item 11[1]	Exp	763	4.18	3.27	-.916*
	Control	699	4.17	3.30	-.872*
Item 13	Exp	758	3.53	3.52	-.012
	Control	699	3.49	3.42	-.067
Item 14	Exp	759	3.89	3.87	-.017
	Control	699	3.82	3.91	+.084
Item 15	Exp	759	3.69	3.58	-.109*
	Control	698	3.68	3.63	-.049
Item 16[1]	Exp	764	3.55	3.22	-.331*
	Control	697	3.56	3.31	-.248*
Item 17	Exp	758	3.75	3.74	-.004
	Control	696	3.77	3.79	+.002
Item 18	Exp	764	4.34	4.35	+.014
	Control	700	4.37	4.38	+.003
Item 19[1]	Exp	763	4.00	3.40	-.600*
	Control	701	3.97	3.38	-.582*

[1]Modified from Fall Survey by changing "mathematics" to "algebra"

*Significant at α = .001

TABLE 16

MEANS AND t-VALUES BY TREATMENT FOR THE ALGEBRA I QUESTIONNAIRE

Topic	Experimental	Control	t-value	sig
Solving linear equations:				
easy	3.76	3.97	-2.62	<.009
like	3.10	3.28	-2.39	<.017
useful	3.18	3.24	-0.85	
Solving word problems:				
easy	2.72	2.42	3.62	<.001
like	2.44	2.20	3.06	<.002
useful	3.94	4.03	-1.30	
Simplifying expressions:				
easy	3.26	3.33	-0.75	
like	2.94	2.98	-0.44	
useful	3.05	3.05	-0.08	
Factoring trinomials:				
easy	3.52	4.02	-5.99	<.001
like	3.08	3.52	-5.31	<.001
useful	3.15	3.04	1.45	
Solving inequalities:				
easy	3.31	3.27	0.47	
like	2.94	2.75	2.24	<.025
useful	3.03	2.92	1.41	
Determining slope, y-intercept, graph:				
easy	3.41	3.69	-3.09	<.002
like	3.08	3.27	-2.08	<.038
useful	3.20	3.21	-0.14	
Powers and roots:				
easy	3.69	3.53	1.91	
like	3.38	3.29	1.31	
useful	3.77	3.66	1.42	
Solving systems:				
easy	3.59	3.75	-1.77	
like	3.21	3.24	-0.36	
useful	3.06	3.16	-1.23	
Working with functions:				
easy	2.93	3.55	-5.72	<.001
like	2.56	3.05	-4.73	<.001
useful	2.88	3.06	-2.01	<.045

TABLE 16 -- Continued

Topic	Experimental	Control	t-value	sig
Use quadratic formula:				
easy	3.08	2.70	3.42	<.001
like	3.00	2.75	2.33	<.020
useful	3.02	2.91	1.24	
Working with integers:				
easy	4.58	4.67	-1.71	
like	4.01	4.17	-2.18	<.030
useful	4.26	4.27	-0.24	
Calculating probabilities:				
easy	3.01	2.89	1.09	
like	2.70	2.81	-1.07	
useful	3.75	3.70	0.48	
Translating word to algebra:				
easy	3.22	3.34	-1.26	
like	2.73	2.96	-2.55	<.011
useful	3.33	3.51	-2.25	<.025

ences in opinions do not reflect corresponding differences in achievement. For example, the experimental group was not particularly good at solving inequalities and the control group did significantly poorer on graphs of linear equations than the experimental group.

Both groups found working with positive and negative numbers easiest and solving word problems hardest. The control group, however, found word problems significantly harder. Both groups liked working with positive and negative numbers best and solving word problems least. The control group liked working with integers significantly more while the experimental group disliked word problems significantly less. Both groups also thought positive and negative numbers would be most useful after high school. However, they disagreed on the least useful topic, with the experimental group identifying functions and the control group the quadratic formula. This disagreement most likely reflects the fact that few experimental students studied functions and that few control students reached the quadratic formula.

Consumer test data

The Consumer Test is a compilation of items measuring performance on specific consumer objectives. As such, analysis of the Consumer Test data was conducted across schools by item and by treatment. Fall data were analyzed only for those 21 of the 28 items retained in the Spring and only for those students who were administered the items both in the Fall and again in the Spring. For students who were administered the Consumer Test in the Fall, 874 received Form A in the Spring and 616 received Form B. The discrepancy between the numbers receiving Form A and Form B reflects the discrepancy in drop-out rates between the schools in the study that did not become apparent until after the Spring testing program was completed.

Table 17 presents the percent of each group responding correctly to each item in Form A and Form B. The Chi Square statistic was used to test for each item whether the number of correct and incorrect responses

a) differed significantly for the experimental versus the control group in the Fall;
b) differed significantly for the experimental versus the control group in the Spring;
c) increased significantly from Fall to Spring for the experimental group;
d) increased significantly from Fall to Spring for the control group; and
e) increased significantly more for the experimental than the control group.

In all cases, a .05 level of significance was used.

Of all 21 items, only one item (1A) showed a significant difference by treatment in the Fall. The experimental group performed significantly better on this item having to do with computing salary from hourly wages. Since at a .05 level significant differences on one in 21 items should be expected, this overall lack of differences attests to the comparability of the two groups at the onset of the evaluation.

In the Spring, two items (6A, 10B) showed a significant difference by treatment, each favoring the experimental group.

TABLE 17

PERCENT OF CORRECT RESPONSES BY ITEM
FOR CONSUMER TEST

FORM A

Item	FALL		SPRING		% GAIN	
	exp	cont	exp	cont	exp	cont
1A	81.4**	73.8	89.5	92.1	8.1*	18.3* **
2A	59.6	56.8	70.4	65.9	10.1*	9.1*
3A	73.1	71.0	80.9	83.6	7.8*	12.6* **
4A	38.1	37.9	50.4	46.3	12.3*	8.4*
5A	11.2	15.7	19.7	21.7	8.5*	6.0*
6A	15.2	13.8	26.2**	16.1	11.0* **	2.3
7A	50.2	53.7	62.6	66.1	12.4*	12.4*
8A	58.1	53.5	66.1	65.7	8.0*	12.2*
9A	42.6	48.1	63.2	64.0	20.6*	15.9*
10A	14.1	18.2	24.7	22.4	10.6* **	4.2
11A	28.9	33.4	51.6	48.1	22.7* **	14.7*

FORM B

Item	FALL exp	FALL cont	SPRING exp	SPRING cont	% GAIN exp	% GAIN cont
1B	67.6	69.9	74.6	77.5	7.0	7.6
2B	37.0	36.7	46.8	45.3	9.8	8.6
3B	19.9	26.0	31.5	36.0	11.6*	10.0
4B	39.4	35.6	48.6	46.7	9.2	11.1*
5B	50.2	44.6	38.8	42.2	-11.4* **	- 2.4
6B	39.4	36.3	44.3	39.4	4.9	3.1
7B	14.4	14.2	10.7	11.8	- 3.7	- 2.4
8B	49.2	46.7	57.2	54.0	8.0	7.3
9B	52.3	55.4	62.7	59.9	10.4* **	4.5
10B	46.8	52.9	54.1**	44.3	7.3 **	- 8.6

* Significant difference (α = .05) between Fall and Spring

** Significant difference (α = .05) between experimental and control

The content covered by both items, compound interest and average speed, were explicitly presented in the experimental materials.

In comparing performance from Fall to Spring on the items, significant gains were observed for both groups on 9 items (1A, 2A, 3A, 4A, 5A, 7A, 8A, 9A, 11A), for the experimental group on 4 additional items (6A, 10A, 3B, 9B), and for the control group on 1 additional item (4B). A significant decline in achievement was observed for the experimental group on Item 5B. The better performance for both groups on Form A items as compared to Form B items has no apparent explanation.

Comparison of gains from Fall to Spring by treatment reveals the experimental group making a significantly greater gain on 5 items (6A, 10A, 11A, 9B, 10B) and the control on 2 items (1A, 3A). The decline on Item 5B observed for both groups was significantly greater for the experimental group than for the control.

Of the five items on which the experimental group showed a significantly greater gain from Fall to Spring over the control group, three items (6A, 11A, 10B) were explicitly presented in the experimental materials. The significantly greater gain of the experimental group on Items 6A and 10B reflects the significant improvement made by them from Fall to Spring on these items, presumably as a result of learning. On item 11A both groups improved significantly from Fall to Spring, but the experimental group more so. Items 10A (calculating square yards of carpet) and 9B (determining best payment plan) were not explicitly presented in the experimental materials. The significantly greater gain from Fall to Spring on these two items for the experimental over the control group [illegible] the significant gain from Fall to

Spring for the experimental group on these items.

Item 1A, one of the two items on which the control group showed significantly greater gains over the experimental group, is the one item on which the two groups showed a significant difference in the Fall, favoring the experimental group then. Since there was no difference between the two groups in the Spring on this item, the significant increase over the experimental group reflects a leveling of performance. The second, Item 3A, concerns computing parking lot charges. There was no significant difference between the two groups either in the Fall or in the Spring. Both groups made significant gains from Fall to Spring, with the control group gaining more than the experimental group.

Item 5B on computing monthly salary from annual salary showed a decline of 11.4% for the experimental group. Analysis of the responses indicates that one foil, correct except for a misplaced decimal point, accounts for most of the incorrect responses.

Textbook

Data concerning the textbook were collected from three sources--teachers, students, and an independent evaluator. For the teachers, two instruments were used to collect information regarding the textbook: End-of-Chapter Report forms; and a year-end Textbook Evaluation Form.

End-of-Chapter Reports

Each experimental teacher in the study was asked to complete an End-of-Chapter Report (Appendix C) for each chapter in the materials. The teacher was asked to supplement this form with any extra materials used by the teacher in conjunction with the particular chapter. Because the comments on these forms are particularly relevant to the selection of changes to be made in the materials, the completed forms with identifying information removed were sent to the project director. It is his analysis that is presented here.

Table 18 lists, for each chapter, the number of teachers returning a form, the average number of days spent on that chapter, and the range of days reported.

TABLE 18

END-OF-CHAPTER REPORTS SUMMARY

Chapter	Number of responses	Mean number of days spent	Range of days spent
1	18	14*	9-23*
2	17	14	9-22
3	17	11	6-18
4	15	10	5-15
5	15	8	5-13
6	13	15	10-22
7	13	10	7-13
8	12	12	6-18
9	12	14	10-22
10	15	10	6-20
11	9	9	4-14
12	9	10	7-14
13	4	7	6-8
14	6	9	6-13
15	8	9	5-20
16	0	-	-

Several bits of information are necessary to interpret the table. The asterisk (*) by Chapter 1 is to note that some teachers included the first testing days as being part of that chapter.

Thus, the mean number of days spent may be slightly less and the range may extend to 20, not 23. The mean number of days spent in later chapters is less because some teachers reported rushing through these chapters in order to get through the material. Not all teachers did each chapter; one reported skipping Chapter 16; it is probable that almost all classes did not cover this chapter. That smaller numbers of teachers responded to later chapters (and none to Chapter 16) is an indication as much of the length of the textbook as of the willingness of teachers to complete the forms.

One observation from the table is that there was a range in the pacing of the classes. This probably reflects both student and teacher characteristics. In the first eight chapters, there was a pattern of the slowest paced class taking two to three times the number of days to complete a chapter as the fastest paced class. This pattern is not as strong in later chapters as a result of the influence of other factors upon time.

It is typical, in the first year of use of new materials, for the pace to be slower. The teacher is not sure what will be important, what will not be, and accordingly does not skip any material. All content tends to be given the same emphasis. To moderate this trend, suggested teaching times were given for each lesson in each chapter. To acknowledge differences in student background, three paces were suggested. These paces, contained in the <u>Notes to the Teacher</u>, were formulated as a result of the trial experiences with the materials, and are given in Table 19.

Comparing Tables 18 and 19, it appears that the suggested schedule did not take into account the vast differences among classes. Some classes can go more quickly than estimated; other

TABLE 19

SUGGESTED NUMBER OF DAYS TO BE SPENT ON EACH CHAPTER

Chapter	Slow	Average	Fast
1	14	12	10
2	14	12	10
3	13	11	10
4	10	9	8
5	8	7	6
6	16*	14*	15
7	13	12	10
8	14	12	11
9	17	16	12
10	12	11	8
11	15	12	11
12	14	13	12
13	11	10	8
14	11*	10*	11
15	7*	12*	13
16	-	12	12

* In some chapters it is expected that slow-paced or average-paced classes would have to skip certain lessons.

classes take more time. On the average, the first eight chapters took a total of 4 days longer than recommended in the teacher's notes for average classes. However, the last eight chapters took 3 days longer each than recommended, despite the rushing of many teachers and the likelihood that reports in later chapters are from better classes.

From the anecdotal comments of the teachers, it is the earlier chapters that cause the problem in overall length. The discrepancy between the data and the anecdotal reports seems due to the influence of the traditional priorities in algebra. The later chapters cover primarily traditional content and time spent is considered reasonable and worthwhile. The earlier chapters cover much of the newer content and even meeting the schedule struck some teachers as a nonproductive use of time.

Corresponding data were not collected from control teachers. In previous studies (Usiskin 1969, 1972), teachers of control classes have skipped as much of their books (due to lack of time) as teachers of experimental classes. The fundamental difference in this study is that the material skipped in control classes is characteristically skipped in algebra classes and does not bother teachers, while the later chapters skipped by experimental classes contain some of the content usually considered basic to first-year algebra.

Two questions were asked for each chapter -- "Which lessons went particularly well?" and "Which lessons did not go well?". The paired questions were designed for the purpose of improving the materials. The responses overall resemble the following set, taken from Chapter 2. In that chapter, two teachers responded that all lessons went well. The other 15 teachers' responses are summarized in Table 20.

TABLE 20

CHAPTER 2 LESSON RESPONSES

Lesson	1	2	3	4	5	6	7	8	9
Number of teachers indicating went well	4	8	4	2	4	2	3	9	6
Number of teachers indicating did not go well	5	0	3	2	2	7	4	1	1

In Chapter 2, judging from the table, Lessons 1, 3, 4, and 7 and possibly 5 yield conflicting results, some indicating that the lesson should be pointed out as particularly good (and thus one should hesitate to modify it) while a like number pointed out that the lesson did not go well (and thus should be modified). Similar conflicting results were found for lessons in every other chapter as well.

Interpretation of these conflicting reports is not clear. For instance, Lesson 1 of Chapter 2 is designed to set up the next few lessons of the chapter, if not the entire volume. But it is very open-ended. Is the conclusion to be made that the lesson is too difficult to be taught, or that the ideas are too vague? Quite similar responses are given for Lesson 7, a lesson covering standard content, albeit in a non-standard way. Three of the 4 negative responses for this lesson come from teachers who liked Lesson 1.

The first 15 chapters contain 115 lessons. To make analysis reasonable, several criteria have been arbitrarily established. No lesson provoking fewer than 5 responses has been included.

Lessons which went particularly well for over 70% of teachers naming the lesson

Chapter 1:	Lesson 1,	Some Uses of Numbers
	Lesson 7,	Negative Numbers - Numbers to Indicate Direction
	Lesson 8,	The Decimal System
	Lesson 9,	The Metric System
Chapter 2:	Lesson 2,	Words and Symbols of Arithmetic
	Lesson 8,	Ordered Pairs
	Lesson 9,	Subscripts

Chapter 3: Lesson 2, The Slide Model for Addition

Lesson 3, The Assemblage Property of Addition

Lesson 4, Zero and Addition

Lesson 5, Subtraction

Lesson 7, A Statistic - the Mean

Lesson 9, Distance

Chapter 4: Lesson 1, The Repeated Addition Model for Multiplication

Lesson 2, The Ordered Pair Model for Multiplication

Lesson 4, The Assemblage Properties of Multiplication

Chapter 5: Lesson 2, Probabilities of Outcomes

Lesson 3, Probabilities of Events

Chapter 7: Lesson 1, Situations Leading to Linear Expressions

Lesson 3, An Algorithm for Solving ax + b = c

Lesson 4, An Algorithm for Solving ax + b < c

Lesson 5, The Distributive Property

Chapter 8: Lesson 6, Decision-Making Using Sentences

Chapter 9: Lesson 1, Types of Graphs

Lesson 2, Equations for Graphs

Chapter 10: Lesson 1, The Repeated Multiplication Model of Powering

Lesson 3, Powering and Order of Operations

Lesson 5, The Power Property

Chapter 11: Lesson 1, Adding Powers

Lesson 3, Dividing Powers

Lesson 5, Powers of Products and Quotients

Chapter 12: Lesson 6, Multiplying Binomials
Lesson 8, Geometric Shortcuts and the Pythagorean Theorem

(There are not enough comments in the later chapters to qualify.)

Lessons which did not go well for over 70% of teachers naming the lesson

Chapter 1: Lesson 3, Number Lines and Bar Graphs
Lesson 4, Rational Numbers and Comparison
Chapter 2: Lesson 6, Replacement Sets for Variables
Chapter 4: Lesson 3, The Area Model for Multiplication
Chapter 8: Lesson 1, Models, Postulates, and Theorems
Chapter 9: Lesson 7, The Finding of the Fahrenheit-Celsius Conversion Formula
Chapter 10: Lesson 6, Negative Exponents

There were more positive than negative comments on lessons. However, a lesson should probably be examined even if only 40% of teachers find it not to go well, perhaps even if 25%. The number of lessons which need improvement is greater than the above list would indicate.

Several lessons were skipped by some teachers. When these occur in the first chapter, it can only be due to philosophy, not to time pressure. For example, one teacher skipped lessons 5, 6, and 9 of that chapter, another skipped lesson 5, and one of these and a third teacher reported skipping the newspaper activity. Clearly these teachers were, at least at this stage of the course, trying to teach their old algebra content from what was proving to be an unhelpful text. There seem to be two types of lessons that were skipped: the first type contains probability and statistics and is easy to explain; the second contains exactly the material most crucial to the development of a sense and motivation for applications (e.g., Chapter 9: Lesson 8, the Evolution of the Mile Record). What effect skipping these kinds of lessons would have on both student attitudes and performance is difficult to assess. It seems unlikely, however, that the effect of skipping is a positive one.

Several teachers reported a necessity to supplement. This was particularly noticeable in Chapter 13, Quadratic Equations, which seems to be the most disliked chapter in the book. If more responses had been received about this chapter, there would have been a few lessons from this chapter that would have been noted as not going well.

Textbook Evaluation Forms

Textbook Evaluation Forms were sent to both experimental and control teachers. Different forms were used for the two groups but some comparable items were included on both forms. Fifteen of the experimental teachers and 16 of the control teachers returned the form. Copies of both forms and the tally of the responses to each question are included in Appendix E.

Of the 17 experimental teachers whose classes were included in the achievement data analysis, 15 returned the Textbook Evaluation Form. This group was split in its reaction to the experimental materials. In response to Item 4, "Would you recommend the use of this text for an average first year algebra class?", 7 responded favorably, 7 responded unfavorably, and 1 was indifferent. Of the 7 unfavorable evaluations, further analysis reveals that 6 reflect the teachers' opinions that the materials were inappropriate for their students. Either they were perceived as too difficult for inner-city or senior high students or too non-traditional for the brighter college-bound students. Hence, only 1 of the unfavorable reactions was a reflection of the quality of the materials.

Not only were the experimental teachers split on the question of their recommendation of the materials, but they were also split on a number of other key issues addressed by the form. Furthermore, there exists an apparent relationship between student achievement and teachers' opinions concerning the experimental materials. Of the 7 who would recommend the textbook, 5 were from the 8 schools in which the experimental group scored significantly higher on an achievement post-test. By comparison, only 2 of the 7 who would not recommend the textbook came from such a school. However, the achievement of the control group was essentially the same among these two groups of schools. Because of these contrasts, the responses to the Textbook Evaluation Form were analyzed by comparison between the 7 favorable teachers and the 7 unfavorable teachers.

In addition to Item 4, the two groups split on three other items asking for a global evaluation of the materials. The favorable teachers all thought the textbook appropriate for the average first-year algebra student (Item 1); most thought that it was easier to read and understand (Item 2) and that the exercises were at about the same level as other algebra books (Item 3). By contrast, the unfavorable teachers voiced no consensus on these three items.

In addition to soliciting global reactions to the experimental textbook, the form solicited teachers' reactions to the development of 10 specific topics common to both the experimental and traditional course (Items 6-15). Once again the contrast between the responses of the two groups of teachers is revealing. The 7 favorable teachers cast 34 votes for "The development is the nicest I've seen" as compared with only 13 such votes from the unfavorable teachers. Similarly, the favorable teachers cast only 1 vote for "I know of a more effective development" compared with 19 such votes from the unfavorable teachers.

The favorable teachers were most complimentary about (in order) the approach to beginning sentence solution (Ch. 6), systems (Ch. 14) and the approach to variables (Ch. 2). They also indicated that with the exception of systems, these topics were easiest for their students while slope (Ch. 9) and square roots (Ch. 12) were most difficult. The unfavorable teachers were least complimentary about (in order) the approach to negative exponents (Ch. 10), systems (Ch. 14), square roots (Ch. 12), and work with properties (Ch. 3-5, 7-8). Their perception of student difficulty with topics reflects a comparable ranking of these topics. On the average the favorable teachers ranked all topics relatively easier for their students than the unfavorable teachers.

Items 16-20 solicited teachers' opinions concerning topics and emphasis unique to the experimental materials. For the most part, teachers chosen to participate in the study had limited or no knowledge of the content change and emphasis of the experimental material, particularly as the material deviated from a traditional first-year algebra course. The contrast between the favorable and unfavorable teachers is again apparent. The notion of models of operation (Item 16) is central to the development of the experimental approach. The favorable teachers generally indicated that they found them useful and recommended that they be kept. By comparison, the unfavorable teachers indicated more frequently that they were not useful, that they were skipped or not emphasized, and that they should be dropped. The unfavorable teachers account for 45 of the 65 "not useful" responses and 43 of the 53 "drop" responses. Less marked contrasts were found between the two groups on Items 17-20.

Some traditional first-year algebra topics were either deleted from the experimental textbook or not emphasized to the same extent as in the traditional textbook. Items 21-28 queried teachers on these omissions. Again, contrasts between the favorable and unfavorable teachers emerged. All of the unfavorable teachers indicated some degree of disagreement with the omission of three standard topics--factoring trinomials, adding fractional expressions requiring a common denominator, and simplification of fractional expressions involving factoring trinomials (Items 21-23). By contrast, the majority of the favorable teachers indicated that they did not mind the omissions. In fact, on all 8 items queried, the majority of the favorable teachers responded positively to the omissions. The majority of the unfavorable teachers responded positively on only 2 omissions--formal logic and coin problems. None of the 15 reporting teachers minded the omission of formal logic (Item 24). Item 45, on the amount of supplementing done, offers another interesting contrast. Six of the 7 unfavorable teachers indicated that they had to supplement more than usual, presumably to cover omitted topics. Only 1 of the favorable teachers indicated a need for more supplementing with the experimental materials.

With respect to the applications in the experimental textbook (Items 29-30), favorable teachers generally indicated that they were interesting to most students and promoted valuable discussion. The unfavorable teachers more often indicated that they were interesting to only a few students and that the arithmetic was too difficult. Only unfavorable teachers felt that there were too many applications or that traditional word problems are better. They also account for 7 of the 9 "didn't do" responses on Item 30.

Textbook Evaluation Forms were also sent to control teachers. The 16 reporting control teachers were more positive about their textbook. Most would recommend their textbook (Item 4). Most feel that it is suitable for the average student (Item 1), is as easy or easier to read than other textbooks (Item 2), and has comparable exercises (Item 3). With the exception of formal logic, all the traditional topics deleted or deemphasized in the experimental textbook received overwhelming support from the control teachers (Items 6-13). In response to Item 23, "Which topics or ideas are generally hardest for your students to understand?", 7 responded translating word problems. Not one experimental teacher lists this response to the comparable item on his/her form. Included in the material they would like to see added to their textbook, the control teachers listed probability and statistics, the metric system, more real-life verbal problems, and word problems with fractions for answers.

In addition to the schools in the study, 18 schools used the materials but were not part of the formal study. Each of these schools was sent the Textbook Evaluation Form (Experimental). Of the estimated 21 teachers using the materials, 12 returned the forms. Their responses are summarized in Appendix E. Unlike their counterparts in the formal study, these teachers were for the most part true volunteers. Their responses are generally more favorable than those experimental teachers in the formal study. But like their counterparts in the formal study, there are marked contrasts between the favorable and unfavorable groups. In response to Item 4, concerning the recommendation one would give the textbook, one teacher strongly recommended against the use of the textbook for average first-year algebra classes. An additional one would not recommend the textbook. These two of the 12 teachers agreed on many other responses -- for example, they are the only two who felt that the textbook was for above-average students (Item 1) and they were the only two who felt that the textbook was harder to read and understand than other first-year algebra textbooks (Item 2). They were two of the three teachers who felt that the exercises were more difficult (Item 3).

These two teachers also constituted almost all of the minority block on some other questions. They account for 19 of the 22 "drop" responses and 16 of the 21 "not useful" responses in Item 16. They are the two who felt that calculating statistics and probability (Items 19-20) should be dropped. In Item 29, they felt that the situations were interesting to only a few students, were too involved or complex for most students, and the traditional word problems are better. In Item 30, these two included the one who felt that the mile run was not interesting and the one who did not do the storm example. They included the one who definitely had trouble with the mathematics or the applications in this course (Item 36). Their views toward first-year algebra or the teaching of applications were not changed (Item 38). They had to supplement more than usual (Item 45). In Item 49, these two teachers responded that a typical student was seldom or never able to understand the lesson from the reading -- no other teacher felt this way. These teachers reported in Item 47 that they assigned reading to some (less than half) of the lessons. (Only one other teacher assigned so little.) Yet they felt that a student should be expected to read in a mathematics textbook frequently (Item 50). They all guessed that a small percentage of their students (10%-30%) had access to a calculator.

In summary, these two teachers felt that the textbook was much too difficult. One thought the reading level was above 12th grade, and the other that the textbook is "way out of line for our type of kid."

The views contrast quite strikingly with those of the other ten teachers. Nine of those would recommend or strongly recommend the textbook; the tenth was not sure, needing "another year to decide," but reported that the textbook might be for the below-average student. On almost every general question of opinion, this 83% majority almost unanimously differs with the two who did not like the textbook. Items 1-3, 19-20, 25-29, 36, and 47-49 demonstrate the biggest differences. This is not to imply that the split is complete or that those ten in the majority are unanimously in agreement with all of the developments used. The response tabulation clearly shows many problem areas. What is striking is the similarity of the split between the favorable and unfavorable teachers whether they were in the formal study or not.

Reading level evaluation

A reading level comparison of the experimental textbook with two other widely-used first-year algebra textbooks was conducted by Dr. Gerald Kulm of Purdue University. The textbooks used for comparison were Holt Algebra I (Holt, Rinehart and Winston, 1974) and Algebra Structure and Method, Book 1 (Houghton-Mifflin, 1976). Four readability measures were employed to rank 20 comparable passages from each of the three textbooks. These measures included two readability formulas, teacher's judgment, and an information-content-level formula. The passages included 15 from the exposition and 5 from problem sets. Table 21 presents the means for each textbook on the four readability measures. The experimental materials ranked between the other two textbooks on three of the four readability measures used and ranked most difficult on the fourth measure. On the basis of his analysis, Kulm concluded that the experimental materials are written at a suitable level for first-year algebra students when compared with these two widely-used textbooks. His complete analysis is included in the Appendix G.

TABLE 21

MEAN READABILITY SCORES

	Kane formula*	Kulm formula*	Teacher judgment	Info levels*
Grand Mean	31.14	18.39	4.07	0.116
Interval	3.00	2.00	0.75	0.050
Algebra Through Applications	31.32	19.76	3.85	.1190
Holt Algebra	32.23	17.50	4.52	.1365
Houghton-Mifflin	29.87	17.91	3.84	.0915

*formulas given in Appendix F

Student evaluations

On the Spring Opinion Survey, five items (5, 12, 20, 23, and 24) dealt specifically with student opinions concerning their algebra textbook. As discussed earlier (see "Attitude Data"), on all five items there was a significant difference in favor of the experimental group. Overall, the experimental group found their textbook more interesting and readable and the explanations more useful than those students using a standard algebra textbook. The experimental group also indicated that they read explanations in their textbook significantly more often than the control group.

Mastery learning materials

During the second and final year of the development of the experimental material, a workbook was created. Using a quasi-mastery learning strategy, the workbook was designed to give attention to the development of skills. At the end of the second year and the beginning of the formal evaluation, the workbook was still in first draft. Extensive revisions and elaborations were needed. The author's attentions were necessarily focused on the final revisions of the experimental textbook. Hence, the workbook was not available for testing with the experimental textbook. One copy of the first draft of the workbook was sent to each experimental teacher to use for supplementary problems or for test items. Both the developer and the teachers noted that the materials were not in an easy-to-use format. However, on the Textbook Evaluation Form (Item 42), almost all of the experimental teachers indicated that they used it for review problems and test problems. Individual teachers indicated that it was very helpful and that copies for each student would have been a valuable addition.

V. DISCUSSION AND RECOMMENDATIONS

Discussion

The study evaluated the effectiveness of the Algebra Through Applications materials as compared with traditional first-year algebra materials. Effectiveness was defined to encompass student achievement on traditional objectives, student achievement on experimental objectives, transfer by students to consumer problems, student attitude, textbook readability, and teacher judgment of the materials. Eight specific questions concerning the materials were addressed by the evaluation.

1. *To what extent do students who study these materials understand the concepts considered standard in first-year algebra as compared to other first-year algebra students?*

The ETS Cooperative Algebra Test was used as a measure of achievement on standard first-year algebra concepts. With this instrument, the students in the standard first-year algebra classes performed better than students in the experimental classes. In 8 of the 17 schools, the control group scored significantly higher on the Spring ETS Test. In the remaining 9 schools there were no significant differences.

On 24 of the 40 items in the ETS Test, the performance of the experimental group was comparable to that of the control group. However, on 16 of the 40 items, the control group performed significantly better. These items assessed such topics as integer arithmetic, evaluation of exponential expressions, multiplication of algebraic expressions and simple monomial factoring. While some of the traditional algebra skills were deliberately omitted in the experimental materials, these were not. Thus, there is an apparent weakness in the experimental materials in the area of the development of traditional algebraic skills. Yet the two groups were comparable on items for translating algebraic expressions and solving linear equations. The experimental materials, however, place more emphasis on the development of these topics. The results do not reflect these differences in emphasis. The absence of individual mastery learning materials (workbooks), the brevity of the skill exercise sets, and the lack of exercises in the chapter reviews in the experimental textbook might account for the apparent weakness in skill development.

2. *To what extent do students who study these materials understand the concepts considered unique to these materials?*

The First Year Algebra Test was developed as a measure of achievement on concepts unique to the experimental materials or on concepts common to both, yet not measured by the ETS Test. Using this instrument, the students using the experimental materials performed better than students in standard first-year algebra classes. In 8 of the 17 schools the experimental group scored significantly higher on the First Year Algebra Test. In the remaining 9 schools, there were no significant differences.

Only 12 of the 33 items on the First Year Algebra Test measured concepts unique to the experimental materials. The experimental group performed significantly better on 9 of these items, including all questions dealing with relative frequency and probability and on questions dealing with the metric system, models for multiplication, rate of change as slope, percentage decrease, and compound interest. Many teachers in their first time through the experimental materials either did not get to the later chapters (particularly the one on probability) or skipped some of the topics unique to the materials. Consequently, the performance of the experimental group on topics unique to these materials speaks well of the integration of probability and applications throughout the experimental textbook.

Overall, in 6 of the 8 schools in which the experimental group performed significantly better than the control group on the First Year Algebra Test, there was simultaneously no significant difference between the two groups on their performance on the ETS Test. This would indicate that the experimental materials can be used successfully in a variety of schools situations, comparing favorably with the traditional first-year algebra materials.

3. *To what extent do these materials help in solving applied problems from real-life situations?*

The Consumer Test data indicate that both groups improved in their applied problem-solving skills in the course of the year. This may be a function of either maturation or first-year algebra. The experimental group showed somewhat more improvement than the control group, particularly on topics explicitly covered in the materials. Whether this advantage will be maintained over time is unknown. The study provides evidence that consumer problem-solving skills would be improved with wider attention to real-life applications throughout the school mathematics curriculum. If consumer problem-solving skills are a primary objective of instruction, then such skills should be explicitly taught.

4. *Is the reading level of the materials comparable with other first-year algebra materials?*

Three sources of information on the readability were tapped: teachers, students, and an independent reading evaluator. The three were not in concert in their evaluation of the experimental materials. Throughout the year, the readability of the experimental textbook (both the amount and the level of difficulty) was called into question by some of the teachers using the materials. But on the Spring Opinion Survey, the experimental students indicated that they found the explanations in their textbook helpful, their textbook more interesting than most, and the materials readable. In fact, there were significant differences favoring the experimental treatment on all items on the Spring Opinion Survey dealing with the textbook. In support of the students' opinion, the reading evaluator, using four evaluation techniques, found the materials comparable to two other widely used algebra textbooks.

There are several possible explanations for the discrepancies noted above. The typed copy and unpolished format of the experimental materials tend to give an impression of more and difficult reading. Also, most traditional materials include more numerical examples and skill exercises. The data indicate that the reading level of the experimental materials is comparable with other first-year algebra materials. Commercial publication and the extension of the skill exercises might give the materials a more traditional algebra textbook format as well as strike a more acceptable balance between reading and other activities. However, there is no indication that the reading should be reduced or simplified.

