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ABSTRACT

This SMSG study guide is intended to provide teachers who use "Intermediate Mathematics," as a textbook with references to materials which will help them to gain a better understanding of the mathematics contained in the text. For each chapter of the text a brief resume of its content is followed by a list of annotated references which are classified as elementary or advanced. Chapter topics include: (1) number systems: (2) coordinate geometry: (3) functions: (4) complex number systems: (5) equations in two variables: (6) systems of equations: (7) logarithms: (8) trigonometry: (9) vectors: (10) polar form: (11) sequences: (12) permutations: and (13) algebraic structures. (MP)

SCHOOL
MATHEMATICS
STUDY GROUP

INTERMEDIATE MATHEMATICS STUDY GUIDE

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INTERMEDIATE MATHEMATICS

This study guide is intended to provide teachers who use *Intermediate Mathem..tics* as a text book with references to materials which will help them to gain a better understanding of the mathematics contained in the text. These references may also be useful in collecting library materials and preparing material for in-service and self study programs.

The mathematics presented in *Intermediate Mathematics* is significant mathematics, both for its own sake and as a preparation for collegiate mathematics, and the expositions are pedagogically sound. The experience of many teachers who have used the text has shown that the material in it is teachable to high school students. The extent to which the text is effective in stimulating students' interest in the study of mathematics depends, in large measure, on how thoroughly the teacher understands the mathematics in the text.

Some of the features of the text which serve to emphasize the importance of a genuinely strong background in mathematics on the part of the teacher are given below.

(1) The text presents many mathematical ideas which require somewhat more sophisticated expositions than those found in traditional texts.

(2) The text puts considerable emphasis on proofs.

(3) The authors of the text were not content merely to give rules but sought to provide a logical basis for understanding rules. In order to do this, they often had to forego the luxury of a short and easy presentation in favor of a careful, methodical, and lengthier development.

It must not be assumed that the use of the guide will afford the teacher the same kind of opportunity for formal study as that which is afforded by summer, academic year, or in-service institutes, or other formal programs. It is hoped, however, that this guide will serve as an effective supplement to more formal study.

1. READING MATHEMATICS

The student of mathematics must accept the fact that the technique for reading mathematics differs from that for reading a newspaper or novel. The compactness of mathematical symbolism makes special demands on the reader. In reading a newspaper or novel, one can often skim superficially over sentences or even paragraphs without losing the thread of the story; in reading mathematics, such a procedure may result in a complete loss of understanding.

In mathematics one must read slowly and carefully. After a first reading to get the general idea, and a second reading to come to grips with details, still further reading may be necessary to gain mastery.

To read mathematics successfully requires paper and pencil. Calculations and arguments in the text should be checked by the reader to ensure complete understanding.

2. ORGANIZATION OF THE STUDY GUIDE

The main purpose of this study guide is to list and organize suitable references for study. The organization of topics is by chapter headings. First there is a brief resume of the content of the chapter of the text *Intermediate Mathematics.* Then follows a list of annotated references which are classified as elementary and advanced.

(a) An *elementary* reference is one in which the level of the exposition is comparable to that in the text *Intermediate Mathematics.* Topics covered do not correspond completely in all cases.

(b) An *advanced* reference is one in which concepts are treated at a more advanced level than in *Intermediate Mathematics.* In some cases fewer topics are developed in the advanced reference but they are developed more rigorously and with greater sophistication. Others provide significant variations and extensions of the topics treated in *Intermediate Mathematics.*

Although these lists of references were obtained by a close examination of many books, the lists are not exhaustive and suggestions for appropriate additions are welcome. At the end of the study guide there is a complete bibliography for the entire guide. This bibliography is arranged alphabetically by authors.

Chapter 1 — NUMBER SYSTEMS

Four important systems of numbers are considered in this chapter; the natural numbers, the integers, the rational numbers, and the real numbers. Strong emphasis is placed on the algebraic structure of these number systems, with stress on the "basic properties" of equality, addition, multiplication, and order relations. These properties are identified as being "basic" because other algebraic properties of each system are then deduced from them.

The structural properties common to all of these systems are emphasized by parallel treatments of them — rather than by a sequential treatment in which each system is constructed from its predecessors.

The chapter ends with a review of radicals, factorization of polynomials, and the manipulation of rational expressions.

