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ABSTRACT

These background papers are written to provide information for the teacher on technical problems that might arise as students carry on various USMES investigations. Background papers are also written which include information about the types of investigations which may provide good opportunities for learning in specific areas. This set includes papers directly related to: biology, design problems, electric circuits, geometry and shapes, group dynamics, graphical representation, measurement, probability and statistics, ratios, proportion and scaling, and simulation activities. (MK)

ED 176996

UNIFIED SCIENCE AND MATHEMATICS FOR ELEMENTARY SCHOOLS

USMES BACKGROUND PAPERS

JULY 1975

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EDUCATION DEVELOPMENT CENTER
55 CHAPEL STREET
NEWTON, MASSACHUSETTS 02160

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PREFACE

The Unified Science and Mathematics for Elementary Schools (USMES) project was formed in response to the recommendations of the 1967 Cambridge Conference on the Correlation of Science and Mathematics in the Schools.* Since its inception in 1970, USMES has been funded by the National Science Foundation to develop and carry out implementation trials of 32 interdisciplinary units centered on long-range investigations of real and practical problems taken from the local school/community environment. School planners can use these units to design a flexible curriculum for grades one through eight in which real problem solving plays an important role.

The development and trial implementation work is carried on by classroom teachers with the assistance of university specialists at workshops and at occasional meetings during the year. This work is coordinated by a staff at the Education Development Center in Newton, Massachusetts. In addition, the staff at EDC coordinates a widespread implementation program involving districts and colleges which are carrying out local USMES implementation programs for teachers and schools in their area.

The following units are currently available for widespread implementation:

- Protecting Property (Burglar Alarm Design)
- Pedestrian Crossings
- Lunch Lines
- Play Area Design and Use
- Describing People
- Designing for Human Proportions
- Dice Design
- Weather Predictions
- Getting There
- Classroom Management
- Orientation
- Traffic Flow
- Consumer Research
- Soft Drink Design
- Manufacturing
- Advertising
- Classroom Design
- School Zoo
- Ways to Learn
- Bicycle Transportation
- Growing Plants

Since all activities in USMES units are initiated by the students in response to a long-range challenge, the students and teachers often have need of resource materials; USMES materials provide some of these resources. The Design Lab

*See Goals for the Correlation of Elementary Science and Mathematics, Houghton Mifflin Co., Boston, 1969.

or its classroom equivalent is a resource for students; using the tools and supplies available, children can follow through on their ideas by constructing measuring tools, testing apparatus, models, etc. The "How To" Cards are another resource for students. Each set of cards gives information about a specific problem; the students use a set only when they want help on that particular problem.

Several types of resources are available for teachers: the USMES Guide, a Resource Book for each challenge, Background Papers, and a Design Lab Manual. A complete set of all these written materials comprise what is called the USMES library. The library, which should be available in each school using USMES units, contains the following:

1. The USMES Guide

   The USMES Guide is a compilation of materials which may be used for long-range planning of a curriculum that incorporates the USMES program. In addition to basic information about the project and the challenges, it contains charts assessing the strengths of the various challenges in terms of their possible math, science, social science, and language arts content.

2. Teacher Resource Books for each challenge

   Each book contains the following sections:

   A. USMES Philosophy--a description of the USMES approach to real problem-solving activities,
   B. General Papers--information about the particular unit,
   C. Documentation--edited logs of class activities,
   D. References--other written materials relevant to the unit, and

   Appendix--charts which indicate the skills, concepts and processes which may be learned and practiced when students become engaged in certain possible activities.

3. Design Lab Manual

   Because many "hands-on" activities may take place in the classroom, the Design Lab Manual should be made available to each USMES teacher.

It contains sections on the style of Design Lab activities, safety considerations, and an inventory of tools and supplies.*

4. "How To" Cards

These short sets of cards provide information to students about specific problems that may arise during USMES units. Solutions for particular computation, graphing and construction problems are explained. A complete list of the "How To" Cards can be found in the USMES Guide.

5. Background Papers

These Background Papers are written to provide information for the teachers on technical problems that might arise as students carry on various investigations. As USMES units are developed and student responses to the challenge become known, teachers express the need for certain kinds of background information not readily accessible elsewhere. Background papers are also written which include information about the types of investigations which may provide good opportunities for learning in specific areas. A complete, annotated list of the Background Papers can be found in the USMES Guide.

The preceding materials are described in brief in the USMES brochure, which can be used by teachers and administrators to disseminate information about the project to the local community. In addition, the Curriculum Correlation Guide is presently being developed and preliminary sections are available. A variety of other dissemination and implementation materials are also available for individuals and groups involved in local implementation programs. Preparing People for USMES, an informational and training resource, provides information on conducting in-service workshops for teachers to prepare them for using

*Because Tri-Wall was the most readily available brand of three-layered cardboard at the time the project began, USMES has used it at workshops and in schools; consequently, references to Tri-Wall can be found throughout the Teacher Resource Books. There are several other brands of three-layered cardboard available and the addresses of the companies that supply three-layered cardboard can be found in the Design Lab Manual.

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USMES in the classroom. Written materials are also available on conducting USMES informational meetings and Design Lab manager training. Other materials include the USMES slide/tape show, videotapes of classroom activities, a general report on evaluation results, a map of implementation locations, a list of experienced USMES teachers and university consultants, and newspaper and magazine articles.

HOW TO LOVE FROGS

by

Abe Flexer

When you first collect frog eggs or tadpoles you can cut away one side of a milk carton and use this as a tank for the first week or two. As the tads get larger, they will need a larger container. Almost anything will do: aquaria, plastic dish pans, even a cardboard box lined with sheet plastic.

Food

Tadpoles are born without mouths and don't eat during the first few weeks of life (nourishment comes from the yolk in the egg). Until the mouths do open, a sprig of any healthy, leafy plant will provide some color and be available as food for the precocious feeders. When the tads have started to make mouths and begin eating (the plant will look ragged), you may feed them in several different ways:

1. Put one or two food pellets of the kind used to feed mice and gerbils into the container of tads. Put these in the container in the morning but remove them at the end of the school day--otherwise the water may become fouled. Leave one pellet for the weekend.

2. Put one or two square inches of parboiled lettuce in the container in the morning; remove at night. Parboiled lettuce is made by dropping bits of lettuce into boiling water and cooking for one to two minutes (ONLY). Parboiled lettuce will keep in the refrigerator for two-three days.

3. Use a few aquarium plants (from a pet supply store; Elodea is usually good). These can be left in as long as they continue to look healthy (green).

4. As the tadpoles get older they appreciate a small piece of raw meat or liver. This should be removed after a few hours. Tying a thread to the meat makes removal easy.

Water

Most tapwater (including "aged" and "treated" tapwater) will kill tadpoles within twenty-four hours. There are three solutions:

1. Use water from a nearby pond if one is nearby.
2. Collect rain water, unless air pollution is bad.
3. Make your own pond water: place any waterproof container (jar, aquarium, cardboard box lined with sheet plastic) near a window; add a 1"-2" layer of good, rich soil; add tap water to cover the soil to a depth of 2½"-3"; allow to sit for three-four days with occasional light stirring.

Whatever kind of water you use, it will be different from what the tadpoles are used to. So, add water to the tank gradually, over a period of days. Never add more than ½ of the volume of water in the tank at one time. Fortunately, tadpoles prefer shallow (1"-2") water. If the water in the tank gets murky, you can remove the gunk from the bottom with a turkey baster.

Even with the best of care, many of your tads will die. Do not be alarmed; this is why the world is not overrun with frogs. The death of a few tads might be a lead-in to a discussion of such questions as: Why do some animals lay so many eggs and others only a few? What would happen if all eggs survived to adulthood? Whenever you see a dead tadpole, remove it with a spoon or turkey baster and discard it as soon as possible.

Development

The time it will take frog eggs to develop into tadpoles and the tadpoles into frogs depends on many things. Some of these are: the kind of frog, the temperature, the kind and amount of food. It is very hard to predict or control development, so play it by ear and be very patient.

In most cases, eggs will hatch within 2 to 6 days; tads will develop mouths about 2 to 15 days after hatching; rear legs usually appear 10 to 30 days after the mouth; front legs appear 10-20 days later and the young adults begin to crawl up onto land shortly thereafter.

This pattern of development involves two major changes in life style:

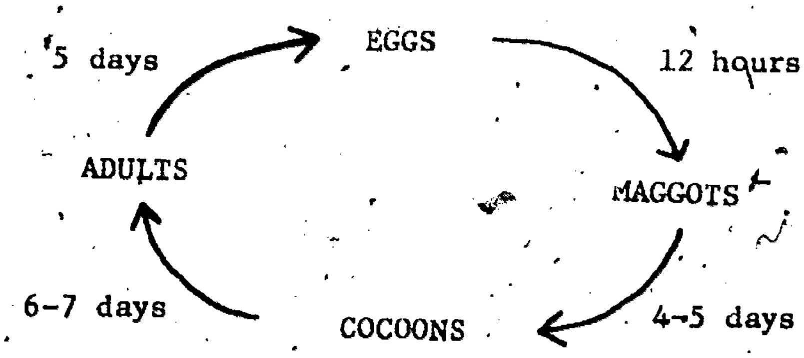
Tadpoles	Frogs
water breathers	air breathers
vegetarians	insect eaters

When front legs appear, be sure to provide a place for the baby frogs to get out of the water, otherwise they might drown. A floating bit of wood or a floating water plant could provide a platform. Cover the aquarium with fine mesh screen and introduce one or two flies. When these disappear, begin to feed the frogs several flies at a time and figure out (by rate of disappearance) how many flies to feed your frogs each day.

RAISING HOUSEFLIES

by Abe Flexer*

Houseflies are very easy to rear in the classroom and may be put to many uses. In the School Zoo unit they may be used as food for many reptiles and amphibians. Fly cultures are clean, odorless and require little attention or specialized equipment. The life cycle takes about 16 days (egg-to-egg) and involves four easily observed stages (egg, maggot, cocoon and adult):



Starting a Colony

The easiest way to start your own breeding colony is to order some houseflies from a biological supply house. Carolina Biological Supply sells 150 cocoons for $2.50 (their catalog number L932) and other houses have similar listings. You may also start a colony with flies that you collect yourself. The fly accumulator suggested below, if carefully lowered over an unwary fly, will be surprisingly effective. A note of warning: flies collected from nature often produce far more females

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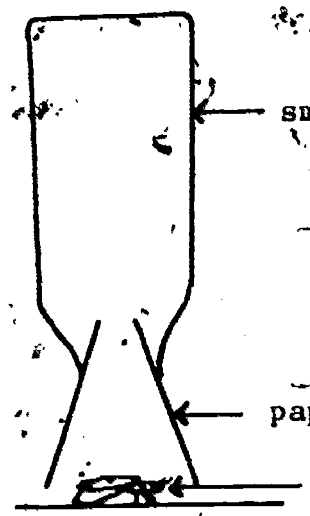
* Based on two articles by Paul D. Merrick: Fly Culturing, Science and Children, 4 (8), May, 1967; The Fly Cycle, WIMSA, Webster College, Webster Groves, Mo., Feb., 1968.

than males, in some cases, there will be only 10% males -- and the proportion is inherited from generation to generation. If you want the flies only for food, pay no attention to this shortage of males. Female flies will taste as good as males.



Fly cages may be made by using a rubber band to attach the wide end of a nylon stocking to the mouth of a wide-mouth gallon jar. Such jars are usually discarded by school cafeterias in large numbers. If the jar used for the cage is cylindrical, it should be taped or tied to a board or blocks of wood to prevent rolling. Access to the interior is through the cut-away toe of the stocking; at other times, flies are kept in the cage by knotting the toe of the stocking.

Adult flies will live for 3-4 weeks if provided with some dry powdered milk and a source of water. These should be provided in separate containers inside the cage.

Adult flies begin to mate 2 or 3 days after they emerge from the cocoons.) By the fifth day after emerging, females that have mated will begin to deposit eggs. Here is a convenient way to collect eggs. Fill a paper or plastic cup (1" deep X 1 1/2 - 2" across) loosely with bran flakes. Add to this an amount of diluted milk (1 part unsweetened, canned milk + 4 parts water) that would half-fill the cup. Mix well. Place one or more of these egg-cups in a cage of adult flies until you think some eggs have been deposited (20 min. - 1 hr). Remove the egg-cup from the cage and search just underneath the surface for the eggs which are tiny, pale, cigar-shaped objects. A cage of 15-20 mated females may produce as many as 2000 eggs per sitting; a single mated female will lay 600-800 eggs over a 3-4 week period.

To maintain a continual supply of new adults, collect eggs to begin a new culture every week or ten days. Eggs can be collected every day if a large number are needed as a food source.

There are two approaches to raising the eggs to the adult stage. The plate method is a little trickier to set up but is more reliable. The jar method is easier to set up but is less reliable. Have your students compare the two methods to determine which is more convenient in your classroom.

Jar Method

Get a short, wide container that you can see through: jars from peanut butter, cocktail herring, or even a short highball glass. Any transparent container will do as a maggot jar as long as it fits through the stocking that covers the opening of the fly cage. Place a plastic bag inside the maggot jar, mouth up; the bag should not reach more than 3/4 of the way up the side of the jar. Add enough of the bran/

diluted milk mixture to the bag to fill it nearly full. Transfer several hundred eggs from an egg cup to the surface of the medium. Cover the mouth of the jar with a few layers of paper towels held in place with a rubber band. Place the jar in a shaded spot away from all heat sources.

The eggs will hatch within 12 hours or so to form tiny maggots. These will move about, feed and grow to full size over a period of 4 to 5 days. Do not worry if at first you can't see the young maggots, as they tend to avoid light and remain under the medium. When the maggots have reached full size, they will crawl up out of the medium and over the edge of the bag seeking a dry place in which to build cocoons. Wrinkles in the plastic bag will form spaces between the bag and the wall of the maggot jar. The maggots will accumulate in these spaces where the entire process of cocoon-building will be easily observed.

The cocoons will hatch in 6 to 7 days at which time the adults should be transferred to the fly cage. Push the maggot jar through the stocking and into the cage, pop off the rubber band and paper towels and allow the adults to escape over a several hour period. Later, remove jar, towels and rubber band. Alternatively, the cocoons may be removed from the maggot jar by hand and placed in a terrarium where they will hatch and become food for whatever is in the terrarium. Remember to reserve a few cocoons to start the next generation. It should be possible to store new cocoons in a refrigerator for several weeks if you want to.

Plate Method*

Flies seem to have little trouble perpetuating themselves in nature, but the laboratory culture of larvae (maggots) can be subject to some complications. Molds are the biggest problem...We have developed a technique which is almost foolproof. A deep plastic petri plate (100mm X 20mm) is filled loosely with the bran-milk mixture. 100-150 eggs are inoculated (and) the plate is then covered with the lid. For best results, the air space between lid and medium should be very small. Molds require a rich air supply for good growth, while fly larvae do very well in "airs" of low oxygen content. But the larvae require some oxygen, however, and one has to be careful that a water seal (from condensation) does not form around the inside of the lid. If this happens, the larvae come to the surface and die.....

...The larvae (in a healthy culture) are almost always there (just below the surface) just avoiding light..

...There are many subtle factors which affect larval cultures. If too many eggs are used, (many) larvae will leave the culture.... On the other hand, if too few eggs are put in,... the cultures will often be blank. Too high a temperature slows down the development of the culture. The best "rule of thumb" is to place the culture away from any cold spot and lift the cover off periodically...

...Just before the onset of (cocoon-building) the fat maggots stop feeding and move "endlessly" around the periphery of the culture

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* Exerpted from Fly Culturing by Paul D. Merrick. Reproduced with permission from Science and Children, March, 1971. Copyright 1971 by the National Science Teachers Association, 1201 Sixteenth Street, N. W., Washington, D.C. 20036.

plate... About the fourth day after egg inoculation, we set the petri dish in a larger bowl or plastic container and remove the lid. As soon as they have the "urge", the maggots crawl over the edge of the petri plate and drop to the floor of the bowl where they form (cocoons). It is a good idea to put something between the bottom of the petri plate and the bowl. Otherwise, the energetic maggots "burrow" between the plate and bowl and "make cocoons". If the maggots are permitted to (make cocoons) under the plate, the (cocoons) are apt to be flattened and produce few flies.

USMES

IDENTIFYING ORGANISMS

by Abe Flexer

As your students collect plants and animals for classroom activities, the need to identify what they collect may arise. For example, identifying a plant or animal makes it easier to do library research on how to grow and maintain the organism, to learn about its life cycle, etc. Such identifications can be made through the use of various keys and field guides.

Keys come in many different levels of complexity and sophistication. Most differences concern the amount of technical vocabulary used in stating the questions, the range of organisms covered, and the precision with which specimens are identified. For the purposes of most classroom activities, the Golden Nature Guides (Golden Press, New York, available at most book stores) will be more than adequate. In addition to simple but accurate keys that use lots of pictures, each booklet contains a few pages about when and how to collect the organisms covered by the key, hints on how to maintain them in captivity, their natural history, and their distribution in the continental U.S. At about $1.50 per copy (paperbound) they are excellent values. Some titles from the series: Birds; Trees; Mammals; Flowers; Insects; Butterflies and Moths; Gamebirds; Fishes; Reptiles and Amphibians; Non-Flowering Plants (algae, fungi, mosses, ferns); The Southeast; The Southwest; The Pacific Northwest; The Rocky Mountains; Seashores; Pond Life; Zoology; Zoo Animals; Spiders.

These can be supplemented with keys and guides made by local naturalists for the plants and animals of your region. Such local or regional guides and keys, available through branches of the Audubon Society, Sierra Club, or through the nearest museum of natural history, are usually inexpensive (often free to schools), simple to use, and loaded with illustrations. Best of all, these will give you some idea of what to look for in your local area -- and when.

How to Use a Key

Keys and field guides consist of series of questions about a specimen which, if correctly answered, will identify the specimen with a common name and a scientific name. The latter is information sometimes needed to conduct a library study; the former is usually a more convenient label for daily use. The questions usually come in pairs: 1/1', 2/2', etc.; 1a/1b, 2a/2b, etc.; A/AA, B/BB, etc. The questions in a pair usually describe mutually exclusive categories and each question sends you to a different part of the rest of the key. For example:

1a. Animal has four or fewer legs . . . . go to 2
1b. Animal has more than four legs. . . . . . . go to 6

2a. Animal has four legs. . . . . go to 3
2b. Animal has two legs . . . . go to 5

As you progress through the key, the questions become more and more specific. Eventually, one of the questions will yield a specific identification.

Here, for example, is an artificial key constructed for an arbitrary collection of the following 11 organisms:

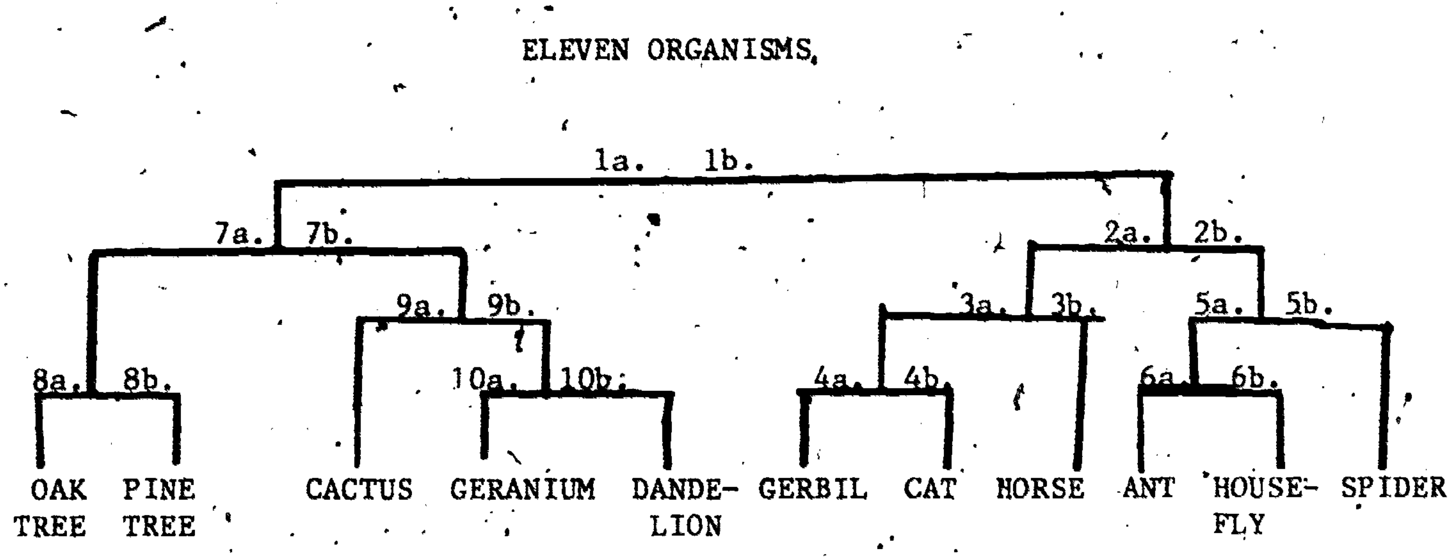
ant	gerbil	cactus	cat
housefly	oak tree	horse	dandelion
pine tree	spider	geranium	

Pretend that you have a specimen of one of these in front of you and that you don't know what it is. Work through the key, step-by-step until you come to the name of the specimen. Do this a few times with different selections from the list above until you see how the key works.

1a.	Obviously a plant	go to 7
1b.	Obviously an animal	go to 2
2a.	Has only 4 legs	3
2b.	Has 6 or 8 legs	5
3a.	Has fur and claws	4
3b.	Has mane and hooves	HORSE
4a.	Small and rat-like; fixed claws	GERBIL
4b.	Larger, with canine teeth and retractable claws	CAT
5a.	Has 6 legs.	6
5b.	Has 8 legs.	SPIDER
6a.	Has pinching jaws, no wings	ANT
6b.	Has wings and large eyes.	HOUSEFLY
7a.	Large and woody	8
7b.	Small and leafy	9
8a.	Produces leaves and acorns.	OAK TREE
8b.	Produces needles and cones.	PINE TREE
9a.	Stem fleshy; without leaves; many spines.	CACTUS
9b.	Stem not fleshy; many leaves.	10
10a.	Leaf surface fuzzy; red flowers	GERANIUM
10b.	Leaf surface smooth; yellow flowers . . .	DANDELION

This is a simple and not-too-efficient key (there could have been fewer questions). Notice that in several cases the two questions in a pair are not exact opposites: 3, 4, 6. This was possible because the key dealt with only a limited number of choices. This will often be true of even the most sophisticated keys -- not all conceivable possibilities actually exist.

Keys really define smaller and smaller sub-sets of the set of all plants, or the set of all trees, or the set of all pink flowers, etc. Here is a diagram that shows how the key given above works:



HINTS FOR GROWING PLANTS

by Jay E. Anderson

A green plant can be thought of as nature's factory. A number of raw materials go in, foods are manufactured, and some by-products are given off. The basic ideas, which we shall consider briefly here, are really quite easy to understand. But, it is humbling to note that scientists still do not fully understand all of the intricate details of the manufacturing process. The prospects of man duplicating the food-making process of green plants still lies in the distant future.

Perhaps you remember studying the equation

$$\text{Carbon dioxide} + \text{Water} + \underset{\text{(energy)}}{\text{Light}} \xrightarrow{\text{Chlorophyll}} \text{Sugar} + \text{Oxygen}$$

If your classes were anything like mine, you no doubt spent more time trying to balance the equation than considering its significance. The basic raw materials are carbon dioxide (CO_2) and water (H_2O). Plants absorb carbon dioxide from the air, and take in water from the soil through their roots. The energy for the manufacturing process comes from light. Sunlight (or light from an artificial source) is trapped by the green substance, chlorophyll, in the leaves. The trapped energy is then used to drive a whole series of chemical reactions that result in the production of simple sugars (for example, glucose, $C_6H_{12}O_6$). Some of the energy is stored as chemical energy in the sugars, and this energy can be released and used later when the sugars are burned (by the plant or by an animal that has eaten it).

In addition to simple sugars, plants manufacture a large variety of other substances. Using nutrients such as nitrogen and phosphorous compounds and various minerals like calcium and potassium, they make proteins, fats, vitamins, enzymes, and chlorophyll. These, too, are all made using the sun's energy, but herein lies another common misconception: People often have the idea that plants don't use any food. However, they burn food and use energy (respiration) just as we do. In order to manufacture all of the substances needed for growth, and to transport materials around, reproduce, and so on, plants use some of the energy they have stored in sugars or starch.

Soil

In addition to anchoring the plant and supplying some 16 to 18 different chemicals needed for plant growth, the soil also must be capable of holding water for the plant. However, water must not be allowed to stand in the spaces between the soil particles since this will prevent air circulation and deprive the plant's root system of oxygen. Thus, for most common plants, the soil must provide both drainage and porosity.

Natural top soils form complex physical structures that enhance their porosity and drainage. However, most natural soils are too "heavy" because of high clay content or a lack of organic matter to be good for raising plants indoors in pots or other small containers. Most soils often become very compacted and almost impermeable when used in pots. Unless your class has access to an exceptionally good quality loam soil having a high organic content, you must either use a commercially prepared potting soil or prepare your own.

The simplest solution is to use a commercial potting soil. Many brands are available, but many are also unsatisfactory because they are about 99% peat moss. I highly recommend "Baccto" brand. I have used it for a number of years in my research, and have had excellent results growing a large variety of plants. "Baccto" seems to be of consistent quality, bag after bag, and is available in most parts of the United States. "Pot of Gold" brand seems to be of about the same quality as "Baccto". The current (summer 1975) price for these brands is $3.99 per 50 lb. bag. Smaller amounts are much more expensive per pound. A couple of 50 lb. bags will go a long ways in a classroom.

A good potting soil can be prepared by mixing top soil, peat moss, and sand in equal portions by volume. Peat moss is the breakdown product of sphagnum moss, which grows in swampy areas of northern forests (sphagnum bogs). Peat moss will hold about twenty times its weight in water and, at the same time, will serve to increase the porosity of the soil and keep it from compacting. Sand also increases porosity and improves drainage. Any sand will do, but that collected from roadsides may have a high salt content that could be toxic to plants. Rinsing the sand thoroughly with water several times in a large bucket, a tub, or a wheelbarrow should prevent any problems.

If the top soil is quite high in clay (when wetted, it will readily form a ball and be molded), you should also add perlite. Perlite is a quarried volcanic stone that helps to increase porosity and prevent compacting. In this case, mix equal volumes of top soil, sand, peat moss, and perlite to make your potting soil.

Peat moss and perlite are relatively inexpensive when purchased in bulk. Peat moss is readily available at supermarkets or discount stores, but you may have to go to a nursery or greenhouse to purchase perlite in bulk. Current prices for four cubic feet of each is about $7 for peat moss and $5 for perlite.

Perlite contains fluoride that will cause "burning" of a few plants including palms, cane plants (Dracaena) and a few other members of the lily family. This is not likely to be a problem for most plants that students will choose to raise, but if it should arise, fir bark, which is also available commercially, is a good substitute.

Vermiculite should not be used to prepare potting soil (contrary to suggestions found in many source books). It tends to clog and compact the soil and results in waterlogging which is exactly the problem you are trying to overcome. Vermiculite may also harbor bacteria and fungi that will be harmful to plant growth.

Beware of the so-called "soil substitutes", which are advertised to do everything that a soil does and then some. Most of these are vermiculite-based and become too compacted and waterlogged in use. There really isn't a substitute for good soil without going to elaborate, sophisticated aerated nutrient solutions (hydroponic culture).

If, in response to the challenge, your students suggest testing different kinds of soil in an effort to see if some are better than others, by all means encourage them to do so. Students may need patient help in understanding the need to keep other conditions constant if they are to compare different soils. That is, the plants, containers, lighting, watering, fertilizing, etc., all have to be the same to compare the effects of different soils on the plants' growth.

Pots and Other Containers

The best single container for growing plants is the red clay pot. It is porous, which allows air to diffuse into the soil. Also, water diffuses out, helping to prevent overwatering. Clay pots should not be painted, glazed, or varnished because this destroys their porosity and produces the equivalent of a plastic pot.

Clay pots are expensive, especially if purchased retail. However, a school should be able to buy them wholesale, and if there is a clay works in the vicinity where they are manufactured, you should be able to obtain them from the factory at a reasonable price. The current factory price in Denver is $8.50 per hundred for four inch pots and $12.50 per hundred for the six inch size. That's about one-fifth of the local retail price!

If your class is raising large numbers of plants for sale, gifts, or transplanting, it is certainly not necessary to use clay pots. Plastic pots are usually less expensive than the clay ones, but offer only aesthetic advantages over the suggestions that follow. The bottom three to four inches of quart, pint, or half pint milk cartons will work fine. Other alternatives include frozen juice containers, various shapes and sizes of plastic jugs, and styrofoam or paper cups. With all of these it is critical that holes be punched in the bottoms to provide drainage. Tin cans are not good because they rust and stain any objects upon which they are placed.

Half gallon milk cartons or similarly sized plastic containers can be halved lengthwise to make planters for several plants. A great idea for starting plants in classrooms was published recently in Heloise's newspaper column, submitted by Carol, age 12:

> "...Take a styrofoam egg carton and cut the lid apart from the bottom. Poke a hole in the bottom of each cup for drainage and fill each section with dirt or potting soil and plant the seeds. The top of the carton can be used on the bottom to catch the drainage.
>
> When they are ready to be transplanted, just push the bottom of the cups and the plants will pop right out without breaking any roots."

Planting and Watering

When planting in any type of container, first insure adequate drainage. Use chips from broken pots or small, flat rocks to loosely cover holes in the bottoms of the containers. This will prevent the soil from compacting in the hole and clogging it. If you're using some of the alternatives to commercial pots, such as those described in the previous section, punch holes about the diameter of a pencil in the bottoms and cover the holes as above. Do not put sand, gravel, charcoal, peat, marbles, or the like in the container. Besides being unnecessary, these waste valuable soil space.

Add soil as necessary depending upon whether you are planting seeds or transplanting. In either case, leave enough space at the top of the container so that when you water you can add enough water that it will drain <u>clear through the soil</u>. This means leaving at least one-half inch of space at the top of small containers (three to four inches in diameter), about an inch in six-inch diameter containers, and so on. This is critical to keep salts from building up to toxic concentrations within the soil, and also insures thorough watering.

Finally, be sure that drainage is not impaired by a saucer or tray placed under the plant containers to catch excess water. If saucers or trays are used, fill them with gravel to a depth about equal to their own depth and then set the plant containers on the gravel. This way, excess water can stand in the gravelly bottom without touching the plant's roots and waterlogging them.

Although many plants can be watered daily as long as the containers have good drainage, more plants are probably harmed by overwatering than underwatering. Plants are more likely to recover from a mild wilting than from root rot. As a rule of thumb, if the top half-inch of soil feels moist, don't water. When it begins to dry, water thoroughly, insuring that water drains out the bottom of the container. The soil in small containers can dry out rapidly, and should be thoroughly watered before weekends (see the section "Classroom Greenhouses" for suggestions on how to keep plants from drying out over long weekends).

I do not recommend using wicks for watering. They often do not adequately wet the entire soil bulk in the container, and they do not allow rinsing of toxic salts from the soil.

If your students' plants should wilt severely, encourage the students to rewater them and wait a week or so to see if they might recover. Often, wilting will kill the leaves but not the plant, and new leaves will begin to sprout within a week or two.

Fertilizer

If your class is using a good quality soil, it should be unnecessary to fertilize for the first four to six weeks. Then one of the numerous commercial "plant foods" can be used according to the manufacturer's directions. Most of the commercial fertilizers supply only the three major elements that plants need: nitrogen, phosphorous, and potassium. If the plants will be grown indoors in the same soil for extended periods (more than two or three months), it is important to use a fertilizer that also supplies the elements that plants need in tiny amounts, the "trace elements". "Miracl-Gro", a product of Stern's Nurseries, Geneva, New York, is the only brand I have found that supplies the trace elements. It is generally available in plant shops, supermarkets, and hardware stores.

Some students may be concerned about "organic" ways of growing plants, and should certainly not be discouraged from experimenting with compost soils, etc. Plants growing in pots or other small containers can be rather easily "burned" by adding manure or rich compost, however. It is also important to realize that all plants use and depend upon inorganic ions such as phosphates, nitrates, potassium, magnesium, iron, and so on. These all occur naturally and are chemically identical to the ions found in commercial fertilizers. Thus, it's certainly not any more unnatural to use commercial fertilizers than raising the plant in an artificial container.

I do not recommend using fish emulsion fertilizers. They often encourage the growth of undesirable fungi or bacteria and are not very good sources of the minerals that plants need. Similarly, despite the fact that many persons will swear by it, I do not recommend using water from soaking eggshells. This encourages the growth of a harmful albumin fungus, and about the only mineral that is supplied is calcium.

Lighting

Plants obtain the energy needed to carry on all of their life processes from light. The light energy that they use to manufacture food by photosynthesis roughly corresponds to the wavelengths that we can see, i.e., the visible spectrum. Plants also use light to time the length of day and night. Day length in temperate regions is a reliable environmental signal that plants use to "know" when to break dormancy, when to flower, when to become dormant, and so on. To time the length of day and night, plants use red and "far red" light. Far red is light having wavelengths just longer than the red light that we can see; it is just off the visible spectrum. Sunlight, of course, contains all the wavelengths that plants need. If artificial lights are used, they must supply light for photosynthesis and for day-length timing.

If your classroom has windows facing south, southeast, or southwest which are not shaded by trees or other buildings, you may not need a supplemental light source for growing plants. In some cases, light reflected from a light-colored wall into northerly-facing windows will be sufficient. At temperate latitudes, however, days in the fall and winter are so short that many plants will not grow well unless artificial light is used. There are two reasons for this. Some plants will go dormant or attempt to go dormant because of the long nights. Secondly, light energy is available to the plants for such a short time each day that they are unable to make sufficient food for good growth.

Artificial lights need not be elaborate or expensive. Ordinary cool-white fluorescent bulbs are as good as any artificial light source for plant growth. Since they do not emit the red wavelengths needed for day-length timing, they must be supplemented with ordinary incandescent bulbs. This combination (cool-white fluorescent and incandescent) is used in most controlled environment chambers for plant research.

A four-foot-long fixture having two fluorescent bulbs will light a growing area approximately six or seven feet long by three to four feet wide. The fixture should be suspended about 18 inches to two feet above the plants. A 60 watt incandescent bulb near each end of the fluorescent fixture will supply ample red light. Hardware stores sell incandescent fixtures that can be clipped (with a spring clip) to the ends or sides of the fluorescent fixture. An inexpensive household electric timer, also available at hardward stores, can be used to automatically control the length of illumination. I would suggest using 16 hours of illumination, or a period approximately equal to the summertime daylength in your area.

I do not recommend using the special plant growth lights that are available. These are both fluorescent and incandescent bulbs marketed under a variety of trade names which are purported to enhance plant growth. They are attempts to provide all of the wavelengths needed for growth and day-length timing in one bulb. In so doing, intensity is invariably sacrificed; and, too little intensity is often the problem if plants are not doing well indoors. To get sufficient intensity, these special bulbs have to be placed very close to plants. They also produce unnatural colors, and are more expensive than regular bulbs. In short, I fail to see any advantage in using special bulbs.

Some plants are much more shade tolerant than others, and some plants will be damaged by too high a light intensity. Requirements for specific plants can be obtained from your local plant shop or from numerous reference books. Tall, spindly growth is a symptom of too little light. For plants that are normally grown outside in full sunlight (e.g., vegetables, bedding plants, flowers), you needn't worry about having too much light in the classroom. High temperatures may be a problem in a sunny window, however. Direct sunlight may be damaging to plants that are normally grown indoors, but it is unlikely that the intensity provided by artificial lights as described above would be too high.

Classroom Greenhouses

Small classroom greenhouses, constructed by covering wooden or wire frames with clear plastic, can be useful to keep planted seeds or small seedlings from drying out or for maintaining high humidities during rooting of plants. They are also helpful for keeping plants from drying out over weekends or longer holiday periods. Caution: they will become too hot if placed in direct sunlight.

Greenhouses may improve the growth of some plants because of the high humidity, but if your class's plants are kept adequately watered, greenhouses are certainly not necessary. Since the plastic will reduce light intensity, most plants will grow better without a plastic enclosure unless it is artificially illuminated. Thus, I would not recommend growing plants in small classroom greenhouses unless it is artificially lighted or you have an exceptionally bright classroom.

Local Resource Persons

Every county (or township) is served by a County Agent (Agricultural Extension Agent) who is a local representative of the Department of Agriculture and works through colleges and universities to encourage the use of new ideas in community development and agricultural practice. Kinds of information typically available from the local Extension Office that may be useful to your students include:

-planting times for fruits and vegetables in your area

-when to start plants indoors to be later sold or transplanted outdoors

-recommended varieties of plants for your area.

The Extension Office can also assist in testing soils and making fertilizer recommendations for local soils. They may also be able to suggest sources of soil or other materials in your area and can help solve problems with pests. You can find the number and location of the local County Agent in the Yellow Pages under "Government, County".

Local floriculturists and nurserymen are also valuable sources of information: They are usually knowledgeable about the needs and care of specific ornamental plants, and can recommend fast-growing, fast-rooting, or fast-flowering species. Some plant shops will donate unsold seeds to schools at the end of the season.

Many communities have active garden clubs whose members are usually more than willing to share their experience and expertise with students.

Larger communities often have bontanical gardens. Their employees are usually very knowledgeable about the culture and propagation of plants. Botanical gardens may also donate cuttings or seeds to schools.

Suggested References

These are but a few of the numerous excellent reference books that are available on plant care and culture. The first five are highly recommended for a classroom or school library. They all include illustrated descriptions of techniques for preparing soil, potting, starting seeds, transplanting, making terrariums, etc., which students should find helpful. I strongly recommend that you have the Baylis book available for students to use in your classroom. The last two books are a bit more technical and are excellent resources for teachers.

Baylis, Maggie. 1973. *House Plants for the Purple Thumb.* 101 Productions, San Francisco. 192 pp.

Crockett, James U. and the Editors of Time-Life Books. *Flowering House Plants.* The Time-Life Encyclopedia of Gardening. Time-Life Books, New York. 160 pp.

Crockett, James U. and the Editors of Time-Life Books. *Foliage House Plants.* The Time-Life Encyclopedia of Gardening. Time-Life Books, New York. 160 pp.

Editorial Staff of Ortho Books. 1974. *House Plants Indoors/ Outdoors.* Ortho Book Division, Chevron Chemical Company.

Doty, Walter L. 1973. *All About Vegetables.* Ortho Book Division, Chevron Chemical Company.

Graf, A. B. 1973. *Exotic House Plants.* Roehrs Company, E. Rutherford, N. J. 178 pp.

Cruso, Thalassa. 1969. *Making Things Grow: A Practical Guide for the Indoor Gardner.* A Borzio Book, Knopf, New York.

HOW TO AVOID CROP FAILURE AND DISASTER: REDUNDANCY

by
Jay E. Anderson

Much of the United States' success in manned space flights can be attributed to redundancy; that is, back-up systems for critical functions such as steering, communications, control of cabin temperature and atmospheric composition, etc. New automobiles sold in the United States are required to have duplicate brake systems, just in case one fails. As another example, each headlight of a car is on a separate circuit. Thus, if a fuse blows, both lights won't be lost.

You can use the idea of back-up systems to help insure success in "Growing Plants." A few ideas are discussed below. No doubt, you will be able to think of others for your particular situation. The important thing is to recognize the need for back-up systems before the disaster.

First, don't fall into the trap of letting each child have his or her own plant. Begin by assuming that some plants are going to die or otherwise be lost. If the children suggest that they each grow one plant, simply ask them what will happen if some plants die. This should suffice to make them see the need for starting more than one plant each.

The problem of "crop" failure", which includes plant loss, failure to grow well, or failure to produce flowers or edible vegetables, can be solved in a number of ways, and the one that students choose will depend upon the nature of the challenge itself. Primary children are more likely to "want my own plant"; whereas, older children are more likely to suggest mass production without having individual plants identified with individual people.

In cases in which the students decide to have individual plants, each student could have several to help insure success. Alternatively, the class might raise spares from which replacements could be made. Note that I said the class might raise spares, not the teacher. It is important that the class as a whole solve the problem presented by the challenge. Part of that problem involves recognizing the need for a back-up system or "spare parts."

In addition to having several plants grown under the same conditions to serve as "spares,", it may be equally important to have alternative sets of conditions under which the plants are grown. This is especially true if the class is not sure of the requirements (light, water, temperature, etc.) for the particular kinds of plants selected. For example, if the students decided to place the plants in the direct sunlight of windows having a southern exposure, you might want to ask what would happen if it were too warm there or if the sun were too bright. With a little encouragement, the class might decide to place a second group of plants in an area having indirect light and compare growth in the two areas. Similarly, students might wish to try different watering schedules, kinds of soil, or growing containers. The need for finding optimal growth conditions is especially urgent in classes that are mass producing plants.

What If The Crop Fails?

Despite back-up systems and the best efforts of class and teacher, "crops" are occasionally going to fail. Seedlings may dry out over a long weekend, light levels may be too low for the plants to produce tubers, bulbs, flowers or whatever in the required time, mice may eat planted seeds (this actually happened in one of the development classrooms), or pests (insects or fungi) may wipe out the crop. What should happen in such a case?

The purpose of a challenge is to present a real problem to children to foster the development of problem solving skills and the application of all sorts of other skills to the solution of that problem. If students fail to meet the challenge, what is learned? "I couldn't do it" or "We tried, but it didn't work (like lots of other things in school)" will probably be the take home lesson if the unit simply stops there--but, it shouldn't. The failure of the plants to grow (flower, produce, etc.) presents another challenge. And this new challenge--to find out what happened and overcome the problem or difficulties--is likely to be viewed by the students as even more real and more important than the original challenge. Thus, if at all possible, the students should be encouraged to pursue the new challenge presented by the "crop failure", thereby continuing to pursue the original challenge.

A few questions such as "What do you suppose happened?" or "Why do you think the plants didn't ____?", followed by "How could we test that idea?" or "Would you like to try some of your ideas...?" should serve to get things rolling again.

HOW TO KEEP FROM, ALSO RAISING APHIDS, RED SPIDER MITES, WHITE FLIES, ETC. WHEN GROWING PLANTS

Jay E. Anderson

Once you start rearing large numbers of different kinds of plants in your classroom, you are likely to run into problems with insects and other pests. It's perfectly natural, of course, for insects or mites to eat plants. But, in the absence of natural population checks such as predators or severe weather, the pests may overpopulate and destroy their habitat - and your class's crop. The most common pests include:

> Aphids - Aphids can be recognized by their pear-shaped bodies that are about the size of the lower case letter "o" in newsprint. These are sucking insects that will often be found along green stems and petioles (leaf stems). They may be winged or wingless, and come in a variety of colors. They appear rather long-legged.
>
> White flies - These delta-shaped, pure white flies are easily recognized. Usually, they reside on the underside of leaves, and they will fly when disturbed.
>
> Mealybugs - Clusters of mealybugs look like small pieces of cotton and are often seen in the angles between the leaf petioles and the stem.
>
> Red spider mites - Usually first detected by web-like filaments on stems and leaves, these tiny relatives of spiders are probably the worst indoor plant pests. They are so tiny they can barely be seen without a hand lens. They are oval shaped, and have eight legs. The egg cases look like salt sprinkled on the bottoms of the leaves.

As you probably know, plant shops, supermarkets, and hardware stores have all sorts of chemical sprays for killing pesky critters. The problem is that these products are designed to kill living things, and are therefore dangerous. Reading the warning labels on several insecticide containers should convince anyone of that. Many commercial insecticides are close chemical relatives of nerve gas, the dreaded chemical warfare agent designed to kill people. These insecticides are the so-called organophosphates. Malathion and carbaryl are examples. If the warning label lists atropine as an antidote, the active ingredient is a nerve gas. These compounds are absorbed through the skin, and can be fatal if they get into eyes or an open cut. I believe that most of the commercial pesticides are too dangerous to spray on plants (and their surroundings) in a classroom where students may handle the sprayed objects. Let's consider some alternatives.

Some pest populations can be controlled by periodically rinsing the plants thoroughly with water. A kitchen sink sprayer works well. Weather permitting, the plants can be taken outside and sprayed with a hose. If potted plants have a well-developed root system, they can be supported upside down with the leaves and stem submerged in a sink or bathtub full of water. Boards placed across the sink or bathtub will support the pot and plant, and can be placed so that they will keep the root/soil ball (and the plant top) from falling even if the roots and soil become dislodged from the pot. The plant top should be left submerged for about two hours to drown the pests.

I have been told that rinsing is especially effective against red spider mites, and I have been told that it is effective against almost everything except red spider mites. My recommendation is to try water first. If rinsing or submerging doesn't completely destroy the pests, these methods may keep the pest populations down to tolerable levels so that more severe treatment is unnecessary.

If water doesn't work, try rotenone or pyrethrum dust, which you will find on the market shelves with other pesticides. These can also be obtained from a veterinarian, because they are used to dust animals. Check the label to be sure that rotenone and/or pyrethrums (or other plant resins) are the only active ingredients. These are natural plant products and are effective against a variety of pests while being quite safe. We have found rotenone to be effective against young tomato hornworms, for example, and it can be safely applied to fruits and vegetables within a day of harvest. It comes in powder form; thus spray drift is not much of a problem. It can be safely applied by children under teacher supervision. Students should wash their hands thoroughly after handling containers or dusted plants. Fruits or vegetables should also be washed thoroughly before being eaten.

One word of caution: Rotenone is very toxic to fish, so take care if you have aquaria in your classroom. Covering the aquaria or moving them from the vicinity of the plants you are dusting should prevent any problems. Care should also be taken to prevent getting the powder on birds or other small animals.

Mealy bugs can be controlled by swabbing them off the plants with a Q-tip dipped in alcohol. Vodka is the best alcohol to use because it doesn't contain chemicals that will harm the plants. Rubbing alcohol or denatured alcohol may be toxic - try a leaf or two and wait a few days before swabbing down the whole plant.

For caterpillar-type pests, which are likely to be more of a problem in outdoor gardens, there is a new bacterial toxin marketed under such names as "Thuricide" or "Biological Worm Spray". This is a safe, natural product that paralyzes the digestive tract of caterpillars. Because it is specific for caterpillars, it can be used without fear of killing desirable predatory insects.

The best means of pest control from both an environmental and an educational standpoint is to use natural predators. Lady bugs and praying mantises are available commercially and may be obtained from plant or garden shops in your area. They can also be purchased from biological supply houses. It may be somewhat difficult to keep predator populations established indoors, but if your class is raising a lot of plants, it's definitely worth a try. Of course if you are trying to encourage natural predators, don't use any pesticides because they will kill the desirable predators as well as pests.

If none of the above methods is effective, you might want to solicit specific advice from local plant shops or an entomologist at a nearby university. Keep in mind, however, that the usual chemical sprays are most likely to be recommended. In such cases, I would recommend the "last resort" method described below. Another possibility is to check the journal *Organic Gardening* for suggestions. I'm told that you can write the editor of *Organic Gardening* for specific recommendations.

<u>The Last Resort</u>

Insecticide strips, such as the Shell No-Pest Strip, are a most effective alternative to spraying when the plants are in (or can be placed in) a relatively small enclosure. For example, one overnight treatment was sufficient to eliminate white flies in a small greenhouse with all windows closed. Similarly, hanging the strip in a plant growth chamber overnight completely wiped out an aphid problem. Individual plants can be treated by placing the plant and the strip in a plastic bag overnight. The key is having the plants in a small enough enclosure so that the insecticide will be effective in a few hours in an area occupied only by the plants and their unwanted guests.

The active ingredient in insecticide stips is nerve gas, and they are dangerous. The fact that it is unlawful to use such strips in restaurants or other places serving food indicates their toxicity. They will kill birds, fish, and other small animals if used in the same room. Insecticide strips should not be used in school and they should not be handled by children. Teachers can take plants home and carry out the treatment in a garage or basement. Despite the inherent dangers, I believe this method to be more effective and far less dangerous than spraying the same sort of compounds on exposed plants.

For treatment, the plants could be placed in a large cardboard box sealed with tape or plastic, in a large plastic bag, in a plastic "greenhouse", or in a relatively tight cabinet or closet. The strip should not be unwrapped until the plants are enclosed and ready for treatment. Following treatment, the stip should be placed in an airtight plastic bag (use several if you are unsure of the seal) for storage until needed again. Take care not to breathe the vapors or to get the materials on hands or clothing. Wash your hands thoroughly immediately after handling the strip. Perhaps these precautions sound a bit scary, but they are really no different than those you should follow when using any insecticide.

Alan Holden July 28, 1971

Highway Intersections

Intersections of Streets

When two streets cross each other, automobiles arrive at the crossing from four directions - two directions along each street (Fig. 1). By putting a stop-light there, the traffic along one of these streets is stopped for awhile so that traffic along the other street can proceed. But even so there is some confusion, because there are always some cars wishing to turn from one street into the other. The cars wishing to turn right may have little trouble. But those wishing to turn left must cross the line of traffic that is moving in the opposite direction, and they may block their own line of traffic while they wait for a chance to turn.

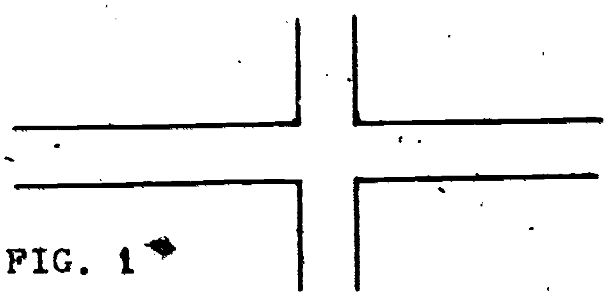


FIG. 1

There are several ways of reducing the severity of this problem. Sometimes the stop-light is made more complicated, providing a little time in which only left-turning cars can proceed. But these must be only the left-turners from one direction on the street; otherwise they would tangle with left-turners from the other direction on the same street. By separating so many stretches of time, each devoted to a single sort of traffic, the delays at the intersection can become quite large.

Sometimes an intersection is opened out into a "traffic circle" (Fig. 2). This arrangement spreads the problem of making turns over a long enough stretch of roadway to permit cars to

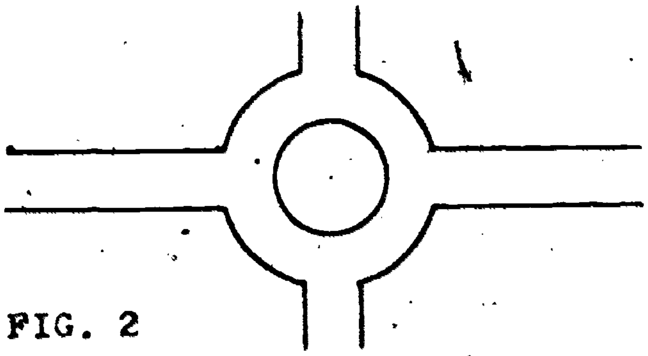


FIG. 2

thread their ways between one another satisfactorily. Nevertheless it is sometimes necessary to provide stop-lights at the entrances to the circle. And in a city a traffic circle requires more space than is usually available. The space in the center of the circle is hard to use effectively because the traffic cuts off pedestrians unless they are given a bridge or a tunnel.

In a city that does not afford space for circles at major street-intersections, a "no-left-turn" rule may help. A car wishing to make a left turn can accomplish its purpose by proceeding straight through the intersection and then making three right turns in succession (Fig. 3). But if all intersections have stop-lights, the car may be delayed by five lights instead of one. If the car wishes to make a U-turn, it may be delayed by as many as nine lights (Fig. 4).

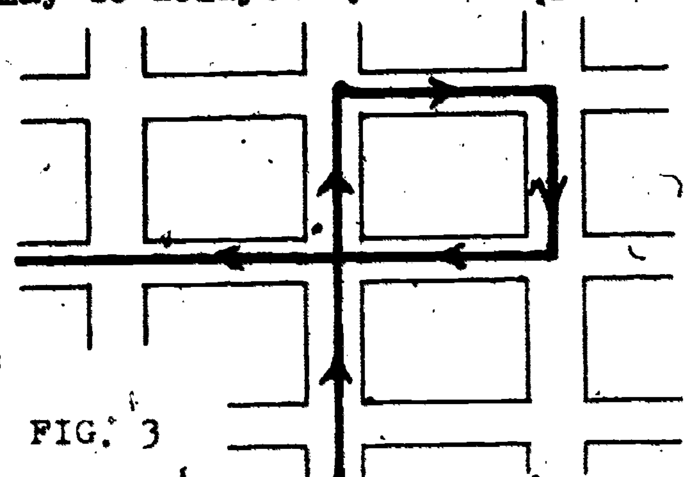


FIG. 3

When a major street is sufficiently broad, as is Commonwealth Avenue in Boston, an important gain can be made by passing it over or under an "intersecting" street such as Massachusetts Avenue, and interconnecting the two by short lengths of parallel side-street (Fig. 5). Then through traffic on Commonwealth Avenue need not

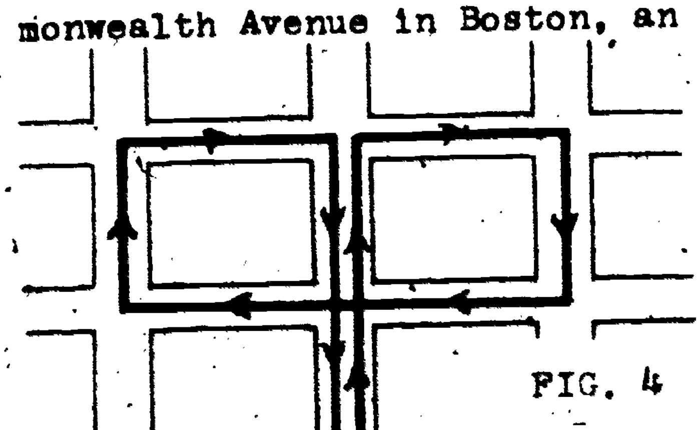


FIG. 4

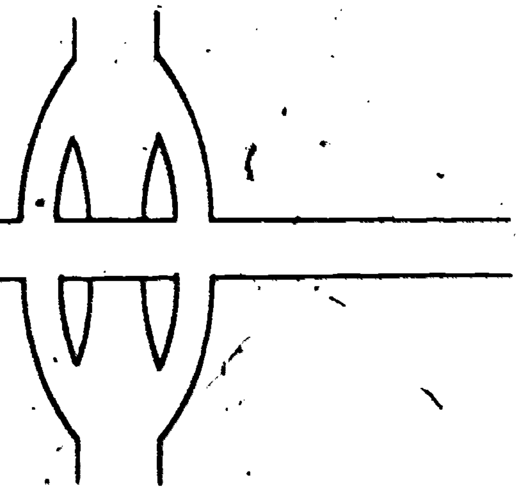


FIG.5

be delayed by a light. But lights are probably necessary on Massachusetts Avenue in order to admit the turners from Commonwealth Avenue at the exits from the side-streets.

From a "one-way street" a left turn is no more difficult than a right turn, and densely traveled cities such as New York may reserve their streets and avenues for one-way traffic. Stop lights are necessary to alternate the flow of cars through north-south and east-west streets. Sometimes the lights are reversed on a staggered schedule so that a car need not be stopped on a major street. But even a driver familiar with the city may have difficulty finding the most suitable route to his destination (Fig. 6).

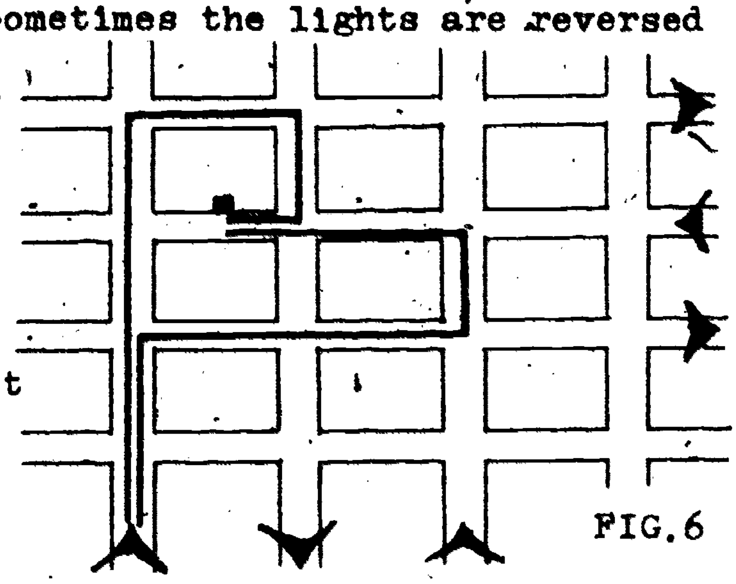


FIG.6

A driver unfamiliar with such a city and wishing to reach "254 East 67th Street" has the task of finding out, by observation as he proceeds, what streets run in what directions and what block on East 67th Street includes #254. Can you suggest a good strategy for him to employ?

<u>The Cloverleaf</u>

When two major highways intersect in more open country, there may be enough space to provide a "highway cloverleaf" (Fig.7). This device is expensive in its use of space and in its require-

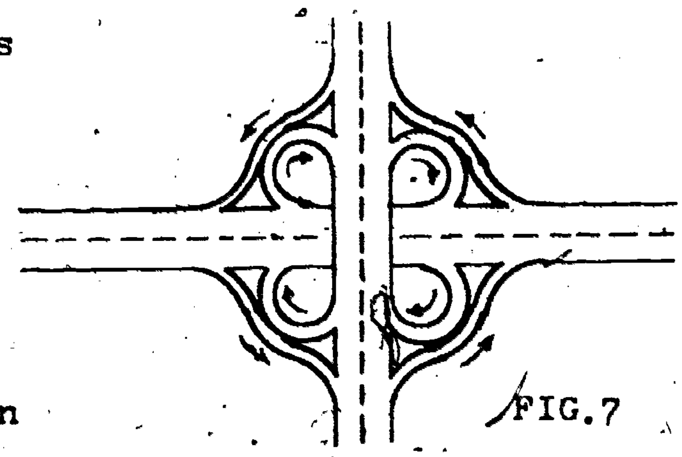


FIG.7

ments of construction, but it has two outstanding virtues: (1) it employs no stop-lights, permitting traffic to proceed without interruption, and (2) it permits a car proceeding in any direction to transfer to any other direction without making a turn to the left across a highway. The cloverleaf carries one highway across the other on a bridge, so that through-traffic can proceed at full speed in both directions on both highways. And it provides connections between the highways, so arranged that a car turning into or out of a connection always turns to the right.

How many connections must the cloverleaf provide? Each road goes in two directions. There should be a connection leading from each direction on one road to each direction on the other road, requiring a total of four connections leading out of the first road. Since these are one-way connections, the second road needs similar connections with the first road, and therefore there must be eight connections all told. The usual cloverleaf provides all eight, as Fig. 7 shows.

There is another instructive way of analyzing this requirement. The two roads provide four directions. Each direction needs a connection with the other three. Hence there should be three-times-four connections - three connections for each of the four directions. This number, twelve, is larger than the number, eight, previously obtained: what connections were previously left out? Clearly the additional four connections are those that would permit a car to turn around and go the other way - the

U-turn connections. Then how can a car make a U-turn on the usual cloverleaf that has only eight connections? Answer: it can use two of the connections in succession (Fig. 8).

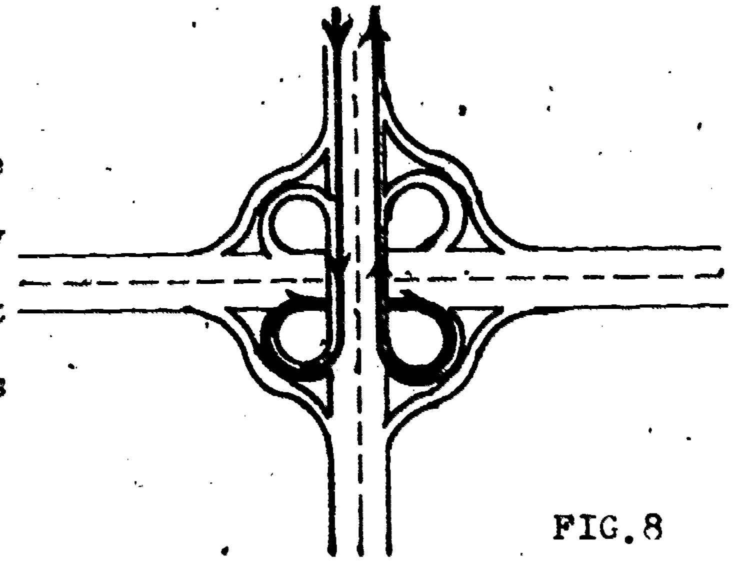


FIG.8

Non-Redundant Cloverleafs

The idea that a car might use several connections in succession in order to get where it wants to go has an important extension. Suppose that a car proceeding north at a cloverleaf wishes to turn east. There is a direct connection that it can use. But perhaps the car is traveling fast and its driver doesn't see the turn-off until too late. The car can still turn east nevertheless by using three connections in succession (Fig. 9). Hence you might say that the north-to-east connection isn't really necessary. But of course it is extremely convenient. And if it wasn't there, and a great many cars wanted to turn from north to east, the cars might jam up the connections and the bits of highway between them.

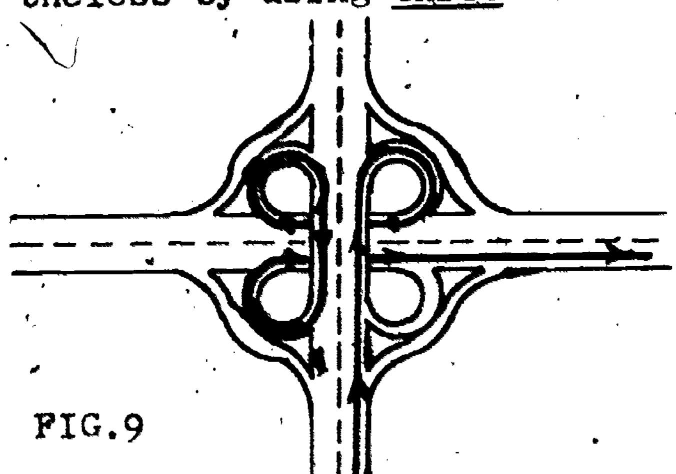


FIG.9

This raises the interesting question, how many connections are necessary at a cloverleaf to permit a car to go in any direction without turning left across a highway? The question can be answered this way. (1) The two highways provide four directions. (2) The connections must provide at least one entrance to each direction and one exit from it, and hence each of the four highway-directions must make contact with two connections. (3) If you concluded from this that eight connections are necessary, however, you would be counting each connection twice, because each connection provides both an exit from one direction and an entrance to another. (4) Hence you conclude that at least four connections are <u>necessary</u>.

But are four connections <u>sufficient</u> - in other words, is it possible to satisfy the requirement with four connections? The best way to answer this question is to show how four sufficient connections can be constructed. Write N, S, E, W. for the four directions, at the corners of a square. Then think of the sides of the square as the symbols for the four connections. Since only one direction of travel is permitted on any connection, put an arrowhead on each connection. If you can travel completely around the square of connections by following the arrows, then clearly you can get from any direction to any other by traveling over one, two, or three connections (Fig. 10).

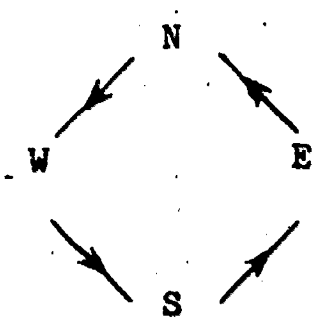


FIG.10

In the stripped-down cloverleaf of Fig.11 the four remaining connections have the arrangement symbolized in Fig. 10. You can see, from the earlier discussion of the U-turn (Fig. 8) and the north-to-east turn (Fig. 9), that the cloverleaf of Fig. 11 accomplishes what you set out to do. You may find it interesting to compare how cars must move through

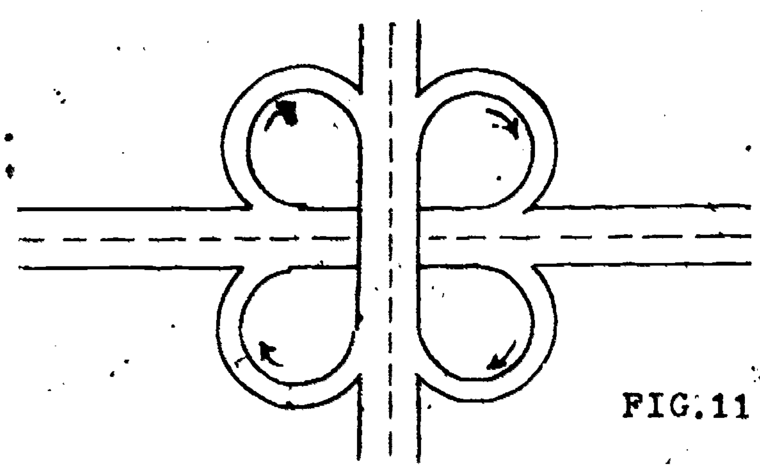


FIG.11

this cloverleaf with the way they must move through the streets of a "no-left-turn" city (Fig. 12).

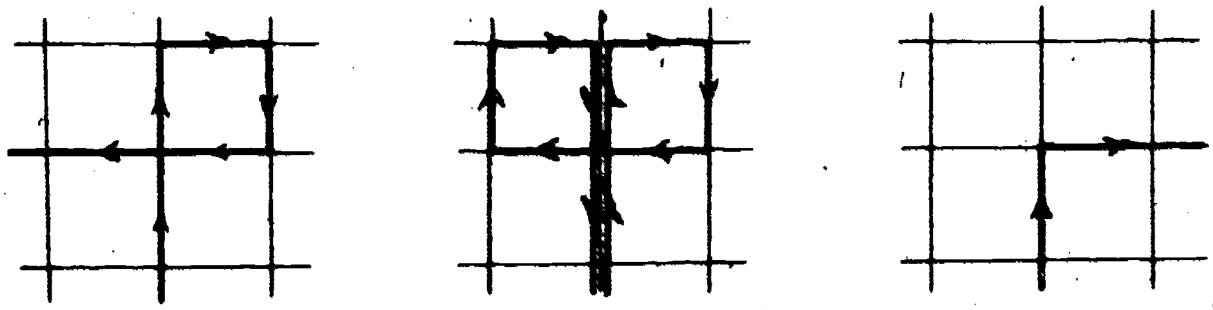
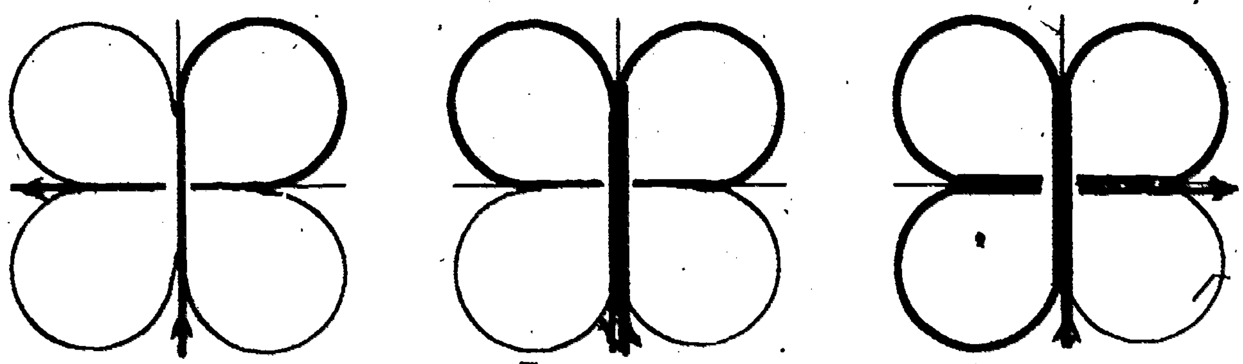


FIGURE 12



In Fig. 10 the four directions were put on the page as they would appear on a conventional map: N is up, E is to the right, etc. But our reasoning about them did not require that we diagram them in that way; we might have used the arrangement in Fig. 13. Then when the directions symbolized by the letters are restored to their directions on a conventional map, the cloverleaf corresponding with Fig. 13 would look like Fig. 14. But,

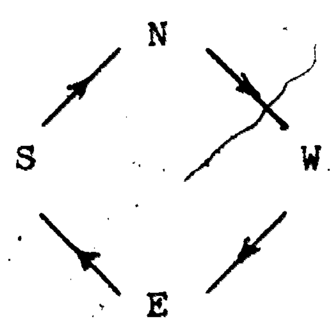


FIG. 13

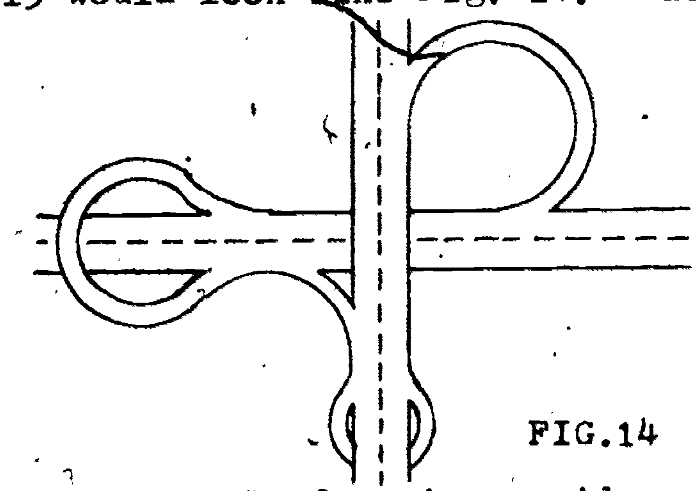


FIG. 14

in agreement with our analysis, this cloverleaf again provides a way for cars to proceed from any of the four directions to any other.

At each of the exits to the connections there must be a sign naming the cities that a car can reach by taking that exit. It may help to give concreteness to the foregoing discussion by designing appropriate signs and showing where they must be put. Suppose for example that one of the highways runs east-and-west between Boston and Albany and the other highway runs north-and-south between Springfield and New York. What cities are named on each of the four exit signs for some one of these cloverleafs? Answer: each sign names the three cities that are not in the direction in which the car has been traveling.

In England traffic proceeds along the left side of any road instead of the right. Could any or all of these cloverleafs be used in England? If not, what changes must be made? Answer: each structure is usable in England, but the direction of travel along each connection is reversed, and so the exits of the American structure become the entrances of the English structure and vice versa.

Three Intersecting Highways

When more than two major highways intersect, the problem of providing connections between them having the virtues of the cloverleaf becomes more complicated. The consequent interweaving of highways and connections often provides fascinating engineering structures -- and the problem of traversing them in a car often intimidates its driver. Drivers might have more confidence if they had taken some thought to the general problems faced by highway engineers so that they could anticipate to some extent the general outlines of their solutions.

Three highways provide six directions of travel. Connecting each directly with all the others would require twenty-four one-way connections if U-turns are not included, and thirty if they are. It is tempting to reduce the costs of land and construction by designing arrangements in which some cars use more than one connection to get where they want to go. We can begin by using the same sort of analysis that we used before to determine how many connections are necessary and sufficient.

When the six directions are written at the corners of a hexagon, its sides furnish arrows symbolizing six one-way connections by which any direction can be reached from any other (Fig. 15). Some cars will have to use five connections in order

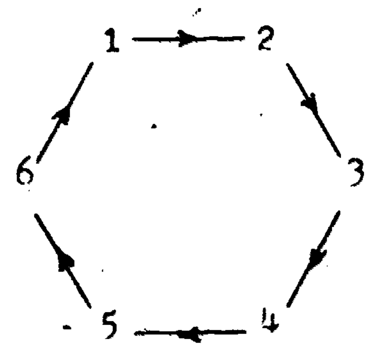


FIG.15

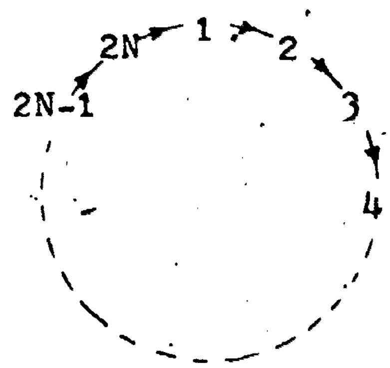


FIG.16

to reach the direction in which they want to travel. Indeed this method of analysis makes clear that, at an intersection of N highways, 2N connections are necessary and sufficient, and some cars must traverse 2N-1 connections (Fig. 16).

As in the earlier case, the directions can be permuted on the diagram without changing these conclusions; and again each permutation will prescribe a different cloverleaf. In designing each cloverleaf you must remember that the entrance to a direction on a highway must precede the exit from that direction.

Some arrangements that might result from this procedure appear in Fig's. 17, 18, 19 and 20. By imagining a car moving through one of these arrangements, you can check that it can reach any direction from any other. Notice that, in obedience

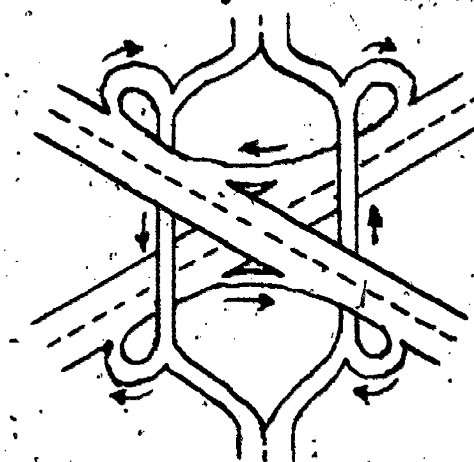


FIG.17

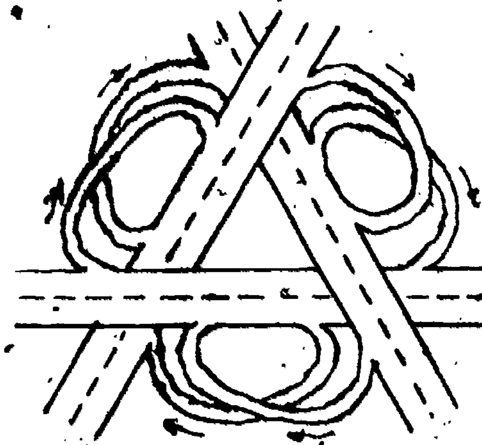


FIG.18

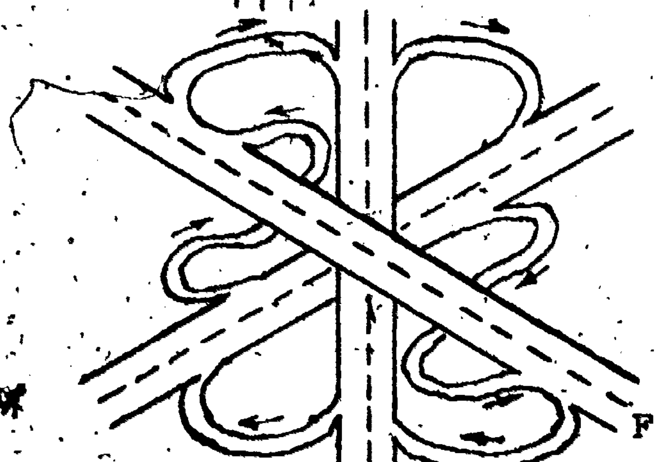


FIG.19

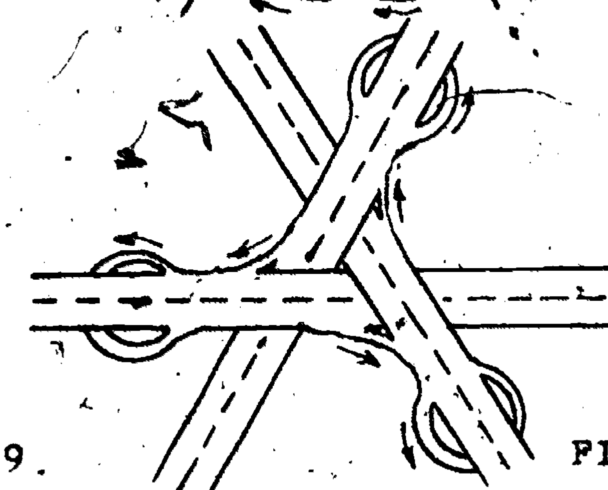


FIG.20

with your method of analyzing the problem (Fig. 15), a complete check arises if you find that a car starting from any direction returns to that direction when and only when it has traversed all six connections.

It is interesting to notice that there is a systematic way of arranging connections to provide a non-redundant clover-leaf at an intersection of any number of highways. That way is symbolized in Fig. 21 for two, three, and four highways. But

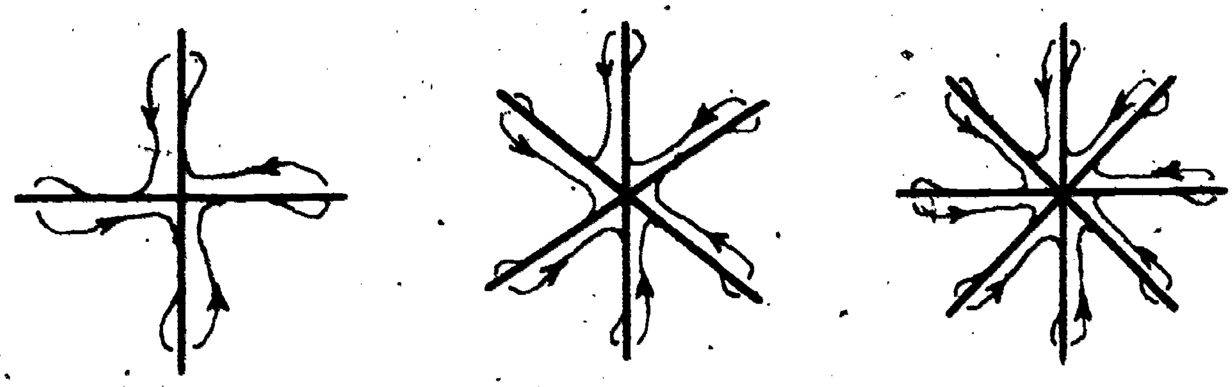


FIG.21

that way might not always be the most suitable for the land available; and it has the engineering disadvantage of requiring that each connection pass over or under a highway.

Deliberate Redundancy

Non-redundant cloverleafs connecting more than two highways would certainly be intolerable in real life. Drivers required to traverse five connections at an intersection of three highways would be angered, and the segment of each highway between exit and entrance would be jammed with cars, most of which intended to use no more than that small bit of the highway. It is interesting, therefore, to consider how much additional connections would alleviate this distress without returning all the way to a completely redundant cloverleaf.

The problem can be made definite by asking, for example, "what is the smallest number of additional connections required to reduce from five to four the maximum number of connections to be traversed by any car where three highways meet?" The easiest way to tackle the problem is to return to Fig. 15 and trace what happens when more connections are added.

After a few trials you find that there is a way of adding only two more connections that will enable all cars to reach their desired directions by using no more than four connections (Fig. 22). If the newly connected directions are the opposite directions on one highway, the two added connections are U-turns on that highway. But remember that the newly connected directions can be chosen as any

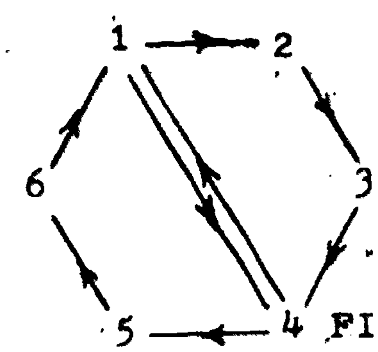


FIG. 22

two of the six directions without affecting the validity of the analysis.

Similarly you can find a way of adding three connections that will reduce the maximum of required traversals from five to three (Fig. 23). And six added connections can reduce

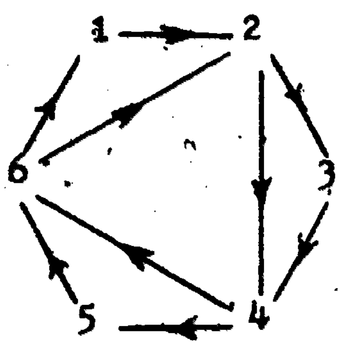


FIG.23

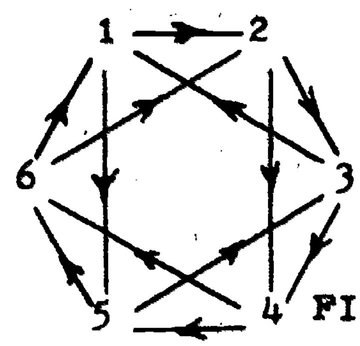


FIG.24

that maximum from five to two (Fig. 24). Perhaps this last result - a three-highway cloverleaf with twelve connections and a maximum traversal requirement of two - would provide an acceptable compromise for many cases in real life. The reduction in the number of connections from thirty (for complete redundancy) to twelve would effect a large simplification in engineering.

When the intersecting highways are "divided highways", whose two directions of traffic are carried on two separate parallel roads, it is especially easy to construct U-turns. Could the six additional connections just discussed be the six U-turns for the three highways? The proposal can be diagramed as in Fig. 25; you find that some cars must traverse three connections; the arrangement does not reduce the maximum to two.

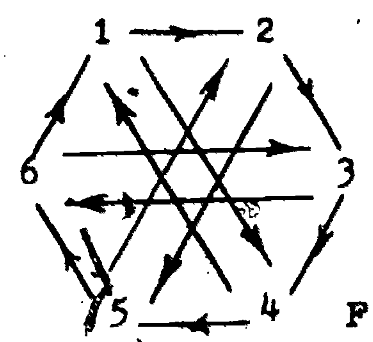


FIG.25

Nevertheless the arrangement of Fig. 25 might please more drivers than that of Fig. 24. Many drivers may find that, despite their best effort, they end up in the wrong direction. These drivers will want to find a clear-cut U-turn so that they can return to the cloverleaf and tackle the wretched thing again.

Youngsters may be interested not only in designing some highway interchanges but also in examining some in the real world. Bird's eye views of some fairly complicated interchanges have been photographed from airplanes. They sometimes appear in advertisements, especially by oil companies. Analyzing and diagraming such a photograph is an instructive exercise.

It is also possible, and much more difficult, to analyse a complicated interchange from the "worm's eye views" obtained by traversing it in a car. After becoming acquainted with some of the ideas discussed above, a class of youngsters might be interested to plan a strategy for making enough direct observations, from buses or private cars, to enable them to diagram an interchange. Then from direct experience they could discuss the effectiveness of its design.

The following exercises, and others like them, may help to give insight into the foregoing analysis and thus into the problems of designing and traversing highway interchanges.

#1. Put numbers 1,2,3,4,5,6 on the directions in Fig's.17-20 so that the connections in these diagrams connect the directions in the way diagramed in Fig.15. Often the simplest way to examine more complicated interchanges is first to find a simple circuit of this sort, diagram it in this way, and then add the redundant connections to the diagram.

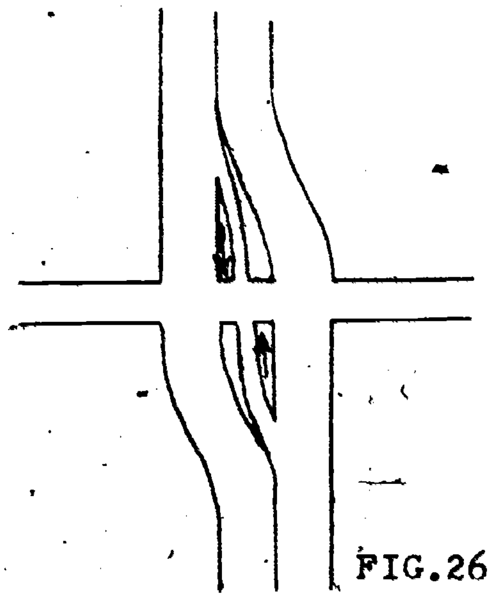


FIG.26

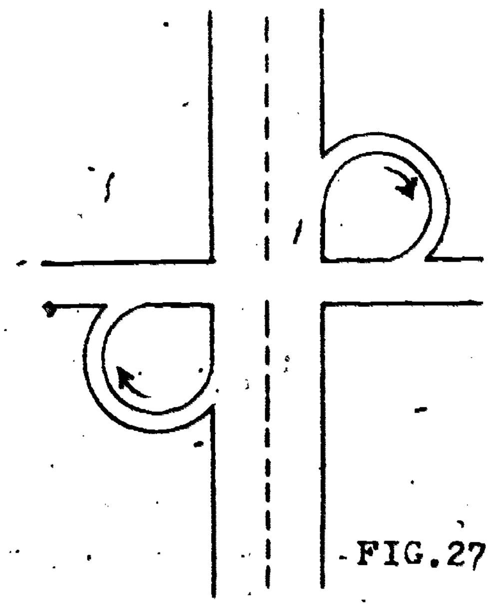


FIG.27

#2. Two familiar methods of connecting a major highway with a minor road (the "waiting line", and the "jug handle") appear in Fig's. 26 and 27. In both arrangements a traffic light stops traffic on the highway for occasional short stretches of time. After examining the courses permitted to cars while the light is red and while it is green, discuss which arrangement you prefer.

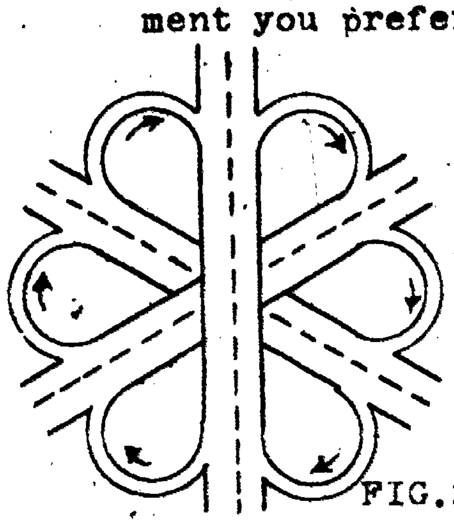


FIG.28

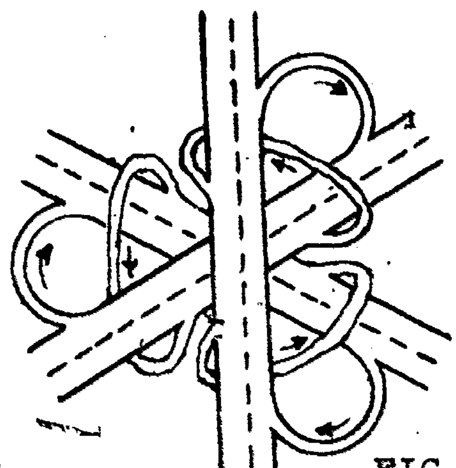


FIG.29

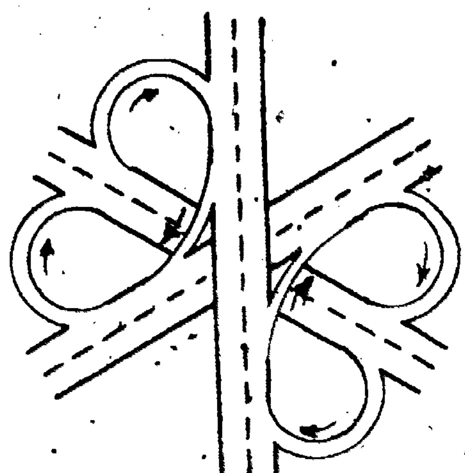


FIG.30

#3. Which, if any, of the six-connection interchanges in Fig's. 28, 29 and 30 is sufficient?

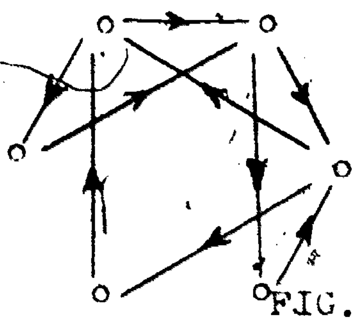


FIG.31

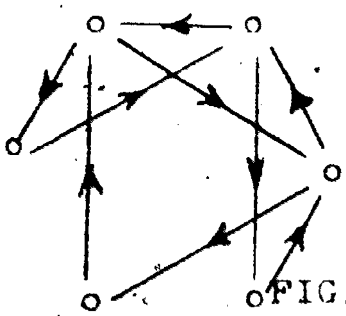


FIG.32

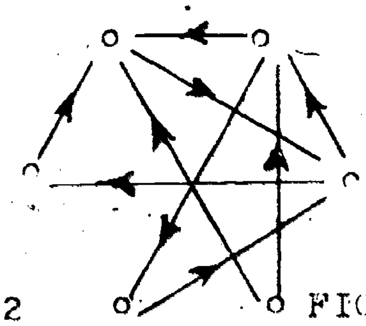


FIG.33

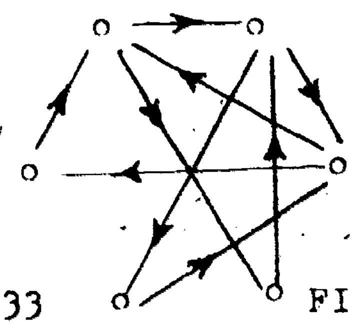


FIG.34

#4. The four interconnection diagrams of Fig's.31,32,33, and 34 form two pairs, in which the two members of each pair are permutations of the same diagram and are therefore equivalent.
 (1) Pair the diagrams.
 (2) Determine the maximum number of connections any car must traverse in an interchange designed according to each diagram.

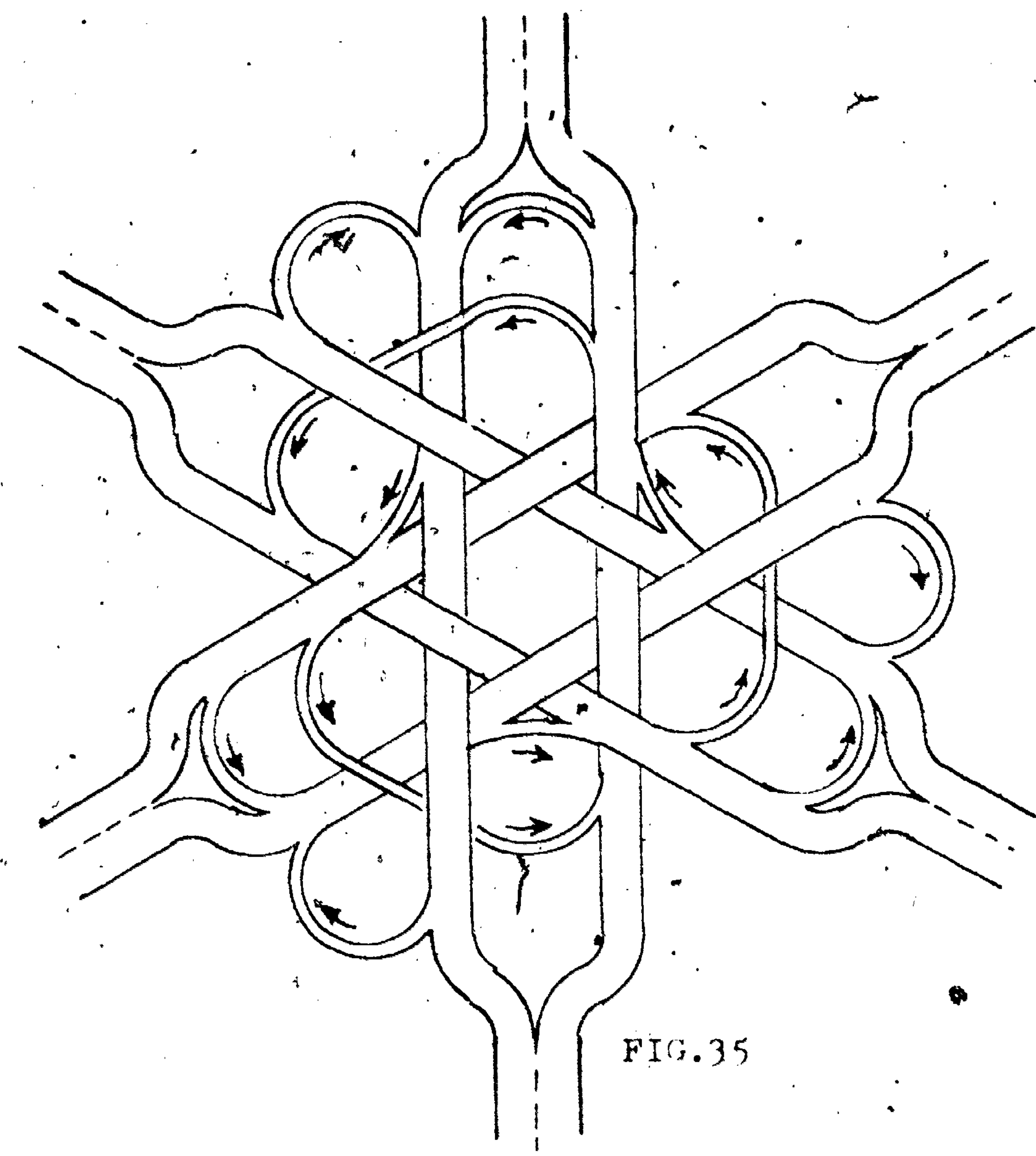


FIG.35

#5. Determine the maximum number of connections that any car must traverse in the twelve-connection interchange of Fig.35, by the method suggested in exercise #1. Notice that, when each direction has several entrances and exits, the problem of placing the entrances ahead of the exits becomes more difficult.

SOME CONSIDERATIONS ON THE CURVATURE OF AN EXIT OR ENTRANCE ROAD

by

Earle Lomon

Introduction

Material contained in this background paper is fairly advanced for elementary school students and teachers. The purpose of this short introduction is to provide an intuitive basis for the proposition that entrance and exit ramps and other connectors must be designed in accordance with certain limitations on curvature.

People riding in a closed car at a constant speed on a straight, smooth road do not experience any forces acting on them. However, when the car goes around a turn, people are conscious of the side of the car pushing on them. The force is called a centripetal force. A car will not turn around a corner unless it is pushed sidewise by the road. Similarly, if you were standing on a fast merry-go-round, you would feel as if a force were pulling you outward, away from the center. However, what you are experiencing is not a real force in the sense of an object exerting a push or pull on another object. What you feel is sometimes erroneously labelled a centrifugal force, but it is merely an inertial reaction to the centripetal force acting on the merry-go-round. If the merry-go-round stopped suddenly, you would go flying in a direction tangent to the circle at that moment. With respect to cars, the centripetal force that turns the car arises either from the friction of the tires on the road surface or from the banking of the roadway. When the maximum frictional force which the tires can exert is not sufficient to turn the car the required amount, the car skids off the road.

When a curve is banked, the outside of the curve is higher in elevation than the inside of the curve; the road is sloped in a crosswise manner. The purpose of this banking is to help the road exert a force to push the car into its turn. Since the force of friction is limited, a car will skid without the push of the banked road when the frictional force is not big enough to maintain the necessary turn. From your own experience driving a car or riding a bicycle, you know that:

a. The greater the speed at which you enter a curve, the more banking necessary to prevent skidding.

b. The sharper the corner (i.e., the smaller the radius of curvature) the more banking necessary.

Finally, it is worth mentioning that, in practice, the amount of banking is limited; on a road too sharply banked, a slowly moving vehicle, or one at rest, will either slide across the road or roll over.

To confirm some of the ideas just explained, try this experiment. Run or ride a bicycle in a circle. Note that you must lean "into" the curve to keep your balance. If your speed is too great, you will slip or skid.

The design of the path to be taken by a connector between highways is a compromise of several important attributes. The most critical are:

1. cost factors relating to the amount of land used, and the amount of paving to be done,
2. control of vehicles changing speed and direction.

Item 1 implies short connectors turning sharply from one direction to the other to keep cost down, while item 2 implies long slowly-turning connectors to allow reasonable speed without sliding. In this paper we consider the relation between car speed and road curvature and some investigations that can be made by students.

A car must leave a highway at nearly its "cruising" speed if it is not to slow down cars behind it in the right hand lane (which are not turning off). Consequently the beginning of the exit ramp must permit high speeds for a sufficiently long distance for smooth braking to be effective. See fig. 1.

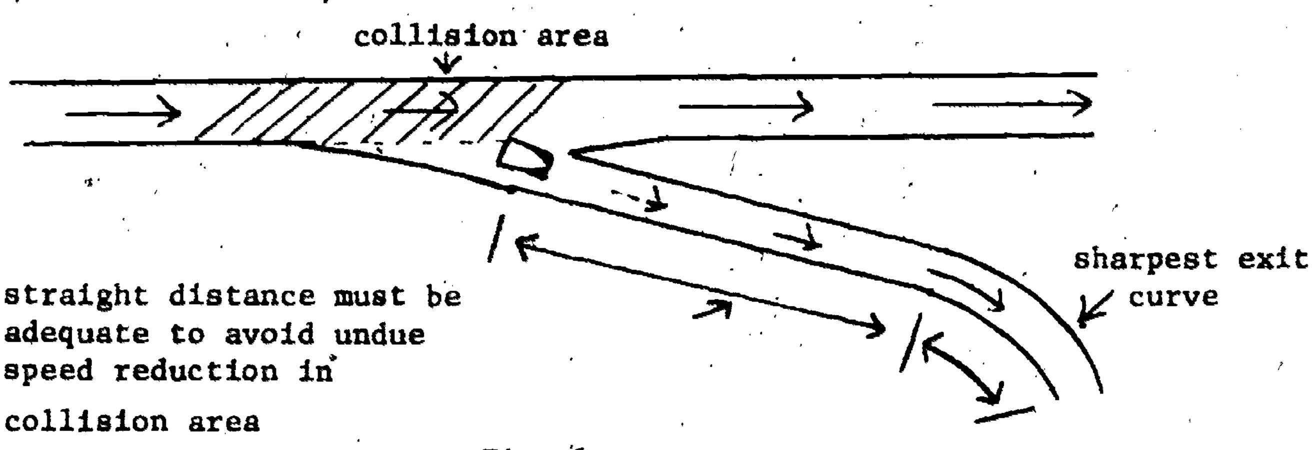


Fig. 1

Fast braking and turning may cause loss of control due to slow response time by the driver, but we are more concerned here with the tendency to slide or slip under these conditions. Sliding occurs when the force of friction between tires and road is inadequate for the required acceleration or deceleration of the car. The harder one brakes, the bigger the deceleration and the more grip is required from the tires. There is a maximum grip the tires can apply; after that the car slides and no larger deceleration can be attained.

In order for a car to travel around a curve it must be pushed toward the inside of the curve.* This push is produced by the road exerting (through friction) a force on the car pushing it toward the inside of the curve. If no other force is exerted on the car, it will travel around the curve at constant speed.

The push toward the inside of the curve has to be bigger if the curvature of the road is larger, if the car's speed is greater, or if the car weighs more. A car slides sideways on curves when the force of friction between the tires and the road is insufficient to produce the push required to keep the car travelling in the curved path. On good surfaces with properly inflated tires, the maximum force of friction is given by the weight of the car multiplied by a coefficient of friction which varies from 2/3 to 9/10 depending on the nature of the tire and road surfaces and whether the road is wet or dry. On icy roads the coefficient of friction may be as low as 1/10. It has been found that the area of contact between tire and road has little effect on the frictional force.

The sliding can be held off to some extent by banking the curve. Extensive banking is not customary because very careful control of speed (neither too large nor too small) is required. Of course, the required

*See Appendix A for a more detailed description.

push toward the center of the curve may be reduced by reducing the speed of the car or making the road's curvature smaller, that is, making the road straighter.

The experimental and mathematical complexities inhibit a thoroughly quantitative approach to these design problems in the field by elementary school students. In the following pages we suggest:

A. Some observations that can be made in the field.

B. Related observations that can be made using model cars and roadways in the Design Lab or classroom.

C. A procedure for measuring the straightness of a curve in the field.

D. A way to use data from tables to make scale drawings or models of interchange designs.

E. Experiments with model cars and roadways which, combined with graphing, will give values for critical speeds according to the straightness of different curves.

A. Some Qualitative Outside Observations

On an overpass or from a nearby hill the students can observe the shape of connectors at a traffic interchange. The problem of the length and curvature of connectors will arise naturally as different interchange designs are compared for area of land used. Students can drive along connectors and note details of structure and of the car's motion. Fig. 2 illustrates the features they are likely to see.

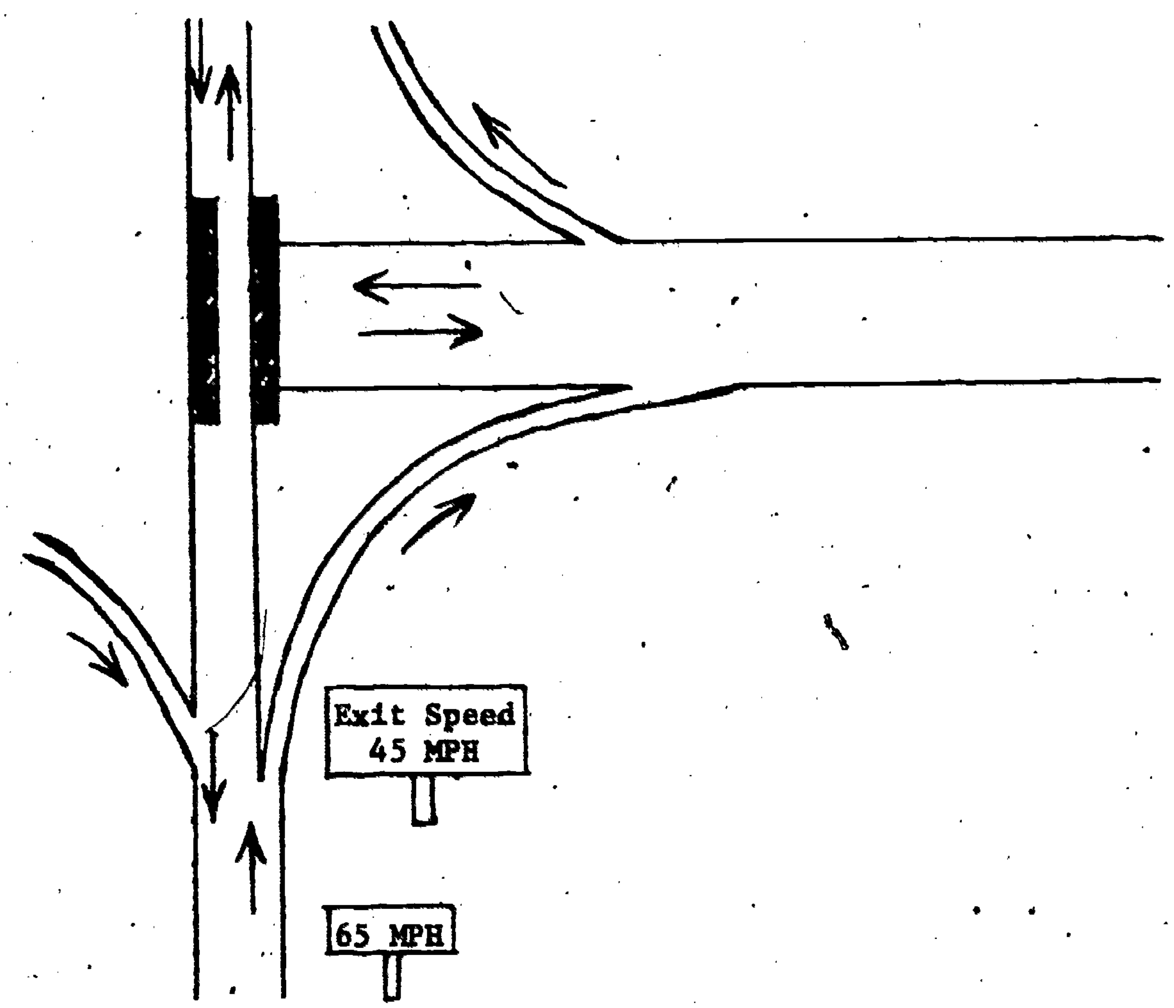


Fig. 2

They will find that they need to make the following observations.

Does the exit road leave the main road at a small angle (nearly parallel), a moderate angle, or a large angle (nearly perpendicular)?

Is the exit road straight or curved?

Is it curved more near the ends or near the middle? Are the curves banked? Very much?

Is the entrance or exit end nearly straight?

If there are straight sections at either end, is one straight section longer than another?

Is the longer one at the faster or at the slower road?

Once they become aware of the relation of connector shape to car control safety they will seek answers to:

Should cars slow down before they get into the connector. Should they slow down after getting in? Where should they do most of their slowing down?

Should cars begin to speed up before they leave the connector? Should they speed up after getting on their next road? Where should they do most of their speeding up?

What speeds should signs permit on each road? On the exit and entrance ramp?

Some observations that might be made on family trips, school outings or specially-planned rides are as follows:

What speeds are the cars you are in travelling? Where do they go the fastest? The slowest?

Is there ever any trouble in slowing down enough or speeding up enough?

What are the unsafe features of leaving or entering a busy highway?

B. Observations with Model Setups

The children can also discover the qualitative relation between speed and straightness of curves by experimenting with models. The speed of a model car can be varied by starting it at different points on a hill.

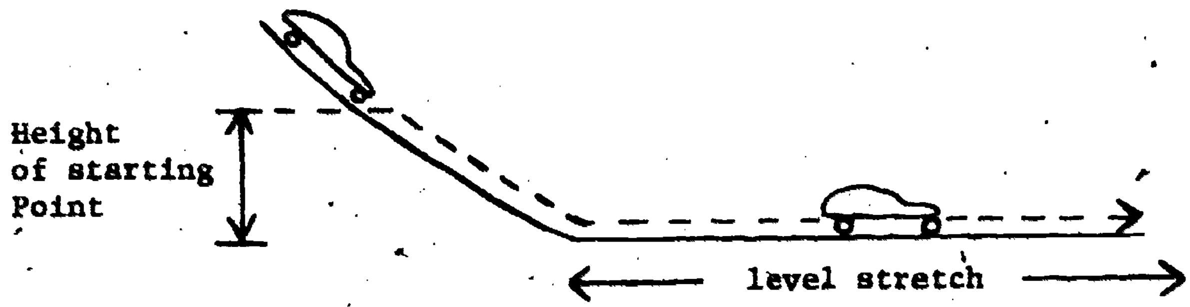


Fig. 3

If a model car starts from rest at a certain height on a hill, it will have a definite speed at the beginning of the level stretch of roadway. The speed will be the same for each trial since it depends only on the height of the starting point and the car used. If the car wheels roll easily, the resultant speed will be nearly the same for all such cars and will depend mostly on the height and not much on the steepness or straightness of the hill.

Curves may be placed on a level stretch near the bottom of the hill. For cars to travel on a curve (without a track or lip to hold them on the curve) they have to be steered, i.e. their front wheels held at an angle. However, the steering may be held fixed throughout a run. Then the downhill in Figure 3 would have to be broad enough for the car to come down on a curve as shown in Figure 4.

Fig. 4

The children will see that a model car will travel around a certain curve easily at low speed. If the car is travelling at a higher speed (starting it higher on the hill) it will slide sideways across the road as it goes around the same curve.

If different curves are used, the children will see that a car that travelled easily at a certain speed around a curve will skid sideways when it goes at the same speed around a curve that is less straight. (See Fig. 5)

Car travelling at same speed

Fig. 5

Data might be collected at this time which several students may want to use later in more extensive experiments with the model setup. They will need to know the height on the hill which produces a speed at which the car first starts to slide sideways on a certain curve. This critical height can be found after a series of trials at different heights. The trials should be repeated with other curves having different amounts of straightness so that the critical height is found for each curve.

The children should build their own design of a roadway in the Design Lab.

C. Quantitative Outside Observations

Students can make the following outside measurements and then calculate the straightness of a section of roadway or how far ahead one goes for a certain amount of turning. With a trundle wheel, a tape measure or a long rod, distances may be measured along the connector. Not only should the total length be measured, but the straight (or very slightly curved) sections at each end should be separately measured, and also the strongly curved section. The width of the connector can be measured at the same time.

The straightness of the strongly curved section can be measured by combining the distance measurement with a measurement of the difference of direction (θ) between two parts of the road. (See Fig. 3).

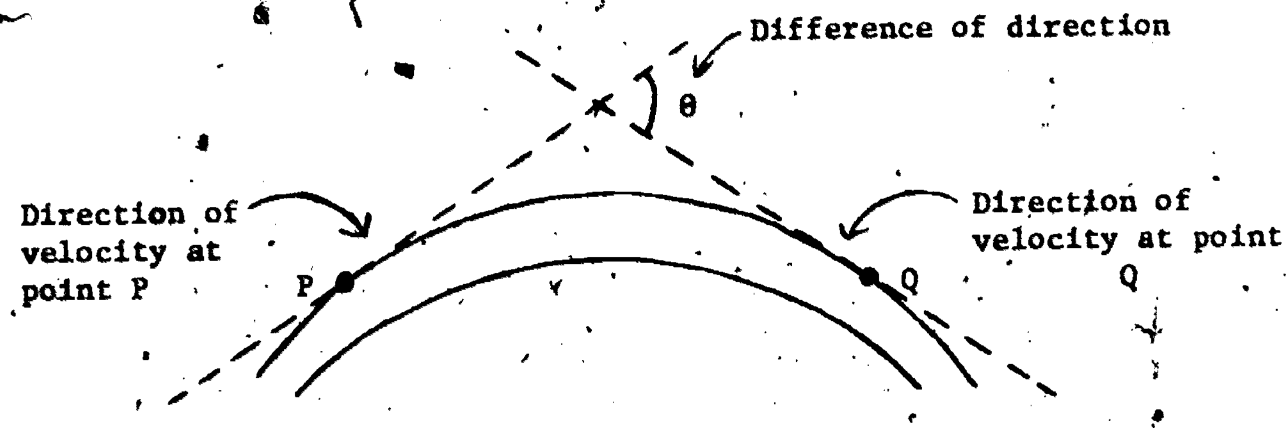


Fig. 6

If the straightness of the road does not seem to change very much between P and Q (see Fig. 7) then a two armed instrument with a simple (large) protractor fixed to one arm may be used to measure θ.

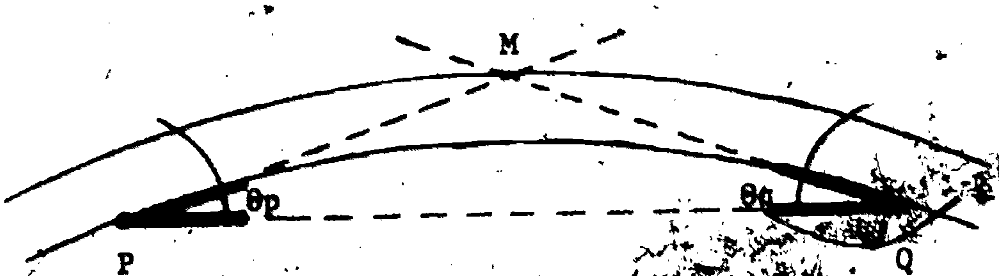


Fig. 7

The student at P holds his instrument horizontally pointing one arm "straight along the road" and the other arm at the student at position Q. The student at Q points one arm "straight (back) along the road" and the other arm at P. The student at P reads his protractor, getting θ_p; the student at Q gets θ_Q. To help point "straight along" they may find it helpful to have a student stand near the middle of the arc (point M) at the outside curve (on the road or a little out in the field).

It turns out that $\theta_P + \theta_Q = \theta$. The students satisfy themselves on this point back in the classroom using a blackboard or piece of paper. One draws any base line (see fig. 8) and makes a line at angle θ_P at one end, and one at angle θ_Q at the other end. The change in direction of the road is shown by the arrows and is measured by the angle θ. One can measure θ directly with a protractor and compare it to $\theta_P + \theta_Q$.

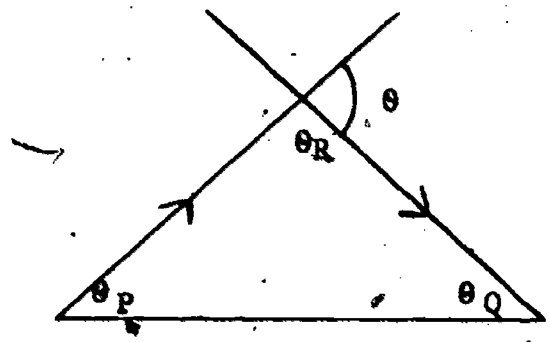


Fig. 8

The comparison can be made for different choice of θ_P and θ_Q confirming each time that $\theta = \theta_P = \theta_Q$. When the students are convinced that $\theta = \theta_P + \theta_Q$ they can just add θ_P and θ_Q in the field and no longer need to draw the triangle of Fig. 8.*

The straightness is the distance travelled per unit angle turned:

$S = \frac{A}{\theta}$ in feet per degree, yards per degree, meters per degree, etc. depending on units used.

For straighter parts of the road, the distance will be greater per degree, so S will be a larger number. A check of posted ramp speeds versus ramp straightness may show that the recommended speed goes down as the straightness or S decreases.

* The students may want to discuss their result in terms of the sum of all the angles in a triangle = $\theta_P+\theta_Q+\theta_R$. It is clear that $\theta + \theta_R$ = a "straight angle" or 180° on the protractor. Using $\theta = \theta_P + \theta_Q$ it follows that $\theta_P+\theta_Q+\theta_R = 180°$ for any triangle.

D. Using Data from Tables

Now that they have measured the straightness of a curve and have an understanding of the relation between speed and straightness of curves, they can understand the data in following tables and use it in designing their interchanges.

Design Lengths of Deceleration Lanes

Design Speed of Highway, mph.	Length of Deceleration Lane, in Feet, for Following Design Speed of Exit Curve, mph. 20	30	40	50
40	250	175		
50	350	250	200	
60	400	350	250	200
70	450	400	350	250

Design Lengths of Acceleration Lanes

Design Speed of Highway, mph.	Length of Acceleration Lane, in Feet, for Following Design Speed of Entrance Curve 20	30	40	50
40	400	250		
50	650	500	250	
60	950	800	550	250
70	1200	1000	800	500

Minimum Safe Straightness for Intersection Curves*

Design Speed of Curve, mph.	15	20	25	30	35	40
Minimum Safe Straightness (feet per degree of turning)	4/5	1 3/5	2 7/10	4	5 1/2	7 1/2

* This table allows for a larger friction factor on less straight curves and a larger banking of curves for higher design speeds.

For example, if they want to plan an exit from a highway with a posted speed of 60 miles per hour, they will see from the tables that they need a deceleration lane of 350 feet before a curve designed for 30 miles per hour speed. The curve must have a straightness of about 4 feet per degree turned.

The children can make scale diagrams of their interchange design. A scale of one inch per 100 feet might be convenient to start with. The deceleration lane would be 3 1/2 inches long. The curve can be found by using a narrow plastic strip placed between the arms of a square. For example, 4 feet per degree would be 360 feet or about 3 1/2 inches for a 90° turn. A plastic strip 3 1/2 inches long could be fit between the arms of a square angle so that the arms run tangent to the ends of the curve. (See Fig. 9)

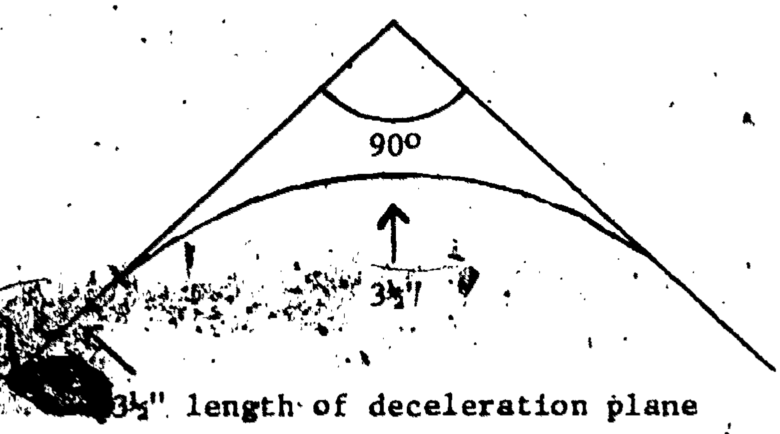


Fig. 9

E. Measurements on Models

It is important for most of these measurements to be able to produce a definite speed and vary it easily. A simple way to do this is to provide a downhill stretch of roadway such as indicated in Fig. 10 or in Fig. 4. If a given model vehicle starts from rest at a fixed height (H), it will have a definite velocity (V) at the beginning of the level postion which depends only on the height and choice of vehicle.

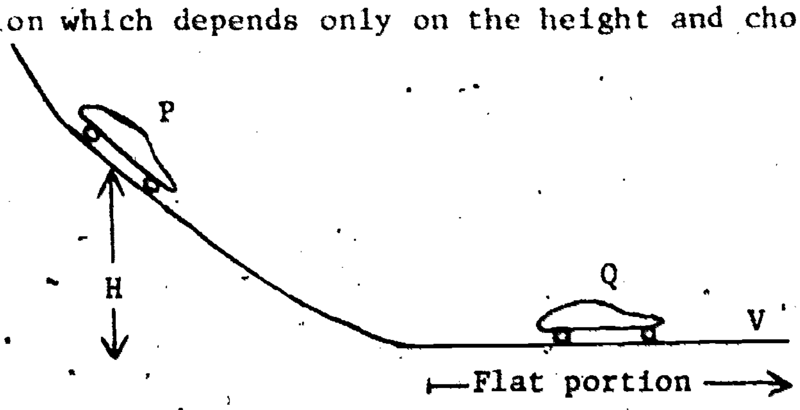


Fig. 10

The velocity is "reproducible" that is, if we repeat the experiment, starting the vehicle from rest at P, it will have the same velocity (V) as before when it reaches Q. Furthermore, if the wheels roll easily, then the resultant velocity for different vehicles will be nearly the same when started from P, so that one does not have to start all over again when one changes the model. Also, if the wheels roll easily, V will depend mostly on H and not much on the shape of the hill (steeper or more gradual, curved, etc.) or on whether the car comes "straight down" or on a crossways curve which will enable results on different roadways to be compared. See Appendix B for a test on how frictionless the rolling is.

Under the above conditions a reproducible definite velocity can be obtained. Before the students measure the velocity, they should find the critical height (H_c) for which the car slides sideways on a specific curve. After a series of trials at different heights, they will find the height at which the car first starts to slide sideways in the curve. The height can be measured in feet and the straightness in feet per 90° turn (1/4 of a circle). See Page 12 for a description of measuring straightness. After critical heights have been found for several curves, a graph can be made of height vs. straightness. It will resemble this:

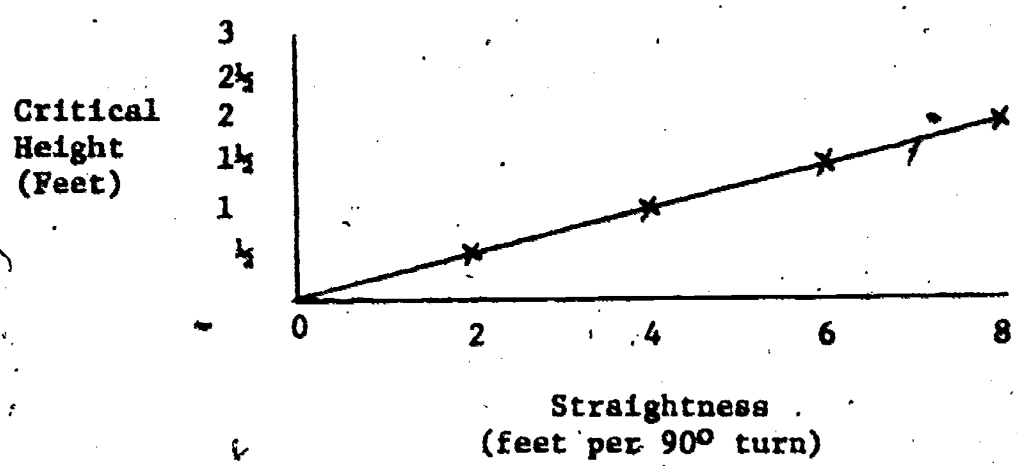


Fig. 11

The next step is to measure the velocity produced at each critical height. To measure the approximate velocity the car must be timed as it travels along a level section of road at the bottom of the hill. The longer the section is, the more accurate the timing will be, but the car may slow down more during the timing. See Appendix C for a test of the amount of slowing down of the velocity.

When the students try different speeds on different curves, the car will be travelling at least several feet a second when it slides sideways on the larger (straighter) model curves. With practice, the students may be able to time down to a half second using a stop watch. This may be done more easily by having one student watch the stop watch while another student watches the car and taps him on the shoulder as the car passes each of two points. This eliminates the time taken to start and stop the watch. The time should be measured several times, starting the car at the same point on the hill, and the median time selected. The use of a bell, clapper, and carbon paper will allow timing down to about 1/50 of a second.

A graph should be made of velocity vs. height for several different values of H. The graph will resemble the following:

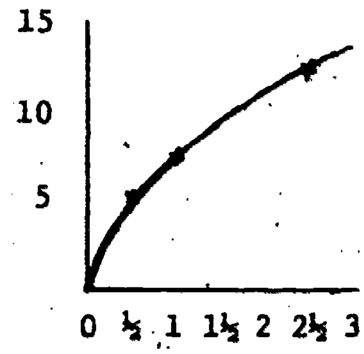


Fig. 12

The velocity increases with the height, but not as fast as would be shown by a straight line graph. See Appendix D for the equation of the graph.

The relation between velocity and straightness can now be found by combining the two sets of data. A graph can be constructed by finding for each critical height the corresponding velocity and the corresponding straightness. A graph is then made which might look like this.

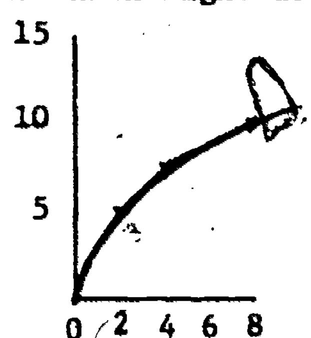


Fig. 13

The measurement obtained from the model may be used by the children to design curves for their interchanges.* Or, they can use the data in the tables on Page 10. How do they change feet per second to miles per hour? Some older children may be able to figure that out if they know that there are 5,280 feet in a mile and 3,600 (60 x 60) seconds in an hour. Others may use (and maybe make) a conversion table starting with 60 mph = 88 ft/sec. The children may find that the speeds they measured were small in comparison to normal highway speeds. They can extend the lines on the graphs in Figure 9 and 10 to obtain data for larger speeds. This may not be necessary if a timing device is used, permitting timing for higher speeds.

* The critical speed for sliding, V_c, does not scale with the length or straightness. Instead $(V_c)^2$ scales with the lengths. A scale of one foot per hundred feet will then lead to a $(V_c)^2$ one hundred times smaller, but V_c only ten times smaller.

APPENDIX A

Notes on Acceleration (Change in Velocity)

What is not so obvious is that when one is turning at constant speed the car is accelerating. That is because the direction of its velocity is changing, so that it is accelerating across the road towards the inside of the curve (centripetal acceleration).

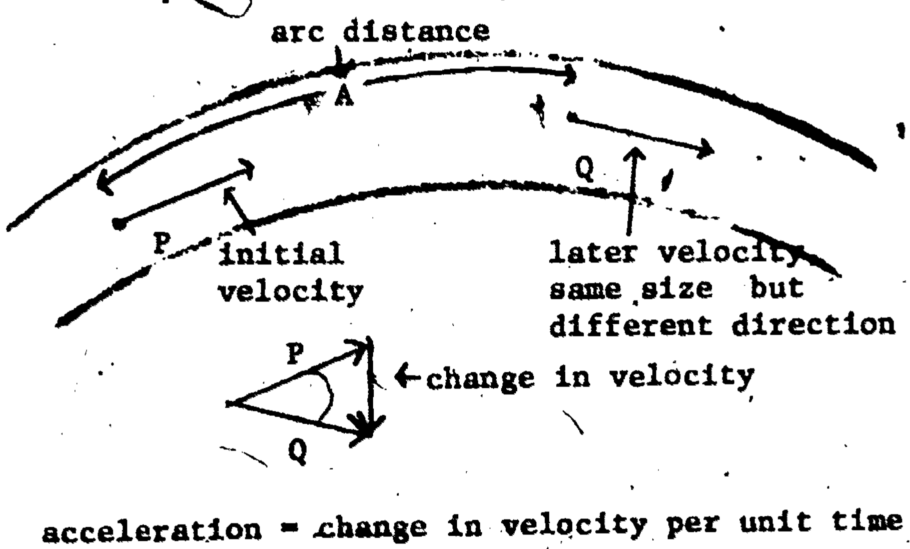


Fig. 1

For this purpose the road must exert through friction a force on the car pushing it towards the inside curve. The force has to be bigger in proportion to the centripetal acceleration. The acceleration is bigger if there is more curvature or if the car's speed is bigger. (You can check that with Figure 1.) The acceleration (change in velocity) is bigger if either the turning angle is bigger or the velocity arrows are longer. The magnitude of the acceleration is $V^2\theta/A$. The force of the road on the tires must then be the mass of the curve times the acceleration to be imparted, i.e. $F=MV^2\theta/A$. The maximum force of friction is given by a "coefficient" of friction (depending on the rubbing surfaces, like tire against road) multiplied by the weight, i.e. $F_M = \mu M g$. It follows that $V^2\theta/A$ must be less than μg to avoid slipping sideways. On good road surfaces with properly inflated tires μ varies between about 2/3 and 9/10, depending on the nature of the surface and whether it is wet or dry. g is 32 feet per sec. per sec.

APPENDIX B

Test for Rolling Friction

A test of how frictionless the rolling is can be made very easily. A down and up hill as in Figure 2 can be constructed

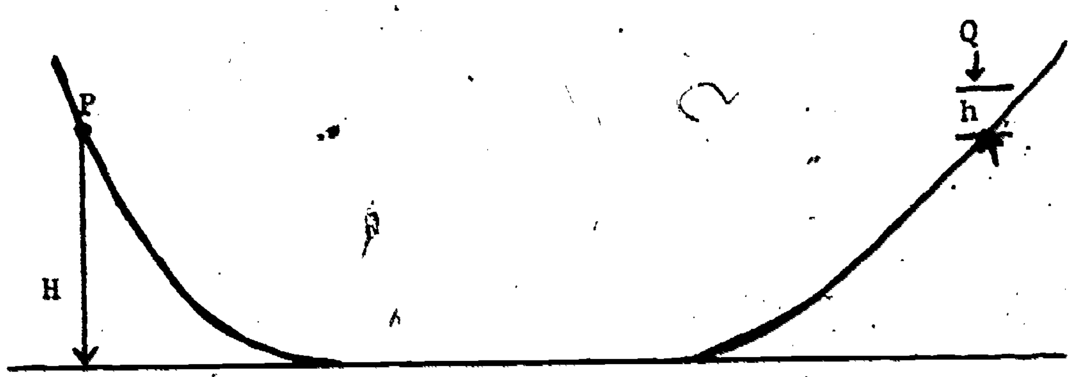


Fig. 2

and a car released at P. The car will roll down the hill, gathering speed and then go up the other side losing speed. It will reach its highest point at Q, at a height lower than at P, by an amount h. If the motion is perfectly frictionless, then h = 0. Of course, h will not be zero, but may be much smaller than H. If h/H is about 1/5 or smaller, the velocities of different vehicles in the valley will be within 5% of each other.

APPENDIX C

Distance for Speed Measurement

To measure the approximate velocity there should be at the bottom of the hill a straight section a few feet in length. The longer it is, the more accurate the timing, but the more slowing down will occur during the timing. Running it up a hill on the other side will be a check on the amount of slowing down. If h grows from 10% of H (for a short straight section to about 30% of H, then the velocity changes by about 10% from the beginning to the end of the straight section. This 10% variation isn't serious.

APPENDIX D

Notes on Figure 12

If friction can be neglected, then the force of gravity will increase the speed of a vehicle from rest as it rolls downhill. The resultant velocity is given by

$$V = \sqrt{2gH}$$

independently of the mass of the vehicle. g is the acceleration due to gravity and is equal to 32 feet/sec/sec or 980 centimeters/sec/sec. H is the vertical height of the starting point on the hill.

DETERMINING TASTE FACTORS FOR SOFT DRINK DESIGN

by USMES Staff*

In responding to the challenge in the Soft Drink Design unit, "Invent a new soft drink that would be popular and produce at a low cost," students often take a survey of people's preferences on soft drinks. In discussing this data, however, students come to the conclusion that a person's preferences might be based on advertising impact, the drinks available, or perhaps color preference. They question whether a preference based on an objective comparison of several different drinks might be different.

This paper outlines several ways in which quantitative, rather that subjective, data can be obtained. Also included are illustrations, using real data, which show a method for recording the data on a chart called a confusion matrix and a way to graphically represent the findings on a map called a drink difference map. The next section suggests ways in which certain taste factors in the drinks tested can be determined from the map. In the final section the proportion of the taste factors which should be included in a new soft drink design is derived from the results of the taste factor analysis combined with preference data.

One of the questions students frequently ask is whether people can really tell differences amoung several cola flavored drinks. This is an important point especially if their opinion poll includes more than one cola drink. If a taste test shows that people are not able to corrsctly identify the different cola drinks, then the preferences for different cola drinks on their poll can be combined to indicate a general preference for a cola drink.

*based on suggestions by Henry Pollak

After discussing the factors involved, students can usually design an appropriate taste test. They generally suggest blindfolding the participants so that any difference in the drink's color will not be a factor in the identification. In addition, they may see that the sequence of the drinks tasted must vary so that the participant doesn't know which drink might be offered next. The students might use a random sequence of drinks by picking numbers out of a hat or they might think of another system for mixing up the selection of drinks. In any case, participants know that there is always the possibility that the same drink will be offered two or three times in a row.

The Confusion Matrix

The results of participants' indentifications of the drinks can be recorded on a matrix which is called a confusion matrix because it clearly shows the number of times a drink is incorrectly identified or "confused." The confusion matrix in figure 1 shows the results of a blindfold taste test for coke and pepsi conducted by a 5th grade class. In recording the results, the students tallied a total of 43 wrong guesses and 54 correct guesses. Their conclusion was that people could indentify the two drinks correctly only 55% of the time.

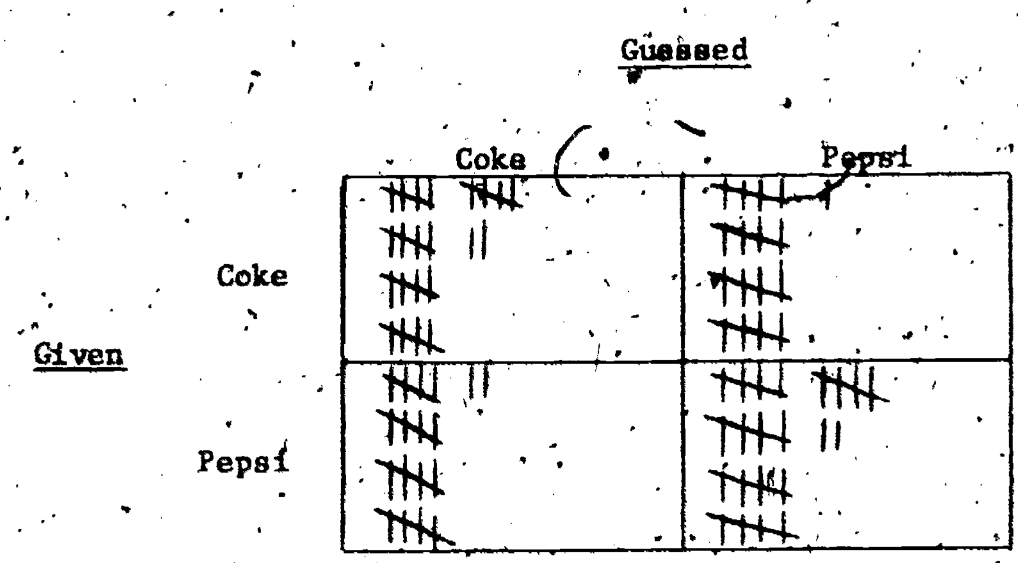


Figure 1

Students should check to see whether the drinks were tested an equal number of times. If not, the tallied results have to be ajusted before a just comparison can be made. In the data shown in figure 1, pepsi was tested only one more time than coke. Students would probably recognize that this is a difference to alter the results significantly.

The Drink Difference Map

If confusion data if gathered on three or more drinks, the results can be used to determine some of the taste factors in the drinks. This is accomplished by analyzing a map drawn from the results of the taste test. Figures 2 and 3 show data collected on five drinks and the drink difference map of the data. Each point on a drink difference map represents one of the tested drinks; the distance from each point to every other point represents the similarity between the drinks.

Drink Difference Map Drawn from Confusion Data

Since points for similar drinks should be close together on the drink difference map, the distances between the points must be inversely proportionate to the number of confusions. For example, if pepsi and coke are confused 10 times and pepsi and root beer are confused only 5 times, the distance between the points for pepsi and root beer must be twice as great as the distance bewteen the pepsi and coke points. We have found that children in the 5th through 8th grades can understand intuitively the idea of inverse proportions in the context of the real data they have collected. In lower grades we suggest another method for collecting data which asks students to establish a similarity rating for pairs of drinks. This method eliminates the problem of inverse proportions and is described beginning on page DP3-4.

In one blindfold tasting experiment students tried to identify each drink as grape, orange, mountain dew, pepsi or root beer. The following is a list of the drinks confused and the number of times they were confused with each other.

Drinks	Confusions
orange and pepsi	1
mountain dew and pepsi	1
mountain dew and root beer	2
pepsi and root beer	3

Before the drink difference map is started, it helps to list the distances apart the points should be placed. These are:

Drinks	Distance apart
orange and pepsi	3 units
mountain dew and pepsi	3 units
mountain dew and root beer	1 1/2 units
pepsi and root beer	1 unit

In this case the points can be plotted approximately the right distance apart on the two-dimensional map shown in figure 2. However, with additional data, the construction of a more complicated two-dimensional or even a three-dimensional map might be necessary. (Figure 4 shows a three-dimensional map of other data.)

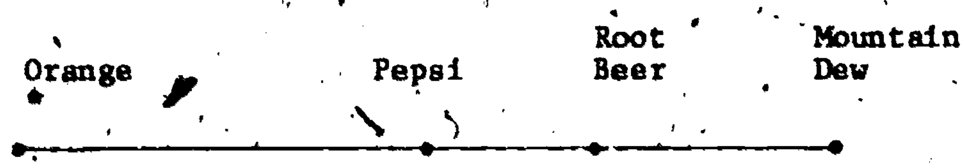


Figure 2

Drink Difference Map Drawn From Similarity Ratings

A drink difference map can also be constructed from data compiled from similarity ratings of pairs of drinks. Students first determine all the different ways the four drinks can be paired. In one class pairings for coke, pepsi, 7-up and sprite were determined to be:

coke and pepsi
coke and 7-up
coke and sprite
pepsi and 7-up
pepsi and sprite
sprite and 7-up

The students then rated each pair as very similar, slightly different or very different. This could be done by blindfold tasting, if necessary, or by occasional tasting to refresh a person's memory on the tastes of the drinks. The chart in figure 3 shows the tallies recorded by a group of 8 children.

Similarity Rating

Pairs of Drinks	very similar	slightly different	very different
coke - pepsi	III	~~IIII~~	
coke - 7-up		I	~~IIII~~ II
coke - sprite	II		~~IIII~~ I
pepsi - 7-up	I		~~IIII~~ II
pepsi - sprite		III	~~IIII~~
sprite - 7-up	III	~~IIII~~	

Figure 3

An approximate overall similarity rating for each pair was then obtained by giving 1 point to a very similar rating, 2 points to a slightly different rating, and 3 points for very different. The overall similarity of the pairs was calculated as:

Pairs of Drinks	Degree of Similarity
coke - pepsi	approx. 13
coke - 7-up	" 23
coke - sprite	" 20
pepsi - 7-up	" 22
sprite - 7-up	" 13

A drink difference map can now be constructed so that the distances between the drink points represent the similarity ratings. Figure 5 show such a map. It was found that a three-dimensional representation was necessary in order to locate the points the appropriate distances apart.

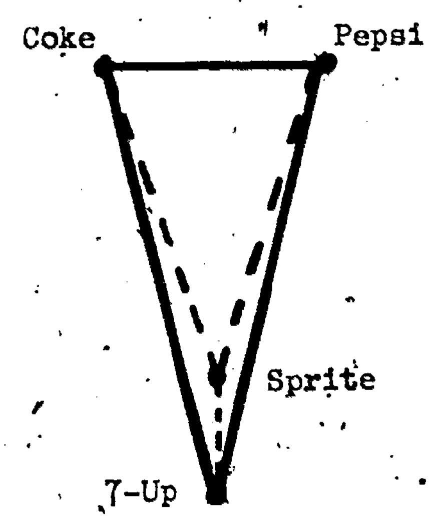


Figure 4

Children can try various methods of constructing a three-demensional map. One suggestion is to use Q-tips of varying lengths and glue the cotton tips together to make a drink point. The different joints might then be dyed different colors to show clearly the different drinks.

Using A Drink Difference Map to Determine Taste Factors and Design a New Drink

After the map has been constructed the students can discuss how the position of points might show certain taste factors. This discussion is facilitated if the students think about the various planes that might intersect the map.

One possible analysis of the map is shown in figure 5. One plane might represent a factor of sweetness, with sweeter drinks appearing at one end of the plane and less sweet drinks at the other. Another plane might show the tartness factor, while still another the difference between cola and fruit tastes. The determination of these factors is subjective. However, students can check their hypotheses by conducting another test that rates the four drinks according to the factors they think are important.

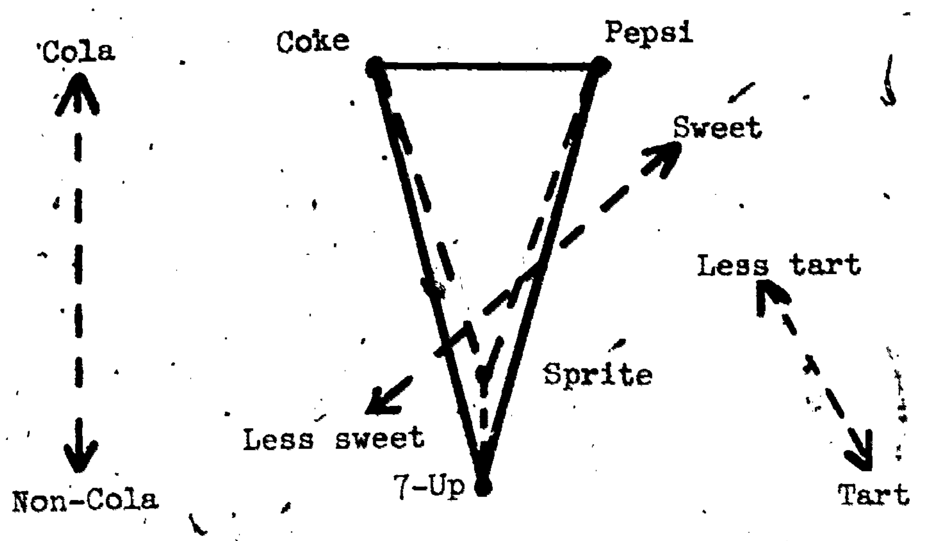


Figure 5

The children might want to add a fifth drink to their map. This would help to identify the taste factors involved.

The final step in using the drink difference map to help in the design of a new soft drink is to show the results of the preference poll on the map and indicate the region of maximum preference. On a preference poll, real data on 1st choices for the four drinks illustrated was found to be:

Drink	1st choice
coke	11
pepsi	6
7-up	11
sprite	2

This data is shown on the map in figure 6 with the region of maximum preference also indicated. The children try to design a drink with a combination of factors which would locate the new drink's point within the area of preference indicated on their map.

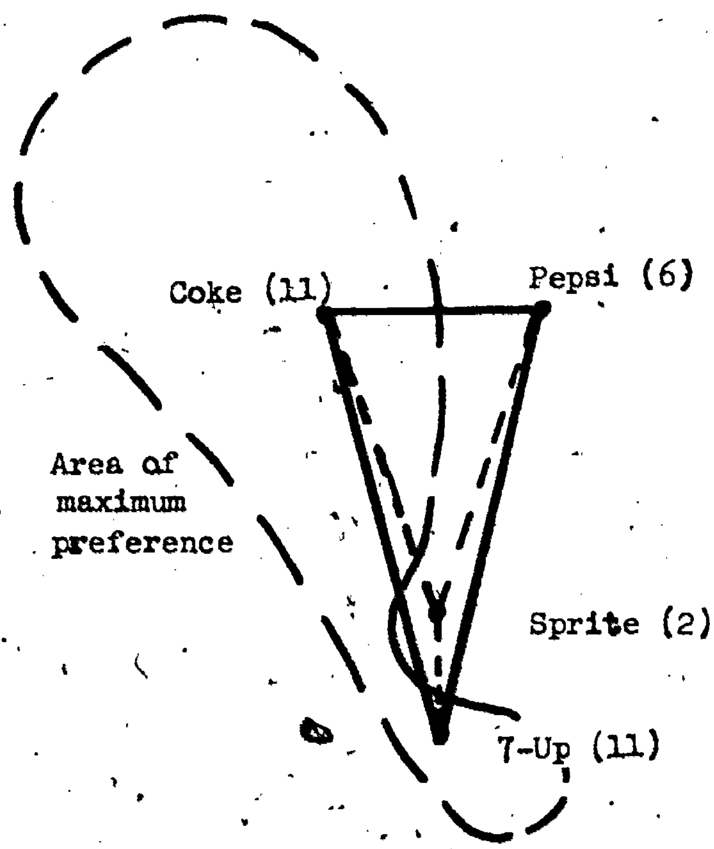


Figure 6

ELECTROMAGNET DESIGN

by

Earle Lomon

Introduction

Material in this background paper is fairly advanced for elementary school teachers and students. The strength of an electromagnet is determined by two related factors, the design of the electromagnet and the amount of current going through it. To complicate things further, the current is determined not only by the design of the electromagnet but also by the total voltage of the batteries being used. The drain on batteries is another factor that should be considered.

The most important thing for children to realize is that any change in the design of an electromagnet must be assessed not only in terms of the strength of the magnet but also in terms of the wear on batteries. They can discover by experimentation that the best balance between the two factors usually achieved by winding more layers of a thinner wire on a shorter coil.

The first section of the paper describes several ways in which the design of an electromagnet can be changed. Experiments that children can conduct to determine the effect of these changes on the strength of an electromagnet are also described. Real data collected in similar experiments is included; in some cases it is represented graphically.

For teachers who are interested in the mathematics of coil design, an appendix is included that gives the basic formulas and descriptions of the effects of variations in the various design factors.

An electromagnet is usually constructed with the following characteristics: a thin, very long, conducting wire is wound uniformly around a cylindrical core, always turning in the same direction. The core is often soft iron. Several layers of wire may be wound on the core going back and forth along its length. A battery is connected to the two ends of the wire.

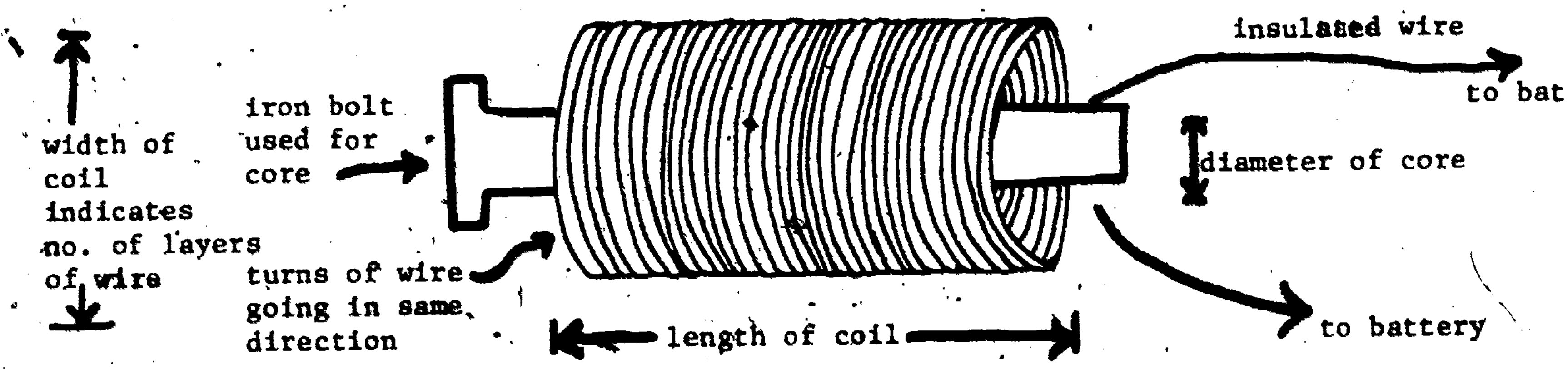


Figure 1

When children examine the different electromagnets they have constructed, they may notice that the electromagnets differ in several respects as follows:

1. length of coil
2. number of layers
3. size of wire used (diameter)
4. thickness of wire insulation
5. size of core

Children may also use more than one battery in their tests.

The effect that each factor has on the strength of the electromagnet can be determined by conducting simple experiments. In conducting their experiments children must make sure that they use iron objects with easily comparable weights; for example, heavy duty paper clips strung together or different lengths of iron rods. They must also make sure that the object has only one point of contact with the core of the electromagnet.

The pull of the electromagnet on iron objects is strongest if the object is placed inside the core. However, since it is usually impossible to put an object inside the core (because the core is solid or the hollow space is too small), it is usually placed at one end of the core. The pull on the object then decreases rapidly as the object is moved away from the end of the coil.

As iron has a magnetic permeability hundreds of times larger than any other materials, it is clear that one wants an iron (or steel) core for sheer strength. However, there are disadvantages to the use of iron cores for certain applications. The only drawback of much consequence for USMES application is that of residual magnetism. When the electromagnet is turned off, the iron will retain some of its magnetism, having become a "permanent" magnet. That can cause difficulty if it is desired that all objects, even small ones, drop off the electromagnet when the current is turned off. Also, the addition of permanent magnetism may confuse the results of the experiments. Soft iron does not retain as much magnetism as hard steel, so the use of soft iron minimizes these problems. If necessary, the core can be demagnetized by reversing the current for a short period; or by turning the core around, if it is loose.

In addition to the above constraints, nickel-cadmium batteries must be used in quantitative experiments because this is the only type of battery which maintains a constant voltage until the moment when the current has been completely drained and the voltage is zero.

In searching for a way to make stronger electromagnets, children should decide first which variable they think is important, and then make some change in that factor while keeping everything else the same. Some possible graphs that they can construct from their data are as follows:

1. strength of magnet as shown by length of rod (number of metal squares or weight of irregular object) vs. number of batteries in series,
2. strength vs. length of wire (same number of batteries, same length of coil, more layers of wire),
3. battery life vs. length of wire (all else equal),
4. strength vs. diameter of wire (same length of wire and same length of coil),
5. battery life vs. diameter of wire (all else equal),
6. strength vs. thickness of wire insulation (same diameter and length of wire, same length of coil),
7. strength vs. neatness of winding (all else equal),
8. strength vs. length of coil (same core, same diameter and length of wire),
9. strength vs. core diameter (all else equal),
10. strength vs. core material (all else equal),
11. proportionate strength vs. area of object in contact with magnet (all else equal).

The various factors in the design of an electromagnet that affect its strength are described in the following sections.

1. Length of Coil

It is not the total number of turns of wire that makes the pull stronger, but the number of turns per unit length. Therefore a stronger electromagnet can be made by winding a certain length of wire in many layers on a shorter coil rather than in one layer on a long coil. There will be some gain in strength if the coil is made as long as the average radius, but little will be gained by making it longer. The optimal dimensions for a coil are shown below in fig. 2.

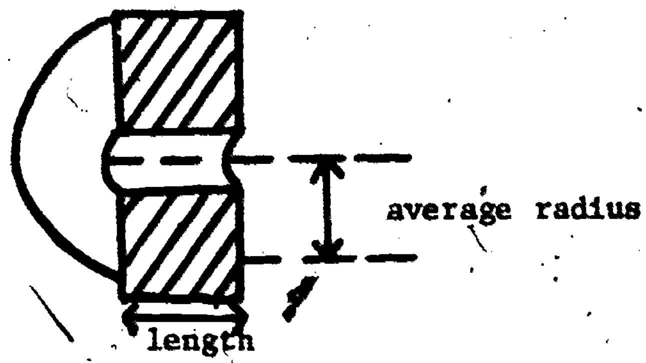


Figure 2

2. Number of Layers of Wire

Many turns of wire must be wound on a coil to achieve the above dimensions. This increases the turns per unit length and the strength of the magnet. However, as each additional layer is added more wire is required to make one turn. This will add more resistance, reducing the current and the strength of the magnet. The best balance between these two factors is found when very thin wire is used.

There is, however, an advantage to increasing the length of any diameter wire until about 10 feet of wire is being used. The following data and the graph in figure 3 shows that 10 feet of #22 or #26 wire wound in a one-inch long coils produced greater pulls than coils made with either less or more wire.

length of wire in feet	wire size (diameter in inches)	length of rod in inches
5	#22 (.025)	43
10	#22 (.025)	54
20	#22 (.025)	48
40	#22 (.025)	44
5	#26 (.016)	23
10	#26 (.016)	30
20	#26 (.016)	24
40	#26 (.016)	20

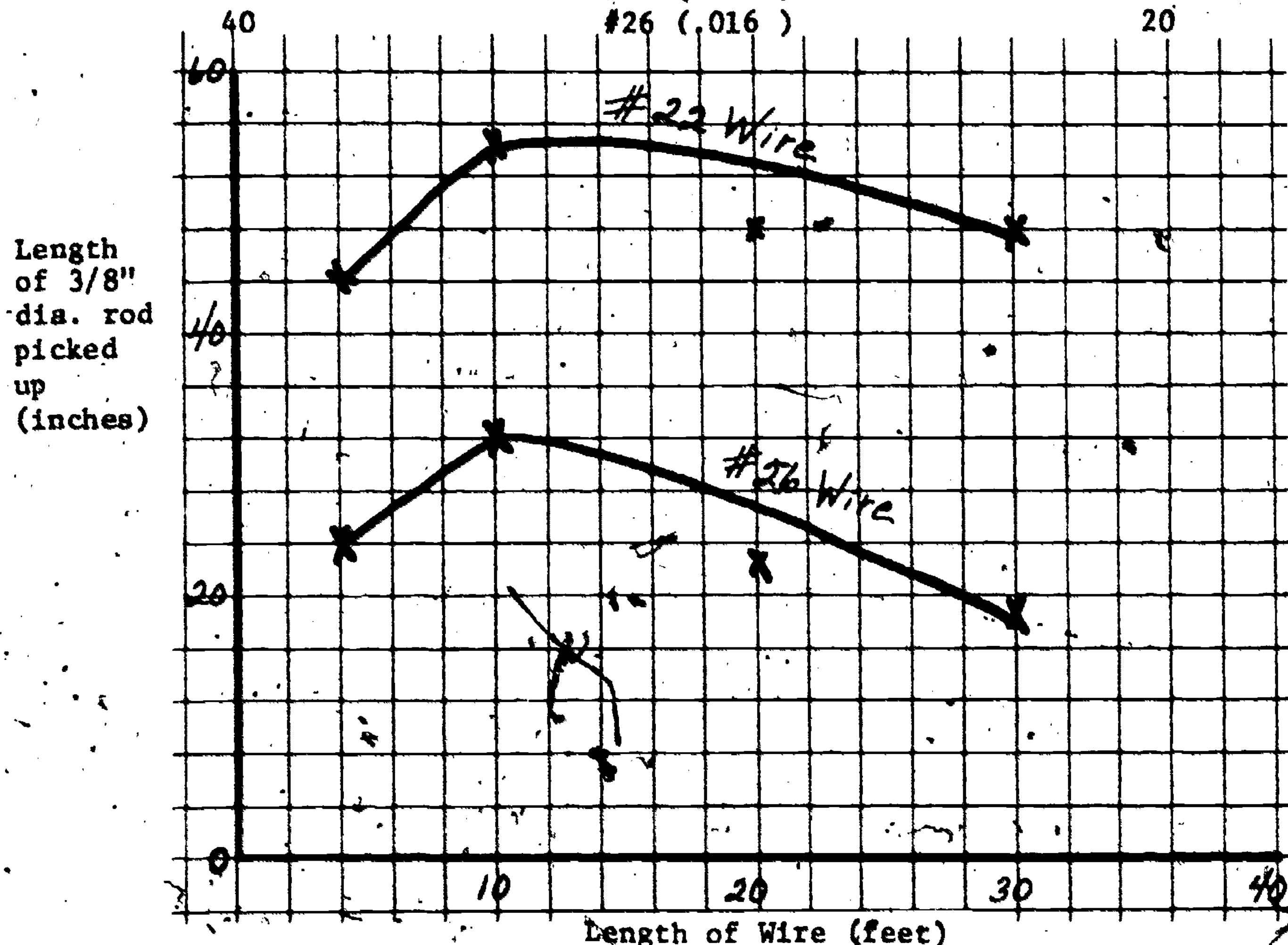


Figure 3

Increasing the length of wire even more will pay if one is trying to economize on batteries. Using twice as much wire will halve the current and double the battery life. The magnet will lose a little strength since the effect of decreasing the current is partially overcome by an increase in the number of turns of wire per unit length. The data below shows that an electromagnet wound with 60 feet of wire held up a maximum weight almost twice as long as an electromagnet wound with 40 feet of wire.

length of wire in feet	size wire (diameter in inches)	length of time rod was held up in min.
20	#28 (.013)	93
40	#28 (.013)	142
60	#28 (.013)	274

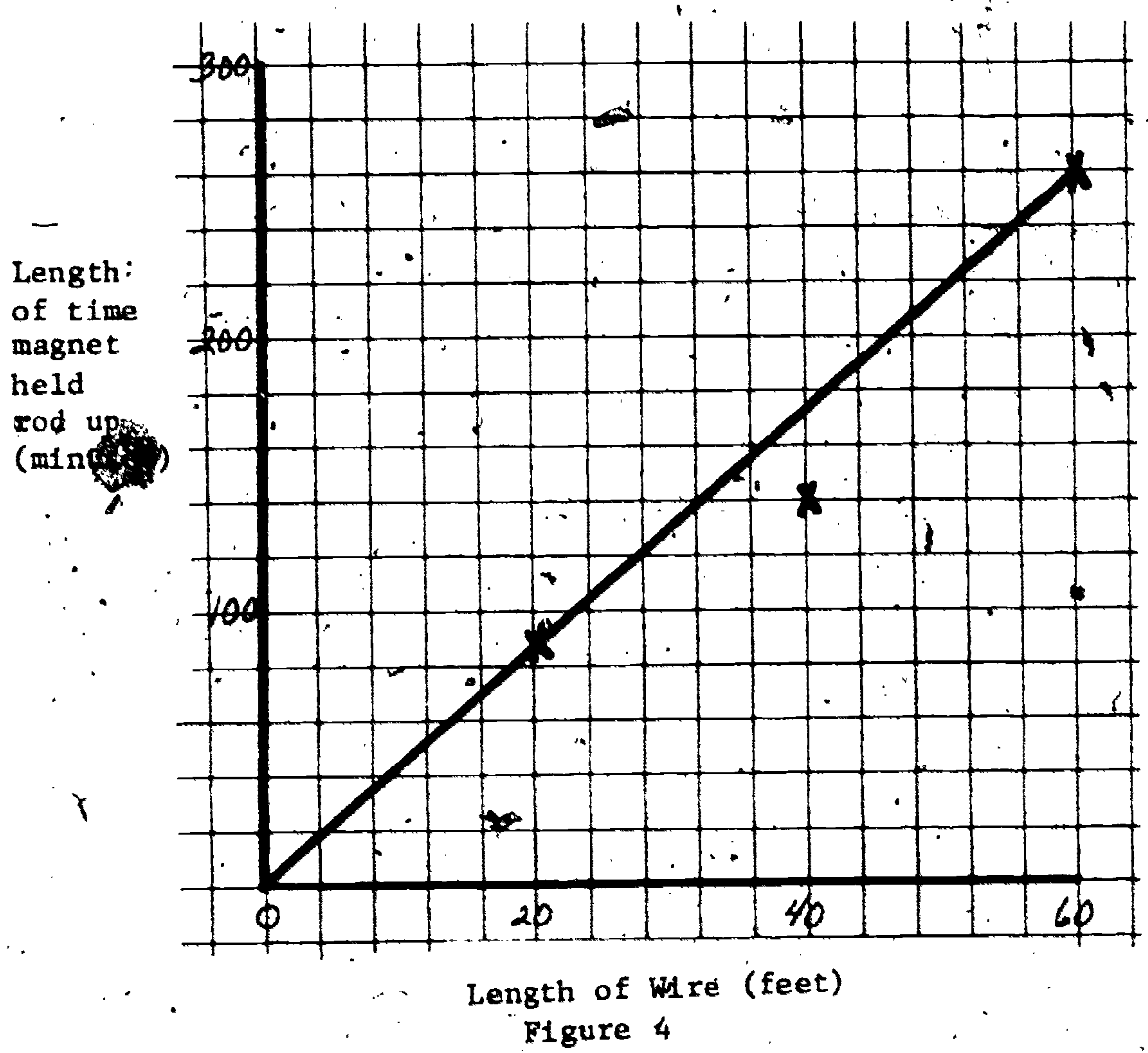


Figure 4

3. Size of Wire Used

If a smaller diameter wire is used, one can wind more turns of wire in the same length and width of coil increasing the magnetic pull. However, this also increases the length of the wire used and reduces the magnetic pull by reducing the current. If a wire with half the diameter is used, the pull of the magnet will be reduced to one quarter of what it was.