5. *To what extent do the mastery learning materials help in improving skills?*

The mastery learning materials (or workbook) were still in the developmental stage and unavailable for the formal evaluation. Preliminary studies by the developer of the experimental materials, indicated that more attention to algebraic skill development was needed in the materials and that the addition of the mastery learning materials appeared to be addressing that need. The present study substantiated this need. What influence the easy availability of accompanying mastery materials might have had on student achievement is unknown. Evidence from the development phase strongly suggests that student achievement would have been improved.

6. *What are the difficulties, if any, of implementing these materials into the school curriculum?*

The present study sought to assess the implementability of the experimental materials indirectly. Such an approach proved naive. The logistic of the evaluation interfered with many of the usual steps in the adoption and implementation of a new textbook. For example, teachers chosen to participate in the study had, at best, only limited information about the experimental materials. Since materials were not yet available when commitments to participate were solicited, teachers did not have an opportunity to review the materials. To avoid bias in the evaluation, a nonintervention policy was adopted. In-service was not provided. The developer of the materials did not visit participating schools. The two evaluators tried to maintain a neutral rather than advocate position. Furthermore, rather than school adoption, the experimental teacher was chosen at random and isolated without the usual colleague-support system. Hence, in the attempt to obtain an unbiased assessment of the experimental materials, a realistic assessment of their implementability was unobtainable.

Although the study did not provide a reasonable assessment of the ease or difficulty of implementing these materials, some informal observations can be made. The test data indicate that even without a reasonable preview or adequate preparation and support, the experimental materials can be used effectively in many situations. On the Textbook Evaluation Form, only 2 of the 15 reporting teachers indicated that they had any trouble with the mathematics or the applications in the materials. In the beginning, 5 indicated that they felt that they might. Although the mathematics does not seem to provide an impediment to implementation, other factors might. The divergence of the experimental materials from the typical first-year algebra syllabus may pose a problem for the use of these materials by traditional teachers without appropriate in-service. Furthermore, this variance gives rise to concerns over student performance in subsequent mathematics courses, particularly second-year algebra. While the scope of the present study prohibited a systematic study of this issue, evidence from the developmental phase (in which the materials were used in the same school for three years) does not substantiate a cause for concern. There is no evidence to suggest that the materials are inappropriate for students planning to take second-year algebra. The lack of adequate drill exercise or available supplemental exercises is another factor that may pose a problem for the use of the materials by the sympathetic teacher.

In addition to the six questions specifically listed in the renewal proposal by the project director, the study also addressed two additional questions.

7. *To what extent are student attitudes about the enjoyment and usefulness of mathematics effected through an applications approach to first-year algebra?*

On the Fall Opinion Survey, there was an overall lack of significant differences between the two groups with respect to their attitudes about the enjoyment or usefulness of mathematics. A general decline in attitude for both groups from Fall to Spring was observed. For the 7 items on which both groups registered a significant decline, 6 were among the 7 items which were modified to read "algebra" in place of "mathematics." The apparent change in attitude might be due to this restriction. Perhaps algebra is viewed as less helpful, necessary and important in everyday life; less interesting and more symbolic; and harder to learn than mathematics in general. However, the non-modified item on which both groups showed a decline addressed the usefulness of mathematics in solving everyday problems. It would seem that the study of algebra, whether through an applications approach or not, does not enhance students' view of the value of mathematics for the real world.

The significant difference between the two groups on individual items lends support to the argument that the integration of word problems into each lesson is more effective than their isolation in separate lessons, or, in some textbooks, separate chapters. The data also suggest that the development of the algebra <u>out of</u> problems rather than the application of algebra <u>to</u> problems makes working these problems more enjoyable.

8. *To what extent are these materials designed for the average first-year algebra student?*

The achievement of the "average students" was comparable to that of the total group. Students in the control group did better on the test and items which emphasized traditional algebra concepts and skills, while students in the experimental group did better on the test and items covering topics emphasized by the experimental materials. Nine of the 15 reporting teachers indicated that they thought the materials appropriate for the average first-year algebra student.

Limited data were obtained on the material's appropriateness for the lowest-ability algebra student. One inner-city senior high school teacher found the materials inappropriate for the lowest-ability students due to the emphasis on reading. However, 2 experimental teachers thought the material appropriate for below-average students. Overall, significantly fewer students dropped out of the experimental treatment than the control treatment. This could be interpreted as indicating that the experimental materials are more successful than traditional materials with marginal students. It would seem that these materials are suitable for average students and that their appropriateness for the lowest-ability student should be evaluated further.

The Algebra Through Applications materials offer a unique approach to first-year algebra; the field evaluation of the materials indicated that they can be used effectively in a variety of school settings. These materials are responsive to the criticism of school mathematics as irrelevant to the real world. As such they represent a serious departure from the traditional first-year algebra course with its emphasis on skill development. As a prototype of an applications approach to first-year algebra, the materials can be used by those who are familiar with them and share their point of view. They also stand as a source of relevant applications for the traditional first-year algebra course and as a point of departure for the development of a more traditional course with an applications orientation.

Limitations

Although schools and teachers were selected from volunteers, care was exercised to assure that those schools selected were reasonably representative of a variety of the nation's schools. However, neither the selection of participating schools nor teachers within schools was random. Yet, random assignments or teachers, and, insofar as possible, students to treatments within schools, were made. Hence, extrapolation beyond the present study should be made with caution.

Many participating teachers endeavored to present the experimental materials in a manner consistent with the material's intent. However, uncertainty in using materials for the first time could have limited their effectiveness. Particularly, some of the topics unique to or heavily emphasized in the experimental materials (*i.e.*, probability, models of operations, and some applications) were inadvertantly omitted or never reached.

Other participating teachers, unable to reconcile the experimental approach with the traditional approach, presented instead a traditional algebra course using materials not designed for that purpose. The absence of any in-service during the year or available coordinators compounded this situation. The extent and effect of unsympathetic teachers could not be quantified. However, interpretation of the findings should be made cognizant of the impact of teacher attitude on the effectiveness of curriculum materials.

The duration of the study was only one year. Hence, comparisons were made between the first year of teaching from new materials and the teaching of familiar materials for which topic to be stressed, time tables, and worksheets have already been developed. Also, the comparative achievement of the experimental group in subsequent mathematics courses could not be investigated.

The one-year duration of the project did not allow for extensive pilot-testing and validation of the project-developed tests. However, care was exercised in their development and limited pilot-testing conducted.

Testing in the schools could not be monitored by the project staff. All tests were administered by participating teachers following directions and schedules provided by the project staff. Subsequent reports were filed by the teachers. No specific problems were identified. However, it is recognized that the testing situations were not uniform across schools. Some teachers counted the post-test score as part of the student's grade; others did not. Since the project did not have the prerogative of establishing grading policies, only consistency within schools was requested. Consequently, the use of student test data as a measure of achievement is subject to these limitations.

Finally, it is recognized that the assumptions of the statistical models utilized in the analysis of the data could not always be completely verified.

Recommendations

Concerning the materials

The evaluation of the <u>Algebra Through Applications</u> materials provided useful feedback on the materials, ranging from identification of typographical errors to suggestions for reorganization. Most of the suggestions coming from teachers using the materials were specific to a particular lesson and will not be made here. Since the evaluation indicated that the materials can be used successfully in a variety of situations, publications with minor editorial revisions would make the materials readily available for immediate use.

Should more substantial revisions be undertaken at a later date, its objective should be to make the materials easily usable, without special preparations, by teachers who share the orientation to first-year algebra they present. In addition to the suggestions from individual teachers for the improvement of particular lessons or topics, the following recommendations for future revisions are made:

a) Additional exercises aimed at the development of algebraic skills are needed. This need might be met with the extension of the existing exercise sets to include more routine manipulative problems, or by the inclusion of mastery materials either in the text or in any accompanying workbook. More examples in the exposition might also prove helpful.

b) Review exercises should be added to the chapter reviews as well as sample chapter test items in the Notes to the Teacher.

c) In terms of format, a revised edition should include colorful illustrations, an index, and lesson titles which reflect both the algebraic and applications content of the lesson.

The evaluation also produced suggestions concerning the development of a more traditional first-year algebra course which retains the philosophy and applications orientation of the present materials. Should further development work be undertaken with the present materials, the following recommendations are made, in addition to those listed above:

a) Although the development of models for operations is central to the development of the materials, the emphasis, particularly in the early chapters, should be reconsidered with the view toward simplification or consolidation.

b) In order to assure and facilitate the coverage of topics important to the objectives and development of the materials (*i.e.*, probability, models, etc.), a reorganization of the materials should be considered. In particular, a reorganization of the materials placing quadratics earlier in the development might encourage the coverage rather than the omission of some of the topics unique to the applications approach.

c) While there was some criticism of exercises devoted to the understanding of the lesson expositions, "Questions covering the reading," such exercises should be retained. However, their integration into a single exercise set for each lesson should be considered.

d) Further developmental work might best be conducted by a team of authors, including experienced classroom algebra teachers.

Concerning implementation

The field evaluation indicates that under sympathetic circumstances, the materials can be used effectively. Schools or teachers considering their use should first review the materials to determine if the materials reflect their own instructional objectives. Particular attention should be focused on the differences, in content and emphasis, between these materials and the traditional first-year algebra course. Schools reviewing the materials should view the objectives of their own mathematics curriculum in a content broader than that of the next course or existing textbooks.

The divergence of the present materials from the usual first-year algebra course is a potential source of difficulty for traditional classroom teachers. Even teachers who agree in principle with the philosophy of the materials may have difficulty omitting traditional topics in order to implement the new materials. Schools using the materials may wish to conduct a faculty seminar on the materials. An in-service program which focuses on the role of the various models of operations and those applications central to the development of the course might facilitate the implementation of the spirit of the materials. In any event, teachers using the materials should expect to supplement the existing problem sets with skill exercises, either their own or others.

Concerning evaluation

Fundamental to any curriculum evaluation are a number of basic issues in educational research concerning a fair yet unbiased testing of experimental materials. The experience obtained in the evaluation of the Algebra Through Applications materials suggests recommendations concerning this issue.

The use of volunteer teachers and the provision of coordinators, in-service or other special supports, have been criticized for producing an unrealistic setting for the evaluation of experimental materials. In the present study, teachers chosen to participate had for the most part only limited knowledge about the experimental materials. They received no external moral or technical support in the form of in-service, local coordinators or consultants, visits by the developer, or repeated site visits by the evaluators. The control teachers, on the other hand, were supported by a long-standing tradition of what is and ought to be first-year algebra. Such a long-standing, even venerated, tradition is a significant bias working against the objective evaluation of non-traditional materials in traditional (typical) classrooms. Likewise, extraordinary intervention on behalf of the experimental materials would have produced a bias in their favor. Such intervention would have also obstructed the objective evaluation of new materials in typical classrooms with typical teachers. However, experimental controls for in-service or other support for the experimental teachers could have been introduced in an effort to quantify this bias. Such controls were not within the scope of the present study. Funding of the evaluation of experimental materials should be at a level sufficient to provide reasonable support for the experimental teachers to assure that the content is accurately presented in the spirit in which it was designed. At the same time, care should be taken to assure that the in-service or support provided balances, rather than outweighs, tradition.

The present study, from planning to post-testing, was conducted within a 14-month period. Recognizing the difficulties inherent in an extended evaluation, a period of three years would have allowed for (1) adequate planning and test development, (2) a follow-up study of students' achievement in subsequent mathematics courses, and (3) a second year of evaluation of the experimental materials with teachers who have experience with the materials.

Concerning additional funding

Current plans call for publication of the materials, Algebra Through Applications, with minor editorial revisions. Once published, the materials will be available as a substitute for the traditional first-year algebra materials, as a source of applications to supplement the traditional materials, or as a point of departure for the commercial development of a more traditional course with an applications orientation. Since the materials can be used effectively in their present form, programs to acquaint teachers and supervisors with the materials should be supported in order that information concerning the product of the First-Year Algebra via Applications Development Project will be communicated to the public. Two activities should be considered. One- or two-day awareness workshops which acquaint teachers or supervisors with the philosophy and content of the materials for the purpose of making judgments about their appropriateness for their school should be offered. Such workshops, lead by the developer of the materials or persons trained by him, should focus on the differences, in content and emphasis, between these materials and the traditional first-year algebra materials. Secondly, one-week workshops which provide appropriate in-service training for teachers planning to use the materials should also be offered. The variance of the experimental materials from the usual first-year algebra course presently threatens their acceptance among traditional classroom teachers without an in-service program comparable to those provided by other experimental curriculum developmental projects. Such workshops could be conducted at a central location by the project staff or in local school districts by a trained teacher or coordinator. The workshop should provide a rationale for the applications approach and an overview of the textbook. Some time should also be devoted to the models of operations and to those applications fundamental to the development of the course.

In addition to support for dissemination activities, support for subsequent development work or substantial revisions, in line with the recommendations made earlier, should be considered. Progress in education demands not only creative approaches but sustained and supported efforts.

REFERENCES

Stanley, Julian and Hopkins, Kenneth. Educational and Psychological Measurement and Evaluation. Englewood Cliffs, N.J.: Prentice Hall, 1972.

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Usiskin, Zalman P. and Bernhold, Jeanne. "Three Reports on Study of 11th Grade Mathematics." Unpublished report of 1971-72 field test of Intermediate Mathematics, University of Chicago, 1973.

APPENDICES

APPENDIX A

SCHOOL AND TEACHER INFORMATION

A1 List of Participating Schools

A2 Distribution of Participating Schools by Geographic Region and Community Size

A3 Information Form for Participating Schools

A4 List of Participating Teachers

A5 Teacher Information Form

APPENDIX A1

LIST OF PARTICIPATING SCHOOLS

Bay Senior High School
Panama City, Florida

Elizabeth City Junior High
Elizabeth City, North Carolina

Fort Mill High School
Fort Mill, South Carolina

Fraser High School
Fraser, Michigan

Fritsche Junior High School
Milwaukee, Wisconsin

Harper High School
Chicago, Illinois

John Adams High School
Ozone Park, New York

Los Alamos High School
Los Alamos, New Mexico

Marcus Whitman Junior High
Port Orchard, Washington

McLean High School
McLean, Virginia

Northwest Whitfield High School
Tunnel Hill, Georgia

Okemos High School
Okemos, Michigan

Olney High School
Philadelphia, Pennsylvania

Owen J. Roberts High School
Pottstown, Pennsylvania

Sequoia High School
Redwood City, California

South Miami Senior High School
Miami, Florida

South Shore High School
Brooklyn, New York

Walter Reed Junior High School
North Hollywood, California

Walton High School
Marietta, Georgia

Wasson Senior High School
Colorado Springs, Colorado

APPENDIX A2

DISTRIBUTION OF PARTICIPATING SCHOOLS BY GEOGRAPHIC REGION AND COMMUNITY SIZE

Region	Community Size		
	Urban (total 8)	Suburban (total 6)	Small town/rural (total 6)
Northeast (total 4)	John Adams HS Ozone Park, NY Olney HS Philadelphia, PA South Shore HS Brooklyn, NY	Owen J. Roberts HS Pottstown, PA (Philadelphia)	
Southeast (total 7)	South Miami HS Miami, FL	McLean HS McLean, VA (Washington, D.C.) Walton HS Marietta, GA (Atlanta)	Bay HS Panama City, FL Elizabeth City JH Elizabeth City, NC Fort Mill HS Fort Mill, SC N.W. Whitfield HS Tunnel Hill, GA
Central (total 4)	Fritsche JH Milwaukee, WI Harper HS Chicago, IL	Fraser HS Fraser, MI (Detroit) Okemos HS Okemos, MI (Lansing)	
West (total 5)	Walter Reed JH N. Hollywood, CA Wasson HS Colorado Sps., CO	Sequoia HS Redwood City, CA (San Francisco)	Los Alamos HS Los Alamos, NM Marcus Whitman JH Port Orchard, WA

INFORMATION FORM FOR PARTICIPATING SCHOOLS

Evaluation Study of *Algebra Through Applications*

1. Name of participating school: ______________________

   Address of school: ______________________

   ____

   Phone at school: AC-______________________

2. Your name or name of primary contact: ______________________

   Business address: ______________________

   ____

   Business phone : ______________________

   Summer or home address: ______________________

   ____

   Summer or home phone : ______________________

3. Participating Teachers (total of 2)

   a) Teacher A

   Name: ______________________

   Summer address: ______________________

   ____

   ____

   Summer phone: ______________________

   b) Teacher B

   Name: ______________________

   Summer address: ______________________

   ____

   ____

   Summer phone: ______________________

4. Participating Classes (total of 4)

   a) Expected enrollment in each class (be as specific as possible)

   Teacher A: class 1 __________ class 2 __________

   Teacher B: class 1 __________ class 2 __________

   b) Describe the students in the four participating classes in terms of their mathematical achievement &/or ability levels.

c) What textbook and special materials will be used with the control classes?

5. School Calendar

a) Date of 1st full day of class in the fall: ______________

b) Date of last full day of class in the spring: ______________

c) If the school normally has a final exam period in the spring, give the (tentative) dates and schedule for it.

6. Testing Program: Describe the school's testing program that would normally involve the students in the four participating classes. Give the name and form of any published tests given and the (tentative) dates for the administration of the tests.

7. Address to which the experimental textbooks are to be shipped:

____

____

____

____

Please complete and return to: Dr. Jane O. Swafford; Department of Mathematics: Northern Michigan University: Marquette, MI 49855

THANK YOU

APPENDIX A4

PARTICIPATING TEACHERS

School	Experimental Teacher	Control Teacher
Bay High	Juanita Bowers	Sadie Williams
Elizabeth City HS	Emily Jackson	Ann Nowell
Fort Mill HS	Katie Culp	Rita Cater
Fraser HS	Carl Vaara	Susan Hill
Fritsche JH	Joan Smith	Harold Rife
Harper HS	Mary O'Neill	Janet Gerut
John Adams HS	Lawrence Lane	Patricia Lane
Los Alamos HS	Margaret Barrett	Elvira Aragon
McLean HS	Edith Elliott	Robert Hayler
N.W. Whitfield HS	Linda Finnell	Geneva Trammell
Okemos HS	Elaine Cowen	Larry Cockroft
Olney HS	Pearl Cohen	Helen Mintzes
Owen J. Roberts HS	Barbara Garrett	Michael Mehle
Sequoia HS	Michael Doody	Charles Paine
South Miami HS	Ruth Williams	Mary Alice Pennington
South Shore HS	Rachelle Ben-David	Morton Newman
Walter Reed JH	Vera Helpern	Dorothy Moser
Walton HS	Myra Medford	Charlyn Shepard
Wasson HS	Robert Zecha	Robert Hiltner

APPENDIX A5

TEACHER INFORMATION FORM

Name ____________________ School ____________________

Address to which postage refund should be sent:

____

____

Address to which results of study should be sent:

____

____

School phone number __________

Home phone number __________

1. age ______ 2. sex ______ 3. highest degree ______

4. No. of courses beyond highest degree ________

5. Including this past year, for how many years have you taught secondary or college mathematics? __________

6. Including this past year, for how many years have you taught first year algebra? __________

7. Have you ever had a mathematics course in which the following topics were studied? (Answer each choice with "Yes" or "No.")

   a) probability
   b) mean, mode, median
   c) mean absolute deviation
   d) standard deviation
   e) chi square
   f) sampling
   g) mathematical applications

(MORE)

8. At what hour of the day does each of the two classes in the study meet? __________ and __________.

9. How long is each period per day in minutes? ______________

10. At what grade level are the students in the study (8th, 9th, 10th, mixed)?
(For each mixed class, specify the number at each grade level.)

11. (Control Teachers only) List text and other materials you plan to use with these classes and describe briefly your teaching method or methods. For the latter indicate the proportion of class time usually devoted to the various methods.

Please complete and return immediately in the enclosed stamped envelope to Dr. Jane O. Swafford, Associate Professor, Department of Mathematics, Northern Michigan University, Marquette, Michigan 49855.

APPENDIX B

TESTS

B1 First Year Algebra Test

B2 Item Content Classification for ETS Test and First Year Algebra Test

B3 Item by Objective Classification for First Year Algebra Test

B4 Fall Consumer Test

B5 Spring Consumer Test: Form A

B6 Spring Consumer Test: Form B

B7 Consensus Objectives in Consumer Applications of Mathematics

B8 Item by Objective Classification for Consumer Test

B9 Fall Opinion Survey

B10 Spring Opinion Survey

B11 Item by Attitude Dimension Classification for Opinion Survey

B12 Algebra I Questionnaire

APPENDIX B1

FIRST YEAR ALGEBRA TEST

Directions: Do not begin until you are told to do so.

Each question has only one correct response. Cross out the corresponding letter on your answer sheet. Do not write on this test.

1. $x - 7x =$

   A $-7x$
   B $-6x$
   C -7
   D -6
   E None of the above.

2. What quantity is one-half of y ?

   A $\frac{y}{\frac{1}{2}}$
   B $\frac{1}{2} - y$
   C $\frac{1}{2} + y$
   D $\frac{1}{2y}$
   E None of the above.

3. If x is randomly selected from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the probability that x is greater than 5 is

   A .20
   B .30
   C $.33\overline{3}$
   D .40
   E .50

4. A restaurant offers 3 salads, 5 main dishes and 4 desserts on its menu. How many different meals can be ordered which have a salad, a main dish, and a dessert ?

   A 3
   B 12
   C 15
   D 20
   E 60

5. When $4x - 7 = 9x + 17$, then $x =$

   A $\frac{-24}{5}$
   B $\frac{-24}{13}$
   C $\frac{10}{13}$
   D 2
   E None of the above.

6. The average weight of freshman girl[s] is closest to

   A 50 kg
   B 20 kg
   C 500 kg
   D 200 kg
   E 100 kg

7. If $x_1 = 4$ and $x_2 = -5$, then $2x_1 - 3$[...]

   A 7
   B 5
   C 4
   D -7
   E None of the above.

8. What is the relative frequency of a[n] event which occurs n times in 1000 tries ?

   A $1000 + n$
   B $1000 - n$
   C $\frac{n}{1000}$
   D $\frac{1000}{n}$
   E $1000n$

PLEASE TURN OVER

9. $24 + 3 \cdot 2 - 6 \div 3 =$

A 52
B 28
C 16
D 8
E None of the above.

10. $\sqrt{50} \approx$

A $5\sqrt{2}$
B $2\sqrt{5}$
C $2\sqrt{25}$
D $25\sqrt{2}$
E 25

11. Suppose two normal fair dice were tossed. What is the probability that both dice show a 5 ?

A 1/5
B 1/6
C 1/10
D 1/25
E 1/36

12. The change in a stock is $1\frac{3}{8}$ one day and $-2\frac{1}{4}$ the next day. In the two days, the net change is

A $-1\frac{1}{8}$
B $-\frac{7}{8}$
C $1\frac{1}{8}$
D $\frac{7}{8}$
E None of the above.

13. If you have 1000 dollars and spend b dollars a week, then how many dollars do you have left after t weeks ?

A 1000b - t
B t(1000 - b)
C 1000 - tb
D (1000 - t)b
E bt - 1000

14. The solutions to $3x^2 + 9x + 5 = 0$ are

A $\frac{9 \pm \sqrt{141}}{6}$
B $\frac{-9 \pm \sqrt{141}}{6}$
C $\frac{-9 \pm \sqrt{21}}{6}$
D $\frac{9 \pm \sqrt{21}}{6}$
E None of the above

15. A coat regularly sold for C dollars is now advertised on sale at 30% off. The sale price of the coat is

A C - .30
B C - .30C
C C + .30C
D .30C
E None of the above.

16. The expression $50x^3 + 50x^2 + 50$ simplifies to

A $150x^6$
B $250x^5$
C $50x^5$
D $125000x^5$
E None of the above.

17. The line with y-intercept 2 and slope 3 has equation

A $x = 2y + 3$
B $x = 3y + 2$
C $y = 2x + 3$
D $y = 3x + 2$
E None of the above.

18. When $x = 2$, then $|x - 5| =$

A 3
B -3
C 7
D -7
E 9

PLEASE GO TO THE NEXT PAGE

19. Assume your team is expected to beat another team 2/3 of the time. If the teams play 3 times during the season, what is the probability your team will win all 3 games ?

A 2
B 2/3
C 3/8
D 2/9
E 8/27

20. If $-3x > 13$, then

A $x > -\frac{3}{13}$

B $x < -\frac{3}{13}$

C $x > -\frac{13}{3}$

D $x < -\frac{13}{3}$

E $x > 16$

21. If you invest \$100 at 6% yearly interest for 5 years, then how many dollars will you have at the end of that time ?

A $100(1.30)$

B $100(.06)^5$

C $(106)^5$

D $100(1.06)^5$

E $100 + 5(.06)(100)$

22. For what values of x is the statement $(5 - x)(x + 3) = 0$ true ?

A $x = 0$
B $x = 3$ or $x = -5$
C $x = -3$ or $x = 5$
D $x = -3$ or $x = -5$
E None of the above.

23. Two towns use water at the same rate per person. Town S, with 430 people uses x gallons per day. Town T, with 690 people, uses y gallons per day. Which is true ?

A $\frac{x}{690} = \frac{430}{y}$

B $\frac{x}{430} = \frac{y}{690}$

C $\frac{x}{690} = \frac{y}{430}$

D $x + y = 1120$

E $430x + 690y = 1120$

24. $(3r - 4s)^2 =$

A $3r^2 - 4s^2$

B $9r^2 - 16s^2$

C $9r^2 - 12rs + 16s^2$

D $9r^2 - 24rs + 16s^2$

E None of the above.

25. When $x < y$ and z is negative, which of the following is true ?

A $x + z > y$

B $x + z > y + z$

C $x - z > y - z$

D $xz > yz$

E $x + \frac{1}{z} > y + \frac{1}{z}$

26. Suppose b boys receive x dollars apiece and g girls receive x dollars apiece. Together the boys and girls have received

A bxgx dollars

B bgx dollars

C x^{bg} dollars

D x^{b+g} dollars

E bx + gx dollars

PLEASE TURN OVER

27. Ms. Smith drove n miles to town. Then she drove 2.8 miles on business in town. Then she drove home. Her total mileage was 11.3 miles. Which sentence best describes the situation ?

A $n + 2.8 = 11.3$
B $2.8 + 11.3 = 2n$
C $2.8 + 2n = 11.3$
D $2.8n = 11.3$
E None of the above.

28. Order the numbers $\frac{5}{7}$, $.\overline{57}$ and $\frac{57}{100}$.

A $\frac{5}{7} < .\overline{57} < \frac{57}{100}$

B $\frac{57}{100} < .\overline{57} < \frac{5}{7}$

C $\frac{57}{100} < \frac{5}{7} < .\overline{57}$

D $.\overline{57} < \frac{57}{100} < \frac{5}{7}$

E None of the above.

29. In 1960 the U. S. population was about 170 million; in 1970 the population was 203 million. When you calculate the change in population per year you are calculating

A a slope
B a y-intercept
C a factor
D a relative frequency
E a probability

30. $\frac{2x + 8}{4} =$

A $\frac{x}{2} + 2$

B $\frac{x}{2} + 4$

C $\frac{x}{2} + 8$

D $2x + 4$

E $2x + 2$

31. 1000 centimeters is the same as

A $\frac{1}{10}$ meter

B 1 meter
C 10 meters
D 100 meters
E None of the above.

32. A car rents for \$15 plus 18 cents per mile. If the cost in dollars is c and the miles driven is m, then

A $m = 15c + .18$
B $m = .18c + 15$
C $c = 15m + .18$
D $c = .18m + 15$
E None of the above.

33. $\frac{x^2y^3}{x^4y^{-2}} =$

A $\frac{1}{x^2y}$

B x^2y

C $\frac{y}{x^2}$

D $\frac{y^5}{x^2}$

E $\frac{y}{x^6}$

LOOK OVER YOUR WORK ON THIS TEST

ITEM CONTENT CLASSIFICATION FOR ETS TEST AND FIRST YEAR ALGEBRA TEST (FYAT)

Content	Item ETS	Item FYAT
Terminology	7	
Combining Terms	3, 5, 10	1, 9, 16
Translation from verbal to algebraic expressions	2, 6, 19, 30	2, 12, 13, 15, 21, 23, 26, 27, 32
Solution of linear equations	1, 13, 23, 38	5
Substitution in algebraic expressions and equations	4, 8, 20, 26	7
Solution of literal equations	16, 39	
Exponents and roots	9, 14, 22	10
Algebraic multiplication and division	15, 21, 25, 28	24, 30, 33
Averages	17	
Systems of linear equations	11, 12, 31	
Graphs of linear functions	32	17, 29
Order and linear inequalities	29, 33, 35, 40	20, 25, 28
Factoring and quadratic equations	18, 24, 34, 37	14, 22
Division by zero	24	
Variation	36	
Relative frequency and probability		3, 8, 11, 19
Metric System		6, 3
Models for operations		4
Absolute value		18

ITEM BY OBJECTIVE CLASSIFICATION FOR FIRST YEAR ALGEBRA TEST

Item	Objective: The student should be able to ________.
1	add or subtract like terms.
2, 4	recognize when to use multiplication in a situation.
3, 11	calculate the probability of an event when given an experiment with easily countable random outcomes.
5	solve linear equations of the form $ax + c = bx + d$.
6	approximate conversion from English to metric system.
7	substitute into and evaluate expressions with subscripted variables.
8	calculate relative frequency from given data.
9	evaluate an expression with knowledge of the order of operations.
10	use the property $\sqrt{xy} = \sqrt{x}\sqrt{y}$ in simplifying expressions.
11, 19	calculate the probability of independent events all occurring.
12	distinguish direction and assign positive or negative numbers when given a situation with two directions.
13	translate situations involving constant gain or loss in mathematical expressions.
14	solve $ax^2 + bx + c = 0$ using quadratic formula.
15	give symbolic answers to percentage increases or decreases.
16	add expressions involving sums of powers of the same variable.
17	determine slope, y-intercept, or a linear equation given sufficient information.
18	evaluate an expression involving absolute value.
20, 25	solve linear inequalities.
21	calculate or give expression for amount accrued with annual compound interest.

Item	Objective: The student should be able to
22	apply the Zero Product Theorem.
23	recognize when to use division in a situation.
24	multiply two binomials.
26, 27, 32	translate situations involving sums of products into mathematical expressions.
28	compare two numbers in decimal or rational form.
29	recognize rate of change in a real situation as the same as "slope".
30	apply the distributive property to simplify expressions.
31	make conversions within the metric system.
33	calculate powers, including power of product or quotients.

CONSUMER TEST

Directions: Do not begin until you are told to do so.

Each question has only one correct answer. Cross out the corresponding letter on your answer sheet.

1. Suppose it costs 20 cents per mile to operate a car. Then how much would it cost to operate that car on a 1200-mile vacation?

   A. $600
   B. $60
   C. $2400
   D. $240
   E. None of the above.

2. In a certain high school, 13 students from the sophomore class of 200 have parents who are farmers. If the high school has 700 students in all, about how many students in the entire school would you expect to have parents who are farmers?

   A. 91
   B. 45
   C. 39
   D. 26
   E. 20

3. Mrs. Johnson buys groceries which cost $23.50. The sales tax in her state is 4%. What is the total price she must pay?

   A. $58.75
   B. $32.90
   C. $27.50
   D. $23.54
   E. None of the above.

4. According to the graph at right, what percentage of U.S. families have an income of $10,000 or more?

   A. 52.4%
   B. 47.6%
   C. 43.6%
   D. 21.2%
   E. None of the above.

5. In a state lottery, there is a 1 in 10 chance that you will match each digit named. What is the chance that you will match two digits in a row?

   A. 1 in 5
   B. 1 in 10
   C. 1 in 20
   D. 1 in 50
   E. 1 in 100

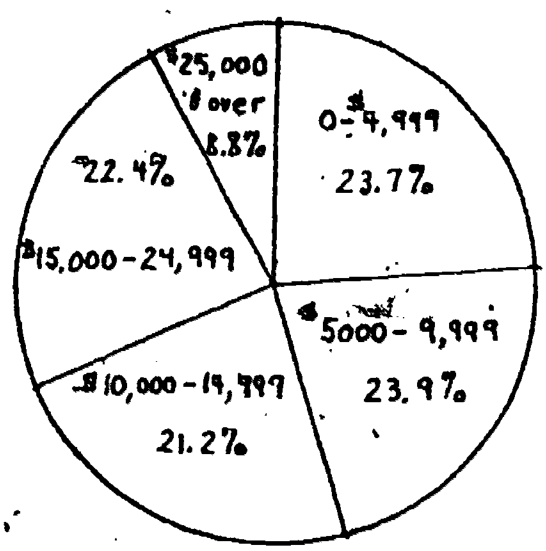
6. A basketball player scores 15, 12 and 24 points in the first three games of the season. What is his scoring average?

   A. 13 2/3 points
   B. 15 points
   C. 17 points
   D. 18 points
   E. None of the above.

7. If you travel 18 kilometers in 12 minutes, what is your average speed?

   A. 2/3 kilometers per minute
   B. 1 1/2 kilometers per minute
   C. 40 kilometers per minute
   D. 90 kilometers per minute
   E. None of the above.

Distribution of U.S. Families By
By Income Class - 1974



8. Which package of breakfast food has the lowest prices per ounce?

A. 16 ounces for 98 cents.
B. 10 ounces for 59 cents.
C. 6 ounces for 45 cents.
D. Exactly two of these are the same.
E. All three are the same.

9. The picture of an insect in a guidebook is one-fourth actual size. The wingspan in the picture is one-half inch. What is the actual wingspan of this insect?

A. one-eighth inch
B. one-fourth inch
C. three-fourths inch
D. 4 inches
E. None of the above.

10. A clerk starts work at 8:45 am. The clerk does not take lunch time and goes home at 3:30 pm. How long does the clerk work?