Elementary References

ADLER, **The New Mathematics**

A popularly written book with emphasis upon the group, ring, and field structure of the subsystems of the real number system.

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** (Revised), Ch. 1-2, pp. 1-86

A treatment of the real number system and its subsystems, including proofs of familiar algebraic laws as consequences of the basic properties of the systems.

COURANT AND ROBBINS, **What is Mathematics?** pp. 1-4, 52-72

A good survey of the real number system in which many of the details of the development are skipped and in which geometrical representation of numbers is employed. The treatment is in much the same spirit as the one in *Intermediate Mathematics.*

MARIA, **The Structure of Arithmetic and Algebra**

An axiomatic development of the real number system written particularly for the secondary school teacher or non-science student. The real number system is considered as an ordered field and there is a discussion of the various subsystems and a somewhat rambling treatment of peripheral topics.

Advanced References

BATES AND KIOKEMEISTER, **The Real Number System**

This book considers the real number system as a "complete ordered field" and provides detailed treatment of its subsystems similar to that of *Intermediate Mathematics*, but it carries the study much further.

HOFFMAN, **Basic Analysis,** Ch. 1, pp. 1-30

A discussion of the four number systems quite similar to the treatment in *Intermediate Mathematics.*

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, **Insights into Modern Mathematics, Twenty-third Yearbook,** Ch. 2, pp. 7-35

A brief treatment of real numbers based on the extension of the various subsystems from the natural numbers. It includes a discussion of mathematical induction, isomorphic systems, and integers as ordered pairs of natural numbers.

OLMSTED, **The Real Number System**

This book considers the real number system as a "complete ordered field" and provides detailed treatment of its subsystems similar to that of *Intermediate Mathematics*, but it carries the study much further.

ROBERTS, **The Real Numbers in an Algebraic Setting**

This reference provides sequential developments of the number systems, beginning with the natural numbers and proceeding to the construction of each of the others in a much more sophisticated treatment than that of *Intermediate Mathematics*.

THURSTON, **The Number System**

A detailed construction of the real number system from Peano's axioms. The first half of the book consists of explanatory comments bearing on and preliminary to the careful construction which follows in the second half.

WAISMANN, **Introduction to Mathematical Thinking**

A careful and thorough discussion of number systems, the history of their development, and of relevant logical and philosophical problems. It goes far beyond the treatment of Chapter 1 of *Intermediate Mathematics*.

Chapter 2 — COORDINATE GEOMETRY

The coordinate geometry in *Intermediate Mathematics* is essentially developed in three chapters. Chapter 2 is devoted to an introduction to coordinate geometry in the plane, Chapter 6 treats the straight line and the conic section, and Chapter 8 includes a limited introduction to coordinate geometry in three dimensions.

In Chapter 2 a coordinate system in the plane is essentially defined to be a "one-to-one correspondence between the set of all points in the plane and the set of all ordered pairs of real numbers." The formula for the distance between two points and the conditions for perpendicularity and parallelism of lines is developed.

Considerable attention is given to describing techniques of sketching graphs of equations and inequalities in two variables. Careful definitions are stated for the symmetry of two points with respect to a line, and the symmetry of two points with respect to a third point. A starred section is devoted to proofs of theorems from synthetic geometry using the methods of coordinate geometry. Finally, there is a section which concentrates on finding algebraic descriptions (equations or inequalities) of sets of points determined by specific geometric conditions.

Elementary References

FULLER, **Analytic Geometry**, Ch. 1, pp. 1-77

This reference treats about the same topics which appear in *Intermediate Mathematics*. The entire book may be considered a rather thorough reference for the analytic geometry that is prerequisite to a study of calculus.

KLINE, OSTERLE AND WILLSON, **Foundations of Advanced Mathematics,** Chs. 16-18, pp. 192-235, 248-254

A standard introductory treatment of the usual topics from coordinate geometry.

RICHARDSON, **Fundamentals of Mathematics** (Revised Edition), Ch. 9, pp. 224-264

A treatment of all of the major topics included in Chapter 2 of *Intermediate Mathematics*. Contained also are some interesting enrichment materials.

SMAIL, **Analytic Geometry and Calculus,** Chs. 1-3, pp. 1-69

This reference contains a standard introduction to rectangular coordinates in the plane, a development of the analytic geometry of the straight line, and a rather thorough treatment of equations of curves.