However, by using a thinner wire we are reducing the current and the resultant drain on the battery. A magnet one quarter as strong will drain the battery at only one-sixteenth of the previous rate. Figure 5 shows that an electromagnet wound with 20 feet of thin wire held up a maximum weight more than 10 times longer than a similar coil wound with thicker wire.

size wire (diameter in inches)	time rod was held up in minutes
#22 (.025)	9
#28 (.013)	93

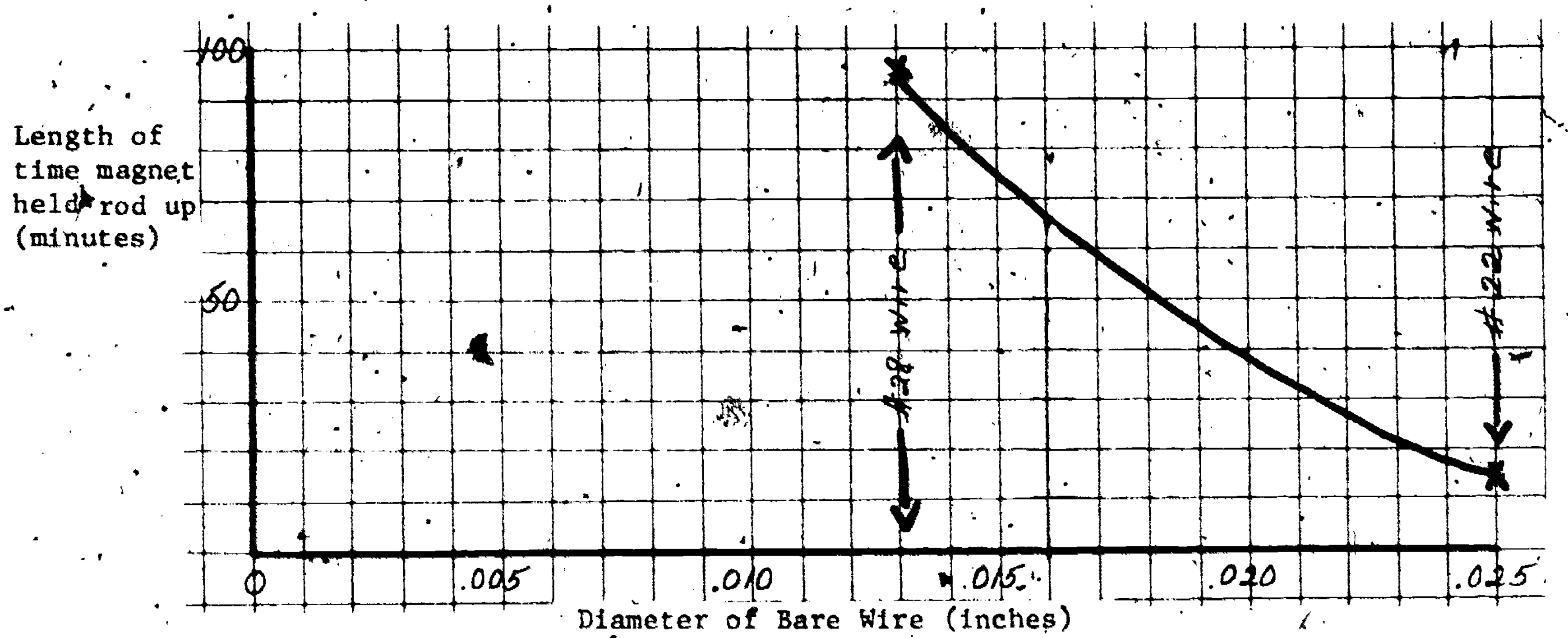


Figure 5

The strength of the thin-wire coil used in the above experiment was only 1/3 the strength of the other coil. If we put enough batteries in series to make up for the loss in current, producing the original magnet strength, each battery will drain 1/3 as fast. This is an advantage for many applications, e.g., the device will run three times as long as the same total cost without somebody replacing batteries.

4. Thickness of Wire Insulation

Wire used for electromagnets must be insulated. Otherwise short circuits will occur and the electricity will not flow around all the loops. Thick insulation will keep down the number of turns per unit length. Therefore, wire with thin insulation such as varnish is preferred for magnet construction. The following data shows how the use of insulated wire reduces magnet strength.

type wire	length rod held up (in inches)
#24 - enamel insulation	36
#24 - plastic insulation	29

5. Size of Core

When an electromagnet is wound on a larger core, the pull of the magnet at a single point of contact will be decreased. One reason for this is that the length of the wire must be increased in order to keep the number of turns per unit length of coil the same - this reduces the current and the pull of the magnet. However, if the iron object being picked up is large compared to the size of the core the total pull will be larger.

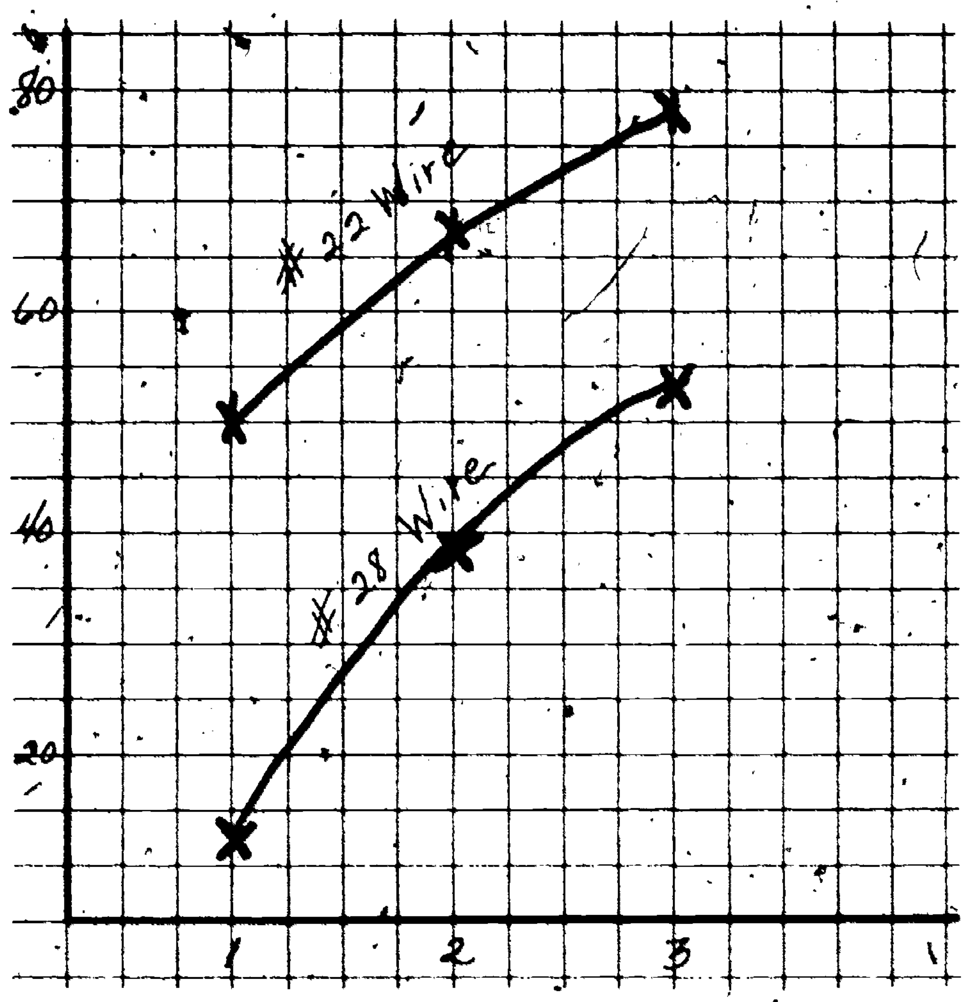
Therefore, one gains by making the core larger until it becomes larger than the object being picked up. Any further increase in core size results in a decreased total pull.

6. Increasing the Voltage

The voltage is readily changed by using different batteries or by using several in series. If the voltage is doubled, this will approximately double the current and the strength of the magnetic pull. Figure 6 shows the increase in strength as the voltage is doubled and tripled by putting batteries in series. Twenty feet of #22 and #28 wire were wound on 2" long coils, which were then tested for strength at each of the three voltages.

no. of batteries in series	wire size (diameter in inches)	length of rod in inches
1	#22 (.025)	49
2	#22 (.025)	66
3	#22 (.025)	76
1	#28 (.013)	15
2	#28 (.013)	37
3	#28 (.013)	52

Length of 3/8" dia. rod picked up (inches)



Number of batteries in series
Figure 6

The utility of putting in more voltage will be limited by the power dissipated. At some point, the magnet is going to get too hot for safety or for its insulation. Also, the batteries are draining proportionately to the current and the number of batteries in series. So twice as much magnetic force costs four times as much in batteries. This method of increasing magnetic strength costs money!

APPENDIX A

Formula for Magnetic Field Around a Solenoid

The force on an iron object is proportional to the magnetic field it is in. That magnetic field for a solenoid (which is the name given to the coil of wire described above) is along the axis.

$$B \sim \mu\, n\, I\, (\cos\alpha + \cos\beta) \qquad (1)$$

where μ (the magnetic permeability) depends on the material of the core, n is the number of turns of the wire per unit length, I is the current in the wire, and α and β are the angles subtended at each end of the coil. The cosine of the angles are determined by the length of the coil (L), the radius of the coil (r), the distance of the iron object is from one end of the coil (d), and the distance it is from the other end of the coil (L - d). (See Fig. 1 and equations 2 and 3).

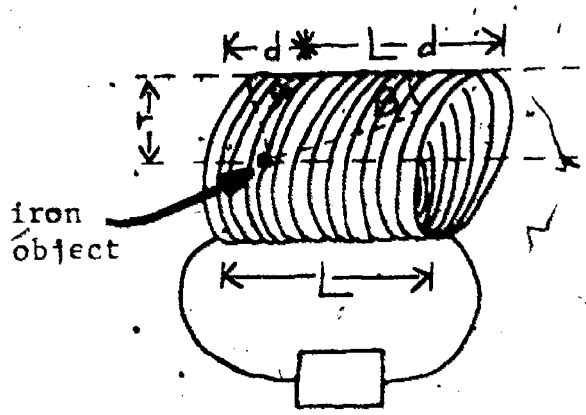


Figure 1.

$$\cos\alpha = d\Big/\sqrt{r^2 + d^2} \qquad (2)$$

$$\cos\beta = (L-d)\Big/\sqrt{r^2 + (L-d)^2} \qquad (3)$$

Notice that at the ends of the coil d = o or d = L and:

$$(\cos\alpha + \cos\beta)_{end} = L\Big/\sqrt{r^2 + L^2} \qquad (4)$$

While at the center of the coil d = 1/2 L and:

$$(\cos\alpha + \cos\beta)_{center} = L\Big/\sqrt{r^2 + L^2/4} \qquad (5)$$

At most, one can only double the strength by trying to get <u>inside</u> the core. This factor of two is obtainable only with a very thin coil ($\sim$ small compared to L). Then:

$$\frac{(\cos\alpha + \cos\beta)\,\text{center}}{(\cos\alpha + \cos\beta)\,\text{end}} = 2 \qquad (6)$$

However, with a thick coil (L small compared to r):

$$\frac{(\cos\alpha + \cos\beta)\,\text{center}}{(\cos\alpha + \cos\beta)\,\text{end}} = 1 \qquad (7)$$

and consequently no advantage is gained by going inside the coil.

[Note also that if one is outside the coil then d must be considered as negative, so that $\cos\alpha$ is negative and subtracts from $\cos\beta$. At large distances from the coil the two terms cancel. Thus the magnet field, B, decreases rapidly as one moves away from the end of the coil].

APPENDIX B

Length of Coil

The length of the coil comes in through the factor $(\cos\alpha + \cos\beta)$ in equation 1 (Appendix A). Assuming that the iron object is being held at the end of the coil:

$$(\cos\alpha + \cos\beta)_{end} = L/\sqrt{r^2 + L^2}$$

If the length (L) is made very small, then $(\cos\alpha + \cos\beta)$ becomes small. However, if the length equals the radius (L = r), then $(\cos\alpha + \cos\beta) = 1/\sqrt{2}$ or approximately 0.7.

No matter how long the coil gets, $(\cos\alpha + \cos\beta)$ can never become as large as 1. Therefore, little can be gained by making the length larger than the radius. In fact as L goes from 1/2 r to r to 2 r, $(\cos\alpha + \cos\beta)$ goes from 0.45 to 0.7 to 0.9. Each increase is less than a factor of 2. But when the length of the coil is doubled, the number of turns per unit length is halved. If the length of wire is doubled to keep the number of turns per unit length the same, the current is halved. Either way we lose more magnetic field strength than we gained by increasing $(\cos\alpha + \cos\beta)$.

If when the length of coil is doubled one also doubles the length of wire (to keep the number of turns per unit length as large as it was) <u>and</u> doubles the voltage (to keep the current as large as it was), then the magnetic field is increased by the increase in $(\cos\alpha + \cos\beta)$, but less than a factor of 2. But the battery cost is twice as great (the same current from twice as many batteries). Thus the magnetic field strength gain is not proportional to the cost.

Until the length of coil equals its radius the gain in strength due to an increase in coil length is nearly proportional to the cost and will be worth it if more strength is still needed.

APPENDIX C

Increasing the Voltage

The current is proportional to the magnetic pull (See equation 1). It depends on the voltage and the resistance of the circuit. This is shown in equation 8 where I is the current, V is the battery voltage (or of a series of batteries) and R is the total resistance of the circuit.

$$I = \frac{V}{R} \tag{8}$$

The total resistance is the sum of the wire resistance (R_w) and the internal resistance of the batteries (R_B).

$$R = R_w + R_B \tag{9}$$

The current is bigger as the voltage is bigger, or if the battery resistance and/or the wire resistance is smaller.

When adding batteries (or using one with a larger voltage) one will also be changing the resistance in the circuit due to the internal resistance of the batteries. For strong electromagnets, however, one usually has enough wire so that the battery resistance is only a small part of the total resistance. When that is true, doubling the voltage will approximately double the current and the pull of the magnet.

The rate of using power is proportional to the voltage that the charges go through and to the number of charges per second (i.e. the current) going through that voltage. The energy loss per second (power loss) is:

$$P = I\,V \tag{10}$$

Using equation 8 we see that:

$$p = \frac{V^2}{R} \quad \text{and} \quad P = I^2 R \tag{11}$$

If we double the voltage keeping the same wire, then the current doubles and equations 10 and 11 show that the power has quadrupled (double voltage x double current).

APPENDIX D

Length of Wire Used

When the length of wire is changed, the total resistance of the wire is changed. The resistance of wire is given in equation 12 below where ℓ is the length of wire, ρ is the resistivity of the metal used, and A is the cross-sectional area of the wire.

$$R = \ell\rho/A \tag{12}$$

The area is proportional to the square of the diameter of the wire.

The total resistance in a circuit is the sum of the wire resistance and the internal resistance of the battery. The current is determined by the voltage of the battery and the circuit resistance as shown in equation 13 below where I is the current, V is the voltage, R_W is the wire resistance, and R_B is the battery resistance.

$$I = \frac{V}{R_W + R_B} \tag{13}$$

When the wire resistance is not large compared to the battery resistance then the total resistance is not increasing proportionately to the length of wire. So there is an advance to increasing the wire length until the wire resistance is several times the battery resistance.

The more layers of wire on the core, the larger the radius of each turn. Therefore the number of turns per unit length does not increase quite as fast as the length of wire, while the resistance does increase that fast. Therefore one will not want to increase the length of the wire much more once its resistance is several times the battery resistance, if one is trying to get the maximum strength. However, if one is trying to economize on batteries and is content with a little less strength, then using twice as much wire will about halve the current and about double the battery life, while increasing the radius (and therefore decreasing the added number of turns) by much less than a factor of 2.

APPENDIX E

Wire Diameter

As the cross-sectional area of the wire is proportional to the square of the diameter, decreasing the diameter by a factor of two will increase the resistance by a factor of 4, hence decreasing the current (if the battery resistance is unimportant) by a factor of 4. On the other hand, one can now wind four times as many turns in close packing in the same length and thickness of coil. Of course the wire would then also be four times as long; so the resistance would be sixteen times greater (four times due to decrease in diameter and four times due to increase in length of wire). With the same voltage the current would be down by a factor of nearly 16. As there are four times as many turns per unit length the magnetic field will be 4/16 = 1/4 of what it was.

So we shouldn't decrease the diameter of the wire? On the contrary: with one sixteenth of the current from the same battery we are draining the battery at one sixteenth of the previous rate, getting a magnet only one fourth as strong. If we put two batteries in series we will double the current and get back to one half of our original magnetic strength. The power use will go up four times (twice the current through twice as many batteries) which is still only one fourth of the original power use for one half of the original magnetic strength.

What if we double the voltage again to four batteries in series? Then we have the original magnetic strength (four times the original number of turns per unit length, at one fourth of the original current) with the original cost in use of batteries (four times as many batteries draining one quarter as fast). There still is an advantage for many applications - the device will run four times as long without somebody replacing batteries.

APPENDIX F

Diameter of the Core

For a fixed length of coil the factor $(\cos\alpha + \cos\beta)$, decreases slowly as the radius of the core increases, thereby decreasing the magnetic field. (See Appendix B*). The magnetic field also is decreased by an increase in the length of the wire. The length of wire must be increased in proportion to the increase in the core radius (for a thin winding) if the number of turns per unit length are to remain the same. If the winding is already thick (many-layered), then increasing the wire radius leads to less than a proportionate increase in the length of wire (for a fixed number of turns per unit length).

However, there is a compensating effect if we are picking up an iron object that is large compared to the core cross-section. The magnetic field induces magnetism in the iron object being picked up over the whole area within the field. Thus if the piece is larger than the core, as the core area increases the induced magnetic area increases, and the pull increases in proportion. The area covered by the core is πr^2 so for large objects one gains by going to larger core. Once the core cross-section is bigger than the piece to be picked up one only loses by increasing further the diameter of the core.

---

* The r in equation (4) applies to the core radius especially when the magnetic field at the end of the core is being investigated.

APPENDIX G

Use of Iron Cores

When the electromagnet is turned off, the iron core will retain some residual magnetism which may effect magnet experiments.

For instance, if two batteries are used in series [illegible] and then one battery is removed, then Equation 1 (Appendix A) leads one to expect about half the magnetic pull. If the battery resistance is not negligible compared to the resistance of the wire, then the expectation is a little more than half. (See Appendix C). But the double current may have caused a high residual magnetism which would change the predicted ratio by increasing the magnetic pull.

Soft iron does not keep as much magnetism as hard steel, so the use of soft iron minimizes those problems. The core can be demagnetized by reversing the current for a short period; or by turning the core around, if it is loose.

IMPORTANT ASPECTS OF PLAYGROUND DESIGN

by

Playground Design Group—1972 Boston Summer Workshop

General Considerations

Should the focus of the unit be on the design of new equipment, the actual construction of the playground, the layout of the grounds, or a combination of these ideas? It would be ideal if every stage of development resulting in a finished playground could be experienced. The time required for the complete process is a relatively unimportant factor.

What materials are available? Perhaps the children could find scroungable items that are native to their area (i.e. an old boat or cargo net in a coastal region). The children should have an opportunity to explore materials before beginning construction to help them get ideas about the variability of uses for the materials.

How much adult support will be available to help with the procurement and construction of materials? Teachers and the school administration might be asked to back the project. Human relations problems arising in connection with the project might be resolved early if the kids could discuss provisions for water play, safety, construction and esthetics, etc. with school personnel and/or parents.

What kinds of playgrounds do children in other countries with the same climate have? What about in countries with different climates?

Two possible pitfalls of the unit might be 1.) the failure to invest the time and interest necessary to make detailed models, and 2.) the failure to make use of natural environmental situations.

Topography - Surface Considerations

a. Blacktop - underground pipes, composition

b. Pea gravel - depth and composition

c. Grass

d. Sand

e. Woodchips

f. Wood planks, ties

g. Astro Turf

h. Indoor-outdoor carpet-linoleum

i. Garden - Botanical, vegetable, rock

j. Water

k. Plastic - Rubber composition

l. Dirt

You can make a longer, steeper slide on a hillside without dangerous height.

Shelters

It has been our observation that children not only need shelter from inclement weather and the hot sun but also enjoy playing in sheltered areas.

In the playgrounds built with elevated platforms, the children's favorite places were under the platforms not on top. Also, the lower the better.

Possible types of shelters:

Tunnels: made from culverts, pipes, tires, railroad ties, cement blocks, bricks

Culverts: upright or horizontal

Platforms: elevated (maximum height 4')

Tents: canvas, parachutes

Treehouse

Playhouse (girls) / Clubhouse (Boys)

Packing Boxes Crates Spools

Vehicles - Constructions

Equipment* (Used at workshop to introduce challenge)

Rope, hemp, nylon

Tires

Pulley

Wrenches (box, socket, Wescott wrench, rachet, extension bar)

Nuts, bolts, washers

Electric drill, drill bits (metal boring)

Tape measure

Ruler

Pencil and paper

Physical Development of Children

Large/small muscle - Activities

jumping
tumbling
pulling
climbing
hanging
leaping
crawling
twisting
running
stretching
bending
squatting
rolling

Mathematical Skills

Basic math skills (addition, subtraction, division, fractions, conversions)

Measurement (weight, linear, volume, human proportions, area circles, density, ratio, scaling, time)

Correlation

Simulation

*See Micro-teaching session/outdoor activity/video taped

General Science - Physics

- study of pendulums
- strengths of materials (rope, etc.)
- speed and distance

friction, energy, gravity, force, inertia, equalibrium point, angles, planes

Graphing

Estimation - prediction

Cause-Effect relationships

Spin Off

Economics

Ecology

Urban Planning

Architecture

Resources

Scrounge Materials (ecological implications)

Utilities Company, e.g. Telephone Co. (cable spools, ladders, wire, poles, etc.)

Gas Stations (Tires)

Auto Junk Yard (Steering wheels, auto parts)

Lumber Yards

Local Contractors (scrap materials, big tires)

EDC (references)

Paper and Box Company (Tubes from International Paper Company)

Plastic Company

Shipyards - cargoe nets - old boat

Wine Distributors (wine kegs)

Recreation and Park Departments

State Institutions - concrete culverts, etc. (Highway Department, Penitentiary)

City - old culverts, timbers

People - labor unions, voc-tech schools, retired people, carpenters, etc

College industrial arts classes

MATERIAL FOR BUILDING EQUIPMENT

Utility Poles

Use only on vertical applications, too many slivers on horizontal. Rule of thumb: sink poles 1/2 of total exposed distance, extend pole 12' above ground, 6' below.

Planks for Cross Members

2" x 6" utility grade (usually are fir, if you have a choice, use hemlock or spruce) cover with sealer.

Cable for Tramlines, etc.

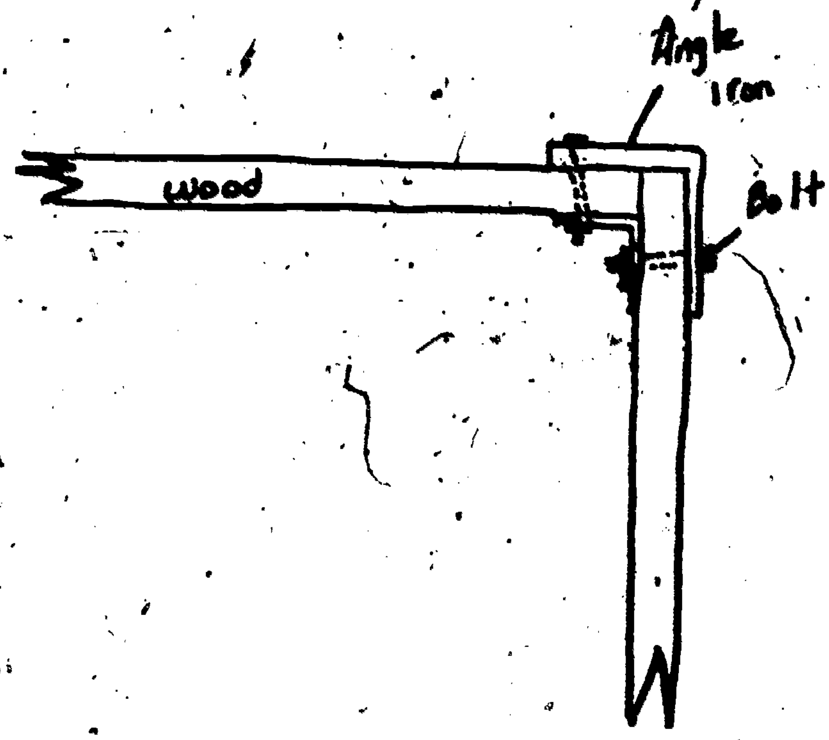
Use aircraft cable 1/4". Aluminum 3/8" cable available from power company. Good because it is made from large strands, soft, easier to work with, doesn't rust or need treatment.

RECOMMENDATIONS

We found that using a high speed (metal) drill bit was much more effective than using the power wood boring bit or auger bit. Also, it made a smaller/cleaner hole.

Substitute ratchet, extension, and socket set-up for box/open end wrenches. They are quicker/easier to work with.

Do not nail lap joints. Use lag bolts or two pieces of angle iron and bolts.



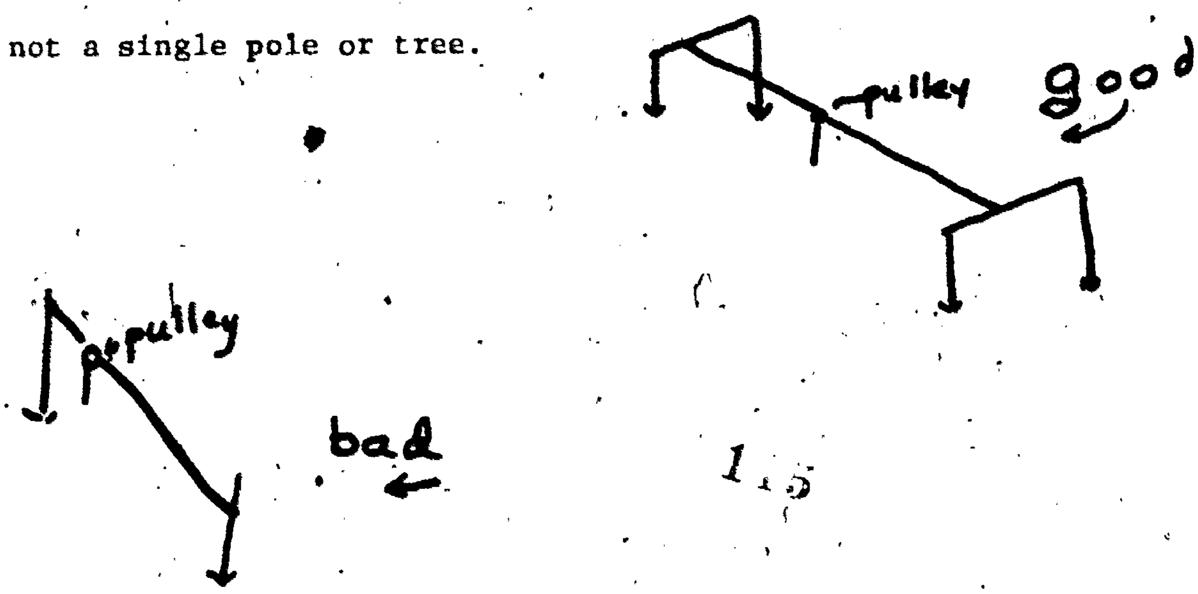
Use wood screws, lag bolts or some other means of fastening two pieces of wood that might possibly be pushed apart. Once pushed back, the nails become exposed.

Cable spools should be given at least two coats of heavy paint to seal against splinters. Boards used in platform construction should be coated with a sealer.

Do not have "low spots" for water accumulation in tunnels.

3/4" hemp rope is difficult to tie knots with. Also, it gives splinters and rope burns.

Be aware of the end point of a tramline. Should be an "H-frame" not a single pole or tree.



- A swivel should be used on any rotating device to prevent finger pinching.

When using bolts in construction, be sure you don't leave any ends exposed or protruding in such a way that a child might run into them. Round off the corners of protruding objects.

Placement of Equipment

Areas underneath sliding poles, swinging ropes, jumping platforms, etc. should be a cushioning material, i.e. sand, wood chips, pea gravel. A retaining wall, such as those constructed from utility poles or tires, are very useful for saving the sand and also setting up a boundary to keep other children from running into the paths of jumpers and swingers.

- With a swinging rope (what can it swing into?) or "vine", be certain that the take off and landing points do not pose as dangerous obstacles if a child misses it or lands incorrectly.

Do not leave any tall constructions a child may climb up and fall off.

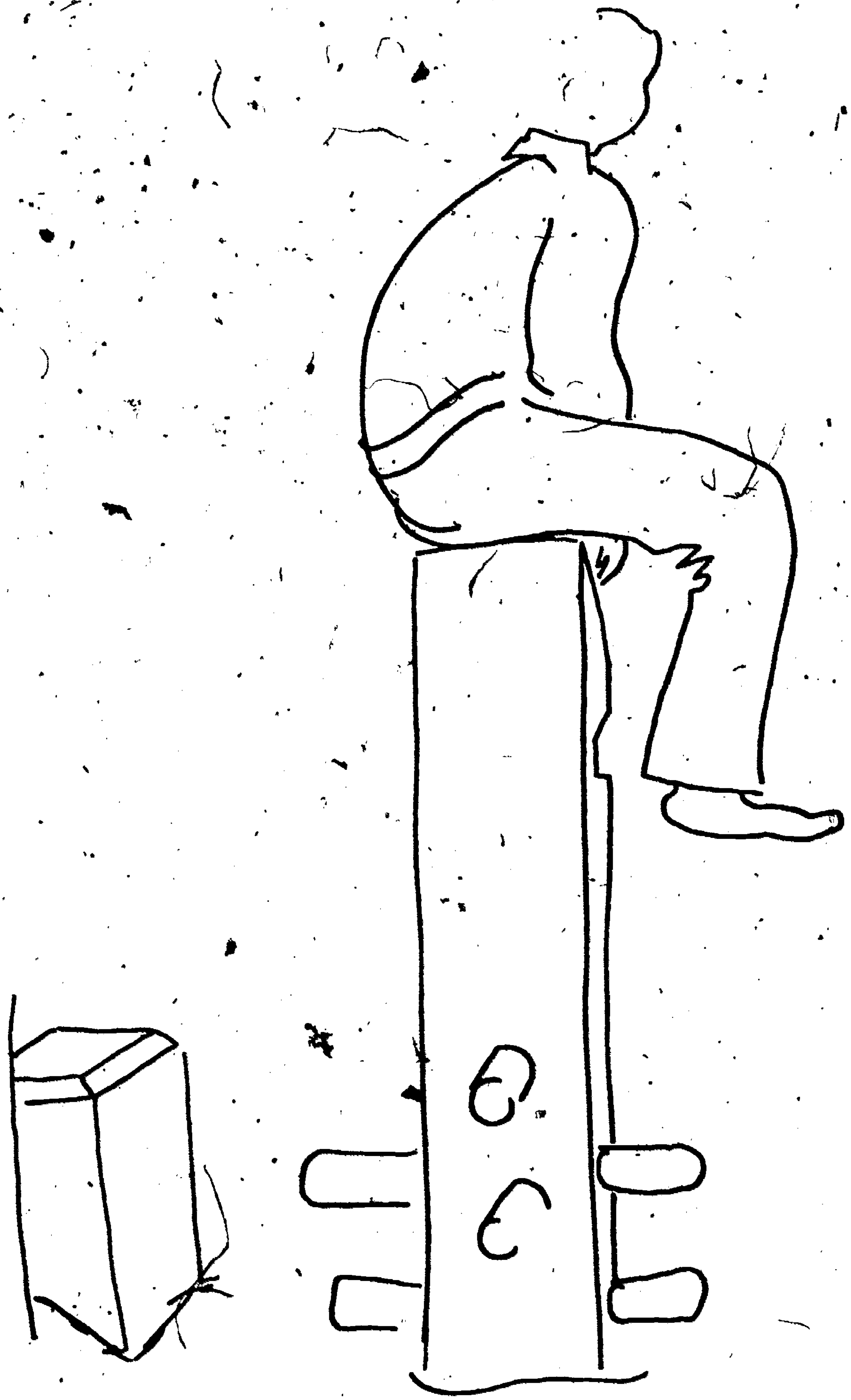
Plan the placement of equipment in such a way as to prevent collisions, i.e. do not have two jumping platforms facing each other.

NOTES ON PLAYGROUNDS VISITED

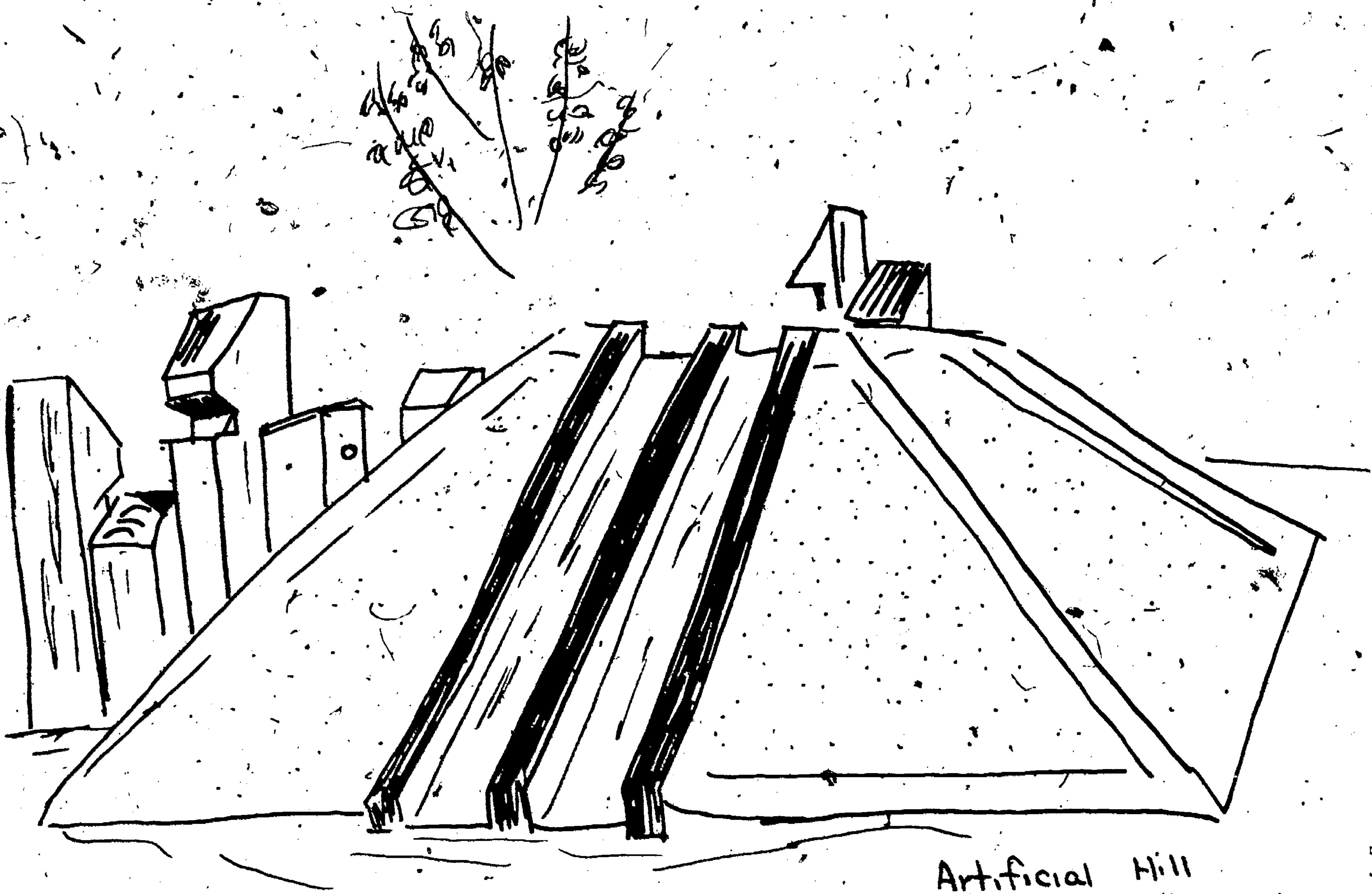
Sherman Street Playground, Cambridge

This was a commercial "kit" which was expensive. There were several hazards because of the small space used, yet this provided good integration of activities and age groups. We noticed the children making use of "underneath" spaces for privacy. Noticed imaginary play activities going on (playing house and store). Kids used sand (which was the base surface of whole area) and flat spaces cut in wood "pilings" to carry on these make-believe sessions. There were provisions for almost any kind of physical activity in groups and privately.

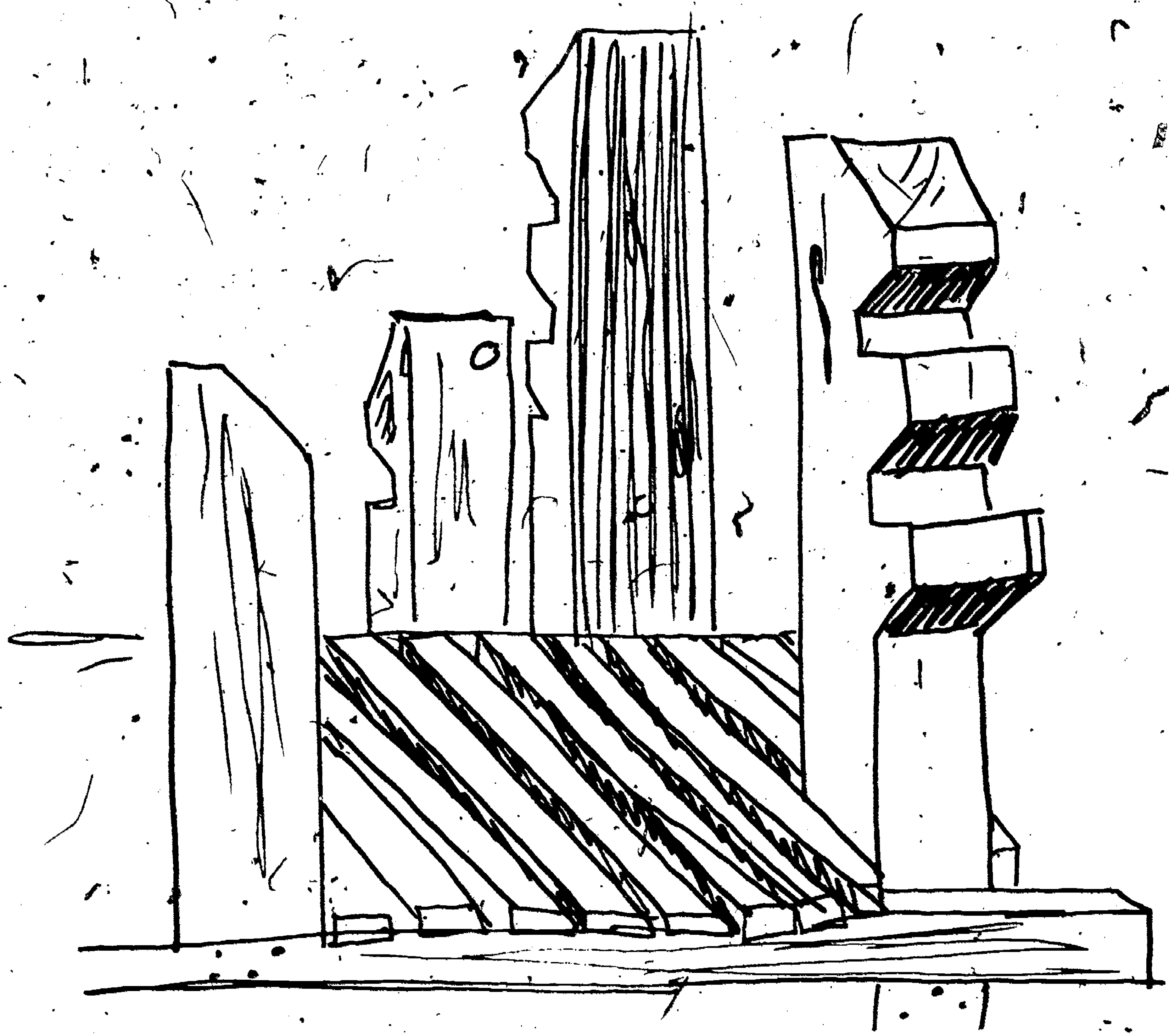
DP5-8



Climbing Pole
Sherman St. Playground - Cambridge



Artificial Hill
with Stainless
Steel Slide



Climbing Poles
Sherman St
Playground -

DP5-10

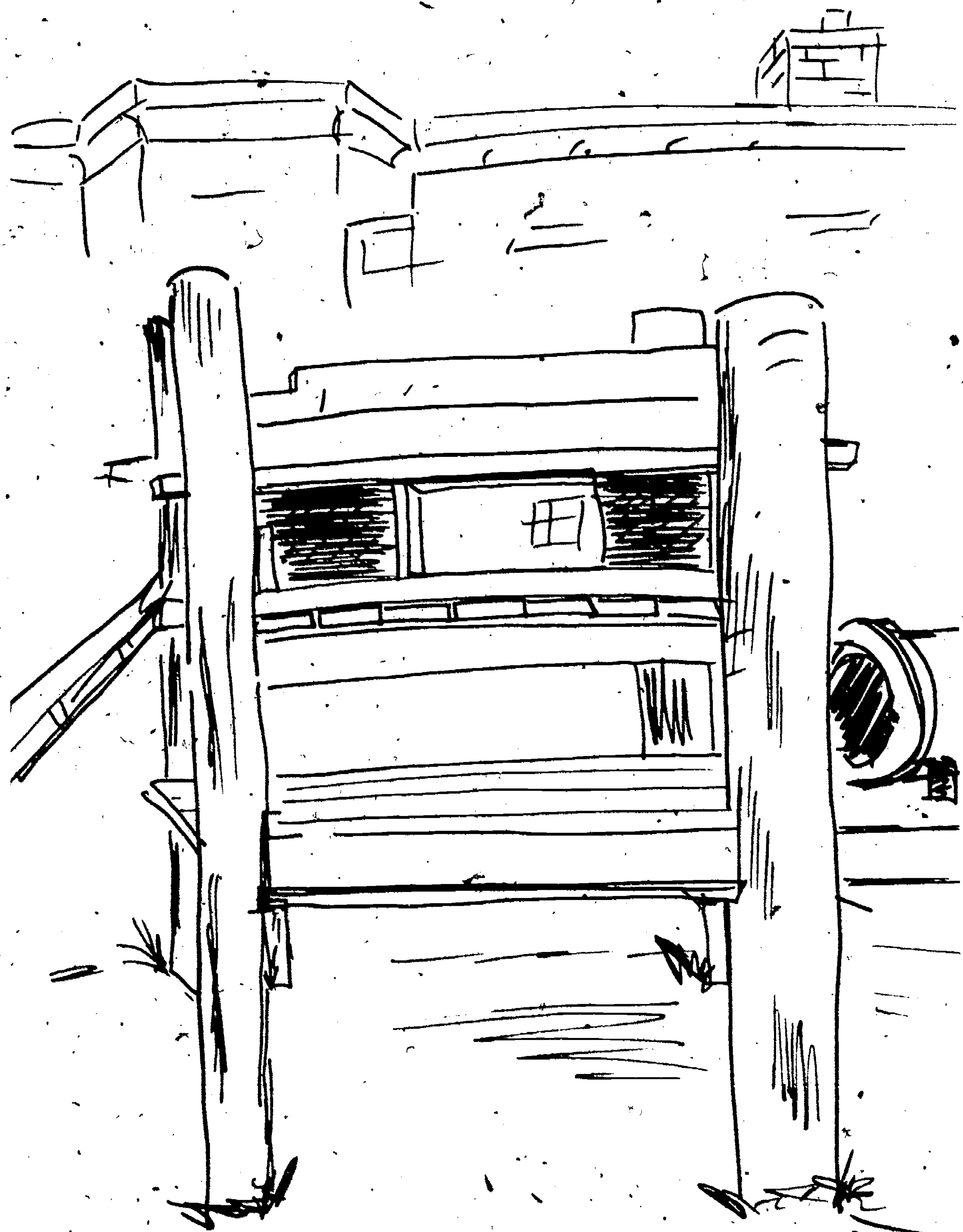
1_0

Central School Playground, Cambridge

Very small area, but good use was made of it.

- An integrated tower, slide, tire climber, shelter, platform. A small (3 ft. long) tunnel was built on one platform.

- Large 7 inch tire swing and single tire swing. Noticed the chain was bolted on in a way that will eventually tear out.



Telephone Pole
and Scrap Lumber Tower + Slide
Central School - Cambridge.

Tire Climber
added in
front

Raymond Street Playground, Cambridge

This was designed by an architect but constructed with non-commercial materials.

- tower slide with pole in center (two levels)
- three hilsides with crisscrossed rail ties, other parts of hill just dirt
- high planks from hill to tower
- provisions for kids sitting in a group (sand boxes, seats, water pool.)
- hard surfaced shelter area for games and other sit-down activities.
- low slide off hill for young children
- low tower of pilings for young children
- under/over movement

A very well integrated playground. Good provisions for meeting social and physical needs.

Problem - equipment (wood parts) in ill repair. Points need reinforcing. Apparently the people who put up the playground didn't care about keeping it up. It wasn't built to protect it from vandalism.

Raymond Street Playground Cambridge Mass Artificial Hill with Railroad Ties

WHAT CAN YOU DO WITH TIRES and ROPE?

USMES
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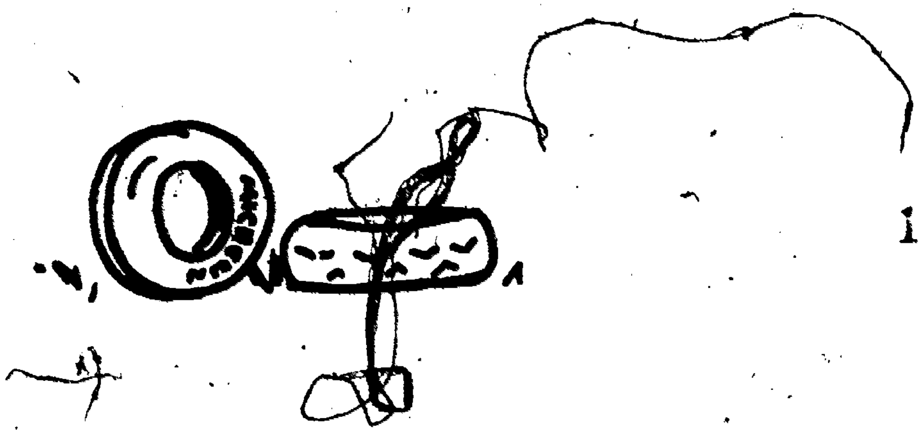
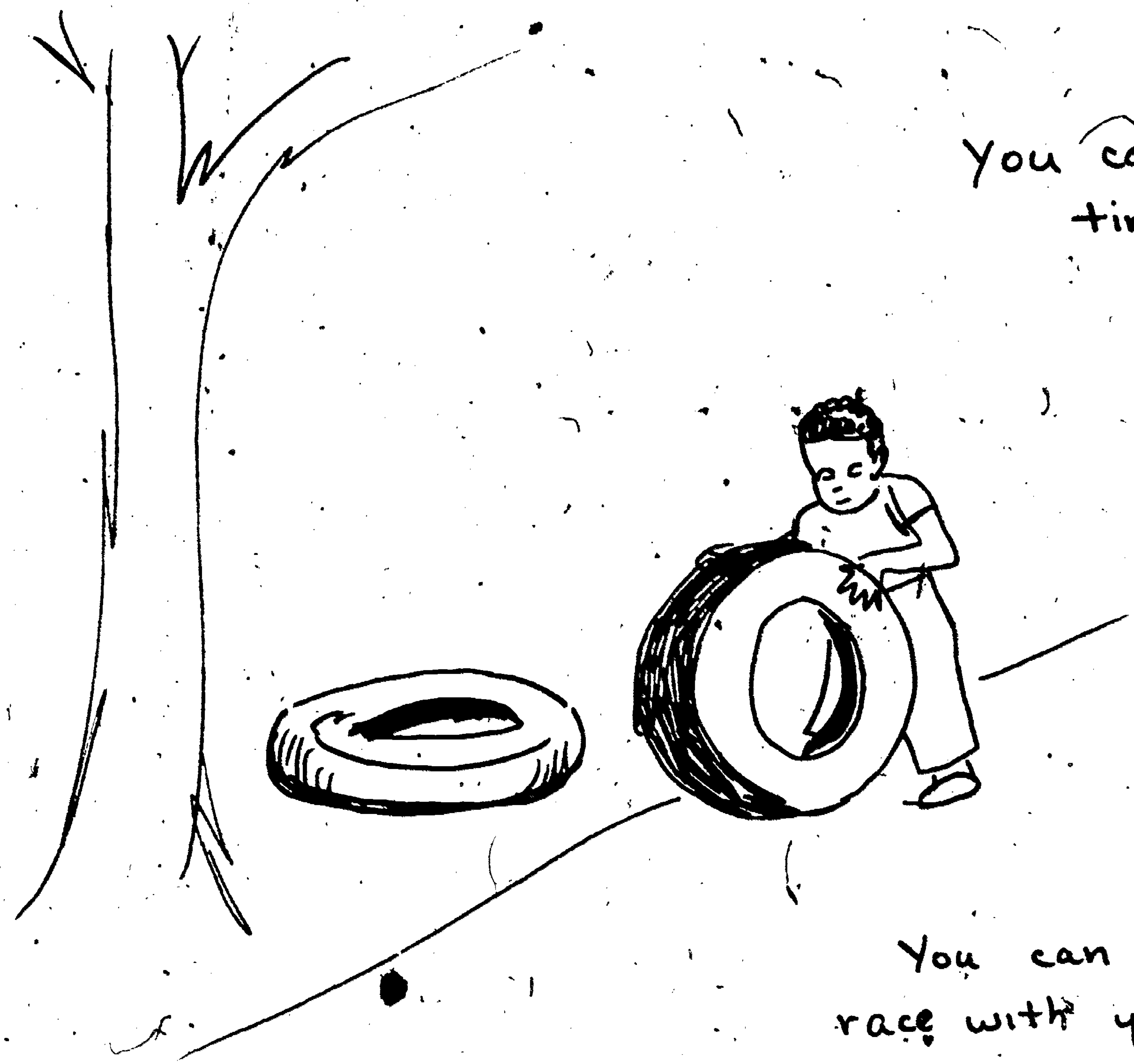
Some Ideas from Playground Design

USMES Summer Workshop, 1972

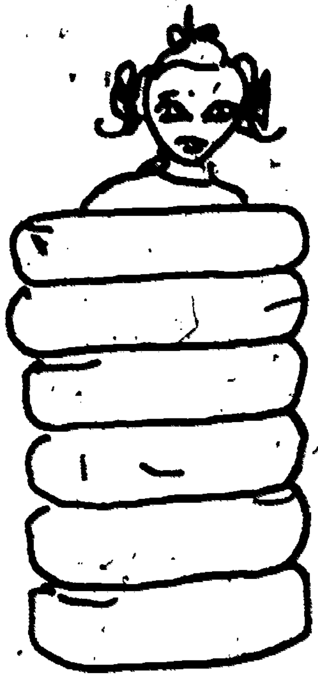




You can roll tires down a hill.

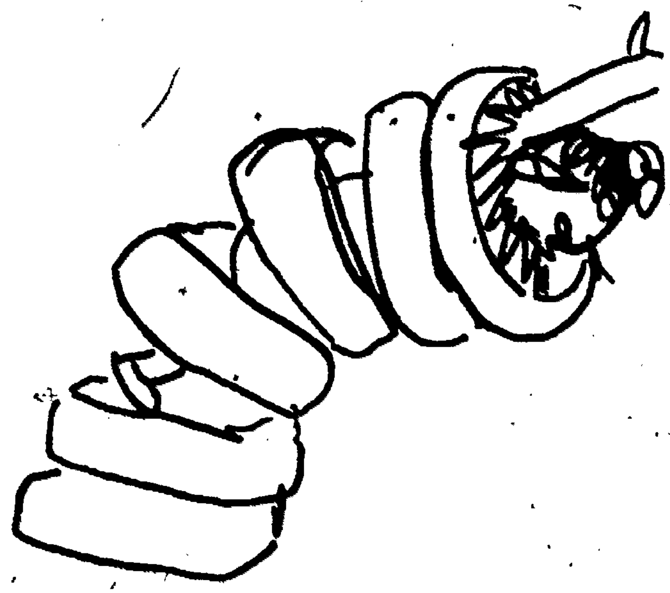
You can have a race with your friends,

DP6-4



You can pile up the tires as high as you are and

Fall over

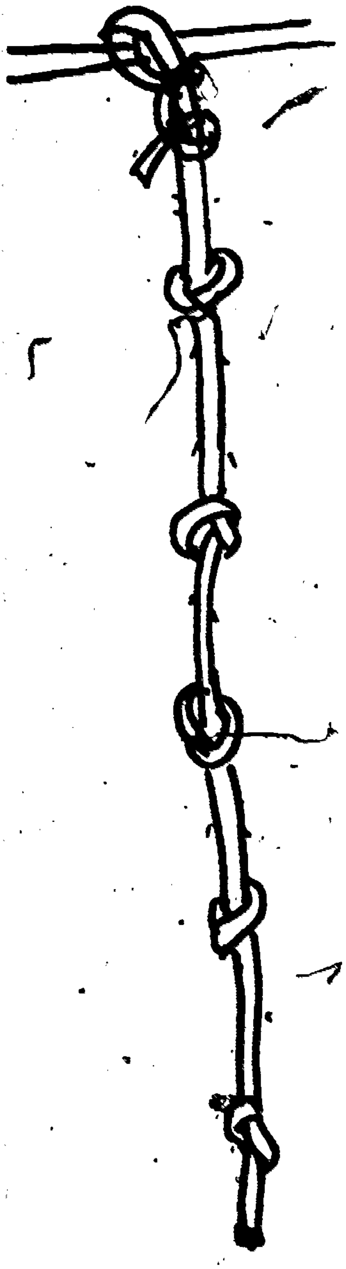


you need a friend to put the top ones on —

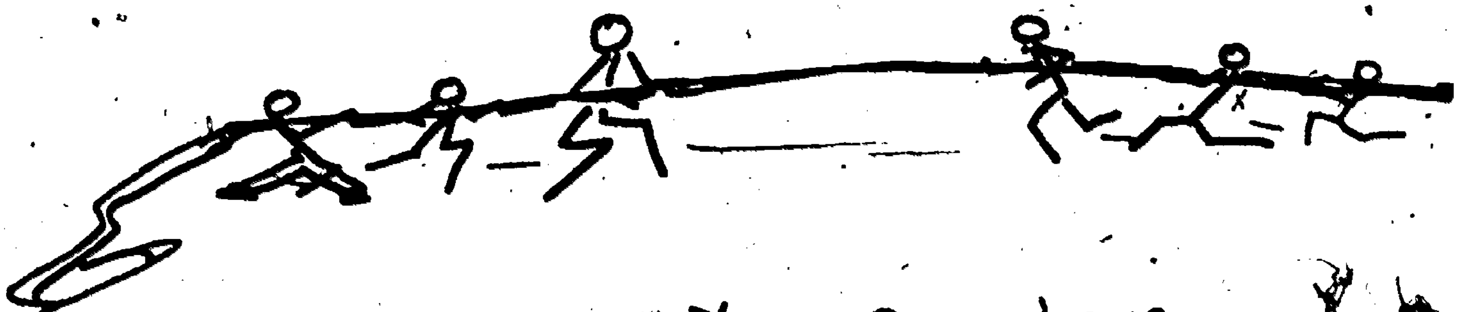
1331

you can make a fort.

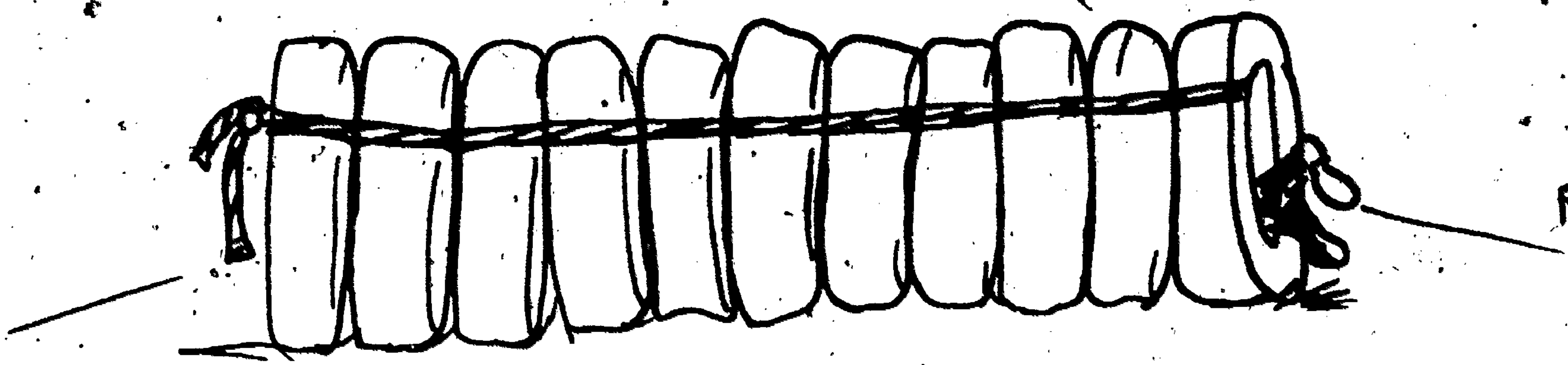
DP6-5



You can make a climbing rope.

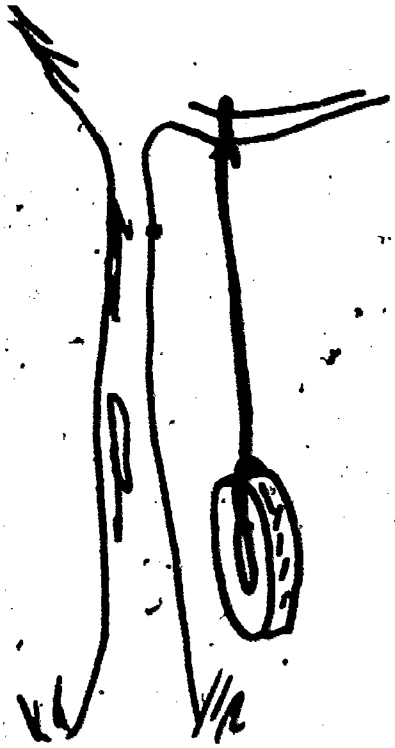


You can have a tug-of-war

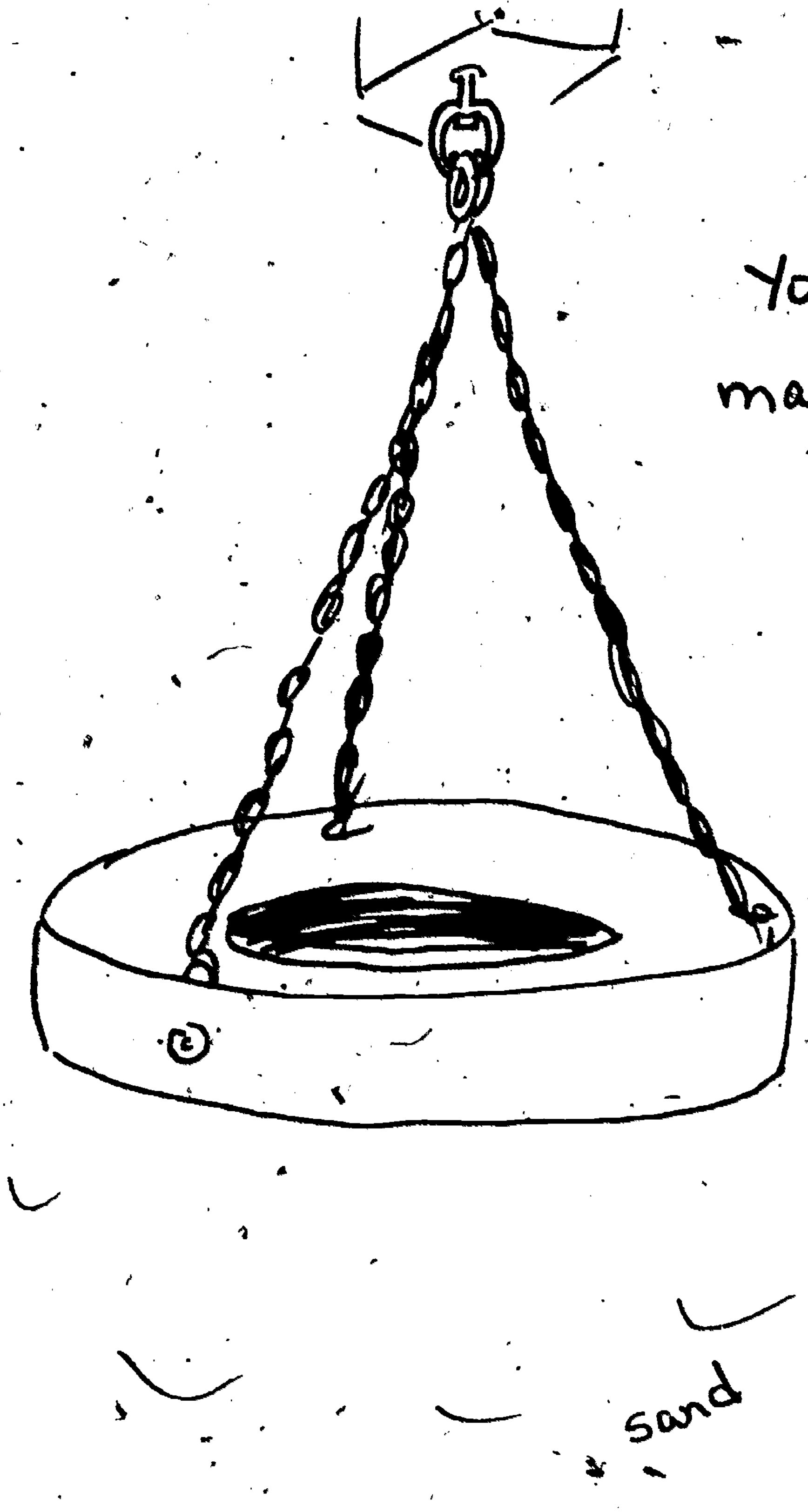


You can make a
tunnel to crawl
through out
of tires and
rope.
You can bolt
the tares
together
for
permanence.

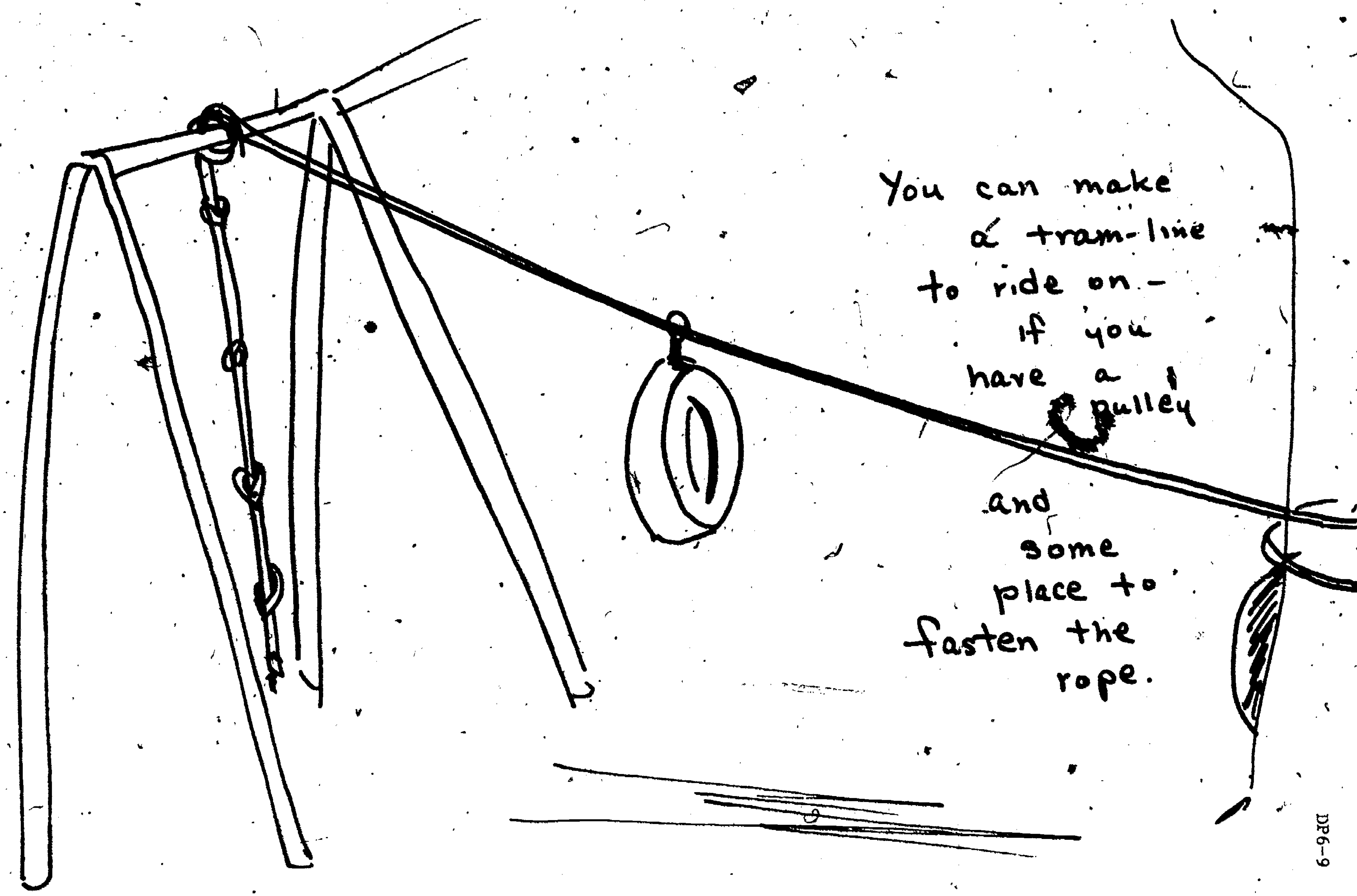
DP6-7



This kind is easier to make.



You can make a swing.


You can make a tram-line to ride on - if you have a pulley and some place to fasten the rope.
DP6-9

DP6-10

what else can you think of — ?

Show us —

TRAFFIC CONGESTION

by

James Kneafsey

I. Introduction

Traffic congestion exists as a problem when vehicular volume or density affects average driver speed to the point where normal traffic flow is reduced sharply. Optimum operating conditions may be assumed to exist, as far as uninterrupted flow is concerned, when traffic is operating in volumes which permit free moving conditions on the roadway. Traffic congestion then results in the two main problems of (1) time loss in travel, and (2) driver inconvenience. It is important to notice that on passing the critical density at which congestion begins, a small increase in traffic results in a large increase in time loss; the increase in density slows the traffic which in turn further increases the density.

II. Peak Hour Conditions in Urban Areas

Within urban areas, traffic congestion is at its worst during peak morning and afternoon hours. These contrast in direction of flow with morning traffic moving into the city and afternoon traffic leaving the city. Afternoon congestion is normally complicated by additional non-working drivers who would not be present in the morning rush.

III. Automobile Trip Purposes

Vehicular congestion is the product of a number of travel objectives other than daily transportation to and from work sites. These trips include shopping, various recreational activities, as well as "pleasure" rides. The choice of auto usage for these purposes (due to private preferences for time and route flexibility, comfort, and privacy) creates added congestion which might be eliminated if alternative forms of mass transit were more readily available.

IV. Externalities of Auto Travel

Traffic congestion has a number of obvious reverberations including the impact on drivers, on roads and on the environment. The more autos on the road, the greater are the chances that they are getting in one another's way.

A) Impact on Drivers - A great majority of auto drivers will avoid congestion at all costs. These avoidance tactics include driving only during non-peak hours or, if necessary, choosing to travel longer routes to avoid known bottlenecks.

B) Impact on Road Maintenance - Congestion on heavily traveled roads impedes road maintenance since repair or paving activities block lanes, thereby increasing auto congestion.

C) Environmental Issues - Congestion in urban areas compounds the negative effects of normal traffic conditions, such as noise and air pollution and increased energy usage in the form of fuel. This is an especially crucial problem in light of the current gasoline shortage situation. Perhaps children might [illegible] to examine the need for smaller, more efficient autos compared to the array of monsters now produced in Detroit.

V. Land Use Requirements

Growing auto travel needs for additional roads or additional lanes in existing roads have raised serious problems for land usage, especially in urban areas where spatiality is a crucial factor determining growth or deterioration. High speed urban highways, in the form of expressways, require more land space both for the road itself and also supporting areas than do city streets. Although the argument in favor of increased urban highways is a reduction of congestion, high speed roads also serve to increase the volume of traffic entering the city. The parking lots and garages necessary to handle this increased volume of entering traffic also necessitate an increase in different amounts and types of land usage.

In many cities at the present time, elaborate mass transit projects are underway (or in the planning stages) in order to provide attractive alternatives to highway transportation. The BART (Bay Area Rapid Transit) System in San Francisco - Oakland represents the most advanced rapid transit system in the United States today. Cities which are constructing or planning mass transit systems include Washington, Baltimore, Pittsburgh, Atlanta, St. Louis, Philadelphia, and Los Angeles, among others. The impacts of these new systems on land use and on existing traffic flow patterns will be dramatic and hopefully beneficial to the growth and quality of the cities.

TRAFFIC FLOW AT PEDESTRIAN CROSSINGS

by

James Kneafsey

I. INTRODUCTION

Traffic flow at the site of pedestrian crossings may be generally depicted in two main categories by virtue of the traffic control equipment required: first, traffic flow which is subjected to a controlled situation, such as urban and suburban intersections with traffic lights, signs and/or specific crossing lines for pedestrians; and, second, traffic which intersects pedestrian crossings in areas or situations not directed by traffic control equipment, as in rural areas.

II. CONTROLLED CROSSINGS

Traffic flow within controlled traffic situations is a function of the variables of volume, vehicle speed, intersection structure as given by the number of road lanes and the timing of control equipment.

Vehicular speed in crossing an intersection is largely a function of the volume of traffic present at the time; auto speed increases as the volume of traffic decreases. The density of traffic within an intersection depends on structural variations of the site in terms of the number of lanes and the width of the intersection area. A given volume of traffic will be less dense and will flow more freely in a wide intersection than will the same volume of traffic in an intersection with a smaller number of lanes.

Traffic flow within intersection areas is most obviously affected by variations in traffic signal controls. Flow is impeded by lights for pedestrians which stop traffic altogether. The timing of traffic controls is also an important factor in the determination of traffic flow with longer green lights accommodating heavier traffic flow.

III. UNCONTROLLED CROSSINGS

Uncontrolled rural pedestrian crossings, including such situations as school and church zones and cattle crossings, are usually marked just by signs. Traffic may slow but does not usually stop completely unless necessary. The absence of traffic control equipment then facilitates traffic flow at the expense of pedestrian safety. Traffic flow is maintained only in conjunction with the increased probability of pedestrian injury. In school and church crossing areas, personnel such as safety patrols are needed to insure safe crossing for children. In these cases, traffic flow should be impeded only as long as necessary for safe crossings to be completed!

Traffic flow is a function of the volume and speed of traffic in a given situation. In the uncontrolled sites of rural areas, the effect of pedestrian crossing on traffic flow may be established by determining the variables of vehicle gap and sight times, of road lay-out and the frequency of pedestrian crossings. Counts may be used to determine the volume of traffic in each direction for a given time span and stopwatches used to measure gap and sight times as well as pedestrian crossing times. A comparison of median gap times with crossing frequency in the form of graphs and histograms should yield an indication of the effect of pedestrian crossings on traffic flow.

TRAFFIC FLOW UNDER ALTERNATIVE STRUCTURAL CONDITIONS

by

James Kneafsey

Variation in traffic flow facility is in direct causal relation to road structure. Factors of road structure variance include:

1. the number of lanes,
2. variations in grades and curvature of roads, and
3. specific conditions which create traffic bottlenecks, such as road construction, signals, bridges, and tunnels.

Traffic congestion on roads is largely the result of an adverse combination of these factors and can only be alleviated by initiating some type of change in the structure of the road.

I. Variation in the Number of Lanes

Variation in the number of lanes is perhaps the most obvious factor affecting traffic congestion in urban areas. These areas are less amenable to road change by construction, in the form of either division or widening than are rural areas, and more prone to traffic congestion due to population density. Many urban streets are formed of two lanes; one-way streets are found predominantly in downtown areas, with parking reducing traffic flow to one or two lanes during working hours. More widely used urban streets are four or six lanes, with separate lanes for right and left turns, resulting in reduced congestion. Limited access roads within urban areas include four lane expressways, which are intended to reduce morning and afternoon peak hour congestion, and interstate links of four or more lanes. Both expressways and interstates, by increasing the possibility of collision, serve at times to heighten traffic congestion rather than decrease it. Additional highway construction (in the form of additional high-speed roads) intended to reduce congestion may serve instead only to increase the volume of traffic.

Variation in the number of road lanes is less directly related to traffic flow in rural areas where congestion is not so general a problem. Existing congestion is more situationally oriented in rural areas, more the result of school and church zones, cattle crossings and the presence of such items as tractors or wagons. Limited access roads in rural areas are generally uncongested unless affected by specific conditions such as road construction or inclement weather conditions.

II. Variation in the Curvature and Grades of Roads

Road curvature directly affects vehicle speed on roadways, with maximum safe speed decreasing as curvature or grade increases. Roads with many curves and bends will slow traffic, resulting in greater congestion than would be present on the same (size) road, without curvature - level curvature tends to prohibit traffic speed more acutely than curves which have been banked. Variation in curvature affects congestion more profoundly on higher speed roads with four or more lanes such as limited access expressways and freeways than on smaller two lane roads which require lower speeds.

III. Traffic Bottlenecks

Traffic bottlenecks are usually the result of specific situations which force a given volume of traffic into a spatially smaller area, thereby creating or worsening congestion. These specific situations include temporary road construction, traffic signals, bridges and tunnels.

This category excludes urban congestion which is pervasively present despite the bottlenecks described below:

(A) Construction -

Road construction takes the form of repair, resurfacing, clearing or the widening of roads and creates congestion by reducing the number of road lanes in use.

(B) Traffic Signals -

Traffic signals create bottlenecks by blocking the flow of traffic, slowing traffic or stopping it altogether in order to accommodate cross flow of pedestrians or vehicles, or left-hand turns.

(C) Bridges and Tunnels -

In areas where bodies of water are present and bridges and tunnels offer the only means of crossing, bottlenecks result as traffic must merge into a smaller number of lanes. This congestion usually diminishes once the bridge or tunnel is passed.

(D) Weather Conditions

In every geographic area of the country, weather conditions pay a role in affecting traffic flow at some time or another during the year. The extent to which operating conditions on highways and local roads are influenced by weather conditions and the remedies to protect against or prevent them depends on the actions of state and local highway officials.

THE NEED FOR TRAFFIC SIGNAL SYNCHRONIZATION IN URBAN AREAS

by

James Kneafsey

I. Introduction

The lack of synchronization of traffic signals is perhaps the greatest impediment to traffic flow in urban areas besides congestion and contributes to much of peak-hour congestion by preventing the free movement of traffic.

II. Synchronization

In a grid city laid out in a systematic pattern of standard blocks, the advantages of synchronized lights should be obvious. Under such a system, autos travelling across the city could cover distances much more easily if traffic signals governing flow in that direction were timed to facilitate flow in that direction. Washington, Baltimore, Columbus and Los Angeles are examples of such cities. This would be especially advantageous during so-called peak hours in the morning and afternoon when the bulk of traffic is moving in one main direction: either inbound or outbound.

In a similar vein, one-way streets could also be used to facilitate flow of traffic during peak hours. This has already been implemented in many cities; in Philadelphia, for instance, the Benjamin Franklin bridge has lanes which can be changed in direction according to the bulk flow of traffic. In a city planned under the grid pattern, this system is relatively simple to implement and contributes vastly to an improved flow of traffic during off-peak hours. In cities with older core downtown areas, such as Boston, where the grid pattern of blocks does not exist, synchronization is more difficult but could be accomplished to ease urban traffic flows during peak hours on selected routes. Experiments of this type are certainly worth the effort especially since the costs of most synchronized systems are not unreasonable.

III. Types of Synchronization

The general types of synchronization encompass

(1) manual controls,

(2) automatic mechanized controls, or

(3) some combination of these two.

There are both advantages and disadvantages to each type. Most urban areas utilize all of them separately and in combination. The "success" of traffic flow within urban areas is largely the product of the effectiveness of the controls utilized.

Manual controls are maintained by police officers who direct traffic movement at intersections. The advantages of employing such personnel are manifest in their discretion or ability to determine how to alleviate congestion while it exists. One major drawback to their widespread use is the underlying labor cost. Also, services of these personnel usually are needed in other capacities such as for emergencies in urban areas.

Among automatic or mechanized traffic controls are two main types: (A) devices which are timed, and (B) those which are directed by radar monitoring of traffic needs. The automatic - constant time devices have the disadvantage of maintaining the same travel time for each direction of traffic, independent of demand, During peak hours when the flow is decidedly heavier in one direction, constant light time impedes flow which might be less congested were manual controls used. Synchronization may take the form of radar sensors, located overhead or in treadles or even as ground-level eye instruments. Radar sensors are able to determine flow in the form of the build-up of "waiting" traffic at a control when it occurs and by changing lights to release this build-up of waiting traffic. Another type of synchronization involves some combination of automatic - constant time controls with sensors which have the advantage of assessing traffic volumes and direction by means of sensors and alleviating congestion by lengthening light time to handle the actual volumes.

IV. Summary

Many cities have adopted various kinds of traffic signal devices in order to improve the flow of traffic through their areas. While manual controls and automatic (constant time) traffic lights operate fairly well during off-peak hours, the stringencies of peak hour traffic volumes require more sophisticated types of traffic synchronization and control equipment, like overhead radar sensors, ground treadles, and ground-level eye sensors.

IMPACT OF PARKING RESTRICTIONS ON TRAFFIC FLOW IN URBAN AREAS DURING PEAK PERIODS

By
JAMES KNEAFSEY

I. Introduction

Due to increased traffic congestion during peak morning and afternoon hours, many urban areas have initiated parking restrictions along main arteries flowing into and out of the city. These restrictions have the purpose of facilitating traffic flow by freeing an additional traffic lane which would otherwise be used for parking. Since the impact of these restrictions affect both traffic flow and city revenues, questions as to their length and location naturally arise, for instance, should restrictions occur on one side of the street or both?

II, Impact on Traffic Flow

Various studies have indicated to traffic authorities that it is usually more effective to coordinate parking restrictions with the direction of traffic flow so that in some cases restrictions occur on one side of the street during morning peak hours and on the other side during the afternoon. On larger, more heavily travelled city arteries, restrictions may last as long as three or four hours, the entire length of the peak period. On smaller lower density streets the restrictions are shorter. As observed by motorists, these restrictions greatly enhance traffic flow; usually strict enforcement in the form of parking tickets and accompanying fines will insure their implementation.

One-way streets can have diagonal as well as parallel parking restrictions. Diagonal parking usually occurs in lower density, wider street areas along one side of the street only; restrictions on this type of parking leaves a wide street area virtually clear for traffic flow. Since the street area can then handle an additional volume of traffic easily and since this volume moves in one direction only, congestion is immediately alleviated. Along more narrow one-way streets where parallel parking is permitted, restrictions of parking along both sides of the street has a similar effect as the elimination of diagonal parking on one side of the street, i.e., the street travel area is widened and congested peak-hour traffic is permitted easier access to main arteries. The length of restrictions along one-way streets for both diagonal and parallel parking is directly related to local needs. More heavily travelled smaller streets will require longer hours of parking restrictions than less frequented or wider streets along which traffic flows unimpeded.

III. The Important Tradeoffs:

Important tradeoffs exist in the implementation of parking restrictions between the problem of congestion and its alleviation on the one hand, and on the other hand, the impact of these restrictions in terms of 1) loss of meter revenues, 2) the effect on merchants' businesses along restricted areas, 3) the necessity created for other parking areas on the other hand, and 4) reallocation of tax monies.

A. Impact of Loss of Meter Revenues:

Parking meters are a large source of income in urban areas; a loss of income due to parking restrictions is significant on a yearly basis and is seriously considered in the planning of location and length of these restrictions. The loss of city revenues, however, is not considered so vital a problem to the majority of urban inhabitants as the problem of prolonged traffic congestion during peak-hour periods. Almost all urban dwellers and many suburbanites who work in the city encounter on a daily basis the problem of congestion and its effect of increased travel time and discomfort. The extent to which the cities subsidize suburban homeowners is a complex, contemporary issue which warrants more attention.

B. Impact on Merchants' Businesses

Parking restrictions along streets where shopping and business areas are located will necessarily affect these businesses by forcing prospective customers to either forego their shopping needs or to seek parking spaces elsewhere in the area during peak-hour periods. Although merchants' losses are taken into due consideration in the planning of parking restrictions, once again, they are usually made subservient to the more prevalent city-wide problem of congestion and traffic flow during peak hours. On the other hand, it may also be noted that parking restrictions along main city streets occur in part both before merchants' opening hours in the morning and after closing hours in the afternoon so that the total time which restrictions would affect merchants' business might be minimal.

C. Additional Parking Areas

Another important impact of parking restrictions along city streets during congested hours is the necessity created for additional parking areas in lieu of the eliminated on-street parking. The necessity for additional parking areas is met by the construction of garages and the use of lots for this purpose. Additional parking spaces are both time-consuming and costly to build and has the added disadvantage of requiring a great deal of land space in an area where spatiality is of utmost importance. The end result is a sizeable expenditure.

D. Reallocation of Tax Monies

Parking restrictions on during peak hour periods have the effect of increasing traffic flow in urban areas. This increased flow of traffic, with its decrease in travel time, serves as an inducement for auto owners to drive rather than use other forms of transportation for their trip purposes. The net effect may be a reallocation of tax monies in favor of auto owners at the expense of other transport users. Suffice it to say that a redistribution of benefits from taxes are an intricate problem in the economics of urban areas.

TRAFFIC FLOW AT ROTARIES

by

James Kneafsey

I. Introduction

The original intent of rotaries was to provide a means for entering vehicles to join the main flow of traffic without disturbing it. In theory, rotaries were meant to assimilate additional autos from various directions without either stopping the main flow or necessitating the use of traffic control equipment. While this has been largely true during non-peak times, rotaries may be potentially dangerous.

II. Problems and Dangers Inherent in the Traffic System

The following is a list of problems and dangers which children may easily observe:

(1) Maximum safe speed is a factor of rotary size and traffic congestion, with maximum speed becoming lower as traffic increases. Even during non-peak hours rotaries slow the main flow of traffic since vehicles must decrease speed to travel the rotary.

(2) Especially during non-peak hours, many vehicles will attempt to enter the rotary at the same speed as the main road, increasing the probability of collision. This probability is heightened during inclement weather, particularly during the winter.

(3) Since many vehicles do not signal their intent to exit from the rotary the probability of collision increases (See diagram I). A potential collision situation occurs if vehicle A wishes to exit at point 1 in Diagram I and fails to signal vehicle B, which

wishes to exit at a distant point and plans to continue to travel the rotary.

(4) In a strictly visual sense, many more signs are needed in a rotary than in an intersection situation, serving to distract drivers unfamiliar with either the area or the rotary system.

(5) There is always the chance that an auto may miss the appropriate exit road and circle the rotary more than once, requiring a greater amount of time to execute a turn than would be necessary to cross an intersection.

(6) During peak hours congestion is worsened by the presence of a rotary since already sluggish traffic is slowed by the unnecessary perimeter of the rotary to be travelled. In a rotary situation (see 1, Diagram II) auto A must travel three quarters of the rotary in order to accomplish what is essentially a left turn. In comparison, by making the same left turn in an intersection situation (see 2, Diagram II) auto B would save travel distance. Congestion is also worsened during peak hours by the fact that a rotary with moving traffic cannot as easily assimilate entering traffic as can a controlled intersection.

(7) Whereas during peak hours in an intersection an accident would tie up traffic moving in one direction only, an accident in or outside a rotary would stop traffic flow altogether, (See diagram III). If an accident occurs at point a, outside the actual rotary, traffic will be stopped at point b within the rotary as a certainty and in all probability at points c, and d. Traffic blocked at points b and d will also block traffic at other points

which would exit at point c. Should the possibility exist that traffic at points h, g, f or e wishes to exit at either of points a or c, all traffic will be completely stopped within the rotary and on outlying exit roads as a result.

(8) Rotaries use more land space than an intersection, a factor of more obvious importance in urban than rural areas.

(9) Within the rotary area, a high collision possibility exists between vehicles which are changing lanes in order to exit, (See diagram IV). Auto A, having entered at point 2, and wishing to exit at point 4, encounters auto B, which has entered point 1 and wishes to exit at point 3. Auto A must then cross either ahead or behind auto B creating collision potential. A variant of this type of congestion occurs when both autos A and B wish to exit at point 3 and must merge to do so.

III. <u>Advantages of Rotary Systems</u>

Certain advantages exist to make traffic rotaries a feasible system in theory. These are:

(1) Traffic control equipment needed in intersection systems is not required for a rotary. This may result in a considerable financial savings in local, state, or federal funding programs.

(2) Autos may enter the traffic flow without stopping, given favorable conditions. Even when they stop, autos may start again at the first opportunity. (Some traffic controls, such as lights, will hold up traffic when there may be gaps available for entry.

(3) Traffic on some high speed roads may be slowed down simply by existence of rotary. In this sense, a rotary is a type of traffic controlling instrument, since its mere presence requires vehicles

to reduce speeds both at the rotary and in the road areas approaching the rotary.

(4) Rotary systems are more aesthetically pleasing than intersections since they allow grass plots and eliminate control equipment.

IV. <u>Presentation in the Classroom</u>

Both the advantages and disadvantages of traffic rotaries may be introduced to children first by means of observation. After children have drawn diagrams, certain simple technical procedures may be used by children during subsequent observation times to substantiate the problems at hand.

To factualize their observations of slow-moving rotary traffic, children may use stopwatches to calculate and compare speeds of autos inside and outside the rotary. Within the classroom, models may be made of rotary systems (possibly with Design Lab materials) and used to demonstrate the necessary decrease in speed. Videotapes of actual systems may be made, although this procedure requires additional equipment.

The tendency of vehicles to travel the rotary without reducing speed, the failure to use signals, and the necessity of additional signs at rotaries may be observed by children on location. Speeds may be calculated and compared with safer, slower speeds. Videotapes of non-signaling vehicles may reveal potentially hazardous situations (2,3,4)*.

Children may be stationed at specific points in the rotary to count cars which circle the rotary more than once. Videotapes may be used to supplement these observations. (5)

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* The numbers in parenthesis here refer to the sections described above under <u>Problems and Dangers Inherent in the Traffic Rotary System</u>.

Peak hour congestion may be observed, photographed and videotaped. Counts may be taken of cars entering and leaving the rotary during a given congested time span and compared to counts taken during non-peak hours (6,7). Accident conditions in a rotary also may be presented by means of models, diagrams and videotapes (8). Unnecessary land use may be shown and measured by means of comparative diagrams of intersection and rotaries.

The more obvious advantageous characteristics of rotaries, i.e., the lack of traffic equipment, increased aesthetic appeal, and assimilation of traffic without stops may be observed. The relation between the advantages and problems of traffic rotaries should also be pointed out by means of the photos, tapes and observations of problematic situations.

V. <u>Summary</u>:

When traffic rotaries were first designed in the 1930's by well-intentioned civil engineers, they were considered panaceas of their time. For reasonable and moderate traffic volumes, the rotaries actually served well. Unfortunately, conditions changed rapidly over the next three decades such that many rotaries now are simply huge bottlenecks, especially during peak-hour traffic flows; yet rotaries do serve a useful function in some cases. The above discussion was intended to highlight the conditions under which rotaries are effective and those under which serious congestion would occur.

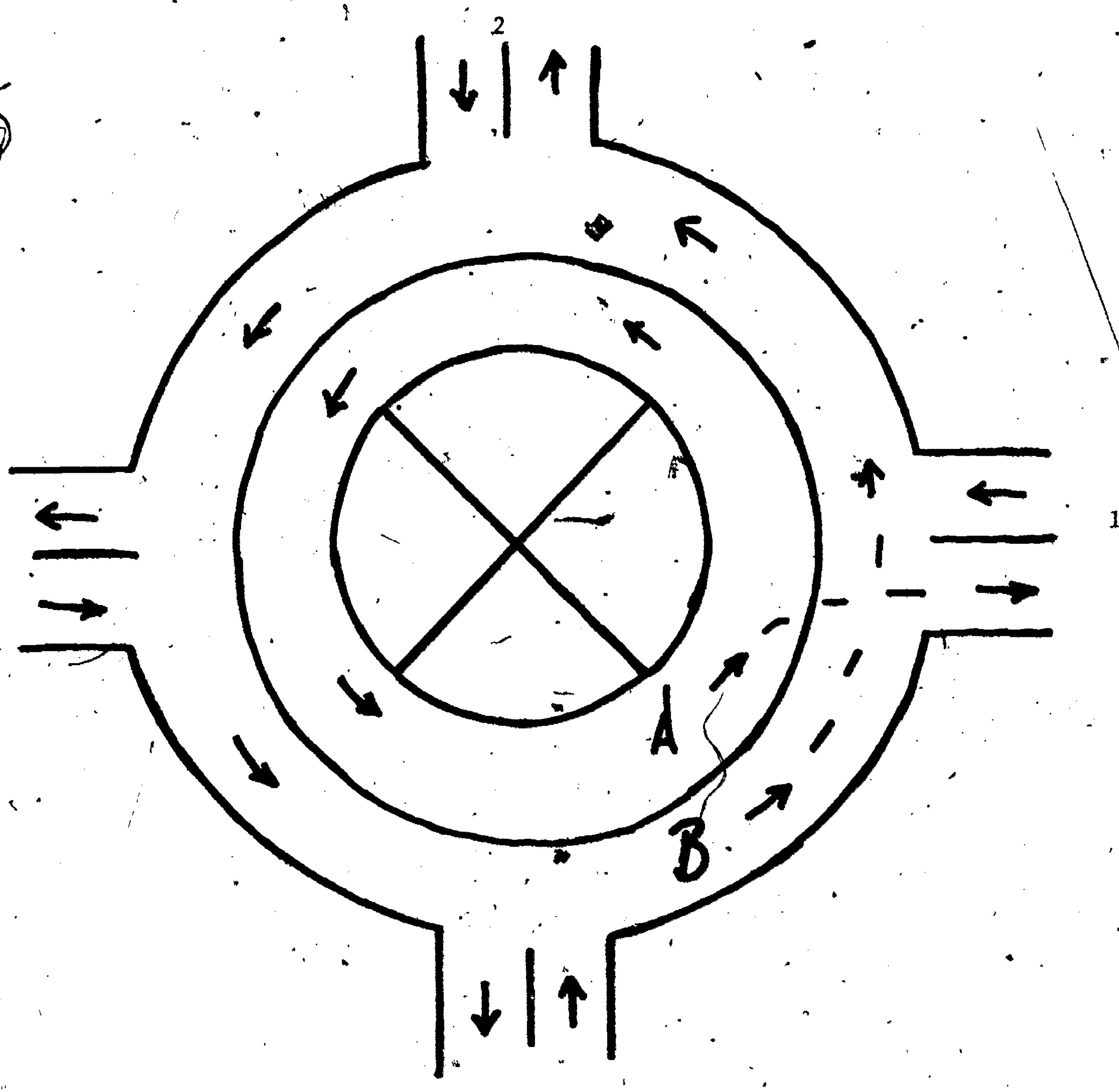
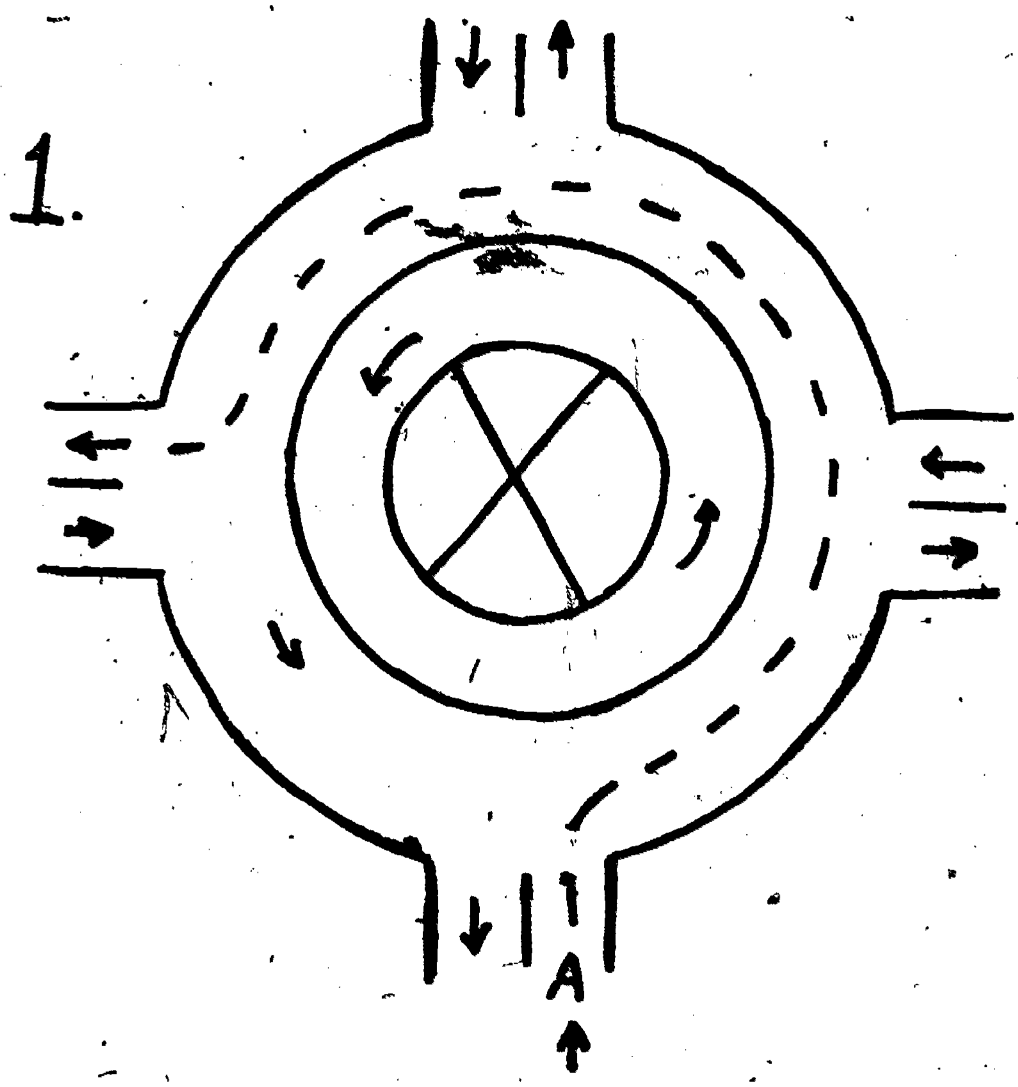


Diagram I.



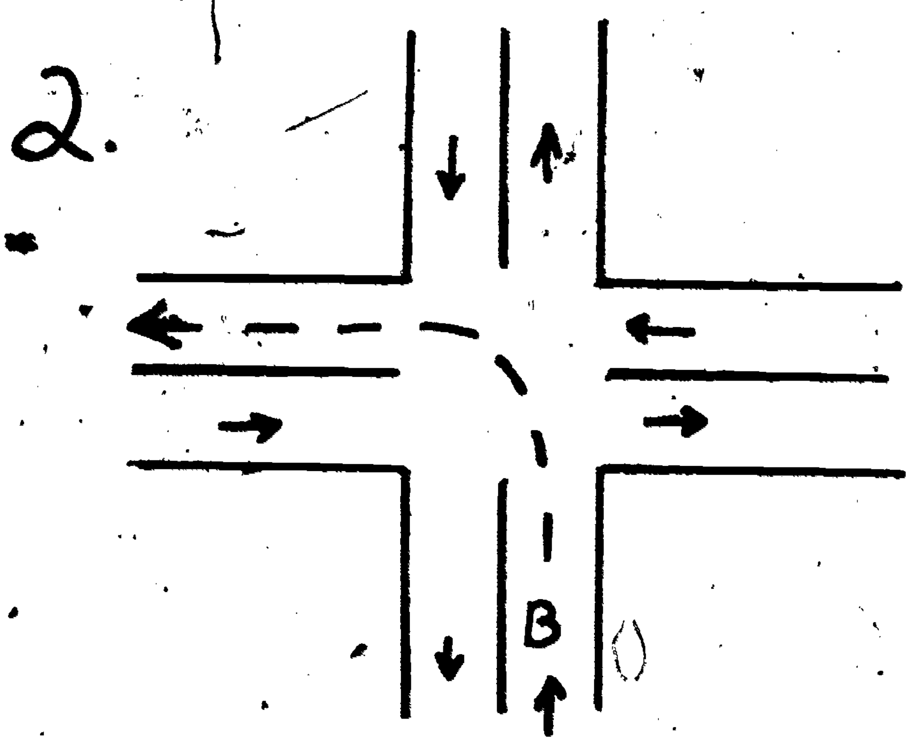


Diagram II.

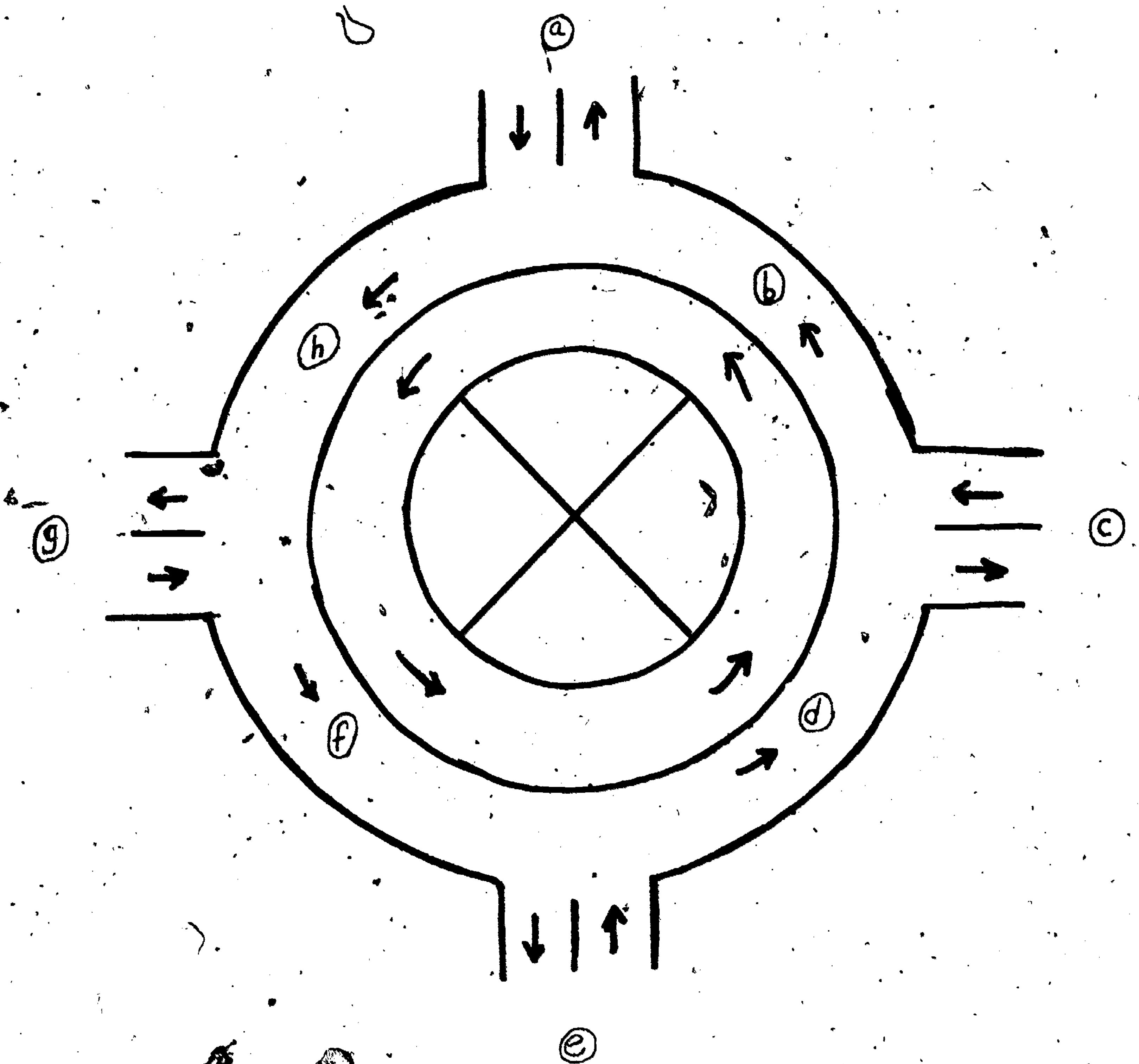


Diagram III.

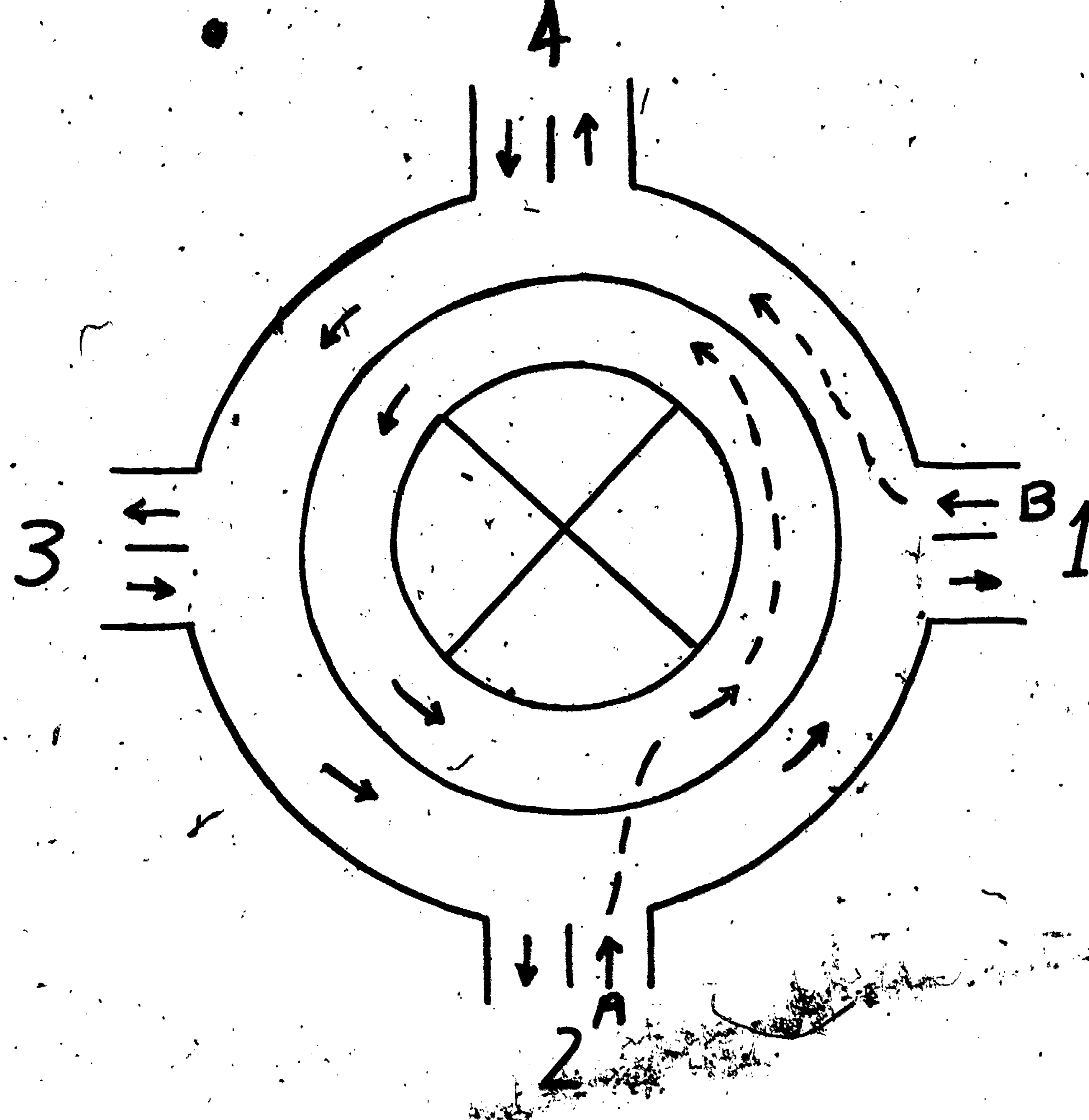


Diagram IV.

PEOPLE AND SPACE

by

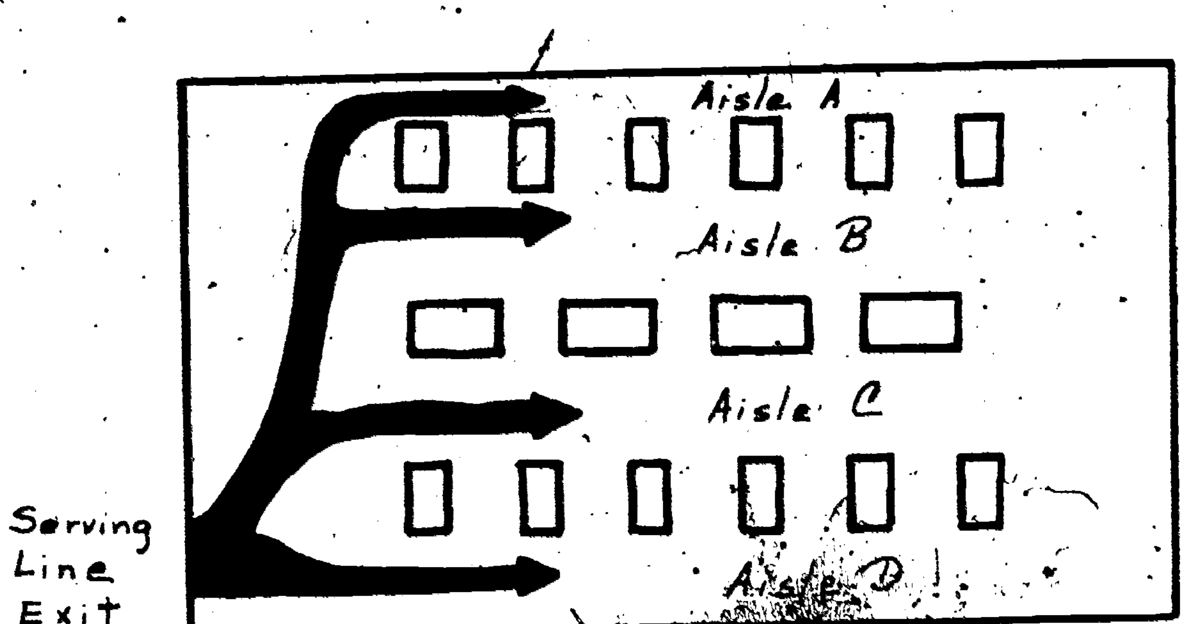
Gorman Gilbert

In several USMES units students have often encountered the problem of understanding how people use space and are influenced by spatial arrangements. In Lunch Lines, for example, at least two teachers (Jeanette Lea and Mary Szlachetka) have found their classes focusing on the physical arrangements of their lunchrooms. In both cases the objective was to increase the efficiency (capacity and speed of service) of the lunchroom. However, in other units, the use of space may be important for different reasons. In Playground Design, spatial relations may determine how often an apparatus is used, and in Classroom Design they may be an important influence on how well each portion of a classroom fulfills its purpose. Also, in all group work spatial relations can affect how often a person talks or who talks to whom.

However, the knowledge that space affects a person's behavior is not the same thing as predicting what these effects will be. In the case of Lunch Lines students have made scale models of lunchroom layouts and have suggested changes in these layouts so as to improve the operation of the lunchroom. Yet, how can the students know that their arrangement will improve lunchroom operations? Or, in the case of Classroom Design, how can students determine, for example, the best arrangement of tables, chairs, and desks? These questions all require that students be able to measure how space is used by people.

DP13-2

Depending upon the objectives of the students, there are a variety of possible measures of space utilization. Again in the case of Lunch Lines, the students may decide that the lunchroom problems are due not to delays in serving the food but to congestion of people going from the serving line to their seats or from their seats to a waste collection area. Students could time persons from when they leave the serving line to the time they sit down at a table. They might also prepare flow diagrams showing the number of people using each aisle to get to their seats. An easy way to do this is to plot on a lunchroom diagram the flow of traffic letting the width of lines indicate the number of persons using an aisle. For example, [illegible] following diagram represents hypothetical data collected by students:



Total Students Leaving Serving Line = 100

Students Using Aisle A = 5

Students Using Aisle B = 35

Students Using Aisle C = 50

Students Using Aisle D = 10

Clearly the diagram can be extended to cover the entire length of each aisle as well as other aisles between tables. Such a diagram might also be color coded to distinguish which direction the persons are welking when using the aisle, and separate flow diagrams could be drawn for different grade levels or classrooms.

There are other spatial measures as well that may be useful in Lunch Lines. Students may measure conflicting traffic patterns by counting the number of times a person passes someone going in the opposite direction. Or, they may determine how wide an aisle should be by counting the number of "bumps" between people or between people and tables, chairs, or walls for a given aisle.* Likewise, students may plot on a lunchroom diagram the order in which seats are taken or the number of seats at each table which are not used. All of these measures allow the students to compare a new lunchroom layout with a previous one to test whether their solution works.

May of these measures are also applicable to units other than Lunch Lines. In Playground Design the students may count the number of times an

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* Jeanette [illegible] did try to determine how wide a lunchroom aisle needs to be.

DP13-4

apparatus is used, the number of students of various ages who use it, and the number of boys who use it compared to the number of girls who use it. Again, these data could be used to see whether the arrangement of the playground equipment affects how the equipment is used. Similarly, in Classroom Design the students might count the number of persons sitting at desks rather than at tables (if there is such a choice), the number of persons using a portion of the classroom at any given time, or the number of persons talking instead of, say, reading in different parts of the classroom.

The last measure is an example of a social consideration as opposed to the efficiency considerations discussed in the case of Lunch Lines. In Classroom Design, the students may not care how <u>many</u> persons use a space but rather <u>how</u> they use it. Thus they may want to measure not only the number of persons engaged in different activities (e.g., talking) in a given space, but also perhaps how the persons are arranged within the space. For example, how far apart do persons sit while talking or while reading? Or, does the shape of a table and a person's position at the table influence how often he talks during a conversation? Both of these questions can be answered by students by measuring distances and counting how often individuals contribute to a conversation.

An interesting illustration of how space influences social behavior is an experiment conducted by Jon Emerson's class. Jon divided his class in two parts and sent one out of the room. While they were gone, the remaining students arranged the chairs and tables in a particular fashion and predicted who would sit where when the other students came back into the room and were invited to sit down. They then observed how often each person spoke after the students sat down. The experiment was then repeated using a different arrangement of chairs and tables with dramatic differences in the number of times given students spoke. While this experiment was not done in conjunction with a particular USMES unit, it might have been. For example, such an experiment might be performed by students in Classroom Design in order to answer a question such as: "What is the best arrangement of the desks and tables in the classroom?"

Thus far the discussion has largely focused upon the individual's needs for space. Yet in Classroom Design, for example, the students may want to know how much total space to allot for different activities within a classroom The first approximation can, of course, be easily found by multiplying the space requirements of an individual engaged in an activity (e.g., reading) by the total number of persons who would be doing that activity at one time. The children might also calculate the ideal size classroom which would enable them to participate in all desired activities with adequate space between the activity areas.

It should be mentioned that there are space standards used by architects in designing buildings and laying out rooms within buildings. Many of these standards are available in architectural reference books. Thus, at some point in the students' investigation of space requirements it might be interesting to ask a local architect to visit the classroom to explain how he allocates space within buildings.

It should be clear that there are many ways by which students can measure the effects of physical arrangements upon human behavior. A few of these have been mentioned here with the hope of stimulating further classroom attention to the spatial aspects of several USMES units.

USMES

SPEED, TRAVEL TIME, VOLUME AND DENSITY RELATIONSHIPS IN TRAFFIC FLOW*

Speed, travel time, volume, and density are four important factors in any traffic flow problem. The relationship of these variables and the changes occurring during free-flow and congestion conditions may suggest possible solutions to the problem.

A series of typical flow relationships is depicted in Figure 1. Six different relationships are shown, namely the relationship between:

1) Speed and Volume,
2) Travel Time and Volume,
3) Speed and Density,
4) Travel Time and Density,
5) Volume and Density, and
6) Travel Time and Speed.

In their observations of traffic flow, children may encounter one or several of these relationships. Each relationship suggests what will happen to one of the variables when the other changes and is demonstrated under two types of traffic conditions: a) free-flow traffic (solid portions of the curve); and b) congestion conditions (the dotted portions of the curve).

Consider relationship (graph 1) between speed and volume. This says that, in a free-flow traffic situation, speed will decrease as volume increases, therefore the declining solid line. However, in a congestion situation, a severe bottleneck would result in low volumes and (of course) slow speeds; light congestion would result in slightly higher speeds and increased volume, hence the positive slope to the dotted line in Figure 1.

Graph 1 illustrates the importance of avoiding congested conditions: for the <u>same volume</u> v, as marked, the traffic can move at speed s_2 in free-flow traffic,

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*Based on suggestions by James Kneafsey

but only at the much reduced speed s_1 in congested traffic. In non-congested conditions the same amount of traffic is moving along the road but at much better (higher) speeds, which means much better (shorter) travel times. Figure 2 illustrates the latter point. Travel times (t) are calculated over the same distance (d). There, at volume v, $t_1 = \frac{d}{s_1}$ and $t_2 = \frac{d}{s_2}$. At volume v, in congested conditions, traffic takes time t_1 to travel distance d, while it takes a much shorter time t_2 during non-congested conditions.

What is happening is that as the volume of traffic entering a road goes up the density (number of cars per mile) of traffic increases. This causes the cars to slow down (increased travel time) which decreases the volume of traffic going past any point (Volume = average speed x density). Consequently cars are "backed up," causing even greater density, which in turn slows them down more, etc. When this vicious circle of conditions exists, one has congested flow. Under free flow there is enough space between cars so that an increase in density only causes a slight slowing down in speed, hence allows a bigger volume to be handled with only a slight increase in travel time.

Another interesting feature of graphs 1 and 2 is that there is a maximum volume, v_{max}. One cannot get more than a certain number of cars through per minute. If one tries, one quickly changes to congested conditions with much greater density and much slower speeds--thus actually decreasing the volume of traffic handled. This is illustrated by graph 5.

As graph 3 shows, the speed always decreases as the density increases. (One can get figure 3 from figure 1 by using density = volume/speed.) What one notices on this graph is that under congested conditions the speed drops more rapidly with increasing density.

As another example, consider relationship (graph 6) between travel time and speed: in a congested situation, speeds (average) are very low resulting in very

TYPICAL RELATIONSHIP CURVES FOR TRAFFIC FLOW DATA:

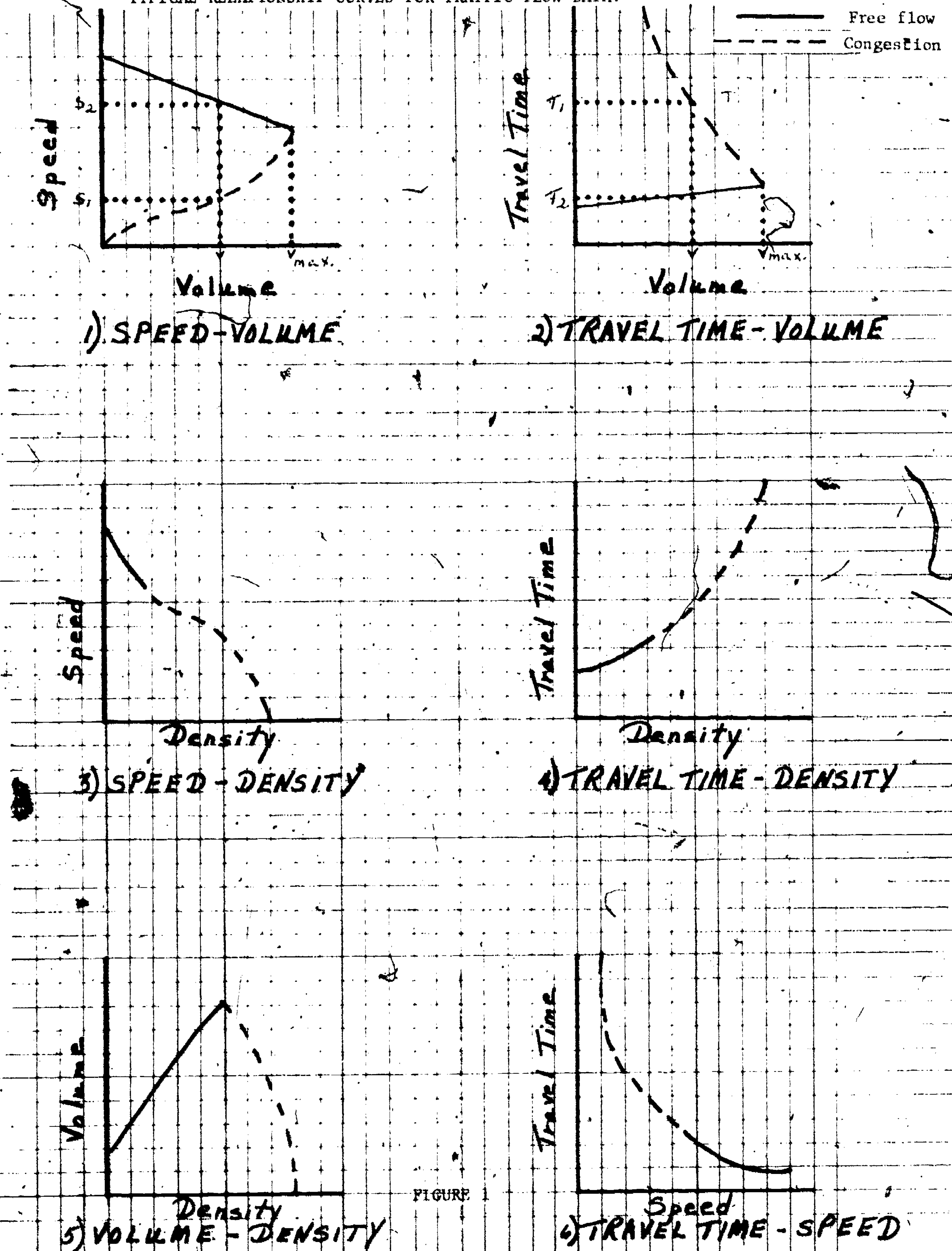

FIGURE 1

high travel times, but as the road becomes less congested, speed increases and travel time decreases--hence the inverse (negative slope) relationship between travel time and speed. Expressed another way: in order for travel times over a given distance to be reduced or shortened, speed must be increases. But that is only possible if densities are low enough.

Speed - density relationships have been developed for the Merritt Parkway in New York State. The data was compiled into one minute intervals. For each one-minute group the mean speed and density were calculated and plotted in Figure 2. A curve may be fitted to the scatter of points. The data does not include any gathered during congestion conditions, consequently it ressembles the solid portion of the speed-density graph in Figure 1.

Children will probably find the variables of volume and speed more meaningful than density. They might also be able to collect data on those two

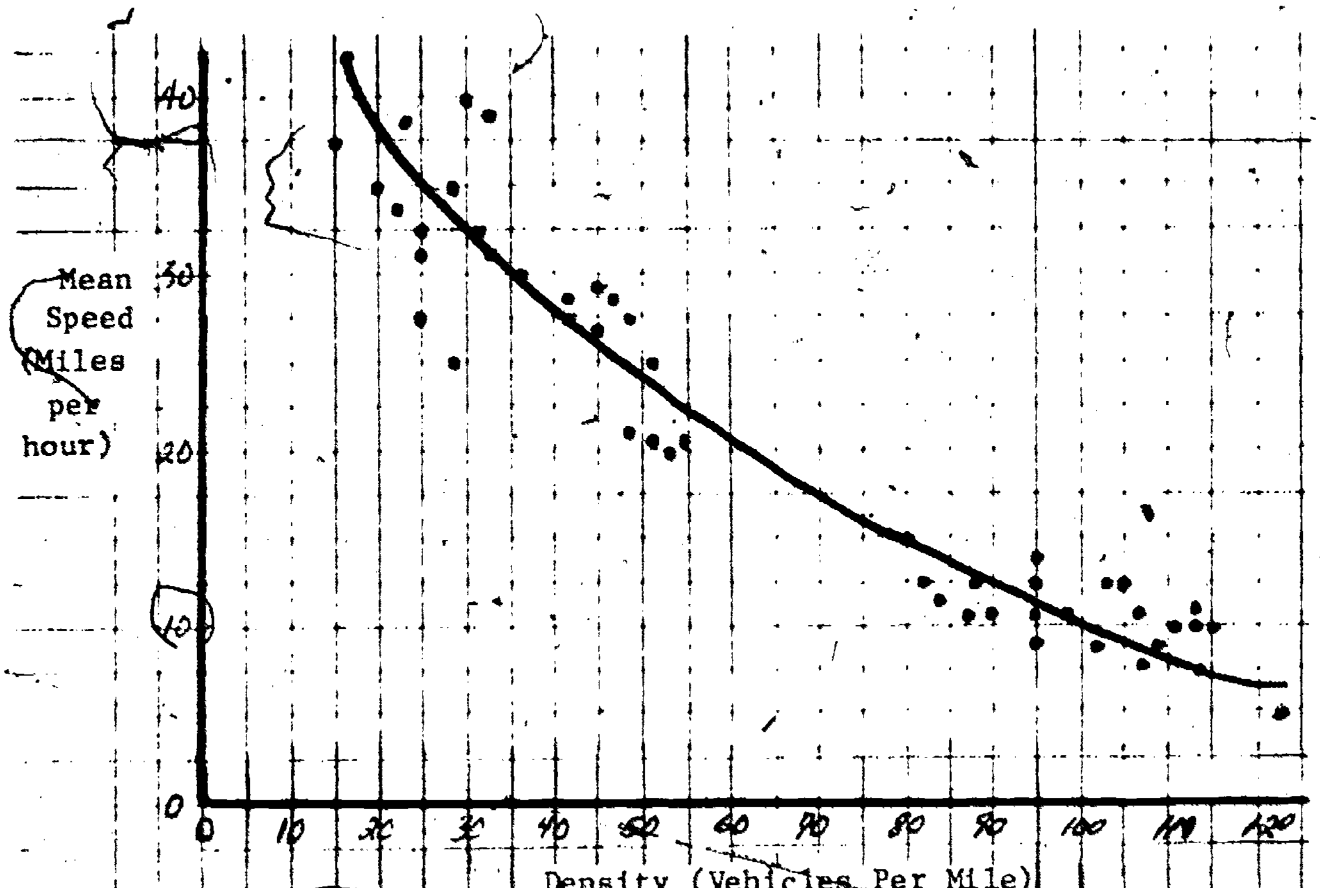

Figure 2

variables at the problem location. The speed data would have to be collected on a sample of cars.* Volume data can be acquired by the children counting cars at a specific location for a definite time interval at various times of day.

If the children can make a quick count of cars that are within the distance being observed and then immediately start counting cars going in and out of the roadway being studied, density data at specific times can be later constructed from their volume data. The following paragraphs describe a field study conducted by traffic engineers in Chicago, Illinois during 1957. If the length of roadway being studied by the children has no entrances or exits, the investigation is simplified considerably from the Chicago study.

The site for the Chicago Area speed-density study is a quarter-mile section of Washington Boulevard between Menard Avenue and Austion Boulevard in the City of Chicago. The section consists of three full blocks with a fixed-time signal at the western end. A uniform width of 40 feet, undivided, with parking removed on the north side from 4:00 p.m. to 6:00 p.m., provides two moving lanes for the heavy westbound commuter movement. Only this westbound movement was studied. There is no commercial traffic except buses, and these make stops in each block. Figure 3 shows a sketch of the plan.

The study was made from 4:30 p.m. to 6:00 p.m. on Friday, March 8, 1957. The road was dry and the weather clear. Observers A and C, stationed at each end of the section, made manual cumulative counts of entering and leaving vehicles, respectively. The observers' watches were synchronized and the cumulative counts were noted at the end of each one-minute period. The purpose was to keep a running record of the number of vehicles in the traffic stream between observers A and C at any time. A third observer, B, recorded vehicles entering or leaving the section between A and C. Note that in the threeblock section traffic can enter west bound

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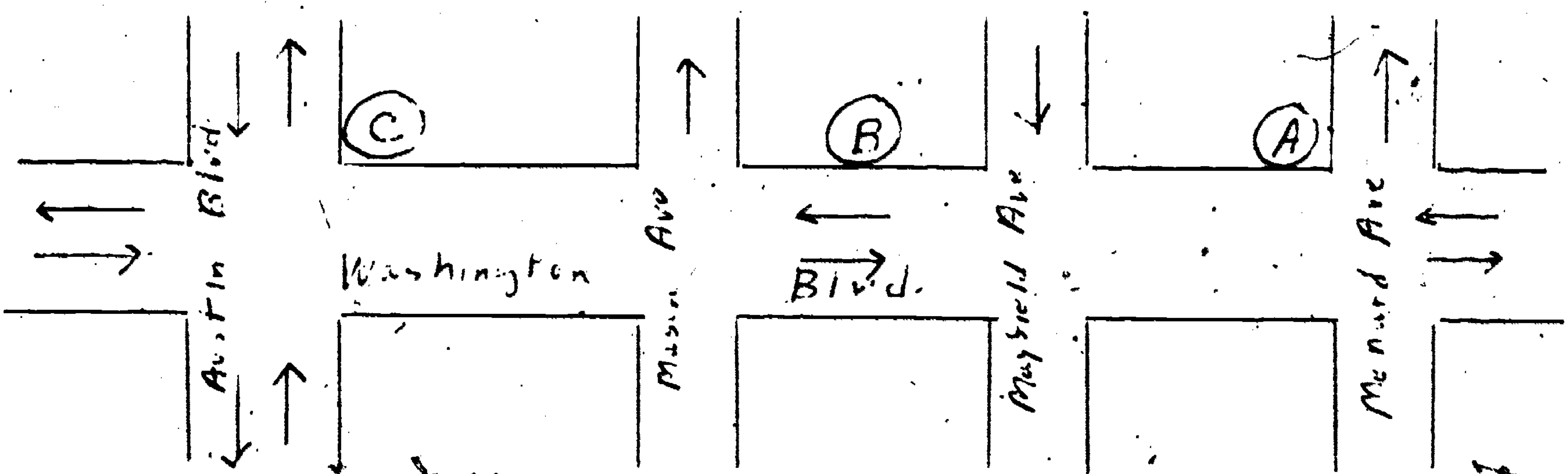
* see log by Bernard Walsh for description of this activity

traffic from only one street. At the end of the test period, the observers made a check on the cumulative counts.

Traffic during the period of the study ranged from moderate to heavy, but the congestion during the heaviest traffic periods was not so severe as to cause a breakdown of traffic flow.



Washington Boulevard, Chicago, Illinois
Figure 3

The following information is readily obtained from an initial density count plus differences in cumulative counts for any specific length observation time.

a) output volume for time interval between cumulative counts (difference between successive output cumulative counts).

b) input volume for time interval between cumulative counts (difference between successive input cumulative counts).

c) instantaneous density at times of cumulative counts (initial density + or - difference between output and input cumulative counts).

Owing to differences in the times to which they apply, density and volume values cannot be correlated exactly. In the Chicago experiment, when travel times were less than 2 minutes (with one minute cumulative counts) the following relationships for average density and average volume were adopted.

Average density at time t = 1/4 (density at time t-1 + 2x density at time + density at time t + 1)

Average volume at time t + 1/4 (input volume over period t-2, t-1 + input volume over period t-1, t + output volume over period t, t + 1 + output volume over period t + 1, t + 2)

The effect of vehicles leaving the section at intermediate points must be considered. Since vehicles which leave the section are counted out during the interval in which they leave, the density value always represents the true number of vehicles in the section. However, the intermediate counts do not appear in the volume formula. A vehicle leaving by a side street is counted only as part of the input volume and therefore has only half of the effect that a through vehicle has on the average volume. This is considered to be reasonable.

BICYCLE TEST COURSE

by

Frank O'Brien

A bicycle driving test course might be laid out by students working on the Bicycle Transportation challenge. Preliminary tests would have to be passed by potential course users in order to qualify for making course runs. These tests could include a road rules test and an inspection test for bikes.

1. Road rules test: (Role of bicyclist on road)

   a) Which side of the road do you use when driving a bike?

   b) What traffic regulations (signals, signs, one-way streets, never passing on a curve or on the left, etc.) do you observe?

   c) What signals do you use when you turn right...left...stop?

   d) What do you do when you approach an intersection with a traffic light that just turned from green to yellow?

   e) What safety equipment should a bike have when driving at night?

   f) Does a bicycle require a license plate?

   g) Who has the right of way under various circumstances, etc.?

2. Inspection test for bikes:

   a) Good tires.

   b) Good brakes.

   c) Tight handlebars.

   d) Fenders.

   e) Check seat for looseness.

   f) Check for broken spokes or bent wheels.

   g) Chain guard.

h) Tight wheel axle nuts.

i) Check for horn or bell.

j) Lights.

k) License plate, etc.

The above safety check for bikes can be made by kids at the school whether they want to run the bikes on the course or not; parents may consider this as a useful service that is necessary to keep their kids' bikes in a safe running condition. A point system can be devised for this safety inspection. An "OK" inspection "sticker" can be created and attached to bikes that pass the inspection.

The students can discuss what sorts of things should be included on the course that would show a safe driver from a not-so-safe driver. A list of situations that kids actually have to cope with during their real bicycle driving experiences follows:

1. Tight turns, left and right (slalom course).
2. Steep uphill grades.
3. Steep downhill grades.
4. Through narrow passage.
5. Over narrow plank.
6. Through wide puddle of water.
7. Across sand and/or mud.
8. Inclined plane "ski" jump.
9. Between parked cars.
10. Speed run on a straight-away.
11. Braking test strip.
12. Traffic light stops and starts.

13. With traffic flow.

14. Against traffic flow.

15. Across traffic flow.

etc., etc., others?

The students should keep safety in mind at all times when laying out the course. A scaled-down sketch of the course can be made on cardboard or large paper before being built. Distances can be determined and various "events" placed according to the terraine, etc. One possible layout is shown on the following page.

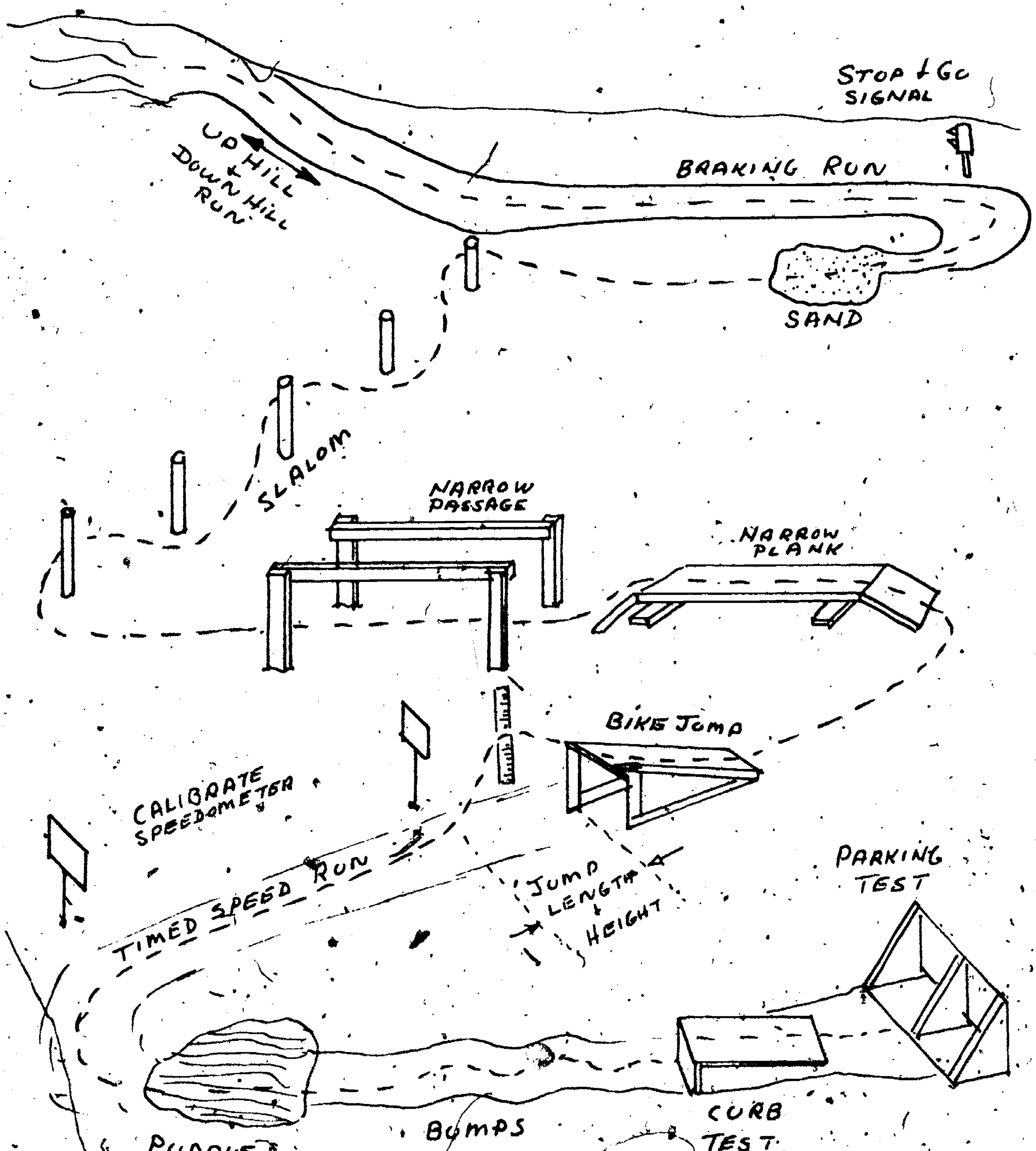
Best time around course plus highest number of points for performance at each test point could be used as a standard for "best drivers" or "safest drivers." A scoring system will have to be devised based on how difficult various aspects of the course prove to be.

Parts of the course can be used for specific activities. For example, the braking run can be used to test braking distance at various speeds. The students could try out 3-speed, 10-speed, etc., bikes on the uphill run and check the time taken and the energy expended in getting up the hill. Speedometers can be checked on a level part of the run by timing the rider over a set distance.

In constructing and using a bicycle test course students will learn and practice many skills such as addition, subtraction, multiplication, division, plotting, measuring distances, angles, measuring time, graphing, scaling, computations of angles, ratios, horse power, mechanical advantage, friction, speed, data taking, value judgment, analyzing data, and forming conclusions based on data taken.

BICYCLE PROVING GROUND

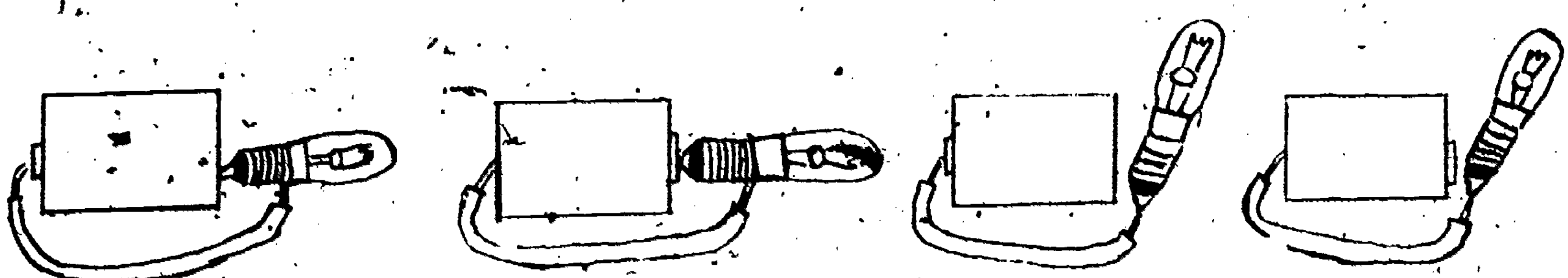
DP17-4



BASIC CIRCUITS*

Simple First Circuit - The first circuit can be built with only three elements: a battery, a bulb, and a wire or some other conducting material such as aluminum foil.

Four methods of lighting a bulb with just one battery and one wire are shown below. It is not necessary for USMES purposes to ask the children to spend time discovering more than one method. However, the class should collect and sketch all methods as they are discovered so that they will be able to use whichever way may be better adapted to one of their later design problems.

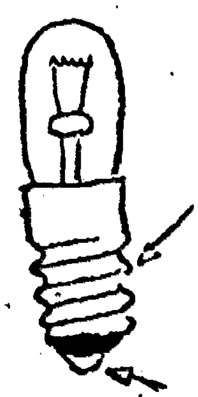


Keeping Records - It's important for children to draw accurate diagrams of their circuits not only to obtain a better understanding but also for the purpose of reassembly and retrial at a later date. The students should keep all diagrams and reports in a special folder constructed for the unit. One design for a folder is illustrated in the set of "How To" cards, "How To Make a Folder To Hold Your Data, Drawings and Reports." If a student's diagram is incorrect, then the teacher should assemble the circuit in accordance with the defective diagram. Usually the student will respond with a statement to the effect that, "That's not what I meant!" To which

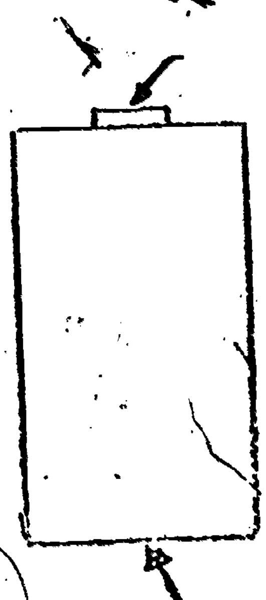
* Based on suggestions by Thacher Robinson.

the teacher may reply, "Then draw a diagram which shows clearly what you did mean."

The student's first important diagram shows clearly the connections made in a one bulb, one wire, one battery circuit.



Two connections to the bulb are necessary for it to light. One of these connections can be made anywhere on the threaded part of the base, and the other connection must be made to the silvery metal blob on the bottom of the bulb.



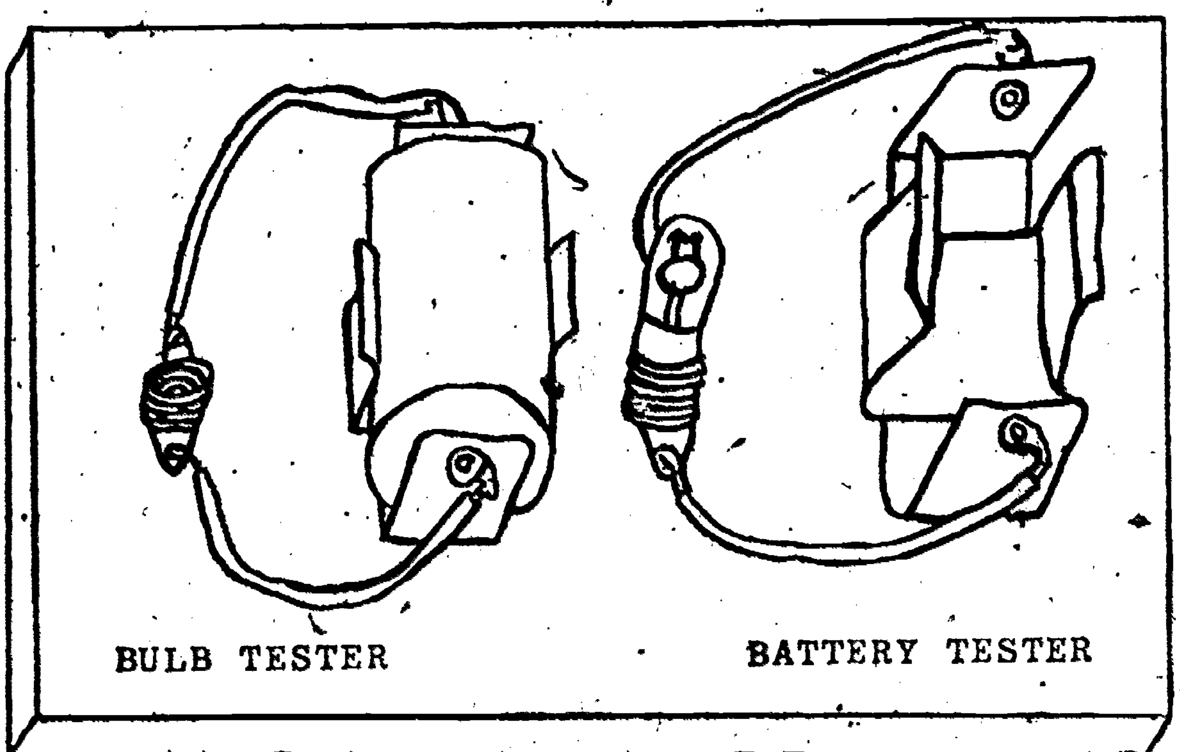
Two connections to the battery are necessary for it to light a bulb. One of these connections must be made to the button, and the other connection must be to the bottom of the battery.

It is important that children become clear about these points by trying out various possibilities, for example, that the bulb will <u>not</u> light if both connections are made to the solder tip of the bulb.

<u>Hardware Necessities</u> - Two-wire circuits are difficult to manage with two hands without using battery holders and/or bulb sockets equipped with Fahnestock clips. After their initial exploratory work, little is gained by having children struggle with bad connections resulting from the use of rubber bands or other makeshift connectors. Alligator clips can be used in place of Fahnestock clips. However, sometimes a wire lays between the teeth not making a good contact. This can be frustrating to children if it happens often. The children should be encouraged to make their own battery holders and bulb sockets in the Design Lab. A shortage of supplies can slow

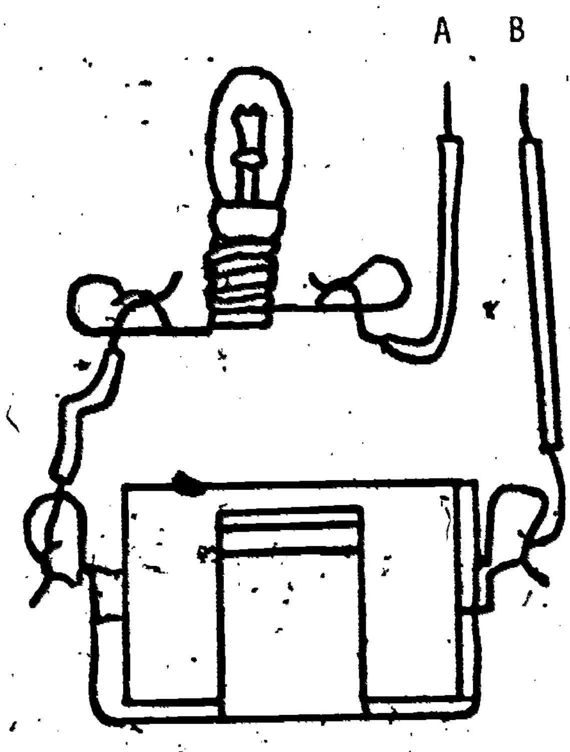
down the children's progress in designing an alarm, and in some cases makes a child discard a good idea. A set of "How To" cards includes possible designs for a bulb socket and a battery holder.

Circuits for Testing - It is convenient to have a permanent battery and bulb tester around. It can be mounted on a small (4" x 9") board as shown below. Solder all connections using flexible (stranded) wire to allow for deformation of battery holder upon battery insertion. Bad connections make a good battery or a good bulb look bad. Check soldered connections to make sure they are bright and shiney, not granular or cracked.



Children should be aware of the necessity of testing the tester, which is accomplished by temporarily putting the tester battery and the tester bulb together in each circuit. They can then use the testers to separate good batteries and bulbs from bad ones. The task is more interesting if there are only one or two good batteries and bulbs among a considerable number of defective ones, since this necessitates a relatively systematic trial of all possible battery and bulb pairs until one gets a light. This determines one definitely good battery and one definitely good bulb, which should be so labeled. This good battery can then be used to test all of the remaining bulbs; and the good bulb can be used to test all of the remaining batteries.

Conductors and Insulators. - One popular early type of burglar alarm is activated when a piece of insulating material is pulled from between two conductors in a circuit. The children can discover which materials are conductors and which are insulators by using a test circuit such as the one shown below. To test the test circuit it is only necessary to touch wires A and B together. If the light lights, then the test circuit is working properly. Buzzers can be used in place of lights in all test circuits.



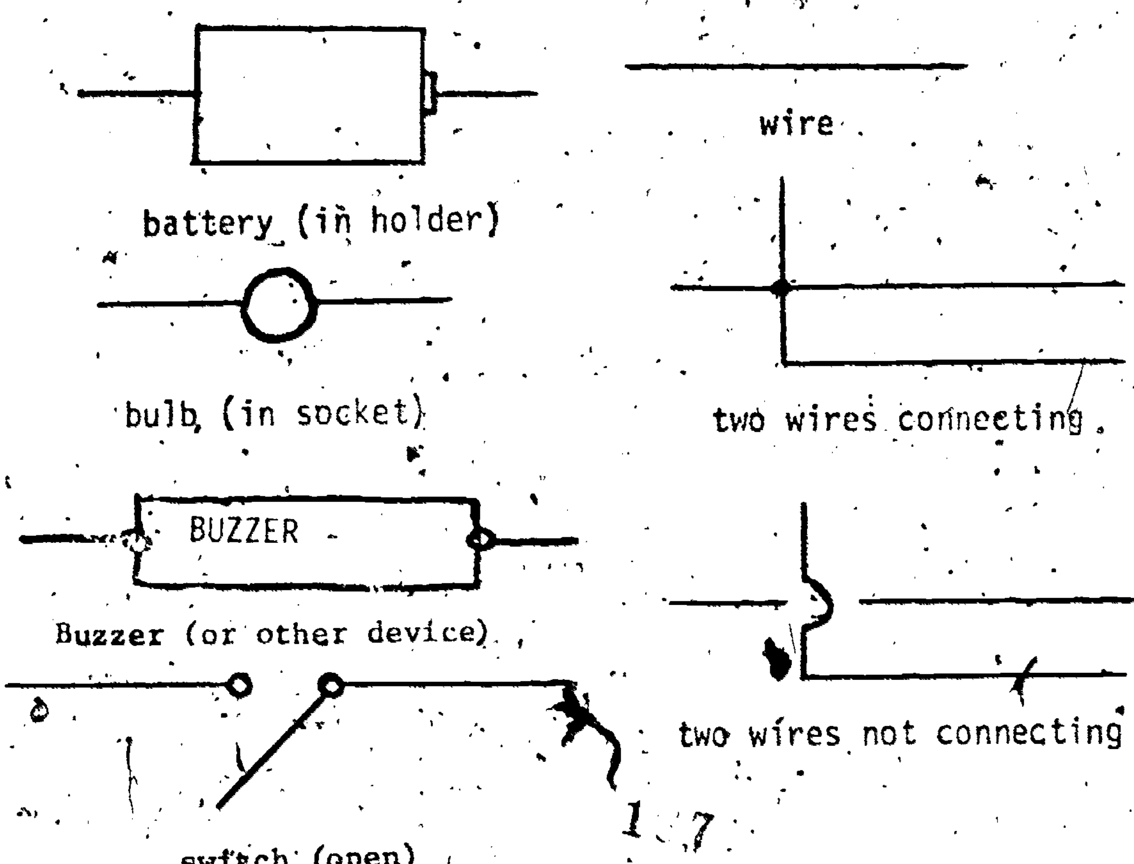
CONDUCTOR TEST CIRCUIT

The children can connect different materials between A and B. If the bulb lights, the material is a conductor with properties that permit an electric current to pass through it. If the bulb does not light, the material is an insulator or is too poor a conductor to permit enough electricity flow to light the bulb.

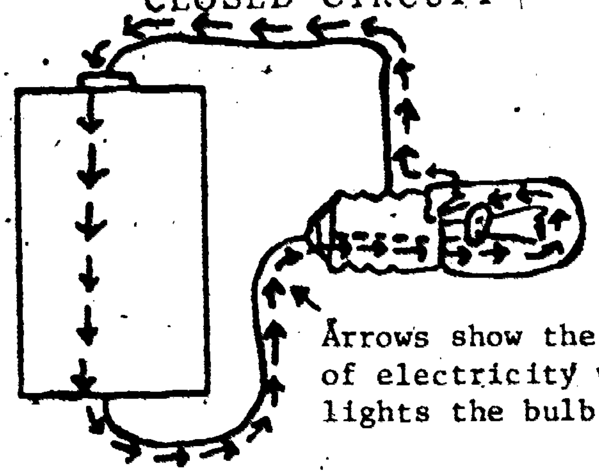
CONDUCTORS:	INSULATORS:
nail	finger
metal spoon	plastics generally
tin can	eraser
unpainted metal in general	water
screw driver	wood or linoleum floor
	chalk

Air is an insulator. Air like most insulators will conduct electricity generated at very high voltages. This shows up as sparks or lightening.

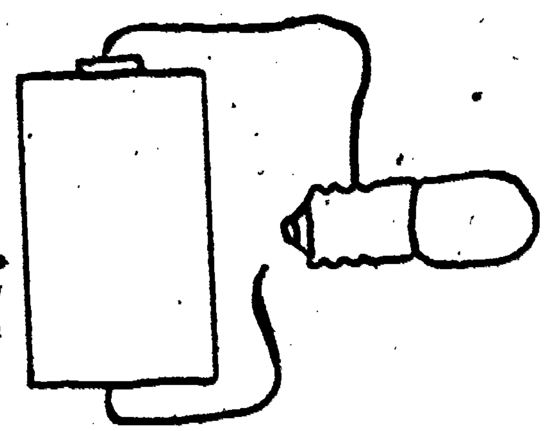
Drawing Schematics - In order to save time in sketching their circuits, the children should learn to use symbols for the different circuit elements. Some symbols easily understood by young children are shown below. They should try drawing a schematic of a circuit and then assemble it by following their drawing. The following forms are simple and sufficiently explicit.



Closed and Open Circuits - The terms "closed" and "open" are often confused but are essential for easy verbalization of circuit conditions. The following sketches illustrate these two conditions.

