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ABSTRACT

This booklet consists of three units selected from the SMSG text materials for grade 4, "Mathematics for the Elementary School." These three units were selected as being necessary for students about to undertake the grade 5 SMSG text materials. Chapter topics include concept of sets, sets of points, and concept of fractional numbers. (MP)

E. Alberty

SCHOOL MATHEMATICS STUDY GROUP

MATHEMATICS FOR THE ELEMENTARY SCHOOL

SELECTED UNITS E-4150

(preliminary edition)

SMSG

MATHEMATICS FOR THE ELEMENTARY SCHOOL

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(Revised edition)

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PREFACE

As one of its contributions to the improvement of mathematics in the schools of this country, the School Mathematics Study Group has prepared a series of sample text materials for grades 4, 5 and 6 under the title "Mathematics for the Elementary School". These are designed to illustrate the kind of mathematics curriculum that we believe appropriate for elementary schools.

As its title indicates, this booklet consists of three units selected from "Grade 4" of "Mathematics for the Elementary School".

To teachers and schools who desire to use these three selected units, we would say that it is imperative that they have the textbook, "Mathematics for the Elementary School, Grade 4", and the Teachers' Commentary for this textbook. The complete textbook and the related teachers' commentary provide a means by which a teacher may gain an understanding of the sequence of units to which these selected three are related. The commentary is essential to any teacher teaching one or more of these units.

There are two important reasons for selecting these particular units. First, pupils who have not completed "Mathematics for the Elementary School, Grade 4" but are to undertake "Grade 5" should as minimum preparation have completed these three units along with a conventional grade 4 program. Second, it is obvious that they are useful in augmenting a conventional program. However, their use with conventional materials will require special attention to the use and growth of the ideas once introduced.

The three units included in this selection are as follows:

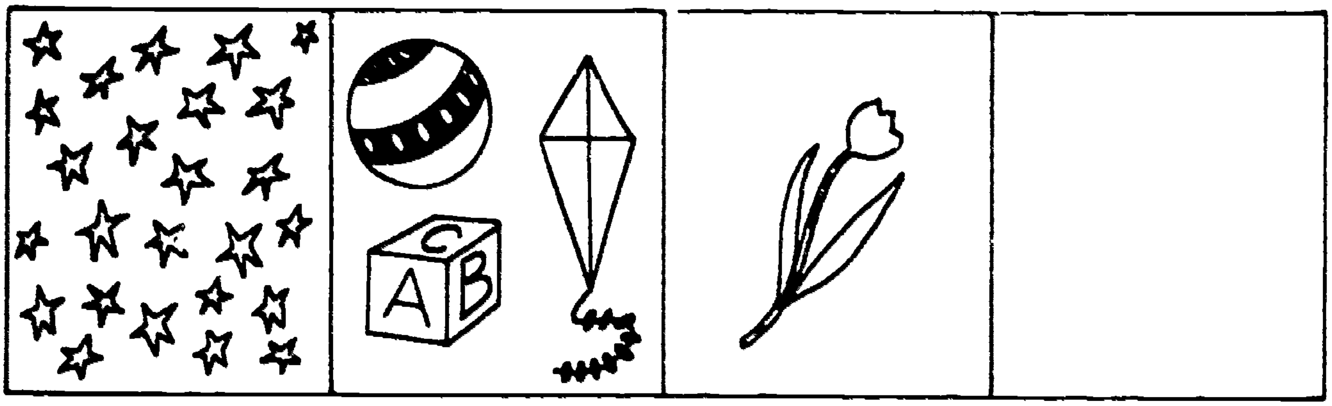
Concept of Sets	Chapter 1
Sets of Points	Chapter 5
Concept of Fractional Numbers	Chapter 10

Chapter 1

CONCEPT OF SETS

THINKING ABOUT SETS

These are pictures of sets.



A Set of Stars   A Set of Toys   A Set of Flowers   An Empty Set

You can think of many sets of things--

The set of children in your school;

The set of children in your class;

The set of numbers, 1, 2, 3, 4, 5, and so on;

The set of numbers, 1, 3, 5, 7, 9, 11, and so on;

The set of numbers, 2, 4, 6, 8, 10, 12, and so on;

The set of letters in the alphabet;

The set of boys in your class who are ten feet tall.

A set is a collection of things. Some of these collections can be sets of objects, sets of people, sets of pictures, and sets of numbers. Think of some examples of sets of things.

A thing that belongs to a set is a member of that set. Each of the letters, b, r, s, t, y, is a member of the set of letters in our alphabet. You are a member of the set of children in your school.

There are sets that have only one member. The set of letters in our alphabet between d and f has only one member. It is the letter e.

There are sets that have no members. The set of children in your class, who are less than four years old, has no members. If a set has no members, it is called the empty set.

We use capital letters for names of sets.
You may use any capital letter you wish.
The letter you choose may help you remember the set.
The states New York and California are members of the set of states of the U. S. A.
We may call this set, Set C. We write

$$C = \{\text{New York, California}\}$$

The counting numbers between 4 and 8 are 5, 6, 7.
We may call this set, Set N. We write:

$$N = \{5, 6, 7\}$$

Exercise Set 1

Name the members of each set:

1. The first five letters of the alphabet.

2. The numbers that you use when you count the first five children in your classroom.

3. The numbers counting by 2's, beginning with 1 and ending with 9.

4. The numbers counting by 2's, beginning with 6 and ending with 16.

5. The letters in your first name.

   (A letter may appear in your first name more than once. Use it only once in the set.)

6. The days of the week whose names begin with "M".

7. The boys in your class less than six years old.

8. The months of the year whose names begin with letter J.

9. The numbers between 30 and 40 that are larger than 50.

10. BRAINTWISTER: The letters which are in the name of your school and not in your last name.

NUMBERS

When you first learned to count, you began with 1. You counted 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and so on. You can count much farther than 12 now. No matter how far you can count, there are still more numbers. If you knew how to count them, you could keep on counting as long as you live. Then, there would still be more numbers. These numbers used in counting are called <u>counting numbers</u>.

In arithmetic there is the set of numbers called the <u>set of whole numbers</u>. These numbers are 0, 1, 2, 3, 4, 5, 6, and so on. We may write the set of whole numbers this way:

$$W = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$$

We may write the set of counting numbers this way:

$$C = \{1, 2, 3, 4, 5, 6, \ldots\}$$

We cannot write all the whole numbers. We use the three dots, ... to mean that there are more numbers than we can write.
The number 0 is the first one written in Set W.
The number 6 is the last number written in the Set W.
But, the number 6 is not the last whole number.
It is just the last number written in Set W.

We write:

$W = \{0, 1, 2, 3, \ldots\}$

$W = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, \ldots\}$

We have used two different ways to name the same set.

Count "by 2's" beginning with the number 0.

You count 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, . . .

These numbers that you name are called <u>even numbers</u>.

The numbers 0, 2, 4, 6, 8, . . ., are called even numbers.

The numerals 38, 54, 76, 128, 100, 200, 1352, are names of some even numbers.

Count "by 2's" beginning with the number 1.

You count 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, . . .

The whole numbers that you name when you count "by 2's" beginning with 1 are called <u>odd numbers</u>.

The numerals 2[illegible], 37, 41, 53, 101, 421, 1247, are names of some odd numbers.

Here are some more sets of things.

Set A	Set B
Mary	a
tree	b
pen	c
car	d
picture	

Set A is a set of words.

The number of words in Set A is 5.

Set B is a set of letters.

The number of letters in Set B is 4.

The number of odd numbers in the set of counting numbers between 1 and 20 is 9. The members of this set are 3, 5, 7, 9, 11, 13, 15, 17, 19.

The number of words in the set { } is 0.

There are no members of this set.

We call this set the empty set.

The number of the empty set is zero.

We use numbers to tell how many members are in a set.

Exercise Set 2

Here are some sets in 1, 2, 3, 4, and 5. Below each set are groups of words. Which best describes each set? Is it a), b), or c)? Write your answer. Then tell how many members are in that set.

1. {1, 3, 5, 7, 9}
   - a) a set of small numbers
   - b) the set of all odd numbers
   - c) the set of odd numbers less than 10

2. {Tuesday, Thursday}
   - a) the set of school days
   - b) the set of the last two days in the week
   - c) the set of days in the week whose names begin with T

3. {10, 20, 30, 40}

a) the set of numbers less than 50

b) the counting numbers less than 50 whose numerals end in zero

c) the set of even numbers less than 50

4. {chalk, book, eraser, pencil}

a) a set of things you find in a schoolroom

b) a set of school furniture

c) a set of things to read

5. {bus, train, automobile, airplane}

a) a set of things you see in the sky

b) a set of things you find in a garage

c) a set of things people may use when they travel

6. Here are some things: potato, 9, Bobby, celery, 3, rock, 5, George, 15, e, 4, bacon, 6, Mary, u, David, a, candy, o, 7, i, key

Select the things that are:

a) a set of boys' names

b) the set of whole numbers larger than 2 and less than 8

c) the set of vowels

d) a set of things to eat

e) a set of things to read

SETS WITHIN SETS

We had some coins in a piggy-bank.

We poured them out on the table.

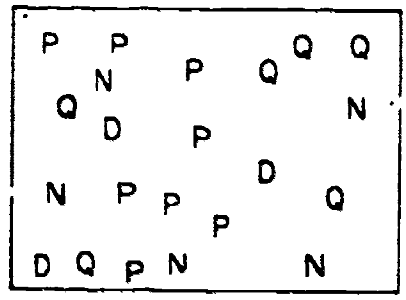
This picture shows the way they fell.

Each N shows a nickel.

Each P shows a penny.

Each D shows a dime.

Each Q shows a quarter.

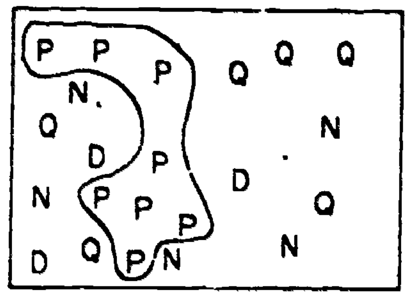


In the set of coins there is a set of pennies.

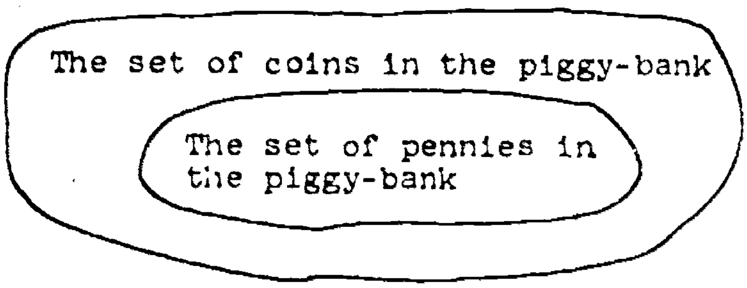
A fence is around all the pennies.

All pennies are inside the fence.

All other coins are outside the fence.



There is another way to show that the set of pennies is within the set of coins. We can show it like this.

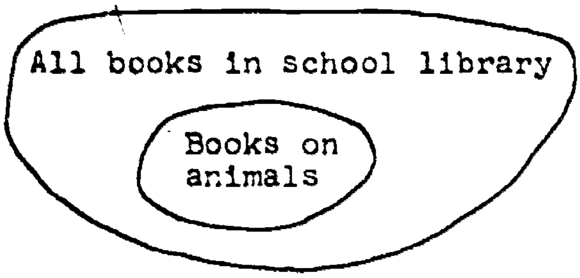


How do we show the pennies in the picture?

They are inside the small ring.

Where are the other coins that are not pennies?

They are outside the small ring but inside the big ring.



This picture shows another set within a set. The set of books on animals is within the set of all books in the school library.

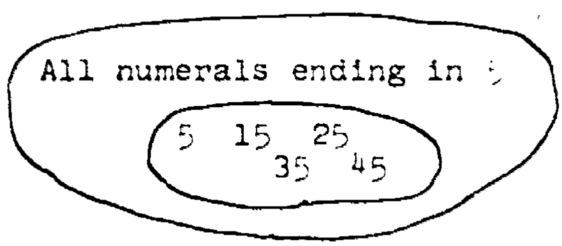
We can say that the set of books on animals is a <u>subset</u> of the set of all books in the library.

Exercise Set 3

1. The set of all pupils in your school is within the set of all pupils in your state. Draw a picture to show this idea.

2. Make a drawing to show that the set of all even numbers is within the set of all whole numbers.

3. The drawing below shows that the set of numerals 5, 15, 25, 35, 45 is within the set of all numerals ending in 5.



Make a drawing to show that the numbers 10, 20, 30, 40 are within the set of all whole numbers.

4. A set of girls in the fourth grade is Mary, Martha, Karen, Kathy, Marian, Sue. Call this set, Set S.

S = {Mary, Martha, Karen, Kathy, Marian, Sue}

Here is some information about this set of girls. Use it in answering the questions in this problem.

Name	Color of Eyes	Color of Hair	Age
Mary	blue	blonde	9
Martha	brown	brown	10
Karen	gray	black	9
Kathy	brown	black	9
Marian	blue	brown	10
Sue	brown	brown	9

a) Write the members of the set of girls who are 10 years old. Call this set, Set B. Is Set B in Set S?

b) Write the members of the set of girls who have gray eyes. Call this set, Set C. Is Set C in Set S?

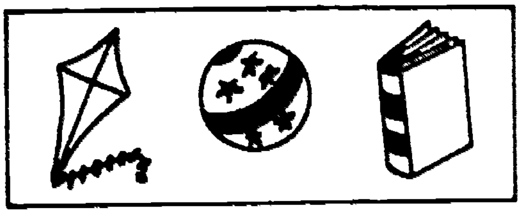
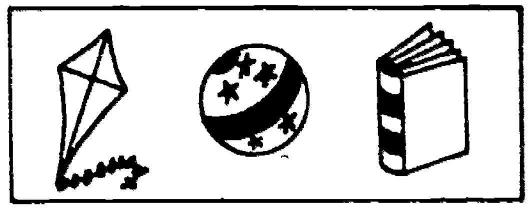
c) Write the members of the set of girls who have black hair and who are 9 years old. Call this set, Set X. Is Set X in Set S?

d) Is Set C in Set X?

5. BRAINTWISTER: Make a drawing to show that the numbers 2, 4, 6, 8, 10 are within the set of even numbers and the even numbers are within the set of whole numbers.

EQUAL SETS

Here are two sets of pictures.

The members of the two sets are the same.

If two sets have the same members, the two sets are equal.

The members of equal sets do not have to be in the same order.

Here are some other sets.

A = {apple, pencil}

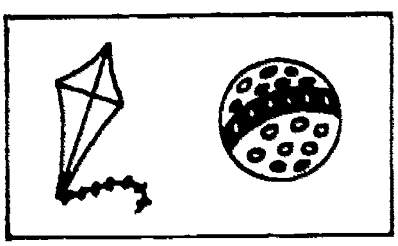
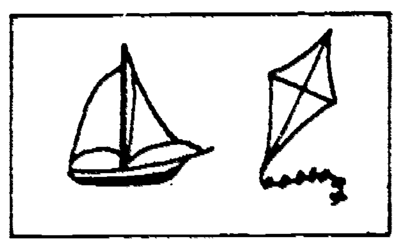
B = {pencil, apple}

We can say that Set A equals Set B. We write: Set A = Set B

M = {5, 1, 3}

N = {1, 3, 5}

Does Set M = Set N? Why?

Here are two sets of pictures.

These sets do not have the same members.

They do have the same number of members.

G = {apple, pencil, house}

H = {dog, car, hat}

Set G is not equal to Set H. We write: Set G $\neq$ Set H

R = {0, 1, 2, 3}

P = {1, 2, 3}

Set R is not equal to Set P. We write: Set R $\neq$ Set P.

Exercise Set 4

A = {4, 5, 7}   B = {5, 4, 7}   C = {7, 4, 5}

1. Does Set A = Set B?
2. Does Set C = Set A?
3. Does Set C = Set B?

X = {b, a, c, k}   Y = {c, b, k, a}   Z = {k, c, t, a}

4. Does Set X = Set Y?
5. Does Set Z = Set X?
6. Does Set Z = Set Y?

R = {6, 10, 8}   S = {10, 7, 8}   T = {8, 6, 10}

7. Does Set R = Set S?
8. Does Set T = Set R?
9. Does Set T = Set S?

Here are some sets: (Use these to answer questions 10, 11, 12, 13.)

$A = \{3, 5, 7, 4, 2\}$

$B = \{2, 3, 4, 5, 6\}$

$C = \{2, 3, 4, 5, 7\}$

$D = \{6, 5, 4, 3, 2\}$

$E = \{5, 3, 7, 2, 4\}$

$F = \{2, 4, 6, 3, 5\}$

10. Set A is equal to what sets?

11. Set C is equal to what sets?

12. Are Set C, Set E, and Set A equal sets?

13. Which sets are equal to Set D?

14. $X = \{t, s, r, d\}$.

    Think of a set that has the same number of members as Set X but is not equal to Set X.

    Call it Set Z.

    Copy and finish: $Z = \{ \qquad\qquad \}$.

15. $B = \{3, 7, 9, 5\}$.

    Set E is the set of all odd numbers less than 10.

    Which is correct? Set B = Set E or Set B $\neq$ Set E.

16. Set A is the set of all whole numbers greater than 5 but less than 10.

    Set B is equal to Set A.

    Name the members of Set B.

17. $D = \{\frac{1}{2}, \frac{1}{4}, \frac{3}{4}\}$

$E = \{\frac{1}{4}, \frac{3}{4}, \frac{1}{2}\}$

Which is correct? Set $D \neq$ Set E or Set $D =$ Set E.

THE UNION OF SETS

John took a hike with his mother and father.

John kept a record of the different birds his mother saw.

He kept a record of the different birds his father saw.

Set A is the set of different birds John's mother saw.

robin	crow	sparrow

Set A

Set B is the set of different birds John's father saw.

hawk	bluejay
wren	eagle

Set B

To find all the different birds John's parents saw, we put Set A and Set B together. Our set is now

robin, crow, sparrow, hawk, wren, bluejay, eagle

This set is the union of Set A and Set B.

We write:

$A \cup B = \{$robin, crow, sparrow, hawk, wren, bluejay, eagle$\}$

We read $A \cup B$: the union of Set A and Set B.

Your class chose some committees for a party.

The committee to select the games was Set G.

The committee to buy the prizes was Set P.

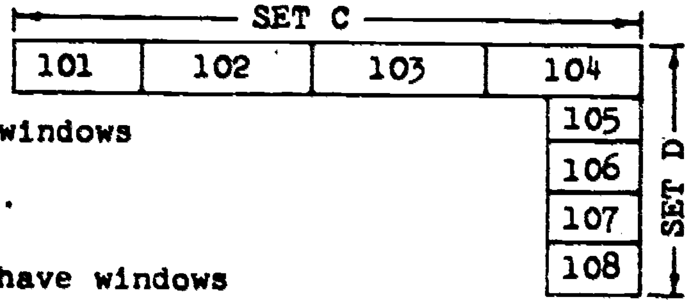
$G = \{$John, James, Helen, Susan$\}$

$P = \{$John, Irene, Phyllis, Samuel$\}$

The two committees met together. What pupils attended the meeting?

$G \cup P = \{$John, James, Helen, Susan, Irene, Phyllis, Samuel$\}$

The picture at the right shows rooms in Jane's school.



Rooms 101, 102, 103, 104 have windows along one side of the building.

Rooms 104, 105, 106, 107, 108 have windows along another side of the building.

$$C = \{101, 102, 103, 104\}$$

$$D = \{104, 105, 106, 107, 108\}$$

We write: $C \cup D = \{101, 102, 103, 104, 105, 106, 107, 108\}$

We read: The union of Set C and Set D is the set whose members are 101, 102, 103, 104, 105, 106, 107, 108.

Eddie is learning to play the trumpet and the piano.

His practice schedule looks like this.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		TRUMPET	TRUMPET	TRUMPET	TRUMPET	
	PIANO	PIANO	PIANO	PIANO		

Set T is the set of days that Eddie practices the trumpet.

Set P is the set of days that Eddie practices the piano.

T = {Tuesday, Wednesday, Thursday, Friday}

P = {Monday, Tuesday, Wednesday, Thursday}

The union of Set T and Set P is the set of days in the week when Eddie practices the trumpet or the piano or both.

We write:

$T \cup P$ = {Monday, Tuesday, Wednesday, Thursday, Friday}

Exercise Set 5

1. A = {cat, dog, cow, horse}

   B = {duck, horse, pig}

   Which one of these sets is the union of Set A and Set B?

   X = {cat, cow, dog, duck, horse, pig}

   Y = {cow, horse, duck, horse, pig}

   Z = {cat, dog, cow, hen, duck, horse, pig}

   Answer. Set X is the union of Set A and Set B. We write

   $A \cup B = X$

2. R = {10, 20, 30, 40, 50}

   S = {60, 70, 80, 90, 100}

   Which one of these sets is the union of Set R and Set S?

   M = {70, 90, 110, 130, 150}

   N = {100, 90, 80, 70, 60, 50, 40, 30, 20, 10}

   Copy and finish: $R \cup S =$

3. G = {a, t, z, r, m, j}

   H = {r, q, z, t}

   Which one of these sets is the union of Set G and Set H?

   M = {q, a, m, j}

   N = {a, t, z, r, m, j, q, r, z, t}

   L = {a, j, m, q, r, t, z}

   Copy and finish: $G \cup H =$

4. J = {white, blue}

K = {red, blue}

Copy and finish: $J \cup K$ =

5. V = {18, 21, 24}

W = {15, 18, 21, 24, 27}

Copy and finish: $V \cup W$ =

6. N = {s, o, a, p}

O = {w, a, t, e, r}

Copy and finish: $N \cup O$ =

7. Set P is the set of odd numbers between 6 and 12. Copy and finish: P =

Set Q is the set of odd numbers less than 7. Copy and finish: Q = and $P \cup Q$ =

8. Set R is the set of even numbers between 90 and 100. Copy and finish: R =

Set S is the set of whole numbers greater than 94 and less than 96.

Copy and finish: S = and $R \cup S$ =

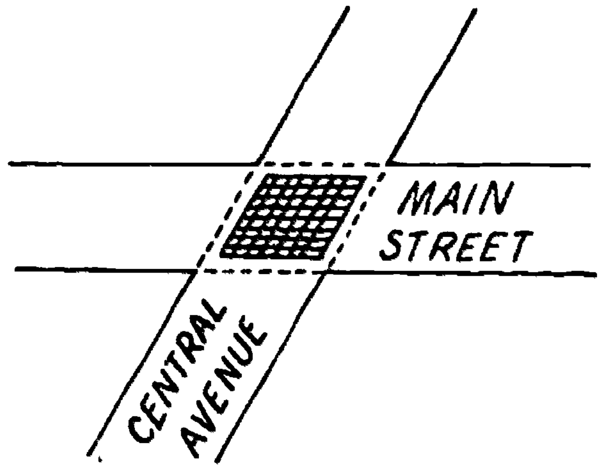
9. Set T is the set of whole numbers between 65 and 66. Copy and finish: T =

Set W is the set of whole numbers larger than 9 and less than 11. Copy and finish: W = and $T \cup W$ =

10. Set X is the set of counting numbers between 25 and 30. Copy and finish: X =

Set Y is the set of even numbers between 25 and 31. Copy and finish: Y = and $X \cup Y$ =

THE INTERSECTION OF SETS



Look at the picture at the right.
Main Street and Central Avenue cross each other. A part of one street is also a part of the other street.
It has been shaded in the picture.
This part belongs to both streets.
It is the intersection of the two streets.

Look at these two sets:

Set A	Set B
Alice	Ellen
Betty	Ken
Ken	Joe
Sue	Sue
Tom	Wendy

Some children are members of both sets.
The children who are members of both sets are Ken and Sue.
This set may be written {Ken, Sue}.
This set is called the intersection of Set A and Set B.
We write: $A \cap B = \{Ken, Sue\}$.
We read $A \cap B$: the intersection of Set A and Set B.
The symbol $\cap$ means "the intersection of."

Here are some more sets:

Set X is the set of numbers we use when we count by fives, starting with 5 and ending with 30.

$$X = \{5, 10, 15, 20, 25, 30\}$$

Set Y is the set of numbers we use when we count by tens, starting with 10 and ending with 50.

$$Y = \{10, 20, 30, 40, 50\}$$

The numbers that are members of both sets X and Y are 10, 20, and 30.

The intersection of Set X and Set Y is the set $\{10, 20, 30\}$.

We write: $X \cap Y = \{10, 20, 30\}$.

$$J = \{0, 2, 4, 6, 8, 10, 12, 14, 16\}$$

$$K = \{1, 3, 5, 7, 9, 11, 13, 15\}$$

Set J is the set of even numbers less than 17.

Set K is the set of odd numbers less than 17.

There are no numbers that are members of both Set J and Set K.

The intersection of Set J and Set K is the set $\{\ \}$.

We write: $J \cap K = \{\quad\}$.

Exercise Set 6

1. A = {car, train, taxi, boat}.

   B = {wagon, boat, airplane, train, bicycle}.

   Which one of these sets is the intersection of Set A and Set B?

   M = {car, taxi, wagon, airplane, bicycle}

   R = {boat, train}

   S = { }

   Answer. Set R is the intersection of Set A and Set B. We write $A \cap B = R$.

2. D = {13, 17, 19, 23}

   E = {9, 11, 13, 15, 17, 18, 21}

   Which one of these sets is the intersection of Set D and Set E?

   P = {13, 17, 19, 23, 9, 11, 13, 15, 17, 19, 21}

   Q = {9, 11, 15, 19, 21}

   R = {17, 13}

   Copy and finish: $D \cap E =$

3. G = {d, x, p, r, q, m}

   H = {t, b, s, n, a}

   Which one of these sets is the intersection of Set G and Set H?

   I = {a, b, d, m, n, p, q, r, s, t, x}

   J = {d, b, p, q, n, m}

   K = { }

   Copy and finish: $G \cap H =$

4. J = {dress, shoe, hat, coat}

K = {shoe, cap, coat, dress}

Copy and finish: $J \cap K =$

5. L = {g, r, a, n, d}

M = {p, i, a, n, o}

Copy and finish: $L \cap M =$

6. N = {73, 59, 8, 81, 63}

O = {104, 49, 73, 58, 18, 95}

Copy and finish: $N \cap O =$

7. Set P is the set of whole numbers less than 7.

Copy and finish: P =

Set Q is the set of whole numbers between 5 and 12.