SMITH, GALE AND NEELEY, **New Analytic Geometry,** Chs. 2-3, pp. 6-45

This reference provides a standard exposition of cartesian coordinates and equations of curves. The development includes a discussion of all major topics treated in Chapter 2 of *Intermediate Mathematics*.

SCHOOL MATHEMATICS STUDY GROUP, **Supplementary and Enrichment Series, Plane Coordinate Geometry**

This reference contains an enlightening introduction to the topic of coordinate geomery. It contains developments of the notion of slope of a line, the distance and mid-point formulas, equations of lines and circles, and also includes some proofs of theorems from geometry using the methods of coordinate geometry.

Advanced References

COMBELLACK, **Introduction to Elementary Functions,** Ch. 3, pp. 45-66; Ch. 6, pp. 152-189

This reference includes a treatment of the same topics that are treated in *Intermediate Mathematics*, but they are presented within a framework that includes a number of additional topics.

HOFFMAN, **Basic Analysis,** Ch. 3, pp. 60-99

A plane coordinate system is defined in the same way as in *Intermediate Mathematics* and essentially the same ideas are developed but from a more rigorous and formal point of view.

SCHOOL MATHEMATICS STUDY GROUP, **Analytic Geometry,** Chs. 1-5, pp. 1-194

This reference will provide the reader with a broad background for the work in Chapter 2 of *Intermediate Mathematics*. Reading of the complete three-part text is highly recommended.

Chapter 3 — THE FUNCTION CONCEPT AND THE LINEAR FUNCTION

The definition of a function is given in terms of two sets and a rule of correspondence. Various examples of functions are given, and functional notation is introduced and used extensively throughout the chapter. The graph of a function is defined as the set of all ordered pairs produced by the rule of the function. If the domain and range of a function consist of real numbers, the expression "the graph of the function" is also used to denote the geometric figure resulting from plotting the ordered pairs as points. Various means of defining (or describing) specific functions are given. There is a brief introduction to composition of functions and inverses of functions.

The linear function is singled out for special attention. It is a function defined by an equation $y = ax + b$, where a and b are real numbers and $a = 0$. Thus in this text "linear function" is synonomous with "first degree function." It is shown that a linear function sets up a one-one correspondence between the set of all real numbers and the set of all real numbers and that, therefore, its inverse exists, which is also a linear function.

Elementary References

DOLCIANI, BERMAN AND WOOTON, **Modern Algebra and Trigonometry,** Ch. 6, pp. 203-220

This treatment differs from that in *Intermediate Mathematics* in that the discussion of functions is preceded by a section on relations. A function is defined accordingly as a set of ordered pairs. Set-builder notation is used.

HAYDEN AND FISCHER, **Algebra II,** Ch. 1, pp. 1-6; Ch. 4, pp. 130-166

The introductory treatment of functions in Chapter 1 is very brief and contains few examples. Chapter 4, on the other hand, has an extensive coverage of the linear function. Abundant use is made of set terminology. There is much greater emphasis on the geometrical considerations of a linear function than in *Intermediate Mathematics*.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra, Second Course,** Ch. 8, pp. 266-297

The treatment parallels that of *Intermediate Mathematics*. Set-builder notation is used extensively to designate domains of functions and to express sets of ordered pairs.

Advanced References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** Second Edition, Ch. 6, pp. 187-218

The treatment of functions goes beyond that of *Intermediate Mathematics* in content. Both the usual functional notation and the "arrow" notation are used.

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry,** Ch. 2, pp. 37-74

An excellent discussion of the function concept. The content is comparable to that in *Intermediate Mathematics*, but the level of exposition is somewhat higher. Constant functions, as well as first degree functions, are embraced by the term "linear function."

SCHOOL MATHEMATICS STUDY GROUP, **Supplementary and Enrichment Series: Functions**

This is a very readable treatise on functions, giving many examples of functions and their graphs. Included in this study are constant functions, linear functions, the absolute-value function, composition of functions, and inversion of functions. In the last section functions are considered as sets of ordered pairs.

TRIMBLE AND LOTT, **Elementary Analysis,** Ch. 4, pp. 95-118

A rather complete discussion of the linear function. The approach to functions in this text is through relations.