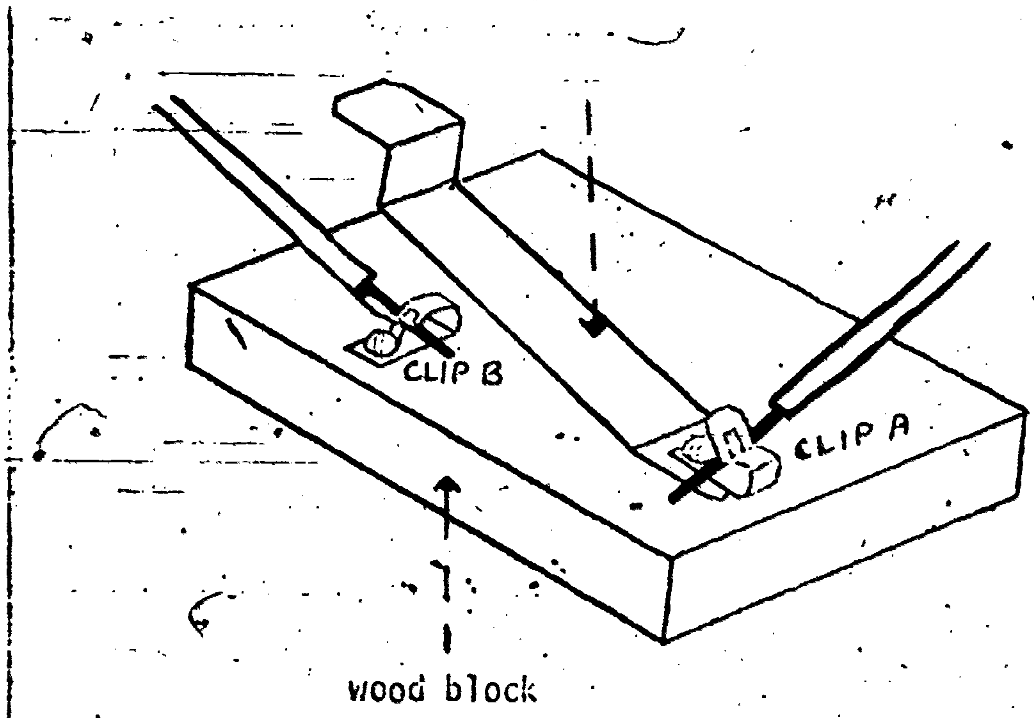


Electricity goes all the way around closed circuits.

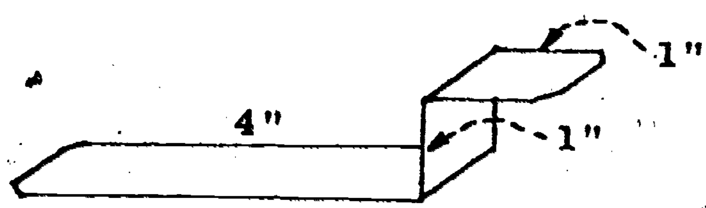


Electricity can NOT flow across OPEN circuits.

Switches - The children will find that switches are an important part of their improved alarm circuits. The sketch on the next page shows a switch which can be easily constructed by the children from metal strips cut from coffee cans.

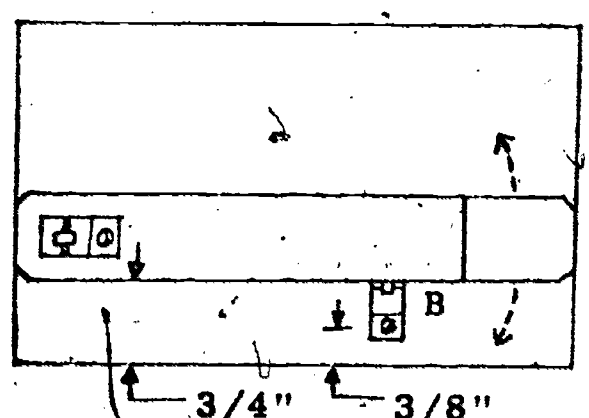


(1) Cut and bend a piece of 3/4" x 6" strip from a tin can as follows:



(2) Attach the strip with Fahnestock clip A and a No. 6x 5/8" roundhead wood screw to a 3" x 5" x 3/4" pine board. It should be 3/4" from the edge, as shown in the picture below.

The screw holding the strip should be just loose enough so that it can be moved stiffly in the directions of the arrows:

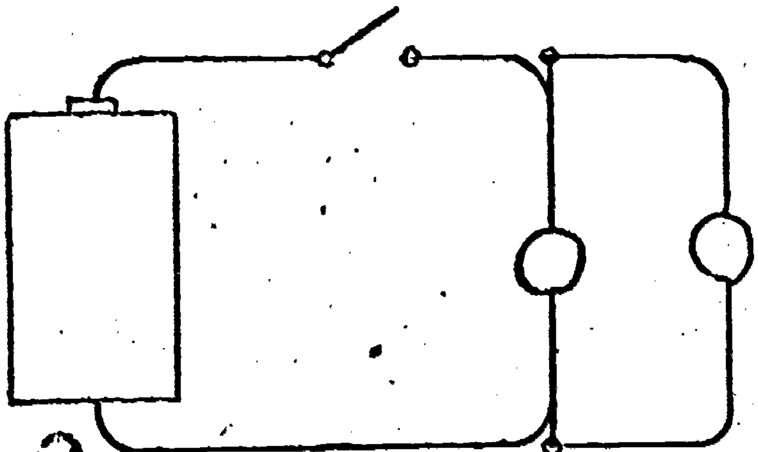


(3) Attach Fahnestock clip B to the pine board with another screw, as shown.

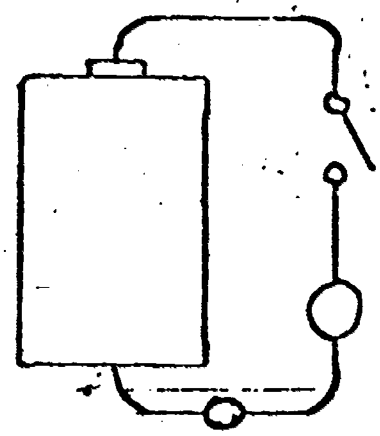
(4) The strip should be bent up so that it remains about ¼" above clip B.

The distances given here are important if you want to be able to add certain parts to your switch later for more complicated experiments.

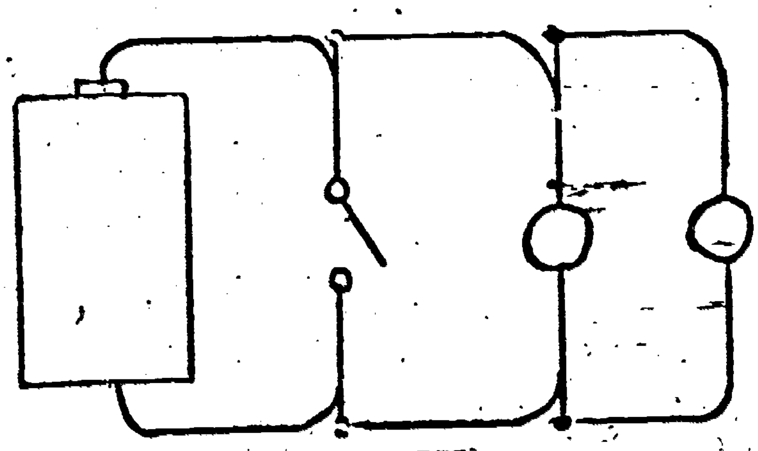
The children should try to use one switch to turn two bulbs on and off at the same time. Several possible circuits are shown below.



CIRCUIT I



CIRCUIT II



CIRCUIT III

The switches in circuits I and II will turn on both bulbs when they are closed. The bulbs in circuit I will be brighter than the bulbs in circuit II since they are connected in parallel instead of in series. Children can refer to the set of "How To" cards on connecting several things to one battery if they need the information for their alarms.

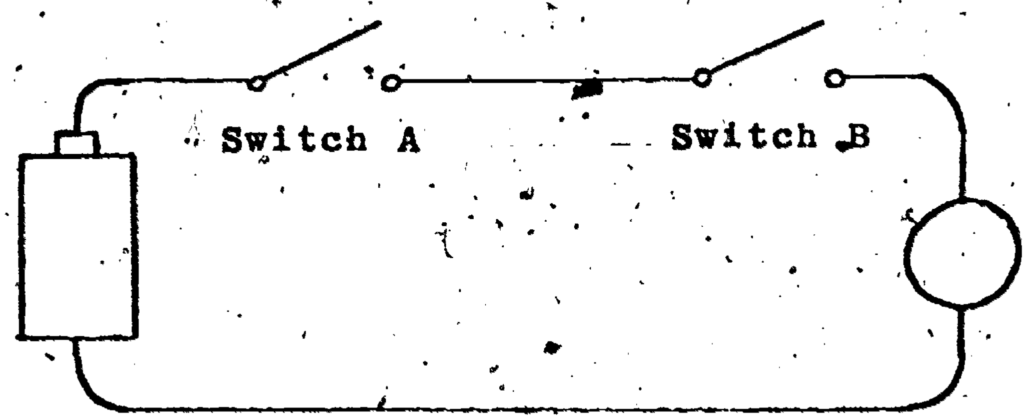
If the children connect the switch in circuit III, they will make a "short-circuit" which will turn off the light and cause the wires between the switch and the battery to become hot when the switch is closed. Students may be referred to the trouble-shooting sequence, "Why Do Wires Get Hot?"

This is not a safe circuit because of the danger of a fire being started by the hot wires when the switch is closed. It will also discharge the battery very rapidly when the light is not lit. Many children discover the short circuit when they look for a way to make a light go on when a burglar breaks the foil in a window, thus opening the circuit. However, this type of alarm is quickly discarded when the children find out that the batteries do not last very long.

Several Switches in a Circuit: The "And" and "Or" Circuits, Possibility Trees, Puzzle Circuits*

Many children are stalled in the design of a relay-type alarm until they realize the potential of using two switches in a circuit (or a double switch in two circuits). The introduction of "And" and "Or" circuits and puzzle circuits may lead to experimentation along these lines.

The following circuit is called an "And" circuit because both switch A and switch B must be closed for the bulb to light.

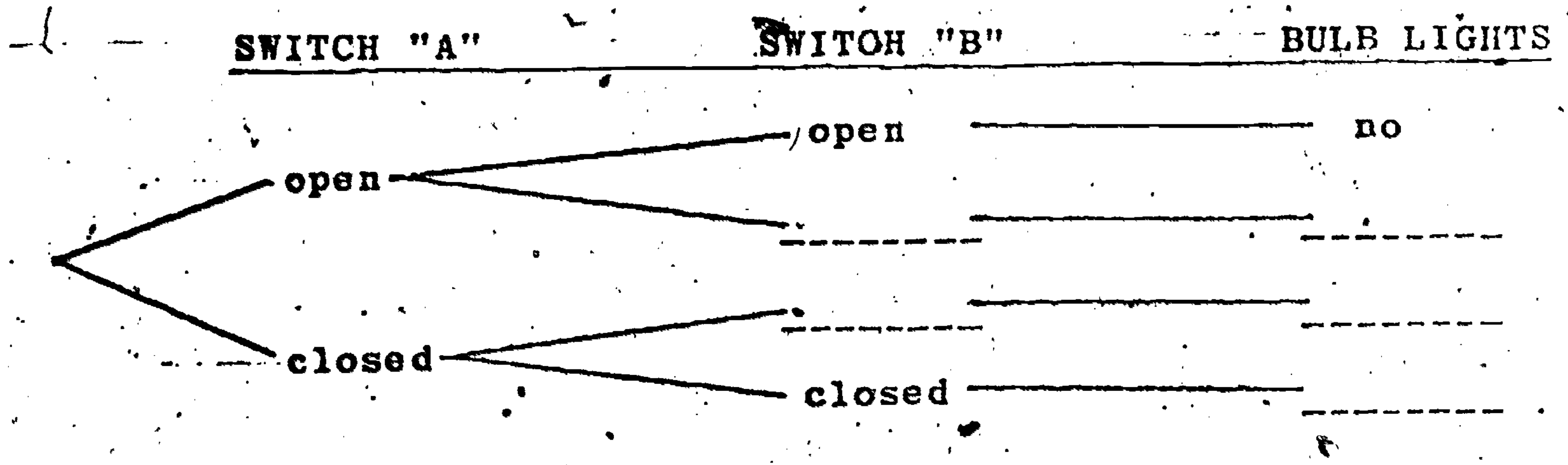


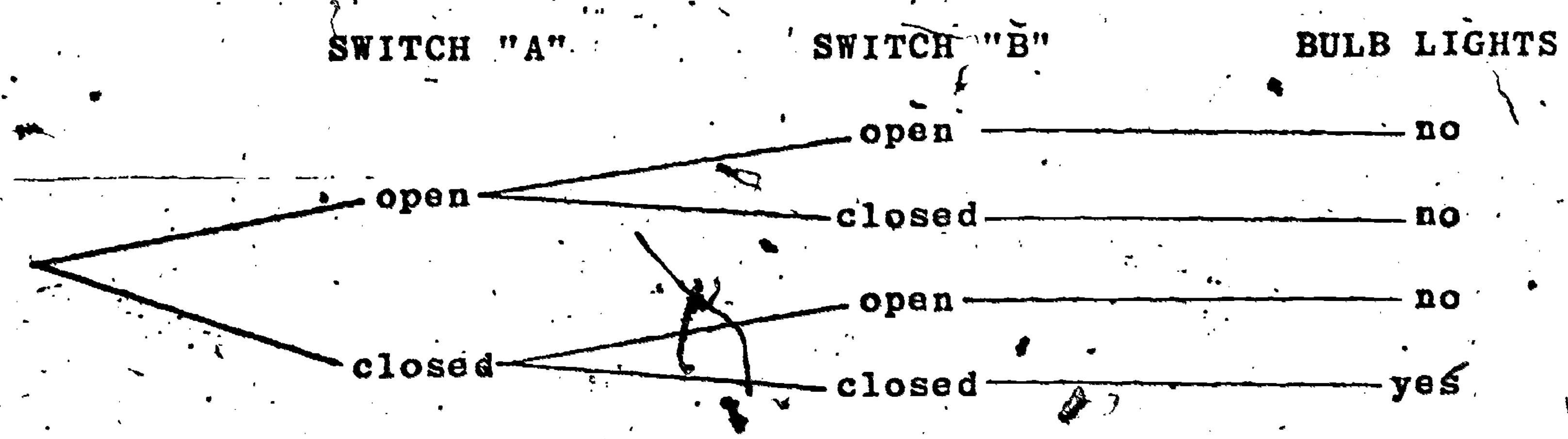
"And" Circuit

The children can list different possible circuits in a Possibility Tree.

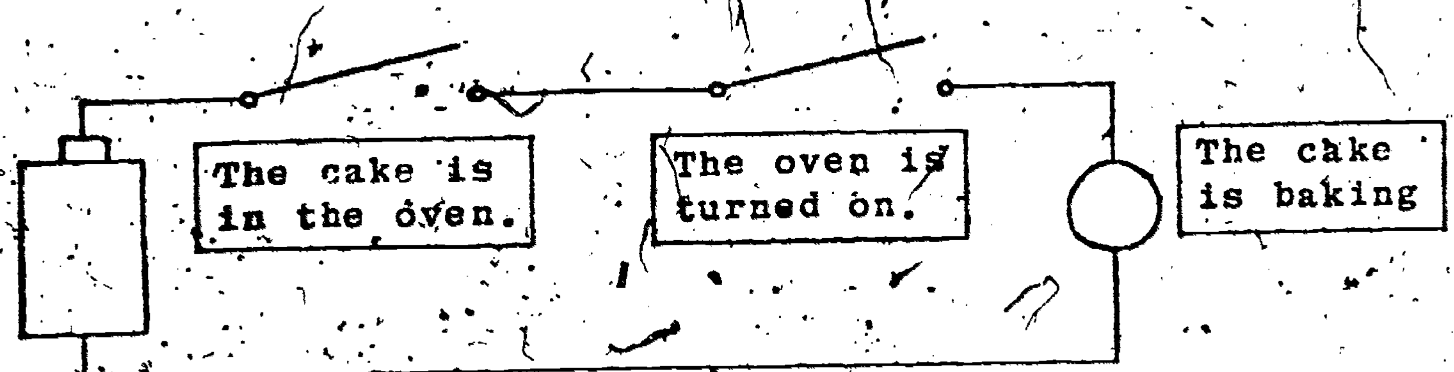
*Note: The examples in the next 10 pages represent supplementary activities in which elementary circuit design can be explored. In themselves, they do not represent a practical challenge. However, especially for primary grade children they may be interesting parallel activities which will feed skills and ideas into the children's continuing activities in USMES units.

A tree for them to fill out is shown below, followed by a completed tree.





The children may also like to make up sentences which will fit the "And" circuit. They could put removable labels on their diagrams in the proper places. Shown below is one labelled diagram and other examples of sentences for the "And" circuit.



	FOR SWITCH "A"	FOR SWITCH "B"	FOR THE BULB
1.	Johnny is on the train.	The train is moving.	Johnny is moving
2.	It is raining	I am outdoors	I am getting wet
3.	Bill is running down the hall past Susan's room.	Susan opens her door suddenly.	Bill runs into the door.
4.	Mary is very interested in her project.	The teacher gives Mary useful help.	Mary's project will succeed.
5.	It is cold outside.	The window is open.	There is a draft.

The following circuit is called an "Or" circuit because the bulb will light if either switch A is closed <u>or</u> swtich B is closed.

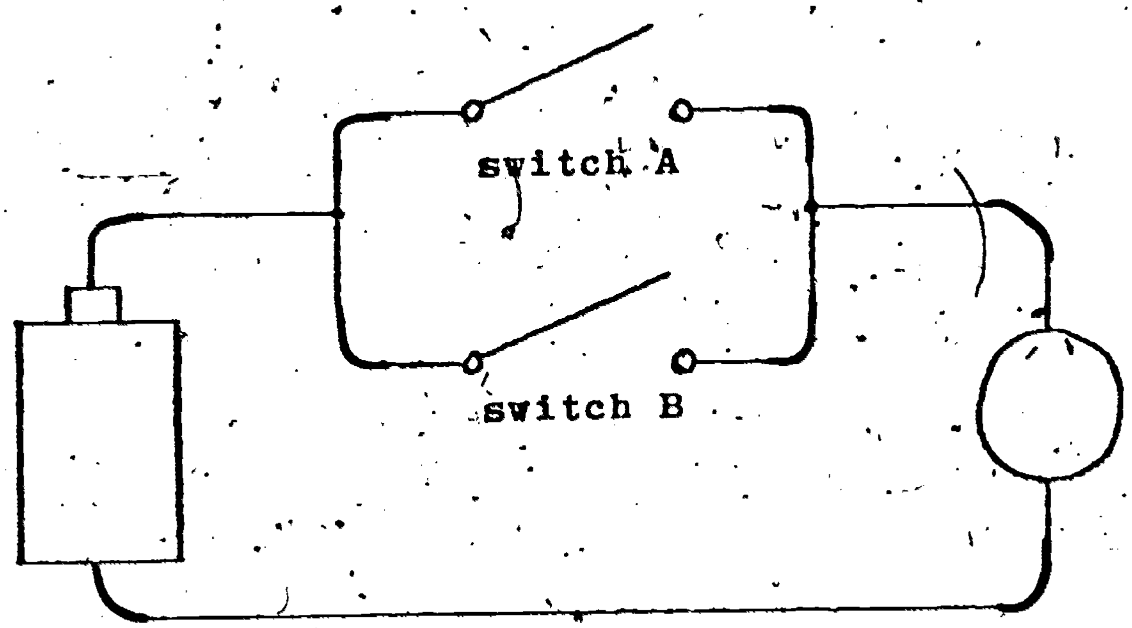
Astute children may comment that the bulb will also light in this circuit if <u>both</u> switch A <u>and</u> switch B are closed. If this happens, it may be worthwhile to point out that English uses "or" in <u>two</u> senses:

<u>Inclusive</u>: "A <u>or</u> B <u>or both</u>," sometimes written "A and/or B.

<u>Exclusive</u>: "A <u>or</u> B <u>but not both</u>."

Often the context determines which meaning is intended.

In most situations, the <u>inclusive</u> "or" is more natural and convenient. For this reason logicians and mathematicians have taken the <u>inclusive</u> as the primary meaning of "or". The simplicity of our "or" circuit is also one of the reasons for this choice.



"Or" Circuit

The children can fill in a Tree of Possibility for the "Or" circuit. The completed chart is shown below.

SWITCH "A"	SWITCH "B"	BULB LIGHTS
open	open	no
open	closed	yes
closed	open	yes
closed	closed	yes

The following are some sentences which make sense in an "Or" circuit. The children may invent many more.

	FOR SWITCH "A"	FOR SWITCH "B"	FOR THE BULB
1.	It is Saturday.	It is Christmas.	There is no school today.
2.	John was up late watching TV.	John isn't interested in the classwork.	John isn't learning much today.
3.	Mary has a sprained ankle.	Mary is absent today.	Mary won't run in the race tod
4.	It is raining.	The snow is melting.	The grass is we
5.	Bill has no money.	Bill is an ecology freak.	Bill won't buy a car.

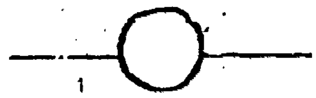
In order to complete puzzle circuits, the children must learn symbols which show whether a switch is definitely closed or definitely open. The regular schematic symbol for a switch does not tell us the condition of a switch and is therefore used as the incomplete symbol in a puzzle circuit. Therefore, in puzzle circuits we use the SYMBOLS:

 **switch, condition NOT DEFINITE**

 **switch, DEFINITELY OPEN**

 **switch, DEFINITELY CLOSED**

Similarly, the schematic for a bulb in a socket is:

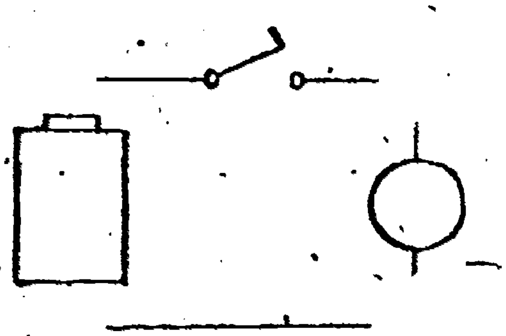


However, this picture doesn't tell whether the bulb is lighted or not. If we want to say definitely what the condition of the bulb is, we can use the following symbolic code:

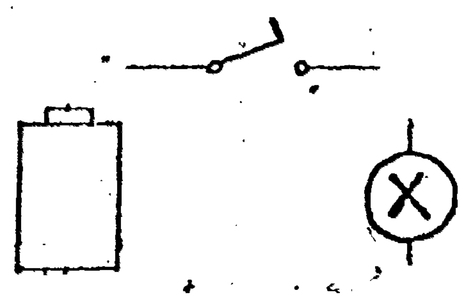
bulb LIGHTED

bulb NOT lighted

Several examples of puzzle circuits are shown below. The first picture shows the condition of the bulb is not definite. However, the switch is shown as definitely open. Therefore, the picture can be completed only as shown by making the bulb as definitely not lighted.

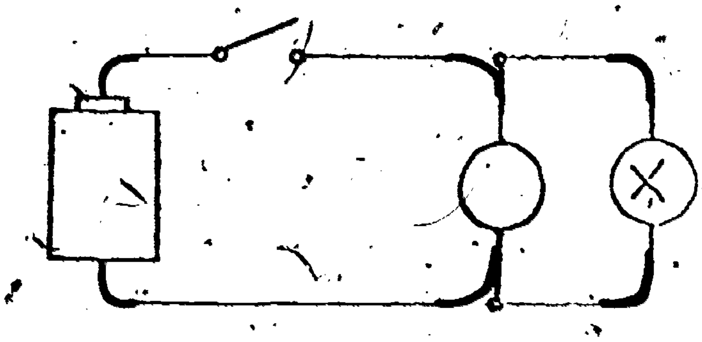


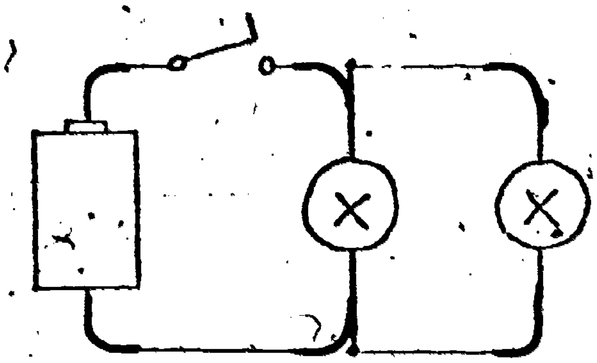
Puzzle Circuit



Complete Diagram

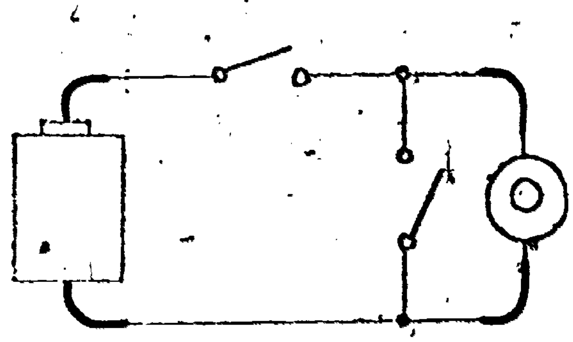
In the first picture below, since one of the lights is shown as not lit, the switch must be open. Then as a consequence, the other light must also be unlit. This circuit and the ones following show parallel connections. See Pages for a discussion of series and parallel connections.





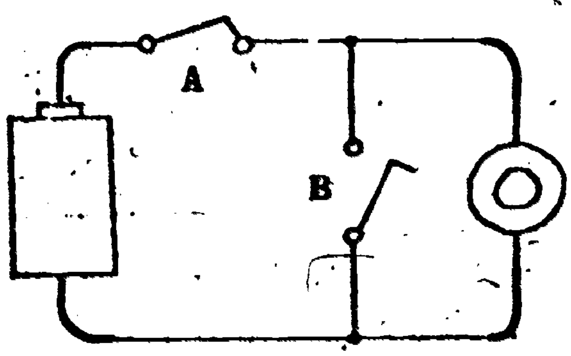
The children may enjoy inventing puzzle circuits, but they should be encouraged to continue with alarm designs as soon as they desire. The use of several switches in an alarm may be seen only as a result of further investigation of simple circuits. Some children see quickly the use of an electromagnet in their alarm circuit and a way to make a connection in a second circuit by dropping ametal piece. However, they may not immediately see the electromagnet as a type of switch. The USMES approach of "If it works, it's right" should be followed without undue emphasis on terminology . Shown below are three other puzzle circuits with completed diagrams and explanations.

A.



Puzzle Circuit

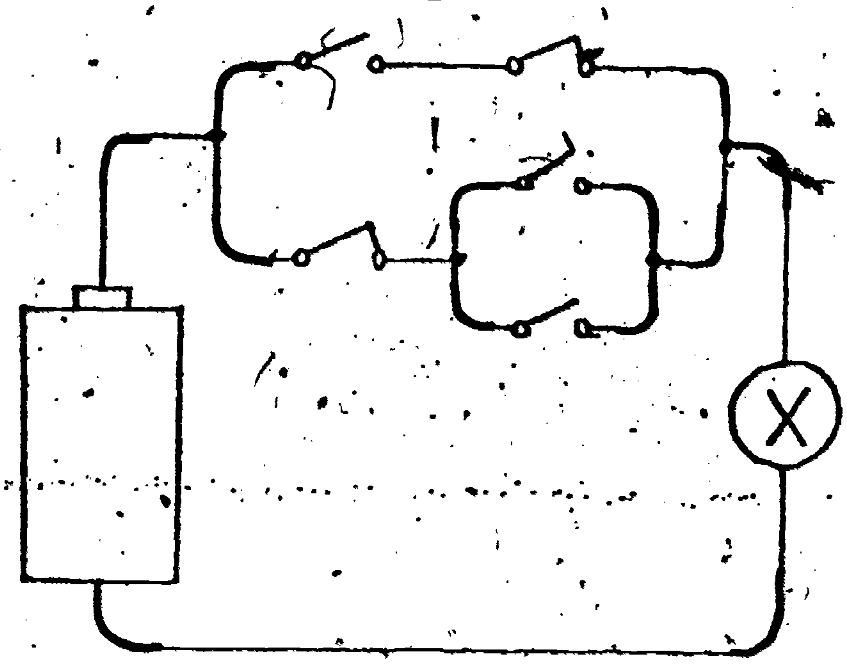
A.



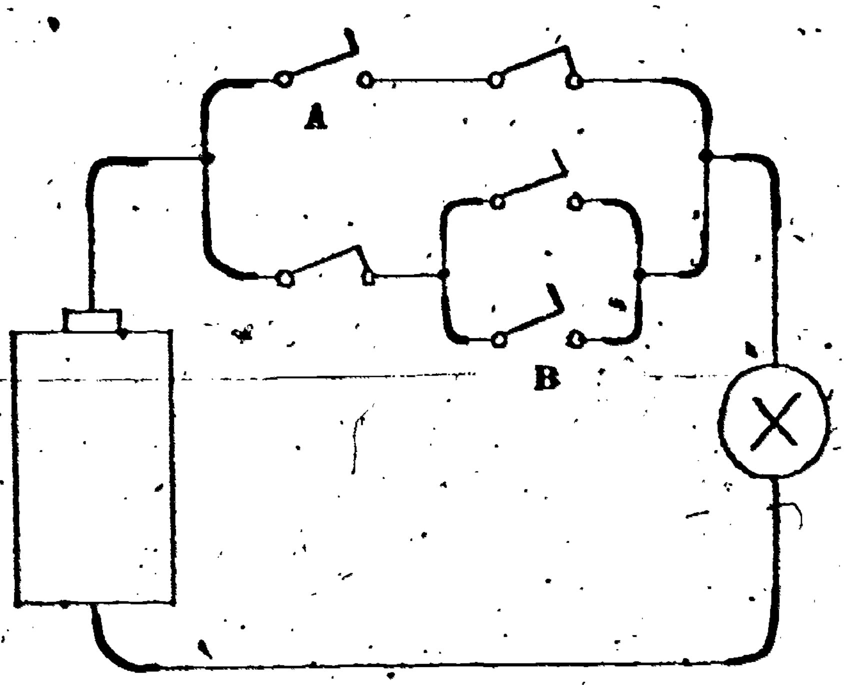
Completed Diagram

The light is shown as lit, therefore switch "A" must be closed. Switch "B" must be open, since otherwise it would create a short circuit across the bulb, which would prevent most of the current from going through the bulb.

B.



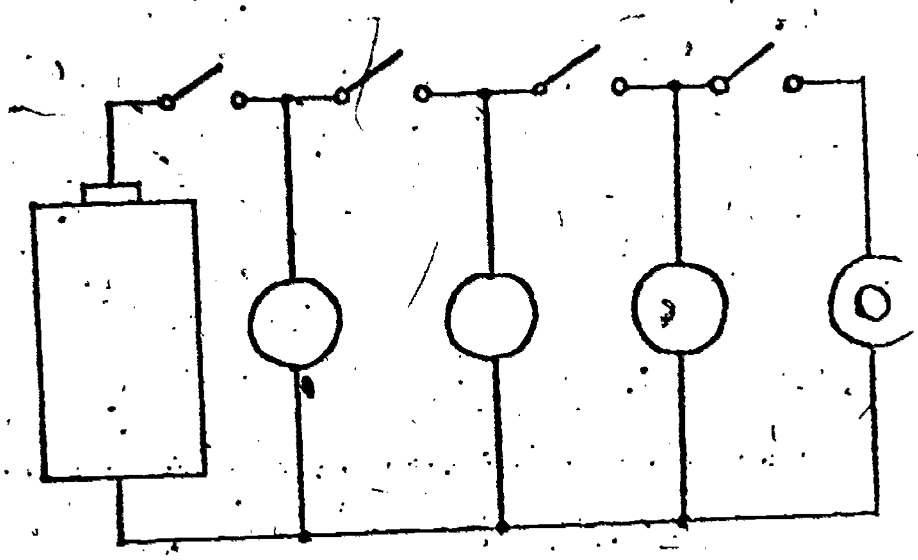
Puzzle Circuit



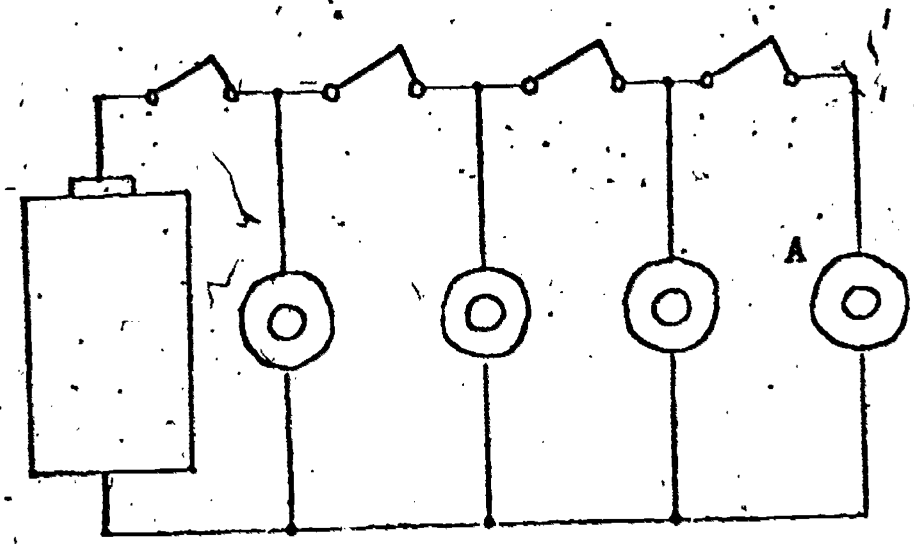
If either switch "A" or switch "B" were closed, there would a closed circuit which would light the bulb. Therefore, since the bulb is specified as unlit, neither "A" nor "B" can be closed.

Completed Diagram

C.



Puzzle Circuit

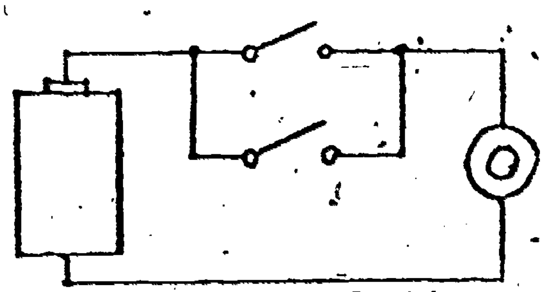


Since bulb "A" is specified as lit, all of the switches must be closed. But then all of the bulbs will light

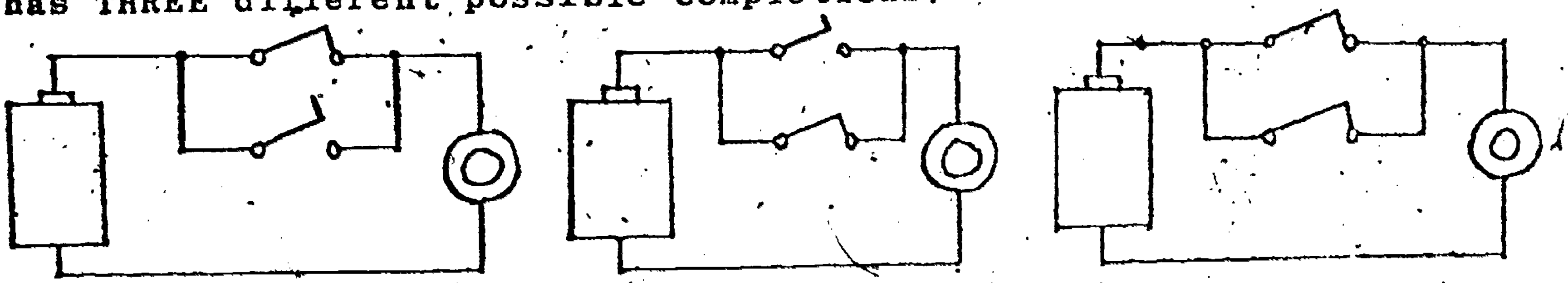
Completed Diagram

Some puzzle circuits have more than one possible completion.

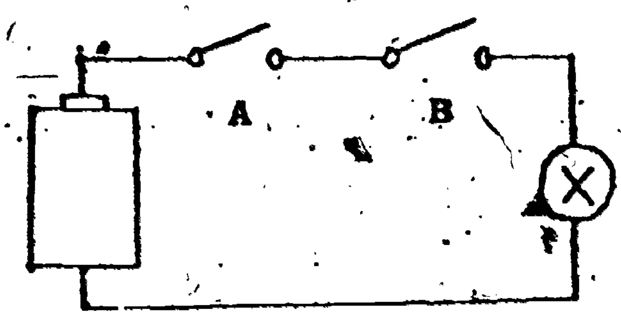
The puzzle circuit



has THREE different possible completions:



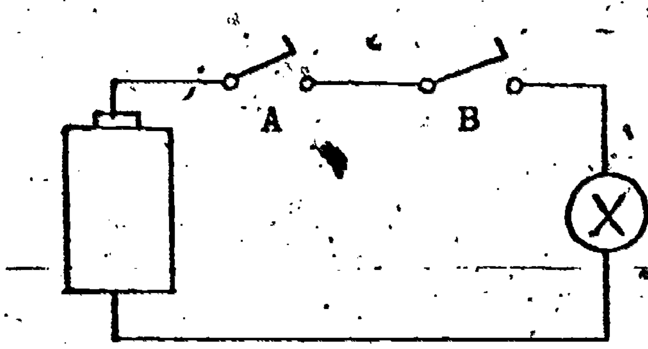
The following puzzle circuit also has three possible completions

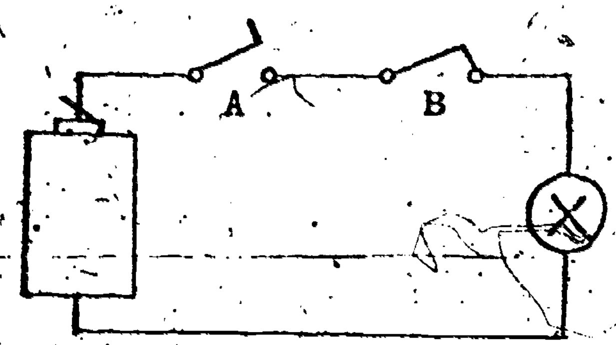


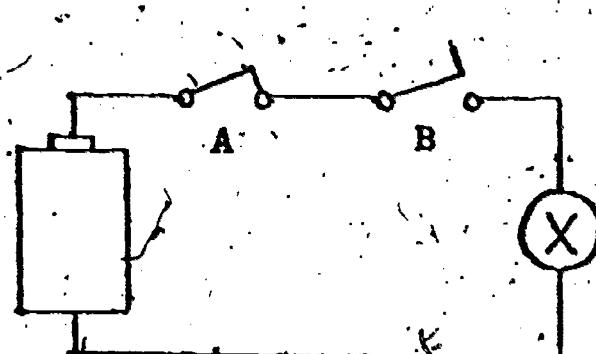
It may help to make a tree of possibilities for the switch conditions and resulting bulb conditions. The completed tree is shown below:

Switch A	Switch B	Bulb Lights
open	open	no
open	closed	no
closed	open	no
closed	closed	yes

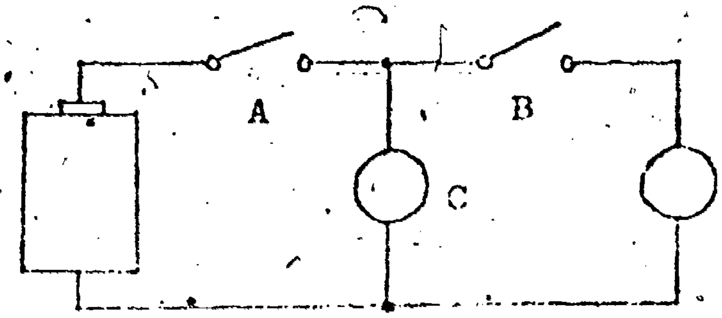
The top three branches correspond to all the possible completions of the given puzzle circuit which specifies the bulb not lit. The three completed diagrams are shown below.





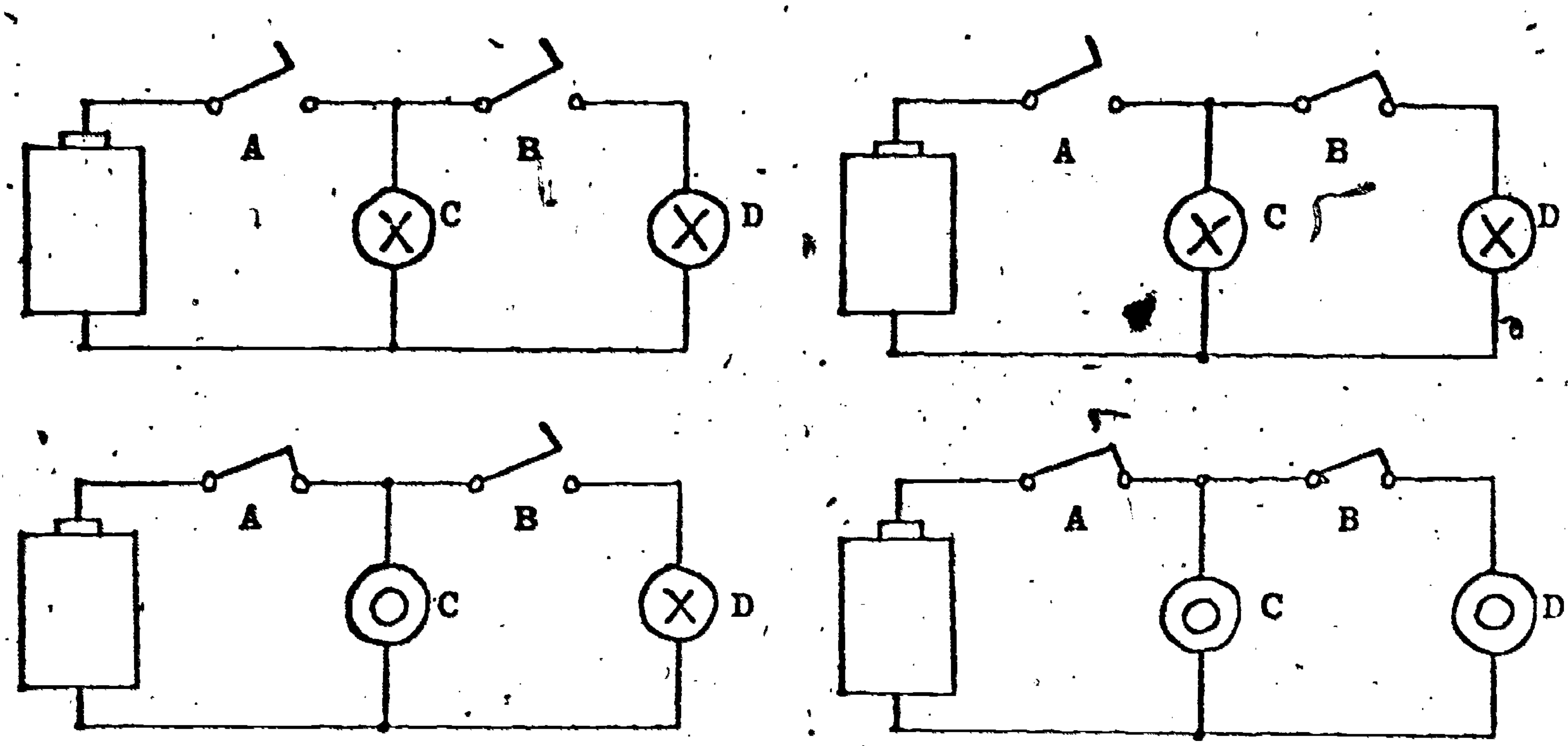


A puzzle circuit with four elements is shown below.



The tree of possibilities shows that all four branches correspond to possible completions of the circuit.

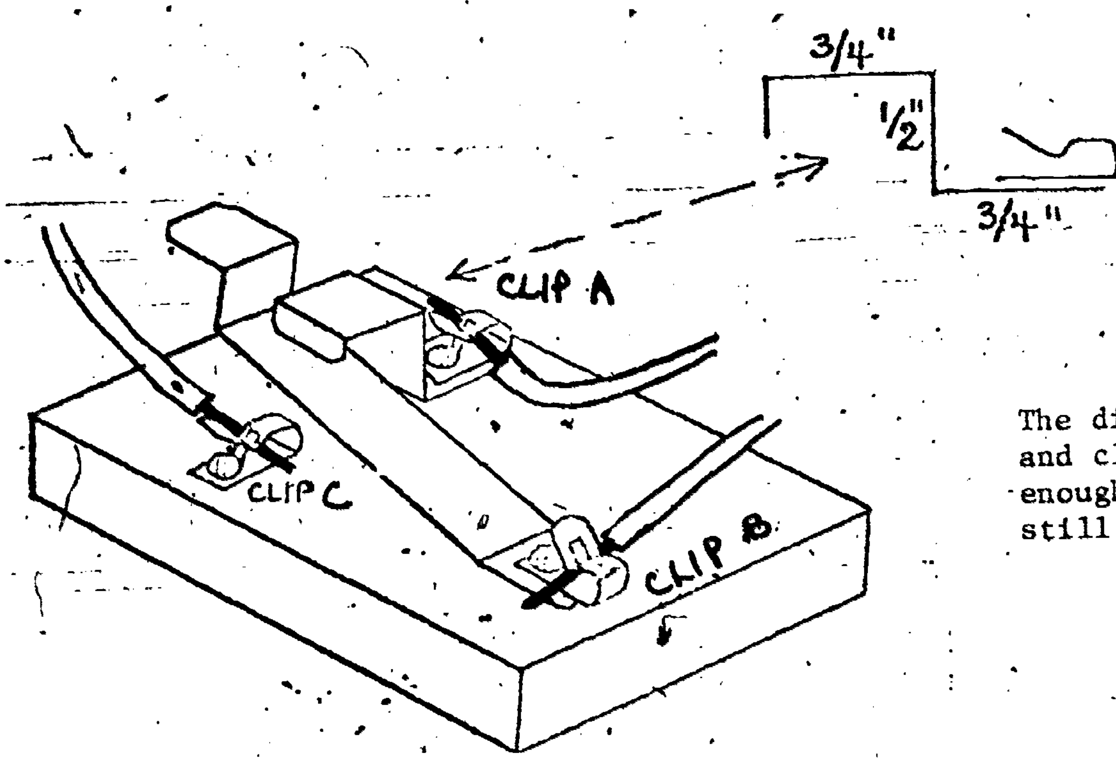
Switch A	Switch B	C Lights	D Lights
open	open	no	no
	closed	no	no
closed	open	yes	no
	closed	yes	yes



Completed Diagrams

Switches that Open the Circuit When You Push the Handle

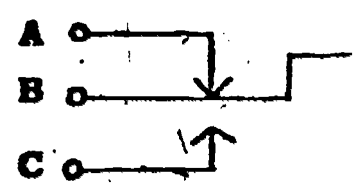
The students' first switch can be converted into a switch with more interesting possibilities by adding an upper contact like this:



Cut from a 3/4" tin strip

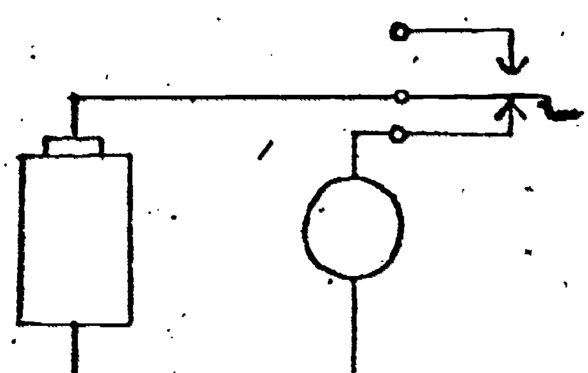
The distance between clip A and clip B should be great enough so that the strip can still be slipped under clip C

The schematic representation of this switch is:



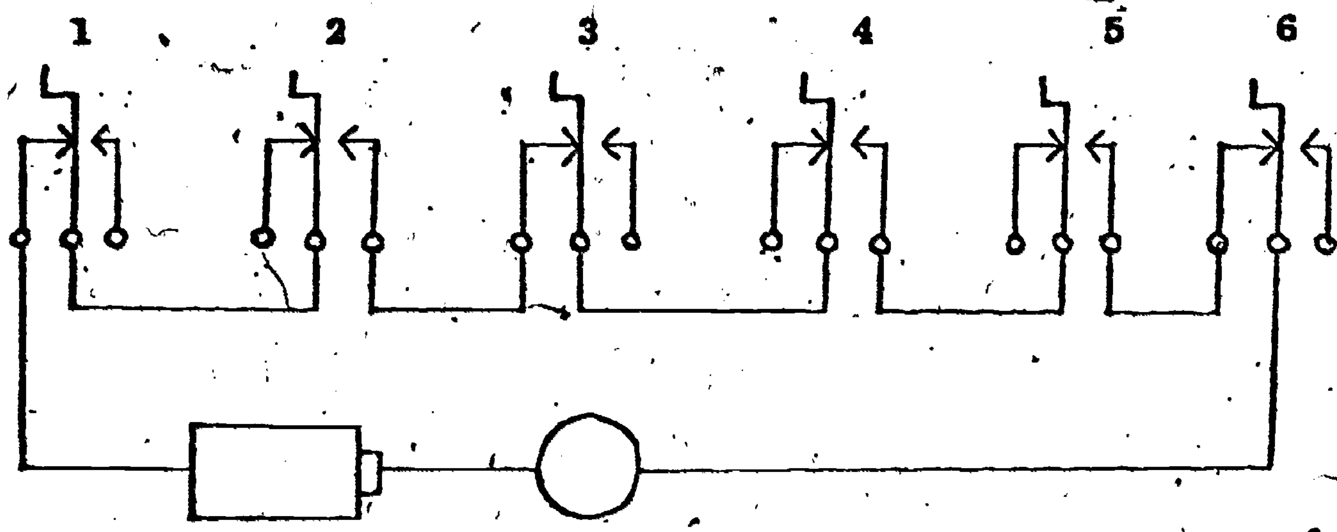
It is customary for such schematics to show the switch in its normal (up) position.

The switch can be connected in the following circuit so that a light is lit at all times except when the handle is pushed down.



Circuit Normally Closed

The children may be interested in looking for a way to connect more than three of these switches in a circuit so that a buzzer of light will operate only when some of them are pushed and others are not pushed. This is accomplished by connecting the switches so that some close only when they are pushed and others are closed only when they are not pushed. See following diagram.



Bulb will light only if switches 2, 4, and 5 are pushed down (i.e. to the right) and if switches 1, 3 and 6 are not pushed.

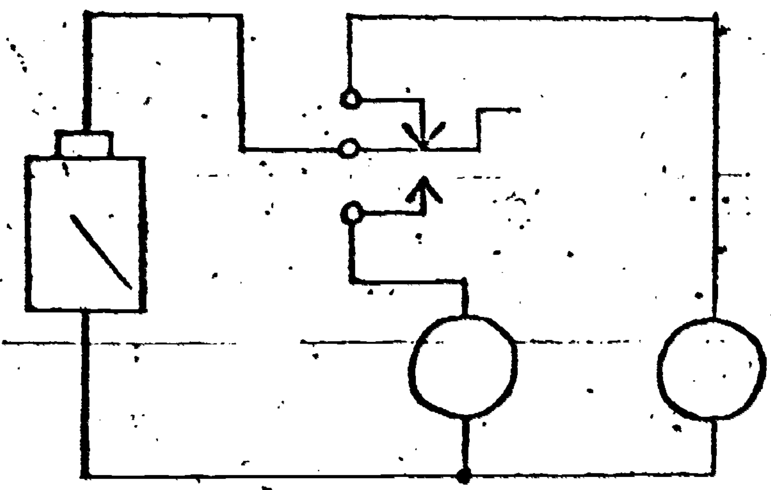
Each branch in this tree does correspond to one way one can wire the combination lock, and the one combination which will close the circuit so wired. Each additional switch in the lock doubles the number of branches in the tree of possibilities; so that we obtain a table like this:

No. of switches in lock	No. of possible combinations
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

In general, n switches give 2^n combinations. (If you include the trivial ones in which the circuit is closed when you push no switches, or all switches.)

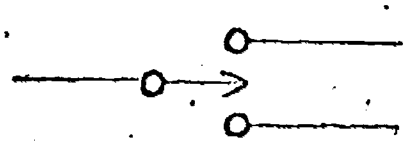
Switches that Alternately Connect Several Different Gadgets

The switch shown on page D1-23can be connected so that the light switches back and forth between two bulbs as the handle is pushed up and down. See diagram below.

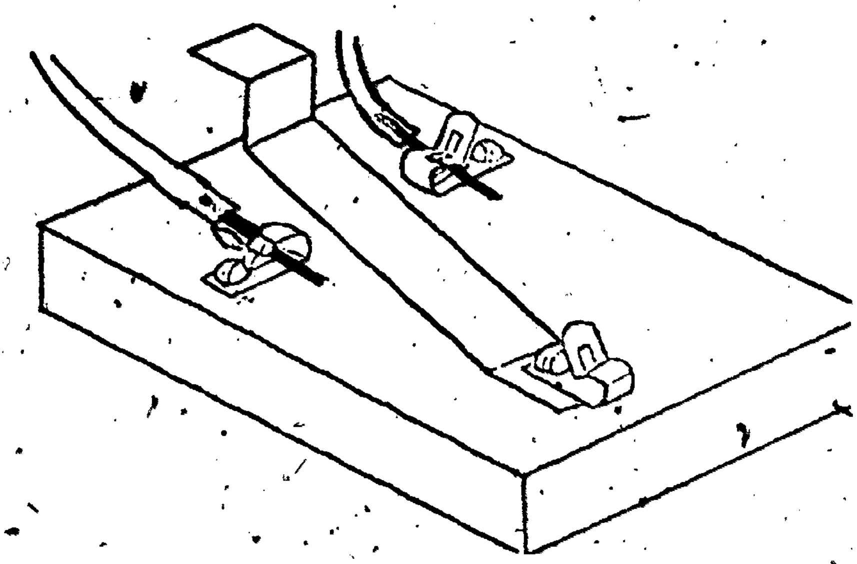


Circuit For Alternate Operation.

Non-Momentary Alternating Switches - Switches that will stay equally easily in each of the two positions can be constructed according to the following diagram.

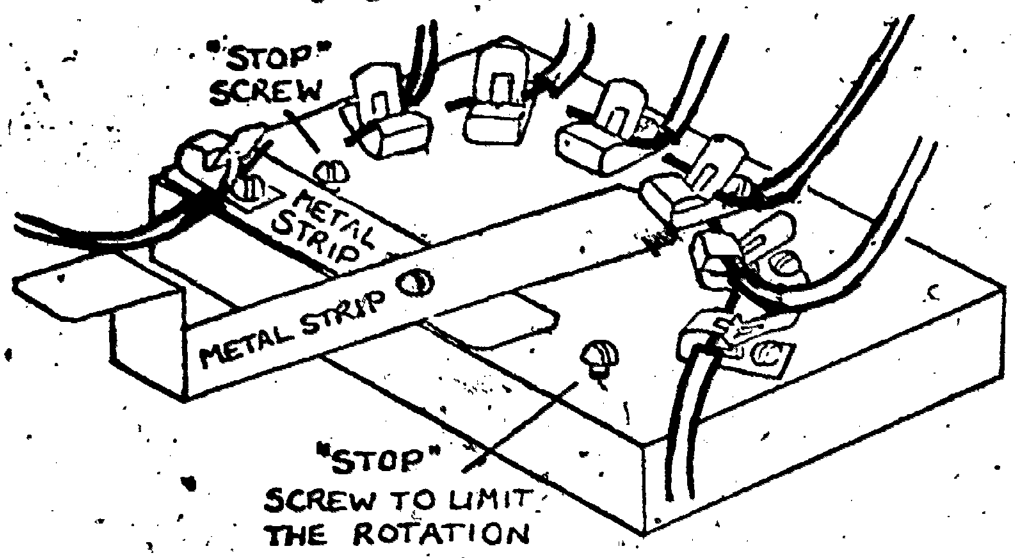


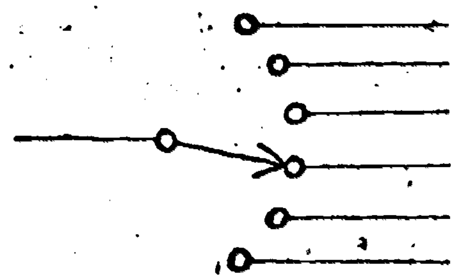
Schematic symbol



Double Pole Switch

A rotary switch is a variation of the above which can control more than two gadgets as follows:

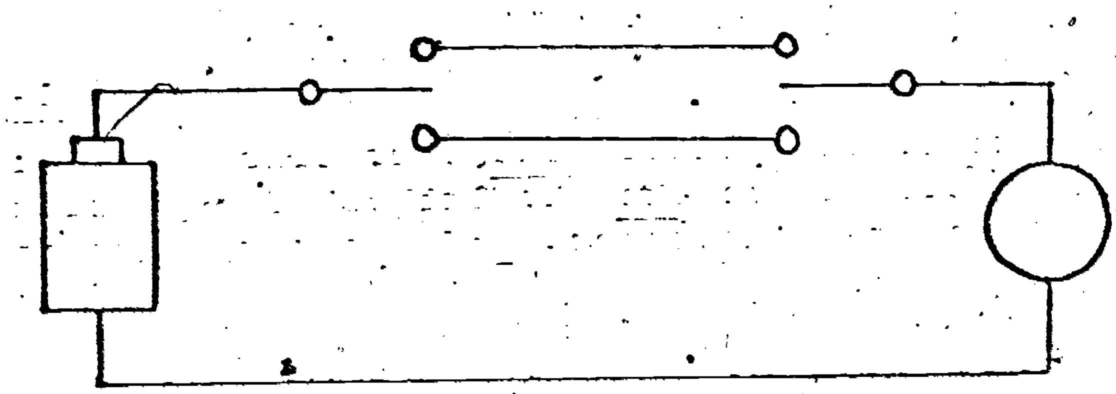




Schematic symbol

Rotary Switch

Students might be interested in investigating how the wiring is accomplished when a light can be turned both on and off at two different places, for example, at the top and bottom of stairs. Two switches can be used with two alternate current paths between the switches. The schematic diagram is shown below. (Each switch is sprung so that it can only rest in one of the contact positions - never in between.)



Switches Connected for Independent Control

Several varieties of switching mechanisms have been improvised for use in alarm circuits. Some ideas which have proved successful are:

1. Metal strap which pushes against another metal strap when door opens, and springs away again when door closes.

2. Hinged and spring-mounted floor board which drops a short distance to close a contact when stepped on, and springs up again afterward.

3. String tied to door, working through pulleys against a weight, to open and close a switch.

4. Permanent magnet attached to door or window which, when pulled away, allows a switch to spring close.

5. One very popular solution consists of two springy pieces of metal mounted so that they snap together when a cardboard or thin wood separator is pulled out from between them by a string connected to door or window. This has the advantage that a long or sharp pull on the string won't damage the switch.

6. One can make an electromagnet with many turns of wire (to reduce current consumption) which, when de-energized, by its circuit being opened by breaking a window, etc., allows a springy piece of metal to snap back to close a second circuit which activates the alarm.

A set of "How To" cards on "How to Use Electromagnets to Turn Things in Electric Circuits On and Off" develops the last idea. Notes on these cards follow.

Using Electromagnets as Switches

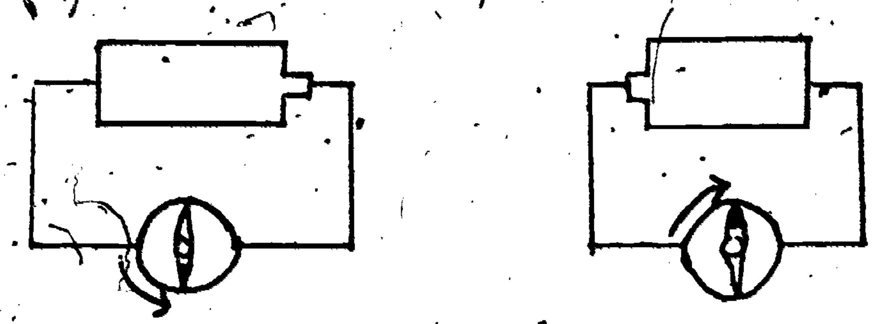
In their investigations of burglar alarms, especially broken window alarms, the children will be looking for a way to activate a buzzer or bell when a circuit is opened by having a foil connector broken. They may see how it can be done with a momentary relay-type switch (See Card 3 in "How to Turn Things in Electric Circuits On and Off"). The problem then is to find a way to operate the switch automatically.

The use of magnets may be viewed as one way to have two pieces of metal (iron or steel) come together automatically. Different groups may want to investigate bar magnets and electromagnets. The following paragraphs suggest the experiments that can be carried out by the students.

Bar Magnets - By bringing three bar magnets together in different ways, the students can establish that like poles repel and unlike poles attract. They first mark one end of one magnet with an X. The ends of the other two magnets which are repelled are marked with O's. They then try those ends together. In this way they find that like ends repel. After labeling the unmarked ends of the three magnets, they can check their assumption. They will also establish the fact that all magnets have two unlike poles.

Magnetic Compasses - By using a bar magnet with a compass needle, they can discover that a compass needle is a bar magnet. The compass needle, because of its mobility can then be used to observe the magnetic field around a single current-carrying wire.

The compass is placed directly over an unconnected wire so that the needle points along the wire. When the ends of the wire are connected to a battery, the compass needle will turn about 90° from the original direction. When the connections to the battery are reversed, the needle will turn 90° in the opposite direction as shown in the following diagram.

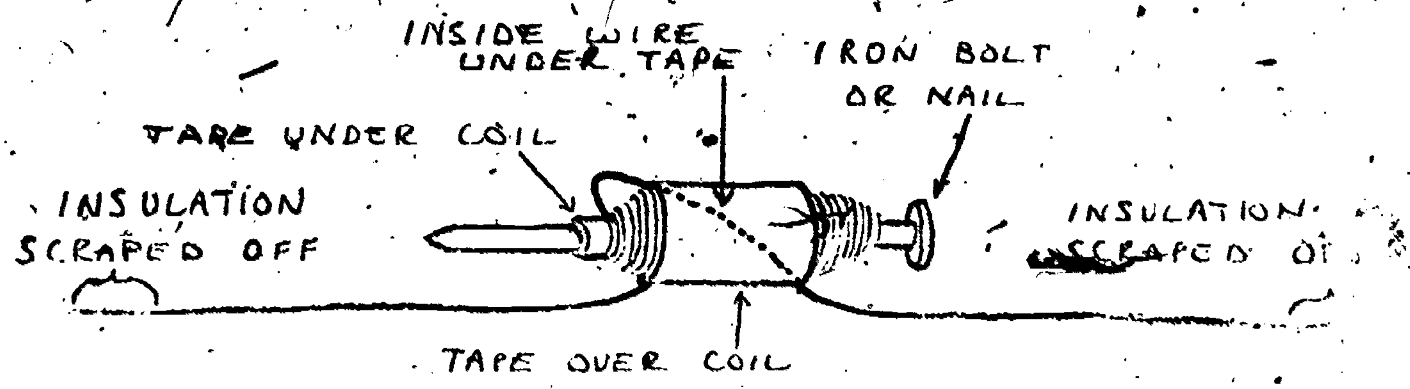


When asked to find how a bar magnet can produce the same motion, they can discover that they must put a bar magnet perpendicular to the direction of the wire and alternate ends pointing to the needle.

Electromagnets - The kids may be asked why a current-carrying wire will not attract iron pieces while a bar magnet will. They may suggest that (the magnetic pull) is too weak. If the wire is wound in a coil, the current will pass near one place many times and build up the magnetic force. The children can wind ten feet of wire around a two inch length of 1/2" diameter plastic tube. To test the strength, they will have to push the tip of a nail (1/2" long) in the end of the tube; the magnetic field is strongest at the center of the coil.

An iron core will multiply the force about 500 times, making the force at the ends large enough to attract a nail placed a short distance away. The children can discover this by placing several different objects such as pencils, chalk, wood dowels, and iron nails or bolts inside the tube. Only iron objects will affect the strength.

Wire can be wound directly on an iron nail or bolt if it is covered with masking tape first. The following steps can be followed, but the children will feel more free to investigate different magnets if allowed to develop their own construction methods. They will better recognize and remember the principles involved if they proceed by trial and error to test their own ideas.



1. Wrap masking tape around the bolt.

2. Measure off 20 feet of number 26 magnet wire. Wrap it around something to keep it from getting messed up.

3. Wind about a foot of the wire at one end of the bolt. Hold it in place with masking tape.

4. Now wind the rest of the wire as smoothly as possible on the bolt in two inch long layers.

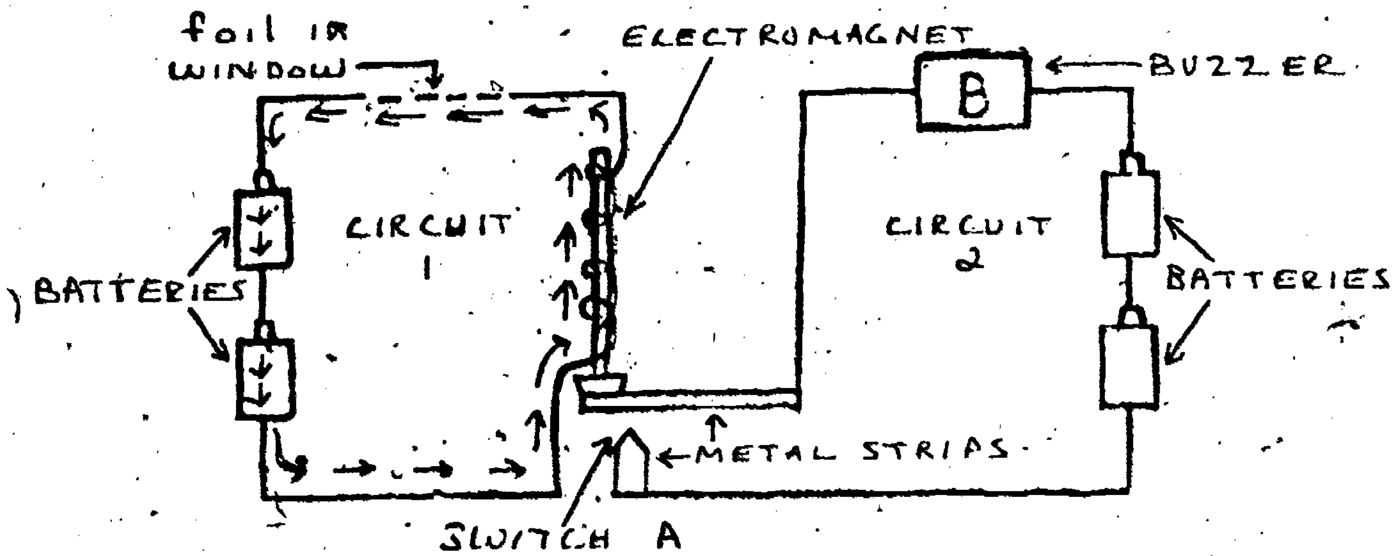
5. Unwind inside end of wire and place over last layer.

6. Wrap tape over the coil, leaving enough wire free to connect to battery.

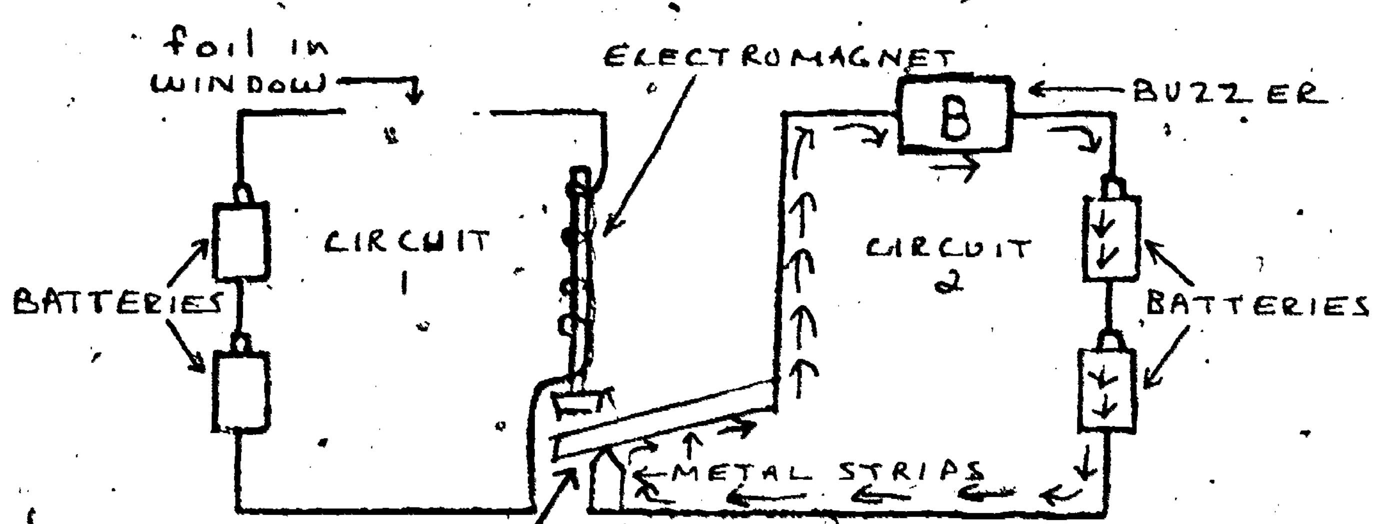
7. Scrape off insulation from each end of the wire.

Experience in development classes has shown that number 28 wire not only breaks easily but also is easily kinked, with the insulation cracking when the kinks are pulled out. Number 18 wire is too thick to wind easily. Therefore, the best sizes of wire to use for electromagnet experiments are #22, #24, and #26. Not all sizes need to be tried for each experiment. Once the children find that batteries do not last long when the thicker wire is used, they will probably decide to use only #26 wire for the other experiments.

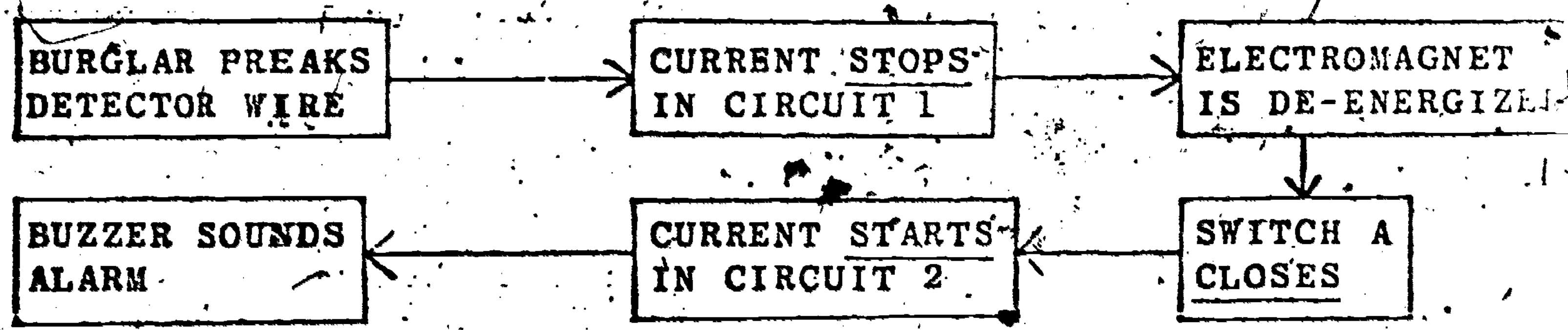
Hopefully the children will recognize the fact that an electromagnet can be used as a switch which controls two circuits. When a piece of iron is attracted to the magnet, it completes one circuit. When this circuit is opened by a break in some connection, the iron piece drops, making contact with another metal piece to close the buzzer circuit. Card 4 in the "How to Use Electromagnet..." set hints as this setup by asking the children to trace the current path in such an arrangement before and after foil in the circuit is broken. Their diagrams may show the current paths as follows: Before foil is broken:



After foil is broken:

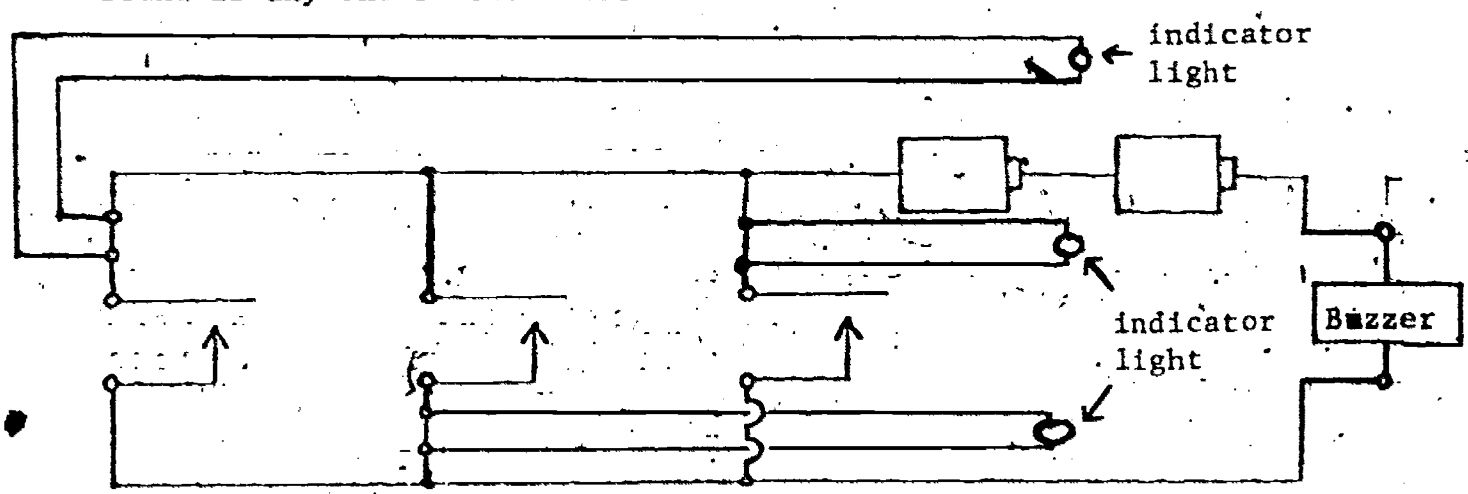


A flow diagram may be helpful:



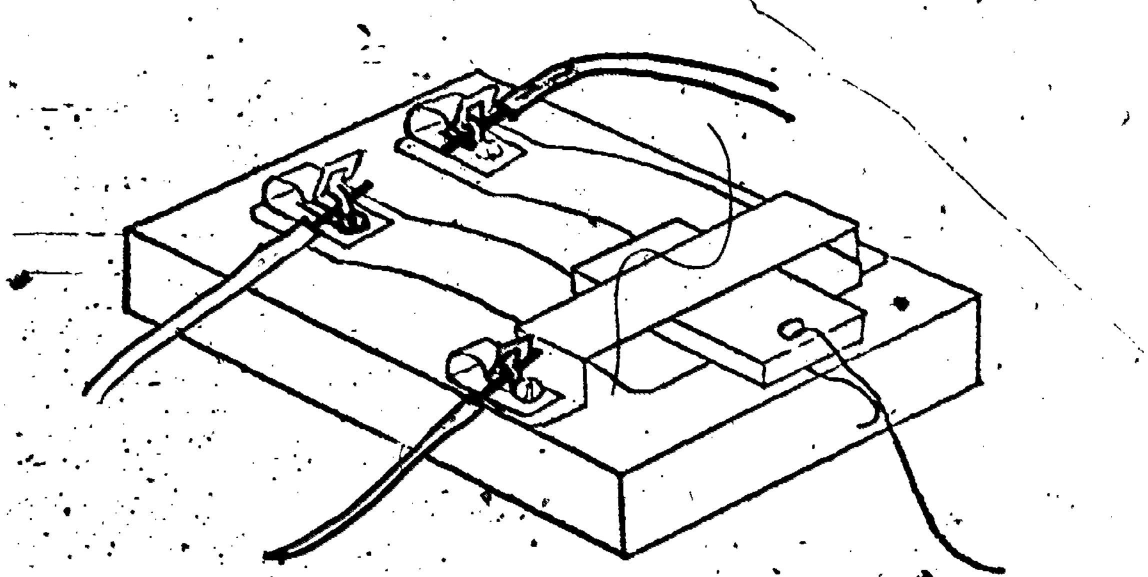
The electromagnet and its associated switch constitute a form of relay, which in this case function as a "not" circuit, in the sense that current flows in circuit 2 if and only if current is not flowing in circuit 1. Compare with "and" and "or" circuits --- pages D1-12 - D1-15.

Some children may want to investigate elaborate hookups for their alarms. They might be interested in a long distance warning system, or in a set-up which will give a warning if a "break" is attempted in any one of several places. The following diagram shows three burglar detectors connected in a generalized "or" circuit so that the buzzer will sound if any one of the switches is closed.

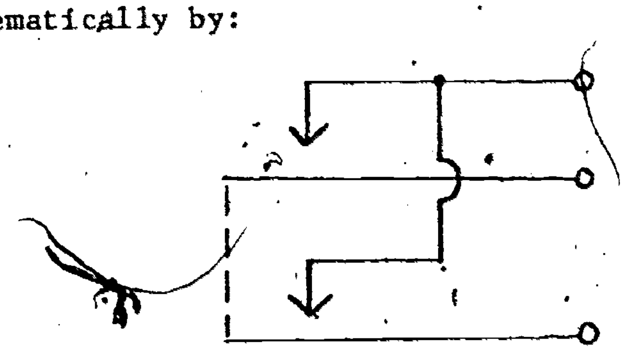


They may want to refine the above system so that a light will indicate at which place the alarm has been activated. This can be done by inserting a light near each switch as indicated. Otherwise it can be done with a type of burglar detector switch which closes two separate circuits at the same time when it is activated. One circuit sounds the general alarm; the other circuit lights the individual position indicator light.

Such switches can be made in many ways. One example is:

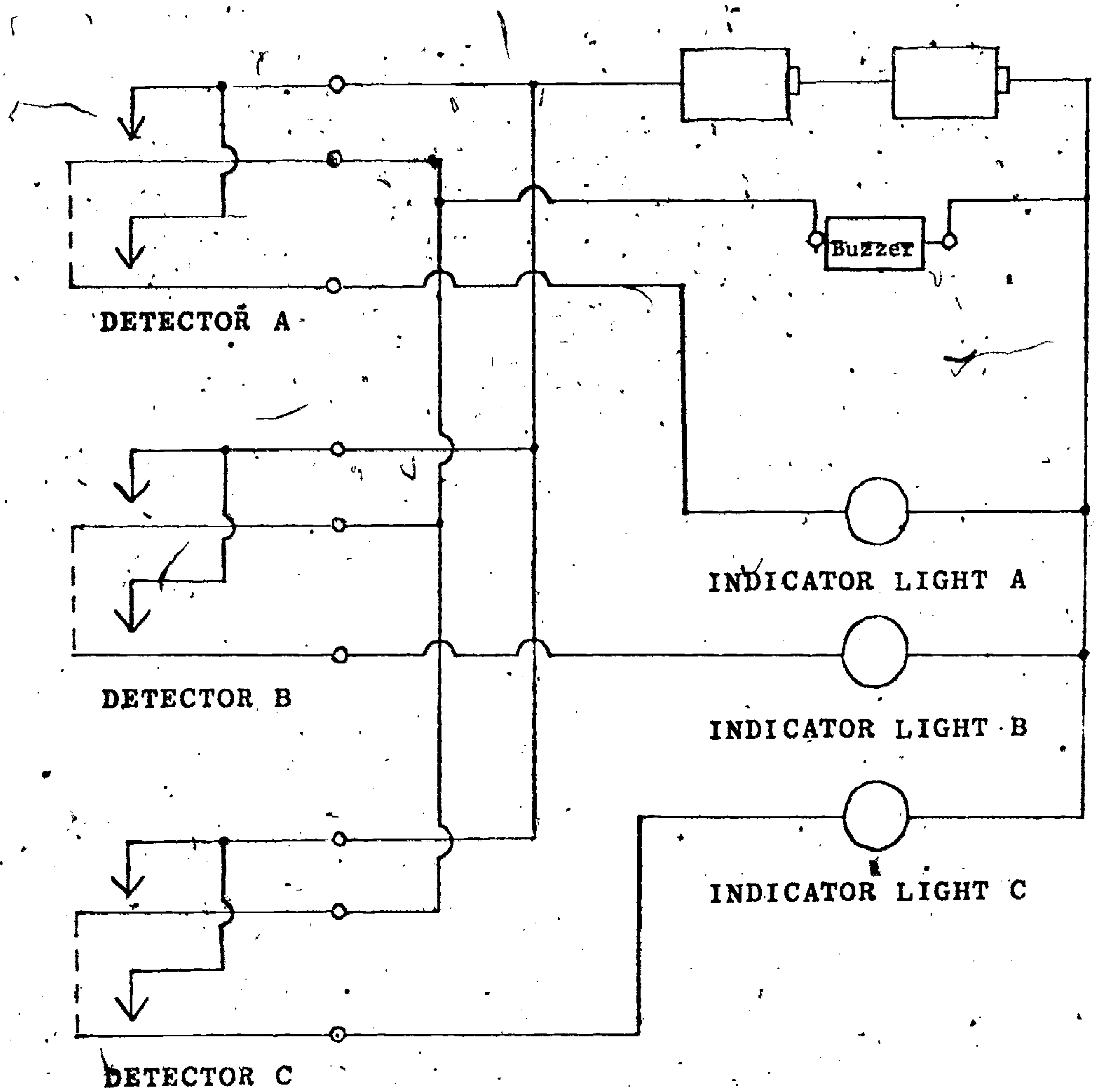


(It is activated by pulling the string.) We can represent such a switch schematically by:



(The dotted line indicates that both switch sections operate at the same time.)

Three such burglar detectors could be connected up as shown on the next page.



Note that if any detector switch is closed, then the circuit for the alarm buzzer is closed, and also the circuit for the individual indicator light corresponding to that particular burglar detector. (This particular circuit and style of burglar detector switch is the entirely original creation of a sixth grade child in the Washington School, Champaign, Illinois.)

TROUBLE SHOOTING ON ELECTRIC CIRCUITS*

As the children build electric circuits ranging from the simple one bulb, one battery, one wire circuit to elaborate electromagnet alarm circuits, they will encounter several problems. The things that are likely to cause trouble are:

(1) Dead batteries

(2) Burned-out bulbs

(3) Other possible defective gadgets: broken wires, (sometimes they break inside the insulation, which can be felt, but not seen easily), battery holders and bulb sockets, switches, magnets, etc.

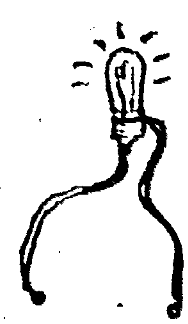
(4) Loose connections, for example:
  (a) Wires not tight in Fahnestock clips.
  (b) Bulbs not screwed into sockets all the way.
  (c) Contacts not tight against battery ends in battery holder.
  (d) Possible other kinds of loose connections.

(5) Wrong connections. This also includes short circuits and problems with connecting several things to one circuit.

With respect to the first four items it is desirable to have children acquire the skill of systematically examining the various possibilities. In particular, the following are two important strategies for locating the causes of troubles.

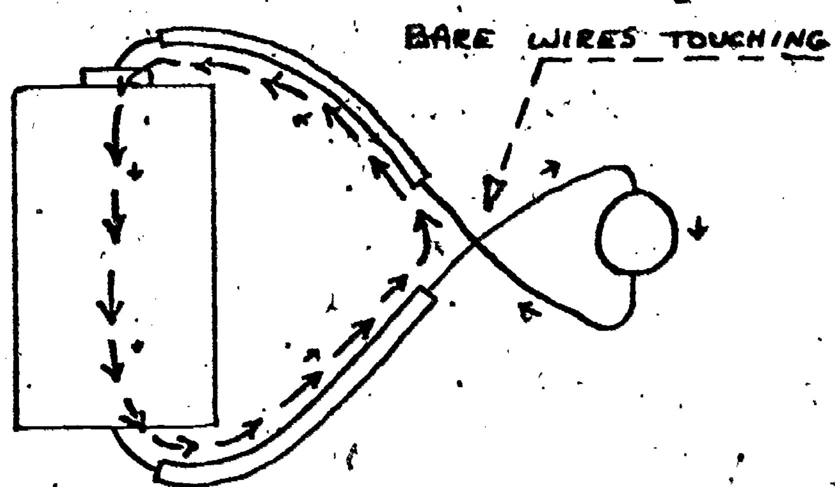
Substitution: Components from a defective system are substituted one at a time for components in another, functioning system (often simpler than the defective system). The functioning system will cease to function properly when a defective component is substituted in it.

Tracing: In any system where something goes into one part of the system and comes out of another part of the system, it may be possible to trace that "something" from the input, through the system, to the output, to find out where it first disappears. This will localize the difficulty. The children could use a bulb with two leads as a circuit tester to carry out this strategy.



*Note: Based on suggestions by Thacher Robinson.

Short Circuits - Many times the children have trouble getting a circuit to work because bare wires are touching in some place. If the two wires from a battery to a bulb, buzzer, or other gadget touch each other without insulation, like this

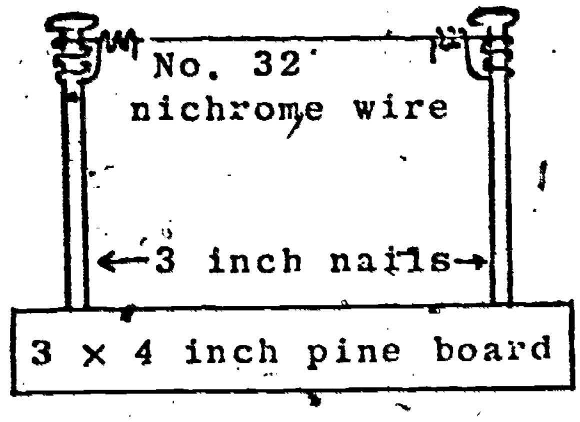


then the battery will push most of the electricity through the short circuit, and it can happen that not enough electricity will get to the bulb or other gadget to operate it properly.

Often, so much electricity will flow in a short circuit that the wires will get hot. This might be dangerous, since it could start a fire.

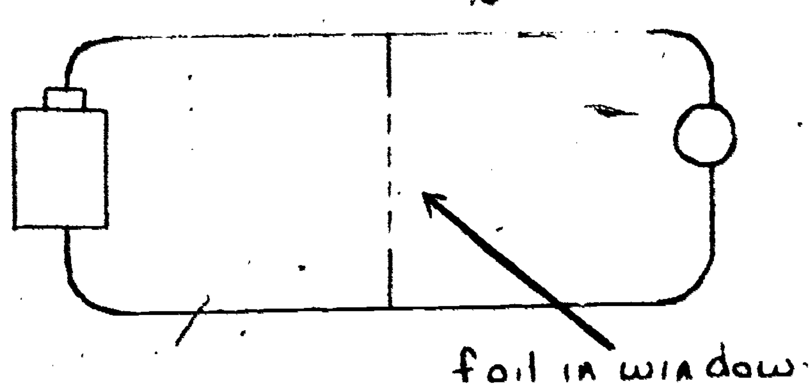
Thus, hot wires may indicate the existence of a short circuit. The term "short circuit" comes from the idea that the path of the current falls "short" of getting to the components which were intended to be activated. However, children usually think of a "short circuit" as being short in length.

The children can investigate heat produced by short circuits by first feeling how much heat is produced by the electricity flowing through a short length (12") of insulated wire. The wire will get hot quickly, and be equally hot all along its length. They can then build an apparatus to hold nichrome (high resistance) wire, shown in the following diagram:



The heat produced is varied by having the electricity flow through different lengths of the wire. This is accomplished by putting ends of wires which are attached to two batteries in series at different distances apart on the nichrome wire. The heat produced is greater the shorter the effective length of the nichrome wire becomes; as the resistance of the wire is decreased the current increases. The nichrome wire gets red hot, then white hot, finally melting as less of it is connected in the circuit.

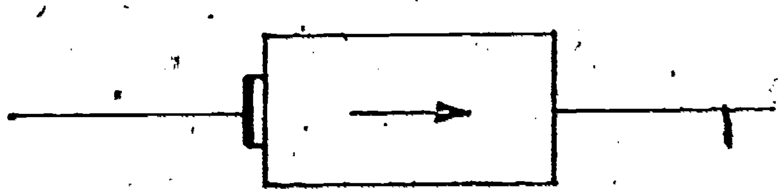
In a circuit containing both thick and thin wires in series only the thin wire gets hot. Much less electricity will heat a thin (high resistance) wire, and at the same time not much electricity can get through it. Therefore there is insufficient current in the circuit to heat up the thick wire. Some children build the following curcuit to make a light go on when a connection is broken. They soon realize that this is not a good alarm circuit not only because the wires get hot but also because the batteries do not last long.



Why Bulbs Burn Brighter or Dimmer

When the children try to put batteries into a circuit in series they may connect one or more so that a direct connection is made between two positive terminals or two negative terminals. To find out why the bulb gets dim or goes out the children can construct several circuits and compare the brightness of the lights.

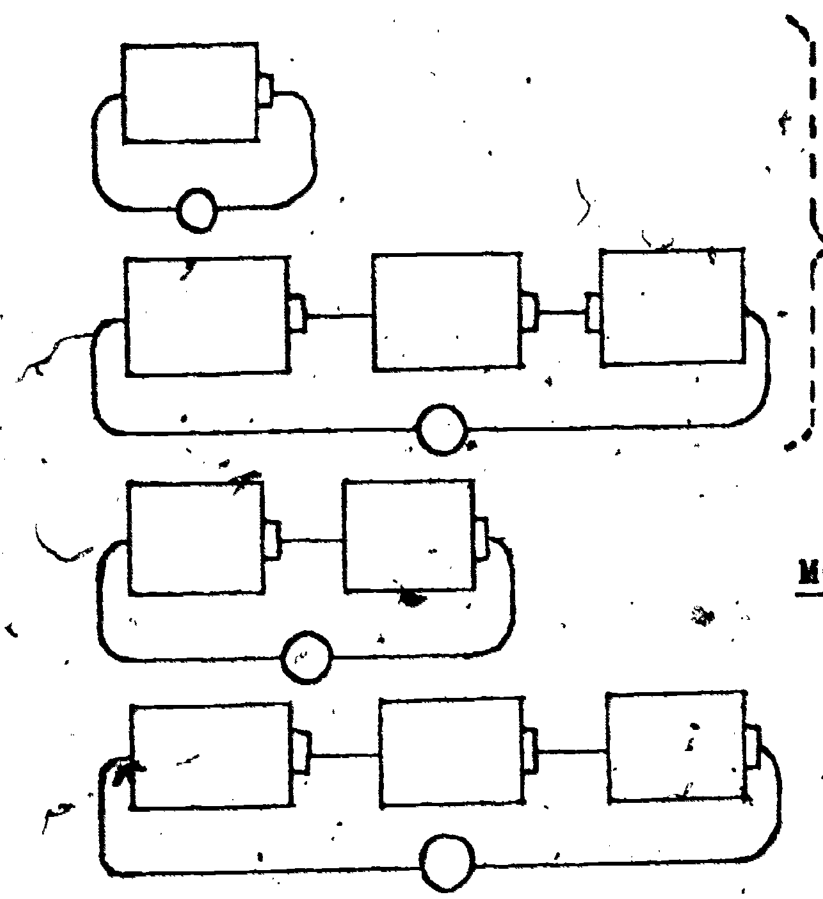
A battery pushes electricity in a definite direction around a closed circuit (electrons coming out of the flat end):



The amount of this push is measured in volts:

Nickel-cadmium battery: 1¼ volts
Carbon-zinc battery: 1½ volts
Lead-sulfuric acid battery: 2 volts

The effective amount of push can be determined by comparing the brightness of the bulbs in the circuits shown below.



	Nickel-cadmium	Carbon-zinc
Dim	1-1/4 volts	1-1/2 volts
Medium	2-1/2 volts	3 volts
Bright	3-3/4 volts	4-1/2 vols

Children will try to make the bulbs burn as brightly as possible in spite of being warned that too much electricity will burn the bulbs out. Some bulbs will burn out with less electricity because they are designed for use in low current circuits. It is best to use number 41 bulbs in alarm circuits. The following chart shows the maximum number of batteries to be connected in series to operate certain gadgets.

GADGET	MAXIMUM number of batteries to be connected in series to this gadget
No.41 bulb	2; or 3 for very short periods of time
Buzzer	4
Homemade electromagnets	Usually 2; or up to 4 for very short periods of time

Connecting Several Gadgets to One Source of Electricity

Sometimes children connect several bulbs or a buzzer and a bulb in a circuit the same way they connected batteries: in series. They then discover that two bulbs burn less lightly than one bulb and perhaps the buzzer won't work at all.

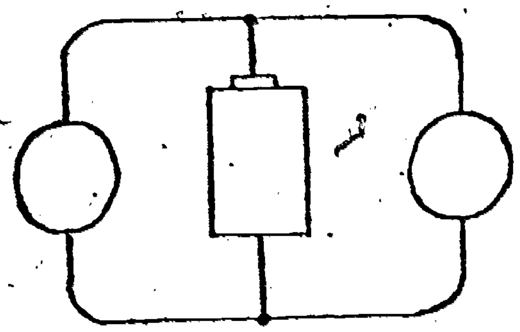
The amount of electric current which flows in a closed electric circuit depends on two factors:

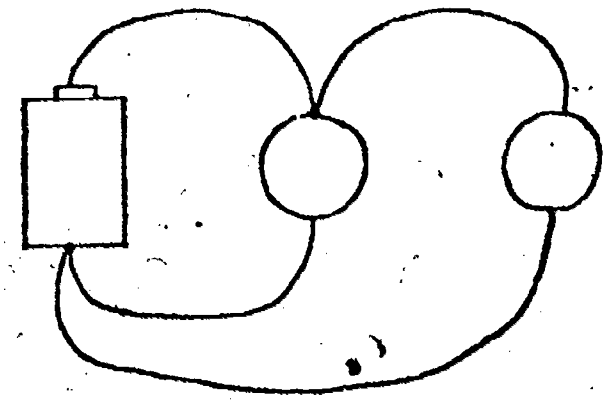
(a) The amount of voltage pushing, and
(b) The resistance to the flow of electricity of all of the gadgets in the circuit.

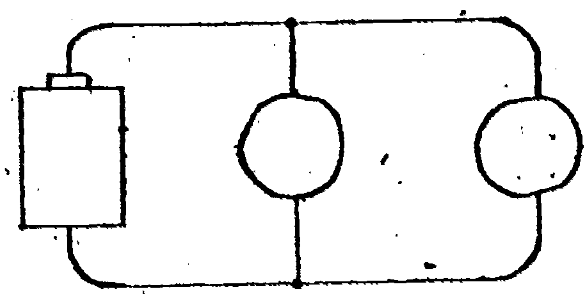
Two bulbs connected in series present more resistance to the flow of electricity than a single bulb.

By trying different connections the children will discover a way to put two bulbs in parallel, thus maintaining their brightness.

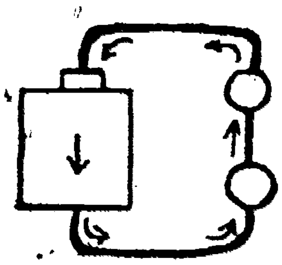
Several possible ways are shown below.



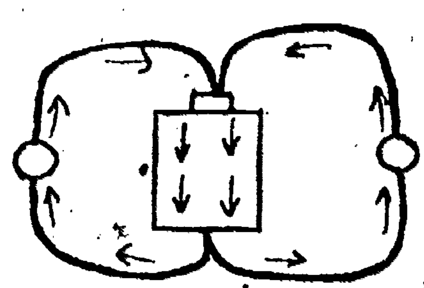




By drawing arrows on their diagrams to show the current paths, the children will discover that the current from the battery divides into separate streams, one for each bulb. After passing through the bulbs the paths recombine. This can be compared to the current path in circuits with bulbs connected in series. The same path passes through each bulb connected in series. See the diagrams below:



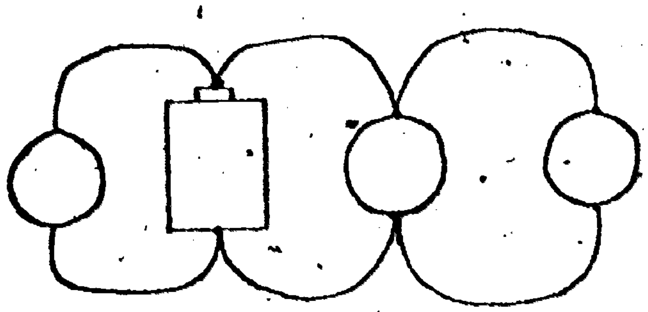
Bulbs Connected in Series

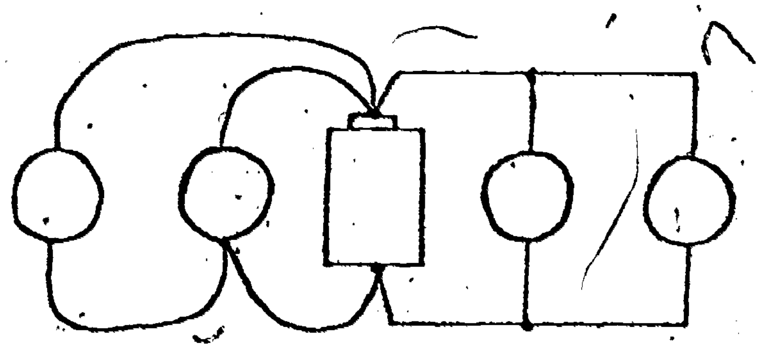


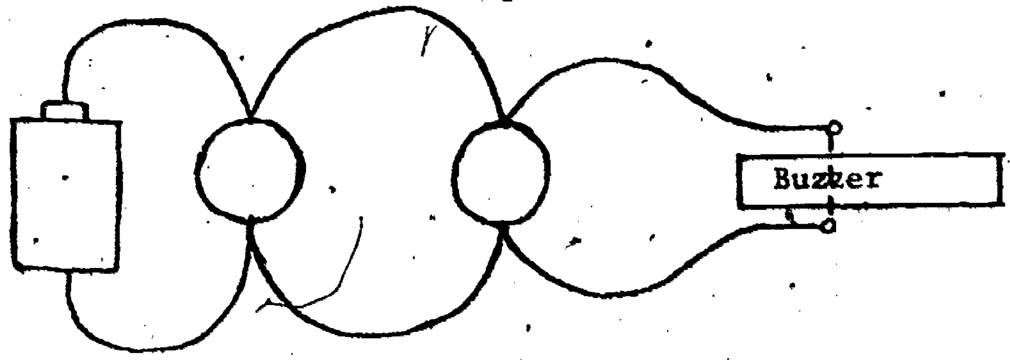
Bulbs Connected in Parallel

Each bulb presents a resistance to the electricity thus reducing the amount of electricity passing through other bulbs in the same path.

The children will discover various ways to put three or four bulbs, or two bulbs and a buzzer in a circuit. Some of the possibilities are:

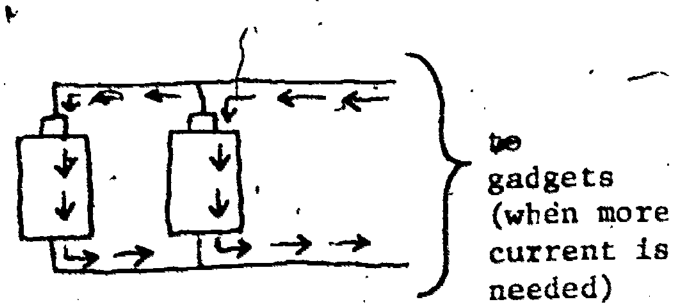




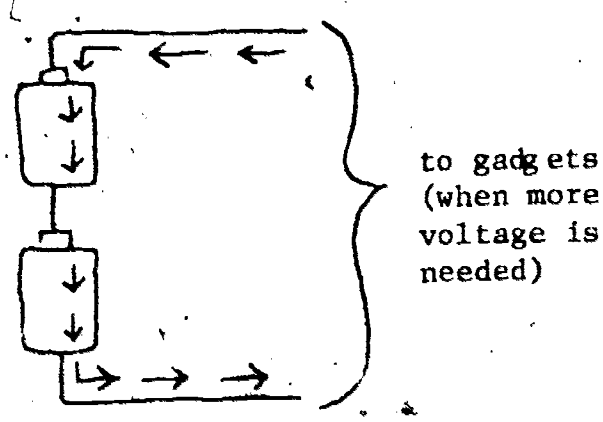


They may find that the bulbs burn less brightly when three or four are connected in parallel. Adding additional batteries in parallel will provide more electricity for the different paths. (A).

When more voltage is needed to operate some gadget such as a buzzer, additional batteries may be connected in series as shown (B):



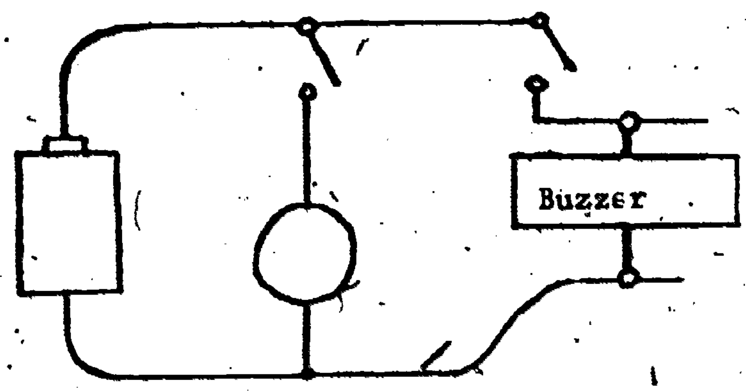
A



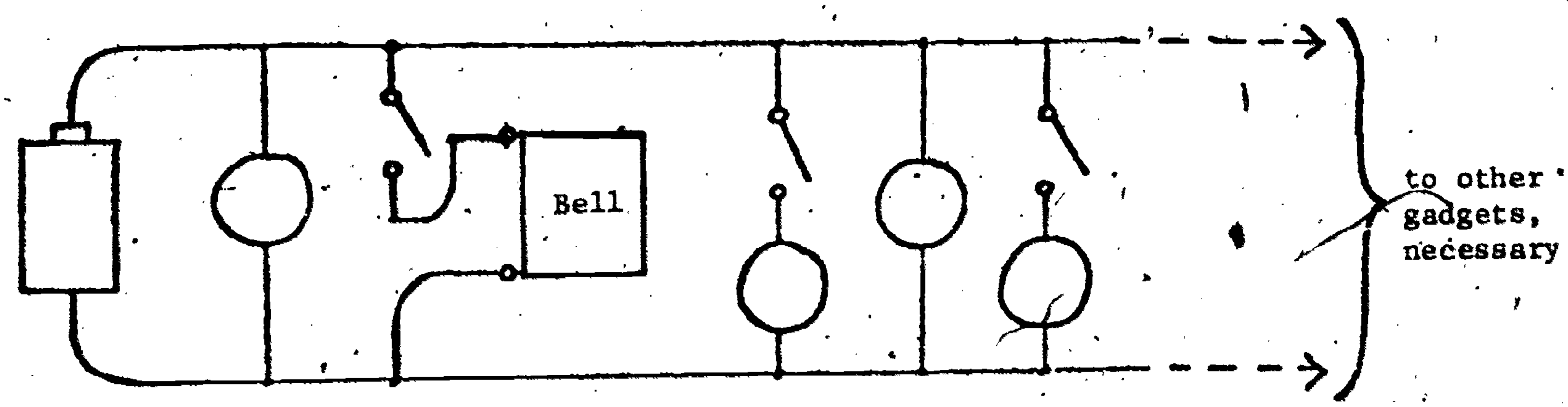
B

Independent Control of Bulbs

In constructing alarms the students may want to turn a bulb off and on without affecting the buzzer or turn the buzzer off without affecting the bulb. This can be accomplished as follows:



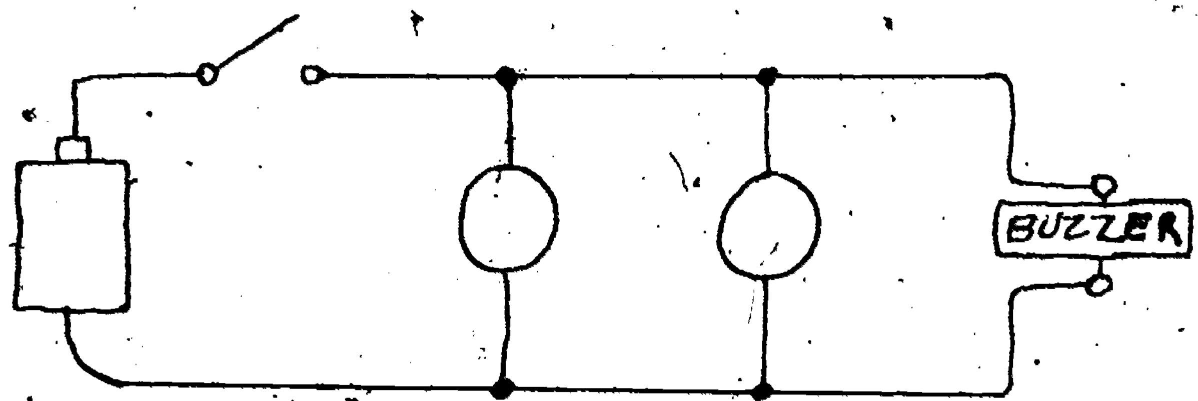
When many gadgets have to be run from one electric source, it is common to run a pair of wires from the source



and to connect the various gadgets across between the two wires. Switches giving independent control of the various gadgets can be inserted where desired.

Switching Control for All Gadgets

It is possible to have one master switch which turns all the gadgets connected to the battery on and off at the same time. The schematic diagram is shown below.



USMES

EXPERIENCES IN WORKING WITH CHILDREN ON ELECTRIC CIRCUIT DESIGN

by

Thacher Robinson

We have been working on electricity projects since January, 1971 in two sixth-grade classrooms and one third-grade classroom at the Washington School in Champaign, Illinois. The necessity of realizing some real classroom interactions for the purposes of the Washington School Project has forced us to proceed with the existing "Electricity and Reasoning" cards for the time being; and has not yet allowed us to prepare the macro-open program envisioned by USMES. Nonetheless, we are observing the children's responses to the strengths and weaknesses of the present card sequence, with an eye to optimum planning of the USMES sequences. Although the population varied with time, there were about eight children in each of these classrooms working with close supervision for approximately one hour a day, five days a week. They also worked off and on with little supervision for an equal amount of time during this period.

It was judged that almost none of the children knew enough about even such basics as the function of switches, and the necessity of closed circuits, to begin with design problems at the very outset; so that all children agreed to work through a set of "Electricity and Reasoning" cards which cover the fundamentals of electricity and reasoning before embarking on elaborate design projects. However, it was agreed that the children could spend up to about half of their time "messing around" with the available equipment in any way which seemed interesting to them, providing they spent the rest of the time working through the cards in a relatively disciplined manner.

One of the major problems in permitting unsupervised experimentation is that the children become involved with electrical circuits which are so complicated that, although things happen in these circuits which they find fascinating, it is impossible for them to sort out the relevant relationships in a way which contributes to their understanding in the sense of increasing their design capabilities. Also, even though the student works through the various levels of a technical subject with a minimum amount of understanding at each level, he will find his comprehension growing steadily fuzzier until there comes a point where he has no idea at all as to what is actually going on. These two factors result in a self-reinforcing tendency to produce dilettantes who are constantly acquiring new interests and starting new projects which never seem to come to much.*

The requirement of written and diagramatic reports seems to be a generally good way to integrate science and reasoning with the development of communication skills. As an example of this style of interaction, the diagram which the children are asked to draw of a circuit made with one battery, one bulb, and one piece of wire very often looks

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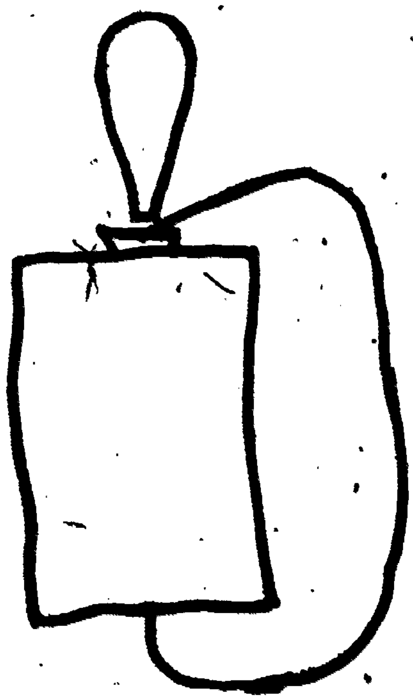
* NOTE: Class discussions about the different designs for burglar alarms will provide a focus for the children's experimentation and reduce the amount of time spent on random over-their-head experiments.

like the following:*



When this happens, I say, "Well, let's see about this!" Then I get a battery, a bulb, and a wire and connect them up as shown, with the wire between the bulb and the battery. This generates a hotwire short circuit and no light. Then the child usually says, "That's not what I meant!" and shows me how to light the bulb with the wire touching the threaded side of the bulb. At this point I usually say, "Then make your picture so that it shows clearly what you meant." After several such tries, the child will come up with a picture of a bulb which clearly shows the various contact parts of its base for the two necessary connections.

I am convinced that it is just this sort of representational distinguishing which really crystalizes an understanding which was previously only

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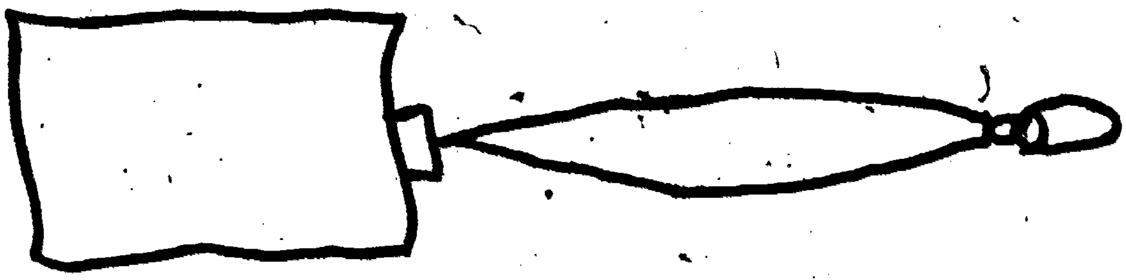
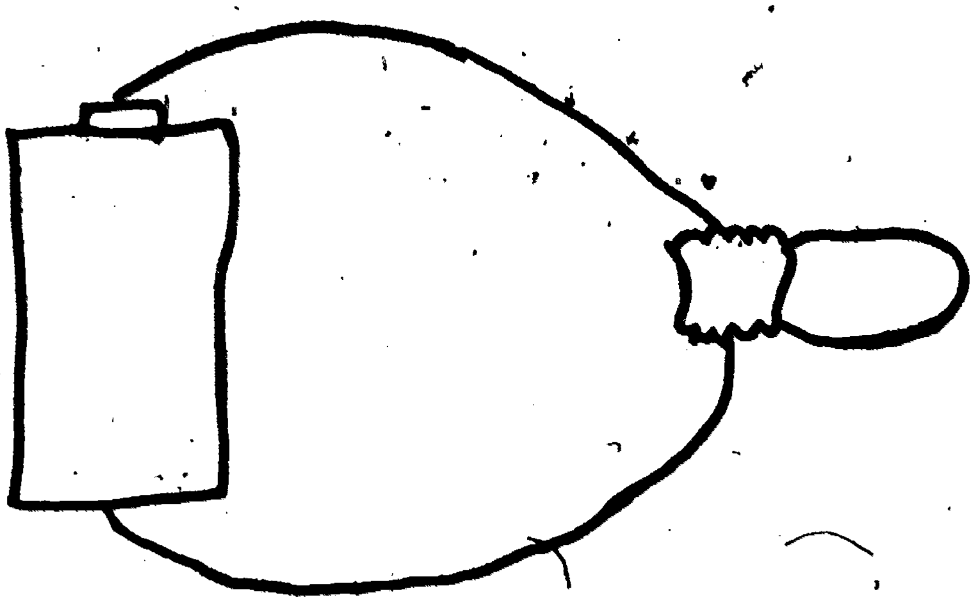
*NOTE: The need to explain to his classmates his alarm design might provide a student with the incentive to write adequate descriptions and make clear sketches. In general there is more emphasis in this trial on explicit verbalization of concepts than is usually sought for in an USMES unit. The point of view taken by USMES is that a child in response to a problem may develop at an early educational stage effective reactions which he can use in later life. These concrete actions also serve as a basis for later abstraction if the student continues such academic interests. Many people are excellent problem solvers and decision makers without being specialists on the psychology of how it is done. In other trials of this unit, the students made considerable progress with much less explicit verbalization.

intuitive; and it can, of course, be applied as easily to written descriptions as to diagrammatic representations.

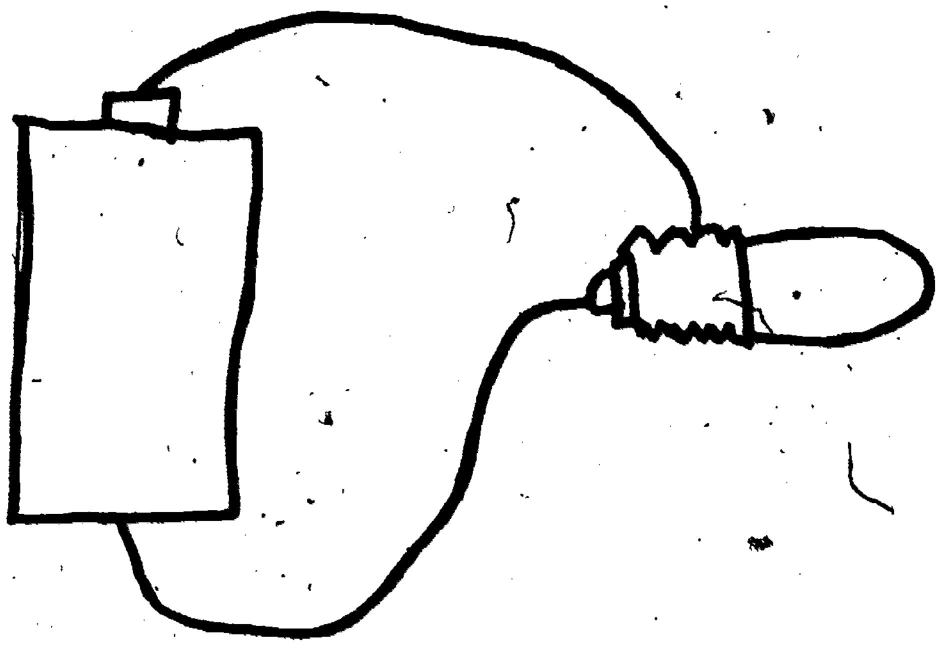
A child presented me with the following drawing of his solution to the problem: make a bulb light using one battery, one bulb and two pieces of wire without having the bulb touch the battery.



I think that it is safe to conjecture that this was pure theorizing on the child's part, uncontaminated with real-world experience. So I simply said, "Let's try it!", and we did; and of course it didn't work. Then the child said, "Oh! I know! One of the wires has to go to the bottom of the battery!" So I told him to draw another diagram showing what he meant. Five minutes later, he came back with this: (Note the much improved drawing of the bulb.)



So we tried it again, I holding the battery connections and he holding the bulb connections. Both of us got our fingers burned and the bulb didn't light. Then the child told me that he suddenly realized that one of the wires had to go to the bottom of the bulb. His final, acceptable picture looked like this:



I have the feeling that this child for the first time in his life began to think really seriously about the fact that his reports were really supposed to reflect some kind of reality in the world, in a very definite sort of way. It is certainly the case that his work was much more careful subsequently.

Several difficulties have arisen during the course of the children's work with basic circuitry.

1. The task of locating a battery and a bulb that are definitely O.K. entails the combinatorial and operational problem of how to try every possible battery with every possible bulb. One student found the bad bulbs by looking for broken filaments and guessed at the batteries by newness of appearance. When I asked her what she would do if these differences were not observable, she shrugged and said, "I'd guess." It seems more feasible to begin a discussion of this process with a group discussion rather than an activity card. Further, the student should be encouraged to label the "standards" so that they don't get mixed up with the components whose condition is uncertain; the "standards" could then be used to test the remaining batteries and bulbs.

2. When children are asked to test a battery and bulb tester, the distinction should be made between using the tester to test other components, and testing the tester itself; i.e., the reliability of the tester depends not only upon the condition of a bulb and a battery, but also of associated wiring, sockets, etc.

3. Several children have difficulty with the definition of a connection. They have the preconceived idea that a connection involves a wire, so that a battery touching a bulb is not a connection. It would seem that the logical way to teach them what a connection is, is to let them learn it from context.

4. Resistance is confusing to many children. A solution might be to ask the children if they can determine which has greater resistance, a wire (or switch) or a bulb?

5. Buzzers should be used as current indicators instead of lights.

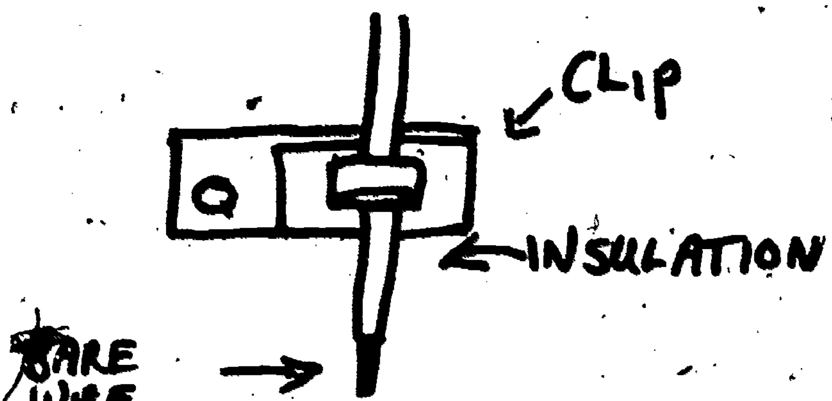
6. Buzzers and other equipment are introduced as devices first and "dissected" later.

7. Nickel-cadmium batteries should be used since their voltage remains constant over a long period (then decays rapidly).

During the first part of March, we suggested to the children that they make burglar alarms for windows, doors and desks in their classroom. Two groups of boys did extensive work on door alarms. Desk alarms seemed to be more popular however, perhaps because they are the sort of things one person works on alone, and the mechanisms used are concealed from other students, so that many people can work on them separately at their desk.

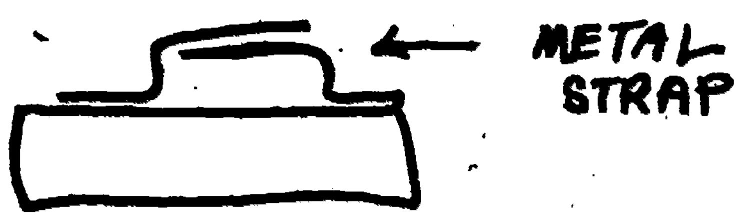
The main problem with alarms is making a circuit that will close when something else opens. This should be accomplished first by making an electrical connection by mechanical means. The children find this idea hard to accept and, in their first attempts, examine nothing but the possibility of using the metal parts of the door, window or desk as a part of the electrical circuit, but have no luck. We suggested over and over that a string be tied to the door and to some part of a switch, but the children ignored this until they were fully convinced that nothing else would work with basic circuit components. A distance operation can be accomplished later by a relay which makes the connection in a second circuit by electro-magnetic means.

One of the girls working on the window alarm tried something like this where opening the window would pull on the wire till the bare part touched the clip:



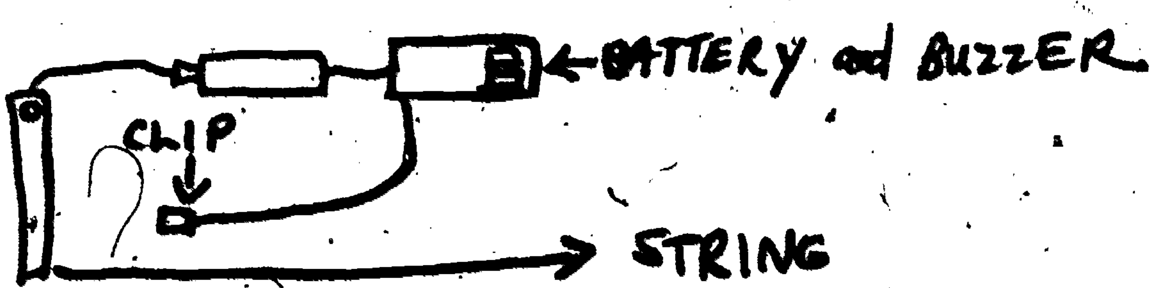
This presented two problems: if the wire were pulled too far it would come out of the clip altogether and break the circuit; and the clip gripped the insulation so tightly that some part of the system would be apt to break before the wire would slide.

A group of boys struggled with this problem until one of them picked up a switch we had made, which is closed in its normal state.



He then put a piece of paper between the metal strips and pulled it out. The others realized the principle immediately and constructed an alarm.

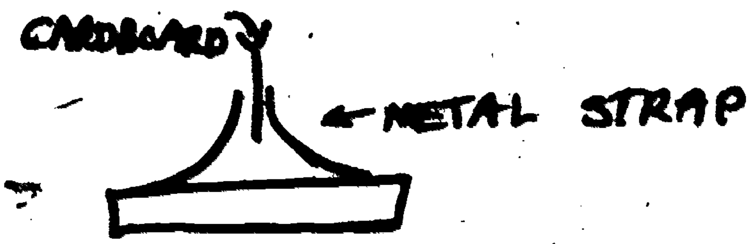
A door alarm design was developed which had too much wrong with it to work. But it is significant because its designers realized that all the alarms thus far built involve an insulator being pulled out from between two conductors, and they attempted something else. Their switch worked something like this:



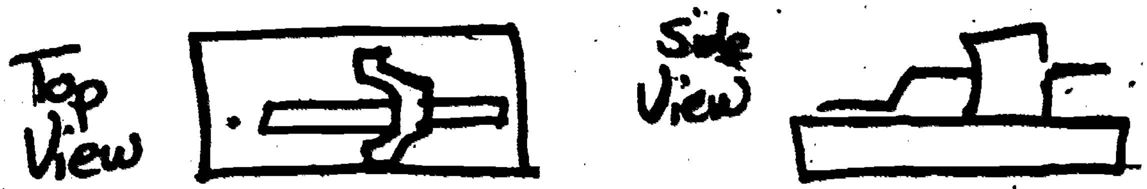
The string pulls the switch in an arc till it connects with the clip, closing the circuit. One problem is that the switch and the knob the string is tied to are at different levels and the string pulls the switch up so that it is no longer on the same plane with the clip and does not connect. Also, because the string is firmly tied to both the door and the switch, pulling on the door must break something.

Various other devices were used to booby trap desks. The most prevalent design was very similar to that used on door alarms. A switch like this was used with cardboard between the two metal parts:

The cardboard is tied to the top of the desk by a string. When the desk top is lifted, the string pulls out the insulator and the circuit closes. This type switch is somewhat inefficient since the cardboard is placed in horizontally, but must be pulled out from above, which tends to dislodge the entire circuit from its place in the desk. A better design that none of the students thought of would be:

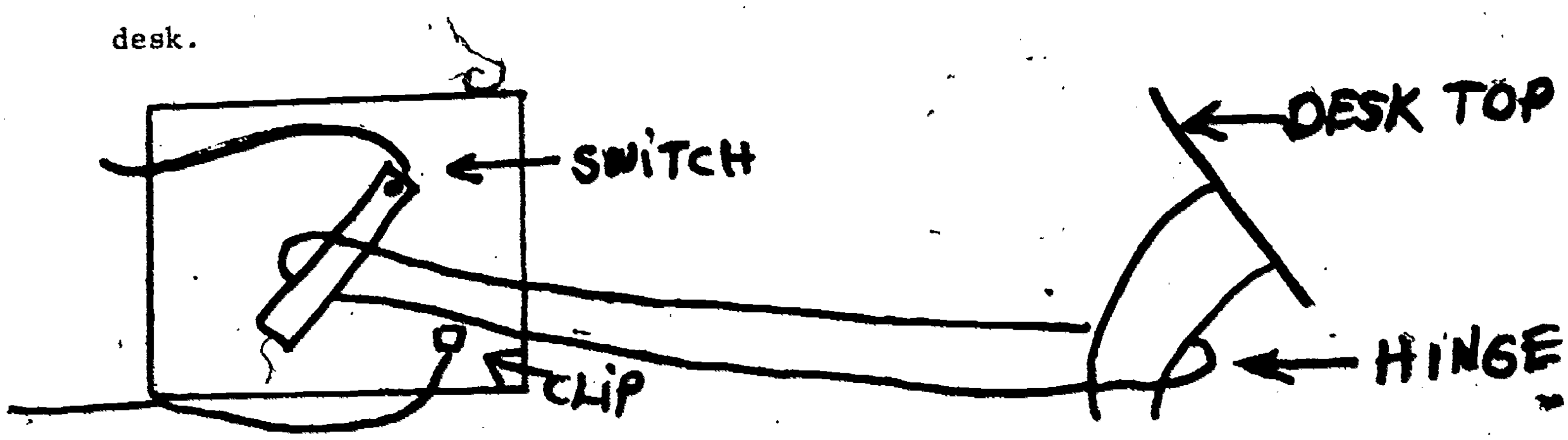


A second desk alarm design worked with a switch like this:



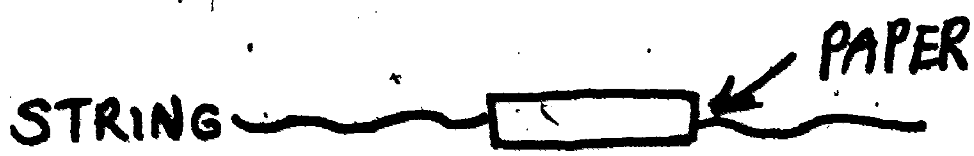
The two metal straps do not touch unless the part designated above is pulled up. This is accomplished again by tying a string to the top of the desk. This has the obvious fault that opening the desk too far will break the string, and the circuit will open.

A third and most interesting design involved the hinges of one boy's desk.

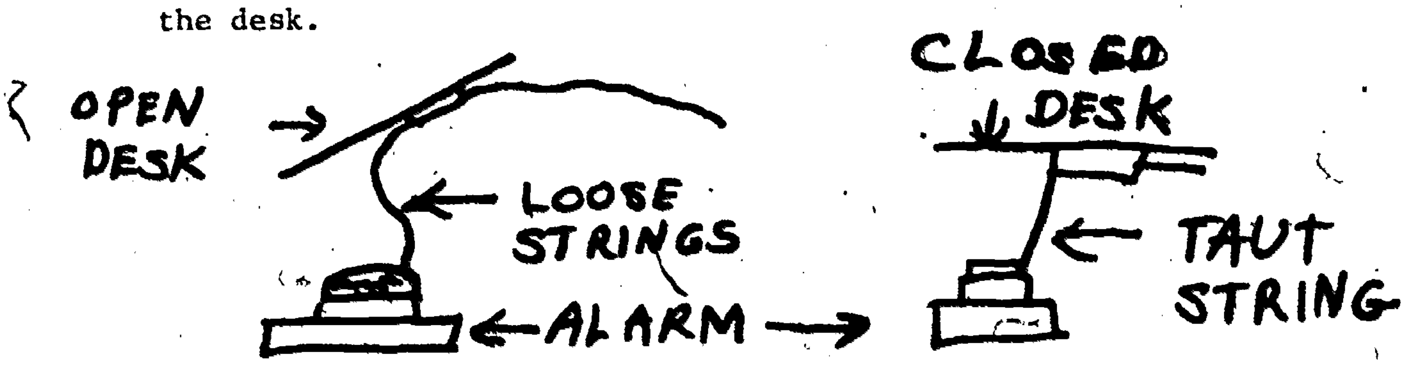


When the desk top opens, a loop tied around the hinge and the switch moves just enough to bring the switch in contact with the clip. Surprisingly, opening the desk top further does not pull at the loop any more, and there is no problem with the string breaking.

Still another boy made quite an innovation in the mechanical workings of burglar alarms. In the standard design alarm the way the string was tied to the desk and insulation made it necessary to open the desk almost all the way before the alarm would go off. This boy attached a loop of paper to his desk top tight enough to hold a string in place, but loose enough that he could pull the string through it.



When arming the alarm he first put the insulator between the metal straps, threaded the string through the paper loop and, as he lowered the desk top, he pulled on the string. This way he could pull the string tight from outside the desk.



The same boy made another student's dollhouse burglar proof. He set up alarms behind the windows and doors. Eventually he plans to wire a few other boxes like this and have all these "houses" hooked up to a police station. This will consist of a map of the houses with light bulbs behind it. When a house is burglarized the buzzer in the house will go on, and at the police station the appropriate light will go on to inform the police where the crime is taking place.

MAKING POLYHEDRA

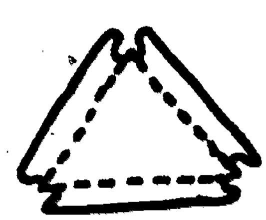
by Alan Holden, Sept. 1969

Many of the units of instruction that I hope to describe require making polyhedra. Three general ways of doing this can be distinguished, each with ~~advantages and~~ disadvantages: (1) assembling a polyhedron out of its separate component faces, joined at their edges by some removable form of attachment, (2) folding a polyhedron from a flat pattern, on which tabs are left to be glued at those edges that are not already represented at the folds, and (3) assembling a polyhedron out of its component faces by permanent glueing at the edges.

Method 1

Decomposable assembly has the advantage that the pieces can be used again for another polyhedron. But more importantly, it facilitates experimenting with schemes of assembly. For example, "the irregular tetrahedron puzzle" of the 1967 Goals Report, p. 142-3, would surely be best implemented with such pieces.

One form for such pieces is represented in the British materials now marketed in this country by Selected Educational Equipment, Inc. Each piece bears a tab on each edge, notched so that the tabs on adjacent faces can be joined by slipping a rubber band around them after they have been scored and folded.



Unfortunately, the most visually conspicuous feature of the result of assembling such pieces is the projecting tabs and their rubber bands. In the only trial with children that I have heard about, the children were displeased with such messy looking products. So am I.

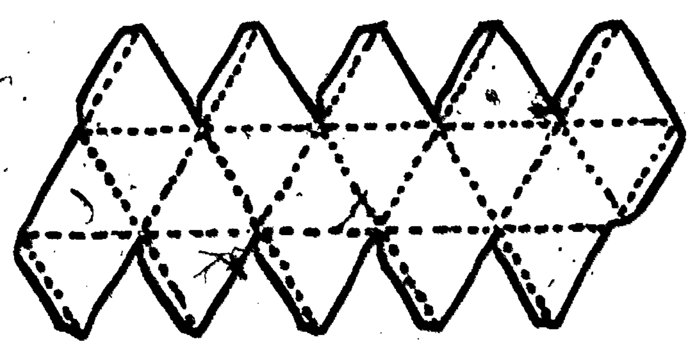
Creative Playthings sells relatively large pieces whose edges are notched so that they can be joined fairly securely without folding or additional fastening. The resulting edges are ragged, the notching is likely to become bent out of shape in a few uses, the the method is hard to embody suitably in smaller pieces.

I have experimented with ways of making contact-adhesion edges on pieces without tabs, but my efforts were unrewarding. The edges picked up lint and could not be used twice. They stuck together either too well or not well enough, and hence, gave manipulative trouble in switching edges.

The problem of decomposable assembly still invites the attention of an industrial designer, in my opinion.

Method 2

Folding flat patterns has long been the most usual way of making polyhedra. With care it affords quite elegant products. The products show one feature irrelevant to their geometry--i.e. a distinction between the folded edges and the tabbed edges--but this is a negligible defect. In pursuing this method, it is common to provide only one tab for each of the edges to be joined, as on this pattern for the regular icosahedron.



The design of a flat pattern, or "net", for a particular polyhedron is a geometric problem fascinating in itself. By offering this problem, the method can be regarded as either advantageous or disadvantageous--advantageous in exhibiting "routes from two to three dimensions" or disadvantageous in presenting a problem peripheral to the structure of the final polyhedron.

Suitable material for polyhedra to be folded from patterns is provided by file cards--3" x 5" or 5" x 8". When the pattern has been laid out with pencil on the material, it is cut out with scissors, folded, and glued with each tab inside the edge joined to it.

Folding along the straight edge of a desk ensures a straight fold but not a sharp one. It is well to score the material along the line of the intended fold with a knife or a razor blade before folding. Such scoring, however, introduces the risk of cutting the material apart; even at best it may seriously weaken the material along the edge.

The best scoring device for minimizing these defects that I have found is the toothed wheel in a handle, called the seamstress' pattern marker; engraved on the handle of mine is "Dritz tracing wheel". It is cheap, at any store carrying sewing supplies. You can roll it smoothly against the edge of a ruler, while you push down on it, with the pattern resting on a thin pad of newspaper.

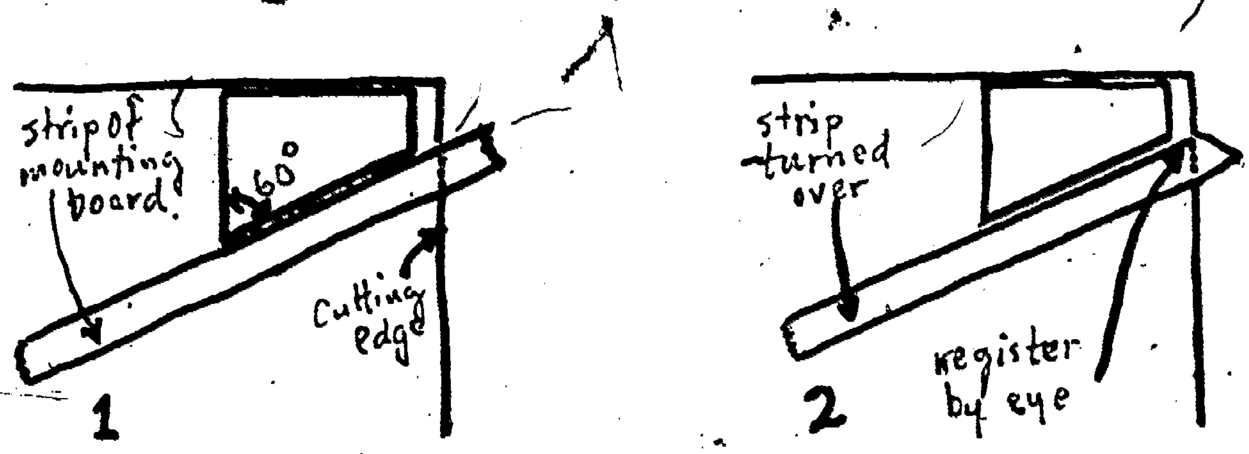
Method 3

The best of method of making polyhedra that I have found is to assemble them from individually cut flat faces, glued together at the edges with Elmer's Gluall, or any other "white glue". The edges of the products become pleasantly accented as dark lines (unlike the folded edges) but are not obtrusive (unlike the decomposable edges).

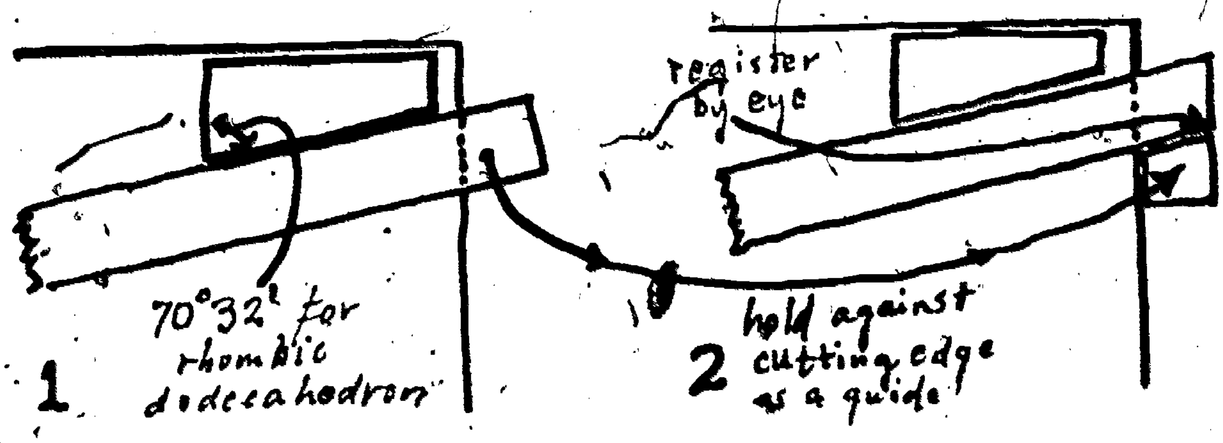
For this method the best materials are "mounting board" (used for mounting photographs, and hence obtainable at photographic supply shops) and "illustration board" (used for charcoal or crayon illustrations, and hence obtainable at artists' supply shops). The former is the lighter in weight, is much the cheaper, and is just as satisfactory unless the products will be treated roughly (e.g. dice). File cards and bristol board give too narrow an edge for reliable glueing.

The faces to be assembled are best cut with mounted shears (whose many uses include trimming photographs and which are obtainable again at photographic supply shops and stationery stores). The most useful models of this tool have a movable barrier or "fence" which can be clamped parallel to the cutting edge at any distance from it.

Cutting a large number of congruent faces is best carried out by first cutting strips of uniform width, and then sliding a strip along the cutting board with one edge against a guide that establishes the desired cutting angle. Equilateral triangles, for example, are made from strips whose width is the altitude of the desired triangle. The guide is a draftsman's 30-60-90 triangle, or a corresponding piece of cardboard. After one cut, the strip is turned over and the vertex is registered by eye over the lower blade of the shears, to establish the second cut. A repetition of this operation over the length of the strip produces a collection of congruent triangles.



Regular hexagons are best cut by first cutting equilateral triangle- and then truncating the vertices of the triangles with the aid of a fence set back from the cutting edge 2/3 of the altitude of the triangles. Squares are cut by using the fence first to cut strips and leaving it in the same position to cut squares from the strips. Cutting other polygons requires the use of a guide made of heavier cardboard with two edges meeting in the desired angle.



The faces are usually best assembled first in pairs. A little white glue is squeezed out of the squeeze-bottle to form a small pool on a 3" x 5" card. A toothpick is dipped in the glue and used to smear the edge of a polygonal face. The edge of the polygon to be attached is pushed against the glue-bearing edge and slid along it to distribute the glue. The pair is then placed on a table with the glued joint standing up. More such joints are made, and after five minutes (more or less;

depending on the condition of the glue and the relative humidity of the atmosphere) the first-made joint has set sufficiently to permit further work with the pair. The joint remains flexible for several hours, however, and hence the dihedral angle is adjustable.

In making an octahedron, for example, the appropriate edges of one pair of triangles are smeared with glue, and the two pairs to be joined are slid toward each other along a flat surface, to form a square pyramid. Two such square pyramids are then joined at their bases to complete the octahedron. In making a rhombic dodecahedron, it is best to glue one rhombus to a pair of rhombuses, to provide three rhombuses meeting at a 3-fold axial corner. This operation fixes the dihedral angles. Four such triples can then be joined successively to complete the polyhedron.

In making more complicated polyhedra, both rigidity and ease of assembly are promoted by assembling the external parts on a simpler polyhedral base whenever possible. Thus, Kepler's "stella octangula" can be made of eight trigonal pyramids attached to the faces of a regular octahedron.

Elegant wire models, defining polyhedra by their edges, can be made by using such polyhedra as those just described to provide the jigs for holding the wires to be joined. The wire (appropriately #16 B. & S. gauge tinned copper) is cut to the lengths of the edges, and each piece is glued to an edge of the cardboard polyhedron with white glue extending a quarter inch short of the vertices. Then the vertices are soldered, and trimmed if necessary with a flat file before removing the cardboard.

The assembly is then soaked in water long enough to soften the cardboard and the faces are pushed inward in succession and removed.

Only older children, with a taste for craftsmanship and some experience with a soldering iron, will care to make such wire models. But the exploration of polyhedra defined by their edges can be pursued by quite young children, using toothpicks stuck into gumdrops or little balls of modelling clay.

More expensively, kits of rods and rubber connectors are sold by Edmunds Scientific Co. among others. A kit of this sort is also marketed as a toy, "Doodle-Stix" (Kay Products Co., 906 Main Street, Cincinnati, Ohio).

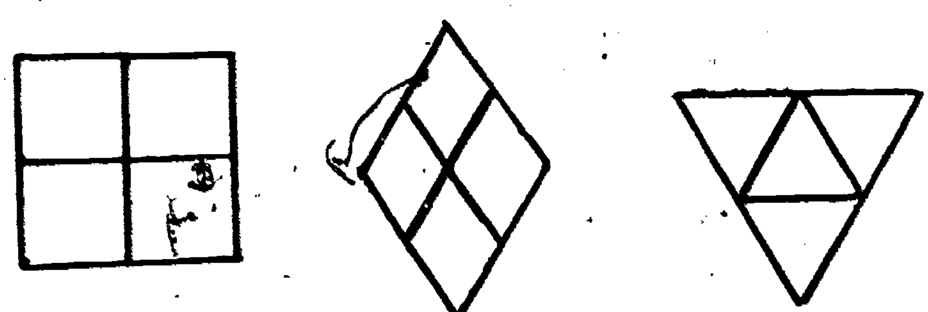
SOLIDS MADE OF EQUILATERAL TRIANGLES

by Alan Holden, Sept. 1969

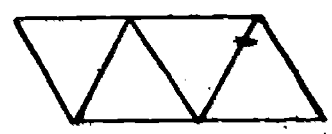
A - If you set yourself the task of making different solid shapes with flat faces of a single sort, joined at their edges, you may conclude that the equilateral triangle is the most versatile face-shape that you can pick. Notice that any side of one triangle will join with any side of another. Furthermore, any number of triangles can share a common vortex if you permit them to fold either way along the edges that meet at the vertex. (Such polyhedra are often called "deltahedra", after the shape of the Greek capital letter, delta.)

B - Of course, any equilateral figure, such as the square, shares these two virtues. But a little free play with triangles, and comparable play with squares and with rhombuses, will give a sense of the triangle's versatility and of the problems encountered in attempting to make closed, solid shapes. Notice as you go that when only three flat pieces join at a vertex, the result is rigid; and when more than three join, the result is floppy. You can gaily attach triangle to triangle for a while in all sorts of arrangements, but you meet the nitty-gritty when you try to close up the whole business, and soon a little forethought and intuitive geometric discipline come into the game. (Two 24-sided deltahedra are enclosed.)

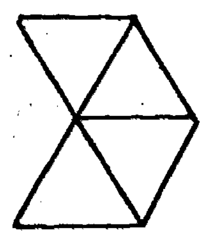
C - Four squares (or four rhombuses or four triangles) can be joined to make a square (or a rhombus, or a triangle) that is twice as big.



In each case the shape is the same, even though the size is different. In the case of the four triangles, however, notice that the three outer triangles can be turned up and joined, to form a regular tetrahedron. In other words, the flat arrangement of four triangles is a suitable flat pattern, or "net", for the tetradedron. Another suitable net for the tetrahedron is the strip-arrangement.

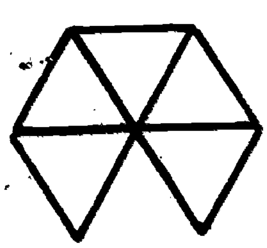


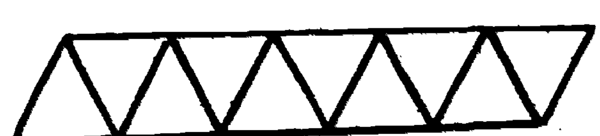
Are there any others? Such work makes contact with the external-wall-papering unit developed by Lyn McLane. Notice that not all arrangements of four triangles will fold into a tetrahedron. For example:



D - The trouble with the last pattern is that four triangles meet at a common vertex, whereas in the tetrahedron only three triangles meet at each vertex. But you can make a solid in which four triangles meet at every vertex--the regular octahedron. For example, you can make it by folding two patterns like the last above and joining the open bases of the two resulting pyramids.

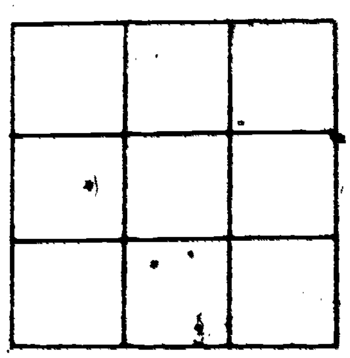
You can go on to make a solid in which five triangles meet at each vertex--the regular icosahedron. The simplest way to make it is to make two pentagonal pyramids and a cylindrical band of ten triangles, and then attach the pyramids to the top and bottom of the cylinder.

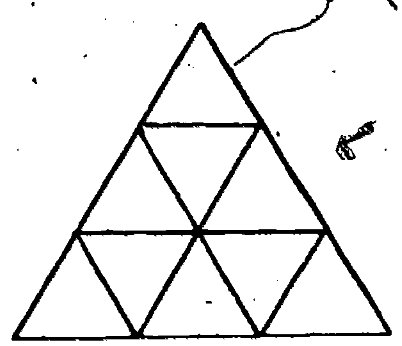




E - Six triangles with one common ve tex lie flat, and form a new flat shape, the regular hexagon. What solids can you make out of it? Clearly, you can't make a closed solid out of this shape alone, because in any closed solid at least three faces must join at each vertex, and when three regular hexagons join at a vertex, the result still lies flat. But if you substitute hexagons for triangles in the net for the tetrahedron, and fold up the net, you get a tetrahedron whose corners are cut off. You can cover the open places with little triangles (making a "truncated tetrahedron") or with little tetrahedra (making a big tetrahedron).

F - You can attach more squares or rhombuses or triangles to the patterns in C, building a square or rhombus or triangle three times as big out of nine of the little units. But hold on: what do you mean by "times as big"? Is the result three or nine "times as big"? Examining this question may provide reenforcement for the idea that an area scales with the numerical square of a length. That idea is commonly introduced by using squares and rectangles; the use of triangles as well may serve to broaden it.





G - Notice that, unlike the net of four triangles (C) the net of nine triangles (F) cannot be folded into a solid shape with no holes. Is there any way of arranging the nine triangles in a net that will fold into a solid shape? A little experimenting will not reveal a way, and might motivate the following reasoning.

If a closed, solid shape is made of N triangles, the component triangles have 3N sides before they are joined. After they are joined, each of these sides is shared by two of the triangles to form an edge of the solid. Hence, the solid has half as many edges as the total number of sides of the original triangles: $E = 3/2\ N$. But E must be a whole number, and thus N must be a multiple of 2. In other words, any closed solid made of equilateral triangles must have an <u>even</u> number of faces, and 9 is an <u>odd</u> number. Here is another simple instance of an "impossibility proof", additional to that already described in a polyhedral context in the 1967 Goals Report, p. 144-5.

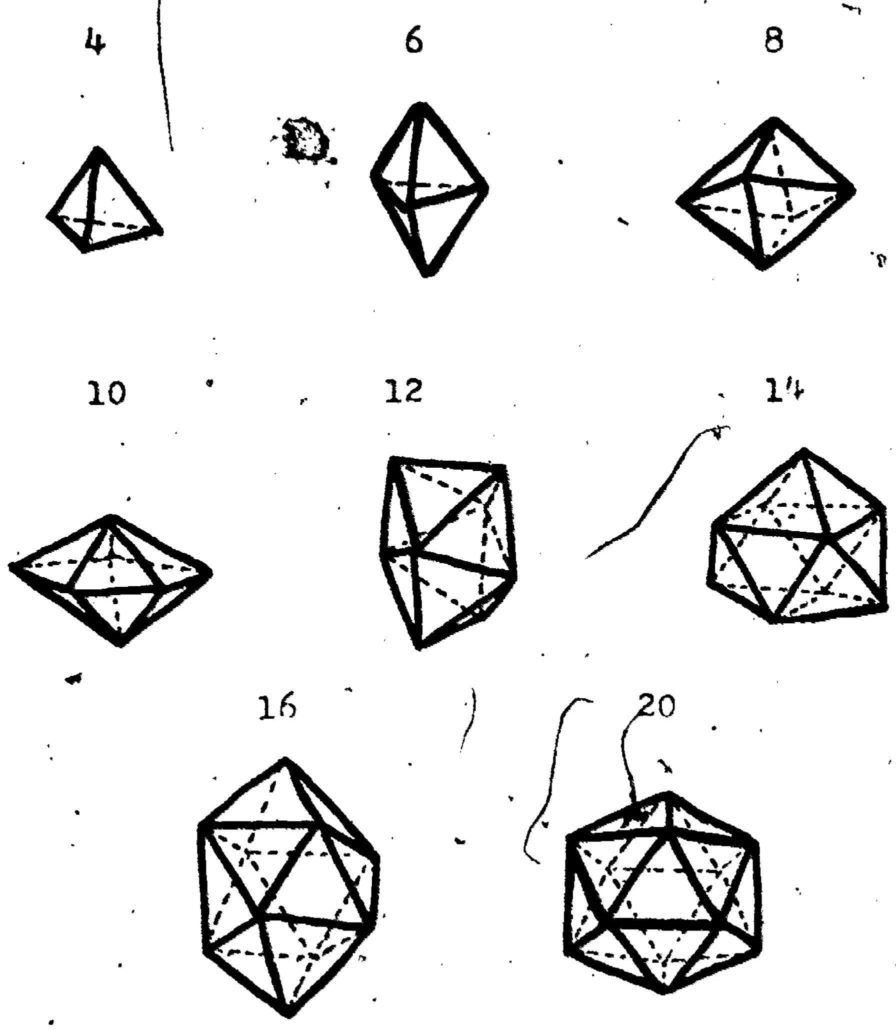
H - Consider now the following game: How many different convex solids can you make whose faces are equilateral triangles? "Convex" means that all vertices and all edges stick out--none turns in, and no pair of faces lies flat. Before jumping ahead and trying, it may be worth while doing some thinking.

In the first place, at least three faces must join at any vertex, in order to retain convexity. When <u>three</u> join at <u>every</u> vertex, we get the <u>smallest</u> possible solid that fits the rules of the game--the regular tetrahedron (C). When <u>five</u> join at <u>every</u> vertex, we get the <u>largest</u>

possible solid in the game--the regular icosahedron (D). Hence, the number of faces on any solid in the game must lie between 4 and 20. Furthermore, we just saw (G) that the number of faces must be even. Thus, it is worth while to hunt only for convex solids made of 4, 6, 8, 10, 12, 14, 16, 18, and 20 triangles.

In fact, there is one (and only one) example of eight of these nine solids. The solid with 18 faces cannot be made. Experiment will convince you of this; rigorous proof that there are eight convex solids whose faces are equilateral triangles did not come until 1947 (Freudenthal and van der Waerden, Simon Stevin 25, 115-121).

In case experiment fails, you may wish to resort to these diagrams.



THE FIVE REGULAR SOLIDS

by Alan Holden, Oct. 1969

A - Take three sticks of equal length (whose size might run anywhere from toothpicks to 3-foot lengths of dowelling) and arrange them in sequence on a piece of paper so that an end of each stick touches an end of the next. When you fiddle them so that an end of the third touches an end of the first, the sticks define the sides of an equilateral triangle. If you draw pencil lines carefully just outside the sticks, you get a map of them. If you cut out the triangle-map with scissors, you can lay it on another piece of paper and make more maps by drawing around it. Notice that you can turn the cut-out triangle around on its map into new positions in which it just fits its map. In fact, there are three such positions, all told.

B - Now, take four sticks and arrange them in a similar sequence so that the fourth joins onto the first. This time you can fiddle them into a lot of different shapes. If you make a map of one of these shapes and repeat what you did for the triangle, you will probably find that there are just two positions in which the cut-out fits its map. But if you have fiddled the sticks into a square, there are four such positions.

C - Try five sticks. You can make a great variety of shapes, some of them even having a re-entrant angle. But there is one way of arranging them so that a cut-out will fit its map in five different positions. And by now, perhaps, you can see that, with any number of sticks of equal length, you could always make one arrangement whose cut-out would fit its map in the same number of positions as the number of sticks that you used. Such a shape is called a regular polygon".

D - Certainly, the square is the most familiar of these regular polygons. And there is an equally familiar solid figure, the cube, which is a box whose six sides are all squares. Can you make other shapes of boxes out of the square? By sticking squares together edge to edge, you can make oblongs, and L-shaped pieces, and out of them you can make oblong boxes and L-shaped boxes. But some of the faces of these boxes are oblong or L-shaped and not square. The cube is the only box that you can make out of the square whose flat faces are all square.

E - What about boxes made of other regular polygons? You can make an especially simple box out of four equilateral triangles--the regular tetrahedron. It is especially simple because it has only four flat faces. Can you imagine any kind of box that has only three flat faces? Four is clearly the minimum number of faces for a flat-faced box.

F - Notice that on each of these boxes--the cube and the tetrahedron--three faces come together at each corner of the box. Try making a box out of regular pentagons (C) by making three faces come together at each corner. You can do it by using twelve pentagons, to make the regular dodecahedron. There is a widely sold desk-calendar and paper-weight having this shape, in which the days of each of the twelve months of the year are stamped on the twelve faces.

G - Can you go on to make a box out of regular hexagons? When you join three of them at a corner, the three hexagons lie flat. The only way to make a box out of regular hexagons is to leave holes in the box; a closed box can't be made. (See G2, Solids Made of Equilateral Triangles, paragraph E.) If you try regular polygons with even more sides, you can't make three join at all. So there are just three kinds of boxes of which each is made out of a single kind of regular polygon and in which three polygons meet at each corner.

H - Now, try making more than three polygons meet at a corner. Using the pentagons, you can only do it by puckering the arrangement as you go around the corner. But that would make a box with dents in it. Using the square, you can make boxes with dents or boxes with oblong or L-shaped faces (D), but no simple boxes whose faces are squares when four meet at a corner.

I - Going back to equilateral triangles, however, you can do better. When four triangles meet at each corner, you can make a box of eight triangles--the regular octahedron. When five triangles meet at each corner, you can close up a box with twenty faces--the regular icosahedron. When six triangles meet at a corner, they lie flat (unless you pucker them) and make a regular hexagon. So you can say that, if you don't want a box with dents in it, the only boxes that you can make with faces that are equilateral triangles, and with corners that are all alike, are the tetrahedron, octahedron, and icosahedron. And what's more, you can say that the only undented boxes, with corners all alike, that you can make out of any single sort of regular polygon are the tetrahedron, cube, octahedron, dodecahedron, and icosahedron.