A. 12 hours and 15 minutes
B. 7 hours and 45 minutes
C. 6 hours and 45 minutes.
D. 5 hours and 15 minutes
E. None of the above.

11. Ruth plans to buy tires for her car during the sale listed below. The car needs four FR78-14 tires. What will be the total cost, including federal tax?

Tire size	Save	Reg.	Sale	+fed. tax
AR78-13	14.00	42.00	28.00	2.02
BR70-13	15.67	47.00	31.33	2.32
BR70-14	17.67	53.00	35.33	2.80
FR78-14	18.67	56.00	37.33	3.01
GR70-14	20.33	61.00	40.67	3.18
GR70-15	22.00	66.00	44.00	3.17
HR70-15	23.00	69.00	46.00	3.36

A. $40.34
B. $152.33
C. $161.36
D. $236.04
E. None of the above.

12. Ms. Hart earns $14,500 annually as an office manager. What is her monthly salary?

A. $120.83
B. $278.85
C. $483.33
D. $1450.00
E. None of the above.

13. AMTRAK TIMETABLE

Read Down

Time	Miles	Location
5 10p	0	Chicago, IL
7 50p	129	Champaign, IL
10 50p	310	Carbondale, IL
3 20a	529	Memphis, TN
7 17a	740	Jackson, MS
11 59a	923	New Orleans, LA

How long is the train trip from Champaign to Memphis according to the schedule?

A. 4 hours and 30 minutes
B. 7 hours and 30 minutes
C. 8 hours and 30 minutes
D. 11 hours and 10 minutes
E. None of the above.

14. What is the sale price of an $80 coat that is marked 40% OFF?

A. $76.80
B. $48.00
C. $40.00
D. $32.00
E. None of the above.

15. In the U.S., the probability that a girl will be born in a single birth is about .52. What is the probability that a boy will be born?

A. about .48
B. about .50
C. about .52
D. not enough information is given
E. None of the above.

16. Suppose a phone bill shows that a 20-minute call to Town X costs $9.15 and a 26-minute call to Town X costs $11.85. Both calls were made at the same time. In phone calls, after the first minute, each additional minute costs the same. What would a 30-minute call cost?

    A. $13.72
    B. $13.67
    C. $13.65
    D. $13.62
    E. None of the above.

17. The student council wants to paint the student lounge. Each of the four walls measures 48 feet by 10 feet. How many gallons of paint will be needed if one gallon covers 300 square feet of surface?

    A. 4 gallons
    B. 5 gallons
    C. 6 gallons
    D. 7 gallons
    E. 8 gallons

18. A refrigerator sells for $300 cash. It can also be purchased for $100 down payment and $10 a month for two years. Which statement is true?

    A. The two-year plan costs $40 more.
    B. The two-year plan costs $20 more.
    C. The cash sale costs $60 more.
    D. The price is the same either way.
    E. None of the above.

19. City Parking Lot Rates

    75¢ First Hour
    50¢ Each Additional Hour

    What does it cost to park for 8 hours?

    A. $4
    B. $4.25
    C. $4.75
    D. $6.00
    E. None of the above.

20. Marion earns $2.30 an hour as a playground supervisor. How much will Marion earn for working 24 hours this week?

    A. $55.20
    B. $54.00
    C. $48.30
    D. $10.44
    E. None of the above.

21. The dimensions of a 10 gallon rectangular fish tank are 14 by 20 by 8 inches. Which of the tanks, with dimensions given below, will hold 20 gallons of water when full?

    A. 28 by 40 by 16 inches.
    B. 28 by 20 by 16 inches.
    C. 28 by 20 by 8 inches.
    D. 14 by 10 by 16 inches.
    E. None of the above.

22. If you have $100 in a bank which gives 5% interest each year and you keep the interest in the account, how much will you have after two years?

    A. $100.00
    B. $101.00
    C. $110.00
    D. $110.25
    E. None of the above.

23. Tahitian Punch (40 servings):

    2 quarts carbonated lemon-lime beverage
    3 1/2 quarts pineapple-grapefruit juice
    1 pint lemon or lime sherbet

    How much lemon-lime beverage is needed to make 30 servings?

    A. 2 quarts
    B. 1 3/4 quarts
    C. 1 1/2 quarts
    D. 1 1/4 quarts
    E. None of the above.

24. Television sets are on sale at two stores.

Store 1	Store 2
10% Discount	15% Discount

How much MORE can you save at Store 2 on a set that is regularly priced at $400?

A. $20
B. $15
C. $10
D. $5
E. None of the above.

25. You toss a perfectly balanced coin nine times. All nine tosses are tails. The next toss

A. will definitely be tails.
B. will definitely be heads.
C. will most likely be tails.
D. will most likely be heads.
E. is equally likely to be tails or heads.

26. Partners Anderson and Briggs agree to share their business profits in the ratio of 2 to 3. What is Anderson's income on a $30,000 profit?

A. $12,000
B. $15,000
C. $18,000
D. $20,000
E. None of the above.

27. A door-to-door salesperson makes a 20% commission on everything sold. How much must the person sell to earn a commission of $50?

A. $50
B. $250
C. $400
D. $1000
E. None of the above.

28. A 12 foot by 15 foot living room is to be carpeted. How many square yards of carpet must be bought for the living room?

A. 180 square yards
B. 90 square yards
C. 60 square yards
D. 20 square yards
E. None of the above.

CONSUMER TEST FORM A

Directions: Do not begin until you are told to do so.

Each question has only one correct answer. Darken the corresponding oval on your answer card. Do not write on test.

1. Marion earns $2.30 an hour as a playground supervisor. How much will Marion earn for working 24 hours this week?

A $55.20
B $54.00
C $48.30
D $10.44
E None of the above.

2. In the U.S., the probability that a boy will be born in a single birth is about .52. What is the probability that a girl will be born?

A about .48
B about .50
C about .52
D not enough information is given
E None of the above.

3. City Parking Lot Rates

75¢ First Hour
50¢ Each Additional Hour

What does it cost to park for 8 hours?

A $4.00
B $4.25
C $4.75
D $6.00
E None of the above

4. A door-to-door salesperson makes a 20% commission on everything sold. How much must the person sell to earn a commission of $50?

A $50
B $250
C $400
D $1000
E None of the above.

5. In a lottery, there is a 1 in 10 chance that you will match each digit named. What is the chance that you will match two digits in a row?

A 1 in 5
B 1 in 10
C 1 in 20
D 1 in 50
E 1 in 100

6. If you have $100 in a bank which gives 5% interest each year and you keep the interest in the account, how much will you have after two years?

A $100.00
B $101.00
C $110.00
D $110.25
E None of the above.

7. A clerk starts work at 8:45 a m. The clerk does not take lunch time and goes home at 3:30 p.m. How long does the clerk work?

A 12 hours and 15 minutes
B 7 hours and 45 minutes
C 6 hours and 45 minutes
D 5 hours and 15 minutes
E None of the above.

8. In a certain high school, 13 students from the sophomore class of 200 have parents who are farmers. If the high school has 700 students in all, about how many students in the entire school would you expect to have parents who are farmers?

A 91
B 45
C 39
D 26
E 20

PLEASE TURN OVER

9. Suppose it costs 20 cents per mile to operate a car. Then how much would it cost to operate that car on a 1200-mile vacation?

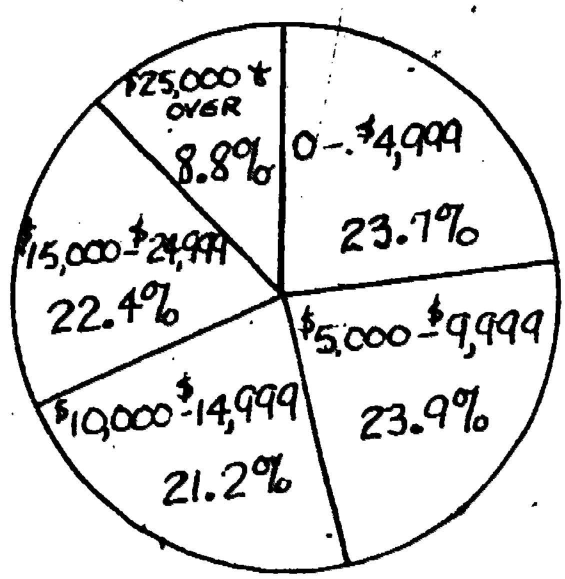
A $600
B $60
C $2400
D $240
E None of the above.

10. A 12 foot by 15 foot living room is to be carpeted. How many square yards of carpet must be bought for the living room?

A 180 square yards
B 90 square yards
C 60 square yards
D 20 square yards
E None of the above.

11. According to the graph at right, what percentage of U.S. families have an income of $10,000 or more?

A 52.4%
B 47.6%
C 43.6%
D 21.2%
E None of the above.



END

APPENDIX B6

CONSUMER TEST · FORM B

Directions: Do not begin until you are told to do so.

Each question has only one correct answer. Darken the corresponding oval on your answer card. Do not write on test.

1. You toss a perfectly balanced coin nine times. All nine tosses are tails. The next toss

   A will definitely be tails.
   B will definitely be heads.
   C will most likely be tails.
   D will most likely be heads.
   E is equally likely to be tails or heads.

2. What is the sale price of an $80 coat that is marked 40% OFF?

   A $76.80
   B $48.00
   C $40.00
   D $32.00
   E None of the above.

3. Partners Anderson and Briggs agree to share their business profits in the ratio of 2 to 3. What is Anderson's share of a $30,000 profit?

   A $12,000
   B $15,000
   C $18,000
   D $20,000
   E None of the above.

4. TAHITIAN PUNCH (40 servings):

2 quarts	carbonated lemon-lime beverage
3½ quarts	pineapple-grapefruit juice
1 pint	lemon or lime sherbet

   How much lemon-lime beverage is needed to make 30 servings?

   A 2 quarts
   B 1 3/4 quarts
   C 1 1/2 quarts
   D 1 1/4 quarts
   E None of the above.

5. Ms. Hart earns $14,500 annually as an office manager. What is her monthly salary?

   A $120.83
   B $278.85
   C $483.33
   D $1450.00
   E None of the above.

6. Which package of breakfast food has the lowest price per ounce?

   A 16 ounces for 98 cents.
   B 10 ounces for 59 cents.
   C 6 ounces for 45 cents.
   D Exactly two of these are the same.
   E All three are the same.

7. The dimensions of a 10 gallon rectangular fish tank are 14 by 20 by 8 inches. Which of the tanks, with dimensions given below, will hold 20 gallons of water when full?

   A 28 by 40 by 16 inches.
   B 28 by 20 by 16 inches.
   C 28 by 20 by 8 inches.
   D 14 by 10 by 16 inches.
   E None of the above.

8. Mrs. Johnson buys groceries which cost $23.50. The sales tax in her state is 4%. What is the total price she must pay?

   A $58.75
   B $32.90
   C $27.50
   D $23.54
   E None of the above.

PLEASE TURN OVER

9. A refrigerator sells for $300 cash. It can also be purchased for $100 down payment and $10 a month for two years. Which statement is true?

   A The two-year plan costs $40 more.
   B The two-year plan costs $20 more.
   C The cash sale costs $60 more.
   D The price is the same either way.
   E None of the above.

10. If you travel 18 kilometers in 12 minutes, what is your average speed?

   A 2/3 kilometers per minute
   B $1\frac{1}{2}$ kilometers per minute
   C 40 kilometers per minute
   D 90 kilometers per minute
   E None of the above.

END

APPENDIX B7

Consensus Objectives in Consumer Application of Mathematics

The student should be able to:

1. interpret circle, bar or polygonal graphs.
2. interpret data in chart or table form.
3. calculate elapsed time given beginning and ending times (in hours and minutes).
4. compare two rates, one given as a fraction and one as a percent.
5. determine the better buy through calculating and comparing unit prices for two products of the same quality, but of different size and price.
6. calculate the cost of pricing a specific distance given the cost per mile.
7. calculate a paycheck, before deductions, given information on rates and time worked.
8. calculate percent problems given specific information such as original price, sale price percent of increase or reduction.
9. calculate the sales tax and total cost for an item or series of items.
10. calculate difference in buying an item for cash or by a time payment plan.
11. determine new quantities when a given mixture is to be made in a different amount.
12. determine how many would do or say something out of z people given that x out of y people (or x% of a group) does or says that.
13. given a timetable, determine the time of departure, time of arrival or traveling time.
14. interpret scale diagrams or maps.
15. calculate the quantity of a substance needed to paint or carpet a surface.
16. calculate the mean as an average.
17. determine linear extrapolation, e.g., with population, inflation, etc.
18. determine variation of area and volume with variations in linear dimensions (e.g., how many square feet in a square yard).
19. calculate simple probability.

20. determine probability of occurrence of two or more consecutive independent events.

21. calculate compound interest.

22. calculate postage, phone, or car rental rates of the form "something + so much per _____."

23. the notion of variability in random-type situations and non-random type situations (e.g., toss a coin 10 times and it will not always give 5 heads).

24. determine rate of speed given distance and time.

25. calculate partner income based on ratio and total income.

APPENDIX B8

ITEM BY OBJECT CLASSIFICATION FOR CONSUMER TEST (FALL, 1976)

Item	Objective	Spring Item Number
1	Objective 6	9A
2	Objective 12	8A
3	Objective 9	8B
4	Objective 1	11A
5	Objective 21	5A
6	Objective 16	
7	Objective 26	10B
8	Objective 5	6B
9	Objective 14	
10	Objective 3	7A
11	Objective 2	
12	Objective 7	5B
13	Objective 13	
14	Objective 8	2B
15	Objective 20	2A
16	Objective 18	
17	Objective 15	
18	Objective 10	9B
19	Objective 24	3A
20	Objective 7	1A
21	Objective 19	7B
22	Objective 23	6A
23	Objective 11	4B
24	Objective 4	
25	Objective 25	1B
26	Objective 27	3B
27	Objective 8	4A
28	Objective 13	10A

Fall, 1976

OPINION SURVEY

Name ______________________ Sex(circle) M F

Age ________ Year in school(circle) 8 9 10 11 12

School ______________________

Teacher ______________________ Period ______

Date ______________________

DIRECTIONS: Each of the statements on this opinion survey expresses a feeling or belief which a person might have towards mathematics. You are to express how much you agree with the belief or feeling given in each statement. The five choices are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly agree (SA). Circle the letter which best indicates how closely you agree or disagree with the feeling or belief expressed in each statement as it concerns you. Answer the way you feel. There are no right or wrong answers.

1.	Mathematics is an interesting subject.	SD	D	U	A	SA
2.	Mathematics is not important in everyday life.	SD	D	U	A	SA
3.	I do not like mathematics.	SD	D	U	A	SA
4.	Mathematics makes me feel stupid.	SD	D	U	A	SA
5.	There is nothing creative about mathematics; it's just memorizing formulas and things.	SD	D	U	A	SA
6.	Most mathematics is too concerned with ideas to be really useful.	SD	D	U	A	SA
7.	Mathematics is something I enjoy a great deal.	SD	D	U	A	SA
8.	Guessing plays a role in doing mathematics.	SD	D	U	A	SA
9.	One value of mathematics is its usefulness in solving everyday problems.	SD	D	U	A	SA

10.	I think knowing some algebra will help me get a good job later.	SD	D	U	A	SA
11.	Working math problems can be fun.	SD	D	U	A	SA
12.	Mathematics is needed in order to keep the world running.	SD	D	U	A	SA
13.	It is boring to work on math puzzles.	SD	D	U	A	SA
14.	Mathematics plays an important role in modern society.	SD	D	U	A	SA
15.	Algebra is only important for science or advanced mathematics.	SD	D	U	A	SA
16.	Learning mathematics is more understanding than memorizing.	SD	D	U	A	SA
17.	Mathematics is easy for me.	SD	D	U	A	SA
18.	There are lots of uses for algebra in the real world.	SD	D	U	A	SA
19.	Mathematics is a dull and boring subject.	SD	D	U	A	SA
20.	Outside of science and engineering there is little need for mathematics in jobs.	SD	D	U	A	SA
21.	A knowledge of mathematics is helpful in understanding today's world.	SD	D	U	A	SA
22.	There is no place for originality in mathematics.	SD	D	U	A	SA
23.	Mathematics is not very useful for solving world problems.	SD	D	U	A	SA
24.	Mathematics is more for boys than for girls.	SD	D	U	A	SA
25.	I plan to take another mathematics course after this one.	Yes	No	Not sure		

APPENDIX B10
OPINION SURVEY

Directions: Each of the statements on this opinion survey expresses a feeling or belief which a person might have toward mathematics. You are to indicate how much you agree with the belief or feeling given in each statement by marking one of the choices on the answer card. The five choices are: SA-Strongly agree, A-Agree, U-Undecided, D-Disagree, SD-Strongly disagree. Darken the oval on the answer card which best shows how much you agree or disagree with the statement. Answer the way you feel. There are no right or wrong answers.

1.	Algebra is an interesting subject.	(a) SA	(b) A	(c) U	(d) D	(e) SD
2.	Algebra is not important in everyday life.	(a) SA	(b) A	(c) U	(d) D	(e) SD
3.	I do not like mathematics.	(a) SA	(b) A	(c) U	(d) D	(e) SD
4.	Algebra is confusing to me.	(a) SA	(b) A	(c) U	(d) D	(e) SD
5.	Explanations in my algebra book helped me to understand algebra.	(a) SA	(b) A	(c) U	(d) D	(e) SD
6.	Algebra is too concerned with symbols to be really useful.	(a) SA	(b) A	(c) U	(d) D	(e) SD
7.	Algebra is easy for me.	(a) SA	(b) A	(c) U	(d) D	(e) SD
8.	One value of mathematics is its usefulness in solving everyday problems.	(a) SA	(b) A	(c) U	(d) D	(e) SD
9.	I think knowing some algebra will help me get a good job later.	(a) SA	(b) A	(c) U	(d) D	(e) SD
10.	I enjoy working word problems.	(a) SA	(b) A	(c) U	(d) D	(e) SD
11.	Algebra is needed in order to keep the world running.	(a) SA	(b) A	(c) U	(d) D	(e) SD
12.	Explanations in my algebra book were of no help in doing the problems.	(a) SA	(b) A	(c) U	(d) D	(e) SD
13.	Algebra is only important for science or advanced mathematics.	(a) SA	(b) A	(c) U	(d) D	(e) SD
14.	Learning mathematics is more understanding than memorizing.	(a) SA	(b) A	(c) U	(d) D	(e) SD
15.	There are lots of uses for algebra in the real world.	(a) SA	(b) A	(c) U	(d) D	(e) SD

PLEASE TURN OVER

16. Algebra is a dull and boring subject. (a) SA (b) A (c) U (d) D (e) SD

17. Mathematics is not very useful for solving world problems. (a) SA (b) A (c) U (d) D (e) SD

18. Mathematics is more for boys than for girls. (a) SA (b) A (c) U (d) D (e) SD

19. A knowledge of algebra is helpful in understanding today's world. (a) SA (b) A (c) U (d) D (e) SD

20. The math book we used this year was

(a) more interesting than most math books.
(b) less interesting than most math books.
(c) neither more nor less interesting than most math books.

21. Algebra is

(a) harder than arithmetic.
(b) easier than arithmetic.
(c) neither easier nor harder than arithmetic.

22. Which best describes you?

(a) I enjoy arithmetic but <u>not</u> algebra.
(b) I enjoy algebra but <u>not</u> arithmetic.
(c) I enjoy <u>both</u> arithmetic and algebra.
(d) I enjoy <u>neither</u> arithmetic nor algebra.

23. I read the explanations in my math book

(a) almost always.
(b) most of the time.
(c) about ½ the time.
(d) some of the time.
(e) almost never.

24. The math book we used this year was

(a) very difficult to read and understand.
(b) moderately difficult to read and understand.
(c) neither easy nor difficult to read and understand.
(d) moderately easy to read and understand.
(e) very easy to read and understand.

25. I plan to take another mathematics course after this one.

(a) yes
(b) no
(c) not sure

END

APPENDIX B11

ITEM BY ATTITUDE DIMENSION CLASSIFICATION FOR FALL (F) AND SPRING (S) OPINION SURVEY

Attitude dimension	Item number
Enjoyment of mathematics	1F ,3F,7F,11F, ,13F,19F ,25F 1S*,3S, , ,10S, ,16S*,25S,22S
Value of mathematics	2F ,9F,10F,12F ,14F,15F,18F,20F,21F ,23F 2S*,8S, 9S,11S*, ,13S,15S, ,19S*,17S
Nature of mathematics	5F,6F ,8F,16F,22F,24F 6S*, ,14S, ,18S,21S
Self-concept	4F, ,17F 4S, 7S*
Textbook	no fall items 5S,12S,20S,23S,24S

*Spring item modified to replace "mathematics" with "algebra"

APPENDIX B12

ALGEBRA I QUESTIONNAIRE

Directions: Each Algebra I topic below is followed by three questions. FIRST, how easy was the topic for you to learn? SECOND, how do you like to do these kinds of problems? THIRD, do you feel the topic will be useful for you to know after you leave high school?

For each topic, indicate your feelings by darkening one oval on your answer card for each number. If you did not study a topic or do not recognize it, mark option (d).

Solving linear equations like $y + 6 = 7 - 3y$

1.	(a) easy to learn	(b) hard to learn	(c) neither	(d) did not study
2.	(a) like to do	(b) dislike to do	(c) neutral	(d) did not study
3.	(a) useful after high school	(b) useless after high school	(c) don't know	(d) did not study

Solving word or application problems

4.	(a) easy	(b) hard	(c) neither	(d) did not study
5.	(a) like	(b) dislike	(c) neutral	(d) did not study
6.	(a) useful	(b) useless	(c) don't know	(d) did not study

Simplifying expressions like $3x^2 - 5x + 2(8 - x)$

7.	(a) easy	(b) hard	(c) neither	(d) did not study
8.	(a) like	(b) dislike	(c) neutral	(d) did not study
9.	(a) useful	(b) useless	(c) don't know	(d) did not study

Factoring expressions like $x^2 - 2x + 3$

10.	(a) easy	(b) hard	(c) neither	(d) did not study
11.	(a) like	(b) dislike	(c) neutral	(d) did not study
12.	(a) useful	(b) useless	(c) don't know	(d) did not study

Solving inequalities like $-2x + 4 \leq 10 + x$

13.	(a) easy	(b) hard	(c) neither	(d) did not study
14.	(a) like	(b) dislike	(c) neutral	(d) did not study
15.	(a) useful	(b) useless	(c) don't know	(d) did not study

Determining the slope, y-intercept or graph of a linear equation

16.	(a) easy	(b) hard	(c) neither	(d) did not study
17.	(a) like	(b) dislike	(c) neutral	(d) did not study
18.	(a) useful	(b) useless	(c) don't know	(d) did not study

PLEASE TURN OVER

Working with expressions involving powers or roots

19. (a) easy (b) hard (c) neither (d) did not study
20. (a) like (b) dislike (c) neutral (d) did not study
21. (a) useful (b) useless (c) don't know (d) did not study

Solving systems of linear equations like $3x + 2y = 13$, $2x - y = 4$

22. (a) easy (b) hard (c) neither (d) did not study
23. (a) like (b) dislike (c) neutral (d) did not study
24. (a) useful (b) useless (c) don't know (d) did not study

Working with functions like $f(x) = 5x + 3$. Find $f(2)$.

25. (a) easy (b) hard (c) neither (d) did not study
26. (a) like (b) dislike (c) neutral (d) did not study
27. (a) useful (b) useless (c) don't know (d) did not study

Using the Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

28. (a) easy (b) hard (c) neither (d) did not study
29. (a) like (b) dislike (c) neutral (d) did not study
30. (a) useful (b) useless (c) don't know (d) did not study

Working with positive and negative numbers

31. (a) easy (b) hard (c) neither (d) did not study
32. (a) like (b) dislike (c) neutral (d) did not study
33. (a) useful (b) useless (c) don't know (d) did not study

Calculating probabilities

34. (a) easy (b) hard (c) neither (d) did not study
35. (a) like (b) dislike (c) neutral (d) did not study
36. (a) useful (b) useless (c) don't know (d) did not study

Translating words into algebraic expressions like "8 more than twice a numb

37. (a) easy (b) hard (c) neither (d) did not study
38. (a) like (b) dislike (c) neutral (d) did not study
39. (a) useful (b) useless (c) don't know (d) did not study

APPENDIX C

TEACHER REPORT FORMS

C1 End-of-Chapter Reports

C2 Textbook Evaluation Form (Experimental)

C3 Textbook Evaluation Form (Control)

END-OF-CHAPTER REPORT

Chapter_____ Name______________________________

School______________________________

1. Number of (school) days including test days spent on chapter __________.

2. Which lessons went particularly well?

3. Which lessons did not go well? If possible identify the source of the difficulty and/or offer suggestions for improvement.

4. Please enclose a copy of your chapter test if one was given. If not, check below.

   ☐ No chapter test given.

Please return this form immediately upon completion of the chapter to Dr. Jane O. Swafford, Department of Mathematics, Northern Michigan University, Marquette, MI 49855.

THANK YOU

APPENDIX C2

TEXTBOOK EVALUATION FORM (Experimental)

Name: ______________________ School: ______________________

Please answer the following questions as they apply to ALGEBRA THROUGH APPLICATIONS by Usiskin.

1. In general, I feel that the book is most appropriate for
   (a) the above average 1st year algebra student
   (b) the average 1st year algebra student
   (c) the below average 1st year algebra student. ______

2. Compared to other 1st year algebra books I have taught from, this book is
   (a) easier to read and understand
   (b) harder to read and understand
   (c) at about the same level ______

3. Compared to other 1st year algebra books I have taught from, the exercises are
   (a) easier
   (b) more difficult
   (c) at about the same level ______

4. I
   (a) would strongly recommend
   (b) would recommend
   (c) am indifferent to
   (d) would not recommend
   (e) would strongly recommend against
   the use of this text for an average 1st year algebra class. ______

5. Before this year, were you dissatisfied with the 1st year algebra course as outlined in most commercial texts?
   (a) very much (b) only slightly (c) no ______

Questions 6-15 ask you to compare the development of certain topics in the experimental text with those you are familiar with in other texts by choosing a letter and a number which best describes your feelings.

Letter choices:
(a) The development is the nicest I've seen.
(b) The development is about as nice as others I know.
(c) I know a more effective development.
(d) I cannot compare with other developments.

Number choices:
(1) The development was rather easy for my students to understand.
(2) The development was about average difficulty for my students.
(3) The development was hard for my students to understand.

	Letter	Number
6. approach to variables (Ch. 2)	______	______
7. subscripts (Ch. 2)	______	______
8. properties (Ch. 3-5)	______	______
9. approach to beginning sentence solving (Ch. 6)	______	______
10. work with distributive property (Ch. 7-8)	______	______
11. approach to slope (Ch. 9)	______	______
12. graphing linear sentences (Ch. 9)	______	______
13. negative exponents (Ch. 10)	______	______
14. square roots (Ch. 12)	______	______
15. systems (Ch. 14)	______	______

16. Check all that apply

Models	Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
Addition:						
union						
joining						
slide						
Subtraction:						
take away						
cutting off						
directed distance						
Multiplication:						
ordered pair						
area						
scale change						
repeated add.						
Division:						
splitting up						
rate						
scale compar.						
Powering:						
repeated mult.						
growth						

17-20 Check all that apply

	Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
17. assemblage property						
18. metric system						
19. calculating statistics						
20. calculating probabilities						

21-28. Each of the following topics is not in the text, or is not in it to the extent that most other texts have it. Choose the most appropriate response.

(a) I did not mind not having to teach this topic.
(b) I would have liked to have taught this topic, but did not.
(c) I taught this topic even though it was not in the book.
(d) I did not teach this topic, but would next year if this book were used.

21. factoring expressions like $3x^2 - 10x + 7$ ______

22. adding fractional expressions requiring getting a least common denominator ______

23. multiplying or dividing fractional expressions where factoring of trinomials is required ______

24. formal logic ______

25. age problems ______

26. digit problems ______

27. coin problems ______

28. distance-rate-time problems ______

29. Check all that apply. What do you think about the nature of the applications in this book?

_____ (a) interesting to most students
_____ (b) interesting to only a few students
_____ (c) interesting to none of the students
_____ (d) arithmetic too difficult in most
_____ (e) situations too involved (complex) for most students
_____ (f) too easy for most students
_____ (g) some are socially too controversial
_____ (h) they promote valuable discussion
_____ (i) they promote wasteful discussion
_____ (j) too many
_____ (k) the traditional word problems are better

30. Here are some applications. Check all that apply.

	Interesting	Not Int.	Easy	Hard	Keep	Drop	Didn't Do
mile run (p. 434)							
storms (p. 315)							
hamburger (p. 316)							
wildlife (p. 257)							
Manhattan (p. 409)							
newspaper (p. 27)							
scouts (p. 610)							

31. What topics or ideas were hardest for your students?

32. What is the furthest lesson you covered in the book: Ch. ____ Lesson ____

33. What lessons (or chapters) did you skip? ____________________

34. What did you find yourself skipping that you would like to have covered?

35. When you first began this course did you feel that you would have trouble with the mathematics or the applications?
(a) definitely (b) somewhat (c) not really ________

36. Did you have trouble with the mathematics or the applications in this course?
(a) definitely (b) somewhat (c) not really ________

37. What topics or applications in this course were hardest for you to understand? How hard was it to understand?

38. Has teaching this course changed your attitudes about 1st year algebra or the teaching of applications? ______ If so, how?

39. I read and used the "Notes to the Teacher"
(a) for each section (c) sometimes (e) never
(b) often (d) seldom ______

40. Answer all that apply. For tests, I would liked to have had
______ (a) complete chapter tests
______ (b) suggested test items from which a test could be made
______ (c) a mastery workbook for each student
______ (d) no tests or test items

41. My tests this year were
(a) identical to those I have given in the past in 1st year algebra
(b) very similar but with some modifications
(c) very different but with some similar problems
(d) completely different from those I have given in the past ______

42. Answer all that apply: With regard to the mastery workbook,
______ (a) I never used it.
______ (b) I used the problems for tests.
______ (c) I used the problems for review.
______ (d) I used the problems often.
______ (e) I used the workbook some way not mentioned above. (Explain be

43. I used the answers to exercises
(a) for each lesson (c) sometimes (e) never
(b) often (d) seldom ______

44. Should answers to exercises be included in the student text?
(a) no
(b) to odd exercises only
(c) to "Questions covering the reading" only
(d) to "Questions covering the reading" and other selected problems
(e) to all exercises
(f) to other (Explain) ______

45. Compared with other 1st year algebra books I have taught from, with this book I
(a) had to supplement more than usual
(b) had to supplement less than usual
(c) supplemented about the same as usual ______

46. What changes in the exercises would you recommend?

47. How often did you make reading assignments from the text?
(a) every lesson
(b) most lessons
(c) about half the lessons
(d) some of the lessons
(e) never ______

48. How many students did reading when you assigned it?
(a) almost all (c) about half (e) almost none
(b) most (d) some ______

49. How often do you feel a typical student was able to understand the lesson from the reading without your reading or explaining it?
(a) almost always (c) sometimes (e) never
(b) often (d) seldom ______

50. How often do you think students should be expected to read in a mathematics text?
(a) frequently
(b) only as a group
(c) only when class explanation is not enough
(d) never ______

51. Check all that apply. Did you allow the use of electronic or mechanical calculators?
______ (a) on all homework problems
______ (b) only on those homework problems marked "C"
______ (c) on a few designated homework problems
______ (d) on no homework problems
______ (e) Students were allowed to bring calculators to class and use them in class work.
______ (f) There was a school or teacher-owned calculator available in the classroom for student use.
______ (g) Calculators could not be used on any tests.
______ (h) Calculators were allowed on some tests.
______ (i) Calculators could be used on all tests.
______ (j) The use of calculators was never considered. ______

52. A calculator for each student
(a) is a necessity with this book
(b) helps but is not necessary
(c) makes no difference with this book ______

53. What percentage of your students have access to a calculator at home? ______ This is my guess. ______ I asked all my students. ______

54. If you had three suggestions to make to improve this book, what are they?

55. If you had a choice would you participate in a study similar to this in the future? __________

56. Please indicate any other comments you might have.

Thank you very much for taking the time to complete this very long form.

APPENDIX C3

TEXTBOOK EVALUATION FORM (Control)

Name: ______________________ School: ______________________

Please answer the following questions as they apply to the book that you are presently using with your 1st year algebra ~~classes~~.

Name of book: ______________________ Authors: ______________________

* 1. In general, I feel that the book is most appropriate for
(a) the above average 1st year algebra student
(b) the average 1st year algebra student
(c) the below average 1st year algebra student ______

* 2. Compared to other 1st year algebra books I have taught from, this book is
(a) easier to read and understand
(b) harder to read and understand
(c) at about the same level ______

*3. Compared to other 1st year algebra books I have taught from, the exercises are
(a) easier
(b) more difficult
(c) at about the same level ______

*4. I
(a) would strongly recommend
(b) would recommend
(c) am indifferent to
(d) would not recommend
(e) would strongly recommend against
the use of this text for an average 1st year algebra class. ______

*5. Are you dissatisfied with the 1st year algebra course as outlined in most commercial texts?
(a) very much (b) only slightly (c) no ______

*6-13. Each of the following topics are covered by most 1st year algebra texts. Choose the most appropriate response.
(a) I do not mind teaching this topic.
(b) I would have liked to have taught this topic, but did not.
(c) I did not teach this topic even though it was in the book.
(d) I did teach this topic, but would rather see it deleted from 1st year algebra.