Chapter 4 — QUADRATIC FUNCTIONS AND QUADRATIC EQUATIONS

Quadratic functions are studied by considering a succession of special types, starting with the function defined by the equation $y = x^2$ and culminating with the general quadratic function defined by the equation $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$. A study of the graphs of the various types of quadratic functions is made.

Various methods of solving quadratic equations are given and the relationship of the roots to the coefficients is studied. Included also are equations that are transformable to quadratic equations, a study of quadratic inequalities and a section on applications.

Elementary References

BRUMFIEL, EICHOLZ AND SHANKS, **Algebra II,** Ch. 4, pp. 101-115

Solution of quadratic equations by factoring is discussed first, then the method of completing squares and the quadratic formula. Graphs of quadratic functions are then discussed briefly.

DOLCIANI, BERMAN AND WOOTON, **Modern Algebra and Trigonometry,** Ch. 6, pp. 220-244; Ch. 7, pp. 268-280

The portion in Chapter 6 is devoted to quadratic functions, the various types being studied in much the same way as in *Intermediate Mathematics*. In Chapter 7 the solution of quadratic equations is taken up, and the role of the discriminant is explained for the cases where the coefficients of the quadratic equation are real numbers.

HAYDEN AND FISCHER, **Algebra Two,** Ch. 5, pp. 167-190

A discussion of quadratic functions which is strongly oriented toward geometrical considerations. Intercepts on the coordinate axes and symmetry with respect to a line are given prominence. The content is somewhat limited in scope.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics, Algebra Two and Trigonometry,** Ch. 7, pp. 211-224

A fairly concise and readable treatment of quadratic functions. The authors choose to talk about zeros of quadratic functions instead of roots of quadratic equations. In content this section falls short of Chapter 4 of *Intermediate Mathematics*.

VANNATTA, GOODWIN AND FAWCETT, **Algebra Two, A Modern Course,** Ch. 6, pp. 163-182

An abbreviated discussion of quadratic functions but a detailed and very readable discourse on the methods of solving quadratic equations. There is a section on equations in quadratic form and also a section on verbal problems involving quadratic equations.

Advanced References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** Ch. 5, pp. 152-162
A very concise and formal discourse on quadratic equations and quadratic inequalities.

BECKENBACH, DROOYAN AND WOOTON, **College Algebra,** Ch. 4, pp. 100-107; Ch. 6, pp. 163-169, pp. 183-186
A concise treatment of quadratic equations. Graphs of quadratic functions and graphs of quadratic inequalities are discussed in separate sections of Chapter 6.

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry,** Ch. 6, pp. 227-237
This college freshman text has a concise discussion of quadratic functions under the heading "Polynominals of the Second Degree." In content it falls short of *Intermediate Mathematics.*

TRIMBLE AND LOTT, **Elementary Analysis, A Modern Approach,** Ch. 6, pp. 229-246
A careful developmental approach to the solution of quadratic equations and quadratic inequalities. Graphs are used freely in the discussion.

Chapter 5 — THE COMPLEX NUMBER SYSTEM

In this chapter the system of complex numbers is introduced as the extension of the system of real numbers which has the field properties and contains an element i for which $i^2 = -1$. From these properties the rules of calculation with complex numbers are deduced. The algebraic and geometric properties of the system which do not depend on trigonometry for their expression are discussed. Finally, it is shown that the system of ordered pairs of real numbers with addition and multiplication defined as suggested by the earlier discussion, has all the properties required of the system of complex numbers.

Elementary References

BRUMFIEL, EICHOLZ AND SHANKS, **Algebra II,** Ch. 5, pp. 127-146
In this refernce the complex number system is introduced as in Chapter 5 of *Intermediate Mathematics,* but with emphasis on statement and verification of properties, rather than development.

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry,** Ch. 5, pp. 199-207
Here complex numbers are introduced as ordered pairs of real numbers, for which equality, addition and multiplication are defined. The exposition is at a somewhat higher level than in *Intermediate Mathematics,* but fewer topics are covered here.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra — Second Course**, Ch. 4, pp. 126-163

A discussion of the complex number system that proceeds along similar lines to that in *Intermediate Mathematics*. There are differences, however, in the details. Pure imaginary numbers are discussed first at some length and then they get "wedded" to the real numbers. The "ordered pair" approach is not mentioned.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics — Algebra Two and Trigonometry**, Ch. 8, pp. 229-253

The complex number system is "built" by extending the real number system through adjunction of the element i. In the development of the new number system much emphasis is placed on following familiar patterns.