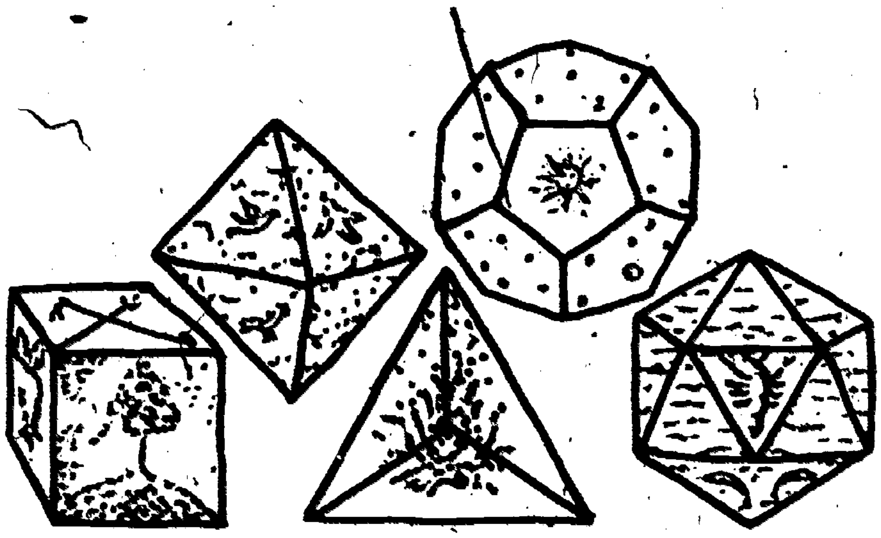
J - These five "convex regular polyhedra" are sometimes called the "Platonic solids", because they were first discovered by a member of

Plato's Academy (probably Thaetetus) more than 2000 years ago. It has been suggested that Euclid designed his famous treatise "The Elements", as a text on regular figures, because it begins with the equilateral triangle (the simplest of the regular polygons) and ends with the regular dodecahedron and icosahedron (the most complex of the regular polyhedra).

The fact that there are just five of these solids stimulated the imaginations of mystically-minded Greeks. They found a correspondence between these solids, on the one hand, and Aristotle's four Elements (Fire, Water, Earth, and Air) and the Universe, on the other. Seventeen hundred years later, Johann Kepler's unique blend of science and mysticism led him to justify the correspondence in the following way:

> Of the five solids, the tetrahedron has the smallest volume for its surface, and the icosahedron has the largest. They, therefore, represent the qualities of dryness and wetness and correspond with Fire and Water. The cube stands firmly on its base, and its stability corresponds with that of Earth. The octahedron, rotating freely when held by two opposite corners, corresponds with the mobile Air. The dodecahedron corresponds with the Universe because the zodizc has twelve signs.

To illustrate these correspondences, Kepler drew a bonfire on his tetrahedron; a lobster and fishes on his icosahedron; a tree, a carrot and gardening tools on his cube; birds and clouds on his octahedron; and the sun, moon, and stars on his dodecahedron.

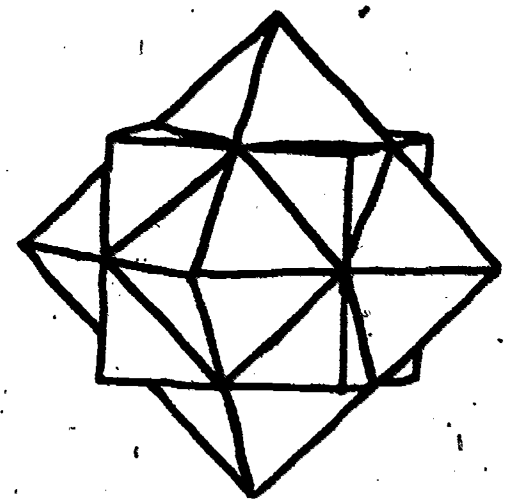


K - Now, count the number of corners, edges, and faces on each of these five polyhedra and make a table of the numbers that you get:

	corners	edges	faces
tetrahedron	4	6	4
cube	8	12	6
octahedron	6	12	8
dodecahedron	20	30	12
icosahedron	12	30	20

Notice that the table suggests pairing the cube with the octahedron, and the dodecahedron with the icosahedron. The members of a pair have the same number of edges, and the numbers of corners and faces are simply interchanged.

L - This interchange of the numbers suggests that you try making a direct comparison of the corners and faces on the members of a pair. For example, if you punch holes in the centers of the faces of a cube and put a stick into each hole, you can call the end of each projecting stick a new corner. You can connect these new corners to one another by rubber bands or string, and find that you have outlined an octahedron with a corner of the cube in each face. A similar operation on a regular dodecahedron will produce the outlines of an icosahedron with the corners of the dodecahedron in the middle of each face. A particularly graphic way of making clear these relationships is to make a polyhedron that represents the two solids <u>interpenetrating</u>.



Two polyhedra that are related by the interchange of corners and faces are said to be "dual" to each other. The tetrahedron is said to be "self-dual."

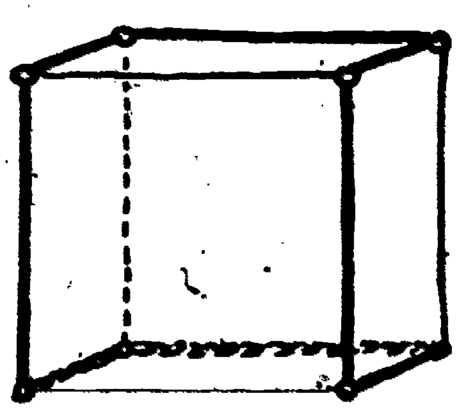
M - With regular polyhedra, you can play an interesting game invented by Sir William Rowan Hamilton about 1850. He suggested that the corners of one of these solids might represent the towns on a small planet, and the edges of the solid might represent the roads between the towns. In his game you begin at one town and travel on roads to all the rest of the towns in succession, visiting each town only once and ending at your starting place. An easy way to search for such a "Hamilton circuit" is to put little sticks along the edges of the solid one by one, using a dab of rubber cement or a tick of adhesive tape to hold each stick in place, so that you can remove it if you want to change your route.

There is another similar game you can play: can you find a route on the solid from face to face across the edge that joins them, by which you visit each face just once, and finally return to the st rting face? You might call such a route a "closed route".

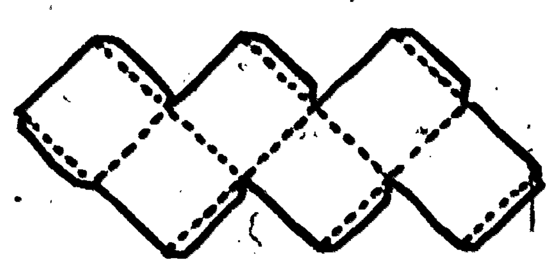
N - Notice now that if you have found a Hamilton circuit on a solid, that is the same thing as finding a closed route on the dual solid. A Hamilton circuit on a cube describes a closed route on an octahedron, etc.

There is an interesting property of a closed route that you can check by papering the solid (using Lyn McLane's techniques) and attaching the faces edge to edge in the sequence given by the closed route. When this closed sequence of faces is opened at any one edge, the faces can be laid out in a strip. Hence, such a strip forms a "strip-pattern" for constructing the solid by folding.

Diagram A shows a Hamilton circuit on a cube: can you lay out the corresponding strip-pattern for an octahedron? Diagram B shows a strip pattern for a cube: what is the corresponding Hamilton circuit on an octahedron?



A



B

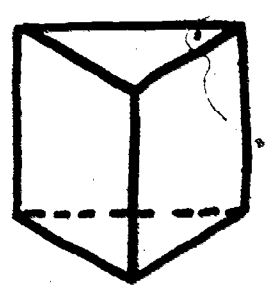
SEMI-REGULAR SOLIDS

by Alan Holden, Oct. 1969

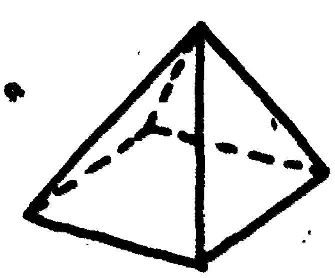
A - As you have seen in Solids Made of Equilateral Triangles, if you choose a single shape, the equilateral triangle, for making the faces of solids, you can make a great variety of solids. But if you set up a rule that the solids should be convex, you can make only eight different solids out of equilateral triangles. And, if you set up an additional rule that the corners should all be alike on any one of them, you find that only three of those eight solids fit both rules—the tetrahedron, octahedron, and icosahedron.

Now try widening your sights: what solids can you make out of regular polygons when you don't require them to be all of one kind? Since you can make so many out of triangles alone, you can expect to be able to make even more by using, say, mixtures of triangles and squares, and so indeed you can.

B - There is an interesting task that you might set yourself. Remember that all the solids which you have made so far have an even number of faces. Using a mixture of regular polygons, can you make a solid with an odd number of faces? First try for five faces. You'll find that you can make two different solids with five faces that are a mixture of triangles and squares. One (A) uses three squares and two triangles, and the other (B) uses one square and four triangles.



A



B

In fact, these are the only five-faced solids made out of regular polygons. You can see why that might be so when you think of using polygons with more than four sides. A regular pentagon, for example, would require five more polygons to join it, one at each of its sides, and the solid would therefore have at least six faces.

C - There is an important difference between these two five-faced solids, having to do with their corners. Three polygons meet at each of the corners of solid A--two squares and a triangle. But on solid B four triangles meet at one of the corners, and two triangles and a square meet at each of the other four corners. When a solid is made out of several kinds of regular polygons but all its corners are alike, as on solid A, the solid is called "semi-regular". Solid B, with two different kinds of corners, is not semi-regular.

On more complicated solids the surest way to tell whether the corners are all alike is to travel a circuit around a corner and notice the order in which you meet the various sorts of faces. On solid A that order is square-square-triangle. Calling a square "4" and a triangle "3", you can write 443 as a shortland "formula for the corner". Then, if all the corners have the same formula and the faces are all regular polygons, the solid is semi-regular.

D - The structure of solid A suggests a way of making other semi-regular solids by analogy. You can think of solid A as made of a band of three squares that form a cylinder capped at both ends by equilateral triangles. Then you can go on to make a band of four squares capped by two squares, a band of five squares capped by two regular pentagons, and so on. In a solid made by a band of N squares capped by two regular N-gons, the formula for each corner is 44N. Notice that the band of four squares capped by squares is the familiar cube; but the others are new solids, each semi-regular, called the "Archimedean prisms".

E - Another family of semi-regular solids will emerge when you examine carefully the regular icosahedron (construction shown in G2 Solids Made of Equilateral Triangles). Instead of attaching pentagonal pyramids to the cylindrical band of ten triangles, you might cap the cylinder with regular pentagons. Then you would have a solid made of ten triangles and two pentagons, and all its corners would be alike, with the formula 3335. Clearly, you could make bands with more or fewer triangles and cap them with regular polygons. In a solid made of a band of 2N triangles capped by two regular N-gons, the formula for each corner is 333N. When N is 3, the solid is the regular octahedron; but again the others are new semi-regular solids, called the "Archimedean antiprisms."

Notice that when you stand a "prism" (D) on one of its caps and look down on it, the corners of the upper cap are exactly above the corners of the lower cap. When you look down an an "antiprism", however, the corners of the upper cap are halfway between the corners of the lower cap.

F - Still another class of semi-regular solids comes into view when you examine the "truncated tetrahedron" (see Solids Made of Equilateral Triangles). You can imagine making that solid by chopping off the corners of a regular tetrahedron. If you chopped off just the right amount, the triangles forming the faces of the tetrahedron would become regular hexagons. Since the holes left at the corners are shaped as equilateral triangles, they can be covered with such triangles. The resulting solid is semi-regular, and the formula for its corners is 366. The imagined chopping is called "truncation".

G - Clearly, the other four regular solids (see G3 The Five Regular Solids) invite similar truncation. The regular octahedron can be truncated so that its triangular faces become regular hexagons and its corners are replaced by squares. Truncating the regular icosahedron to yield hexagonal faces replaces its corners by regular pentagons. Truncating the cube converts its square faces into regular octagons and replaces its corners by equilateral triangles. Similarly, truncating the regular dodecahedron yields a solid composed of regular decagons and equilateral triangles.

Notice that in these chopping operations the original faces have been replaced by regular polygons with twice as many sides, and the original corners have been replaced by regular polygons with as many sides as the number of faces that originally met at a corner. The numbers of faces on these semi-regular solids are easily counted by remembering the numbers of faces and corners on the original regular solids.

In order to count the number of <u>corners</u> on these solids, notice that the act of truncation has put two corners on each of the edges of the original regular solid. Since none of the original corners survives, the new corners are the only corners. Thus the octahedron, with twelve edges, yields a truncated octahedron with twenty-four corners.

The number of edges on these solids will be the sum of two easily found numbers. In the first place, a central fragment of each of the original edges survives. In addition, there is a new edge for each of the sides of the polygons that replace the original corners. In the case of the truncated octahedron, the twelve original edges are supplemented by the twenty-four edges contributed by the four sides of the six new squares, and hence the solid has thirty-six edges.

H - If you chop down the corners of these four regular solids still further, you replace their original faces by new faces of the same shape but smaller. For example, you can chop the corners of a regular octahedron to get a semi-regular solid made of eight triangles and six squares. Notice that you can also get the same solid by chopping the corners of the cube further than before. Similarly, further chopping of either the icosahedron or the dodecahedron will produce a solid made of twenty triangles and twelve pentagons.

Here is a reflection of the duality relation between pairs of regular solids. The members of a dual pair can both be truncated to yield the same solid. In testimony to this relationship, these two semi-regular solids are called respectively the "cuboctahedron" and the "icosideodecahedron."

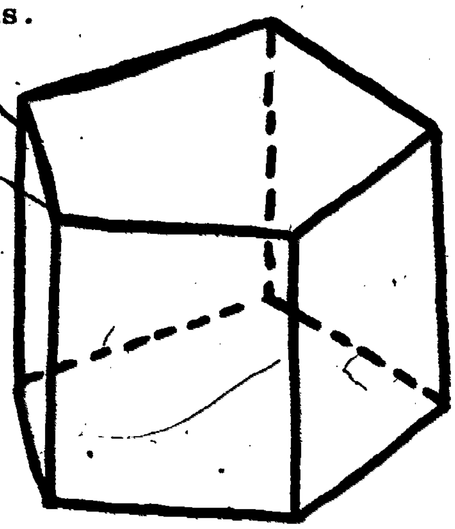
On these solids you can still count the faces by adding the numbers of faces and corners on the parental solid. Here, however, the two corners produced on each edge by truncation have coalesced to form a single corner, and thus the solid has the same number of corners as the number of edges on its parent. Since that coalescence has obliterated the edges of the parent, the only edges on the new solids are those contributed by the sides of the new polygons. In summary, the cuboctahedron has 14 faces (6 + 8), 12 corners, and 24 edges (6 x 4).

The coalescence of corners just mentioned suggests an interesting way of looking at these two solids. The new corners arise at the midpoints of the obliterated edges of the parental regular solids. Thus, the new solids can be outlined by stretching strings between the midpoints of the edges of the parent.

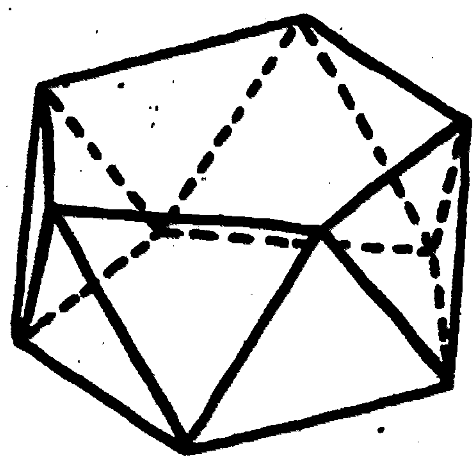
I - Why stop? Chop, chop! If you continue to chop the octahedron beyond the cuboctahedron point, you will soon get a truncated cube; and finally all relics of the octahedron's faces will disappear and you will end with a little cube. Similarly, if you truncate a cube to the bitter end, you will get a little octahedron. The sequence octahedron, truncated octahedron, cuboctahedron, truncated cube, cube, can be traversed in either direction by truncation. This is an especially dramatic result of the fact that the cube and the octahedron are dual to each other.

J - Truncation of regular solids has now yielded you seven semi-regular solids, described in F (one solid), G (four solids), and H (two solids). Are there more? The question seems to have interested Archimedes. He found six more, closely related to these seven, but not obtainable by truncation. Three of them are made of three different kinds of regular polygons, not just two kinds. Archimedes is believed to have written a book about all thirteen of these solids, but it has been lost, and our knowledge of the ancient Greek concern with them is largely due to Pappus' great work, "The Collection". In more recent times it has been proved that these thirteen, and the prisms and antiprisms, are the only possible convex semi-regular solids. They are diagrammed and named below.

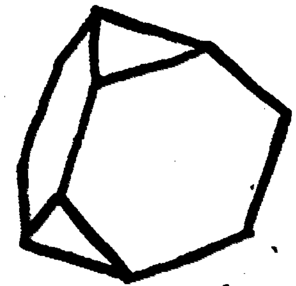
K - Two of these solids have an especially interesting property. The snub cube and the snub dodecahedron can each be made in two forms, which are mirror images of each other and are not superposable. The two forms are analogous to a right hand and a left hand. You might say, "They are alike, but not the same". Indeed, perhaps you should say that there are fifteen, rather than thirteen, of these semi-regular solids.



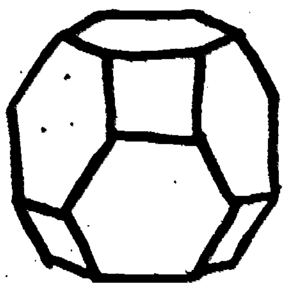
Pentagonal
Archimedean
prism



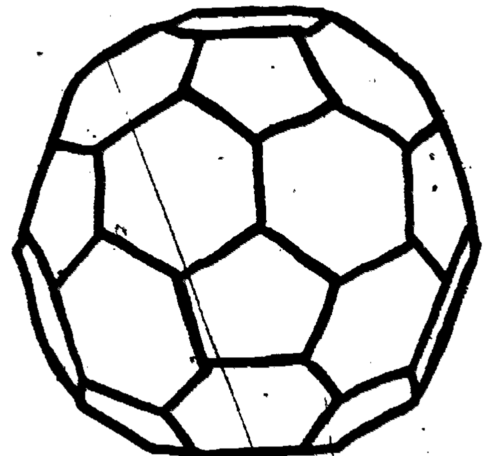
Pentagonal
Archimedean
antiprism



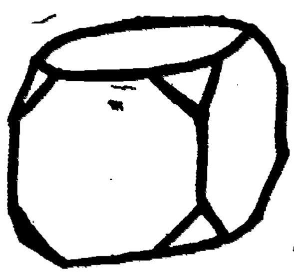
Truncated tetrahedron



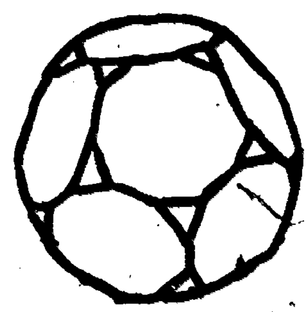
Truncated octahedron



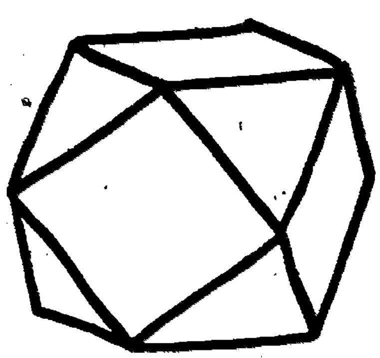
Truncated icosahedron



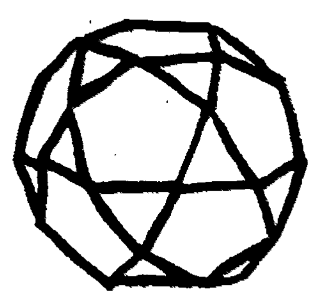
Truncated cube



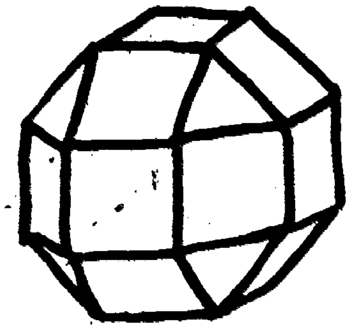
Truncated dodecahedron



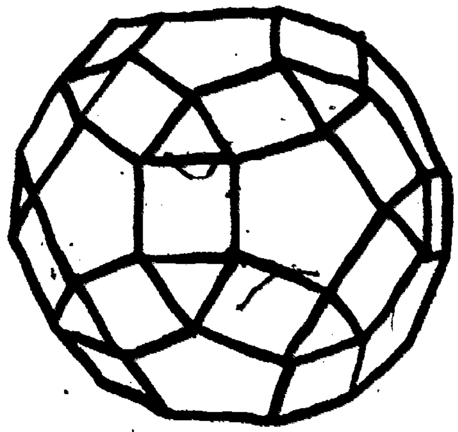
Cuboctahedron



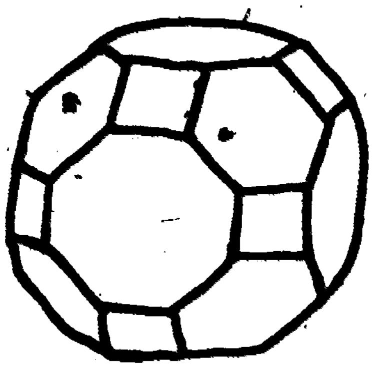
Icosidodecahedron



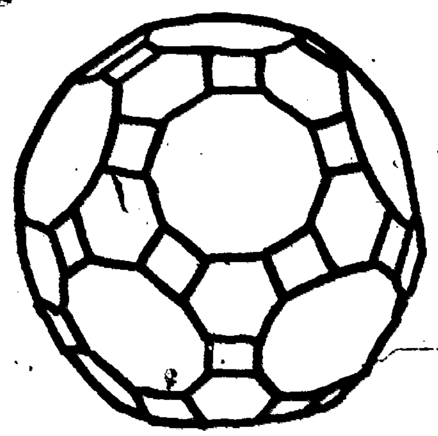
Small
rhombicuboctahedron



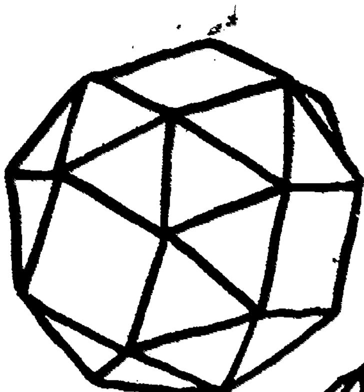
Small
rhombicosidodecahedron



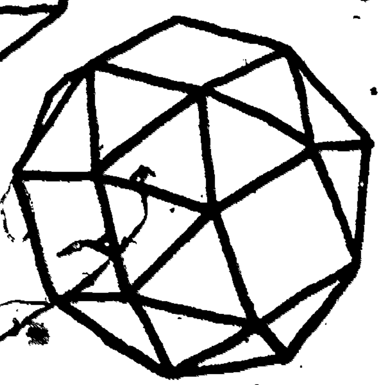
Great
rhombicuboctahedron

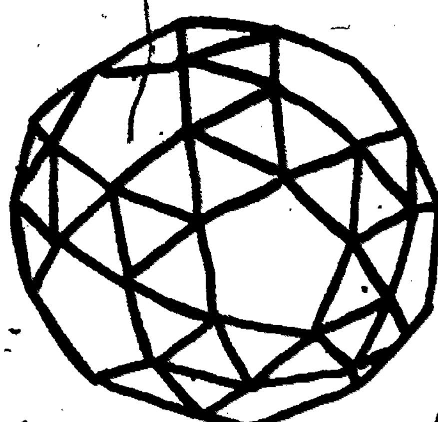


Great
rhombicosidodecahedron

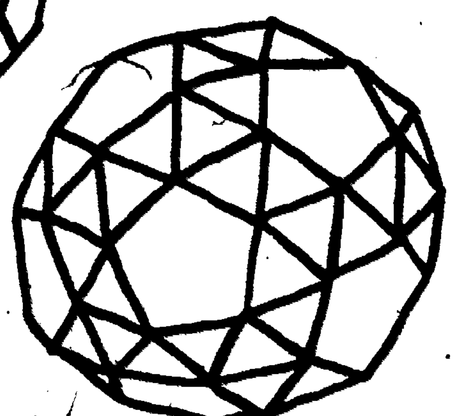


Snub
cube





Snub
dodecahedron



"FAIR" AND "REGULAR" POLYHEDRONS

by

Professor Earle Lomon

Polyhedrons: A polyhedron is a closed three-dimensional figure bounded by intersecting plane surfaces. The plane surfaces form the faces of the polyhedron. Pairs of plane surfaces intersect in straight lines which form the edges of the polyhedron. These edges bound areas of the plane surfaces which are the faces of the polyhedron. As the plane faces are bounded by straight edges, they form polygons. Edges are common to two polygons in that the length of edges of polygons have to match in pairs. Where three or more faces meet at a point they form a corner of the polyhedron. This is also a corner of each of the adjoining faces, and a point of intersection of three or more edges. Equal number of faces and edges meet at that corner.

Convex Polyhedrons: If the angle made by the two faces at every edge points outward then the polyhedron is called convex. Every face of a convex polyhedron can be put in complete contact with a large flat surface, such as a table. None of the faces are tucked inside (in concave sections). The faces only touch at their edges and do not intersect elsewhere.

Regular Polyhedrons: If a polyhedron is regular then every face is like (congruent to) every other face, every edge is like (same length and same angle as) every other edge, and every corner is like (same number of faces and edges coming in at the same angle as) every other corner.

Fair Polyhedrons: When rolling a convex polyhedron as a die it will eventually stop with a face in contact with the table. If the polyhedron is concave it may stop on some edges enclosing three of more faces in a hollow. From the point of view of dicing this is as if there was a face or faces covering the projecting edges (enclosing the concavities in the smallest convex polyhedron). This is equivalent to a convex polyhedron, possibly with "loaded" weight distributions in the interior. Consequently we will refer only to convex shapes below.

Fair Dice: Assuming that one and only one face of a polyhedron is assigned to each player, then each player has an equal chance of his side coming to rest on the table if every face is the same in all respects as any other face, (presuming the die to be made of an entirely homogeneous material). This means more than that every face is congruent to every other face. It also means that faces meeting adjacent faces at congruent edges do so at the same angle. It further means that congruent corners of faces meet at the same type of corner of the polyhedron. But it does not mean that all edges are congruent or that all corners are alike. A face of a polyhedron may have differing edges and corners; but every other face must have the same distribution of edges and corners. Every regular polyhedron is fair but not every fair polyhedron is regular.

For example, the regular tetrahedron is made out of four equilateral triangles, three equal angles and edges meeting at each corner. But a tetrahedron can also be constructed out of four scalene triangles, the three different angles and edges meeting at each corner. The latter tetrahedron is fair but not regular because not every edge is like every other edge.

There are also simple examples of fair polyhedrons in which corners as well as edges differ (but their distribution is the same for every face). An example is often constructed by youngsters by sticking together two regular tetrahedrons at a face of each. At the "equator" where the faces are stuck together, four triangles come to a corner. The remaining two corners at the "poles" are the normal tetrahedron three-sided corners. Isosceles triangles could be used. A more complicated example is the rhombic dodecahedron.

Partial Fair Dice: There are also partial fair dice which can well be used for fair games. These are polyhedrons in which some faces are the same in all respects as each other, but the remainder differ from the first set (and maybe from each other also). An example is a regular prism, in which all the "sides" are the same, but the "ends" differ from the sides. For instance, for a triangular right prism the ends are two congruent equilateral triangles and the three sides are rectangles. The three rectangles are fair for three players. If the prism ever does land on one of its ends, then that throw is discounted and a new throw is made.

Exploration and Discovery of Regular Polyhedrons: A regular polyhedron is necessarily made out of regular polygons (as it has all equal edges and angles) so one may consider each regular polygon one at a time. Starting with the polygon with fewest edges, the equilateral triangle, one tries how many can be put together to form a corner of a tetrahedron. Two polygons of any type will be stuck together making a two-dimensional figure unless spaces by a third polygon. Hence we need at least three triangles at a corner. When three are put together the base forms another such equilateral triangle. Placing a fourth triangle over the base completes the regular tetrahedron (the regular polyhedron with the least number of faces).

Putting four equilateral triangles around a corner makes a "flatter" corner. Make another such four-sided corner and stick them together at their bases. Around the equator four triangles come together at each corner just as at the poles! This is the regular octahedron.

Putting five triangles together makes a still flatter corner. If we stick two such together by their bases, the corners on the equator will have four, not five, faces around them. This would be fair but not regular. One could make a similar fair figure out of isosceles triangles. However one can make a girdle of ten equilateral triangles in a row for the equator capping it on each side by the five-sided corners mentioned above. Three triangles from the girdle meet at a point with two triangles from a cap, hence five-sided corners around the girdle just as at the poles! This is the regular icosahedron.

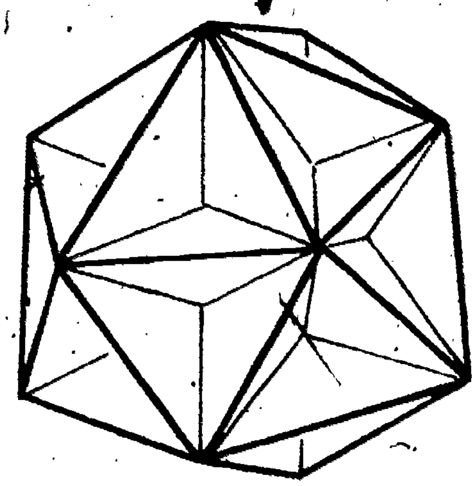
Well, now, let's put six triangles together at a corner. It is completely flat, hence not leading to a convex shape. $6 \times 60^\circ = 360^\circ$ The six triangles can be laid out on the table filling in all around the point. If one keeps adding triangles around the outside of this cluster, one can fill in an infinite plane. Seven triangles put together will not even lie flat without folding the triangles in and out. Therefore, triangles lead to only the above three regular polyhedrons -- tetrahedron, octahedron, and icosahedron. It is beginning to be apparent that there are not to be many convex regular polyhedrons.

Next, one can try the faces of the four-sided regular polygon, the familiar square. Putting the minimum of three together to form a corner, one finds that two such corners fit together to form the familiar <u>cube</u>. Try putting four squares together at a corner, and it is already flat! Five or more squares would have to fold in and out.

The next polygon is the regular pentagon. Three of them can be put to a corner. Two such corners will not fit together. But a band of six pentagons can form an equator into which the two polar caps will fit. This makes the dodecahedron. Four pentagons at a corner require folding.

Next try the hexagon. Three of these at a corner make a flat surface, and more require folding. Three seven-sided regular polygons need folding. All the convex regular polyhedrons have been found! There are five.

<u>A note on non-convex regular figures:</u> In non-convex figures, some plane surfaces will intersect in the interior of the polygonal surfaces as well as at their edges. If one only counts the bounding intersections as "edges" of the polyhedron, then one can have cases where all of these exterior "edges" and their associated corners are the same. Then the figure is called regular, although the interior edges and corners will differ from the exterior ones. A case is shown below.



In this case five regular pentagons are interlaced. The exterior edges are heavily outlined and show the regularity of the figure. Note that the whole thing could be covered by 20 triangles forming a convex regular polyhedron, the icosahedron. For dicing purposes only the icosahedron counts.

MASS PRODUCTION OF EQUILATERAL TRIANGLES AND SQUARES

by

Louise Buckner and Frank O'Brien

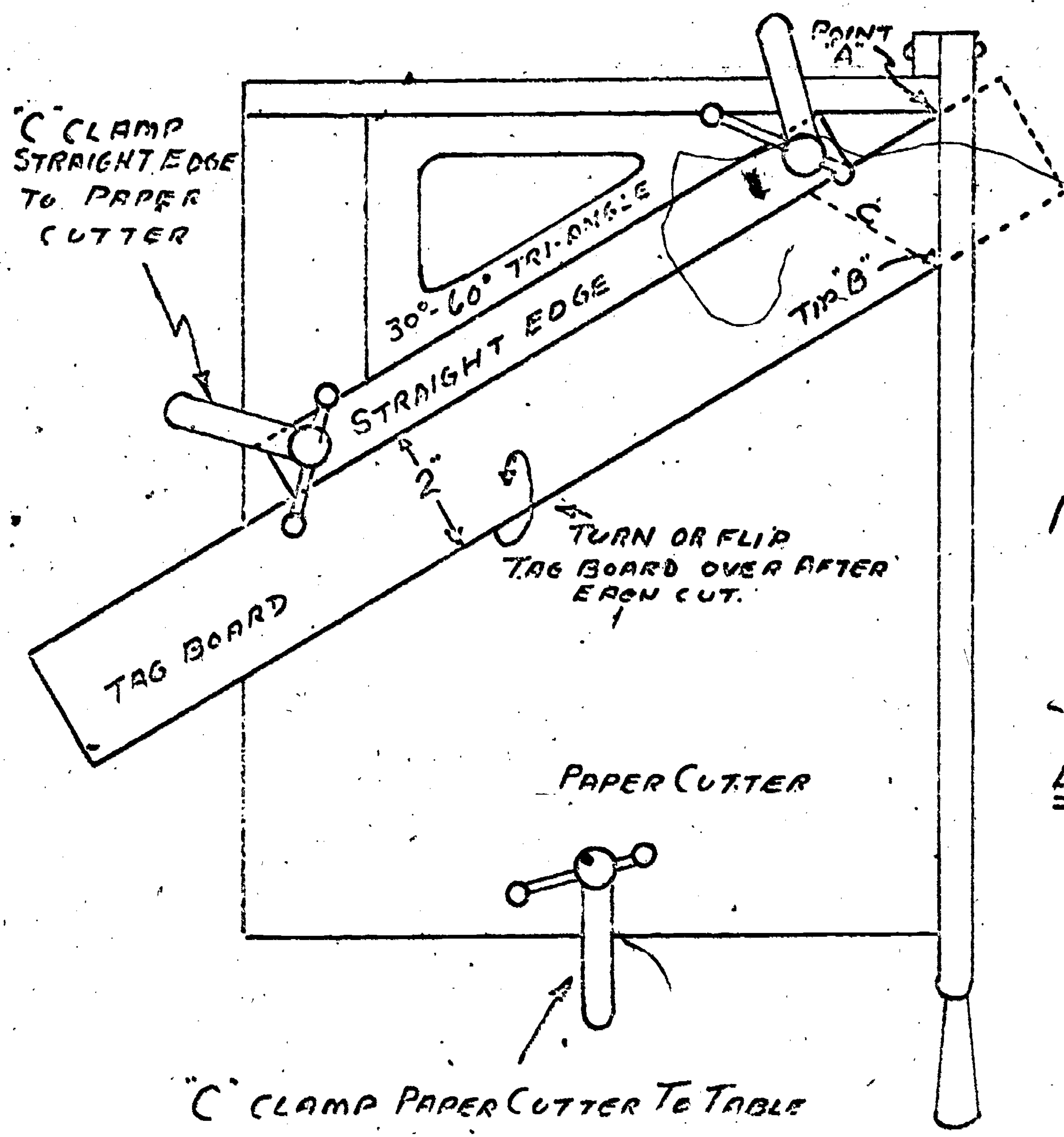
Equilateral Triangles

Equipment:

1. Paper cutter
2. Tagboard cut in pieces no longer than cutter blade length
3. Three "C" clamps
4. One 30°-60° plastic triangle
5. Straight edge

Procedure:

1. Determine the desired length of side, in this case 2".
2. Cut strips of tagboard 2" wide.
3. Clamp straight edge to cutter board according to sketch page 32.
4. Place a tagboard strip along the straight edge.
5. Pull cutter blade down; discard the first piece of tagboard cut.
6. Flip tagboard strip over and cut again (see sketch).
7. Repeat the above process flipping the tagboard over before each cut until strip is used up. This could be called a flip-flop procedure.



1- FIRST CUT IS WASTED
2- FLIP TAG BD. OVER AND PLACE TIP B RIGHT UP TO POINT "A" ON PAPER CUTTER
3- YOU WILL THEN BE CUTTING ALONG DOTTED LINE "C".
4- KEEP FLIPPING AND CUTTING IN THIS MANNER UNTIL PIECE IS USED UP.

NOTE:
USE 30°-60° TRIANGLE TO SET ST. EDGE TO PROPER ANGLE

CUT TAG BD. INTO 2" WIDE STRIPS.

EQUILATERAL TRI-ANGLES

MANY PIECES CAN BE CUT UNIFORMLY ON A PAPER CUTTER.

(ISOSCELES TRI-ANGLES ARE CUT IN THE SAME MANNER. JUST SET STRAIGHT EDGE TO PROPER ANGLE

Squares

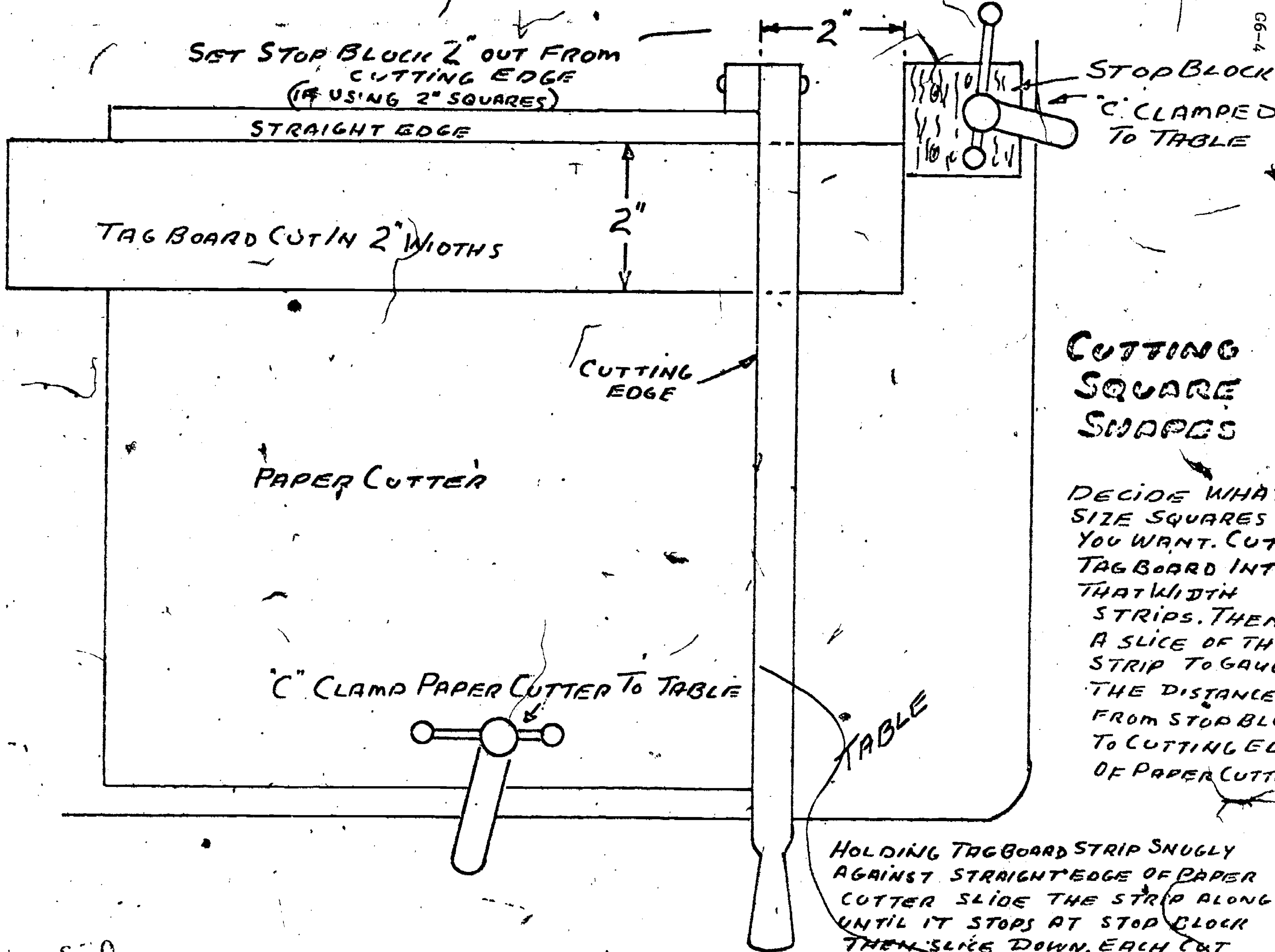
Equipment:

1. Paper cutter
2. Two "C" clamps
3. One 2" x 4" x 4" wood block for stop
4. Tagboard cut in pieces no longer than length of paper cutter blade
5. Yardstick

Procedure:

1. Determine the desired length of side, in this case 2".
2. Clamp wood stop to cutter board or table according to sketch page 34.
3. Place pieces of tagboard on cutter board against stop and cut 2" wide strips.
4. Place long edge of tagboard snugly against the straight edge on paper cutter (see sketch) and simultaneously place short edge of tagboard snugly against the stop and slice off 2" x 2" shapes.
5. Continue cutting procedure until strip is used up.

G6-4



CUTTING SQUARE SHAPES

DECIDE WHAT SIZE SQUARES YOU WANT. CUT TAGBOARD INTO THAT WIDTH STRIPS. THEN US A SLICE OF THAT STRIP TO GAUGE THE DISTANCE FROM STOP BLOCK TO CUTTING EDGE OF PAPER CUTTER

HOLDING TAGBOARD STRIP SNUGLY AGAINST STRAIGHT EDGE OF PAPER CUTTER SLIDE THE STRIP ALONG UNTIL IT STOPS AT STOP BLOCK THEN SLICE DOWN. EACH CUT WILL YIELD A UNIFORM SQUARE.

A VOTING PROCEDURE COMPARISON THAT MAY ARISE IN USMES ACTIVITIES

by

Earle L. Lomon

Voting is used to make group decisions. There are in many cases a variety of voting procedures that seem reasonable but can lead to different decisions. When this is the case a group may wish to investigate the different possibilities to find out which voting procedure leads to the decision that will most satisfy the members of the group. A method for rating group satisfaction is outlined on page 6. The voting procedure with the highest satisfaction rating will probably be accepted as the fairest.

In this paper we describe a situation which may arise often in USMES units:

> After investigating several ways of handling a problem such as a lunchline arrangement, a school crossing, a bus schedule, etc. a group needs to decide which alternative it will recommend to the principal, parents, town government, etc. Although they have found out many facts, they must still make a value judgment. One way may be more convenient but also more expensive.

Let us assume that there are at least three ways that have enough good features so that someone in the group prefers it. In fact the discussion below will apply to any situation where a group must decide between more than two alternatives.

If they take a vote as to which alternative is the favorite the plurality* may go to the case which is strongly disliked by the majority. The number of strongly dissatisfied people may be greater than for another choice. A different voting procedure would be to vote for elimination for one alternative,

* A number of votes greater than the number cast for any other option but less than half the total votes cast.

then another, and so on until only one is left. This will lead to a compromise choice, often with more overall satisfaction. A detailed analysis of the two procedures will be made below for a particular case. There are other ways of voting, such as separating the alternatives into two sets and then voting to determine the favorite set. Then subdividing the winning set into two sets, and so on. In some classes that may be suggested and the children will want to investigate the effectiveness of such procedures.

An example:

Preference Assumptions

For the sake of definiteness, we assume that the group has been working on the Pedestrian Crossing unit. The reader will have no difficulty in imagining parallel circumstances in other units.

Suppose that a class doing the Pedestrian Crossing unit has done its investigation and is now deciding to recommend one of the following to the town traffic department:

A. Build an overhead crosswalk.

B. Put a police officer on duty.

C. Install a walk light.

D. Paint white stripes to mark a crossing and install a stop sign.

E. Mark the crossing with a caution sign only.

F. Do nothing.

[A,B,C,D,E,F represent the set of outcomes.]

To go further with a specific example we must now specify the preferences of the group of children; not only their first preference, but their second, third, etc. For those whose first preference is not extreme, we may assume that the order of their preferences is for their nearest neighbors, etc. However, among their nearest neighbors, they have a choice and they can either be "right leaning" or "left leaning". Therefore, those who like A best probably will have a "preference order" (A B C D E F). Those who like F best have a "preference order" (F E D C B A), while those who like D best have a "preference order" of either (D E C F B A) or (D C E B F A). The set of preference orders of a group, together with the voting procedure chosen, determines the outcome.

Now, of course, many individuals may have a different preference order than those assumed above. That will in no way hinder the class from evaluating voting procedures by the same method used below. If their preference orders are very different from the assumed case it may even change the result as to the most acceptable voting system. However, chances are that voting preference order will be similar enough to the above to validate the same voting procedure.

Further for the sake of explicitness, we will assume a certain distribution of first preferences in the group of twenty-five children involved. As the class had uncovered evidence of at least some danger at the crossing, it will be assumed that none vote for F and only three for E. With varying opinions about how much money should be spent versus how much care and waiting should be expected of the pedestrian, we find the vote totals as follows:

Votes for:	Recommendation
7	A (overhead crosswalk)
5	B (police officer)
5	C (walk light)
5	D (painted crosswalk + stop sign)
3	E (caution sign)
0	F (do nothing)

A is by far the most expensive and in this case the amount of pedestrian and automobile traffic can be adequately handled by the other means. Nevertheless, it is not unlikely that a substantial number of the class, even a plurality, will be prone to choose the most dramatic solution. Because of the variety of choices of roughly equal worth (B C D), the minority voting for the less thoughtful choice (A) are a plurality! What will be the outcome of voting and the group's overall satisfaction with that voting?

The Vote

First, let us check the outcome for the two different voting procedures. In the first procedure, each one votes for his favorite choice. A gets seven votes and wins by the plurality rule. A is the outcome of the first procedure.

In the second procedure the class decides to eliminate one possibility at a time by each person voting against the choice he likes least. In the first round the seven persons preferring choice A, the five B preferrers, and the five C preferrers will all vote against F, the most distant choice from all of them. The five D preferrers and the three E preferrers will vote against A. Therefore, the vote totals are as follows:

Votes against:	Recommendation
8	A (overhead crosswalk)
17	F (do nothing)

F is eliminated. A, B, C, D, E remain.

In the next round the seven A and five B preferrers vote against E, while the three E and five D preferrers vote against A. The five C preferrers are likely to be split against A and E. But even if they give three votes against A and only two vote against F, the vote will be as follows:

Votes against:	Recommendation
11	A (overhead crosswalk)
14	E (caution sign)

E is eliminated. A, B, C, D remain.

In the next round the three E, the five D and the five C preferrers will all vote against A, the furthest remaining choice. The seven A and the five B preferrers will vote against D.

Votes against:	Recommendation
13	A (overhead crosswalk)
12	D (painted crosswalk + stop sign)

A is eliminated! B, C, D remain.

In the next round the seven A and five B preferrers vote against D, while the five D and three E preferrers vote against B. The five C preferrers will split their vote. Even if they put three of their votes against B and only two against D, the totals will be:

Votes against:	Recommendation
11	B (police officer)
14	D (painted crosswalk + stop sign)

D is eliminated. B, C remain.

In the final round, the seven A and five B preferrers will vote against C, but the five C and five D and three E preferrers will vote against B.

Votes against:	Recommendation
13	B (police officer)
12	C (walk light)

B is eliminated. C is the victor of the second voting procedure.

Satisfaction Rating

In deciding which voting procedure satisfies the class most, the children may decide to each give a voting procedure a rating of five points if its outcome is that child's favorite choice, four points if it leads to a second favorite choice, three points for the third favorite choice, two points for the fourth favorite, one point for the fifth favorite, and no points for the least favorite. What will be the total point score for each voting procedure?

In the first procedure the outcome is A, so the A preferrers will each give the procedure a rating of five. Three of the B preferrers will give the A outcome a four rating, and two will give it a three rating. Conversely, two of the C preferrers will give a two rating and three a one rating. The D and E preferrers will each give the choice of A a zero rating. The total satisfaction rating for A is:

A preferrers:	7 x 5	=	35
B preferrers:	3 x 4	=	12
	2 x 3	=	6
C preferrers:	2 x 2	=	4
	3 x 1	=	3
D preferrers:			0
E preferrers:			0
			60

In the second procedure the outcome is C. The A preferrers give this outcome a rating of three. The B preferrers give it a rating of four or three. The C preferrers rate it five, of course. The D preferrers rate it four or three, and the E preferrers rate it two. The total satisfaction rating for C is:

A preferrers:	7 x 3	=	21
B preferrers:	2 x 4	=	8
	3 x 3	=	9
C preferrers:	5 x 5	=	25
D preferrers:	2 x 4	=	8
	3 x 3	=	9
E preferrers:	2 x 3	=	6
			86

The second voting procedure leads to nearly 50% more satisfaction than the first procedure!

Some Remarks on Content

The social science content of the above type of activity is very strong. Fair, satisfying voting procedures are the essence of stable democracy. The very fact that outcomes can change with the choice of different procedures will surprise many students, but is used by every leader and politician to make it more likely that his view will prevail. The criteria for the "best" procedures involve basic value questions of humanity, fairness, and stability.

The processes of considering different possibilities, making sets of hypotheses, trying different procedures and evaluating results are all involved. This is a good example of the scientific process at work.

In terms of mathematics, there is of course, multiplication, addition, and comparison. There is also an important amount of logical ordering and systematic organization of the type that is at the heart of much creative mathematics. In particular voting analysis provides a real use of the nomenclature, methods, and ideas of set theory.

The first time the students compare voting procedures and discuss voting models it is not a good idea to encumber the straightforward examination with the definitions and general concepts of set theory. Some of the language may come in in a natural way. But if the students go on to consider a large variety of voting procedures on several issues, they may find it relevant to formulate the general aspects. This will bring them into contact with sets and their properties in a meaningful way.

In an intensive survey of voting methods and results, it will be helpful to be explicit about:

1. the set of possible outcomes,
2. the set of all possible preference orders,

3. the subset of preference orders likely to be followed by the voters*,
4. the set of voting procedures to be studied (which may be divided into classes such as those which depend on pluralities, or those in which only one vote is taken, etc.),
5. the set of outcomes obtained by the various voting procedures using the stated preferences.

The voting procedure considered may become quite complex, including successive subdivisions of the set of outcomes, or the use of amendment procedure. Other technical papers will address those topics.

* Or a probability distribution of such subsets.

NOTES ON THE USE OF HISTOGRAMS

FOR PEDESTRIAN CROSSING PROBLEMS

by

Prof. Percy Pierre and Prof. Donald Coleman

Two important kinds of data for a pedestrian crossing are the time it takes a student to cross and the time between the passage of two consecutive cars. If the time between the passage of two consecutive cars is large enough compared to the time it takes a person to cross, then the intersection is "safe". The crucial uncertainty in the last statement is what do we mean by large enough. The use of histograms is one means of dealing with this question.

Consider the following experiment at an actual intersection. Using a stop watch, measure and record the actual times it takes many different people to go from one side of the street to the other. These numbers are called crossing times. The data may be recorded as follows:

Crossing Times

1st person	3 sec.
2nd person	4 sec.
3rd. person	1 sec.
4th person	4 sec.

In order to conserve space, an alternative way of listing the data is: 3, 4, 1, 4, 5, 2, 1, 3, 6, 2, 6, 3, 4, 6, 4, 7, 5, 4, 4, 3, 3, 1, 2, 4, 3, 5, 2, 5, 4, 3, 1, 9, 2, 3, 5, 6, 4, 5, 7, 2 seconds

We also need data on the times between the passage of cars. One way of doing this is for the timer to start his stop watch when the first car passes. When the second car passes he calls out the time and continues to call out the times of each succeeding car until the experiment is ended.

The raw data would look like:

1st car	0 min.	0 sec.
2nd car	0 min.	6 sec.
3rd car	0 min.	7 sec.
4th car	0 min.	10 sec.
5th car	0 min.	16 sec.
.	.	.
.	.	.
14th car	1 min.	3 sec.
.	.	.
.	.	.
.	.	.

The gap times are the times between cars. They are obtained from the previous data as:

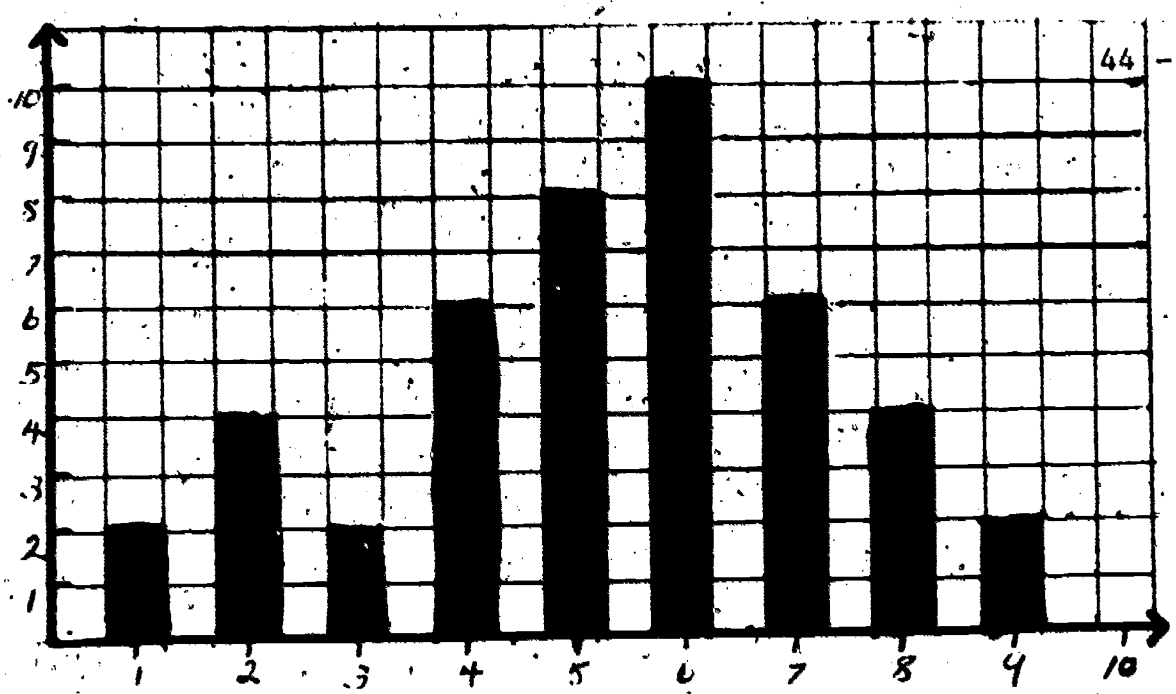
1st & 2nd car	6 sec.
2nd & 3rd car	1 sec.
3rd & 4th car	3 sec.
.	.
.	.
.	.

In order to conserve space we list the gap times as follows: 6, 1, 3, 6, 3, 6, 4, 5, 7, 6, 1, 5, 6, 4, 8, 5, 9, 8, 9, 8, 5, 7, 2, 4, 5, 5, 4, 7, 2, 7, 4, 6, 4, 7, 5, 2, 7, 6, 8, 6, 2, 5, 6, 6, seconds

One way of visually presenting data and making visual comparisons is with histograms. Histograms for the two sets of data are presented below in Figs. 1 & 2.

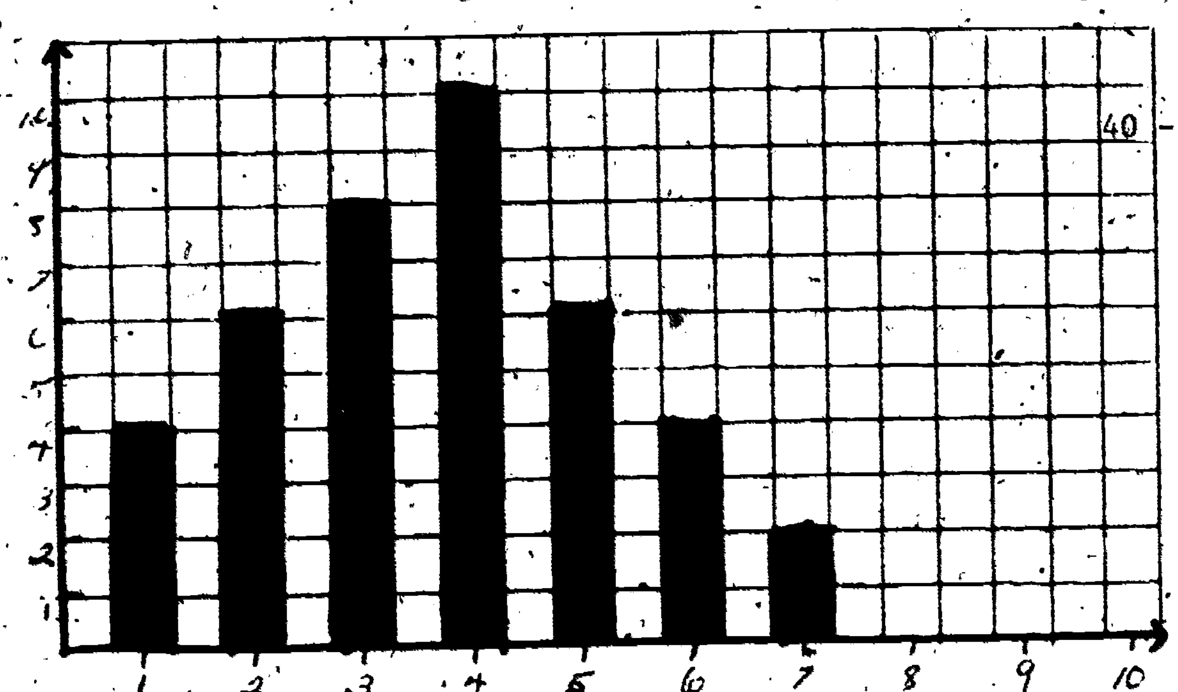
Gap Histogram

Fig. 1



Crossing Time

Fig. 2

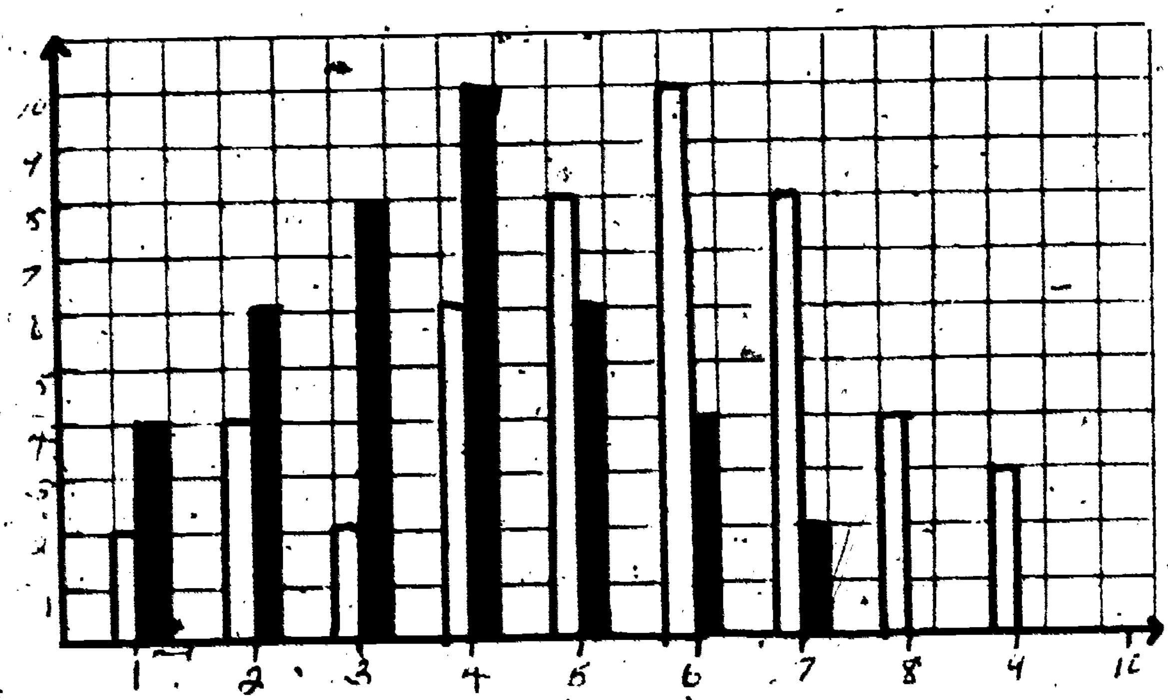


A third graph is drawn in which the two histograms are shown on the same sheet. (See Fig. 3) It is clear from just looking at the data that the gap times are frequently large while the crossing times are frequently small. If data was collected at some other intersections you may find that the gap times are more frequently smaller than crossing times.

Another more mathematical way of comparing the two sets of data is to compute the means of the two sets of data and compare. We can also compare the medians or the modes. Such comparisons will give us some feel for the situation but it may not be enough. Even if the mean gap time is larger than the mean

Combined Histograms

Fig. 3



The dark columns are crossing times.
The light columns are gap times.

crossing time, some slow people will still have difficulty crossing, and even if the mean gap times are smaller than the mean crossing time, some fast people will still manage to cross. In order to do more precise comparisons we can use the cumulative distribution diagram Fig. 4. Note that 85% of the children have crossing times of less than 5 sec. This means that gap times of 5 sec. or more will allow 85% of the children to cross.

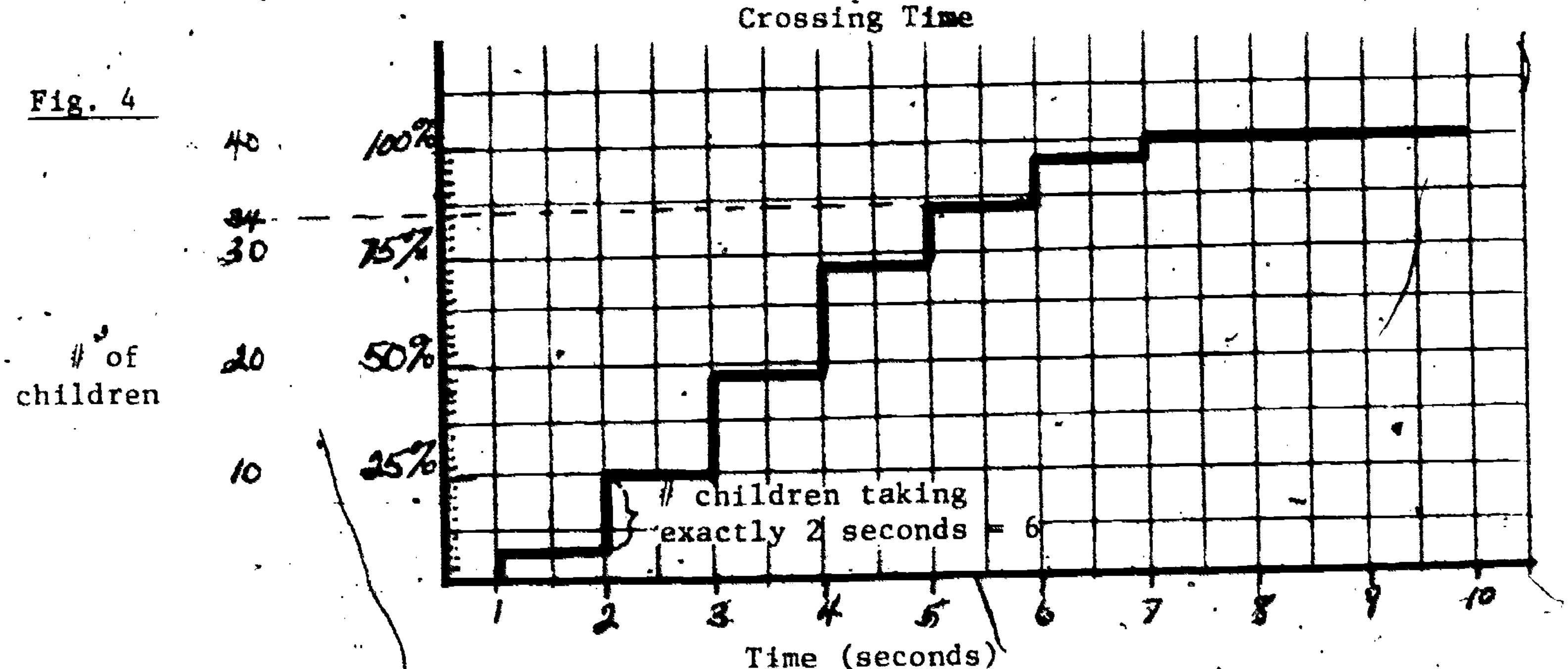
If one looked at the histogram (Fig. 1), one would see that only 30 of the 44 gap times are 5 sec. or larger. This means that for 85% of the people only 70% of the gaps are large enough. This last statement is as precise as one would want to get.

In summary, we would say that a safe intersection can be defined in many different ways. Most reasonable ways will compare the two sets of data on gap times and crossing times. An appropriate measure of comparison is to a large extent arbitrary.

Cumulative Distribution

Crossing Time

Fig. 4



Note that 34 of the children (85%) have a crossing time of 5 sec. of less.

USMES

NOTES ON DATA HANDLING

by

Dr. Percy Pierre

Suppose we take 50 pieces of data each on the time it takes children to cross the street and the time of the gaps between cars.

For example:

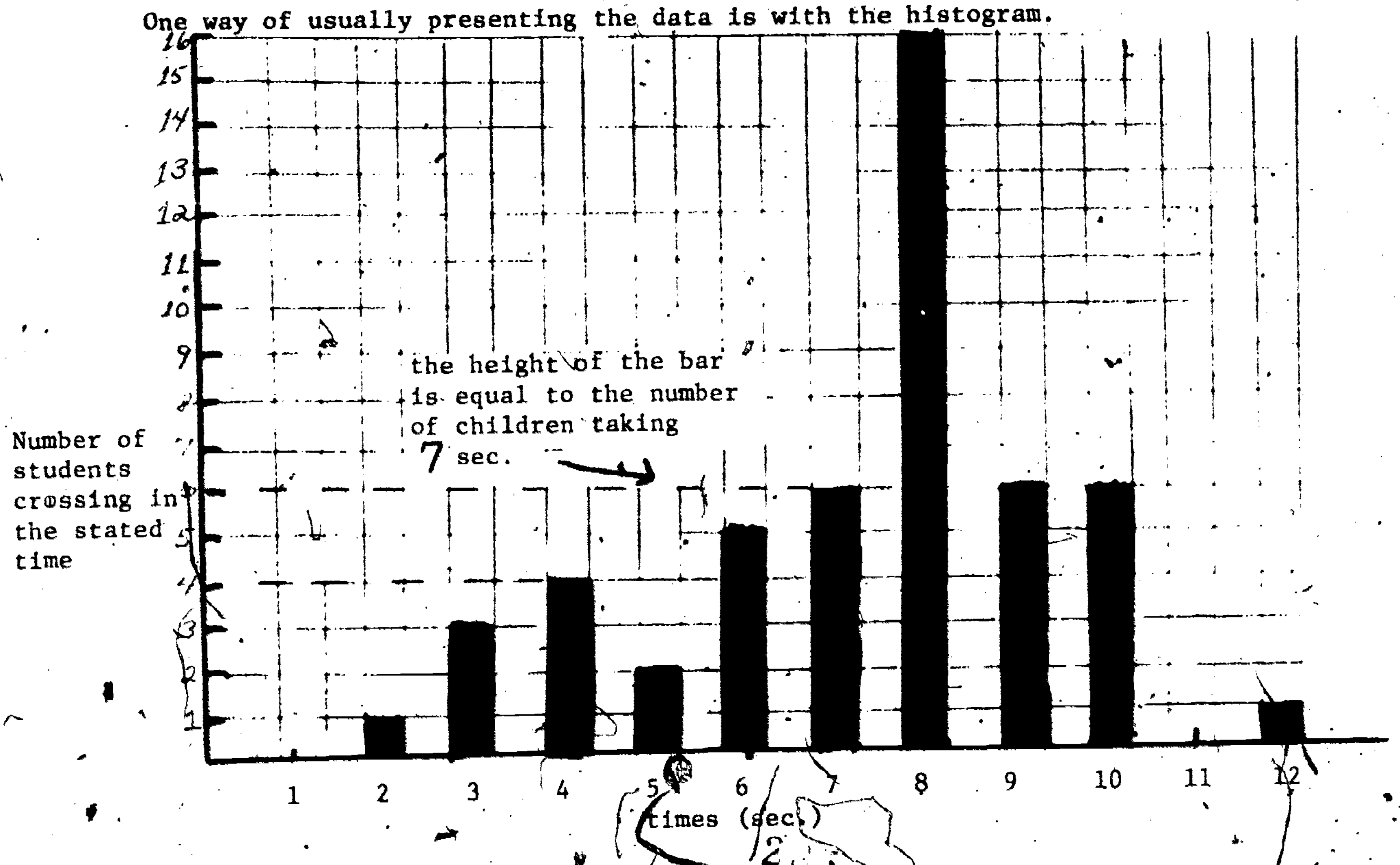
<u>Crossing times (sec.)</u>

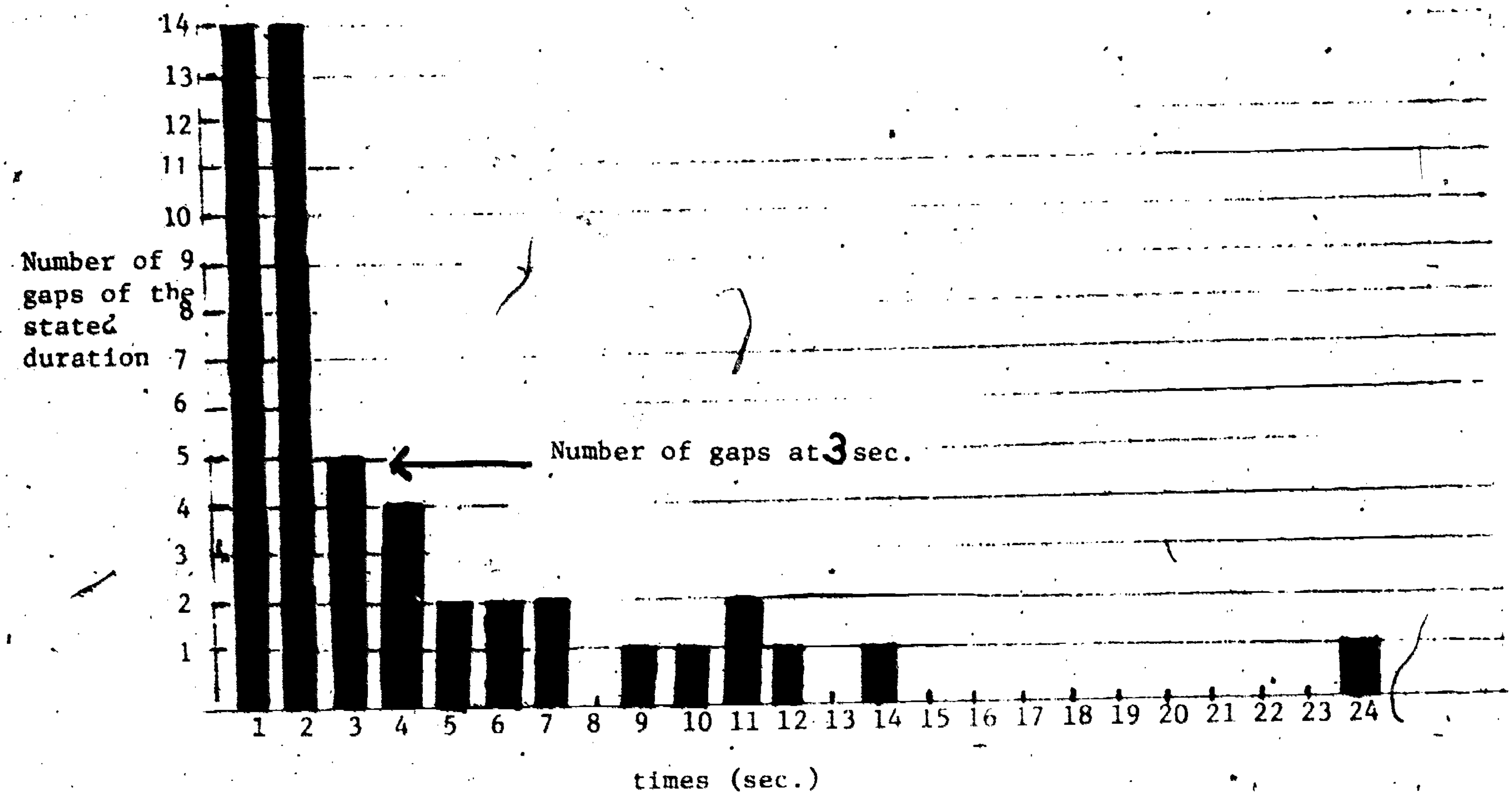
8, 5, 9, 10, 4, 9, 6, 8, 6, 6, 8, 3, 9, 6, 8, 10, 3, 5, 2, 10, 7, 8, 8, 9, 7, 8, 10, 7, 8, 9, 4, 8, 8, 6, 7, 3, 10, 9, 8, 10, 12, 4, 4, 8, 8, 8, 8, 8, 7, 7

<u>Gap times (sec.)</u>

2, 2, 11, 2, 2, 4, 7, 2, 1, 1, 1, 2, 2, 6, 3, 4, 2, 5, 3, 2, 1, 7, 12, 3, 1, 1, 1, 1, 9, 14, 1, 1, 2, 2, 1, 3, 6, 1, 11, 2, 1, 2, 4, 1, 4, 2, 24, 10, 5, 3

One way of usually presenting the data is with the histogram.





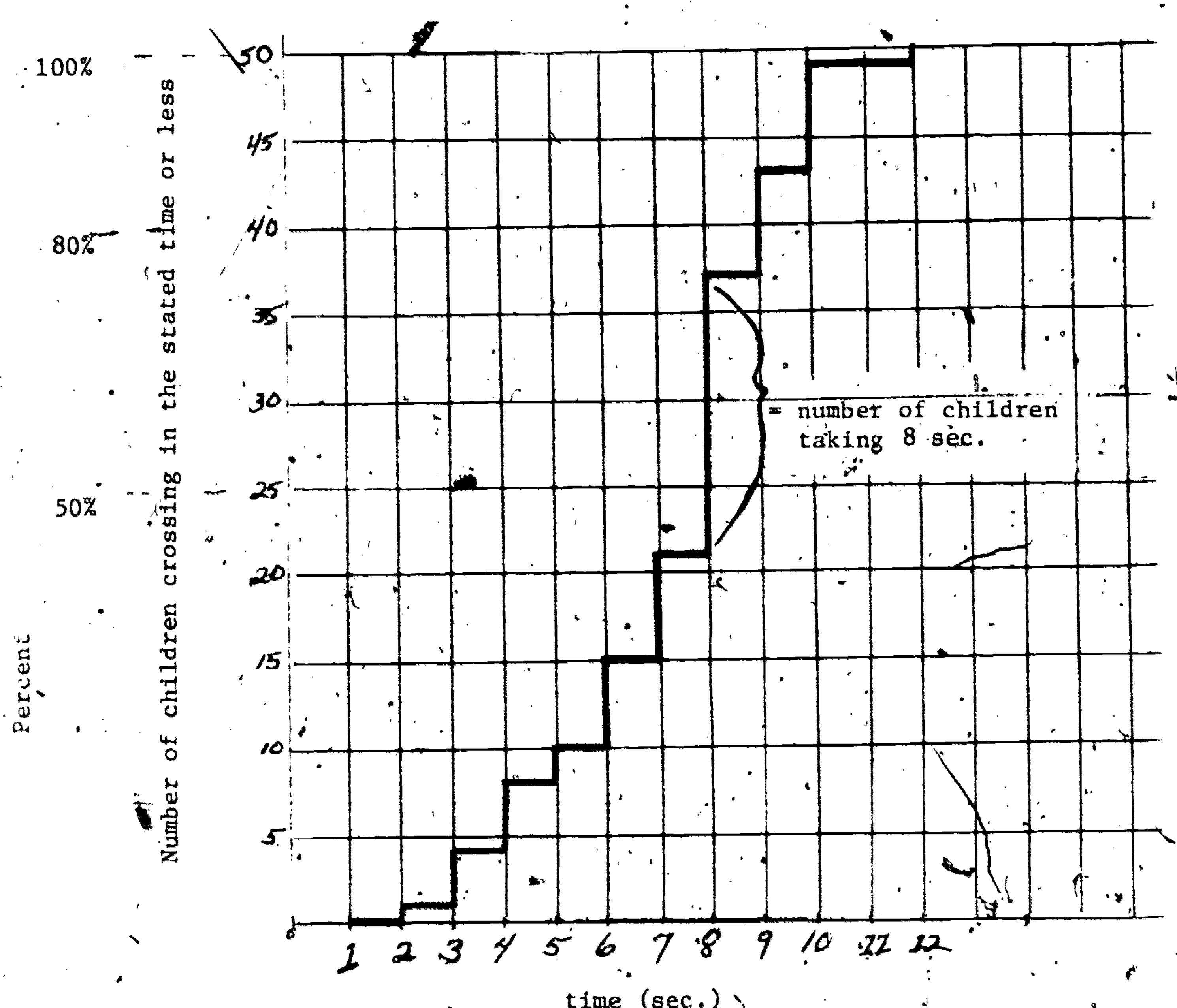
You might compute average crossing time and average gap time. If the average gap time is much bigger than the average crossing time, the intersection would seem to be safe. Otherwise it is not.

Instead of comparing averages, you may want to compare medians. The median of a group of numbers is a number (in the above case the number of seconds) such that half of all the events entered in the histogram are a number (time) less than or equal to it and half are greater than or equal to it. If the group has 51 events, then the 26th biggest number is the median. If the group has 50 events, then we usually take the 26th number as the median. Very often the 25th and 26th number will be the same.

The medians of the above two sets of data are 8 seconds and 2 seconds respectively.

In order to discuss a safe gap in traffic, there is another way of presenting the data in graphical form. It is called the cumulative distribution graph. Cumulative graphs show:

1. The range of values of the observations along the horizontal scale.
2. The value 0 to the total number of samples (children) observed along the vertical scale.
3. A height above each value corresponding to the total number of samples observed less than or equal to the value shown on the horizontal scale.
4. Lines drawn "over and up" showing the increment at each step. (The children can find the cumulative values by starting at the smallest value and drawing the increment lines at each successive value.)



Note that 80% (40) of the children would cross in 9 sec. or less.

A question which the children might discuss:

What is a safe gap?

a. A gap long enough for all of the children to cross (12 sec.)

b. A gap long enough for 80% of the children to cross (9 sec.)

c. A gap twice as long as it takes the average child to cross (16 sec.)

by

Earle Lomon

The eye is very fast at seeing relationships and trends in a geometric form. It can often pick out features long before the brain can pick them out by sorting through a list of numbers. For instance, look for about 30 seconds at the following table of numbers representing the New York Times weekly closing index over a year (1971): 478, 491, 499, 503, 512, 518, 507, 508, 518, 517, 526, 516, 522, 533, 545, 553, 551, 548, 549, 539, 532, 545, 541, 527, 519, 525, 531, 520, 518, 504, 498, 505, 541, 557, 562, 563, 560, 542, 544, 546, 535, 512, 508, 512, 499, 489, 491, 523, 525, 541, 552, 552.

What can you say about the movement of the stock market in that year? On the other hand, look at the same data as it is organized in the middle part of the graph in Figure 1. Note that the small horizontal lines represent the data on closing prices.

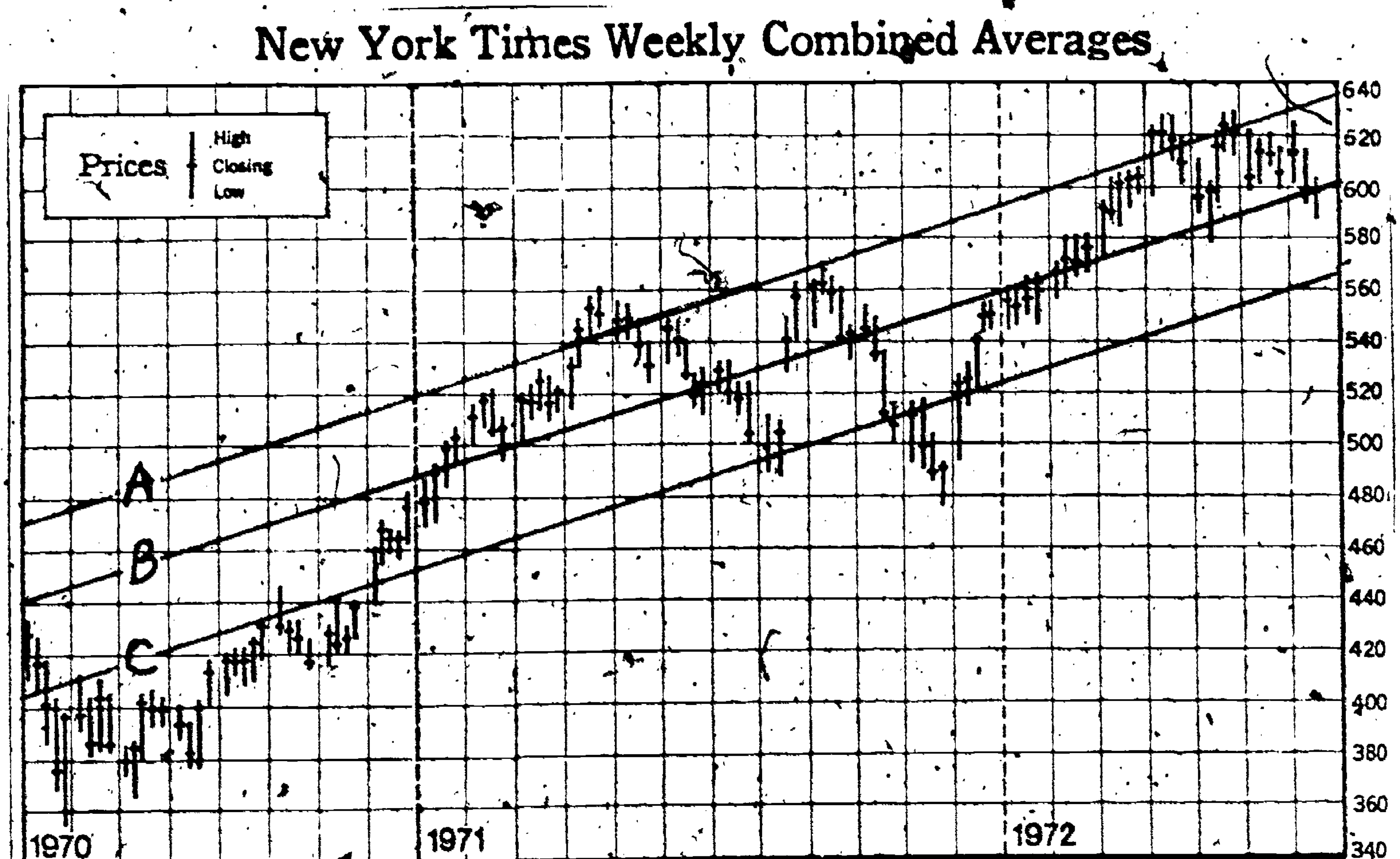


Figure 1

In a few seconds one can draw free-hand (it can be done more precisely by some simple calculations) a line (line B) representing the general trend throughout the year, and two sattelite lines (lines A and C) representing the range of fluctuations from the central trend. We can say quickly that the stock index rose about 70 points in 52 weeks, or about 1 1/3 points per week on the average. We can see immediately that the fluctuations were about 60 points, considerably larger than the average weekly rise. Both these observations, made in a few seconds, can be of tremendous help to the investor. It shows him that although the market may be rising, the fluctuations are much bigger than the rising trend per week. Thus, he may have to keep his money invested in a sample of stocks over a period of a year or more before the rising trend is likely to pay off. On the other hand, if his stocks make an up-swing as big as 30 points in one week, he may be just as well off to sell then as the trend may change and never bring him higher. Notice by the way that the average trend and fluctuations are pretty much the same in the shown portions of 1970 and 1972, as in 1971.

What is it in the structure of the graph which makes the result so graphic? Why are the inferences from the graph useful? The main object in graphing is to represent numerical relationships as geometrical relationships. In this case, the vertical distance represents the amount the New York Times index exceeds 340 points, while the horizontal distance represents the number of months from the beginning of May 1970. As one goes up it represents an increasing index (i.e. the <u>order</u> of the positions upward is significant) and equal jumps up represent equal increases in the index (i.e. there is a constant scale). Similarly the months are marked <u>in order</u> along the horizontal axis, each month being represented by the same width.

It is not always necessary that both directions in a graph have a numerical significance or a meaningful order. For instance, one may be graphing heights of students in class. A bar graph may be made with the horizontal axis being proportional to the height, but with the students arranged arbitrarily on the vertical axis. Even if they were arranged alphabetically, the vertical distribution would probably have no significance, as no one expects any correlation between initials and height.

In one USMES class the children constructed the graph (Fig. 2) of heights vs. names.

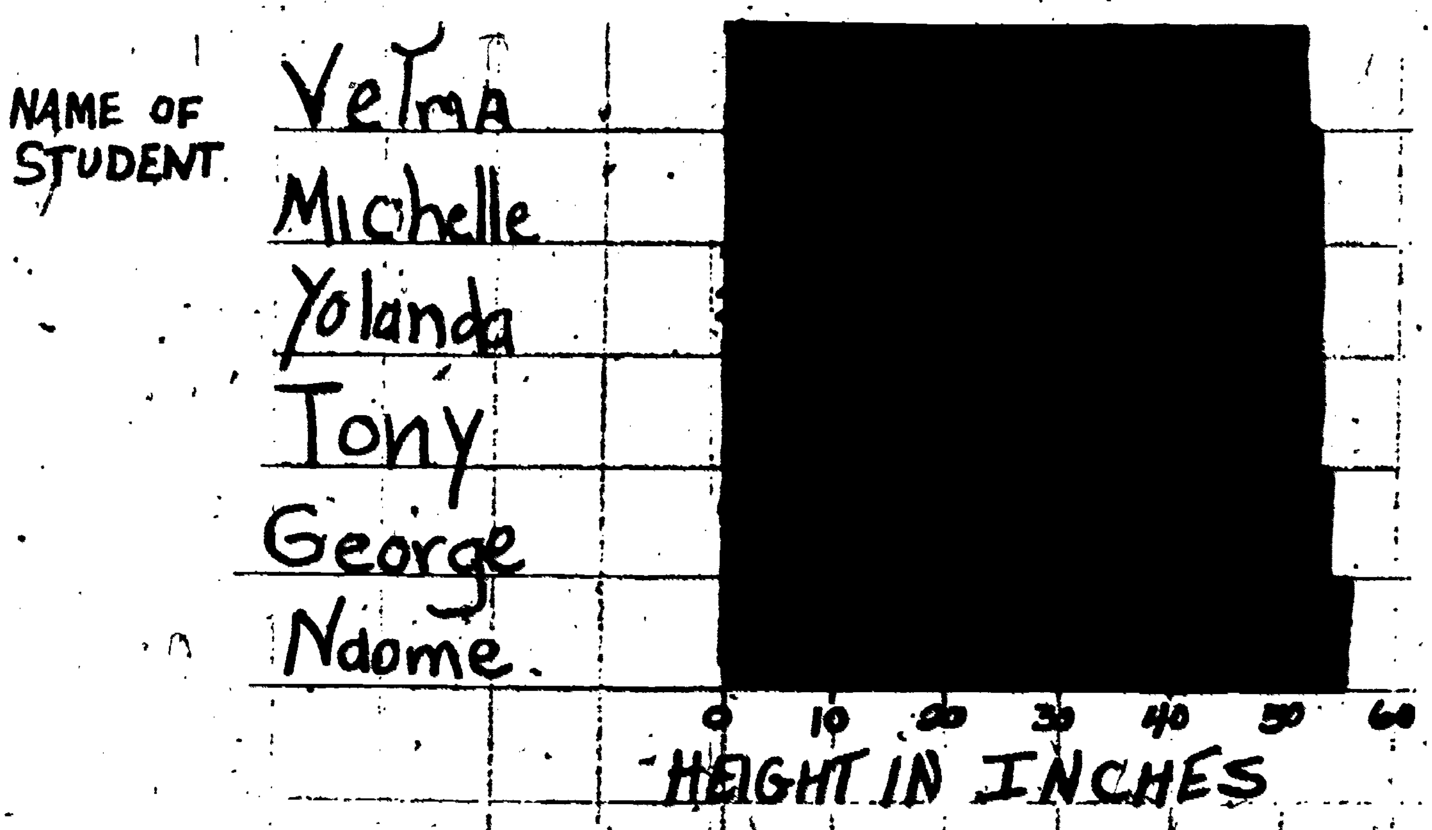
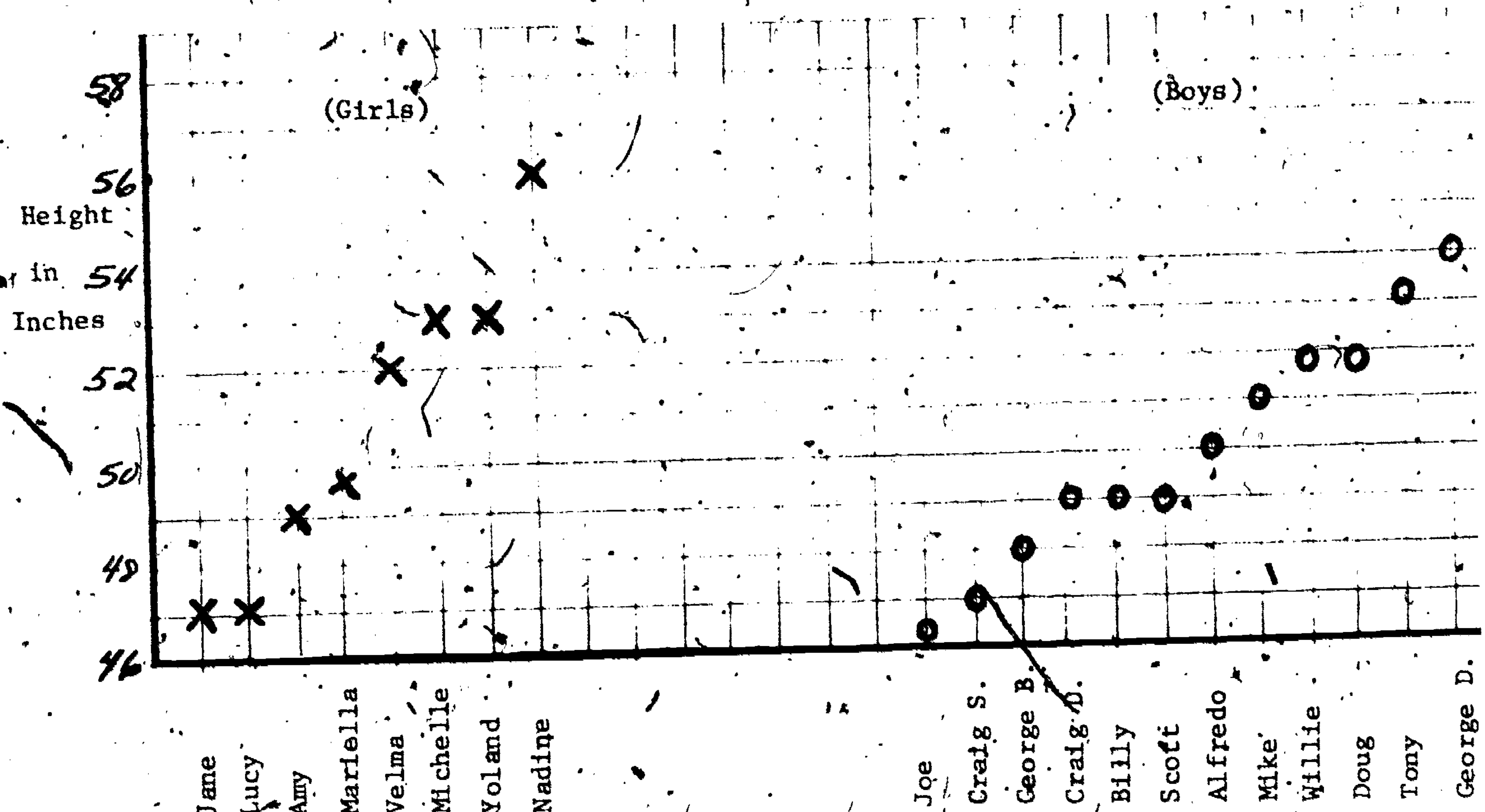


Figure 2

On the other hand, it may be informative to arrange the students horizontally by age. Is there a noticeable trend towards increased height with age? If so, about how many inches per year? We are unable to present an illustrative graph because no data has been received which gives us information about heights vs. ages in years, months and days. The growth in inches per year can be inferred by drawing a line, similar to line B in Fig. 1, on the graph of height vs. age. When a <u>meaningful</u> quantitative order is used on both axes, more information can be gained.

The same data can be graphed in more than one way to obtain information on different things. The heights of all the girls could be graphed starting from the left, and the heights of all the boys after that to the right. This would at a glance indicate if the girls or boys tended to be taller.

Fig. 3 shows a graph of students' heights with girls put first (on the left) on the horizontal axis, with boys following. The students' names are ranked by height but not age.



Students in Second Grade Class
Figure 3

If in the same graph the girls were put in order of age, and then the boys in order of their age, one might get the average growth rate of each sex. If the difference of slope was large enough, one would be able to tell if boys or girls of that grade level were growing faster. The differences in height and growth between boys and girls would stand out even better if their heights were overlayed on the same age axis (see paper "Representing Several Sets of Data on One Graph".)

5. The New York Times' graph illustrates other features of importance in designing easily read and understood graphs. Among these, the choice of vertical and haorizontal "scale" is one of the most rudimentary and generally needed. By "scale" is meant the interval size per number. It is generally determined by the range of data to be supplied and the size available for the graph. In this case, the financial editor wished to display more than two years of prices and, with all the other things he wanted to get on the page, decided he could only give it a three-column spread. In the approximately 8 inches available he wanted to mark off 27 months, so that he had just a little over 1/4" per month and made vertical lines accordingly. The data is further subdivided into weeks, but the eye can easily pick out whether a mark is the first, second, third, fourth or fifth between vertical lines.

The vertical scale was chosen to spread all of the prices in those years over the maximum vertical space he could afford, approximately five inches. The lowest number he needed to plot was 356, and the highest was 629, or a spread of 273. He wanted to distribute this spread over approximately 15 intervals of a little more than 1/4" each that he could get it into the 5". Although $\frac{273}{15} = 18$, it is much easier for people to visualize parts of 20, so each horizontal line was chosen to mark off a 20¢ interval. The lowest line on the graph stands for 340¢ and the highest for 640¢. If he had started with 0¢, there would have been a great deal of blank space in his graph and the data we are interested in would be crowded into less than the upper half of the graph. This would make it considerably harder to read and interpret at a glance. On the other hand, the reader must note that the scale does not start at zero. Otherwise, he would get the erroneous impression of a very large rise in price. So clarity of presentation sometimes involves the risk of misreading by someone who is unaware that graphs do not always start at zero.

There are times when including the zero helps in understanding even if the data begins considerably higher. For instance, if a graph is being made of the speed a model car goes on the flat versus the height of the hill it has rolled down (see paper "Some Considerations on the Curvature of an Exit or Entrance Road" Manual), one may get something like:

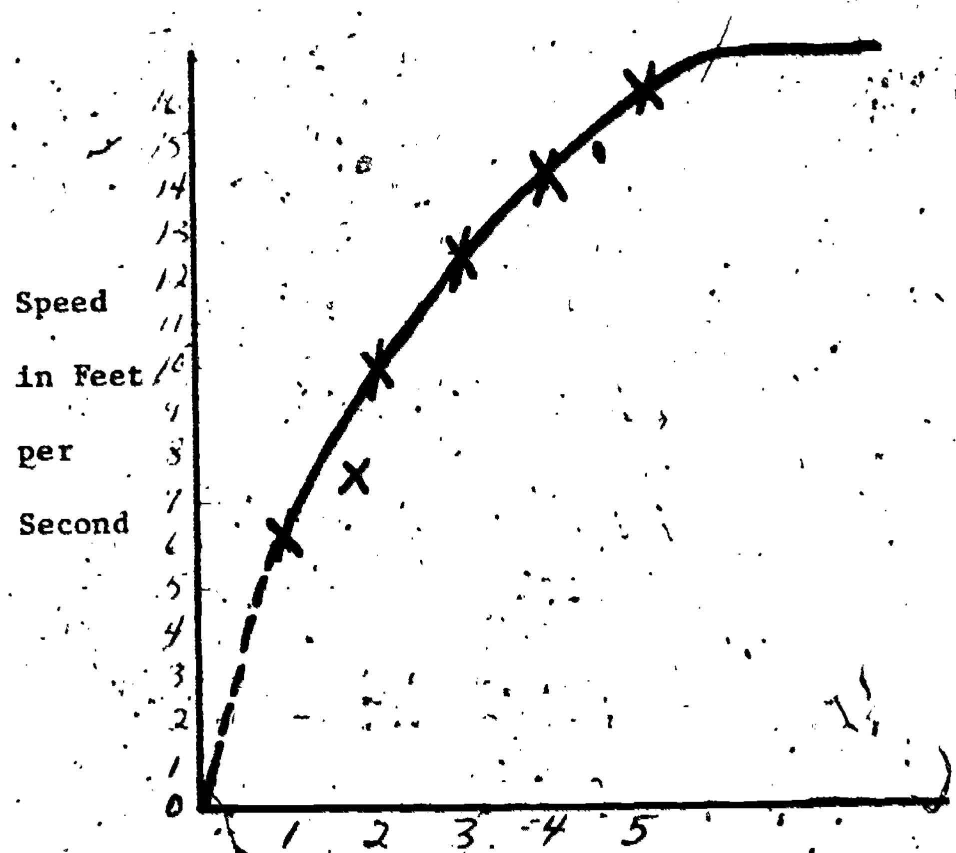


Figure 4

The curve is not very smooth because of the errors made in this rather difficult measurement. But one would like to be able to extrapolate the curve to larger and smaller hills. It looks like the curve may be going towards zero speed for zero height. Intuition can be used to assume it does and make a smooth curve (as shown) going through zero. Not only does this help extend the curve down to smaller heights; but because it makes it quite evident that the curve isn't straight, but is bending over, it also helps make a sensible extension to greater heights. Actually because of friction, the car will stop rolling before the hill goes away altogether. The dotted line represents that behavior. But in either case, the meaning of the curve is clarified by noticing where the "zero speed - zero height" point is located.

We return once again to the New York Times price index graph to emphasize another important aspect of many graphs - they often compile much data into single points, rather than displaying every separate datum. This often provides a summing up or averaging which would be hard to see in a straight representation of the raw data. Each little horizontal bar in the New York Times graph is an average of end of day (closing) price index over the week. Each day's closing price index is itself an average over the closing price of many stocks. The weekly average of the closing price index gives people an idea of how the stock market is going, removing the confusion of changes from Monday to Friday due to opening and closing of the stock market, daily news events, or the fortunes or misfortunes of a particular company. Of course, each day's closing price index of individual stocks can be relevant for someone "playing the market" daily, or looking for a single investment that may give unusually high profits. But for the long-range investor, or someone assessing how well the economy is doing, the New York Times graph showing the averages of many stocks is more useful.

There are many familiar cases in USMES units in which the compiling of data is important but often is not accomplished by the students. In displaying the results of a questionnaire, making a bar graph of "yes", "no", or "no answer" versus each respondent is not very much more revealing than the list from which the graph is made (see Fig. 5).

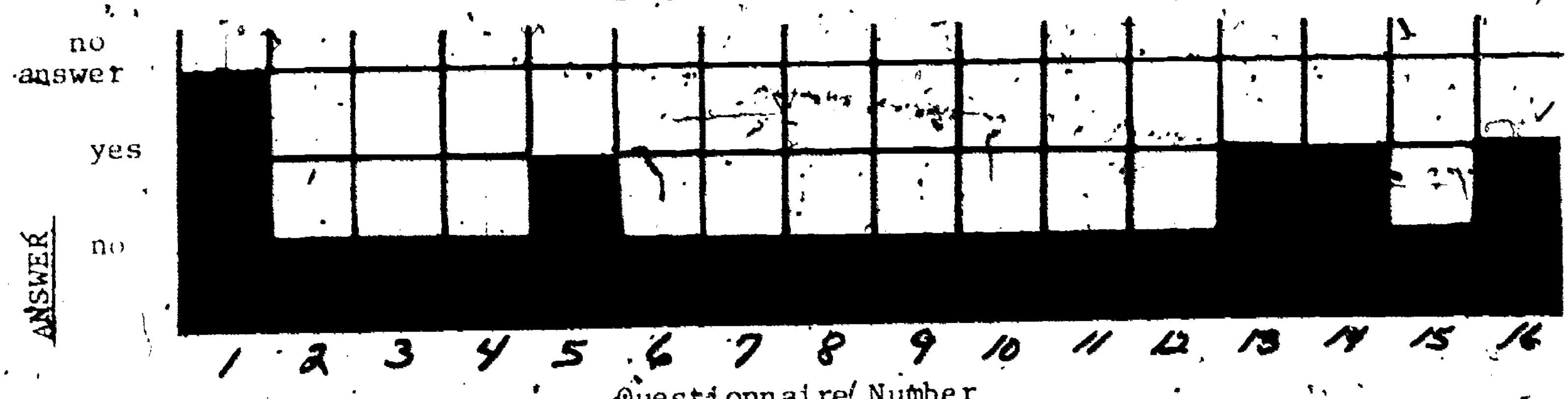


Figure 5

However, collecting the number of "yes" votes for each question and plotting that number vertically, versus the question horizontally will reveal immediately which questions elecited an unusually strong positive or negative reaction. The number of "no" and "no answer" responses for each question may be plotted on the other graphs. Or the ratio of "yes" to "no" answers may be plotted.

The data from the class which represented the results of their questionnaire as in Fig. 5 is compiled to show the number of "yes" answers for each question (see Fig. 6).

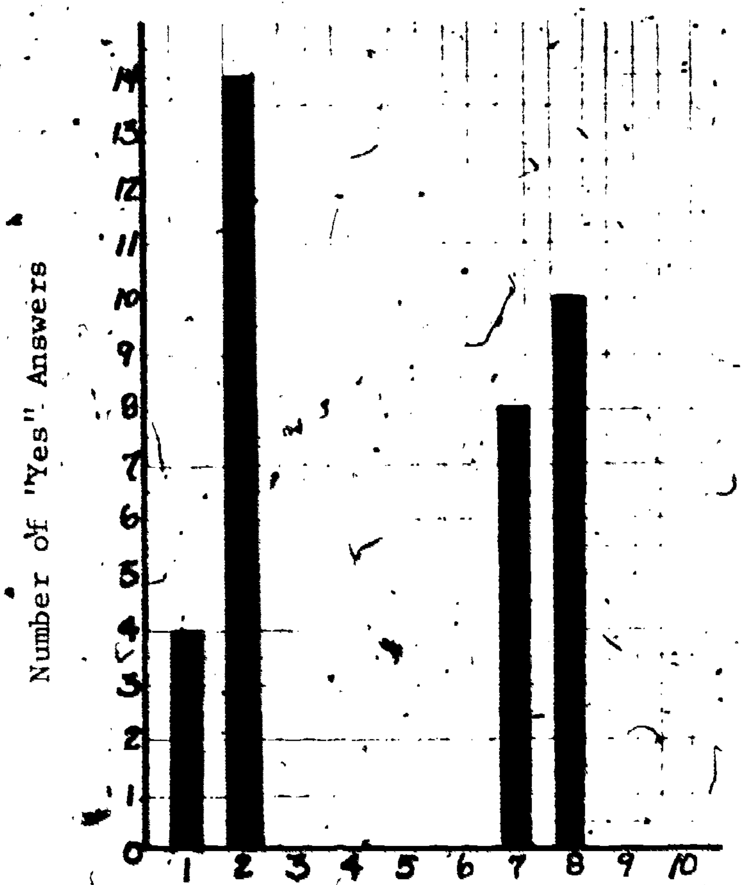


Figure 6

Another common USMES example arises in repeated observations, such as tossing a die or a coin over and over again, in measuring something several times to check on accuracy, or in taking a long series of gap times in traffic. Merely making a graph of the number of heads out of ten tosses for each sample set of ten (or the length obtained for each measurement, or the gap time between each successive pair of cars) will enable the eye to get a rough idea of the range of results and how it fluctuates. However, a frequency histogram will tell much more. For example, one may compile and then graph the number of times that out of ten tosses 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 heads come up (or the number of times the measurement was between 45"-46", or 46"-47", etc. or the number of gap times of 1, 2, 3, 4, 5, 6, 7, or 8 seconds).

Fig. 7 shows a histogram constructed by children of the number of children crossing in a stated length of time. Fig. 8 shows the results of tossing a penny in samples of 10 tosses. Such graphs quickly enable one to read off the mode and median of the distributors which measure the probability of the event, and the range which shows the statistical variation involved. More than that, it shows clearly the odds of any particular number of heads showing up (or the likelihood of the time measurement being a given amount from the median or of a gap time of any desirable amount appearing).

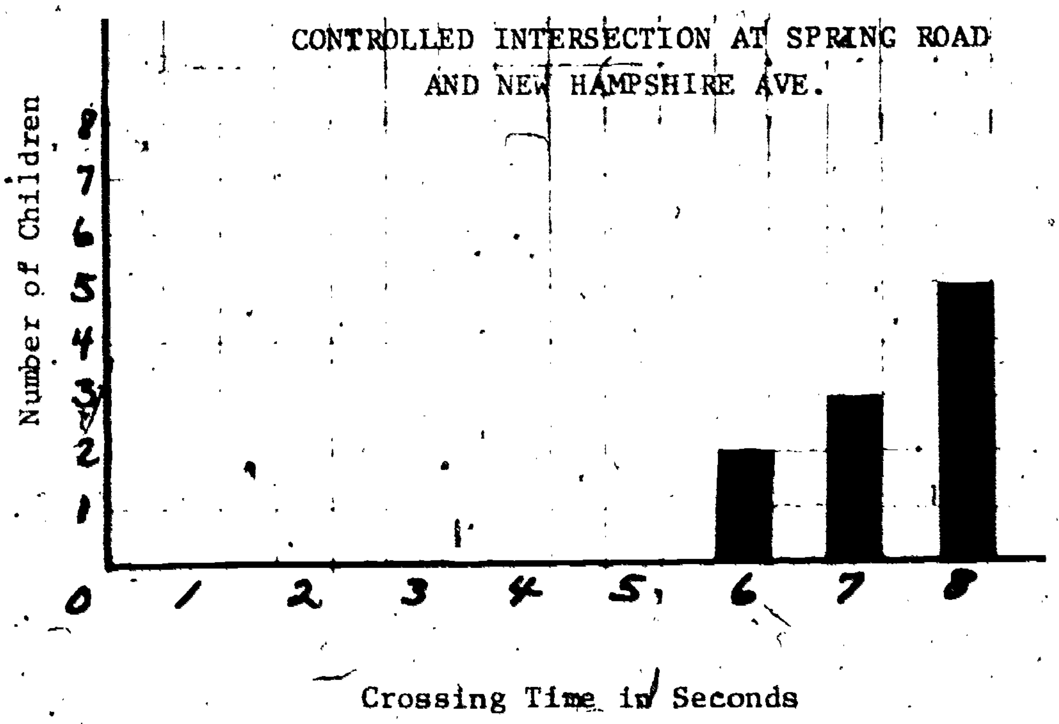
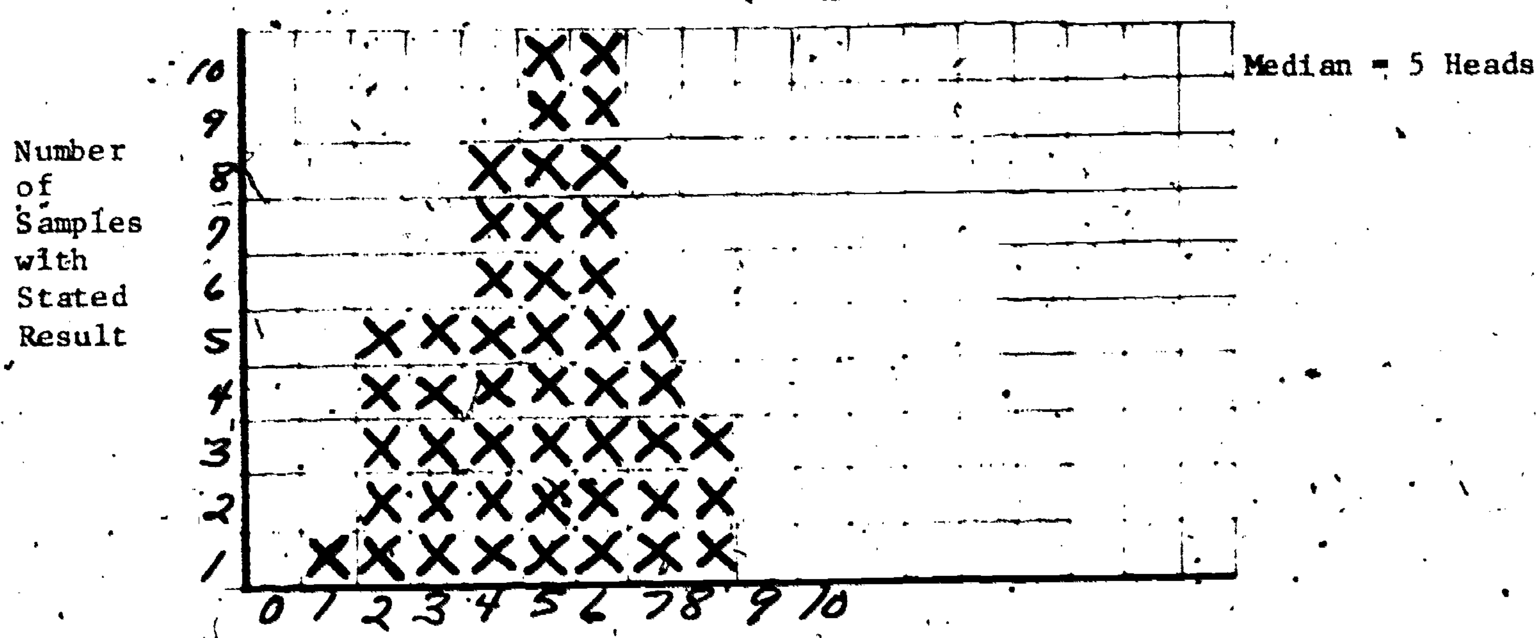


Figure 7



Number of "Heads" in Samples of ten tosses

Figure 8

In USMES logs you will find many examples of children making more and more useful versions of graphs of their data. You will also see some examples of children switching from one set of data to another without making much progress in the understanding or presentation of such data. They do not get the full benefit of thier observations. Those who critically examine their graphs and reorganize them to advantage are rewarded by the value of the graphs in forming their recommendations and convincing others.

USMES

REPRESENTING SEVERAL SETS OF DATA ON ONE GRAPH

by

Betty Beck

As the children collect many sets of data in USMES classes, occasions arise when it is useful to represent more than one set of data on a graph. For example, in the Lunch Lines Unit, the children may have data collected on different days of the week on the number of children from the different grades eating lunch. In the Pedestrian Crossings Unit, they may have collected data on the children's crossing times for several streets. They may wind magnets with different size wire and different lengths of the same wire for the Electromagnet Device Design Unit. Or, for the Consumer Research Unit, they may have tested the battery life of several different brands of batteries.

One way of representing two sets of data on one graph is to construct a bar graph with adjacent columns representing the two sets. The graph in Figure 1 was constructed by children in Pamela Fazzini's Lunch Lines class.

The graph in Fig. 1 shows two sets of data: The clerk's count of milk sold each day, and the children's count of the same thing. The data as shown on adjacent columns can be compared quite easily. However, when the bar graph form is used to represent additional sets of data, the results can become quite difficult to analyze. The graph in Fig. 2 shows three sets of data: The teacher's count, the clerk's count and the children's count of the number of lunches to be served on each day.

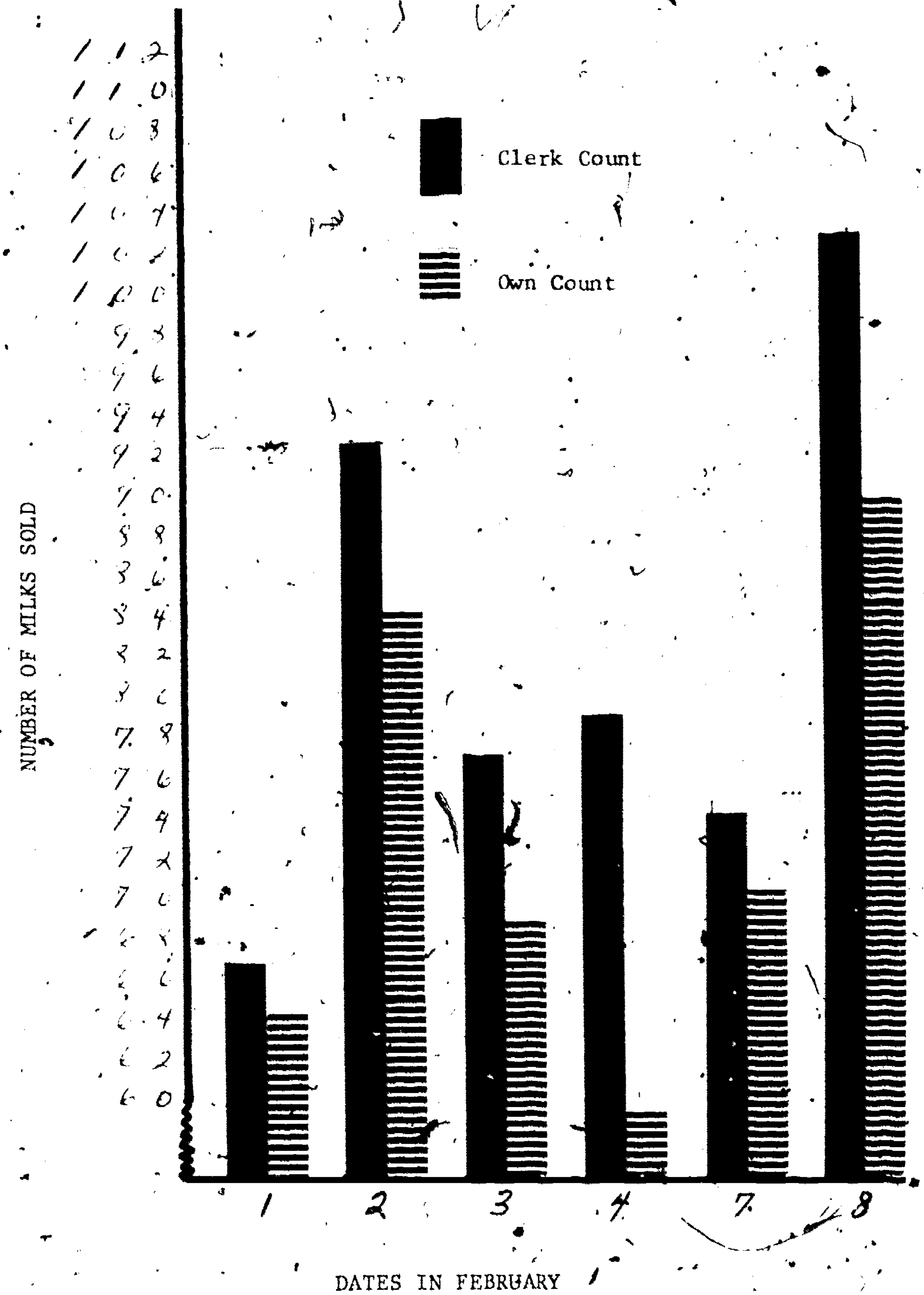
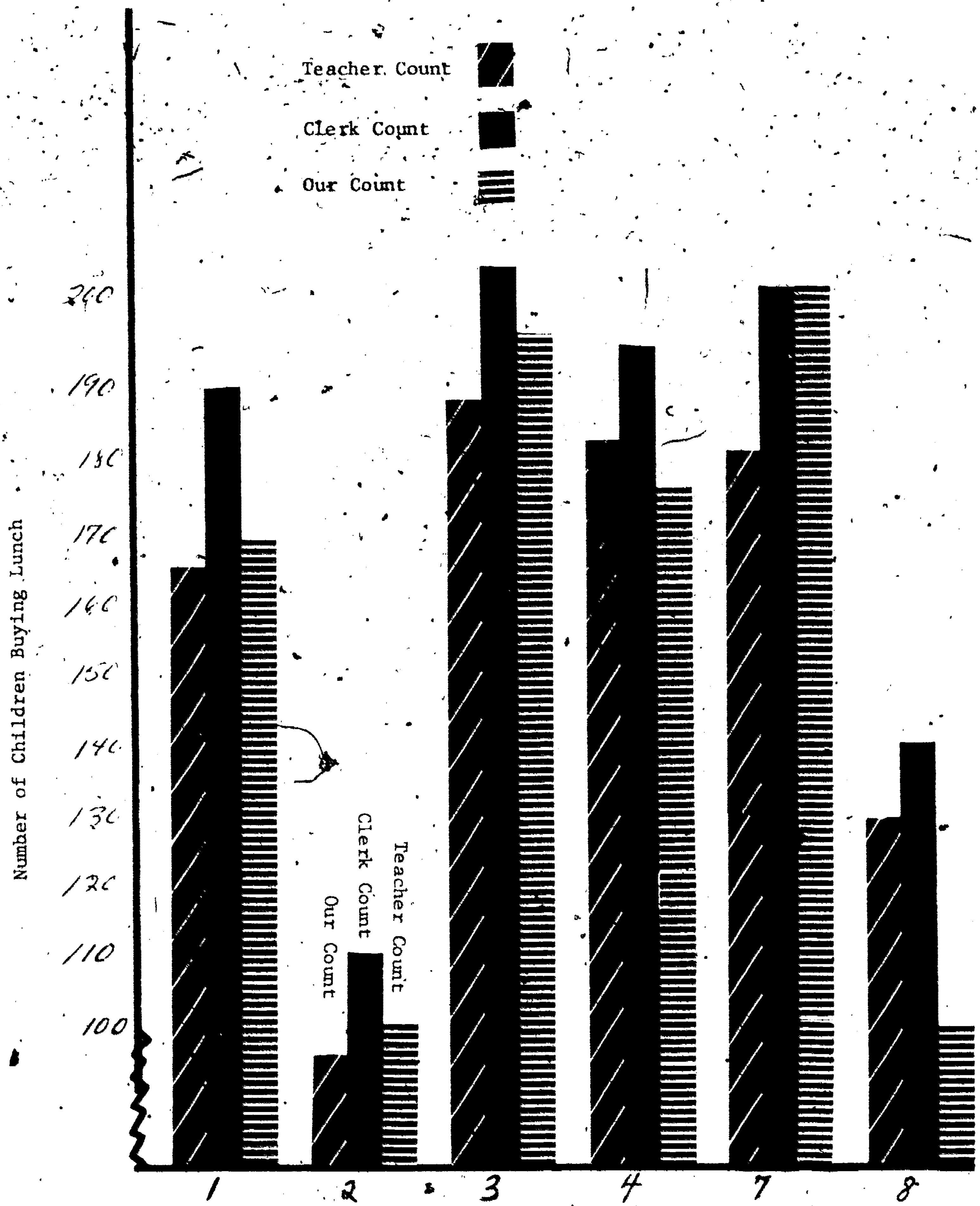

Clerk Count
Own Count
NUMBER OF MILKS SOLD
112
110
108
106
104
102
100
98
96
94
92
90
88
86
84
82
80
78
76
74
72
70
68
66
64
62
60
1
2
3
4
7
8
DATES IN FEBRUARY

Figure 1



DATES IN FEBRUARY

Figure 2

Both of the above graphs could have been drawn as line graphs instead of bar graphs. The lines connect the tops of the columns. See Fig. 3 for a line graph of the three sets of data on lunch counts.

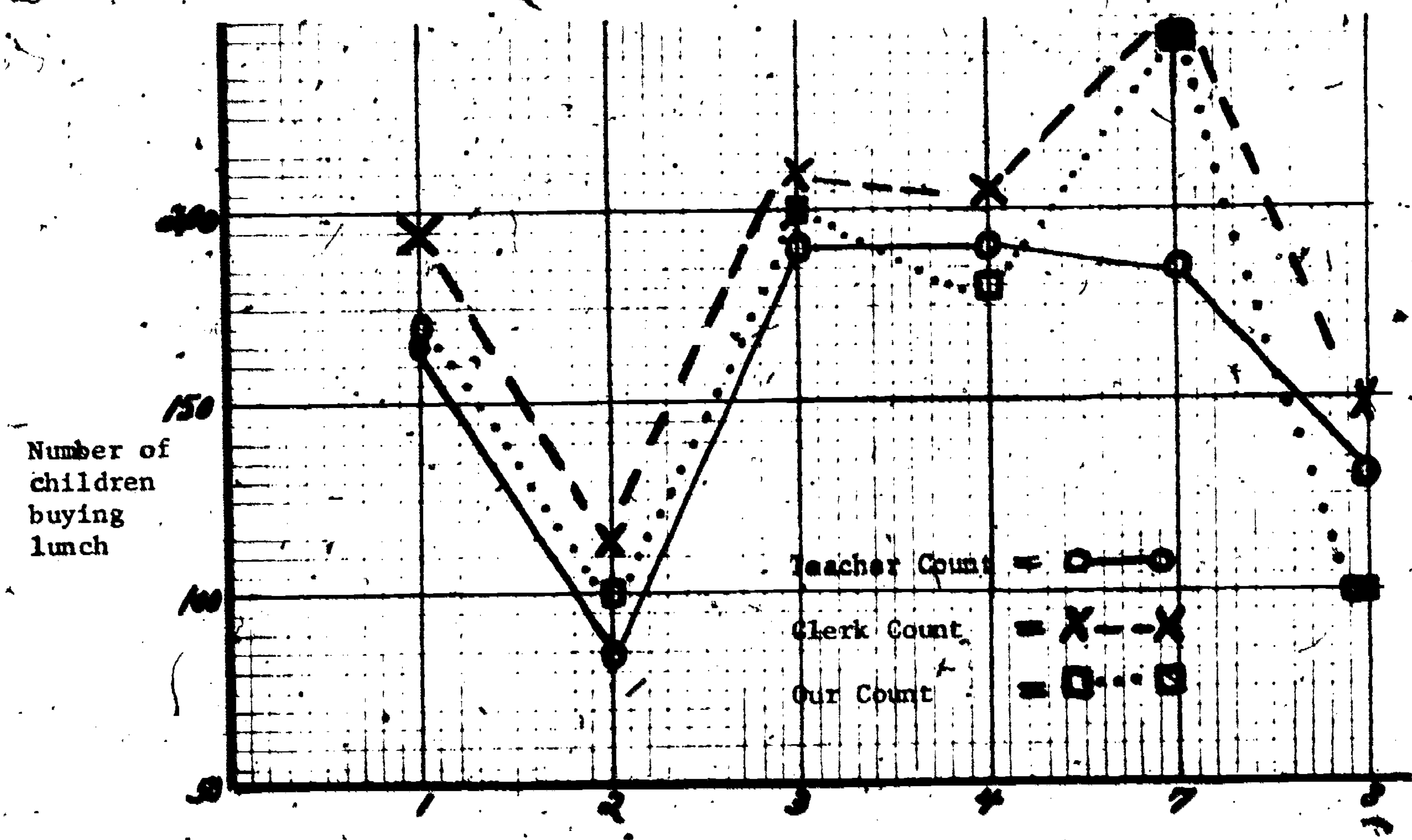


Figure 3

If a line graph is drawn, the data can be more easily compared with the following questions asked:

Why is the clerk's count of the number of lunches sold consistently higher than the children's count?

Why is the teacher's count on the number of lunches served sometimes more than the student's count and sometimes less?

Why is the difference on lunch counts greater on some days than on others?

The above sets of data on milk counts and lunch counts can also be used to compare graphically the difference between the students and clerk's counts of the two items. Fig. 4 shows a line graph of the differences in the counts for milk and for lunch. The children could ask what there is different about the way the milk counts are taken and the way the lunches are counted, that would explain the large difference in counts some days and a small difference in counts on other days.

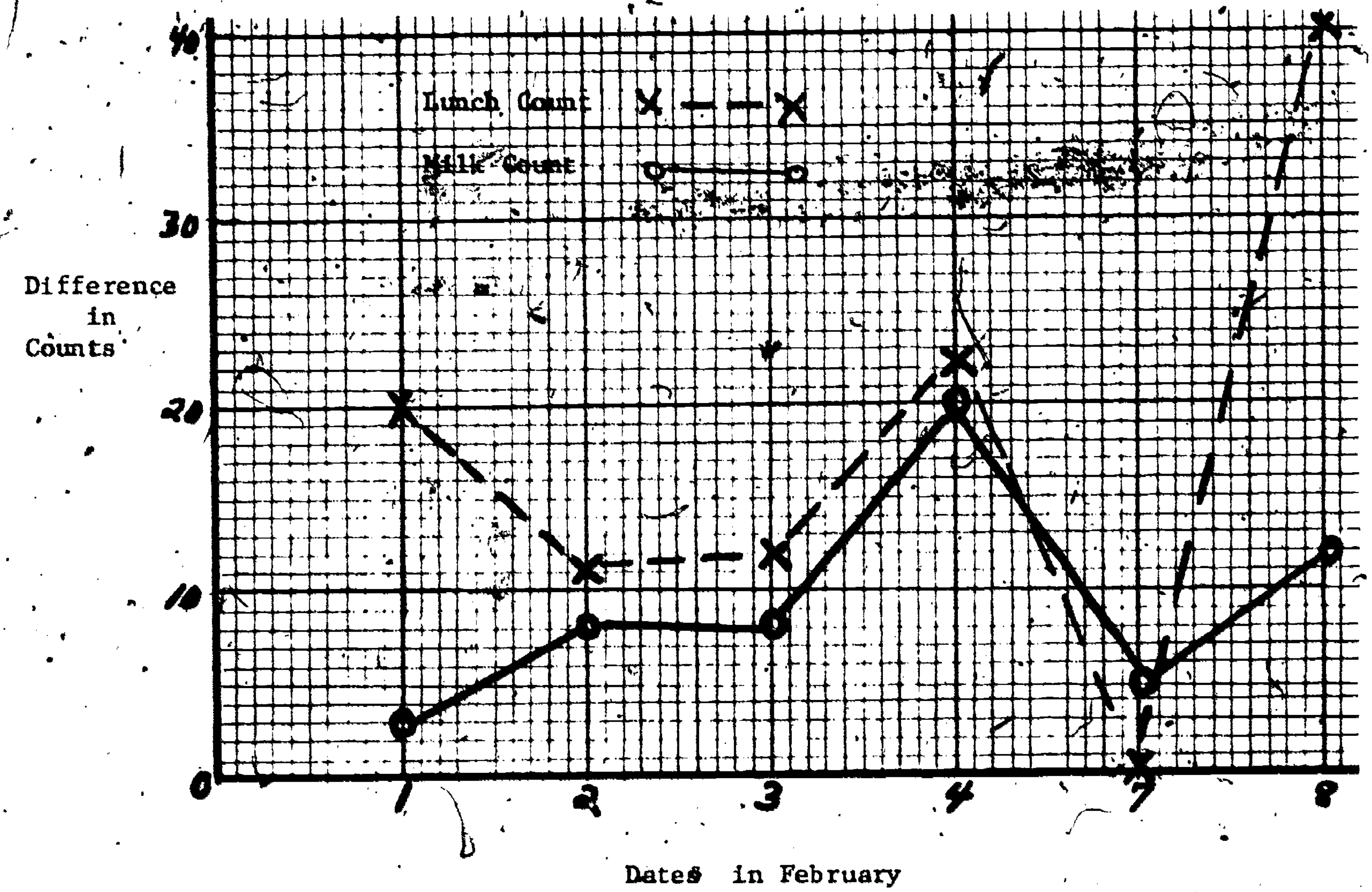


Figure 4

Many times the representation of two sets of data on the graph will provide new insights, suggest new approaches to the challenge, or show gaps in the types of data being collected. For example, the children in Bob Farias' Pedestrian Crossings class collected data on crossing times at four streets and constructed bar graphs for each street which show the number of students who crossed in a specific time (see Fig. 5). The graphs (and the medians) show that most students took longer crossing Follen Rd. and Pleasant St. than they did crossing Mass. Ave and at the traffic light.

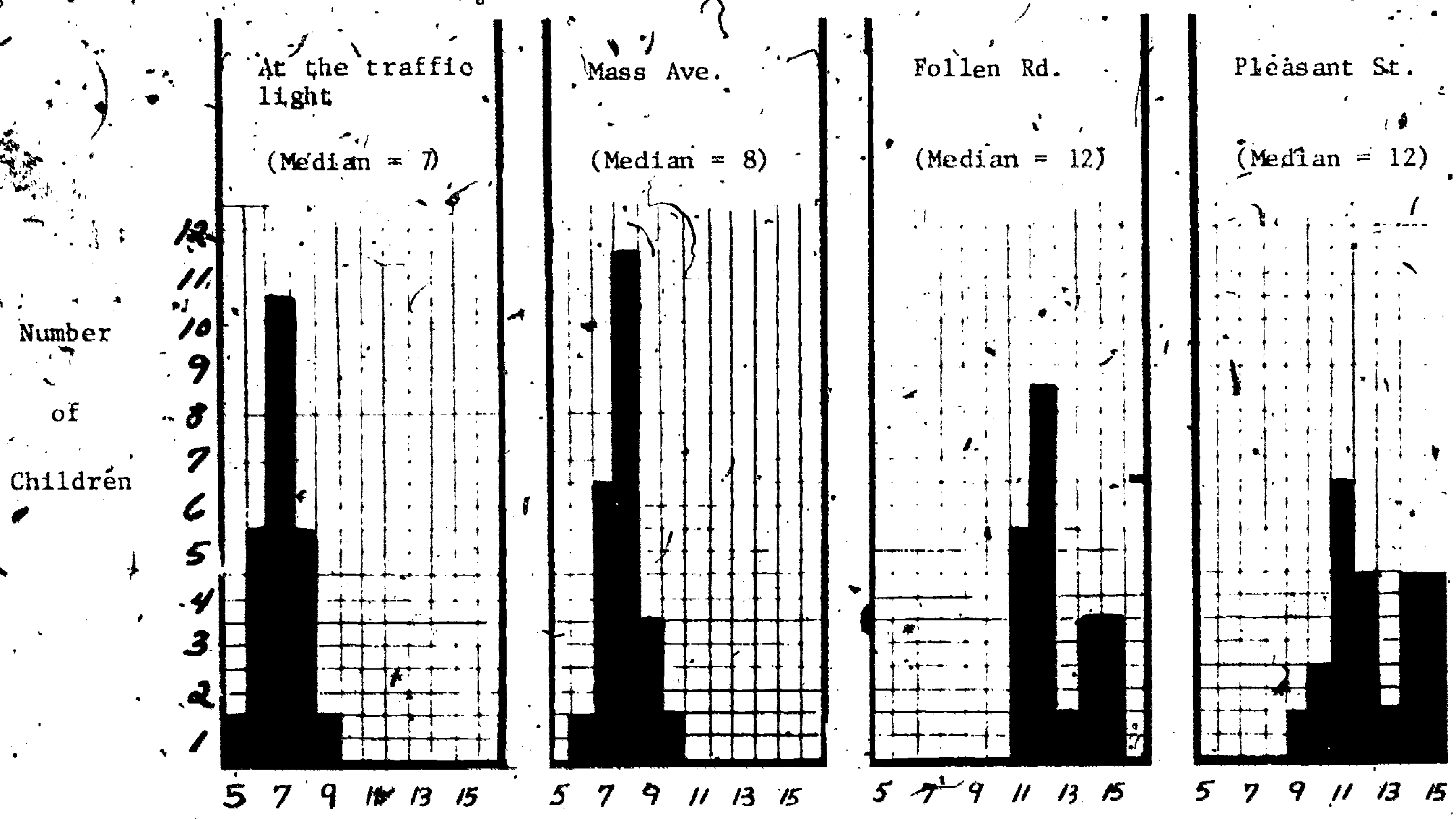


Figure 5

However, if each students' data is represented on a line graph, some additional insights might be drawn. The graph for the crossing times of four students is shown below in Fig. 7. The kids could ask why Kim and Leslie took longer to cross Pleasant St. than Follen Rd., while John and Tommy took longer to cross Follen Rd., than Pleasant St. Why did Kim take so long crossing Mass. Ave, when he crossed at the traffic light faster than the others? Perhaps some factor influencing crossing time had been overlooked, or perhaps the data gathering procedured varied. The kids could construct other graphs which would show Follen Rd. and Pleasant St. in reverse order on the horizontal axis. If this was done, Tommy's line would be almost straight; John's line would go up slighly; Kim and Leslie's lines would go down slightly. In the new graph, the student crossing times for thetwo streets would not appear to be as different as they appear in Fig.6.

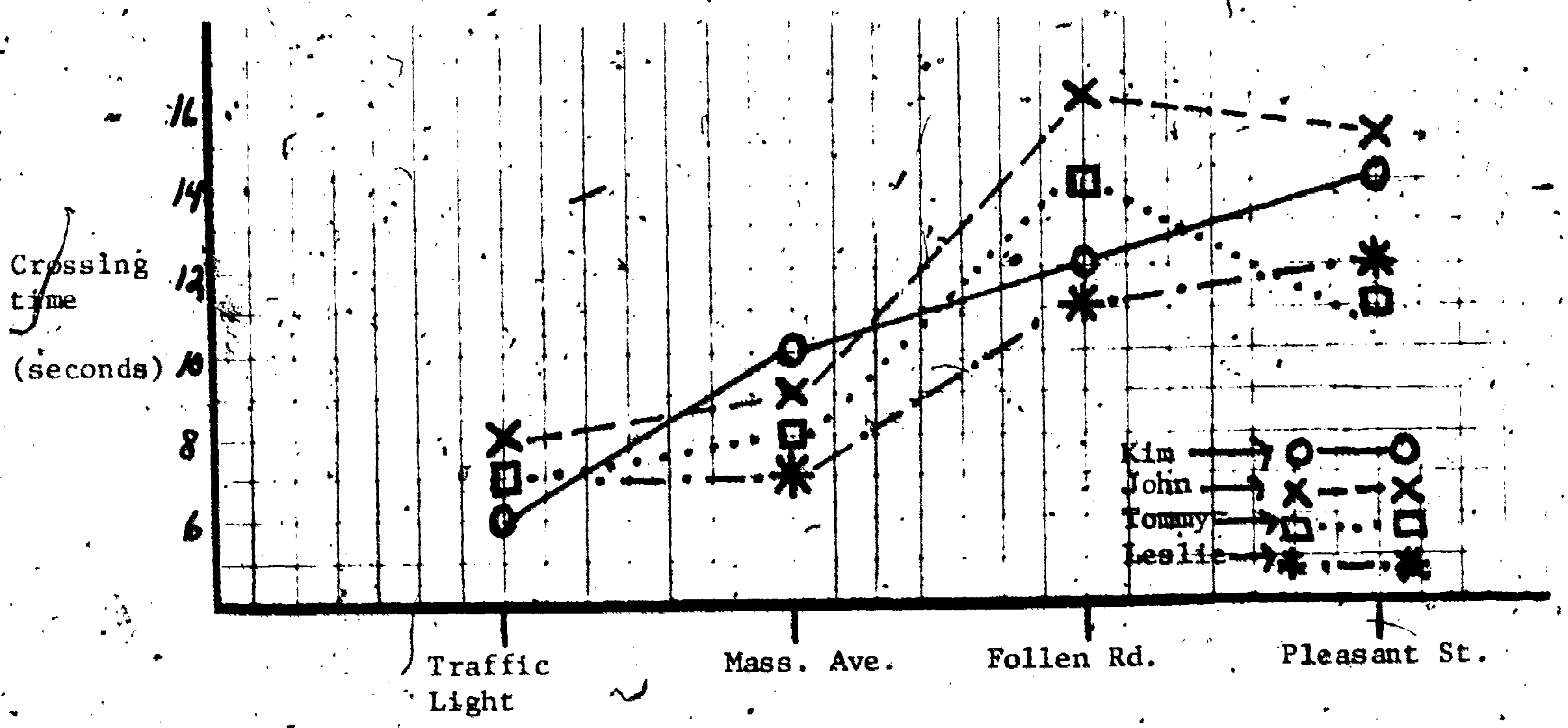
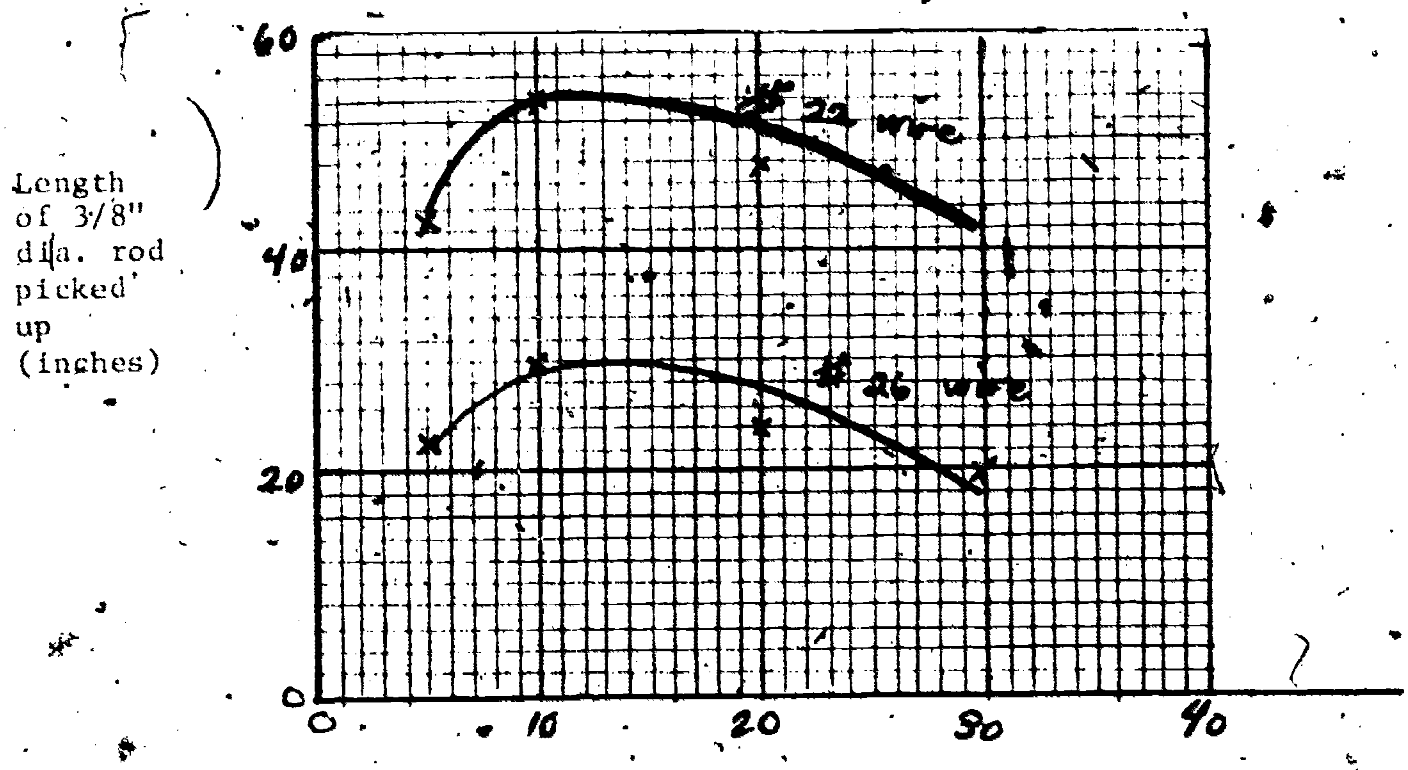


Figure 6

The line graphs in Figs. 3,4 and 6 have been constructed with very obvious data points. This is a good way to show that the variable on the horizontal axis is not continuous - the spots on the axis between the numbers (or items) indicated have no meaning. If the variable shown on the horizontal axis is continuous, then the line can be drawn straight through points which do not stand out.

Many experiments carried out in different USMES units produce data about a factor which is a continuous variable - times (distances, lengths). The spots on the horizontal axis between the times (distances, lengths) for which the data is collected have meaning. For example, the children can determine the strength of a magnet by determining what length of a 3/8" diameter steel rod the magnet will hold. They test the magnet's strength when it is wound with 5, 10, 20, and 30 foot lengths of wire. The line graph of their data for several magnets each wound with either #22 or #26 wire is shown in Fig. 7. One can assume that the strength of a magnet wound with 7, 18, or 24 feet of #22 wire is represented by a point on the line representing #22 wire. One can also assume that if magnets were wound with #24 wire, the results could be shown on a line between the two lines of the graph.



Length of Wire (feet)
Figure 7

In the example on page 7, if the children had measured the widths of the four streets, they could have plotted each student's crossing time vs. the width of the street. One can assume that the students' crossing times can be then found on the graph for other streets having other widths. If the distance across the street was the only factor involved, each student's line would be a straight line, assuming he walked at the same rate across each street. The absence of a straight line on the graph would point to the presence of other factors than the width of the street. Did certain students hurry across certain streets? Did they have to wait part-way across other streets

Data was collected by students in one Dice Design class by tossing thumbtacks in samples of 5 tosses. The data was then grouped in samples of 20 tosses and the results of both sample sizes graphed as shown in Fig. 10. In order to represent different sized samples on one graph, proportionate results must be shown on the horizontal axis. For example, if a thumbtack lands point up two times out of 5 tosses, the proportionate result is .4. With the larger sample, point up results of 8 or 9 times in 20 tosses are also graphed as a proportionate result of .4.

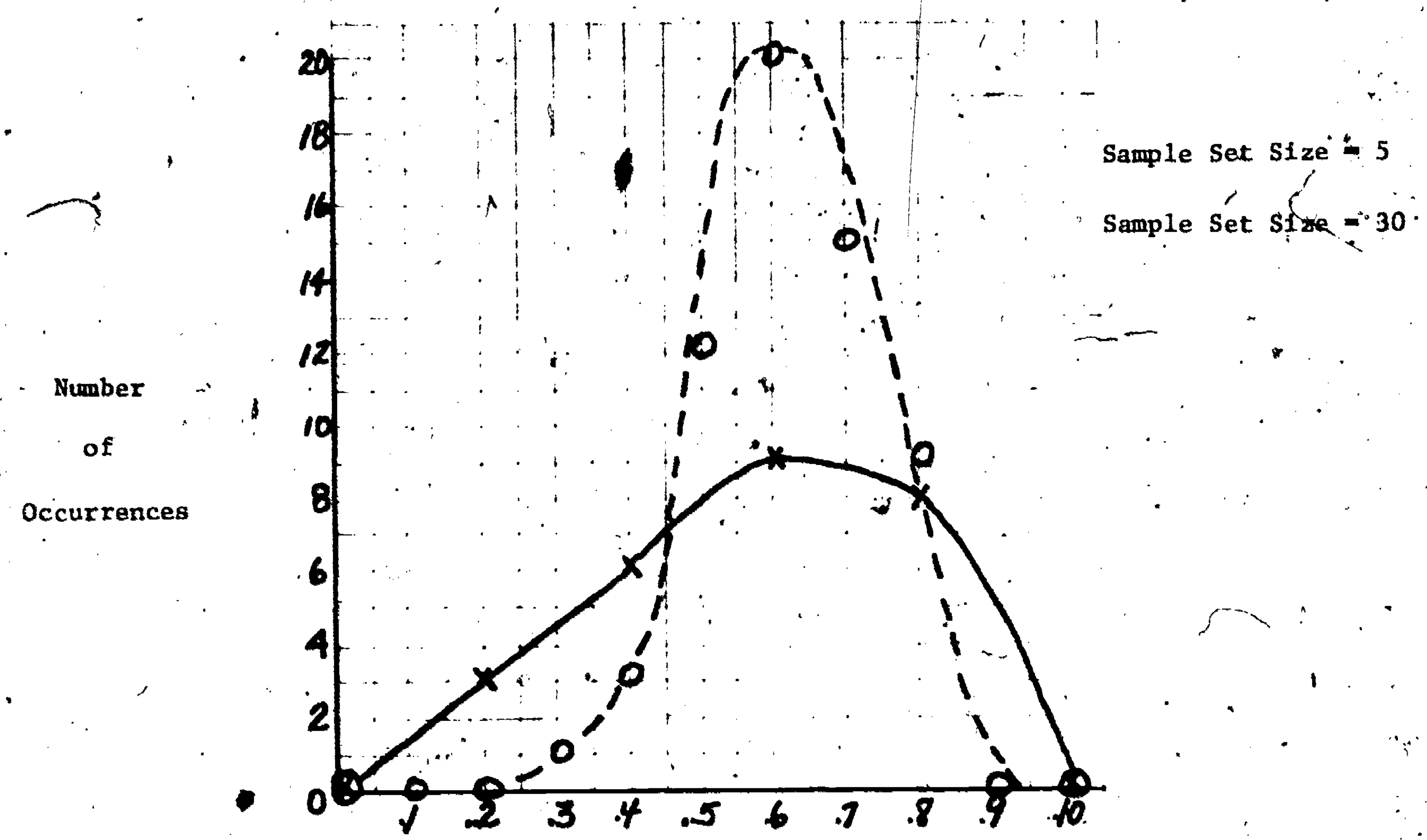


Figure 9

The activities described in the teacher logs suggest that the following graphs could be constructed.

1. Milk counts according to classes with different lines showing different days of the week.

2. Car counts in certain time periods during the day with different lines showing different days of the week or different types of weather.

3. Median crossing times at different crossings with different lines showing data at morning, noon and school dismissal times.

Many other opportunities will arise for the children to show several sets of their data on one graph.

PLOTTING WEATHER PREDICTIONS DATA ON THREE-DIMENSIONAL PEGBOARD GRAPHS

(Based on suggestions by Jack Borsting and Leland Webb)

The Weather Predictions unit provides the opportunity and need for the children to construct and use various types of graphs. In addition to standard-type graphs, three-dimensional pegboard graphs can be constructed to plot data and to correlate various variables as they pertain to weather predictions. Regression lines can also be established making it possible to spot trends between changes in weather variables and the actual weather. The following paragraphs discuss several types of graphs which can be useful tools in making more accurate weather predictions.

1. A graph with no observations or measurements, merely the outcome of guesses. The children would guess what they thought would happen the following day and a tally would be kept as to correct and incorrect predictions. Each tally would represent a day. One possible format follows:

correct	incorrect
𝍸 ||	𝍸 𝍸

The children could then compare the results of their straight guesses with predictions based on observations and/or measurements of different variables. Climatology data could also be used. Hopefully, the guessing method would not be as accurate as the other methods.

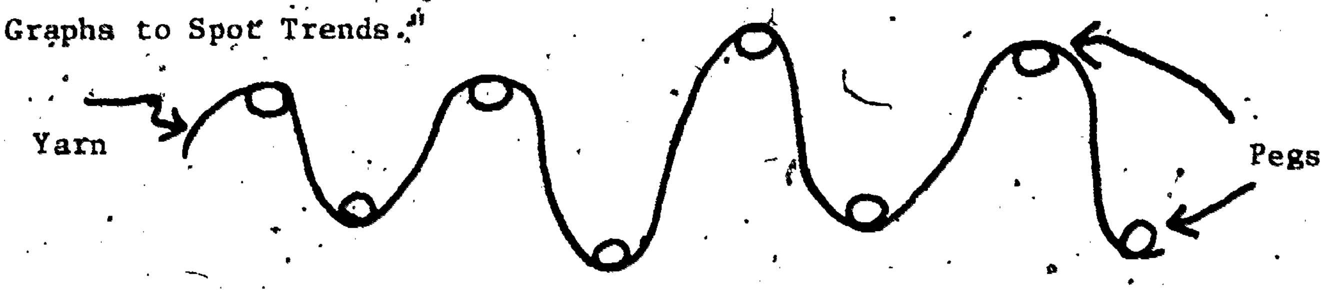
2. The children might use a large pegboard to plot the data they collect each day on different variables such as pressure, temperature and

relative humidity. Different vertical scales are placed along the sides of the pegboard for the different variables. Then, a variety of different-colored, and/or different-shaped, blocks representing the different variables are placed each day over dowels inserted into the appropriate holes in the pegboard. Figure 1 shows a picture of this type of pegboard representation of data gathered in Boston, Massachusetts, during the month of January, 1972. The data used was obtained from the Superintendent of Documents, U.S. Government Printing Office and represents daily averages in most cases. The data sheets are reproduced on pages GR5-10 and 11; ordering information is shown on the first page of the sheets.

Similar data may be compiled from the daily weather report in the local newspaper or on a local TV station. This will usually provide the barometric pressure at one (or two) times during the day along with high and low temperatures, the relative humidity, and the rainfall for the past 24 hours. The 24-hours change in pressure can then be easily obtained. It should be noted, however, that these are relatively long-term variations and that short-term variations that could signal a weather change can be missed. The only way to avoid this is to use frequent readings of instruments.

On the pegboard shown in Figure 1, squares show average relative humidity, circles show average barometric pressure and triangles represent average temperatures. Precipitation is shown in two ways: 1) the amount is shown across the top of the chart and 2) dark squares, circles and triangles are used on days when the amount of rain was .01" or more. (Figure 1 is on page GR5-4.)

This pegboard graph can also be used to show the correlation between the variables and the actual weather. Different colored yarn can be used to connect points representing relative humidity data, ... pressure data or temperature data. Sometimes connecting the points with string or yarn is useful for interpretation of data. At other times it may serve only to confuse. For example, if the data is such that the line formed by the yarn is ragged with no discernible trend, as shown below, then the pegboard graph is confusing. When this occurs, it is better, perhaps, to find a regression line. This is discussed in the background paper "Using Scatter Graphs to Spot Trends."



Figures 2, 3, and 4 show a solid line for relative humidity, pressure and temperature, respectively. A quick glance at Figure 2 shows that the relative humidity was high on rainy days. An examination of the pressure line in Figure 3 shows in many cases a decrease in pressure from the preceding day whenever it rained. However, no correlation is evident in Figure 4 between change in temperature and rain. Of course, some changes in the variables are unknown because of the absence of more frequent measurements. Figures 2, 3, and 4 are on pages GR5-5, 6, and 7, respectively.