Copy and finish: Q =

Copy and finish: $P \cap Q =$

and $P \cup Q =$

8. Set R is the set of whole numbers larger than 38 and less than 44.

Copy and finish: R =

Set S is the set of numbers between 36 and 46 that are not even numbers.

Copy and finish: S =

Copy and finish: $R \cap S =$

$R \cup S =$

7. SUPPLEMENTARY PRACTICE EXERCISES

THINKING ABOUT SETS -- Exercise Set 7

Write the members of each of these sets.

1. The set of even numbers less than 12.

2. The set of counting numbers less than 20 and larger than 10.

3. The set of odd numbers between 10 and 20.

4. The set of whole numbers less than 17 and larger than 15.

5. The set of numbers between 30 and 40 that are larger than 60.

6. How many members are there in the set of letters of our alphabet?

7. Here is a set: {Tuesday, Thursday}. Describe this set by writing on your paper: This is a set of ______________________

   ____

8. Name two sets that have no members.

9. Make a picture of a set. Then describe the set by saying: This is a set of: ______________________

10. Describe this set in your own words:

$$A = \{5, 10, 15, 20, 25\}$$

SETS WITHIN SETS -- <u>Exercise</u> <u>Set 8</u>

1. Draw a picture to show that the set of dimes is a set within the set of United States coins.

2. A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   Set B = the set of all counting numbers.

   Draw a picture to show that Set A is a set within Set B.

   Make up one <u>subset</u> (set within a set) for each of the following:

3. {a, b, c, d, e, f}

4. {Ford, Chevrolet, Plymouth, Rambler, Cadillac}

5. {The set of holidays in a year}

6. Write the set of vowels. Call this set Set A. Now write a subset of Set A.

7. Set C = the set of all states in the United States.

   Write a subset of Set C.

8. Set X = the set of dimes. We can say Set X is a set within the set of all ______________________________

9. Set Y = the set of counting numbers.

   Z = {3, 8, 15, 93, 173}

   Set Z is a ______________ of Set Y.

10. Set C is the set of all odd numbers. Make up a subset of Set C.

EQUAL SETS -- <u>Exercise</u> <u>Set 9</u>

1. A = {9, 4, 3}   B = {4, 3, 9}   Does Set A = Set B?

2. M = {10, 11, 12}   N = {10, 11, 13}
   Does Set M = Set N?

3. X = {dog, cat, mouse}   Y = {mouse, cat, horse, dog}
   Does Set X equal Set Y?

4. Set D is the set of whole numbers greater than 7 and less than 12. Set E is equal to Set D. Name the members of Set E.

Here are some sets:
F = {2, 4, 6, 8}
G = {4, 2, 8, 6}
H = {8, 6, 1, 4}
K = {8, 6, 1, 2}
L = {4, 8, 6, 1}
M = {6, 2, 8, 4}

5. Set F is equal to what sets?

6. Are Set H and Set L equal sets?

7. Which sets are equal to Set K?

8. A = {2, 4, 6, 8} Set B is the set of all even numbers less than 20. Which is correct? Set A = Set B or Set A ≠ Set B?

9. Make up a set. Call this set Set X. Now make up a set that is equal to Set X.

10. Set J is the set of the first five letters of the alphabet.

Set K is the set of the last five letters of the alphabet.

Does Set J = Set K?

THE UNION OF SETS -- <u>Exercise</u> <u>Set 10</u>

1. A = {1, 2, 3, 4}   B = {5, 6, 7, 8}

   The union of Set A and Set B is the set:

2. C = {c, a, n, d, y}   D = {c, o, k, e}

   Copy and finish: $C \cup D =$

3. E = {5, 10, 15, 20, 25}   F = {30, 35, 40, 45}

   Which one of these sets is the union of Set E and Set F?

   M = {10, 20, 25, 45, 50, 55, 60, 65, 70}

   N = {45, 40, 35, 30, 25, 20, 15, 10, 5}

4. H = {2, 4, 6, 8, 10}   J = {3, 6, 9, 12}

   Write the members of the union of Set H and Set J.

5. Set K is the set of odd numbers between 10 and 20.

   Set L is the set of even numbers between 10 and 20.

   Copy and finish: $K \cup L =$

6. Set R is the set of whole numbers between 47 and 48.

   Set S is the set of whole numbers larger than 15 and less than 17.

   Copy and finish: $R \cup S =$

7. X = {B, E}   Y = {A, O, C, E}

   Copy and finish: $X \cup Y =$

8. $T = \{\frac{1}{2}, \frac{1}{5}, \frac{1}{4}, \frac{3}{4}\}$ $V = \{\frac{1}{3}, \frac{1}{5}, \frac{3}{4}, \frac{1}{2}, \frac{1}{4}\}$

Set W is the union of Set T and Set V. Write the members of Set W.

9. $C = \{1, 6, 7, 8, 3\}$ $D = \{2, 6, 7, 8, 1\}$

Set E is the union of Set C and Set D. Write the members of Set E.

10. Set M is the set of whole numbers between 20 and 21. Set N is the set of even numbers between 6 and 8. Copy and finish: $M \cup N =$

11. $A = \{9, 18, 27, 36\}$ Set B has 5 members.

$A \cup B = \{3, 6, 9, 12, 15, 18, 27, 36\}$

$A \cap B = \{9\}$

$B =$

THE INTERSECTION OF SETS -- Exercise Set 11

1. A = {1, 2, 3, 4, 5} B = {2, 4, 6, 8, 1}

   Which one of these sets is the intersection of Set A and Set B?

   M = {5, 6, 1, 2}

   N = {1, 2, 4}

   S = {1, 4, 6}

2. C = {a, e, i, o, u} D = {a, b, c, d, e}

   Set E = the intersection of Set C and Set D. Write the members of Set E.

3. R = {5, 10, 15, 20, 25} S = {10, 20, 30, 40}

   Set T is the intersection of Set R and Set S. Write the members of Set T.

4. Set X is the set of the first five counting numbers. Set Y is the set of odd numbers between 4 and 12. Set Z is the intersection of Set X and Set Y. Write the members of Set Z.

5. K = {dogs, cats, mice} L = {pigs, dogs, cats, mice}

   M = {horses, cows, dogs}

   Copy and finish:

   $K \cap L =$

   $K \cap M =$

   $L \cap M =$

6. H = {s, t, u, d, y} J = {h, a, r, d}

   Copy and finish: $H \cap J =$

7. A = {desk, chair, pencil} B = {eraser, chalk, chair}

C = {book, tablet, desk}

Copy and finish:

$A \cap C =$

$A \cap B =$

$B \cap C =$

$C \cup A =$

$C \cup B =$

$B \cup A =$

8. J = {12, 18, 24, 30}. Set K has two members. One member is 6. $K \cap J = 18$. Copy and finish: K =

9. Set R is the set of counting numbers less than 6. Copy and finish: R =

Set S is the set of counting numbers between 5 and 11.

Copy and finish: S =

Copy and finish:

$R \cap S =$

$R \cup S =$

10. Set P is the set of all even numbers between 1 and 7.

Set Q is the set of all odd numbers between 1 and 7.

Copy and finish:

$P \cap Q =$

$P \cup Q =$

Chapter 5

SETS OF POINTS

LEARNING ABOUT SPACE

We are living in the space age. Man has already traveled in space and more space exploration will be done.

Suppose we were to plan a trip to Mars. Our space ship will have to follow a path which leads to Mars. Mars is moving all the time. To reach it, we must know its location in space, its speed in space, and its direction of travel.

The study of space and location is part of mathematics. This part of mathematics is called geometry. The things that we have been learning about the number system and about addition and subtraction belong to the part of mathematics called arithmetic.

To study geometry we need good imaginations. We make models and draw pictures to help us learn about things we cannot see. But our imaginations must help us too. Is your "imaginer" ready?

Geometry is not new. Thousands of years ago the Egyptians and Babylonians used ideas from geometry. It helped them to plan pyramids, lay out their fields, and study about the moon, stars, and planets.

The first geometry book was written about 2200 years ago. It had most of the ideas we still use in studying about space. However, many new ideas about geometry are still being discovered. Maybe you will be one to discover a new idea.

At first "geometry" meant "earth measure." But now geometry also uses ideas which do not involve measurements. In this unit called Sets of Points we are going to study some of these ideas.

We know that a "set" is a collection of things. Can you guess what a set of "points" would be? First you would need to know what a "point" is. In this unit we will learn about points, space, curves, line segments, rays, circles, polygons, and angles.

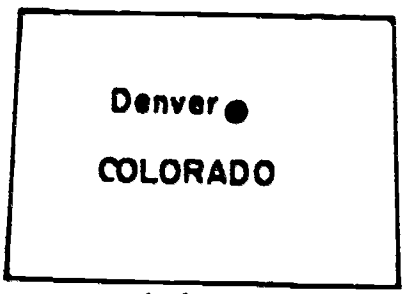
POINTS

What is a point? Is it the end of a sharp pencil? Is it the end of a needle? Is it the dot a pencil makes? Let's see what "point" means in geometry.

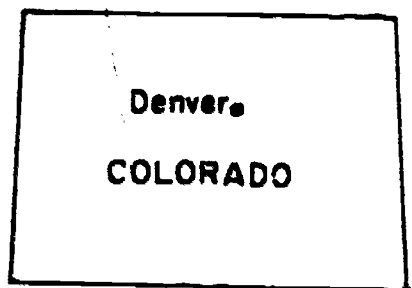
Working Together

1. Use your sharpest pencil to make a dot near the center of a sheet of paper. Now make a dot with a crayon. Next make a dot with a dull pencil. Do these dots look alike? In what way are they different? In what way are they the same?

2. Which of these maps of Colorado best shows the location of the state capital? Why?



(a)



(b)

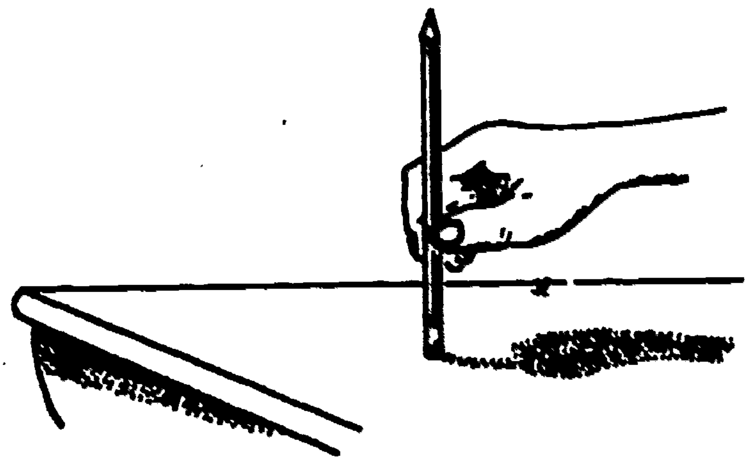
The dots you made in the first example and the dots in the maps of Colorado are attempts to show an exact location. The small dot marks the location more exactly. In geometry we often let a small dot represent a *point*. However, the dot is not the *point* any more than a picture of a cow *is* a cow.

A point in geometry means an exact location in space. Can you imagine something so small that you cannot see it? A point is so small it has no size at all.

Unless you have a microscope, you cannot see a germ. However, a germ covers many points as we think of them. If you were going to mark on a sheet of paper the locations covered by one germ, you would need a very sharp pencil. The dot made with the pencil would cover all these locations.

A very small dot is used to represent or stand for a point although a point is smaller than any dot which can be made.

3. Hold your pencil with its tip above your desk as in the drawing.



Could the sharpened tip of the pencil show a point? Move your pencil to another part of your desk. Does the tip now show a different point? Points do not move. They always stay in the same location.

In geometry we usually name pictures of points with capital letters like this:

A. B. C.

The points represented by A, B, and C can be called a "set of points." We will learn many interesting things about sets of points.

4. Describe a set of three points in your classroom.

5. Describe a set of two points in your classroom.

6. Describe a set of eight points in your classroom.

Exercise Set 1

1. Which of these is the best representation of a point?

A• B• C● D●

2. On your paper mark a set of five points using small dots. Label these pictures of points using the first five letters of the alphabet.

3. Write the letter of the best answer.
A dot made with a pencil covers
    a) one point
    b) one hundred points
    c) several points
    d) more points than can be counted

4. Which of these best describes a point?
    a) a mark made with a pencil
    b) a very very small dot
    c) an exact location in space
    d) a dot

SPACE

What is space? Is it air? Is it an empty place? Is it the distance from Earth to Mars? It is not any of these as we think of "space" in geometry.

Here are some examples of things which occupy sets of points in space:

The eraser on your pencil

The door to your classroom

Your little finger

Working Together

1. Now can you guess what "space" is? Which answer would you choose?

(a) Space is something hollow.

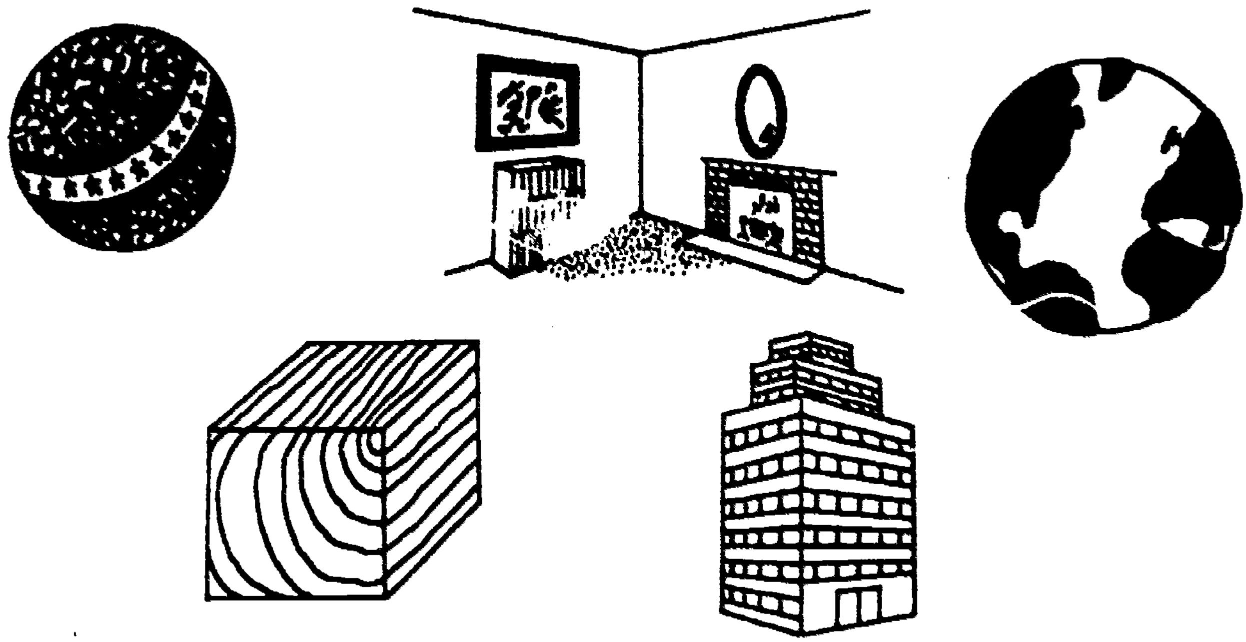
(b) Space is an object like a door or a finger.

(c) Space is the set of all the exact locations everywhere.

If you chose answer (c) you were correct. <u>Space is the set of all points</u>.

This means all exact locations everywhere. All the locations on the head of a pin, in your home, in your city and the sky above, in your country, in the world, and in the entire universe are points in space.

Space as we now picture it is probably very different from the idea you had. Any object you can think of covers or occupies lots of points of space. For example, a ball, a block of wood, a room, a building, the earth are all occupying parts of space.



2. Must a part of space be filled with air only?

3. Does a block of wood contain one point of space, a thousand points of space, or more points of space than can be counted?

4. Place a cup on a desk. It represents many points. Move the cup to some other place. Does it now represent the same set of points as before?

5. Place a block of wood on a desk. It represents many points. Move it to some other place. Does it now represent the same set of points as before?

Exercise Set 2

Write the letter of the right answer.

1. Which of these best tells what space is?

   a) Space is all empty places.

   b) Space is a set of points.

   c) Space is the set of all points.

   d) Space is the air around the earth.

2. In a truck load of grain, there are

   a) just as many points as there are grains.

   b) more points than there are grains.

3. Which ones of these represent a part of space?

   a) A mark you make on your paper.

   b) The idea of truth.

   c) Your teacher.

   d) A tree.

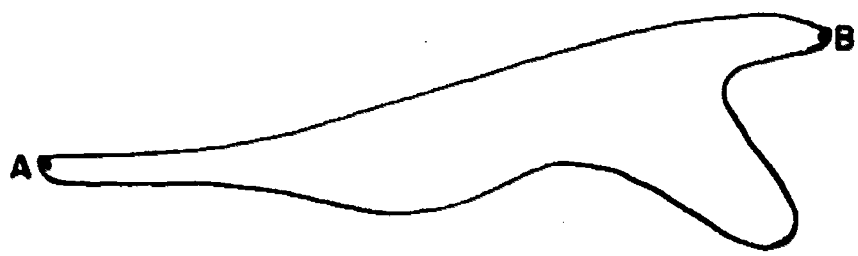
   e) The idea of beauty.

   f) The crease in a piece of paper.

Working Together

1. Use two small bits of paper to mark two points on your desk. Trace with your finger to show ways you could go from one point to the other. How many different paths could you follow in going from one point to the other? Can you trace the most direct way to go from one point to the other?

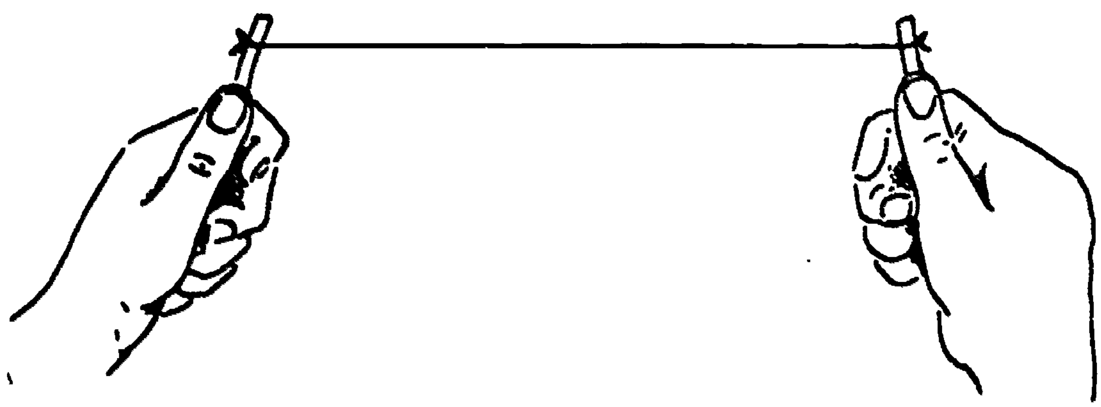
2. There are many ways of going from A to B. We see a picture of two ways.



Mark two points on a sheet of paper. Label them A and B. Show 5 ways of going from A to B on your paper. We do not have to stay on the paper. Think how you can go from A to B and touch the paper only at the dots.

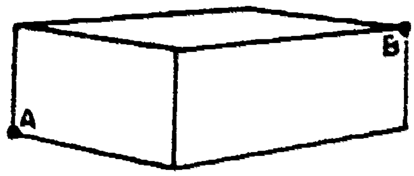
In going from one point to another, you have traced a curve with the tip of your finger or with your pencil. We think of a curve as a set of points. It is all the different locations your finger tip or pencil passes through in going from one point to another.

3. Let us think a little more about curves between two points. Suppose we use a piece of string tied to two pencils at the eraser ends to help us. We can let the erasers of the pencils mark two locations. Let us locate these points as far apart as the string will let us put our pencils. Does the string show the most direct path?



This direct path is a way of showing a special type of curve. We call it a <u>line segment</u>. Put dots on this string using chalk, pencil, or a pen. These dots mark points for us. We think of a <u>line segment</u> as a set of points. It is the set of all the points we have marked and all other points on our tightly stretched string. It also contains the two points represented by the erasers. We can show line segments in other ways.

4. Mark on a block of wood the points A and B as shown in the picture.



Draw two curves on the block from point A to point B using two colors of crayon.

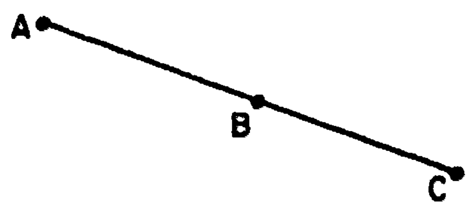
Did either curve you drew contain any line segments?

One excellent way to show a line segment is to draw a picture of one with a ruler and pencil. On your paper draw a segment connecting the two dots as shown in the figure below. We shall represent a line segment in this way.

We name this "line segment AB." A short way to write "the line segment AB" is $\overline{AB}$. $\overline{AB}$ means line segment AB. The line segment ends at points A and B. Therefore points A and B are called <u>endpoints</u>.

5. Think of the corner of your classroom as representing a point. What three things suggest line segments with this point as one endpoint?

6. Name all the line segments you see represented in this figure with endpoints in the set of points {A, B, C}.



7. Mark a point on your paper. Call it point A. How many different line segments can you draw with A as an endpoint?

8. Give some examples of representation of line segments suggested by objects in your classroom.

9. Does your state have line segments as a part of its boundary?

10. Mark a point on your paper. Would you call your mark a line segment?

11. Mark something like this 

on your paper. Is it a line segment?

12. Mark something on your paper which does not represent a line segment.

Exercise Set 3

1. Mark two points on your paper as is shown here.

A B

Draw three different curves from A to B.

Write the letter of the best answer.

Each curve between these two points A and B goes through:

   a) one point

   b) three points

   c) many points

   d) more points than can be counted.

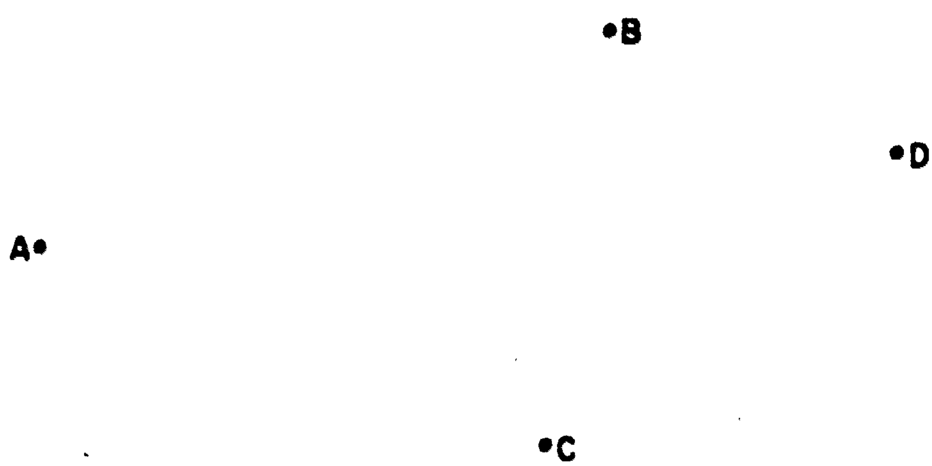
2. The set of points {A, B} is marked below. Copy this set on your paper. Draw as many line segments as possible having both endpoints in this set. How many are there?

A

B

3. Copy the set of points {A, B, C} on your paper. Draw all the line segments having both endpoints in {A, B, C}. How many line segments are there?

4. Copy the set of points {A, B, C, D} on your paper. How many line segments can you draw, each having two of the points named as endpoints? Be sure to draw all the line segments. Name the line segments you drew.



5. Name all the line segments you see in this figure. Both endpoints must be in the set, {A, B, C}.

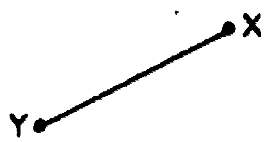
6. Mark a point on your paper and label it A as shown below. Draw pictures of two line segments having A as an endpoint.

•A

7. Mark a point on your paper and label it P. Draw pictures of three line segments having P as an endpoint.

P•

8. Complete these statements on your sheet of paper.



a) This is a picture of a ____?____ ____?____.

b) We write its name ____?____.

Working Together

1. On your paper use a ruler to draw a line segment like $\overline{AB}$.

Draw a longer line segment which contains point A and point B by extending $\overline{AB}$ in both directions. Label the endpoints of this segment with the letters C and D. Does your drawing look something like this?

Is $\overline{AB}$ contained in $\overline{CD}$?

2. Draw an even longer line segment which contains points A and B by extending $\overline{CD}$ in both directions. Label the endpoints of this segment with the letters E and F. Your drawing might now look like this:

Is $\overline{AB}$ contained in $\overline{EF}$?

If you had a larger piece of paper and a longer ruler, you could draw a still longer line segment which would contain point A and point B. Think how this line segment would look if you were to draw longer and longer segments which contain points A and B.

Can you imagine how your drawing would look if it were extended without end? This is what we think of when we think of <u>line</u>. A <u>line</u> has no endpoints. It contains line segments of longer and longer length.

3. Below is a picture of a line.

The arrows are used to show that it goes on and on in both directions without end. Only part of the line can be pictured on this page. We can call the line pictured, line AB. A short way of writing line AB is $\overleftrightarrow{AB}$.

Both A and B name points on the line. We know C and D name other points on the line. We could also call this line, line CD, or line AC, or line AD.

Line AB is the same as line BA. What other names can this line have? Use just the points named.

4. Here is a picture of $\overline{KS}$. M, P, L, and R, name other points of this line segment.

Copy $\overline{KS}$ and its labeled points.

a) Draw a line segment which pictures still more of the line PS and also includes line segment KS. Can you draw a complete picture of the line PL?

b) Draw a picture of a line AB. On your paper show a short way of writing line AB.

Remember that a line segment is a set of many points and a line also is a set of many points.

5. Follow these instructions carefully.

a) Mark a point on your paper and label it A. Draw one line through point A.

b) Now draw a different line through point A.

c) Next draw three more different lines through point A.

d) Mark one point different from point A on each of the lines you have drawn. Label the points with the letters B, C, D, E, F.

e) Can we draw more lines through point A?

f) Which is the correct ending to the sentence below. Through point A we can draw:

one line.

more lines than you can count.

g) Describe the position of a line segment through A which is not on your sheet of paper.

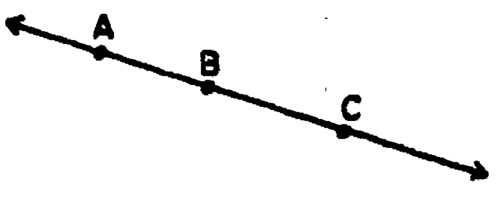
6. On your paper mark two points, A and B.

a) How many line segments can you draw with endpoints A and B?

b) How many line segments can you draw which pass through both A and B?

c) How many <u>lines</u> are there that contain both A and B?