6. factoring expressions like $3x^2 - 10x + 7$ ______
7. adding fractional expressions requiring getting a least common denominator ______
8. multiplying or dividing fractional expressions where factoring of trinomials is required ______
9. formal logic ______
10. age problems ______
11. digit problems ______
12. coin problems ______
13. distance-rate-time problems ______

* *Item comparable to item on Experimental Form*

14-20. Were any of the following mentioned or studied in your 1st year algebra class this year?

14. probability: Mentioned? ______ Studied? ______
15. statistics: Mentioned? ______ Studied? ______
16. metric system: Mentioned? ______ Studied? ______
17. functions: Mentioned? ______ Studied? ______
18. real world word problems: Mentioned? ______ Studied? ______
19. graphing of real data: Mentioned? ______ Studied? ______
20. quadratic formula: Mentioned? ______ Studied? ______

21. What material could be deleted from your text without disturbing you?

22. What material would you like to see added to your text?

*23. Which topics or ideas are generally hardest for your students to understand?

24. Which topics or ideas are generally easiest for your students to understand?

*25. What topics did you find yourself skipping that you would like to have covered?

*26. When you first taught from this book did you feel that you would have trouble with the mathematics? ______
(a) definitely (b) somewhat (c) not really

*27. How often do you use the Teacher's Commentary or Notes? ______
(a) for each section (b) often (c) sometimes (d) seldom (e) never

28. Are chapter tests available with this book? ______
If so, how often do you use them (Perhaps with Modifications)? ______
(a) for each chapter (b) often (c) sometimes (d) seldom (e) never

29. How often do you use the "Answers to Exercises" or Solution Key?
(a) for each assignment (c) sometimes (e) never
(b) often (d) seldom ______

*30. Do you feel that answers to exercises should be included as part of the student's text?
(a) to odd exercises only
(b) only to the easiest exercises
(c) only to the most difficult exercises
(d) to all exercises
(e) not to any exercises
(f) other (Explain) ______________________________ ______

31. When (if ever) have you found it necessary or convenient to use other sources than the book for assignments?
(a) for each assignment (c) sometimes (e) never
(b) often (d) seldom ______

32. How often did you make reading assignments from the text?
(a) every lesson
(b) most lessons
(c) about half the lessons
(d) some of the lessons
(e) never ______

33. How many students did reading when you assigned it?
(a) almost all (c) about half (e) almost none
(b) most (d) seldom ______

34. How often do you feel a typical student was able to understand the lesson from the reading without your reading or explaining it?
(a) almost always (c) sometimes (e) never
(b) often (d) seldom ______

*35. How often do you think students should be expected to read in a mathematics text?
(a) frequently
(b) only as a group
(c) only when class explanation is not enough
(d) never ______

36. Check all that apply. Did you allow the use of electronic or mechanical calculators?
______ (a) on all homework problems
______ (b) only on those homework problems marked "C"
______ (c) on a few designated homework problems
______ (d) on no homework problems
______ (e) Students were allowed to bring calculators to class and use them in class work.
______ (f) There was a school or teacher-owned calculator available in the classroom for student use.
______ (g) Calculators could not be used on any tests.
______ (h) Calculators were allowed on some tests.
______ (i) Calculators could be used on all tests.
______ (j) The use of calculators was never considered. ______

37. A calculator for each student
(a) is a necessity with this book
(b) helps but is not necessary
(c) makes no difference with this book

38. What percentage of your students have access to a calculator at home? ________ This is my guess. ________ I asked all my students. ________

39. If you had a choice would you participate in a study similar to this in the future? ________

40 Please indicate any other comments you might have.

Thank you very much for taking the time to complete this very long form.

APPENDIX D

SITE VISIT FORMS

D1 Instruction for Site Visits (Without Student Interviews)

D2 Instruction for Site Visits (With Student Interviews)

D3 Classroom Observation Form

D4 Teacher Interview Form

D5 Principal or Chairman Interview Form

D6 Student Interview Form

D7 List of Site Visitors

APPENDIX D1

Site Visits

I. Classroom observations

Observe in at least one control class and one experimental class. If the schedule does not dictate which, let the teachers choose the class they would prefer to have observed. On the observation form, note the size and composition of the class, what goes on, and your general impression about the class, the teacher, and how the algebra is going. In each case try to compare the control and experimental classes and note any significant differences between the two.

II. Teacher interviews

Talk to each teacher privately, preferably after the observation. Either complete the questionnaire with them, or take notes and fill it in later.

III. Principal and/or department chairman interview

Chat with the principal and/or department chairman. Try to find out two things. Are there any problems with the materials? Are there any problems with the study, specifically the testing program? Summarize your observations on the observation form.

APPENDIX D2

Site Visits

I. Classroom observations

Observe in at least one control class and one experimental class. If the schedule does not dictate which, let the teachers choose the class they would prefer to have observed. On the observation form, note the size and composition of the class, what goes on, and your general impression about the class, the teacher, and how the algebra is going. In each case, try to compare the control and experimental classes and note any significant differences between the two.

II. Teacher interviews

Talk to each teacher privately, preferably after the observation. Either complete the questionnaire with them, or take notes and fill it in later.

III. Principal and/or department chairman interview

Chat with the principal and/or department chairman. Try to find out two things. Are there any problems with the materials? Are there any problems with the study, specifically the testing program? Summarize your observations on the observation form.

IV. Student Interviews

Choose five students at random using the page of random digits enclosed. During the "work on assignment" portion of the class period, circulate around the class offering assistance. Informally interview the selected students. You may not have time for all five. Immediately note their responses on the enclosed form.

Classroom Observation Form

Name of teacher______________________________ Control_____

Experimental_____

Class observed____________________ (period/hour) Date________

1. Size and composition (grade level, sex, race) of class.

2. Outline of day's activities. (Include name of control text and pages of day's assignment.)

3. General impressions and comments.

Teacher Interview Form

1. Name____________________________________ Date______________

2. Classes in study____

3. How do the two classes compare and/or contrast?
   (Type of students, ability, collective personality, etc.)

4. What are you doing different between the two classes?

5. Have students asked questions like, "What good is all this?"

6. How did the Fall Testing Program go? (Problems or suggestions)

7. Are there any problems or complaints with the experimental materials? Specifically check

   a) difficulties with readability or amount of reading in experimental text as compared with the usual algebra test.

   b) difficulties with "model" for operations.

   c) need for and use of supplemental materials as compared with the usual algebra text.

8. Other concerns or comments.

Principal or Chairman Interview

Name and title ______________________________

Date ______________________

1. Are there any problems or concerns with the experimental materials?

2. Are there any problems or concerns with the evaluation project, specifically the testing program?

Student Interviews Form

Instructor:____________________ No. of students interviewed________

Date________________________________

Ask each question of every student interviewed and summarize their responses.

1. Are you enjoying algebra? Why or why not?

2. Do you think you will ever use the mathematics you are learning in algebra? If yes, when or what for? If no, why not?

3. Does algebra deal with the real world? or Do you think most people use algebra in their jobs or everyday life? If yes, give an example. If no, what does algebra deal with?

4. How do you like the textbook? What exactly do you like or dislike?

5. How much reading of the textbook do you do? Is it difficult to read? Is there too much reading in the book to suit you?

APPENDIX D7

LIST OF SITE VISITS

School	Observer	Date Visited
Bay High	Dale Underwood	March 11, 1977
*Elizabeth City	Henry Kepner	May 2, 1977
*Fort Mill	Henry Kepner	April 29, 1977
Fraser	Terrance Coburn	March 3, 1977
*Fritsche	Henry Kepner	December 8, 1976
Harper	John Easton	March 30, 1977
*John Adams	Sandra Clarkson	March 25, 1977
Los Alamos	Sid Humble	March 7, 1977
McLean	James Fey	March 28, 1977
*N.W. Whitfield	Jane Swafford	January 7, 1977
*Okemos	Jane Swafford	February 3, 1977
Olney	Bruce Burt	March 11, 1977
Owen J. Roberts	Bruce Burt	March 28, 1977
*Sequoia	Jane Swafford	February 22, 1977
*S. Miami	Edwin McClintock	February 18, 1977
*S. Shore	Sandra Clarkson	March 29, 1977
*Walter Reed	Jane Swafford	February 23, 1977
*Walton	Jane Swafford	January 8, 1977
Wasson	Max Bell	February 24, 1977

*School in which student interviews were also conducted.

APPENDIX E

ADDITIONAL DATA AND STATISTICS

TWO-WAY ANALYSES OF VARIANCE FOR "AVERAGE" STUDENTS

Arithmetic Test

Source	ss	df	ms	F	sig
Main effect	733.81	17	43.17	4.81	<.000
treatment	21.99	1	21.99	2.45	<.118
school	722.88	16	45.18	5.03	<.000
Interaction	170.29	16	10.64	1.19	<.274
Explained	904.10	33	27.40	3.05	<.000
Residual	6258.83	697	8.98		
Total	7162.93	730			

Fall ETS

Source	ss	df	ms	F	sig
Main Effect	1406.26	17	82.72	6.34	<.000
treatment	49.88	1	49.88	3.82	<.051
school	1322.49	16	82.65	6.34	<.000
Interaction	204.41	16	12.78	0.98	<.478
Explained	1610.68	33	48.81	3.74	<.000
Residual	9093.05	697	13.05		
Total	10703.73	730			

Spring ETS

Source	ss	df	ms	F	sig
Main effect	9094.20	17	534.95	24.59	<.000
treatment	755.58	1	755.58	34.73	<.000
school	8270.32	16	516.90	23.76	<.000
Interaction	859.00	16	53.69	2.47	<.001
Explained	9953.20	33	301.61	13.86	<.000
Residual	15164.50	697	21.76		
Total	25117.70	730	34.41		

First Year Algebra Test

Source	ss	df	ms	F	sig
Main effect	6433.20	17	378.42	21.39	<.000
treatment	721.64	1	721.64	40.78	<.000
school	5696.05	16	356.00	20.12	<.000
Interaction	690.16	16	43.14	2.44	<.001
Explained	7123.36	33	215.86	12.20	<.000
Residual	12333.76	697	17.70		
Total	19457.13	730	26.65		

APPENDIX E2

TEXTBOOK EVALUATION FORM (Experimental)

Name: Summary N=15 School: unfavorable and neutral teachers in ( )

Please answer the following questions as they apply to ALGEBRA THROUGH APPLICATIONS by Usiskin.

1. In general, I feel that the book is most appropriate for
   (4) (a) the above average 1st year algebra student
   7 (2) (b) the average 1st year algebra student
   (2) (c) the below average 1st year algebra student ______

2. Compared to other 1st year algebra books I have taught from, this book is
   4 (3) (a) easier to read and understand
   1 (4) (b) harder to read and understand
   1 (2) (c) at about the same level ______

3. Compared to other 1st year algebra books I have taught from, the exercises are
   1 (2) (a) easier
   1 (3) (b) more difficult
   4 (2) (c) at about the same level ______

4. I
   1 (a) would strongly recommend
   6 (b) would recommend
   (1) (c) am indifferent to
   (6) (d) would not recommend
   (1) (e) would strongly recommend against
   the use of this text for an average 1st year algebra class. ______

5. Before this year, were you dissatisfied with the 1st year algebra course as outlined in most commercial texts?
   1 (3) (a) very much 3 (3) (b) only slightly 3 (2) (c) no ______

Questions 6-15 ask you to compare the development of certain topics in the experimental text with those you are familiar with in other texts by choosing a letter and a number which best describes your feelings.

Letter choices:
(a) The development is the nicest I've seen.
(b) The development is about as nice as others I know.
(c) I know a more effective development.
(d) I cannot compare with other developments.

Number choices:
(1) The development was rather easy for my students to understand.
(2) The development was about average difficulty for my students.
(3) The development was hard for my students to understand.

	Letter a	b	c	d	Number 1	2	3
6. approach to variables (Ch. 2)	4 (2)	2 (5)			(3)	(3)	(1)
7. subscripts (Ch. 2)	4 (2)	1 (4)	(1)	1	(4)	(2)	(1)
8. properties (Ch. 3-5)	1 (2)	4 (2)	1 (3)		(2)	(2)	(4)
9. approach to beginning sentence solving (Ch. 6)	7 (2)	(3)	(2)		(1)	(4)	(2)
10. work with distributive property (Ch. 7-8)	1	6 (5)	(2)			(4)	(3)
11. approach to slope (Ch. 9)	3 (4)	3 (2)	(1)	1	(1)	(3)	(3)
12. graphing linear sentences (Ch. 9)	3 (2)	4 (3)	(2)		(1)	(4)	(2)
13. negative exponents (Ch. 10)	3	4 (3)	(3)	(1)	(1)	2	(1)
14. square roots (Ch. 12)	3 (1)	4 (2)	(3)	(1)		(4)	(3)
15. systems (Ch. 14)	3	2 (3)	(2)	(1)	(1)	(5)	

AV = 1.44 fav
AV = 2.16 unfav

16. Check all that apply

Models	Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
Addition:						
union	4 (5)	2 (3)		3 (4)	5 (5)	(3)
joining	4 (2)	2 (5)	(1)	3 (4)	4 (2)	1 (5)
slide	4 (4)	1 (3)	(1)	3 (4)	5 (3)	1 (4)
Subtraction:						
take away	4 (4)	2 (4)	(1)	3 (2)	3 (4)	(3)
cutting off	4 (2)	2 (6)	(1)	3 (2)	3 (2)	(4)
directed distance	6 (5)	(3)	(1)	3 (4)	4 (5)	(3)
Multiplication:						
ordered pair	6 (4)	1 (3)	(1)	3 (3)	4 (4)	(4)
area	5 (5)	2 (3)	1	2 (4)	2 (4)	2 (3)
scale change	5 (4)	(3)	2 (1)	2 (4)	4 (4)	1 (4)
repeated add.	6 (7)	1 (1)	(1)	3 (3)	4 (6)	(1)
Division:						
splitting up	4 (5)	3 (3)	1	3 (4)	4 (5)	1 (2)
rate	5 (7)	1 (1)	1	3 (4)	4 (6)	1 (1)
scale compar.	4 (4)	2 (4)	1	3 (4)	4 (4)	2 (3)
Powering:						
repeated mult.	7 (6)		(2)	4 (5)	4 (5)	
growth	4 (4)	1 (3)	1 (4)	3 (2)	3 (3)	1 (3)

17-20 Check all that apply

		Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
17.	assemblage property	5 (5)	2 (4)	2	4 (4)	4 (4)	1 (1)
18.	metric system	7 (5)	(1)	2 (3)	3 (2)	6 (3)	(2)
19.	calculating statistics	3 (5)	2 (1)	3 (2)	3 (3)	3 (4)	1 (2)
20.	calculating probabilities	7 (6)		2 (1)	3 (4)	6 (4)	1

21-28. Each of the following topics is not in the text, or is not in it to the extent that most other texts have it. Choose the most appropriate response.

(a) I did not mind not having to teach this topic.
(b) I would have liked to have taught this topic, but did not.
(c) I taught this topic even though it was not in the book.
(d) I did not teach this topic, but would next year if this book were used.

(a) (b) (c)

21. factoring expressions like $3x^2 - 10x + 7$ — 4 4 / (2)(5)

22. adding fractional expressions requiring getting a least common denominator — 5 1 / (5)(1)

23. multiplying or dividing fractional expressions where factoring of trinomials is required — 4 1 / (5)(2)(1)

24. formal logic — 7 / (3)

25. age problems — 6 / (4)(2)(2)

26. digit problems — 5 / (3)(2)(2)

27. coin problems — 5 / (3)(2)(3)

28. distance-rate-time problems — 5 / (1)(4)(3) 2

29. Check all that apply. What do you think about the nature of the applications in this book?

- 5 (2) (a) interesting to most students
- 3 (4) (b) interesting to only a few students
- ____ (c) interesting to none of the students
- (4) (d) arithmetic too difficult in most
- (3) (e) situations too involved (complex) for most students
- ____ (f) too easy for most students
- ____ (g) some are socially too controversial
- 5 (4) (h) they promote valuable discussion
- ____ (i) they promote wasteful discussion
- (3) (j) too many
- (2) (k) the traditional word problems are better

30. Here are some applications. Check all that apply.

	Interesting	Not Int.	Easy	Hard	Keep	Drop	Didn't Do
mile run (p. 434)	6 (6)	1	3 (1)	2 (2)	4 (5)	2	(1)
storms (p. 315)	4 (4)	3 (3)	1 (1)	1 (3)	2 (2)	2 (4)	(1)
hamburger (p. 316)	7 (6)		6 (6)	(1)	6 (6)		
wildlife (p. 257)	7 (6)		3 (4)	2	5 (5)	1	(1)
Manhattan (p. 409)	6 (4)	1 (2)	4	1 (3)	3 (3)	2 (4)	
newspaper (p. 27)	5 (3)	1 (3)	4 (1)	1	4 (2)	(2)	(2)
scouts (p. 610)	6 (1)	(2)	5 (1)	(1)	4 (3)	(1)	(2) (2)

31. What topics or ideas were hardest for your students?

models 4
properties 2
prob 2
patterns 2
prob. dev. 2

slope 2
equations 2
exponents
metric

systems
amt. of reading 1

32. What is the furthest lesson you covered in the book: Ch. ____ Lesson ____

Range Ch 8 – End of Book
Ch 8 (1) Ch 10 (2) Ch 11 (2) Ch 13 (3)

33. What lessons (or chapters) did you skip? Ch 13 ||||

34. What did you find yourself skipping that you would like to have covered?

last chapter
Some applications (3)
prob. and stat. (3)

35. When you first began this course did you feel that you would have trouble with the mathematics or the applications?

(a) definitely (1) (b) somewhat 2 (2) (c) not really 5 (5) ____

36. Did you have trouble with the mathematics or the applications in this course?

(a) definitely (1) (b) somewhat 1 (1) (c) not really 6 (7) ____

37. What topics or applications in this course were hardest for you to understand? How hard was it to understand?

models (6)
subscripts
mean abs. dev.
statistics
assemblage prop

All teachers changed "hard to understand" to "hard to teach"

38. Has teaching this course changed your attitudes about 1st year algebra or the teaching of applications? ______ If so, how?

yes 2 (2)
no 3 (5)
"I will use some of applications" (2)

39. I read and used the "Notes to the Teacher"
2 (3) (a) for each section 2 (4) (c) sometimes (e) never
3 (3) (b) often 1 (d) seldom ______

40. Answer all that apply. For tests, I would liked to have had
4 (3) (a) complete chapter tests
6 (3) (b) suggested test items from which a test could be made
2 (4) (c) a mastery workbook for each student
1 (1) (d) no tests or test items

41. My tests this year were
(a) identical to those I have given in the past in 1st year algebra
2 (2) (b) very similar but with some modifications
2 (5) (c) very different but with some similar problems
1 (1) (d) completely different from those I have given in the past ______

42. Answer all that apply: With regard to the mastery workbook,
______ (a) I never used it.
7 (5) (b) I used the problems for tests.
7 (7) (c) I used the problems for review.
2 (3) (d) I used the problems often.
(3) (e) I used the workbook some way not mentioned above. (Explain bel

43. I used the answers to exercises
(1) (a) for each lesson 4 (c) sometimes (4) (e) never
1 (1) (b) often 2 (1) (d) seldom ______

44. Should answers to exercises be included in the student text?
4 (1) (a) no
3 (4) (b) to odd exercises only
(c) to "Questions covering the reading" only
(2 (d) to "Questions covering the reading" and other selected problems
(1) (e) to all exercises
(f) to other (Explain) ______

45. Compared with other 1st year algebra books I have taught from, with this book I
1 (1) (a) ~~had~~ to supplement more than usual
2 (b) had to supplement less than usual
3 (2) (c) supplemented about the same as usual

46. What changes in the exercises would you recommend?

MORE 5 (4)

47. How often did you make reading assignments from the text?
1 (2) (a) every lesson
2 (2) (b) most lessons
1 (1) (c) about half the lessons
3 (3) (d) some of the lessons
(e) never

48. How many students did reading when you assigned it?
(1) (a) almost all 1 (2) (c) about half (1) (e) almost none
4 (2) (b) most 2 (2) (d) some

49. How often do you feel a typical student was able to understand the lesson from the reading without your reading or explaining it?
(a) almost always 5 (3) (c) sometimes (1) (e) never
2 (1) (b) often (3) (d) seldom

50. How often do you think students should be expected to read in a mathematics text?
5 (7) (a) frequently
(b) only as a group
(1) (c) only when class explanation is not enough
(d) never
2 OCCASIONALLY

51. Check all that apply. Did you allow the use of electronic or mechanical calculators?
4 (3) (a) on all homework problems
1 (1) (b) only on those homework problems marked "C"
____ (c) on a few designated homework problems
____ (d) on no homework problems
5 (5) (e) Students were allowed to bring calculators to class and use them in class work.
2 (1) (f) There was a school or teacher-owned calculator available in the classroom for student use.
4 (2) (g) Calculators could not be used on any tests.
3 (1) (h) Calculators were allowed on some tests.
(2) (i) Calculators could be used on all tests.
1 (2) (j) The use of calculators was never considered.

52. A calculator for each student
(a) is a necessity with this book
(b) helps but is not necessary
(c) makes no difference with this book

53. What percentage of your students have access to a calculator at home? 1/2 - 75% This is my guess / 14 I asked all my students. 1 $\bar{X} = 30\%$

54. If you had three suggestions to make to improve this book, what are they?

colorful illustrations
enlarge metric
less reading
less emphasis on models
more practice problems in arith, alg manip, solving eq
more application for slopes and graphing
include exercises in chapter review
leave ans off mastery question
more emphasis in graphing linear eq
less statistics and probability
simply exercise reading to 7th grade
more examples, figures, charts
more mechanical exercises
correct errors (esp in ans book)
include index
rename sections to reflect both alg and applications covered
needs to be coordinated with alg II
shorten
consolidate probability
shorten in order to get to quadratic
exercises harder than examples

55. If you had a choice would you participate in a study similar to this in the future? yes 7 (4) NO 1 (3)

56. Please indicate any other comments you might have.

It has been en exciting experience for me.
Need to teach it again before making more suggestions
I like what was attempted but don't think it appropriate for students who have had pre-alg.
Approach very good and very interesting to most students. Student often said now see connection between math and actual problems
10, 11, 12th graders probably need 1½ years to cover book. 9th could do it with ease.
I got nervous about so much traditional algebra being either intro late or not at all.
Not appropriate for senior high reading at 5th grade level
I enjoyed the many interesting examples--but I would not use it again.
Great examples and interesting appl. but too slow for good students
Too much of traditional alg omitted to allow time for statistics.
Should have had inservice--quite a change from traditional--might have done better job.
Not for inner city students, arith skill poor, conditioned for dull (Note: kids didn't have calculators)

Thank you very much for taking the time to complete this very long form.

APPENDIX E3

TEXTBOOK EVALUATION FORM (Control)

Name: N=16 School: ______

*Please answer the following questions as they apply to the book that you are presently using with your 1st year algebra classes.

Name of book: ______ Authors: ______ ______

★ 1. In general, I feel that the book is most appropriate for
2 (a) the above average 1st year algebra student
13 (b) the average 1st year algebra student
1 (c) the below average 1st year algebra student ______

★ 2. Compared to other 1st year algebra books I have taught from, this book is
6 (a) easier to read and understand
4 (b) harder to read and understand
6 (c) at about the same level ______

★ 3. Compared to other 1st year algebra books I have taught from, the exercises are
4 (a) easier
1 (b) more difficult
10 (c) at about the same level ______

★ 4. I
2 (a) would strongly recommend
9 (b) would recommend
2 (c) am indifferent to
2 (d) would not recommend
1 (e) would strongly recommend against
the use of this text for an average 1st year algebra class. ______

★ 5. Are you dissatisfied with the 1st year algebra course as outlined in most commercial texts?
3 (a) very much 4 (b) only slightly 9 (c) no ______

★ 6-13. Each of the following topics are covered by most 1st year algebra texts. Choose the most appropriate response.
(a) I do not mind teaching this topic.
(b) I would have liked to have taught this topic, but did not.
(c) I did not teach this topic even though it was in the book.
(d) I did teach this topic, but would rather see it deleted from 1st year algebra.

6. factoring expressions like $3x^2 - 10x + 7$ 14a, 2d

7. adding fractional expressions requiring getting a least common denominator 16a

8. multiplying or dividing fractional expressions where factoring of trinomials is required 14a, 1b, 1c

9. formal logic 4a, 6b, 3c

10. age problems 16a

11. digit problems 14a 2c

12. coin problems 15a 1c

13. distance-rate-time problems 16a

★ Items comparable to Items on Experimental Form.

14-20. Were any of the following mentioned or studied in your 1st year algebra class this year?

14. probability: Mentioned? 5 Studied? ____
15. statistics: Mentioned? 5 Studied? ____
16. ~~metric~~ system: Mentioned? 7 Studied? 3
17. functions: Mentioned? 6 Studied? 8
18. real world word problems: Mentioned? 6 Studied? 10
19. graphing of real data: Mentioned? 9 Studied? 4
20. quadratic formula: Mentioned? 9 Studied? 11

21. What material could be deleted from your text without disturbing you?
trigonometry (4)
set theory (2)
some factoring
other comments (13)

22. What material would you like to see added to your text?
probability and statistics (3)
metric system (2)
more real-life word problems with fractions for answers (2)

★ 23. Which topics or ideas are generally hardest for your students to understand?
translating words into algebra (7)
fractions (11)
slope, finding equations, or graphs of linear equations (7)
factoring (3) radicals (2) others (6)

24. Which topics or ideas are generally easiest for your students to understand?
equations (5)
factoring (3)
graphing ordered pairs (6)
integers (2)
others (10)

★ 25. What topics did you find yourself skipping that you would like to have covered?
functions (2)
quadratic
word problems
systems
negative exponents
radicals

★ 26. When you first taught from this book did you feel that you would have trouble with the mathematics?
(a) definitely (b) somewhat 16 (c) not really ____

★ 27. How often do you use the Teacher's Commentary or Notes?
1 (a) for each section 5 (c) sometimes 4 (e) never
1 (b) often 5 (d) seldom ____

28. Are chapter tests available with this book?
If so, how often do you use them (Perhaps with Modifications)?
3 (a) for each chapter 1 (c) sometimes 9 (e) never
2 (b) often 1 (d) seldom ____

*29. How often do you use the "Answers to Exercises" or Solution Key?
4 (a) for each assignment 2 (c) sometimes 3 (e) never
3 (b) often 4 (d) seldom ______

*30. Do you feel that answers to exercises should be included as part of the student's text?
10 (a) to odd exercises only
1 (b) only to the easiest exercises
1 (c) only to the most difficult exercises
1 (d) to all exercises
2 (e) not to any exercises
1 (f) other (Explain) IN SEPARATE MANUAL ______

*31. When (if ever) have you found it necessary or convenient to use other sources than the book for assignments?
(a) for each assignment 5 (c) sometimes 3 (e) never
5 (b) often 3 (d) seldom ______

*32. How often did you make reading assignments from the text?
1 (a) every lesson
3 (b) most lessons
(c) about half the lessons
6 (d) some of the lessons
6 (e) never ______

*33. How many students did reading when you assigned it?
1 (a) almost all 5 (c) about half 4 (e) almost none
1 (b) most 1 (d) seldom ______

*34. How often do you feel a typical student was able to understand the lesson from the reading without your reading or explaining it?
(a) almost always 5 (c) sometimes (e) never
1 (b) often 9 (d) seldom ______

*35. How often do you think students should be expected to read in a mathematics text?
9 (a) frequently
(b) only as a group
5 (c) only when class explanation is not enough
(d) never ______

*36. Check all that apply. Did you allow the use of electronic or mechanical calculators?
4 (a) on all homework problems
____ (b) only on those homework problems marked "C"
2 (c) on a few designated homework problems
4 (d) on no homework problems
4 (e) Students were allowed to bring calculators to class and use them in class work.
1 (f) There was a school or teacher-owned calculator available in the classroom for student use.
9 (g) Calculators could not be used on any tests.
2 (h) Calculators were allowed on some tests.
____ (i) Calculators could be used on all tests.
____ (j) The use of calculators was never considered. ______

*37. A calculator for each student
(a) is a necessity with this book
4 (b) helps but is not necessary
11 (c) makes no difference with this book
1 (e) SERVES NO USEFUL PURPOSE

* 38. What percentage of your students have access to a calculator at home? 0%-100% This is my guess. 13 I asked all my students. 2 $\bar{X} = 45\%$

* 39. If you had a choice would you participate in a study similar to this in the future? yes (11) no (3) Depends (2)

* 40 Please indicate any other comments you might have.

Thank you very much for taking the time to complete this very long form.

APPENDIX E4

TEXTBOOK EVALUATIC[illegible] FORM (Experimental)

Name: SUMMARY N=12 School: Not in formal study

Please answer the following questions as they apply to ALGEBRA THROUGH APPLICATIONS by Usiskin.

1. In general, I feel that the book is most appropriate for
   (a) the above average 1st year algebra student (2)
   (b) the average 1st year algebra student (8)
   (c) the below average 1st year algebra student (1)

2. Compared to other 1st year algebra books I have taught from, this book is
   (a) easier to read and understand (1)
   (b) harder to read and understand (2)
   (c) at about the same level (1)

3. Compared to other 1st year algebra books I have taught from, the exercises are
   (a) easier (1)
   (b) more difficult (8)
   (c) at about the same level (8)

4. I
   (a) would strongly recommend (2)
   (b) would recommend (7)
   (c) am indifferent to (1)
   (d) would not recommend (1)
   (e) would strongly recommend against (1)
   the use of this text for an average 1st year algebra class.

5. Before this year, were you dissatisfied with the 1st year algebra course as outlined in most commercial texts?
   (4) (a) very much (4) (b) only slightly (c) no

Questions 6-15 ask you to compare the development of certain topics in the experimental text with those you are familiar with in other texts by choosing a letter and a number which best describes your feelings.

Letter choices:
(a) The development is the nicest I've seen.
(b) The development is about as nice as others I know.
(c) I know a more effective development.
(d) I cannot compare with other developments.

Number choices:
(1) The development was rather easy for my students to understand.
(2) The development was about average difficulty for my students.
(3) The development was hard for my students to understand.

		Letter (a)	Letter (b)	Letter (c)	Letter (d)	Number (1)	Number (2)	Number (3)
6.	approach to variables (Ch. 2)	6	[illegible]	[illegible]		[illegible]	[illegible]	
7.	subscripts (Ch. 2)	7	[illegible]		1	7	[illegible]	
8.	properties (Ch. 3-5)	[illegible]	[illegible]	2		[illegible]	[illegible]	1
9.	approach to beginning sentence solving (Ch. 6)	6	[illegible]	1		1	[illegible]	2
10.	work with distributive property (Ch. 7-8)	7	[illegible]	2		1	[illegible]	[illegible]
11.	approach to slope (Ch. 9)	7	1	1		[illegible]	[illegible]	[illegible]
12.	graphing linear sentences (Ch. 9)	3	[illegible]	2			[illegible]	6
13.	negative exponents (Ch. 10)	7	[illegible]	1		5	[illegible]	[illegible]
14.	square roots (Ch. 12)	6	[illegible]	[illegible]		1	[illegible]	[illegible]
15.	systems (Ch. 14)	2	[illegible]			[illegible]	[illegible]	[illegible]

16. Check all that apply

Models	Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
Addition:						
union	8	1	1	4	7	1
joining	7	1	2	T 4	6	1
slide	7	2	2	5	6	2
Subtraction:						
take away	7	1	2	4	6	1
cutting off	7	2	1	4	6	2
directed distance	9	1	1	5	3	1
Multiplication:						
ordered pair	3	1		5	7	1
area	9	1		5	8	1
scale change	3	1		5	7	2
repeated add.	7	2	1	4	6	2
Division:						
splitting up	7	2	1	4	7	1
rate	9	1	1	5	[illegible]	1
scale compar.	7	1		6	7	2
Powering:						
repeated mult.	3	1		5	7	1
growth	7	3	1	5	6	3

17-20 Check all that apply

	Useful	Not Useful	Skipped or not emphasized	Emphasized	Keep	Drop
17. assemblage property	7	4	2	5	7	3
18. metric system	7	1	2	4	[illegible]	1
19. calculating statistics	3	1	3	3	3	2
20. calculating probabilities	9	1	1	9	7	2

21-28. Each of the following topics is not in the text, or is not in it to the extent that most other texts have it. Choose the most appropriate response.

(a) I did not mind not having to teach this topic.

(b) I would have liked to have taught this topic, but did not.

(c) I taught this topic even though it was not in the book.

(d) I did not teach this topic, but would next year if this book were used.