Figure 1

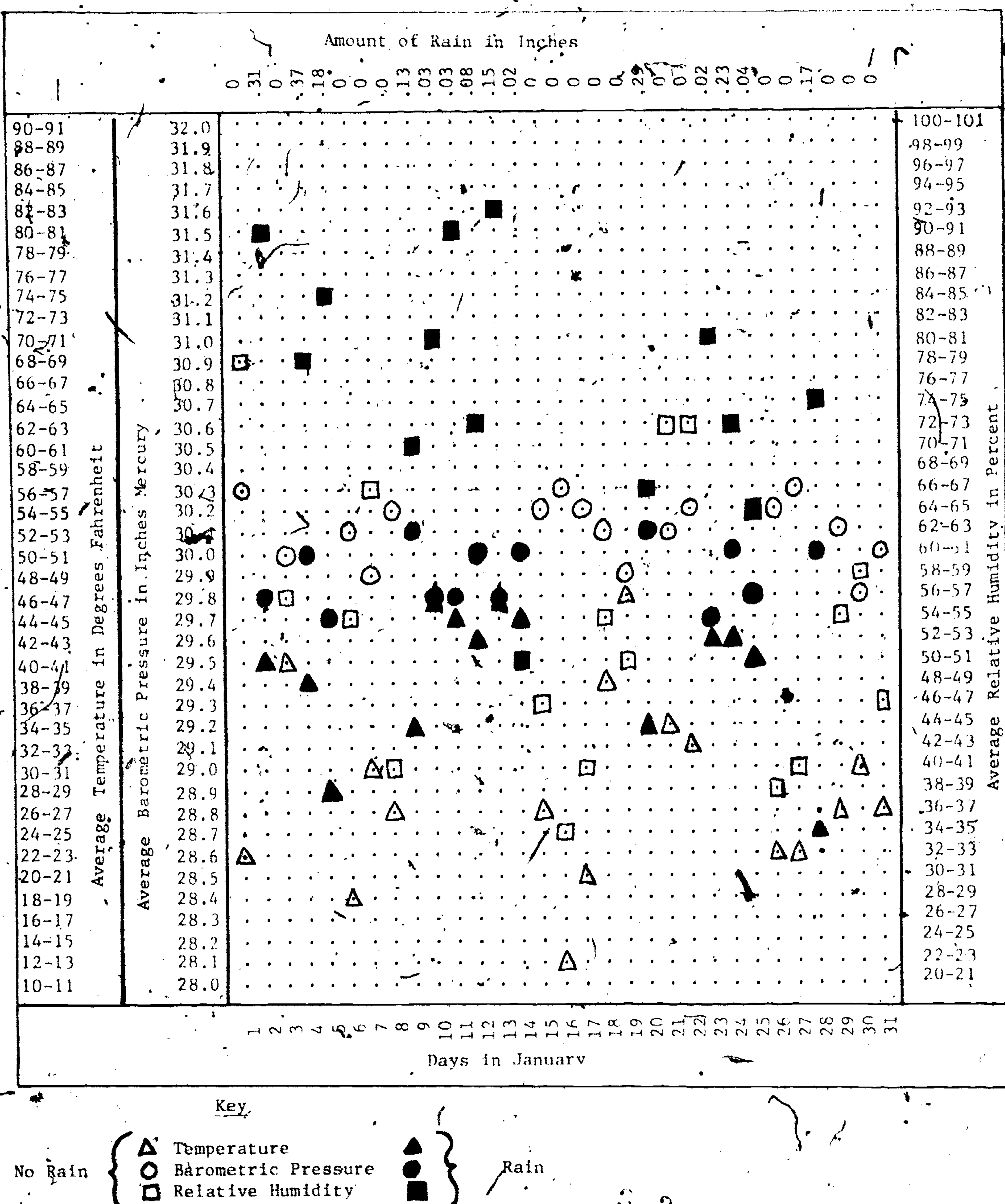
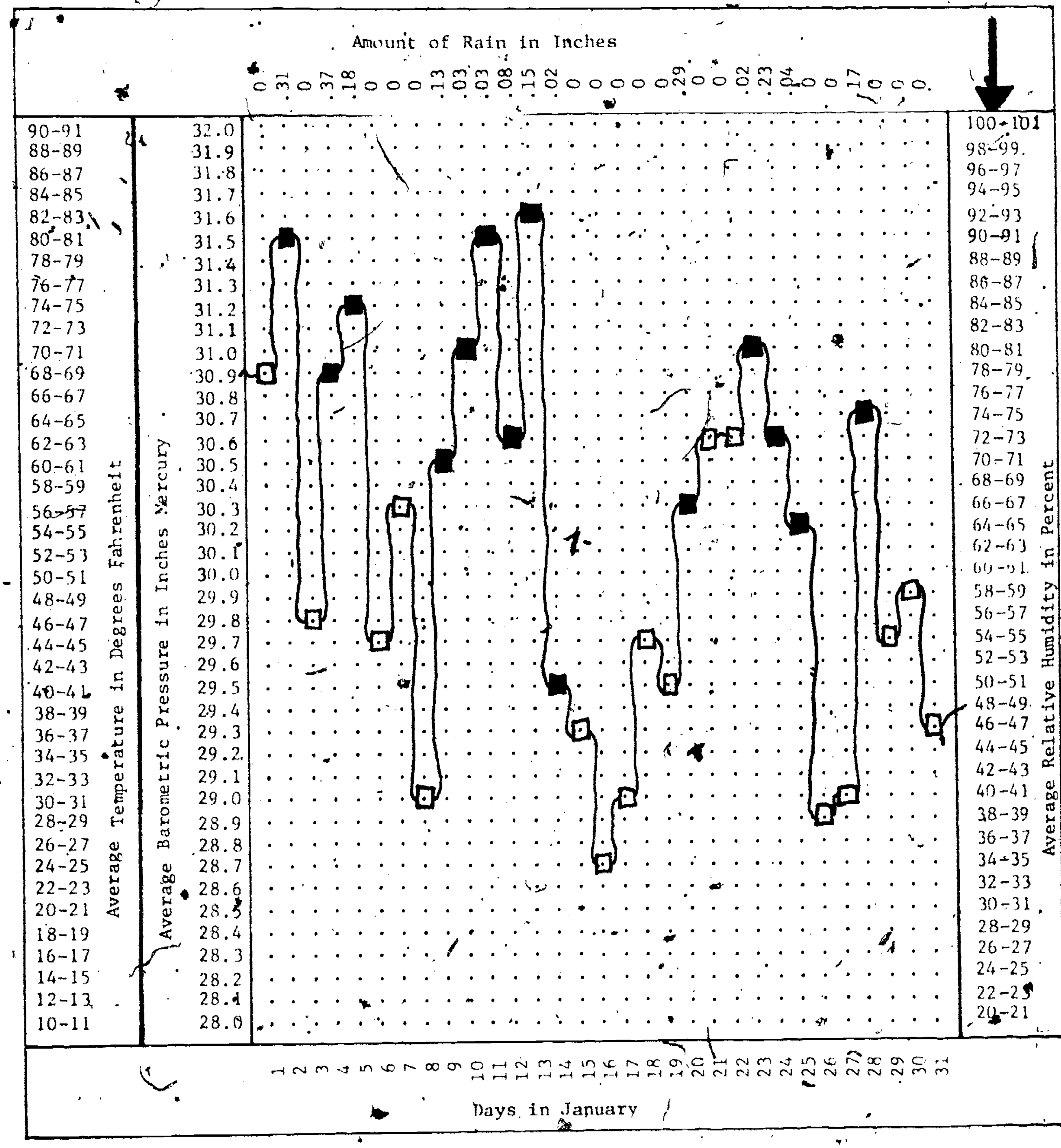

Amount of Rain in Inches
Average Temperature in Degrees Fahrenheit
Average Barometric Pressure in Inches Mercury
Average Relative Humidity in Percent
Days in January
Key
No Rain
Temperature
Barometric Pressure
Relative Humidity
Rain

Figure 2

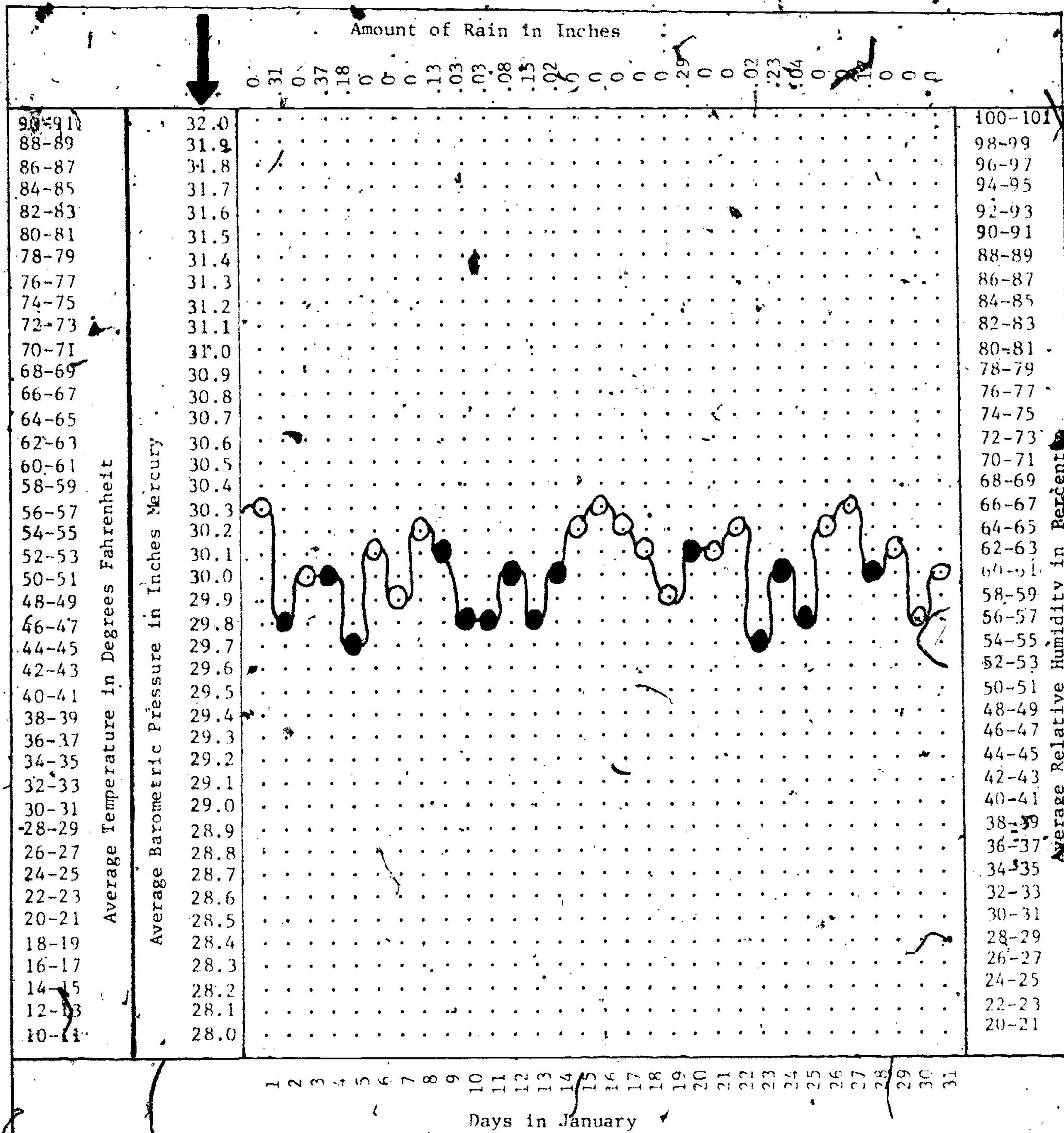


Key

No Rain { ☐ Relative Humidity ■ } Rain

GR5-6

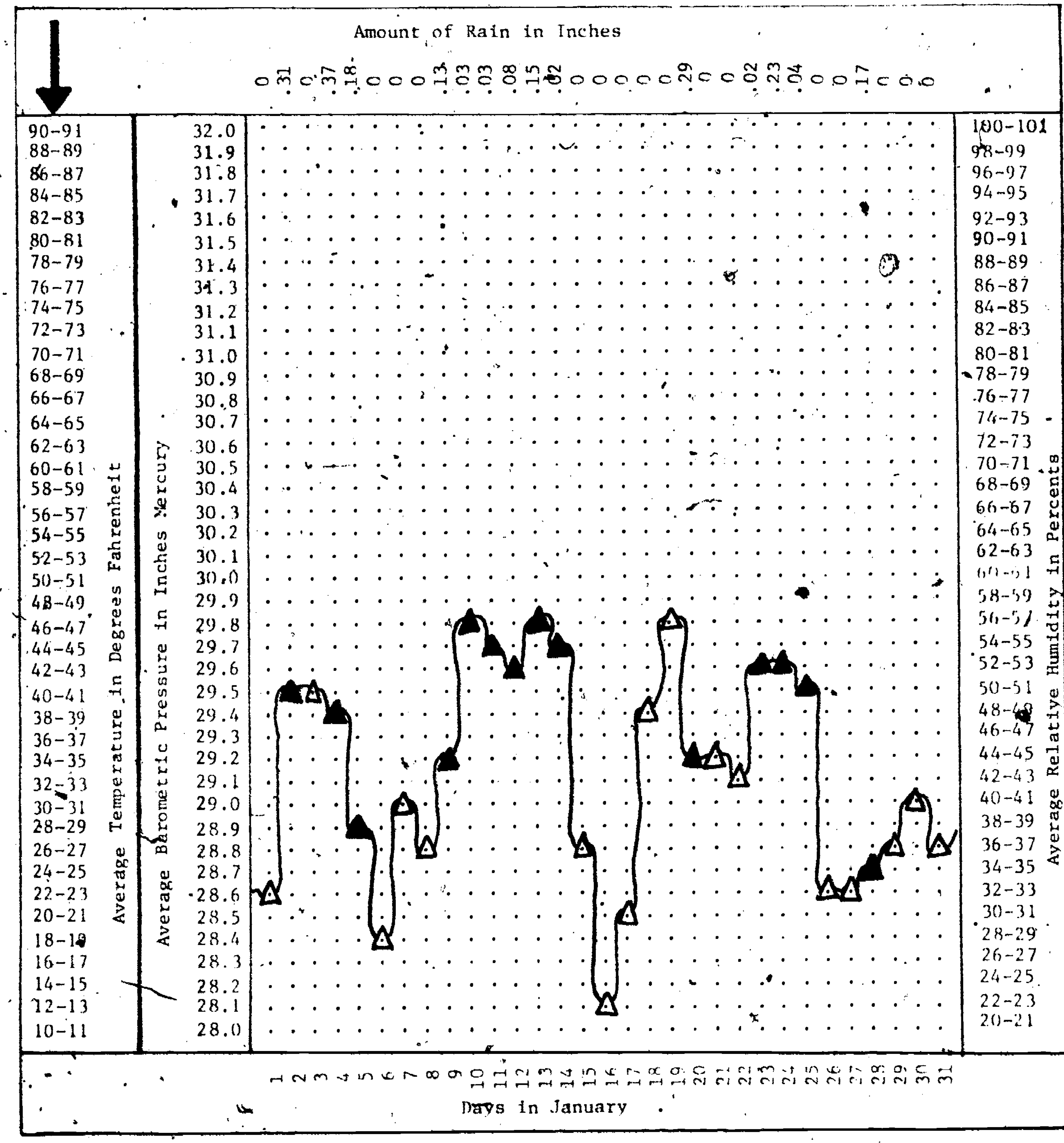
Figure 3



Key

No Rain { O Barometric Pressure ● } Rain

Figure 4



Key

No Rain { △ Temperature ▲ } Rain

3. The correlation between change in pressure and amount of rain can be seen on another type of three-dimensional pegboard graph. Figure 5 shows a graph of different amounts of rain vs. change in pressure for the month of January, 1972. (If the pressure changes from 30.0 inches of mercury to 29.9, the change is -0.1; to 29.8, the change is -0.2, etc.) A quick glance at the graph shows that the larger amounts of rain occur when the pressure drop is greater. This trend can be seen more clearly when the median amount of rain is found for a change in pressure in graduations of 0.2 inches each. Figure 6 shows these median amounts of rain circled and a line drawn between circles. The background paper "Using Scatter Graphs to Spot Trends" gives details on this technique. As children collect data, they can establish this trend, as well as others. They may also find that there is no trend at all.

The children can use the data shown on the pegboard graph to make predictions in terms of probability of rain. By counting blocks, the children can see that the pressure dropped on 13 days during September; it rained 5 of these days. The children may decide that the probability of rain is about 30% on days when the pressure has dropped. On the other hand, on the 16 days when the pressure remained the same or increased, it rained only twice. On those days the probability of rain might be stated as only 10%. As more data is collected, the children can make better predictions of the weather with less guessing required.

Figure 5

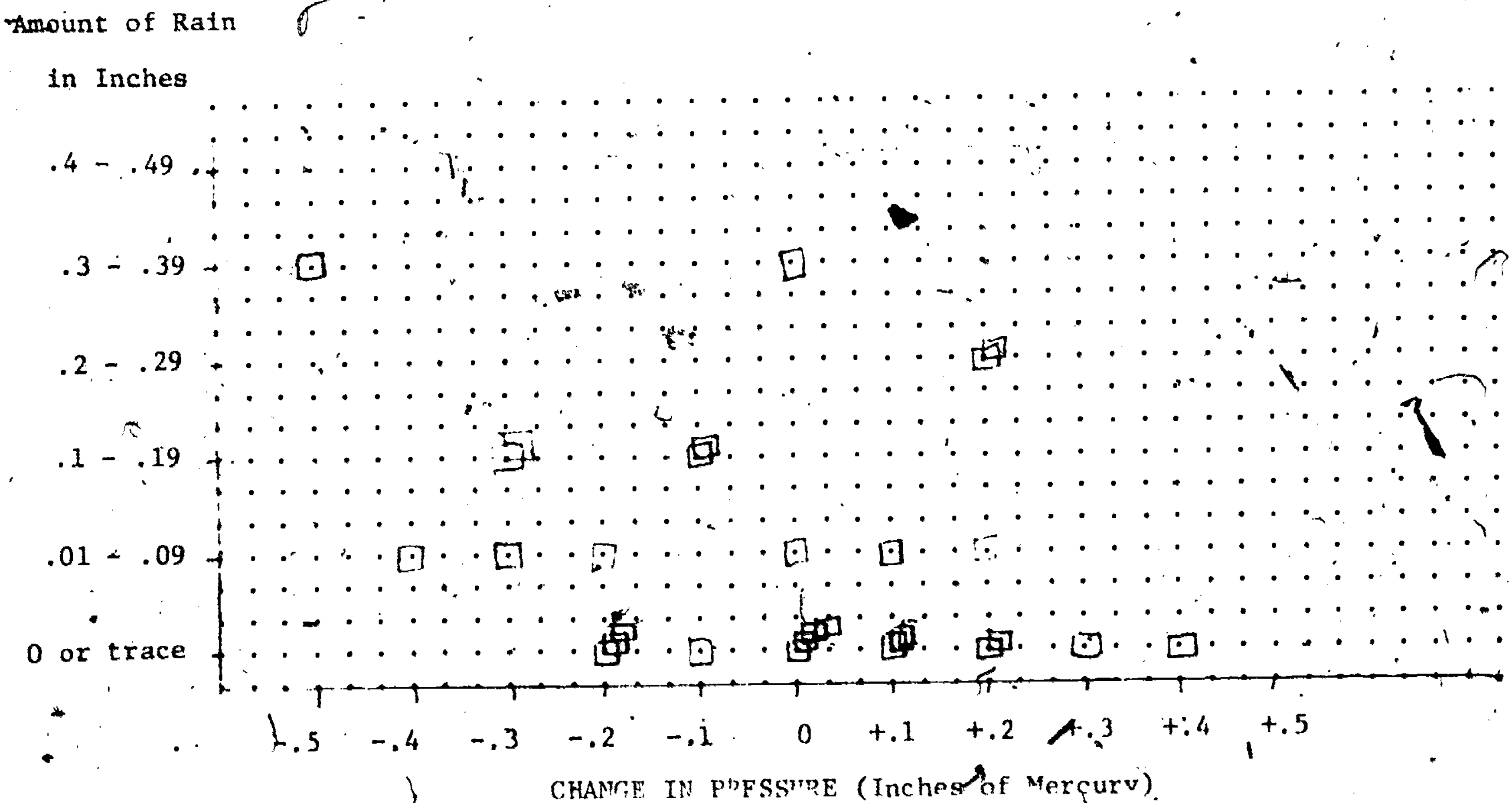
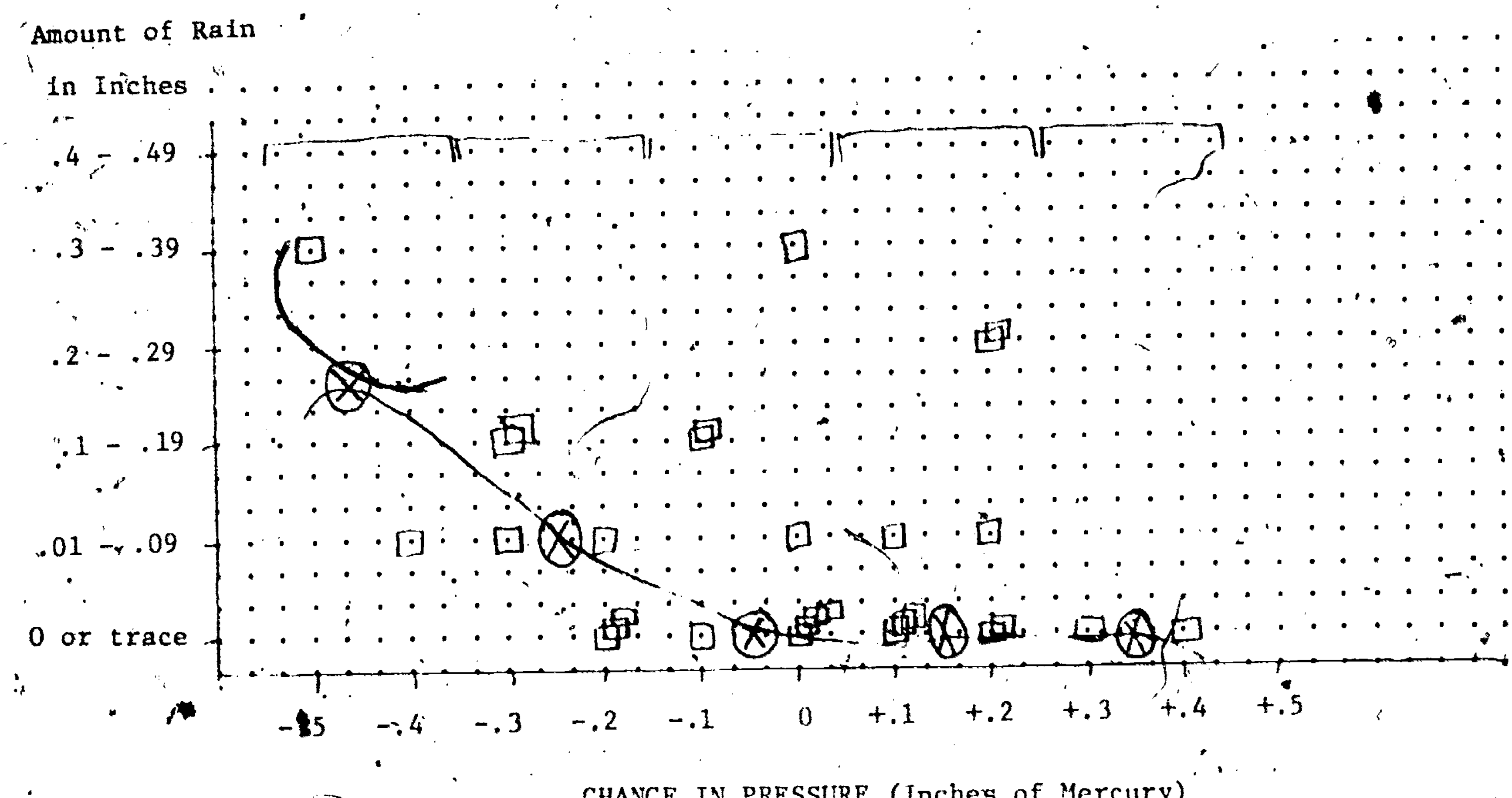


Figure 6



GR5-10

LOCAL CLIMATOLOGICAL DATA

U.S. DEPARTMENT OF COMMERCE
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
ENVIRONMENTAL DATA SERVICE

BOSTON, MASSACHUSETTS
GEN LOGAN INTERNATIONAL AP
JANUARY 1972

Latitude 42° 22' N Longitude 71° 02' W Elevation (ground) [illegible] Standard time used EASTERN WBAN #14739

Average Relative Humidity in Percents | Change in Pressure — Inches of Mercury

Date	Temperature °F Maximum (2)	Minimum (3)	Average (4)	Departure from normal (5)	Average dew point (6)	Degree days Base 65° Heating (7A)	Cooling (7B)	Weather types on dates of occurrence (8)
1	32	13	23	-7	17	42	0	
2	51	31	41	11	36	24	0	1
3	43	36	40	10	25	25	0	
4	42	35	39	9	33	26	0	1
5	41	16	29	-1	26	36	0	1
6	25	11	18	-12	4	47	0	8
7	36	23	30	0	19	35	0	8
8	34	18	26	-4	5	39	0	
9	51	18	35	5	26	30	0	
10	55	39	47	17	41	18	0	
11	50	38	44	14	40	21	0	2 8
12	49	35	42	12	34	23	0	1 8
13	58	35	47*	17	46	18	0	1 2 8
14	59*	29	44	14	20	21	0	1 4
15	34	19	27	-3	11	38	0	
16	19	5	12*	-18	-11	53	0	
17	34	5*	20	-10	-1	45	0	
18	49	28	39	9	23	26	0	
19	57	35	46	16	28	19	0	
20	38	32	35	5	24	30	0	1 4
21	41	28	35	5	28	30	0	1
22	38	26	32	2	24	33	0	1 8
23	51	35	43	13	36	22	0	2
24	48	37	43	13	32	22	0	1
25	56	25	41	11	29	24	0	1
26	25	19	22	-8	-1	43	0	
27	28	17	23	-7	1	42	0	
28	27	22	25	-5	16	40	0	
29	31	20	26	-4	11	39	0	
30	36	23	30	0	16	35	0	
31	31	21	26	-5	8	39	0	
	Sum 1269 Avg 40.9	Sum 774 Avg 25.0	Avg 33.0	Dep 3.1	Avg 21	Total 985 Dep -103	Total 0 Dep 0	
						Season to date Total 2426 Dep -333	Total 0 Dep	

Date (22)	Snow, ice pellets or ice on ground at 07AM In. (9)	Precipitation Water equivalent In. (10)	Snow, ice pellets In. (11)	Average station pressure In. Elev. 29 feet m.s.l. (12)	Resultant direction (13)	Resultant speed m.p.h. (14)	Average speed m.p.h. (15)	Fastest mile Speed m.p.h. (16)	Fastest mile Direction (17)	Sunshine Hours and tenths (18)	Sunshine Percent of possible (19)	Sky cover Sunrise to sunset (20)	Sky cover Midnight to midnight (21)
1	2	T	T	30.32	28	4.6	9.2	15	SW	5.4	59	9	8
2	2	.31	T	29.77	22	6.0	11.7	24	W	0.0	0	10	9
3	T	0	0	29.99	27	15.6	16.1	30	NW	8.0	87	4	2
4	T	.37	0	30.01	23	3.2	9.9	14	SW	0.0	0	10	9
5	T	.18	1.8	29.74	35	11.7	12.2	19	N	0.0	0	10	10
6	2	0	0	30.11	31	7.0	7.6	14	NW	7.6	83	4	6
7	1	0	0	29.87	25	8.4	10.6	21	W	7.0	76	6	5
8	T	0	0	30.21	31	15.9	16.3	26	NW	9.1	98	0	0
9	T	.13	0	30.09	21	17.0	12.7	30	SW	1.8	20	10	9
10	0	.03	0	29.81	25	5.0	8.3	16	SW	7.8	83	5	6
11	0	.03	0	29.83	33	1.8	5.0	10	NW	0.5	5	10	10
12	0	.08	0	29.98	25	5.5	8.5	18	NW	8.9	95	3	4
13	0	.15	0	29.85	18	9.7	10.8	76	SW	0.0	0	10	10
14	0	.07	T	30.00	31	11.0	13.5	29	NW	3.0	32	10	10
15	0	0	0	30.25	34	9.6	9.8	24	NW	1.5	16	10	9
16	0	0	0	30.30	32	18.6	18.8	29	NW	8.7	92	3	4
17	0	0	0	30.25	25	13.6	15.2	30	SW	4.0	42	10	5
18	0	0	0	30.13	23	11.1	11.5	24	SW	7.8	82	4	2
19	0	0	0	29.89	26	14.2	16.0	35	W	8.4	88	6	4
20	0	.29	3.0	30.13	20	4.3	10.5	18	SE	5.8	60	8	6
21	2	0	0	30.09	34	5.4	8.6	14	NW	6.3	65	8	8
22	T	T	0	30.19	03	3.8	6.6	9	E	0.0	0	10	10
23	0	.02	0	29.74	28	7.1	9.1	16	W	5.1	53	7	9
24	0	.23	T	29.97	32	3.8	8.2	19	NW	7.1	73	5	7
25	0	.04	T	29.78	22	15.5	24.3	49	W	3.4	35	7	6
26	0	0	0	30.24	29	23.1	23.4	32	W	9.1	93	1	1
27	0	0	0	30.26	27	16.4	16.8	25	W	9.3	94	1	3
28	2	.17	3.0	29.97	32	3.9	6.8	[illegible]	SW	0.6	6	9	8
29	2	0	0	30.09	26	[illegible]	11.2	21	W	9.6	97	1	2
30	1	T	T	29.84	26	7.5	11.2	26	W	6.0	61	7	7
31	1	0	0	29.97	28	14.5	14.7	20	W	9.8	99	10	0
		Total 2.05 Dep -1.89	Total 7.8	For the month: 30.02	28	7.5	12.1	49	W Date 25	Total 161.6 Possible month 294.2	55	Sum 198 Avg 6.4	Sum 189 Avg 5.[illegible]

Weather types: 1 Fog; 2 Heavy fog X; 3 Thunderstorm; 4 Ice pellets; 5 Hail; 6 Glaze; 7 Duststorm; 8 Smoke, Haze; 9 Blowing snow

Number of days			
Maximum Temp ≥ 90°	≤ 32°	Minimum Temp ≤ 32°	≤ 0°
0	8	22	0

Number of days: Precipitation ≥ .01 inch 14; Snow, ice pellets ≥ 1.0 inch 3; Thunderstorms 0; Heavy fog X 3; Clear 7 Partly cloudy 10 Cloudy 14

Greatest in 24 hours and dates: Precipitation .50 4-5; Snow, ice pellets 3.0 28+

Greatest depth on ground of snow, ice pellets or ice and date: 3 28+

HOURLY PRECIPITATION (Water equivalent in inches)

Date	A.M. 1	2	3	4	5	6	7	8	9	10	11	12	P.M. 1	2	3	4	5	6	7	8	9	10	11	12
1	T	T						T	T	T	T		T	.02	.02	.02	.13	.06	.05	.01	T			
2																								
3													T	.05	.02	.03	.01	.09	.07	.07	.03	T		
4																								
5	T	T	T	.03	.03	.02	T	T	.01	T	.01	.01	.02	.01	.01	.02	.01	T						
6																								
7																								
8													T	T	T	T	T	T	.02	.02	.01	.03	.03	.02
9																								
10	.02	.01	T	T																				T
11							.01	.02	T	T														
12		.01	.05	.02													.10	.05	T		T	T	T	
13								T	T	T	T	T	T	T										
14			.02	T				T			T													
15																								
16																								
17																								
18																								
19																T	T	T	.06	.12	.08	.01	.01	.01
20																								
21																							T	
22																								
23	.01	T	T	.01	T	T																		
24				.05	.06	.01	.08	.02	.01	T														
25						T	.01	T	.01	.01	.01	T							T					
26																								
27																								
28		T	.01	.02	.02	.03	.02	.01	.01	.01	.01	.02	.01	T	T	T								
29																								
30				T	T	T	T	T	T	T														
31																								

* Extreme temperatures for the month May be the last of more than one occurrence
\- Below zero temperature or negative departure from normal
≥ 70° at Alaskan stations
* Also on an earlier date or dates
X Heavy fog restricts visibility to ¼ mile or less
T In the Hourly Precipitation table and in columns 9, 10, and 11 indicates an amount too small to measure
The season for degree days begins with July for heating and with January for cooling
Data in columns 6, 12, 13, 14 and 15 are based on 8 observations per day at 3-hour intervals
Wind directions are those from which the wind blows. Resultant wind is the vector sum of wind directions and speeds divided by the number of observations. Figures for directions are tens of degrees from true North, i.e. 09 = East, 18 = South, 27 = West, 36 = North and 00 = Calm. When directions are in tens of degrees in Col 17 entries in Col 16 are fastest observed 1-minute speeds. If the / appears in Col 17 speeds are gusts
Any errors detected will be corrected and changes in summary data will be annotated in the annual summary.

Subscription Price: Local Climatological Data $1.00 per year including annual summary if published. Single copy: 10 cents for monthly summary, 15 cents for annual summary. Checks or money orders should be made payable and remittances and correspondence should be sent to the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.

I certify that this is an official publication of the National Oceanic and Atmospheric Administration, and is compiled from records on file at the National Climatic Center, Asheville, North Carolina 28801.

William H. Haggard
Director, National Climatic Center

SUMMARY BY HOURS

Hour Local time	AVERAGES Sky cover tenths	Station pressure In.	Temperature Air °F	Wet bulb °F	Dew Pt. °F	Relative humidity %	Wind speed m.p.h.	Resultant wind Direction	Speed m.p.h.
01	6	30.01	31	28	20	67	12.1	27	7.9
04	6	30.02	30	27	20	67	11.9	28	8.1
07	6	30.04	29	2[illegible]	20	69	11.2	29	6.3
10	6	30.06	32	29	21	64	11.5	28	7.8
13	6	29.98	37	32	23	59	12.8	27	7.5
16	6	29.99	37	32	21	56	13.0	28	8.0
19	6	30.02	34	30	21	62	11.9	27	7.1
22	6	30.03	32	29	21	64	12.4	28	8.3

USCOMM — NOAA — ASHEVILLE 925

GR5-11

OBSERVATIONS AT 3-HOUR INTERVALS

DAY 01	77.6
DAY 02	90.1
DAY 03	55.6
DAY 04	79.3
DAY 05	84.6 (?)
DAY 06	54.3
DAY 07	62.2
DAY 08	40.1
DAY 09	79.1
DAY 10	80.1
DAY 11	82.2 (?)
DAY 12	72.2
DAY 13	92
DAY 14	49.7
DAY 15	46.4
DAY 16	34.2
DAY 17	40.3
DAY 18	55.3
DAY 19	53.2
DAY 20	67.7 (?)
DAY 21	72
DAY 22	72.1
DAY 23	80.2
DAY 24	71.5 (?)
DAY 25	63.7
DAY 26	37.7
DAY 27	41.3
DAY 28	75.1
DAY 29	55.2
DAY 30	58.4
DAY 31	47.4

NOTES

CEILING COLUMN

UNL indicates an unlimited ceiling.

WEATHER COLUMN

- • Tornado
- T Thunderstorm
- Q Squall
- R Rain
- RW Rain showers
- ZR Freezing rain
- L Drizzle
- ZL Freezing drizzle
- S Snow
- SP Snow pellets
- IC Ice crystals
- SW Snow showers
- SG Snow grains
- IP Ice pellets
- A Hail
- F Fog
- IF Ice fog
- GF Ground fog
- BD Blowing dust
- BN Blowing sand
- BS Blowing snow
- BY Blowing spray
- K Smoke
- H Haze
- D Dust

WIND COLUMNS

Directions are those from which the wind blows indicated in tens of degrees from true North, i.e., 09 for East, 18 for South, 27 for West. Entry of 00 in the direction column indicates calm.

Speed is expressed in knots; multiply by 1.15 to convert to miles per hour.

ADDITIONAL DATA

Other observational data contained in records on file can be furnished at cost via microfilm, microfiche, or paper copies of the original records. Inquiries as to availability and costs should be addressed to: Director, National Climatic Center, Federal Building, Asheville, North Carolina 28801.

STATION BOSTON MASS   YEAR & MONTH 72 01

U.S. DEPARTMENT OF COMMERCE
NATIONAL CLIMATIC CENTER
FEDERAL BUILDING
ASHEVILLE, N.C. 28801

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Note: Handwritten figures represent the average relative humidity.

USING SCATTER GRAPHS TO SPOT TRENDS

Earle Lomon

In many USMES units the relationship between dependent variables is useful information for finding good solutions to real problems. Sometimes that relationship is rather definite and precise. For instance, the velocity, v, of a nearly frictionless cart that has rolled down a hill of height, h, is approximately given by $v(\text{ft. area}) = 8\sqrt{h(\text{feet})}$ and a plot of reasonably good measurements will be well fitted by a smooth parabolic curve (see technical paper on roadway design). A plot of a few experimental points can then be used for interpolation to predict the velocity obtained for heights between those measured.

There are other cases in which the relationship is not definite but in which knowing the trend of the variables with each other is important. An example is shown in Figure 1. Each dot represents an individual's chest width (vertical axis) and collar bone to waist distance (horizontal axis). The relationship is certainly not definite. For instance, for the 24 people with the collar bone to waist distance of 13 inches, the chest width varies from 6 1/2" to 12". Nevertheless, there is a clear trend for the chest width to get larger as the collar bone to waist distance increases. The children quantified this relationship by boxing off the points into clusters (See Fig.) and then using a middle point as representative of the cluster.

The resulting pairs of measurements at each of the five mid-points provide a relationbetween the variables which the children expressed in a table.

A. Chest Width = 7"
Collar Bone to Waist = 9"

B. Chest Width = 8"
Collar Bone to Waist = 11"

C. Chest Width = 8"
Collar Bone to Waist = 13"

D. Chest Width = 9"
Collar Bone to Waist = 15"

E. Chest Width = 11"
Collar Bone to Waist = 17"

This shows that for successive increments of 2 inches in "collar-bone-to-waist" there is a 1", zero, 1" and 2" increment in "chest width." This is an average of 1" of chest width increase for each 2" of "collar-bone-to-waist." That quantitative result can be used for interpolation. (A chest width of about 10" goes with a "collar-bone-to-waist" average of about 16".) It can also be used for extrapolation. (A chest width of 12" goes with an average "collar-bone-to-waist" of about 19".) Of course, extrapolation must be used only for short extensions: one might otherwise conclude that a chest width of 2" implies a "collar-bone-to-waist" of (-1)"!

The children had a very direct use of their clustering and table. They surmised that aprons made "perfect" for any of the above 5 sets of measurements would be comfortable for everyone in the cluster, because the variation in a cluster was small enough. Thus, they concluded that they only needed at most those five choices to satisfy customers for the aprons. It is most important to note what the cluster graph told them they did not need. Although there are people with chest widths near 7" and people with "collar-bone-to-waist" of 15", no one in their sample had both measurements simultaneously. Thus, the demand for aprons suited for that combination would be very low and they made none. In fact, they had reduced a possible 5 x 5 = 25 apron sizes to only 5! This was because the scatter graph had shown them the <u>correlation</u> between one variable and another. This is another way of speaking of the <u>trend</u> of one with the other.

The children "eye-balled" the graph and drew free-hand boxes to isolate the clusters. They then found the center of a cluster by eye. Are there less arbitrary and perhaps more precise ways of finding a trend in a scatter graph? Sometimes the "eye-balling" is completely inconclusive, as it was for the "waist-to-knee" or "waist" scatter graph of the same class. (See Figure 2 on following page)

Let us apply a procedure of finding the median value of "waist-to-knee" for equal bands of waist measurements. Let us first choose 1" bands, i.e. 21" to 21 1/2", 22" to 22 1/2", etc. (See Figure 3, page .) The first such band (we will call it the 21" band) has 9 points in it. Counting up five points from the bottom (or from the top) we come to "waist-to-knee" - 19 1/2" and have marked it with an <u>X</u>. The 23" band contains 20 points, so we mark an <u>X</u> midway between the 10th and 11th points, and so on, for all the other bands. The <u>X</u>'s cover a much more narrow range than the points, but they jiggle around a lot, showing no obvious trend. Yet, the jiggling may obscure a trend. For instance, if one ignores the first point, then one may infer a possible upward trend.

The jiggling of the <u>X</u>'s is about as large as any overall trend. Thus, further averaging is needed to establish any correlation. We can find the median points of 2" bands (21" to 22 1/2", 23" to 24 1/2", etc.). These medians are shown by circles on Figure 4. Now the jiggle is much less, adjacent circles being within an inch of each other, except for the last circle. With the exception of the last circle again, all the circles are 18" + 1/2". As 1/2" is an irrelevant amount with respect to "waist-to-knee" comfort, we have now established that there is no significant trend, if any.

What about the last circle? It could be just a fluctuation, especially as there is so little (only 6 points) data in the 33" - 34 1/2" band. However, it could be the beginning of a region with a stronger trend (very large waists go with larger "waist-to-knee." We simply can't tell from this data, and it would be dangerous to draw important inferences from it. Is there any other useful information to be gleaned from this graph? One notes that there are no "waist-to-knees" of under 16" for "waists" of over 29". Thus, one might decide to provide fewer aprons with the largest waist measurement and smallest "waist-to-knee" measurement.

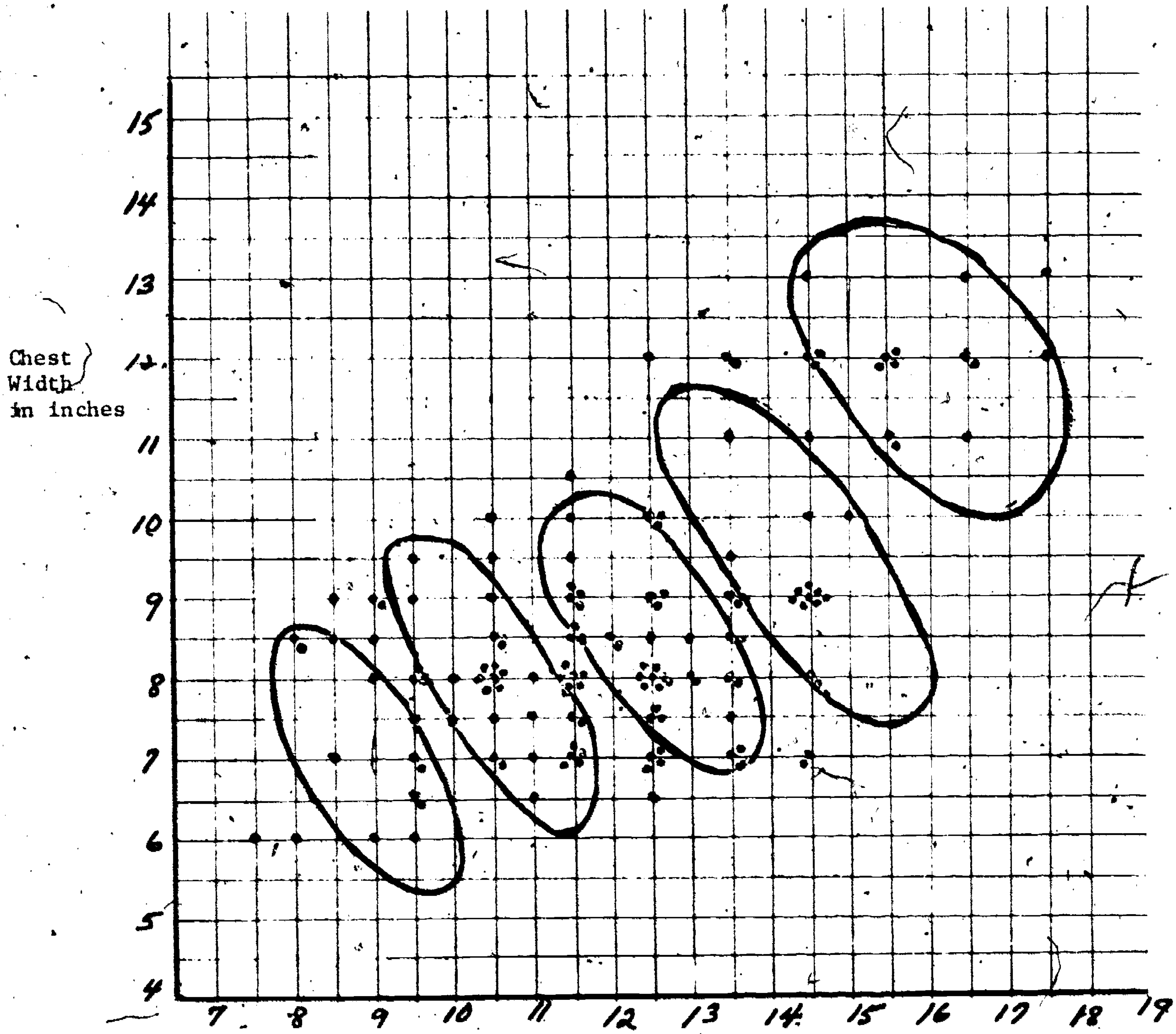
Means within each band could be used instead of medians. There will be some differences in the jiggling of means and the jiggling of medians. But when the bands are big enough to reduce the jiggling of one to an insignificant amount, it will likely reduce the jiggling of the other similarly. The trends shown may be a little different if the points don't tend to be roughly symmetric about the median point.

The above paragraphs describe the procedure for spotting a trend in the "Y" variable (waist-to-knee measurement) with respect to the "X" variable (waist measurement). However, the scatter graph might have been drawn with the variables represented on the opposite axis. The reader might try turning the graph around and drawing equal bands for waist-to-knee measurement and find the median waist measurement for each band. Do the results confirm the lack of any significant trend?

The reader might also find medians (or means) for the chest width vs. collar bone to waist scatter graph (Figure 1). Are the results the same as those obtained by the eye-balling method?

Triangle Graphs and Scatter Graphs

Triangle graphs basically contain the same information as scatter graphs. They show a relation between two measurements which is designated by a point. In the technical paper, "Geometric Comparison of Ratios," a case studied is trunk length (vertical axis) versus leg length (horizontal axis). In Figure 6 of that paper, points are marked that show the relation of those two measurements for different people. The only difference from a scatter graph is that lines are drawn from the origin (zero trunk length and zero leg length) to each of those points. The reason for those lines is that their steepness is a geometric description of the "ratio" or "proportion" of those two measurements. Thus, in a triangle graph we also learn something about the scatter, median, etc. of the ratio of the two measurements. Nevertheless, all triangle graphs can be used as scatter graphs, by ignoring the lines. The graph can give information on trends, size clusters, etc.



Collar Bone to Waist in Inches

Figure 1

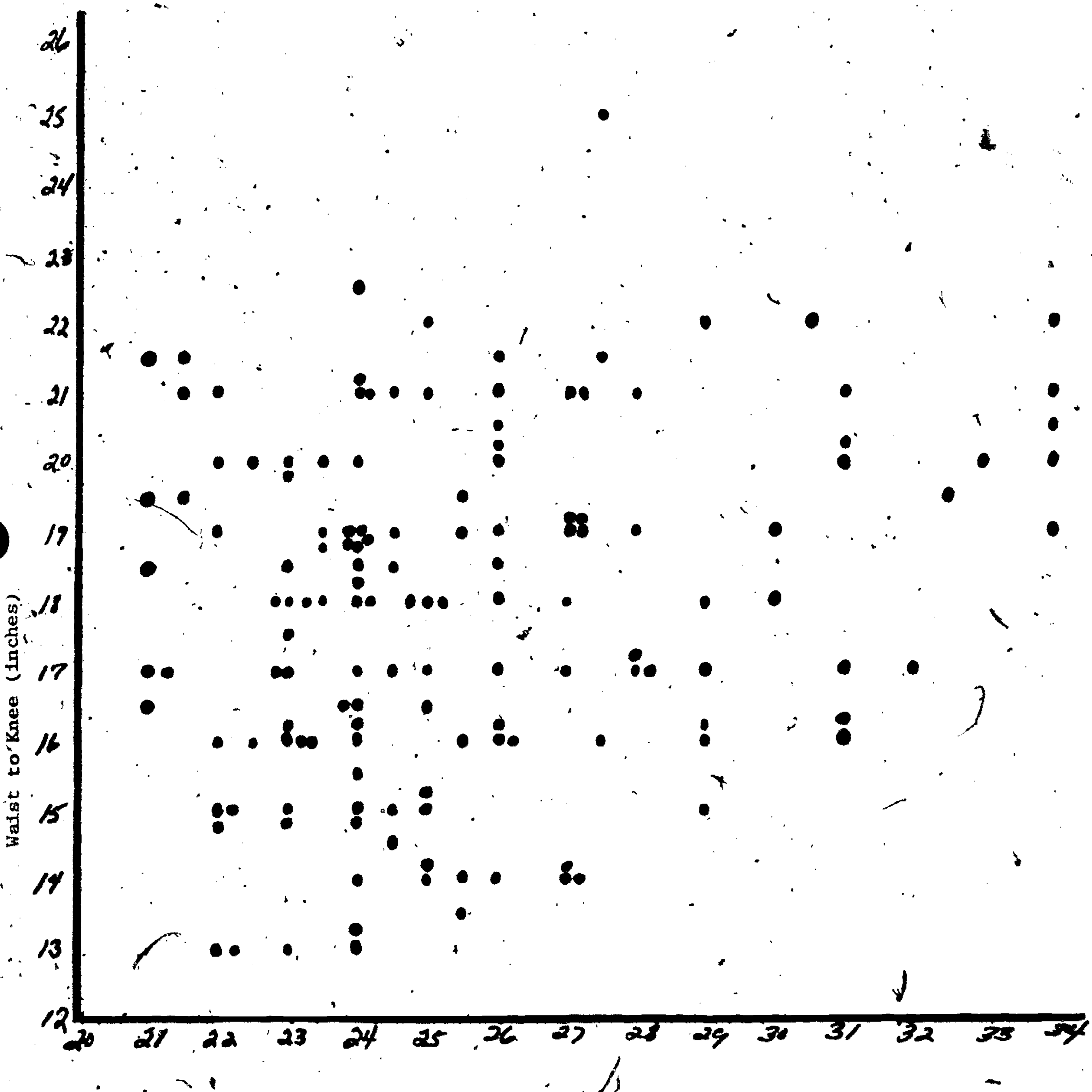

Waist to Knee (inches)
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
Waist (in inches)

Figure 2

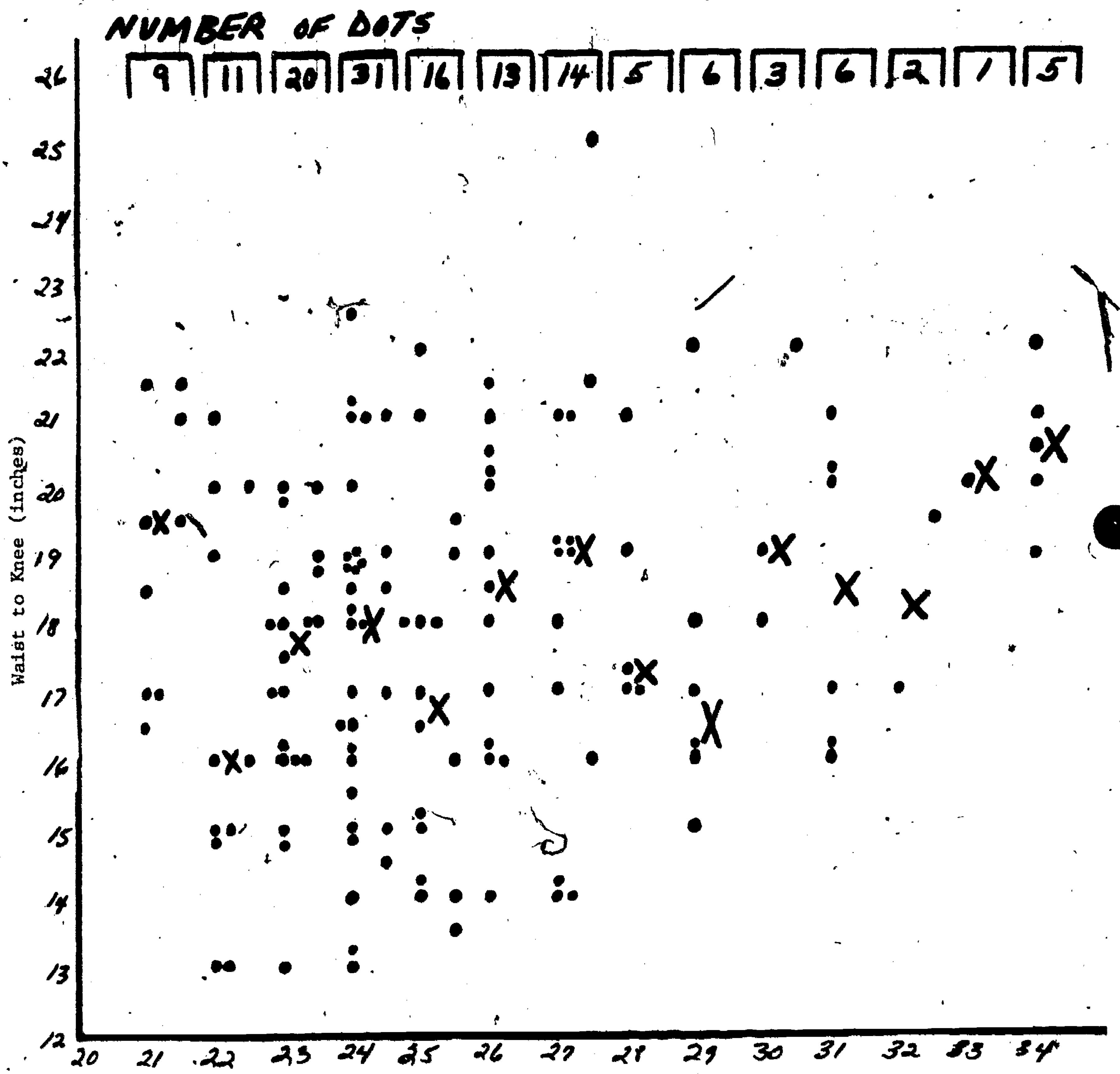

NUMBER OF DOTS
9
11
20
31
16
13
14
5
6
3
6
2
1
5
Waist to Knee (inches)
Waist (in inches)

Figure 3

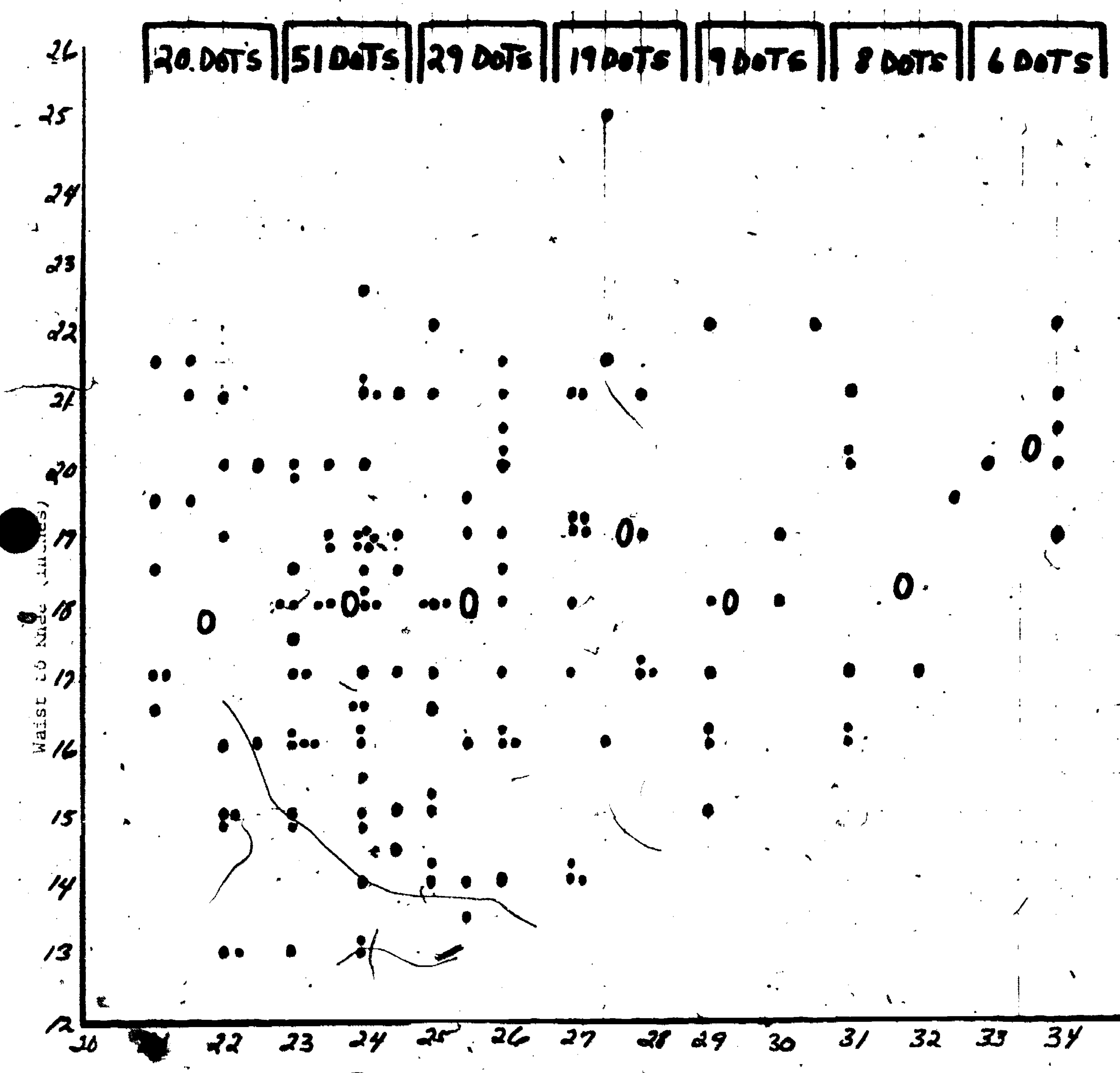

20 DOTS
51 DOTS
29 DOTS
19 DOTS
9 DOTS
8 DOTS
6 DOTS
Waist to Knee (inches)
Waist (in inches

Figure 4

DATA GATHERING AND GENERATING HISTOGRAMS AT THE SAME TIME

(STACK 'EM AND GRAPH 'EM AT ONE FELL SWOOP!)

by
Ed Liddle

In several USMES units, the histogram occupies a revered place in data analysis and communication. However, in working with children and the teachers of children I am frequently left with the feeling that, while an acceptable graph can be guided, cajoled, or coerced from the students, the meaning of the graph has not come through. The major concept, that of the graph being a 2-D representation of the data gathered from a 3-D physical world, is oft overlooked or ignored. This paper looks at ways in which the analysis of the data and the gathering of the data can be conjuncted. That is, what are ways in which the realness of the data can be preserved through the analysis and interpretation stages.

In considering this problem from a practical perspective, I see two major problems in the way many classes utilize histograms. The first is the conceptual distance between the data and the histogram; the second is the time delay. Many times data are gathered and days intervene between this collection and the organization and graphing process. This time lag adds to the problem of conceptually relating the histogram to the physical data. Detailed records of what was done and what happened in the experimental procedure would facilitate the recalling of the physical context in which the data occurred but the specificity children use is often insufficient to be of much help. In addition, if young children are not at a stage of development which allows them to function well at an abstract level, the use of recorded data presents even more of a problem.

What, then, can be done to alleviate these problems? The general theme of the following is to have (1) the data gathering and histogram construction occur as close together in time as possible and, (2) to have the format used in the data gathering, analysis and display processes be as closely related in physical characteristics to the objects in the system of interest as possible. For example, if one is gathering data on people preferences for red, green or yellow pop (soda or tonic!) three plastic tubes fixed in a piece of wood into which marbles could be dropped would be used in conducting the survey. The first tube would be for red marbles, the second for green marbles and the third for yellow marbles. As people indicate their preference a marble of the appropriate color is dropped in the proper tube. After completing the survey, one ends up with three stacks of marbes, the color representing the color of pop. By viewing the three tubes horizontally one has a histogram! There is no need to transcribe written responses to a sheet of graph paper. The histogram has been constructed when the data were gathered. (See figure 1.)

If one wishes to teach the idea of a two dimensional histogram, obtain a slide projector or other light source. Shine the light horizontally on the marble and tube array. The shadow (really light with holes in it!) can be projected on a sheet of paper. This can be traced. The idea is that a two dimensional graph is really a projection of the 3-D world onto a 2-D sheet of paper.

Other such histogram-data collectors are possible. The simplest device consists of a board through which nails have been driven. Any uniform objects--wooden beads, nuts (the kind that go on bolts!), wooden discs, etc. are then used to record responses by piling them up on the appropriate nail. (See figure 2.)

In working with 1st grade children an even more basic approach is suggested. For example, consider a cookie preference survey. Instead of recording the data with pencil and paper in the standard way, the cookies themselves can be placed in a bag (sack) or box. For example, if Oreos are preferred by an individual, he/she drops an Oreo into the box for Oreos. After completing the survey, each type of cookie can be placed in a stack. This won't work, however, if the different cookies vary in thickness. In that case, the children can use cut-outs (Tri-Wall?) of the cookies, thus obtaining a uniform thickness for each type of cookie.

Figure 1

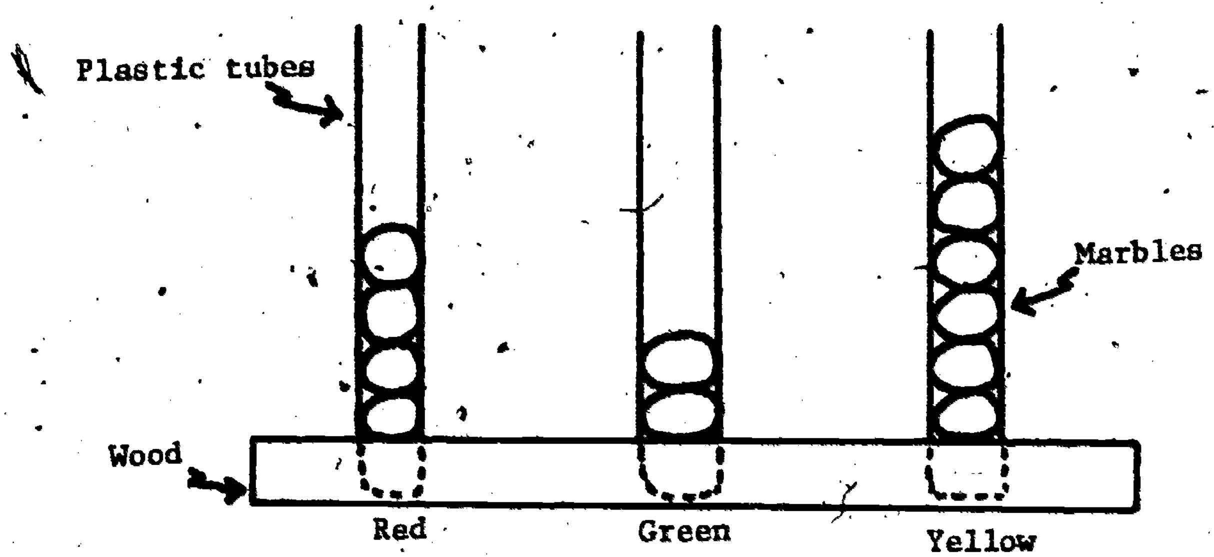
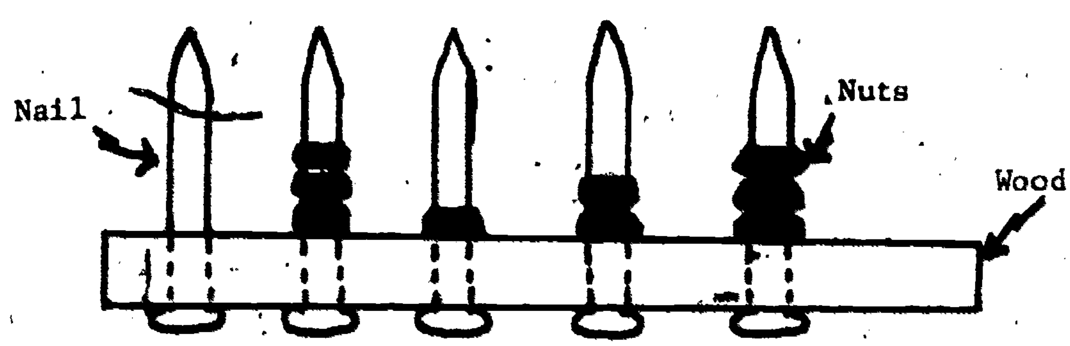


Figure 2



GULLIVER'S TRAVELS ACTIVITY*

by

Abe Flexer

<u>Possible Teaser</u>: Discuss J. Swift's <u>Gulliver's Travels</u> and read the following excerpt from Chapter 6 of "A Voyage to Lilliput":

> "Then they (Lilliputian tailors) measured my right thumb, and desired no more; for by a mathematical computation, that twice round the thumb is once round the wrist, and so on to the neck and the waist, and by the help of my old shirt, which I displayed on the ground before them for a pattern, they fitted me exactly."

<u>First Challenge</u>: "Were the Lilliputians accurate? How can we find out?"

Have small groups devise ways to evaluate the Lilliputian approximations. They will need to discuss:

1. How to make measurements (techniques, units);
2. How to record and interpret data (charts, tables, graphs).

Allow the groups to act with minimal guidance. Eventually, at least one group should devise scatter plots of thumb circumference <u>vs</u>. wrist circumference, wrist <u>vs</u>. neck, neck <u>vs</u>. waist. See Fig. 1 for a scatter plot of weight <u>vs</u>. height.

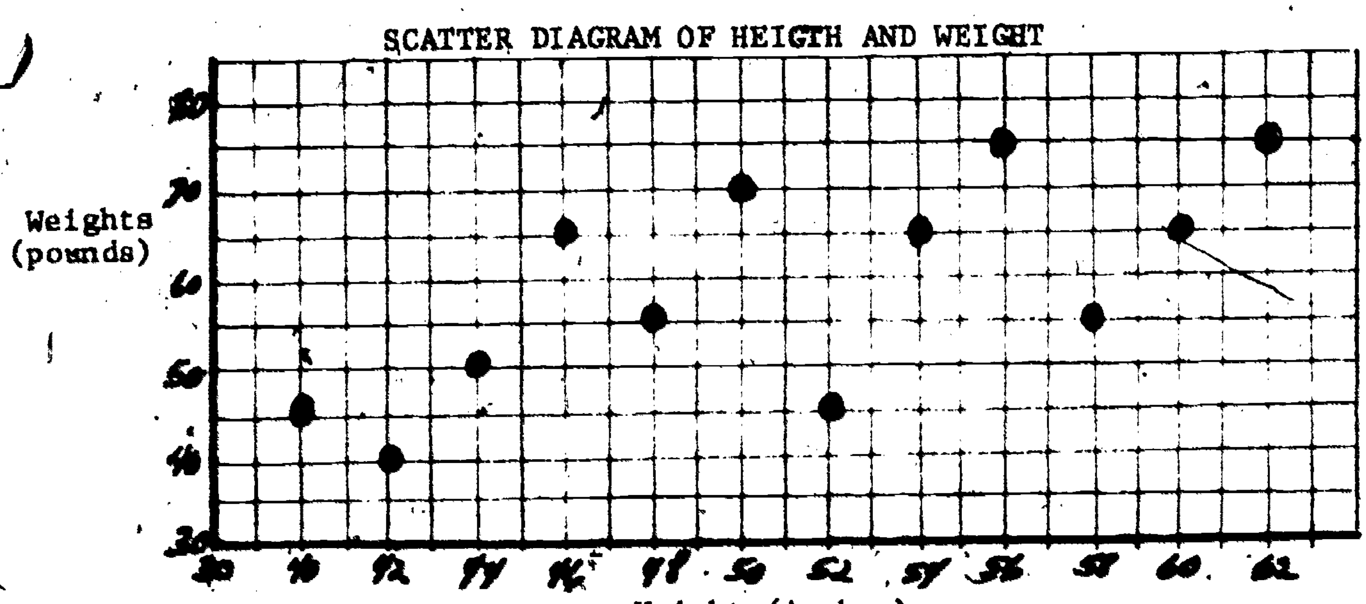


Figure 1

Convene a class discussion when groups have made some progress to compare their recommendations. Adopt one or several of the recommendations and have small groups gather data for the class with each group using the recommendation of its choice.

Employ another class discussion to evaluate each recommendation and the method used by the Lilliputians. Triangle diagrams might be constructed from scatter plots by drawing straight lines from the points to the origin. These lines represent the ratios of the two measurements and might be compared with plots of known numerical ratios, e.g. 2:1, 3:1, 4:1. See Fig. 2 and 3, and technical papers on the subject.

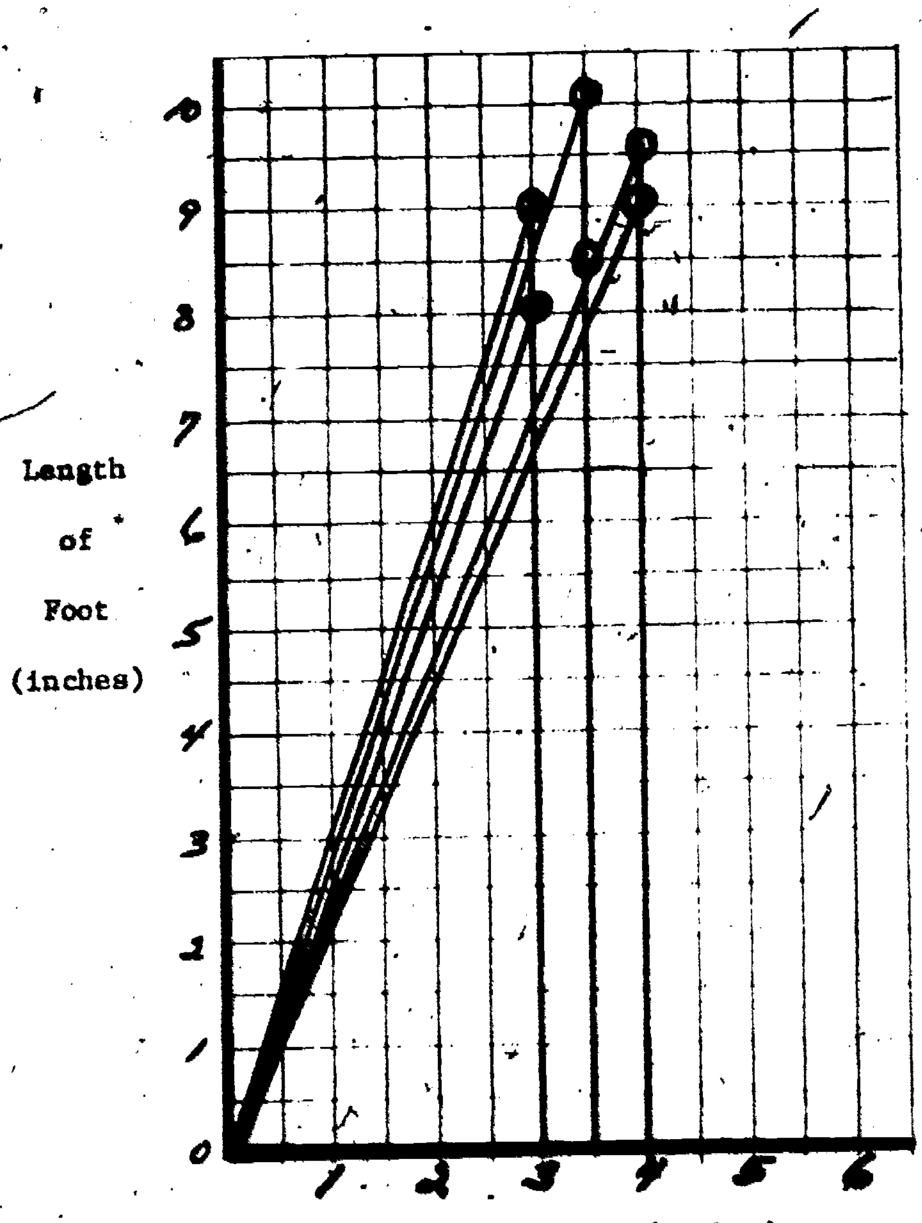


Figure 2

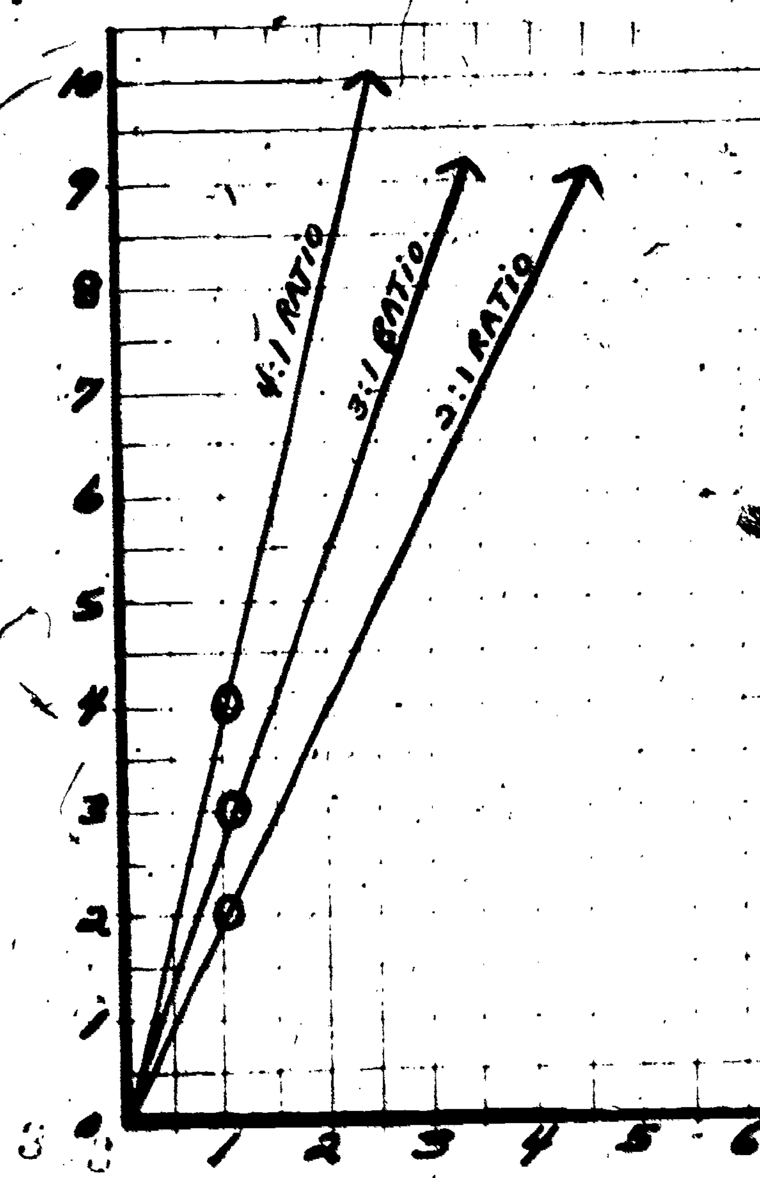


Figure 3

<u>Designing for Human Proportions Challenge</u>: "Can you discover other correlations among parts of the body? How might they be used for clothing patterns or furniture design?" Again, work should be done in small groups with an occasional class discussion to compare the conclusions of the small groups. Your students will suggest many correlations and should be encouraged to test as many as time and their patience permit. Some of the correlations that may be discovered:

- width of hand (or palm) <u>vs</u>. distance from elbow to wrist (hockey gloves)
- width of foot <u>vs</u>. length of foot (number of shoe widths needed for each shoe size)
- arm length <u>vs</u>. height (placement of chalkboards, light switches, etc.)
- seat student on floor against wall and compare distance from floor to top of head <u>vs</u>. distance from wall to heel (ratio of jacket length to trouser length, ratio of back height to seat depth for chairs, etc.).

"Do the correlations you have discovered hold true for people of all ages and of both sexes? How can the information be used (number of furniture or clothing sizes needed, best height for chalkboards, puppet sizes)?" This question should be raised during the final class discussion in connection with the second challenge. Students should set up a program for gathering data from other classes, other grades, families and friends. Data should be pooled, but analysis of the data should again be carried out in small groups that report in class discussions. Encourage students to summarize their discoveries on a graph, sketch or short summary that can be explained and posted for all to see. Perhaps the most striking discovery your students will make is that the comparison of length of head vs. height varies dramatically with age (from about 1:4 among infants to about 1:7 among adults.) Fig. 4 shows sets of data on hand length and leg length for both adults and children on one diagram. The median trunk length/leg length ratio for children is 12/22 and for adults, 36/40.

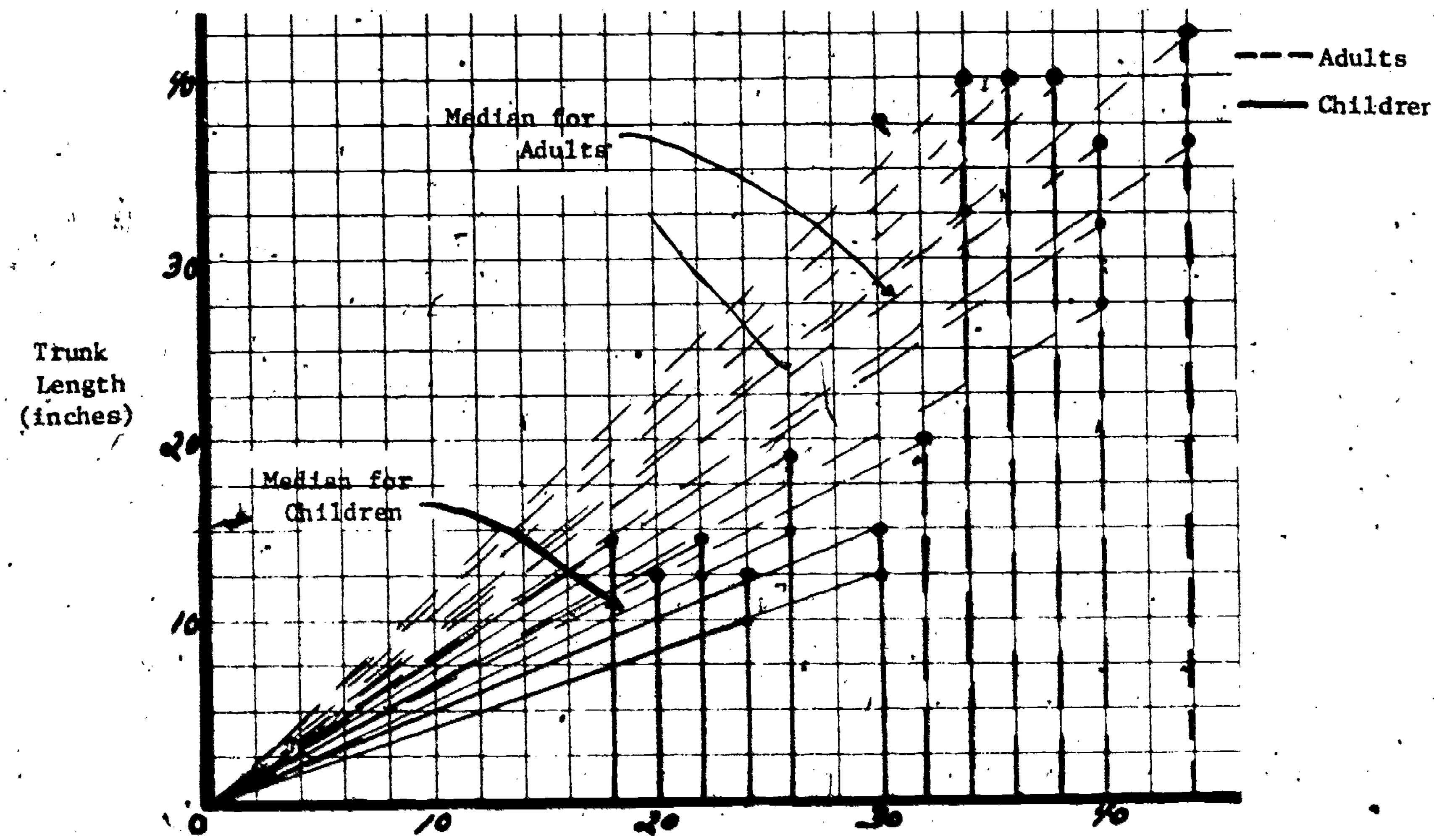


Figure 4

Subsequent Activities: The Designing for Human Proportions activities can lead into a variety of other investigations that offer many opportunities for your students to discover further uses for the skills and insights developed during the unit. Students should work in pairs or very small groups and should be encouraged to select an activity of particular interest to them.

Geometric Correlations (Taken from Biggs & MacLean, "Freedom to Learn" which should be consulted for additional ideas.)

-graph circumference vs. diameter for various circles

-graph perimeter vs. area for various squares

-graph perimeter vs. side for various squares

-graph area vs. side for various squares

-graph area vs. width for rectangles of constant perimeter

-graph volume vs. circumference for cylinders of constant height

-encourage your students to conduct original investigations of other, similar correlations -- there are many!

Estimation Problems (Based on ESI's "Particles and Peas" and the techniques developed therein; your students will grasp these techniques quickly if they have worked on the unit.)

-estimate the number of:
beans in a jar;
bricks in a wall;
hairs on someone's head;
people in a crowd;
kernels on an ear of corn;
etc.

-estimate the weight of:
a grain of rice;
a grain of sand;
a grain of salt

-estimate the pressure of:
your foot on the ground;
an elephant's foot on the ground;
a knife edge on something you are cutting;
a pencil point you are using on a sheet of paper, etc.

<u>Additional Scaling Problems</u>: These problems are most similar to those faced by Gulliver and his hosts and will provide interesting challenges for your students, particularly if they verify their estimates by constructing some of the objects listed among the following suggestions:

1. What are the dimensions of the smallest useful: ruler; blackboard eraser; pencil eraser; soup-spoon; knife; thumbtack; bookend, etc.

2. What are the dimensions of the largest useful: ruler; blackboard eraser; pencil eraser, etc.

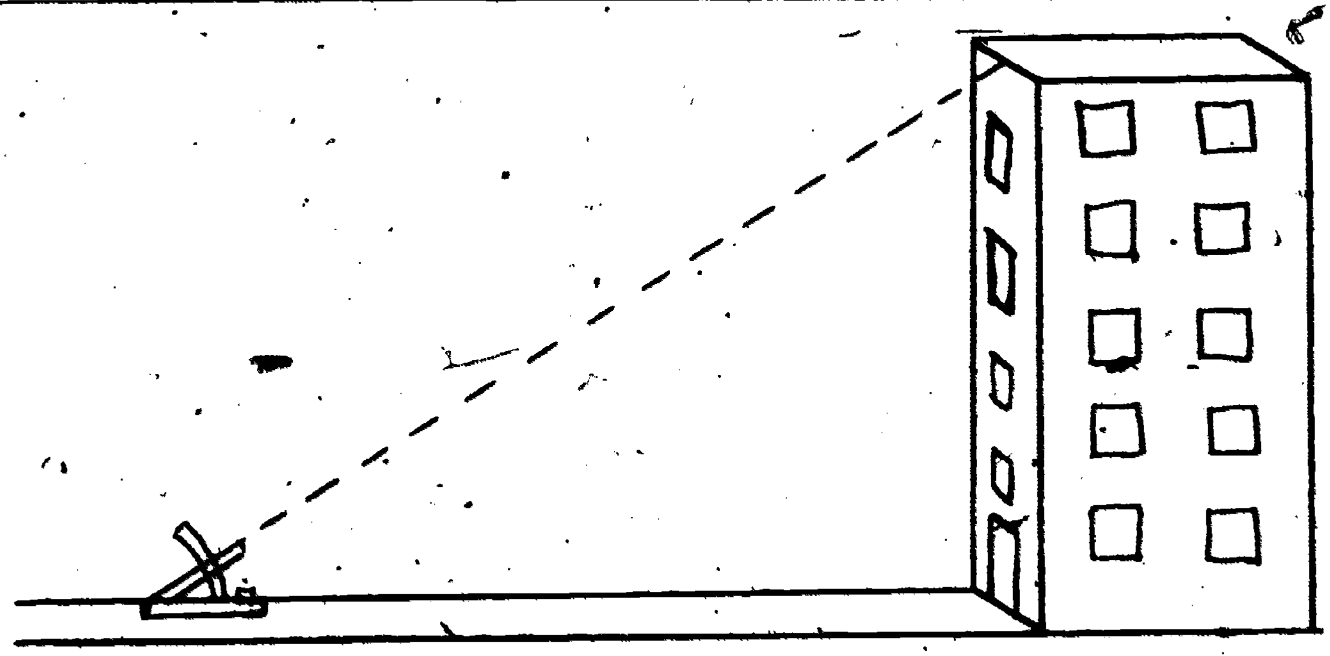
MEASURING HEIGHTS OF TREES AND BUILDINGS

by
Earle Lomon

Very often it is desirable to measure the height of some large structure (tree, building, hill, B747, etc.) for which it is difficult or impossible to use a direct measurement such as dropping a tape measure from the top to the ground. Various methods of "triangulation" are usually employed in such cases. School children can discover how to apply such methods through measurement of a series of paper or blackboard size triangles and find ways to obtain an adequate sighting angle measurement on a tall, distant object.

We shall concentrate first on a simple approach for the frequent case (such as most buildings or trees) in which: 1.) the tall object rises nearly vertically from the ground, 2.) a nearly level approach to the base of the object is available and 3.) the level approach is at least as long as the object is tall. This will then be modified to allow for a shorter approach. Finally, we will discuss techniques when the simplifications due to verticality and clear level approaches are not possible. Discovery and measurement techniques may progress more naturally if the students first have experience with the simpler cases.

Measuring the Height of a Tall Building with an Open Level Approach

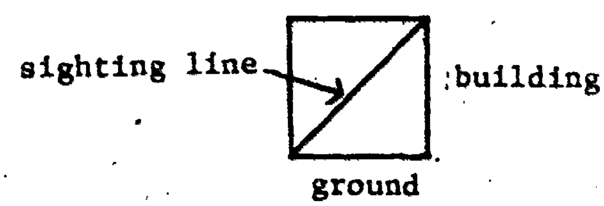


A student holds one arm horizontal (of an instrument he designed and built himself)* and points the other end at the top of the building. The horizontal arm is also pointed at the building at a place which is about the same height as the instrument is held. He may have a level fixed on the horizontal arm if he wants more accuracy (or if there is a slope to the ground). To point the upper arm accurately he may have put sighting notches on it. He moves towards or away from the building until he finds that the sighting arm is a 45° (or one-eighth of a full circle) to the horizontal arm. The 45° angle is marked on a piece of cardboard or wood attached to the horizontal arm. It can be found by dividing a right angle in two by compass construction, or by using a protractor. The student then measures the distance from that point to the base of the building. After some investigation of right triangles he will be able to conclude that that same distance is the height of the building.

How does the student learn that a triangle with one right angle (between building and ground) and one 45° angle (between ground and sighting line) is isosceles? He may have already learned this from paper and pencil or geo-board activities. On the other hand he may have found it out in response to the challenge to measure the height of a building or tree. He draws a vertical line on a blackboard to represent the building. He then asks what will be the angle between his line of sight and a horizontal line at a point the same distance from the building as its height. He draws the horizontal distance, and then measures the angle (with a protractor, or tries a series of 30°, 45°, 60° angles previously made to see which one fits).

*See page 6 for one possible design.

He finds about 45°. He then tries a "building" of a different height and gets approximately the same result. After several trials he may decide it will be the same for a very large triangle, even if he can't test the vertical side. Just in case, a test can be arranged by dropping a long rope from a roof. Or the height of one story could be measured, and the number of stories counted. Sometimes a student in drawing all these triangles may stumble upon a simple proof: such as by constructing another vertical and horizontal side to complete a square. The symmetry indicates

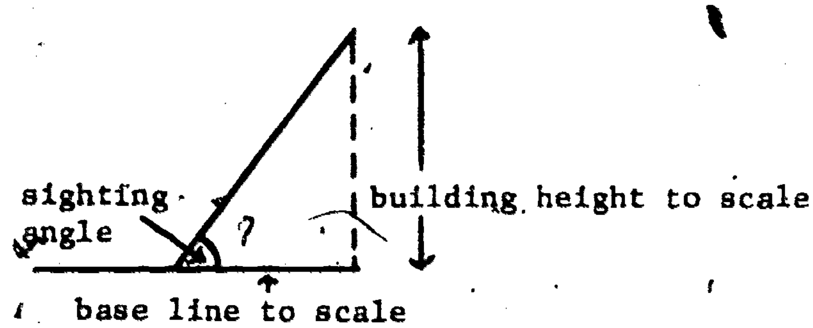


that the sighting line must bisect the right angle.

Measuring the Height When Retreat from the Building is Limited

What if there is not sufficient level space? The student finds for example that he can only back up to where the sighting line is 60° from the horizontal.* The following are two ways for a student to solve this problem.

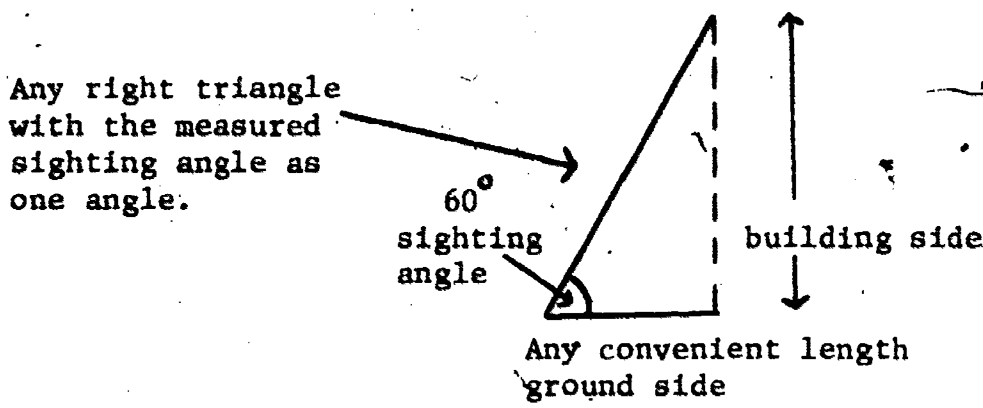
A. He can draw the base line to scale and use the measured "sighting" angle to construct a triangle as follows:



*It should be noted that accuracy of the horizontal and the angle become more important at a close approach.

By constructing a vertical line at the end of the base, he then has the height to the same scale. He can measure the length of the line and calculate the actual height of the building.*

B. He can draw a right angled triangle with a 60° angle and find the ratio of the "building" side to the "ground" side. (If he has chosen the "ground" side to be a few units on his ruler, he need only divide by a one digit number.)



$$\frac{\text{building side}}{\text{ground side}} = \frac{\text{building height}}{\text{ground distance}}$$

The ratio is not 1, as it was for the 45° case, but about 1.7 (1 3/4). Will it stay the same ratio, as it did for all 45° right triangles? He checks with several sizes and always gets the same ratio. He decides that the building is about 1.7 (1 3/4) times taller than the level distance from his sighting point to base of the building.

After a time a student will decide that ratios of sides of right angled triangles remain constant if another angle in the triangle is fixed. Then he will simply measure the sighting angle from any convenient point and measure the distance from that point to the base of the building. He will:

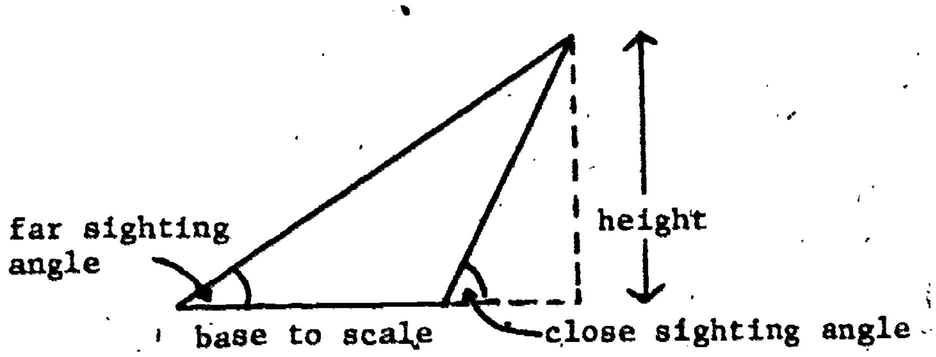
a. draw a small right triangle with the same base angle as measured,

b. calculate the ratio of sides measured on the small triangle,

c. calculate the building height.

Measuring Height when the Point Under the Height Cannot be Reached

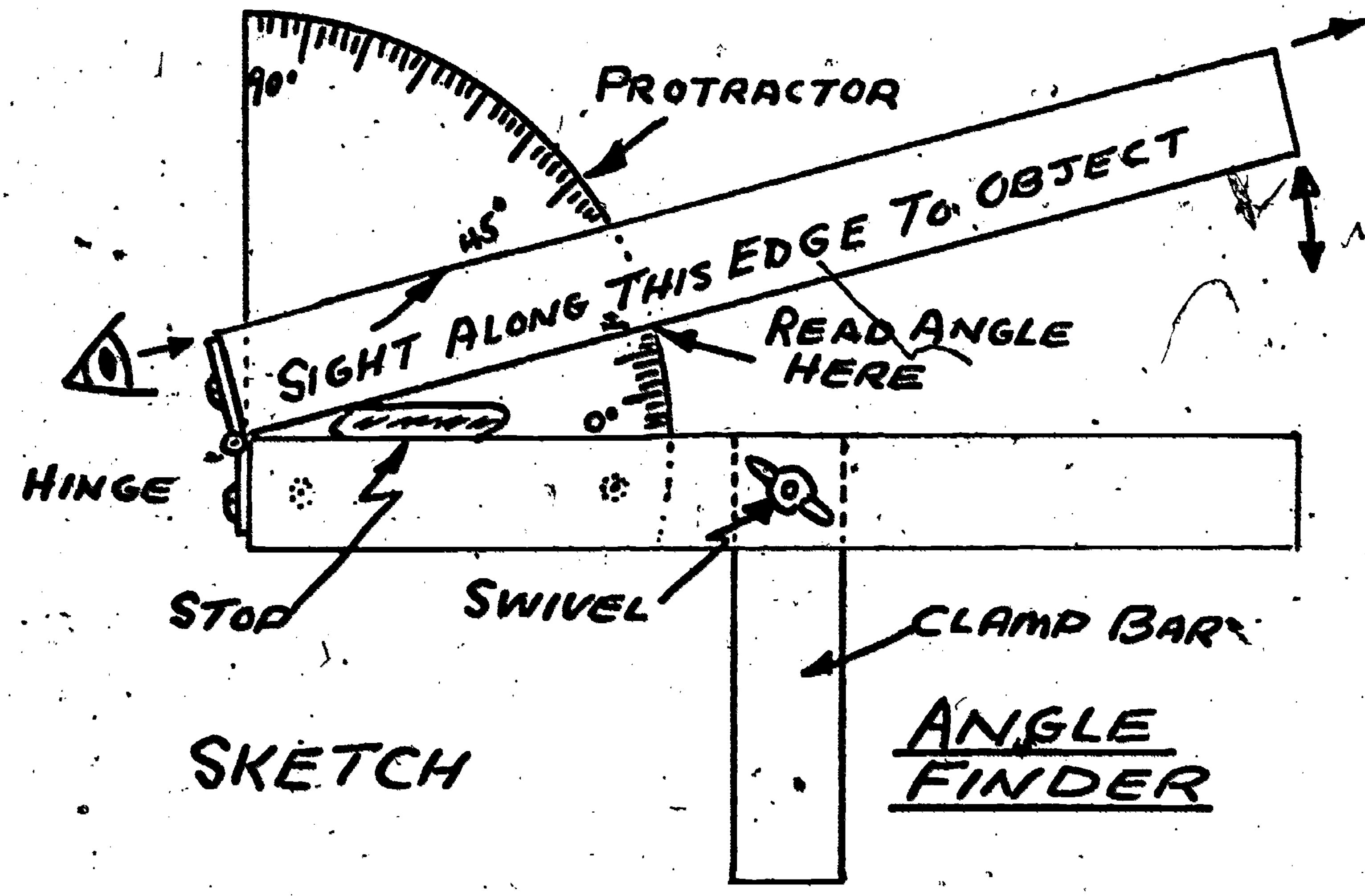
With that experience a student may discover what to do when he cannot measure to a point directly below the height to be measured. This happens for instance when measuring the height of a hill, or a tree across a pond. He may decide to take two angle sightings, one from a close approach distance (as close as they can get on fairly level ground) and one from a good distance. The distance between the two sightings is measured. He then draws this base line to scale, and using both angles constructs a triangle.



Dropping a verticle from the tip, he then has the height to the same scale. He can measure the length of the line and calculate the actual height.

*As in the previous section the measurement from a close approach requires more accuracy. In addition in this case the answer depends strongly on the difference between the two measured angles. Hence it is important that the horizontal base line be very nearly the same in both cases. A level fixed to the horizontal arm may be necessary, unless a very flat stretch of ground is available.

This is one possible design for an angle finder.



You sight along the top of the arm but read the angle along the bottom of the arm. This difference is not important if the building or tree is tall.

(This paper replaces the old paper M3
Redundancy and Distribution in Measurement.)

DETERMINING THE BEST INSTRUMENT TO USE FOR A CERTAIN MEASUREMENT

by USMES Staff

In many classes children decide to use certain measuring instruments (rulers, yardsticks, tape measures, string) in an arbitrary manner according to what is available. However, the accuracy of each instrument and each suggested procedure can be determined by collecting and analyzing data from repeated measurements. The decision as to which tool should be used can then be based on the analysis of data rather than on a haphazard choice.

The teacher may first suggest that a number of independent measurements be made using the same instrument, either by different children or by the same child several different times. The results of these various measurements of the same quantity can then be compared. First graders, for example, may find out that most measurements of the distance across a room that are made by placing one foot in front of the other will disagree by eight or ten "feet." The children may then attempt to discover methods of measurement which are more precise—for example, by using a uniform ruler or a length of string or by carefully making pencil marks on the floor before moving a yardstick.

Even after making more careful measurements, the children will see that, in general, measurements produce <u>distributions</u> rather than <u>exact results</u>. The range of measurements they obtain from repeated measurements with an instrument is a good indication either of the inherent accuracy of that instrument or of their method of using it. In addition, by comparing the ranges of measurements obtained by using different instruments, they can find the one which gives the best accuracy (i.e., the smallest range).

The following histograms show the results of measuring the width of a room twenty times with each of three tools--a ruler, a yardstick, and a ten-foot steel tape measure. Since it was decided that it was desirable to have a rug come to within one-quarter inch of the wall, the measurements are grouped in columns representing one-half inch differences.

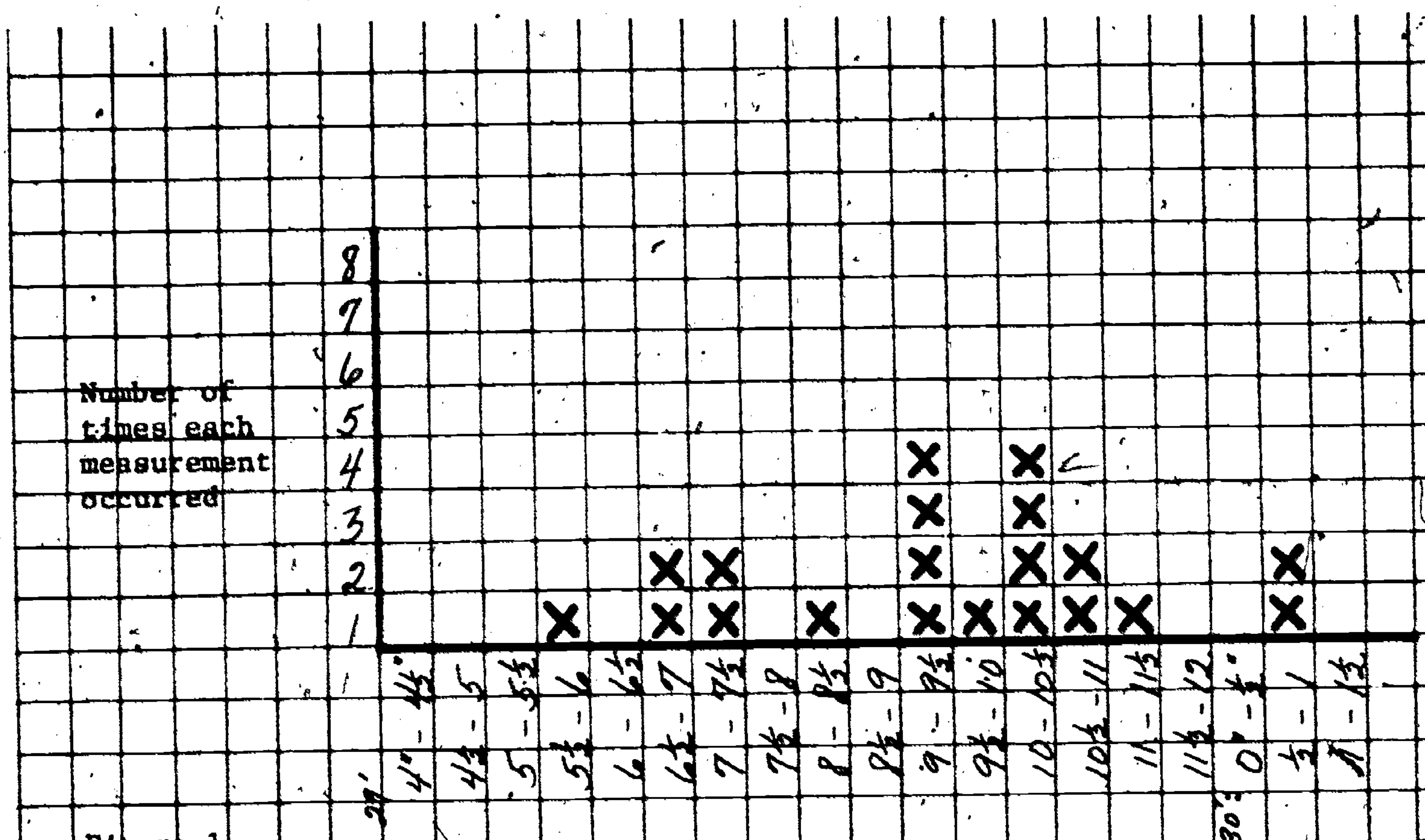


Figure 1 Room Width Using Ruler

Figure 2

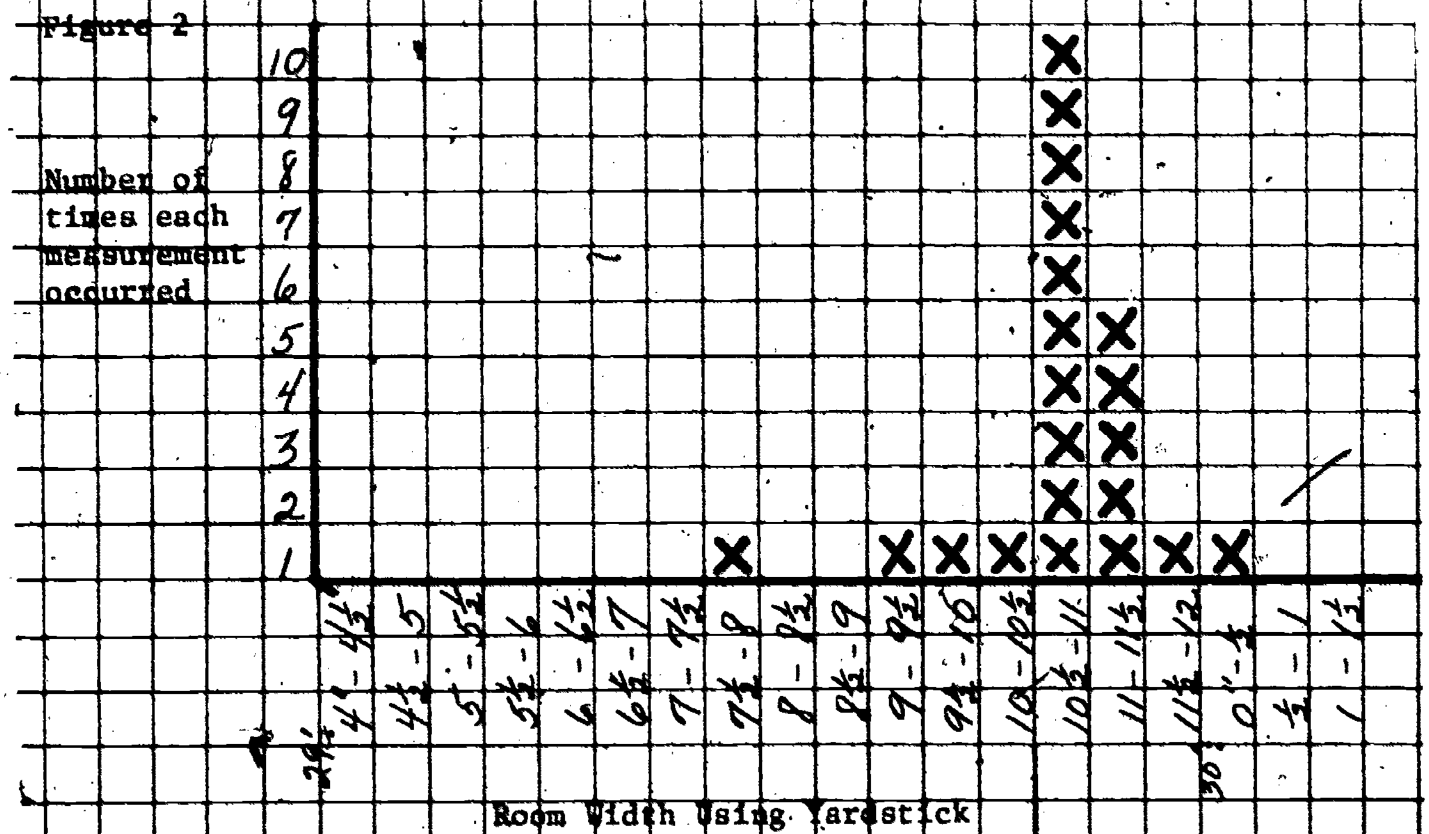
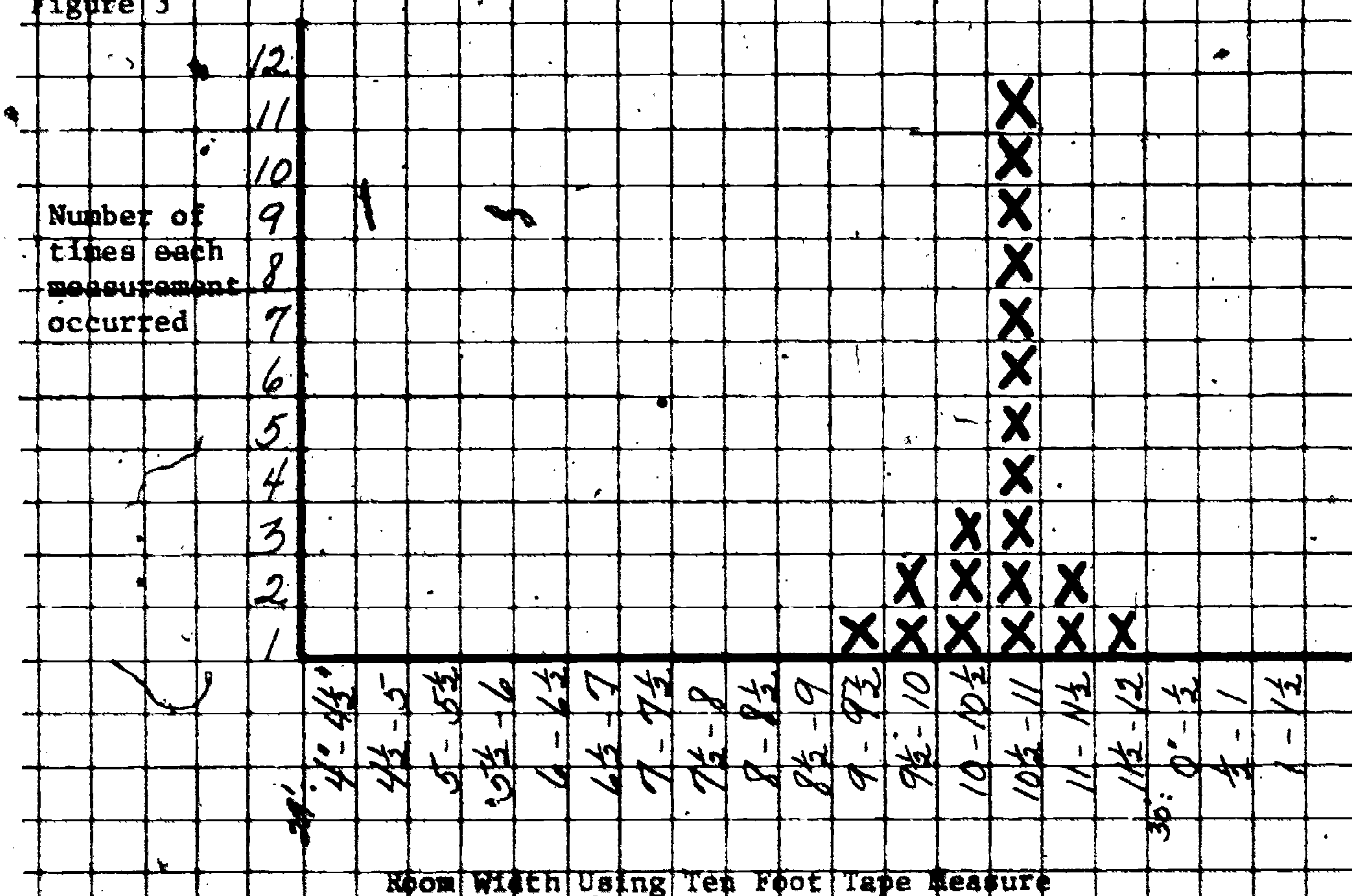


Figure 3



As shown in the preceding histograms, the ranges of measurements for the different tools are as follows:

tool	range
ruler	7-1/2"
yardstick	5"
tape measure	3-1/2"

The children may see that the range of measurements would be greater if an error was made in counting the number of times a tool was moved. In discussing the need either for remeasuring or for ignoring some measurements, they may decide that it takes less time to ignore certain measurements than to remeasure. A good way to accomplish this is to calculate a range that is narrower than the full range; one possible trimmed range that could be found is the range encompassing the middle 80% of the data. To find this range, ten per cent of the measurements at each end of the histogram are crossed off. For example, in the above histograms, which show twenty measurements, the two largest and two smallest would be cancelled. (The children could decide whether this is a reasonable number of measurements to discount.) The 80% ranges from measurements taken with the ruler, yardstick, and tape measure are as follows:

tool	80% range
ruler	5"
yardstick	2"
tapemeasure	2"

We can decide by looking at these measurements that the best accuracy is obtained by using either a yardstick or a long tape measure. (Measurements of much longer or much shorter distances may show that other tools are more accurate).

The children can compare tools for convenience in measuring as well as for accuracy. One example is that a trundle wheel is better to use than a yardstick when measuring the width of a busy street. Another example is that one person can easily use a yardstick, but it is quite difficult for one person to manage a ten-foot steel rule.* If two people can work together on each measurement, it would probably be quicker to use a ten-foot steel rule; if each measurement has to be made by one person, the yardstick would be the better instrument to use.

The children may discuss when a measurement is likely to be accurate enough for a particular purpose. For example, if the purpose is to place tables in four rows in a lunchroom, then knowing the length of the room to within three of four inches should be sufficent. (Since there are likely to be three aisles between the rows of tables, an inch more or less for each of the aisles will make little difference.) However, if an object such as a trash can is to be fit into a small space, a difference of 1/4 inch may be important.

*The range for the ten-foot rule shown in the data would probably have been less if two people had worked together to make each measurement.

MEASURING THE SPEED OF CARS

by

Earle L. Lomon

In either of the Pedestrian Crossings or Traffic Flow units the children often decide to measure the speed of cars. There are two basic approaches to such a measurement:

1. Measuring the time the vehicle takes to cover a pre-set distance.
2. Measuring the distance it covers during a pre-set time period.

Pre-set Distance Method

One advantage to choosing the strip of roadway in advance is that the maximum visibility distance from the pedestrian crossing point or intersection may be selected. The time taken by the faster cars to cover this distance is the "safe" time one has to cross the road if no car is in sight when starting across. For safety that time should be longer than the time it usually takes to cross the road. Thus the measurement of the time taken for a car to cover the sight distance is a valuable piece of information by itself, even if it is not used to determine speed.

Another advantage is that the time can be accurately measured by two, or less accurately by one, observer. At one end of the fixed distance a student can raise his hand or a flag as the vehicle crosses a marked line. The student at the other end will start a stopwatch at the instant of the signal, and will stop the stopwatch as the vehicle crosses a line at his position. As the distance is relatively long, the time is also, with the stopping and starting errors small in proportion.

The sight distance can be measured several times, the accuracy of different instruments compared, and the median found of the most valid measurements.

How long an interval does the stopwatch have to measure? If the car is going 30 m.p.h. (that is 44 ft/sec.*) across 300 ft. (a football field length), it will take 300/44 seconds or 6.8 seconds to transit. With practice the children should be able to start or stop a watch within about 1/2 second of a signal. The error will probably be in the same direction at both ends so that the total error is likely to be less than 0.5 seconds, or 7% of the total time of 6.8 seconds. This in turn could lead to about 7% error in the speed, that is to an error of about ± 3 ft/sec. or ± 2 m.p.h. with a 30 m.p.h. speed. That is quite tolerable for practical purposes.

What are the disadvantages of this approach? The most usual is that many of the children doing the unit can not divide well. The activity could be used as motivation to learn to divide*, but in some cases the children may not be ready to learn more than division by single digit numbers. In this method one must divide whatever time comes up, maybe 6-1/2 seconds, into the fixed distance.

Another disadvantage is that it does not allow for any change in speed by the vehicle. The car may be going 40 m.p.h. coming over the hill, but then reduce its speed to 25 m.p.h. on approaching the intersection. The above method only gives the average speed for that case. The observer would not know the car's speed in the critical region near the intersection.

<u>Pre-Set Time Method</u>

When the measurement to be made is the distance a car travels in a certain time period the stopwatch handler can stand near the starting line and start his instrument as the vehicle crosses. Then a fixed time

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* 60 m.p.h. = $\frac{60 \times 5280}{60 \times 60}$ ft/sec. = 88 ft/sec. or 1 m.p.h. = $\frac{88}{60}$ ft/sec. = 1.5 ft/sec.

* The children could start with the triangle graph technique described in "Geometric Comparison of Ratio". This would give them the relative speed of cars, the median car, etc. By comparing on the same graph speed triangles with the triangles of simple fractions, they could get approximate speeds.

later he will signal observers down the road to observe the position of the car. If the time is fixed to be 10 seconds then dividing to find the speed will be particularly easy. However, this may correspond to a rather long distance for signalling under some conditions (440 ft. at 30 m.p.h.). Also, unless on the highway, a car would tend to change its speed over such distances. Perhaps five seconds could be used. If dividing by five is not easy for those students it would be an opportunity to discuss the relation between dividing by five or instead dividing by ten and then multiplying by two.

The observation of where the car is after the fixed time may require more than one person near the end of the measurement zone, but is otherwise simple. A series of chalk marks, stones, or posts can be put along the curb or roadside. They can be placed for example at 6 foot intervals in the region where the car is likely to be. Then enough observers are needed so that all the markers can be seen by at least one of them. When the stopwatch keeper signals, the observer nearest the vehicle notes which marker the vehicle is at.*

If the cars are travelling between 20 m.p.h. and 40 m.p.h. (that is 29 ft/sec/ to 59 ft/sec.) then after 5 seconds their positions will be spread between 145 feet and 295 feet from the starting point. The interval of 150 feet could be covered by 26 markers. The important thing is that for sufficient accuracy the children need only note the nearest marker, so that they do not have to be very close. Perhaps three observers could watch closely enough. Perhaps 16 markers placed 10 feet apart would suffice.

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* The children may think of a way to use the marble chute counter to help them in their quick appraisal of a car's location at the end of the time period. The marble could be started on signal, ringing a bell when the time period is over, or a more sophisticated system could be worked out.

More accurate placement of the final position of the car is unnecessary because an error of 5 feet out of 145 feet is only a 3% error. With time measurements they are likely to have errors of about 0.5 seconds out of 5 seconds, a 10% timing error. A total error of even 15 % (20 feet out of 150 feet) with a car going 40 m.p.h. is ± 6 m.p.h., just about within tolerable limits. A 15% difference will be just sufficient for their purpose of knowing whether the car is speeding badly and if there is a safe time to cross (15% of a 7 sec. crossing is about 1 sec.).

Of course if a car does not reach the markers, they will know it is going unusually slowly. If it goes by all the markers they will know it is speeding. In either case they can make some estimate of how far it has gone, perhaps by noting landmarks, or putting up extra markers at 50 ft. intervals.

ELECTRIC TRUNDLE WHEEL
by
Charles Donahoe

Since I needed a trundle wheel made for a measuring test at EDC, I asked five boys if they would like to undertake such a project. They said yes and we started in right away. I had brought in a trundle wheel that was not very well made as an example of what a trundle wheel looked like and how it worked. My hope was that they would prefer to make a better one of their own design. One of the boys got a screwdriver, tightened a screw, and everyone agreed it seemed to work fine. I asked the group how the wheel could work so that you would know every time it made a complete turn. Jim suggested stopping every so often to check the line on the wheel. Tony said, "I know, put a light bulb on it that lights up every time it makes a complete turn." The rest of the group agreed this was a really good idea. We discussed ways that this might be done but did not resolve the problem.

Two days later when the group again gathered, the boys decided to make another wheel out of tri-wall. The problem was what size the wheel should be for maximum effectiveness. I suggested 36" might be a nice circumference. Theodore said he thought that was an awfully large wheel and he wasn't sure the circle cutter would be large enough. I asked if the circle had to be 36" across. Everyone agreed that it would. I asked why the trundle wheel was rolled across the floor. Skeeter answered it was to measure the distance. "Is the distance the same around as the distance across?" "No." "Which one has to be 36"?" "The around distance." I suggested someone cut out circles and see what distance around they could get. Unfortunately, everyone but Theodore had to leave. He and I decided to work on this ourselves and we played with different size circles and decided to use the 11" diameter size for the wheel. Since the circle cutter was broken the next day, the boys decided to use the saber saw and make the wheel out of wood instead of Tri-wall. They used a compass, measured 5-1/2", and cut out their wheel. It was a little bumpy but came out rather well. Jim sanded it down to even it off more.

The boys used the handle from the old trundle wheel and drilled a hole through both the handle and the wheel. A bolt was inserted and the trundle wheel turned out to be very wobbly. We all realized that the wheel would not make a very good measurement with the erratic path it was following. Tony and Theodore talked over the problem and decided that another handle was needed on the wheel. That way there would be support on both sides. (See Fig. 1)

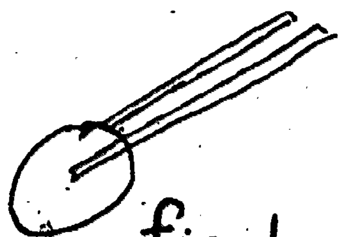


fig. 1

They soon found that two handles made the trundlewheel too unwieldy to push easily. Jim suggested cutting one handle in half. Willy said this wouldn't work because it would just fall down. Tony suggested nailing them together but when Theodore held them as if nailed, the wheel was hard to move. Tony suggested a piece of wood in between the two. This was tried and it worked. The boys decided to cut it down even further and leave about a 5" or 6" clearance between the block and the wheel. (See Fig. 2.) This turned out to be very important in the making of the switch, although none of us realized it at the time.

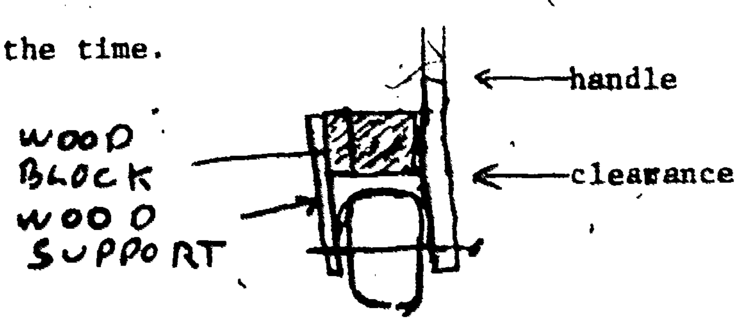


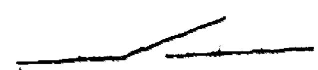
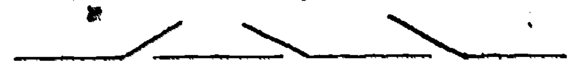
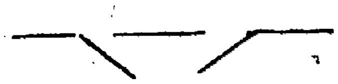
fig. 2

The next problem was to follow up on Tony's suggestion about putting a light bulb on the trundle wheel to signal each complete turn. It took a number of class sessions and a lot of hard thinking and trial of several arrangements to arrive at the final setup.

The boys knew how to make the type of switch that has two metal parts which are pushed together to make a complete circuit. They assumed that the same type of switch would work on the trundle wheel. Their first idea was to use a switch similar to the one on the door alarm. (Two metal strips forced together by the pressure of a foot)) Theodore taped the two metal pieces to the wheel in this fashion.

The wheel worked well with the switch attached. Just one problem - How can the wires be attached? The support on one side of the wheel and the handle on the other side made it impossible to attach the battery and bulb to the wheel. Jim suggested attaching the battery and bulb on the handle above the wheel. Fae and Skeeter tried to explain how this would work. I asked them what would happen to the wire. They thought about this and then realized the wire would go around with the wheel. I asked them if there was a place the wire could go through so it would not go around. They realized the center of the wheel was the only spot. They decided to try this and pushed the wire through the hole in the handle and the wheel center and connected it to Theodore's switch. When this arrangement was tested, the wire kept getting tangled on the bolt or pulling enough on the side of the hole so it disconnected from the switch.

The boys worked on this problem a long time. At first every new idea involved running a wire on the wheel. One idea of this type came very close to the final switch, i.e. the suggestion of hanging a piece of tin from the wood block and having it touch a piece of metal on the wheel. Everyone saw this wouldn't work for the same reason the other arrangements hadn't worked. The boys were about ready to give up and there was some talk about the problem being impossible to solve.

I asked them to try to think of some sort of switch that didn't require putting any wires on the wheel. They admitted they hadn't thought of this but really couldn't see how such a thing could work. Tony suggested that a flat piece of metal could be placed on the wheel instead of a switch. The next step seemed quite obvious to me - but not to the boys. I thought a little more help was needed. I drew the familiar switch on the board. 
I asked the boys what other switches might look like. "Can there be more than two pieces to a switch?" Jim suggested four. 
I asked how about three.  Still nothing. Without saying anything, I drew the switch upside down.  No reaction.

Two or three of the boys were holding the trundle wheel and trying to think of some way. Tony was holding the two loose wires that needed to be connected somewhere and was touching them together making the light go off and on. I didn't see what he had done but all at once the boys announced they had "figured it out." Tony demonstrated for me. He put one wire on each side of the block and slowly turned the wheel which still had the metal piece on the rim. The light came on. He did it two or three times and it worked. Success.

I asked the boys how the wires should be connected, suggesting the use of something beside the bare wire hanging down. Jim suggested using the same type of material we used on other projects - tin can pieces and fahnestock clips. When this work was completed, including the cutting of a longer piece of tin for the wheel, everyone watched as Tony made the first run. The light didn't go on. Depression. The connections were checked, the battery was checked, and the switch was tested. Everything seemed all right. When the strips were in contact with the metal strip on the motionless wheel, the light came on. Theodore realized what was causing the problem. One battery did not provide enough power. Two batteries were put on instead of one and the light worked. Later we added two more batteries to make the light brighter and to make sure a contact was made on each revolution of the model. The final design is shown in Figs. 3 and 4.

REFINING CHILDREN'S INVESTIGATIONS OF CONSUMER PRODUCTS

by

USMES Staff

Often the prospect of producing test results which might effect some change in consumer buying or manufacturer marketting behavior is very exciting to a child. If he realizes that these test results need to be accurate and valid to induce this change, it is likely that he will try to refine as much as possible the quality of his quantitative experimentation and data analysis techniques. In particular, he might seek to improve:

1. his selection of important characteristics to test,
2. his design of test apparatus,
3. his awareness and subsequent control of important variables,
4. his understanding of the need to take repeated measures and of the significance of the error range in each measurement.

In discussing prospective test products, the children might consider who uses each product. Is it important for use in the Design Lab? Do parents use it? Do kids? Two or three products which the children believe should be tested might be singled out for further study. The class might then break into small groups each of which is responsible for performing one test on the different brands of a product. Evaluations of test results, methods of testing and analysis, and the needs for new tests or retesting might be carried on with larger product groups or with the whole class.

With respect to each product, they should thoroughly discuss the following points:

a) Have we looked at the most important properties for the usual uses of this product?

b) Have we noted the prices? Do we want to know the price per length, ounces, sheet, tube, etc.? Or do we need to find out how much is needed for a particular purpose (some glues may absorb more easily for instance, or spread thicker) and calculate the price per use?

c) For each important property of the product have we tested in a way which allows a quantitative comparison? Have we tested accurately enough? Has more than one group checked each measurement? If we do the test several times, what do the results tell us about our accuracy. Is finding the mean the only way to use a set of measurmments of the same thing which differ a little bit from each other?

I list here as examples some of the important properties and possible tests that may be done for some representative products.

Glue (important for Design Lab use):

Important properties are:

1. the cost per job,
2. the strength for holding together a few often-used materials,
3. the drying time,
4. the effect of moisture,
5. whether it dries transparent, light, or dark colored.

For testing they may stick together specified areas of wood, tri-wall, etc. They could hang weights from one piece until it breaks at the glue, or the materials breaks. Having allowed more than enough time for drying the first time, they may then make a series of samples and test them at shorter and shorter times with a weight that was held when thoroughly dry. They can try applying a thinner or thicker layer of the same glue to see which works best (and determine costs). They can soak the dried samples in water for different lengths of time.

Paper Toweling (important to parents):

Having decided on a size piece to use in tests, they can calculate cost per unit. They can then see how well that unit mops up (volume or oil or water soaked up), how well it scrubs on table top or sinks, (distance scrubbed before tearing when wet or dry), or how soft it is if used as napkins. The kids could decide if a subjective judgement is sufficient for the last characteristic.

Bubble Gum (important to kids):

Cost per mouthful and lasting time of flavor and size of bubbles may be the only characteristics of importance. The children may be finished after finding an average value for those three quantities. Price per chewing time may be the most meaningful cost.

Ball Point Pens or Felt Tip Pens (important to kids):

Important properties are:

1. cost for certain amount of use,
2. writing performance - amount of skipping as well as effectiveness on a few often-used surfaces and ability to write at different angles,
3. rate at which the point dries out.

Before testing, the kids might discuss how long they think a pen will last. How many minutes per day do they use a pen before the ink is gone? In designing a lifetime test they can discuss whether it is better to move the paper (perhaps adding machine tape) or the pen. Which method would allow the point to exert a certain continuous pressure on the surface?

Batteries (important for Design Lab):

Important properties are:

1. cost for certain amount of use,
2. length of time during which the performance is adequate for a certain purpose.

Before testing, the kids might discuss how much a battery is worth to them when it produces less than a maximum voltage (a less bright light, a weaker electromagnet, etc.) When is a light bright enough or a magnet strong enough for their purpose?

The lifetime at this performance level can be tested by either using squares of colored acetate to measure brightness of a light or by finding the length of time an electromagnet will hold up a specific length of steel rod. If the kids prefer the rod test (which is a more accurate indication of voltage) and have a range of acceptable brightness levels, they can graph brightness vs. length of rod and pick the rod length which is held by the same battery that just gives acceptable brightness as their standard test for acceptable voltage level. They will also find how brightness varies with voltage (rod length).

In calculating most of rechargeable batteries, the kids can consider the cost of the charger as well as the number of times a battery can be recharged.

<u>Tape</u> (important for Design Lab use):

Important properties are:

1. cost per job,
2. holding strength as pull is applied at different angles,
3. strength of tape,
4. effect of time, cold, heat, and moisture.

Before testing the kids can decide the specific purpose of the tape. Is it to be used to hang pictures on certain wall surfaces? Is it to be used on packages? They can determine which angle of pull is critical for a specific use. The holding strength can be retested after the tape (in its use) has been subjected for different lengths of time to cold, heat, and moisture.

Plastic Wrap (important to parents):

Important properties are:

1. cost per job,
2. retention of moisture,
3. clinging strength,
4. resistance to tearing, puncturing, etc.,
5. effect of time, cold, heat, and moisture.

Before testing, the kids can decide specific purposes of wrap - to cover bowls?...to wrap fruit? Pieces of fruit can be weighed before and after being wrapped for different lengths of time in different wraps. Weights can be used to exert force as determined by the particular test, e.g. weights hung on wrap as it covers a pointed bottle top for puncture test.

Pencils (important to kids):

Important properties are:

1. cost for certain amount of use,
2. performance - effectiveness on different often-used writing surfaces; width, darkness and consistancy of mark; ease of erasing.

For testing kids could find number of turns of pencil sharpener necessary for a sharp point and amount of pencil used up in those number of turns. Performance can be tested using same apparatus as designed for testing pens. In calculating cost kids should decide how short a pencil can get before it is discarded.

WEATHER FACTORS AND THEIR MEASUREMENT

by Ray Brady, Jr.

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I. THE RAIN GAUGE

One of the oldest and simplest of the meteorological instruments is the rain gauge. The weather man measures rainfall (or snowfall) by the depth of water that would lie on the ground if none of it ran away (or soaked into the ground).

The official rain gauge of the U.S. Weather Bureau is shown below. It is a funnel with an 8-inch diameter top with a 2-inch high rim to prevent water from splashing off the funnel. Precipitation flows into a measuring tube with an area 1/10 that of the funnel mouth (This means the measuring tube has a diameter of 2.53 inches.). The depth of the water in the tube is then measured. The result is divided by 10 to obtain the true rainfall. Thus, 10 inches of water in the measuring tube equals one inch of rain.

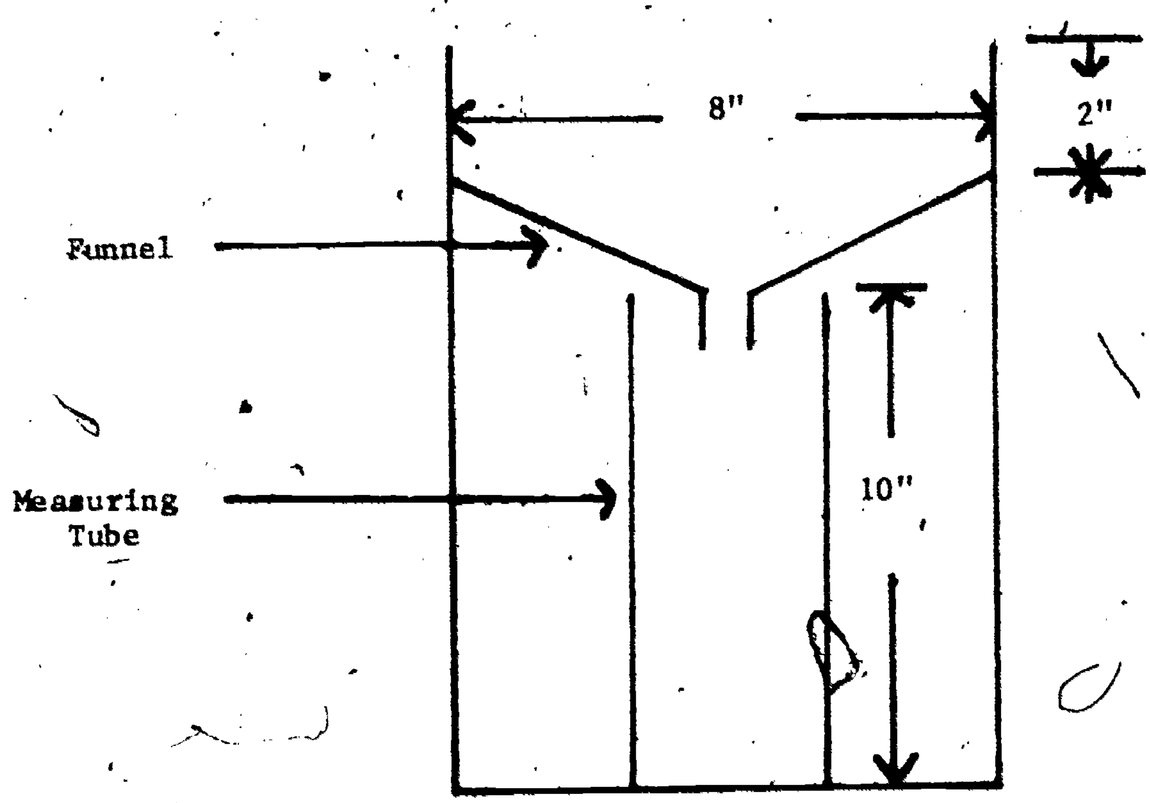


Figure 1

Rain Gauge

Rather than constructing one so elaborate, the children may simply use a coffee can to collect the rain (or snow) and a tall thin jar to measure it. The jar is easily calibrated by putting 1 inch of water in the coffee can and pouring it into the jar. Mark the height exactly and divide the distance from the bottom to this mark into 10 equal parts. Each of these intermediate marks then represents 1/10 of an inch of rainfall.

The rainfall should be measured as soon as possible after it stops to avoid any loss due to evaporation. Snowfall or sleet is measured by slowly melting it indoors and measuring the depth of the melted water. Ten inches of snow will melt into one inch of water. This is due to the large amount of air trapped in the snow.

The measurements will be more accurate if the gauge is about two feet off the ground and away from buildings and trees that might shield it.

Funnels and measuring tubes of different sizes can also be used; however, the funnel should have a fairly large diameter. A general calibration which relates the funnel diameter to the measuring tube diameter can be easily found. The diameter of the funnel is A, and the diameter of the measuring tube is B. The rainfall is denoted by R, and the height of the water in the measuring tube is denoted by H. (See Figure 2.)

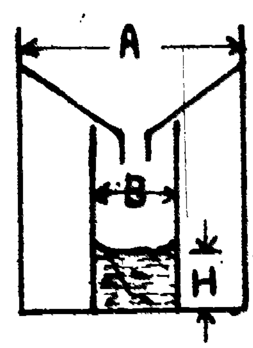


Figure 2

The volume of water which would fall into the funnel is given by the volume of a cylinder whose height is equal to the rainfall:

$$V = \pi\left(\frac{A^2}{4}\right)R$$

However, this is the same as the volume of the water in the measuring tube, which is given by

$$V = \pi\left(\frac{B^2}{4}\right)H\,.$$

Since the two volumes must be the same, we equate them to find

$$\pi\left(\frac{A^2}{4}\right)R = \pi\left(\frac{B^2}{4}\right)H,$$

from which we find

$$R = \left(\frac{B}{A}\right)^2 H.$$

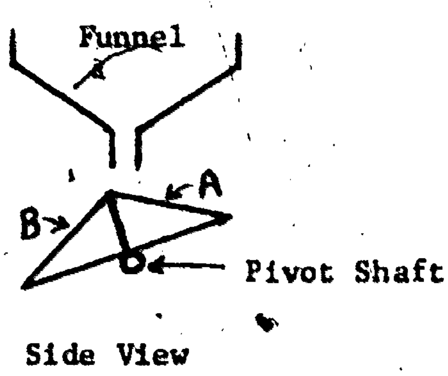
With this relation the gauge is calibrated. For example, suppose the funnel has a diameter of 10 inches and the measuring tube has a diameter of 2 inches. Further, suppose the measured depth of water in the measuring tube is 1 inch. The rainfall measurement is then

$$R = \left(\frac{2\text{ inches}}{10\text{ inches}}\right)^2 1\text{ inch}$$

which becomes $= \frac{1}{25}$ inch

and $R = 0.04$ inch.

One may readily note that the official gauge shown in Figure 1 would overflow if the rainfall were more than one inch. The U.S. Weather Services uses an automatic rain gauge to avoid this problem. In this device, shown below in Figure 3, the collecting pans tip when the amount of rain in them reaches a certain weight. This empties the full pan and brings an empty pan underneath the funnel. A device to count the number of turns is attached to the shaft.



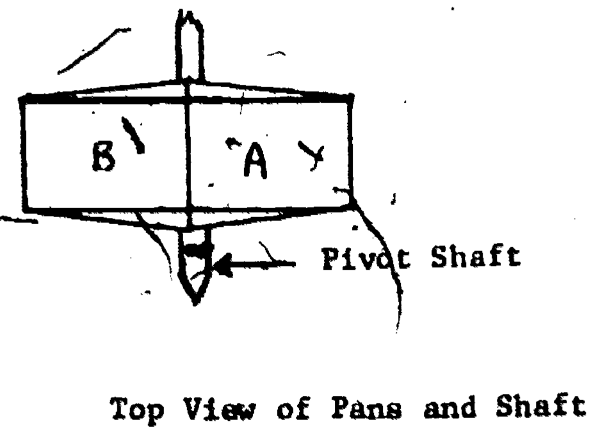


Figure 3

Automatic Rain Gauge

Pan A collects water until the weight is enough to tip it so that pan B is beneath the funnel spout and begins to fill. A recording device is attached to the shaft and records each emptying of a pan. This gauge is generally designed so that if the area of the collecting funnel is F, then 1/F inches of rain will tip the collecting pan. Thus, if the funnel mouth is a 10"X10" square, the area is 100 square inches, and each pan will tip and empty for every 0.01 inches of rainfall.

II. RELATIVE HUMIDITY

Relative humidity is the amount of water vapor in the air at a given temperature compared to the maximum amount of water vapor the air can hold at that temperature. It is expressed as a percent. When the amount of water vapor reaches the maximum, the relative humidity is 100 percent, and we say the air is saturated. The temperature at this point is the dew point temperature. If the temperature should drop any lower, some vapor would condense out of the air as droplets because the air cannot hold the moisture. The lower the temperature, the less moisture the air can hold.

The condensation may occur as dew when the temperature of the surface drops below the dew point. If, in addition, the dew point is below freezing, the condensation will occur as frost. This effect occurs outside on the ground, as well as on window panes. The water vapor condenses directly as tiny ice crystals to form frost, rather than condensing as water first. Then the dew point is referred to as the frost point. The condensation may also occur as fog in valleys and over water after the surface has radiated away some of its heat. The warm damp air above the ground is chilled below the dew point and condensation occurs on tiny airborne particles.

Condensation also occurs as clouds when warm, moist air rises and cools by expansion. The water vapor condenses on tiny particles in the atmosphere as small water droplets to form a cloud. These droplets are so small, some are only .0004 inch in diameter, that very light air movement holds them up.

Relative humidity can be measured in two ways - with a psychrometer or a hygrometer. Both are relatively easy to make, although the psychrometer may be more accurate.

The psychrometer consists of two thermometers: one, a regular dry-bulb thermometer; the other, a wet-bulb thermometer. As moisture evaporates from the wet-bulb thermometer, its temperature is lowered relative to the dry-bulb thermometer. The lower the humidity, the more water that can evaporate, and the lower the wet-bulb thermometer will read. Using a table, one can then measure the relative humidity by this temperature difference due to the cooling process of evaporation. In fact, psychrometer means, literally, to measure by cooling.

The psychrometer shown below in Figure 4 is described on a "How To" card. It is made from two inexpensive thermometers and a shoelace or a short piece of gauze to keep the bulb wet. If a shoelace is used, it should be washed well and boiled to remove impurities. The thermometers should both read the <u>same</u> temperature when dry. If they do not, one must always remember that part of the temperature difference is due to this error and <u>not</u> to the humidity.

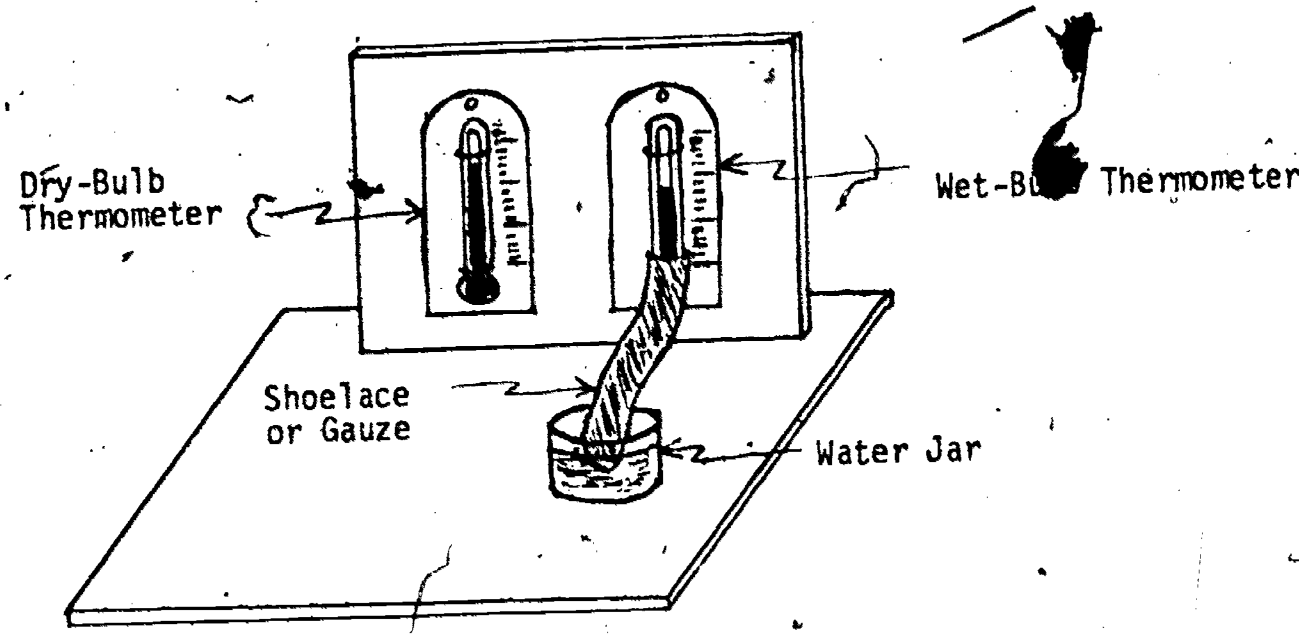


Figure 4

To measure the relative humidity, fan (do not blow) the wet-bulb thermometer until it stops going down. Read the two thermometers and find the relative humidity from Table 1. For example, suppose the dry-bulb temperature is 50°F., and the wet-bulb temperature is 45°F. The difference is 5°F. Read across the top of the table to 5°F. and down the side to 50°F. The relative humidity as given by the table is 67 percent.

RELATIVE HUMIDITY TABLE

Dry bulb thermometer reading	Difference between dry bulb and wet bulb thermometers in degrees Fahrenheit (°F.)																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0°	67	33																		
5°	73	46	20																	
10°	78	56	34	13																
15°	82	64	46	29	11															
20°	85	70	55	40	25	12														
25°	87	74	62	49	37	25	13	1												
30°	89	78	67	56	45	36	26	16	5											
35°	91	81	72	63	54	45	36	27	19	10	2									
40°	92	83	75	68	60	52	45	37	29	22	15	7								
45°	93	86	78	71	64	57	51	44	38	31	25	18	12	6						
50°	93	87	80	74	67	61	55	49	43	38	32	27	21	16	10	5				
55°	94	88	82	76	70	65	59	54	49	43	38	33	28	23	19	11	9	5		
60°	94	89	83	78	73	68	63	58	53	48	43	39	34	30	26	21	17	13	9	5
65°	95	90	85	80	75	70	66	61	56	52	48	44	39	35	31	27	24	20	16	12
70°	95	90	86	81	77	72	68	64	59	55	51	48	44	40	36	33	29	25	22	19
75°	96	91	86	82	78	74	70	66	62	58	53	51	47	44	40	37	34	30	27	24
80°	96	91	87	83	79	75	72	68	64	61	57	54	50	47	44	41	38	35	32	29
85°	96	92	88	84	81	77	73	70	66	63	59	57	53	50	47	44	41	38	36	33
90°	96	92	89	85	81	78	74	71	68	65	61	58	55	52	49	47	44	41	39	36
95°	96	93	89	86	82	79	76	73	69	66	63	61	58	55	52	50	47	44	42	39
100°	96	93	89	86	83	80	77	73	70	68	65	62	59	56	54	51	49	46	44	41

Table 1

It is interesting to note that the height of the bottom of a cumulus cloud can be easily determined with the psychrometer and a dew point chart, which is shown on the next page. It gives the dew point (in °F.) for a given difference in wet- and dry-bulb thermometer readings. (This chart is, of course, related to the relative humidity table since relative humidity determines the dew point.) The following method works only with a cumulus cloud because a cumulus cloud is formed by the cooling of water vapor directly over the area from which the water evaporated rather than further away.

First, use the chart to determine the dew point. In order for a cumulus cloud to form, the air must cool from the surface temperature reading to the dew point. It is known that the temperature of the air drops off by 5 1/2 degrees F. for each 1000 feet of rise in altitude. Thus, one can readily find out how high the water vapor has to rise before the dew point is reached.

For example, suppose the air temperature is 91 degrees and the dew point is 80 degrees (relative humidity of 71%). The air must cool by 11 degrees to reach the dew point. This will happen if it rises about 2000 feet. The base of any cumulus cloud must be about 2000 feet up.

DEWPOINT CHART

Air Temp.	Difference between wet and dry bulb thermometers (degrees)																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	-7	-20																							
5	-1	-9	-24																						
10	5	-2	-10	-27																					
15	11	6	0	-9																					
20	16	12	8	2	(-)	-21																			
25	22	19	15	10	(-)	-3	(-)																		
30	27	25	21	18	(-)	8	(-)	-7																	
35	33	30	28	25	21	17	13	7	0	-11	-41														
40	38	35	33	30	28	25	21	18	13	7	-1	-14													
45	43	41	38	36	34	31	28	25	22	18	13	7	-1	-14											
50	48	46	44	42	40	37	34	32	29	26	22	18	13	8	0	-13									
55	53	51	50	48	45	43	41	38	36	33	30	27	24	20	15	9	1	-12	-59						
60	58	57	55	53	51	49	47	45	43	40	38	35	32	29	25	21	17	11	4	-8	-36				
65	63	62	60	59	57	55	53	51	49	47	45	42	40	37	34	31	27	24	19	14	7	-3	-22		
70	69	67	65	64	62	61	59	57	55	53	51	49	47	44	42	39	36	33	30	26	22	17	11	2	-11
75	74	72	71	69	68	66	64	63	61	59	57	55	54	51	49	47	44	42	39	36	32	29	25	21	15
80	79	77	76	74	73	72	70	68	67	65	63	62	60	58	56	54	52	50	47	44	42	39	36	32	28
85	84	82	81	80	78	77	75	74	72	71	69	68	66	64	62	61	59	57	54	52	50	48	45	42	39
90	89	87	86	85	83	82	81	79	78	76	75	73	72	70	69	67	65	63	61	59	57	55	53	51	48
95	94	93	91	90	89	87	86	85	83	82	80	79	78	76	74	73	71	70	68	66	64	62	60	58	56
100	99	98	96	95	94	93	91	90	89	87	86	85	83	82	80	79	77	76	74	72	71	69	67	65	63

Table 2

The hygrometer* uses a long hair to measure relative humidity. As the moisture in the air increases, the hair absorbs some of it and stretches very slightly. When the moisture decreases, the hair contracts slightly. Hygrometer means to measure wetness.

One simple hygrometer is shown below in Figure 5.

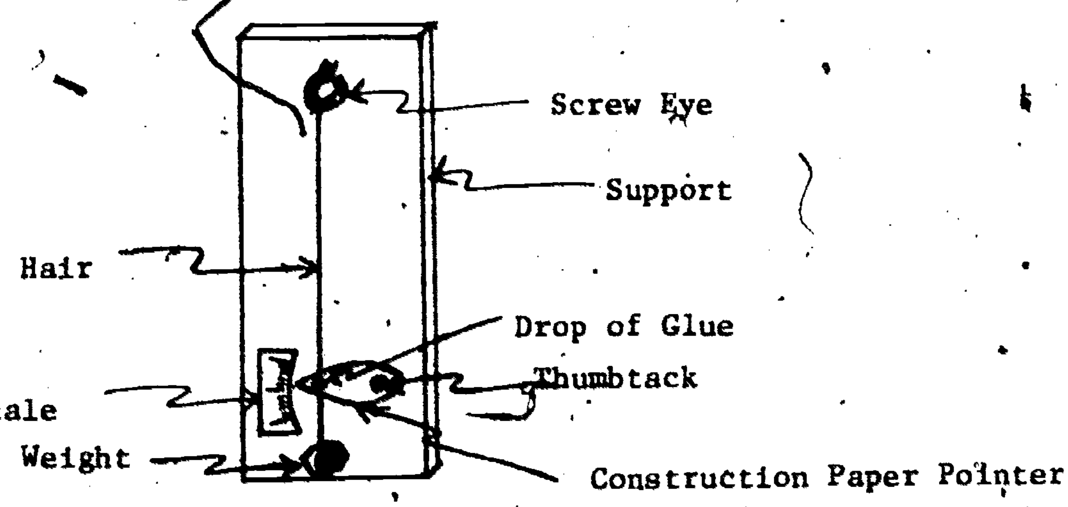


Figure 5

Obtain a fairly long hair - 18" should be sufficient, although a shorter one will certainly work. (The longer the hair the more movement will be seen in the pointer.) The hair should be washed in alchol to remove the natural oil.

Insert a screweye into the Tri-Wall (any other type of support will serve as well) and tie one end of the hair to it. Tie the other end of the hair to the weight. Put a drop of glue near the end of the pointer and glue the lower end of the hair to it. When the glue is dry, attach the pointer to the support with a thumbtack, making sure the pointer turns freely. Draw a scale on a white card and glue it to the support. The device can then be calibrated using a psychrometer or another (calibrated) hygrometer.

* This instrument should not be confused with a hydrometer, which is an instrument to measure the density of liquids. A service station attendant uses a hydrometer to determine the amount of antifreeze in a car's radiator.

A slightly more elaborate hygrometer is shown below in Figure 6.

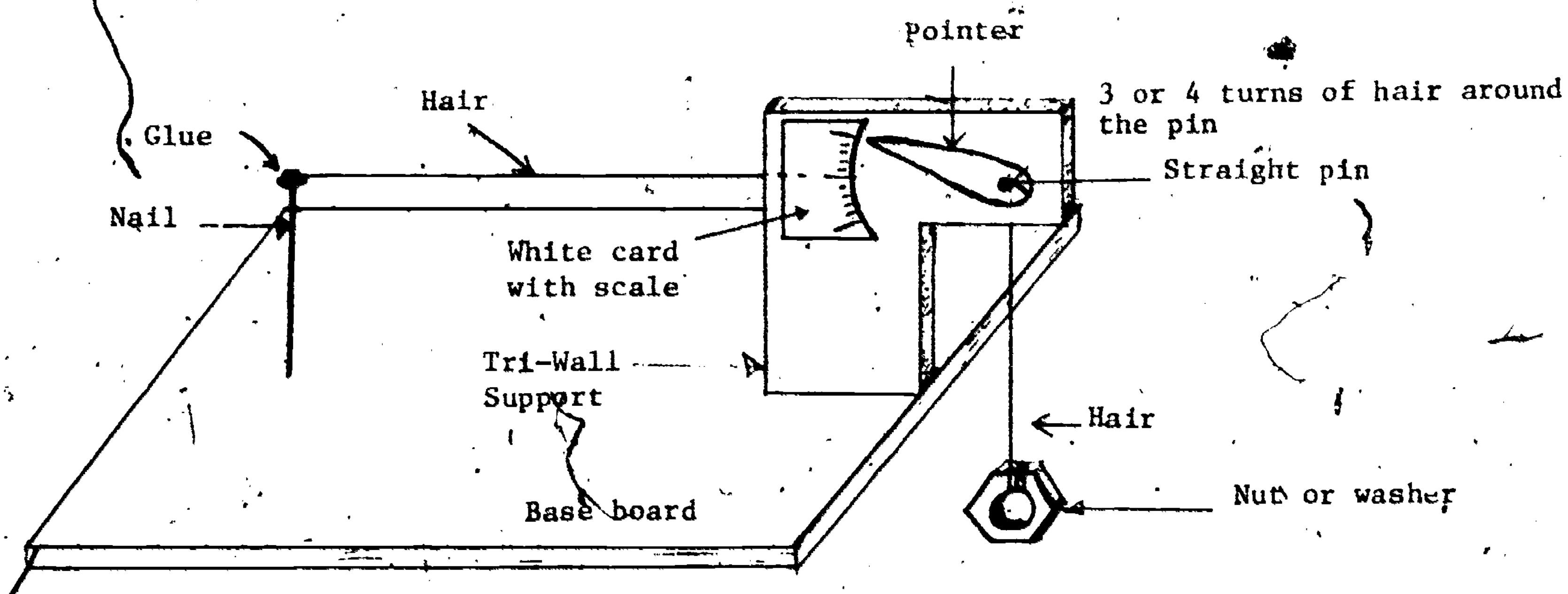


Figure 6

Fasten the hair to a nail driven into the board. Fasten a lightweight pointer to a straight pin. Mount a piece of Tri-Wall on the other end of the board. Push the pin through the Tri-Wall and make sure the pointer rotates easily. Tie a washer, nut, or fish sinker to the other end of the hair and wrap 3 or 4 turns of hair around the pin.

Draw a scale on a white card and glue it to the Tri-Wall. This hygrometer is then calibrated using a psychrometer or other (calibrated) hygrometer.

III. PRECIPITATION AND FOG

Evaporation, condensation, and precipitation are the three parts of the atmospheric water cycle. Condensation has been previously mentioned in the section on relative humidity, but the difference between precipitation and condensation is not readily appreciated. The condensation process consists of an accumulation of water molecules into tiny droplets. Precipitation, on the other hand, consists of an accumulation of these tiny droplets into larger drops (or clumps of ice crystals) of the size we call rain (or snow). The average cloud droplet has a radius of less than 20μ (microns - one micron is 0.0001 centimeters or 0.000034 inches), while the average raindrop has a radius of about 1000μ, nearly 50 times larger. Since the volume is proportional to the cube of the radius, we can see that a raindrop is made up of nearly a million droplets.

We have previously mentioned dew as a form of condensation occurring when the temperature drops below the dewpoint. We have also mentioned hoar frost as another type of condensation which occurs when the dewpoint is below freezing. It consists of ice crystals in the form of scales, needles, feathers, and fans. The water vapor freezes into ice crystals without going through a liquid form, a process known as sublimation. Another type of condensation similar to frost is rime. Rime consists of layers of ice crystals and forms when the droplets of supercooled clouds and fogs touch objects. In high mountains and over polar ice and snow fields, rime deposits may be appreciable. In the supercooling process the cloud or fog droplets are cooled below the freezing point but do not freeze into ice crystals. Such a condition is highly unstable, and when the droplets touch a cold surface, they instantly freeze into a deposit of rime.

Drizzle is a form of precipitation consisting of a fine sprinkle of numerous, very small, uniform water drops with a diameter of less than 0.5 millimeters (=500μ=0.017 inches). Because the drops are so small and light, they seem to float about and follow the air currents. An umbrella is virtually useless since the drops dance about rather than falling vertically. Drizzle forms in stratus clouds not far from the ground. These clouds must be in a shallow layer (as clouds go) since the drops had little chance to collide with other drops to form larger drops of rain. Thus, drizzle may be described as either an oversized fog or an undersized rain.

Rain is the precipitation of liquid water in drops larger than in a drizzle. Sometimes the drops may be the same size as in a drizzle but not numerous enough to qualify as a drizzle. This generally signifies rain forming at great height and then falling through a layer of dry air so that some drops evaporate completely and other shrink on the way down. Sometimes, rain is seen to fall from a cloud and evaporate before it strikes the ground.

Snow is the precipitation of solid water in the form of hexagonal crystals or stars. Even at temperatures well below freezing, the crystals carry a thin film of water so they stick together when they collide. At very low temperatures the crystals are dry and only small flakes are seen.

Sleet has two different definitions. The British definition is that sleet is a mixture of snow and rain or melting snow. In North America, sleet is defined as rain from warm air aloft which falls through a layer of cold air near the ground. The drops do not turn into snow but freeze into grains of ice. Thus, sleet is formed from rain plus low-level freezing plus warm surface temperatures.

Glaze, or freezing rain, occurs when rain falls through a layer of cold air at ground level and freezes when it strikes the cold ground, trees, power lines, etc. Glaze is formed from rain plus ground-level freezing.

Granular snow is the frozen counterpart of drizzle. It is composed of small ice grains falling from stratus clouds.

Hail is precipitation of balls or pieces of ice with diameters usually ranging from 0.2 inches to 2 inches or more, falling either separately or fused into irregular lumps. The largest reported hailstone measured almost 5½ inches in diameter and weighed one and a half pounds. During this storm in Potter, Nebraska in 1928, hailstones fell so hard that many were buried in the ground.

A cross section of a hailstone usually shows concentric layers of clear ice alternating with snow. They are a peculiar product of thunderclouds. According to the most prevalent theory, hailstones form when ice crystals are caught in the great updrafts and downdrafts inside the towering thunderclouds. The crystal is carried from the raining bottom to the freezing top, a distance of some six to ten miles, several times by the vertical wind currents. It alternately melts and freezes, collecting more water each time it passes through the bottom of the cloud, until it is too heavy for the updraft to support, and it falls to the ground. Each concentric layer is a record of a journey to the top of the cloud. Because of their weight and the severity of the storm they spring from, hailstones do much damage to farm crops every year.