7. Below we have represented a line and three points of the line.

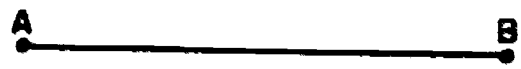


Shall we label this line $\overleftrightarrow{AB}$ or $\overleftrightarrow{AC}$?

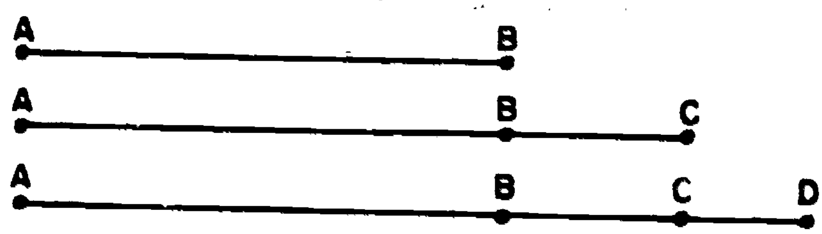
In problem 6 we saw that with a ruler and a pencil only one line could be drawn through points A and B. From now on think of this statement as a fact; there is exactly one line that can be drawn through the two points A and B.

Working Together

1. Use a ruler to draw a line segment AB on your paper.



Now suppose we make longer and longer line segments but always keep A as one of the endpoints, as,



Then suppose we do not have a second endpoint, as in the picture below.

This gives us an idea for what is called a <u>ray</u>.

We can show a picture of only part of a ray on this page. We can name this ray, ray AB. Both A and B name points of the ray.

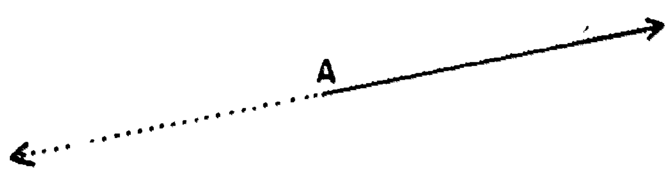
A ray has one endpoint. A is the name of the endpoint of the ray. A short way of writing ray AB is $\overrightarrow{AB}$. The endpoint is named first.

This is a picture of ray BA. What is its endpoint?

Ray BA is not the same as ray AB. Can you tell why? The endpoint of $\overrightarrow{BA}$ is B. What is the endpoint of $\overrightarrow{AB}$?

We can say that a <u>ray</u> is the union of the endpoint and all points on a line in one direction from this point.

For example, look at the line represented below and the point on it labeled A. One ray is represented by the solid part of the line. The other ray is represented by the dotted part of the line. The point A belongs to both rays represented and is called the <u>endpoint</u> of either ray.



A ray is always part of a line. A set of rays is nicely represented by a beam of light from a flashlight. Each starts at the flashlight and extends in one direction without end.

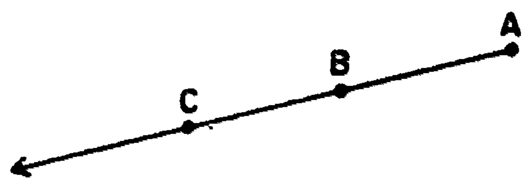
2. $\overrightarrow{AB}$ is represented below

   a) Is $\overrightarrow{AC}$ another name for this ray?

   b) Is $\overrightarrow{BC}$ another name for this ray?

   c) Is the ray $\overrightarrow{BA}$ represented?

   d) Is the ray $\overrightarrow{BC}$ represented?



Exercise Set 4

1. a) On your paper copy points F and E.

   Draw a picture of line segment FE.

   b) Mark two points on your paper and name them G and H. Draw a picture of the line through G and H.

   c) Draw a picture of a ray. Name it $\overrightarrow{KL}$.

   d) Write the symbol for line segment FE; for line GH; for ray KL.

   e) What is the endpoint of $\overrightarrow{KL}$?

   f) Is $\overrightarrow{KL}$ the same ray as $\overrightarrow{LK}$? Why?

2. Let A be the name of a point of the line below. How many rays which are part of this line can have A as an endpoint?

3. Draw a picture of a line on your paper. Let A be a point of this line.

   a) Choose a point of the line different from A in one direction and label it B.

   b) Choose another point of the line in the other direction from A and label it C.

   c) Name two rays with endpoint A which are part of this line.

   d) Are there any more rays on this line which have A as an endpoint?

4. Label a point on your paper as A.

   a) Draw one ray with endpoint A.

   b) Draw another ray with endpoint A.

   c) Draw four more rays with endpoint A.

   d) How many rays could there be with A as endpoint?

5. On your paper draw a ray. Label it $\overrightarrow{AB}$. What is its endpoint? How many rays are there with endpoint A that contain point B?

6. On your paper draw a ray with endpoint A.

   a) Choose a point on the ray different from A and label it B.

   b) Is $\overline{AB}$ contained in $\overrightarrow{AB}$?

   c) How many segments could there be on $\overrightarrow{AB}$ which have A as endpoint?

7. Mark two points on your paper and label them R and S.

•S

R•

   a) How many lines can you draw which contain point R?

   b) How many lines can you draw which contain point S?

   c) How many lines can you draw which contain both R and S?

Which is the correct ending?

8. A line has

   a) exactly 1 endpoint.

   b) 2 endpoints.

   c) no endpoints.

9. A ray has

   a) exactly 1 endpoint.

   b) 2 endpoints.

   c) no endpoints.

10. A line segment has

   a) exactly 1 endpoint.

   b) 2 endpoints.

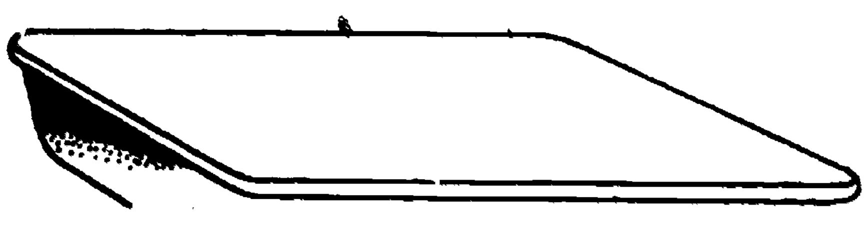
   c) no endpoints.

Working Together

1. Can you find some flat surfaces in your classroom? Name as many as you can.

Do you know the geometric name for a set of points suggested by a flat surface? It is <u>plane</u>. Each flat surface you have named represents <u>part of a plane</u>.

2. Put your finger on a point on the top of your desk. Now put it on a different point. How many different points can you find on the flat top of your desk? How many points do you think there are in a plane?

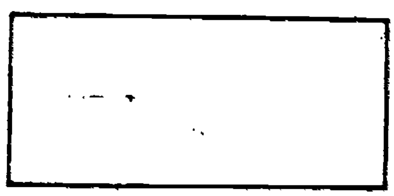


3. Put your finger at a point above the top of your desk, then at a point below the top of your desk. Are there many points which are not in the plane represented by the top of your desk?

From now on we shall think of _a_ _part_ _of_ _a_ _plane_ as a set of points in space. It is the kind of set suggested by all points on a flat table top, or on a wall, or on the floor. A piece of paper lying flat on your desk also suggests a part of a plane.

4. To get a better idea of what we mean by a plane, follow these directions.

a) See the picture of the figure below. Draw one like it near the center of a piece of paper.



b) Trace the figure with a (red) crayon. Color the part of the plane inside of the figure the same color. Give this colored part the name A. Is this colored region a picture of part of a plane?

c) Draw a bigger figure which encloses the colored region.

A. Color the larger figure and its inside (red) also. Name this new colored region, B.

Does the new colored region picture a part of the same plane as A?

Does colored region A or colored region B picture more of this plane?

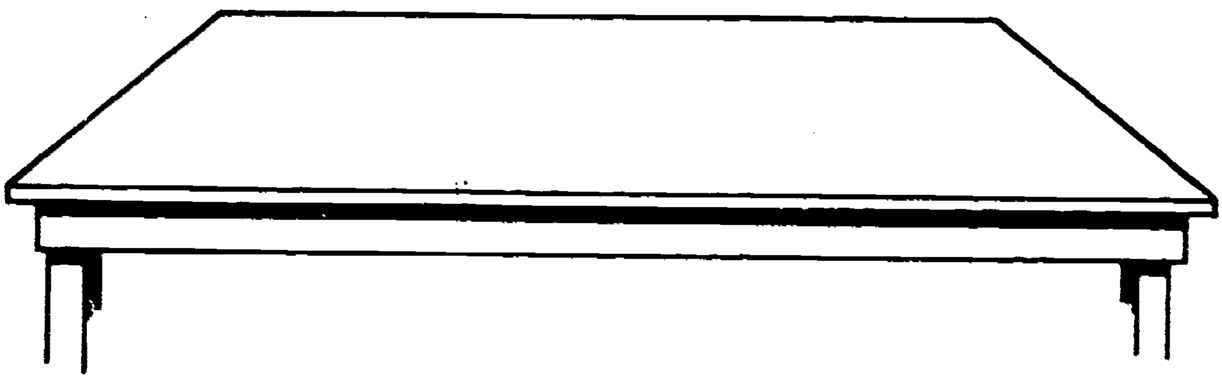
d) Draw a third figure which encloses the colored region B. Color this figure and its inside (red). Name this new colored region, C.

Does this new colored region picture a part of the same plane that A did?

Does colored region A or colored region B or colored region C picture more of this plane?

e) Can you draw a picture of the complete plane suggested by regions A, B, and C?

As we think of a line containing longer and longer segments s. shall we think of <u>a whole plane</u> as containing larger and larger flat surfaces. Imagine your table top growing larger and larger on all sides. You would then have a table top upon which you could walk as far as you wished in any direction.



5. Does the set of points represented by the table top move when the table is moved?

6. Name some other objects which represent parts of planes.

7. Is there more than one plane in space?

We shall often use a sheet of paper placed on a flat table or desk top to represent a part of a plane. The table top itself may be thought of as containing even more points of this same plane.

Exercise Set 5

1. Does a plane as we shall think of it contain one point or more points than can be counted?

2. Take a sheet of paper. Think of it as part of a plane. Is it possible to draw more than one line in this plane?

   If so, draw two lines.

   Now draw three more lines.

   Draw ten more lines.

3. Does a plane contain one line, two lines, or more lines than can be counted?

4. Think of the top of your desk as a part of a plane. Describe the locatior of a point not on this plane. Describe the location of a line not on this plane.

5. Does a plane contain all points of space?

6. Does a plane contain some lines but not all lines in space?

7. Take a sheet of paper. Think of it as part of a plane. Describe a line which is not on this plane. Draw a ray which is on the plane. Describe a ray which is not on this plane.

8. If the endpoint of a ray is not on a certain plane, is the ray on that plane?

9. If the endpoint of a ray is on a certain plane, then must the ray be on that plane?

10. If two points of a ray are on the plane, then must the ray be on the plane?

Working Together

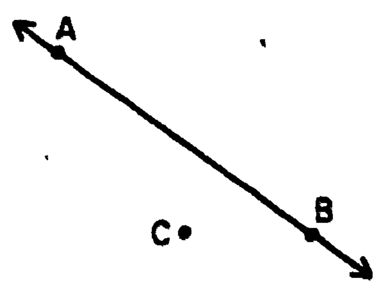
Let us think about a line and a plane. Suppose the line has two of its points in the plane. For example, look at the points A and B represented on this page. The page suggests part of a plane which contains A and B.

1. Answer these questions carefully.

a) How many lines contain both points A and B?

b) Are all of the points of $\overleftrightarrow{AB}$ contained in a plane which contains A and B?

c) Think of a third point in the plane and label it C. Draw line CA. Is $\overleftrightarrow{CA}$ in the plane?



d) Draw line CB. Is $\overleftrightarrow{CB}$ in the plane?

Suppose we have two points, A and B. Suppose we have a plane called E. If point A is in plane E and point B is in plane E, then the entire line AB is in plane E.

2. Which is the correct ending:

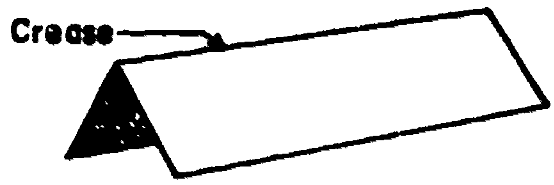
A line with two of its points contained in a plane

a) has some, but not all, of its points contained in that plane.

b) has all of its points contained in that plane.

3. Can there be more than one plane containing two points? If there is more than one plane, give an example. Remember there is just one line containing these two points.

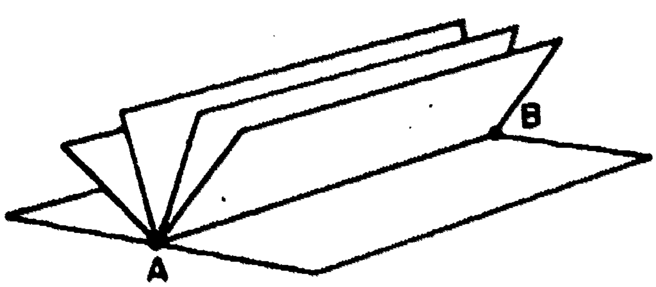
4. Fold a piece of paper in half. We think of the crease as a line segment. Stand the folded paper on your desk so that the crease does not touch it. The paper makes a tent.



Does this suggest parts of two planes which contain the line segment represented by the crease? If so, show them.

5. Give an example of two points and three planes passing through them.

6. Open a thin book so that you see the pages as in the figure below. The spine of the book suggests a segment. Name it $\overline{AB}$.



a) What does each page suggest?

b) Does each page pass through the spine of the book?

7. Choose two points in space. Now how many planes do you think contain the same two points?

8. Choose a line segment in space. How many planes do you think contain this line segment?

9. Choose a line in space. How many planes do you think contain this line?

10. Which is the correct ending?

a) Two points in space are contained in

1) only one plane.

2) many, many planes, but we could count them.

3) more planes than can be counted.

b) A line segment is contained in

1) only one plane.

2) many, many planes, but we could count them.

3) more planes than can be counted.

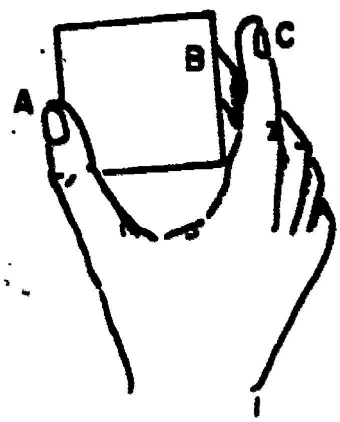
c) A line is contained in

1) only one plane.

2) many, many planes, but we could count them.

3) more planes than can be counted.

11. **We are now going to make a very important observation. Hold a piece of cardboard at the middle of the side edges by the thumb and middle finger as shown below.**



Without moving the thumb and middle finger we are able to use our other hand to rotate the cardboard to many positions showing many planes through $\overleftrightarrow{AB}$.

a) Now rotate the cardboard until it touches the tip of your index finger. Think of the tip of this finger as point C.

The card now represents a plane passing through the points A, B, and C.

Does there seem to be another plane passing through points A, B, and C?

This suggests that through three points not on a line there passes just one plane. We shall think of this as a geometric fact.

b) Think of a door representing part of a plane and the hinges representing two points. As the door swings open, does it suggest many planes through these points?

Now hold a finger tip against the door. Your finger tip represents a third point which holds the door open in one position.

This again suggests that through three points not on a line there passes just one plane.

12. Review

a) A plane contains more points and more lines than can be counted.

b) If two points of a line are contained in a plane, the whole line is contained in the plane.

c) Through two points there passes more planes than can be counted.

d) Through three points not on a line there passes one and only one plane.

INTERSECTIONS OF LINES AND PLANES

Working Together

Do you recall what we mean by the intersection of two sets?

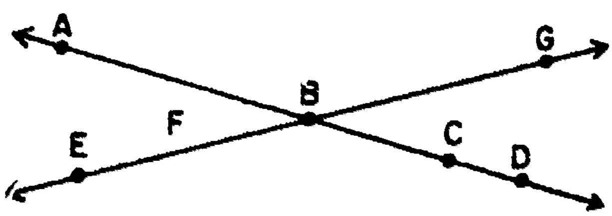
1. Set A = {2, 3, 5, 9} Set B = {1, 2, 3, 4, 9}
The intersection of A and B is { }.

2. Set R = {M, A, R, Y} Set S = {C, A, N, D, Y}
The intersection of R and S is { }.
The union of R and S is { }.

3. Set E = {5, 6, 7, 8} Set F = {9, 10, 11, 12}
The intersection of E and F is { }.
The union of E and F is { }.

You know that a line is a set of points and a plane is a set of points. Let us find the intersection of two lines.

Look at the points named on the lines in the picture below.



4. What points of $\overleftrightarrow{AC}$ are labeled? Of $\overleftrightarrow{EG}$?

5. Is there any point that is on both lines?

6. What is the intersection of $\overrightarrow{AC}$ and $\overrightarrow{EG}$?
What is the intersection of $\overrightarrow{BA}$ and $\overrightarrow{BG}$?
What is the intersection of $\overrightarrow{FE}$ and $\overrightarrow{BG}$?

7. How many points are in the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$?

8. Can you hold two pencils to represent lines so that no point is on both lines? Can you do this in more than one way?

9. Use a card to represent a plane and a pencil to represent a line. Can you hold them to make their intersection

a) one point?

b) no points?

c) just two points?

d) many points?

10. Use two cards to represent two planes. Can you hold them so the intersection of the planes they represent is

a) just one point?

b) just two points?

c) more than two points?

d) no points?

11. Which of these pictures show that:

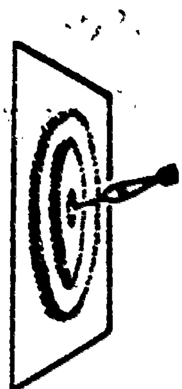
a) the intersection of a line and a plane is one point?

b) the intersection of a line and a plane is a line?

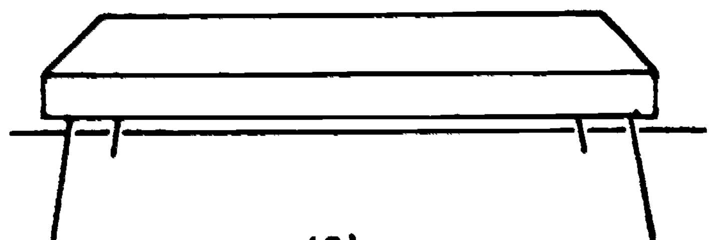
c) the intersection of two planes is a line?

d) the intersection of two lines is a point?

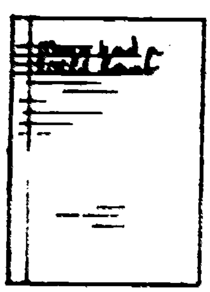
e) the intersection of two lines is the empty set?



(1)



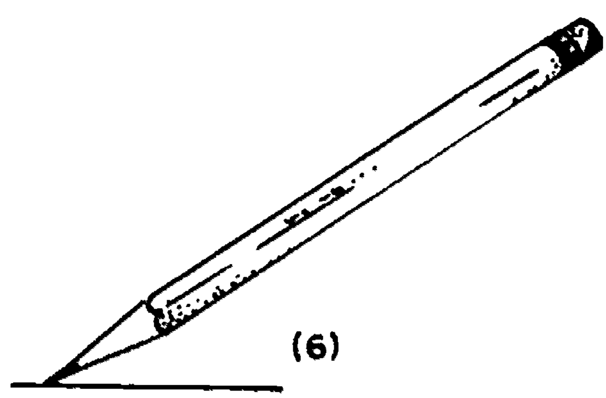
(2)



(3)

(4)

(5)



(6)

Exercise Set 6

1. Mark a point on your paper and label it A. Draw two different lines through A. What is the intersection of the two lines?

2. Mark two different points on your paper and call them B and C. Can you draw two different lines, both through B and C? Can you draw one line through both B and C?

3. What word will make this a true sentence? If two different lines in a plane cross, their intersection is ___?___ point.

4. Can you draw a picture to represent two lines whose intersection seems to be the empty set? If so, draw it.

5. Look at the walls, floor, and ceiling of your classroom. Which represent pairs of planes which cross?

   a) the side wall and front wall

   b) the floor and ceiling

   c) the back wall and front wall

   d) the front wall and ceiling

6. Which of the walls, floor, and ceiling represent planes which do not cross?

   a) the floor and side wall

   b) the floor and ceiling

   c) the back wall and front wall

   d) the front wall and ceiling

7. Imagine you have folded a sheet of paper and opened it to form a tent as we did before. Does the folded sheet suggest two intersecting planes? What is the intersection in this case?

8. Complete this sentence. Two intersecting planes in space intersect in a _____?_____.

9. If three different points of a line are in a plane, what can you say about the line and the plane?

10. Review

    We have learned the following facts.

    a) If two different lines in a plane cross, their intersection is one point.

    b) If two different planes in space cross, their intersection is one line.

    c) If a line and a plane cross, the intersection is either one point or the entire line.

SIMPLE CLOSED CURVES

Working Together

In the section on curves, we drew a path from a point A to a point B. We called the set of points the tip of the pencil passed through a <u>curve</u>.

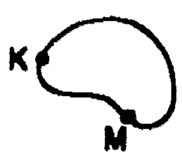
1. Mark two points on your paper and name them T and R. Draw on your paper a picture of a curve from T to R.

2. Mark two points F and H. Draw $\overline{FH}$. We also call $\overline{FH}$ a curve.

3. a) Mark a point K. Draw a curve that starts at K and comes back to K along a different path. Could you draw the curve using line segments?

Since your curve begins and ends at the same point, it is a <u>closed curve</u>.

b) Mark a different point on the curve that contains K and call it M. Can you start at M and trace the curve and come back to M? Did you trace every point of the curve?



4. Mark a point A and a point B. Draw a curve that begins at A and passes through B and then comes back to A without crossing itself.

Your curve through A and B is called a simple closed curve. It starts at one point and comes back to this point without intersecting itself. All the points of a simple closed curve are in the same plane.

5. Mark a point C and a point D. Draw a curve that starts at C and passes through D twice and then comes back to C.

Your curve through C and D is not a simple closed curve because it intersects itself at D.

6. The curve below does not intersect itself. Why is it not a simple closed curve?

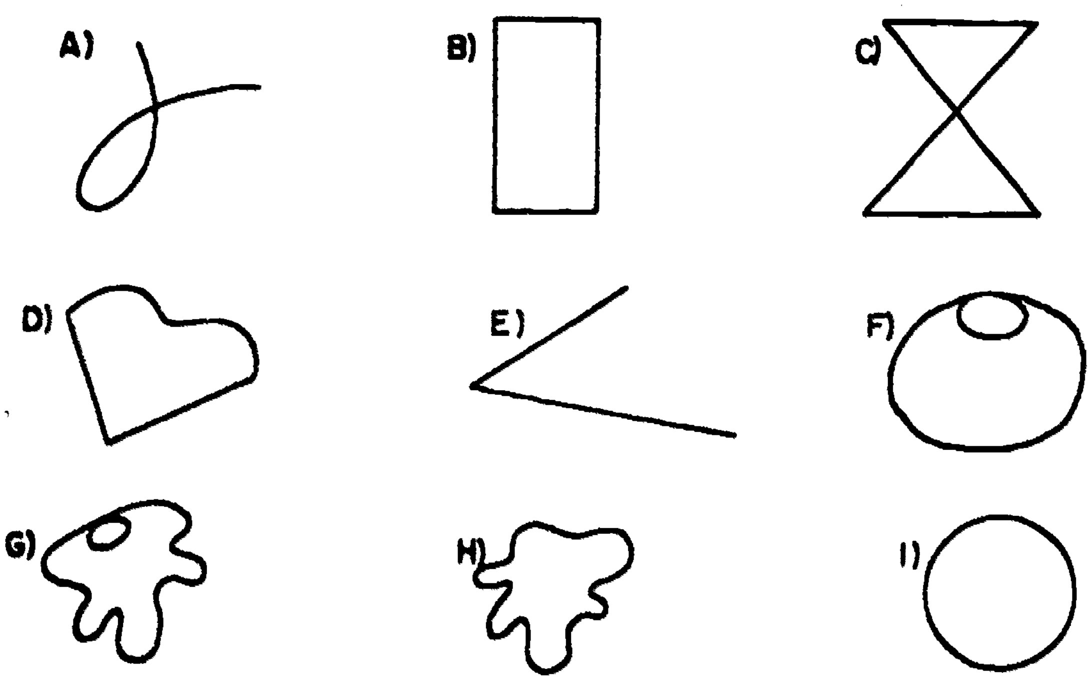
7. Is the figure below a simple closed curve? Why?

8. A frame around a picture suggests a simple closed curve. Name some other things which suggest simple closed curves.

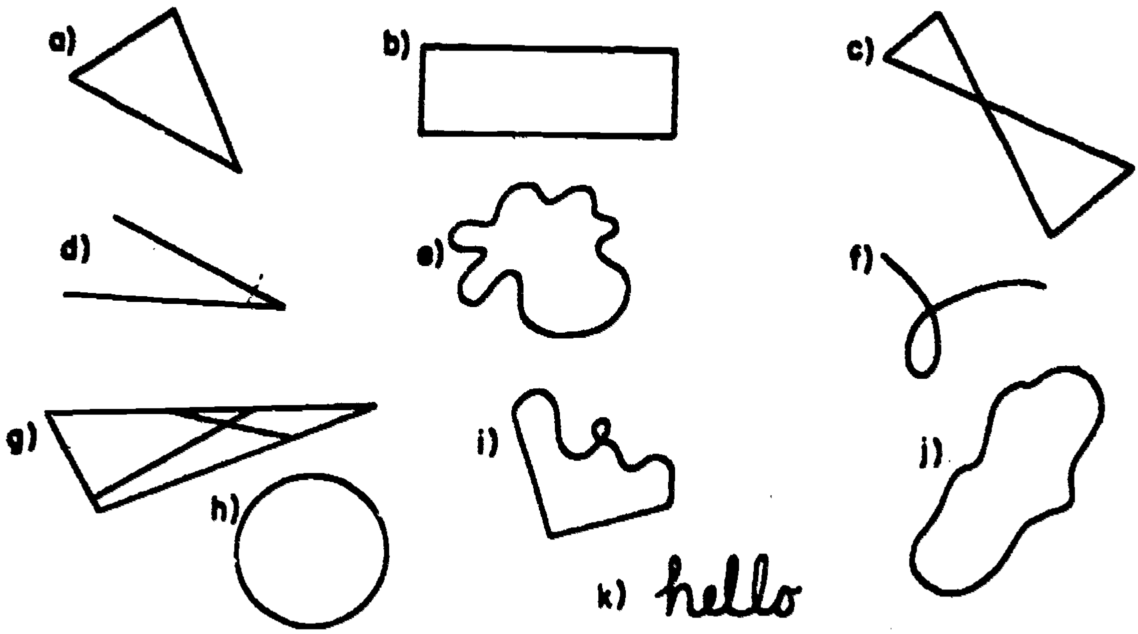
Exercise Set 7

1. Draw a simple closed curve on your paper. Draw it with a blue crayon. Color red the part of the plane inside the curve. Color green the part of the plane outside the curve. (Can you color all of this plane?)