		(a)	(b)	(c)
21.	factoring expressions like $3x^2 - 10x + 7$	4	2	
22.	adding fractional expressions requiring getting a least common denominator	5	3	2
23.	multiplying or dividing fractional expressions where factoring of trinomials is required	6	1	3

		(a)	(b)	(c)	(d)
24.	formal logic	10	2		
25.	age problems	10	2		
26.	digit problems	9	3		
27.	coin problems	10	2		
28.	distance-rate-time problems	5	2	2	

29. Check all that apply. What do you think about the nature of the applications in this book?

- 3 (a) interesting to most students
- 4 (b) interesting to only a few students
- ___ (c) interesting to none of the students
- 3 (d) arithmetic too difficult in most
- 3 (e) situations too involved (complex) for most students
- ___ (f) too easy for most students
- ___ (g) some are socially too controversial
- 2 (h) they promote valuable discussion
- ___ (i) they promote wasteful discussion
- ___ (j) too many
- 1 (k) the traditional word problems are better
- 1 (l) not enough

30. Here are some applications. Check all that apply.

	Interesting	Not Int.	Easy	Hard	Keep	Drop	Didn't Do
mile run (p. 434)	10	1	1	6	9	2	1
storms (p. 315)	7	3	3	4	7	3	1
hamburger (p. 316)	9	1	8		7	2	
wildlife (p. 257)	11		3	5	10	1	
Manhattan (p. 409)	4	5	4	4	3	6	1
newspaper (p. 27)	6	3	4		7		2
scouts (p. 610)	4	2	2		5		4

31. What topics or ideas were hardest for your students?

Slope 2   Chapters 7, 14 1
Equation for line 2   Models 1
Quadratics 1   Graphing lines 2
Working with [illegible] 1   Ch. 15 2

32. What is the furthest lesson you covered in the book: Ch. ____ Lesson ____

33. What lessons (or chapters) did you skip? ____________

34. What did you find yourself skipping that you would like to have covered?

Ch. 12 1   Ch. 15 5
13 3   15 (end) 2
14 2   16 6

35. When you first began this course did you feel that you would have trouble with the mathematics or the applications?
(a) definitely (2) (b) somewhat (3) (c) not really (7) ____________

36. Did you have trouble with the mathematics or the applications in this course?
(a) definitely (1) (b) somewhat (3) (c) not really (7) ____________

37. What topics or applications in this course were hardest for you to understand? How hard was it to understand?

Probability - Statistics 2
Models 2
Trig 1
Area models 1
Newspaper 1

38. Has teaching this course changed your attitudes about 1st year algebra or the teaching of applications? ______ If so, how?

Yes 5 No 4.

Like models 1 - Reinforced earlier dissatisfaction 1
have real applications now 2
more meaning 2

39. I read and used the "Notes to the Teacher"
4 (a) for each section 2 (c) sometimes (e) never
4 (b) often 2 (d) seldom

40. Answer all that apply. For tests, I would liked to have had
__3__ (a) complete chapter tests
__7__ (b) suggested test items from which a test could be made
__7__ (c) a mastery workbook for each student
_____ (d) no tests or test items

41. My tests this year were
(a) identical to those I have given in the past in 1st year algebra
3 (b) very similar but with some modifications
6 (c) very different but with some similar problems
4 (d) completely different from those I have given in the past

42. Answer all that apply: With regard to the mastery workbook,
__1__ (a) I never used it.
__4__ (b) I used the problems for tests.
__7__ (c) I used the problems for review.
__3__ (d) I used the problems often.
__1__ (e) I used the workbook some way not mentioned above. (Explain b

43. I used the answers to exercises
1 (a) for each lesson 7 (c) sometimes (e) never
2 (b) often 2 (d) seldom

44. Should answers to exercises be included in the student text?
6 (a) no
1 (b) to odd exercises only
1 (c) to "Questions covering the reading" only
1 (d) to "Questions covering the reading" and other selected problems
1 (e) to all exercises
2 (f) to other (Explain) Selected QCR and SAR
Not QCR, but all others

45. Compared with other 1st year algebra books I have taught from, with this book I
5 (a) had to supplement more than usual
4 (b) had to supplement less than usual
3 (c) supplemented about the same as usual

46. What changes in the exercises would you recommend?

Ease arithmetic in examples 2 | Clarify explanations 2
problems 1
More 1
More challenging 1

47. How often did you make reading assignments from the text?
4 (a) every lesson
4 (b) most lessons
1 (c) about half the lessons
3 (d) some of the lessons
(e) never

48. How many students did reading when you assigned it?
3 (a) almost all | 6 (c) about half | (e) almost none
2 (b) most | (d) some

49. How often do you feel a typical student was able to understand the lesson from the reading without your reading or explaining it?
(a) almost always | 6 (c) sometimes | 1 (e) never
4 (b) often | 1 (d) seldom

50. How often do you think students should be expected to read in a mathematics text?
11 (a) frequently
1 (b) only as a group
(c) only when class explanation is not enough
(d) never

51. Check all that apply. Did you allow the use of electronic or mechanical calculators?
6 (a) on all homework problems
2 (b) only on those homework problems marked "C"
(c) on a few designated homework problems
(d) on no homework problems
11 (e) Students were allowed to bring calculators to class and use them in class work.
3 (f) There was a school or teacher-owned calculator available in the classroom for student use.
6 (g) Calculators could not be used on any tests.
? (h) Calculators were allowed on some tests.
1 (i) Calculators could be used on all tests.
1 (j) The use of calculators was never considered.

52. A calculator for each student
1 (a) is a necessity with this book
6 (b) helps but is not necessary
1 (c) makes no difference with this book

53. What percentage of your students have access to a calculator at home? ______ This is my guess. ______ I asked all my students. ______

Guess 40-60 | Asked 50-75
30 | 75
80 | 60
10 | 25
10 |
70 |
20 | 75%

54. If you had three suggestions to make to improve this book, what are they?

55. If you had a choice would you participate in a study similar to this in the future? No I Yes II

56. Please indicate any other comments you might have.

Thank you very much for taking the time to complete this very long form.

APPENDIX F

READING LEVEL EVALUATION OF ALGEBRA THROUGH APPLICATIONS

GERALD KULM, PURDUE UNIVERSITY

Reading Level Evaluation of
Algebra Through Applications

Gerald Kulm, Purdue University

Present methods of readability measurement for mathematics textbooks do not provide for making grade level estimates. However, a number of methods have been developed and validated for comparing the readability of mathematics textbook passages in a rank order fashion. These are summarized briefly as follows:

a) Kulm Readability Formula.

This formula was developed especially for 9th grade algebra materials and accounts for approximately 30 percent of the variance in the reading difficulty of explanatory material (Kulm, 1971). The formula was validated by having algebra students complete cloze tests on 100 explanatory and illustrative passages from a variety of algebra textbooks. The formula is:

$Y = 26.3 - .16X_1 + .05X_2 - .14X_3 - .08X_4$

Where Y = predicted cloze score

X_1 = percent of math symbols

X_2 = percent of reader-directed sentences

X_3 = average sentence length

X_4 = percent of math vocabulary words

b) Kane Readability Formula.

This formula was developed for material primarily at the 6th-9th grade levels and accounts for approximately 30 percent of the variance in reading difficulty (Kane, Byrne & Hater, 1974). The formula is:

$$Y = 35.52 - .15X_1 + .10X_2 - 42X_3 - .17X_4$$

Where Y = predicted cloze score

X_1 = number of words not on Dale 3000 word list and not on 80% math list

X_2 = number of changes from symbol to word and vice-versa

X_3 = number of words not on 80% math list plus number of symbols not on 90% symbol list

X_4 = number of question marks

c) Teacher Judgement.

This procedure uses a list of criteria on which teacher evaluate the overall difficulty of a passage. The average rating provides a means of ranking mathematics passages according to reading difficulty. Comparisons obtained by this method correlate .70 with comprehension test scores and .65 with cloze scores (Loehrlein, 1974). The list of criteria items for judgement is given in the directions that are appended to the report.

d) Information Content.

Recent work in the area of software science has indicated that the information content and language level of technical prose can be measured (Kulm, 1975; Halstead, 1977). The measures are objectively determined from the words and symbols of a passage and provide for assessment of the informational complexity of text material.

The formulas are:

$$V = N \log_2 n$$

$$L = \frac{2}{n_1} \times \frac{n_2}{N_2}$$

$$I = \frac{VL}{N}$$

Where N = total number of words and sumbols

n = number of different words and symbols

n_1 = number of different function words and sumbols

n_2 = number of different content words and symbols

V = volume of information

L = language level

I = information content per word

Each of these measures by itself has not been applied widely enough to warrant a decision about readability to be made with complete confidence. On the other hand, there is evidence that the measures do differentiate among grade levels of matheamtics text (Kulm, 1975).

Procedures

Textbook samples: The Algebra Through Applications text was compared with two widely used algebra textbooks (Holt Algebra and Houghton-Mifflin Red Algebra). First, fifteen sample passages were selected from the ATA text; one from approximately every 30 pages of text. Each passage was chosen from explanatory material and each covered a complete topic. The samples did not include exercises, tests, or optional material. The next step was to select passages from each of the other two textbooks which covered the same topics as the ATA samples. Finally, five samples of word problems were selected from each of the three texts, representing a cross-section of the problems in each text. The explanatory passages were each approximately 300 tokens in length. Table 1 presents the topic, page numbers and token length of each sample passage. For longer passages, a maximum of 300 tokens was analyzed. For word problems, each sample consisted of several problems with a total length of 200 tokens.

A photostatic copy was made of each passage, eliminating the use of color as a variable in judging their difficulty. <u>Readability data</u>: Four measures of readability were obtained for each of the 45 explanatory passages and three measures were determined for the 15 word problem samples. The Kane and Kulm readability formulas were applied to each of the 60 samples using the procedures described in the attached directions. Teacher judgements were also obtained for each of the 60 passages. The teachers were enrolled in either a graduate mathematics methods course or a graduate course in teaching reading in secondary content areas. The directions for teacher judgement are attached. Finally, the language level of each explanatory passage was calculated. It was believed that the language level measure was not applicable to the word problems, since each problem was a separate topic. The language level measure is intended to be a measure of a single entity of text.

Analysis and Results

The predicted cloze scores were calculated for each passage using both the Kane and the Kulm readability formulas. Some of the passages were shorter than 300 tokens so it was necessary to adjust the Kane formula variables. This correction was done by computing an estimate for each independent variable as follows:

$$\hat{X}_i = \frac{300}{n} X_i$$

where n = actual passage length and

X_i = actual variable value

The estimates were then used in the formula to obtain the predicted cloze score.

The information content values were used to calculate the language level for each passage. This variable was selected from the available information measures because it is similar to the psycholinguistic concept of type-token ratio and because it was found in previous work to be related to reading level of mathematics text material (Kulm, 1975).

The teacher judgement ratings were assigned values 1 through 7 corresponding to extremely low to extremely high reading complexity, respectively. The values were averaged, resulting in a mean teacher judgement score for each passage.

For each passage, a profile of reading complexity was prepared by using the four measures. The score for each of the 20 triples of passages covering similar content was plotted on the same graph for each measure. The profiles provide comparisons of a) the reading level of each passage with the mean for all passages on each measure and b) the rank of each passages with the other two passages for each measure. The profiles for the twenty triples and for the means of all passages from each text are given in figures 1 through 21.

A second type of reading complexity evaluation was obtained by computing a composite readability score for each explanatory passage. This score was computed as follows: a) within each triple, the three passages were ranked for each readability measure (3=highest, 2=middle, 1=lowest, in the case of a tie the average of the ranks was assigned to each); b) the ranks on the four measures were summed, producing a minimum score of 4 and a maximum of 12. The composite scores are given in Table 5. The ATA textbook had the best composite readability score on seven of the fifteen explanatory passages and was lowest on three passages. The ATA text also has the highest mean composite score.

Conclusions

Based on the analyses completed, it can be stated with some confidence, that the Algebra Through Applications text is written at a suitable level for ninth graders. Compared with two widely used algebra texts, the ATA text falls between the two on three of the four readability measures used and is above both on the other measure, when the means over all sample passages are considred. It may be worth noting that the measure on which ATA is best, the Kulm formula, was developed especially for elementary algebra texts. It is also significant that the means of the measures were fairly consistent in ranking the texts and there were no great variations.

As expected there were considerable within-text and between-topic variations among the four readability measures. For the Kane and Kulm formulas, these variations are easy to explain.

In the Kulm formula, a high perdentage of symbolism results in lower reading ease, whereas the Kane formula is more sensitive to the mathematics vocabulary complexity. The Holt text, for example, was ranked consistently low and the ATA text consistently high by the Kulm formula due to the high and low relative percentages of symbolism, respectively, in the two texts. The Kane formula consistently favored the Holt text since symbolism was not a factor and few words are used in the text.

The mean teacher judgement scores on all passages were similar for the three texts, with the Holt book slightly higher. In examining the individual passages which were rated easy or difficult by teachers, it was possible to discern a few patterns. Generally, teachers rated as difficult those passages that dealt with difficult vocabulary and/or especially complex topics, new symbolism, or passages that contained a combination of tables or graphs. Passages were rated as easier to read when they contained few words, used numerical examples (rather than general variables), or contained repeated examples of a process or principle. In general, it appeared that teachers were somewhat-content oriented in judging reading ease rather than using criteria that were strictly related to readability. This finding is especially true for the word problem samples. A comparison on passage ranks revealed that teacher judgement ratings agreed most closely with Kane formula ranks (24 of 60) than with the Kulm or Language Level measures. The explanation for this is probably related to the preference of teachers for passages with few words and numerical examples and their low rating of passages with difficult vocabulary. In any case, the ATA text lies very close to the mean in terms of teacher judgement, with only one or two passages being judged as especially difficult.

The Language Level measure did not produce a great deal of variation between texts. On the other hand, there was a 30 percent agreement on the ranks of passages between Language Level and the Kane formula, indicating that the measure may provide a somewhat useful criterion for reading ease. A post hoc analysis of the information variable V/L which according to Halstead (1977) provides a measure of the "intelligence quotient" of a message, produced a 35 percent agreement with teacher judgement. The mean of all sample passages on both L and V/L for the ATA text was between the other two texts, indicating that the information content of the text is at an appropriate level. Further work is necessary before making more definitive judgements on these measures.

In summary, it appears that on the basis of the best available measures of readability for mathematics materials, the ATA text does compare favorably in reading ease with popular texts currently in use. The consistency in the measures in ranking the texts provides support for the validity of this conclusion. The ATA text is, therefore, judged in the evaluator's opinion written at a suitable level for elementary algebra students.

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Directions for Counting Word and Symbol Tokens

For

Kulm and Kane Readability Formulas

What tokens to count.

Begin the sample with a title. Count the title and all words and symbols on the page. Include: (a) numerals and letters for ordering, (b) words or symbols on graphs, (c) symbols for figures such as Δ in Δ ABC, (d) placeholders, and (e) any punctuation symbol when used with a special meaning in mathematics (for example, the colon in $\{x:x > 2\}$). Do not count pictures or arrows, geometric figures, and punctuation which does not have a special mathematical meaning.

What are tokens?

Word tokens: Most word tokens are simply written words. Examples: *number, the, is, follow, answer, triangle.* In general a word token is separated from surrounding material by spaces. The number of word tokens in an abbreviation is determined by the number of different word tokens which the abbreviation replaces which have some representation in the abbreviated form. *U.S.A.* has three word tokens; *cm.* has one word token. Hyphenated word tokens are counted as one or more word tokens according to whether the parts can be used alone with meaning. *Non-linear* in one word token; *one-to-one* is three word tokens.

Math tokens: these are signs which appear in the language of mathematics which are not word tokens, punctuation, or drawings such as $\sqrt{\ }$, 2, +, ^{2}, and %. They are the smallest units which can be used independently to convey the intended meaning of that part of the written material. These rules may be helpful: (1) A graphic sign in which all parts are connected is at most one math token. For example, x is one math token. (2) A graphic sign in which all parts are not connected is more than one math token if two or more parts independently convey the intended meaning of part of the material or if the parts are separated by other tokens. For example: = and } are each one math token; $\geq$, x^2, 35, are each two math tokens; (a) is three math tokens.

Order in which tokens are counted.

Tokens when written on a math page are not always ordered from left to right as in ordinary English. The order imposed by vocalization should be followed. A vocalization is not necessarily unique but it should be consistent across uses of the formula on passages that will be compared. The expression $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ has tokens ordered as follows:

x, =, –, b, +, –, $\sqrt{}$, b, ^{2}, –, 4, a, c, –, 2, a .

Kane Readability Formula

Directions for Measuring Variables

Counting variable A: The number of words not on the Dale list of 3000 Familiar Words that are not on the List of Mathematics Words Familiar to 7th-8th grade students.

1. Circle all word tokens in the sample that are not on the Dale List of 3000 Familiar Words. Circle all words, even if they occur more than once. Do not consider numerals or symbols in this count. The Dale list and directions for using it are found in Appendix D.
2. Consider each of the circled words. If a word is not on the List of Familiar Mathematics Words, put a line through the word. The List of Familiar Mathematics Words is given in Appendix E.

3. Count the words that have both a circle and a line drawn through them. This count is variable A and should be entered on the worksheet.

Counting variable B. The number of changes from a word token to a math token and vice-versa. Letters as such, whether used for ordering, a variable, of a figure name, are considered math tokens for purposes of counting variable B. The count of variable B in the example below would be 7.

Example: In ① $\downarrow$ ΔABC, ② $\downarrow$ if ③ $\downarrow$ $\angle B=45^{\circ}$ ④ $\downarrow$ and ⑤ $\downarrow$ $\angle C=90^{\circ}$, ⑥ $\downarrow$

How many degrees are there in ⑦ $\downarrow$ $\angle A$?

Note: The arrows shown in this example indicate changes from word to math token or vice versa, and are not part of the original passage.

A figure should be considered in itself. That is, only count changes that occur in the figure itself. There is no count at the beginning or ending of a figure. The count for variable B should be entered on the worksheet.

Counting variable C: The number of different mathematics words on the List of Mathematics Words Not Familiar to 7th-8th grade students plus the number of different mathematics symbols not on the List of Mathematics Symbols Known by 7th-8th grade students.

Appendix F contains the List of Unfamiliar Mathematics Words. This list contains words that were tested with students in 1970 and which less than 80% of those tested said they knew.

Appendix G contains the List of Familiar Mathematics Symbols.

How to count math tokens was described above. It is important to note the difference between Appendices F and G. Appendix F contains unfamiliar words; Appendix G contains familiar symbols. Thus there will be a difference in methods of counting.

For each sample:

1. Count the number of word tokens in the sample that are on the List of Unfamiliar Mathematics Words found in Appendix F. Directions for making the count are in Appendix F.

Count the number of these word tokens that are different. That is, if 'commutative' were used four times in the sample it would only count as *one* unfamiliar word in this count. However, if different word forms are used they are counted as different word tokens in this count. Thus 'commutative' and 'commutativity' would both be counted. This is count C1 and should be entered on the worksheet.

2. Count the number of math tokens in the sample that are not on the List of Familiar Mathematics Symbols found in Appendix G.

Count the number of these mathematics tokens that are different. The 2 in x^2 and b_2 are counted as different because of position. A symbol is considered familiar only if it is in a context similar to that given in the appendix. for example, - in 8 - 3 is familiar, but - in $^-4$ is not familiar. This is count C2. It should be entered on the worksheet. The sum of variables C1 and C2 is variable C and should be entered on the worksheet.

Counting variable D: The number of question marks. Merely count the number of '?' in the sample and record this number on the worksheet.

Kulm Readability Formula

Directions for Measuring Variables

Counting Variable X1: The percentage of mathematical symbols (tokens). Count the total number of math tokens. Divide by the total number of all tokens, and multiply by 100 to obtain the percentage of math tokens, to the nearest unit.

Counting Variable X2: The percentage of reader-directed sentences. Count the number of sentences that contain a form of the pronoun "you," or that are imperative, or that are questions. Count each sentence only once, even if it has more than one of these properties. Divide by the total number of sentences and multiply by 100. Note: The last complete sentence is counted as the last sentence.

Counting Variable X3: Average sentence length. Count the total number of tokens included up to the last complete sentence. Divide by the number of complete sentences. Equations are counted as sentences unless they are included within a sentence that has word tokens, in which case they are counted as tokens of that sentence.

Counting Variable X4: The percentage of math vocabulary words. Count the total number of words that are on the List of Mathematical Terms (Kane, et al, Appendix A). Count a word every time it appears. Divide by the total number of all tokens, and multiply by 100.

Teacher Judgement

Directions: You are asked to evaluate the reading difficulty of several passages from elementary algebra books. First read each passage, one at a time, with attention to the criteria listed below.

After reading a passage, please use the rating scale provided to indicate your judgement of the passage for elementary algebra students, grades 8-10. This rating should be your overall rating of the passage according to difficulty of comprehension. Note that "high" means high difficulty and "low" means low difficulty of reading level.

Interest level of this mathematical topic

Comprehensibility of author's writing style

Quality of examples - effectiveness in making the point clear

Author's anticipation of readers' questions

Average sentence length

Average word length

Average complexity of sentence structure

Overall difficulty of math symbols

Overall difficulty of math vocabulary words

Appropriateness of illustrations (includes graphs, lists, tables, figres, pictures, diagrams, etc.)

Ease with which illustrations can be understood

Number of illustrations

Number of examples

Number of questions

Number of math symbols

Number of vocabulary words

Number if different words having 3 or more syllables

Number of math concepts contained in the passage

TABLE 1

Topic, Page Numbers, and Token Length for Textbook Samples

Passage	Topic	ATA[a] pages	Holt pages	Holt tokens	H-M[b] pages
1	Variables	79-80	4-5	229	4-5
2	Adding Integers on No. line	128	21-22	300	35-36
3	Division	217	31-32	300	78-79
4	Solving $x+a=b$	280-281	59-60	285	48-49
5	Powers of monomials	500	122	300	122-23
6	Scientific Notation	511	140	251	148
7	Ratio	257	261	211	201
8	Inequality	286	94-95	196	255
9	Function	690	317	279	287
10	Linear Functions	699	276	270	311-12
11	Solving linear systems	622	293	300	346
12	Square roots	518	384-385	300	382
13	Quadratic functions	709	320	194	426
14	Solving $ax+b=c$	319	85	300	61
15	Meaning of $a \cdot b=0$	670	158	300	180
16	Problem set[c]	136	82&86	200	105-106
17	Problem set	278	209	200	204
18	Problem set	372	304-305	200	331
19	Problem set	508	355&367	200	393
20	Problem set	619&621	433	200	268

[a] passages 1-15 were 300 tokens long.

[b] passages 1-15 were 300 tokens long except for passage 2(200) and passage 11(290).

[c] passages 16-20 were 200 tokens long for all texts.

TABLE 2

Values for Reading Complexity Measures for
Algebra Through Applications

Passage	Kane Formula	Kulm Formula	Teacher Judge	Language Level
1	28.82	19.48	4.66	.1139
2	32.73	19.58	4.25	.1477
3	31.08	22.06	4.00	.1266
4	34.64	19.04	4.66	.0980
5	27.74	16.57	3.33	.1163
6	26.79	16.45	5.00	.1318
7	26.16	23.95	4.50	.1196
8	29.64	22.03	3.33	.1135
9	28.24	19.79	4.00	.1471
10	23.35	15.12	2.25	.1307
11	30.24	17.23	5.00	.1110
12	34.66	17.07	3.33	.0888
13	28.21	13.92	3.66	.1298
14	31.19	17.71	3.00	.1308
15	33.80	19.45	4.00	.0793
16	33.64	23.75	2.66	–
17	37.24	21.37	5.33	–
18	32.82	22.72	1.75	–
19	39.66	23.71	4.00	–
20	35.68	24.12	4.33	–
Means	31.32	19.76	3.85	.1190

TABLE 3

Values for Reading Complexity Measures for Holt Algebra

Passage	Kane Formula	Kulm Formula	Teacher Judge	Language Level
1	32.79	18.05	5.00	.1254
2	35.51*	17.40	5.00	.1515
3	30.71	13.04	5.75	.2083
4	32.26*	16.50	4.00	.1192
5	29.31	11.91	4.00	.1798
6	27.37*	16.09	5.33	.3330
7	30.50*	18.84	5.75	.0584
8	29.54*	15.62	4.50	.1295
9	29.31*	15.13	4.33	.0809
10	29.79*	14.64	4.00	.1404
11	30.59	14.13	5.50	.0973
12	31.11	12.97	4.00	.1269
13	21.54*	16.62	4.00	.1106
14	32.92	14.58	4.33	.1011
15	34.62	14.03	4.75	.0854
16	37.06	24.43	3.00	–
17	39.12	23.90	4.33	–
18	37.10	22.33	4.66	–
19	35.78	24.84	4.66	–
20	37.71	24.99	3.50	–
Means	32.23	17.50	4.52	.1365

*Corrected for length

TABLE 4

Values for Reading Complexity Measures for
Houghton - Mifflin "Red" Algebra 1

Passage	Kane Formula	Kulm Formula	Teacher Judge	Language Level
1	27.51	18.11	3.50	.0882
2	34.06*	18.46	5.33	.1269
3	30.98	14.04	4.00	.0629
4	29.70	17.73	3.75	.0752
5	30.08	10.38	2.50	.0888
6	23.08	14.09	4.00	.0818
7	29.48	17.53	4.00	.0879
8	36.21	19.07	4.50	.1058
9	27.49	18.71	3.00	.0651
10	29.96	13.13	3.66	.0624
11	24.96*	18.09	4.33	.1181
12	25.55	18.24	4.00	.0908
13	27.75	13.05	3.50	.1073
14	25.16	18.51	4.66	.1199
15	29.74	17.77	3.33	.0909
16	35.65	21.79	3.25	-
17	34.36	21.90	4.00	-
18	32.44	22.90	5.00	-
19	33.14	21.98	4.33	-
20	30.21	22.65	2.25	-
Means	29.87	17.91	3.84	.0915

*Corrected for length

TABLE 5

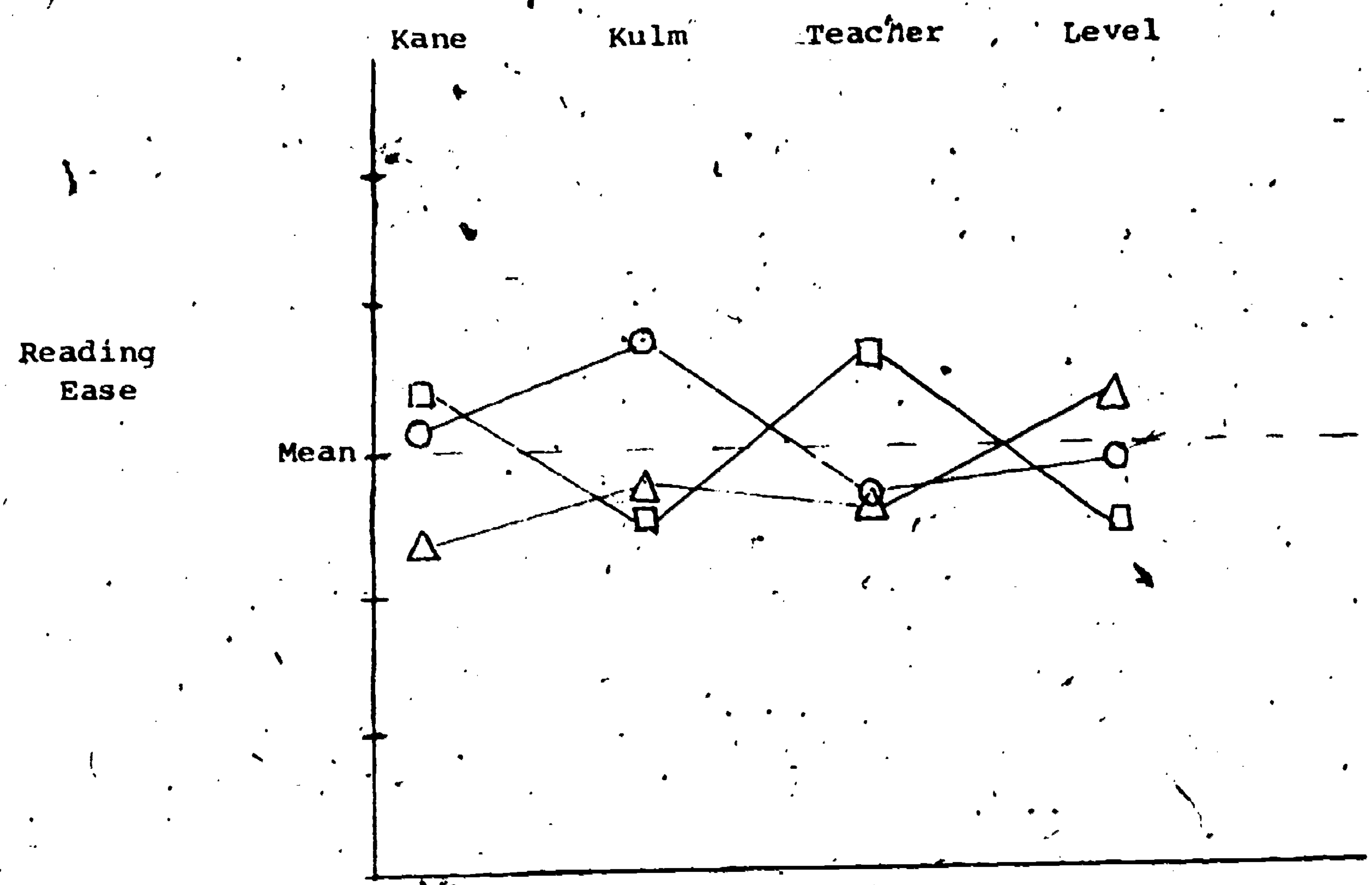
Composite Reading Complexity Score
for Each Passage

Passage	ATA	Holt	H-M
1	9	11*	6
2	7	7	10*
3	9.5*	8	8.5
4	11*	6	7
5	8*	8*	8*
6	9*	9*	6
7	7	11*	6
8	8	5.5	10.5*
9	8	9*	7
10	7	8	9*
11	9*	9*	6
12	9*	6.5	8.5
13	8	9*	7
14	6	9*	9*
15	10*	9	5
Means	8.37	8.33	7.57

*Highest composite score in this topic.