Another common form of condensation is fog which has two main divisions: advection fogs and radiation fogs. Fog is formed by one of two processes:

cooling or evaporation. Radiation fog is usually formed when the earth's surface radiates heat into the night air, thereby cooling the air directly above it. This chilled air, heavier than the air about it, flows downhill into valleys and condensation occurs on tiny airborne particles as fog. How high this type of fog reaches is determined by air movement. In a dead calm, the fog layer may be only a few feet high, while with a light wind it can be quite high and dense. Such a fog will usually "burn off" in the morning sunshine of the next day. Because the temperature of the oceans has little daily variation (about 1°F.), radiation fogs do not develop at sea.

Ocean fog is usually an advection fog. This type of fog is formed when a mass of warm, moist air moves to an area where the temperature is below the dewpoint (for the air mass), and, being cooled by the surface, it condenses into fog. For example, warm, moist air from the Gulf Stream area is blown over the Labrador Current (a cold ocean current), cools and forms a fog. Such fogs are quite common in this region -- the Grand Bank off the New England coast.

A fog which is similar to this type of advection fog occurs over snow when the air is very damp or when it is raining. The surface of the snow is at the freezing point, but the air just above it is warmer and very moist. The cooling at the surface of the snow causes a fog to form. However, if the air temperature is too much above freezing, the snow will melt without the formation of fog.

Advection fogs have two variations: 1) the steam fog, more commonly known as arctic sea smoke and 2) the upslope fog. Arctic sea smoke is formed by evaporation and is quite common along arctic coasts during cold spells. It is especially bothersome in the fjords of northern Norway and along the Alaskan coast. In this type of fog cold air flows over the warm ocean surface (or a warm land surface). The evaporation of water from the surface is so intense that steam pours out of the water to fill the air with fog.

Upslope fog, common along the Pacific Coast of the United States, is formed when moist sea air is blown against hills and cooled by being lifted to colder heights. This type of fog - cloud is quite often seen in mountain areas and in coastal hills and mountains. Many mountains may have a long-lasting cloudcap due to an upslope fog. In this case, the upslope fog is a stratus cloud.

Fogs may also be formed by a mixture of advection and cooling. These are common during the winter along the Pacific Coast. Moist sea air is trapped in the valleys and cooled by advection followed by radiational cooling at night.

IV. MEASURING THE PERCENTAGE OF CLOUD COVER PHOTOGRAPHICALLY

By using a Polaroid camera and a grid, one can obtain a reasonably accurate measure of the amount of cloud cover over a general area. It will not work for the entire sky due to the problem which makes cloud cover farther off look thicker than it actually is. See Figure 7.

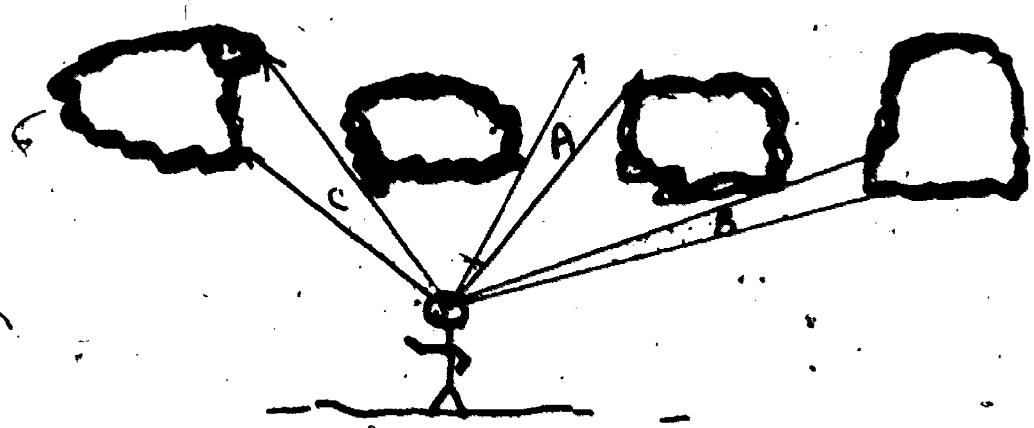


Figure 7.

Observer looking overhead sees some blue sky within the cone labeled A. Looking to the side he sees only clouds within the cones labeled B and C, although the cloud cover is essentially the same.

To measure the percentage of cloud cover, one takes several Polaroid photographs of the sky so as to get a reasonable sampling. (A large cloud directly overhead may give a fake 100 percent cloud cover and may then drift away. A second measurement then may give zero percent cover.) The number of photographs and time interval between each is determined by cloud type and speed. Slow-moving clouds require larger time intervals, perhaps, than fast-moving clouds.

Once the photographs have been obtained, a grid of 100 squares can be constructed. The grid should be exactly the same size as one of the photographs with ten blocks per side.* It should be drawn on transparent plastic, such as that for overhead projectors, so it can be used as an overlay.

To calculate the percentage, one simply counts the total number of squares partially covered as being 1/4, 1/2, or 3/4 covered. The results for each photograph are then averaged to provide an overall measurement. One example is shown below in Figure 8.

Working in rows from left to right, starting at the top, we find a total of 27 rectangles covered out of a total of 100 rectangles. Thus, the cloud cover is about 27 percent.

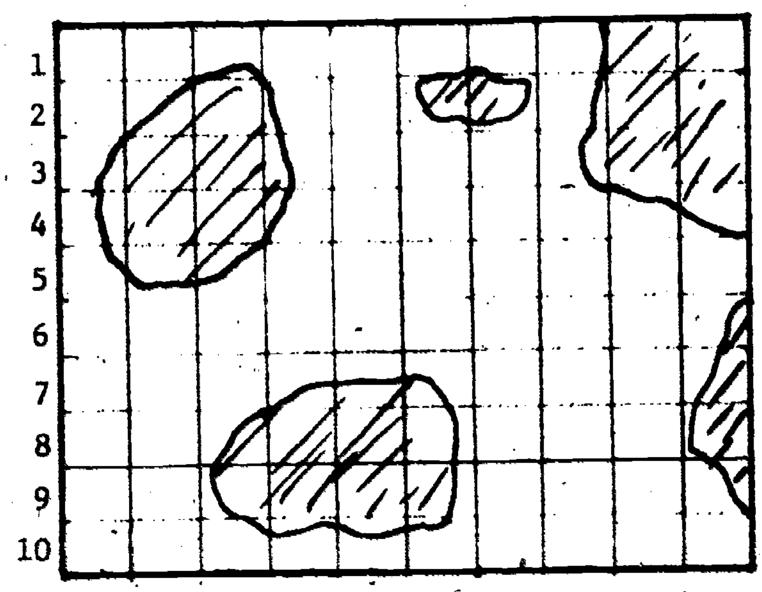


Figure 8

Cloud cover photograph with grid.

		Row Total
Row 1 -	0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 + 1	= 2
2 -	0 + ½ + 1 + ¼ + 0 + ½ + ¾ + ¼ + 1 + 1	= 5¼
3 -	¼ + 1 + 1 + ¼ + 0 + 0 + 0 + ¼ + 1 + 1	= 4 ¾
4 -	½ + 1 + 1 + 0 + 0 + 0 + 0 + 0 + ¼ + ¾	= 3½
5 -	¼ + ¾ + ¼ + 0 + 0 + 0 + 0 + 0 + 0 + 0	= 1¼
6 -	0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + ¼	= ¼
7 -	0 + 0 + 0 + ¼ + ½ + ¼ + 0 + 0 + 0 + ¾	= 1 ¾
8 -	0 + 0 + ½ + 1 + 1 + ¾ + 0 + 0 + 0 + ¾	= 4
9 -	0 + 0 + ¾ + 1 + 1 + ¾ + 0 + 0 + 0 + ¼	= 3 ¾
10 -	0 + 0 + 0 + ¼ + ¼ + 0 + 0 + 0 + 0 + 0	= ½
	Total	= 27

*If the photograph is not square, the grid can be constructed to have sides equal to the shorter side of the photograph. The grid is then placed over the center of the photograph.

When taking the photographs, the camera should have as wide a field of view as possible. Furthermore, it should be pointed directly overhead and not angled to one side. If it doesn't point directly overhead, the data will be false due to the angles as shown in Figure 7.

V. WIND DIRECTION AND SPEED

Wind Direction

Long ago men guessed that the wind foretold the weather. Wind from one direction brought fair weather while wind from another brought storms. So they observed wind direction by watching which way smoke drifted or a tree bent. Then, as today, the wind was named for the direction from which it blew (e.g., a south wind blows from the south to the north). Today we know that factors other than wind foretell the weather, but we know that the direction of the wind is a very important factor. Rainy weather may generally come from one direction while fair weather may come from another.

Wind vanes are among the earliest known weather instruments. Before the Christian Era, rooftops in ancient Rome had wind vanes, indicating an awareness of the relationship of wind and weather.

Shown below in Figure 9 is a simple wind vane described on a "How To" card. It is made from Tri-Wall and wooden dowel rods.

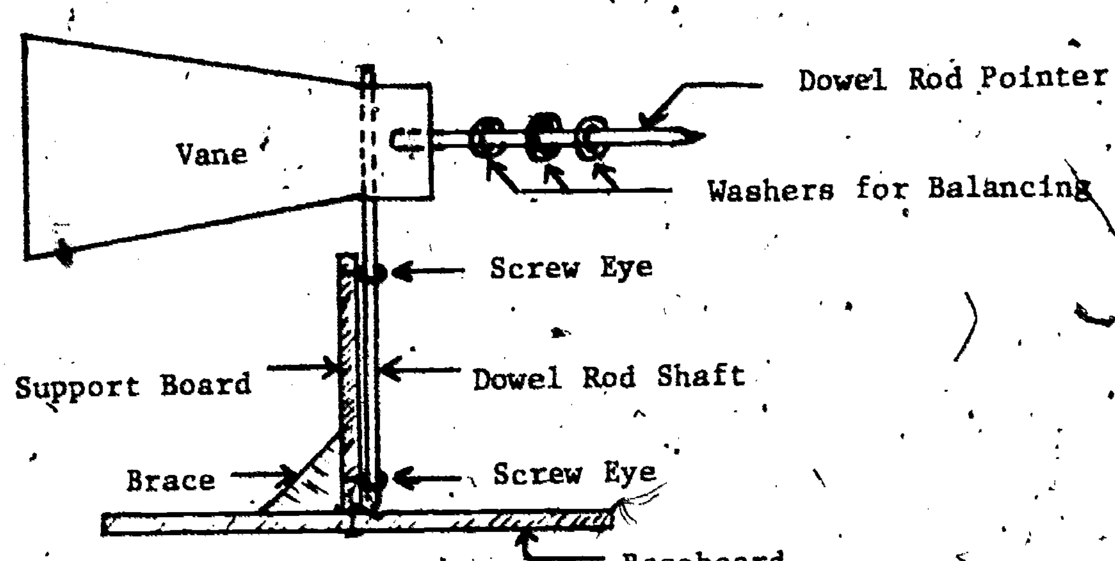


Figure 9

Improvements may readily be made, especially as it is not durable if exposed to much rainy weather. A better one could be made from sheet metal or plywood, always remembering to keep it as well-balanced as possible. A metal rod could be used for the shaft with a nail for a pivot as shown in Figure 10. The main things to remember are that it should turn freely and should present a fairly large surface area to the wind so it will turn in a light wind.

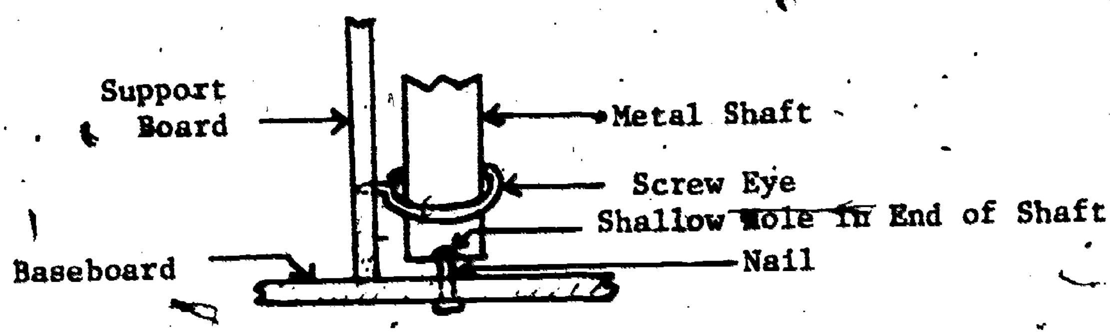


Figure 10
Bearing Detail

To orient the wind vane properly, you must find the direction of true north. This can be done in several ways. The midday shadow points to true north. A vertical stick could be used to find true north from the shadow it casts. The Pole Star (Polaris) is always at true north, also.

Perhaps the easiest way is to use a magnetic compass. Remember, however, that the compass points to the <u>magnetic</u> north pole not the <u>geographic</u> north pole or true north. A correction must be added or subtracted depending on your location. The map in Figure 11 shows the average variation for the United States. Westerly variation means the compass needle points west of true north by the indicated number of degrees, while easterly variations means it points east of true north. Once the direction of the north-south line is determined, local objectives (such as tree, houses, etc.) may be used as a reference to locate it when necessary.

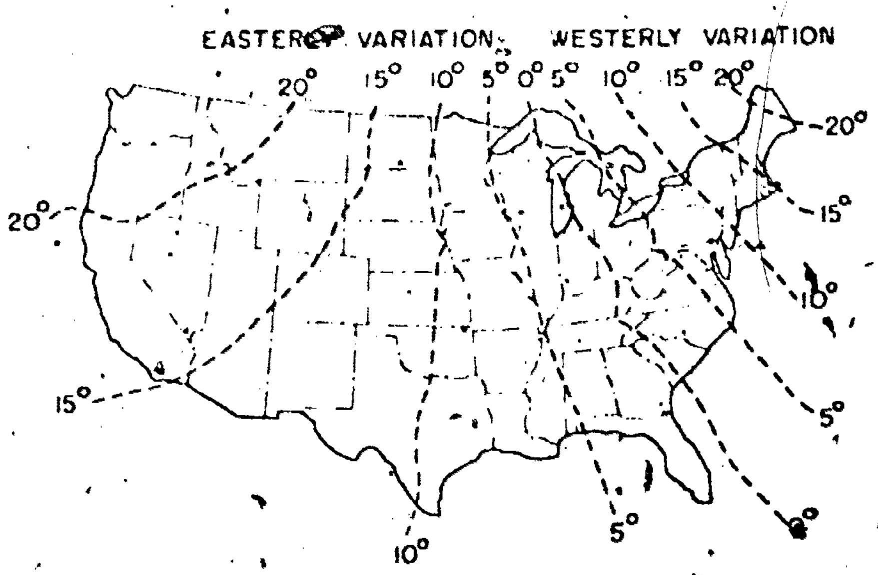


Figure 11

Wind Speed

A simple method for finding wind speed is to use the table shown on the next page (Table 3). This is the Beaufort Scale devised in 1805 by a British admiral, Sir Francis Beaufort. His original scale was intended for sailors of his day and gave wind speeds in terms of its effect on sails, rigging, and waves. It has been modified for use on land and is quite useful for determining wind speed by visual observation.

Various instruments have been developed over the years to measure wind speed. One experimenter even used a device resembling wind chimes. The tone produced was related to the wind speed. A modern cup anemometer consists of 3 or 4 hollow cups mounted at the end of horizontal arms that are at right

Wind Speed in Miles Per Hour	Effects of Wind	Weather Bureau	Beaufort Number
Less than 1	smoke rises straight up	Calm	0
1 to 3	smoke drifts in the wind but wind vanes do not turn	Calm	1
4 to 7	you feel the wind on your face, leaves rustle and wind vanes move	Light	2
8 to 12	the wind extends a little flag and keeps leaves and small twigs in motion	Gentle	3
13 to 18	wind raises dust and loose paper and small branches are kept in motion	Moderate	4
19 to 24	the wind sways small trees in leaf, and little white wavelets form on ponds and lakes	Fresh	5
25 to 31	large branches of trees move, telephone wires whistle, and it is hard to use an umbrella	Fresh	6
32 to 38	whole trees bend, and it is hard to walk against the wind	Strong	7
39 to 46	twigs break off the trees	Strong	8
47 to 54	large limbs are broken off trees and roofs are damaged	A Gale	9
55 to 63	whole trees are uprooted (rarely seen inland)	A Gale	10
64 to 75	damage is very widespread (rare except near oceans	A Whole Gale	11
above 75	tremendous damage is caused	A Hurricane	12

Table 3

Beaufort Scale

Wind speed in terms of visible effects.

angles to a vertical shaft. The wind exerts more force on the hollow side of the cups, causing the shaft to rotate. In some, the turning of the shaft rotates gears which measure the wind speed. In others the shaft rotation generates an electric current which actuates the pointer of a meter calibrated to read miles per hour. Another modern type of anemometer uses an airplane propeller type device to rotate the shaft.

A relatively simple anemometer is shown below in Figure 12. While it may be difficult to use as an accurate measuring device, it does illustrate the working of the anemometer.

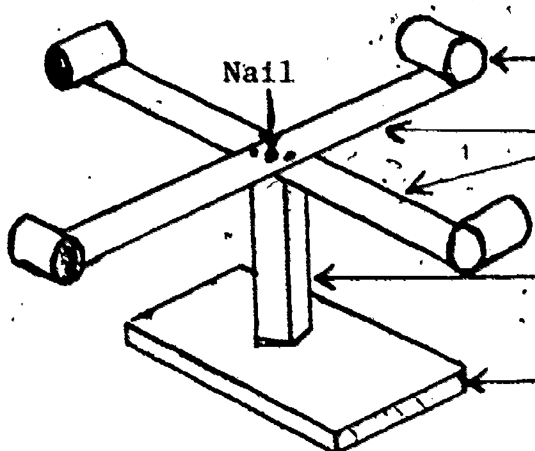


Figure 12
A simple anemometer.

The arms can be made from a yardstick, such as those given out by hardware stores, cut in half. The cups are simply four small tin cans of the same size fastened to the arms with small bolts and nuts.

Cross the two yardstick pieces at right angles and fasten together with a tenpenny nail through their exact centers. Use small wire brads to hold the arms at right angles. (Small, short screws will also work.) Drill a hole in the center of each end of the arms about 1 inch from the end.

Drill a hole in the side of each of the cans. This hole should be located so that the can is balanced from front to back. Fasten the cans to the arms with small nuts and bolts through the holes.

Drill a hole into the end of the piece of wood to be used for the vertical post. It should be slightly shallower than the length of the nail sticking out of the arms and slightly larger in diameter. The point of the nail rests on the bottom of the hole and acts as a bearing. The entire arm assembly turns on this pivot point. This is shown in Figure 13.

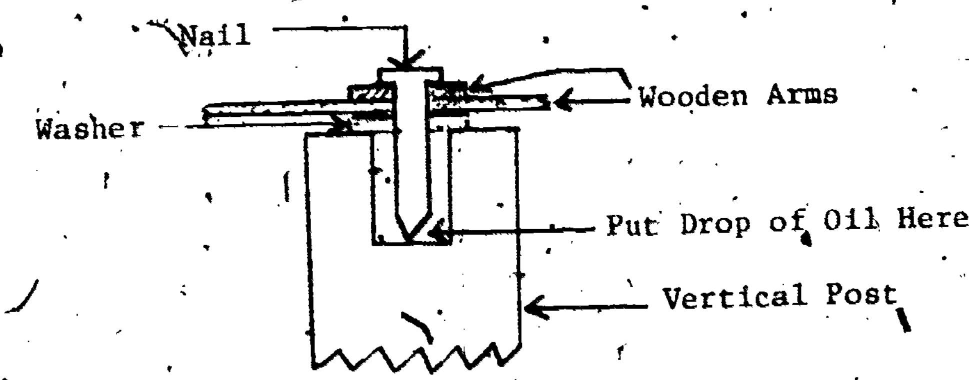


Figure 13
Bearing detail.

When the nail in the arms is inserted into the hole, the arms should just clear the post and should turn freely. The other end of the post is nailed to a baseboard or, if a broomstick is used, stuck into the ground. The anemometer can be painted to make it more durable with one cup painted a different color so turns can be counted easily. A few drops of oil should be put into the hole before putting the nail (in the arms) into the hole.

A somewhat more elaborate anemometer is shown in Figure 14. The use of the eggbeater as a gear reduction device enables faster wind speeds to be observed. When the anemometer is turning very rapidly, individual turns may be difficult to observe. However, the larger gear on the egg-beater will turn only once for several turns of the anemometer cups.

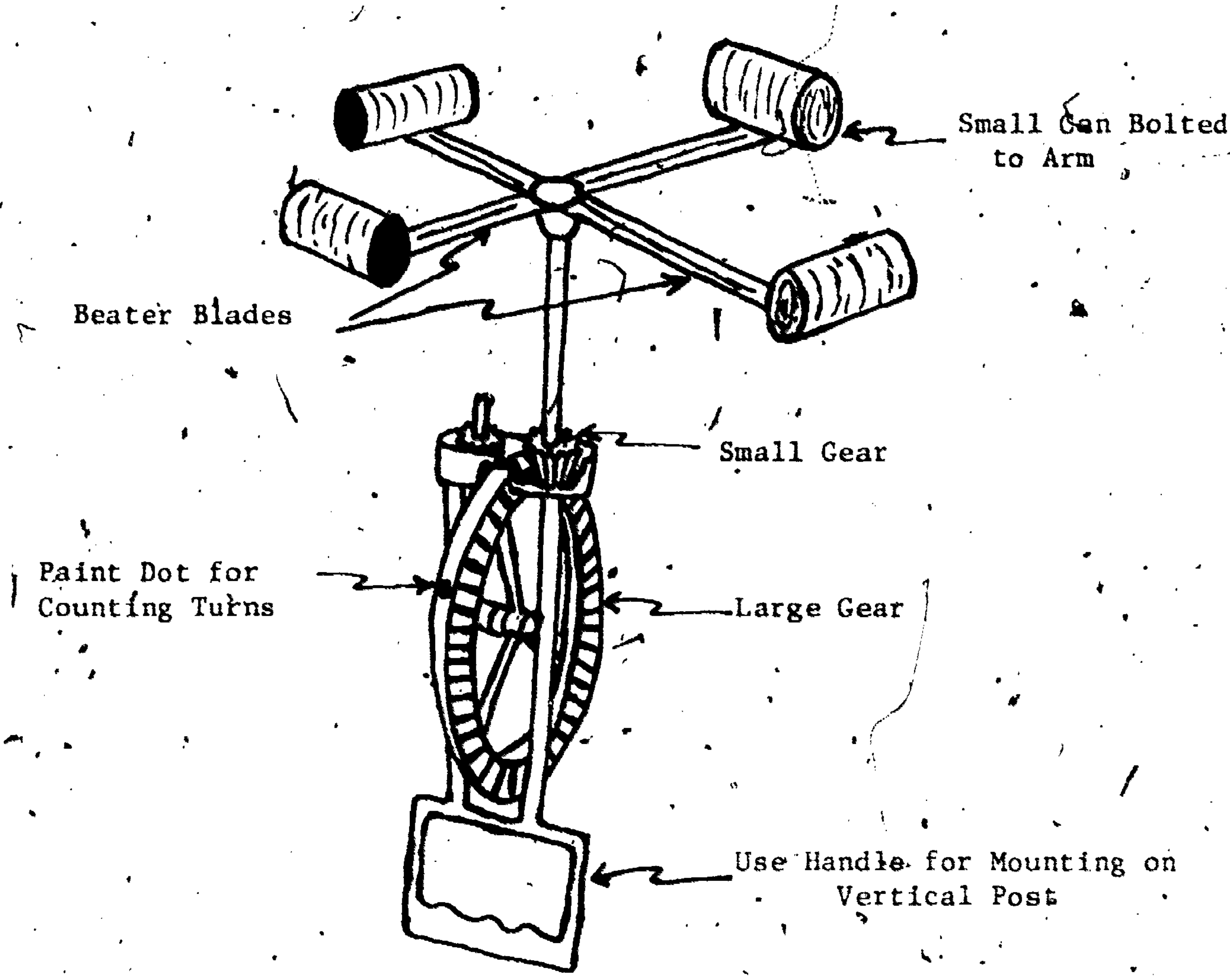


Figure 14

Eggbeater Anemometer

One of the beater shafts is sawed off (use a hacksaw) near the small gear. The blades of the remaining beater are cut apart near the bottom of the beater and carefully straightened. The wire frame around the beaters and the hand crank are also cut away. It should now resemble Figure 14 without the small cans. The four small cans are now fastened to the beater blades by drilling a small hole through each blade and the side of each can and bolting them together as shown in Figure 14. A dot of paint on the edge of the large gear will make it easier to count the number of turns it makes.

With this anemometer, calibration to exact wind speed is difficult. The best method involves taking the finished anemometer to a local weather station and calibrating it with another (already calibrated) anemometer. An alternative is to calibrate it using the Beaufort Scale. A second alternative is to hold the device out a car window and have someone drive at several selected speeds (10 mph, 20 mph, 30 mph, etc.) for a minute while you count the number of turns of the larger gear. However, this last method is not very easy to do accurately. In any case, one needs to know the number of turns per minute of the larger gear for several known wind speeds to calibrate the instrument.

The design and construction of the anemometer may be quite varied, depending on available materials and the age level of the children. Small cans or tablespoons will work for the wind cups. A roller skate wheel can be used in place of the eggbeater as a bearing, but then individual turns must be counted. Also, the arms must be strapped to the wheel as the wheel absorbs too much heat for soldering to be effective. Gears and shafts from old Erector Sets may also be used to construct the anemometer. One thing to keep in mind is that the anemometer should be capable of withstanding strong gusts of wind.

VI. AIR PRESSURE AND THE BAROMETER

Invented by Torricelli in 1643, the barometer was the first of the modern instruments for weather prediction. Renaissance scientists were quickly fascinated by observing that certain kinds of weather often accompanied certain pressure readings. Even today some barometers carry the predictions "Change" at 29.5 inches (of mercury) flanked by "Rain" and "Stormy" at lower pressures and "Fair" and "Very Dry" at higher pressures.

Actually, atmospheric pressure depends on many factors, including altitude, and is far from being a certain indicator of a specific weather condition. Its chief usefulness occurs in indicating that a change is on the way, for a change in pressure usually does indicate a change in the weather. Much more information, such as temperature, relative humidity, wind direction, cloud patterns, weather elsewhere, previous weather conditions, etc., is needed to forecast the weather.

The barometer is an instrument which weighs the total column of air that is pressing down on a given area at a certain time. The word itself means to measure heaviness or weight. This weight of air per unit area is the atmospheric pressure. When the air is heavy, we have high pressure, and when it is light, we have low pressure. We can weigh the air by using a spring scale or by using a balance, except we have to have the air pressing down on one side and no air (a vacuum) pressing down on the other side of the balance. In the balance system, we use a column of mercury to balance a column of air. From the length of the column of mercury, the weight of the column of air at that time can be found. Above the mercury column in the tube is a vacuum. The height of the column in inches gives the atmospheric pressure, and it is this height which is referred to when the weatherman says the pressure is so many <u>inches</u>. Other substances could be used to measure the pressure in this type of barometer, but the column would have to be considerably longer. If water were used, the column would be 34 feet tall! One needs a heavy liquid which won't evaporate. A simple mercury barometer is shown in Figure 15. Refer to Appendix A for a detailed description.

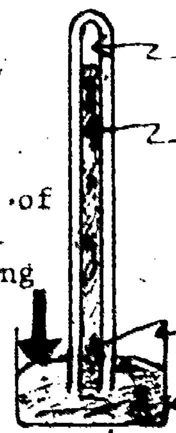


Figure 15
Simple mercury barometer.

The spring balance type of barometer is the <u>aneroid</u> barometer. Here, a sealed metal container from which the air has been evacuated is used. A pointer is attached to one side of the container. As the air pressure changes, the sides of the container move — in, for increasing pressure, and out, for decreasing pressure. This causes the pointer to move across a scale which is calibrated to read inches of mercury. This is the type usually found in homes and offices.

As mentioned previously, air pressure varies with altitude. At sea level the average pressure is 29.92 inches. At 1,000 feet, it is 28.86 inches; at 2,000 feet, it is 27.82 inches; and at 3,000 feet, it is 26.82 inches. The average air pressure in Denver at 5,000 feet would be 24.90 inches. The average normal air pressure at the top of the Empire State Building is about 28.60 inches, nearly 1.3 inches less than at the bottom.

For the barometer whose construction is described in the "How To" cards, mineral oil was chosen as being an adequate liquid to use. Figure 16 shows a cross-sectional view of such a barometer.

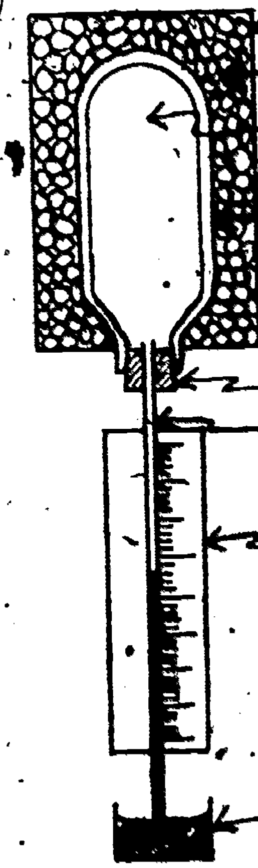
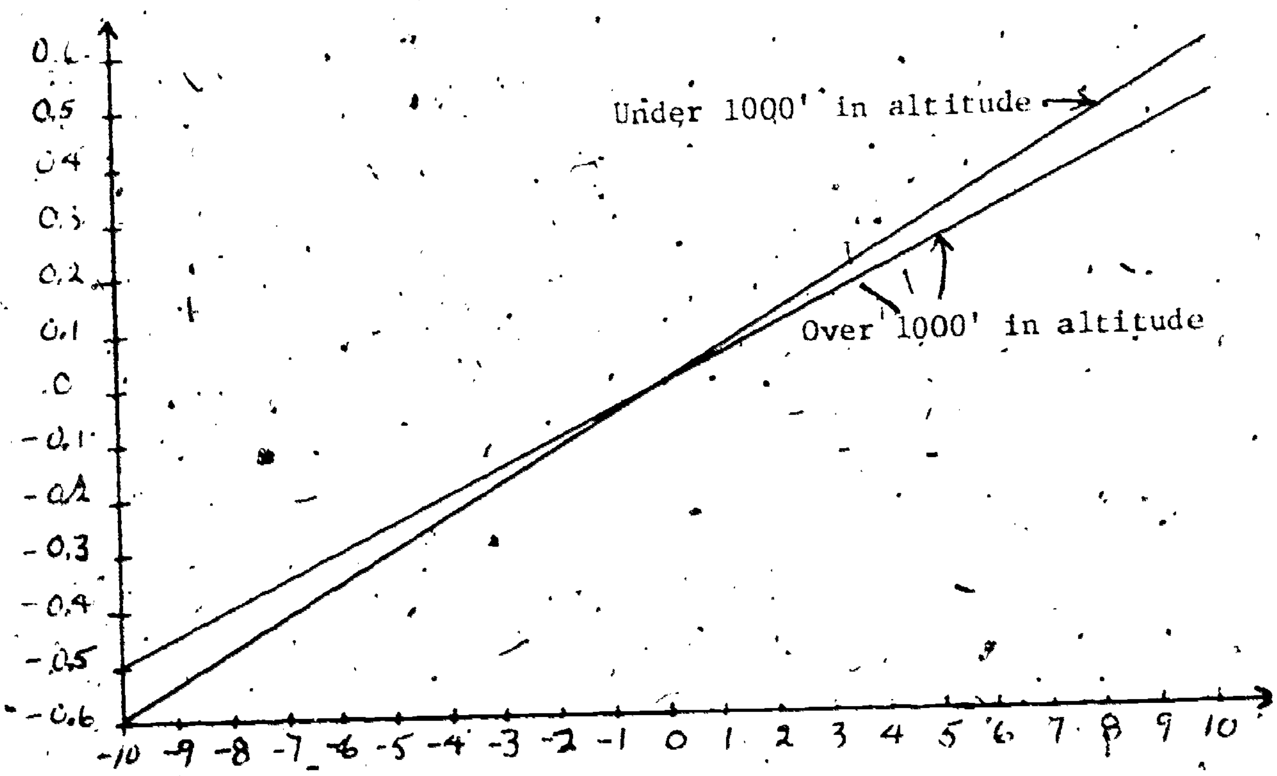


Figure 16

One basic problem which may affect the use of this barometer is the effect of temperature on the measurement. As the temperature rises, it causes the air remaining in the Thermos bottle to expand. This causes the oil in the tube to fall, indicating a drop in atmospheric pressure. This change is spurious, however. If the pressure is read at the same temperature each time, then the readings will be consistent, and the effect of temperature is essentially cancelled. If the temperature does vary, the pressure reading can be corrected for the change by using the graph in

Figure 17. The relation between the temperature change and its effect on the pressure reading is fairly linear but is different for different altitudes. In an attempt to avoid this problem entirely, the barometer should be placed away from sources of heat, such as radiators, sunlight, or incandescent lights. In using the temperature correction with the Thermos bottle barometer, one must also be sure that the temperature inside the Thermos is the same as the outside of the Thermos. It usually takes several hours for the temperature inside the Thermos to equalize.



Change in temperature from calibration temperature (in F.)

Figure 17

Another problem which may arise is that of a sealed building. While it is preferable that the barometer be set up in a spot from which it need not be moved (thereby assuring more accurate readings), it may become necessary to take the barometer outdoors to obtain an accurate reading. Of course, if the students construct a weather station, the barometer may be kept there. If the classroom building is air conditioned, the building may be sealed. Consequently, the air pressure inside may bear no relation to the air pressure outside.

An alternative procedure to avoid the temperature problem is to use a styrofoam ice bucket, or a similar item, such as a styrofoam instrument packing case. The barometer jar is packed in ice to keep the temperature constant. An ice-water mixture will maintain a temperature of 32°F. In this case, a Thermos bottle may not be necessary, and an ordinary glass jar will do. This barometer is shown in Figure 18.

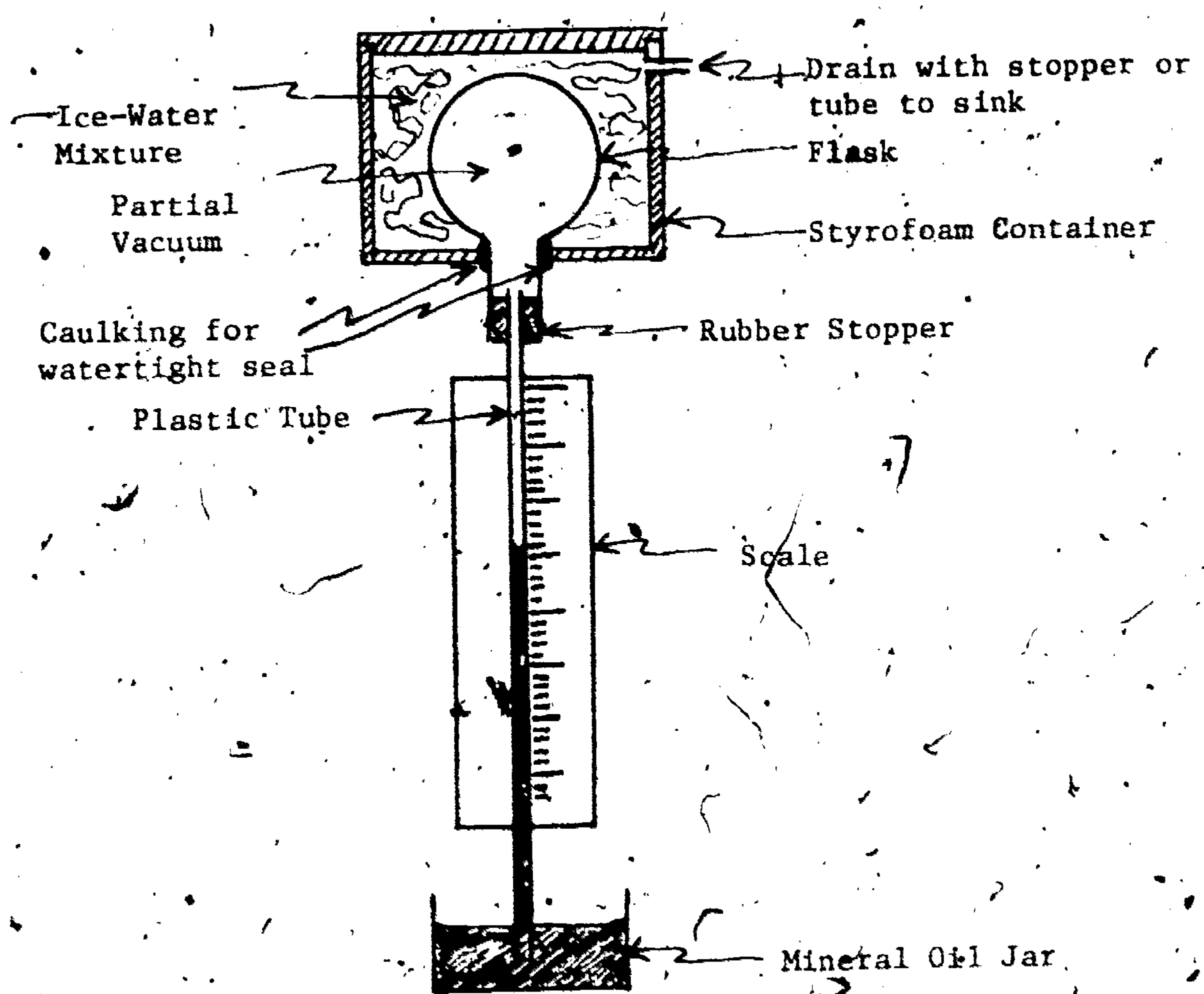


Figure 18

Calibration is most easily done by using another barometer which is already calibrated, such as one from a high school physics or chemistry lab nearby or an aneroid barometer from home. One can also call the nearest U.S. Weather Bureau office, TV station, or amateur weatherman to get the barometric pressure reading. With several different readings one can readily calibrate the scale.

Several problems may arise during the construction of these barometers. Plastic tubing is suggested for the barometer tube, since it is easily cut and will not break. Glass tubing can, of course, be used; but it is more dangerous due to the possibility of shattering. Rubber stoppers can be obtained at hobby shops or, perhaps, from high school labs. A number 6 stopper fits a Thermos bottle quite well. In fact, it seems to make an airtight seal so that wax is not needed for this purpose. If the hole in the stopper is the same as the tubing outer diameter, the tubing will fit tightly and an airtight seal will be formed here also. Some liquid soap on the end of the tubing will lubricate it so that it will go into the stopper easily. Care must be taken not to bend the tubing as it will crack. The tubing should be grasped near the end to be inserted into the stopper. Twisting the tubing back and forth slightly often helps. If glass tubing is being used, the hands should be held so that the tubing will not be jammed into the hand if it should break.

Filling the tube with mineral oil can also present problems. If the Thermos bottle barometer is being constructed, the easiest method is to put warm water in the Thermos for a short while, and then empty it. The stopper, with the tube already inserted in it, is inserted in the mouth of the bottle. The tube is then upended into the mineral oil jar. As the Thermos bottle cools, the air inside contracts, drawing the mineral oil up into the tube. In the order of construction steps, the Thermos bottle can be taped into the cotton - filled milk carton before the filling procedure just described takes place. The milk carton is then fastened to the support. It may take several hours for the Thermos bottle to cool completely and for the mineral oil to rise as far as it can.

If the mineral oil should rise too far, this can be easily remedied. Get a small flexible straw, and put one end of it directly beneath the bottom opening of the barometer tube. (See Figure 19.) Very gently below an air bubble into the barometer tube. Repeat this as many times as is necessary to bring the level of mineral oil in the tube down to a reasonable range. An alternative to this procedure is to use a medicine dropper instead of a straw.

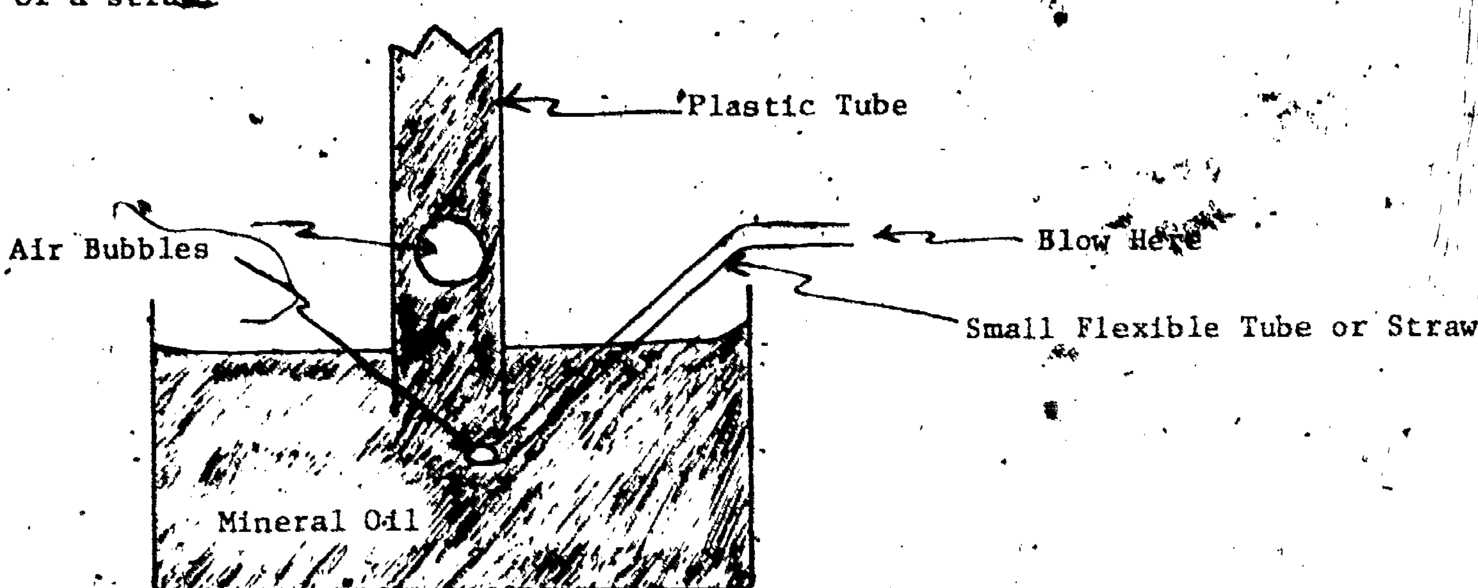


Figure 19

Filling the tube with mineral oil when constructing the ice-water bath type of barometer is much simpler. The barometer is constructed in its entirety, upended into the mineral oil jar, and fastened to the support. When the ice and water are added to the styrofoam chamber on top, the air in the barometer jar (or flask) will contract, drawing the mineral oil up the tube. If the mineral oil should rise too far, it can be lowered by the method previously mentioned.

One thing to note is the pressure when getting the mineral oil into the tube. If the pressure is low for your area, do not let the oil go too far up the tube. Otherwise, when the pressure does rise, the mineral oil will rise all the way into the bottle itself. When the pressure is high, make sure the oil goes up to a reasonable height, or it will all come out when the pressure falls later, and the barometer will have to be recalibrated.

If the rubber stopper and plastic tube do not make an airtight seal, air will slowly leak into the partial vacuum in the Thermos bottle, and the mineral oil will gradually leak out. If this should happen, candle wax may be used to form a seal. First, remove the barometer from the support and let all the mineral oil run out. Next, remove the rubber stopper and reheat the inside of the Thermos bottle with warm water. Pour out the water and insert the rubber stopper. Then, using a lighted candle, drop melted wax around the rubber stopper where it joins the Thermos and where the plastic tube goes in, as shown in Figure 20. Invert the barometer and place the bottom of the tube in the mineral oil. Refasten the barometer to the support. The wax should prevent the barometer from leaking.

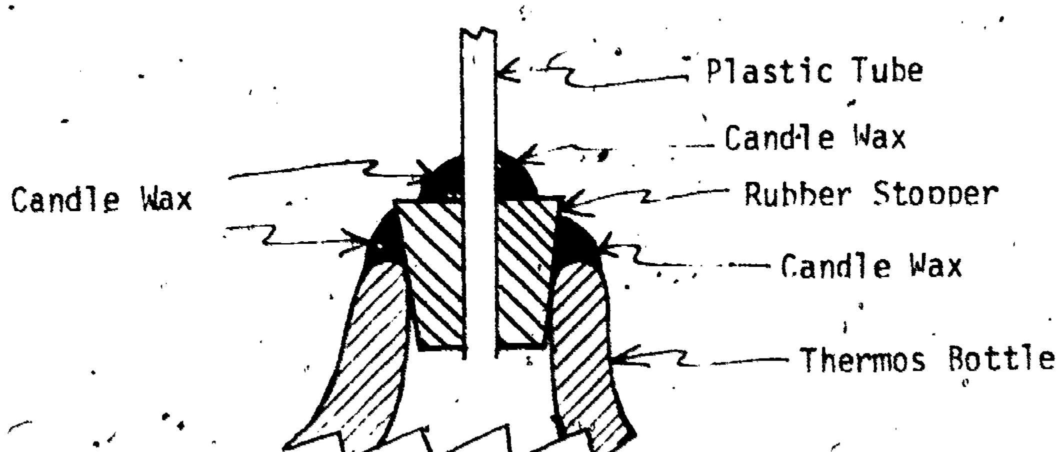


Figure 20

APPENDIX A: Mercury Barometer

In the mercury barometer shown in Figure 15, let the area of the top of the mercury in the jar be A, and let the weight of the air pressing down be W. This means the air pressure at that surface is $P = W/A$ since this is how pressure is defined. The weight of the column of mercury is found by multiplying its density (weight per unit volume), d, by the volume of the column. If the height of the column is h and its area is a, its volume is $(h \times a)$. The weight of the mercury is then $w = h \times a \times d$. The pressure exerted at the surface of the mercury by this column is $p = w/a = h \times d$. At this surface, the pressure of the air column and the pressure of the mercury column must be equal. (If this were not true, mercury would flow either into or out of the tube until it were true.) Thus, we can set the two pressures equal to each other:

$$P = p = h \times d.$$

Normal sea level pressure is given as 29.92 inches of mercury, and the density of mercury is 0.492 pounds per cubic inch. This gives us a normal sea level pressure of $P = 14.7$ pounds per cubic inch.

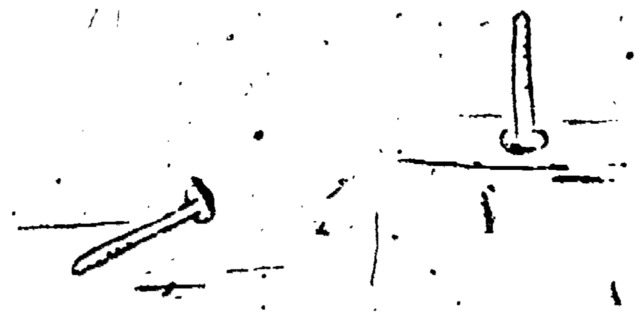
USMES

PROBABILITY: THUMB TACK EXPERIMENT*

Most people find it difficult to think about chance events. Let's see if we can agree on answers to the following questions:

Question 1: If we drop a nail from a height of about 4 ft., will it land in a "point-up" position,

or in a "point-down" position?



Question 2: If we drop a thumbtack from a height of about 4 ft., will it land in a "point-up" position,

or in a "point-down" position?

Question 3: If we drop a nail or a thumbtack from a height of about 4 ft., will it land "point-up" the same number of times in each set of 20 drops?

Question 4: Can you guess the outcome more closely if you use sets of 40 drops, instead of 20?

To find answers to these questions you may want to perform an experiment. To keep hidden factors from entering into the experiment, it is usually desirable to drop the thumbtack in nearly the same way each time. We can do this by setting the tack "point-up" on a desk, and slowly pushing it off the desk by means of the edge of a book. You can think of many other ways to achieve uniformity: for example, you can rest your forearm on a desk and hold the tack in a paper cup, then turn the cup quickly upside down, so that the tack falls out.

*from Cambridge Conference on the Study of Mathematics (CCSM) Feasibility Studies

The way that you record your data is also important. You want to preserve as much of the data as possible, so that you can use it to answer new questions that may arise in the future. One way to do this, in the thumb-tack experiment, is to use the letter "U" to mean "point-up" and to use the letter "D" to mean "point-down." Record each outcome in the order of occurrence, grouping the symbols in groups of 5, so that your record for 20 drops might look like this:

U U U D U
D U U D D
D U U D D
D U U D U

A set of original data is shown below.

Marilyn:
D D D [illegible] U
D D D [illegible] [illegible]
U D D U D
U D U [illegible] U

Jerry:
D U D U U
D U U D U
U U U U D
U U U U U

Harold:
U U D U D
U U D U D
D D U [illegible] U
D U D D D

Ellen:
U D U U D
U U [illegible] U U
D D [illegible] [illegible] U
D D [illegible] U D

Tony:
U U U U D
U D D U U
U U U U D
U D D U U

Richard:
D U D D D
D U D D D
D U U D U
D U D U D

Nancy:
D D U D U
D U U U U
U U U D D
U D U U U

Susan:
U U U D D
D U D U U
U U D D U
U U D D D

Joan:	U U D D U D U U D U U D U D U U U D D U	Jim:	D U U D D D D U U D U D D U U U D D U U
Francis:	D D U D U D U U D D D U D D U D U D D U	Tony:	D D D U D U D U D D U D U D D U D D D U
Marge:	D U D U D U D D D U D D U U U D U D D U	René:	U U U U U D D D D D D D U U U U U D U D
George:	D D D U D U D U U U D D D U D U U U D U	Steve:	U U U D U U U D U D D U U D U U D D D U
Jeff:	U D U D U D U D D D U D D U D D D U D D	Mary:	U D D D D D U U U D D U D U D D U U U D
Ann:	U D U U D U U D U D D D D U U D U U U U	Jake:	D D D D U U U U U U D D D U U D U U U U
Charles:	U D D D D D U U U U U U D U D D D D D U	Walter:	U U U U U U D D D U U D D U U D U U U D
Betty:	D D U D U U D U U U D D U D U D U D U U	Jenny:	D D U D D D D D D U D D D U U D U D U U

You can use the above data to make sets of data which have different numbers of drops in each set. For example, you can make up 12 groups of 5 drops each, using the first five drops from the first 12 students:

Number of "ups" in each group of 5 drops

2, 3, 3, 3, 4, 1, 2, 3, 3, 2, 2, 1

Similarly, you can make 12 groups of 10 drops each, by using the last 10 drops of each of the last 12 students:

Number of "ups" in each group of 10 drops

5, 5, 6, 4, 6, 6, 5, 3, 5, 5, 6, 5

Twelve groups of 20 drops each can be compiled by using the last 10 drops from pairs of students:

Number of "ups" in each group of 20 drops

14, 7, 12, 12, 12, 8, 11, 10, 8, 12, 10, 10

Similarly, 10 groups of 40 drops each are made up by combining the work of pairs of students:

Number of "ups" in each group of 40 drops

23, 21, 21, 24, 22, 15, 20, 22, 16, 24, 22, 18

We need some good methods for studying how much "variation" there is in our sets of data. Here are 3 methods:

I. The Method of "Just Looking"

II. The Method of Graphs

III. The Method of Quartile Ranges

I. The Method of "Just Looking."

For groups of 5, we got these numbers: (counting "Ups")

2, 3, 3, 3, 4, 1, 2, 3, 3, 2, 2, 1

For groups of 20, we got these numbers:

14, 7, 12, 12, 12, 8, 11, 10, 8, 12, 10, 10

By just looking at these numbers, which set of numbers seems to show greater "variation"?

II. The Method of Graphs.

We can show the first set of numbers in a graph like this:

Number of "Ups" in 12 groups of 5 drops

frequency

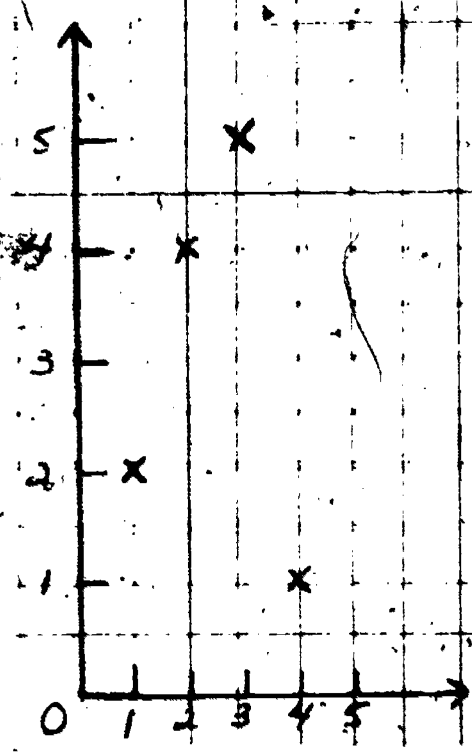


Figure 1

Numbers of "Ups" in each group

PS1-6

The graph of the second set of numbers is:

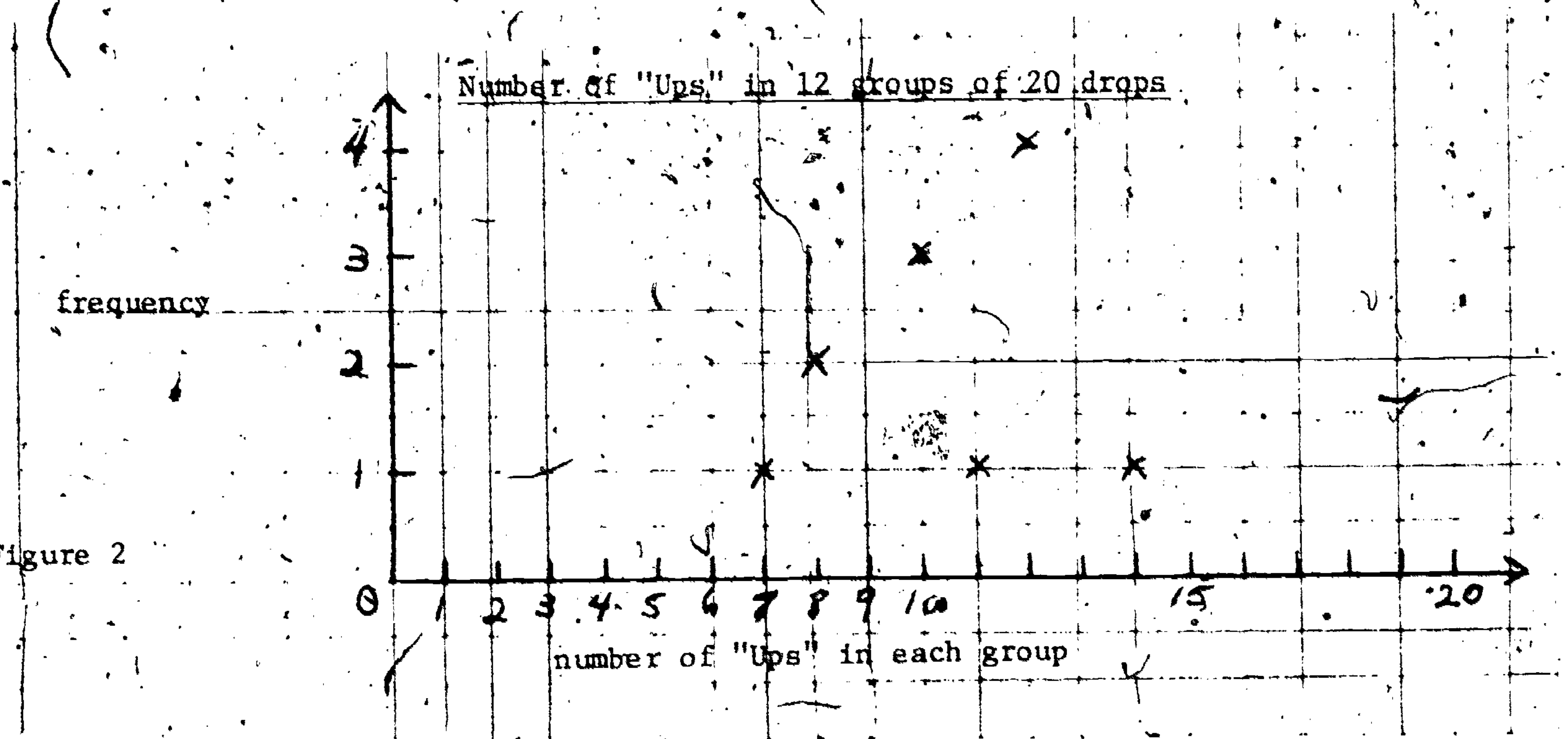


Figure 2

From looking at these two graphs, which set of numbers seems to show greater variability?

III. The Method of Comparing Quartile Ranges

To use this method, we arrange the number of ups per group in order of size for groups of 5:

1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4

Further, for groups of 20:

7, 8, 8, 10, 10, 10, 11, 12, 12, 12, 12, 14

We then find the quartile range of each set by discarding the three "largest" (25%) and the three "smallest" (25%) numbers in each. The numbers left are:

2, 2, 2, 3, 3, 3

10, 10, 10, 11, 12, 12

For these "trimmed" sets of numbers, we compute the ranges:

3 - 2 = 1 quartile range for first set of numbers (groups of 5)

12 - 10 = 2 quartile range for second set of numbers (groups of 20)

From comparing the quartile ranges, which set of numbers seems to show the greater variability?

When we were using 10 groups of 5 drops each, we had a sample size n, equal to 5.

When we were using 10 groups of 20 drops each, we had a sample size n, equal to 20.

In general, when we increase the sample size by making it 4 times as big, the variability of the total number of U's would be expected to increase by a factor of 2. Consequently, mathematicians say that the variability of the total number of U's increases as $\sqrt{n}$.

Even though the second set of numbers (the large sample size) seemed to show more variability, there is <u>some</u> sense in which it really shows <u>less</u> variability. What do you think? How would <u>you</u> suggest we deal with these two sets of numbers?

Even though the total number of U's is harder to predict for larger samples, the proportional occurrence of U's is easier to predict for larger samples.

Let's test this idea by each of our 3 methods for comparing variability. Earlier, we compared the variability of the total number of U's. We now compare the variability of the proportional or fractional number of U's.

Method I:

The fractional number of U's in the 5 drop case can be found by taking the total number of U's:

2, 3, 3, 3, 4, 1, 2, 3, 3, 2, 2, 1

and dividing by the total number of drops (in this case, 5):

$\frac{2}{5}, \frac{3}{5}, \frac{3}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{3}{5}, \frac{2}{5}, \frac{2}{5}, \frac{1}{5}$

To more easily compare with the fractions in the 20 drop set we multiply both numerator and denominator by 4, getting:

$\frac{8}{20}, \frac{12}{20}, \frac{12}{20}, \frac{12}{20}, \frac{16}{20}, \frac{4}{20}, \frac{8}{20}, \frac{12}{20}, \frac{12}{20}, \frac{8}{20}, \frac{8}{20}, \frac{4}{20}$

for the groups of 20 drops we get:

$\frac{14}{20}, \frac{7}{20}, \frac{12}{20}, \frac{12}{20}, \frac{12}{20}, \frac{8}{20}, \frac{11}{20}, \frac{10}{20}, \frac{8}{20}, \frac{12}{20}, \frac{10}{20}, \frac{10}{20}$

Can you tell, by just looking, which set of numbers varies more?

Method II: Comparison by Graphs.

We shall mark both sets of numbers on the same graph, using x's to indicate the 1st set, and 0's to indicate the 2nd set:

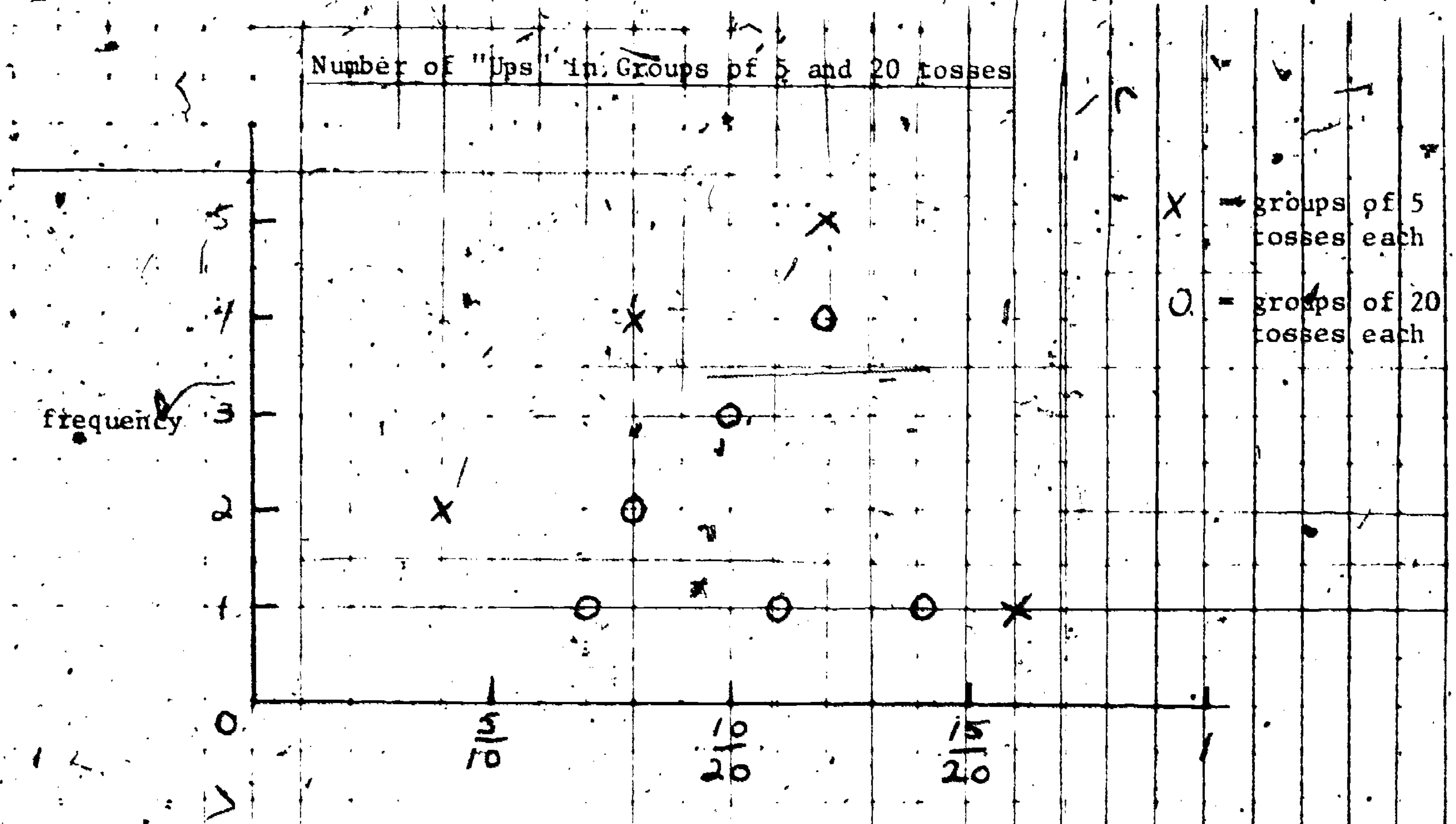


Figure 3

Which set of numbers shows greater consistency? Which shows greater variability? Did it work out the way you expected?

III. The Method of Comparing Quartile Ranges

The 1st set of numbers (proportion of U's in group of 5 drops) in order of size is:

0.2, 0.2, 0.4, 0.4, 0.4, 0.4, 0.6, 0.6, 0.6, 0.6, 0.6, 0.8

We delete the three (25%) largest and the three (25%) smallest to get a "trimmed" set of numbers:

0.4, 0.4, 0.4, 016, 0.6, 0.6

The quartile range is 0.6 - 0.4 = 0.2.

The 2nd set of numbers (proportion of U's in groups of 20 drops) is:

0.35, 0.40, 0.40, 0.50, 0.50, 0.50, 0.55, 0.60, 0.60, 0.60, 0.60, 0.70.

If we omit the three largest and three smallest, we get the "trimmed" set of numbers:

0.50, 0.50, 0.50, 0.55, 0.60, 0.60

The quartile range of this "trimmed" set is 0.60 - 0.50 = 0.10.

Does the total number of U's vary more in large samples or in small samples?

Does the proportion of U's vary more in large samples, or in small samples?

Earlier we showed how the variability of the total number of U's increases as $\sqrt{n}$ where n = sample size. In the fractional proportion of U's, however, the situation is quite different. Here, if we multiply the sample size by 4, the variability of the fractional proportion of U's decreases by a factor of 2. Consequently, mathematicians say that the fractional proportion of U's decreases like $\frac{1}{\sqrt{n}}$.

We can doublecheck the ideas we have discussed earlier by collecting more data.

Each student in a 6th grade class in Lexington, Mass. tossed thumbtacks and recorded the data for 10 groups of 5 tosses each. This provided total results for 240 groups of 5 tosses each. They then combined their data into 60 groups of 20 tosses each. The first two frequency distribution graphs (figs. 4 & 5) show the variation in the total number of "Ups" in groups of 5 and 20 tosses. The third frequency distribution graph shows on one graph (Figure 6) the fractional number of "Ups" in groups of 5 and 20 tosses.

Figure 7 shows the variation in quartile range with sample size. One line (marked with x's) shows that the quartile range of total number of "Ups" increases from about 2 to 4 with increase in sample size from 5 to 20 tosses. This confirms the statement that the variability of the total number of "Ups" increases as $\sqrt{n}$. The other line (marked with o's) shows that quartile range of fractional number of "Ups" decreases from about 4 to 2 with an increase in sample size from 5 to 20 tosses. This, likewise, confirms the statement that variability of fractional numbers of "Ups" decreases like $\frac{1}{\sqrt{n}}$.

Frequency Distribution Graphs

Number of "Ups" in Groups of 5 tosses

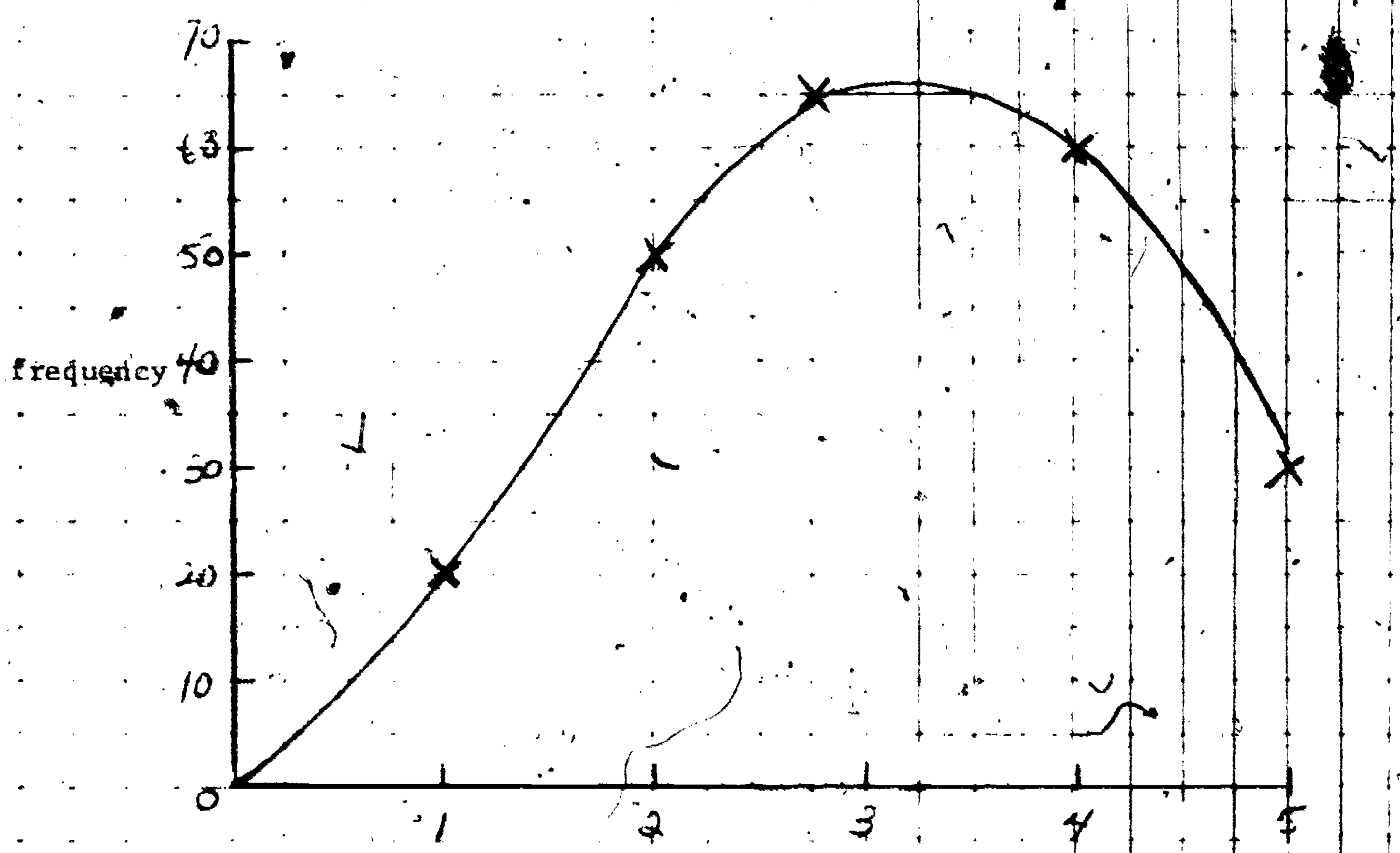
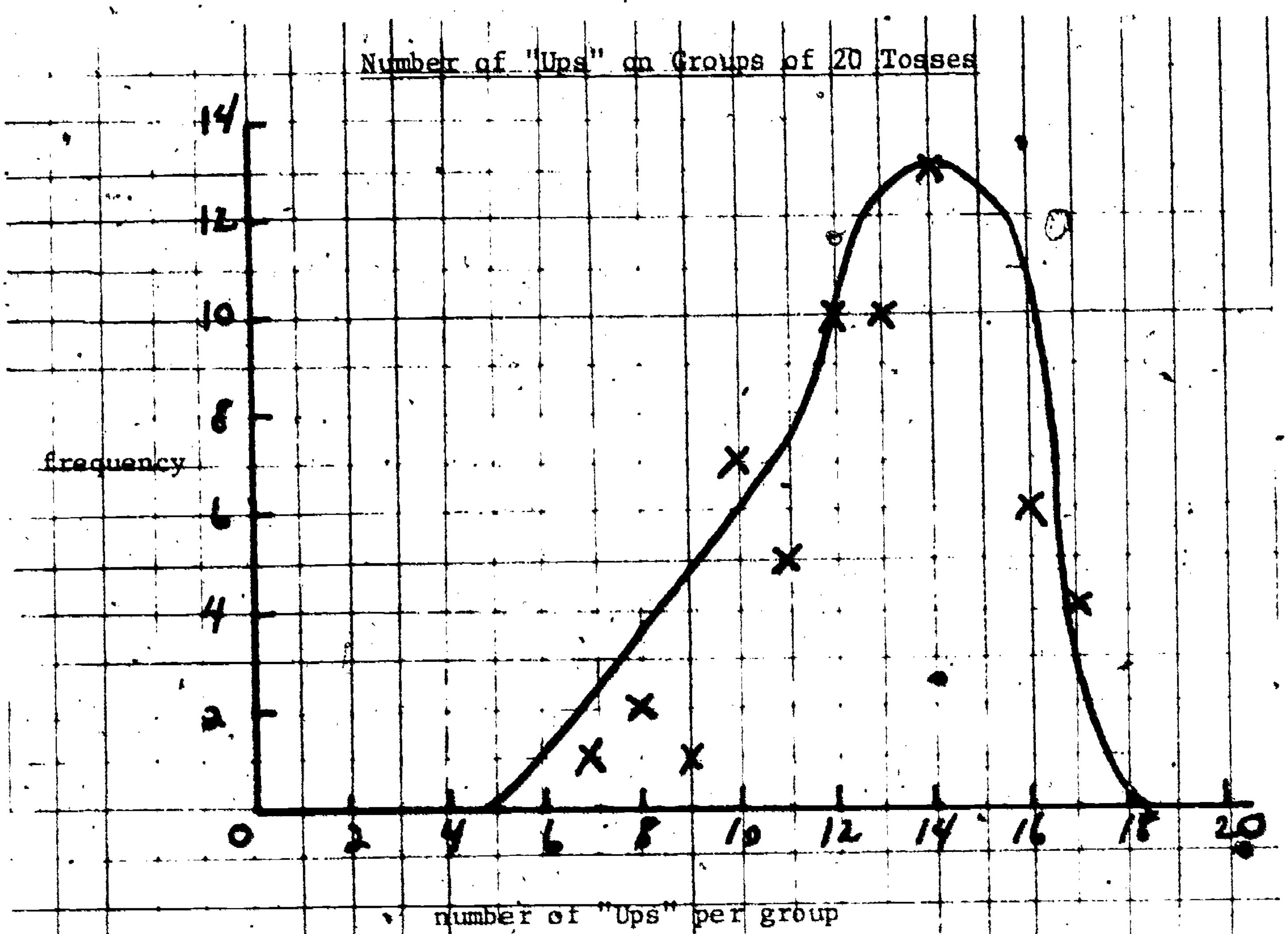
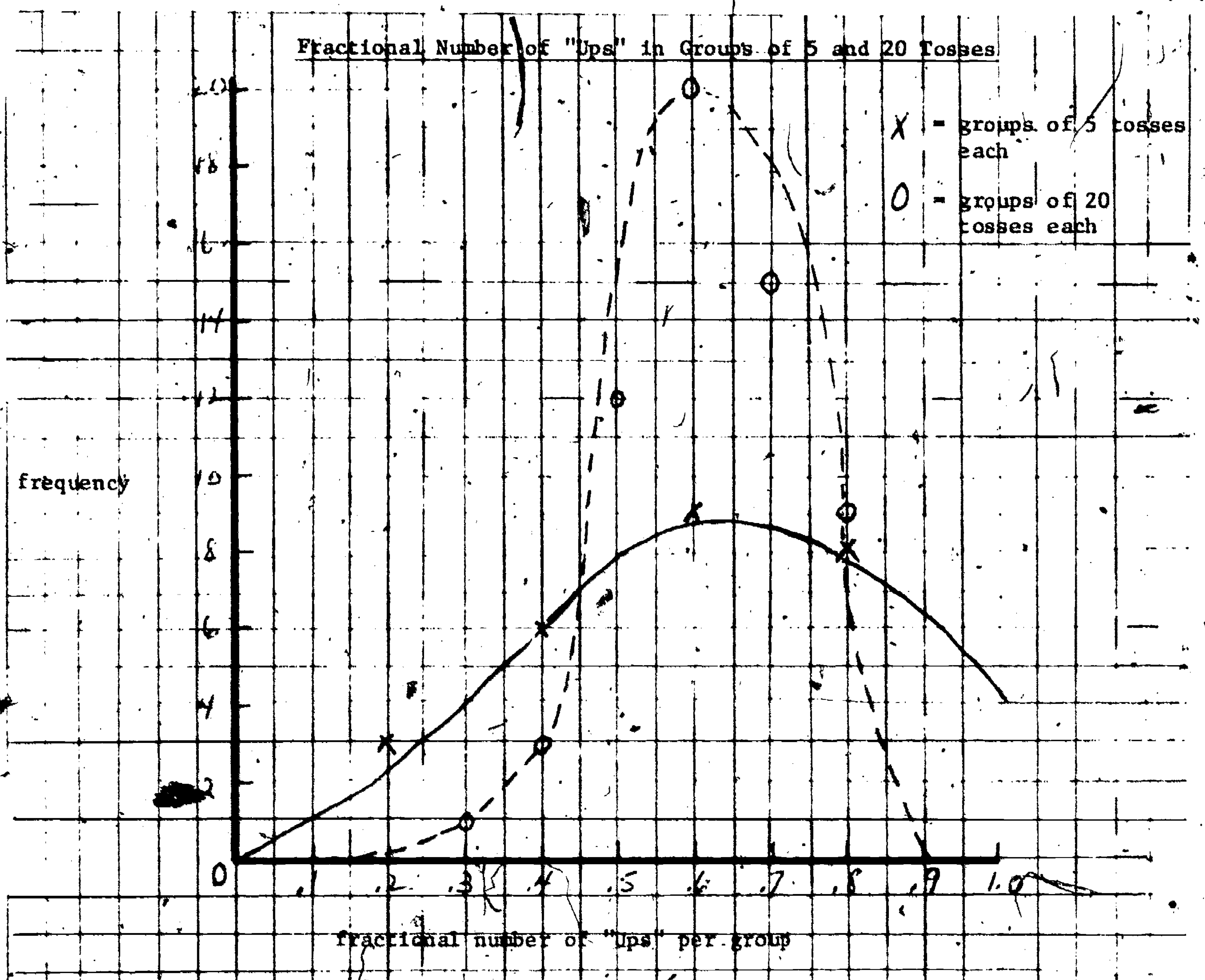


Figure 4



Number of "Ups" on Groups of 20 Tosses
14
12
10
8
6
4
2
frequency
0
2
4
6
8
10
12
14
16
18
20
number of "Ups" per group

Figure 5



Fractional Number of "Ups" in Groups of 5 and 20 Tosses
X = groups of 5 tosses each
O = groups of 20 tosses each
frequency
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
fractional number of "Ups" per group

Figure 6

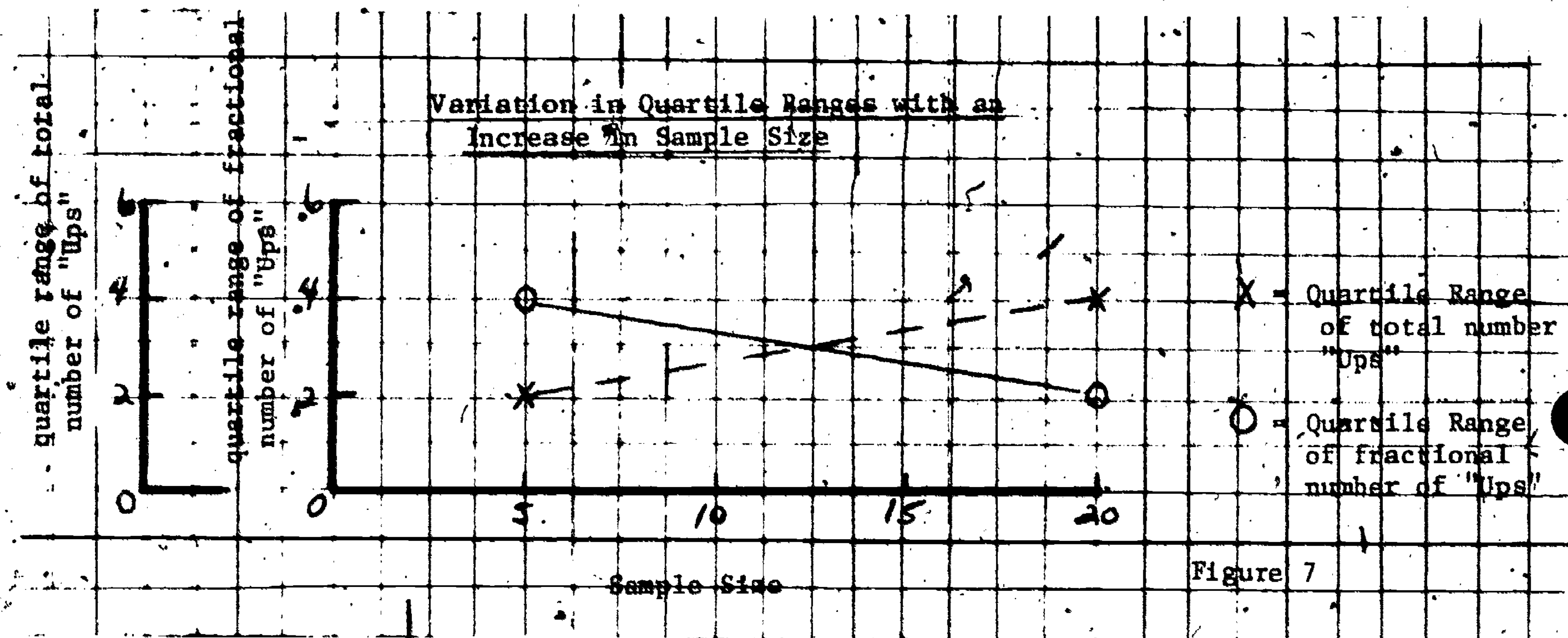

Variation in Quartile Ranges with an Increase in Sample Size
quartile range of total number of "Ups"
quartile range of fractional number of "Ups"
6
4
2
0
5
10
15
20
Sample Size
X = Quartile Range of total number "Ups"
O = Quartile Range of fractional number of "Ups"

Figure 7

USMES

WEATHER PREDICTION

Bob Renard

I. Introduction

Weather Prediction is not new to any of us--child or adult. It comes naturally. An implied weather forecast is made by a child who looks out the window and says, "It looks like rain, Mommy," or by his father who says, "Guess I'll take my heavy coat today; whenever the north wind blows in the morning, it's cold all day." The professional weather forecaster does that same type of thing for a living, having done quite a bit of serious study of the atmosphere. Pages PS3-9 and PS3-10 contain a policy statement of the American Meteorological Society on weather forecasts and the degree of skill (accuracy exceeding levels obtained using persistence or climatology methods) possible for various range forecasts.

But one doesn't need to be a pro to be a success at prediction, as long as the problem is approached properly. Walking before running in this case means forecasting for the short-range period first (i.e. up to 24 hours), then gradually working up to the bigger problems, as medium range (24 to 36 hours) and then all out--2,3,--n days!

One should not try to compete with the professional forecaster, at least initially. Frustration and disinterest are sure to follow. Concentration on scientific approach (via simple mathematics and statistics), using quantitative observations of weather and its changes and anlyses of the interrelations of weather elements should be basic to the forecast made by the students.

II. Short-Range Weather Prediction

The first attempt at forecasting ought to be short-range prediction. For our purposes I'd define this as a period no greater than 24 hours and composed of two parts or stages:

1) within the classroom day plus or minus an hour or two (as 8AM to 6PM) and

2) the night following the classroom day plus or minus an hour or two (as 6PM to 8AM).

The forecasts in these time periods will be mainly ones of predicting the intradiurnal changes (i.e. changes within a 24-hour day). There is nothing more regular in the atmosphere than the intradiurnal change of a weather element[1]--in many cases approaching the regularity of the rising and setting of the sun and, indeed, closely tied to that phenomenon!

A. Temperature Forecasting

As an example of approach, consider surface temperature, an important and easy-to-work-with weather element. Professionally, surface temperature is the air temperature measured in a little white box to keep out the sun's rays[2], sitting on legs six feet above the ground, with louvered sides to let the air pass freely through. These are the temperatures you see in the newspaper or on TV or hear on the radio. Figure 1 shows schematically the temperature as a function of time. Such a curve represents the average intradiurnal temperature profile. One like it can be drawn for any locality. The average of any weather element over a period of years is referred to as the climatological value.

Find out from your local full-service or climatological National Weather Service station what the average intradiurnal temperature curve looks like for each month in your city. In addition (or perhaps as a substitute),

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[1]Major weather elements are temperature, wind (speed and direction pressure, moisture, visibility, sky condition (clear, cloudy, etc.), precipitation (rain, snow, etc.)

[2]An object exposed to the sun will become hotter than the air which lets the sunlight through comparatively easily.

children should gather the data and construct their own curves, covering as much of the day as possible. Time of minimum, although usually close to sunrise, may differ from locale to locale; the time of maximum, although usually around 2 or 3PM for stations in the interior may be closer to 11AM or 12 noon for stations on or near the continental coasts. (Standard times.)

1. Predicting Today's Maximum Temperature. Armed with the intradiurnal temperature curve for your city, you are ready to start simple, but meaningful, prediction. Try the following on forecasting today's maximum temperature.

a. Simplest[3] approach: Assume the average or climatological temperature curve will prevail today and hence represent the best estimate of today's max or any other temperature. A student will soon learn that this is not the best approach and he/she will inquire as to how to improve the forecast accuracy.

b. Persistence: Before doing other things with the intradiurnal curve try the following. Since most of us, in whatever city we are located, experience similar weather over several-day periods, we'll say that yesterday's temperature maximum is the best guess for today's maximum. This approach is called persistence[4]. Again we soon find out that this method leaves something to be desired, although much better than a. above. And, the challenge goes on!

3Simplest means scientifically simplest here. Perhaps you should start with the pure guess. Even the lucky guessers won't stay in business more than a day or two. They will be willing to convert to the forecast methods having some scientific validity and chance of success.

4Note the attached Policy Statement of the American Meteorological Society, a statement issued to indicate to the American public the current state of forecasting. Climatology and persistence are noted in the third paragraph. To earn his keep, a professional forecaster's predictions must be an improvement over climatology and persistence, but this is where the pro starts, too --and many times he never gets much beyond the accuracy of these two basic approaches!

c. More with the intradiurnal curve: Given the temperature at any hour during the school day (you observe it or call the National Weather Service station) and the climatological intradiurnal temperature profile, can you predict the maximum for the day and come closer than a. or b. above? Yes! For example, suppose at 9AM today the temperature is 3^{0}F higher than the average 9AM temperature (see X in Figure 1). Simple prediction: the max will be 3^{0}F higher than the climatological max. (See M in Figure 1.) But, alas, as it turns out the maximum was reached at 4PM and was at M . So we still "blew" the forecast, but only a little!

What next? Can we still stay with the simple prediction approach and yet get a bit more complex? Yes, again! We have just used climatology and we find that it properly suggests the temperature rise and fall as a function of the hour of the day, but each day has its own uniqueness, to the extent that both the time and magnitude of intradiurnal changes differ due to the variation of other weather elements.

d. Temperature versus other weather elements: At this point we can bring in the other weather elements, as sky cover, pressure, winds, moisture, etc. It soon becomes evident that the atmospheric elements are as interrelated as bones in our body. The day on which we "blew" our forecast (in c. above) was cloudy in the morning, clear in the afternoon and winds shifted from north[5] to south between morning and afternoon. That's hardly an average day anyplace so we wouldn't expect the climatological temperature curve to yield a perfect forecast. The situation just presented could be analyzed as follows. Clouds usually keep the temperature lower during the day, but if it were cloudy all night the temperatures would start out higher in the morning hours, especially with any stirring of the air (wind), while clear skies and south winds relate well to higher-than-normal temperatures,

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[5]A north wind is a wind blowing from the north.

and a delay in the insulation due to clouds will delay the time of the maximum. But, you really don't need to know all this--let your mathematical-statistical tools bring out these relationships between weather elements.

Thus, on a pegboard or some such similar correlation device used to record and analyze the relation of one element to another, it would be well to identify the maximum temperature and the time at which it occurs, on clear days in one color, on partly cloudy days in another color, on rainy (snowy) days in another color, etc. Further, the direction from which the wind blows also relates to the maximum temperature. Thus, another pegboard, another set of color codings. It will become obvious very quickly that sky condition and wind direction, among other things, are related to the time and degree of maximum temperature and that the climatological temperature profile is just a good starting point. Therefore, we can better our forecasts by knowing what kind of day it is—clear, cloudy, windy, snowy, etc.

2. Predicting Tonight's Minimum Temperature. Short-range forecasts for the period (approximately) 4PM to 8AM the next morning are also fairly simple. The same rules and tools used for the maximum temperature forecasts apply here. There is the additional benefit of spreading the student's interest to outside classroom hours but still staying within the simple intradiurnal period. If a student looks at the thermometer near daybreak (which is not too hard in the winter), this will be closer to the minimum on many days.

B. Forecasting Other Weather Elements

Approaches similar to that used for temperature can be applied to relative humidity (expresses the ratio of actual moisture in the air to the maximum amount the air can hold at the given temperature)., The intradiurnal relative humidity curve is nearly out of phase with that of temperature. (See Figure 2.)

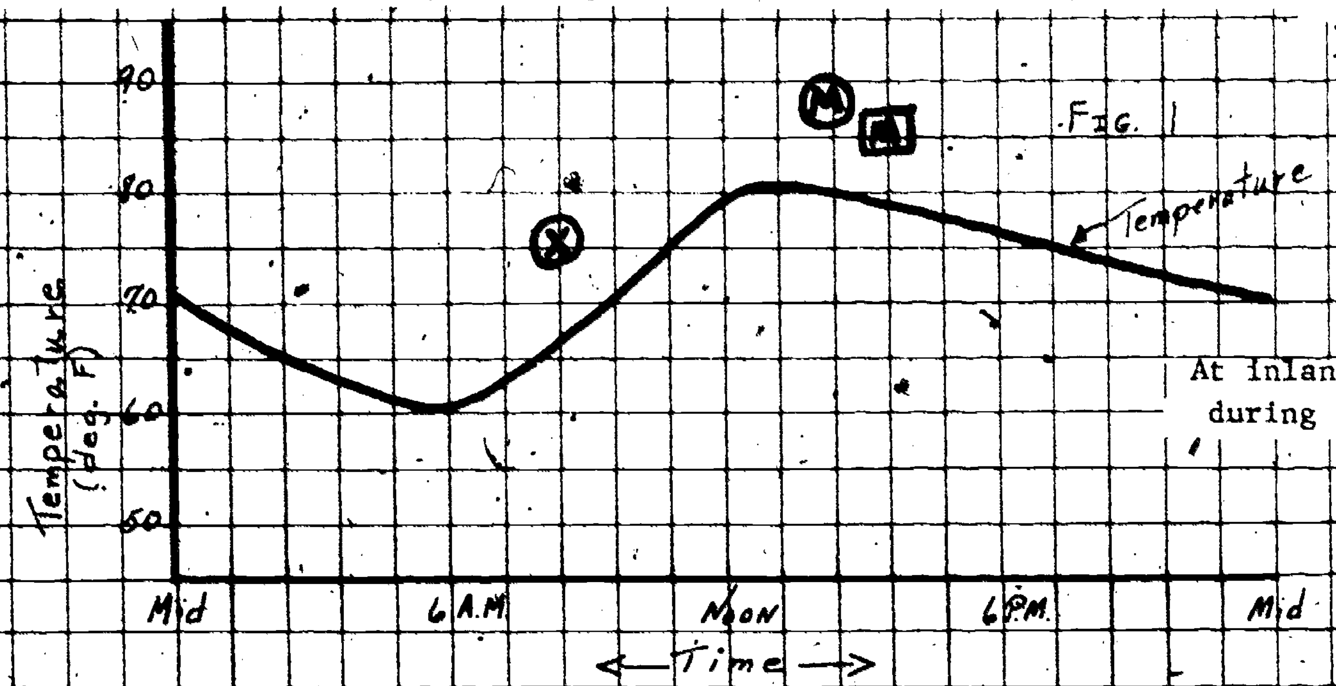
Or, try wind speed, whose intradiurnal curve usually resembles that of temperature (Figure 2), or try pressure, although this element is more complex since the diurnal pressure curve looks like that shown in Figure 3. A further complication with pressure is the hour-by-hour change in pressure due to changes in the weather pattern. The changes in pressure due to the weather pattern are much larger than the diurnal changes. Consequently, the intradiurnal effect is masked by the larger changes, especially in winter and at higher latitudes, making it very difficult to measure or predict diurnal pressure changes. (Note: Changes in the weather pattern are involved in the medium and long-range forecast problem.)

Other weather elements, as cloud, precipitation, visibility, and wind direction have only subtle intradiurnal changes in most areas and at most times of the year, and so are not appropriate for the short-range approach discussed to this point. For these elements, the methods of the medium range apply, even for periods less than 24 hours.

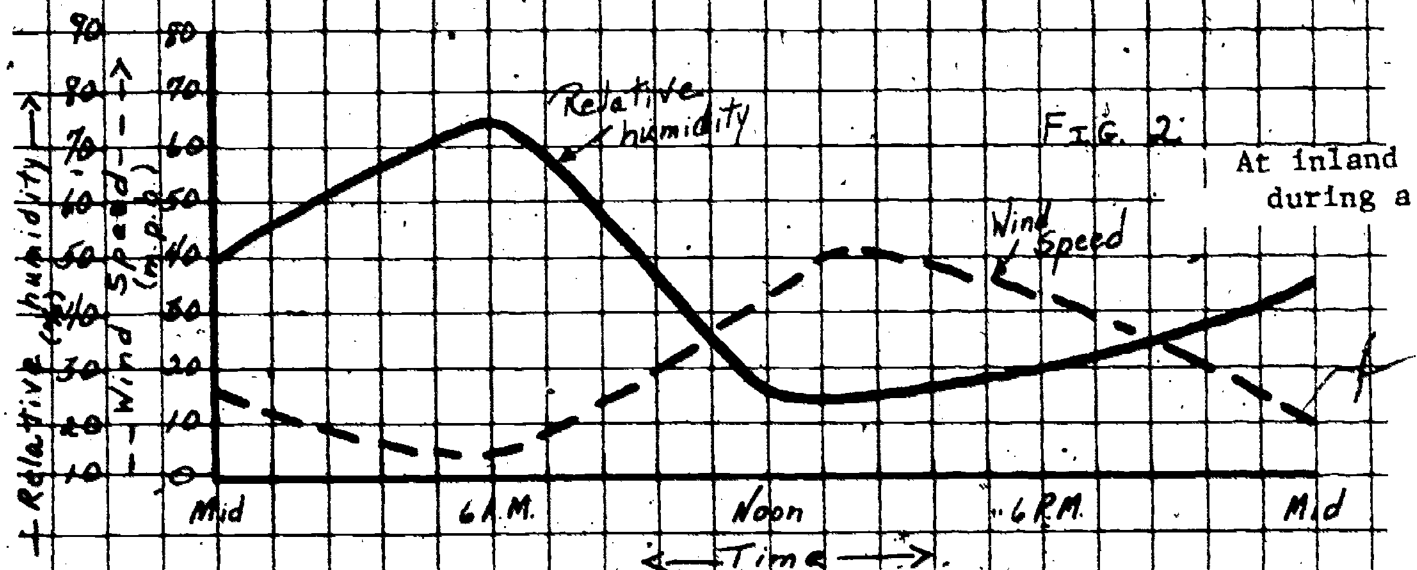
C. Final Remarks

Introduce the simple short-range prediction on days when a small change in weather pattern is not expected. (The teacher should keep abreast of the local forecast from the National Weather Service.) After introduction of the prediction scheme it won't make any difference on which days the scheme is applied. In fact, a day showing an unusual trend of a weather element, like the highest daytime temperature in the morning and lowest in the afternoon is a good opportunity to make the student aware of the fact that a change in temperature over some period of time, such as 12 or 24 hours, represents that part of the change closely related to the time of the day (intradiurnal) plus that part of the change due to changing weather patterns (interdiurnal). Interdiurnal is the change from one day to the next (as from 9AM today to 9AM tomorrow, or the difference between the maximum today and that of tomorrow). If the temperature falls during the day the interdiurnal is masking the intradiurnal changes. Such events do occur. More on this in the sections on medium and long-range forecasting.

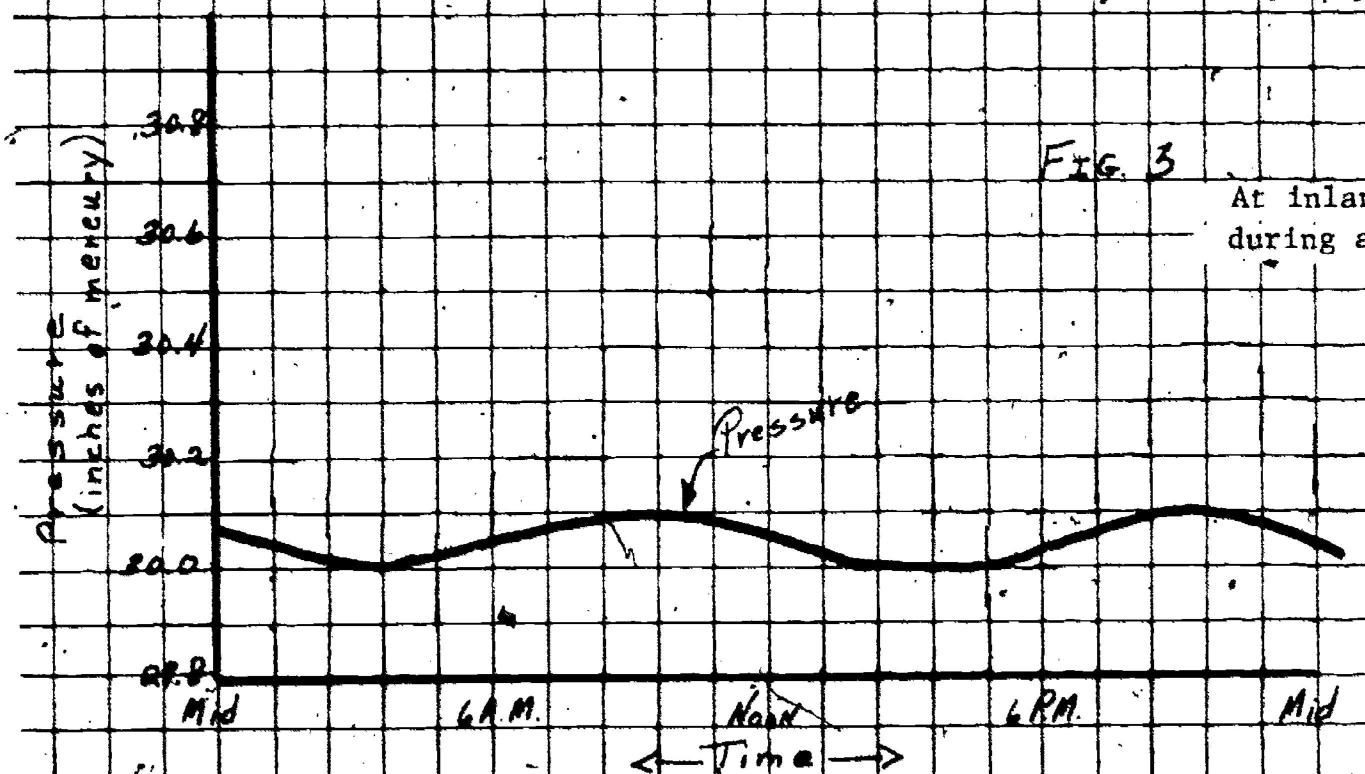
A very satisfying part of the short-range prediction is the chance to verify (determine the accuracy of) the forecast by, or even before, the end of the school day. In other words the student can find out how well he or she is doing in prediction not too long after the prediction is made.



At inland location
during a summer day



At inland location
during a summer day



At inland location
during a summer day

POLICY STATEMENT OF THE AMERICAN METEOROLOGICAL SOCIETY ON WEATHER FORECASTING (Reference: Bulletin of the American Meteorological Society, January 1973)

One of the most important activities in the field of meteorology is the preparation of weather forecasts as a vital service to public and private interests. Weather forecasts are used by individuals to guide their daily living and by industry, agriculture, forestry, commerce, transportation, and government to guide their operations. The widespread need for accurate advice on expected future weather conditions and the critical dependence of public safety and welfare upon the quality of such information make it desirable to describe the present weather forecasting capability of the meteorological profession.

With the introduction of high-speed computers into numerical weather prediction in recent years, along with improved numerical descriptions of the real atmosphere, and the development of modern observational techniques, such as radar and weather satellites, forecast accuracy has shown a significant improvement. Although the national economy directly benefits from increased forecast accuracy, the value of a weather forecast depends not only on its accuracy but also on the manner in which it is utilized and the method by and speed with which it is communicated to users.

Forecast accuracy attained by procedures such as predicting that the weather will remain unchanged (persistence) or by predicting average weather occurrences based upon past weather records (climatology), or simple variations on these procedures, serve as objective bases for measuring forecasting skill. Unless forecast accuracy exceeds levels achieved by basic methods such as these, skill cannot be said to exist. Moreover, skill in weather forecasting varies with the meteorological situation, geographical area, and season.

Weather forecasts prepared by professionally-trained personnel presently achieve the following levels of skill, on the average:

For periods up to 48 hours, weather forecasts of considerable skill and utility are attained. Detailed forecasts of weather and its changes can be made for the first 36 hours. Probability estimates markedly increase the information content of such forecasts, especially with regard to precipitation occurrence. In this period skill is a maximum in predicting the motion and general effects of weather systems having dimensions of five-hundred miles or more. However, small-scale features imbedded in these systems cause hour-to-hour variations in weather which are difficult to predict, especially for local areas with irregular topography. Also, the exact location of certain highly significant weather phenomena, such as severe thunderstorms and tornadoes, cannot be forecast accurately with any degree of skill beyond a few hours, although the general area of severe storm activity may be predicted up to 24 hours in advance. Accurate forecasts for infrequent events such as heavy snow, sleet and damaging winds are usually limited to periods not exceeding 24 hours.

For periods up to 5 days, daily temperature forecasts of moderate skill and usefulness are possible for periods extending to about 5 days. Precipitation forecasts to 3 days, at an equivalent level of skill, can be made, but the skill drops to marginal levels on the fourth and fifth days.

For periods of more than 5 days, average temperature conditions for periods from a week up to a month or season can be predicted with some slight skill. Day-to-day or week-to-week forecasts within this time range have not demonstrated skill. There is some skill in prediction of total precipitation amounts for periods of 5 to 7 days in advance; skill for longer periods is marginal.

Recent theoretical work on atmospheric predictability indicates that the intrinsic properties of the atmosphere, together with the impossibility of observing every detail of atmospheric behavior, impose an upper limit for the prediction of day-to-day weather changes. This period is believed to be about one to two weeks, depending on the criteria used to define a useful forecast. Present day forecasting accuracy, as cited above, falls short of the theoretical limit. There are also limits to the extent of time for which average quantities such as weekly or monthly mean temperatures can be forecast, but theoretical estimates of these limits are not available as yet.

DESIGN OF SURVEYS AND SAMPLES

by

Susan J. Devlin and Anne E. Freeny

In several of the USMES units, a class is likely to decide they need to know something about a group of people in order to carry out their project. For example, in the Human Porportions unit, if they decide to make and sell aprons, they would need to determine what sizes the aprons should be and how many to make of each size. Or in the Soft Drink Design unit, if their goal is to create a new soda, they might decide to find out what sodas are most popular. In both cases, the class will have to decide how to collect the information that they need to know. A procedure which collects opinions or measurements about people or some other item is called a <u>survey</u>. When only some of the people or items one is interested in are measured, it is called a <u>sample survey</u>.

This paper discusses two major areas to consider in assuring that the survey results will be useful. These are survey design and sample design. <u>Survey design</u> includes determining what characteristics to measure, how to measure them, and whom to measure. <u>Sample design</u> is concerned with choosing a fair sampling procedure so that results from the sample will accurately reflect the behavior of the items of interest.

I. Survey Design - What to Measure

The first and most important step is for the class to decide exactly what they need to know. Too often when the objectives of a survey are not well formulated the survey results are less useful than they should be for one or both of the following reasons:

1. Not all the information needed is collected.

2. The wrong information is collected.

The following discussion tries to indicate how what the class needs to know might be expressed in order to make it more likely that the right information will be collected.

For the apron example, the class must go from an objective of determining apron size to a more specific statement of what body lengths are needed to determine the different sizes. If the class simply says that they need to know how many big children and how many small children there are, the resulting measurements in terms of "bigness" might be height and weight. Deciding what to measure for an apron size requires knowing what the pattern for an apron should look like and what measurements are needed to cut this apron to fit a person. Given that the aprons are to be as in the diagram below, then the body measurements needed are

the waist,

the length from neck to waist, and

the length from waist to knees.

Clearly height and weight would not be sufficient to determine an apron size and the three measurements given above are the information which should be collected by a survey. Then how many aprons to make of each size can be decided from the survey results.

In the soda example, problems may be caused by the difficulty in expressing precisely what it is the class needs to know. Suppose the class has decided to find out "what sodas are most popular." Does "most popular" refer to the soda which is liked the best or the one that is bought the most? A soda liked best may not be bought the most because of the costs of different sodas. Does "soda" mean a brand name or a flavor? If the class plans to create a new soda, it would appear that asking about flavor preferences is more relevant to their objective than asking about brand preferences. A precise statement of objective here might be that they need to know what soda flavors people like the best.

II. Survey Design - How to Measure

There are two types of measurements being made in our examples, physical measurements and opinion or preference measurements. Physical measurements are usually well defined by having decided "what to measure" (for example, heights, weights, number of children per family, etc.) and questions about them are easily phrased in terms of the measurement units. (How many inches tall are you, how many ounces of soda are there in this bottle, how many children are there in your family?) Thus in the apron example the class can simply measure the decided-upon lengths in inches.

Preference or opinion measurements, which are unitless quantities, are another matter. Here seemingly straight-forward questions can measure something other than what is desired, and there are real difficulties in determining "how to measure," i.e., what the right questions are. The rest of this section is devoted to using the soda example to show first how certain questions can be inappropriate, and then how opinion questions can be worded to get meaningful results.

Having decided that they are interested in flavor preferences, a class is likely to compile a list of all the names, probably by brand name, of sodas that they can think of and then survey people by asking them to pick from that list which one or several they like the best. Some problems arise with wording a question in this way because it is not clear what the answers really mean. First, is this question finding out brand preferences or flavor preferences? When totaling which drinks are checked most often as favorites, Coke, Pepsi, RC Cola, and other cola drinks may not be among the top favorites even though cola flavored drinks are the single most popular. Even being aware of this, how should brand name results be combined into groups to give information about flavor preference? A second problem is that dislike is confused with having never tried the drink. Third, does "like the best" necessarily mean "taste the best" to everyone? Perhaps some people like a drink not only because it tastes good but also because it is less expensive. Finally, a survey conducted in this way requires recognition of a brand name. A favorite soda's name might not be recognized, and a child may check a less preferred but well advertised brand, so that an advertising effect has influenced the results. All these problems could be alleviated by measuring preference by a taste test. However, since it may be impractical to carry out a survey using soda samples to taste, consider one way a questionnaire can be prepared.

A type of question which is easy to ask but difficult to interpret is that with a yes or no answer choice. For example, "Do you like Coke?" is a simple question with a straight-forward answer. But now consider the questions "Do you love Coke?" and "Do you drink Coke?" Of course the answers to these three phrasings will be quite different.

It is usually better to solicit answers to preference questions in the form of a scale. For example:

What do you think of Coke?

5. I love it.

4. I like it.

3. It is OK.

2. I don't like it.

1. I hate it.

I have not tried it.

Notice that "I have not tried it" is included as a possible answer to separate dislike and unfamiliarity. A score, such as 1 to 5 (see above) can be associated with the other possible answers, giving the most positive answer the highest score. In summarizing the results, the average score for each question for all people answering the question with anything except "I have not tried it" can be found. If there are several questions of this type, such as "What do you think of Orange Crush?", etc., then the average scores for each of these questions can be compared. The most popular brands will be those having the highest average scores, thus establishing a "preference rating" for each brand. Having corrected for "I have not tried it" and having considered each soda separately, then the preference ratings by brand can be combined to indicate flavor preference.

An "unfamiliarity rating" could be established by simply counting the number of "I have not tried it" answers to each question.

It has been found that when using a scale with fewer levels, say 3, there is a tendency to answer using always the middle level, such as "It is OK." A scale with more than 5 levels is generally inadvisable, since in many situations people find it hard to discriminate between very many unit-less levels. Consequently a scale with 5 levels, as in the example above, is a good compromise.

III. Survey Design - Whom to Measure

Exactly what is the group of people or other items about which the class wants to know something? This group is called the target population. For the apron example, the target population should be all people who will buy an apron. But who are these people? Are they the class, the school, the town, etc.? If the sale is to be at the school and the class has decided to make only children's aprons, then one definition of the target population is all children attending the school.

A more subtle problem arises now. Before the apron sale, the class can survey only potential buyers. What if the day of the sale the 5th graders are on a field trip or many younger children are not allowed to carry enough money to buy an apron? Having assumed that everyone enrolled in the school is a potential buyer, the class may be left with too many large or small aprons. This example shows how sometimes the target population cannot be defined as one would like it. Here it must be the entire school, or say the school without the lower grades if they will not have enough money. No matter how carefully one tries to define the target population, in this case it can never be the population of interest, i.e. the people who will actually buy an apron.

The problem in defining the target population for the soda example are similar if the objective of the class is to sell the new soda which they are going to design from the results of the flavor survey. However, if that is not their motivation, they can decide whose taste preferences they are interested in, such as adults, high school students, or children. If they decide that they are interested in children's preferences then they must decide of what ages. Even if they decide on only a very narrow age range, all children in that group cannot be interviewed in the survey, and perhaps not even all children in the town in that age range. They must choose a sample, say the school, to represent the target population. Some of the problems of selecting a sample are discussed in the next section.

IV. Sample Design - Ways to Sample

It is often time consuming and expensive to measure an entire target population so a natural thing to do is to take a sample of the target population, i.e. to take a sample survey. However, just as it is important to define the target population carefully, it is also important to choose a sample which represents the target population fairly. A cook will stir a stew before tasting it to be sure that all ingredients are mixed, so that she is getting a sample which will taste like the whole stew. In the same way a sampling procedure must insure that each item in the target population is equally likely to be chosen. Here are two sampling procedures which meet this requirement if properly done.

The first is random sampling. An example of a good way to take a random sample is to write the names of all the units in the target population on separate pieces of paper and put them in a container, thoroughly mixing the pieces of paper (just as in the stew). The names that are drawn, not replacing them in the container after they are drawn, are then the units to be sampled. Another way is to have all the names on separate cards and shuffle the cards very well. Then draw enough cards from the deck to determine the sample. If the target population is the school, then the units are all the children in that school, and it is their names which would be put in the container or written on the cards to be shuffled.

Sometimes it is impossible or difficult to take a random sample. Consider the case where a sample must be taken from a product coming off an assembly line. The items which come off first must be shipped to stores before all of the items have been manufactured. This means that the target population is never entirely available to be randomly sampled. The second kind of sampling, called systematic sampling, is needed. For systematic sampling it is necessary to line up or list the target population in some order. Then if the sample is to be some fraction of the target population, say one fifth, select every fifth unit from the ordered list. Items on an assembly line will have a natural order imposed by the line itself, and the sample can be drawn by taking every fifth item (or tenth, or twentieth, etc. depending on what fraction of the population is to be sampled) as it comes to the end of the line. And if a systematic sample of size 25 is to be taken from a school which has 125 children, select the sample by counting down all the class name lists, including every fifth name as a pupil to be sampled. Or simply use the sequence in which the children arrive at school as the ordering and include every fifth pupil coming through the door in the sample.

In using systematic sampling, trouble can occur if there is some pattern in the way the target population is ordered. For example, suppose 5 pupils are to be chosen from a class seated in a room. The class size is 25 and are sitting in 5 rows, 5 in each row. If every fifth person, counting up the rows, is systematically selected, the 5 children sitting in the front will be chosen as the sample. Now suppose their teacher has arranged the seating plan so the smallest children are in front and the tallest in the back. The sample will consist of the 5 shortest children. If a sample of pupils to measure for apron sizes is selected in this way, the results would indicate that the class is generally much smaller and less variable in size than it really is. Though most orderings will not cause such a problem, part of planning a systematic sample must be to consider what the ordering of the population will be and to make sure the characteristics determining that order are not related to the characteristics to be measured.

V. Sample Design - Variability

How does sampling affect the "goodness" of the summaries made from the sample survey? Summaries may include such things as the <u>mean</u> (the average value), the <u>median</u> (the middle value of a set of ordered numbers if the sample size is odd, and the average of the two middle values if the sample size is even), the <u>range</u> (the largest value minus the smallest value), and the proportions of the sample measurements at each possible value. Because the target population is sampled, the summary values calculated will not necessarily be the same as they would be if the entire target population was surveyed. This section will demonstrate how much the sample summary can vary from the target population summary and will describe 2 factors affecting this variability. Though the median will be used in the example for the sake of simplicity, what is said is true for any type of sample summary.

Suppose for the apron example the class decides to fit the apron by using only one waist measurement for all aprons to be made and let the apron ties adjust for the actual differences in the children's waist measurements. Then, they could use the sample median as the waist measurement to assume for all apron sizes. Using the median, rather than a smaller or larger measurement, hopefully will mean that none of the aprons will be much too small or much too large. Also there will be as many aprons too big as too small, which seems like a good compromise. A small target population with no duplicate measurements for simplicity will be used to show what can happen to the sample median in different situations.

If the target population is 11 children whose waist measurements are

19", 20", 21", 22", 23", 24", 25", 26", 27", 28", 29",

then the median is 24" for the entire target population. Suppose a random sample of size 3 is taken. If the sample is

23", 26", 28",

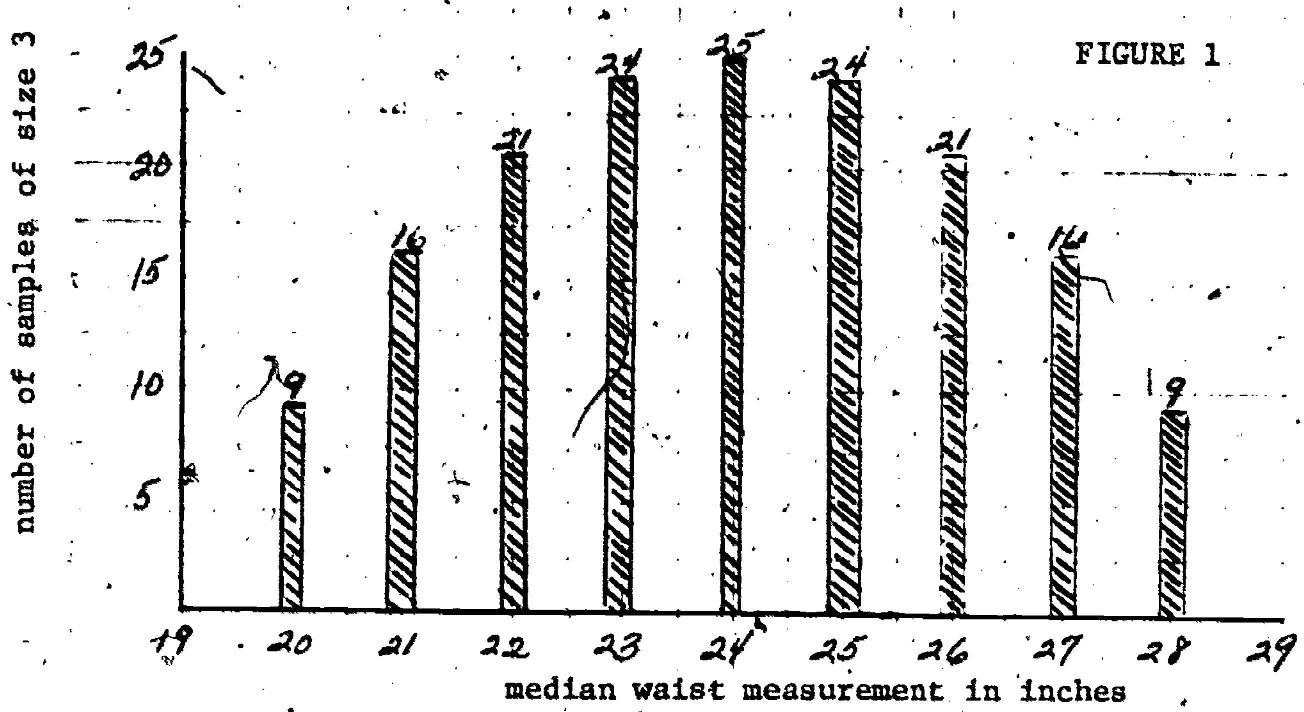
then the median of the sample is 26". If the sample is

20", 25", 27",

then the sample median is 25". This shows that the sample median, which is called an <u>estimate</u> of the population median, is clearly not necessarily equal to the population median. For the first of these samples, the difference between the sample median and the population median is 2", for the second it is 1". If either of these 2 sample medians are used to determine what the overall apron waist size should be, the aprons will fit more awkwardly children with small waist measurements since the sample median is greater than 24", the population median. Consider how often and how much the sample median differs from the population median.

There are 165 different samples with a sample size of 3 which can be drawn from a target population of size 11. (The appendix gives a description of how to determine the number of distinct samples and what they are.) Suppose every possible sample is enumerated and the sample median is determined for each. All the possible median values and how many samples have median value are displayed in Figure 1. Note that the most common value of the sample median

FIGURE 1

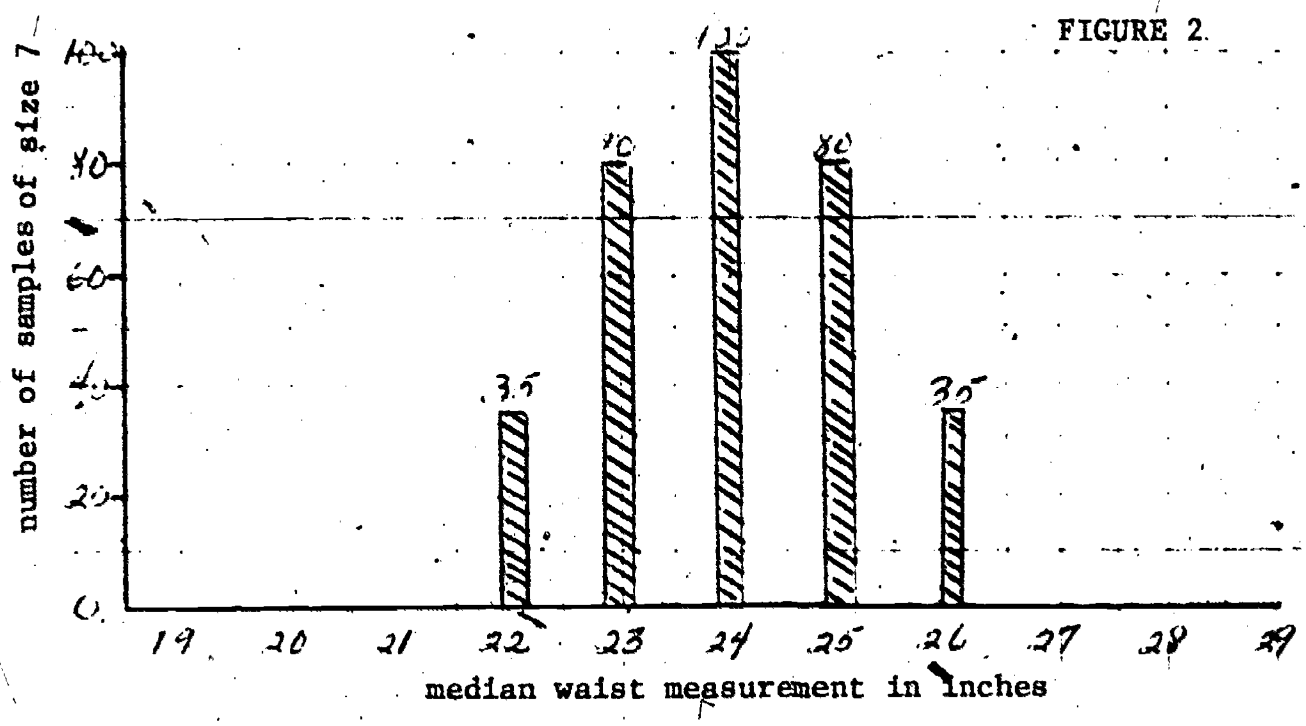


Frequency of Occurrence of Median Value
in all Possible Samples of Size 3

(where the graph peaks) is 24", the same as the population median. However, the sample median actually equals the population median only 25 times out of 165, or for fewer than 1/8 of the samples. 73 times out of 165, or less than 1/2 of the time, the sample median will be off by no more than 1"; however, the sample median can vary as much as 4" from 24". Each of these samples is equally likely to occur if the sampling has been done correctly. What this means in terms of choosing the waist size for the aprons is that less than 1/8 of the time the sample median will say to use a waist measurement of 24". How can we do better?

Two things affect how much the sample summary value, i.e. the estimate, is likely to vary from the population value, or in other words, how precise the estimate is. First is the sample size. The larger the sample size, the more precise the estimate is. That is, for large samples the sample estimate will be more likely to be closer to the population value than it would be for a smaller sample from the same population. To show this, compare the results described above for samples of size 3 to samples of size 7 from the same target population. There are 330 different samples of size 7 that can be drawn. If these are listed and the sample median for each is found, the display of these medians is Figure 2. Here again the graph peaks at 24", the value of the

FIGURE 2



Frequency of Occurrence of Median Value in all Possible Samples of Size 7

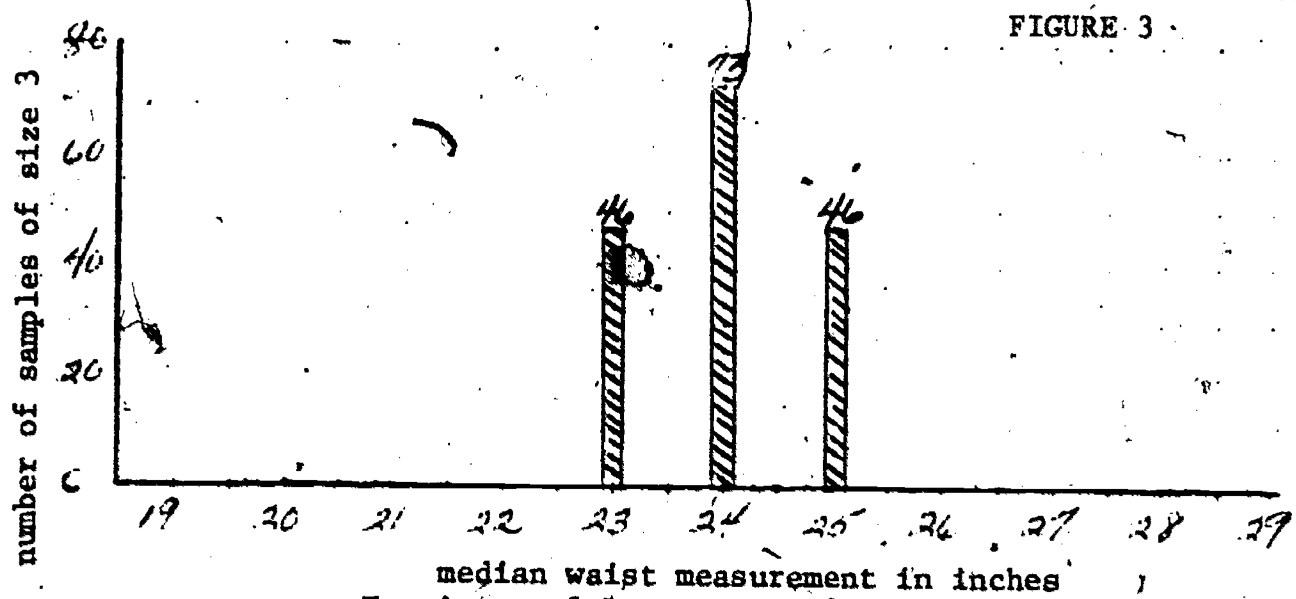
population median. But now the sample median has a value of 24" 100 times out of 330. This is about 1/3 of the possible samples, much larger than 1/8, which was the comparable fraction for all possible samples of size 3 (Figure 1). 260 of the 330 samples, or about 3/4 of the samples, have medians which vary no more than 1" from 24", the target population median. All the possible sample medians differ by no more than 2" from the population median, as compared with up to 4" for the samples of size 3. Thus the estimate is more often equal or close to the population median and does not differ from it by as much, i.e. the estimate is more precise for the samples of size 7 than for the samples of size 3.

A second effect on the precision of an estimated value from the population value is the variability of the characteristic of interest in the population. The range of the measurements in our example is 19" to 29". Consider a second population of the same size which also has a population median of 24", but a narrower range:

23", 23", 23", 23", 24", 24", 24", 25", 25", 25", 25".

Again let us consider the 165 possible samples of size 3. Now compare Figure 3, the diagram presenting all the possible medians for these samples, with Figure 1. Figure 3 demonstrates that the sample median will be 24" 73

FIGURE 3



Frequency of Occurrence of Median Value
in all Possible Samples of Size 3

times out of the 165 samples, or almost 3 times as often as is indicated by Figure 1. Figure 3 also shows that the sample medians never differ from the target population median by more than 1". This demonstrates that an estimate from a sample of a given size will be less precise for a population with larger range than for a population with smaller range. In terms of the apron example, if the children being measured have a fairly narrow range of waist sizes, then the sample median will be more likely to be close to or equal to the target population median, 24". In general if a target population is more homogeneous, that is, if the items of the population are more similar with respect to what is to be measured, the estimated median, or any other estimated value, will more likely be close to the target population value. Thus it is advisable to take a larger sample from a population which is less homogeneous with respect to what is being measured than would be needed if the population is more homogeneous.

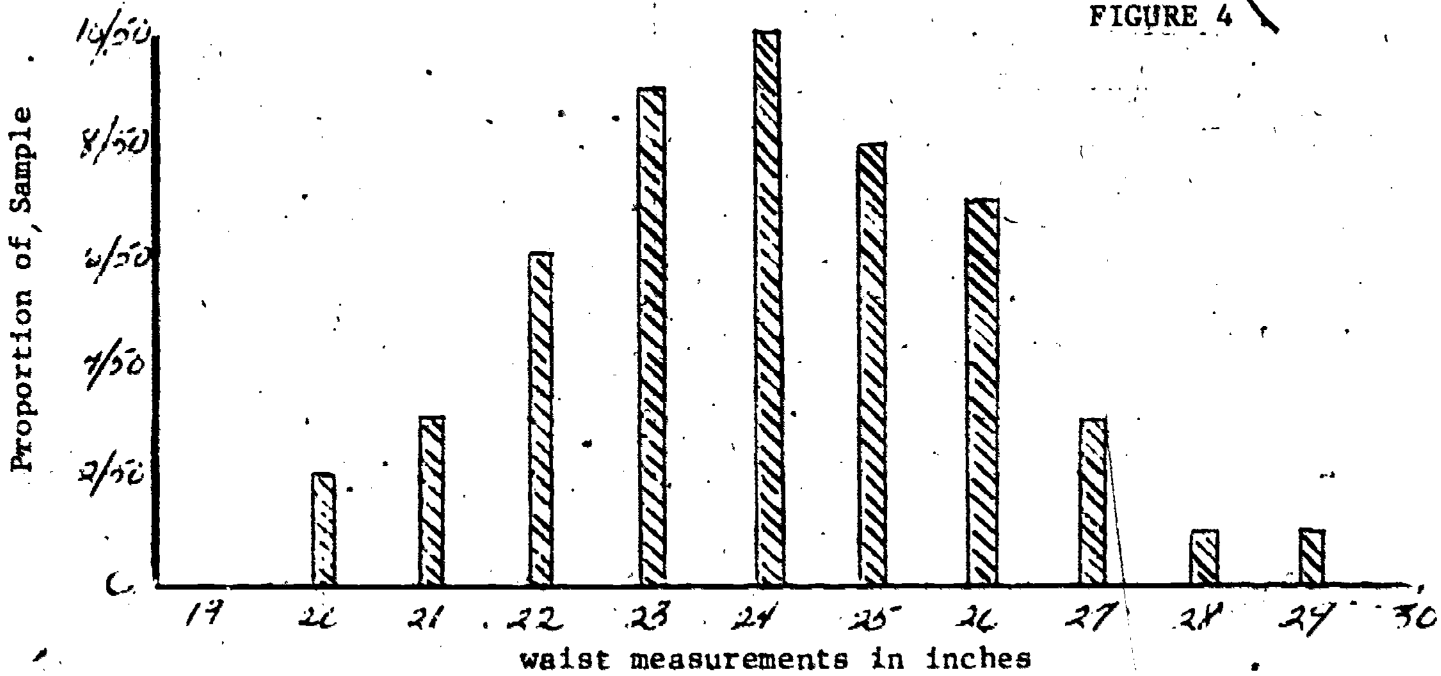
Occasionally some characteristic of the target population will make groups selected by this characteristic more homogeneous with respect to what is being measured. For example, age is probably a good indicator of size. We can take advantage of this knowledge by dividing the target population into relatively homogeneous groups and taking a separate sample from each group. That information on each group is more accurate than information from any completely random grouping is likely to be due to the decrease in variability as discussed in the last paragraph. This idea of initially grouping the target population into relatively homogeneous groups and then drawing samples from each is the basis of a procedure called stratified sampling, with the groups called the strata.

VI. Sample Design - Interpretation of the Results

How does knowledge of sample design influence the interpretation of possible results? Consider a more complicated example. If a single waist measurement will not be sufficient for all apron sizes, then one number such as the median will not be an appropriate summary concerning waist measurements. Now the main objective requires deciding how many, or what proportion, of the potential buyers have waist measurements of 22", 23", 24", etc.* These proportions might be grouped later when the different sizes are defined. Now if 1/3 of the children sampled had waist measurement 24", it would be desirable that about 1/3 of the aprons made fit a child with waist 24".

For this example suppose the target population is a school with an enrollment of 250 children, a more realistic situation. A random or systematic sample of 50 children is selected and the 3 measurements indicated in section I are taken. If 2 of the 50 children have a 20" waist, the sample proportion of children with 20" waists is 2/50 or 1/25. If 10 children have waist 24", that sample proportion is 10/50 or 1/5. If no children have waist 30", that sample proportion is 0. In the same manner all the proportions are determined. These proportions can be depicted as in the bar graph in Figure 4.

FIGURE 4



Proportions Corresponding to Possible Waist Measurements
for a Sample Size 50

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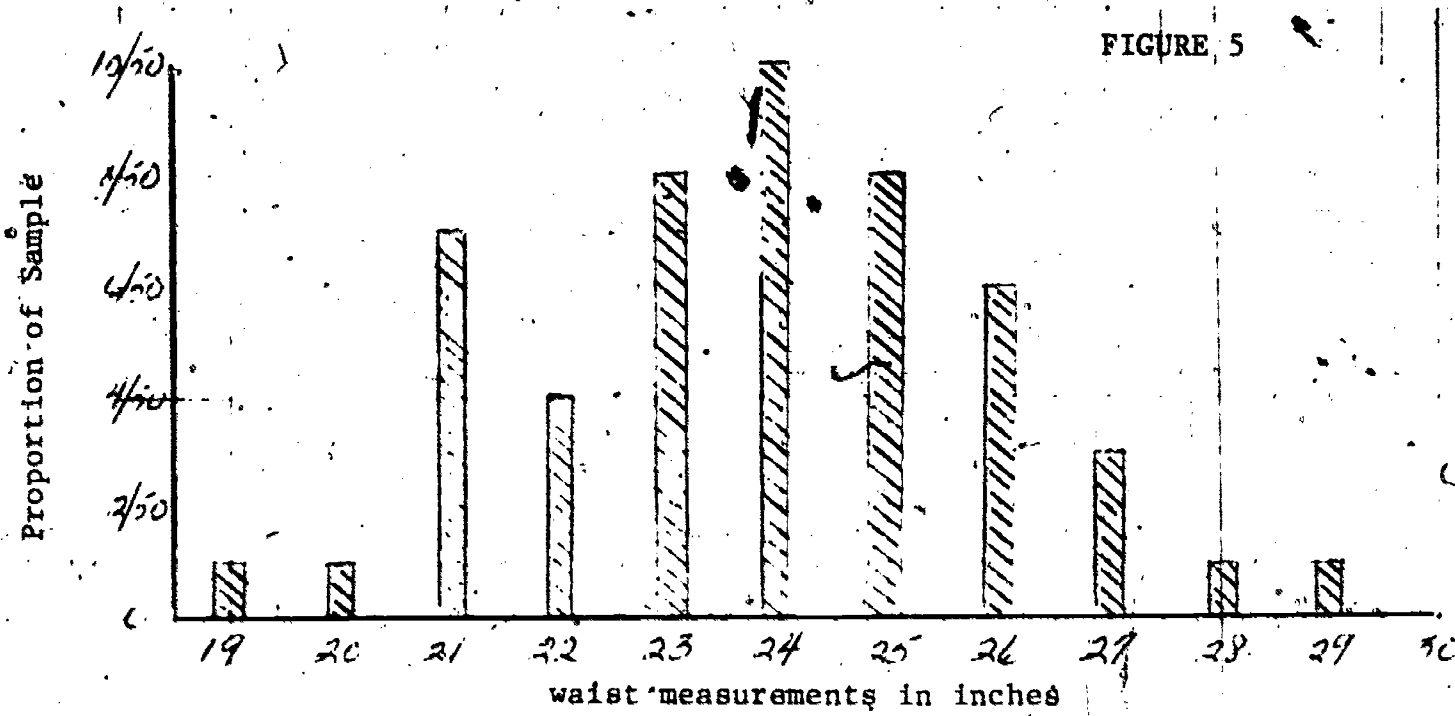
* If the class recorded measurements in fractions of inches, i.e. 23 1/2" or 21 1/4", we will consider all measurements rounded to the nearest integer for this example. Then the 21" proportion refers to waists from 20 1/2" up to but not including 21 1/2". Either 20 1/2 or 21 1/2 could have been included in the definition of 21 as long as what is done is done systematically for each integer definition.

Just as for the median, the sample proportions are estimates of the target population proportions. That is, though the sample proportion of 30" is 0, perhaps the population proportion is 1/100, 1/250, or 0. Since the population proportions are not known, one can only try to assure that the sample is providing a fairly precise estimate of a summary of the target population by considering the way the sample was taken, the sample size, and the homogeneity of the target population.

Consider how these reasons why samples do not exactly reflect the target population affect what we think of the sample survey data. Using the situation just described, suppose the sample proportions of waist measurements are as displayed in Figure 5, rather than as in Figure 4. Where Figure 4 has a peak

FIGURE 5



Proportions Corresponding to Possible Waist Measurements for a Different Sample of Size 50

at 24" with smooth decrease in proportions as the waist gets greater than or less than 24", Figure 5 shows a smaller peak at 21" in addition to a peak at 24". It could be that the target population is correctly reflected in this pattern of 2 peaks. However, the peak at 21" might also be because of sample design or sample variability. Recall the systematic sampling example where a sample consisting of the five shortest children in the class was drawn. If only 1 or 2 teachers had their seating arrangement so the smallest children (these children are likely to be thinner) were in the front, and a systematic sample including all children in the front row was drawn in all classrooms, the resulting sample proportions could be displayed as in Figure 5. Even if the sampling procedure gives a fair sample, there is a chance that the bar graph may have an unexpected peak simply because the estimate (i.e. the sample proportion) is not exactly equal to the actual proportion in the target population.

To summarize, if Figure 5 occurred as a sample survey summary and intuitively something like Figure 4 was expected, 3 possible explanations exist:

1. The intuition is wrong; the population does have a peak at 21".
2. The sampling procedure has given an unfair sample with too many thin children.
3. The peak at 21" is only due to the variability of the estimate.

In studying the sample survey results, there is no cook book procedure that can be used to determine which of the 3 is the case. Using the summary as evidence, it is advisable to review the sampling procedure to see if the conclusions could be uncovering hidden sampling procedure errors. If no errors can be found, explanation 2 is unlikely. Next the reasons for the variability of the estimates should be considered. Is the sample size sufficiently large (considering the homogeneity of the target population) to be confident that the estimate should be fairly precise? Remember that the population proportions are not known and one can only try to be sure that the summary results are not due to errors in the sampling procedure or too small a sample and then use the results as if explanation 1 is correct.

VI. General Remarks

We have discussed how the natural approach of a class to say "Let's ask some people" should be guided into considering the problems of taking a sample survey step by step. In designing the survey it is always a good idea to have them state as precisely as possible, either verbally or in writing, each of the three steps "what to measure," "how to measure it," and "whom to measure" (what is the target population). In trying to do this, they will become more aware of the pitfalls of different approaches and the consequences of different decisions at each of the three steps.* It may be necessary to go back and rethink an earlier step if they decide at a later point that what they have chosen to do will be too difficult to do sensibly or even that it cannot be done at all.

In sampling, we have tried to show the importance of taking a sample which truly represents the target population, using some of the techniques of random and systematic sampling. We have also tried to give a feeling for why sample summary values may not be equal to the target population values and how much they may differ. Here again, the class should decide exactly how they will take their sample, and then consider if the procedure will really give them a

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* Editor's Note: Trials of suggested procedures in the class often help the children refine their judgments on these 3 factors.

fair sample. And even after the sample is taken and they are looking at the results, they should be aware of the reasons why these results will not be exactly the same as if the whole target population had been surveyed.

APPENDIX

In this paper it is stated that there are 165 ways of choosing 3 items from a group of 11 items. This number is found in the following way:

$$\frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$$

Similarly there are 330 ways to choose 7 out of a group of 11:

$$\frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 330$$

Suppose there are only seven items in the population, denoted by

A B C D E F G

There are 35 possible ways of selecting a sample of size 3:

$$\frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35$$

All 35 samples can be systematically written down columnwise in the following way:

ABC	ADF	BDF	CFG
ABD	ADG	BDG	DEF
ABE	AEF	BEF	DEG
ABF	AEG	BEG	DFG
ABG	AFG	BFG	EFG
ACD	BCD	CDE	
ACE	BCE	CDF	
ACF	BCF	CDG	
ACG	BCG	CEF	
ADE	BDE	CEG	

Examining the changes down the columns shows how the samples are generated. Thinking of the items as being ordered from left to right, start by simply choosing the first 3 items, A B C, as the first sample. (Remember that these are not random samples but simply listings of all possible samples.) Next replace the C with D, E, F, and then G to get the next 4 samples. Now all samples containing both A and B have been listed. Next replace B with C and combine these two items with those left (D, E, F, G) to get the next 4 samples. Now replace C with D and combine A and D with E, F, and G successively to get the next 3 samples, and so on. When all samples containing A have been generated, start with B as the first item in the sample and continue in exactly the same way. All samples of any size can be generated following this procedure.

EXAMINING ONE AND TWO SETS OF DATA

PART I: A GENERAL STRATEGY AND ONE-SAMPLE METHODS

by

Lorraine Denby and James M. Landwehr

Bell Telephone Laboratories
Murray Hill, New Jersey 07974

A class of students decided to study the lifetimes of two brands of flashlight batteries. They performed an experiment yielding several measured lifetimes from Brand A batteries and also from Brand B batteries. Another class was interested in two different methods of teaching a unit on fractions. They devised an experiment in which some students were taught by the first method, other students were taught by the second method, and a measure of improvement was recorded for each student in the two groups. In still another class students were concerned with traffic safety in the streets near the school. They conducted an experiment in which the speeds of cars were measured when a crossing guard was, and was not, present.

Many aspects of these three experiments are quite different from one another, but for each experiment one is interested in using the two sets of measurements to help answer certain questions. If one must choose one brand of flashlight batteries, some relevant questions might be the following. Which brand has the longer average lifetime? Is there smaller variability in the lifetimes for one brand or the other? Does one brand have fewer extremely small lifetimes, or more extremely long lifetimes?

What are the relative costs of the batteries? Concerning the teaching example, one might like to know if there is evidence that one method of teaching really gives better results, in some sense, than the other method. Similarly, does the presence of a guard reduce the speeds of the cars, and if so, by how much? Do the speeds of all cars seem to be reduced, or only the speeds of cars driving very fast?

Appropriate questions depend on the particular application. Sometimes the two sets of measurements are so different that the answers are extremely obvious, but sometimes a variety of statistical methods are useful and necessary to interpret the data. This paper develops and gives examples of several techniques for examining one set of measurements and for comparing two sets of measurements. This paper is written in modular form, and different parts can be read separately from one another.

The first section of Part I gives some general comments about an over-all strategy for analyzing data from applications such as those mentioned above. It discusses certain general considerations to keep in mind concerning the analysis, and it also discusses the order in which the statistical methods of presented here and in Parts II and III might be used. The general philosophy is to do the simpler techniques first and proceed to more complicated methods only if the simpler ones do not give clear and sufficient answers from the data. This section is not explicitly required in order to read any of the rest of the paper.

The second section of Part I is a discussion of numerical and graphical methods for examining one set of measurements.

Included are the median, mode, mean, histogram, cumulative distribution function, and measures of variability.

Part II develops a graphical method, known as the q-q plot, for presenting the values from two sets of measurements. The plot is easy to interpret, no special graph paper is required, and construction of the plot requires about the same effort as construction of the one-sample plots discussed below. The one sample material of Part I is required for understanding this part.

Finally, Part III gives a way of seeing if two sets of measurements are significantly different from one another, or whether it may be the case that the two sets of data are essentially the same, apart from random fluctuation. This method is called a permutation test and requires no special mathematical tables. It is somewhat more complicated than the preceding material, but only the definitions of median and mean from Part I are explicitly required for this part.

I. A STRATEGY FOR EXAMINING AND ANALYZING DATA

This section discusses some general points that should be kept in mind when analyzing data. If one has one set of measurements or two related sets of measurements to compare, then one or more of the statistical techniques discussed in this paper may be useful. The order in which these techniques could be used and their relative places in the analysis are discussed here. Situations are described where it is sufficient to use only the simpler techniques, and the more complicated methods are not needed. This section could be used as a guide, or check-list, as one proceeds through the examination of a set of data. However, none of the material in this section is explicitly required to understand any of the techniques presented in the later parts of this paper.

I.1.4 <u>How were the measurements obtained</u>?

Before attempting any analysis of the data, it is important to understand exactly what was measured, how the measurements were obtained, who did the measuring, and perhaps when and where the measurements were performed. For example, if the data are speeds of cars, were they obtained by a sophisticated radar device operated by a policeman, or by stopping the cars and asking the driver how fast he was going, or by children measuring the time (with a stopwatch or wristwatch?) it took the cars to travel a previously measured distance? Answers to questions such as these might indicate obvious, inherent problems and biases in the data; thus these answers could indicate how much effort, if any, is worth putting into analyzing the numbers obtained. A useful rule-of-thumb is that one should be able to clearly and completely explain the process used to produce a typical number in the data before beginning the actual analysis of the values.

This whole area of experimental design -- or exactly how the data were obtained -- can be of crucial importance in determining whether or not the data shed any light on certain questions. Careful design and conscientious execution is needed to generate useful data. It is hard to make broad generalizations concerning good and bad design, but each of the data examples discussed in later sections is preceded by a discussion of the design of the experiment. These discussions illustrate the type of points that should be kept in mind.

I.2. Why are we interested in the data?

It is useful to think of some interesting questions before starting to look at the data. What do we want to learn about? The questions may be fairly obvious, but it is good to have them specifically in mind. For example, if we measured the speeds of cars under two different conditions, we might ask some of the following. Are the speeds about the same under the two conditions? If the cars travel faster in one situation, by how much are they faster? Are they uniformly faster, or is there an overlap in the speeds? Are there some cars that go extremely fast or slow under one or the other condition?

Interesting questions can range from very specific to more general. Moreover, one will often think of additional interesting questions after the analysis has begun; this should be encouraged, and sometimes these questions turn out to be the most useful of all. However, it is desirable to have some questions in mind at the beginning, and these questions can serve as a focus for the analysis. Some of the statistical methods of this paper are directed at specific questions, while others are directed at more general questions.

I.3. Are there any particularly striking features of the data?

Generally the first thing to do with the actual numerical values is to simply look through a listing of them. Are there any values that look so strange and unreasonable, compared with the rest of the values, that it is likely that some errors in recording were made? For example, if most of the car speeds are from 10 to 35 mph, but one is 120 and another is 3.5, one might suspect that an error in recording the decimal point had been made in each of these cases. If there are such outlying values, the work leading to them should be checked in detail to see if errors were made. Sometimes an error will be found which can be corrected, and sometimes it will be clear that there was an error made, but it cannot be corrected. Then this value must be discarded from further consideration. However, sometimes no error will be detected, and then the question remains as to how this value should be treated in the rest of the analysis. No simple general answer is possible, but it is important to realize that there is a possible spurious value (or several values), and to think about the effect this may have on any of the subsequent analyses.

In eyeballing the data one should also keep in mind the questions that are appropriate for these data, as discussed in section I.2 above. Just looking at the numerical values may give easy, obvious and complete answers to all the questions of interest. If so, then there is no need to proceed with more complicated numerical or graphical methods. For most problems, though, at least one numerical and/or graphical technique will be of value beyond an eyeball examination. However, a look at the numerical values is generally simpler and should be done first; if this gives clear and sufficient answers, nothing further is required for these data.

I.4. Do certain one-sample summaries or displays adequately describe the data?

Often an interesting part of the data, or the entire data, consists of a set of measurements under one condition. For example, one may wish to know the distribution of the speeds of cars measured at a certain intersection. In another class, one may wish to know the improvements of students' scores measured under one method of teaching. Summarization of the values by one or more one-sample statistics is usually worthwhile. Several one-sample summary statistics -- such as mean, median, and inter-quartile range -- are discussed below; different statistics are appropriate depending on the sort of questions that are being asked of the data. The discussion points out the type of question that each statistic is directed towards and gives some informal comparison of several statistics that are directed at similar goals. For some problems these simple, one-sample summary statistics are entirely adequate for the questions and purposes at hand, and no further analysis is required.

Several simple graphical methods are available for displaying the values in a set of data. Two methods useful in a variety of situations, the histogram and the empirical cumulative distribution plot, are presented below. For most sets of data these require slightly more effort to construct then do the summary statistics discussed here, but graphical methods often lead to a better understanding of the entire set of values than would be obtained with just summary statistics. Graphical methods often expose some interesting aspect of the values that could be missed if only summary statistics were calculated. Thus, graphical methods are more likely to suggest new questions concerning the data. Generally, it is useful to construct at least one graph from a set of values. The amount of effort required is small compared to the effort required to get a good set of measurements and a good set of summary statistics, and the potential benefit is large.

I.5. <u>How can two sets of measurements be compared?</u>

In many experiments a central purpose is to compare values obtained under two different, yet related, conditions. For example, car speeds could be measured with a policeman present and without a policeman. The performance of children could be measured under two different methods of teaching. The first thing one can do is to summarize each of the two sets of values using some appropriate statistics. That is, separately treat each set, summarizing it as discussed in section 1.4 above. This is sometimes adequate to show that there is clearly a huge and important difference between the measurements obtained under the two conditions, or alternatively that there is an extremely small and negligible difference between the measurements obtained under the two conditions. If answers to the important questions are clear and obvious, more complicated analyses are not required.

Just as in looking at one set of data, graphical methods can be extremely useful in examining and comparing two related sets of measurements. One possibility is to make one-sample graphs, such as histograms, separately for each sample.

However, this does not directly bring out similarities and differences between the two samples in one plot and can lead to faulty comparisons if not drawn carefully. Part II presents a graphical method, called the q-q plot, that is specifically designed for comparing two sets of data. It often shows interesting aspects of the data that could be missed if only numerical summaries or one-sample graphical methods were used. Moreover, it is generally easier to construct a q-q plot than to construct two histograms. Just as in the examination of one set of measurements, when comparing two sets of values it is generally useful to form at least one graph -- preferrably a q-q plot -- from the data. Again, the amount of effort required is small compared to the effort required to get a good set of measurements, and the potential benefit is large.

I.6. <u>How sure can we be that an apparent difference between two sets of measurements is real, and not just due to random fluctuations?</u>

Sometimes initial examination of two sets of data reveals some apparent differences, but not overwhelming, clearly important differences. One wants to know if the apparent differences are real, in some sense. One way of thinking about this question is to ask if a replication of the entire experiment would yield results giving about the same important differences between the two sets of measurements as was observed in the original data. If the answer is yes, then it seems safe to treat the difference as real. However, if the answer is no, then one should realize that this difference might only be due to random fluctuation, rather than to real differences corresponding to the two conditions under which the measurements were obtained.

Often the best way to answer this question is to re-do the experiment and see how the results compare. In any experiment there are many factors that could influence the results.

For example, suppose a class wants to determine which of two methods of teaching a unit on fractions gives better results. An experiment is done whereby different children from the same class study using each method. Some factors that might influence the results are the following: the material prepared by the children; the time of day in which the material was presented; the sex, age, and educational ability of the children in the two groups; and the motivations of the children, etc. One might argue that some of these factors should not make much difference, but the only way to really tell is to replicate the experiment (perhaps more than once). If the entire experiment is done again independently by different groups within the class or by groups in another class, and if the differences between the two methods of teaching turn out about the same as they were originally, then one would feel more confident that the results really should be believed (for children of these backgrounds) than if the replication had not been performed. If the results turn out to be drastically different, then one would like to know why they were different, and further investigation might be required. By re-doing the experiment, we allow not only for changes in the results due to "internal variability," such as the varying performance of the students in this particular class over time, but also for changes in results due to external factors, such as those listed above. Sometimes differences in results due to changing these "external factors" are larger than initially suspected; replicating the entire experiment is a good way to get some idea of their magnitude.

Replication of an experiment can involve a lot of effort, time, and money. Sometimes, before doing this, it is worthwhile to ask a more specific question of the data. Considering only the variability that appears within the two sets of data that we have available, could this observed difference be due merely to chance? That is, considering only the "internal variability," could random fluctuation alone have led to the observed difference between the two sets? If the answer is yes, we might be less enthusiastic about replicating the experiment than if the answer were no. A statistical technique that gives an answer to this question is presented in Part III of this paper. Some additional comments on replicating the experiment also appear in Part III. The statistical method given there is often useful as a first step in trying to decide if the apparent differences between the two sets of data are real and important. Even if the answer is yes, students may want to replicate the experiment to compare the actual results with the indication of this statistical method.

I.7. Summary

The general philosophy of this section is that one should first understand what the data actually measure, then have some important questions in mind, and then proceed from simpler to more complex analyses. If the simple analyses give clear and resounding answers to the questions, then there is no need to use more complicated techniques. However, there are situations where the more complicated techniques do indeed give additional, useful information.

II. METHODS FOR EXAMINING ONE SET OF MEASUREMENTS

In the consumer research unit, classes experiment to learn characteristics and advantages of various brands of products. One class chose to investigate the lifetime of flashlight batteries. After some discussion, the students decided to limit themselves to just two brands, brand A and Brand B, in their initial study and to design an experiment to evaluate the lifetimes of new batteries from these brands. Instead of just using one battery from each brand, putting them into two flashlights that would be turned on simultaneously and then seeing which one burned out first, they decided that they would obtain a more precise answer if they checked several batteries from each brand. Experimenting with several batteries from each brand allowed them to obtain a better idea of the variability of each brand of batteries chosen. Since there were 22 students in the class, they decided to test 11 batteries from each brand -- a total of 22 batteries. They decided to each bring a flashlight from home, since then they could test all the batteries at the same time. This also allowed the batteries to be compared in "typical" household flashlights, rather than new flashlights. Some batteries might perform relatively better in new flashlights while other batteries might perform relatively better in typical flashlights. It seems more useful to compare batteries in typical flashlights rather than new flashlights.

Another aspect of the experiment involved obtaining 11 sets of new batteries for each sample. One factor that might affect the lifetime of a battery is how long the battery has been on the shelf or in the stockroom of the store. Buying all the Brand A batteries from the same store might yield batteries that had been shipped together from the manufacturer and were all unusually fresh or unusually old. They might thus give results that would not be typical of all batteries of this brand that are actually for sale in the community. Thus the class decided to purchase the batteries separately from a variety of stores in the area. The new batteries, 11 sets each of Brand A and Brand B, were assigned haphazardly to the 22 flashlights.

It was necessary to decide precisely when the batteries wore out, so the following system was devised. The 22 flashlights were lined up on a table in a dark closet shining on white cardboard four feet away from each light. Over time the lights got dimmer and dimmer, so it was decided that a flashlight would be defined as out when it gave no visible spot on the cardboard in the dark closet. When a flashlight did go off the bulb was checked to make sure that the batteries had worn out and not the bulb. The flashlights were arranged haphazardly so that the observers would not know if a particular light contained Brand A or Brand B batteries until the light went out; then the flashlight was opened and the time and brand of batteries recorded. The lights were turned on in the morning and off at the end of the school day; during the day the students alternated checking them every 6 minutes, recording and removing lights that had gone out. Thus the lifetimes of the batteries were recorded to the nearest tenth of an hour. The data obtained from this experiment are given below in hours:

Brand A	Brand B
9.5	6.8
.5	4.2
12.8	8.1
7.2	11.4
9.3	5.1
3.1	10.0
15.2	6.3
11.3	11.9
5.0	11.1
7.8	4.6
5.9	8.9

An eventual goal of this experiment was to make some useful statements concerning the relative lifetimes of the two brands of batteries. However, first it is reasonable to consider ways of summarizing and presenting the values separately for each brand. Before explicitly considering ways of comparing two sets of data as presented in Parts II and III, we will first discuss several ways of examining a single set of values. The values obtained for each of Brand A and Brand B will be used to illustrate the one-sample methods discussed below. One simple way of comparing two sets of measurements is to separately summarize each set using appropriate one-sample techniques. Then the summarizations for the two sets of values can be informally compared to each other. This is often a useful first step in comparing two sets of measurements, and for some sets of data it gives clear, sufficient answers to the questions of interest. However, for many sets of data the methods presented in Parts II and III give valuable additional information that might be missed if only one-sample methods were used for comparison; moreover, the method of Part II is particularly useful for clearly displaying the relationships between the values in the two sets of measurements, even if these relationships might be discovered from just using one-sample methods.

We can see from the numbers obtained from this experiment (and probably from the other experiments that you have previously performed) that the values obtained in a sample are not all exactly equal to the same value. There is a distribution of values. One can summarize the distribution of a set of data by calculating a number of summary statistics from the values. In this example a basic question in which the students were interested was whether there is a general, over-all difference between the lifetimes of Brand A batteries and Brand B batteries. A number of characteristics about the distribution of each brand's sample will affect one's thoughts about the question.

First we consider ways of indicating the center of the sample. That is, we want some measure of the location of the set of data -- an indication of what is a "typical" value, in some sense, for the sample. The arithmetic average of all the values, the mean, is one measure of the location of a sample. The mean of a sample is calculated by summing the values of each observation in the sample and then dividing the total by the number of observations in the sample. Totalling the observations for Brand A we obtain 72.6. Now we divide by 11 and obtain 6.6 as the mean of this sample. The mean of brand B batteries is 8.3.

The mean is often calculated and used as a measure of the center of the data, but it has certain drawbacks. Suppose the first observation instead of being 9.5 was 59.5. Then the mean of the sample would be 11.1, which differs greatly from the previous value of 6.6. If one did not look closely at the numbers then he might think that a Brand A battery generally lasts 5 hours longer than a Brand B battery, when actually the average is that high mainly due to the value of one battery. The value for this battery could be so high possibly because of a mistake in recording or possibly because a very small portion of batteries really do last 5 to 10 times longer than most batteries. So, it does not seem reasonable to just rely on the mean to summarize where the center of the data lies. Although the arithmetic mean is often useful, it does not always give a value that reflects the location of the bulk of the data. As we have seen, it can be greatly affected by one observation that happens to be much larger or smaller than all the others.