2. Tell which of the following are pictures of simple closed curves.



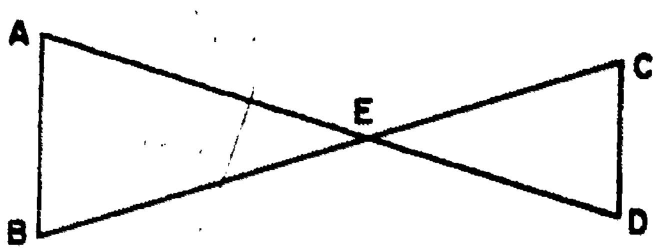
3. Which of the pictures <u>are not</u> simple closed curves?



4. Do the boundaries of most states on a map of the United States represent simple closed curves?

5. Name one state whose boundary on the map of the United States does not represent a simple closed curve. Name another such state.

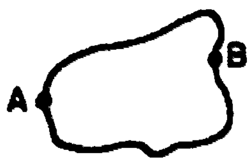
6. Is the figure below a simple closed curve?

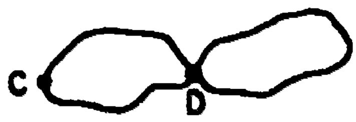


Is it the union of simple closed curves? How many simple closed curves?

7. Look at the curves in Ex. 3. Give other names for some of the simple closed curves.

8. Did your curve for Ex. 4 and 5 of page look something like this?





Working Together

1. Draw a simple closed curve which is the union of

a) three line segments. What is a name for your figure?

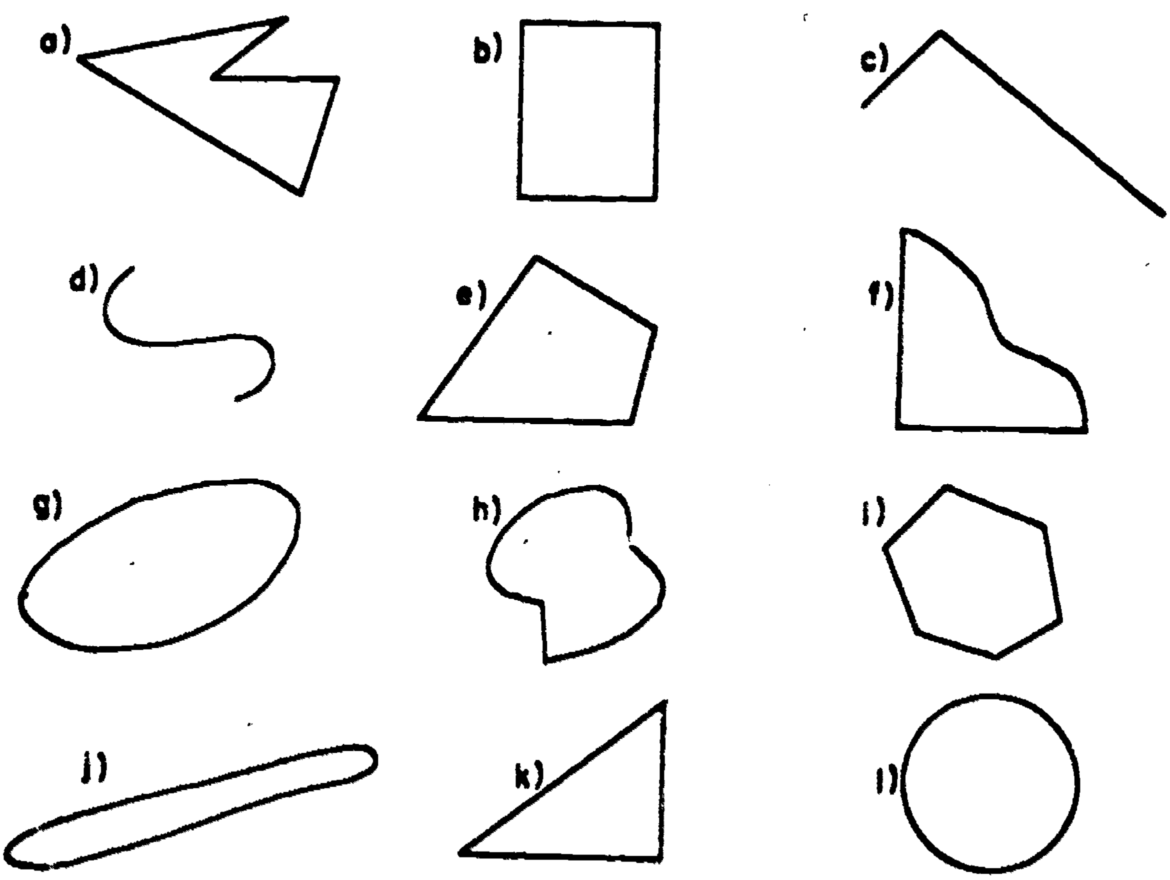
b) four line segments. What is a name for your figure?

c) five line segments. What is a name for your figure?

2. Can you draw a simple closed curve with two line segments? Why?

A simple closed curve which is the union of line segments is called a <u>polygon</u>.

3. Which of these are pictures of simple closed curves? Which are pictures of polygons?



4. Describe some line segments in your classroom which form polygons.

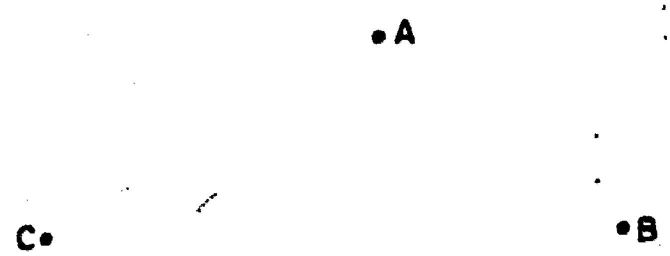
A polygon which is the union of three line segments is a <u>triangle</u>.

A polygon which is the union of four line segments is a <u>quadrilateral</u>.

5. Which of the pictures in Ex. 3 are pictures of triangles?

6. Which of the pictures in Ex. 3 are pictures of quadrilaterals?

7. Mark three points on your paper like these. Draw $\overline{AB}$, $\overline{CB}$, $\overline{AC}$.



a) Does the figure represent a polygon?

b) Does the figure represent a triangle?

c) What words should complete this sentence?

A triangle is made up of ____?____ line segments and these line segments have ____?____ different endpoints.

We usually label the endpoints of the segments in a triangle by capital letters, such as, A, B, C, and use the name or symbol $\triangle ABC$. Another equally good name is $\triangle BAC$.

8. Can you give another name for the triangle?

Exercise Set 8

1. a) Draw a picture of a simple closed curve that is the union of three line segments.

   b) Label the endpoints of the three line segments.

   c) How many different endpoints are there?

   d) What are two names for this kind of simple closed curve?

2. a) Draw a picture of a simple closed curve that is the union of four line segments.

   b) Label the endpoints of the four line segments. How many endpoints are there?

   c) What are two names for this kind of simple closed curve?

3. a) Draw a picture of a simple closed curve that is the union of five line segments.

   b) What is a name for this kind of simple closed curve?

4. Locate on your paper points like these below.

Draw $\overline{AD}$, $\overline{DB}$, $\overline{CB}$, and $\overline{AC}$.

5. Which of these are names for this figure?

a) simple closed curve

b) polygon

c) triangle

d) quadrilateral

6. Draw $\overline{AB}$ in your drawing for Ex. 4. How many triangles do you see? Name them.

7. Now draw $\overline{CD}$ in the same figure. Mark the intersection of $\overline{AB}$ and $\overline{DC}$. Label it E. How many triangles do you see now? Name them.

8. Draw a set of points which is the union of three line segments. Draw a closed curve which is the union of three line segments. Can these drawings be different? Can these drawings be the same?

9. Could a polygon have exactly 8,999 sides?

10. Could a polygon be the union of two line segments and part of the letter O?

11. Is the letter O a polygon?

Working Together

1. Mark a point on your paper and label it A. Mark another point two inches from point A.

2. Mark many other points which are two inches from the point marked A.

3. Mark some more points which are two inches from the point marked A. Be sure they are in all directions from point A.

4. Do the points you marked suggest a simple closed curve? (Do not use the point marked A.) If not, mark some more points which are two inches from A. Now draw a simple closed curve through these points.

5. Mark some more points on your drawing which are also two inches from the point marked A

6. Are these new points on the picture of the simple closed curve you drew? If not, change your simple closed curve so that these new points will be on it.

7. Does your drawing suggest a simple closed curve which has a special name?

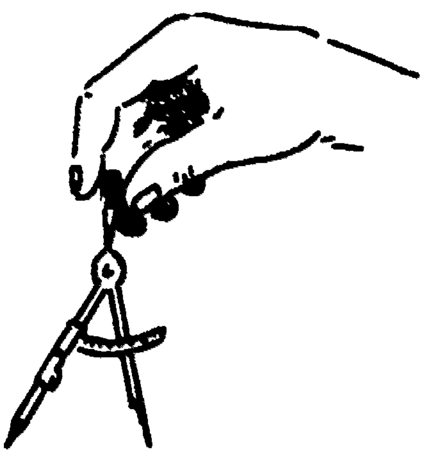
The name of the curve is circle.

A circle cannot be accurately represented by drawing with a pencil and a ruler. A compass is needed.

The easiest way to draw a circle with a compass is to hold the top of the compass between your thumb and index finger. If you press lightly, the compass will work better for you. Press slightly harder on the sharp tip of the compass than you do on the pencil part of the compass.

When you start to draw a circle, do not lift the compass from the paper until the circle is completed. Do not forget to tilt the compass in the direction you are drawing the circle.

Practice using your compass correctly.



8. **Directions for drawing a circle.**

a) Mark a point on your paper and label it B. B will not be part of the circle.

b) Set your compass so that the metal tip is two inches from the pencil tip.

c) Put the metal tip on the point marked B. Now swing the pencil point so that you draw a simple closed curve. Do not let the distance between the pencil point and metal point change while you are drawing.

You have drawn a picture of a circle.

The point marked B is called the <u>center</u> of the circle. Point B is <u>not</u> part of the circle.

9. Mark two points of your circle. Label the points C and D. Draw a picture of $\overline{BC}$.

B marks the center of the circle and C marks a point on the circle.

$\overline{BC}$ is called a <u>radius</u> of the circle.

10. Draw a picture of $\overline{BD}$ in your drawing. $\overline{BD}$ is also a radius of the circle. Why?

a) Can you draw another radius? If so, do. Call it $\overline{BE}$.

b) Can you draw still another radius? If so, do.

c) How many radii does a circle have? (Radii is the plural of radius.)

11. Are the sentences below true? Use your picture to help you decide.

a) A circle has all its points the same distance from a point inside called the center.

b) B marks the center of this circle.

c) All the radii of a circle have the same length.

d) $\overline{BD}$ is a radius of the circle.

e) $\overline{BC}$ and $\overline{BE}$ are also radii of the circle.

Exercise Set 9

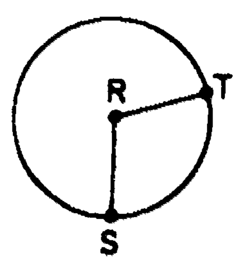
1. a) Mark a point on your paper and label it C. Draw a circle with C as center.

   b) Draw a radius of your circle.

   c) Mark a point of your circle and label it D. Draw $\overline{CD}$.

   d) $\overline{CD}$ is a _____?_____ of the circle.

2. Look at this picture.



   a) Name the center of the circle.

   b) Name a radius of the circle.

   c) What is true about the lengths of $\overline{RT}$ and $\overline{RS}$?

3. Mark two points about two inches apart. Call the points A and B.

   a) Draw a circle with the center at the point A.

   b) Draw a different circle with point A as the center.

   c) Draw a third circle with point B as the center.

   d) Draw a radius of each circle.

4. Mark two points R and S about two inches apart.

   a) Draw a circle with center at point R and passing through point S.

   b) Draw a circle with the center at S and passing through R.

   c) Is $\overline{RS}$ a radius of both circles?

5. Mark two points A and B on your paper.

   a) Draw a circle with the center at A and having $\overline{AB}$ as a radius.

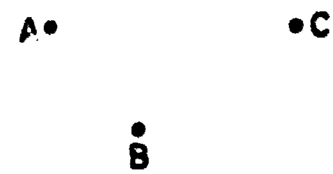
   b) Draw three more radii of this circle.

6. Draw two different circles so that a radius of one has the same length as a radius of the other.

7. Draw two different circles so that one has a radius of different length from the other.

8. Draw two different circles with the same center.

9. Trace the points of $\{A, B, C\}$ on your paper.



a) Draw $\overline{AB}$.

b) Draw the circle with center at A and passing through B.

c) Draw a circle with center at C and a radius equal in length to the length of $\overline{AB}$.

d) Draw a radius of the circle you just made.

e) Is the length of this radius equal to the length of $\overline{AB}$?

10. a) Could the intersection of two circles be the empty set? Draw a figure to show your answer.

b) Could the intersection of two circles be a set with exactly one point? Draw a figure to show your answer.

c) Could the intersection of two circles be a set which has exactly two points? Draw a figure to show your answer.

11. a) Could the intersection of a circle and a line be the empty set? Draw a figure to show your answer.

b) Could the intersection of a circle and a line be a set which has exactly one point? Draw a figure to show your answer.

c) Could the intersection of a circle and a line be a set whi h has exactly two points? Draw a figure to show your answer.

BRAINTWISTER

12. a) Could the intersection of two circles be a set which has exactly three points?

b) Could the intersection of a circle and a line be a set which has exactly 3 points?

REGIONS IN A PLANE

Working Together

1. Draw a picture of a triangle. Trace the triangle with a blue crayon.

2. Color the part of the plane inside the triangle red. The set of points you colored red is called the <u>interior</u> of the triangle.

3. Color the part of the sheet outside the triangle yellow. This set of points which you colored yellow is part of the <u>exterior</u> of the triangle.

The set of points of the triangle is <u>not</u> in the interior and is <u>not</u> in the exterior of the triangle.

4. Use your compass to draw a circle. Trace the circle with a blue crayon.

5. Color the interior of the circle red.

6. Color the exterior of the circle yellow.

7. Mark a point of the circle. Label it A. Is point A in the interior of the circle? Is A in the exterior of the circle? Mark another point which is not in the interior of the circle and is not in the exterior of the circle.

8. Draw a triangle with blue crayon. Color the interior of the triangle blue also.

9. The part of the plane colored blue is the union of two sets of points. What two sets?

The union of a simple closed curve and its interior is called a <u>plane region</u>. The one you colored blue is called a <u>triangular region</u>.

Exercise Set 10

1. a) Draw a triangle. Color the triangle and its interior red.

   b) What is the name of the part of the plane which is red?

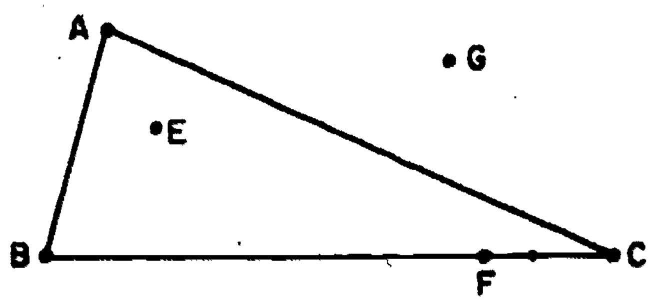
   c) What is the name of the part of the plane which is not red?

2. a) Draw a circle. Color the circle and its interior blue.

   b) What do you think should be the name of the part of the plane which is blue?

   c) What is the name of the part of the plane which is not blue?

3. Look at the figure and the labeled points.



Which sentences are true?

E is: a) a point of the triangle.

b) a point of the interior of the triangle.

c) a point of the exterior of the triangle.

d) a point of the triangular region.

4. F is: a) a point of the triangle.

   b) a point of the interior of the triangle.

   c) a point of the exterior of the triangle.

   d) a point of the triangular region.

5. G is: a) a point of the triangle.
   b) a point of the interior of the triangle.
   c) a point of the exterior of the triangle.
   d) a point of the triangular region.

6. A is: a) a point of the triangle.
   b) a point of the interior of the triangle.
   c) a point of the exterior of the triangle.
   d) a point of the triangular region.

7. Mark a point A and a point B at least two inches from A. Draw a circle with center A and with AB a radius.

   Which endings are correct for the figure in Ex. 7?

8. A is a point of

   a) the circle.
   b) the interior of the circle.
   c) the exterior of the circle.
   d) the circular region.

9. B is a point of

   a) the circle.
   b) the interior of the circle.
   c) the exterior of the circle.
   d) the circular region.

10. Mark a point of the exterior of your circle. Label it C.

11. Mark a point of the circular region. Label it D.

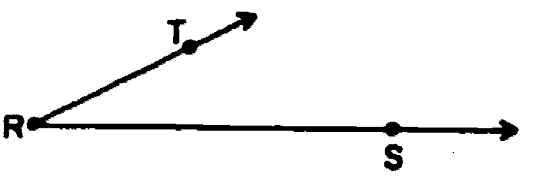
12. Mark a point which is not in the interior and not in the exterior of the circle. Label it E.

Working Together

1. Mark a point R on your paper. Draw a ray with R as endpoint. Mark another point on the ray and label it S.

2. Draw a second ray with R as endpoint. Do not draw it on $\overleftrightarrow{RS}$. Mark a point on this ray and label it T.

Does your drawing look something like this?



This drawing represents a new geometric figure called an angle.

An angle is the union of two rays which have the same endpoint but are not on the same line.

In the figure, R is the vertex of the angle. The endpoint of both rays is called the vertex of the angle.

Each ray is a <u>ray of the angle</u>. $\overrightarrow{RT}$ and $\overrightarrow{RS}$ are rays of the angle in the drawing.

3. Part of an angle is represented by two edges of your desk which meet at a corner.

a) What represents the vertex of the angle?

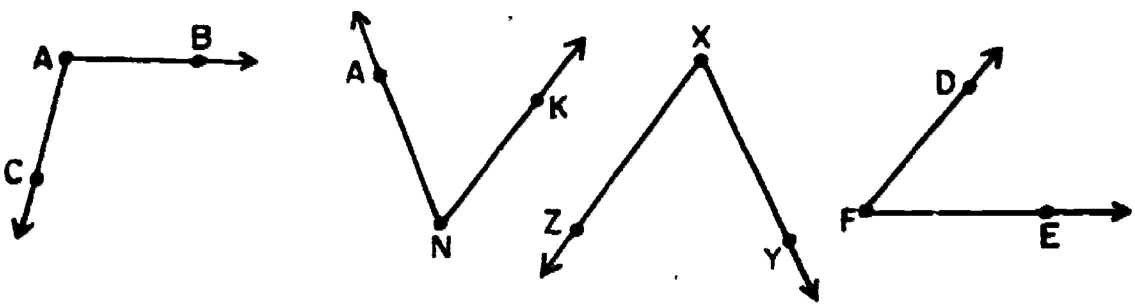
b) What represents the rays of the angle?

c) Why do we say these are only part of the angle?

4. Do the hands of a clock suggest an angle? If so, what represents the vertex? What represents the rays?

5. Describe other things in your classroom which suggest an angle.

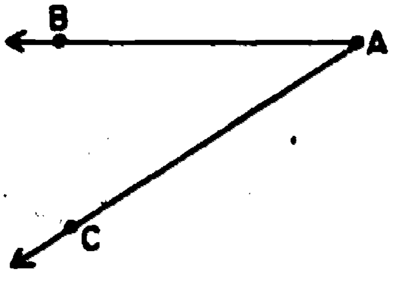
6. In each angle pictured below, name the vertex and the rays.



We name the first angle in the picture $\angle BAC$ or $\angle CAB$. Either is correct. The middle letter must be the label for the vertex.

7. Draw an angle. Label it $\angle SRT$. Did you put the correct letter at the vertex?

8. Below is represented $\angle BAC$. Copy the picture on your paper.



a) Choose a point on $\overrightarrow{AB}$ different from A and B and label it D.

b) Choose a point on $\overrightarrow{AC}$ different from A and C and label it E.

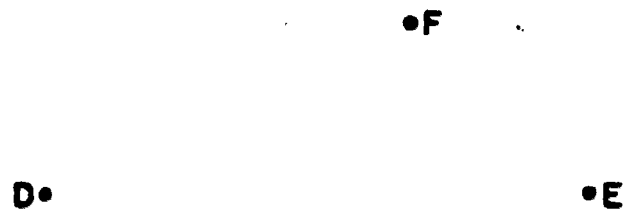
c) Is $\overrightarrow{AB}$ the same ray as $\overrightarrow{AD}$?

d) Is $\overrightarrow{AC}$ the same ray as $\overrightarrow{AE}$?

e) Is $\angle BAC$ the same angle as $\angle DAE$?

No matter how we label an angle, the middle letter always represents the vertex.

9. Three points are shown below.



Write on a sheet of paper the words that complete these sentences.

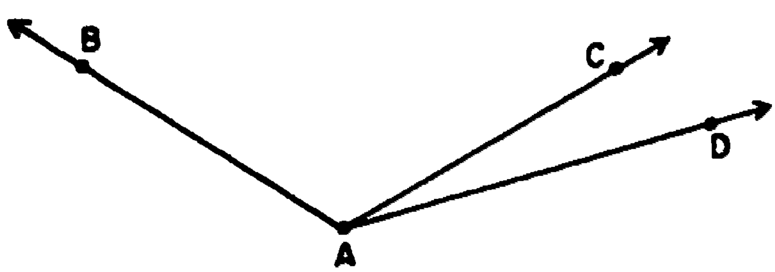
a) There is _____?_____ ray through D and F with endpoint F.

b) There is _____?_____ ray through F and E with endpoint F.

c) There is _____?_____ angle containing D and E with vertex F. This angle is labeled _____?_____ or _____?_____.

Exercise Set 11

1. Here are three rays. Each has the endpoint A. Name three angles.



2. a) Mark a point C on your paper. Draw a picture of two angles which have the point marked C as a vertex.

   b) Name the rays of each angle.

3. a) Mark a point A on your paper. Draw a picture of at least 4 angles which have the point marked A as a vertex. Do this by drawing 5 different rays, not on the same line, with A as endpoint. Choose a point different from A on each ray. Label these points with the capital letters B, C, D, E, and F.

   b) Name the rays of each angle.

   c) Name each angle.

BRAINTWISTERS

4. Try to repeat Ex. 3 by using only 3 rays (no two of them on the same line) with A as endpoint. Did you get a picture of four angles? How many angles does your picture represent?

5. Try to repeat Ex. 3 by using only 4 rays (no two of them on the same line) with A as endpoint. Did you get a picture of exactly four angles? How many angles does your picture represent?

Working Together

1. Look at the points below which are labeled A, B, and C. They are not on the same line. Mark three points like this on your paper and label them.

2. Draw: $\overrightarrow{AB}$

$\overrightarrow{AC}$

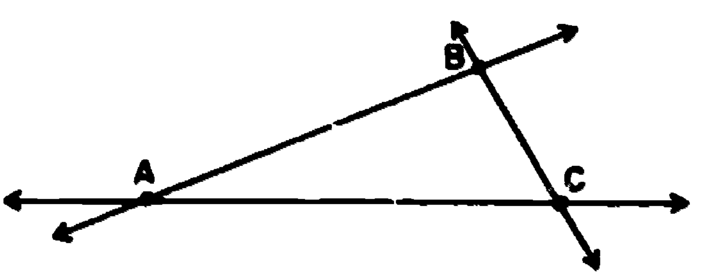
$\overrightarrow{BC}$

$\overrightarrow{BA}$

$\overrightarrow{CB}$

$\overrightarrow{CA}$

3. Does your drawing look something like this?



Write on a sheet of paper the words that complete these sentences.

a) $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$ form a ____?____.

b) The angle with vertex A which contains B and C is called ____?____.

c) The angle with vertex B which contains A and C is called ____?____.

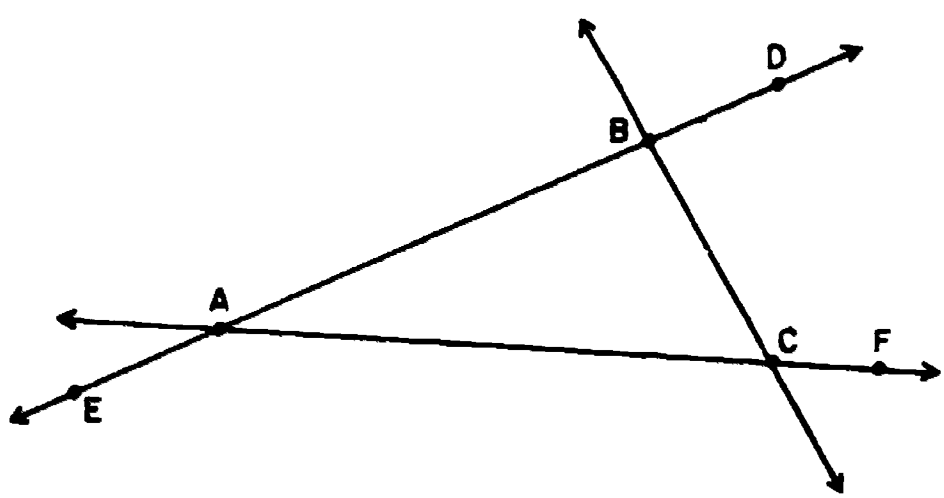
d) The angle with vertex C which contains A and B is called ____?____.

4. Mark a point of $\overrightarrow{AB}$ which is not a point of $\overline{AB}$. Label it D.

5. Mark a point of $\overrightarrow{BA}$ which is not a point of $\overline{AB}$. Label it E.

6. Mark a point of $\overrightarrow{AC}$ which is not a point of $\overline{AC}$. Label it F.

Does your drawing look like this now?