SCHOOL MATHEMATICS STUDY GROUP, **Elementary Functions** (Part 1), Ch. 2, pp. 74-84

Here the Fundamental Theorem of Algebra and the discussion of complex roots of a polynomial are presented in the context of a general discussion of polynomial equations.

VANNATTA, GOODWIN AND FAWCETT, **Algebra Two — A Modern Course**, Ch. 11, pp. 340-350

In this treatment of complex numbers, the real number system is extended by adjunction of the element i.

Advanced References

KNOPP, **Elements of the Theory of Functions**, Ch. 2, pp. 13-27

The complex number system is developed in terms of ordered pairs of real numbers, motivated by geometric interpretation.

LEDERMANN, **Complex Numbers**, Ch. 1, pp. 1-27

The discussion of the structure of the complex number system is on an elementary level. Other topics, however, are presented more extensively and at a higher level than in *Intermediate Mathematics*.

MCCOY, **Introduction to Modern Algebra**, Ch. 7, pp. 112-116.

In this text complex numbers are introduced in terms of ordered pairs of real numbers. There is a brief discussion of other major topics of Chapter 5 of *Intermediate Mathematics*.

Chapter 6 — EQUATIONS OF FIRST AND SECOND DEGREE IN TWO VARIABLES

This chapter provides a systematic treatment of graphs of equations of the first and second degree in two variables, using the techniques of analytic geometry. The theorem is proved that the graph of a linear equation is a straight line and that every straight line is the graph of a linear equation. The equation of the parabola is developed from the usual locus definition with the aid of the distance formula. Subsequently, the general equation of the conic is developed from the ratio definition, and the definitions of the ellipse and hyperbola are given in terms of the ratio definition. Those properties of the ellipse and hyperbola which involve "focal distances" are developed.

Elementary References

COMBELLACK, **Introduction to Elementary Functions,** Ch. 7, pp. 190-227
The standard forms of the equations of the parabola, ellipse, and hyperbola are developed from the ratio definition of a conic as in *Intermediate Mathematics*. The usual "sum of distances" and "difference of distances" properties of the ellipse and hyperbola, respectively, are proved as theorems.

KELLS AND STOTZ, **Analytic Geometry,** Chs. 4-5, pp. 56-85; Ch. 8, pp. 117-150
This reference provides a standard treatment of equations of lines and circles. The equations of the conic sections are developed from the ratio definition as in *Intermediate Mathematics*.

SMITH, GALE AND NEELEY, **New Analytic Geometry,** Chs. 4-6, pp. 46-107
This reference provides a good discussion of the analytic geometry of the line. The developments of equations of parabolas, ellipses, and hyperbolas proceed from the usual locus definitions. Eccentricity is discussed informally in connection with the shapes and ellipses and hyperbolas.

Advanced References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** Ch. 9, pp. 276-319
This reference contains a condensed treatment of the analytic geometry of the line and of the conic sections usually considered essential to a study of the calculus. Direction cosines are used in developing the equations of a line and in proving certain theorems about parallel and perpendicular lines. Polar coordinates and parametric equations are introduced.

FULLER, **Analytic Geometry,** Ch. 2-3, pp. 28-81
This reference contains a thorough treatment of the analytic geometry of the line and the conic sections.

GLICKSMAN AND RUDERMAN, **Fundamentals for Advanced Mathematics,** Ch. 11, pp. 312-324; Ch. 17, pp. 412-431
A treatment of the analytic geometry of the line and the conic sections. The developments are thorough. The equations of the conic sections are developed from the ratio definition as in *Intermediate Mathematics*.

PROTTER AND MORREY, **Calculus with Analytic Geometry,** Ch. 3, pp. 40-65; Ch. 9, pp. 254-262; Ch. 10, pp. 286-330
This reference takes up systematically the analytic geometry of the line, circle and the conics. The equations of the parabola, ellipse and hyperbola are developed from the "focal-distances" definitions. Eccentricity of the ellipse and of the hyperbola are presented as properties of the associated curves.