The median is another measure of location, and it does not suffer from this drawback. The median is the value of the middle point in a sample. It is obtained in the following way.

First, the sample values are ordered from smallest to largest. If the number of values in the sample is odd, then the median is defined as the middle value in this list. If the number of sample values is even, so that there is no single middle value, then the median is defined to be the average of the two numbers in the middle of the list. Thus, in either case the number of observations below the median is the same as the number of observations above the median. Our two samples reorder in the following way:

order number	1	2	3	4	5	6	7	8	9	10	11
Brand A	.5	3.1	5.0	5.9	7.2	7.8	9.3	9.5	11.3	12.8	15.2
Brand B	4.2	4.6	5.1	6.3	6.8	8.1	8.9	10.0	11.1	11.4	11.9

Since there are 11 observations in each sample the 6^{th} largest point is the median, i.e., the median of Brand A is 7.8 and of Brand B is 8.1. Calculating the median does not involve adding long columns of numbers; the median is easier and faster to calculate than the mean. Moreover, there is less chance of making an arithmetic slip in computing the median. Also note that the median remains unchanged if the first Brand A observation is 9.5 or 59.5; the median is unaffected by a few outlying values.

Another measure of location is the mode, defined as the most frequently occurring value in the sample. In each of the Brand A and Brand B lifetimes no value occurs more than once, so the modes are not even defined here. The mode is generally less useful than the median, especially if there is only a moderate number of measurements.

An important feature of the data is the variability of the values in the sample. A measure of variability that can be easily calculated from the ordered sample values is the <u>interquartile range</u>. This is defined as the difference of the point that is greater than 25% of the data from the point that is greater than 75% of the data. Thus it is the length of the interval that includes the middle 50% of the data, excluding 25% on each end. To calculate this we must explicitly find that value that is greater than 75% of the sample, and less than 25% of the sample. This is called the third quartile. If $\frac{3}{4}$ of the number of observations is an integer then the third quartile is the average of the value of the observation with that order number and the value of the next larger observation. If 3/4 of the number of observations is a fraction then the fraction is rounded up to the next whole number larger than it and the value of the third quartile is the observation in the sample whose order number is that whole number. The first quartile (the observation larger than 25% or 1/4 of the sample) is obtained by similar rules applied to the fraction or whole number which is 1/4 of the number of observations. (The second quartile is the median, which was discussed above.) Our measure of variability is obtained by subtracting the first quartile from the third quartile.

Now let's calculate the interquartile range for each of our samples. We have 11 points in each sample so 3/4 of 11 is $\frac{33}{4} = 8\ 1/4$. Therefore, we round 8 1/4 up to 9 and the third quartile is the 9^{th} largest observation (i.e., the observation whose order number is 9). In calculating the first quartile we see that $\frac{1}{4}$ of 11 is $\frac{11}{4}$ = 2 3/4 which is rounded up to 3. So, the first quartile is the third largest observation. For the first sample, the value of the first quartile, the third largest observation, is 5.0 and the third quartile, the ninth largest observation, is 11.3. So, the interquartile range of Brand A is 11.3 - 5.0 = 6.3. For Brand B batteries the interquartile range is 11.1 - 5.1 = 6.0. The interquartile ranges in the two samples are about equal, meaning that there is about the same amount of variability in each of the two samples. Consideration of the interquartile range suggests another statistic that could be calculated or a measure of the location of the sample; this is discussed in Appendix A.

Now let's consider using the above summary statistics, particularly the median and the interquartile range, to compare our two sets of data. Suppose we are interested in whether or not the lifetime as a "typical" Brand A battery is different enough from the lifetime of a "typical" Brand B battery to be concerned about. The median gives one measure of a "typical" value, and we found above that the medians here are 7.8 and 8.1. However, to decide how we should feel about these values we must also consider the variability in each sample.

If each of the two samples had miniscule variability -- such as interquartile ranges of 0.01 and 0.02 -- then we might feel pretty certain that there is a true difference in the locations of the two sets of data. Even so, this difference of 7.8 vs. 8.1 might be small enough so as not to be of any practical interest. If each of the two samples had much larger variability, though, then we might be much less certain about an apparent difference between the locations in the two samples.

Here the interquartile ranges in the two samples -- 6.3 and 6.0 -- are close to each other. These values are large compared to the differences between the two means -- 6.6 and 8.3 -- and also large compared to the differences between the two medians -- 7.8 and 8.1. The Brand B median falls within the interval containing the middle 50% of the Brand A data, and the Brand A median falls within the interval containing the middle 50% of the Brand B data. There is substantial overlap of the two distributions, as could also be seen from examining the raw values. It seems that, even if there is some general, over-all difference between the two samples, this difference is not too large compared to the internal variability in each of the samples.

On certain sets of data the means or medians and the interquartile ranges of the two samples are similar, leading us to feel that the two sets of data are basically the same. However, more detailed examination sometimes discloses interesting features. Suppose the ordered values obtained for the Brand A batteries were

.1 .4 7.3 9.0 11.0 14.2 16.8 17.1 18.0 30. 40.

and for Brand B they were

3.1 4.0 7.3 9.0 11.0 14.2 16.8 17.1 18.0 21 25

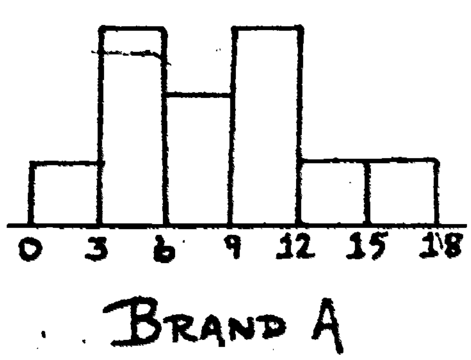
Inspection of the two samples shows that the first and third quartiles are equal; moreover, all the middle values are equal. The medians, interquartile ranges, and mid-means are equal in the two samples. If we were to just stop after comparing these statistics we would probably feel that both brands of batteries had the same distribution of lifetimes, so it doesn't matter which of the two brands is used. However, inspection of the smallest values of Brand A shows that this brand has a possibility of failing almost immediately 2/11 of the time, whereas Brand B lasts a minimum of 3 or 4 hours. So, for example, if we were purchasing batteries to take on a camping trip we might prefer to buy Brand B batteries, believing that they will last at least 3 hours. However, inspection of the 2 largest points of sample A shows that they are much larger than the two largest of sample B. So, for example, if we were buying batteries to use around our house where we can have a supply of them on hand or where it is easy to buy more when they go out, then we might prefer Brand A, hoping to occasionally get a battery that will last substantially longer than any of those from Brand B.

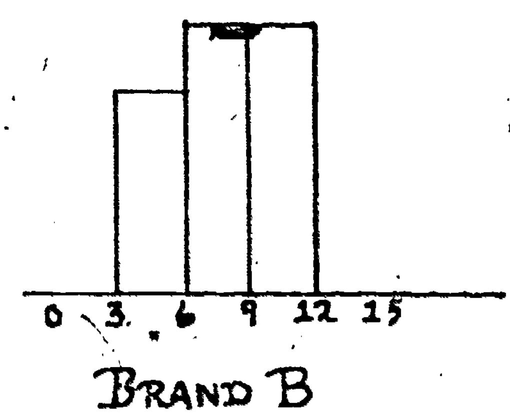
This is an extreme example. Other configurations of the relationship between the small and large points in both samples can occur, and it would be possible to speculate on how we would feel about such configurations. Sometimes, for certain purposes, we really wouldn't care about the relationship of the small and large values in the two samples. However, if we might be interested in the smallest and largest points (especially when there are many observations and it is difficult to just eyeball the list of ordered observations to learn what is going on) then other techniques are often useful in examining the distributions of the two samples.

The preceding discussion has focussed on several numerical statistics for summarizing a set of data. However, it is often useful to simply present the data using one or more graphical methods. Interesting, special features of the data are generally more readily discovered through graphical displays than through numerical summaries. Even when there is nothing "unusual" about a set of data a graph is a useful way of looking at the data and of transmitting the information in the data to others.

A widely used plot for displaying one sample of values is the <u>histogram</u>, or bar graph. This plot has the possible observed values on the horizontal axis; the vertical height of the bar that is drawn above each value indicates the number of times this value has appeared in the sample. Many times the horizontal axis is divided into intervals so that the height of the bar indicates the number of observations falling in each interval. A possible way of comparing two sets of data is to draw the histogram from each sample and then visually compare the two histograms. Figure 1 gives the sample histograms for Brand A and Brand B batteries.

FIGURE 1





One thing we notice is that the shortest and longest lifetimes of Brand A batteries were more extreme than the corresponding lifetimes from Brand B. Each histogram is basically centered about the median for that sample, and the spread of most of the data looks about the same in the two histograms. This agrees with the interquartile ranges being close to one another. The histograms, though, give the additional information that the extreme values appear to differ between the two samples.

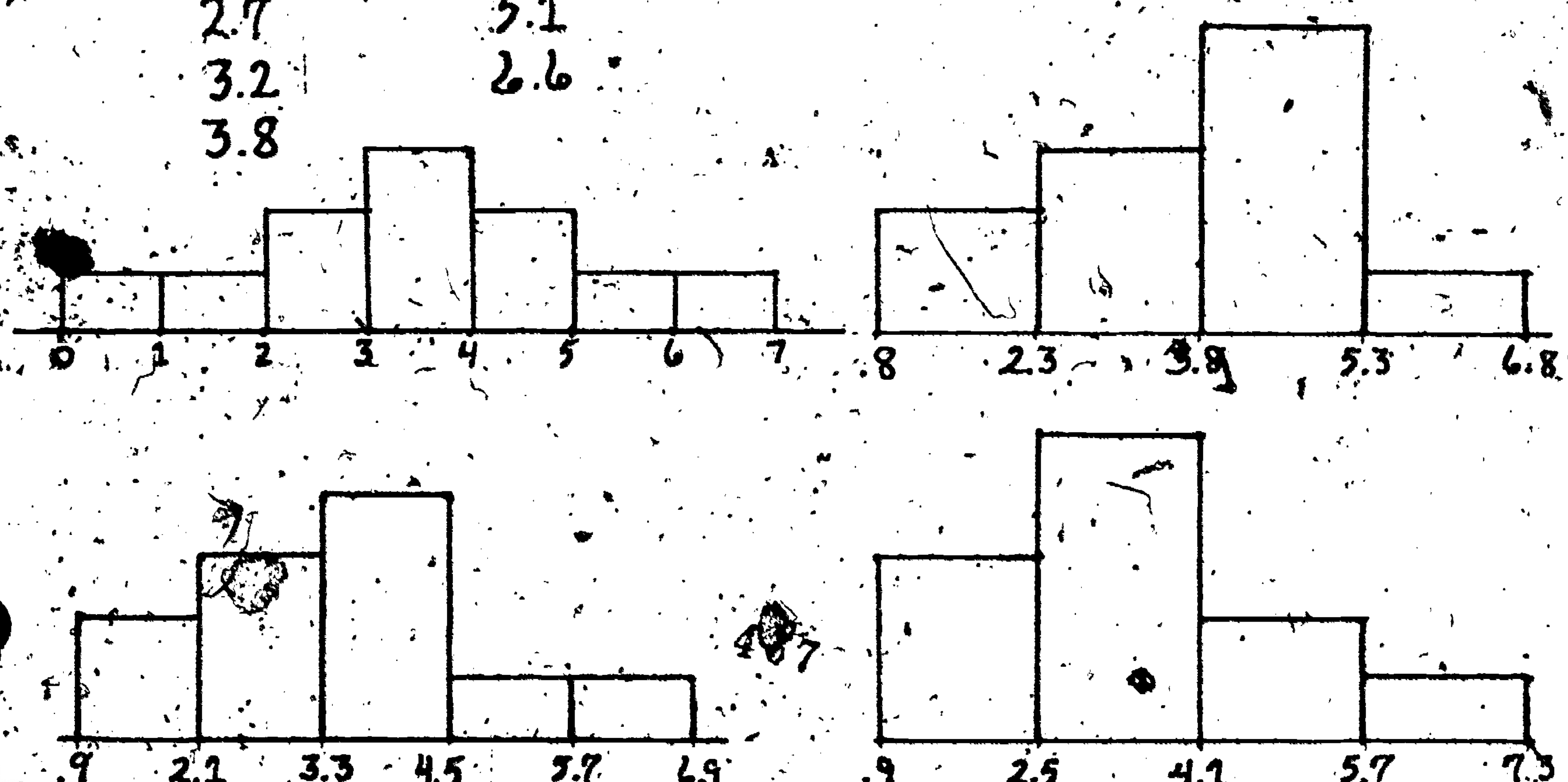
Construction of a histogram involves an inherent problem. Class intervals along the horizontal axis must be chosen, and depending on where one chooses the class intervals the histogram can look very different. In Figure 2 we list a sample of 11 numbers and then create 4 histograms using these numbers.

FIGURE 2

DATA:

.9
1.2
2.4
2.7
3.2
3.8
3.9
4.0
4.2
5.1
6.6

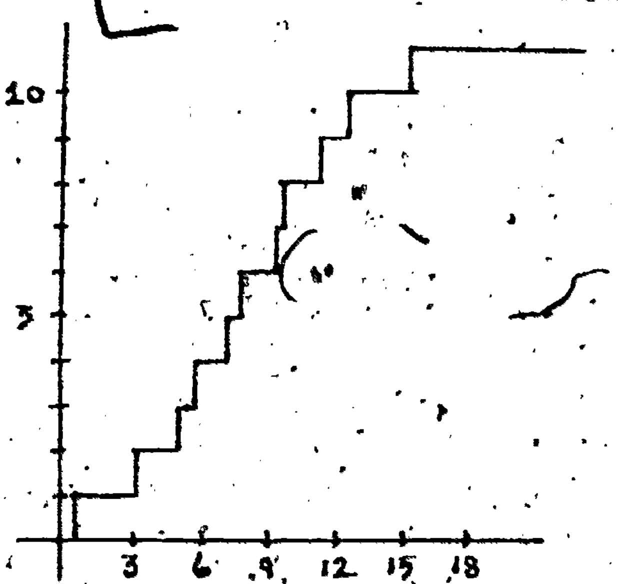
FOUR HISTOGRAMS CONSTRUCTED FROM THE SAME DATA



It is easy to see that as we divide the data into different intervals the shape of the histogram changes greatly. When comparing two samples' histograms we would have to be careful in some way that the intervals we choose do not distort either sample. By inspecting the first two histograms in Figure 2 we see that even though the exact same data was used to create these histograms they tend to indicate two different distributions. This does not seem to be a very desirable quality of this technique.

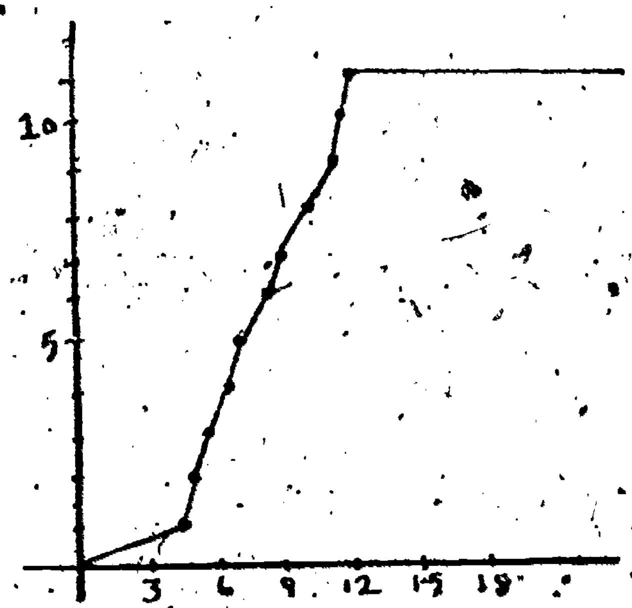
A graphical method for displaying one set of data that gets around this difficulty is the cumulative distribution graph. This is closely related to the histogram and is constructed as follows. Instead of plotting the number of observations equal to a given value or falling into a given interval on the vertical axis, we plot the number of observations less than or equal to each observed value on the vertical axis vs. the observed value on the horizontal axis. There is one point for each observed value. Figure 3 gives such graphs for each of the two sets of battery lifetimes.

FIGURE 3.



BRAND A

FIGURE 3a



BRAND B

FIGURE 3b

Sometimes the plotted points are joined by horizantal and vertical lines as in Figure 3a, giving the appearance of steps. Othertimes the plotted points are joined by straight line segments, as in Figure 3b. The numbers on the vertical axis are often divided by the total sample size, giving the proportion of observations less than or equal to each observed value; thus the median and quartiles can be read from this plot. The proportions start at 0 at the left side of the graph, increase through the middle of the graph, and reach 1 at the right side. Although this graph perhaps does not have the intuitive appeal of the histogram, it does a better job of showing the actual data values and any granularity that is present. Moreover, no choice of class intervals is required to construct the cumulative distribution graph.

Part II presents a graphical method explicitly designed for comparing the distributions from two sets of data. This method permits making a variety of comparisons between the two sets more easily than by looking at separate histograms or cumulative distribution graphs for each sample, and this method does not suffer from the disadvantages of histograms discussed above.

APPENDIX I-A

Suppose we take all observations from the first one larger than the first quartile up to and including the last observation smaller than the third quartile and average these values. Thus we form the arithmetic average of the values in the middle 50% of the data. This is called the mid-mean. It is not affected by extreme outlying values, as is the mean. Yet it is not obtained from just one or two data values, as is the median. Conceptually, it is somewhere between averaging all values -- the mean -- and just taking the middle value -- the median. (However, numerically it need not fall between the computed values for the mean and the median.) It is somewhat more complicated to calculate than the median. For the battery data of Section I there are 11 observations for each brand, so the first and third quartiles are the third largest and the ninth largest observations, respectively. Thus to calculate the mid-mean we average the fourth through the eighth largest observations. For Brand A this gives

$$(5.9 + 7.2 + 7.8 + 9.3 + 9.5)/5 = 39.7/5 = 7.94;$$

for Brand B the mid-mean is

$$(6.3 + 6.8 + 8.1 + 8.9 + 10.0)/5 = 40.1/5 = 8.02.$$

Recall that the median for Brand A and Brand B are 7.8 and 8.1 respectively. Generally the mid-mean will be fairly close to the median, as is the case with both sets of data here.

EXAMINING ONE AND TWO SETS OF DATA

PART II: A GRAPHICAL METHOD FOR COMPARING TWO SAMPLES

by

Lorraine Denby and James M. Landwehr

Bell Telephone Laboratories
Murray Hill, New Jersey 07974

This section presents a graphical method that is useful for displaying and comparing two sets of data. This method displays the information from both samples in a single plot, and it permits clear and easy comparison of many different aspects of the two sets of measurements. First we will explain and present the construction of this plot, using the battery data discussed in Part I. Then we will develop and illustrate a variety of properties of the q-q plot; through use of the q-q plot one can make many distributional comparisons between two sets of data. This section ends with the discussion of a new example.

II.1. Explaining the Q-Q Plot

Consider the battery lifetime experiment that was discussed in Part I. The data consists of 11 measured lifetimes for each of the two brands of batteries. For ease of reference the data are listed again below:

Brand A	9.5	.5	12.8	7.2	9.3	3.1	15.2	11.3	5.0	7.8	5.9
Brand B	6.8	4.2	8.1	11.4	5.1	10.0	6.3	11.9	11.1	4.6	8.9

With only a single set of numbers we know that plots such as the histogram and cumulative distribution graph are useful in presenting the data. With two sets of numbers a natural extension is to try and find effective ways of presenting both sets of data on the same graph. One idea is to plot the values in one set against the values in the other set.

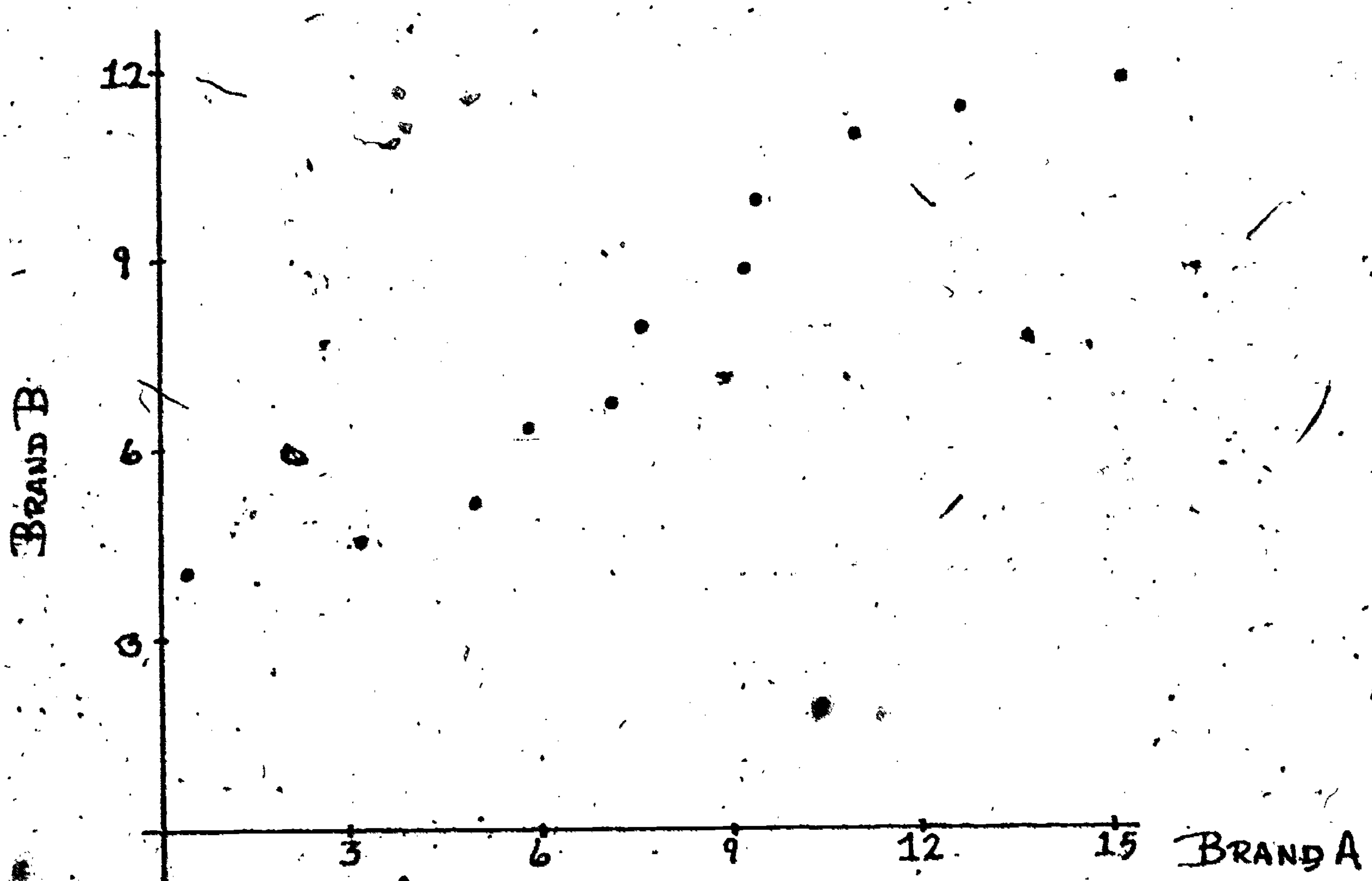
For example, if one set of measurements gives the weights and the other set gives the heights of a class of children, it is natural to think of plotting the heights against the weights. That is, for each child we plot the weight on the horizontal axis and the height on the vertical axis. Each plotted point corresponds to one child, and the total number of points is the same as the number of children whose heights and weights were measured. Examination of such a plot would show the general relationship between height and weight for these children. We would expect to see a general trend that taller children are heavier. We would also look to see whether or not there are certain children who are unusually light or heavy for their height, whether or not there is a distinct subset of children whose height-weight values differ markedly from the rest, etc. Another example of a situation in which such a plot would be useful is the following. Suppose data are available giving, for each city in a state, the total population of the city and the total public park area in the city. A plot of park area against population, with each city corresponding to a point on the plot, would be of interest. We might want to know whether larger cities tend to have more park area. Interesting special features and any general relationships would be exhibited clearly. Each of these plots is an example of a <u>scatter plot</u>.

Can this idea be directly used with our battery data? The answer, unfortunately, is no. There is a crucial difference between the two examples in the above paragraph and the battery example. In both the height-weight and the population-park area examples there is a natural pairing between the two sets of measurements. In the height-weight example the two measurements come from the same children; each height is naturally paired with the weight from the same child. In the second example both measurements are made on the same city. In one example each point on the plot gives the two measurements for some particular child, and in the other example each point gives the two measurements for some particular city. However, in the battery example we do not have this natural pairing of the observations in the two sets of measurements. One set of measurements simply gives the lifetimes for 11 different Brand A batteries, while the other sample gives the lifetimes for 11 different Brand B batteries. There is no natural unit, such as a child or a city, giving for each value in one set a corresponding value in the other set.

It might be useful to plot each value from one set against the "corresponding value" in the other set but with the battery example it is not obvious what the "corresponding value" should be. One way of associating "reasonable" values in the two sets of measurements is to do the following. First, order the observations from smallest to largest in each data set, obtaining values as listed below:

order number	1	2	3	4	5	6	7	8	9	10	11
Brand A	.5	3.1	5.0	5.9	7.2	7.8	9.3	9.5	11.3	12.8	15.2
Brand B	4.2	4.6	5.1	6.3	6.8	8.1	8.9	10.0	11.1	11.4	11.9

Now we plot the smallest observation from Brand B against the smallest observation from Brand A; the second smallest observation from Brand B against the second smallest observation from Brand A; and so on, until we finally plot the largest observation of Brand B vs. the largest observation of Brand A. This plot is given in Figure 1.



Q-Q Plot

Figure 1

This process makes sense here because the sample sizes in the two data sets are equal. There are 11 points on the plot, since there are 11 observations in each data set. Each point does not correspond to a particular person, or a particular city, as in the earlier examples; that type of correspondence is simply not possible for data such as the battery example. However, each point does give a specific, potentially interesting aspect of the data. One point shows the largest values in both samples, another shows the fourth largest values in both sample, another the second smallest values in both samples, etc.

The plot developed above and illustrated in Figure 1 is called a q-q plot. When the sample sizes are equal in both sets of measurements the q-q plot is simply a plot of the ordered values in one sample vs. the corresponding ordered values in the other sample.

Discussion of the properties of this graphical method and interpretations of some example plots begin in Section II.2 below. Two additional ways of motivating the construction of the q-q plot are given in Appendix A. Both approaches lead to exactly the same plot as developed above. One approach makes use of the cumulative distribution graph which was discussed in Part 1. The second approach in Appendix A makes use of the concept of <u>quantiles</u>. The term q-q plot is an abbreviation for quantile-quantile plot. When the sizes of the two data sets are not equal, construction of the q-q plot is slightly more complicated; this is discussed in Appendix B.

II.2. Interpretation When Both Samples Are The Same

We have just discussed what a q-q plot is and how one goes about constructing it. But, what good is it? How does it benefit our analyses over and above one sample summaries? What properties of the distributions of the two samples can we compare by constructing and inspecting a q-q plot?

This section and the following four sections discuss and illustrate how various aspects of the two distributions can be seen in q-q plots. An extremely useful property of the q-q plot is that it enables many different properties to be compared.

First, let's discuss what a q-q plot would look like if there were no differences between the two sets of data and then we will discuss how differences in various characteristics appear on the plot.

Suppose our two samples of data happened to contain exactly the same numbers. This is a rare occurrence but we will discuss it as an example where there could be no question whatsoever that the distributions of the two samples were the same. As our observational values in this case we will use the first 10 whole numbers. The first sample consists of 10 numbers -- 1,2,...,9,10 -- and so does the second sample. Figure 2 displays the q-q plot for this situation. In this case the plotted points fall exactly on the 45° line from the origin.

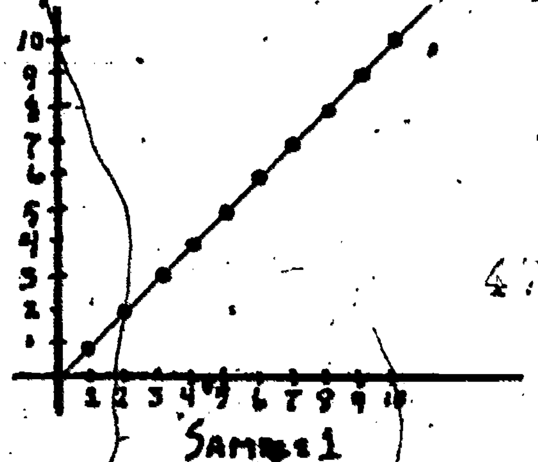
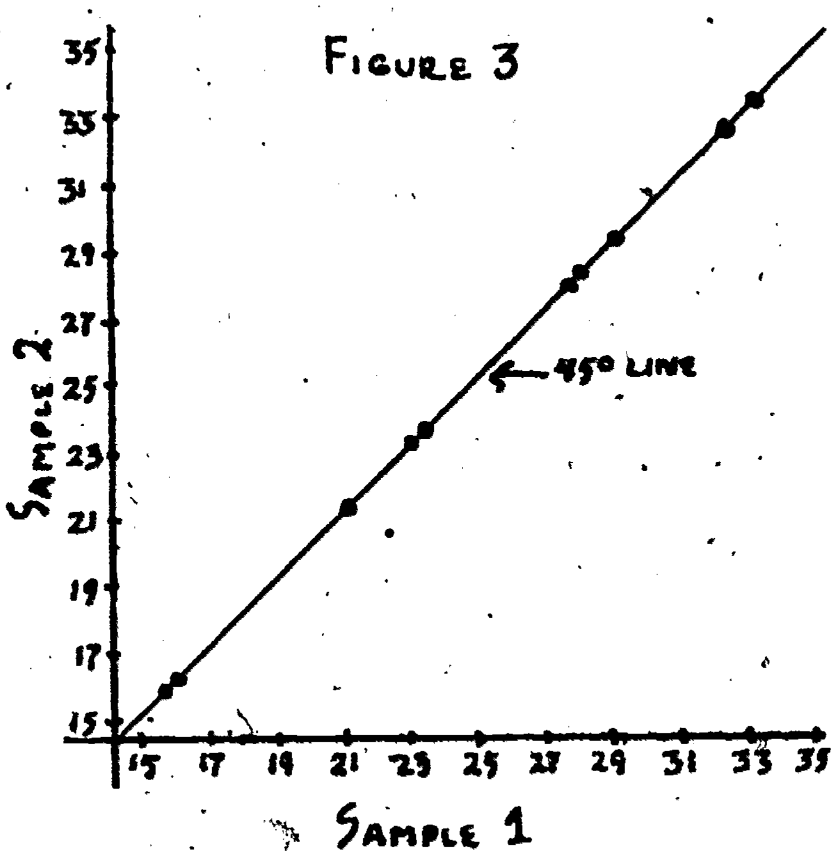


FIGURE 2

Even if the points are not so evenly distributed as are the first ten whole numbers, when two samples contain exactly the same values the points in their q-q plot fall exactly on the 45^{o} line. For example, suppose two samples of size ten contain exactly the same values in both samples and the ordered values are as follows:

16.2 16.3 21.4 23.3 23.9 28.2 28.7 29.5 32.4 33.8

The q-q plot for these data, given in Figure 3, shows that the plotted points fall exactly on the 45^{o} line but are not equally spaced along this line as was the case in Figure 2.



We will name these two samples as our "example samples" since we need to refer to them in the following paragraphs.

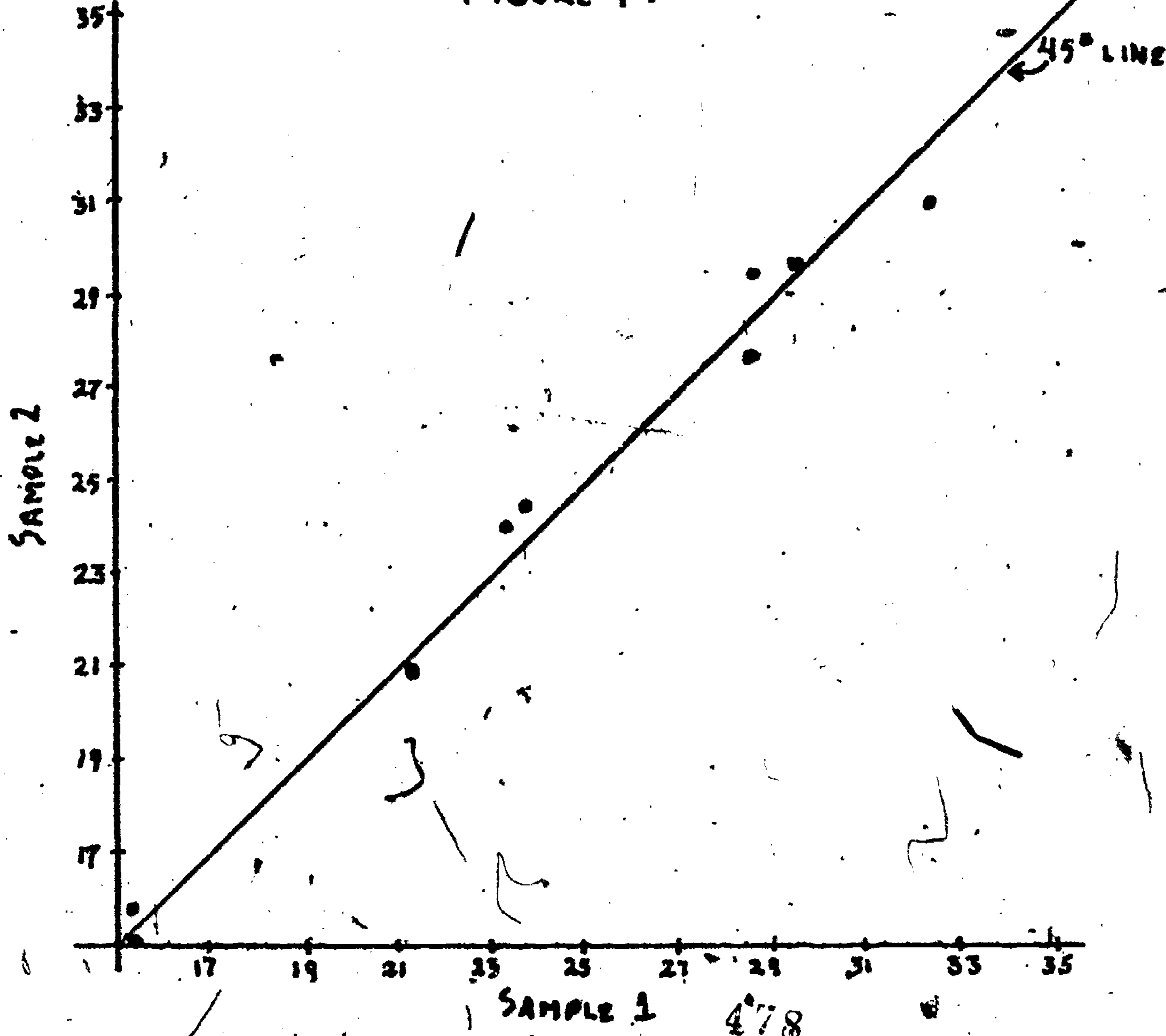
The closer the plotted points lie to the 45^{o} line the more similar the two samples are. Now, suppose instead of both samples containing values that are exactly equal, that they contain points which are nearly equal but not exactly. Each observation in the first sample has a corresponding observation in the second sample which has approximately the same value. For example, suppose our two samples were as shown in Table 1:

Table 1

Sample 1	16.2	16.3	21.4	23.3	23.8	28.2	28.7	29.5	32.4	33.8
Sample 2	16.0	16.8	21.0	23.9	24.4	27.6	29.3	29.6	30.8	36.4

The q-q plot for these two samples can be found in Figure 4.

FIGURE 4



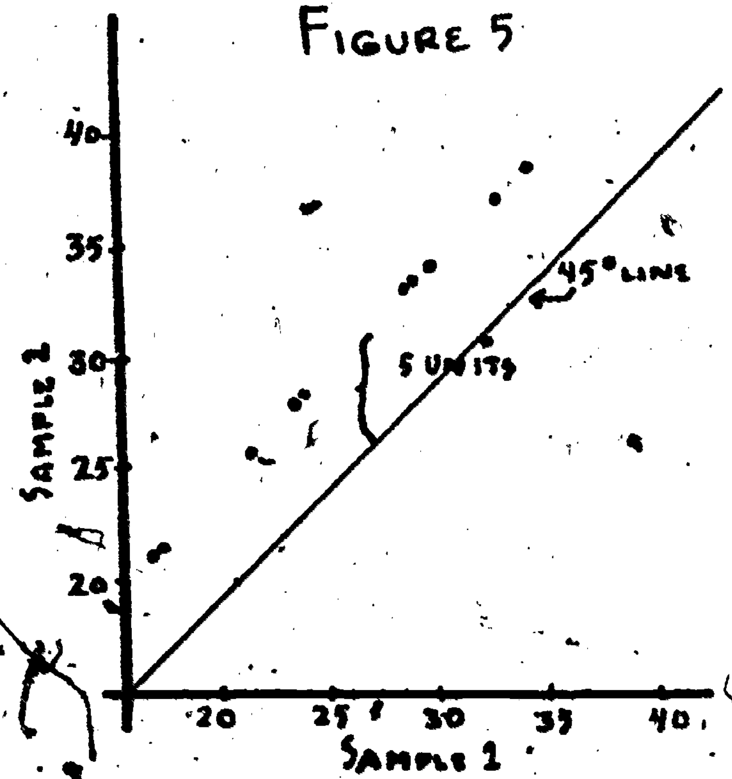
Here the two samples are very nearly equal and this causes the plotted points to be scattered close to the 45° line. If the plotted points are obviously scattered about the 45° line, then the two samples are considered essentially equal. When the scatter is not close to this line or when the scatter is about some other line or even when the plotted points follow some curved line, then there probably are some differences between the two distributions. These situations and their interpretations will be discussed in the following sections.

II.3. Interpretations When The Samples Differ In Location-Median

The first characteristic that we will discuss is the comparison of the medians of the two samples. The median (the middle point of the sample) is one of the measures of location discussed in Part I. If the medians are equal or close then there is no difference in the center of location of the two samples. The middle plotted point on the q-q plot (or in the case of the samples with an even number of observations, the midway point between the two middle plotted points) actually represents the median of both the first and second sample. Its value along the horizontal axis is the median of the first sample and its value along the vertical axis is the median of the second sample. So, therefore, if this middle plotted point lies on or close to the 45° line, then the locations of the samples are the same.

Let's consider what happens when the example sample is shifted by adding a constant to each value. We take the numbers as they stand for the first sample and for the second sample we will add 5 to each one of these numbers. So our first sample's values are 16.2,16.3,21.4,...32.4,33.8 and our second samples's values are 21.2,21.3,26.4,...,37.4,38.8. Figure 5 is the q-q plot of these two samples. We can see that the plotted points lie on a line 5 units higher than the 45° line and parallel to it.

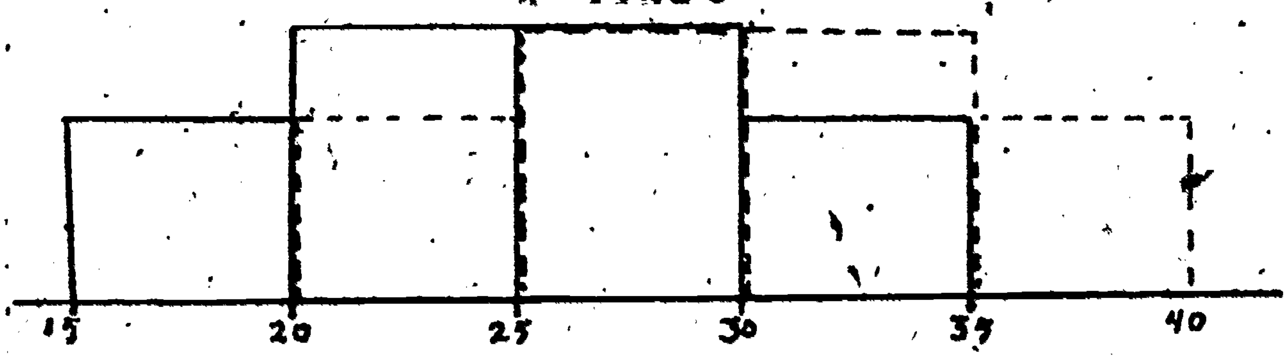
FIGURE 5



Thus we see that if the two samples are the same except for a constant difference in the corresponding observations, then the q-q plot will be shifted parallel to the 45° line. The number of units that the line through the plotted points is above the 45° line is the amount that the second sample, including the median of the second sample, is larger than the corresponding values of the first sample. Similarly, if the line through the plotted points is parallel to but below the 45° line, then the values in the first sample, including the median, are larger than the corresponding values in the second sample by the amount of their shift.

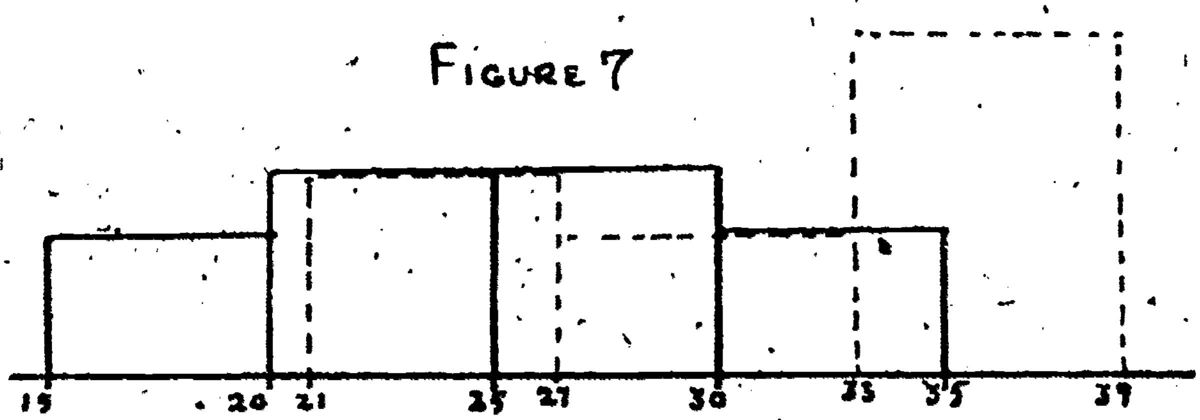
Now, disregard for the moment the difficulties with the use of two histograms and consider what this translation does to the plot of the two histograms. Figure 6 plots the first sample's histogram in a solid line and the second sample's histogram in a dotted line.

Figure 6



We see that the second sample's histogram is 5 units to the right of that of the first sample. Since care was taken in choosing the intervals we see that, other than the fact that the second histogram is moved to the right, the two histograms are identical. Indeed they should be since no other aspect of the distsibutions were affected by the addition. However, Figure 7 shows two histograms of these samples when the interval choices disturbed the shape of the two histograms.

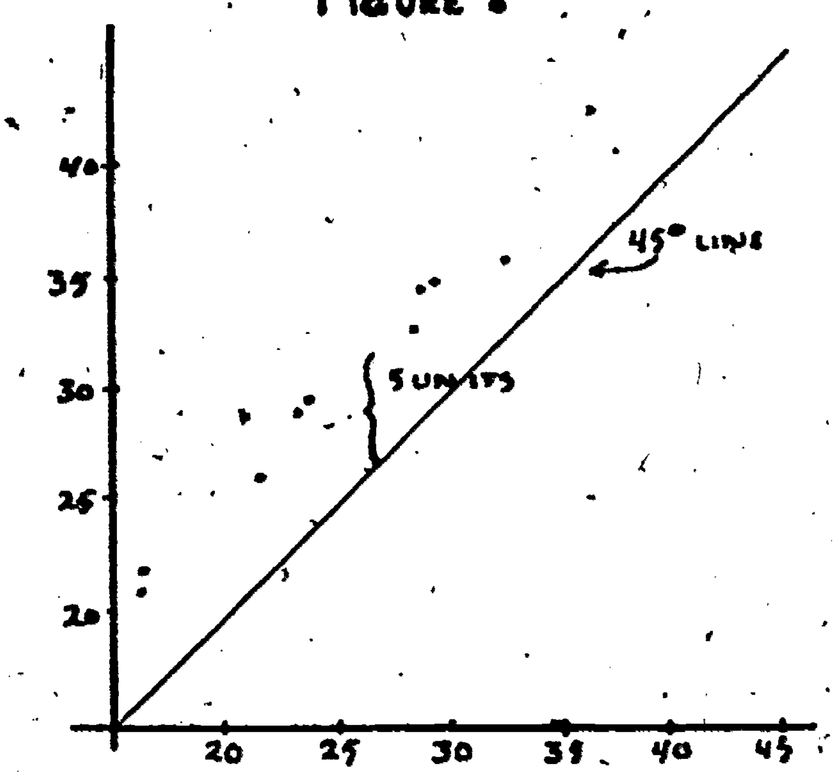
Figure 7



In this case the histograms do not look identical and one might conclude that there are other differences between the samples besides a shift in location. Since we know how the samples were created we know that this is not so. All other distributional aspects are the same except for location. This fact is clearly evident by inspecting the q-q plot in Figure 5.

Now let's look at the two samples in Table 1 which were determined nearly equal by their q-q plot in Figure 4. Recall that these plotted points were scattered near the 45° line, meaning that the two samples were essentially the same in all distributional aspects. If we merely change the location of the second sample by adding 5 to each observation, we do not affect any other characteristic of the distributions. Figure 8 shows the q-q plot for these samples.

FIGURE 8



We see that the plotted points lie around a line parallel to the 45° line and 5 units higher. The configuration of the points is the same as in Figure 4 but just shifted up. If we were to subtract 5 from each value in the second sample then the plotted points in their q-q plot would approximate a line parallel to the 45^{o} line but 5 units below it.

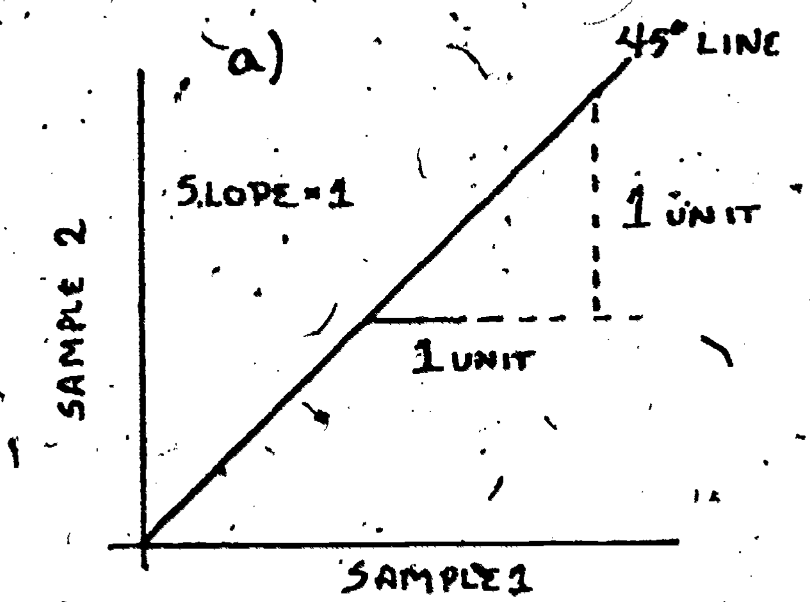
So, in summary, if the configuration of points on the q-q plot is scattered about a straight line parallel to the 45^{o} line then we can conclude that the samples are essentially the same except for a difference in location. This difference in location is indicated by the amount and direction of the shift of the line through the data from the 45^{o} line. If the plotted points exist in some other configuration then there are other differences between the distributions of the two samples. Discussion of the detection of these differences is contained in the following sections.

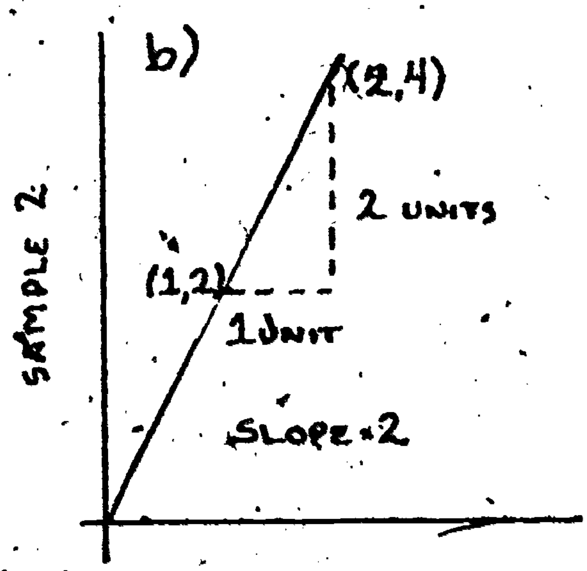
II.4. Interpretation When The Samples Differ In Variability -- Interquartile Range

The interquartile range describes the variability of a single sample. If the interquartile ranges of two samples are nearly equal then the two samples have the same variability. The interquartile range is the difference between the 75^{th} percentile and the 25^{th} percentile. In constructing a q-q plot the 25^{th} percentile of the first sample is plotted against the 25^{th} percentile of the second sample and the 75^{th} percentiles of the samples are plotted against each other (since they are merely given ordered sample observations). In order to interpret the meaning of these points we first need a short digression concerning the slope of a line.

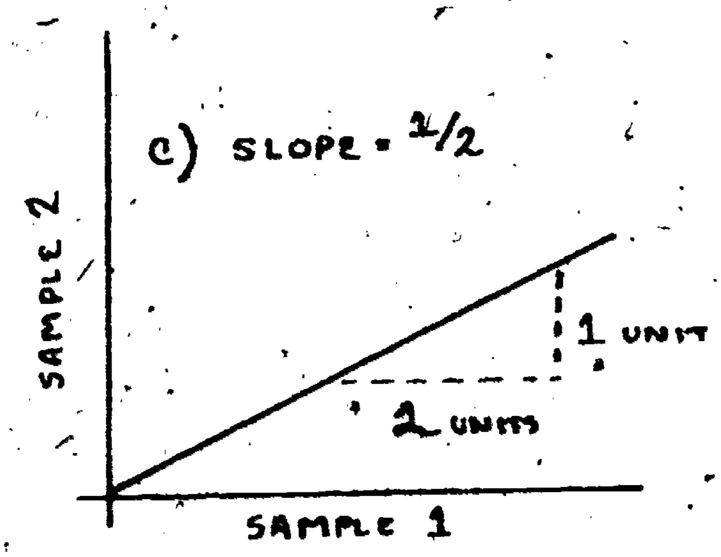
The slope of a line is a term that relates to the steepness of the line. The 45° line has a slope of 1 since for every increase on the horizontal scale of a single unit the value of the corresponding points on the vertical axis increases by a single unit. Lines that appear steeper than the 45° line have a slope greater than one. For example, see Figure 9b.

FIGURE 9





b) LINE IS STEEPER THAN 45° LINE. THUS VARIABILITY OF 2 TIMES LARGER THAN SAMPLE 1.



c) LINE IS SHALLOWER THAN 45° LINE. THUS VARIABILITY OF SAMPLE 2 IS 1/2 THAT OF SAMPLE 1 OR, IN OTHER WORDS, SAMPLE 1'S VARIABILIT IS TWICE THAT OF SAMPLE 2.

Here we picture a line with a slope of 2. For every single unit change on the horizontal axis the corresponding point on the vertical axis increases by two units. Point (1,2) is plotted on this graph. If we move to the right from this point one unit we obtain the value 2 on the horizontal axis and the corresponding point on the vertical axis has a value of 4 -- a two unit increase. Lines which appear shallower than the 45° line have a slope less than one as shown in Figure 9c. For example, if a line has slope 1/2 then a 2 unit increase on the horizontal axis corresponds to a 1 unit increase in the vertical direction.

Now let's see how the slope of the line on the q-q plot between the points corresponding to the first and third quartiles should be interpreted. The horizontal distance between these points indicates the interquartile range in the first sample, while the vertical distance between these points indicates the interquartile range in the second sample. If the interquartile ranges are about equal, then these distances are about equal so the slope of the line joining these points is about 1.

Consider, for instance, our last example. The interquartile range of the samples is the difference in this case between the 3rd and 8th largest points. So the interquartile range of the first sample is 29.5 - 21.4 = 8.1 and of the second sample is 34.6 - 26.0 = 8.6. The interquartile ranges are approximately equal and we saw in Figure 8 that the plotted points surround a line parallel to the 45° line. You could calculate the interquartile ranges of the other samples that we have studied and you would learn that their interquartile ranges would be approximately equal. This corresponds to the q-q plots that have been constructed for each.

Now let's again look at the data pictured in Figure 4 on page 8. We will transform the observations of the second sample to have twice the variability of the first. By simply multiplying each observation of the second sample by 2, this sample's variability is doubled, but so is the value of the median. Therefore, to keep the sample's median the same, the value of the median must be subtracted from each point. In other words, we will transform the second sample in the following way:

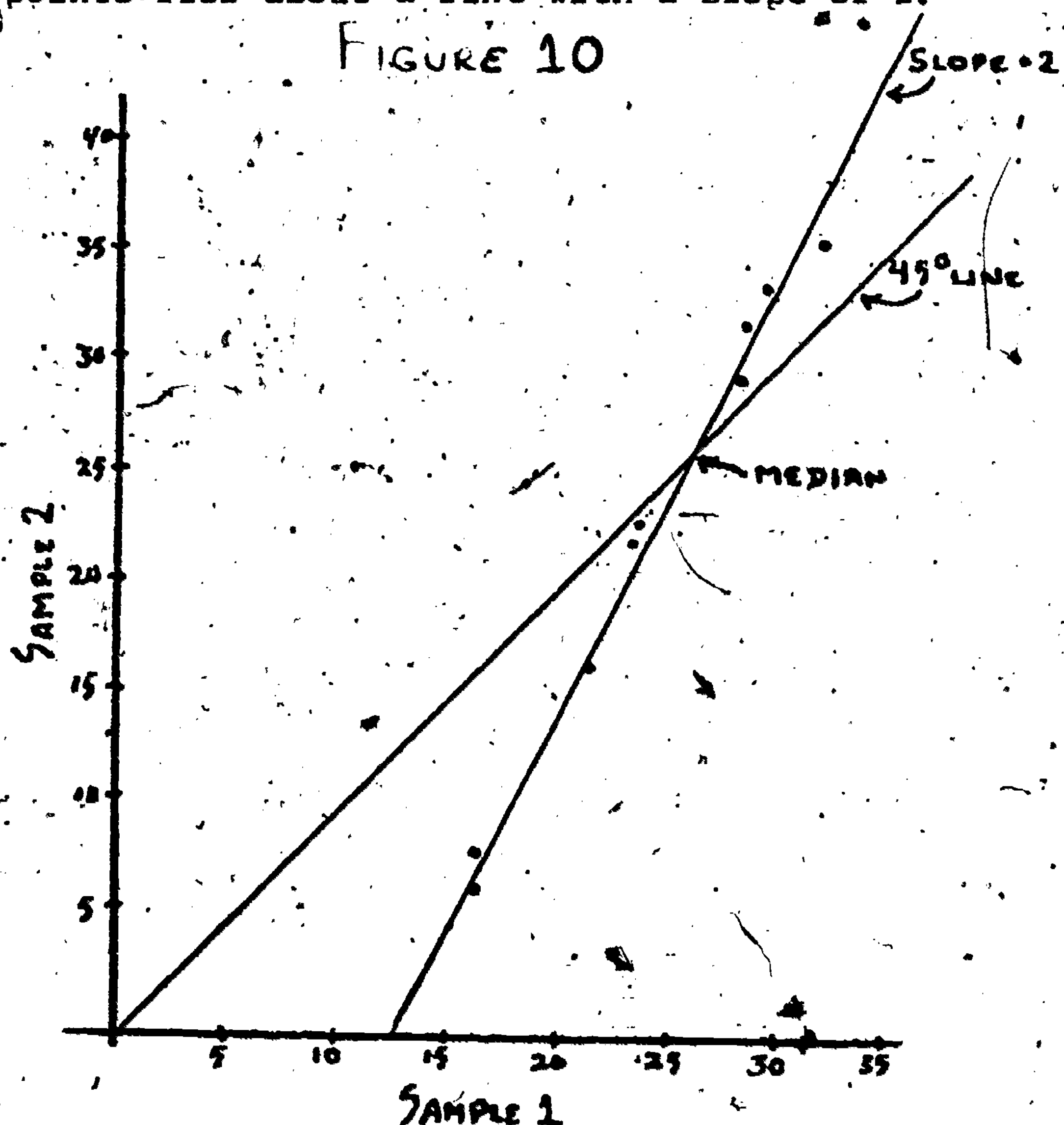
Ordered Second Sample		Transformed Sample
16.0	2 x 16.0 - 26.0 =	6.0
16.8	2 x 16.8 - 26.0 =	7.6
21.0	etc.	16.0
23.9		21.8
24.4		22.8
27.6		29.2
29.3		32.6
29.6		33.2
30.8		35.6
36.4		46.8

median = (24.4+27.6)/2 = 26.0

median transformed sample = (22.8+29.2)/2 = 26.0

The q-q plot of these transformed values and the original first sample is shown in Figure 10. We notice that the median "point" still lies close to the 45° line. However, the set of plotted points lies about a line with a slope of 2.

FIGURE 10



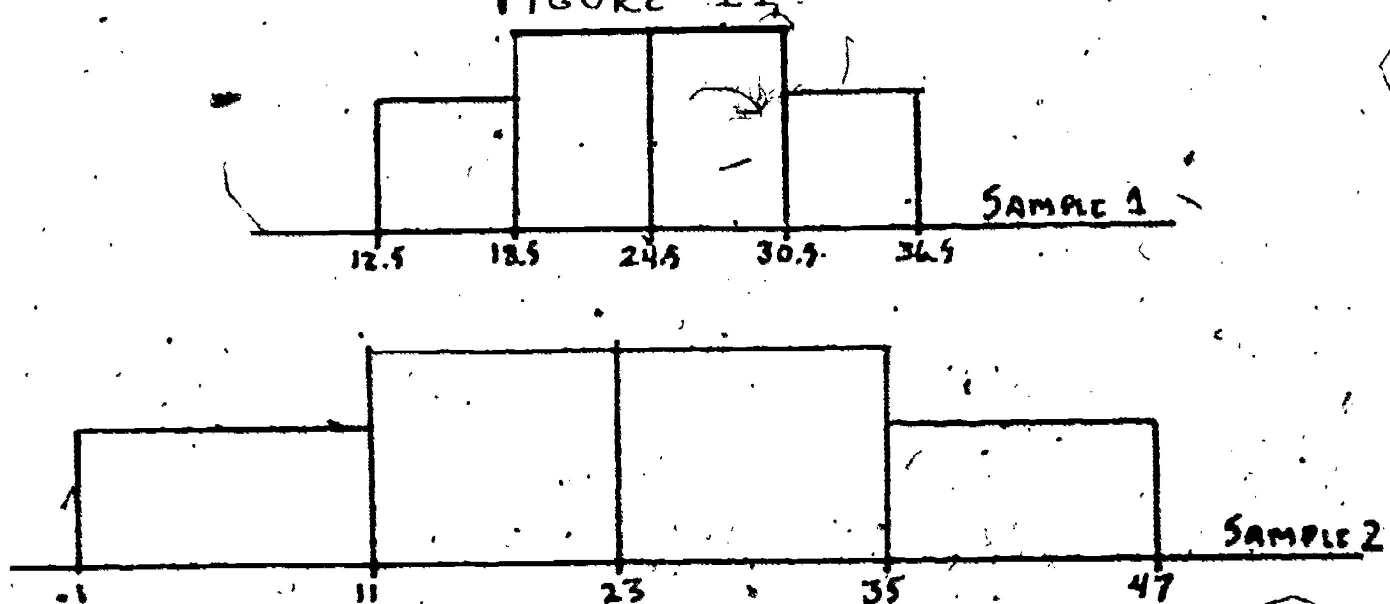
This graphically indicates that the interquartile range for the second sample is twice that of the first sample; i.e., the variability of the second sample is now twice that of the first. Thus there is no difference between the locations of the two samples but they do differ in variability.

In general, if the plotted points between the two quartiles surround a straight line then the slope of this line indicates the relative variabilities in the two sets of data. For example, if the interquartile range for the first sample is twice that of the second sample, then the horizontal distance between the points corresponding to the quartiles on the q-q plot will be twice the vertical distance. Thus the slope on the q-q plot will be about 1/2. Another way of thinking about this is that if the slope is 1/2, then adjacent observations differ in the second sample by about 1/2 as much as adjacent observations differ in the first sample; the variability is about half as large in the second sample as in the first sample. Similarly, if the variability is larger in the second sample, as in Figure 10, then the slope will be greater than 1.

Since the points in Figure 10 surround a straight line, there appears to be no other difference between these two distributions than this difference in variability. If the points were to deviate from a straight line then it would imply that there are other differences, and we need the following sections to interpret the differences correctly.

What happens to the corresponding histogram when we transform the second sample? Figure 11 shows the two histograms before and after the transformation.

FIGURE 11



We see that when the variability is doubled the sample histogram becomes twice as wide. Again, to see this the class intervals have to be chosen correctly. If we were to decrease the sample variability by half, the slope of the q-q plot would be 1/2 as in Figure 9c and the corresponding histogram would become half as wide.

II.5. Interpretation of Extreme Values -- Tails

So far we have seen how the concepts of location, as measured, for example, by the median, and variability, as measured, for example, by the interquartile range, appear in the q-q plot. We know how the q-q plot looks if two sets of data are the same except for differences either in location and/or variability. However, sometimes additional aspects of the data are important and affect the interpretations we should make.

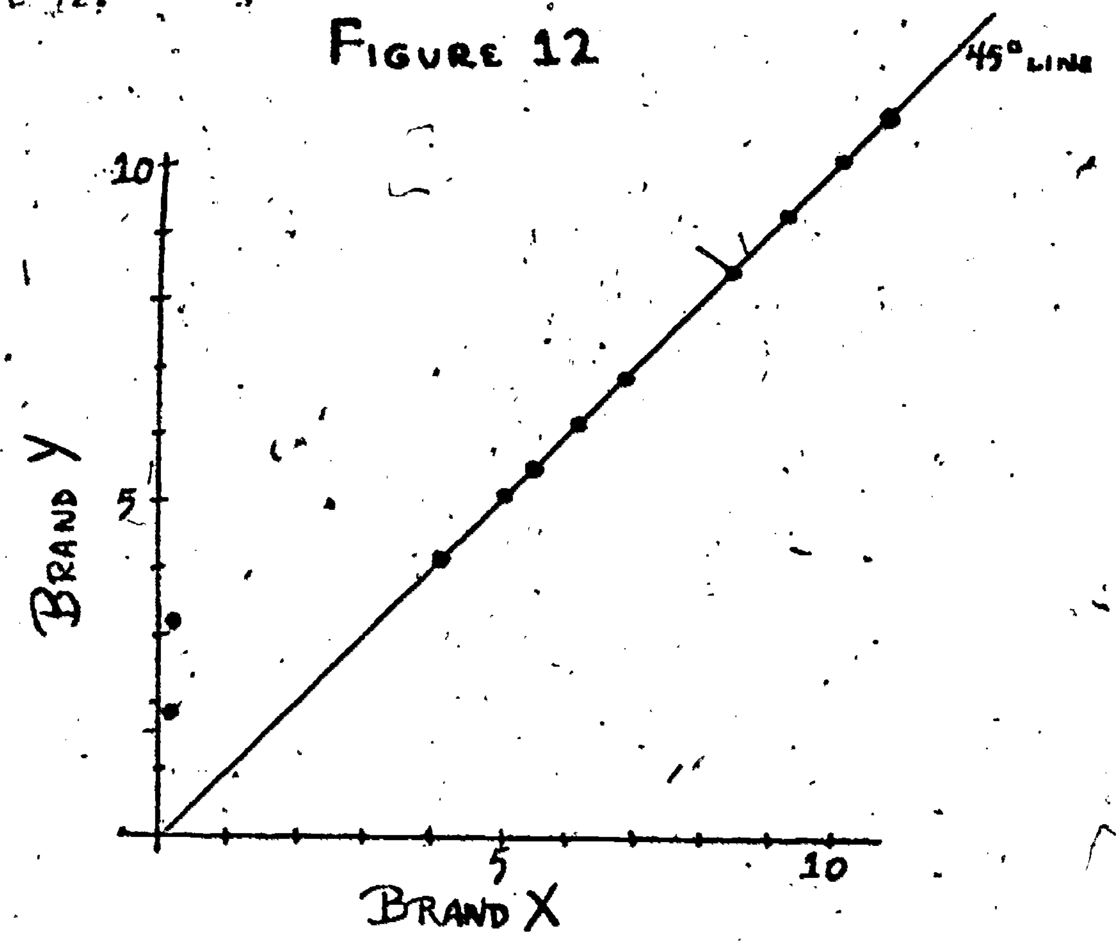
Consider the battery example. The lifetimes from the two brands of batteries might have about the same median and about the same interquartile range, but one brand could still have many failures earlier than the other brand. For example, consider the following artificial example, which is given with the values ordered within each sample.

	1	2	3	4	5	6	7	8	9	10	11
Brand X	0.1	0.2	4.1	5.1	5.4	6.2	6.9	8.4	9.3	10.1	10.8
Brand Y	1.0	3.2	4.1	5.1	5.4	6.2	6.9	8.4	9.3	10.1	10.8

Here the medians are both 6.2, and the interquartile ranges are both 9.3 - 4.1 = 5.2. The values in both samples are the same, except for the two smallest values. But these smallest values do not enter into the calculation of either the median or the interquartile range. However, here one would clearly prefer Brand Y to Brand X, as there appears to be about a 20% chance of getting a very short lifetime with Brand X, and otherwise the two sets of lifetimes are identical. There is an important difference between the values in these two sets of data, and we would not want to miss this difference by concentrating only on the median and interquartile range.

Let's examine the q-q plot for these data; it is given in Figure 12.

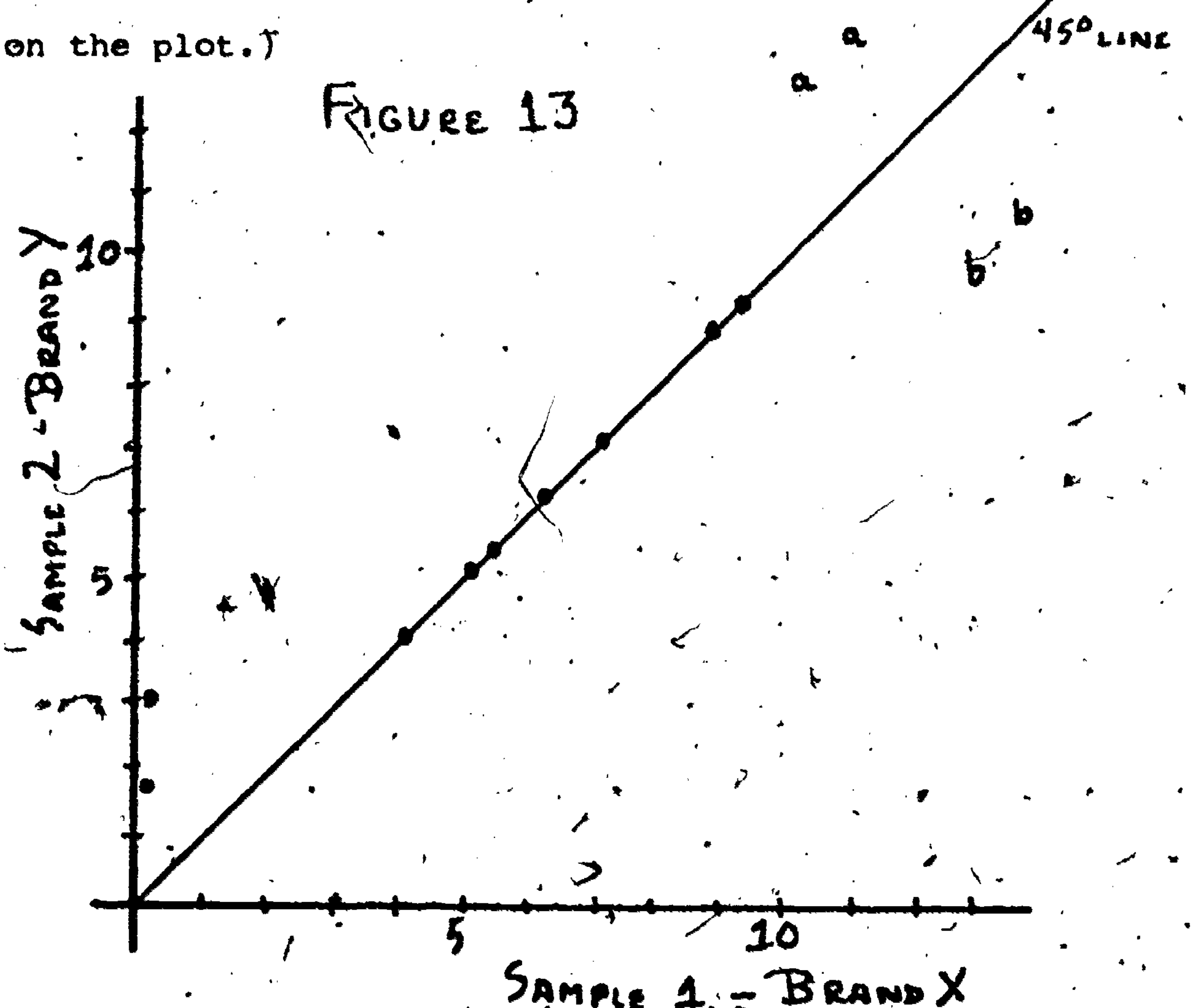
FIGURE 12



Immediately we see that all but two of the points lie on the 45° line through the origin, and our attention is drawn to those two points. For them the Brand Y values are much larger than the corresponding Brand X values. A quick examination of this plot shows these important features of the data very clearly.

The extreme low and the extreme high values of a distribution are often referred to as the tails of the distribution. Sometimes it is important to consider the low and high tails in a set of data, and the relative magnitudes of the tails when comparing two sets of data, as in this battery example.

Suppose the upper tails in this Brand X - Brand Y data are changed. We will make several modifications of the largest two values in the two samples and see how this affects the q-q plot and the interpretation of the data. First suppose that the two largest Brand Y values are 13.5 and 12.9 instead of 10.8 and 10.1; the rest of the Brand Y values and all the Brand X values are as listed above. The q-q plot for these data are given by the dots and a's in Figure 1.3. (For the time being ignore the two b's on the plot.)



This plot shows that the values in the middle of both sets of data are distributed the same, as these points fall on the 45° line; it is also apparent that the values in both the upper and the lower tails are not the same in both sets of data. The lower tails are as discussed before. Concerning the upper tails, the distances that the two a's are above the 45° line shows the amount that these Brand Y values are larger than the corresponding Brand X values. Another way of thinking about this is as follows. In Figure 13 the indicated line passes through the bulk of the data; if the relationships between the two distributions in the tails were the same as the relationship in the central part of the data, then the points in this plot corresponding to the tails would also fall on this straight line. The plot clearly has points above the straight line in both tails, indicating that these Brand Y values are larger than the corresponding Brand X values. Although the medians and interquartile ranges are the same in the two samples, the plot shows clearly and exactly the way in which the Brand Y batteries have longer lifetimes than Brand X batteries. If the cost of the two brands is the same, Brand Y appears preferable to Brand X.

Now suppose that these data were again altered so that the two largest Brand X values are 13.5 and 12.9, while the largest Brand Y values are 10.8 and 10.1, and the rest of the values remain unchanged. Then the q-q plot is given by the dots and b's in Figure 13, ignoring the two a's. The situation in the upper tail is reversed, and the fact that the tail of Brand X contains larger values is indicated by the points (the b's) being on the lower side -- the Brand X side -- of the line through the central part of the data. Here it is not so clear which brand of battery is preferable. Through the middle part of the data, the lifetimes are the same; for a short lifetime Brand Y is better, but if there happens to be a long lifetime Brand X is better. The plot clearly shows all three of these features of the data.

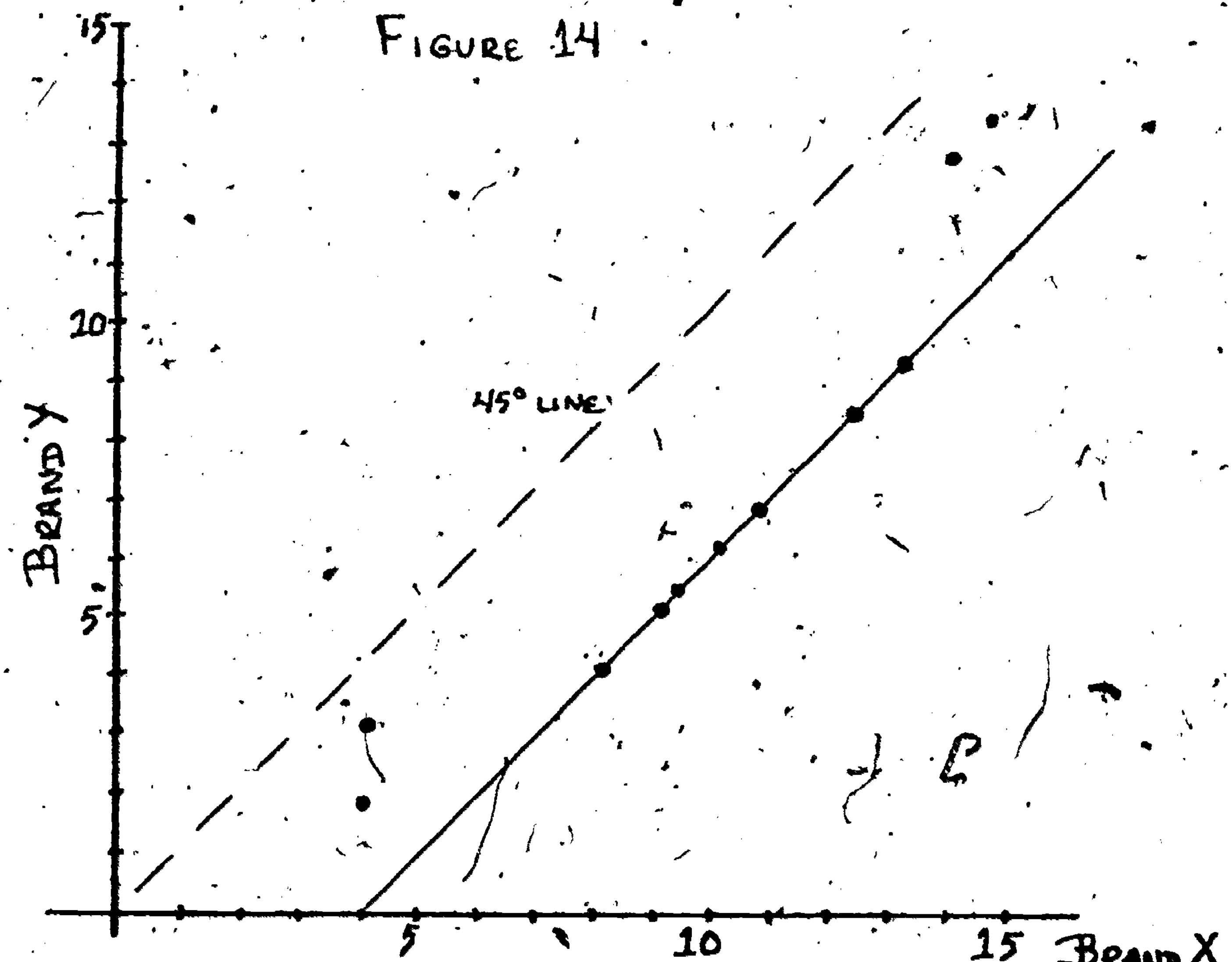
The above discussion has all been in terms of examples where the middle parts of the two distributions are exactly the same. Thus the straight line through this part of the data was the 45° line from the origin. However, the concept of comparing the differences in the tails of two distributions by examining the points at the end of the q-q plot to the straight line through the central points can be extended to situations in which the straight line is not the 45° line. The interpretations then are analogous to the previous examples, but slightly more complicated.

We illustrate with still another modification of the Brand X data. Suppose a 4 is added to each Brand X value and the Brand Y values are as before, with the two largest values 13.5 and 12.9; this gives the following.

Brand X	4.1	4.2	8.1	9.1	9.4	10.2	10.9	12.4	13.3	14.1	14.8
Brand Y	1.9	2.1	4.1	5.1	5.4	6.2	6.9	8.4	9.3	12.9	13.5

The median for Brand X is now 10.2, the interquartile range is 13.3 - 8.1 = 5.2, the same as before. (Indeed, from sections II.3 and II.4 we know that adding a constant to all values changes the median by that constant but leaves the interquartile range unchanged.) The q-q plot for these data is given in Figure 14; the dotted line is the 45° line from the origin, and the solid line passes through the central part of the points.

FIGURE 14



Most of the points are on the solid straight line, but two at each end are above the solid line. The fact that all points are below the dotted line -- on the Brand X side of the 45° line through the origin -- means that at each part of the distribution the Brand X value is larger than the corresponding Brand Y value. If all the points were on the solid straight line, we know from section II.3 that the data could be summarized as follows. The

Brand X values are 4 units larger than the Brand Y values, since the solid line is moved 4 units from the dotted line, indicating a difference in location; and there is no difference in variability, since the solid line is the same slope as the dotted line.

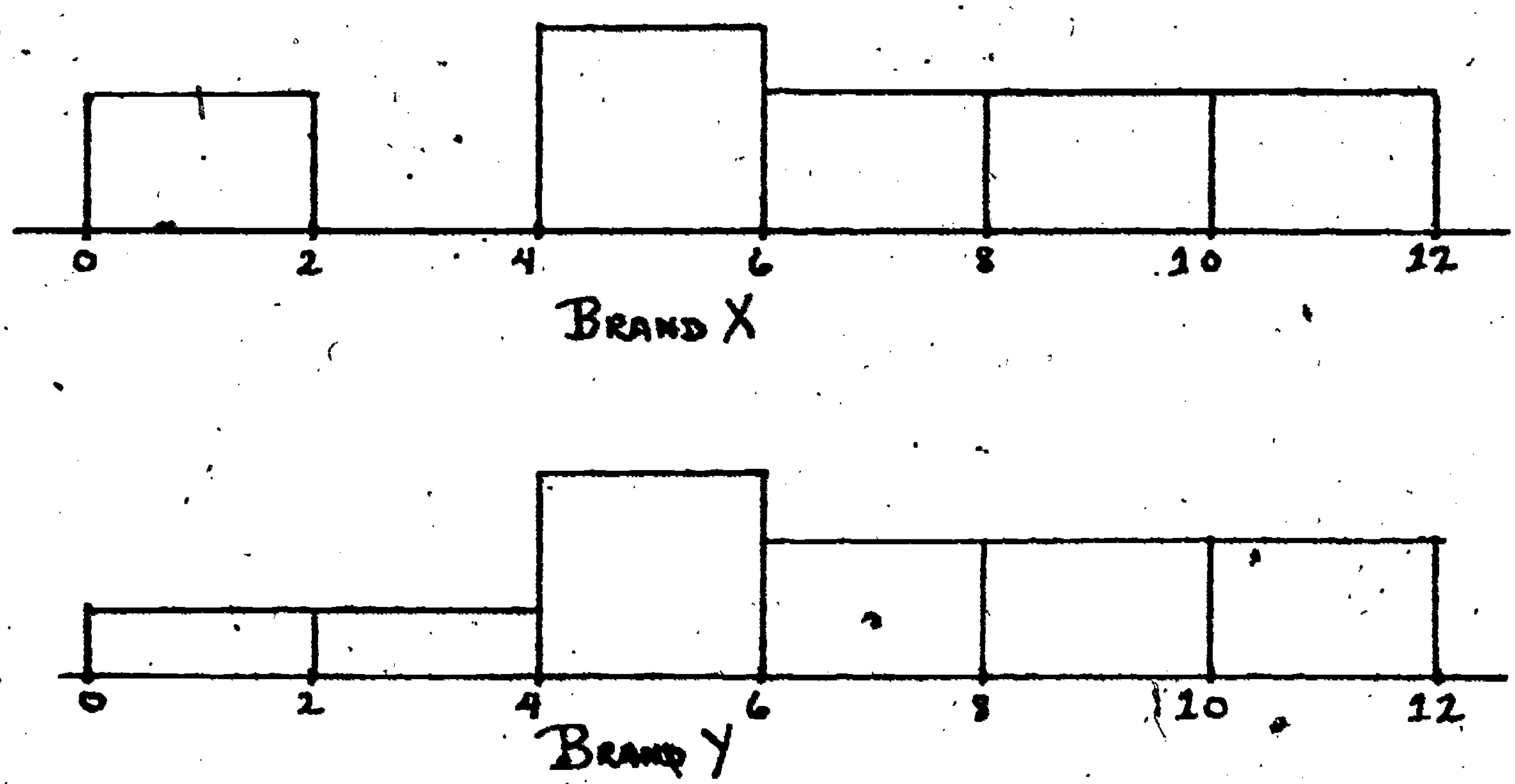
But now how should these 4 points lying well above this solid line affect this interpretation? They indicate that in both the lower and the upper tails the Brand Y values are larger compared to the Brand X values than one would expect from the relationship between the values in the central part of both sets of data. Note that in each tail the Brand X values are actually larger than the corresponding Brand Y values -- the points are on the Brand X side of the dotted line. However, the Brand Y values are larger relative to the relationship that holds between the two data sets for the middle values of the data set since the extreme points are above -- on the Brand Y side -- of the solid line. Another way of saying this is that for most of the data the Brand X values appear to be 4 units larger than the Brand Y values, but this does not hold in the tails; there the Brand Y values are relatively larger than one would expect them to be, even though they are still smaller than the corresponding Brand X values.

The most important feature of this data is likely to be the over-all difference of 4 in the location of the data sets; one would prefer Brand X to Brand Y, if the costs are equal. However, this additional difference in the tails might be of interest. It indicates that the relationship between the two distributions of lifetimes is not the same in the middle as it is in the tails, and it might be interesting to know why Brand Y does relatively better in the tails though it does worse overall. This difference in the tails is most easily spotted using the q-q plot. Such a difference can be seen whether the straight line through the central part of the data is the 45° line, as discussed earlier in this section; or a line with slope 1 shifted from the 45° line, as here; or a line shifted from the 45° line and with slope different from 1. No example is given here for this last situation, but the interpretation would be that there is an over-all difference in location and variability between the two samples as indicated by the straight line, and in addition there is a difference in the tails. This is much like the situation just discussed, where there was an overall difference in location but not variability.

Would these differences in the tails of the two distributions, which have stood out so markedly in Figures 12, 13 and 14, have been missed if q-q plots had not been constructed? For small sets of simple, artificial data such as this a close study of only the numerical values, without using any graphs, would reveal the features discussed above. However, larger sets of real data have fluctuations so that values are not exactly the same and the points of the q-q plot do not lie exactly on a straight line; then it is more difficult to find differences in tails simply through staring at the numerical values. Moreover, if one does not find such a difference, one still cannot be very sure that there really is no difference, as it could easily be missed from a study of the raw numbers. However, the q-q plot shows clearly and directly any differences in the tails, so an important difference would not go unnoticed.

Would construction and comparison of two histograms exhibit the differences in the tails? The answer depends on how the intervals for the two samples are chosen. In Part I we saw that choosing different intervals could definitely alter the appearance of the histogram; similarly, whether or not the two histograms show the differences in the tails depends on how the intervals for both histograms are chosen. Figure 15 gives histograms of the first Brand X - Brand Y data with reasonable choices for intervals.

FIGURE 15



We notice that there is some difference at the lower ends of the distribution, but this plot does not bring out the important features of the data as clearly and as simply as does the q-q plot of Figure 12. Moreover, if we had instead labeled the two end cells as "≤4" and "≥10", then there would be no difference between the two histograms.

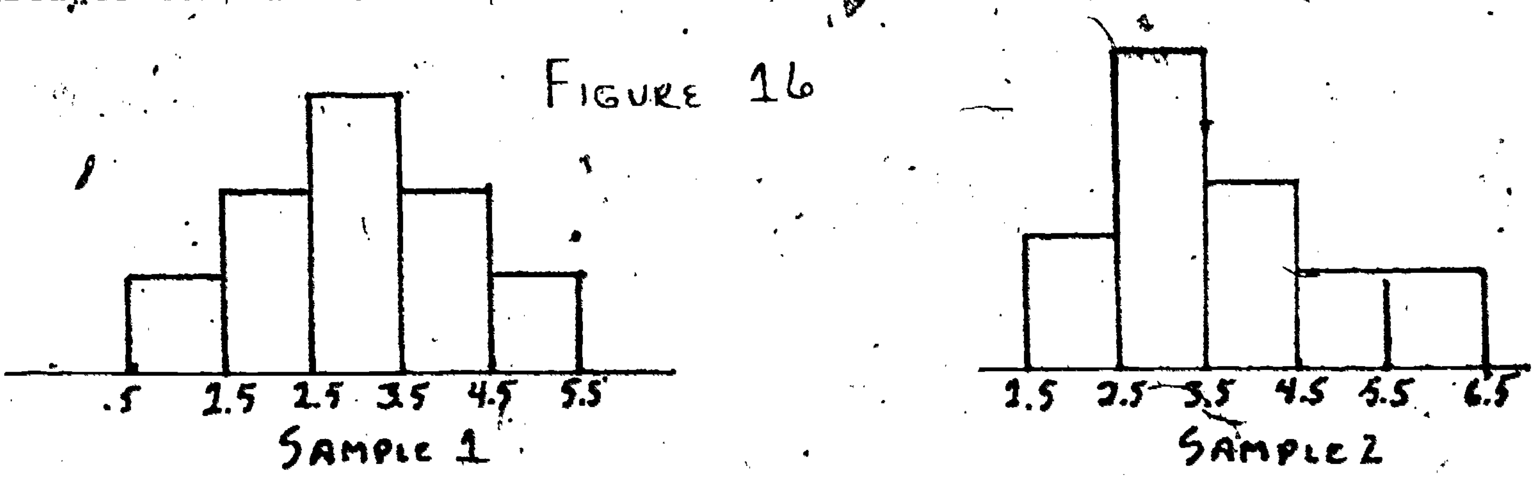
In Sections II.2 and II.3 we learned that if the points in the q-q plot fall basically on a straight line, then the two sets of data can be adequately summarized and compared in terms of differences in location and/or variability, and that these are related to the straight line through the data. Now from this section we see that if the points in the q-q plot fall basically on a straight line except for one or several points at either end of the plot, then in addition there is a difference in the tails of the two distributions. This can be important in certain problems, and it is worth knowing. If the extreme points are above the straight line through most of the data, then the sample plotted on the vertical axis is relatively larger than the other sample in this tail, compared to what would be expected from the central part of the data. Conversly, if the points are below the straight line, then the sample plotted on the horizontal axis is relatively larger. Thus, we can now interpret q-q plots where much of the data falls around some straight line, with the exception of a few outlying values at either end. This type of situation includes very many, if not most, q-q plots of real data.

II.6. <u>Interpretation When The Points Do Not Fall On A Straight Line — Shape</u>

This section discusses a more complicated concept in the comparison of two sets of data and can be skipped on first reading. This material is useful when the points in the q-q plot do not fall near a straight line, even with the exception of a few outlying points. It is also useful when we want to compare two distributions apart from obvious differences in location and/or variability.

Even if two distributions have the same medians, interquartile ranges, and there are not extreme outlying values in the tails, this does not mean that the two distributions are identical. The reason is that there can be differences in the distributional shapes. One way of thinking about shape is that it is the way the histogram of the data appears, apart from its location and variability. The relative shape of two distributions can be important in making comparisons between them. This section discusses several examples and shows how the q-q plot can be used to compare the shapes of two sets of data.

FIGURE 16



Sample 1:

.5
1.2
1.6
1.9
2.1 → 25TH QUANTILE
2.4
2.6
2.8
2.9 → MEDIAN
3.1
3.2
3.4
3.6
3.9 → 75TH QUANTILE
4.1
4.3
4.6
5.2

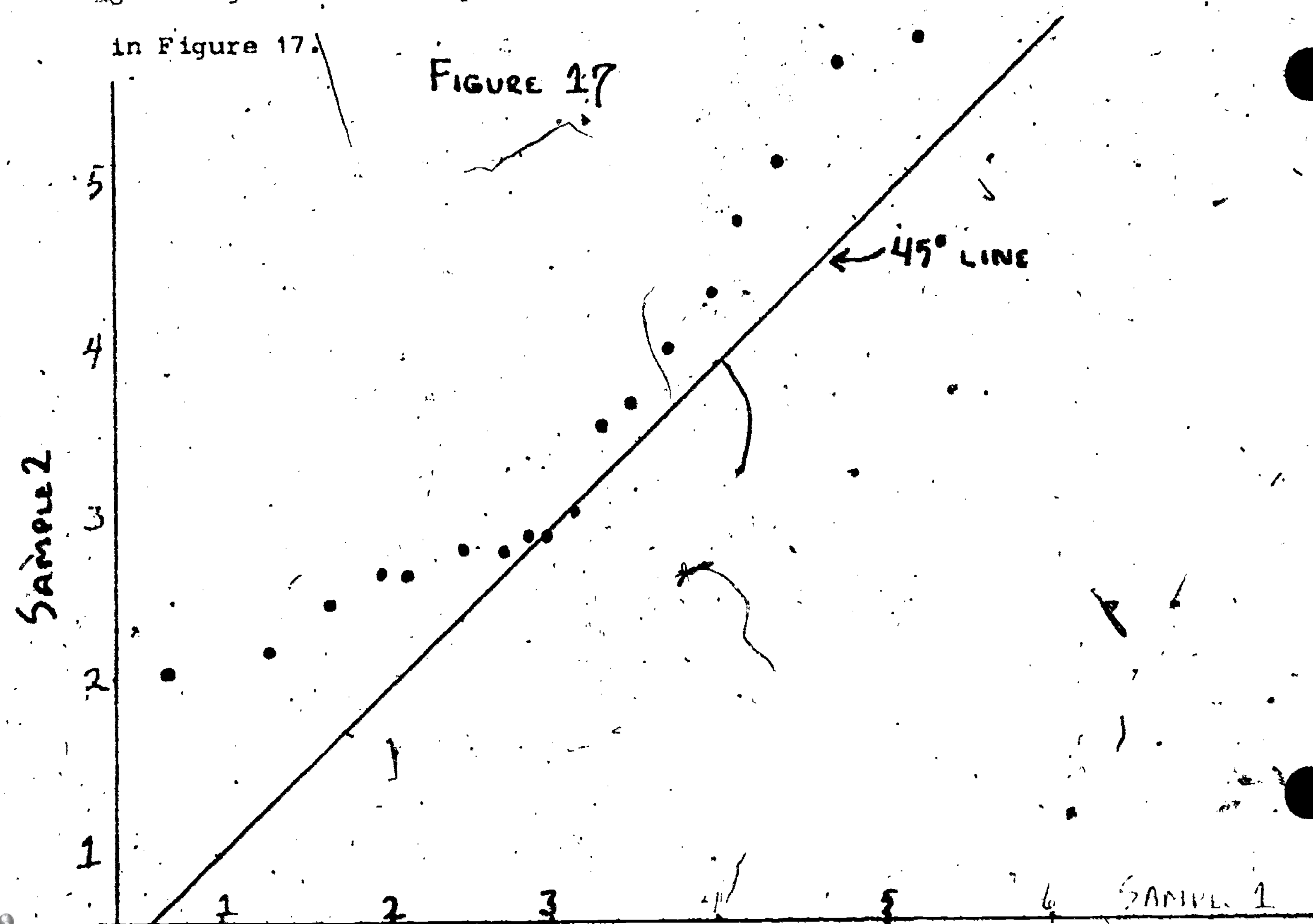
Sample 2:

2.1
2.2
2.4
2.6
2.6 → 25TH QUANTILE
2.8
2.8
2.9
2.9 → MEDIAN
3.1
3.6
3.7
4.1
4.4 → 75TH QUANTILE
4.8
5.2
5.8
5.9

INTERQUARTILE RANGE – 3.9 - 2.1 = 1.8   4.4 - 2.6 = 1.8

Consider Figure 16. Here we have two samples that have equal medians and variabilities; yet even from the histograms we can see that the distributions are not the same. Their distributional shapes are different. If we would take the first sample and fold its histogram around the median value we would see that each half of the histogram is shaped the same. However, the second sample does not have both halves of the same shape. The right hand side is much longer than the left hand side. In this example it is obvious from the histograms that the two samples' shapes differ, but sometimes it is not so obvious. Even here it is not so obvious what else we can say about these data from glancing at the histograms. The q-q plot for these data is given in Figure 17.

FIGURE 17



The points do not all fall near a single straight line, nor do they fall near a straight line except for several outlying points. However, we do see that all points lie above the 45^{o} line from the origin. This means that throughout the entire range of the data each Sample 2 value is somewhat larger than the corresponding Sample 1 value. The amounts by which the Sample 2 values are larger vary over the range of the data, since the points do not fall near a single straight line. The q-q plot shows this difference between the two samples more clearly than does comparing the two histograms. Even though both samples have the same medians, interquartile ranges and there are no extremely outlying values in the tails, there is a definite difference between the two samples; this difference is striking from a glance at the q-q plot. If the data were lifetimes of batteries of equal cost we would prefer Sample 2, since each value is slightly larger than the corresponding value from Sample 1. The difference between the two samples here is qualitatively different from the "tail-differences" discussed in Section II.5, but each type of difference is shown by the q-q plot.

Now let's turn to a different example concerning the relative shapes of two distributions. Figure 18 gives two sets of data and their histograms.

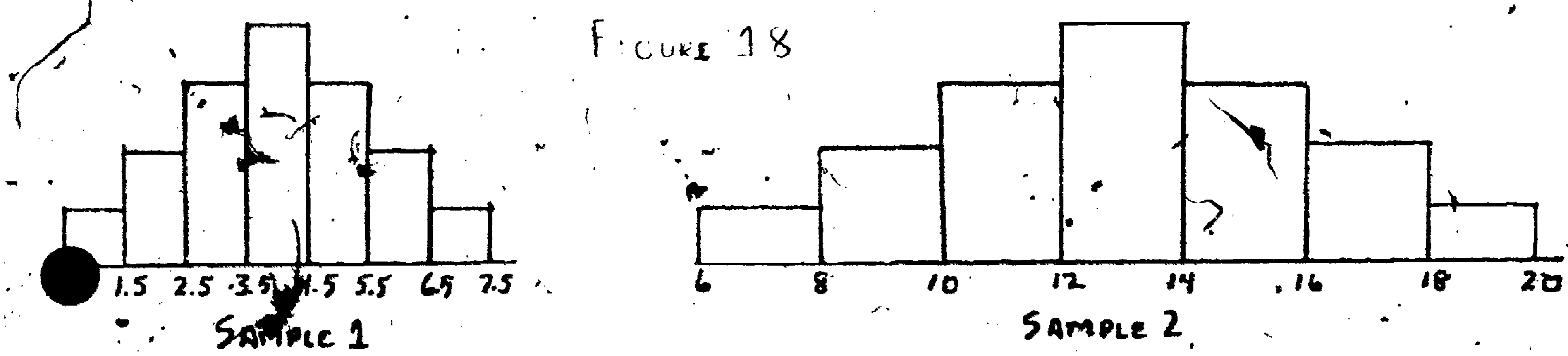
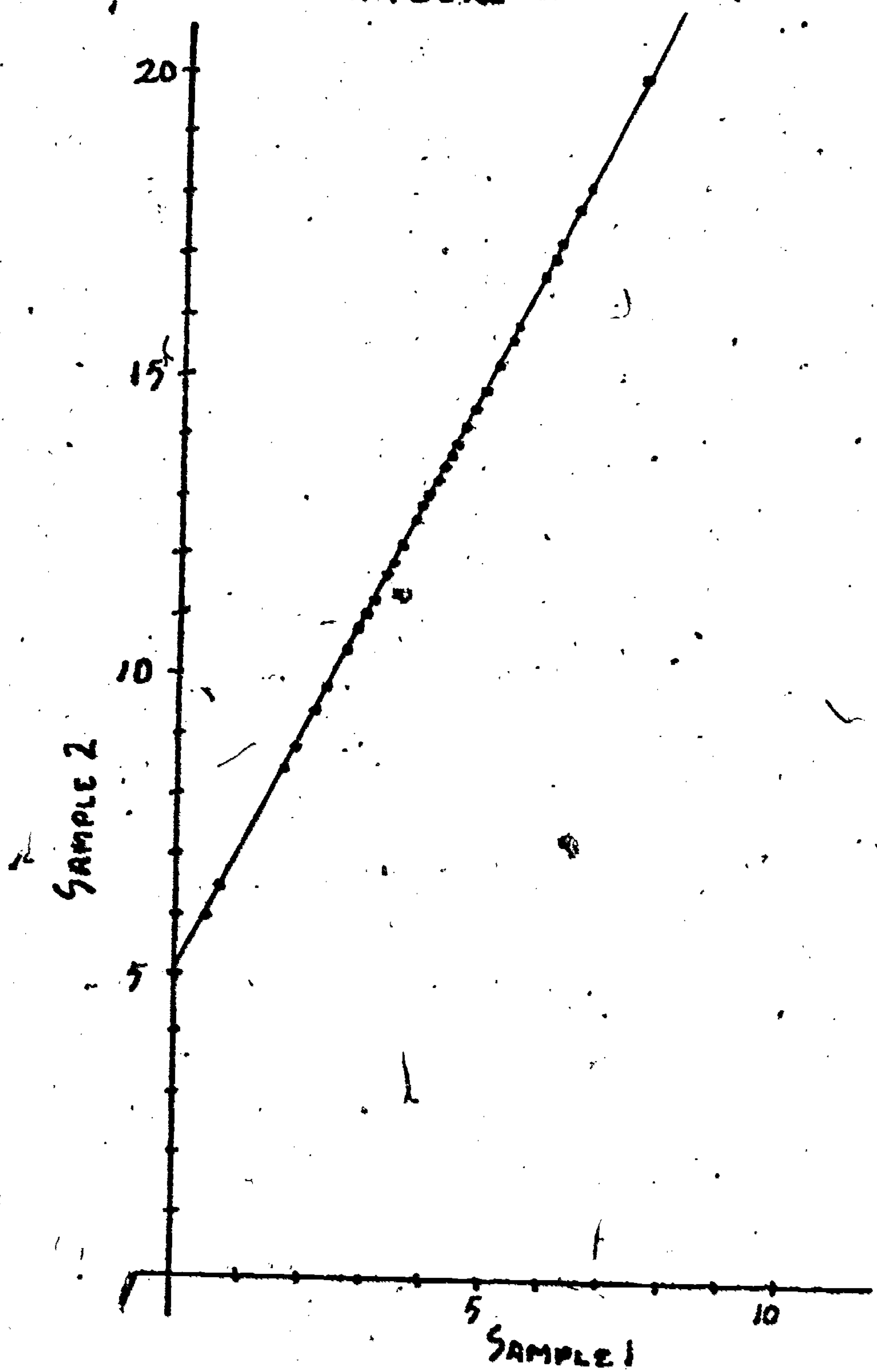


Figure 18

The two sets obviously differ in their locations and variabilities and thus their histograms differ. However, the two histograms do have the same basic shape. The q-q plot is given in Figure 19.

FIGURE 19



The points fall on a straight line. This illustrates a basic property: if two distributions have exactly the same shape then their q-q plot will be a straight line. Similarly, if the two distributions have approximately the same shape, then the points on their q-q plot will fall near some straight line. Now recall that in Section II.4 we learned that if two data sets differ only in their location and variability, then their q-q plot is some straight line; the particular straight line depends on their relative locations and variabilities. Thus, if two distributions have the same shape then their q-q plot is a straight line, and this means that these distributions differ only in location and variability.

It is nice if we can completely summarize the differences between two sets of data by saying that there is only a difference in location and/or variability, and apart from these the distributions are the same. This means that for every aspect of one distribution there is a corresponding aspect of the other distribution, aside from the differences in location and/or variability; i.e., their shapes are the same. The q-q plot is extremely useful for determining if such a summarization is possible. If the points on the q-q plot fall reasonably close to a straight line, then such a summarization is possible. But if the points do not lie near a straight line, then it is not sufficient to say that the differences between the two data sets can be completely summarized by differences in location and/or variability. Then there are other features of the two distributions that also differ.

Even when a straight line does not fit all the points in a q-q plot, often a straight line will pass near many of the points. Then it is useful to draw this line and interpret the differences between the two distributions for the remainder of the points in terms of this straight line. This is basically the idea that was used in discussing tails in Section II.5; there we were concerned with the situation in which only a few points at either end of the data lie quite far from the straight line. However, the principles used there also apply more generally in interpreting the departure of other points on the q-q plot from the straight line.

This paragraph gives a brief discussion of a situation in which an interesting question is whether or not two distributions have the same shape. Suppose we have data on the heights at birth of all 25 boys born in a certain hospital in a certain month, and we also measure the heights of all 25 boys in the 8^{th} grade in a school near the hospital. We would like to know how these two distributions compare; obviously there will be a tremendous difference in location, since an 8^{th} grade boy is much taller than a baby boy. Obviously there will also be a tremendous difference in variability, as the heights of 8^{th} grade boys vary over a much greater range than the heights of baby boys. However, do these differences adequately summarize the differences between the two distributions? Or, for example, are the smallest baby boys relatively smaller compared to the rest of baby boys than are the smallest 8^{th} grade boys? The way to examine these and similar questions is to make a q-q plot of these two distributions and then interpret this plot using the various points discussed in this and the previous sections. (Note that in this example we do not have the baby height and 8th grade height of the same boys. To obtain such data we would have to either find old records or wait 13 years. However, such data would permit the use of other statistical methods to better study the relationships between heights at these two ages than would the data of this example.)

II.7. Interpretation of the Battery Example Q-Q Plot

In Part I we presented an example where a class performed an experiment studying the lifetimes of two brands of batteries. The q-q plot constructed to study this example is presented in Figure 1 on page 4. In Sections II.2-II.6 we have learned how to interpret a q-q plot, so we will now discuss the battery example in terms of the four distributional characteristics. The reader should now go back to Figure 1 and draw the 45° line and a line through the bulk of this data on the plot. These lines will be nearly coincident, and the middle points of the plot closely surround both lines. Therefore, we can consider the two samples' locations and variabilities nearly equal. However, we do notice deviations from this line in both tails. In the lower tail Brand B batteries have higher lifetimes than the corresponding Brand A batteries. In the upper tail Brand A batteries have higher lifetimes than the respective Brand B batteries since the plotted points fall on the Brand A side of the line through the data. As far as shape, since the plotted points basically surround a straight line except for the extreme points we can conclude that the two lifetime distributions have about the same shape except for their tails.

The students concluded from inspecting this q-q plot, that for their purposes the two brands of batteries were essentially the same. However, the differences noted in the tails might be of interest in other more specific questions or possibly even could be the subject of further investigation.

II.8. Another Example of the Use of Q-Q Plotting

The students decided to study whether a crossing guard's appearance affected the speeds of cars at a given intersection. The data consist of measurements of the speeds of cars passing a particular corner between 8:00 and 8:15 on Tuesday and Wednesday of a school week. The weather was clear both days. Every fifth car or truck passing through the intersection was timed, since there were not enough people available to measure every vehicle. Moreover, measuring only every fifth car allows enough of an interval between the vehicles so that their speeds would not necessarily be close to one another. Adjacent vehicles are much more likely to have similar speeds than are vehicles that are separated; if one were told the speed of a car, he could do a lot better job of guessing the speed of the next car than if he weren't told the speed of the first car. Thus, measurements of adjacent vehicles should not be treated as though they are separate, independent quantities. In this problem it seems desirable to be able to treat the measurements as independent of one another, so it is fortunate that there was a sufficient gap between the vehicles whose speeds were recorded. On Wednesday a crossing guard was at the intersection, holding a large sign saying "Caution -- Children Crossing," but on Tuesday no guard was present. On both days the children actually crossed the street by pushing a "Walk" button, waiting for a light to flash red to the traffic, and then crossing the street. On Wednesday the guard did not explicitly slow down the cars, but was merely visible to the driver. The goal was to see if the speeds were significantly lower when the guard was present.

There are 10 values when the guard was present, and 12 values for no guard. The ordered points in each sample are as follows:

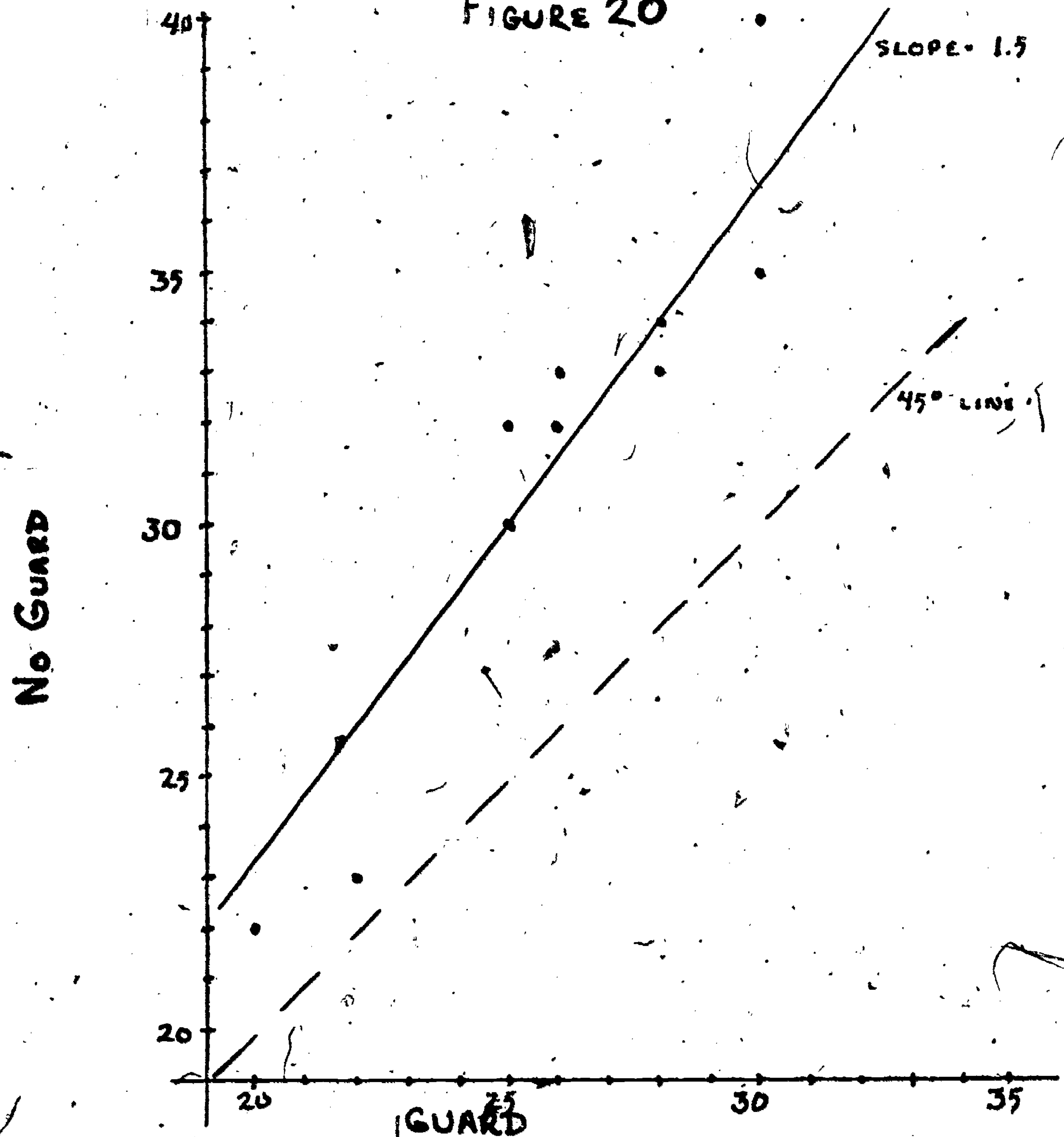
order number	1	2	3	4	5	6	7	8	9	10	11	12
with a guard	20	22	25	25	26	26	28	28	30	30		
no guard	22	23	29	30	32	32	33	33	34	35	35	40

When the samples are of unequal size the q-q plot is a little more difficult to construct than when the two samples have the same number of observations, since we must choose which subset of points from the larger sample to plot against the ordered points of the smaller sample. Appendix B discusses how we choose the values of the larger set of points to be used in the q-q plot. Applying that technique we match the points in the sample of car speeds with a guard to be plotted against the sample points without a guard as follows:

order number	1	2	3	4	5	6	7	8	9	10	11	12
with a guard	20	22	25	25	26	26	28	28	30	30		
no guard	22	23	29	30	32	32	33	33	34	35	35	40

Then the q-q plot is constructed using these pairs of points, giving Figure 20.

FIGURE 20



We will now inspect how well the plotted points surround the 45° line. The 45° line is represented by the dotted line on the plot. All the paired points are located above this line. A solid line which cuts through the plotted points is also drawn on the q-q plot. This solid line does not parallel the dotted line. The solid line's slope is about 1.5. This indicates that the variability with no guard is about 1.5 times larger than when there is a guard. However, we can see that other lines with slightly different slopes could also be drawn near most of the plotted points, so we would not want to conclude that the variability with no guard is exactly 1.5 times as large as when there is a guard. The median of the sample without a guard is about 6 1/2 mph larger than with a guard. This difference appears reasonably large considering the size of the original values. The plotted points lie along the line drawn fairly well, so therefore we can conclude that the two samples have about the same distributional shape and tails. It certainly appears that cars drive faster through this intersection without a guard than when a guard is present.

II.9. Concluding Comments on Q-Q Plots

The above sections have discussed the interpretation of q-q plots in terms of four features of two distributions -- location, variability, tails (extreme values), and shape. Various one-sample statistics, including some discussed in Part I, can be used to summarize each of these properties; by calculating such statistics for both samples we can compare the two sets of data. The discussion and the examples have brought out the point that these properties are not all equally important in all examples; the property or properties that are important in each application depends on the data for that application and the questions of interest.

Graphical methods in general are helpful in examining data, and they often give a useful adjunct to numerical summaries. One graphical method for comparing two samples is to form histograms of each sample and then compare the histograms. Some problems with comparing histograms, though, are the following. It is necessary to choose the class intervals carefully in each histogram or distortions can occur. We must compare two different plots by eye, as the information from both samples is not in one plot. Because of the granularity of histograms, certain detailed characteristics of the data can be missed entirely. If certain obvious differences between the histograms strike you at first, you can easily overlook other subtle differences between the histograms. In general, it may take much effort and careful study to extract all the important information from the data using two histograms. Information extraction is more difficult in this case because of the application of two one sample techniques, rather than using a technique explicitly devised for comparing two samples.

We have seen that q-q plots give a useful graphical way of comparing two sets of data. Some strong points of this technique are summarized below. It is not necessary to pick class intervals, as the data values themselves appear directly on the plot. All the information is summarized in a single plot. In interpreting this plot one is generally comparing points to some straight line; this is a fairly easy thing to do, as deviations of points from a straight line are not hard to spot. If there are no departures from this straight line, one needs only to understand the meaning of the particular straight line to which the comparisons were made to make meaningful comparisons between the two samples. The slope of the line tells one about comparative variability, the height about comparative location. The q-q plot makes it possible to easily pick out interesting aspects of the data that might otherwise be missed. The q-q plot is particularly easy to construct for two equal-sized samples, and it is not too hard to construct for unequal sample sizes. The plot could be made for sample sizes as small as 3 or 4. Examining the plot is exploratory in nature, in that one does not have to decide beforehand the specific aspects of the data to compare; particularly interesting features present themselves. In general, the q-q plot often shows things about the data that might be missed if only other methods were used. Moreover, even for certain properties that are given by other numerical or graphical methods, the q-q plot gives a simple and clear way of presenting and comparing these properties in the two sets of data.

A disadvantage of the q-q plot is that it is not a traditional technique for graphing two sets of data, and it is just presently becoming widely used. Thus the method may be unfamiliar to some, and some explanation of the plot may be required. However, we feel that the advantages of the q-q display are sufficient to warrant the necessary explanation.

A possible danger in examining q-q plots is that of over-interpreting the data. One should not necessarily attach overwhelming importance to every small bump in the plot. It is hard to make general statements about what sort of departures from straight lines are truly important, as the answers depend very much on the particular problem and questions of interest. The possibility of over-interpretation should be kept in mind. Part III presents a technique that is sometimes useful in determining whether or not an apparent difference between two samples is real, or could simply be due to chance deviations.

APPENDIX II-A

This appendix presents two different ways of introducing the q-q plot. Both approaches lead to exactly the same plot as developed in Section II.1.

In Part I we presented the cumulative distribution graph as a useful way of displaying a single set of data. When comparing two sets of data it is natural to consider making two of these graphs, one for each set. Moreover, we could put them both on the same sheet, marking points by x's for one set and by o's for the other set. Figure A1 gives such a plot for the battery data.

This is a useful plot, and we can see many aspects of the data from it. We can see which brand has the larger value, and by how much. General characteristics of one brand having consistently larger or smaller observations across the entire distribution would be apparent, if the x's were consistently either to the right or to the left of the o's.

However, the vertical dimension of this plot is not being used very effectively. It simply gives the order numbers of the observations, spaced equally up the page. Nothing in the vertical dimension is affected by the actual values in the data. Since we would like to use both dimensions in the plot to convey as much information as possible, we are led to consider ways of modifying Figure A1 so as to make the vertical dimension more useful.

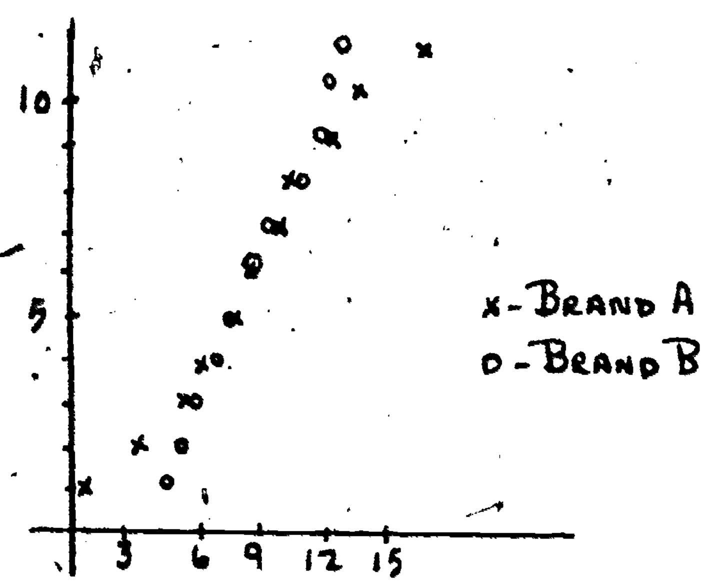
One way of thinking about Figure A1 is that, for each observation order number (1 through 11 here), the values from the two samples are plotted using the horizontal axis. The observation for one data set is indicated by an x and for the other data set by an o. Instead of plotting both values on the horizontal axis we could simply plot a point whose horizontal component is the value from one sample, and whose vertical component is the value from the other sample. That is, for a point x we leave its horizontal component unaltered, but instead of having its vertical component the observation order number, we make the vertical component for x the corresponding value from the other sample, i.e., the value for the o that was previously plotted next to this x. For example, for observation order number 4, Figure 3.2 gives an x at 5.9 and an o at 6.3. Instead, we plot a single point at horizontal distance 5.9 and vertical distance 6.3. For order number 10 we plot a point with horizontal value 12.6 and vertical value 11.4. This is done for each of the 11 ordered observation pairs. The resulting plot has only 11 points rather than the 22 points of Figure 5. Indeed, what we are doing is plotting the smallest value of Brand B vs. the smallest value of Brand A (observations with order number 1), the second smallest value of Brand B vs. the second smallest value of Brand A (observations with order number 2), etc. This is exactly Figure 4, the q-q plot developed in Section II.1.

Thus the above discussion gives a different way of motivating and developing the q-q plot. Again, for equal sized sets of data, we plot the ordered values from one set against the corresponding ordered values from the other set. Still another way of explaining the q-q plot is given below.

First, we need the concept of "quantile." The 30^{th} quantile of a distribution is that value which is larger than 30 percent of the distribution, and smaller than 70 percent. Similarly, the 80^{th} quantile of a distribution is that value which is larger than 80 percent of the distribution, and smaller than 20 percent. The 50^{th} quantile is the median. The q-q plot (an abbreviation for quantile-quantile plot) gives the quantiles of sample A on the horizontal axis, plotted against the quantiles corresponding to the same percentages from sample B on the vertical axis.

Consider what this gives when the two samples have equal numbers of observations, say 11. We might estimate that the smallest value of sample A would be greater than a certain percentage of the population from which the sample was drawn (see resource unit PS4, "Design of Surveys and Samples," for a discussion of "sample" and "population"). The estimated percentage is 1/12 = 8.33%; justification of why 1/12 is a reasonable estimate is not needed here. However, we would also estimate that the smallest value in sample B would be greater than the same percentage of its population, since sample A and sample B are the same size. That is, using reasoning analogous to that for sample A, we would also estimate that the smallest value of sample B would be greater than 8.33% of its population. Thus, the smallest value of sample A and the smallest value of sample B are quantiles corresponding to the same percentage. Note that the point on Figure 4 closest to the bottom left corner is the smallest value in each sample and corresponds to the 8.3 quantile in each sample. Similarly the second smallest value of sample A and the second smallest value of sample B are also quantiles corresponding to the same percentage — here 2/12 = 16.6%. With equal numbers of observations in the two samples, then, plotting the quantiles of sample B vs. the quantiles of sample A corresponding to the same percentages means: plot the smallest value of B vs. the second smallest value of A, etc. We saw in Section II.1 that this gives a q-q plot.

FIGURE A1



APPENDIX II-B

Q-Q Plot With Unequal Sample Sizes

This appendix shows how to construct a q-q plot when the numbers of observations in the two data sets are not equal. The method is still based on plotting observations from one sample vs. corresponding observations from the other sample. Here "corresponding" has to be precisely defined. Since the sample sizes differ, there is not the simple one-to-one relationship between the ordered observations as with equal sample sizes.

The method is illustrated by an example, and all the calculations are given in Figure B1. We will work through the various steps. Suppose the smaller sample has size 7; we use m to denote the size of the smaller sample, so here m = 7. Order the observations from this sample, as before. The size of the larger sample is denoted by n, and in the example this is 12. Also order the observations from this sample. The problem is to match ordered observations in the smaller sample with appropriate ordered observations from the larger sample. Clearly it would not be appropriate to use the smallest 7, or the largest 7, of the 12 observations in the larger set. In order to find the appropriate 7 of these 12 observations do the following. Write the 7 ordered observations and the 7 order numbers as in the left two columns at the bottom of Figure B1. Find m+1; here this is 7 + 1 = 8. Divide each of the m order numbers by (m+1), giving the third column. Now find n+1; here this is 12 + 1 = 13. Multiply each of the fractions in the third column by (n+1), getting the fourth column. Now we round each of the numbers in the fourth column to the closest integer; that is, if the fractional part is greater than 1/2 we round to the next higher number, while if the fractional part is less than 1/2 we round to the next lower number. However, if the fractional part is exactly 1/2, e.g., 6 4/8 = 6 1/2, then we enter this as (6+7)/2. Thus we obtain the fifth column. Each entry in this column will be an integer, unless it is the average of two adjacent integers, as (6+7)/2. The numbers in the fifth column give the order numbers of the observations in the larger sample (of size n) that correspond to each of the m observations of the smaller sample in column 1. Thus the sixth and final column is obtained by going back to the ordered observations in the larger sample. For the value (6+7)/2 we simply average the 6^{th} and 7^{th} observation in the larger sample. Then the q-q plot is formed by plotting the 7(=m) points (2.2,1.7); (3.1,2.4); (3.9,3.3); (4.4,4.2); (4.9,4.8); (6.0,5.7); (6.2,6.4). This is given in Figure B2. Interpretation of this plot follows the exact same principles laid out in Section III for plots constructed from equal sized samples.

Let's step back and examine what has actually been done here. It is helpful to look at the lines near the top of Figure B1 indicating the observation order numbers that are plotted against one another. The median of the smaller sample, order number 4, is plotted against the median of the larger sample, the average of order numbers 6 and 7. This is reasonable. Similarly the extreme observations in the smaller sample are plotted against the more extreme observations of the larger sample.

Another example may also be helpful. Since the method used to form pairs of observations depends only on the sizes of the two samples and not on the actual observed values, this time we omit the observed values. Figure B3 indicates all the calculations required to form a q-q plot when the sample sizes are 6 and 8.

The calculations of column 4 of these tables, (i/m+1)·(n+1), can be somewhat tedious. However, these can be done quickly and accurately with a small calculator if one is available.

Other slightly more complicated methods are sometimes used to form q-q plots from samples of unequal size. These methods differ from this approach in that they require interpolating and averaging two adjacent observations in the larger sample to get columns 5 and 6, unlike the simple rounding done here. However, for most sets of data there is little qualitative difference between the q-q plot obtained using the simple method given here and the q-q plot obtained using a more complicated interpolative scheme. Thus, the method presented in this appendix is adequate for most purposes.

FIGURE B1

Smaller sample size = 7 = m

Observation		2.2	3.1	3.9	4.4	4.9	6.0	6.2				
Order number		1	2	3	4	5	6	7				
Order number	1	2	3	4	5	6	7	8	9	10	11	12
Observation	1.5	1.7	2.4	3.1	3.3	4.1	4.3	4.8	5.6	5.7	6.4	6.7

Larger sample size = 12 = n

Obs.	(Order number) (=i)	(Order number) / m+1	$\left(\frac{i}{m+1}\right)$ (n+1)	Round to	Obs.
2.2	1	1/8	13/8 = 1 5/8	2	1.7
3.1	2	2/8	26/8 = 3 2/8	3	2.4
3.9	3	3/8	39/8 = 4 7/8	5	3.3
4.4	4	4/8	52/8 = 6 4/8	(6+7)/2	4.2
4.9	5	5/8	65/8 = 8 1/8	8	4.8
6.0	6	6/8	78/8 = 9 6/8	10	5.7
6.2	7	7/8	91/8 = 11 3/8	11	6.4

FIGURE B2

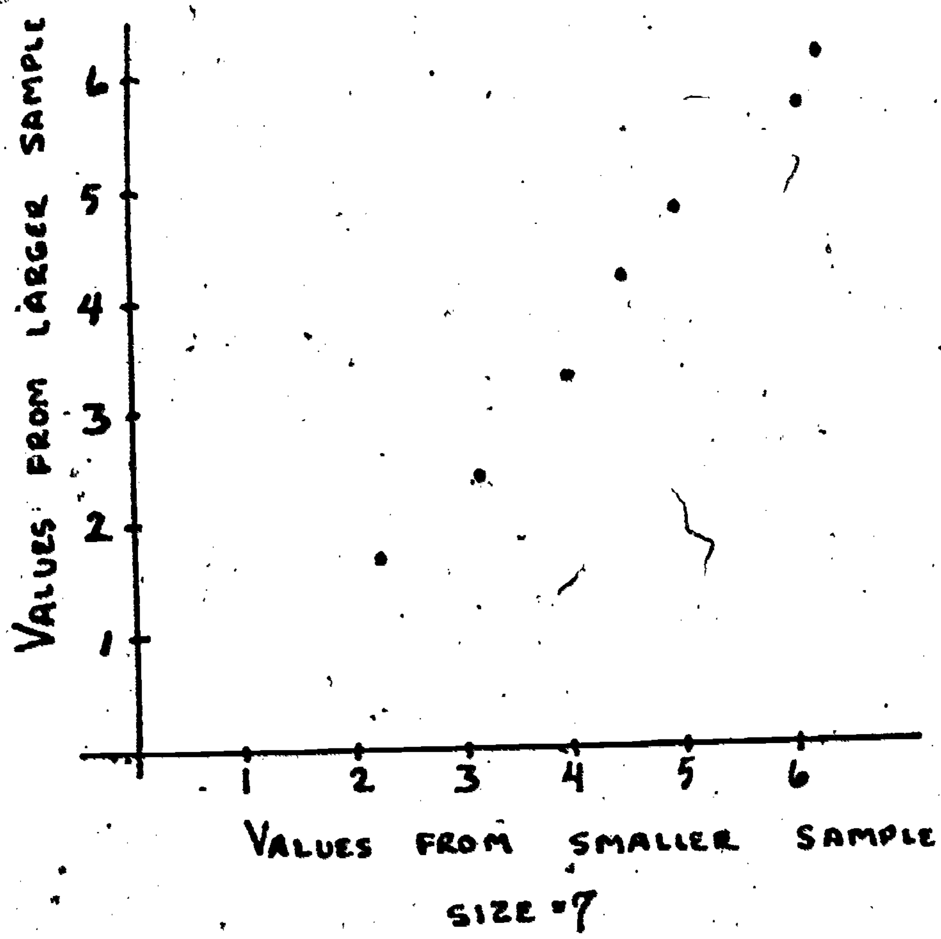

VALUES FROM LARGER SAMPLE
6
5
4
3
2
1
1
2
3
4
5
6
VALUES FROM SMALLER SAMPLE
SIZE = 7

FIGURE B3

Smaller sample size = 6 = m

Order number: 1 2 3 4 5 6

Order number: 1 2 3 4 5 6 7 8

Larger sample size = 8 = n

Obs.	Order number = i	$\frac{i}{m+1}$	$\left(\frac{i}{m+1}\right)(n+1)$	Round to	Obs.
	1	1/7	9/7 = 1 2/7	1	
	2	2/7	18/7 = 2 4/7	3	
	3	3/7	27/7 = 3 6/7	4	
	4	4/7	36/7 = 5 1/7	5	
	5	5/7	45/7 = 6 3/7	6	
	6	6/7	54/7 = 7 5/7	8	

EXAMINING ONE AND TWO SETS OF DATA

PART III: ASSESSING THE SIGNIFICANCE OF THE DIFFERENCES BETWEEN TWO SAMPLES

by

Lorraine Denby and James M. Landwehr

Bell Telephone Laboratories
Murray Hill, New Jersey 07974

Often after looking at the data both with summary statistics and graphical methods we might still have questions as to what this really means. Sometimes it will be obvious that there is a distinct difference between the two samples. You might be sure that this difference is large enough to be concerned with and feel confident that the data have been sufficiently analyzed. Other times the two samples are so similar that you also are sure that there is no difference. But, for the times when you are not certain you may want to use a technique designed to help you put a degree of certainty on your decision. In such situations the permutation test descussed in this part may prove useful.

In fact, if there is some question concerning whether or not an apparent difference observed in the data is "real," one should consider repeating the experiment to see if the same sort of difference occurs again. Reasons for doing this were discussed in Part I. However, replication of the experiment is often expensive and/or time consuming; then it is useful to examine the present data more carefully to see whether or not a difference such as has been observed could be due only to random fluctuation. This part presents a statistical technique for this purpose. But in order to explain why this technique is reasonable we will first consider what sort of analysis we might do if part of the experiment had been replicated. This analysis has certain similarities with the permutation test presented later in this part.

Suppose someone has measured the time it took for three fifth-grade boys and three fifth-grade girls to cross an intersection. The times, in seconds, are boys - 11, 7, 12; and girls - 5, 9, 8. All of the children crossed separately, so their times should not have affected each other. One might assume, until evidence to the contrary is presented, that it takes boys and girls the same amount of time, on the average, to cross the intersection. Do these data give enough evidence so that we should abandon this assumption? Obviously it does not take all boys exactly the same amount of time to cross the street, as the times here -- 7, 11, and 12 -- are not all the same. However, it is still quite possible that the average time for boys -- apart from random differences from boy to boy and from one crossing to the next for the same boy -- is the same as the average time for the girls. It is, of course, possible that the average crossing time depends on the age of the child, but in this example we are considering children of approximately equal age.

If we were to replicate this experiment we would measure crossing times for more sets of boys and girls. However, suppose that for some reason we have obtained crossing times for several sets of three boys but only for one set of three girls. In fact, in replicating the experiment we would want to obtain data both for more sets of boys and more sets of girls, since we would want the variety of "experimental conditions" over which the data were obtained to be comparable for the boys and for the girls. By "experimental conditions" we mean things such as the day of the week, the speed and density of the traffic, the time of day, the weather, etc. The following discussion, concerned with an appropriate analysis if only the boys' data were replicated, is intended merely to help in understanding the permutation test, which is presented later in this part.

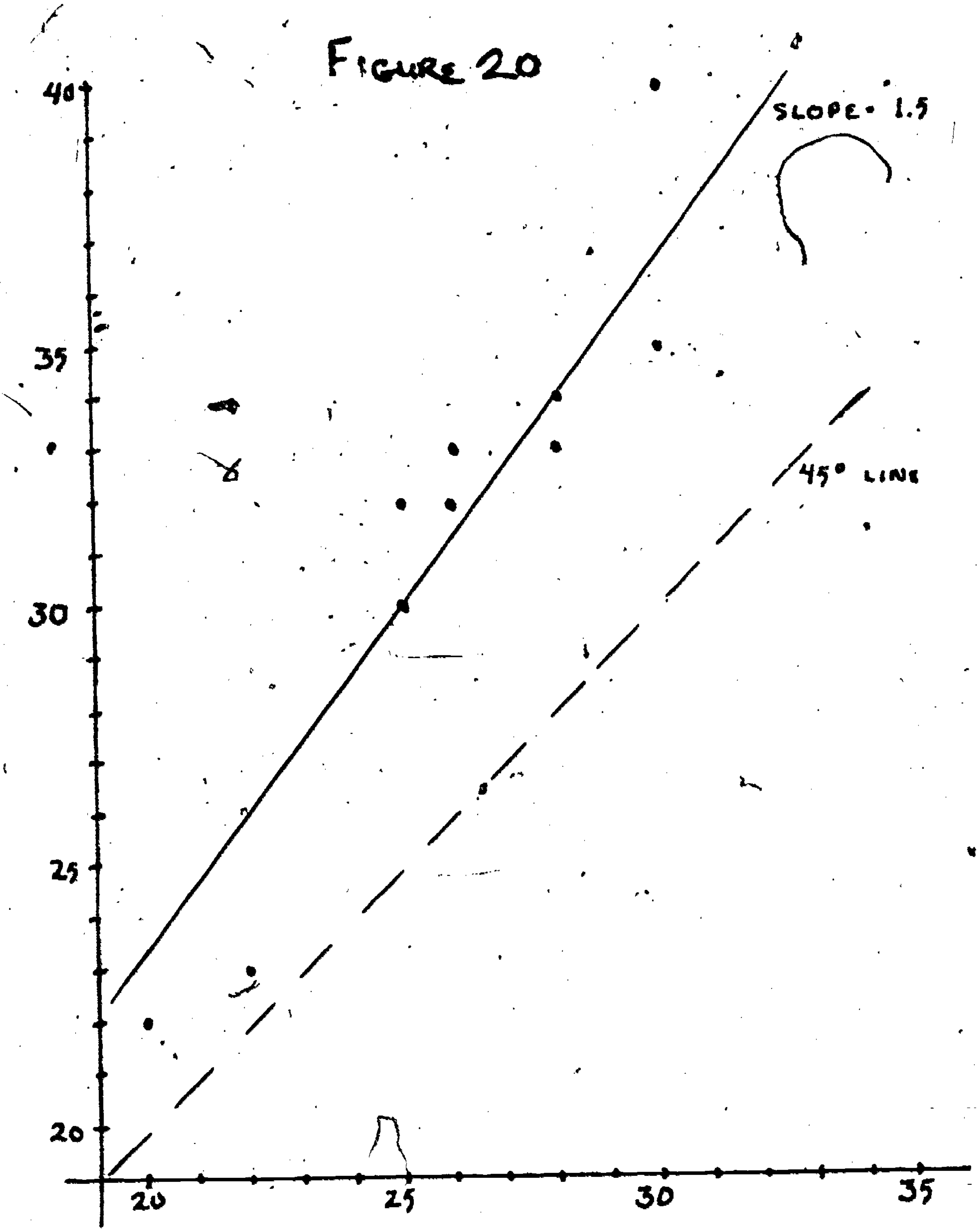
Suppose we measure the crossing times of three girls and for 20 different sets of three boys. Each set of three boys is measured either on a different day or at a different time of the same day. Since we are concerned with the average crossing times for the girls and the average crossing time for the boys, it is reasonable to calculate the average crossing time for each of the twenty sets of three boys and for the set of three girls. Simply because of random fluctuation we would not expect all of the twenty boys' averages to be exactly the same. These averages will vary from one to another and the amount of this variation is an indication of the fluctuation due simply to random chance. This assumes, though, that there is nothing external that causes the boys to cross particularly quickly or slowly on the different days; i.e., the experimental conditions that do change do not influence the results. Then what we would like to do is compare the distribution of boys' averages to the single girls' average crossing time. This could be done graphically by forming a cumulative distribution graph (see Part I) of the twenty boys' average crossing times and seeing where the girls' average falls within this distribution. For example, if the girls' average falls near the middle of this distribution, then we see that it is not unusual to have a boys' average time near the girls' value, or higher or lower. The girls' value is not extreme; thus, the girls' value seems to fit in with the distribution of boys' values. Since the boys' values presumably differ only because of random fluctuations, there is no strong evidence that the girls' value does not also differ only because of random fluctuation; the underlying girls' average could plausibly be the same as the underlying boys' average crossing time. However, suppose the girls' average falls at the extreme of the distribution of the boys' average crossing time. This situation implies that it is not so likely that random fluctuation alone will give a value as extreme as the observed girls' average. Thus here the evidence is stronger that the girls' average crossing time is really different from the boys' average crossing time.

These ideas relate to the permutation test developed below in that "pseudo-replicates," corresponding in a sense to the twenty sets of three boys' crossing times, will be generated from the initial data. Then the distribution of "pseudo-replicates" will be compared to the difference between the boys' and girls' average crossing times in the observed data.

However, we wish to emphasize that the permutation test is not intended as a substitute for replication of the experiment for reasons discussed in Part I. Moreover, replication of only one of the sets of measurements, such as gathering measurements only for more boys in the crossing time example, would only be appropriate if we could be positive that any change in the experimental conditions could not possibly affect the results. Normally this will not be the case and we would collect more of both sets of measurements. In the crossing time example we would want data for both sets of three boys and sets of three girls. The ideas involved in the above discussion, though, are useful in understanding the permutation test.

The permutation test helps decide whether there is a significant difference between the value of a given one sample statistic in the two samples. For example, it can be used with the medians of two samples, or the interquartile ranges or the means. We will be discussing in detail the permutation method as applied to determining whether there is a difference in means. Later we will also use one of the examples to illustrate this technique for comparing medians. This method could also analogously be used with other more complicated statistics but we will leave the extension to the reader.

A more precise way of phrasing the question of interest is the following. Let us hypothesize that the phenomenon underlying the two samples is the same; given this hypothesis, then, the two samples should differ only as a result of random fluctuation -- there should be no systematic differences between the two samples. The question can be stated: "Does the evidence in the data refute the hypothesis that the phenomena underlying the two samples are the same?" Sometimes, as in Figure 20 of Part II, page 38, the answer to this question is not completely obvious.



Nevertheless, this is an important question, and often it can be answered satisfactorily by using the statistical methods that will be presented below.

Before developing statistical methods for answering this question, another short digression is needed in order to develop a concept fundamental to the material that will follow.

Consider a coin that we hypothesize is fair -- that is, we suppose that the coin is equally likely to land heads or tails when tossed. We will toss the coin 100 times and record the total numbers of heads and tails. What sort of results would seem to refute our hypothesis that the coin is fair? If the coin landed heads all 100 times, and never tails, certainly everyone would agree that this is sufficient evidence that the coin is not equally likely to come up heads or tails. In other words, if the coin were fair, it would be virtually impossible for 100 independent tosses to all land heads; therefore, observing all 100 heads leads us to reject the hypothesis that the coin is fair. The evidence is strong enough to refute the hypothesis. Indeed, if the coin landed heads 97 times and tails 3 times, everyone would still agree that this is sufficient evidence that the coin is not equally likely to land heads or tails. Again, the evidence from the 100 tosses is sufficient to reject the hypothesis that the coin is fair. Similarly, results of 100 tails and 0 heads, or 97 tails and 3 heads, are also obviously strong enough to conclude that the coin is not fair.

Now suppose that the 100 tosses give exactly 50 heads and 50 tails. This clearly gives no evidence whatsoever to refute the hypothesis of a fair coin, as the proportions of heads and tails are exactly equal. Now instead suppose that we observe

51 heads and 49 tails; is this evidence enough to refute the hypothesis that the coin is fair? The answer would seem to be no, because chance alone could easily give a fluctuation of this size in 100 tosses. We would not expect a fair coin to necessarily give exactly an equal number of heads and tails in 100 tosses, just as we would not expect exactly the same total number of heads in the next 100 tosses as we observed in the last 100 tosses. Observing a slightly different number of heads in the next 100 tosses would not lead us to conclude that something mysterious had happened to the coin and changed the coin's propensity to land heads. Similarly, observing 51 heads and 49 tails is close enough to the "expectation" of 50 heads and 50 tails so that we would not want to reject our hypothesis that the coin is fair.

However, suppose the tosses give 58 heads and 42 tails, or perhaps 65 heads and 35 tails. Are either of these enough evidence to dismiss the hypothesis that the coin is fair? While everyone's intuition will lead him to the stated conclusions for the previous examples, these new cases are more difficult, and intuition alone does not give a satisfactory answer. One thing that does seem clear is that if results of 58-42 are sufficient to reject the hypothesis of a fair coin, then so should results of 65-35 also be sufficient to reject this hypothesis. Results of 65 heads and 35 tails give stronger evidence against the coin being fair than do results of 58 heads and 42 tails, since 65-35 is a more extreme deviation from the "expected" 50-50 than is 58-42. But are results as extreme as either 65-35 or 58-42 so extreme that the hypothesis of a fair coin is no longer tenable?

If one were to ask several people their views on these questions, most people would have difficulty in reaching a conclusion at all, as opposed to the 97-3, 50-50, and 51-49 cases, where there would be no difficulty. Moreover, some people will say that the evidence is sufficient to conclude that the coin is not fair, while others will say that the evidence is not sufficient. Intuition is not satisfactory here, and some mathematical analysis (using probability theory and the binomial distribution) is helpful, but this is not the place to develop that theory.

The important points from this discussion can be summarized as follows. Very extreme differences of the numbers of heads and tails in 100 tosses of a coin are, intuitively, quite unlikely to occur if the coin is fair. Therefore, if we observe an extreme result we are safe in concluding that the coin is not fair. However, a very even result, such as 51-49, seems quite reasonable if indeed the coin is fair. Therefore, such a result does not enable us to conclude that the coin is not fair. Intuition alone does not allow the formation of conclusions in many intermediate situations, but at least we can order possible results, with more extreme results, such as 65-35, giving more evidence against the hypothesis that the coin is fair than do less extreme results, such as 58-42.

Now we will return to the problem discussed in the beginning of this part -- whether or not the differences between two samples are large enough so that they cannot be attributed to chance alone. This problem will be discussed in terms of the boys' and girls' crossing time example, and the analysis will be related to the concepts in the coin tossing example brought out above.

Let us start, then, with the hypothesis that the time it takes fifth grade boys to cross the street is the same as the time it takes fifth grade girls to cross, apart from random fluctuations. We will now examine the data given earlier in this part in the light of this hypothesis and see if something "very unlikely" has occurred. If something unlikely has happened we will then abandon the hypothesis of equal average crossing time. This whole process is analogous to the coin tossing example, where the hypothesis was that the coin was fair. Given a fair coin, observing 97 heads and 3 tails is clearly "very unlikely." Thus, if we do observe this result, we abandon the hypothesis that the coin is fair. The consequences of the hypothesis (of equal average crossing time for boys and girls) that will be worked out below correspond to the mathematical analysis that would be needed to determine if the 58-42 or 65-35 results in the coin tossing experiment are extreme enough to drop the hypothesis of a fair coin.

Assume that the crossing times are the same for boys and girls, apart from random fluctuations. Suppose someone tells us that crossing times were measured separately for six children, three boys and three girls, and that the six times were 9, 12, 7, 5, 11, 8. However, the names of the children were lost, so we cannot tell which time is for a boy and which time is for a girl. Then would we have any way of inferring which times were boys and which were girls? Under the present assumptions, no. If the times for boys and the times for girls were distributed in exactly the same way, then we have no way of knowing which times were boys and which were girls. Is it more likely that the boys' times were 9, 12, 7 (and thus the girls' times were 5, 11, 8) or that the boys' times were 7, 5, 11 (so the girls' were 9, 12, 8)? Neither is more likely. Our assumption of equal crossing time for boys and girls, apart from random fluctuations, says that any particular value is just as likely to arrive from a boy or from a girl. Then the assignment of any three of the six values -- we know there were three boys -- to the boys is as likely to be correct as the assignment of any other three values.

Consider all possible different assignments of the six values to the three boys. To obtain these, first order the six values: 5, 7, 8, 9, 11, 12. Assigning 8, 11, and 12 to boy_1, boy_2, and boy_3 will be equivalent, for our purposes, to assigning 11, 12, and 8 to boy_1, boy_2, and boy_3, for example. That is, the only important point is which set of three times is assigned to the boys, and not the specific boy to which each time is assigned. Finally, once three times have been assigned to boys, the remaining three times must be assigned to the girls. We know that there were three boys and three girls; we just do not know which specific times were boys and which were girls. All possible assignments are listed in Table 1. Appendix A discusses the construction of this table; it gives a way of ordering the possible assignments so that you can be sure you haven't missed any.

TABLE 1

Assignment Number	boys	girls	average crossing time: boys	average crossing time: girls	difference
1	5,7,8	9,11,12	6 2/3	10 2/3	4
2	5,7,9	8,11,12	7	10 1/3	3 1/3
3	5,7,11	8,9,12	7 2/3	9 2/3	2
4	5,7,12	8,9,11	8	9 1/3	1 1/3
5	5,8,9	7,11,12	7 1/3	10	2 2/3
6	5,8,11	7,9,12	8	9 1/3	1 1/3
7	5,8,12	7,9,11	8 1/3	9	2/3
8	5,9,11	7,8,12	8 1/3	9	2/3
9	5,9,12	7,8,11	8 2/3	8 2/3	0
10	5,11,12	7,8,9	9 1/3	8	1 1/3
11	7,8,9	5,11,12	8	9 1/3	1 1/3
12	7,8,11	5,9,12	8 2/3	8 2/3	0
13	7,8,12	5,9,11	9	8 1/3	2/3
14	7,9,11	5,8,12	9	8 1/3	2/3
15	7,9,12	5,8,11	9 1/3	8	1 1/3
16	7,11,12	5,8,9	10	7 1/3	2 2/3
17	8,9,11	5,7,12	9 1/3	8	1 1/3
18	8,9,12	5,7,11	9 2/3	7 2/3	2
19	8,11,12	5,7,9	10 1/3	7	3 1/3
20	9,11,12	5,7,8	10 2/3	6 2/3	4

Under our assumptions, the argument above implies that any of these assignments is as likely to be correct as any other; they are all equally likely. Since there are 20 possible assignments, each has chance 1/20 = 5% of being correct. This argument was based on our hypothesis that the crossing times were the same for boys and for girls. What we have done here is create 20 "pseudo-replicates" of three boys' crossing times from our original set of six measurements. Now these "pseudo-replicates" will be compared to the actual observed boys' crossing times in a manner analogous to the discussion near the beginning of this part concerning what we would do if we really did have replicates for the boys.

For each assignment, we can calculate the average crossing time for the boys and the average crossing time for the girls. These are given in the right side of Table 1. Under the hypothesis of equal average crossing time for boys and girls, we would expect these numbers to be the same for the true assignment of the six values to the three boys and three girls, apart from random fluctuation. The difference in the average crossing times for boys and girls gives a natural measure of how extreme a given assignment is. This difference is analogous to the number of heads in 100 tosses in the coin tossing example; an average crossing time difference of 0 corresponds to 50 heads.

If the true average crossing times are not equal in this problem, then we don't really know whether the boys would be faster or the girls would be faster. Each possibility is certainly plausible, though different people might have different opinions about which is more likely. Thus here it is reasonable to disregard the sign of the difference between the average crossing times, as is done in the right column of Table 1. Thus two differences that are equal are treated as giving equal evidence against the hypothesis, whether or not the boys' or the girls' average is larger. All possible assignments from Table 1 can be ordered in terms of the magnitude of the resulting differences; this is given in Table 2.

Continuing the analogy with the coin tossing example, we maintain that a more extreme result gives more evidence against the original hypothesis (be it that crossing times are equal or that the coin is fair) than does a less extreme result. In the coin tossing example we only used our intuition in deciding that certain results (such as 99-1) were so extreme that they obviously would cause abandoning the hypothesis of a fair coin. For less extreme results (such as 65-35) we did not have a quantitative measure of how unlikely such results really are. Here, though, the analysis has shown that each of the 20 values in Table 2 is equally likely, as a consequence of our hypothesis that the crossing times are equal. Thus, assuming this hypothesis and looking at the values in Table 2, we can make statements such as the following: the chances are 2 of 20 (10%) that the difference is 4; the chances are 6 of 20 (30%) that the difference is greater than or equal 2 2/3 (i.e., that the difference is 2 2/3 or 3 1/3 or 4); the chances are 14 of 20 (70%) that the difference is greater than or equal 1 1/3 (i.e., that the difference is 1 1/3, 2, 2 2/3, 3 1/3, or 4). Recall again that

TABLE 2

Order of the Differences from the 20 Assignments

Assignment Number	Difference	Rank
1	4	1
20	4	2
2	3 1/3	3
19	3 1/3	4
5	2 2/3	5
16	2 2/3	6
3	2	7
18	2	8
4	1 1/3	9
6	1 1/3	10
11	1 1/3	11
10	1 1/3	12
15	1 1/3	13
17	1 1/3	14
7	2/3	15
8	2/3	16
13	2/3	17
14	2/3	18
9	0	19
12	0	20

all of these statements are explicitly based on, and are consequences of, the assumption that boys have the same distribution of crossing times as girls.

Now we want to use this information to determine how "unlikely" the actual observed crossing times for the three boys and three girls really are. To do this we need to determine what the chances are of obtaining a result at least as extreme as the result that has actually been observed. Then, if the chance of this happening is sufficiently small, we know that we have observed an unlikely event (analogous to 97 heads and 3 tails). Therefore, we would conclude that the hypothesis on which the analysis was based -- that the distributions of boys' and girls' crossing times are the same -- is not correct.

The previous analysis has been performed without using the information specifying which of the three times were, in fact, from boys and which three were from girls. For this data, the boys' times were 7, 11, and 12, giving a difference of 2 2/3. How unlikely is this -- how much evidence does this give against the hypothesis of equal crossing times? Another way of phrasing this is, "What are the chances of such an occurrence when the hypothesis is true?"

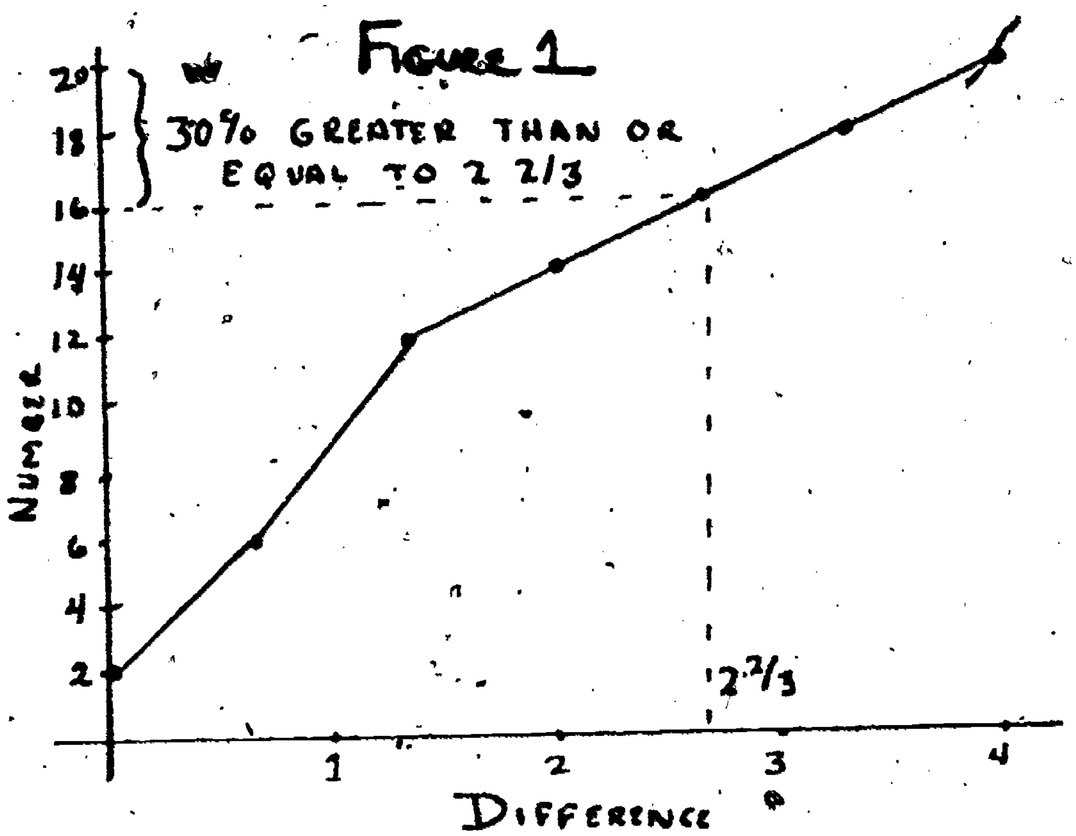
Since a large difference gives more evidence against the hypothesis than does a small difference, the appropriate calculation involves finding the chance of getting as much, or more, evidence against the hypothesis as has been given by the data. It is not reasonable to just calculate the chance that the difference is exactly equal to the observed difference from the data. Such a calculation would imply, for example, that a difference of 0 gives more evidence against the hypothesis of equal crossing times than does a difference of 1 1/3, since the chance of observing 0 is 10% (2 of 20) while the chance of observing 1 1/3 is 30% (6 of 20). This is clearly not reasonable, as a difference of 0 gives the least possible evidence against the hypothesis.

The crucial point in these calculations is that we find the number of differences in Table 2 that are as large as or larger than the observed differences. If the hypothesis of equal crossing time is true, this tells how unlikely it would be to observe a value giving at least as much evidence against the hypothesis as the actual observations give. This is called a "tail" calculation, as we look at the "tail" of the distribution of possible differences, not just at the actual observed difference. This calculation guarantees that a more extreme result (such as 1 1/3) will have a smaller chance than the less extreme result (such as 0), as the calculation for the less extreme result will include those terms that enter into the calculation of the more extreme result. For example, the chance of a difference greater than or equal to 0 is 100% (20 of 20) while the chance of a difference greater than or equal to 1 1/3 is 70% (14 of 20). This is intuitively satisfying, as the more extreme result thus gives more evidence against the hypothesis than the less extreme result.

For the observations in the crossing time example, the difference is 2 2/3, so the chance that the difference of the average girls' time from the average boys' time is at least 2 2/3 is 6 of 20, or 30%. Rephrasing, the chance of getting a difference as large or larger than that observed is 30%, if the hypothesis that the girls' crossing times and the boys' crossing times are the same is indeed true.

A graphical comparison of these differences could be obtained by forming a cumulative distribution graph of the differences in Table 2 as pictured in Figure 1 and then seeing whether or not the observed value of 2 2/3 is extreme in terms of this distribution. This is analogous to the idea in the beginning of this part for comparing the distribution of boys' average crossing time to the single girls' average crossing time.

Figure 1



From this graph we get the visual idea that 2 2/3 is in the middle of the distribution and does not fall in the extremes. There is a 30% chance of obtaining a difference greater than or equal to 2 2/3 if the hypothesis of equal crossing times is true. This agrees with the answer obtained in the previous paragraph.

In this example a sufficiently extreme difference with either a positive or negative sign would intuitively make us reject the hypothesis for the problem; this is the reason we disregarded the sign in the above calculations. However, there are other problems where an extreme difference in one direction would cause us to reject the hypothesis, while an extreme difference in the other direction would intuitively be quite confusing. As an example, consider the car speed example of Part I. The speeds of cars crossing a certain intersection are measured on two consecutive school days. On the first day no police guard is stationed at the intersection, while the second day a guard is present. One might wonder whether or not the data show an effect of the guard on the speeds of the cars. To analyze the data using the methods just presented, we state the hypothesis of "no difference" as follows: we hypothesize that the average speed of the cars is the same when the guard is present as when he is not present. Then we see if the data give evidence to reject this hypothesis. Intuitively, if the speeds of the cars are much slower when the guard is present compared to when he is not present, we would want to reject this hypothesis. However, if the speeds are much faster when the guard is present, we are surprised and confused. It is hard to believe that people would drive substantially faster when they see a police guard than when they do not. If we do observe people driving substantially faster with the guard present, we might look for other explanations.

Was the weather different on the two days, causing a change in speeds? Was the traffic density different? Were the measurements really made correctly? Were the two sets of measurements made at the same time of day? Was there some other mitigating factor nearby, causing a change in driving habits? (Indeed, all of these potential problems should be considered in any case in the analysis of data from this example.) For this example, then, only an extreme difference in one direction -- average speed lower with policeman present -- would intuitively cause us to reject the hypothesis of equal average speed. Such an example is called "one-sided". Problems where either a positive or a negative difference could lead to rejection of the hypothesis are called "two-sided". Slightly different calculations are required in the two types of problems.

In a two-sided problem an extreme deviation in either direction could lead to rejection of the hypothesis. Thus, the sign of the difference can be ignored for these calculations, and the method is exactly as given above. However, in a one-sided problem only extreme events in one direction could lead to rejection of the hypothesis. Then we must retain the signs of the differences and calculate the chance of observing a result as extreme or more extreme as the data, in the direction that would lead to rejection of the hypothesis. An example of this type of calculation is given near the end of this part.