7. Are D, E, and F points of the rays of the angles you named in Ex. 3?

8. a) Are D, E, and F points of the triangle?

   b) Are D, E, and F points of the interior of the triangle?

   c) Are D, E, and F points of the exterior of the triangle?

Ex. 3 shows that a triangle suggests three angles. These angles are not part of the triangle. This is true because a triangle is made up of segments and an angle is made up of rays.

Remember when we studied circles we spoke of the center of a circle. The center is not part of the circle.

In the same way we say $\angle ABC$, $\angle BCA$, and $\angle CAB$ are angles of the triangle although they are not part of the triangle. We call the vertices of these angles the vertices of the triangle. Vertices is the plural of vertex. The vertices of a triangle are the endpoints of the segments of the triangle.

9. Draw a triangle. Label its vertices D, E, F.

a) Name the three angles of the triangle.

b) The three angles of a triangle suggest how many rays?

Exercise Set 12

Make drawings to represent

1. A line
2. A ray
3. A segment
4. A simple closed curve

5. A triangle

6. A circle

7. A polygon

8. Two lines which cross

9. A quadrilateral

10. Three lines which cross but not all in the same point

11. An angle

12. The union of a triangle and one angle suggested by the triangle

13. A triangular region

Using the drawing below name:

14. the intersection of $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

15. three different triangles.

16. a segment which is not a side of a triangle.

17. a point of the interior of some triangle.

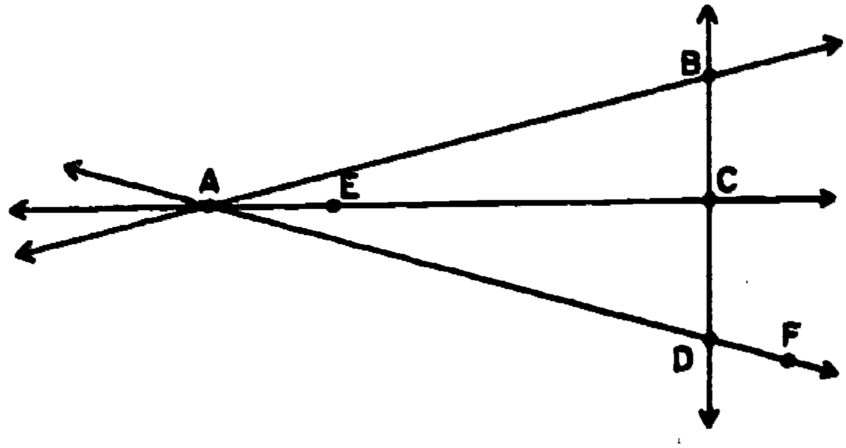
18. a point of the exterior of triangle ABD.

19. the intersection of $\overleftrightarrow{AF}$ and $\overleftrightarrow{BC}$.

20. the intersection of $\overleftrightarrow{AC}$ and $\overrightarrow{DF}$.

21. the intersection of $\overline{AE}$ and $\overline{BD}$.

22. the endpoint of $\overrightarrow{AE}$.



Which sentences are true?

23. The intersection of two different planes may be:

    a) a line.

    b) the empty set.

    c) a set which has exactly one point.

    d) a plane.

24. The intersection of a line and a plane may be:

    a) a set which has exactly two points.

    b) the empty set.

    c) the line.

    d) the plane.

    e) a set which has exactly one point.

BRAINTWISTERS

25. The intersection of a triangle and a plane may be:

    a) a set which has exactly one point.

    b) the empty set.

    c) the triangle.

    d) a set which has exactly three points.

    e) a set which has more points of the triangle than can be counted but not all the points of the triangle.

26. The intersection of a circle and a plane may be:

    a) a set which has exactly one point.

    b) the empty set.

    c) a set which has exactly two points.

    d) a set which has exactly three points.

    e) the circle.

Chapter 10

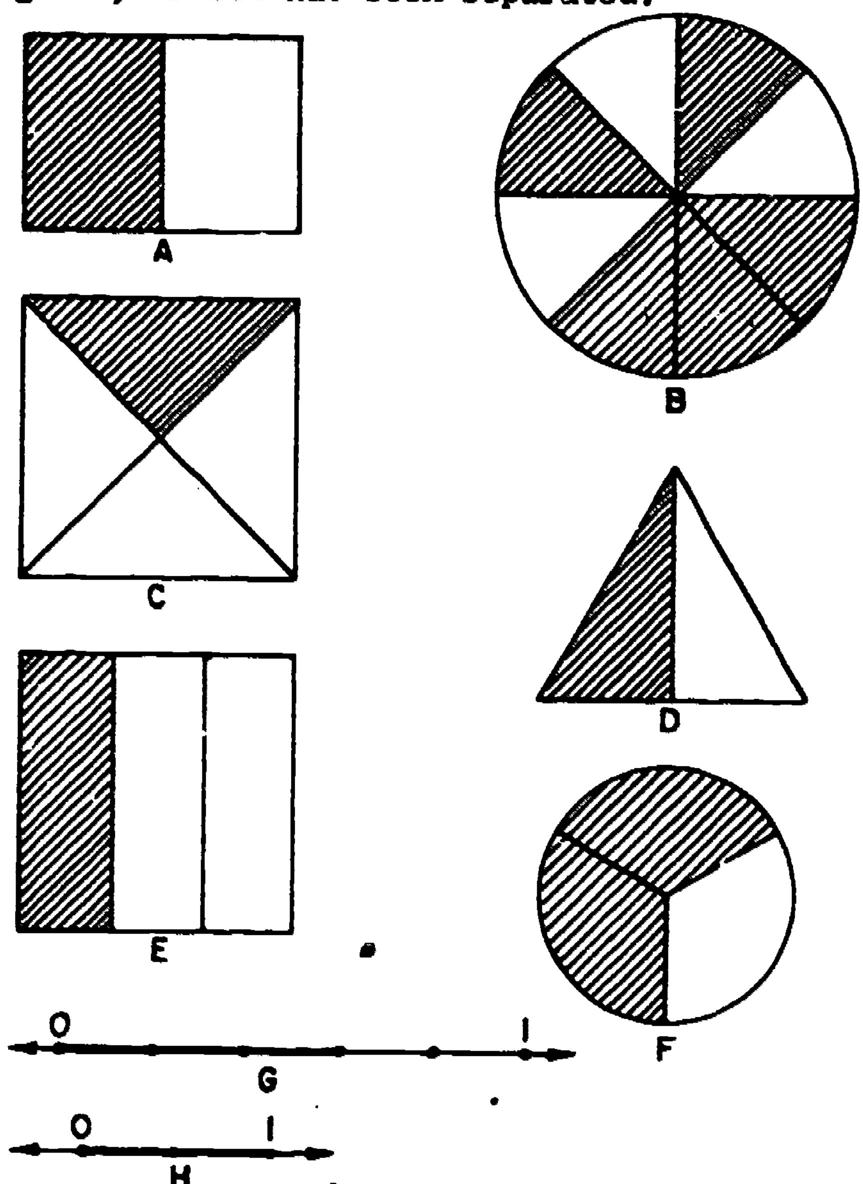
CONCEPT OF RATIONAL NUMBERS

IDEA OF RATIONAL NUMBERS

Exploration

Look at each of the figures on this page.

For each figure, choose a pair of numbers at the right which can be used to talk about the number of parts that are shaded and the number of congruent parts into which each unit region, unit segment, or set has been separated.



Pairs of Numbers

a. 1 and 4

b. 3 and 4

c. 3 and 5

d. 1 and 2

e. 5 and 8

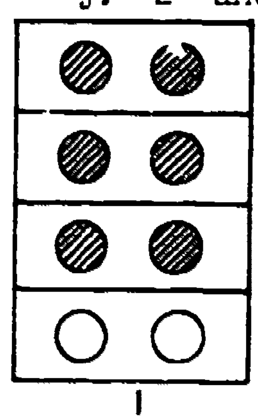
f. 1 and 3

g. 2 and 3

h. 2 and 2

i. 6 and 8

j. 2 and 5

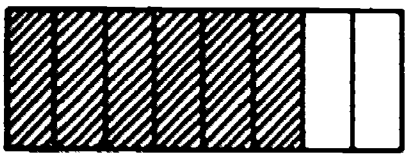


Were you able to find a pair of numbers for each? Did you find these -- A-a; B-e; C-a; D-d; E-f; F-g; G-c; H-h; and I-b?

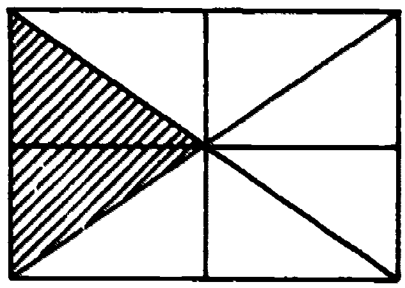
Exercise Set 1

1. Copy the table and complete it, using the figures A, B, C, D, E, F, and G.

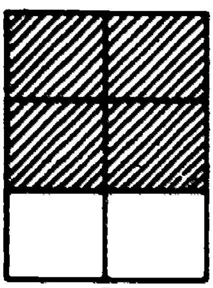
Figure	Parts Shaded	Congruent Parts in Unit
A		
B		
C		
D		
E		
F		



A

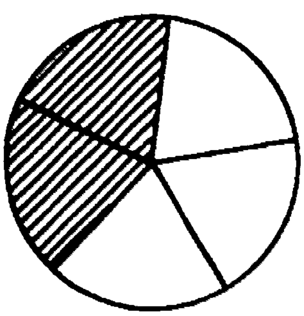


B

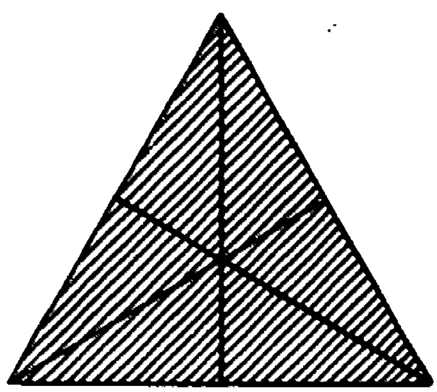


C

D

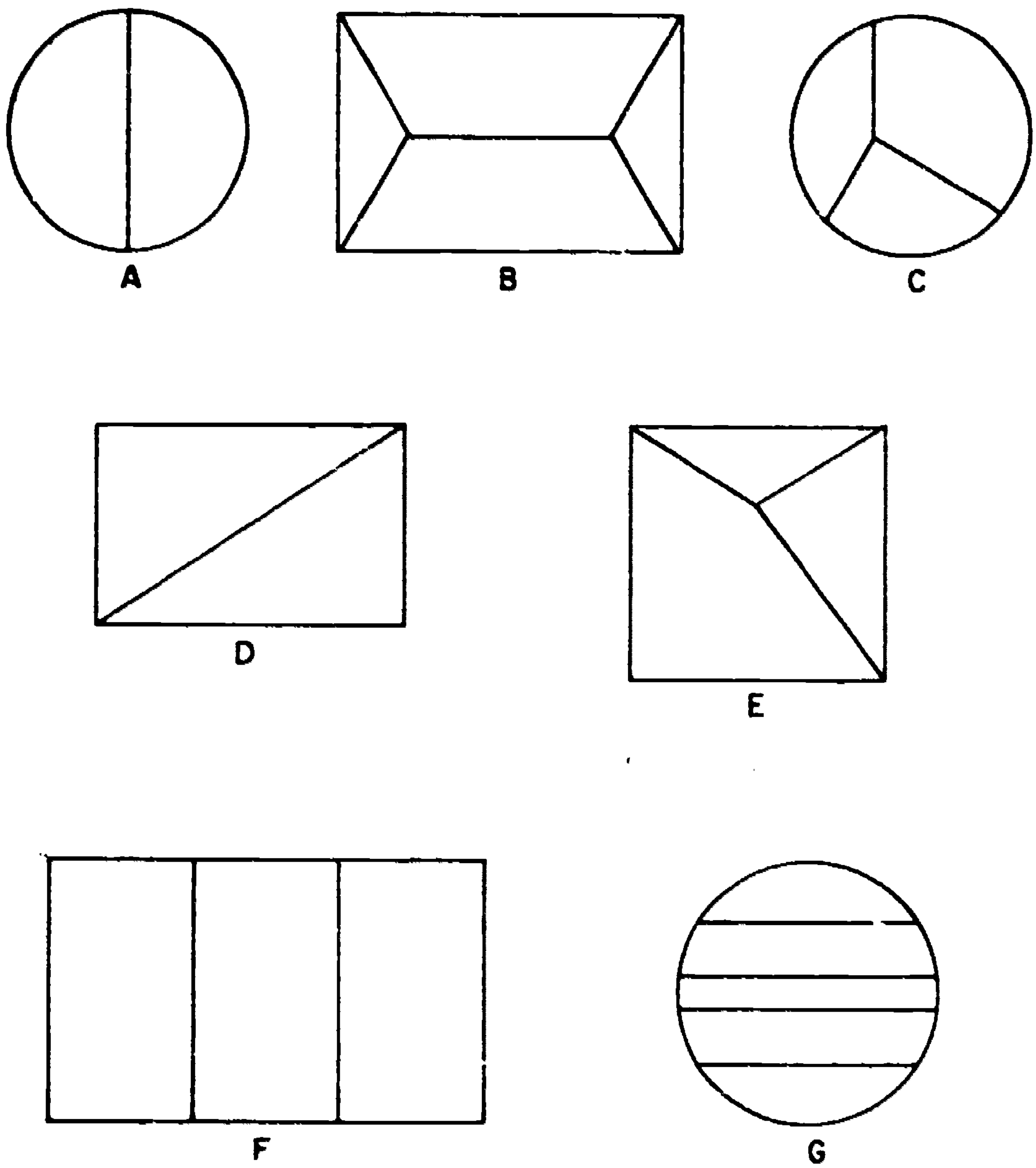


E

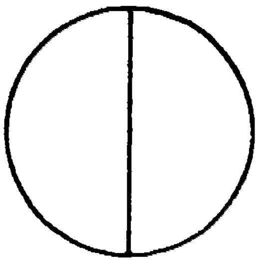


F

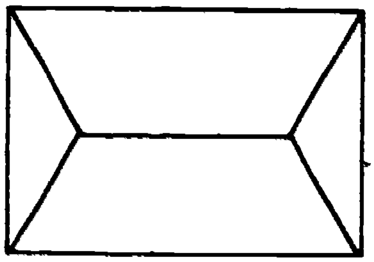
2. Write on your paper the letters from A to G. After each, write Yes if the figure has been partitioned into congruent regions. Write No if the figure has not been partitioned into congruent regions.



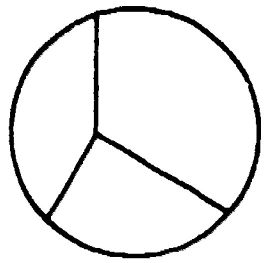
2. Write on your paper the letters from A to G. After each, write *Yes* if the figure *has been* partitioned into congruent regions. Write *No* if the figure *has not been* partitioned into congruent regions.



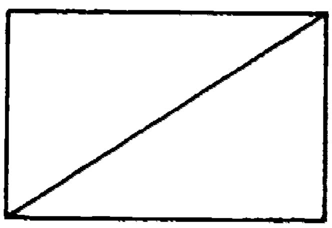
A



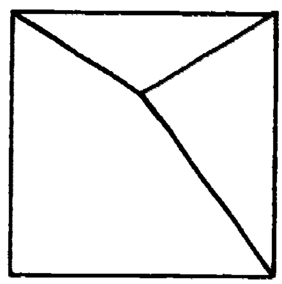
B



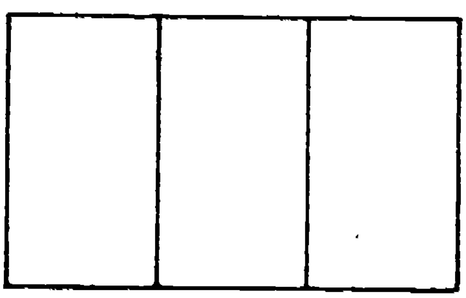
C



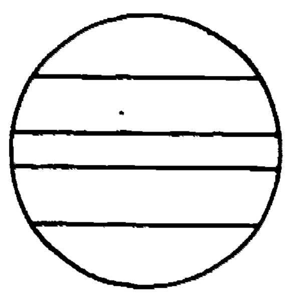
D



E



F

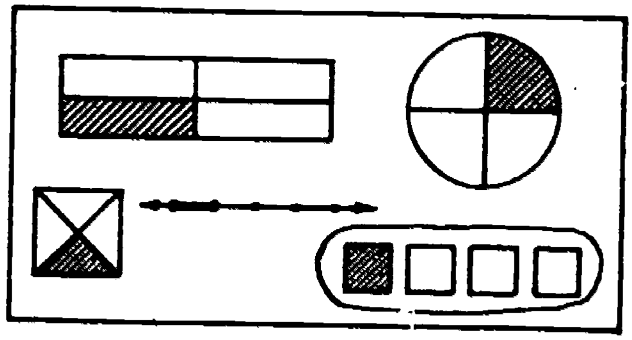


G

A NEW KIND OF NUMBER

Exploration

When a region is partitioned into congruent parts and some of these parts are shaded, we use a new kind of number to describe what we see. These new numbers are called rational numbers. $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$ are rational numbers. They are read, "one-half," "one-fourth," and "three-eighths."



Each of these figures at the right suggests the same rational number. The rational number is one-fourth. The symbol, $\frac{1}{4}$, which names the rational number one-fourth is called a fraction. Fractions are written using two numerals. The two numerals are separated by a horizontal bar.

For example:

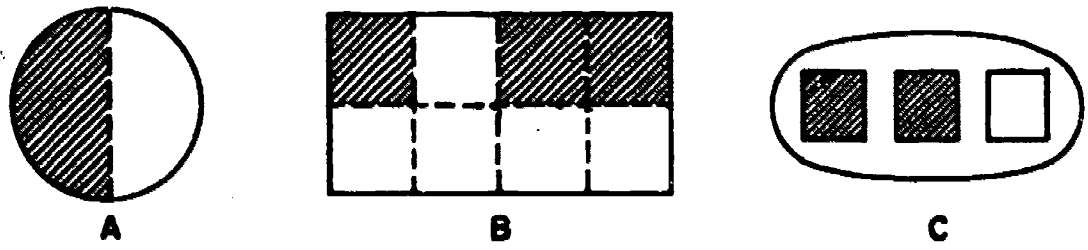
$$\frac{1}{4}$$

The numerals are 1 and 4.

The numeral above the bar tells the number of congruent parts of equivalent subsets described. The number is called the numerator.

The numeral below the bar tells the number of congruent parts into which the set of objects, unit region, or unit segment has been partitioned. The number is called the denominator.

What rational number is suggested by each of these figures below?



What rational number does each of these figures suggest?

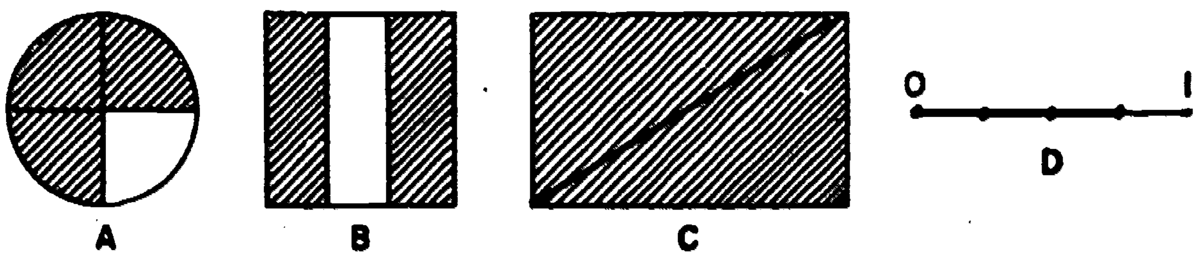


Figure A suggests the rational number, $\frac{3}{4}$, read three-fourths.

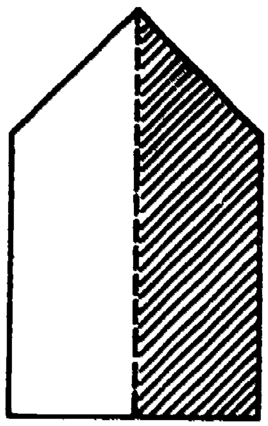
Figure B suggests the rational number, $\frac{2}{3}$, read two-thirds.

Figure C suggests the rational number, $\frac{2}{2}$, read two halves.

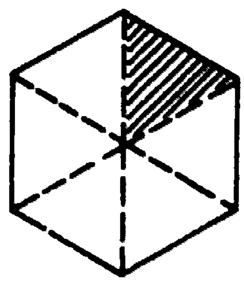
Figure D also suggests the rational number, $\frac{3}{4}$.

Exercise Set 2

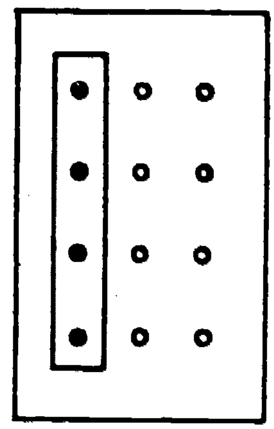
For each figure, write a fraction which names the rational number suggested by the shaded region.



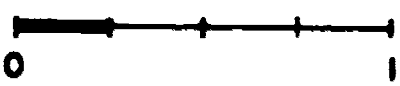
A



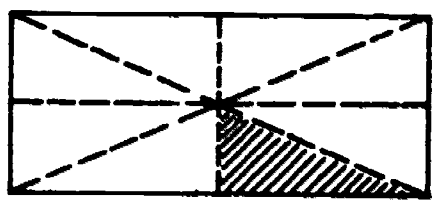
B



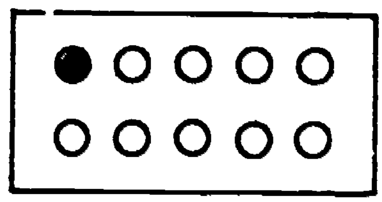
C



D



E



F

2. Write as fractions:

a) one half ______

b) one-third ______

c) one-tenth ______

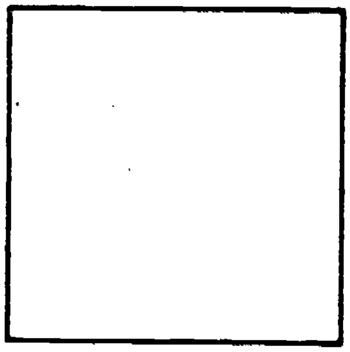
d) one-eighth ______

e) one-sixth ______

f) one-fourth ______

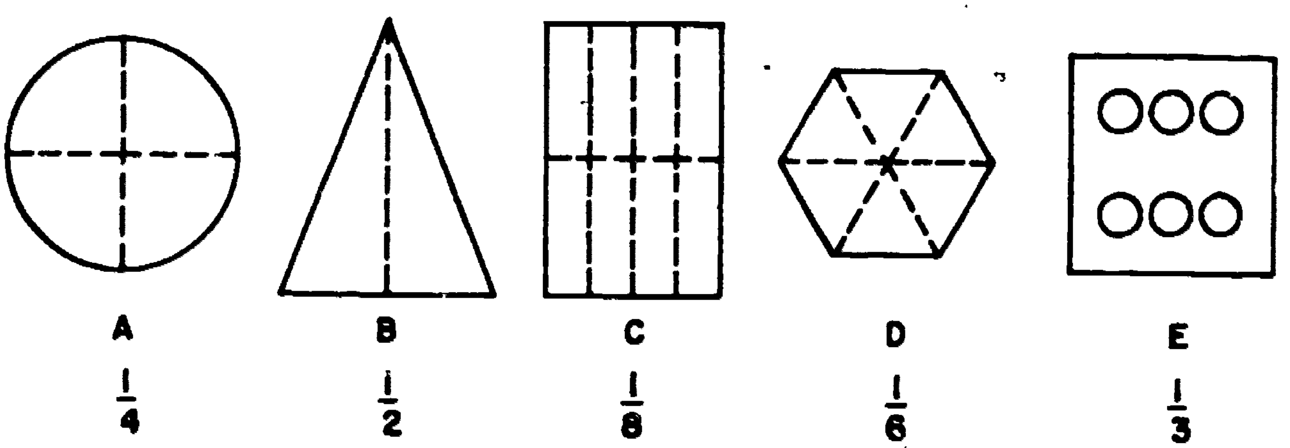
3. Copy the unit square in Figure H at least six times. (Make more copies if you want them.) In how many ways can you separate the unit square to show:

$\frac{1}{2}$? $\frac{1}{4}$? $\frac{1}{8}$?



H

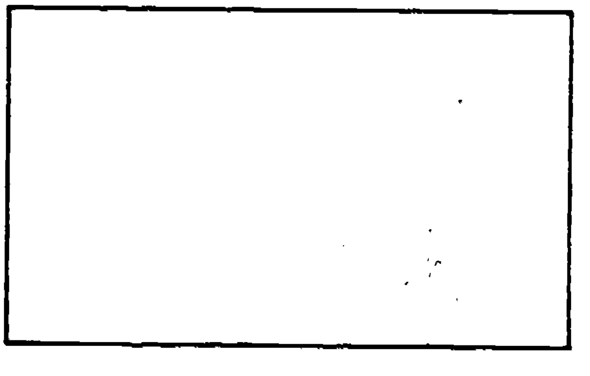
4. Copy and shade the part which is described by the fraction below each figure.



5. Copy and complete this chart.

Fraction	Number of Congruent Parts in Unit	Number of Congruent Parts Counted
$\frac{1}{4}$		
$\frac{1}{2}$		
$\frac{1}{10}$		
$\frac{1}{6}$		
$\frac{1}{3}$		
$\frac{1}{8}$		

6. On your paper, make 6 copies of the unit region shown below. Make the unit regions the same size. Then show a picture that suggests each of the rational numbers named in exercise 5.



Exercise Set 3

1. Use these figures to complete the chart below.

A has been done for you.