SCHOOL MATHEMATICS STUDY GROUP, **Analytic Geometry,** Ch. 7, pp. 267-307
The development in this reference is based on the assumption that the student is familiar with most of Chapter 6 of *Intermediate Mathematics*. Polar equations of the conic sections are developed first, and the corresponding rectangular equations are derived by transformation of coordinates. In using this reference the reader is advised to familiarize himself with

the six chapters that precede the single chapter which is cited. Reading of the complete three-part text is highly recommended.

SMALL, **Analytic Geometry and Calculus**, Ch. 12, pp. 231-277

The equation of the parabola is developed in a standard way. The equations of the ellipse and hyperbola are developed from the "focal-distances" definitions. The ratio properties are stated as theorems.

Chapter 7 — SYSTEMS OF EQUATIONS IN TWO VARIABLES

Solution sets of equations and of systems of equations in two variables are defined in the early part of the chapter. The important concept of equivalence of equations and of systems of equations is carefully developed and is used to find the solution sets of systems of linear equations. Systems of equations other than linear, in two variables are also considered. Finally, there is a discussion of inequalities and systems of inequalities in two variables.

Elementary References

CAMERON, **Algebra and Trigonometry**, (Revised Edition), Ch. 5, pp. 56-61

Solution of systems of two equations in two variables by graphs and by elimination. Emphasis is placed upon equivalence of systems of equations.

DOLCIANI, BERMAN AND WOOTON, **Modern Algebra and Trigonometry**, Ch. 3, pp. 95-100, pp. 102-107

A lucid treatment of systems of equations in two variables and of systems of inequalities in two variables. The method of "linear combinations" of arriving at a system which is equivalent to a given system, is explained very clearly.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra — Second Course**, Ch. 2, pp. 70-84

The notion of equivalence of equations in two variables and of systems of equations in two variables is used as a tool, but the reader is expected to be familiar with the notion beforehand. Systems of inequalities are also discussed.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Matthematics — Algebra Two and Trigonometry**, Ch. 10, pp. 285-292

A brief treatment of systems of equations in two variables. Equivalence of systems is explained in a clear fashion.

VANNATTA, GOODWIN AND FAWCETT, **Algebra Two — A Modern Course**, Ch. 7, pp. 215-222, pp. 227-240

The discussion in this text is somewhat less theoretical than that of *Intermediate Mathematics*, geometrical motivation being used to a much greater degree. Included in this treatment are systems of equations in two variables, systems of linear inequalities in two variables, as well as other systems.

Advanced References

ANDREE, **Selections from Modern Abstract Algebra**, Ch. 6, pp. 145-158

Matrices are used to solve systems of linear equations in two or more variables. If the reader is not familiar with matrices, he will find that studying

Chapter 5 of this reference will give him adequate preparation for Chapter 6.

OLMSTED, **Solid Analytic Geometry**, Ch. III, pp. 56-67

This reference gives a very concise treatment of simultaneous linear equations. Matrix methods of solution are employed. The notion of rank of matrices is used to determine the existence of solutions of systems of equations.

Chapter 8 — SYSTEMS OF FIRST DEGREE EQUATIONS IN THREE VARIABLES

The chapter starts with an introduction to the three dimensional rectangular coordinate system. The formula for the distance between two points is developed and this formula is used to find the equation of a plane each point of which is equidistant from two given points. It is then shown that the equation of every plane is a first degree equation in three variables, and conversely.

Solution sets of equations in three variables and of systems of equations in three variables are discussed. Graphical and algebraic methods of solving systems consisting of two or three linear equations in three variables are given. The algebraic method developed is that of reducing the system to triangular form.

Elementary References

DOLCIANI, BERMAN AND WOOTON, **Modern Algebra and Trigonometry, Book 2**, Teacher's Edition, Ch. 3, Special Section, pp. A-G

This section includes not only algebraic solutions of systems of linear equations (and equations which are linear in form) in three variables, but introduces a three dimensional coordinate system and demonstrates graphically the solution of a system of three linear equations in three unknowns.