Now it is necessary to leave the strictly objective analysis of this data and answer a subjective question. We have observed an event that our results show has a chance of 30%. The number obtained by this whole process - here 30% - is called the attained significance level. If the hypothesis of equal crossing time is correct, then about 30% of the time we will observe an event at least as extreme as the present data. Is this attained significance level, 30%, so small so that we should conclude that something very unlikely has happened? Using the coin tossing analogy, our intuition told us that the chance of observing a result as extreme as 97 heads and 3 tails, if the coin were fair, was so very small that, if we do in fact observe 97 heads and 3 tails, then we must conclude that something is wrong; the hypothesis of a fair coin is not correct. Is the chance here -- 30% -- so small that we should make a similar conclusion for this problem? If we were to decide that 30% is small enough, then the conclusion would be that the hypothesis of equal crossing times for boys and girls is not tenable. The data would be strong enough for us to reject this hypothesis.

How should we actually use the attained significance level to guide us in our consideration of the data? There is no simple, universal answer; appropriate uses depend on the purposes of the analysis and the resources available. However, some general considerations can be outlined. In many problems we will first simply want to summarize, either for ourselves or for others, the degree of evidence that the data give against the initial hypothesis; presenting the attained significance level gives a precise way of doing this. If this number is fairly large, such as 30%, 60%, 45%, or even 10%, then many people would feel that an event has occurred that is not at all surprising; the data have not given them any reason to change their assumption that the initial hypothesis is correct. However, if the attained significance level is quite small, say 1%, 0.04%, or 0.1%, then most people would feel that the data have given them adequate justification for abandoning the initial hypothesis. Many people feel ambivalent concerning numbers in the range of 1% to 10%, feeling that they have fairly strong, but not overwhelming, evidence to reject the hypothesis.

However, many problems ultimately require some action, or decision, on the part of the scientist, and much more has to be considered than simply the attained significance level in order to arrive at an appropriate response. For example, consider the car speed "with guard" and "with no guard" example of Part II. Suppose the attained significance level is so small that no one would still maintain that a guard has no affect on the car speeds. Does this prove that a guard should be hired for this intersection? Not necessarily. A number of questions should still be asked. Does reduction in speed due to the guard contribute enough towards safety to compensate for the cost of the guard? If the guard reduced all speeds from 70 mph to 60 mph, the difference would be extremely significant, in the sense that the guard has a definite effect. However, the contribution to safety would appear minimal, as 60 mph is still so fast as to be very dangerous to pedestrians. Similarly, if all speeds were reduced from exactly 15 mph to exactly 13 mph, the guard would still have a definite effect, but the effect might not be large enough to warrant the cost. Moreover, even if the cost of the guard were considered warranted, is this the best way of spending the money? Alternatively, would speed bumps be more effective, or perhaps a series of signs? Would some other location of the guard, away from the intersection, lead to an even greater reduction in speed? Various questions such as these should be considered in problems where some action or recommendation is contemplated.

In summary, suppose a problem involves comparing two sets of data, and suppose it is reasonable to pose an initial hypothesis that there is no difference between the two sets. Moreover, suppose examination of the data (including graphical examination) indicates that calculation of the attained significance level of the initial hypothesis is worthwhile. Then the attained significance level generally gives additional valuable information, and its interpretation is an important part of the analysis of the data. However, it is by no means the only part of the analysis, and other factors must be considered in order to arrive at an appropriate response or recommendation for the original problem.

The entire method that has been discussed in terms of the crossing-time example could be described as judging the statistical significance of the difference between two samples by using the permutation test. A brief review of the central points follows. The method is really only needed when there is a possible difference between the two samples, but the difference is not so extreme as to be obvious. A hypothesis is stated that there is no difference between the situations underlying the two samples, and the data are examined to see if there is sufficient evidence to reject this hypothesis. The method consists of forming all possible assignments of the observations into two groups of the same size as the observed samples, but disregarding which particular value came from which sample. For each assignment, the difference between the averages of the two groups is calculated and these differences are ordered. The signs of the differences are retained or not depending on whether a one-tailed or two-tailed calculation is appropriate. Then a tail calculation is performed, getting the number of assignments giving a difference as extreme or more extreme as the observed difference. Then this number is divided by the total number of assignments, giving the chance of a result at least as extreme as that observed, if the hypothesis is true. If this chance is very small, an unlikely event has occurred, so the data have given sufficient evidence to reject the hypothesis. However, if the chance is not small, then the data have not given sufficient cause to reject the hypothesis.

A Second Example

This part contains another small example. The calculations required for the permutation test are given in detail, but the rationale of the calculations, as given in the first example, is not repeated. In the Ways To Learn unit several different methods of learning were tried by different parts of the class, and we want to know if the data show that one method is really superior to another. Consider only two of the methods of learning, using games and using business examples. Four children were in each group; pre-study and post-study tests were given all children, so the amount each child learned is measured by the difference of his pre-study from his post-study test result. The data are given in Table 3. The hypothesis is that there is no difference between the effectiveness of teaching with the games method and the business method. List all eight scores in increasing order, so as to ease the compilation of all possible assignments of the data into two groups of four each. The values are: 5, 7, 8, 8, 9, 10, 12, 13. Note that 8 appears twice in the data. In compiling all possible assignments we want to keep these two values distinct, since they did not come from the same child, and the two numbers did not necessarily have to be the same. It was only by chance that two children achieved the same score, and this fact should not affect the basic method used. Thus from now on we write one value as 8 and the other as 8*. All possible assignments of the eight values to two groups of four each are listed in Table 4. The difference of the average score of group 2 from the average score of group 1 is calculated for each assignment.

For this problem, a very large positive value of game average minus business average would seem to give just as good a reason to reject the hypothesis of equal effectiveness as would a very large negative value for this difference. We have no prior reason to feel that either the game method or the business method should be more effective, if the methods are not equally effective. Thus a two-tailed calculation is appropriate. We disregard the signs of the differences and order the values in Table

TABLE 3

Games			Business		
pre	post	score	pre	post	score
5	13	8	1	8	7
0	13	13	0	5	5
0	10	10	3	12	9
0	12	12	2	10	8

TABLE 4

Assignment Number	Group 1	Group 2	Average Score In Group 1 Minus Average Score In Group 2
1	5,7,8,8*	9,10,12,13	-4
2	5,7,8,9,	8*,10,12,13	-3 1/2
3	5,7,8,10	8*,9,12,13	-3
4	5,7,8,12	8*,9,10,13	-2
5	5,7,8,12	8*,9,10,12	-1 1/2
6	5,7,8*,9	8,10,12,13	-3 1/2
7	5,7,8*,10	8,9,12,13	-3
8	5,7,8*,12	8,9,10,13	-2
9	5,7,8*,13	8,9,10,12	-1 1/2
10	5,7,9,10	8,8*,12,13	-2 1/2
11	5,7,9,12	8,8*,10,13	-1 1/2
12	5,7,9,13	8,8*,10,12	-1
13	5,7,10,12	8,8*,9,13	-1
14	5,7,10,13	8,8*,9,12	- 1/2
15	5,7,12,13	8,8*,9,10	1/2
16	5,8,8*,9	7,10,12,13	-3
17	5,8,8*,10	7,9,12,13	-2 1/2
18	5,8,8*,12	7,9,10,13	-1 1/2
19	5,8,8*,13	7,9,10,12	-1
20	5,8,9,10	7,8*,12,13	-2
21	5,8,9,12	7,8*,10,13	-1

Assignment Number	Group 1	Group 2	Average Score In Group 1 Minus Average Score In Group 2
22	5,8,9,13	7,8*,10,12	- 1/2
23	5,8,10,12	7,8*,9,13	- 1/2
24	5,8,10,13	7,8*,9,12	0
25	5,8,12,13	7,8*,9,10	1
26	5,8*,9,10	7,8,12,13	-2
27	5,8*,9,12	7,8,10,13	-1
28	5,8*,9,13	7,8,10,12	- 1/2
29	5,8*,10,12	7,8,9,13	- 1/2
30	5,8*,10,13	7,8,9,12	0
31	5,8*,12,13	7,8,9,10	1
32	5,9,10,12	7,8,8*,13	0
33	5,9,10,13	7,8,8*,12	1/2
34	5,9,12,13	7,8,8*,10	1 1/2
35	5,10,12,13	7,8,8*,9	2
36	7,8,8*,9	5,10,12,13	-2
37	7,8,8*,10	5,9,12,13	-1 1/2
38	7,8,8*,12	5,9,10,13	- 1/2
39	7,8,8*,13	5,9,10,12	0
40	7,8,9,10	5,8*,12,13	-1
41	7,8,9,12	5,8*,10,13	0
42	7,8,9,13	5,8*,10,12	1/2
43	7,8,10,12	5,8*,9,13	1/2
44	7,8,10,13	5,8*,9,12	1

Assignment Number	Group 1	Group 2	Average Score In Group 1 Minus Average Score In Group 2
45	7,8,12,13	5,8*,9,10	2
46	7,8*,9,10	5,8,12,13	-1
47	7,8*,9,12	5,8,10,13	0
48	7,8*,9,13	5,8,10,12	1/2
49	7,8*,10,12	5,8,9,13	1/2
50	7,8*,10,13	5,8,9,12	1
51	7,8*,12,13	5,8,9,10	2
52	7,9,10,12	5,8,8*,13	1
53	7,9,10,13	5,8,8*,12	1 1/2
54	7,9,12,13	5,8,8*,10	2 1/2
55	7,10,12,13	5,8,8*,9	3
56	8,8*,9,10	5,7,12,13	- 1/2
57	8,8*,9,12	5,7,10,13	1/2
58	8,8*,9,13	5,7,10,12	1
59	8,8*,10,12	5,7,9,13	1
60	8,8*,10,13	5,7,9,12	1 1/2
61	8,8*,12,13	5,7,9,10	2 1/2
62	8,9,10,12	5,7,8*,13	1 1/2
63	8,9,10,13	5,7,8*,12	2
64	8,9,12,13	5,7,8*,10	3
65	8,10,12,13	5,7,8*,9	3 1/2
66	8*,9,10,12	5,7,8,13	1 1/2
67	8*,9,10,13	5,7,8,12	2

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Assignment Number	Group 1	Group 2	Average Score In Group 1 Minus Average Score In Group 2
68	8*,9,12,13	5,7,8,10	3
69	8*,10,12,13	5,7,8,9	3 1/2
70	9,10,12,13	5,7,8,8*	4

5. The games average minus the business average is 3 1/2. Table 5 shows that 6 of the 70 assignments give differences greater than or equal 3 1/2. Therefore, the chances of observing a result at least as extreme as the data here is 6 of 70, or about 8.6%, if the hypothesis of equal effectiveness is correct. This is a fairly low number, but it is not extremely low. The data give some evidence, but definitely not overwhelming evidence, for rejecting the hypothesis that the two teaching methods are equally effective. Another experiment with more students in each group might well give a more definite result.

An Example With More Data

This part analyzes the example discussed at the end of the presentation of q-q plots. The speeds of cars were measured when a guard was and was not present. The data are in Table 6, and the q-q plot in in Figure 20 of Part I (see page 6 of this part).

Examining the values and the plot show that the speeds are generally lower when no guard is present, but there is definite random fluctuation. Could such a difference have occurred by chance if the guard has no effect? To answer this, we need to apply the permutation test method. The basic principles presented above will be used in this example, but the large amounts of data present an additional difficulty. Moreover, the statistic used here to summarize the location of each set of data is the median, while the arithmetic mean was used in the above examples. There are 10 values when the guard was present, and 12 values for no guard. The fact that the two samples are of

TABLE 5

Ordering of the Differences from the 70 Assignments, Disregarding the Signs of the Differences

Assg. Number	Diff.	Rank	Assg. Number	Diff.	Rank	Assg. Number	Diff.	Rank
1	4	1	63	2	25	58	1	49
70	4	2	67	2	26	59	1	50
2	3 1/2	3	5	1 1/2	27	14	1/2	51
6	3 1/2	4	9	1 1/2	28	22	1/2	52
65	3 1/2	5	11	1 1/2	29	23	1/2	53
69	3 1/2	6	18	1 1/2	30	28	1/2	54
3	3	7	37	1 1/2	31	29	1/2	55
7	3	8	34	1 1/2	32	38	1/2	56
16	3	9	53	1 1/2	33	56	1/2	57
55	3	10	60	1 1/2	34	15	1/2	58
64	3	11	62	1 1/2	35	33	1/2	59
68	3	12	66	1 1/2	36	42	1/2	60
10	2 1/2	13	12	1	37	43	1/2	61
17	2 1/2	14	13	1	38	48	1/2	62
54	2 1/2	15	19	1	39	49	1/2	63
61	2 1/2	16	21	1	40	57	1/2	64
4	2	17	27	1	41	24	0	65
8	2	18	40	1	42	30	0	66
20	2	19	46	1	43	32	0	67
26	2	20	25	1	44	39	0	68
36	2	21	31	1	45	41	0	69
35	2	22	44	1	46	47	0	70
45	2	23	50	1	47			
51	2	24	52	1	48			

TABLE 6

Tuesday No Guard	Wednesday With Guard
33	25
35	28
22	20
35	22
29	25
23	30
34	30
33	26
40	28
30	26
32	
32	
avg = 31.5	avg = 26.0

difference = -5.5

unequal size differs from the previous examples. However, this does not present a problem. We simply want to find all possible assignments of the 22 values into two groups of sizes 10 and 12. As before, we could list all possible assignments into two groups of these sizes, calculate the median for each group, obtain the difference, order the differences, and so on.

However, the formula at the end of Appendix A shows that with two groups of sizes m=10 and n=12 there are 646,646 possible assignments of the data values into two groups. We do not intend to list anywhere near 646,646 assignments.

Conceptually, what we desire to do is to consider the population of the differences between the two group medians, over all possible assignments. Then we wish to calculate the proportion of this population that is more extreme -- in either one tail or both tails -- than the observed difference. The set of all possible assignments is thought of as a population. Unfortunately, this population is so large that it is impossible to list all of its members. Nevertheless, we need to learn something about this population.

A natural approach is to obtain a sample from this population with the hope that the sample will reflect fairly well the properties of the population that are of interest to us. The resource unit PS4, "Design of Surveys and Samples," discusses concepts related to sampling from a population. Here we would like to obtain a random sample from the population of all 646,646 assignments. A suggestion in the above paper leads to a method for doing this. If the population under consideration consists of all the children in a school and a random sample of size 30, say, from this population is desired, then do the following. Write each child's name on a separate card, shuffle all the cards thoroughly, and then draw the top 30 cards from the deck. Each child named on a card becomes a member of the sample, and the 30 children obtained in this way form the random sample. Of course, repeating the whole process, including reshuffling the cards, will give a different random sample.

Here the problem faced is more complicated. It is impossible to list each member of the population on a separate card, as above, as then there would be 646,646 cards. Instead, write the 22 numbers in Table 6 on 22 cards. Shuffle them thoroughly, and call the top 10 cards group 1 and the bottom 12 group 12. These two groups give one of the 646,646 possible assignments; this method gives a random sample of size one from the population of possible assignments. Note that in the preceeding example, each of the 30 cards chosen was a sample of size one from the population of children in the school, and the 30 cards gave the entire random sample that was desired. Here, though, the order of all 22 cards is required to obtain only one member of the population of all possible assignments. If we wish to obtain a random sample of 20 assignments, this process of shuffling and looking at the top 10 and bottom 12 cards must be repeated 20 different times.

Suppose we obtain 20 assignments by this process. For each assignment, calculate the median for the first group of 10 minus the median for the second group of 12. Note the use of the median here while earlier examples used the average. These 20 differences form a random sample from the population of all 646,646 differences. This was done and the results are given in Table 7. It is reasonable to expect these 20 values to give some information about the population. However, there is probably some of the population smaller than the smallest value and some of the population is probably larger than the largest value. In fact, one can expect about 1/21st of the population to be smaller than the lowest of the 20 values, about 2/21st of the population to be smaller than the second lowest of the 20 values, etc. Thus, about 20/21st of the population should be lower than the largest of the 20 values, so about 1/21st of the population should be greater than the largest of the 20 values. That is, it is reasonable to expect the 20 values of the random sample to break the population into 21 pieces with each piece containing about the same proportion of the population. The values from the random sample should be representative of the population. Of course, repeating this whole process would give another random sample of size 20, undoubtedly with slightly different differences. Thus we cannot expect <u>exactly</u> 1/21st of the population to be smaller than the lowest of the 20 values in the random sample, but only <u>approximately</u> 1/21st of the population. If it were desirable to obtain more information about the population, a larger random sample could be obtained by continuing to shuffle the cards and assigning the top 10 to group 1, etc. If a random sample of 49 were obtained, we would expect about 1/50th = 2% of the population to have values lower than the lowest in the sam-

TABLE 7

20 Randomly Chosen Assignments from the Values in Table

Group 1*										Gp 1 median	Gp 2 median	Diff.
22,	25,	28,	30,	30,	32,	32,	33,	34,	35	31	27	4
22,	26,	28,	30,	32,	33,	33,	35,	35,	40	32.5	27	5.5
22,	26,	26,	28,	30,	30,	32,	33,	34,	35	30	28.5	1.5
20,	23,	25,	26,	26,	28,	29,	30,	32,	35	27	31	-4
22,	22,	23,	25,	30,	32,	32,	33,	35,	40	31	28.5	2.5
22,	22,	28,	30,	30,	32,	32,	33,	34,	35	31	27	4
20,	22,	22,	25,	26,	28,	29,	30,	33,	34	27	31	-4
20,	22,	23,	26,	30,	32,	33,	34,	35,	40	31	28.5	2.5
20,	22,	23,	25,	26,	30,	32,	33,	33,	34	28	29.5	-1.5
20,	22,	23,	26,	26,	28,	30,	34,	35,	40	27	30	-3
22,	23,	25,	26,	28,	28,	29,	30,	33,	34	28	31	-3
22,	23,	25,	26,	29,	30,	32,	33,	34,	40	29.5	29	0.5
23,	25,	26,	28,	29,	30,	30,	33,	34,	35	29.5	29	0.5
20,	22,	25,	26,	28,	30,	30,	32,	33,	34	29	29.5	-0.5
20,	22,	25,	26,	26,	30,	32,	33,	33,	40	28	29.5	-1.5
22,	22,	26,	28,	28,	29,	30,	30,	32,	33	28.5	31	-2.5
22,	22,	25,	25,	26,	29,	30,	33,	33,	35	27.5	30	-2.5
20,	22,	22,	25,	28,	30,	33,	34,	35,	40	29	29.5	-0.5
22,	25,	25,	26,	28,	30,	30,	30,	33,	33	29	30.5	-1.5
20,	22,	22,	28,	29,	30,	30,	32,	34,	35	29.5	29	0.5

* Only the values in Group 1 are given here, to conserve space. The observations were ordered here to ease calculation of the median. The values in Group 2 for each assignment are all those from Table 6 that are not in Group 1.

ple, similar to the above.

The next step in the analysis of this data is to order the 20 differences in Table 7; this is given in Table 8. We estimate that about 1/21st = 4.8% of the population gives differences less than or equal -4. From our actual data in Table 6 we observe the difference of no guard average speed from guard average speed as -6.5. A one-tail calculation appears appropriate for this problem, as we would be quite surprised if cars drove significantly faster with a guard present compared to no guard. Therefore, we would like to make the statement, the chance of obtaining a result as extreme as these data is ??%, if the hypothesis of no effect of the guard is correct. The value ??% would be the proportion of the population less than or equal -6.5. But we do not know this proportion; the best we can do is use the information from Table 8, and estimate that this proportion is less than 4.8%. The resulting statement is that the chance of obtaining a result as extreme as these data is estimated as less than 4.8%, if the hypothesis is correct.

Since the observed difference of -6.5 is substantially smaller than the lowest value in Table 8, -4, intuition suggests that the proportion of the population less than or equal -6.5 may be somewhat smaller than 4.8%. An unlikely event has occurred, so the hypothesis can be rejected. The data give sufficient evidence to conclude that the cars do not travel at the same median speed when the guard is present as when he is not. The cars go significantly slower when the guard is present.

TABLE 8

Ordering of the 20 Differences Obtained in Table

Value	Rank
-4	1
-4	2
-3	3
-3	4
-2.5	5
-2.5	6
-1.5	7
-1.5	8
-1.5	9
-0.5	10
-0.5	11
0.5	12
0.5	13
0.5	14
1.5	15
2.5	16
2.5	17
4	18
4	19
5.5	20

An electronic computer can be used to make the analysis of this sample slightly more precise. The computer can do something equivalent to shuffling the cards by generating a series of random numbers. Many more assignments can be generated this way than could possibly be done by hand shuffling. Thus, the proportion of the population of all assignments giving a difference less than or equal to the observed -6.5 can be estimated much more accurately. This was done, and the estimate obtained was 0.5%. This is much smaller than the value 4.8% obtained above, but would likely lead to similar substantive conclusions from the data. Further information can also be obtained from this data, using a combination of statistical ideas, probability theory, and the capabilities of an electronic computer.

Many additional statistical ideas and methods are discussed in Statistics by Example, edited by F. Mosteller and others (Addison Wesley Publishing Co., 1973). These are four soft-cover books, with many examples of problems and appropriate statistical analyses. The material is intended for high-school students but should be of interest to both older and younger audiences.

APPENDIX III-A

This appendix discusses how to form tables such as Table 1, Table 4, and Table 5. We illustrate the general method by assuming that we have 6 numbers and want to list all possible divisions of these numbers into two sets of 3 numbers each.

Denote the numbers by a, b, c, d, e, f. These symbols could be replaced by the particular values in the sample. We want to list all divisions systematically, so as to not miss any. After we have put three letters into set 1 the remaining three letters must automatically go into set 2; thus it is simpler to just think of the successive sets of elements for set 1.

Table C1 gives all possible divisions into two sets, and we now present the rationale behind constructing the table. This material should be read while looking at the table. Start by putting the first three letters — a, b, c — into set 1. Then replace the last letter, c, by the next letter in the list, d. This gives a, b, d. Again replace the last letter in this set by the following letter giving a, b, e. Doing this again gives a, b, f. This process of incrementing the third letter in the set cannot be carried further, though, since f is the last letter in the list. Then we now go to the second letter. We originally had a, b, c, and now we want to increment the second letter from b to c. This gives a, c, c, which is not legal since the c appears twice. Thus we must also increment c, giving a, c, d. Now we can again increment the third letter in the list, giving a, c, e, and then a, c, f.

At this point we again cannot increment the third letter further, so we go back and increment the second letter again, giving a, d, d. This is not legal, so we write a, d, e. Then a, d, f; incrementing the second letter again gives a, e, f.

Now we can no longer increment the second letter; thus we go back and increment the first letter from a to b. Thus the next value for set 1 is b, c, d. We get this by having the second and third letters as low as possible, given that the first letter is b. Following the same strategy of first incrementing the third letter as much as possible and then the second letter gives b, c, e; b, c, f; b, d, e; b, d, f; b, e, f. Now the first letter must be incremented again from b to c, and the second and third letters are returned to their lowest values possible with first letter c. This gives c, d, e. The remaining values for set 1 follow by continuing this strategy.

There are several things to notice about this way of listing the divisions. First, the reader should convince himself that each possible set of three letters appears somewhere as set 1; no possibility has been missed, and none is listed twice. Each set 1 is listed in alphabetical order. Although each set 2 is simply the letters not in the corresponding set 1, the set 2 list turns out to be exactly the same as the set 1 list read from bottom to top. This can be used as a check that the set 1 list has been constructed correctly.

This paragraph briefly indicates how the above principles are used to construct lists of two sets of four letters from the eight letters a, b, c, d, e, f, g, h. Again, only the elements of set 1 are given. Start with a, b, c, d, then a, b, c, e, etc., to a, b, c, h. Then increment the third letter, giving a, b, d, e. Then a, b, d, f, etc., to a, b, d, h. Then, since the third letter can still be incremented comes a, b, e, f, then a, b, e, g, then a, b, e, h. Continue incrementing the third and fourth letters until we have a, b, g, h. Now increment the second letter, giving a, c, d, e. Then a, c, d, f, etc., to a, c, d, h. Then comes a, c, e, f, etc., until finally we reach a, c, g, h. Then the second letter is incremented, giving a, d, e, f. This whole process continues until a, f, g, h. Then the first letter is incremented, giving b, c, d, e, and the same system is followed until the value e, f, g, h is finally reached. This system was followed to create Table 4.

A formula is available telling the number of possible assignments. If there are n values in one sample, and m in the other, with m smaller than or equal to n, then there are

$$\frac{(m+n)(m+n-1)\cdots(n+2)(n+1)}{m\cdot(m-1)\cdots(2)(1)}$$

possible assignments. In the first example, m = n = 3, and the formula gives $6\cdot5\cdot4/3\cdot2\cdot1 = 20$, which is what we found. For the second example in the text, m = n = 4, and the formula gives $8\cdot7\cdot6\cdot5/4\cdot3\cdot2\cdot1 = 70$, again what we found. For the car speed example m = 10, n = 12, and the formula is

$$\frac{22\cdot21\cdot20\cdot19\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13}{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}$$

equals 646,646.

TABLE CI

All Possible Divisions of a, b, c, d, e, f
into two sets of three each

Assignment	Set 1	Set 2
1	a b c	d e f
2	a b d	c e f
3	a b e	c d f
4	a b f	c d e
5	a c d	b e f
6	a c e	b d f
7	a c f	b d e
8	a d e	b c f
9	a d f	b c e
10	a e f	b c d
11	b c d	a e f
12	b c e	a d f
13	b c f	a d e
14	b d e	a c f
15	b d f	a c e
16	b e f	a c d
17	c d e	a b f
18	c d f	a b e
19	c e f	a b d
20	d e f	a b c

GRAPHIC REPRESENTATION OF FRACTIONS
(Revised May, 1973)

by
Merrill B. Goldberg

Students are required to work with ratios or fractions in many of the USMES Units. They have to compute cost per unit in the Consumer Research Unit, determine representative proportions in Designing for Human Proportions, etc. By representing fractions on graph paper or pegboard, in the manner to be presented here, it is possible to list a collection of several fractions in increasing order thereby determining the least fraction, the greatest fraction, and the median fraction. It should be emphasized that the techniques required to represent a fraction graphically are not isolated. The techniques here provide early experience with determining co-ordinates of a point on a graph. In addition, the concept of the slope of a line recurs in algebra and even later in calculus and so although it is easy enough to present to elementary students, any curious adult can learn from the technique.

While the original problem leading to the technique was to arrange a list of fractions in increasing order, the graphic picture of fractions provides insight into a number of concepts related to the study of fractions. Among these concepts to be explained here are: the numerator and denominator as two numbers that comprise a fraction; equivalent fractions; reducing a fraction to lowest terms; adding two fractions with like denominators; adding two fractions with different denominators; and comparing two fractions to determine which is larger. In addition to presenting the technique, I will include some brief pointers on presenting it to a class.

The background for picturing fractions is either a sheet of graph paper or a pegboard. Since it will be necessary to plot accurately, graph paper with 1/2 inch grids should be used at first. In using a pegboard (about 3 feet by 4 feet in size) it would be advisable at first to use every fourth row of holes as they are spaced quite close together. Chalk lines may be drawn to indicate the rows being used. Golf tees or bolts the size of the holes in the pegboard are used to mark the points of interest. Students can work individually on graph paper and a group of students can work on the pegboard. The dots on the figures in this paper correspond to holes in the pegboard. The crosses marked at specific dots correspond to the placing of a golf tee or bolt in the corresponding hole. The adaptation to graph paper is simply to place a mark on the paper instead of placing a golf tee in a hole of the pegboard. On graph paper, the points of interest are the points where a horizontal and vertical line intersect.

For a first demonstration of the technique, let's work with fractions where the numerator and denominator are both less than or equal to ten. Accordingly, we can use every fourth row of holes in a pegboard. In Figure 1, we plot the fractions: 2/5; 3/5; 4/5; 4/6; 4/7; and 4/8. The first three fractions have the same denominator (5) with the numerators increasing. Thus the first three fractions are in increasing order. The last four fractions in the list have the same numerator (4) with the denominators increasing. Thus the last four fractions are in decreasing order. We will see these facts as well as others in the graph.

The steps in plotting the fractions on pegboard or graph paper are as follows (see Figure 1).

1. Place a golf tee in a hole toward the lower left corner of the pegboard. This will be the "origin" of the graph. From this point we will be counting UP and to the RIGHT in order to represent ratios. It will be helpful to have a horizontal number line (called the "horizontal axis") and a vertical number line (called the "vertical axis") extending from the origin. The origin and the axes (plural of axis) are basic to any graphing activity. Since they are number lines, they can be labeled accordingly as shown in Figure 1. The "origin" is the point that is zero on both number lines. Each hole in the pegboard may now be described in a unique manner by giving its coordinates relative to the two axes. This is done by stating how many holes to count to the RIGHT and how many holes to count UP from the origin.

2. To represent the fraction 2/5 we start at the origin and count to the RIGHT 5 positions - to the number 5 (for fifths) on the horizontal axis. Next, count UP 2 positions (the number of fifths). Place a golf tee in this hole which is 5 places to the RIGHT of the vertical axis and 2 places UP from the horizontal axis. This golf tee just placed is in the position which represents the fraction 2/5 .

3. To represent the fraction 3/5 we again start counting from the origin. Count UP 3 and to the RIGHT 5 (or to the RIGHT 5 and then UP 3 as the order does not matter). Place a golf tee in this hole to represent 3/5 .

4. The other fractions: 4/5; 4/6; 4/7; and 4/8 are represented similarly as shown in Figure 1.

Remember to always start counting from the origin. To represent a fraction, the number on the bottom (the denominator of the fraction) is the number of positions to count to the RIGHT. The top number (the numerator of the fraction) is the number of positions to count UP. It makes no difference whether we count to the RIGHT first and then UP or if we count UP first and then to the RIGHT. As long as we always count UP the number of positions which is the numerator of the fraction and we count to the RIGHT the number of holes which is the denominator of the fraction, we will always arrive at the same position in which to place the golf tee to represent the fraction.

To reason about the numbers represented on the graph it will be helpful to visualize a line segment connecting the point representing the fraction with the origin. These line segments have been included in Figure 1. Also it is important to label each fraction on the graph or on the pegboard. A convenient way to label a fraction on the pegboard is to have available several small pieces of paper about two inches square. Each fraction to be represented should be written on a sheet of paper. Then the paper is pierced at the top center by the golf tee or bolt before inserting it in the hole. That way, each fraction is clearly represented on the pegboard.

The steepness of the lines in figure 1 corresponds to the magnitude of the fraction. The line for the fraction 4/5 is steeper (more counter-clockwise) than the line for the fraction 3/5. In working with fractions, if the denominator stays the same and the numerator is increased, the number represented by the fraction increases. This corresponds to the fact that the lines representing 2/5, 3/5, and 4/5 increase in steepness. If the numerator stays the same and the denominator increases, then the number represented by the fraction decreases. In figure 1, this corresponds to the fact that the lines representing 4/5, 4/6, 4/7, and 4/8 decrease in steepness. This is the key to using the graph to arrange fractions in increasing order or, more simply, to compare two fractions to see which is larger. The length of the lines in the picture does not matter. The important point in determining the size of the fraction is the steepness of the line. In algebra and calculus, this will be referred to as the SLOPE of the line. Since slope is an important concept in higher mathematics and since it is also an intuitive concept, it would be beneficial to use the terminology of slope of the line rather than steepness. To arrange the six fractions: 2/5; 3/5; 4/5; 4/6; 4/7; and 4/8 in increasing order, we need only look at the graph and list the fractions in the order that corresponds to increasing slope of the lines representing the fractions. Listing the fractions in increasing order from smallest to largest we get: 2/5; 4/8; 4/7; 3/5; 4/6; and 4/5.

If pegboard is used, there is an interesting way to list the fractions in increasing order. Tie a loop in one end of a piece of yarn or string and place the loop over a golf tee inserted in the "origin". The string should be long enough to reach all the way to the top right corner of the pegboard. The string forms a line segment originating from the origin which can be rotated to correspond to the various lines determined by the fractions. Start by holding the string on the right so that it lies along the horizontal axis. Rotate the "line of string" counter-clockwise around the origin holding it close to the surface of the pegboard. The first tee it contacts will be the smallest fraction (in this case, 2/5). Next pass the string over this golf tee and continue rotating to determine the next largest fraction. Continuing in this manner, a collection of fractions represented on pegboard can be easily listed in increasing order.

As a means of listing fractions in increasing order, this method is quite useful. To appreciate its usefulness, consider the alternative methods. Converting each fraction in the list to a decimal equivalent can be tedious. Changing the fractions in the collection to equivalent fractions with a common denominator is also tedious. (The least common denominator for the collection presented here and in Figure 1 is 840.)

As mentioned earlier, many concepts of fractions can be interpreted in terms of the graphic representation outlined above. We have already seen the concepts of numerator and denominator as the two numbers that comprise a fraction. The numerator is the number of steps UP - or the co-ordinate relative to the vertical axis. The denominator is the co-ordinate relative to the horizontal axis - the number of steps counted to the RIGHT to locate the position for the fraction. Fractions with the same numerator will lie along a horizontal row (as do 4/5; 4/6; 4/7; and 4/8 in figure 1). Fractions with the same denominator will lie along a vertical column (as do 2/5; 3/5; and 4/5 in figure 1). We have also already seen a method for comparing two fractions to determine which is larger. In fact, by looking at the slopes of the various lines representing the fractions, we can arrange a collection of fractions in increasing order with relative ease. The concept of equivalent fractions is particularly easy to see in the graph.

Look again at Figure 1 - in particular at the line determined by the fraction 4/8. Follow the line from the point which is 8 RIGHT and 4 UP back to the origin. Notice that you pass through three other dots on the way back to the origin. These three dots represent the fractions 3/6; 2/4; and 1/2. All of these points lie on the line joining the origin to the point representing the fraction 4/8 and so the lines connecting each of these points with the origin have the same slope as the line for the fraction 4/8. Indeed all of these fractions (3/6; 2/4; and 1/2) are equivalent to 4/8 - i.e. the numbers they represent are all equal. Of the four equivalent fractions 4/8, 3/6, 2/4, and 1/2, the fraction 1/2 is distinguished by the fact that between the point on the graph in figure 1 which represents 1/2 and the origin there are no other points on the line. That is because the fraction 1/2 is in lowest terms! On Figure 1, look at the line representing the fraction 4/6. Trace it back to the origin. It passes through one other point - the point that would be the fraction 2/3 (3 RIGHT and 2 UP). No further points lie on the line between 2/3 and the origin because 2/3 is in lowest terms. A quick check of Figure 1 shows that all the other fractions pictured (2/5; 3/5; 4/5; and 4/7) are in lowest terms because no other points (holes in the pegboard) lie on the line segment joining each of these points to the origin.

There is likely to be confusion as to what represents the fraction - the point or the line. This corresponds to the idea that several fractions can represent the same number. For example, the fractions 2/4; 3/6; 4/8; and 102/204 are different fractions yet they all represent the same number - the number usually expressed as 1/2. If the fractions 2/4; 3/6; 4/8 and 102/204 were all plotted with golf tees on a pegboard (a large imaginary pegboard to be sure if 102/204 is to be there!), they would all lie on a single line radiating from the origin. This line would pass through other points in addition to those mentioned and if extended beyond 102/204 would pass through even more. So in essence, the "fraction" is the point and the line radiating from the origin through this point is the number. Each point on the line has co-ordinates (a pair of numbers) that specify a fraction. Points on the same line correspond to equivalent fractions which all represent the same number. The hole or dot on the line which is closest to the origin corresponds to the fraction which expresses the number in lowest terms.

It is also possible to demonstrate the addition and subtraction of fractions using this graphic representation. It is exactly the same as adding and subtracting on a number line. The key here is work on a VERTICAL NUMBER LINE. For example, consider the problem 1/5 + 2/5. It's almost like adding 1 orange + 2 oranges, except instead of oranges, we're adding fifths. Picture a vertical number line parallel to the vertical axis which is located above the number 5 (to indicate fifths) on the horizontal axis. See Figure 2. The fraction 1/5 would correspond to the number 1 on this vertical number line. The fraction 2/5 would correspond to the number 2 on this number line. Adding 1 + 2 on this vertical number line results in the number 3 on this vertical number line (of fifths) which corresponds to the fraction 3/5 on the graph. As long as the two fractions to be added have the same denominator, they will be represented by points on the graph which are on the same vertical number line. They then may be added on the vertical number line. Subtraction is similarly carried out as subtraction on a vertical number line.

Addition and subtraction of fractions with different denominators is also easily pictured on the graph. Again the key is to perform the addition or subtraction on a vertical number line. For example, consider the problem 1/4 + 1/2 . Recall that the point which is 2 RIGHT and 1 UP represents the fraction 1/2 while the "number 1/2" is represented by a line radiating from the origin through the point representing 1/2 . All points on this line segment are equivalent fractions which represent the same number as the fraction 1/2 . Since addition must take place on a vertical number line, the required process is to find a vertical number line on which the numbers 1/4 and 1/2 are both conveniently represented. Extend the line from the origin through the point representing the fraction 1/2 as shown in Figure 3. It passes through a point on the vertical number line which would be above the 4 - indicating fourths. The fraction 2/4 is equivalent to the fraction 1/2. Since the fractions 1/4 and 2/4 are on the same "vertical number line", the addition can take place on the vertical number line corresponding to fourths to give the sum: 1/4 + 1/2 = 3/4 .

As a further demonstration of the addition of fractions, consider the problem 1/3 + 1/2 . This problem is shown in Figure 4 and is briefly described as follows. The fractions 1/3 and 1/2 are not on the same vertical number line and so must be relocated on a common vertical number line in order to be added. Extending the lines radiating from the origin through each of these two fractions, it is found that each has an equivalent fraction which is represented by a point on the vertical number line extending up from the 6 on the horizontal axis. The fraction 1/2 is equivalent to the fraction 3/6 located on this vertical number line. The fraction 1/3 is equivalent to the fraction

2/6 located on this vertical number line. The addition of 2 and 3 on the vertical number line for sixths yields a sum of 5/6 for the problem 1/2 + 1/3. Finally, drawing a line back from 5/6 to the origin, no other points (holes in the pegboard) are encountered. So the fraction 5/6 is in lowest terms.

To demonstrate the power of a graphic representation of fractions, consider the problem of arranging the following fractions in increasing order to enable finding the median. The seven fractions to be considered are: 24/32; 23/25; 25/35; 25/23; 28/33; 24/30; and 20/28. Recall that the median is the one in the middle when the list has been arranged in increasing order. Since there are seven fractions, the median will be the fourth one in the list when we have arranged them in increasing order. The lowest common denominator for the seven fractions stated above is 2,833,600 and so the method of graphically representing the fractions is definitely easier than the routine process of changing the fractions to equivalent fractions with a common denominator!

The first step in listing the fractions in increasing order is to graph them. This is done by first establishing an origin in the lower left corner and noting a horizontal and vertical axis. The fraction 24/32 is represented by the position which is 32 RIGHT and 24 UP. The remaining six fractions are represented similarly as shown in Figure 5. The lines radiating from the origin to each of the points for the fractions have been shown. Arranging these lines according to increasing slope arranges the fractions in increasing order. As shown in Figure 5, the lines for 20/28 and 25/35 have the same slope and so represent equivalent fractions (the same number). Using the symbol $<$ to denote "is less than", the fractions arranged in order are: $20/28 = 25/35 < 24/32 < 24/30 < 28/33 < 23/25 < 25/23$. The order of listing 20/28 and 25/35 could be reversed since the numbers represented by these two fractions are equal. Counting over to the fourth fraction in the list determines that the median is 24/30. As a check on this method we could change each fraction into an equivalent fraction expressed in terms of the lowest common denominator: 2,833,600 -- but that seems an unwieldy and thankless task! Alternately, we could express each fraction in terms of a decimal to check our ordering. Computation gives the following: 20/28 = .714; 25/35 = .714; 24/32 = .750; 28/33 = .848; 23/25 = .920; and 25/23 = 1.087. Thus the ordering found by graphing is correct.

In conclusion, a few hints and warnings regarding the graphic representation of fractions: Accuracy of plotting is essential, especially when desiring to reduce to lowest terms. Accuracy is increased by working on a larger scale, as with 1/2 inch graph paper or using every fourth row of holes in the pegboard. Golf tees are cheap and convenient to use, but frequently wobble around in the pegboard. Bolts or large screws the same size diameter as the holes in the pegboard will fit more securely. Be sure to label the fractions as they are plotted. On the pegboard this may be achieved by writing the fraction on a small piece of paper which is then pierced by the golf tee (or bolt) and mounted on the pegboard at the appropriate spot. If you wish to check that a number of fractions have been correctly placed on the graph, it is more quickly done by noting that all fractions in a horizontal row will have the same numerator (the numerator is given by the co-ordinate along the vertical axis) while all the fractions in a vertical column will have the same denominator. Rather than checking each fraction individually, simply check each horizontal row for the correct numerator and then check each vertical column for the correct denominator.

Pieces of yarn or string are a great help in visualizing the line segments of interest. By tying a loop in one end and hooking it over the golf tee at the origin, the string can be stretched to form a line which can then be rotated counter-clockwise to list fractions in increasing order. Also, a string may facilitate visualizing the vertical number lines on which addition and subtraction are to take place.

A final word of warning and encouragement. Some of the arithmetic performed by these methods will lack total accuracy as a result of the thickness of the holes, string, golf tees, etc., however, there should not be any gross inaccuracy and the results should always be approximately correct. The model of representing fractions on a graph is quite "faithful" and so much can be learned by students formulating theories and testing them. The experience of working with a graph and learning a few words of terminology such as horizontal axis, vertical axis, origin, co-ordinates, and slope will be extremely beneficial in later years.

Figure 1

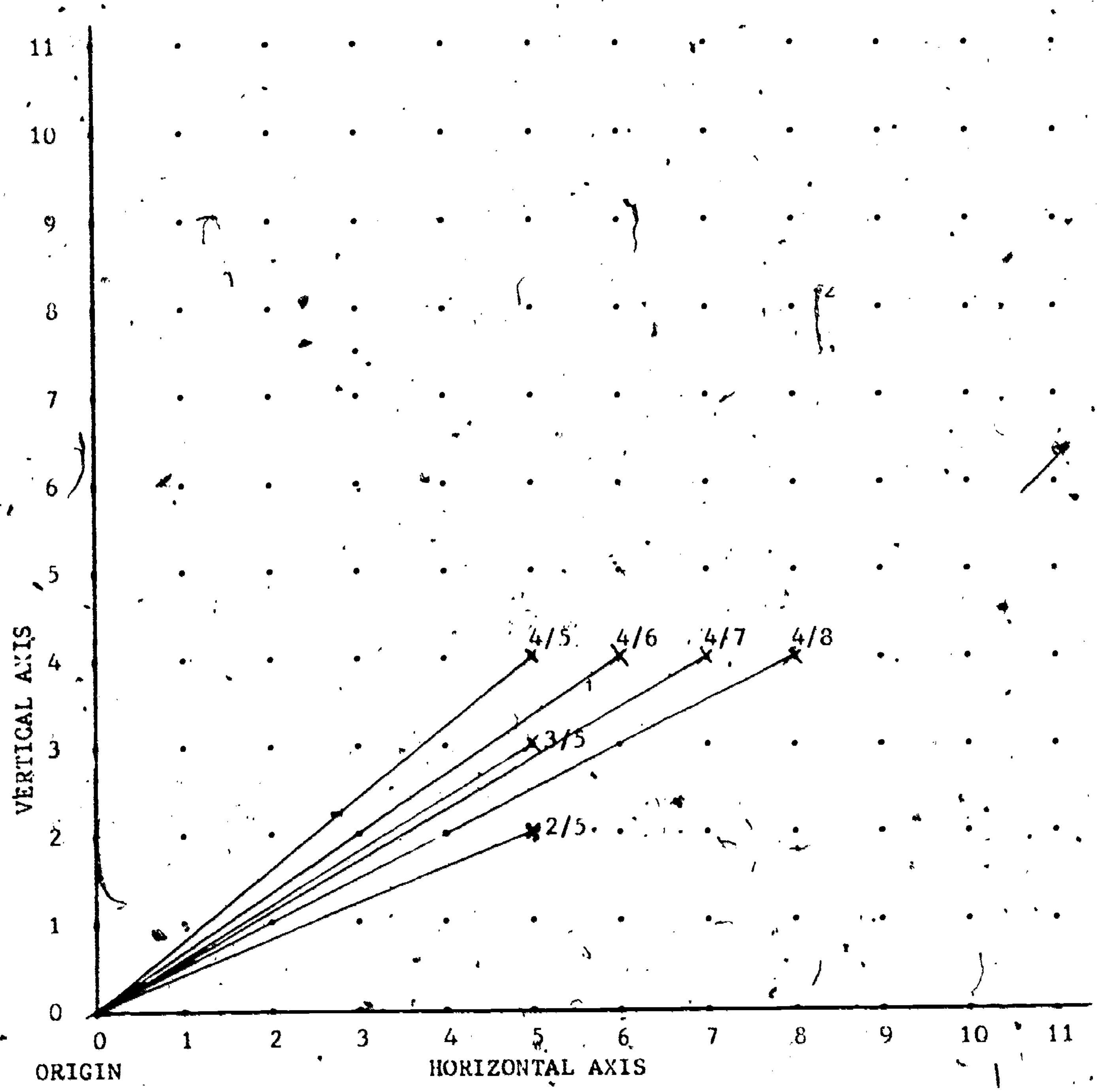


Figure 1 shows the origin in the lower left corner, the horizontal number line called the horizontal axis, and the vertical number line called the vertical axis. The fraction 2/5 is shown as the point which is located 5 RIGHT and 2 UP from the origin. The line through this point and the origin represents the number 2/5. Other fractions shown are: 3/5; 4/5; 4/6; 4/7 and 4/8.

Figure 2

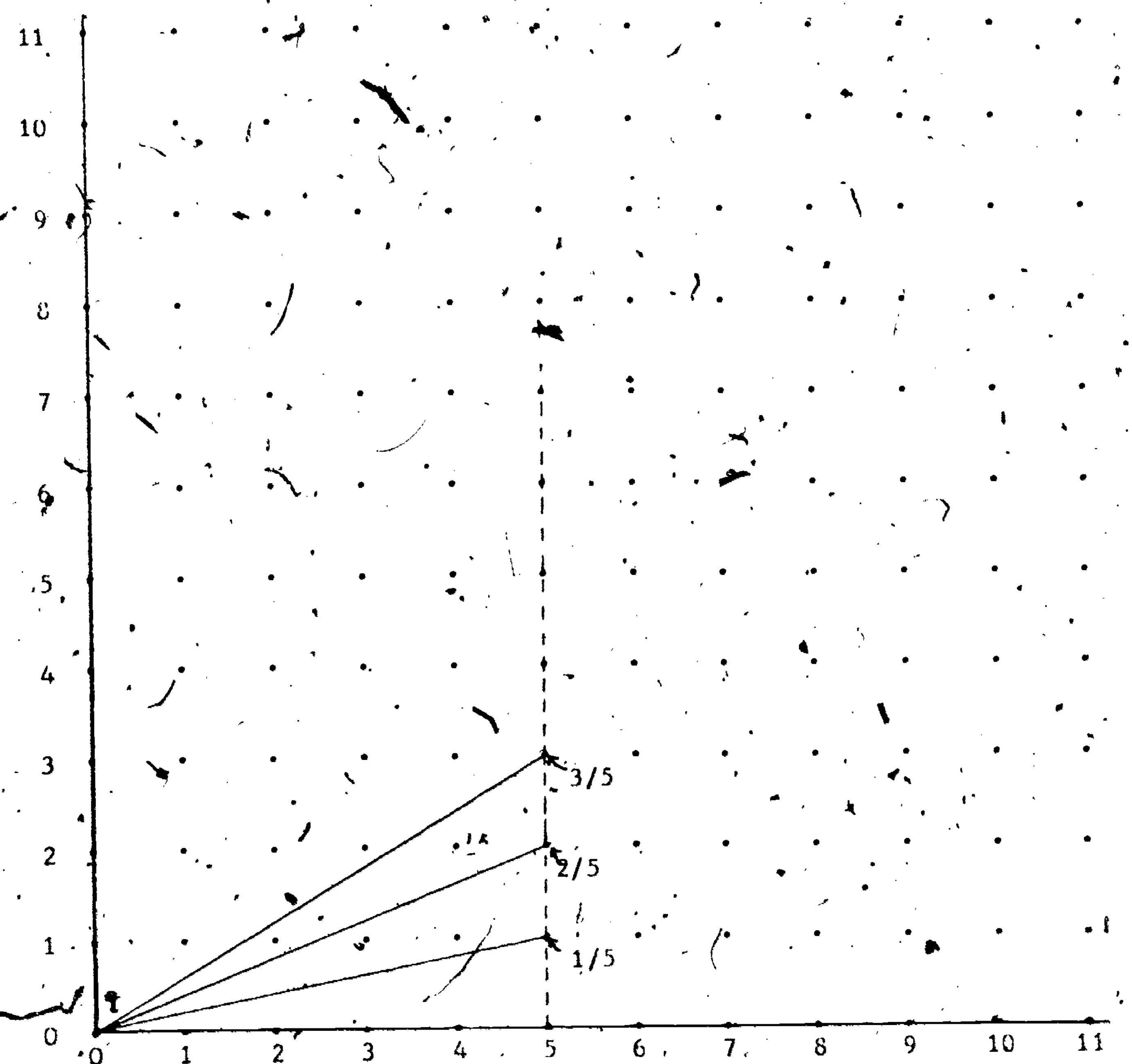


Figure 2 demonstrates how fractions with the same denominator are added on a vertical number line. Here the problem 1/5 + 2/5 = 3/5 is shown as addition along the vertical number line of fifths shown as a broken line.

Figure 3

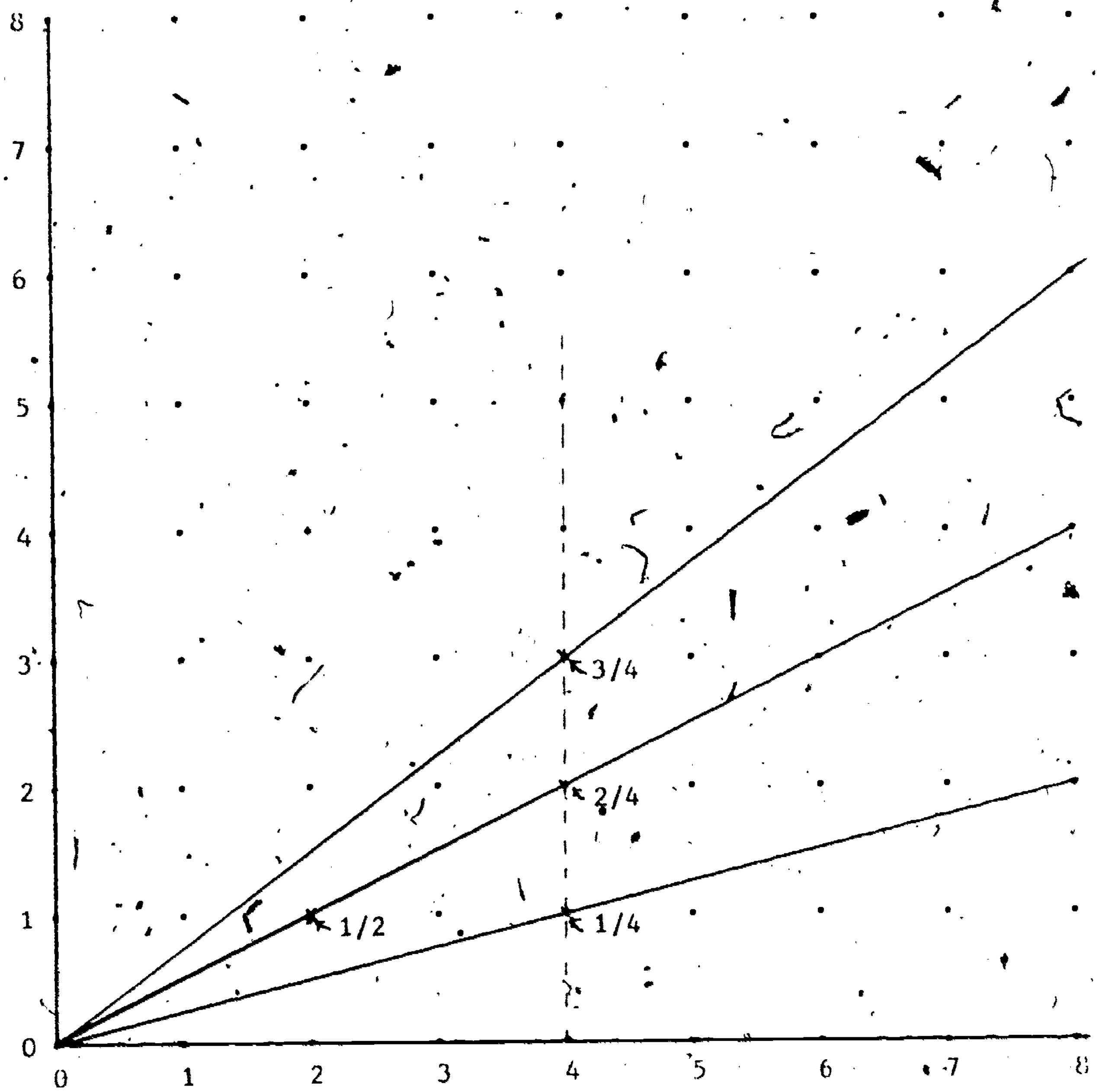


Figure 3 demonstrates how fractions with different denominators are added - again along a vertical number line. Here the addition of 1/2 + 1/4 = 3/4 is shown as addition along the vertical number line of fourths shown as a broken line. Note that the addition could have taken place on a vertical number line of eighths to give the result 6/8 which then passes through the point for the fraction 3/4.

Figure 4

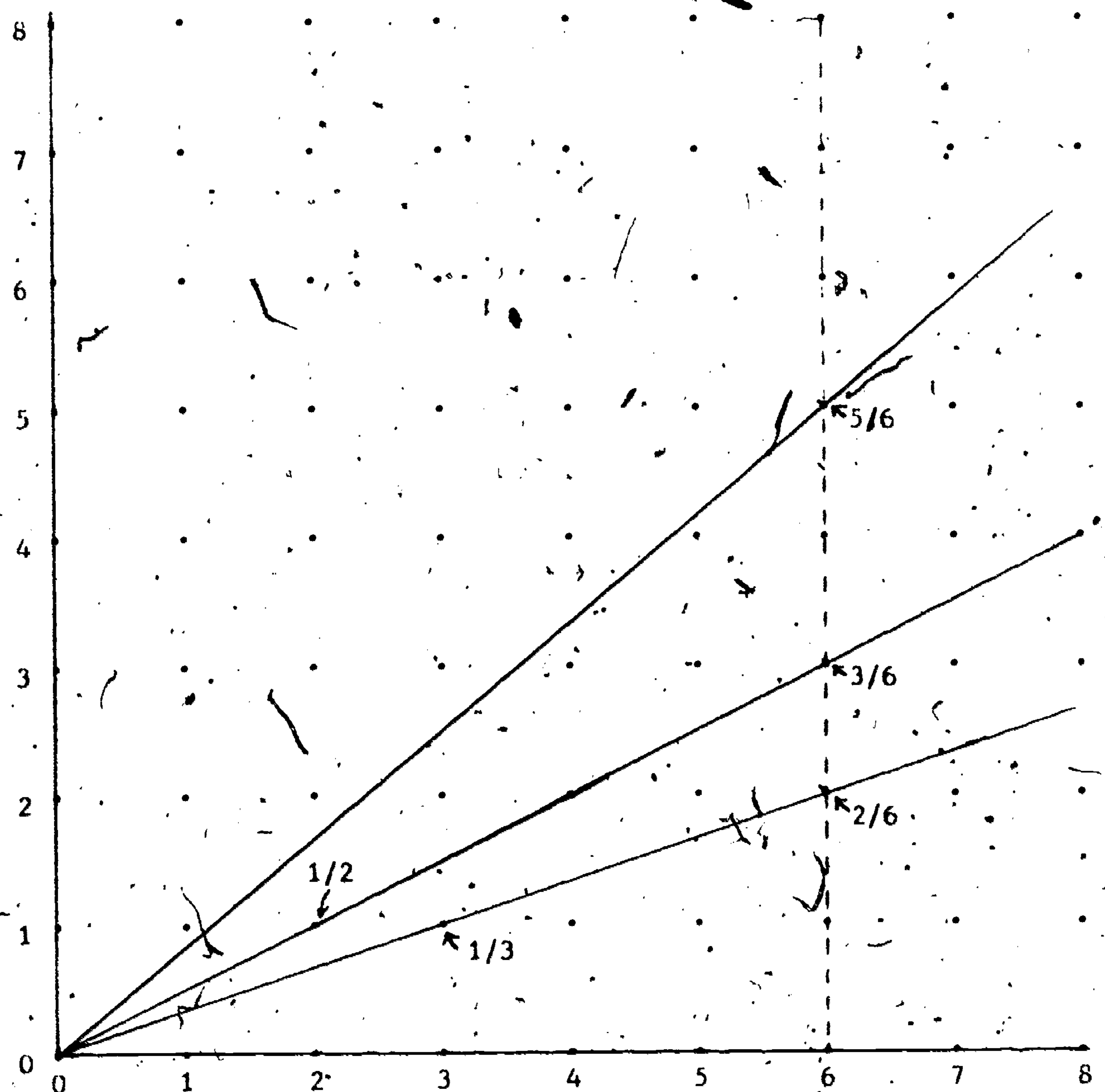


Figure 4 demonstrates the addition of 1/2 + 1/3 = 5/6 taking place as addition on the vertical number line of sixths (shown as a broken line).

Figure 5

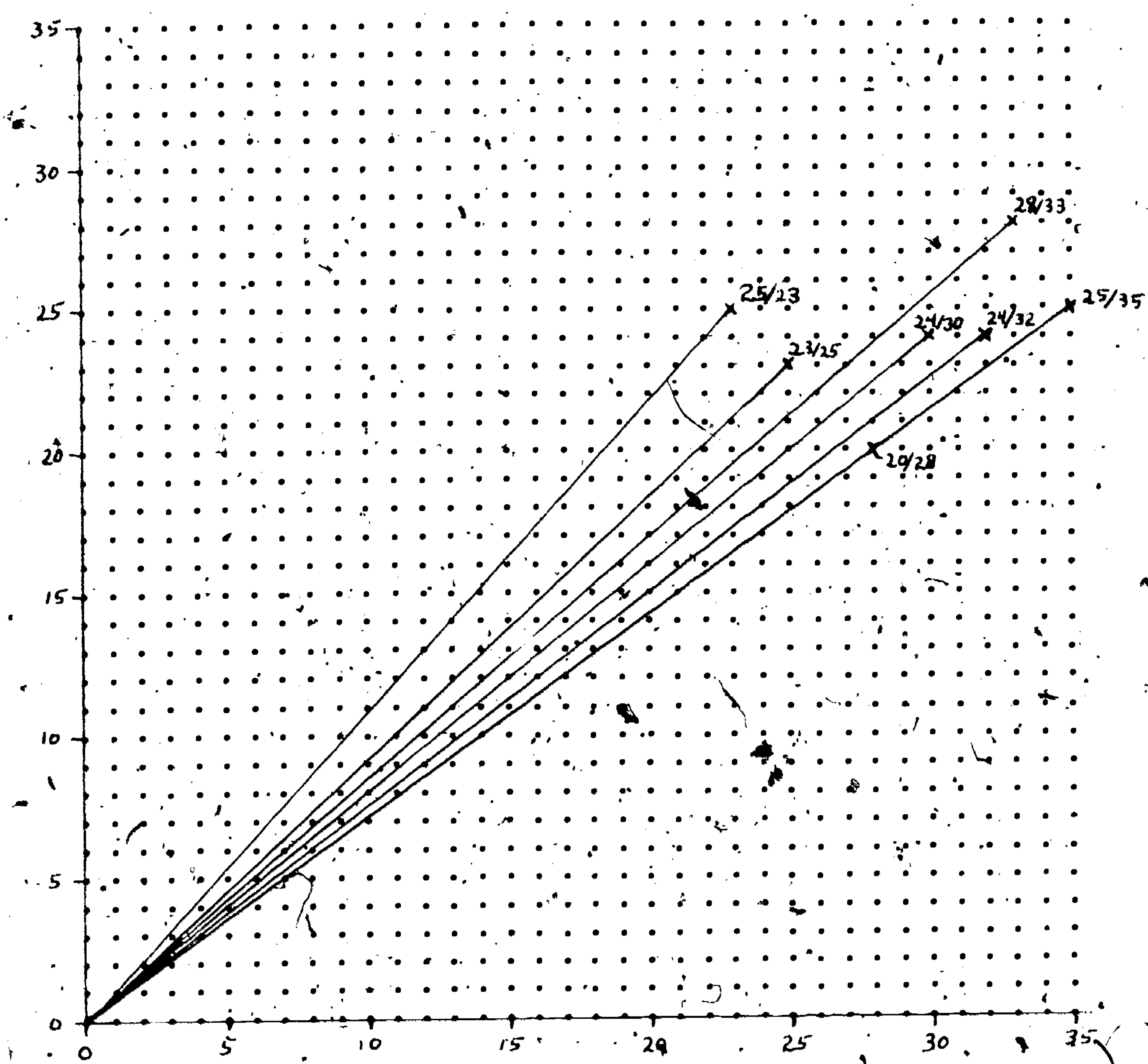


Figure 5 shows how the method of representing fractions on a graph aids in arranging the fractions in increasing order. By noting the slopes of lines, we see 20/28 = 25/35 < 24/32 < 24/30 < 28/33 < 23/25 < 25/23. Therefore the median of these seven fractions is 24/30.

GEOMETRIC COMPARISON OF RATIOS

by
Earle Lomon

The relative value of ratios is important information in many investigations. For instance, the children may want to find answers to the following questions.

> Describing People Unit - "What is the median ratio of trunk length to leg length for 12-year old children?.... for 7-year olds?" How do they differ for the two ages?
>
> Dice Design Unit - "How do the ratios of median (or quartile range) to sample size compare for samples of 10 and 40 tosses?"
>
> Consumer Research Unit - "How do prices per length (volume, weight) compare for different brands of a product?"
>
> Traffic Interchange Unit - "How does the straightness of one curve measured in feet per degree turning compare to the straightness of another curve?"

This paper describes a technique by which ratios can be compared to each other simply by drawing to scale. This avoids division in which only a minority of elementary school children are skilled. We note however that when children are learning to divide or are comparing different fractions this technique can also be used as a representation so that they can better understand such statements as $\frac{2}{5} > \frac{3}{8}$.*

The general approach is to first "picture" the ratio as indicated in figure 1. To represent the ratio $\frac{7}{9}$ for instance, the child draws a horizontal line 9 units long (inches, centimeters or any convenient choice) from

*When children perform division so as to convert fractions into decimal fractions, the latter can be directly compared for size. This will confirm for the student the result obtained from drawing the triangle. For instance, 2/5 = 0.4 and 3/8 = 0.375. As the first non-zero digit is 4 in the case of 2/5 and 3 in the case of 3/8, the first number is bigger.

point A and from its end a vertical line 7 units long. (This may be done on graph paper - perhaps 1/2" squares, eliminating the need for measurement or for making a right angle.) The direction of the hypotenuse from the left hand corner as shown by the arrow, represents the ratio.

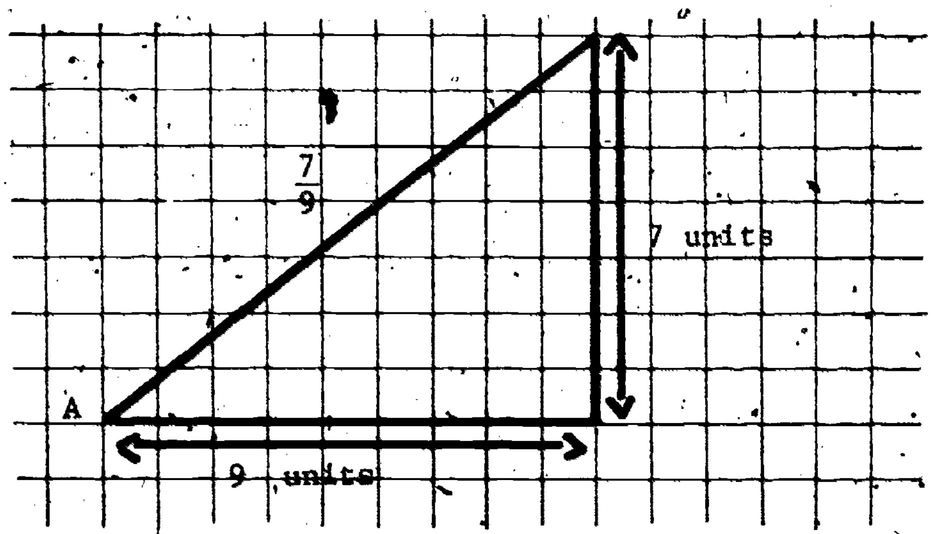


Fig. 1

There is usually no need for the child to measure the angle in any way. If another ratio that is bigger, is so constructed from A, say $\frac{4}{3}$, then the direction of the hypotenuse will be to the more vertical than that of the previous case, as shown in figure 2. If the ratio is smaller, say $\frac{3}{5}$, then the direction will be less vertical as also shown in figure 2. If the ratio is the same as the last, such as $\frac{6}{10}$, then the direction will be the same. This is shown in figure 3.

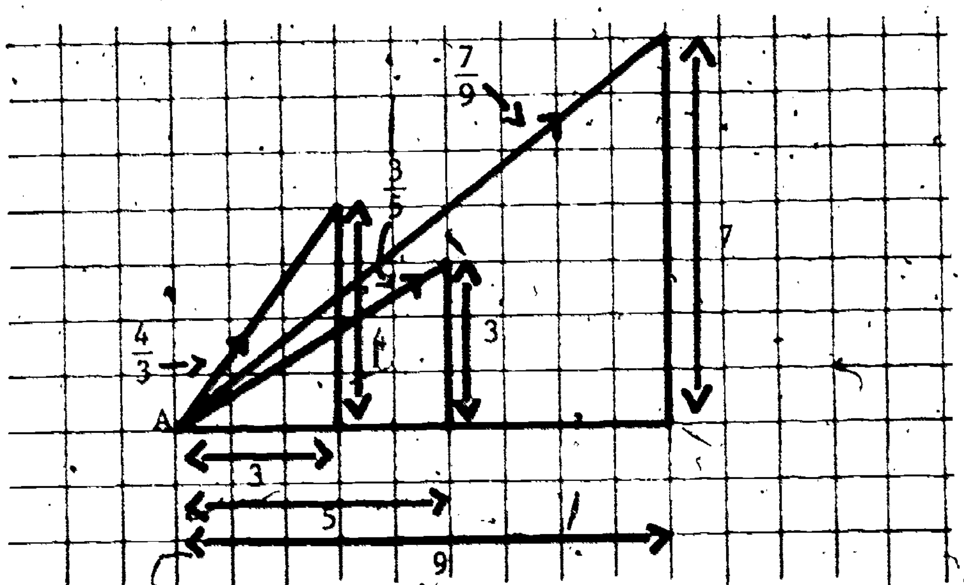


Fig. 2

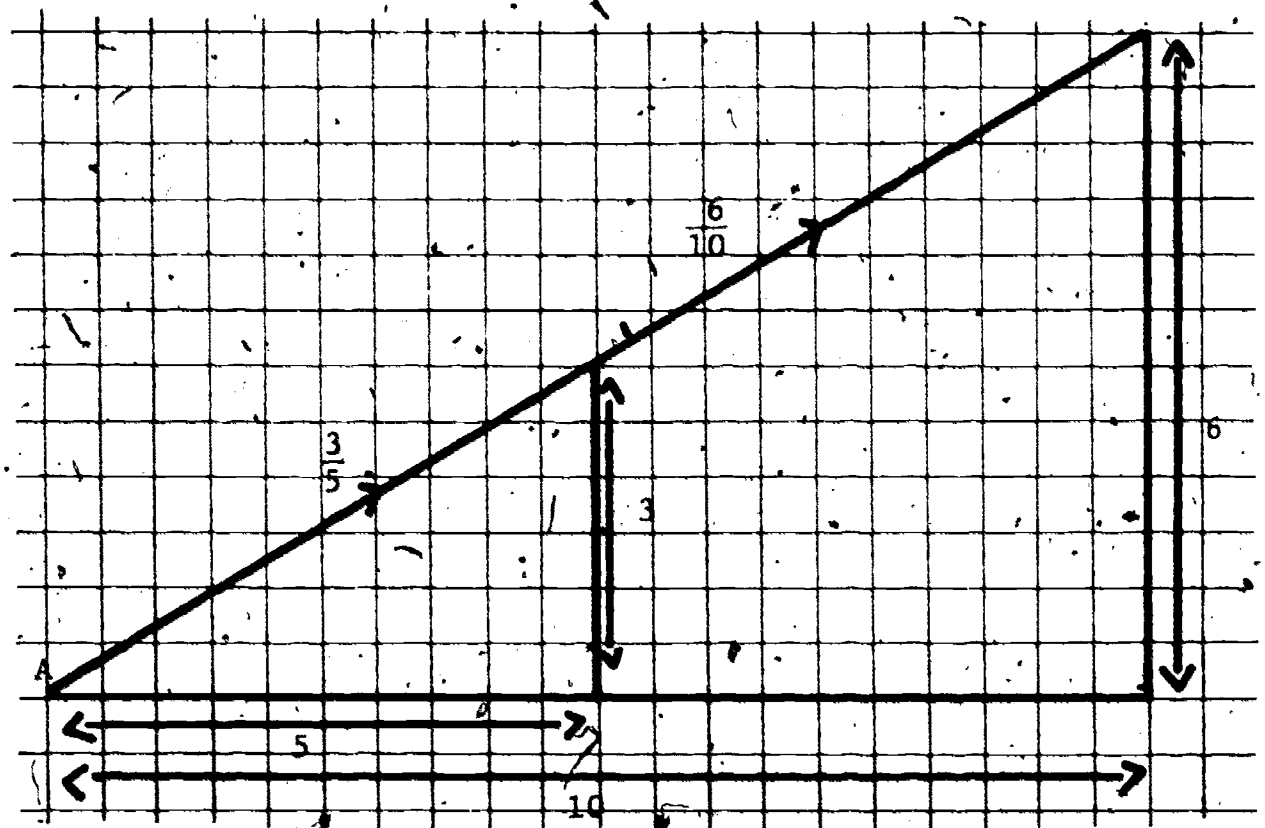


Fig. 3

A child can be introduced to this concept by constructing the triangles for several simple ratios that he knows to be equal, larger or smaller. For instance, even in the primary grades most children will be familiar with the fact that $\frac{2}{4}$ is the same as $\frac{1}{2}$, and that $\frac{1}{2}$ is bigger than $\frac{1}{4}$ but smaller than 1 (or $\frac{1}{1}$). The child's drawing (see Fig. 4) would illustrate how the direction of the hypotenuse tells him whether one ratio is bigger, smaller or equal to another.

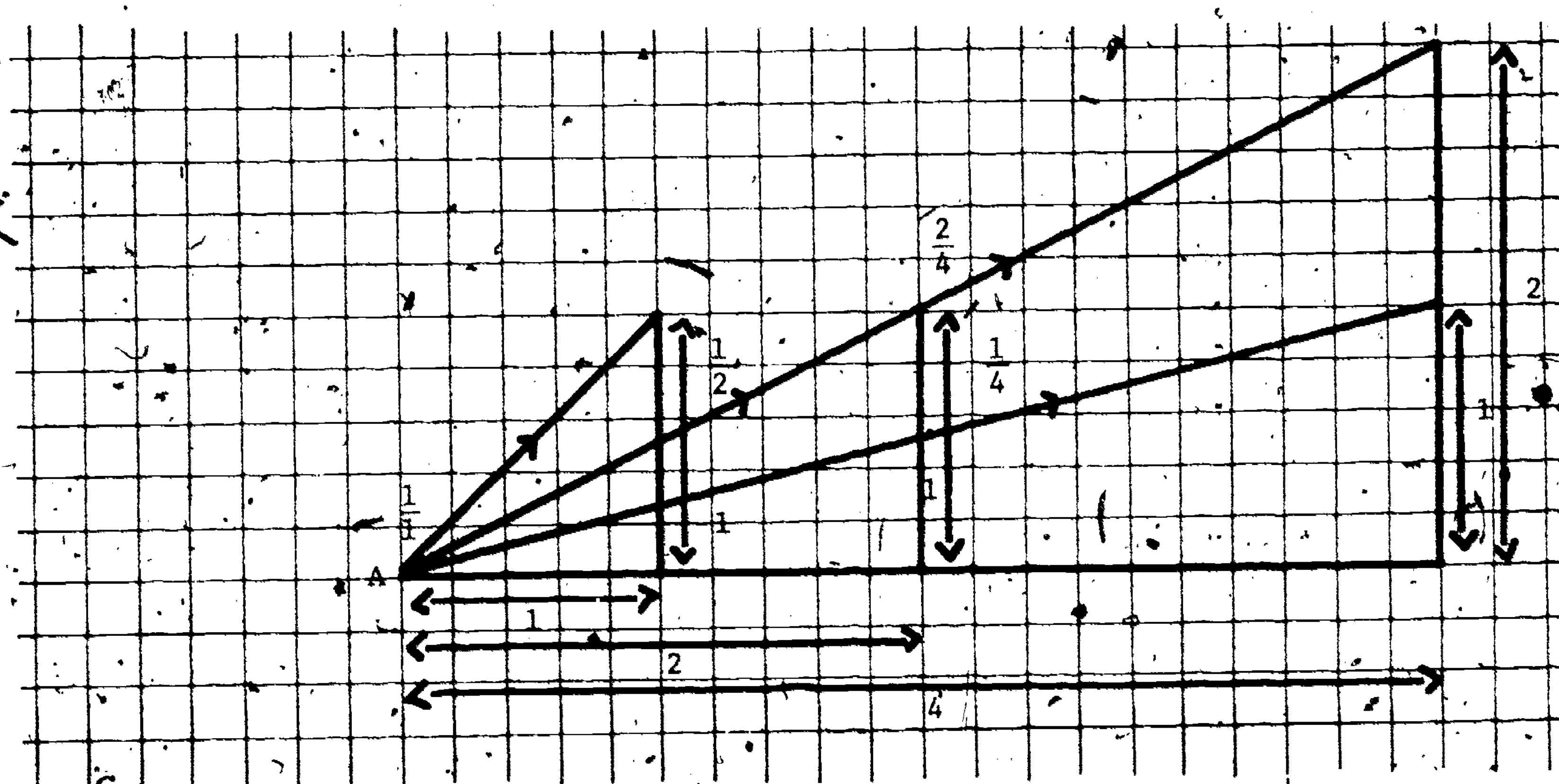


Fig. 4

The children can practice with as many different ratios as they like until they are convinced that ratios they know to be equal give the same direction, etc.

Applications to "Describing People"

How much alike are fifth graders in the ratio of trunk length to leg length? This question may arise when considering the design of dresses or suits, or it may come up in deciding what "long-legged" implies in that age group. How does this ratio for fifth graders compare with the same ratio for first graders or for adults?

These questions can be answered by the above triangle diagrams without any angular measurements. For this particular case one can take advantage of the fact that a person can sit on the floor with legs extended and back up against the wall, thus making the triangle diagram. The direction from his heels to the top of his shoulders (bottom of his neck) represents the ratio of trunk length to leg length as shown in figure 5.

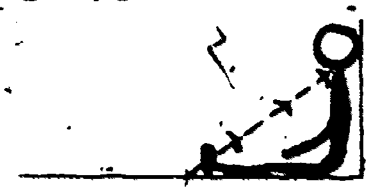


Fig. 5

The younger children could take a large piece of cardboard, put it against the side of a child sitting down as above, and with a yardstick draw an arrow from heels to neck. The older or more skilled child can measure the two lengths and then draw them to scale on graph paper. Even in the latter case the comparison of the "live" case with the scale drawings will reinforce the observation that the hypotenuse direction is the same. Of course the measurements can be made standing up once the concept is clear.

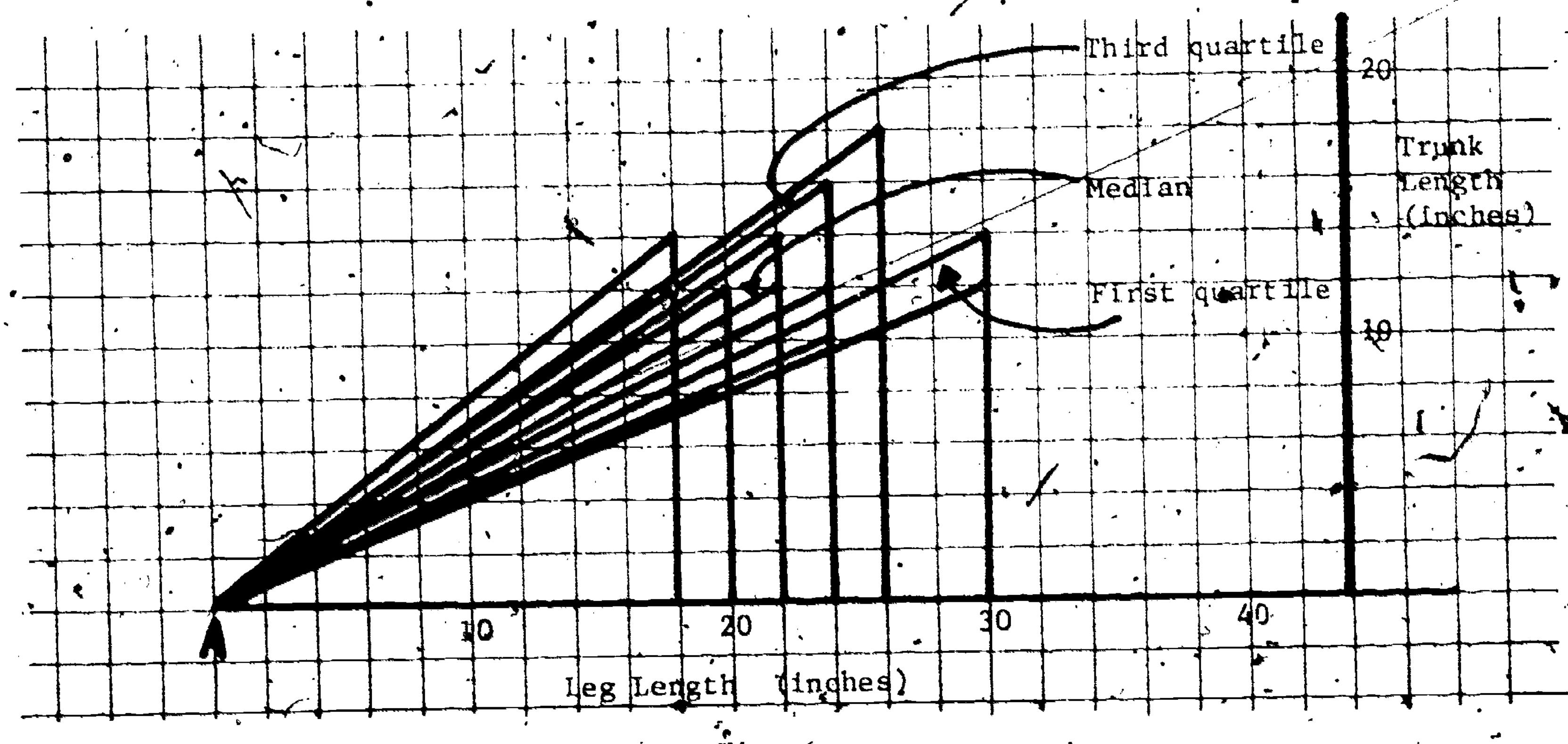


Fig. 6

When the ratio is taken for, say, eleven children and all the triangles drawn from the same point A, the result will look something like figure 6. The median can be found and also the quartiles. One can easily identify either the total range (largest to smallest ratio) or the quartile range. If they are designing clothes then the median will tell them the best ratio of jacket to pants length, and the ranges will indicate how important it will be to have special sizes for long-legged or short-legged peopled. If they are describing someone to be identified in a crowd then they could call those above the 3rd quartile "short-legged" and those below the 1st quartile "long-legged."

On the other hand, the question may be one concerning growth. Do these ratios tend to differ in an adult from a child? To answer this they could make a similar graph for eleven adults and superimpose it on the above graph. (Perhaps they would make the second graph on a transparency using colored lines. The results might be something like figure 7.

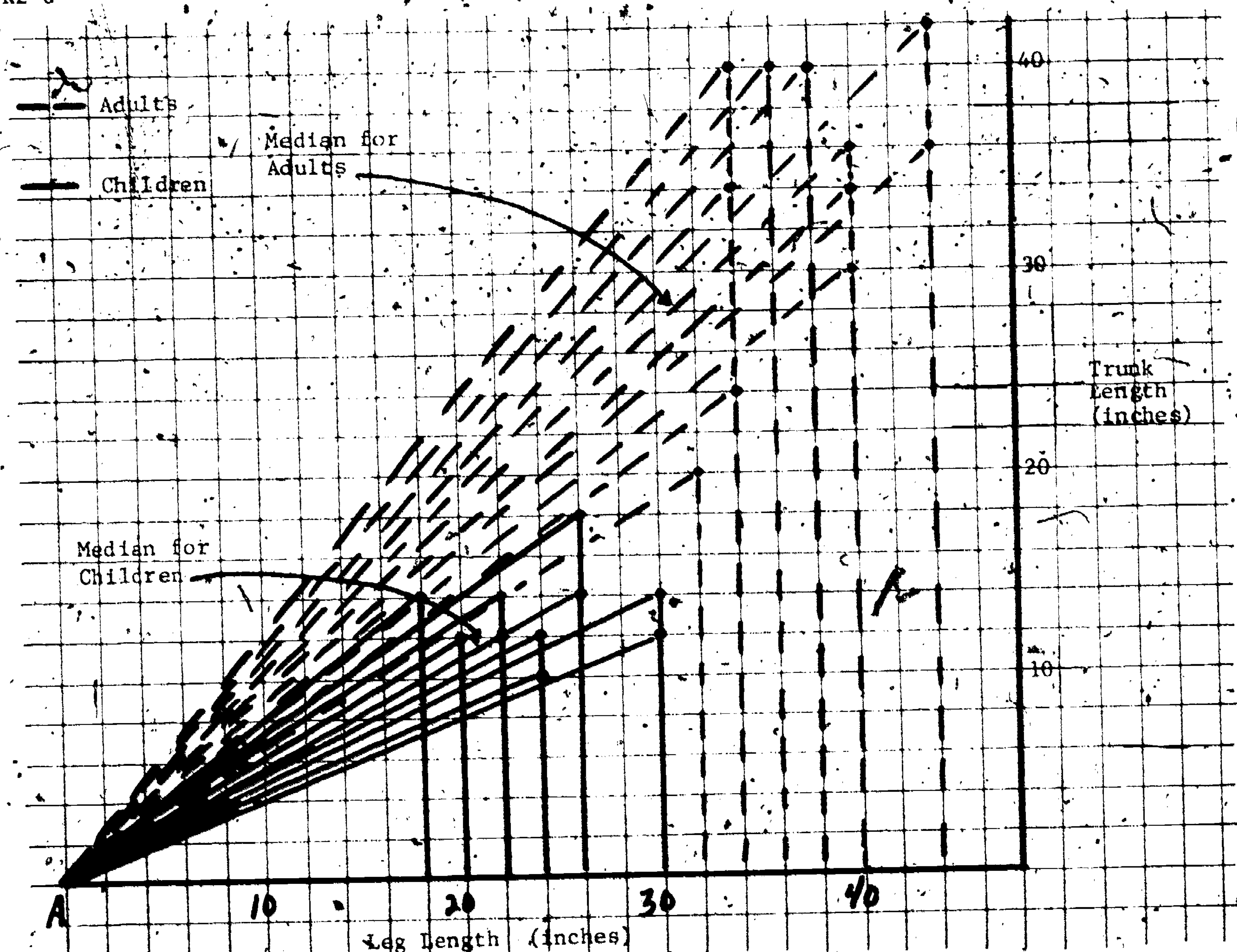


Fig. 7

They could observe that the median direction for adults is more vertical than that for children, that is adults are relatively shorter legged. They may indeed find that the first quartile for adults is more vertical or about the same direction as the third quartile for children; which implies that there is in general a difference between adults and children in that respect and not just a chance variation in the two sample sets.

Other ratios can be compared by the same method; for example, the ratio of wrist circumference to arm length for different children. The children might discuss which measurement should be shown on the vertical line. It doesn't make any difference as long as they are consistent. The meaning of the direction of the hypotenuse should also be discussed with the children. In figure 8 the steeper line represents children with short fat arms, while in figure 9 the steeper line represents children with long thin arms.

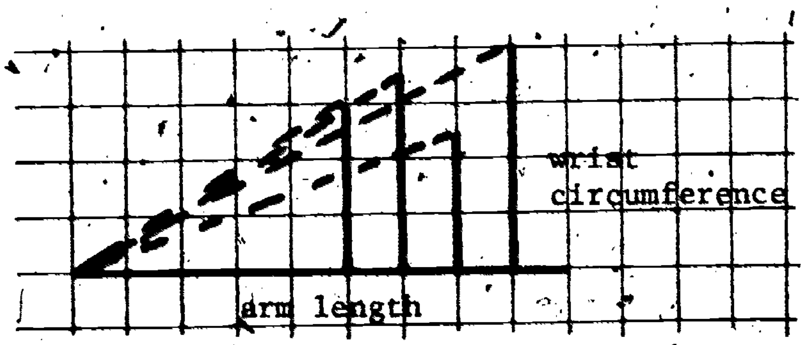


Fig. 8

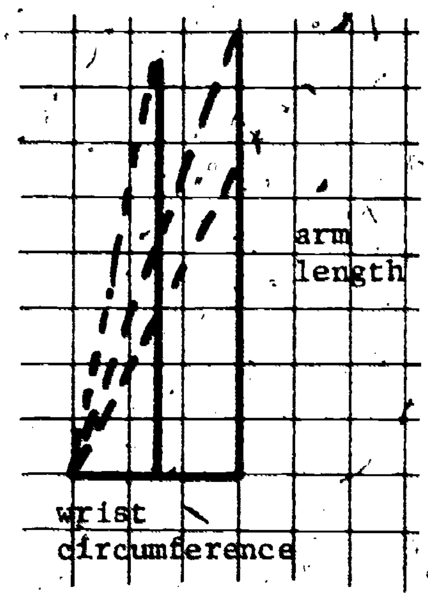


Fig.9

If the directions do not differ much then the children would know that one ratio could be used in making shirt sleeves and that only the arm length would have to be specified as part of a size.

MAKING AND USING A SCALE DRAWING

by

Professor Earle Lomon

Challenged to improve their school lunchroom and its use, students will suggest rearrangements of the equipment in the room — the serving tables, dining tables and benches or chairs, trash cans and so on. This may be coupled with suggestions for different traffic patterns involving changes in entrance and exit, aisles, waiting line position and such things as the number seated at a table. These are all very important things to consider changing, but difficult to test over and over again in the actual lunchroom.

If the students are asked for suggestions as to how they may compare suggested changes in order to select the best for actual try-out, they are likely to offer to make drawings or models. They may want to make a large-scale model with movable pieces so that several people can design different arrangements. When each arrangement is complete, it can be copied onto a smaller scale drawing to keep as a record for future comparisons, and for a few arrangements, as a plan to follow in rearranging the lunchroom itself.

Measurement:

To make a scale layout one must start with measurements of all the relevant objects. Guesses back in the classroom can be in error enough to spoil conjectured arrangements. The class should be asked to make a list of things they need to measure. They will usually think right away of the length and width of the room and the tables. The measuring group may notice other things when in the lunchroom making their measurements. Among these, if not already listed, may be the length and width of benches, the diameter of trash cans, the width of doors, and the dimensions of any fixed serving area such as large counters.

When using their scale layout the children may find that they need some more measurements. Perhaps they forgot to measure how far the doors were along the wall, the distance between benches and table, or the width needed between benches for comfortable passage. A few students can return to get those measurements.

In measuring they will probably use a yardstick (meter stick)* and report their results in yards and inches (meters and centimeters). More than one student should measure each thing, and their results should be compared. When two results are fairly close (say within 12 inches of each other for the length of the room, and within an inch for the width of a table), their difference represents the limitations on measurement accuracy with the tools at hand and the skill of the experimenter. From several such measurements one gets a most likely choice by taking the average of the median. The median is by far easier as it is simply the middle measurement when they are arranged in order of size. If five measurements of the width of the room turn out to be, in increasing order,

9 yd., 32 in. (9 m.)
9 yd., 34 in. (9 m., 5 cm.)
10 yd., 1 in. (9 m., 12 cm.)
10 yd., 2 in. (9 m., 15 cm.)
10 yd., 5 in. (9 m., 22 cm.),

the median is the third one, 10 yd., 1 in. (9 m., 12 cm.). The average in this case is 10 yd., 4 in. (9 m., 11 cm.) which is not significantly different

*Metric unit measurements are provided in parentheses after English unit measurements so that scaling can be done in the metric system if meter sticks are available.

within the achievable accuracy. If four measurements had been made one could choose either the second or third, or take an average of the second or third. It is far easier to take an average of only two measurements (it is the number "in the middle" between the two and can be reasonably guessed at, especially on a number line) than of three or more.

Sometimes one of the measurements is far different from the others. For instance, if a sixth measurement of the above width of the room had come out 9 yd., 2 in. (8 m., 25 cm.), this is likely to have resulted from a gross error such as missing a count on the number of times the yardstick (meter stick) was moved over, or moving it over to the wrong place when distracted. Such gross error measurements should be discarded and not included in finding the median or average. Enough measurements need to be taken to see which cluster together and which are grossly different.

In order not to work too hard one should have some idea of the accuracy needed for design purpose. An error of 6 in. (15 cm.) in the length of a room, when distributed between ten aisles between tables only makes an error of about 1/2 inch (1 1/2 cm.) per aisle, which won't make very much difference to the passage of people. But an error of 2 in. (5 cm.) on the width of tables may cut down actual aisle space by those 2 in. (5 cm.) which may be uncomfortable.

Constructing the large scale model:

It would be convenient for the students working on the layout to have the layout on a table of about 3 ft. x 5 ft. (1 m. x 1 m., 60 cm.). If so, it is very likely that a convenient scale may be about 3 in. per yard (10 cm. per meter). If the lunchroom is, say 30 ft. x 50 ft. (9 m. x 15 m.), then a piece of heavy cardboard of about 34 in. x 55 in. (95 cm. x 155 cm.) would be suitable. If its corners are square, then it can be marked off in inches along the edges. Parallel lines can then be drawn from edge to edge to mark off the board in squares. Every three inches (10 cm.) could be lined off and each square would represent a yard by a yard (a meter by a meter) in the actual lunchroom.

The markings on each edge will be connected by a line drawn against the edge of a yardstick (meter stick), but the board is too long for the yardstick (meter stick) to reach the length of 55 inches (155 cm.). One must also make three inch marks across the width from, say the 30 inch (80 cm.) mark on one long side to the 30 inch (80 cm.) mark on the other long side. This will give intermediate points along the length of the paper which can be connected to points on either far end.

The lines will come out at a slant if the board itself did not have square corners. The corners should be checked before beginning with a right-angles plastic triangle, or at least with the corner of a large note pad.

The students can then mark off lines for the two outside walls at right angles to each other two inches (5 cm.) in from one side, and an adjacent side. The other two walls are then marked off at the correct distance from the first two. From there, walls, doors and so on can be marked in according to their measurements. The lines on the cardboard will give them positions to the nearest yard (meter). When scaling fractions of a yard, the children will obtain some practice in division. Eighteen inches is half a yard and so will become one and one half inches to scale: Nine inches is a fourth of a yard and so will become three quarters of an inch to scale, and so on. They may use dividers and trial and error to divide to thirds and other divisions not on their rulers.

All the fixed distances, walls, doors, pipes, immovable furniture, etc. should be drawn to scale. The movable parts, such as desks, benches and trash cans should be cut to scale <u>in plan view</u> out of lighter cardboard. There is no need to make 3-dimensional models of the furniture, as they will not be arranging any things in the vertical. If benches are partly under tables, the cardboard rectangles for the benches can be slipped partly under the large cardboard table rectangles. Making 3-dimensional models takes a lot of time which would only slow down the lunchroom investigation.

The students can now lay out the pieces according to their ideas, making sure the aisles are wide enough and that doors are not blocked. They may want to put "people pieces" (about one inch diameter circles) on the benches or marching down the aisles. They may combine their timing measurements with this layout also. They can see how the queue forms as arrivals come in at a certain rate and by moving "people pieces" around see if the students may get into each other's way. When they have one set-up all ready it is time to make a small scale sketch of the whole thing.

<u>Making and using the small scale sketches</u>:

Pads of 1/4" or 1/8" (1/10") squared paper can be obtained. By using the 1" spacing to represent a 3" (10 cm.) spacing of the large scale layout, the result will fit on one sheet or perhaps on two sheets taped together. All the lines of the large scale layout should be drawn in on the small scale sketch, and all the movables should be drawn in also. The small scale drawing is meant to be a record of one arrangement and hence it needs no movable pieces. When complete it should be labelled with the designers names, and whether it is their first, second or third, and then put in a folder.

When several designs are ready, the small scale sketches can be presented to the class as a whole for criticism, comparison, and possible improvement. The best ones can then be presented to the principal or lunchroom manager with a request that they be tried.

The small scale drawing of the design that is being tried can then be used as a blueprint for arranging the lunchroom. Several copies may be needed. The large scale layout can also be set up on a table in the lunchroom while the arranging is going on. The small scale sketches can finally be used in the class report of their activities.

THE SIT-DOWN GAME

by

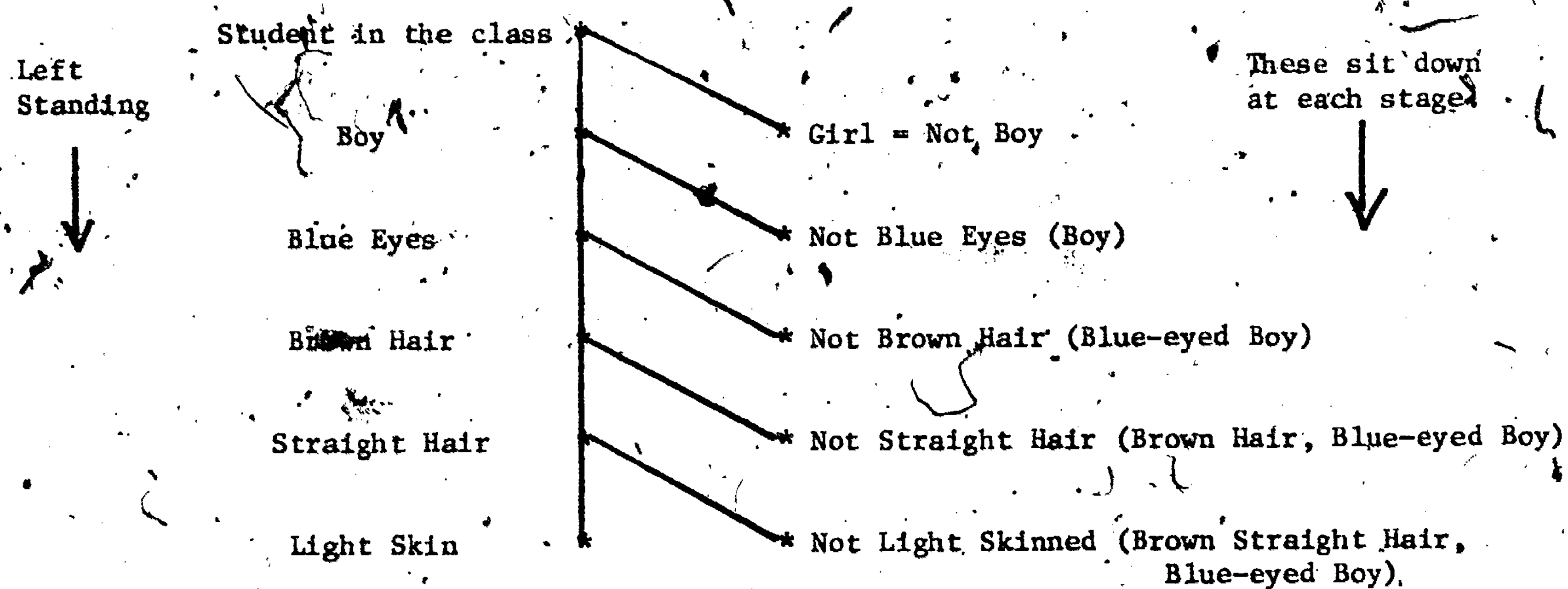
Merrill B. Goldberg

The sit-down game provides a natural vehicle for illustrating ideas which the students may have seen in a study of set theory. Each of these ideas, however, can also be stated without the use of set theory and so a lack of knowledge of set theory should not preclude playing the sit-down game -- complete with a discussion of what is happening. Moreover, this activity can be used to lead to a discovery for the students of set theory concepts and rules.

The basic activity of the sit-down game is the following. The entire class stands. Characteristics such as "has two eyes," "has brown hair," "is a boy," etc. are named and written on the blackboard as they are named. As each characteristic is named, each student standing must decide if he has that characteristic or not. If he does, then he remains standing; if he does not have the characteristic named, then he immediately sits down. The game continues - characteristics are listed until only one student is left standing.

Listing characteristics divides the class into groups or sets. The students standing are those students who have all of the listed characteristics (in other words are in the intersection of all the sets specified). A possible aid to discussing the game is to make a tree diagram as the characteristics are listed, as follows:

(Everyone is standing)



As each characteristic is listed, the vertical line is extended downward to represent the standing students who have ALL of the characteristics listed on the left. Also at each stage, a line is extended diagonally downward to the right to represent those students who sit down at that stage of the game. For example, when the characteristic "brown hair" is named, the students who sit down are the students who are standing and do not have brown hair. Other students may not have brown hair either, but they do not sit down because they are already seated. Thus the students who sit down at that stage are the students who do not have brown hair, but are blue-eyed boys. It may even be worthwhile to have the students sign their name on the blackboard on the right in the appropriate place as they sit down. Then when the game is done, there will be some information on the board about every student except those left standing. They would then sign on the left and be totally described.

While it is not essential that students know the language of set theory in order to play the game, the game does offer several opportunities to use the language and ideas of set theory. As each characteristic is listed, questions should be raised: What do all of the students standing have in common (besides the fact that they are all standing) that distinguishes them from each student that is sitting? Can each seated student find some characteristic in which he differs from everyone who is standing? Is there anyone in the room that has none of the characteristics listed? Which characteristics listed cause more people to sit down? In the language of set theory, each characteristic is thought of as specifying a set, e.g. the set of boys, the set of brown-haired people, the set of people with two eyes, etc. As each set is specified, the students standing decide whether or not they are members of that set. The members of the set (who are standing) remain standing, while the non-members sit down. At each stage, the students standing have in common the characteristic that they are members of ALL of the sets specified, that is they are members of the INTERSECTION of the sets specified on the blackboard. The seated students also have something in common which is a little more difficult to phrase. Each seated student can find one listed characteristic which he does not have -- namely the one which resulted in his sitting down. Since all of the students standing have ALL of the characteristics listed, they have the particular characteristic that caused a seated student to sit down. Therefore, each seated student can find one characteristic that he does not have, but that all of the standing students do have. In the language of set theory, each seated student can find one set of which he is NOT a member, but of which every standing student IS a member. This could also be phrased as "A seated student is one who can find a characteristic on the board which he does NOT have, while a standing student has all of the characteristics listed on the board." Note that the characteristic that distinguishes one seated student from all of the standing students will be different from the characteristic that distinguished another seated student from all of the standing students. The common property shared by all of the seated students is that for each seated student there is at least one of the specified sets of which they are not a member.

Another interesting question arises after only one student is left standing. If the same characteristics are listed in a different order and this new list of characteristics is used to play the game, what will happen? Will anyone be left standing? Will the same person be left standing? Will he be the only one standing? This question is easily resolved by simply playing the game and observing the outcome. After this has been done, it should become apparent that no matter what the order of specifying characteristics, the same people will have them and so the same person will have ALL of them and be left standing, while every other student will not have ALL of them and so will sit down at some point. Again, the language of set theory may be used. Here, we can say that the intersection of two or more sets is the same no matter which order is used to list the sets. For example, $A \cap (B \cap C) = C \cap (A \cap B) = B \cap (C \cap A)$, etc.

The efficiency with which characteristics eliminate students is related to the size of the set which they specify, or in other words how many students have that characteristic. Thus some characteristics such as "has two eyes" will include everyone in the class and will therefore be most inefficient since it eliminates no one. A characteristic such as "is a girl" is possessed by roughly half the class and so eliminates the other half. A characteristic such as eye-color specifies a small set since, in general, less than half the students will have eyes of a specified color. Such a characteristic therefore eliminates a larger set and is therefore more efficient in reducing the standing group to only one student. The efficiency of a characteristic may vary with the order in which it is presented. This is related to the predictive nature of certain characteristics. Specifically, darker skin, eyes, and hair are highly correlated and so once one of them is listed, the others become less efficient - possibly to the point where no one additional sits down after a second or third of these correlated characteristics is specified. The students could no doubt verbalize this correlation.

Finally, many variations exist to the details governing the playing of the sit-down game. The following list is only a beginning and by no means exhausts the possibilities. The students would hopefully wish to make up several different versions and then try to predict the outcome.

VARIATIONS:

1. One student is "it" and he does the listing of characteristics trying to leave only himself standing at the end of the game. Can he do it with fewer questions than another student?

2. A student names the characteristics attempting to leave only a pre-selected other student standing.

3. The class could be split up into teams and then either of the above two versions be played alternating a person from one team with a person from the other team and a score be kept for the entire team. The score being the number of characteristics needed - with low score winning. Alternately, the score could be 10 less the number of characteristics listed with the high score winning.

4. Specify a list of characteristics with no particular student in mind and then use these to play the game.

5. Specify a list of characteristics so that when these are used no one is left standing at the end of play. Characteristics such as "has 17 ears" could reduce this task to nonsense. If the students realize this and specify such characteristics, they should be made to realize that they are describing a set which is disjoint from the set of members of the class. That is the intersection of the set of people with 17 ears and the class is the empty set. They may even wish to say that the set of people with 17 ears is another name for the empty set. To avoid such characteristics, it may be desirable to state that each characteristic in the list must be possessed by at least one member of the class. Then in specifying a list that leaves no one standing, the students may be lead into ideas of correlated characteristics.

6. Finally, there is the variation of listing characteristics so that a selected PAIR of students is left standing and everyone else is seated. That cold easily become extremely difficult.

SET THEORY ACTIVITIES: ROPE CIRCLES AND VENN DIAGRAMS

by

Merrill Goldberg

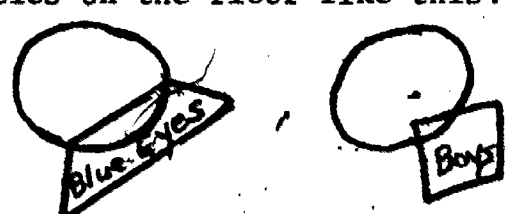
Specifying characteristics for individuals is a natural introduction to set theory. A set is simply a collection of things. Specifying a characteristic, such as "blue eyes" specifies a set - namely the collection of things (in this case students in the class) with blue eyes. The language of set theory is useful (but not necessary) in playing the Sit-Down Game. This set of activities provides an elementary way to introduce the basic idea of a set as a collection of things. Concepts discussed include: class membership (being a member of a set); the complement of a set; and the intersection of sets.

Rope Circles

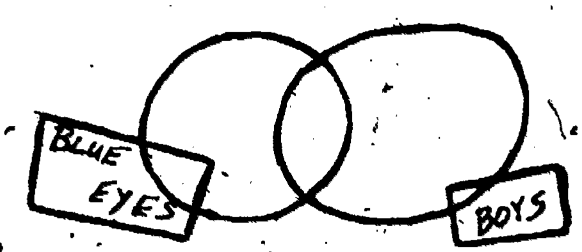
A rope circle is just that. A length of rope, heavy cord, or preferably colored yarn about 20 feet long is tied together at the ends to form a closed loop. This loop can be thought of as the boundary of a more or less circular region on the floor. The loop is placed on the floor and labelled by taping or pinning a sheet of paper to it on which has been placed a characteristic such as "blue eyes". The inside of this circular region represents the set of all students with blue eyes. All the students who belong in this set (i.e. have blue eyes) stand inside the rope circle labelled "blue eyes". These students are members of the set of blue eyed students represented by the rope circle on the floor. Those students who do NOT have blue eyes are NOT members of the set of blue-eyed students. They belong somewhere, too. They belong OUTSIDE the rope circle representing the set of blue-eyed students because they are NOT members of that set - they do NOT have blue eyes. The rope circle labelled "blue eyes" defines a set of blue-eyed students - those who are standing inside the circle. It also defines another set - the set of students who do NOT have blue eyes; that is, those students who are not standing inside the rope circle labelled "blue eyes". The set of students who are NOT inside the rope circle are in the COMPLEMENT of the set defined by the rope circle. In other words, the complement of a set is the set of things which are NOT members of the set. The complement of the set of blue-eyed students is the set of students that do not have blue eyes. The complement of a set defined by a rope circle on the floor can be pictured as the space outside the rope circle. The rope circle labelled "blue eyes" defines two sets: The set of blue-eyed students and the complement of the set of blue-eyed students. These two sets are represented by the inside and the outside of the rope circle. Those students who are members of the set stand inside the rope circle. Those students who are NOT members of the set; i.e. are members of the COMPLEMENT of the set stand outside the rope circle. Thus each time a rope circle is placed on the floor and labelled to specify a set, every student must stand either inside it if he is a member of the set, or outside it if he is not a member of the set.

We have thus far explored the basic idea of a set, including the topics of class membership and the complement of a set. Without mentioning it, we have also seen the concept of a set dividing the universe (all students in the class) into two disjoint sets. More specifically, in setting a rope circle on the floor, all of the students in the class are divided into two groups - those that belong inside the circle and those that belong outside the circle. Every student belongs either inside or outside and no student belongs in both places. He is either a member of the set or he is a member of the complement of the set. He is not a member of both the set and its complement. In other words, each set, together with its complement, divides the entire class into two groups, with everyone belonging to either one of the two groups. Two sets are said to be DISJOINT if they have no members in common – that is, if nothing belongs to both of them. Each characteristic such as blue eyes specifies a set which is DISJOINT from its complement. The set of boys divides the class into two groups: those students who are boys and those students who are not boys, i.e. the set of girls. These are two disjoint sets. It is therefore not necessary to use a rope circle to describe the set of girls if one has already been used to describe the set of boys, since the set of girls is the complement of the set of boys and so the girls would simply be standing outside the rope circle labelled "boys". No additional information would result from specifying an additional circle for the girls – and a fair amount of confusion could arise later.

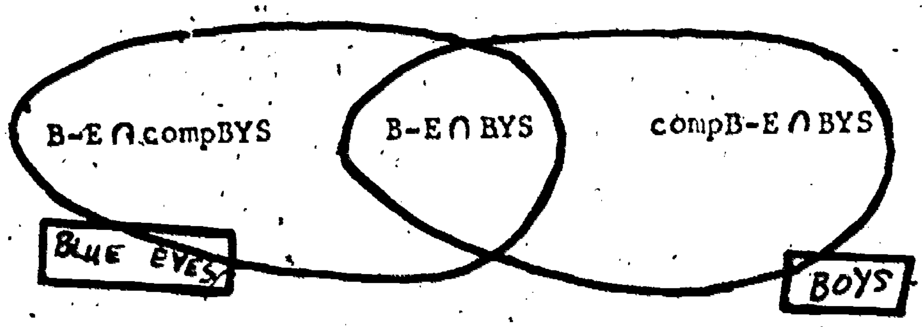
After one circle is set on the floor, and the students place themselves appropriately either inside it our outside it – noting that the set and its complement are indeed disjoint and that everyone either belongs inside it or outside it, pick up the circle and put down a second circle with a different label. Suppose for example that the first circle was labelled "blue eyes" and that this second circle is labelled "boys". After each student has definitely realized whether he belongs inside or outside of each of these circles when placed one at a time on the floor, place both circles on the floor like this:



Ask the students now to stand either inside or outside of each of these circles. That should cause some confusion – especially for the boys with blue eyes! Which circle should they stand in? They belong inside the circle labelled "boys" because they are members of the set of boys. They also belong inside the circle labelled "blue eyes" because they are members of the set of blue-eyed students. They belong to both sets because they are boys AND they have blue eyes. Therefore, they should stand inside both of the circles. The students should quickly rearrange the circles on the floor like this:



It is then possible for each student to stand in one of four appropriate places. Those students who are girls and do not have blue eyes will be standing outside both of the circles - they will not be in either set since they are neither boys nor do they have blue eyes. The boys who do not have blue eyes must stand in the place where they can be inside the circle labelled "boys" but outside the circle labelled "blue eyes". The girls who do have blue eyes must stand so that they are inside the circle labelled "blue eyes" but outside the circle labelled "boys". Finally, the boys with blue eyes will be standing in the section that is part of the inside of both circles since they are members of both the set of boys and the set of blue-eyed students. They are members of the INTERSECTION of the two sets. The intersection of two sets is the set of things which belong to BOTH of them. It is possible to describe all four of the places where students are standing as intersections of various sets. To do this, it will be helpful to have some symbols to use in naming the sets. For example, let BYS be the label for the set of BOYS and let B-E be the label for the set of students with blue eyes. Instead of introducing new labels for the complements of each of these sets, it will be better to introduce a symbol which will mean "complement of". We can introduce the letters "comp" for this purpose and place them immediately before the set whose complement we wish to label. Thus, the set of girls being the complement of the set of boys will be labelled: compBYS. Similarly, the set of students who do not have blue eyes is labelled: compB-E since it is the complement of the set of students with blue eyes, that is, the complement of the set labelled B-E. One more symbol is needed to show intersection. There is a standard mathematical symbol used to indicate the intersection of two sets. It is an upside-down U and it looks like this: $\cap$. To indicate the intersection of two sets, the intersection symbol is placed between the symbols for the sets. Thus, the intersection of the set of boys with the set of blue-eyed students would be written: BYS $\cap$ B-E. The set of girls with blue eyes would be written: compBYS $\cap$ B-E. The symbols: compB-E $\cap$ BYS indicates the set which is the intersection of the sets compB-E (the complement of the set of blue-eyed students) and BYS (the set of boys). This is the set of boys that do not have blue eyes. The four regions that appear when our two circles are placed on the floor can now be labelled with our symbols as follows:



compB-E ∩ compBYS

Now place a third rope circle on the floor defining a third set - say the set of students with brown hair. For this set, we'll use the symbol BR-H. The set BR-H also divides the class into two disjoint sets: those who have brown hair and those who do not have brown hair. The set of boys with blue eyes and brown hair will be the intersection of the three sets specified as the insides of the rope circles. It has the symbol: BYS $\cap$ B-E $\cap$ BR-H. How does it compare to the set with the symbol: B-E $\cap$ BR-H $\cap$ BYS?

The two sets are the same. This is an example of the principle of commutativity for intersecting sets: "The intersection of sets specifies the same set no matter in which order the sets are listed in the intersection." To be more proper mathematically, we should remark that intersection is an operation defined between two sets (like addition is defined between two numbers). The intersection of three sets need not be defined since it can be given in terms of intersecting sets two at a time. In the example preceding, we have denoted a set by:

BYS $\cap$ B-E $\cap$ BR-H. This is an intersection of three sets. It can be viewed in terms of the operation of intersecting two sets by first intersecting the sets BYS and B-E to give the set: BYS $\cap$ B-E. This is now viewed as one set - to aid our viewing it as a single set, place parentheses around it: (BYS $\cap$ B-E). This single set is now intersected with the set BR-H to give: (BYS $\cap$ B-E) $\cap$ BR-H. The associative property of set intersection states that in intersecting more than two sets via a process of intersecting them two at a time and building up as above, the same set will result in the end no matter how the sets have been grouped for intersecting. An example of this is: (BYS $\cap$ B-E) $\cap$ BR-H is the same set as BYS $\cap$ (B-E $\cap$ BR-H).

By adding the third circle to the floor, there now are eight distinct regions which can be specified as set intersections. These are pictured below. Note that the key to the numbered regions identifies them as the intersections of three sets in each case. If the class has studied set theory and is accustomed to using parenthesis and intersecting sets two at a time instead of indicating the intersection of three sets, then a key using that notation would be more appropriate. In all cases, the region is identified by only one intersection of sets. Each such region could also be specified by the intersection of the same sets but in a different order. The same region is specified no matter which order is used to indicate the intersection. All that matters is the sets used. This then is the picture for three rope circles on the floor.

KEY

1 = compBYS ∩ compB-E ∩ compBR-H

2 = BYS ∩ compB-E ∩ compBR-H

3 = compBYS ∩ B-E ∩ compBR-H

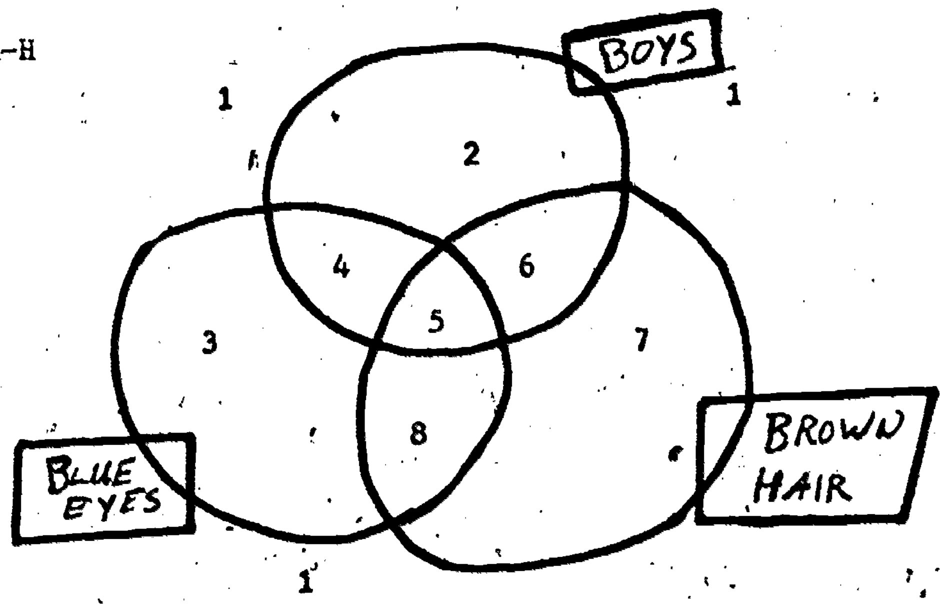
4 = BYS ∩ B-E ∩ compBR-H

5 = BYS ∩ B-E ∩ BR-H

6 = BYS ∩ compB-E ∩ BR-H

7 = compBYS ∩ compB-E ∩ BR-H

8 = compBYS ∩ B-E ∩ BR-H

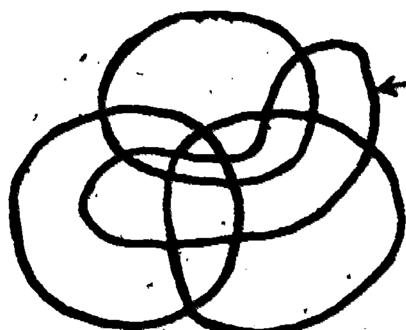


Note that every one of the eight regions in the above diagram is described as the intersection of THREE sets. To describe one of the regions it is not enough to specify which of the three sets: BYS; B-E; BR-H we are in; we must also specify which of the sets we are NOT in. For each of the three sets we must specify whether we are IN it or OUT of it. Thus to specify the region of the picture in which a student belongs we must ask three questions each of which is answered by a "yes" or a "no". These three questions to be asked of the student to be placed in a region are: Is the student in the set of BOYS?; Is the student in the set of students with

BLUE EYES?; and Is the student in the set of students with BROWN HAIR? There is no shortcut possible. All three of these questions must be answered before a student can be properly placed in one of the eight regions. Each time one of these three questions is answered the number of possible regions of the diagram in which to place a student is halved. For example, let's place Sally who has blonde hair and blue eyes in the proper region. At first before answering any of the three questions Sally could belong in any of the eight regions pictured. Asking the first question: Does Sally belong to the set of BOYS? No. Therefore Sally cannot be placed in any of the regions which lie within the circle labelled "BOYS". Thus Sally will be correctly placed in one of the four regions numbered 1, 3, 7, or 8. The possibilities are now four rather than eight. Asking the first question reduced the number of possibilities to just half as many. Asking the second question again halved the number of possibilities: Does Sally belong to the set of students with BLUE EYES? Yes. Therefore Sally must belong in one of the regions which lie within the circle labelled "BLUE EYES". Of the possibilities 1, 3, 7, and 8 which remained after considering the first question only regions 3 and 8 lie within the circle labelled "BLUE EYES". Thus Sally will be correctly placed in one of the two regions numbered 3 or 8. There are now two possibilities - half as many as there were before considering the second question. Asking the third question will again halve the number of possibilities leaving the ONE in which Sally correctly belongs. Does Sally belong to the set of students with BROWN HAIR? No. Therefore of the two regions (3 and 8) Sally belongs in the one which lies outside the circle labelled "BROWN HAIR". This is region 3 which is described as compBYS $\cap$ B-E $\cap$ compBR-H.

Students will probably not want to go through this somewhat lengthy process of separately considering each of the three questions in order to designate the region in which they correctly belong. Yet, before a correct placement can be made, each of these three questions must be answered correctly. To shorten the process somewhat each student could indicate where he thinks he belongs. To check that each student is placing himself correctly, three students (or teams of students) in the class could act as "specialists" - specializing in one of the three sets. There must be at least one "specialist" for each of the three sets. The specialist for the set "BOYS" would concern himself only with whether or not the student has correctly placed himself in a region which is IN or OUT of the set of BOYS. Therefore the "BOYS specialist" would agree with Sally's placement of herself as long as she chose to place herself in regions 1, 3, 7, or 8. The "BLUE EYES specialist" would agree with Sally's placement of herself as long as she chose to place herself in regions 3, 4, 5, or 8 since these are all the regions which lie within the set B-E. The "BROWN HAIR specialist" would agree with Sally's placement of herself as long as she chose to place herself in regions 1, 2, 3 or 4 since these are all the regions that do not lie within the set BR-H. If Sally places herself in region 3 (the correct region for blonde, blue-eyed Sally) then all three specialists will agree with her. If Sally places herself in any of the other seven regions then at least one of the specialists will disagree with her. Assuming that looking at only one characteristic (as a specialist) is easier than considering three and that the specialists can do their job accurately, this method of using "specialists" provides a means of testing the correctness of a student's placement of himself in one of the eight regions.

Adding a fourth circle is quite difficult if it is to be added to the first three so that all possible combinations of sets and their complements will be pictured as a region. To do this, it will be necessary for the fourth circle to be a non-circular closed loop. One possible way of positioning a fourth circle so that all 16 of the possible combinations of the four sets and their complements will have a region is pictured:



There are exactly 16 regions defined by being inside or outside the four circles. Find them.

Adding a fourth circle is sufficiently difficult so that at this stage it should be easier for the students to begin picturing the intersections as just that- intersections of sets. The concept of set intersection is easy to extend to the intersection of five, six, seven or even 100 or more sets. The idea of rope circles gets complicated at four. So this may be a good place to stop picturing the sets as regions on the floor.

VENN DIAGRAMS are circular pictures drawn to represent sets. They are precisely the pictures that are drawn here to represent the rope circles. While rope circles are placed on the floor, circular regions similar to those pictured here may be drawn on the blackboard. As students stand in a rope circle, their initials or name may be placed in the appropriate circular region on the blackboard. In this way, a Venn diagram is put on the board as the rope circles are placed on the floor. The concepts involved are identical, the difference being only one of presentation.

The question will almost certainly arise as to the significance of the line in the diagram or the rope in the rope circle. What does it mean to be on the line? Is that someone who is almost but not quite in the set? NO! A set is a clearly defined object. Everything is either a member of the set or it is not. Thus everything is either in the circle or not in the circle. The line and being on the line is possible in our picture of the set only because it is a picture of the set and not the set itself. The set of boys is the collection of things which are boys and not the part of the floor that is enclosed by the rope labelled "boys". The rope circle and the Venn diagrams are means of representing the concept of sets, they are not the sets themselves.

Another activity using set theory is the sit-down game. Set theoretic concepts involved in that game are discussed in the technical paper describing the game. The students standing are the members of the intersection of all the sets defined by the characteristics on the blackboard.

USING VENN DIAGRAMS TO FIND THE BEST DESCRIPTION

by
USMES Staff

The children in most classes make up individual inventories of characteristics to use in the sit-down game. As they try out their descriptions, they find that some children are identified using the first few characteristics listed. However, in some cases, the child to be identified sits down with others when a certain characteristic is named. In other cases, he is left standing with others when the list is finished. (See technical paper, "The Sit-Down Game", for a more complete description of the sit-down game.)

Venn diagrams can also be used to find out which characteristics identify certain children. After some experience with "living" Venn diagrams (see technical paper, "Set Theory Activities: Rope Circles and Venn Diagrams"), the children can draw Venn diagrams showing two, three or four characteristics. Their drawings will help them:

1. Find a description which will separate each child from his classmates.
2. Find the number of characteristics needed in most descriptions.
3. Find the best characteristics to use in a short description.

Attached at the end of this paper are the inventories of the characteristics of the children in one class. The following paragraphs show how the type of information listed above can be obtained from these inventories.

By drawing a Venn diagram of the set of girls and the set of blue-eyed children, the children will find that Amy is the only girl with blue eyes. A Venn diagram of the set of girls and the set of blond-haired children will show that Amy is also the only girl with blond hair. Either description uses only two characteristics (see Figure 1).

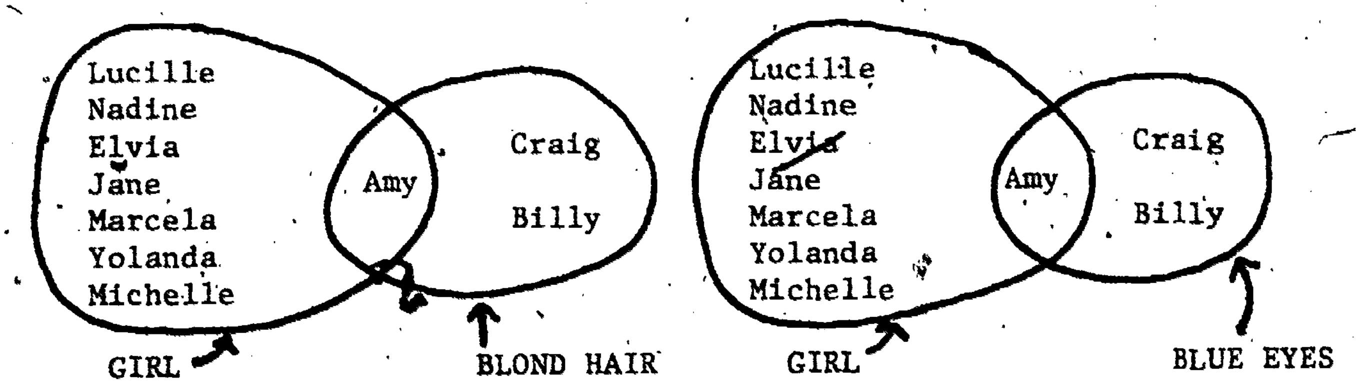


Figure 1

By looking at their lists or by drawing the Venn diagram in Figure 2 the kids would also see that all children in the class with blond hair also have blue eyes.

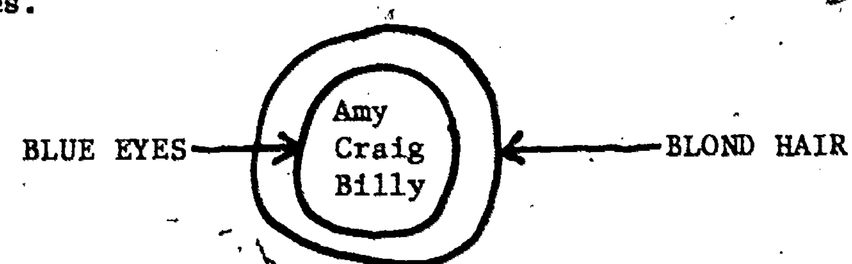


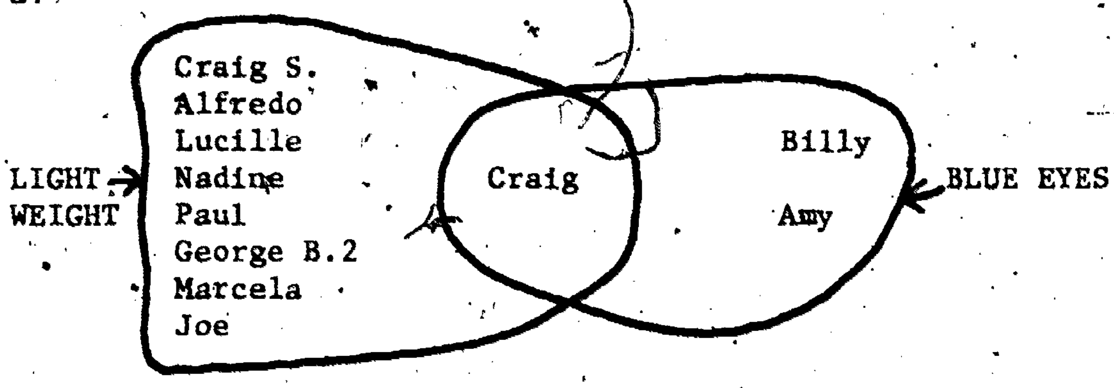
Figure 2

In this class, both characteristics of blond hair and blue eyes are not necessary in a description. The children might check other classes to see if the correlation between the two characteristics is high enough to eliminate one of them as a general rule. If this is the case, they might be asked which characteristic - hair color or eye color - is best to use in a description. Which can be easily seen at a distance? Which can be changed?

As seen in the Venn diagrams in Figure 3, Craig can be specified as:

a. a thin child with blue eyes, or
b. a boy with straight blond hair.

a.



b.

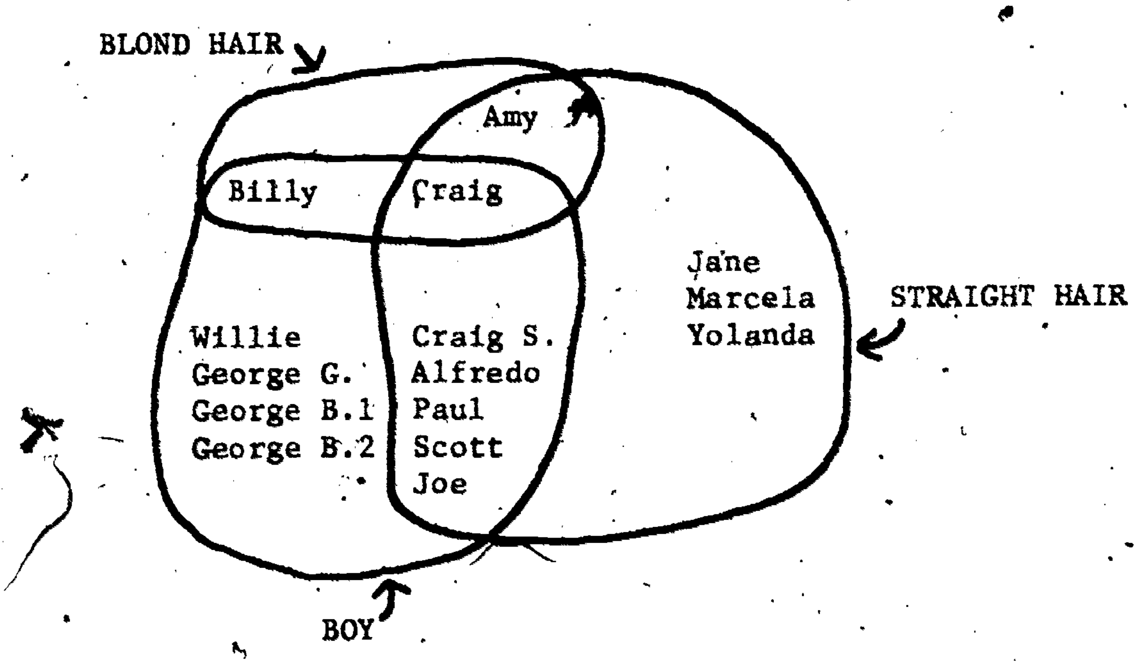


Figure 3

Which description is the best? How many children in the school are light weight with blue eyes? How many boys have blond straight hair? The children can find several descriptions which will identify a person in their class, and then check other classes to find out how large a sample they need to use to correctly determine the best characteristics to use in a description.

A Venn diagram (Figure 4) with three characteristics shows that Alfredo is the only boy in the class with black hair and average height. The same diagram shows that Amy is the only girl who is average height and does not have black hair.

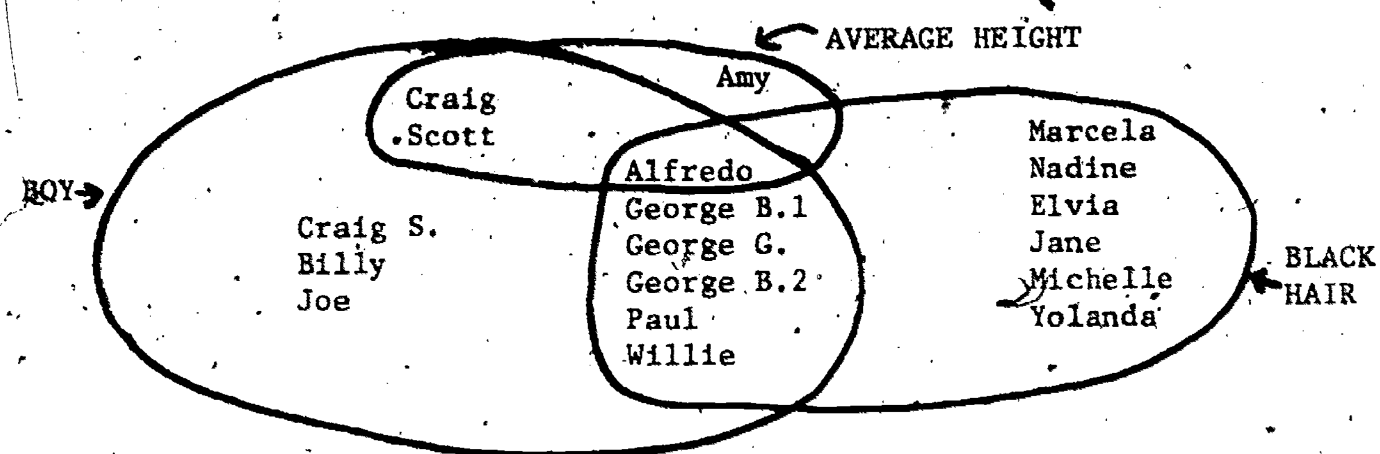


Figure 4

It takes four characteristics - boy, brown eyes, short, straight hair - to identify Paul. See the Venn diagram in Figure 5. The same diagram also isolates Amy, Lucille, Billy, Craig, and George B.2 with different combinations of the four characteristics or their complements.

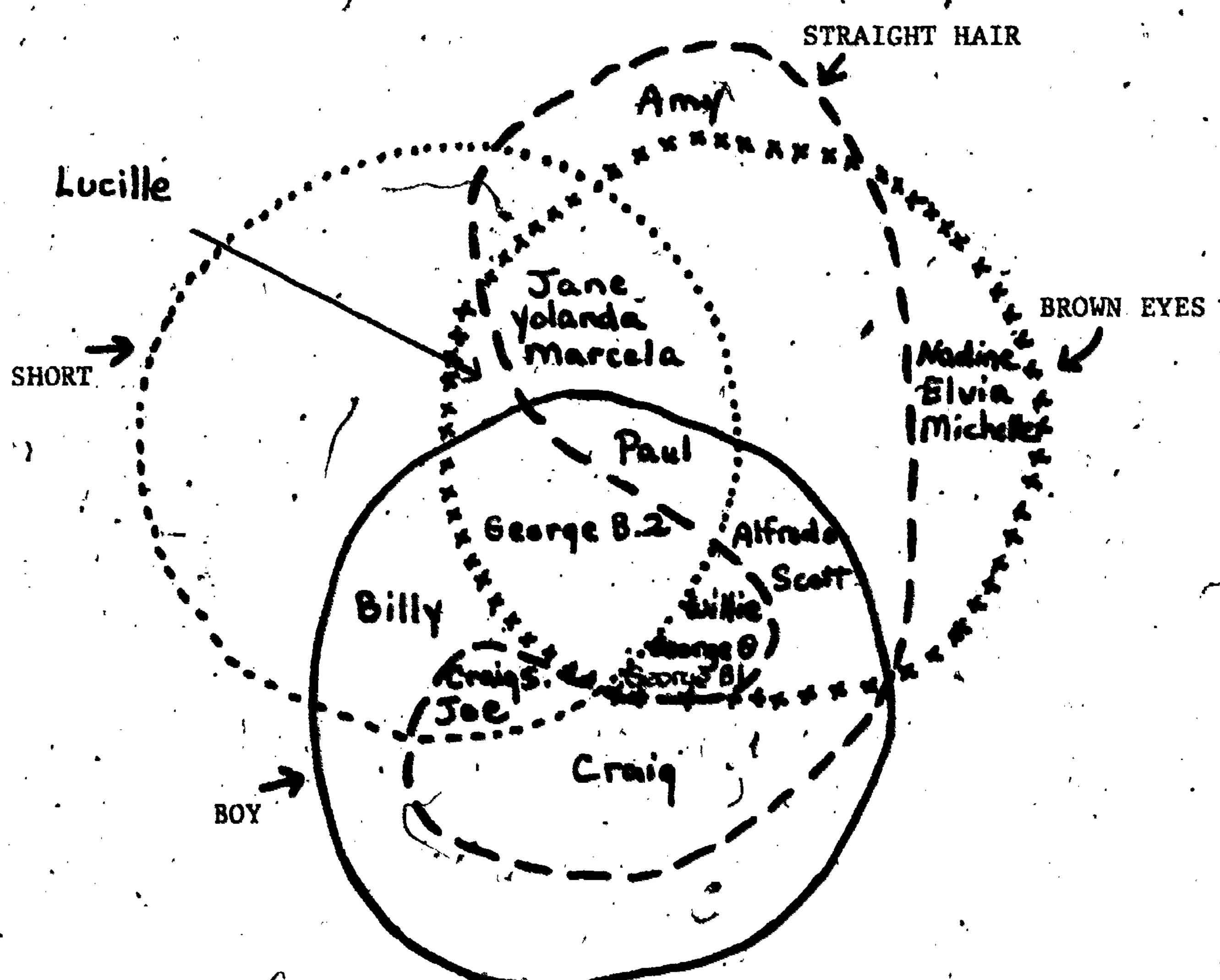


Figure 5

An alternative way of drawing a Venn diagram with four sets (and their complements) is shown below in Figure 6. The same six children are isolated using this type of diagram. In the sit-down game the person left standing will depend upon the order in which the characteristics are named.

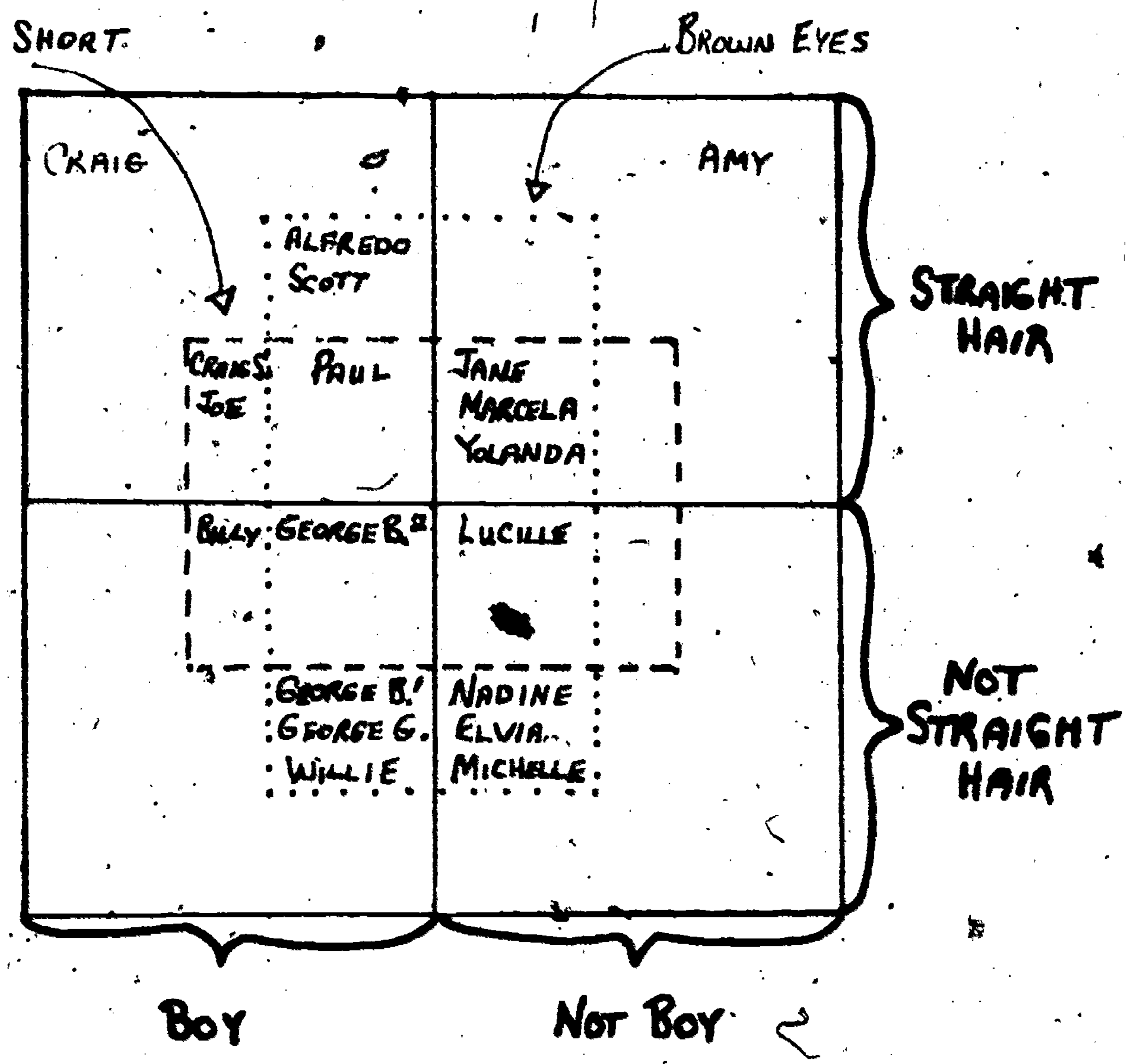


Figure 6

The children may discover that it isn't always easy to identify only one person in a group. Joe and Craig S. have seven identical characteristics. In that case, perhaps an additional characteristic such as shape of face might help.

After the children have been able to identify the children in their class, they can construct a histogram of the number of children versus the minimum number of characteristics needed for identification. The histogram in Figure 7 shows this data for the sample class.

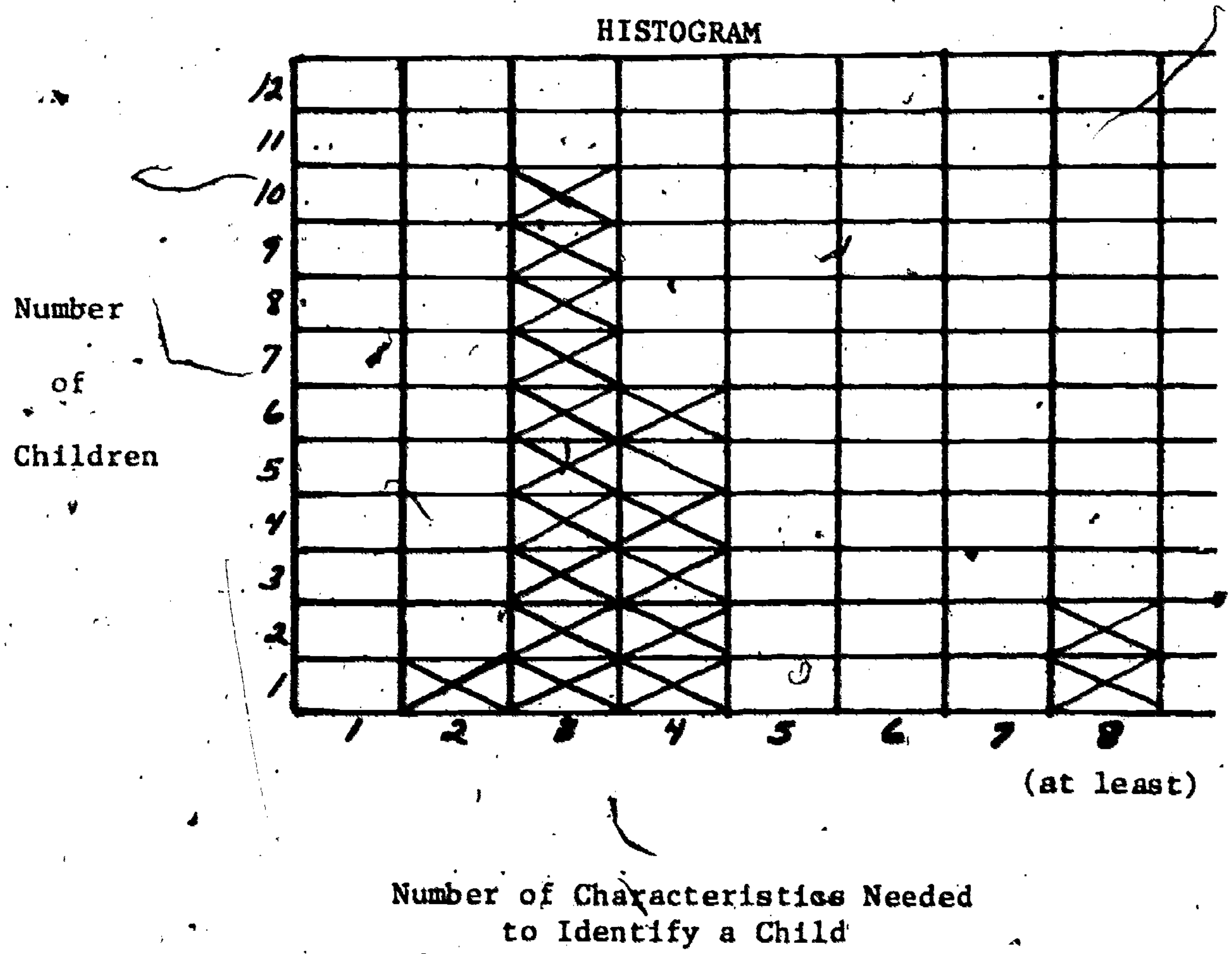
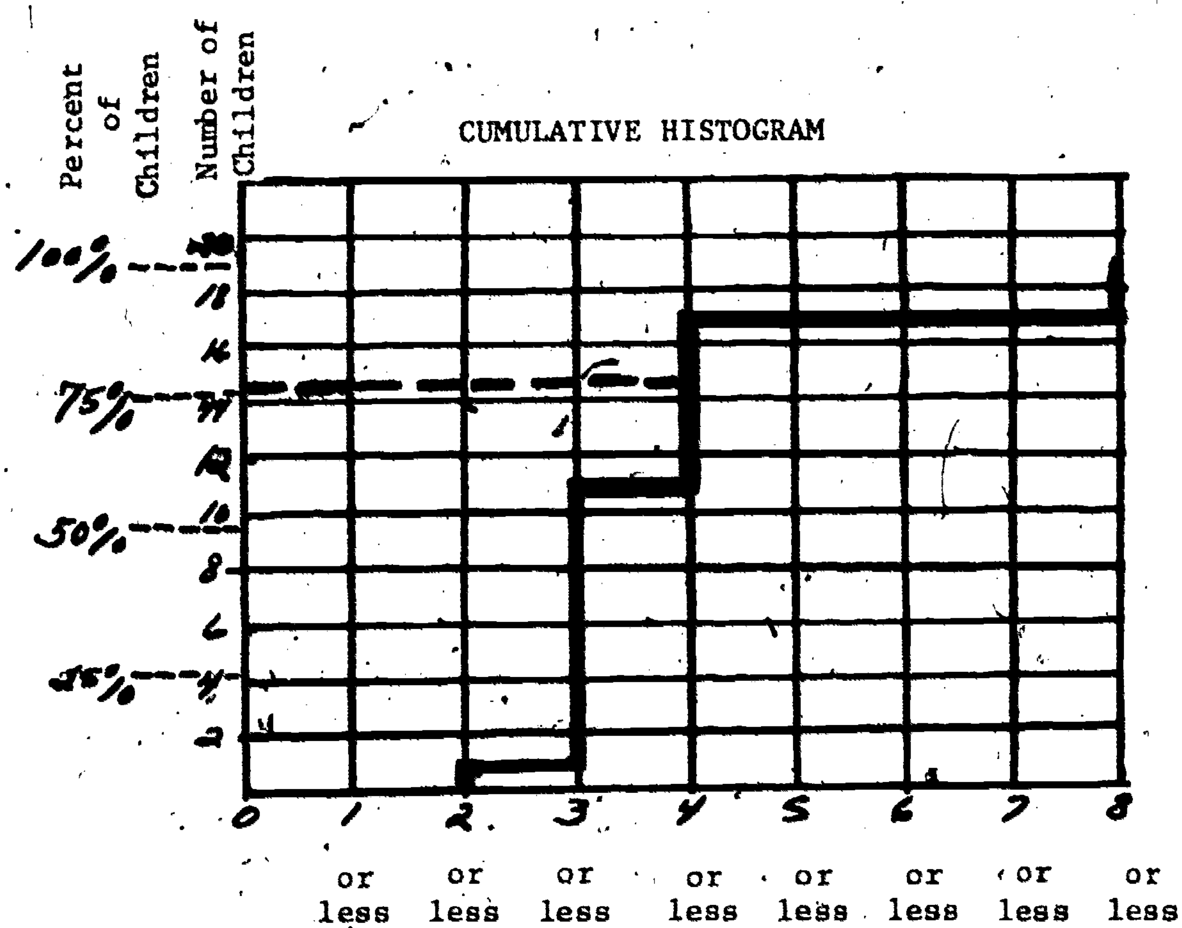


Figure 7

The children might also draw a cumulative histogram using the same data. The number of characteristics needed for correct identification 75% of the time might be taken as a large enough number to include in any description. Figure 8, which is based on the data in Figure 7, shows this to be four characteristics.



Number of Characteristics Needed
to Identify a Child

Figure 8

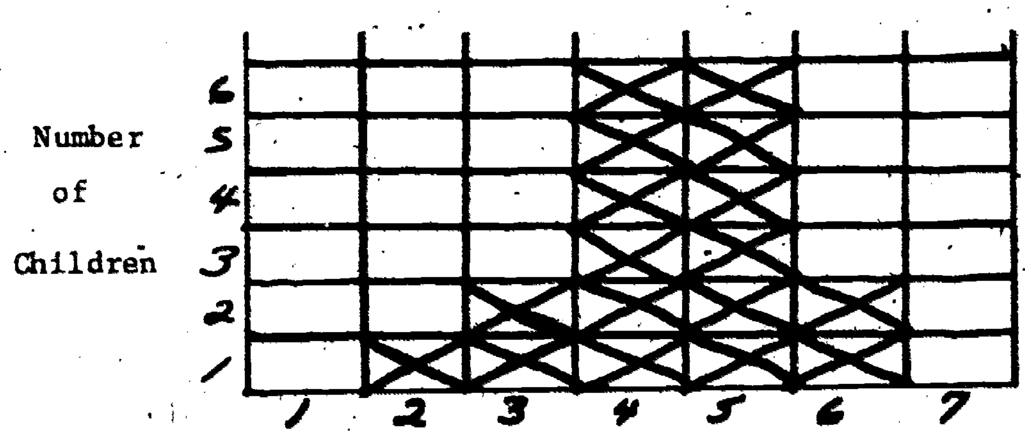
The children will notice that the characteristics used in their "minimum" descriptions are not the same in each description. So, the problem arises: Which four characteristics to use in a description? The children may suggest different combinations as the best. They can check by listing the characteristics in a certain order and drawing a series of Venn diagrams.

The histogram in Figure 9 and cumulative histogram in Figure 10 show the results for the sample class for an order of characteristics as follows:

sex
eye color
height (tall, average or short)
weight (heavy, average or light)
type hair (straight, wavy or curly)
skin color (dark, tan or light)
hair color

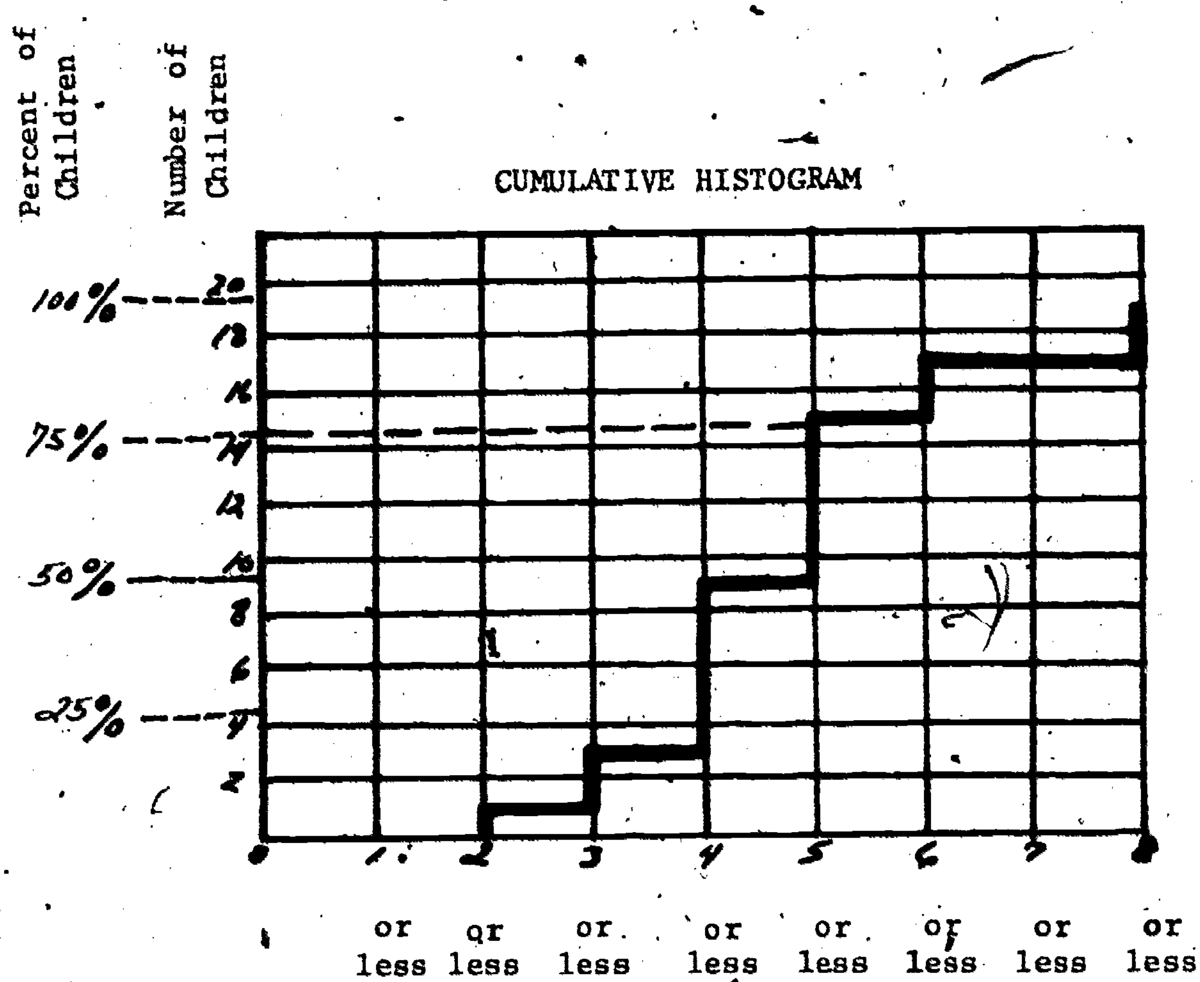
Using this order of characteristics, 75% of the children in the class would be identified using the first five characteristics. These characteristics (or another series) could be tried out in a real life situation such as finding a child on the playground or in the library. The characteristics used in descriptions of lost children can also be investigated.

HISTOGRAM



Number of Characteristics Needed
to Identify a Child

Figure 9



Number of Charateristics Needed to Identify a Child

Figure 10

STUDENTS' CHARACTERISTIC INVENTORIES

Craig S.

boy
lt. brown hair
green eyes
light skin
straight hair
light weight
short

Craig

boy
blonde hair
blue eyes
tan skin
straight hair
light weight
average height

Alfredo

boy
black hair
brown eyes
tan skin
straight hair
light weight
average height

George B.1

boy
black hair
brown eyes
tan skin
wavy hair
heavy weight
tall

George G.

boy
black hair
brown eyes
dark skin
curly hair
heavy weight
tall

Billy

boy
blonde hair
blue eyes
light skin
wavy hair
heavy weight
short

Lucille

girl
lt. brown hair
brown eyes
light skin
wavy hair
light weight
short

Nadine

girl
black hair
brown eyes
dark skin
wavy hair
light weight
tall

Paul

boy
black hair
brown eyes
tan skin
straight hair
light weight
short

George B.2

boy
black hair
brown eyes
tan skin
wavy hair
light weight
short

Elvia H.

girl
black hair
brown eyes
tan skin
wavy hair
average weight
tall

Jane

girl
black hair
brown eyes
tan skin
straight hair
heavy weight
short

Marcela

girl
black hair
brown eyes
light skin
straight hair
light weight
short

Amy

girl
blonde hair
blue eyes
light skin
straight hair
heavy weight
average height

Yolanda

girl
black hair
brown eyes
tan skin
straight hair
average weight
short

Scott

boy
lt. brown hair
brown eyes
light skin
straight hair
heavy weight
average height

Michelle

girl
black hair
brown eyes
dark skin
curly hair
heavy weight
tall

Willie

boy
black hair
brown eyes
tan skin
curly hair
average weight
tall

Joe

boy
light brown hair
green eyes
light skin
straight hair
light weight
short