Readability Profile for Passages

$\overline{A}$___ , $\overline{H}$___ , $\overline{M}$___



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

(A) Algebra through Applications

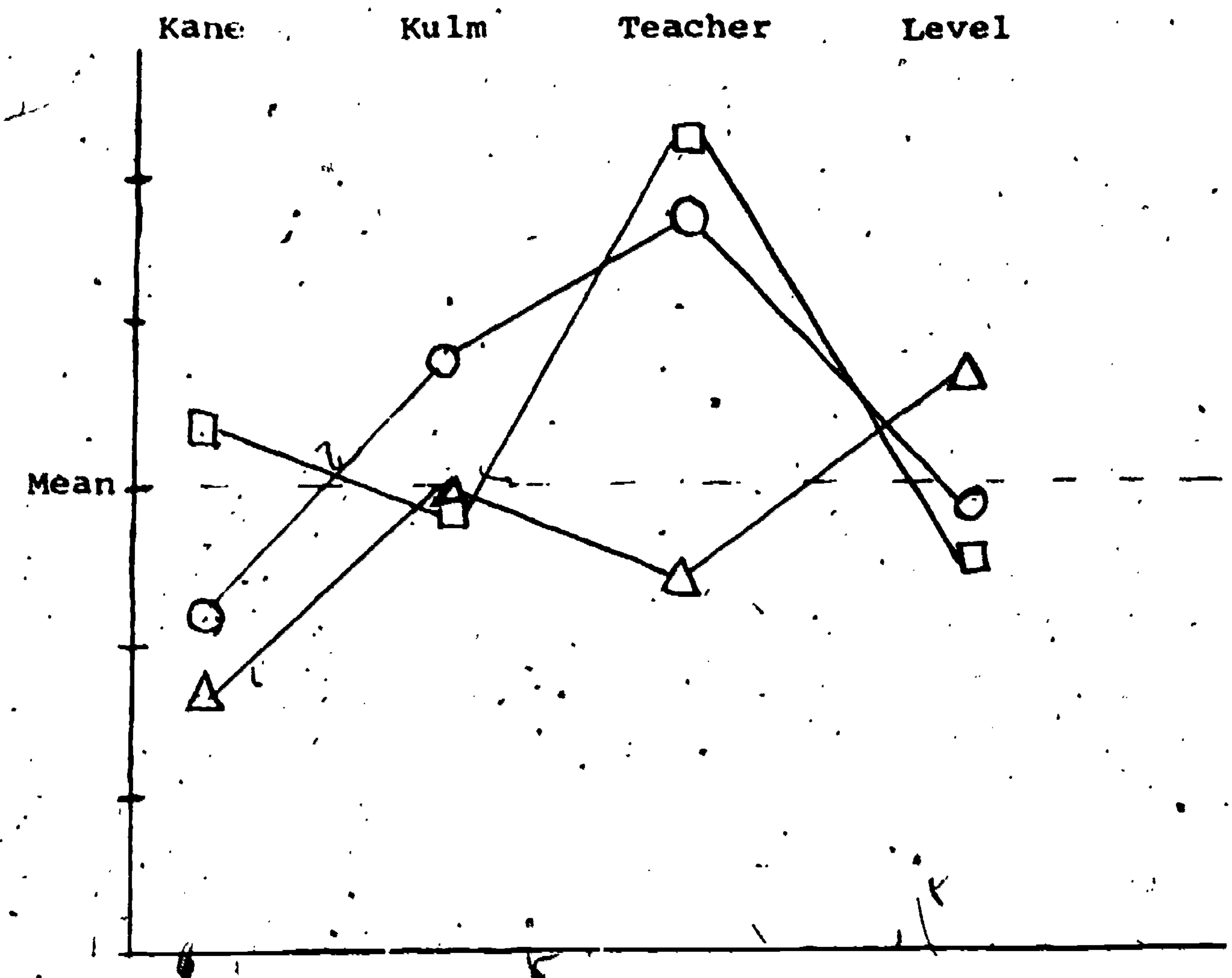
[H] Holt, Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 1 , H 1 , M 1



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

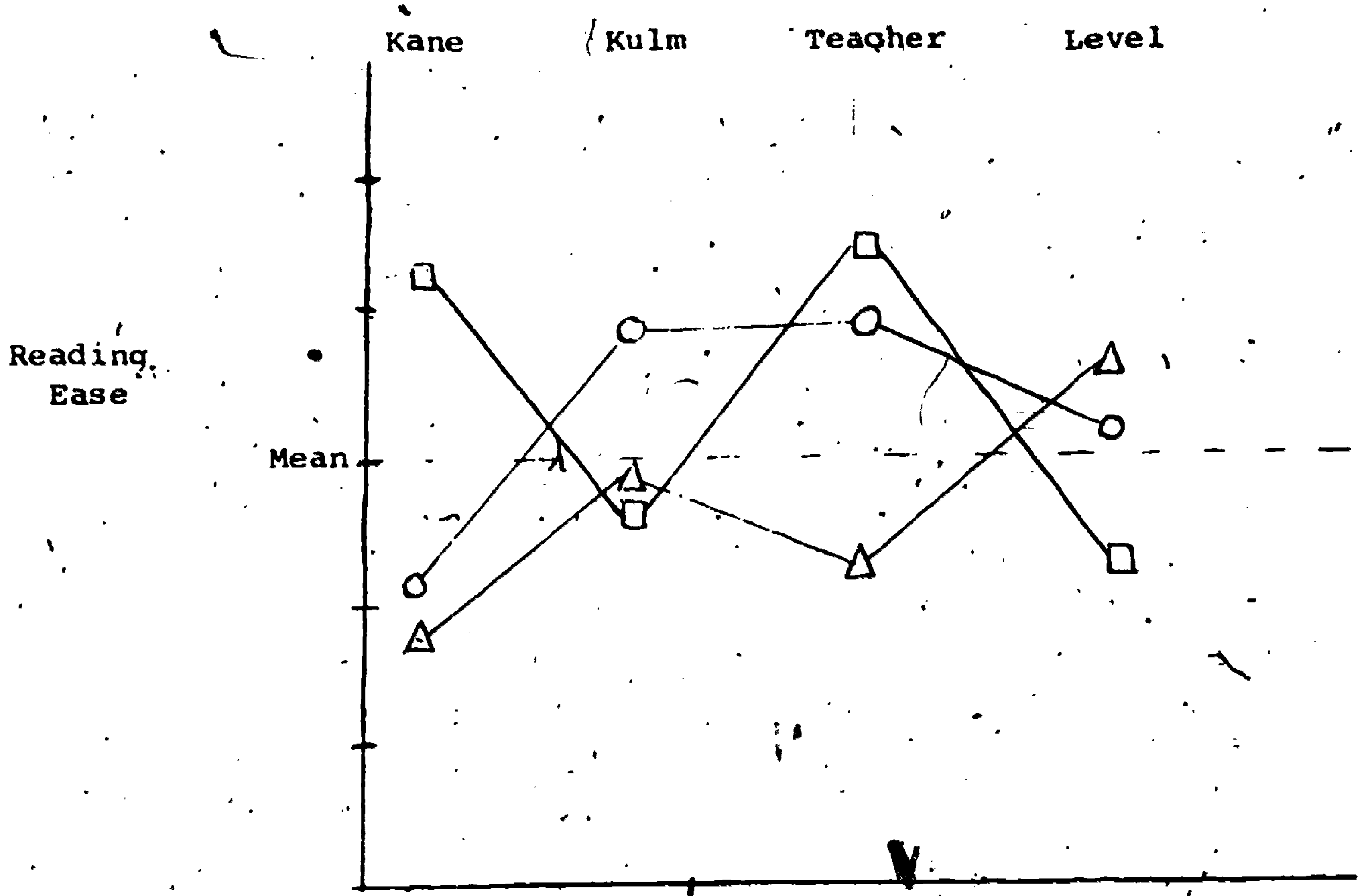
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 2 , H 2 , M 2



Legend

(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

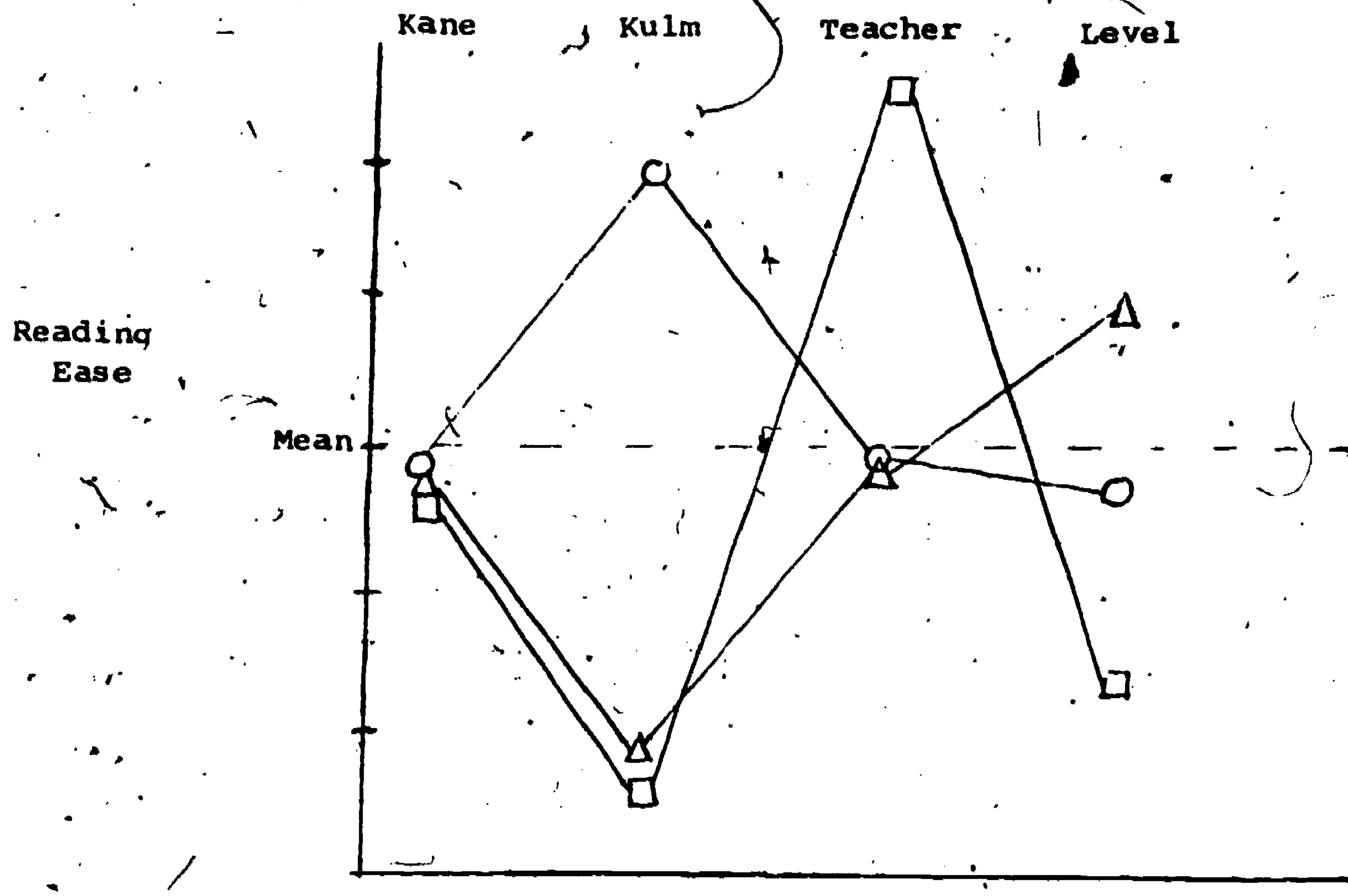
Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 3 , H 3 , M 3



<u>Scales</u>

The means and intervals on the vertical axis correspond to the following values:

<u>Legend</u>

(A) Algebra through Applications

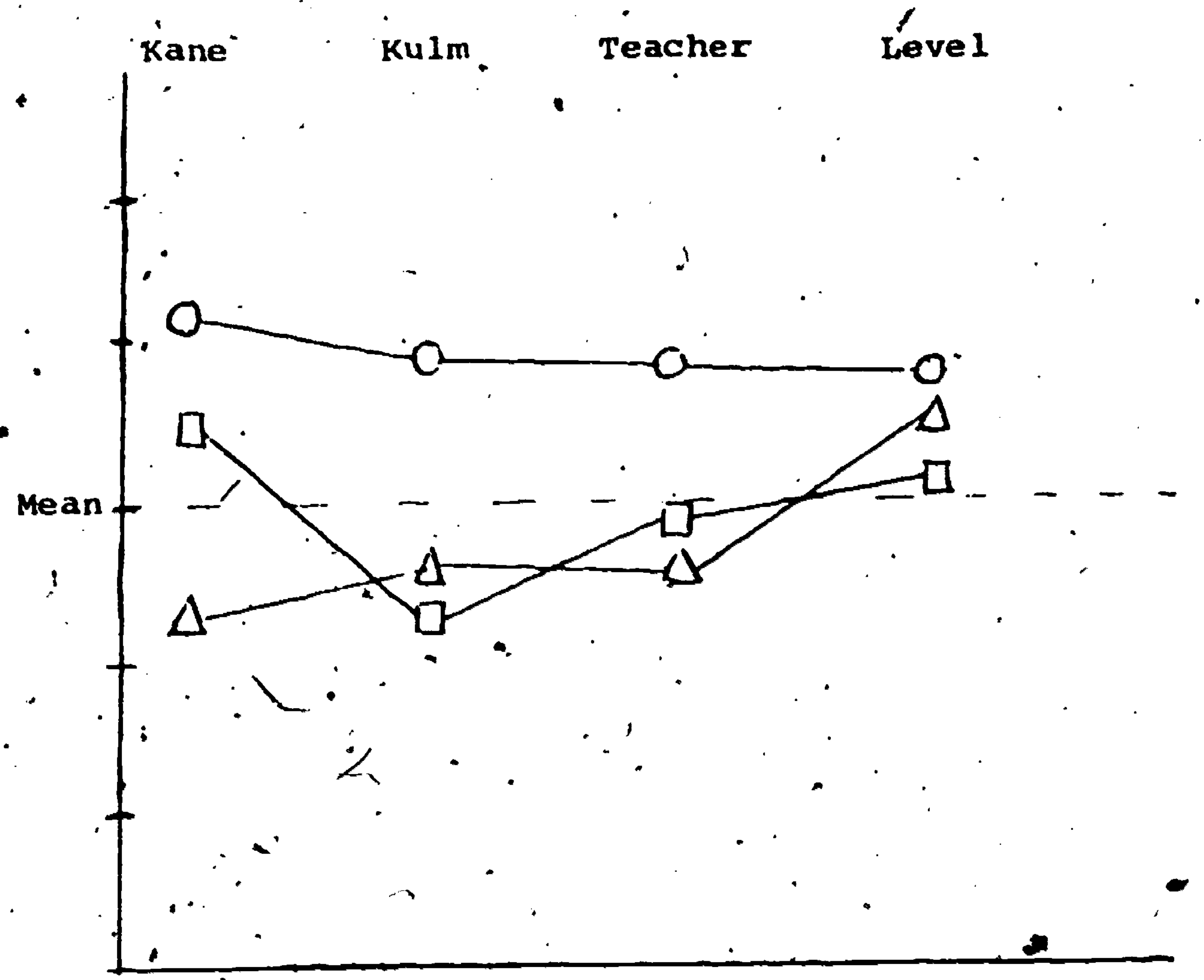
[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 4 , H 4 , M 4



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

(A) Algebra through Applications

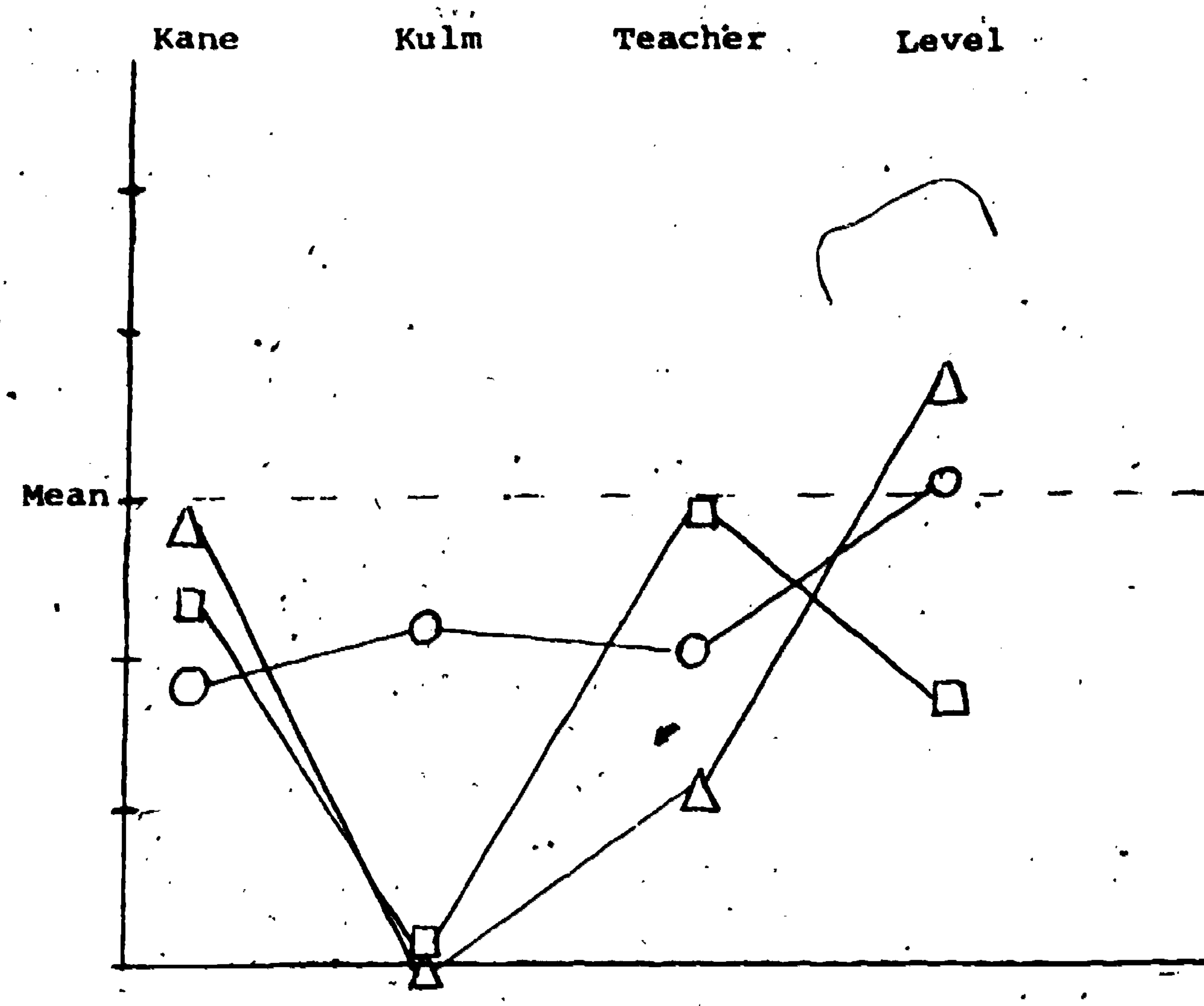
[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 5 , H 5 , M 5



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

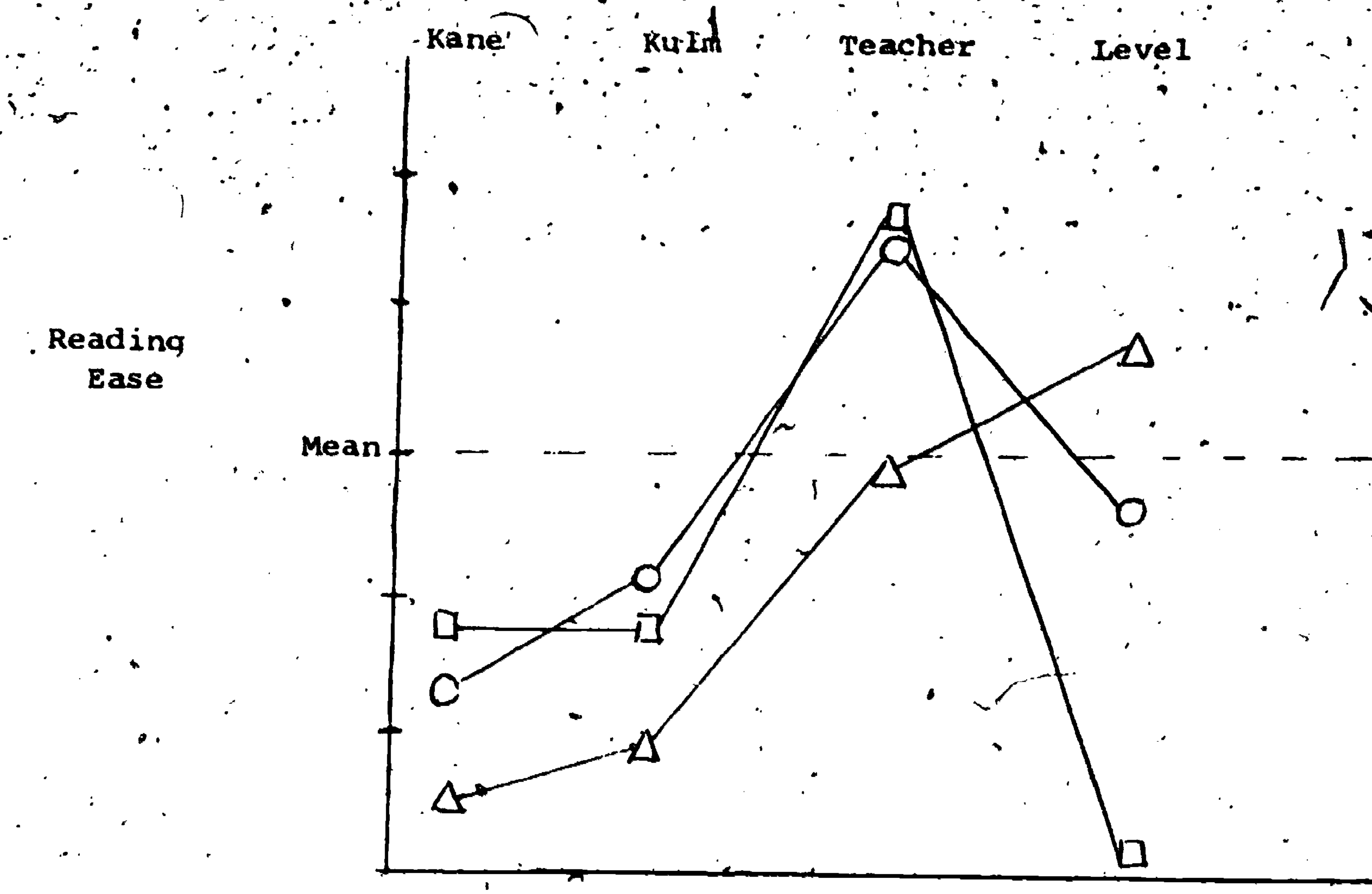
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 6 , H 6 , M 6



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.05[illegible]

Legend

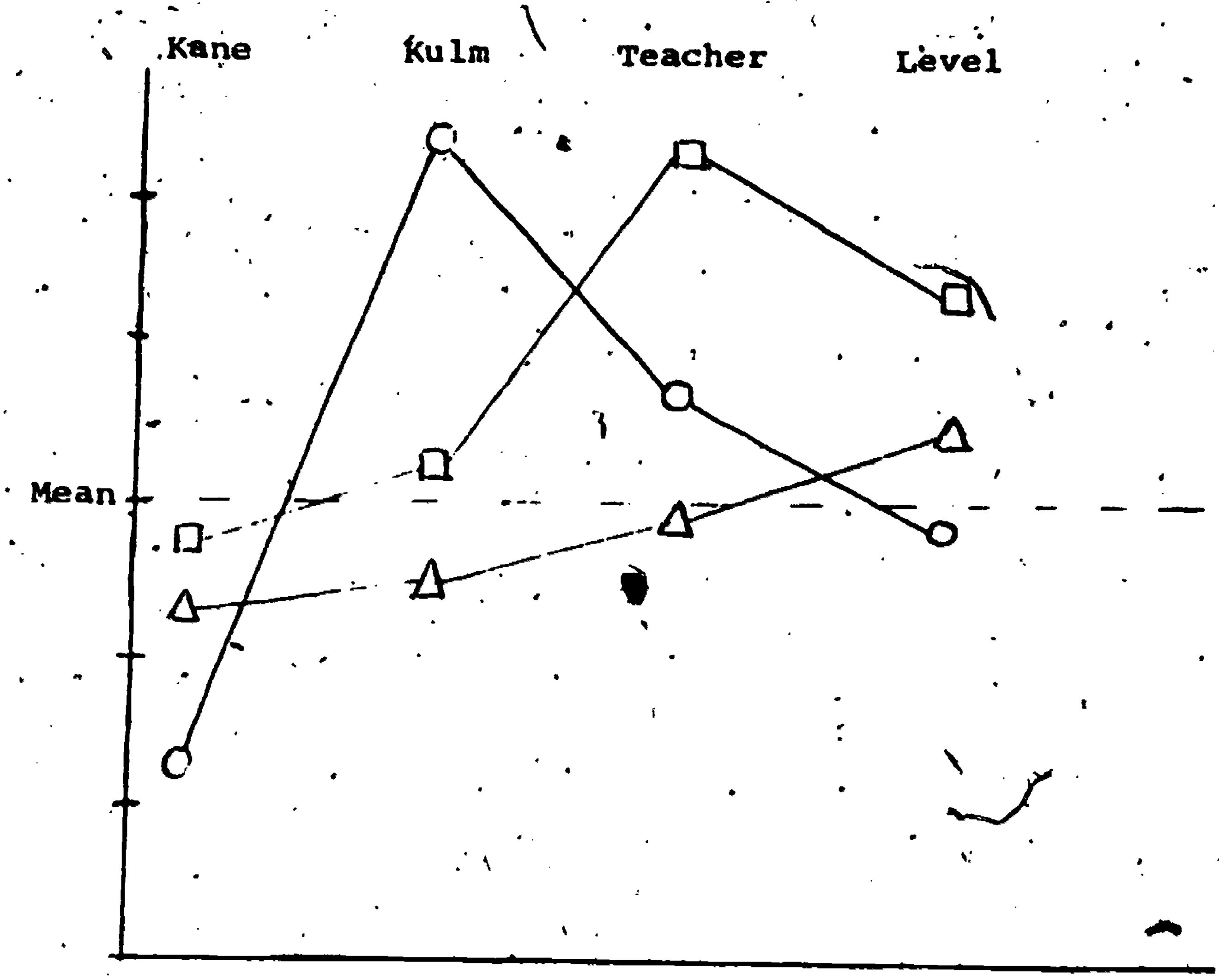
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 7 , H 7 , M 7



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

A Algebra through Applications

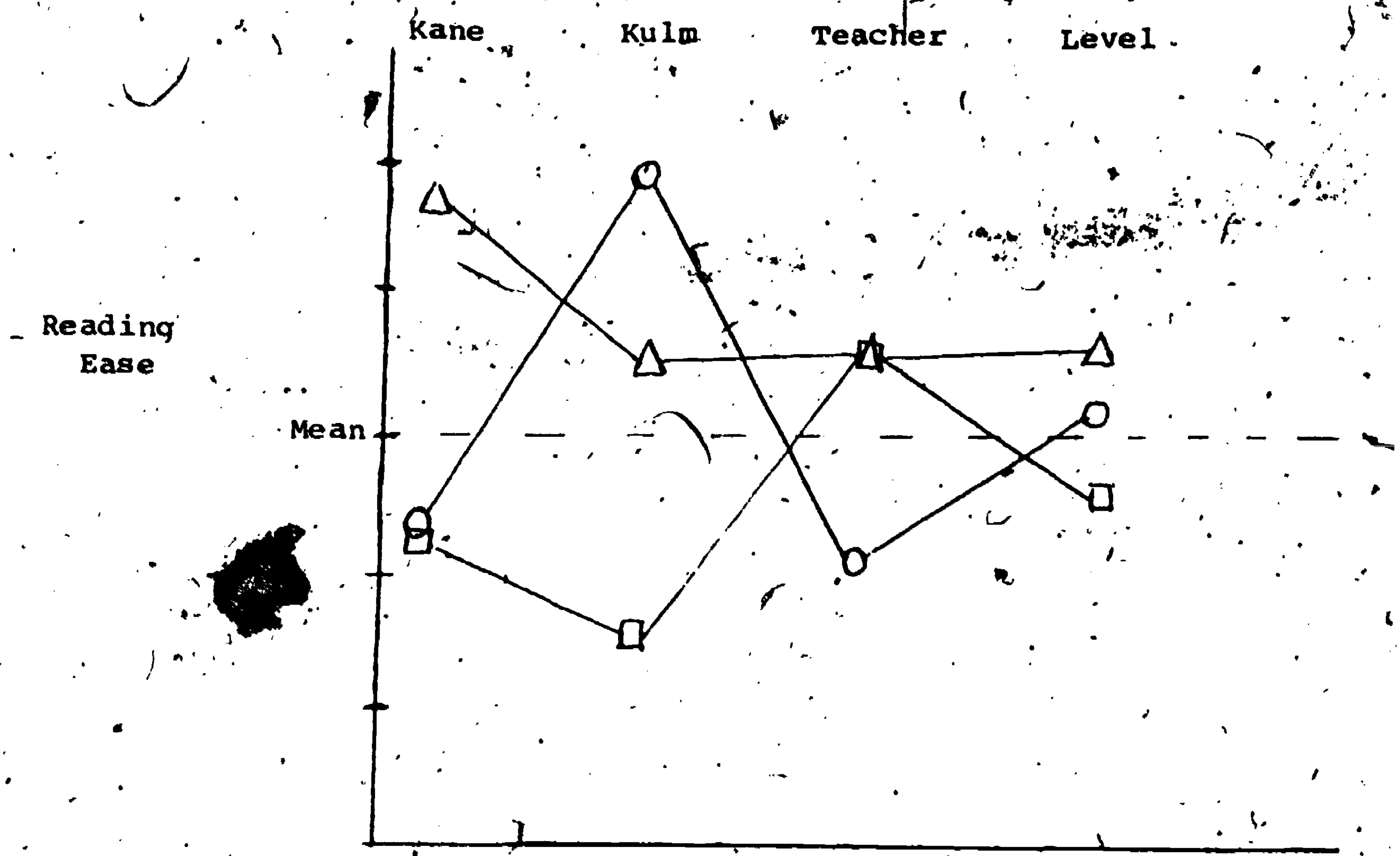
H Holt Algebra I

M Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passage

A 8 , H 8 , M 8



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

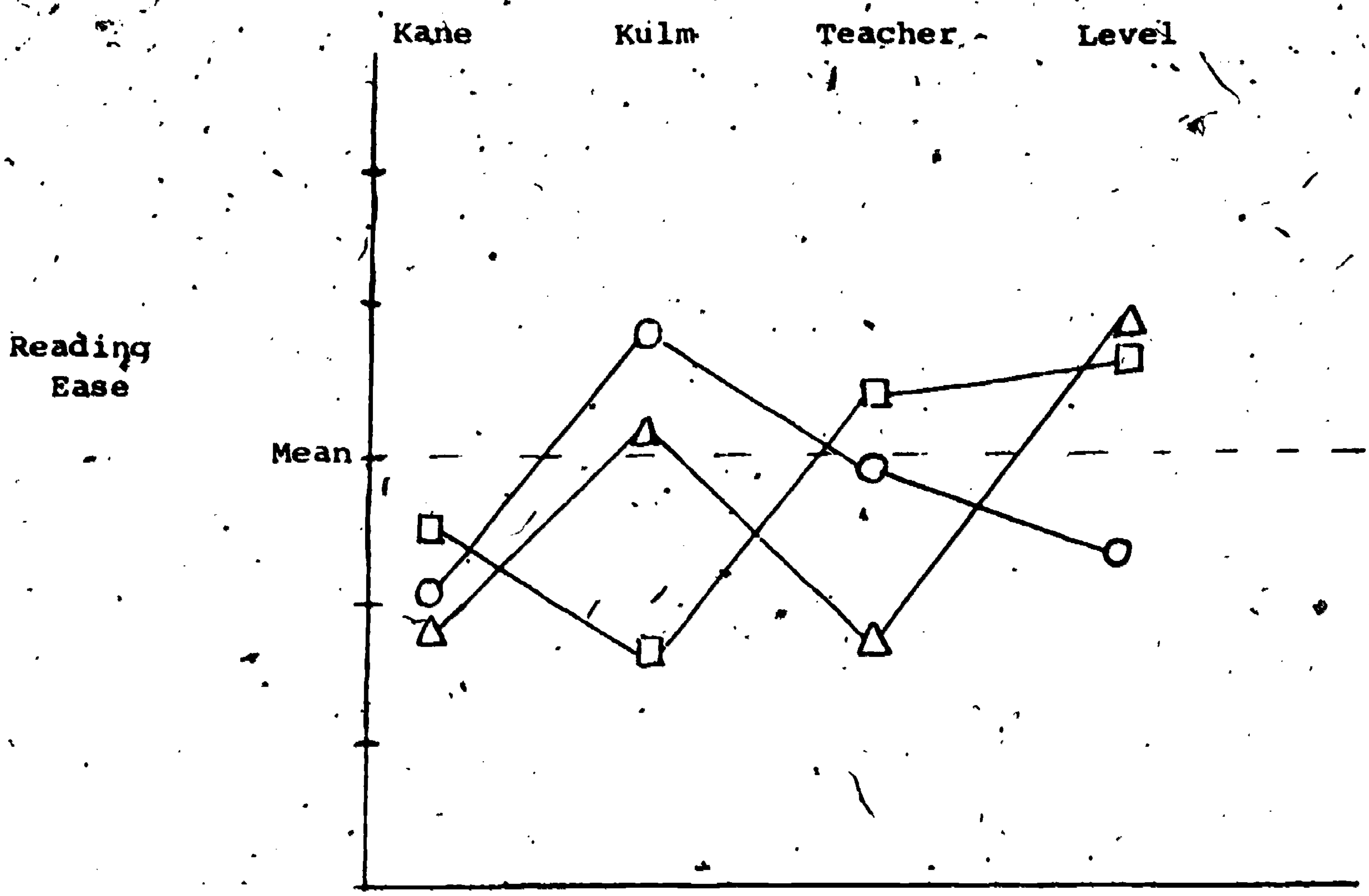
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 9 ; H 9 , M 9



<u>Scales</u>

The means and intervals on the vertical axis correspond to the following values:

<u>Legend</u>

(A) Algebra through Applications

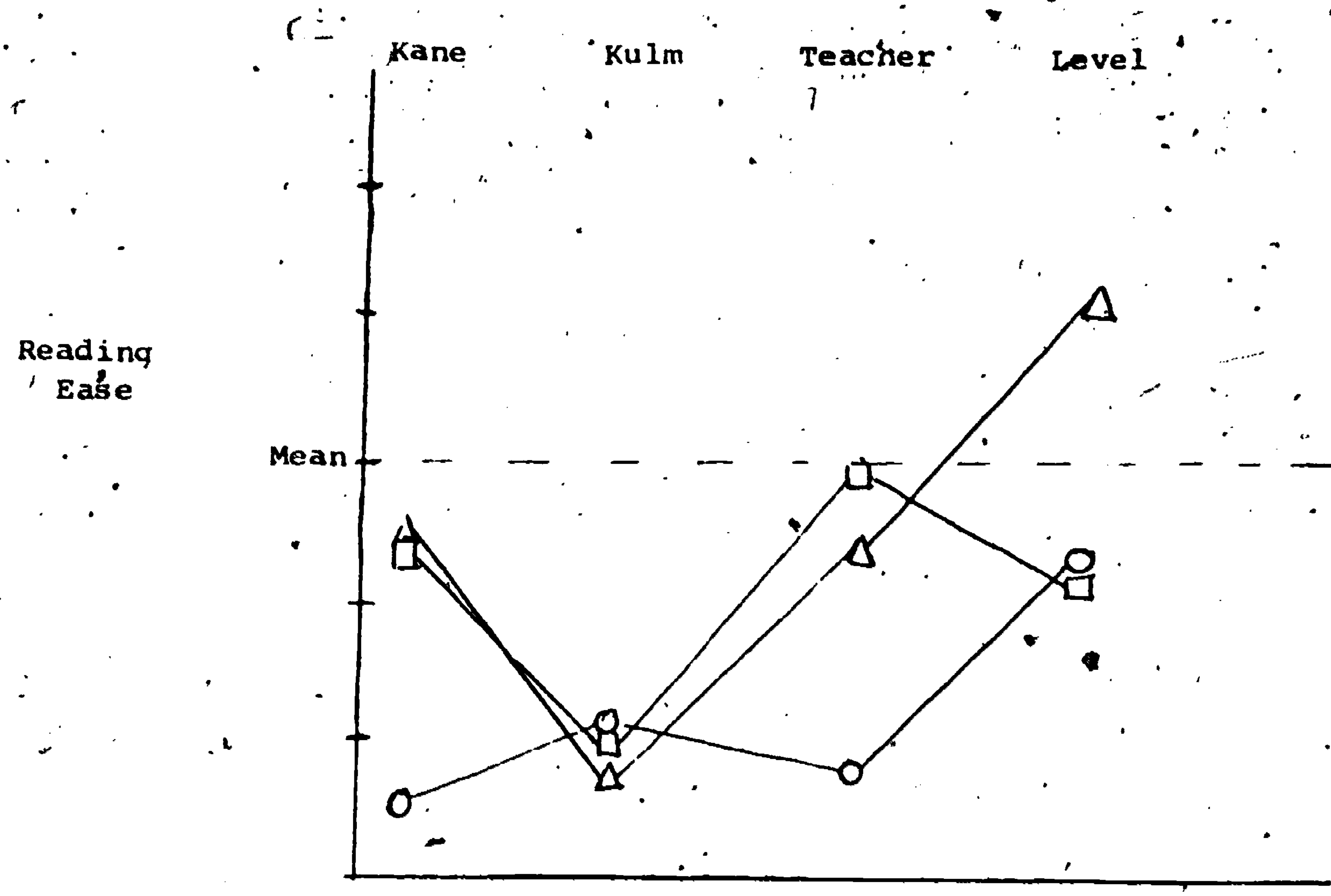
[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 10, H 10, M 10