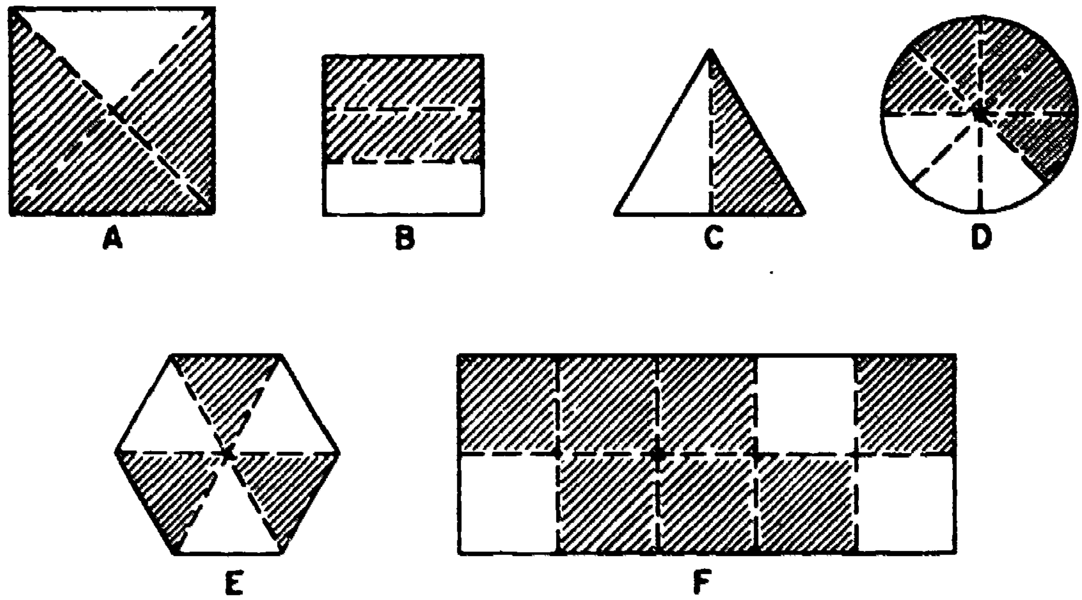


Figure	Number of Congruent Parts in Figure	Number of Shaded Parts	Rational Number Suggested by Shaded Part
A	4	3	$\frac{3}{4}$
B			
C			
D			
E			
F			

2. Using figures A, B, C, D, E, and F of exercise 1, write the name of the rational number suggested by the unshaded part of each figure.

3. Use these figures to complete the sentences below.

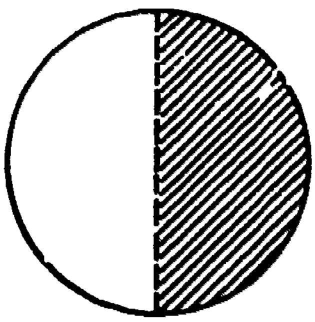


Fig. A

Fig. B

Set C = {1,2,4,6,8,10,12,14}

Fig. C

a) Figure A has been separated into ________ congruent regions. ________ region has been shaded. The shaded region is best described by the rational number named by the fraction ________.

b) Points M, N, and O separate $\overline{XY}$ into _____ congruent segments. m $\overline{XM}$ = ______.

c) Set C has _______ members. _____ member names an odd number. This member is ______ of all the members of Set C.

4. Study your answers to exercises 1, 2, and 3. Copy and then write "above" or "below" in each blank.

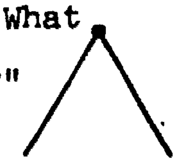
a) The numeral ______ the bar names the number of congruent parts into which the unit has been separated.

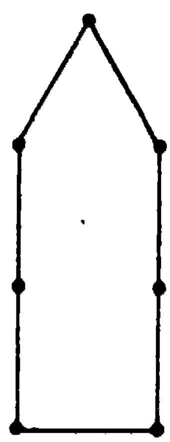
b) The numeral ______ the bar names the number of congruent parts which are described.

5. Ann watched 'levision programs. Each was $\frac{1}{4}$ of an hour long.

a) How long did Ann watch television?

b) How much longer would she need to watch TV to make her total time 1 hour?

6. A figure like the one pictured below was made by laying toothpicks, each the same size, end-to-end. What fractional part of the perimeter is the "roof"  ?

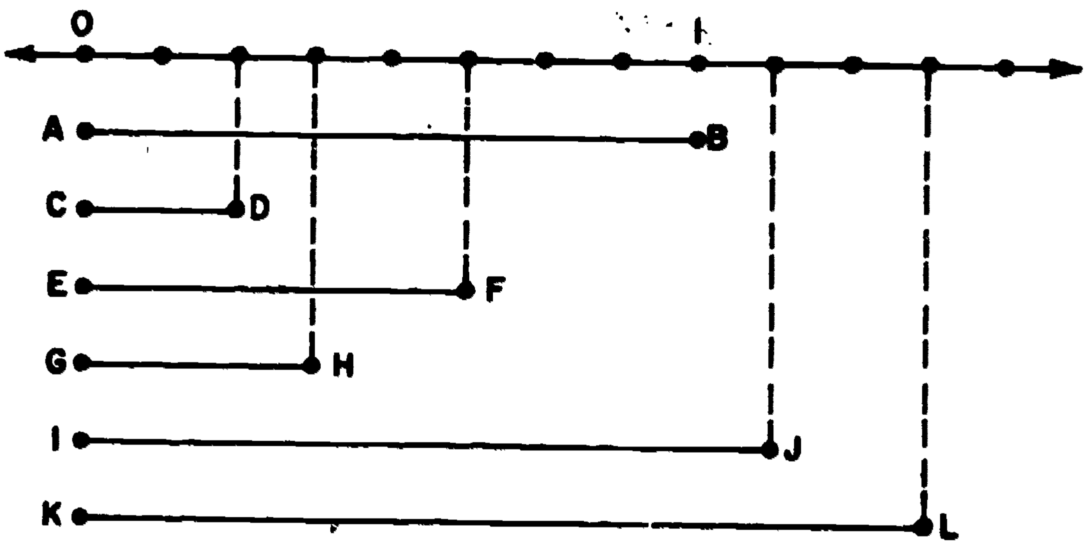


1.

RATIONAL NUMBERS GREATER THAN ONE

Exploration

In the picture below, the line segment AB is 1 unit long.



1. (a) On the number line the unit segment is separable into _____ congruent segments.

   (b) Use a fraction: Each small segment is _____ of the unit segment.

   (c) The measure of $\overline{AB}$ is 1. The measure of $\overline{AB}$ is also ______. (Use a fraction.)

   (d) Is $\frac{8}{8}$ the measure of line segment AB?

   (e) Is $\frac{2}{8}$ the measure of line segment CD?

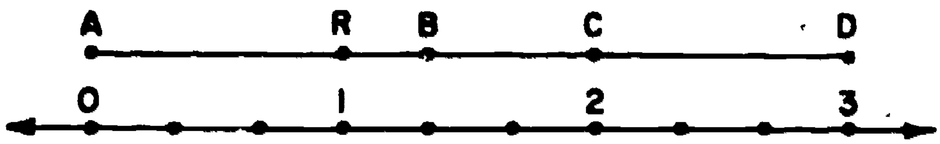
   (f) Is $\frac{5}{8}$ the measure of line segment EF?

   (g) Is $\frac{3}{8}$ the measure of line segment GH?

   (h) Is $\frac{9}{8}$ the measure of line segment IJ?

   (i) Is $\frac{11}{8}$ the measure of line segment KL?

2. Each unit segment of the number line below has been separated into 3 congruent segments. $\overline{AR}$ is the same length as the unit segment.

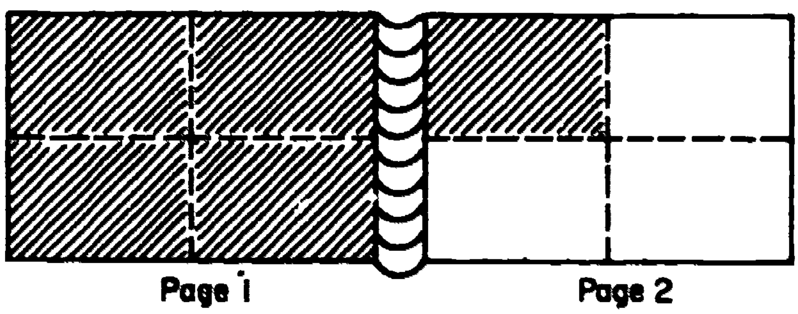


Use this number line to answer the questions.

(a) What fraction names the measure of $\overline{AR}$?

(b) What fraction names the measure of $\overline{AB}$? $\overline{AC}$? $\overline{AD}$?

3. Bill has a photograph album. Each page is separated into 4 congruent parts. On each page he can place 4 pictures.



If Bill pastes 5 pictures in his album, he will cover $\frac{4}{4}$ of one page and $\frac{1}{4}$ of another page. What rational number describes the number of pages covered?

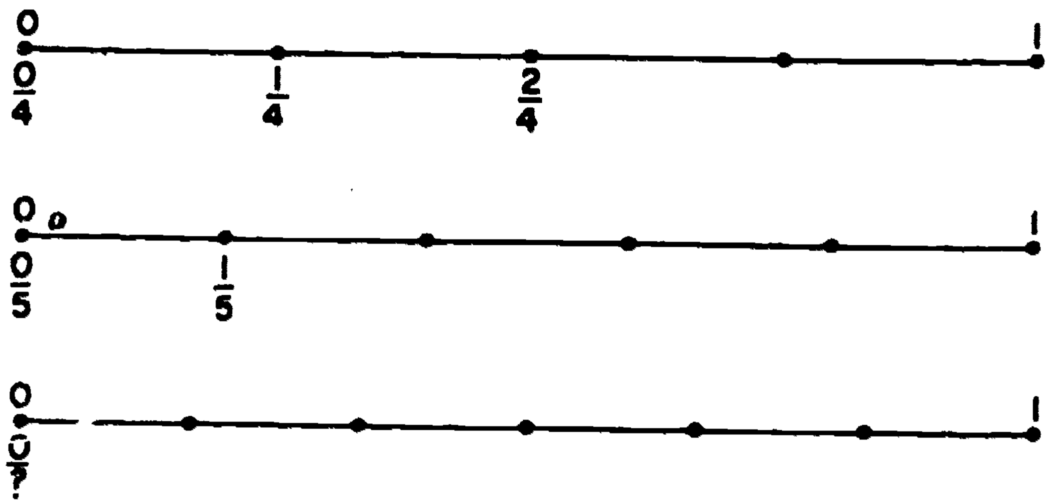
Fractions like $\frac{2}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{4}{5}$ tell us that the measure of a segment or a region is less than 1.

Fractions like $\frac{8}{8}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$ tell us that the measure of a segment or region is exactly 1.

Fractions like $\frac{9}{8}$, $\frac{11}{8}$, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, $\frac{4}{3}$, $\frac{5}{2}$ tell us that the measure of a segment or region is greater than 1.

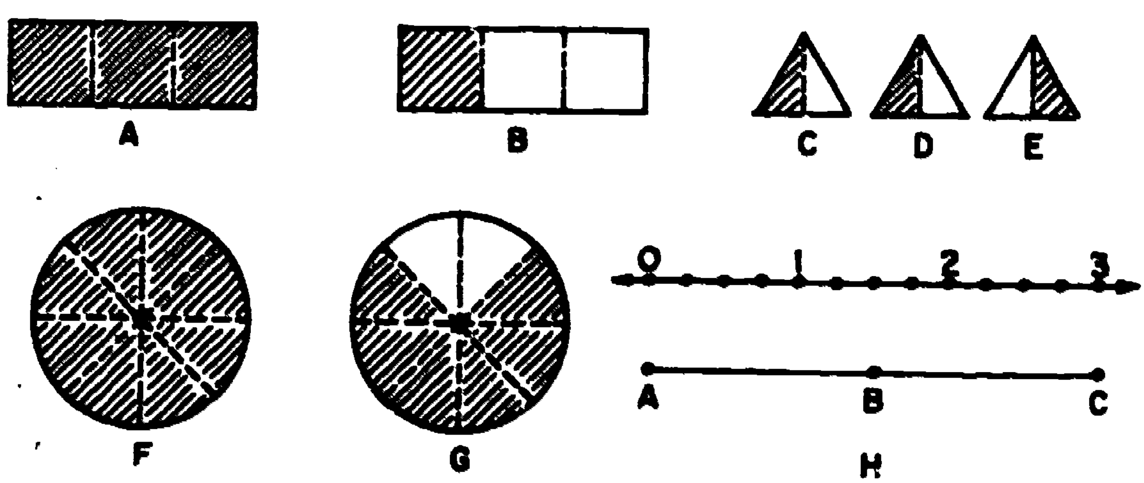
Exercise Set 4

1. Copy the unit segments below. The dots separate each unit segment into smaller, congruent segments. Label each dot correctly.



Each of the figures below represents a unit region or unit segment.

2. Study these diagrams. Then answer the questions on the next page.



a) How many thirds are there in A?

How many thirds are there in B?

How many thirds are shown in A and B together?

What rational fraction is suggested by the shaded region of A and B together?

What rational number is suggested by the unshaded region of A and B together?

b) What rational number is suggested by the shaded region in C? in D? in E?

What rational number is suggested by the unshaded region in C? in D? in E?

What rational number best describes the shaded regions in C, D, and E altogether?

What rational number best describes the unshaded regions in C, D, and E altogether?

c) What rational number is suggested by the shaded region of F and G together?

What rational number is suggested by the unshaded region of F and G together?

d) In Figure H, what rational number is the measure of $\overline{AB}$? of $\overline{AC}$?

3. For each figure, write the fraction that names the rational number suggested by the shaded part.

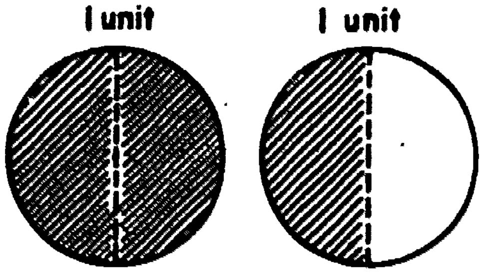


FIG. A

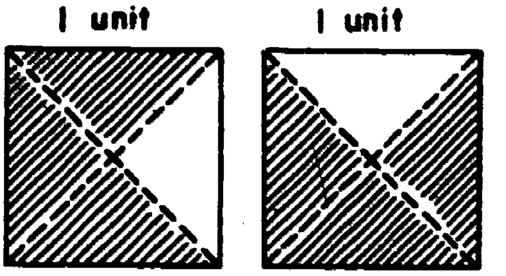


FIG. B

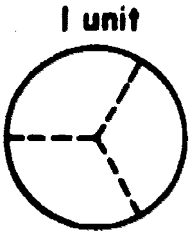


FIG. C

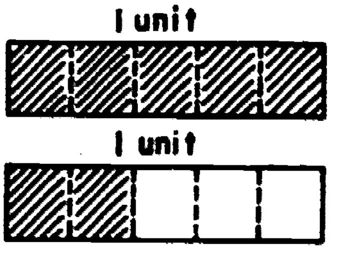


FIG. D

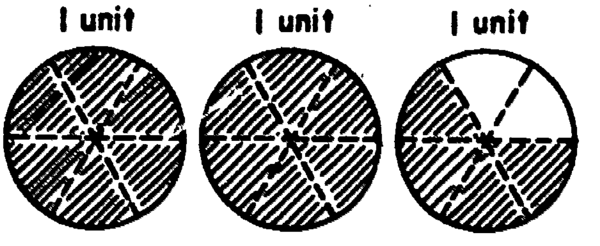
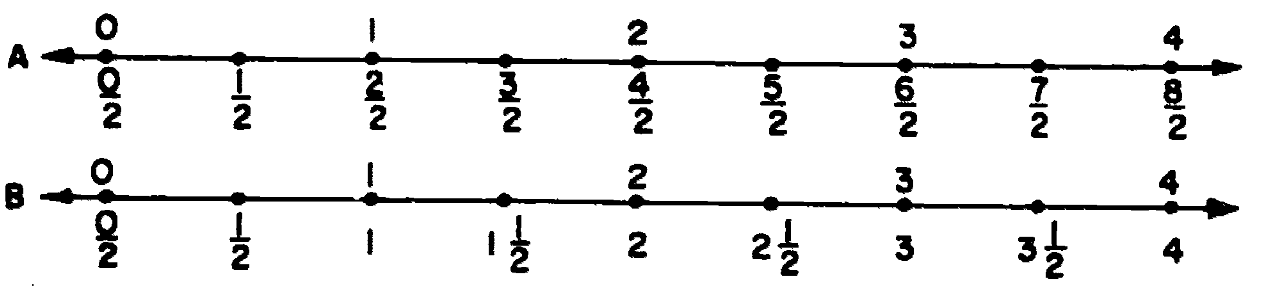


FIG. E

4. Using these number lines, complete the sentences below.



a) 1 one and 1 half = $\frac{\quad}{2}$ or ____

b) 4 = $\frac{\quad}{2}$

c) 3 ones and 1 half = $\frac{\quad}{2}$ or ____

d) 2 ones and 1 half = $\frac{\quad}{2}$ or ____

e) $\frac{3}{2}$ = 1 ____ and 1 ____

f) 2 = $\frac{\quad}{2}$

g) $\frac{2}{2}$ = ____

5. Copy the line segment shown below on your paper.

$\overline{AB}$ is a unit segment.

a) Mark a point D so that $\overline{AD}$ is $\frac{1}{2}$ unit long.

b) Mark a point E so that $\overline{AE}$ is $\frac{3}{2}$ units long.

c) Mark a point F so that $\overline{AF}$ is $\frac{5}{2}$ units long.

6. Copy the line segment below. Notice each unit segment has been separated into 3 congruent segments.

Using a certain unit, the measure of $\overline{XY}$ is $\frac{4}{3}$.

Mark new points U, V, and W so that

a) $\overline{XU}$ is 1 unit long.

b) $\overline{XV}$ is $\frac{2}{3}$ unit long.

c) $\overline{XW}$ is $\frac{5}{3}$ units long.

7. Mark is 4 feet tall. What number gives his height in yards?

8. Ellen watched 5 television programs. How many hours did she watch TV if each program was:

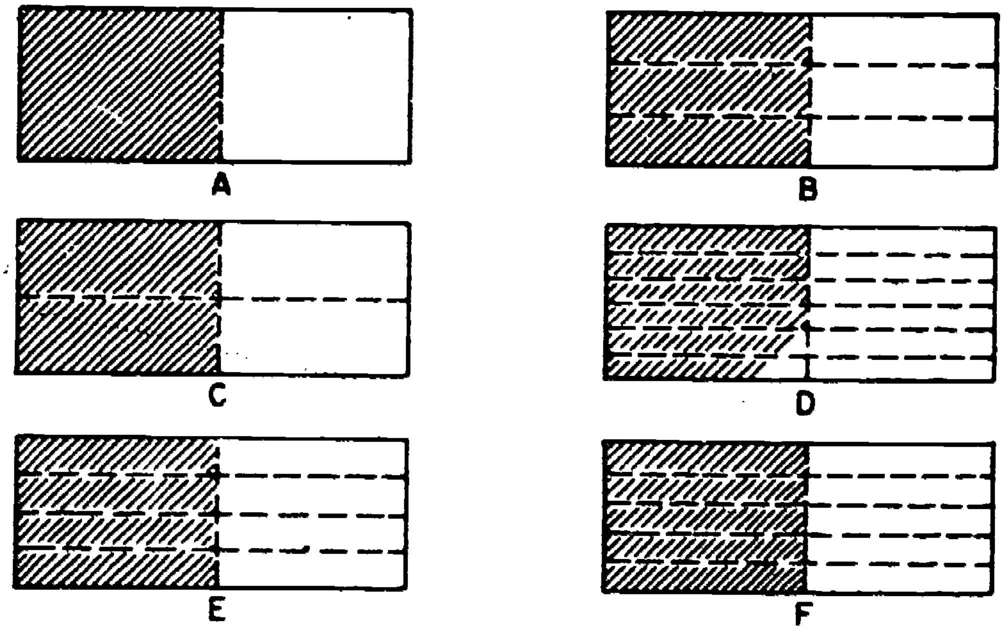
a) $\frac{1}{4}$ of an hour long?

b) $\frac{1}{2}$ of an hour long?

DIFFERENT NAMES FOR THE SAME NUMBER

Exploration

1. The pictures of unit regions below suggest some ways of thinking of one-half.



In A, what fraction names the measure of the shaded region?

In B, what fraction names the measure of the shaded region?

In C, what fraction names the measure of the shaded region?

In D, what fraction names the measure of the shaded region?

In E, what fraction names the measure of the shaded region?

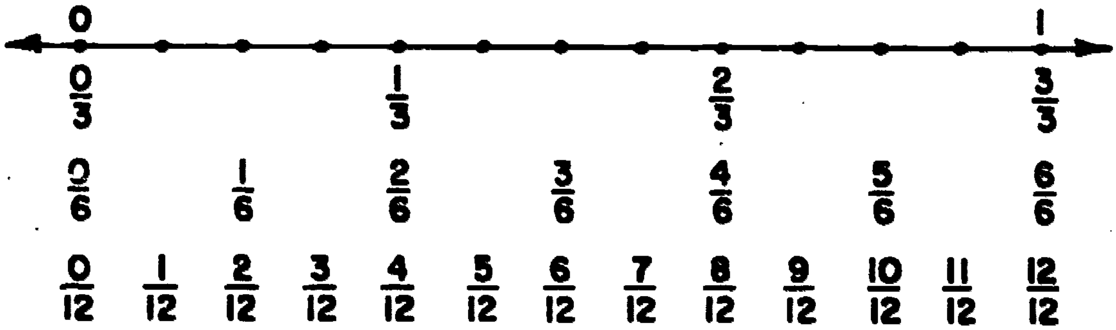
In F, what fraction names the measure of the shaded region?

$\frac{1}{2}$, $\frac{3}{6}$, $\frac{2}{4}$, $\frac{6}{12}$, $\frac{4}{8}$, and $\frac{5}{10}$ are all ways of naming the rational number $\frac{1}{2}$.

We can write: $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$.

What are some other fractions that name this same number?

We say that $\frac{1}{2}$ is the simplest name, or simplest form, for this rational number. Can you tell why?



2. Make true statements by writing a fraction in each blank. Use the number line above to help you.

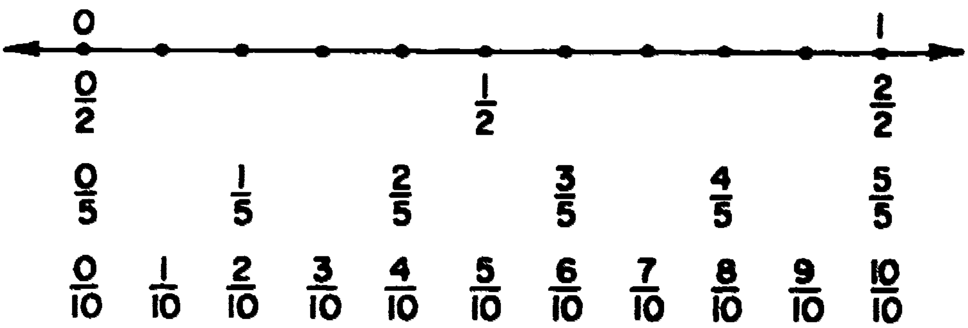
a. $\frac{1}{6}$ = ______

b. $\frac{10}{12}$ = ______

c. $\frac{1}{3}$ = ______ = ______

d. $\frac{4}{6}$ = ______ = ______

e. 1 = ______ = ______ = ______



3. Use the number line above to help you write the missing numerator or denominator.

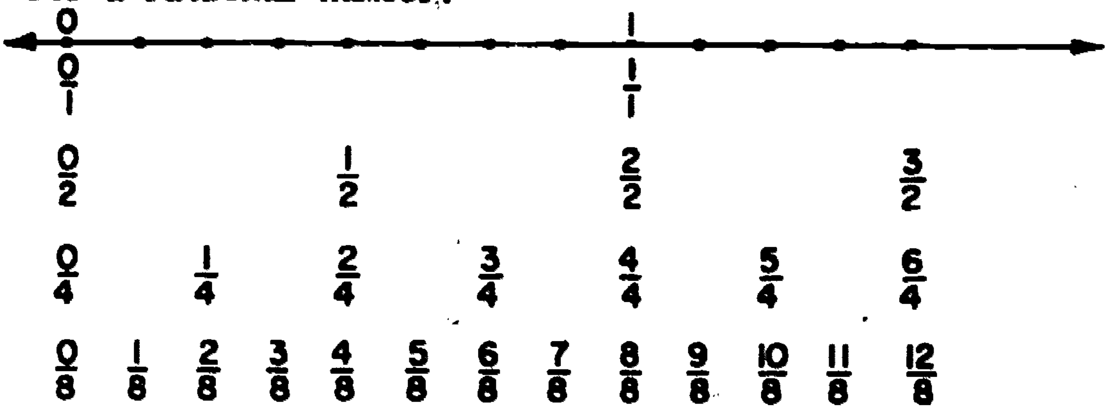
a. $\frac{1}{2} = \frac{n}{10}$

b. $\frac{2}{10} = \frac{1}{n}$

c. $\frac{4}{5} = \frac{n}{10}$

d. $1 = \frac{10}{n} = \frac{8}{5}$

4. Using one number line, we can show many different names for a rational number.



We see that some fractions are names for the same rational number.

What other fractions are names for the rational number $\frac{1}{2}$?

What other fraction is a name for the rational number $\frac{1}{4}$?

What other fraction is a name for the rational number $\frac{3}{4}$?

Can you find other fractions that name the same rational number on this line?

One rational number may be named by many fractions.

The rational number $\frac{1}{4}$ may be named by: $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{6}{24}, \ldots\}$

The rational number $\frac{2}{3}$ may be named by: $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

The rational number $\frac{2}{5}$ may be named by: $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$

The rational number $\frac{1}{10}$ may be named by: $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \ldots\}$

Can you think of other fractions which would name each of these numbers above?

Many fractions can be used to name the same whole numbers.

For example, 1 may be indicated by

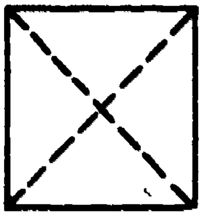
$\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6},$ and so on.

Can you name three other fractions that belong to this set?

Exercise Set 5

Copy each of these figures.

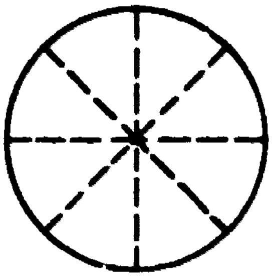
1.



Color $\frac{2}{4}$ of this figure.

$\frac{2}{4}$ is another name for ___.

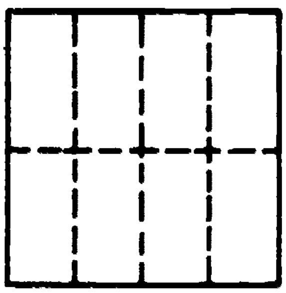
2.