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry**, Ch. 7, pp. 265-269

The triangulation method of solving systems of linear equations in three or more variables is given here.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra, Second Course**, Ch. 2, pp. 84-88

This brief section gives only an algebraic solution of systems of equations in three or more variables.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics, Algebra Two and Trigonometry**, Ch. 10, pp. 292-294

This section discusses only algebraic solutions of systems of linear equations in three variables.

Advanced References

ANDREE, **Selections from Modern Abstract Algebra**, Ch. 6, pp. 145-158

This short chapter on linear systems deals with algebraic solutions of systems of linear equations in three or more variables. A superficial acquaintance with matrices and row operations on matrices is assumed. The reading level is otherwise quite elementary.

CAMERON, **Algebra and Trigonometry**, (Revised Edition), Ch. 5, pp. 61-64, Ch. 6, pp. 69-91

Solution of systems of equations by reduction to equivalent systems in echelon form. Matrices are used in writing and solving systems of equations. There is a study of determinants and their application to solving systems of equations.

EISENHART, **Coordinate Geometry**, Ch. 2, pp. 71-77, pp. 88-93

This college level text has a more traditional treatment of rectangular coordinates in space. There is quite a careful introductory discussion of rectangular coordinates in space and a derivation of the distance formula for two points. The treatment of the equation of a plane is based on a preceding section dealing with the equations of a line in space.

OLMSTED, **Solid Analytic Geometry**, Ch. III, pp. 56-67

This reference gives a very concise treatment of simultaneous linear equations. Matrix methods of solution are employed. The notion of rank of matrices is used to determine the existence of solutions of systems of equations.

SCHOOL MATHEMATICS STUDY GROUP, **Analytic Geometry**, Ch. 8, pp. 309-313, pp. 324-327

The three dimensional rectangular coordinate system is introduced and the formula for the distance between two points is derived. It is shown that the equation of every plane in three-space is a linear equation of the form $ax + by + cz + d = 0$. Some graphs of first degree equations in three variables are shown.

Chapter 9 — LOGARITHMS AND EXPONENTS

In this chapter the logarithm function is defined in terms of the area under the curve $y = \frac{k}{x}$ and over the interval from 1 to x ($k > 0$ and $x > 0$). Proper choices of values of k yield $\log_e x$ and $\log_{10} x$. The properties of the logarithm functions and their graphs are derived from this definition. This is followed by an explanation of how logarithms are used in computation.

The exponential function is defined as the inverse of the logarithm function. While this order of presentation is essentially the reverse of the exponent - logarithm approach found in many current text books, it has the advantage of providing an acceptable definition of a^x when x is an irrational number.

Elementary References

DAVIS, **The Lore of Large Numbers**, pp. 10-32

A discussion of large numbers expressed in exponential form. This is light reading.

GLICKSMAN AND RUDERMAN, **Fundamentals for Advanced Mathematics**, Ch. 18, pp. 432-449

A more formal approach is used to define the exponential function. The existence, for example, of the function $f_R(x) = 2^x$ (domain: all reals), is postulated to satisfy a set of axioms. The usual properties of the exponential and logarithmic functions are derived from these axioms.

HAYDEN AND FISCHER, **Algebra Two,** Ch. 7, pp. 237-277

The more traditional approach in which the exponential function is defined first and then the logarithm function. If b is an irrational number, 10^b is defined to be the limit (which is assumed to exist) which 10^x approaches as x approaches b through a sequence of rational numbers.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra — Second Course,** Ch. 6, pp. 223-265

A treatment of the exponent-logarithm approach which is quite similar to that of Hayden and Fischer, *Algebra Two.*

MARKUSHEVICH, **Areas and Logarithms**

This translation of a Russian monograph provides an elementary treatment of areas and leads to the following definition of the natural logarithm $\ln b = \int_1^b x^{-1}dx$ ($\ln b$ means $\log_e b$). The basic properties of logarithms are then deduced from this definition. The reader need not have had any previous experience with natural logarithms or calculus.

SCHOOL MATHEMATICS STUDY GROUP, **Elementary Functions,** Ch. 4, pp. 180-189

Applications to radio-active decay, compound interest, and laws of cooling are discussed.

TAYLOR AND WADE, **University Freshman Mathematics,** Ch. 9, pp. 220-247

A treatment of the exponent-logarithm approach on an intermediate level of difficulty.