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

(A) Algebra through Applications

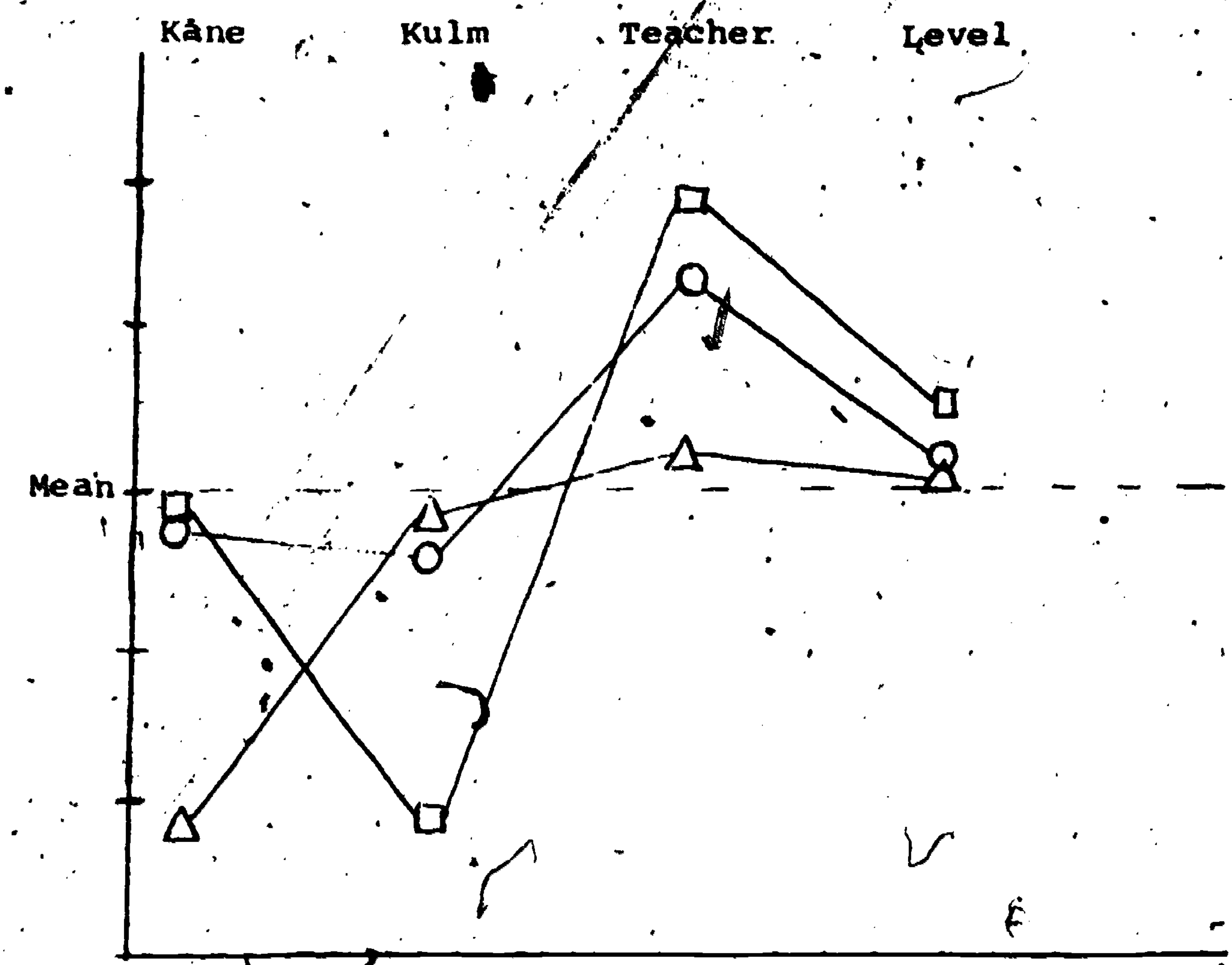
[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 11 , H 11 , M 11



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

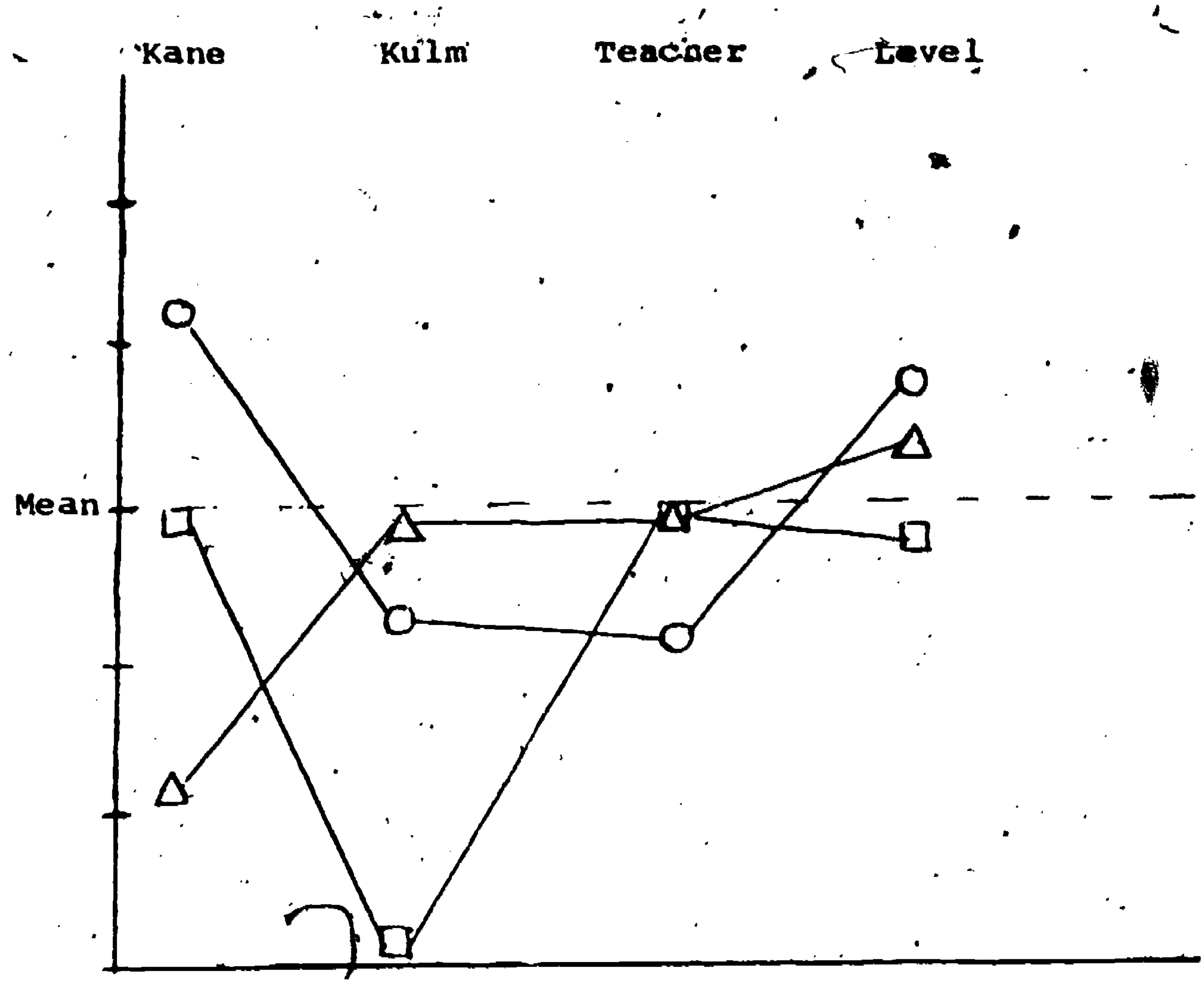
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 12 , H 12 , M 12



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

(A) Algebra through Applications

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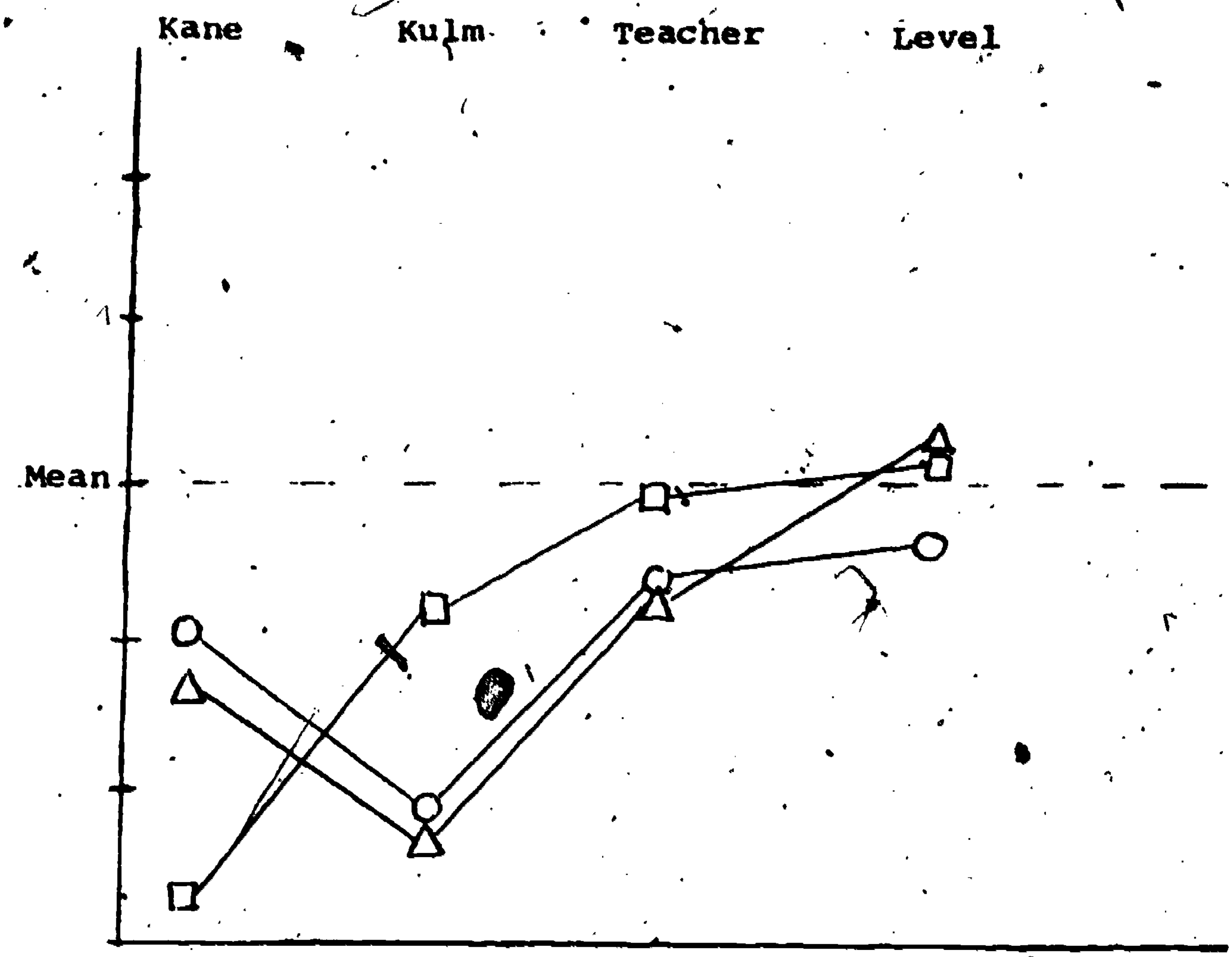
Readability Profile for Passages

A 13 , H 13 , M 13



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

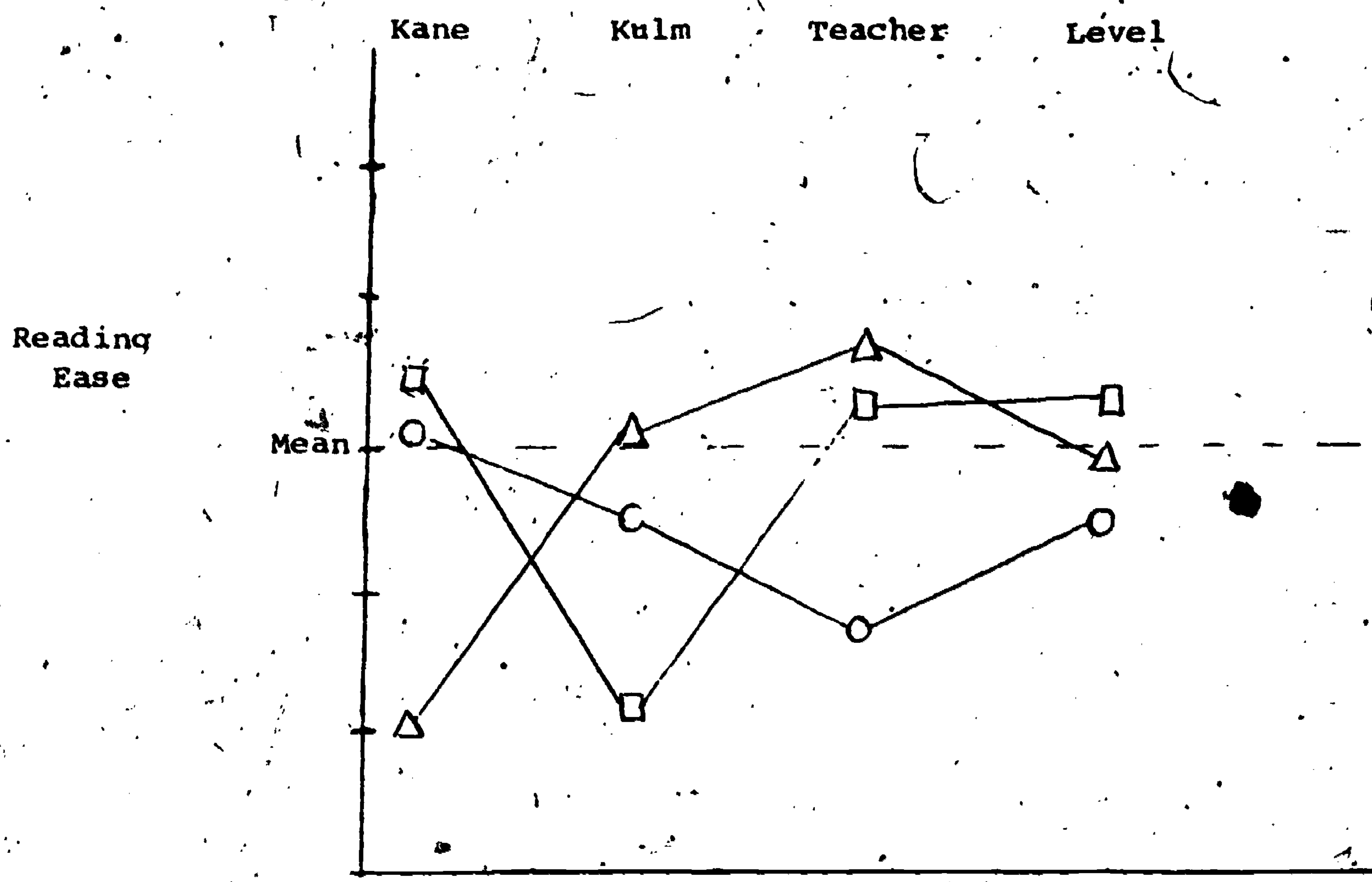
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 14 , H 14 , M 14



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

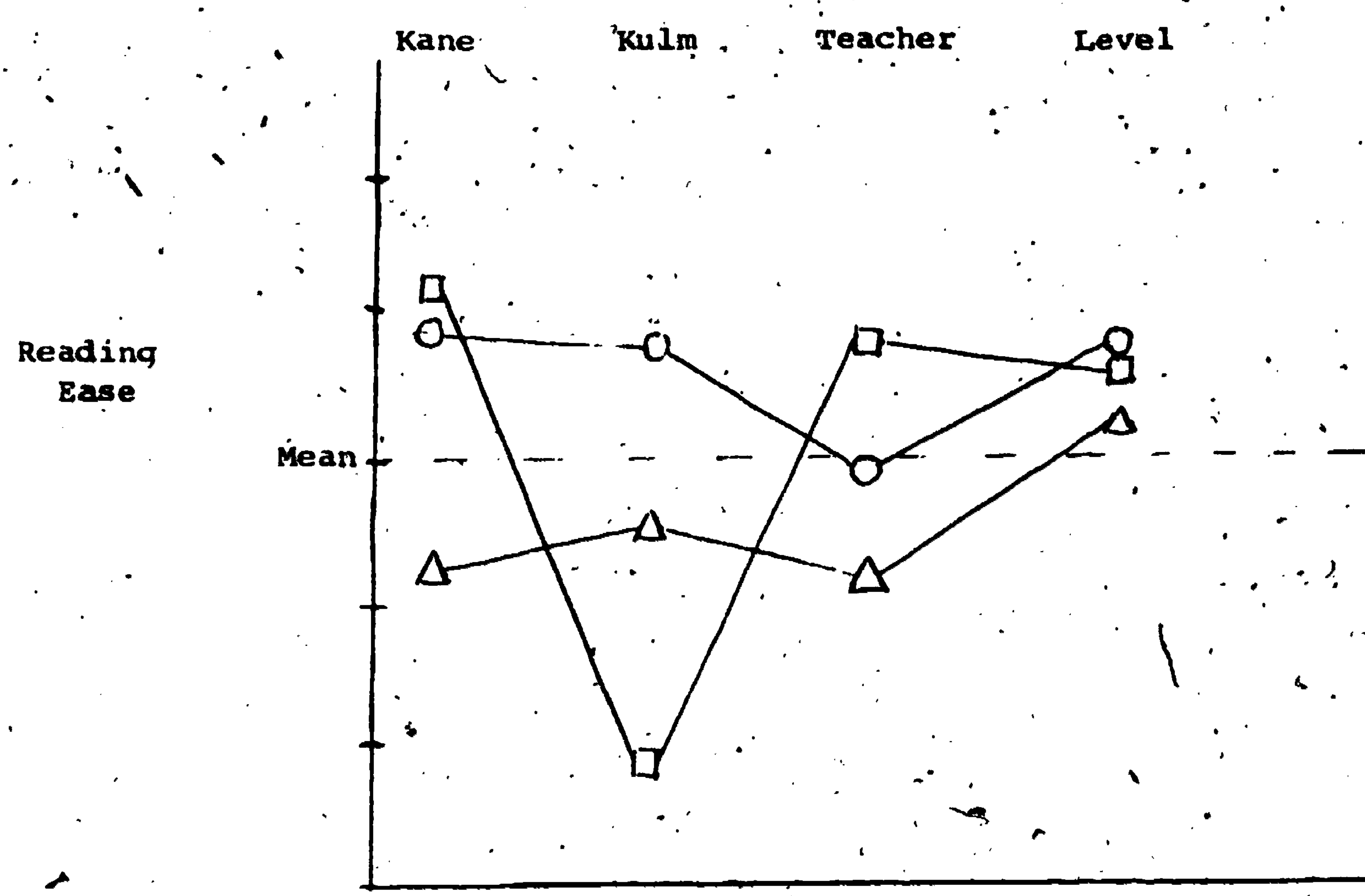
(A) Algebra through Applications

[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 15, H 15, M 15



Scales

The means and intervals on the vertical axis correspond to the following values:

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Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

A Algebra through Applications

H Holt Algebra I

M Houghton-Mifflin Algebra Book I

Readability Profile for Passages

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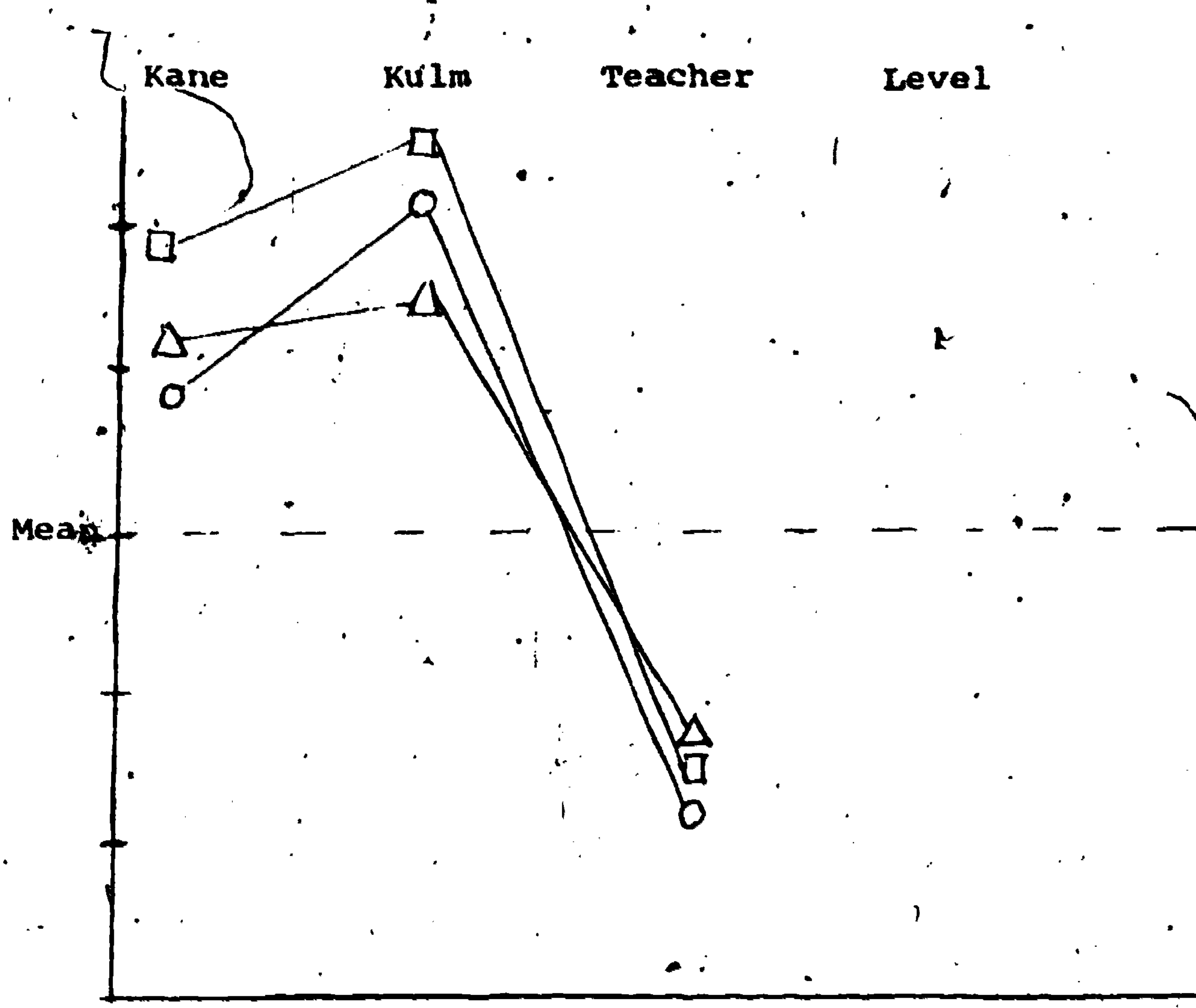


Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

(A) Algebra through Applications

[H] Holt Algebra I

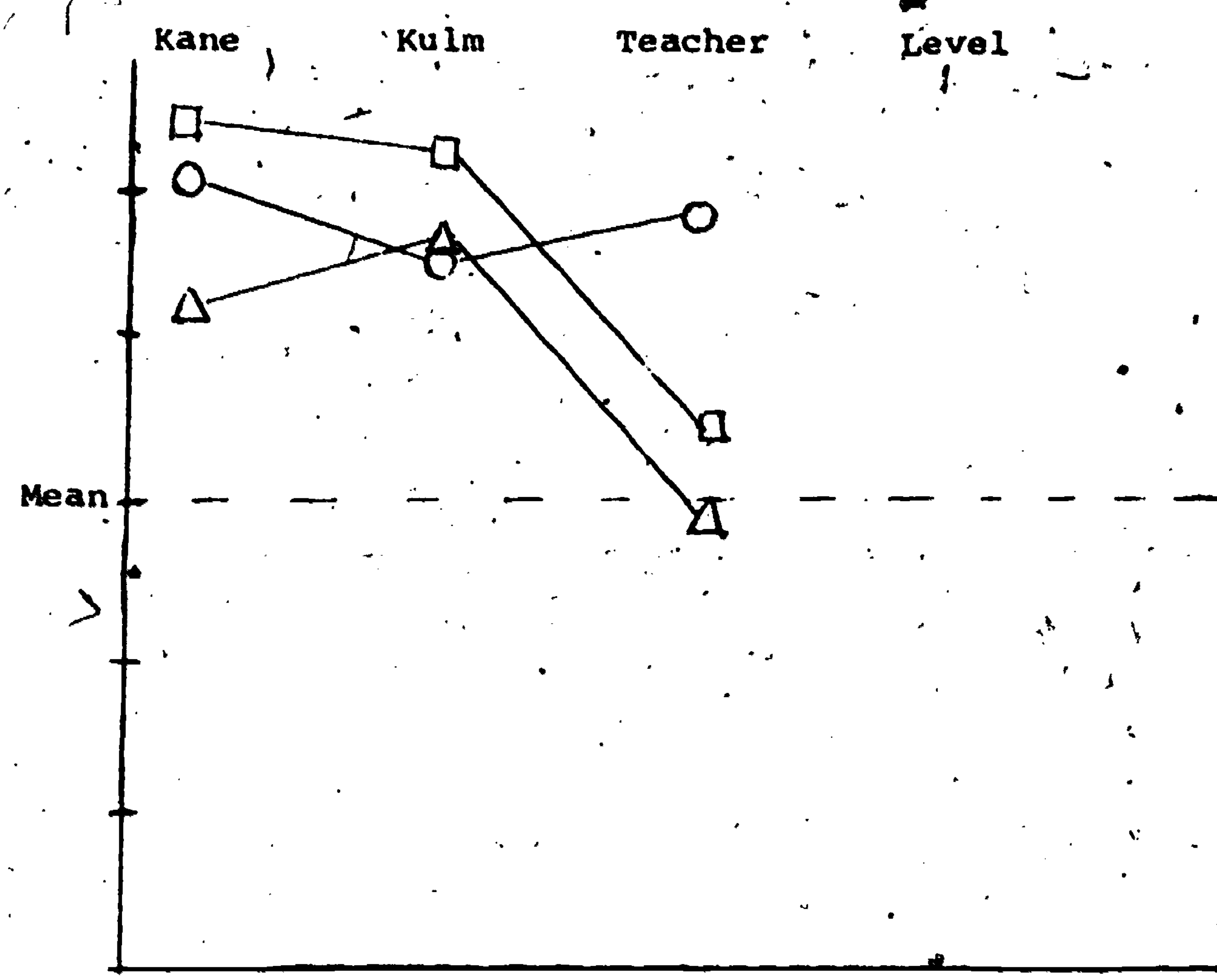
(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 17 , H 17 , M 17

Reading Ease



Scales

The means and intervals on the vertical axis correspond to the following values:

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Legend

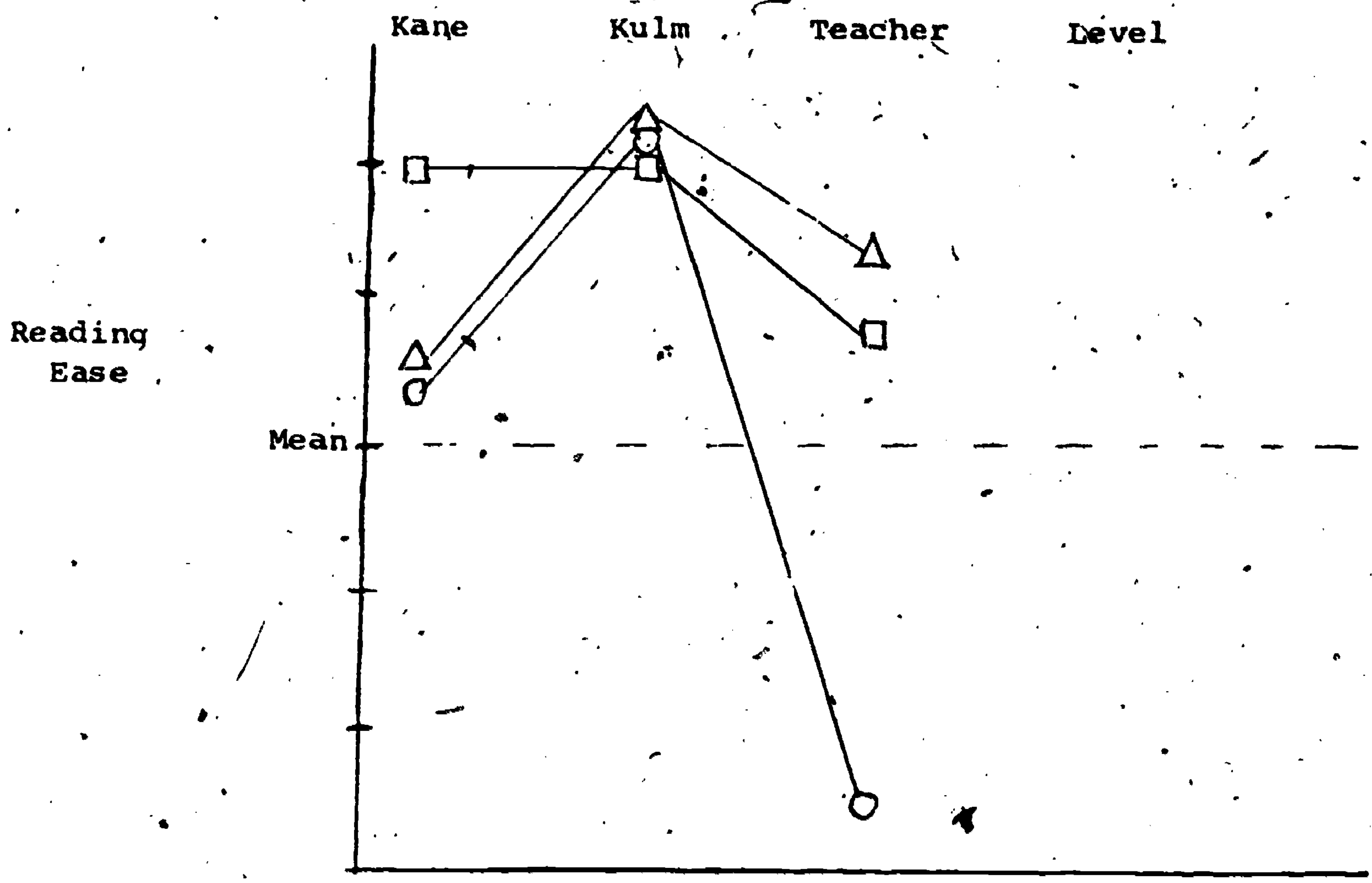
(A) Algebra through Applications

H Holt Algebra I

M Houghton-Mifflin Algebra Book I

Readability Profile for Passages

A 18 , H 18 , M 18.