Color $\frac{6}{8}$ of this figure.

$\frac{6}{8}$ is another name for ___.

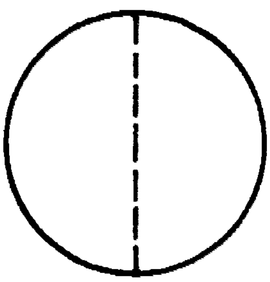
3.



Color $\frac{2}{8}$ of this figure.

$\frac{2}{8}$ is another name for ___.

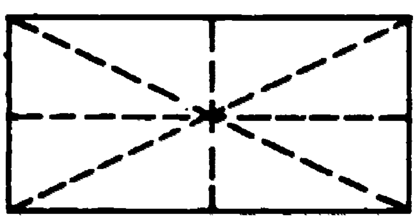
4.



Color $\frac{2}{2}$ of this figure.

$\frac{2}{2}$ is another name for ___.

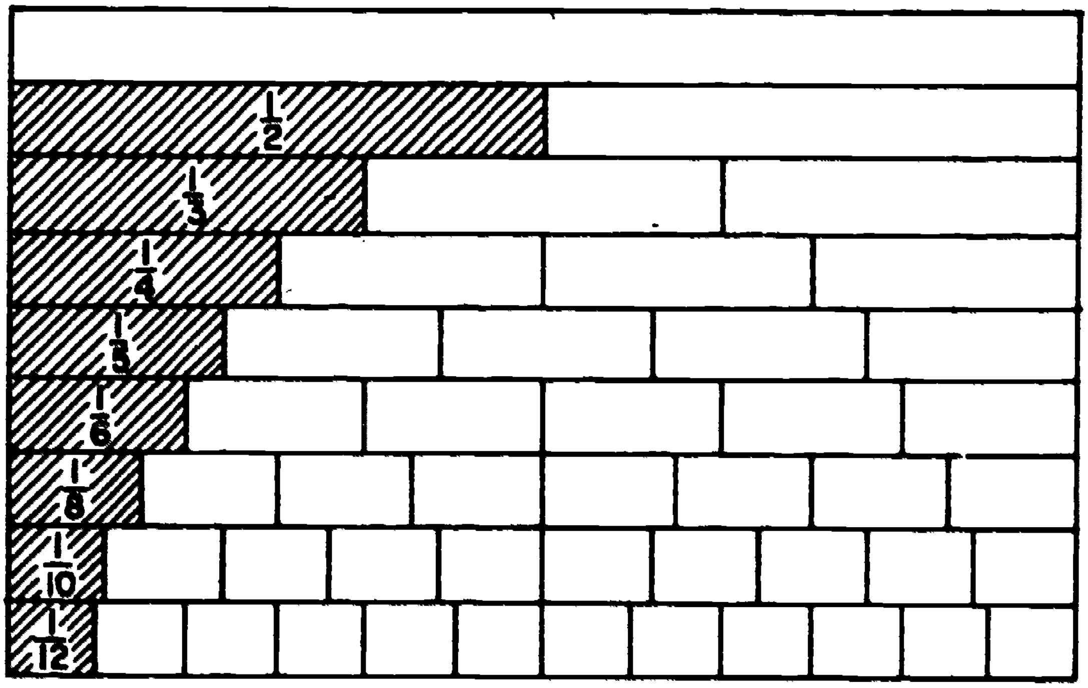
5.



Color $\frac{4}{8}$ of this figure.

$\frac{4}{8}$ is another name for ___.

6. Using this chart, write as many names as you can for



a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{4}$

d) $\frac{1}{5}$

e) $\frac{4}{6}$

f) $\frac{6}{8}$

7. Write at least three other fractions which name each of the following rational numbers. If you can write more than three, do so.

a) $\frac{1}{4}$

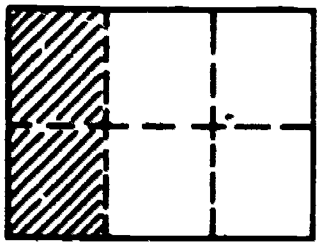
b) $\frac{2}{5}$

c) $\frac{5}{2}$

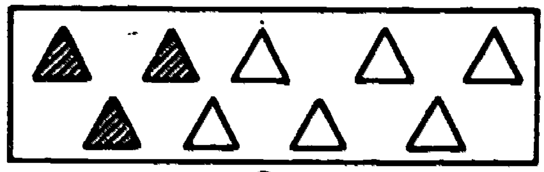
d) $\frac{2}{3}$

e) $\frac{3}{4}$

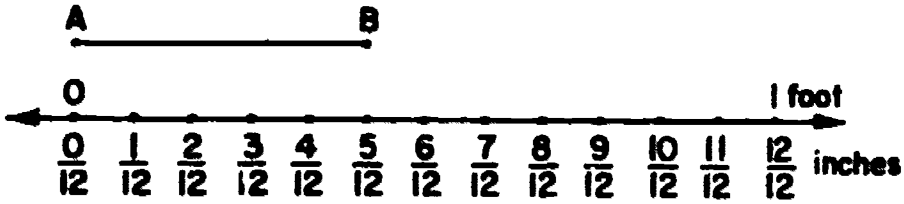
8. The diagrams below suggest three other names for $\frac{1}{5}$. What are they?



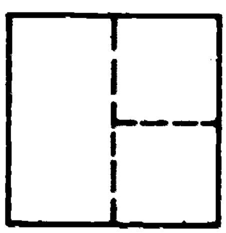
A



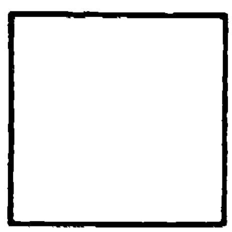
B



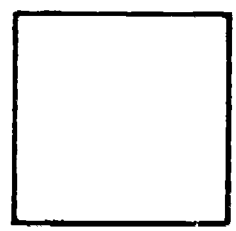
9. Draw 5 boxes like the ones below. Separate each box to show the mathematical sentence written below. The first one is done for you.



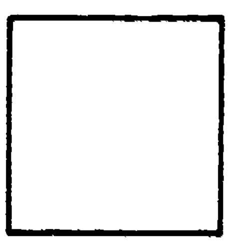
$\frac{1}{2} = \frac{2}{4}$



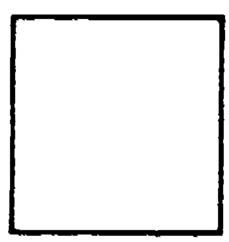
$\frac{4}{8} = \frac{1}{2}$



$\frac{1}{4} = \frac{2}{8}$



$\frac{6}{8} = \frac{3}{4}$



$\frac{2}{6} = \frac{1}{3}$

10. Complete:

a) $1 = \frac{\quad}{2}$

b) $\frac{2}{6} = \frac{\quad}{3}$

c) $\frac{1}{2} = \frac{\quad}{4}$

d) $\frac{1}{4} = \frac{\quad}{8}$

e) $\frac{4}{8} = \frac{\quad}{2}$

f) $\frac{8}{8} = \frac{\quad}{4}$

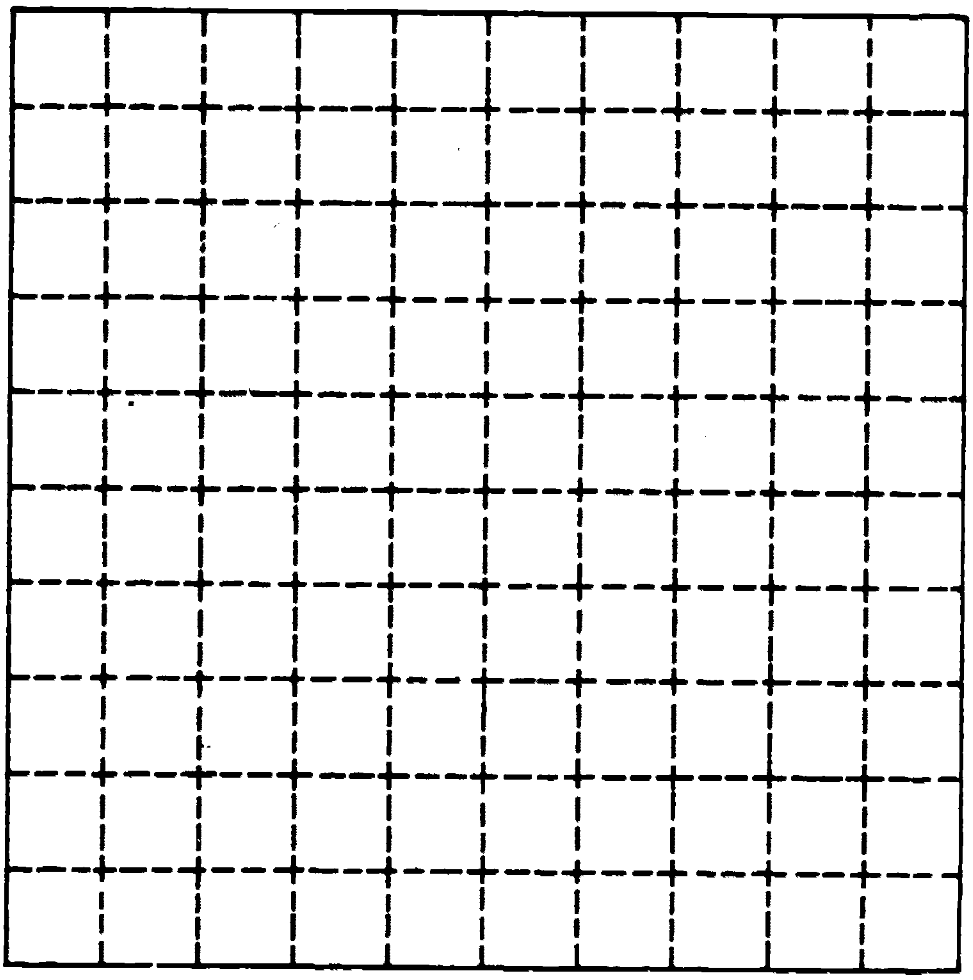
g) $\frac{1}{3} = \frac{\quad}{9}$

h) $\frac{2}{4} = \frac{\quad}{8}$

i) $\frac{6}{8} = \frac{\quad}{4}$

j) $\frac{1}{3} = \frac{\quad}{12}$

11.



The unit square shown on the preceding page has been separated into 100 congruent square regions.

a) Each small square region is what part of the unit square region?

b) Each small square region is what part of 1 row or 1 column of square regions?

c) Each row or each column of square regions is what part of the unit square region?

d) $\frac{4}{10} = \frac{?}{100}$

e) $\frac{7}{10} = \frac{?}{100}$

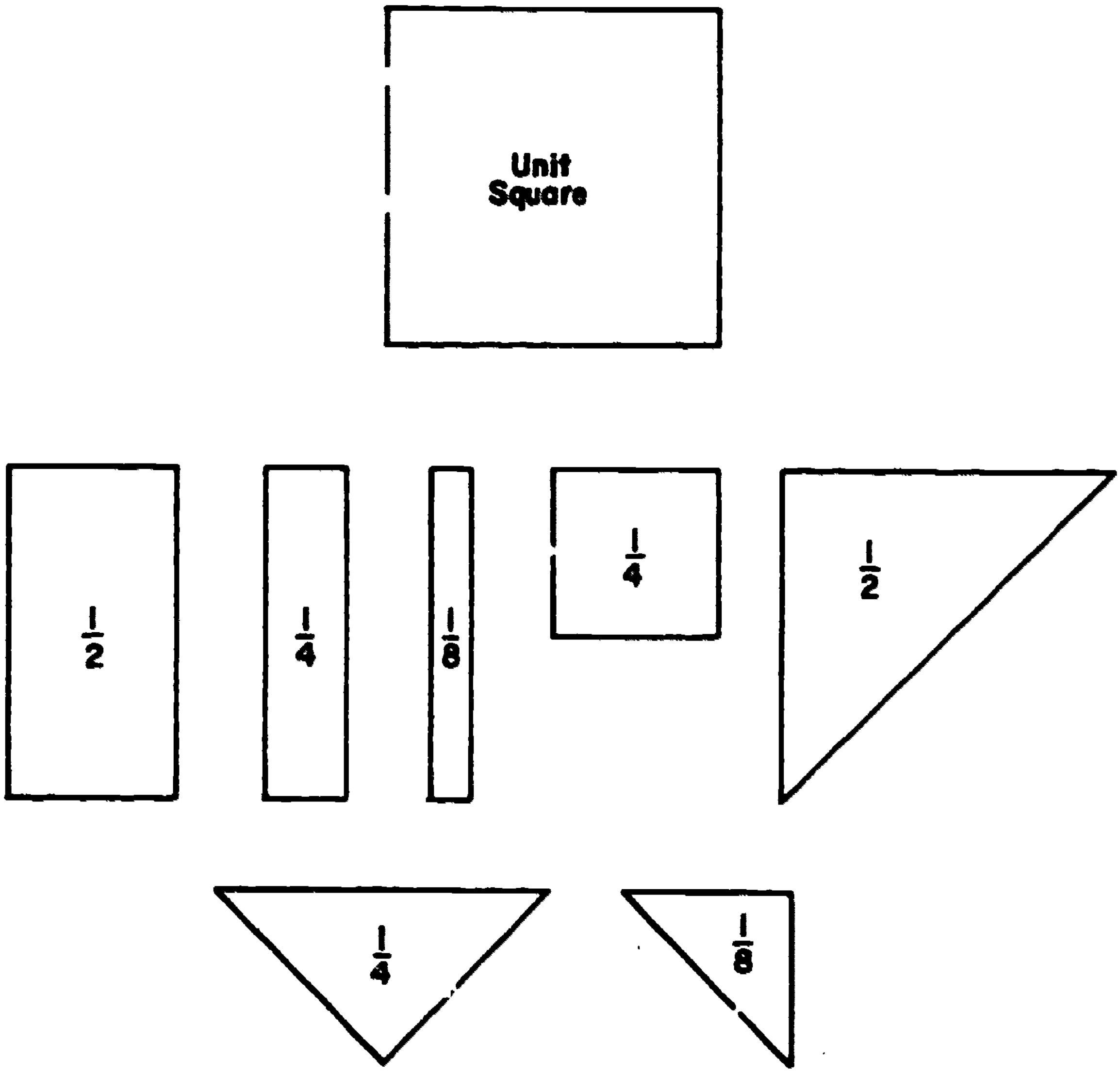
f) $\frac{90}{100} = \frac{?}{10}$

g) $\frac{30}{100} = \frac{?}{10}$

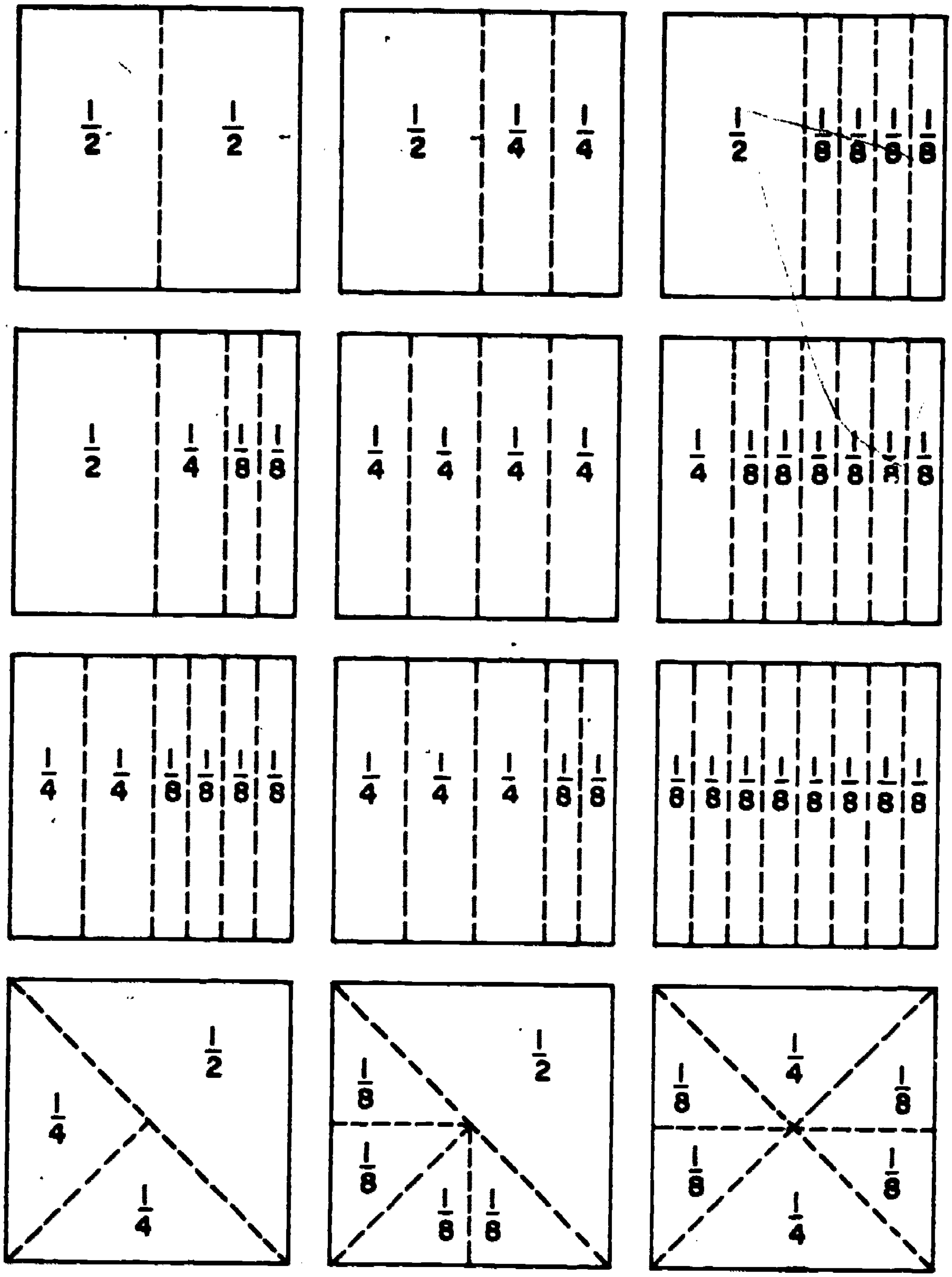
h) How many small square regions should you color if you are to color $\frac{47}{100}$ of the unit square region?

$\frac{83}{100}$? $\frac{100}{100}$? $\frac{1}{10}$? $\frac{7}{10}$? and $\frac{\text{[illegible]}}{10}$?

Puzzle. In how many different ways can you cover the unit square using the fractional pieces shown? Each piece may be used more than once. You may wish to trace, cut out, and make several copies of each model region before you work your puzzle.

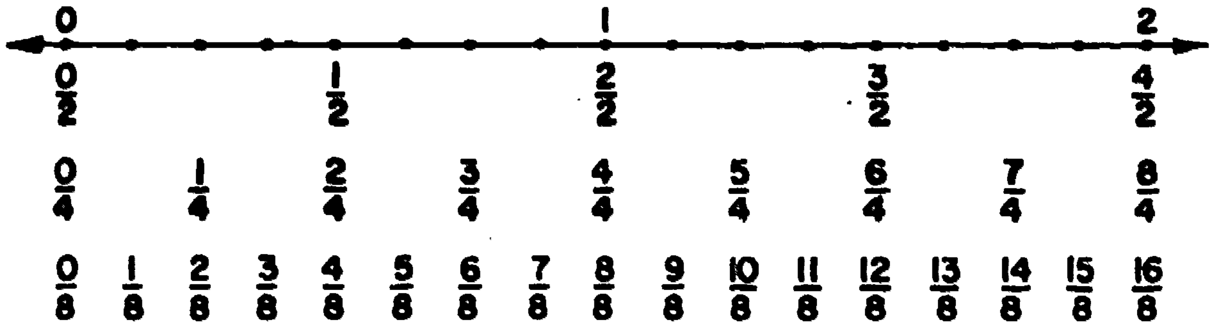


Here are a few solutions to the puzzle on page 541. How many more did you discover?



ORDERING THE RATIONAL NUMBERS

Exploration



Look at the number line.

Is $\frac{1}{2}$ to the right of $\frac{1}{4}$? Is $\frac{1}{2} > \frac{1}{4}$?

Is $\frac{3}{4}$ to the right of $\frac{3}{8}$? Is $\frac{3}{4} > \frac{3}{8}$?

Is $\frac{5}{4}$ to the right of $\frac{4}{2}$? Is $\frac{5}{4} > \frac{4}{2}$?

Is 0 to the left of $\frac{1}{4}$? Is $0 < \frac{1}{4}$?

Is $\frac{2}{4}$ to the left of $\frac{4}{2}$? Is $\frac{2}{4} < \frac{4}{2}$?

Is $\frac{5}{8}$ to the left of $\frac{6}{8}$? Is $\frac{5}{8} < \frac{6}{8}$?

It is easy to see that $\frac{1}{4}$, $\frac{3}{4}$, $\frac{6}{4}$, and $\frac{8}{4}$ are ordered from least to greatest.

Are $\frac{1}{2}$, $\frac{5}{8}$, $\frac{5}{4}$, and $\frac{4}{2}$ ordered from the least to greatest?

It would be easier to decide if we used other fractions for these numbers.

Using other names for these same numbers, we can write them as $\frac{4}{8}$, $\frac{5}{8}$, $\frac{10}{8}$, and $\frac{16}{8}$.

Now we see the numbers are named in order from least to greatest.

As you move to the right along a number line, the rational numbers become greater. As you move to the left, they become less.

Exercise Set 6

1. Use this chart and the symbols $>$ and $<$ to complete the senten-es below.

<table>
<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{1}{2}$</td></tr>
<tr><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td></tr>
<tr><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td></tr>
<tr><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td></tr>
</table>

a) $\frac{1}{2}$ ____ $\frac{1}{4}$ c) $\frac{3}{8}$ ____ $\frac{1}{4}$ e) $\frac{3}{4}$ ____ $\frac{5}{8}$

b) $\frac{1}{8}$ ____ $\frac{1}{4}$ d) $\frac{1}{2}$ ____ $\frac{3}{8}$ f) $\frac{3}{8}$ ____ $\frac{2}{4}$

2. Write the correct answer. The fraction chart above may be used, if needed.

a) Which number is less: $\frac{17}{8}$ or $\frac{16}{8}$?

Which is farther to the left on the number line?

b) Which number is less: $\frac{10}{8}$ or $\frac{12}{8}$?

Which is farther to the left on the number line?

c) Which number is less: $\frac{17}{8}$ or $\frac{15}{8}$?

Which is farther to the left on the number line?

d) Which number is less: $\frac{11}{4}$ or $\frac{4}{2}$?

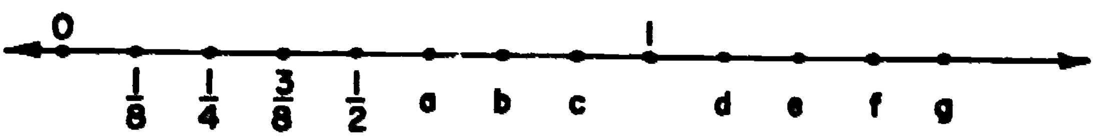
Which is farther to the left on the number line?