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

(A) Algebra through Applications

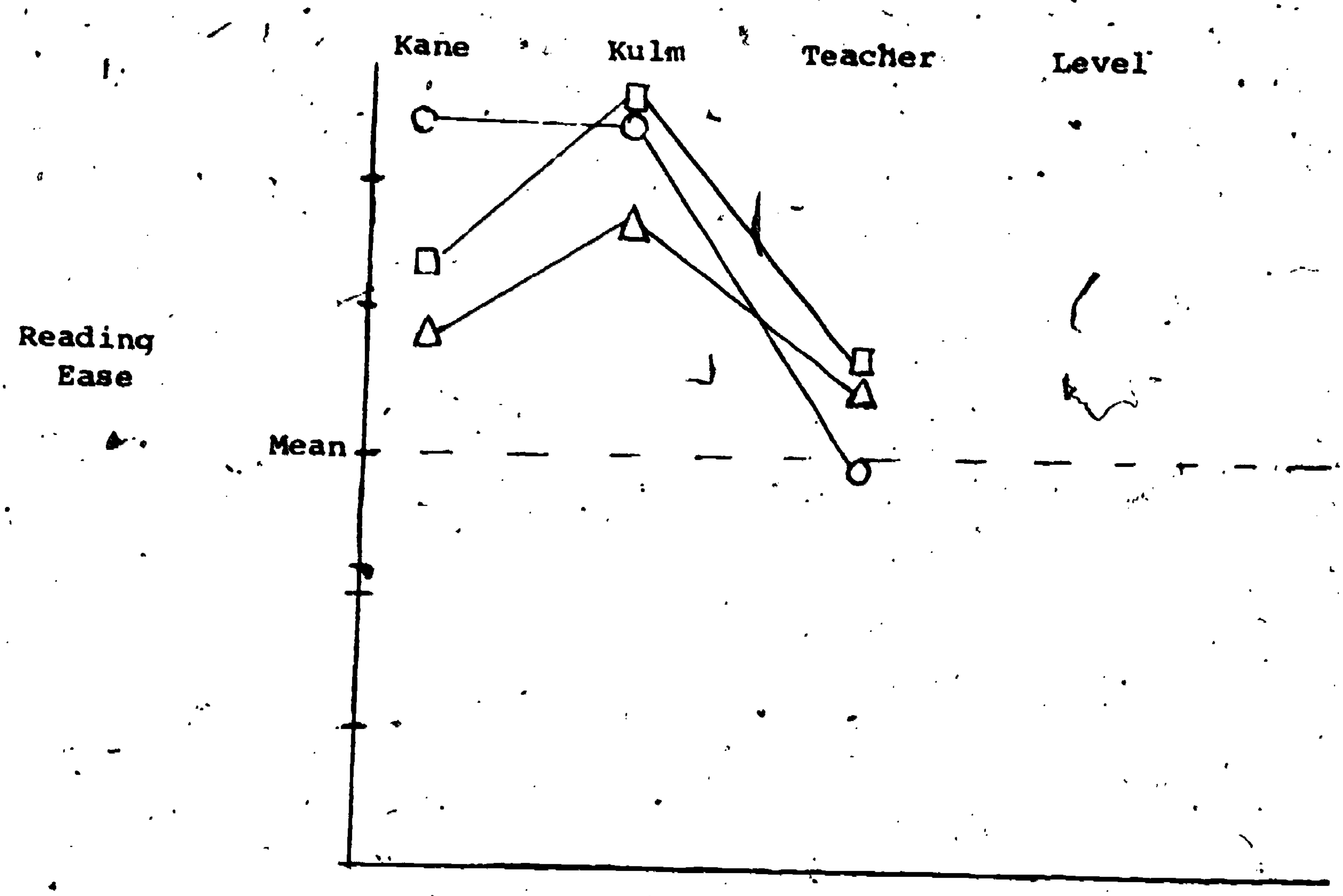
[H] Holt Algebra I

(M) Houghton-Mifflin Algebra Book I

Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 19 , H 19 , M 19



Scales

The means and intervals on the vertical axis correspond to the following values:

Legend

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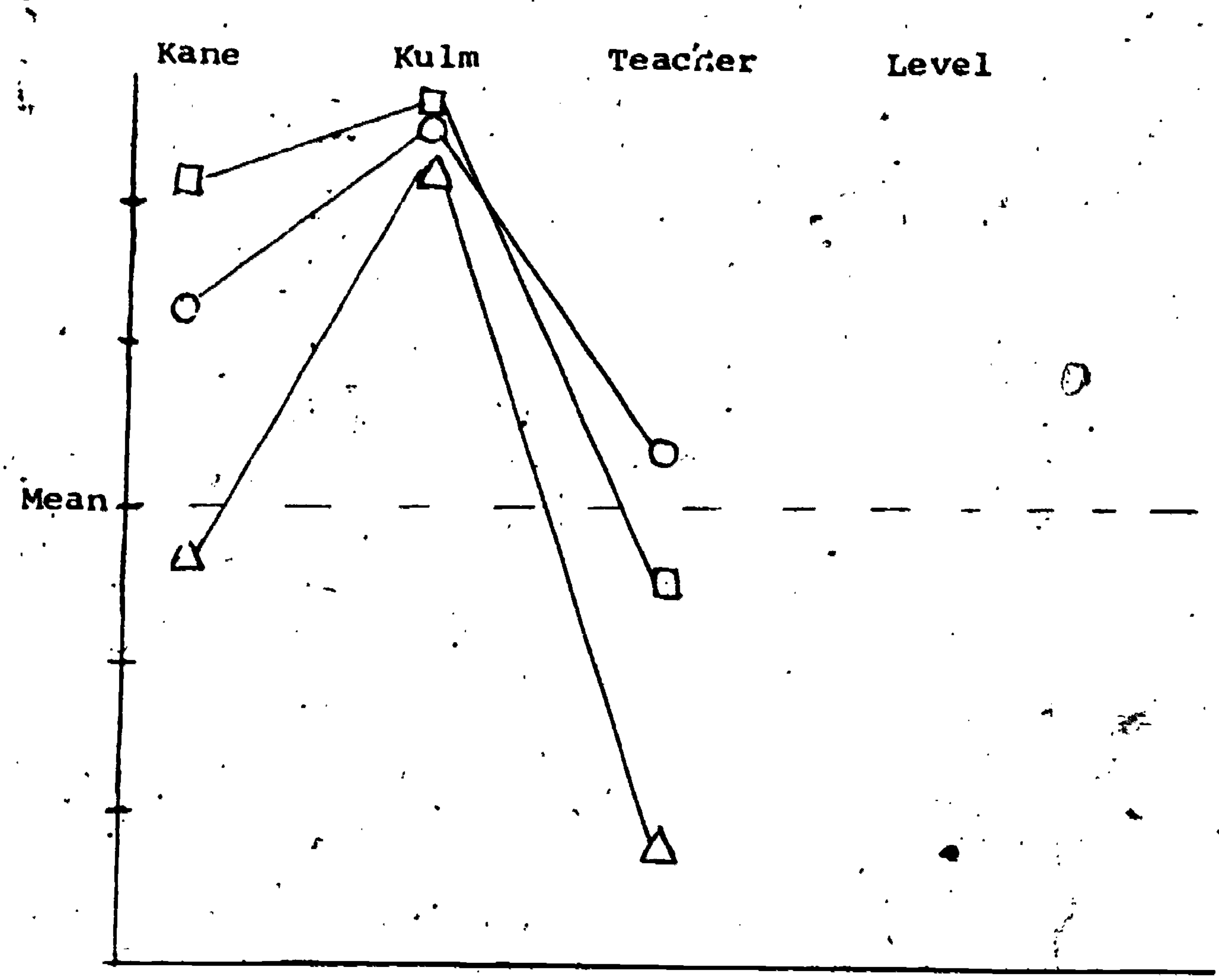
Measure	Grand Mean	Interval
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
Teacher Judgement:	4.07	0.75
Level:	0.116	0.050

Readability Profile for Passages

A 20 , H 20 , M 20



<u>Scales</u>

The means and intervals on the vertical axis correspond to the following values:

<u>Measure</u>	<u>Grand Mean</u>	<u>Interval</u>
Kane formula:	31.14	3.00
Kulm formula:	18.39	2.00
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Level:	0.116	0.050

<u>Legend</u>

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APPENDIX G

NOTIFICATION OF PROPOSED STUDY

NOTIFICATION OF PROPOSED STUDY

ALGEBRA THROUGH APPLICATIONS

The four sheets which follow constitute an attempt to describe the materials of the First-Year Algebra Via Applications Development Project, an NSF-funded project centered at the University of Chicago.

Included in this description are the following:

Sheet 1: Summary description of the course

Sheet 2: Table of Contents of the materials

Sheet 3: A sample lesson from the materials
"Estimating Wildlife Populations" (Ch. 5, Lesson 1)

Sheet 4: A sample lesson from the materials
"Slope" (first four pages) (Ch. 7, Lesson 3)

Sheet 5: Four sample pages from the materials
problems from
"Describing Patterns Using Variables" (p. 2-14)
"The Distributive Property" (p. 6-22)
reading from
"The Power Property" (p. 8-15)
"TV Ratings and Sampling" (p. 1-18)

At this time, the materials themselves exist only in dittoed form and complete sets cannot be distributed. A printed testing version will be available in August, 1976, for the testing planned during the school year 1976-77.

Further information is available from the project director, Zalman Usiskin, Department of Education, University of Chicago, 5835 S. Kimbark Avenue, Chicago, Illinois 60637.

ALGEBRA THROUGH APPLICATIONS

Summary Description of the Course

These materials are designed for average first-year algebra students; the course is designed to be a substitute for the traditional course.

This course has some differences in content, mathematical approach, and pedagogical approach from traditional courses.

Content differences: The traditional skills associated with first-year algebra are present, but with the following exceptions: factoring of polynomials, fractional expressions and simplifications which require factoring, some complicated radical simplifications, and artificial word problems.

In their place, great attention is given to the uses of numbers, operations, linear expressions and relations, powers, and sentence-solving. Common statistical measures, the chi-square test, simple probability, sampling, and randomness are integrated into the course. The very large number of other applications include, wherever possible, problems which involve the analysis of actual data and real situations.

Mathematical approach: In this course, a model of a concept is an extraction of the commonality of many actual applications. (For example, addition has three models: union of sets, joining lengths, and slides.) In a typical course, the development proceeds as follows:

properties ⟶ skills ⟶ word problems

In this course:

real-world situations ⟶ models ⟶ properties
real-world situations ⟶ models ⟶ skills
real-world situations ⟶ models ⟶ other applications

That is, from the models, properties or generalizations of the concept can be seen, there develops motivation for the need to have certain simplification and sentence-solving skills, and other applications of the concept are available if not self-evident.

Pedagogical approach: The materials are designed for standard self-contained classrooms. A mastery learning strategy is being tested for the learning of some of the skills. For this work, the student uses a workbook which is closely tied to the text itself.

Z. Usiskin
April, 1976

Your name ______________________ School name ______________________

Address ______________________ Address ______________________

______________________ ______________________

Name of person to contact at this school (if not the same as above)

____

☐ We are interested in using Algebra Through Applications and being a part of the study. (If you check this box, please answer Questions 1-5 below.)

1. Total number of classes (exp. & control) which might be involved _____

   Total number of students who might be involved _____

   (Half of these classes would use the experimental materials.)

2. What grades does your school include? (Circle.)

   9-12 7-12 10-12 7-9 6-9 Other (Identify) _______

3. Number of students in this school ______________

   Type of school (public, parochial, private, etc.) ______________

   Describe the area in which your school is located.

4. Describe the students in the classes you would want involved in the study.

5. If more than one level of algebra is taught at the school, name these levels and describe how students are selected for them.

☐ Even if not selected for the study, we would like to use Algebra Through Applications. We would probably order about _______ copies of the text.

Please return by May 15th to Professor Jane Swafford, Department of Mathematics, Northern Michigan University, Marquette, Michigan 49855

ALGEBRA THROUGH APPLICATIONS

Availability of Materials for 1976-77 and Summary Description of Testing Program

The materials described on the attached pages will be available to any schools that wish to use them and in any quantity for the school year 1976-77. They will be printed in two volumes, size 6 x 9, and distributed at cost. The cost per student will be between \$3 and \$3.50.

We are looking for 40 experimental classes (using Algebra Through Applications) and 40 control classes (using commercially available texts). No more than 2 of each will be from any one school. Schools will be selected to provide a representative sample of geographic, socio-economic, and cultural areas.

The text materials will be provided free of charge to schools participating in this study. (Other schools who wish to use the materials may do so but will have to pay cost.) In turn, these schools will be expected to administer all of the tests which are part of the study. It is possible that as many as 3 days at the very beginning of the year and 4 days at the end of the year might be devoted to testing. It is also possible that periodic very short tests might be given during the school year.

All results from a given school will be transmitted back to that school. No information will be made public from which a given school's scores could be ascertained.

If you are interested in using Algebra Through Applications, read the information below and the form on the reverse side. This form does not obligate you in any way nor does it constitute an order blank. If you respond positively, you will be sent further information.

Timetable: This form must be returned by May 15th to Jane Swafford in order for a school to be considered for this study.

May 22: Schools will be selected. During the week of May 25th, phone calls will be made to selected schools to confirm possible arrangements.

June 1: Formal notification will be sent to schools participating in the study.

July 1: Orders must be in by this date to insure availability of materials for the fall semester.

August 10: Materials will be shipped to schools. Do not expect arrival before August 20.

(If you fill out the other side of this sheet, you will be sent an order form and detailed information about ordering.)

If you are a supervisor and have a variety of possible participating schools, please duplicate the form on the reverse side as necessary.

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Chapters 1-8 of the Table of Contents to the right are from the 2nd draft of the materials. Chapters 9-13 are from the 1st draft. In that first draft, Chapters 12 and 13 were not fully completed; it is expected that they will be longer in the testing version.

22.

with Probability and Statistics

2nd Pilot Version 1975-76

TABLE OF CONTENTS

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SENTENCE-SOLVING

Lesson 1 - Estimating Wildlife Populations

People hunt rabbits and fish for food or sport. Whalers want oil as well as food. Scientists study migration habits of birds. Conservationists worry about endangered species. The garment industry wants furs. Park and forest rangers want to know what animals are around.

These people need to know how many animals there are of a given type. But animals move around. How can you count birds? What about whales or fish? Rabbits are very small. Each species of animal presents its own problems of counting.

Because direct counting is difficult or impossible, indirect methods are used to <u>estimate</u> animal populations. One indirect method is called the <u>catch-recatch</u> or <u>marking</u> method. Here is how it might work if you wanted to know how many trout were in a certain lake.

Suppose you caught 50 trout and 7 of them had a distinctive red marking. Then $\frac{7}{50}$ is the experimental probability of a trout having that marking. It is natural to think that $\frac{7}{50}$ is close to the ratio of <u>all</u> marked trout to <u>all</u> trout in the lake. That is, you would think that

$$\frac{7}{50} \approx \frac{\text{total number of marked trout}}{\text{total number of trout in the lake}}$$

But we don't know how many trout are in the lake. So what is often done is to handmark some trout (usually with paint) in advance and return them to the lake. If 75 trout were marked, then

$$\frac{7}{50} \approx \frac{75}{\text{total number of trout in the lake}}$$

Le[illegible] stan[illegible] the [illegible] [illegible] trout [illegible] the [illegible]. Th[illegible] T is approximated by the solution to this equation.

$$\frac{7}{50} = \frac{75}{T}$$

Can you see that T is greater than 500 but less than 550?

In general, the marking method is as follows. We wish to find T, the total number of trout. (1) Mark m trout and return them to the lake. (2) Catch C trout a few days later. Let C_m be the total caught that were marked earlier. (3) Assume

$$\left(\begin{array}{c}\text{experimental probability}\\ \text{that trout is marked}\end{array}\right) = \left(\begin{array}{c}\text{ratio of marked trout}\\ \text{to all trout}\end{array}\right)$$

That is, $\frac{\text{no. of marked trout caught}}{\text{no. of caught trout}} = \frac{\text{total no. of marked trout}}{\text{total no. of trout}}$

$$\frac{C_m}{C} = \frac{m}{T}$$

In this process, C_m, C, and m are known. T has to be found.

This is an example where finding the solution to an equation can help solve a real problem. But how do you solve this equation?

$$\frac{7}{50} = \frac{75}{T}$$

The next lessons show how this is done.

<u>Questions covering the reading</u>

1-8. Why might each of the following people be interested in wildlife populations?

1. game hunter	2. fisherman
3. tourist	4. scientist
5. clothier	6. conservationist
7. forest ranger	8. whaler

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9. Suppose you want to count the number of deer in a particular area of a forest. Describe how the marking method might be used.

10. In the marking method, what ratio is assumed equal to the experimental probability that an animal is marked?

11. Suppose 200 trout are marked and replaced in a lake. Later 150 trout are captured and 3 of these are found to be marked. If T is an estimate of the total number of trout, how might T, 200, 150, and 3 be related?

Questions testing understanding of the reading

1-2. In 1970 in Dryden Lake, New York, a marking estimate was done. The first fish were marked on their fins in mid-November. The second catch of fish was done around December 1st. (Fish do not enter this lake from outside sources.) Here is the data that was collected. (For more problems like this, see Question for discussion #6.)

	Marked (Nov.)	Captured (Dec.)	Captured and Previously Marked (Dec.)
Large-mouth bass	213	104	13
Pickerel	232	329	16

Give an equation which could be solved to estimate the number

1. large-mouth bass in the lake
2. pickerel in the lake

3-5. Suppose you wish to estimate t, the total number of marbles in a large bag. So you take 10 marbles out and paint a spot on them, let the paint dry, and return the marbles. You mix the marbles and then later take out 25 marbles. Of these m are marked.

3. What equation estimates a relationship between m, t, 10, and 15?

4. Estimate t if m = 3.

5. Estimate t is m is 2.

6. St. Paul Island in Alaska has 12 fur seal rookeries (breeding places). In 1961, to estimate the fur seal pup population in Gorbatch rookery, 4963 fur seal pups were tagged in early August. In late August a sample of 900 pups was examined and 218 of these were previously tagged. (This data is from the July, 1968, issue of the Transactions of the American Fisheries Society.) Let F be the number of fur seal pups in Gorbatch rookery. Solving what equation gives an estimate for F?

7. Suppose that you have a large number of paper clips. Describe how the marking method could be used to estimate the total number of paper clips.

8. Multiple Choice. In a forest there are D deer and you wish to estimate D. You capture and mark 100 deer. Later you catch 50 deer and all 50 are marked. Which of choices (a)-(d) is not possible?

(a) D = 100 (b) D = 50 (c) D = 300
(d) D = 5000 (e) All are possible.

9. The marking method does not lead to the exact population. It only gives an estimate. Why is this not a particular weakness of the method?

Questions for discussion or exploration

1-4. The marking method makes some assumptions not mentioned in the lesson. How would you try to assure that each of the following happens? Can you make sure that these things happen?

1. The marked animals are not affected by marking and the marks or tags do not come off.

2. The marked animals are mixed in the population.

3. The non-marked animals are just as available for capture as the marked animals.

4. The population of the animals does not change between marking and recapture.

5. Create an experiment at home which uses the marking method. (For example, you might try to estimate the number of hairpins or paper clips or rubber bands or nails in a place where large numbers of these are kept.)

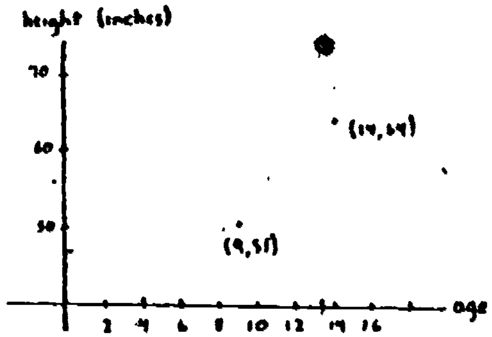
6. For more information about estimating wildlife populations, read "Estimating the Size of Wildlife Population," by S. Chatterjee, in Statistics By Example (Reading, MA: Addison-Wesley, 1973), pp. 99-104.

7. For information on how whale populations are estimated, read "The Plight of the Whales," by D.G. Chapman, in Statistics: A Guide to the Unknown (San Francisco: Holden-Day, 1972), pp. 84-91.

Lesson 3

Slope

In this lesson, we consider a simple idea called rate of change. This leads to the idea of slope.



Suppose a girl is 4'3" tall at the age of 9 and 5'4" tall at the age of 14. How fast has the girl grown?

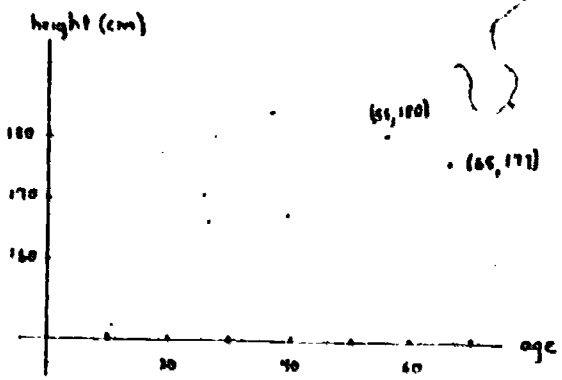
You can easily calculate that the girl has grown 13" in the 5 years. The rate of change of her height is $\frac{13}{5}$ or 2.6 inches per year. Notice that the numbers 13 and 5 are found by subtracting the heights (in inches) and the ages (in years).

Past the age of 50, people tend to lose height as they get older. Suppose a man was 180 cm tall at the age of 55 and is 177 cm tall at the age of 65.

His change in height per year

$$= \frac{\text{change in height}}{\text{change in age}} = \frac{177 - 180}{65 - 55}$$

$$= -\frac{3}{10}$$

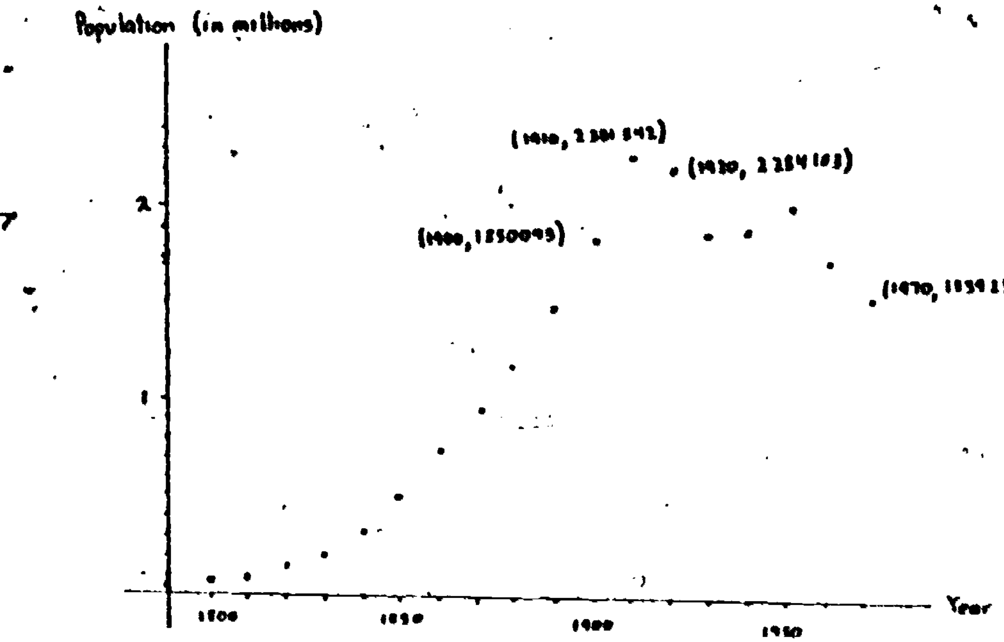


Every rate of change is found by dividing (rate is one model for division) two changes. The changes are directed distances and found by subtraction.

Here is the population of Manhattan Island (part of New York City) according to the ten-year census figures.

Year	Pop	Year	Pop	Year	Pop	Year	Pop
1790	33,131	1840	312,710	1890	1,441,216	1940	1,889,924
1800	60,515	1850	515,547	1900	1,850,093	1950	1,960,101
1810	96,373	1860	813,669	1910	2,331,542	1960	1,698,281
1820	123,706	1870	942,292	1920	2,284,103	1970	1,539,233
1830	202,589	1880	1,164,673	1930	1,867,312		

There is a lot of data. So a graph can help.



A segment connecting the dots from 1900 to 1910 would be the most slanted upward to the right. That is when the largest rate of change of population occurred. That rate is

$$\frac{\text{change in pop.}}{\text{change in time}} = \frac{2{,}331{,}542 - 1{,}850{,}093}{1910 - 1900} = \frac{481{,}449}{10} = 48{,}144.9 \text{ (per year)}$$

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From 1920 to 1970, Manhattan lost population. So the rate of change of population is negative.

$$\frac{\text{change in pop.}}{\text{change in time}} = \frac{1,539,233 - 2,284,103}{1970 - 1920} = \frac{-744,870}{50} = -14,897.4 \text{ (per year)}$$

Thus Manhattan lost an average of about 15,000 people per year in the 50 years from 1920 to 1970.

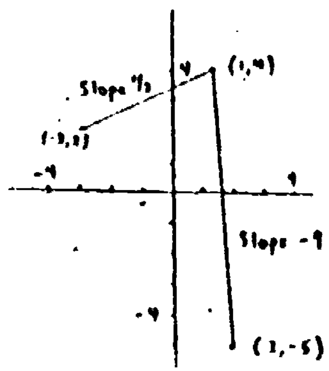
In mathematics, the word <u>slope</u> means "rate of change." It comes from the idea of the slope of a hill.

Definition

> The <u>slope</u> determined by two points is
>
> $$\frac{\text{change in 2nd coordinates}}{\text{change in 1st coordinates}}$$
>
> That is, if the points are (x_1, y_1) and (x_2, y_2), the slope is
>
> $$\frac{y_2 - y_1}{x_2 - x_1}$$

In all of the earlier examples, slopes were being calculated. Here are some other calculations.

1. The slope determined by $(-3,2)$ and $(1,4)$ is $\frac{4-2}{1--3}$, which is $\frac{2}{4}$ or $\frac{1}{2}$. The segment joining these points goes up 2 when going 4 to the right.



2. The slope determined by $(1,4)$ and $(2,-5)$ is $\frac{-5-4}{2-1}$ or -9. The segment joining these points goes down 9 units while going 1 to the right.

Questions covering the reading

1. The change in height per year is change in ____________ divided by change in ____________.

2-5. Here are heights for a girl.

age	9	10	11	12	13	14	15
height	4'3"	4'5"	4'10"	5'1"	5'3"	5'4"	5'5"

Calculate the rate of change in height (per year):

2. from age 9 to age 12
3. from age 10 to age 14
4. from age 10 to age 11
5. from age 12 to age 15

6. Give an example where a rate of change is negative.

7-10. Use the Manhattan populations given on p. 7-12. Calculate the change in population per year:

7. from 1840 to 1940
8. from 1910 to 1960
9. from 1790 to 1890
10. from 1920 to 1930

11-14. Use the information on p. 7-12.

11. When did Manhattan's population have the greatest increase?
12. When did Manhattan's population decrease the most?
13. Do you think Manhattan ever grew by 50,000 people in one year?
14. On the average, did Manhattan lose people at a greater rate from 1960 to 1970 or from 1920 to 1970?
15. "Slope" means ____________.
16. The slope determined by (x_1, y_1) and (x_2, y_2) is ____________.

17-24. Calculate the slope for each pair of points.

17. (3,6), (4,10)
18. (5,100), (7,150)
19. (-1,8), (2,-6)
20. (7,-5), (11,-2)
21. (1.2, -4), (-1.2, -5)
22. (8, -6.3), (1.4, -9.6)
23. $\left(-\frac{1}{5}, \frac{2}{3}\right), \left(-\frac{2}{5}, \frac{4}{3}\right)$
24. $\left(-\frac{3}{7}, -\frac{1}{5}\right), \left(-\frac{2}{7}, \frac{1}{2}\right)$

Questions testing understanding of the reading

1. According to the <u>Guiness Book of World Records</u>, the most freakish rise in temperature ever recorded occurred in Spearfish, South Dakota, January 22, 1943. At 7:30 AM it was -4°F; at 7:32 AM it was 45°F. What was the rate of change in temperature (per minute)?

Lesson 4

TV Ratings and Sampling

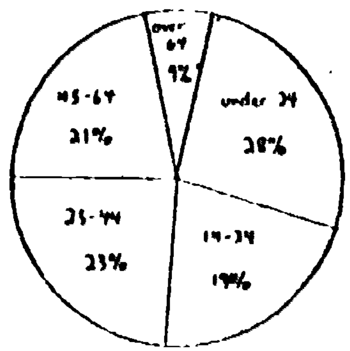
It costs money to make a TV program. Most TV shows are paid for by large companies like IBM, Revlon, Chrysler, and Kellogg's. The companies do this in order to sell their products. So they want lots of viewers.

How does a company know if a lot of people are watching its program? It receives its program's ratings from other companies whose business it is to estimate how many people are watching each program. You know that programs are dropped because of low ratings. So ratings are important to entertainers as well as companies.

How are TV ratings determined? The process that is used is called sampling. Here are the steps in that process.

Step 1. The set of all people who watch TV is described. Age, sex, and location are very important. This set is called the population.

It is not easy to describe the population. But the census helps. At right is a circle graph showing a distribution of ages in the U.S. population in 1970.



Step 2. A subset of the population is very carefully selected. This subset is called a sample. It usually contains people in from 1000 to 1000 families. The sample is designed to be very much like the larger population.

Lesson 4

The Power Property

4. You now know two models which tell you what B^5 means.

First model (Repeated multiplication): $B^5 = B \cdot B \cdot B \cdot B \cdot B$

Second model (Growth): B^5 is what a quantity is multiplied by in 5 years if it is multiplied by B every year.

Each model suggests that powers are closely related to multiplication. So you should expect properties which relate multiplication and powers.

Suppose you wish to multiply 2^7 by 2^4. Is there an easy way? Using the repeated multiplication model:

$$2^7 \cdot 2^4 = \underbrace{2 \cdot 2 \cdot \ldots \cdot 2}_{7 \text{ times}} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ times}}$$

$$= \underbrace{2 \cdot 2 \cdot \ldots \cdot 2}_{11 \text{ times}}$$

$$= 2^{11}$$

A similar problem involves the growth model. Suppose you save money at 6% yearly interest.

In 3 years you will have $(1.06)^3$ times what you have now.

In 5 years you would have $(1.06)^2$ times what you have in 3 years.

This is $(1.06)^2 \cdot (1.06)^3 \cdot$ what you have now.

But in 5 years you would have $(1.06)^5 \cdot$ what you have now.

That is, $(1.06)^2 \cdot (1.06)^3 = (1.06)^5$

These examples suggest the power property, a property which we assume true.

Power Property

For any real numbers for which B^m and B^n have meaning, $B^m \cdot B^n = B^{m+n}$

27-34. Solve:

27. $6x + 2x = 4$

28. $-30 = 7y + -5y$

29. $-3a + -4a = \frac{1}{7}$

30. $2.3B + -3.5B = .24$

31. $3 + 4c + -5c = -20$

32. $-6d - 11 + -5d = 0$

33. $2y + 3y + 8y = -39$

34. $-500x + 28 + 620x = 628$

Questions testing understanding of the reading

1-6. Using the distributive property, each of the following problems can be done easily in your head. Calculators are not allowed. Simplify:

1. $97 \cdot 5 + 3 \cdot 5$

2. $8\frac{1}{2} \cdot 3 + 1\frac{1}{2} \cdot 3$

3. $\frac{1}{4} \cdot \frac{11}{17} + \frac{3}{4} \cdot \frac{11}{17}$

4. $\frac{1}{2} \cdot -947 + \frac{1}{2} \cdot -947$

5. $15 \cdot 73 + -5 \cdot 73$

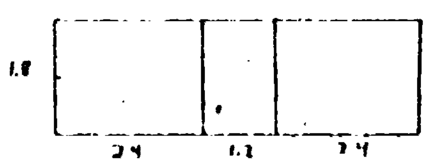
6. $80 \cdot 62 + 10 \cdot 62 + 10 \cdot 62$

7-8. In 1970, according to the U.S. Dept. of Agriculture, approximately 113.7 lbs of beef, 2.9 lbs of veal, 3.3 lbs of lamb or mutton and 66.4 lbs of pork were consumed per person in the U.S. Together beef, veal, lamb, mutton, and pork are called "meats."

7. For a city of 100,000 people, approximately how many pounds of meats were consumed?

8. For each n people, approximately how many pounds of meats were consumed?

C 9-10. Three windows are pictured. Each has the shape of a rectangle. Dimensions are in meters.



9. How much glass is in the two left windows?

10. How much glass is in all three windows?

11-12. In a singing group there are 4 girls and 5 boys. For singing duets, how many boy-girl pairs are possible:

11. If 2 more girls join the group.

12. If g more girls join the group.

5. If m is any real number, then $7 \cdot m - 6 \cdot m = m$.

6. If t is any real number, then $\frac{1}{2} \cdot t + \frac{1}{2} \cdot t = t$

7-18. Some instances of a pattern are given. Use variables to describe the possible general pattern.

7. $6 \cdot 1 = 6$
$14.3 \cdot 1 = 14.3$
$\pi \cdot 1 = \pi$
$0 \cdot 1 = 0$

8. $18 = 0 + 18$
$14 = 0 + 14$
$3677 = 0 + 3677$
$0 = 0 + 0$

9. $7 + 7 = 2 \cdot 7$
$11.3 + 11.3 = 2 \cdot 11.3$
$37 + 37 = 2 \cdot 37$
$66 + 66 = 2 \cdot 66$

10. $4712 - 4712 = 0$
$\frac{1}{6} - \frac{1}{6} = 0$
$4 - 4 = 0$
$.031 - .031 = 0$

11. $5 \cdot 14 = 14 \cdot 5$
$47.2 \cdot 31.6 = 31.6 \cdot 47.2$
$\frac{1}{2} \cdot 100 = 100 \cdot \frac{1}{2}$

12. $6 \cdot 3 + 6 \cdot 4 = 6 \cdot (3 + 4)$
$6 \cdot 11 + 6 \cdot \frac{1}{3} = 6 \cdot (11 + \frac{1}{3})$
$6 \cdot 7 + 6 \cdot 100 = 6(7 + 100)$

13. $\frac{5}{3} - \frac{1}{3} = \frac{5 - 1}{3}$
$\frac{18}{14} - \frac{11}{14} = \frac{18 - 11}{14}$
$\frac{9}{130} - \frac{6}{130} = \frac{9 - 6}{130}$

14. $\frac{1 \cdot 3}{2 \cdot 3} = \frac{1}{2}$
$\frac{46 \cdot 3}{3 \cdot 3} = \frac{46}{3}$
$\frac{11 \cdot 3}{10 \cdot 3} = \frac{11}{10}$

15. 1 tape costs $1 \cdot 4.98$
2 tapes cost $2 \cdot 4.98$
13 tapes cost $13 \cdot 4.98$
6 tapes cost $6 \cdot 4.98$

16. In 2 years, there will be $2 \cdot 500$ more people in the town.
In 14 years, there will be $14 \cdot 500$ more people in the town.
In 5 years, there will be $5 \cdot 500$ more people in the town.

17. One cow has $4 \cdot 1$ legs.
Six cows have $4 \cdot 6$ legs.
80 cows have $4 \cdot 80$ legs.

18. $100m \approx 110$ yd
$2 \cdot 100m \approx 2 \cdot 110$ yd
$14 \cdot 100m \approx 14 \cdot 110$yd

19-20. Given are 4 instances of a pattern. The instances are true.
(a) Give what seems to be the obvious description of this pattern.
(b) Show that your description is not always true.

19. $5 \cdot 2 < 100$
$16 \cdot 2 < 100$
$27 \cdot 2 < 100$
$7\frac{1}{2} \cdot 2 < 100$

20. $17 \cdot 17 > 17$
$3 \cdot 3 > 3$
$105 \cdot 105 > 105$
$2.9 \cdot 2.9 > 2.9$