3. Arrange members of each set in order from léast to greatest. Make diagrams if you need them.

$$A = \{ \tfrac{7}{2}, \tfrac{3}{2}, \tfrac{11}{2}, \tfrac{13}{2}, \tfrac{5}{2} \}$$

$$B = \{ \tfrac{7}{4}, \tfrac{3}{4}, 2, \tfrac{9}{4}, \tfrac{11}{4} \}$$

4. Associate a rational number with points a, b, c, d, e, f, and g in the diagram below.



a = c = e = g =

b = d = f =

5. List in order the numbers used in counting by two-thirds from $\frac{2}{3}$ to 4.

6. List in order the numbers used in counting by three-halves from $\frac{3}{2}$ to 9.

7. Write two other names for each of the following numbers.

a) $\frac{12}{8}$ c) $\frac{10}{4}$

b) $\frac{5}{2}$ d) 3

8. Copy and complete by writing the symbol $>$ or $<$ in each box.

a) $\frac{1}{4}$ ___ $\frac{1}{2}$

b) $\frac{1}{2}$ ___ $\frac{1}{8}$

c) $\frac{1}{10}$ ___ 1

d) 1 ___ $\frac{1}{2}$

e) $\frac{1}{4}$ ___ $\frac{1}{8}$

f) $\frac{1}{6}$ ___ $\frac{1}{3}$

9. Rearrange these numbers in order from least to greatest.

a) $\frac{5}{8}, \frac{3}{8}, \frac{3}{4}, \frac{1}{8}, \frac{1}{4}$

b) $\frac{1}{3}, \frac{1}{5}, \frac{1}{2}, \frac{1}{6}, \frac{1}{4}$

c) $\frac{2}{3}, \frac{5}{6}, \frac{1}{6}, \frac{1}{3}$

d) $\frac{3}{8}, \frac{1}{2}, \frac{3}{4}, \frac{5}{5}$

10. Arrange in order the numbers in each set below. Begin with the greatest.

$A = \left\{\frac{2}{4}, \frac{3}{4}, \frac{1}{4}\right\}$

$B = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{8}\right\}$

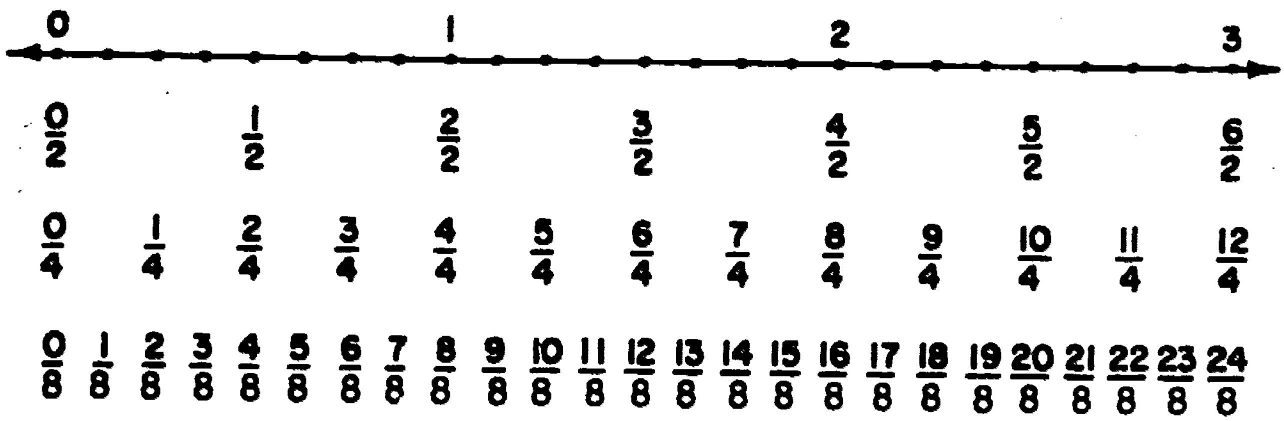
$C = \left\{\frac{1}{2}, \frac{3}{8}, \frac{3}{4}\right\}$

11. Arrange these numbers from least to greatest.

$\frac{1}{2}, \frac{1}{10}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{1}{6}.$

Exercise Set 7

Supplementary Exercises



1. Copy and write $>$, $<$, or $=$ in each blank to make a true sentence. The number line above will help you.

a) $\frac{5}{4}$ ____ $\frac{1}{2}$ e) $\frac{2}{2}$ ____ 1

b) $\frac{8}{4}$ ____ 2 f) $\frac{3}{4}$ ____ $\frac{5}{8}$

c) 3 ____ $\frac{6}{2}$ g) $\frac{7}{4}$ ____ $\frac{11}{8}$

d) $\frac{3}{2}$ ____ $\frac{3}{4}$ h) $\frac{18}{8}$ ____ $\frac{8}{4}$

2. Which fraction of each pair below will be farther to the right on the number line?

a) $\frac{19}{4}$ or $\frac{17}{8}$

b) $\frac{11}{5}$ or $\frac{5}{4}$

c) $\frac{5}{2}$ or $\frac{18}{8}$

d) $\frac{10}{2}$ or $\frac{5}{2}$

e) $\frac{14}{8}$ or $\frac{0}{4}$

f) $\frac{11}{4}$ or $\frac{4}{2}$

g) $\frac{5}{4}$ or $\frac{3}{2}$

h) $\frac{1}{2}$ or $\frac{1}{4}$

3. Rearrange each set. Put members in order from least to greatest.

$A = \left\{\frac{7}{2}, \frac{3}{2}, \frac{11}{2}, \frac{5}{2}\right\}$

$B = \left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{3}, \frac{1}{8}\right\}$

4. Copy and fill in each blank with the symbol $>$, $<$, or $=$.

a) $\frac{3}{3}$ ____ $\frac{1}{3}$

b) 2 ____ $\frac{4}{3}$

c) $\frac{4}{4}$ ____ 1

d) $\frac{4}{5}$ ____ $\frac{2}{5}$

e) 1 ____ 2

f) $\frac{3}{3}$ ____ $\frac{2}{2}$

g) 2 ____ $\frac{7}{4}$

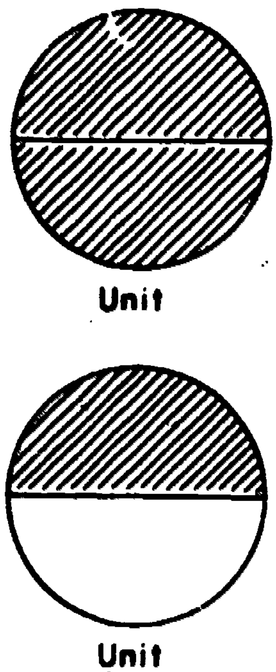
h) $\frac{1}{2}$ ____ 1

5. Look at exercise 4. Which fraction in each pair labels a point farther to the right on the number line?

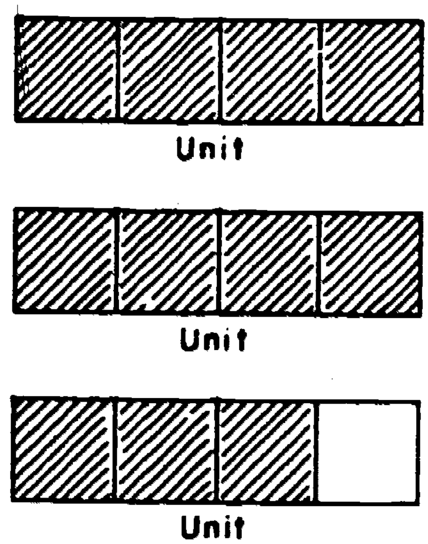
A NEW KIND OF NAME

These pictures help us think about the numbers, $\frac{3}{2}$ and $\frac{11}{4}$.

A.



B.



$\frac{3}{2} = \frac{2}{2}$ and $\frac{1}{2}$

$\frac{3}{2} = 1$ one and 1 half

or,

$\frac{3}{2} = 1\frac{1}{2}$

$\frac{11}{4} = \frac{4}{4}$ and $\frac{4}{4}$ and $\frac{3}{4}$

$\frac{11}{4} = 1$ one and 1 one and 3 fourths

$\frac{11}{4} =$ 2 ones and 3 fourths

or,

$\frac{11}{4} = 2\frac{3}{4}$

Another way of naming $\frac{3}{2}$ is $1\frac{1}{2}$

Another way of naming $\frac{11}{4}$ is $2\frac{3}{4}$

We call $1\frac{1}{2}$ and $2\frac{3}{4}$ mixed forms.

Rational numbers named by fractions like $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{7}{8}$ tell us that the measure of a region, segment, or set is <u>less than</u> 1.

Rational numbers named by fractions like $\frac{2}{2}$, $\frac{8}{8}$, $\frac{4}{4}$, and $\frac{3}{3}$ tell us that the measure of a region, segment, or set is <u>equal</u> to 1.

Rational numbers named by fractions like $\frac{7}{4}$, $\frac{3}{2}$, and $\frac{5}{3}$ tell us that the measure of a region, segment, or set is <u>greater than</u> 1.

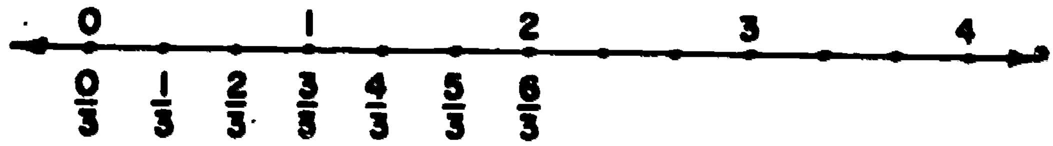
Other names for 1 are $\frac{4}{4}$, $\frac{2}{2}$, and $\frac{3}{3}$.

Since this is true, $\frac{7}{4}$, $\frac{3}{2}$, and $\frac{5}{3}$ may be renamed $1\frac{3}{4}$ $1\frac{1}{2}$, and $1\frac{2}{3}$.

$1\frac{3}{4}$, $1\frac{1}{2}$, and $1\frac{2}{3}$ are read, "one and three-fourths," "one and one-half," and "one and two-thirds." Fractions written in this way are said to be in <u>mixed form</u>.

Exercise Set 8

1. Copy and finish the number line below. Then use it to complete the mathematical sentences so that each will be a true sentence.



a) $\frac{4}{3} = \frac{\_\_}{3}$ and $\frac{1}{3}$

$\frac{4}{3} = \_\_\_\_\_\_$

b) $\frac{6}{3}$ is $\frac{\_\_}{3}$ and $\frac{3}{3}$

$\frac{6}{3} = \_\_\_$

c) $\frac{11}{3} = 3$ ones and $\frac{\_\_}{3}$

$\frac{11}{3} = 3\frac{\_\_}{3}$

d) $2\frac{2}{3} = 2$ and $\frac{2}{3}$

$2\frac{2}{3} = \frac{6}{3}$ and $\frac{2}{3}$

$2\frac{2}{3} = \frac{\_\_}{3}$

e) $4 = \frac{\_\_}{3}$

f) $3\frac{1}{3} = \frac{9}{3}$ and $\frac{\_\_}{3}$

$3\frac{1}{3} = \_\_\_\_\_\_$

2. Arrange the numbers in each of the following sets in order from least to greatest. Use diagrams if you need them.

$A = \{\frac{3}{4}, 0, \frac{7}{4}, 2, \frac{9}{4}, 1\}$

$B = \{\frac{5}{3}, 1, \frac{10}{3}, 4, \frac{7}{3}, 2, \frac{2}{3}, 3\}$

3. Peter has 13 blocks to walk to school. Each block is $\frac{1}{10}$ mile long. How many miles does he have to walk to school?

4. A pound of butter is usually divided into four bars of the same size. Vicky found 7 bars of butter in her refrigerator. How many pounds of butter were in the refrigerator?

Can you do these without any help? Try some of them.

5. Write the mixed form for each of these numbers.

a) $\frac{5}{4} =$	d) $\frac{9}{2} =$
b) $\frac{6}{2} =$	e) $\frac{12}{5} =$
c) $\frac{8}{5} =$	f) $\frac{7}{3} =$

6. Which is greater? Write the name of the greater number in each pair. You may use a number line to help you decide.

a) $\frac{5}{3}$ or $1\frac{1}{2}$	e) $\frac{8}{7}$ or $1\frac{1}{8}$
b) $2\frac{3}{4}$ or $\frac{10}{4}$	f) $3\frac{1}{6}$ or $\frac{5}{2}$
c) $\frac{12}{3}$ or $3\frac{7}{8}$	g) $4\frac{2}{3}$ or $\frac{31}{10}$
d) 6 or $\frac{21}{3}$	h) $1\frac{3}{4}$ or $\frac{8}{4}$

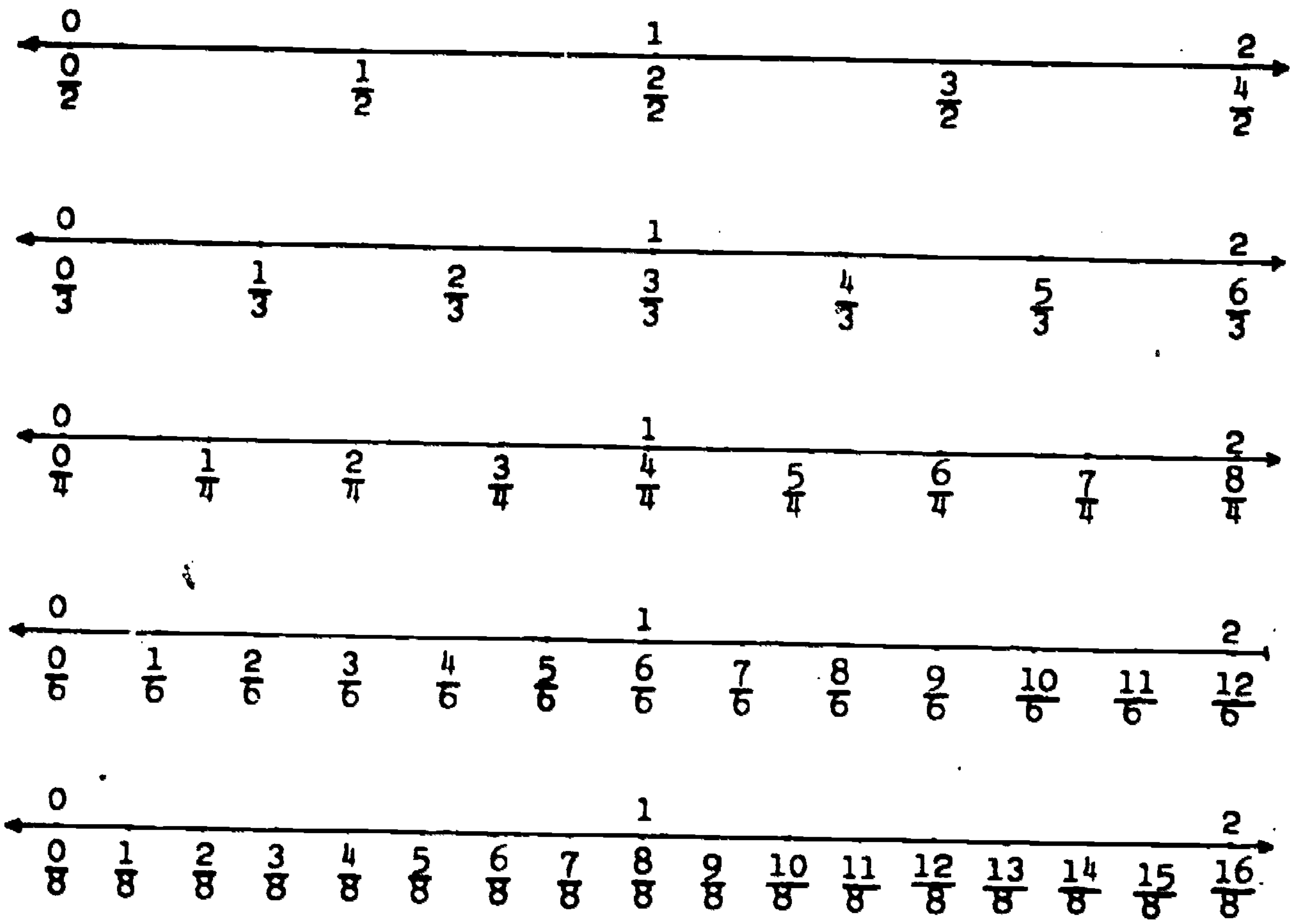
7. Copy and complete. Use diagrams if you need them.

a) $1\frac{2}{5} = \frac{\quad}{5}$	d) $3\frac{1}{2} = \frac{\quad}{2}$
b) $2\frac{1}{3} = \frac{\quad}{3}$	e) $1\frac{5}{6} = \frac{11}{\quad}$
c) $2\frac{3}{4} = \frac{\quad}{4}$	f) $2\frac{3}{8} = \frac{19}{\quad}$

8. Between what two whole numbers on the number line would the following fractions be?

a) $5\frac{1}{3}$ b) $2\frac{3}{4}$ c) $7\frac{1}{2}$ d) $6\frac{9}{10}$

Exercise Set 9



1. Use the number line above. Copy the following mathematical sentences. Write the symbol > or < in each blank to make the sentence true.

a) $\frac{10}{8}$ ____ $\frac{12}{8}$	e) $\frac{3}{6}$ ____ $\frac{9}{8}$
b) 2 ____ $\frac{3}{4}$	f) $\frac{8}{8}$ ____ $\frac{4}{3}$
c) $\frac{5}{6}$ ____ 1	g) $\frac{17}{4}$ ____ $\frac{11}{8}$
d) $\frac{3}{2}$ ____ $\frac{5}{4}$	h) $\frac{4}{3}$ ____ $\frac{7}{6}$

2. Starting at zero, list in order the numbers used in

a) counting by one-half to $\frac{4}{2}$

b) counting by two-thirds to $\frac{6}{3}$

c) counting by three eighths to $\frac{15}{8}$

3. Write 2 other names for each of the following.

a) $\frac{1}{2}$ = ____, ____

b) 1 = ____, ____

c) $1\frac{1}{4}$ = ____, ____

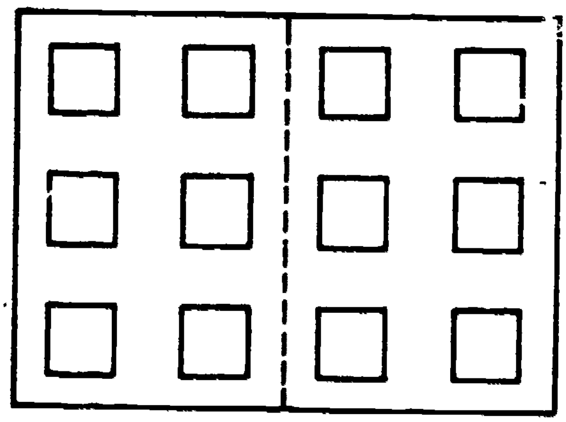
d) $2\frac{1}{2}$ = ____, ____

4. Match each rational number in Column 1 with a fraction that names the same number from Column 2.

	Column 1		Column 2
a)	$\frac{1}{2}$	f)	$\frac{8}{4}$
b)	$\frac{1}{4}$	g)	$\frac{6}{4}$
c)	2	h)	$\frac{2}{4}$
d)	$1\frac{1}{2}$	i)	$\frac{2}{2}$
e)	1	j)	$\frac{2}{8}$

Exploration

Below are pictures of sets of 12 objects.



Set A

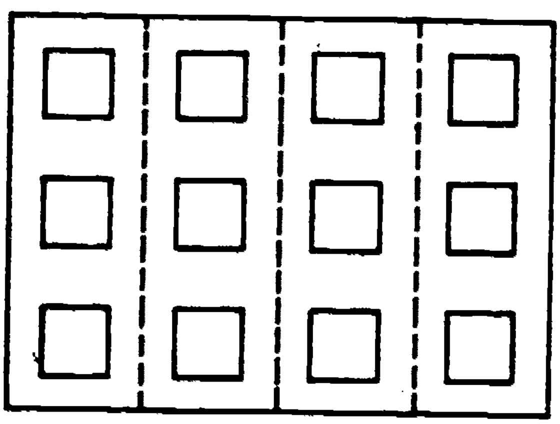
Dotted lines separate the picture of Set A into 2 subsets.

How many objects are there in 1 subset?

How many objects are there in 2 subsets?

Is $\frac{1}{2}$ of 12 objects equal to 6 objects?

Is $\frac{2}{2}$ of 12 objects equal to 12 objects?



Set B

Set B has been separated into 4 subsets.

How many objects are in each subset?

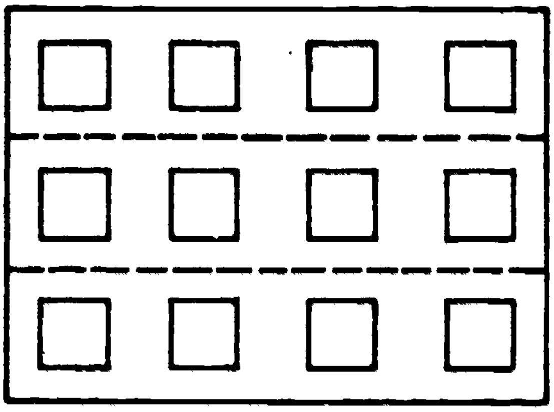
$\frac{1}{4}$ of 12 = ______

$\frac{2}{4}$ of 12 = ______

$\frac{3}{4}$ of 12 = ______

$\frac{4}{4}$ of 12 = ______

Is $\frac{1}{2}$ of 12 = $\frac{2}{4}$ of 12?



Set C

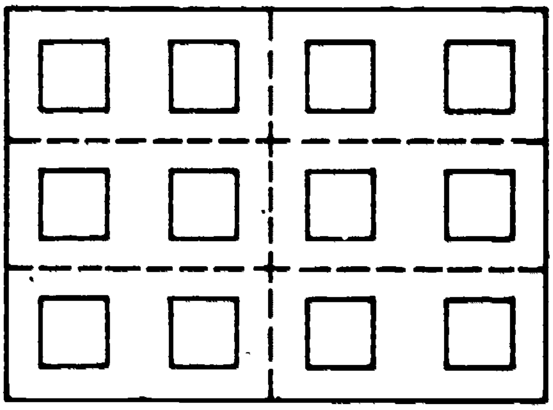
Dotted lines separate Set C into

______ subsets.

What is $\frac{1}{3}$ of 12?

What is $\frac{2}{3}$ of 12?

What is $\frac{3}{3}$ of 12?



Set D

Set D has been separated into

______ subsets.

$2 = \frac{\ \ }{6}$ of 12.

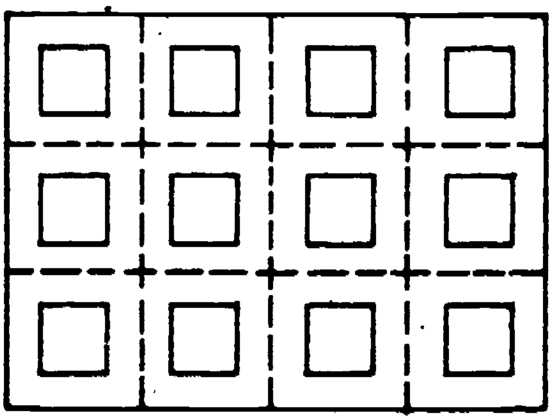
$4 = \frac{\ \ }{6}$ of 12.

$6 = \frac{\ \ }{6}$ of 12.

$8 = \frac{\ \ }{6}$ of 12.

$10 = \frac{\ \ }{6}$ of 12.

$12 = \frac{\ \ }{6}$ of 12.



Set E

Each subset in E shows

______ of 12.

$\frac{3}{12}$ of 12 = _____

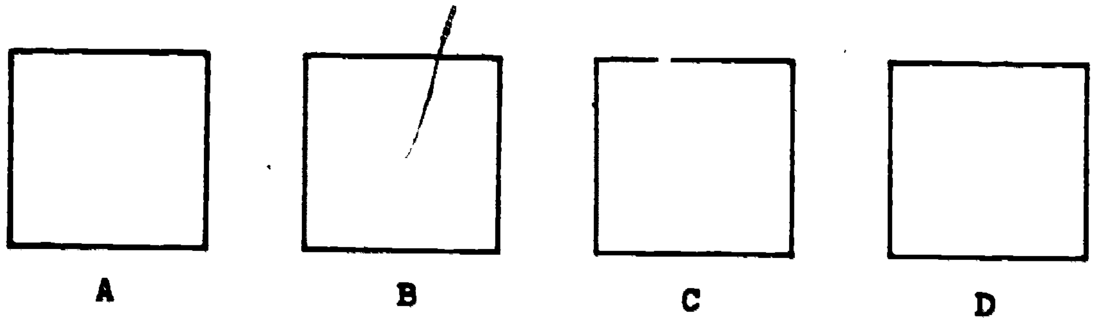
$\frac{4}{12}$ of 12 = _____

$\frac{6}{12}$ of 12 = _____

$\frac{8}{12}$ of 12 = _____

$\frac{9}{12}$ of 12 = _____

Exercise Set 10



1. A, B, C, and D are unit square regions. Copy them on your paper. Separate each one into four equal regions.

   a) Color $\frac{1}{4}$ of A red.

   b) Color $\frac{2}{4}$ of B blue.

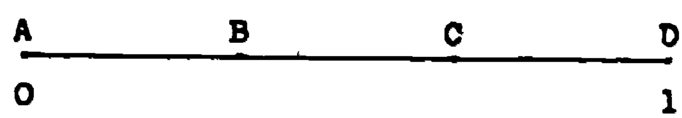
   c) Color $\frac{3}{4}$ of C green.

   d) Color $\frac{4}{4}$ of D green.

   e) $\frac{4}{4}$ is another name for ______.

   f) Write the fraction that best describes the uncolored regions of each unit square region above.

2.



Points B and C separate the unit line segment AD into 3 congruent segments.

a) m $\overline{AB}$ =

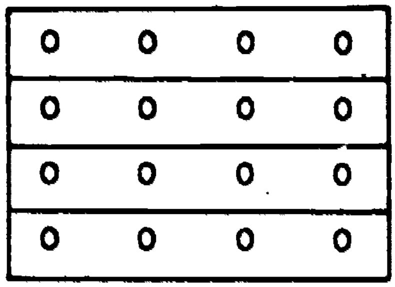
b) m $\overline{AC}$ =

c) m $\overline{AD}$ =

d) $\frac{3}{3}$ = ______

Exercise Set 11

1. Look at the picture of a set of objects below.



It has been partitioned into 4 subsets.

The same number of objects are in each subset.

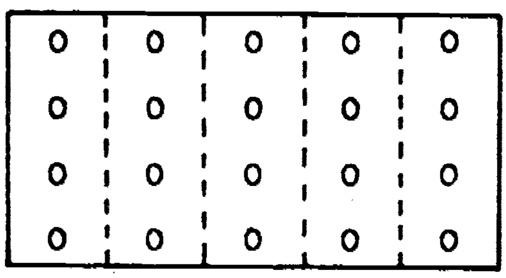
What is $\frac{1}{4}$ of 16?

What is $\frac{2}{4}$ of 16?

What is $\frac{3}{4}$ of 16?

What is $\frac{4}{4}$ of 16?

2. Here is another picture of a set of objects. It has been partitioned into five subsets. The same number of objects are in each subset.



What is $\frac{1}{5}$ of 20?

What is $\frac{2}{5}$ of 20?

What is $\frac{3}{5}$ of 20?

What is $\frac{4}{5}$ of 20?

What is $\frac{5}{5}$ of 20?

3. Complete the following. Use sets of objects if you need them.

a) $\frac{1}{3}$ of 6 is ______	e) $\frac{2}{3}$ of 6 is ______
b) $\frac{1}{2}$ of 4 is ______	f) $\frac{3}{4}$ of 8 is ______
c) $\frac{1}{4}$ of 8 is ______	g) $\frac{2}{3}$ of 9 is ______
d) $\frac{1}{2}$ of 10 is ______	h) $\frac{2}{5}$ of 10 is ______

4. Jane bought six doughnuts. She ate $\frac{1}{3}$ of them. How many doughnuts did Jane eat? How many doughnuts did Jane have left?

5. Bill had twenty marbles. He lost $\frac{1}{4}$ of them. How many marbles did Bill lose? How many did he have left?

6. Alice had 36 jacks. She traded $\frac{1}{4}$ of them to Mary. How many jacks did Alice trade? How many jacks did Alice have left?

7. On the way from the store, Bob dropped a dozen eggs. He looked inside the carton. He found $\frac{3}{4}$ of the eggs broken. How many eggs are there in a dozen? How many eggs were broken? How many eggs were not broken?

BRAINTWISTER

John gave Bill sixteen jelly beans. This was $\frac{1}{2}$ of the number John had. How many did John have at the beginning?

Exercise Set 12

1. There were 20 problems on an arithmetic test. John worked all but $\frac{1}{4}$ of them. How many problems did John finish?

2. $\frac{1}{3}$ of a string of 12 Christmas tree lights had burned out. How many lights had to be replaced?

3. At a sale, books that had been 50¢ were selling for $\frac{1}{2}$ of the regular price. What was the sale price?

4. A box which had contained 24 candy bars was two-thirds full. How many candy bars were in the box?

5. A football game is played in 4 quarters. It takes 1 hour of actual playing time to play a game. How many minutes of actual playing time are gone at the end of the third quarter?

6. There were 6 boys and 3 girls on a softball team. What part of the team were boys?

7. The year is separated into four seasons of equal length. What part of the year is each season?

8. Mary has a collection of 15 dolls. $\frac{2}{3}$ of them represent children from other countries. How many of the dolls represent children from other countries?

9. Jim was making a model of a plane. He needed a single piece of wood $\frac{3}{4}$ of a foot long. He had a piece of wood 8 inches long. Could he use this piece? Why?