Advanced References

BATES AND KIOKEMEISTER, **The Real Number System,** Ch. 5, pp. 52-64

A rigorous discussion of real numbers as exponents.

NIVEN, **Calculus, An Introductory Approach,** Ch. 7, pp. 121-131

A treatment of the logarithm-exponent approach, in which $\log_e x$ is defined as $\int_1^x \frac{dv}{v}$, for $x > 0$, and exponential function is defined as the inverse of the natural logarithm function. This book is recommended for teachers who desire a quick and refreshing reacquaintance with calculus.

PROTTER AND MORREY, **Calculus with Analytical Geometry,** Ch. 11, pp. 351-362

The natural logarithm function is defined by the equation $\ln x = \int_1^x \frac{1}{t}dt$, $x > 0$. Its properties are derived from previously established properties of integrals. The exponential function is defined as the inverse of the logarithm function.

Chapter 10 — INTRODUCTION TO TRIGONOMETRY

The main part of the treatment of trigonometry is presented in Chapter 10. In addition, certain aspects of the subject are used extensively in Chapter 11 which deals with vectors and in Chapter 12 which deals with the polar form of complex numbers. The point of departure in the development presented in Chapter 10 involves a discussion of motions (or paths) on the circumference of a circle. Three units of angle measure are introduced: radiar ,easure, degree measure and revolution. The six trigonometric functions of angles are defined first; then the six trigonometric functions of real numbers are defined. A number of basic properties involving the sine and cosine functions are proved as theorems. Applications of the trigonometric functions are made to solving problems involving triangulation. Graphical representation of the trigonometric functions is used to motivate the concept of periodicity. The Law of Cosines and the Law of Sines are derived and then applied to the solution of the general triangles. Proofs of the usual addition, difference, and double angle formulas as given.

Elementary References

ALLENDOERFER AND OAKLEY, **Fundamentals of Freshman Mathematics,** Chs. 12-13, pp. 233-298

The two chapters in this reference contain standard treatments of trigonometric functions of angles as well as trigonometric functions of real numbers.

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** Ch. 8, pp. 232-275

This reference contains a treatment of the trigonometric functions of real numbers. Included is a section devoted to amplitude and frequency modulation of radio carrier waves.

BRIXEY AND ANDREE, **Modern Trigonometry**

A development of the trigonometric functions of an angle and trigonometric functions of a number. The book also contains considerable review material from algebra.

BRUMFIEL, EICHOLZ AND SHANKS, **Algebra II,** Ch. 11, pp. 306-345

The single chapter in this reference contains a short, yet reasonably complete and rigorous introduction to the trigonometric functions.

CAMERON, **Algebra and Trigonometry,** (Revised Edition), Chs. 9-10, pp. 150-241

Chapter 9 of this reference deals with trigonometric functions of angles. Chapter 10 treats functions of real numbers and concentrates on the analytic aspects of the subject.

DOLCIANI, BERMAN AND WOOTON, **Modern Algebra and Trigonometry,** Chs. 10-11, pp. 373-445

In Chapter 10 of this reference a careful distinction is made between circular functions, whose dc ıain is the set of real numbers, and trigonometric functions, whose domain is a set of angles. In Chapter 11 the usual properties of circular (and trigonometric) functions are developed in such a way that the results apply both to circular and trigonometric functions.

HAYDEN AND FISCHER, **Algebra Two,** Chs. 8-9, pp. 278-341

This reference contains a treatment which, except for details and organization, is similar to the treatment in *Intermediate Mathematics.*

HERBERG AND BRISTOL, **Elementary Mathematical Analysis,** Ch. 7, pp. 214-271

The single chapter contains a complete introductory treatment of trigonometry for high school students. The functions are derived by wrapping a number line around the unit circle. Included is a brief discussion of the representation of sin x and cos x as "nonterminating polynomials." The discussion of graphical methods for representing functions and also the brief discussion of inverse functions should be especially helpful to both students and teachers.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics; Algebra Two and Trigonometry,** Chs. 11-13, pp. 311-435

The three chapters named in this reference constitute a complete introductory treatment to plane trigonometry for high school students. The work is introduced with a development of the concept of the wrapping function. The sine and cosine functions are defined to be functions having the same domain as the wrapping functions. At the end of Chapter 13 there is a brief introduction to vectors.

SCHOOL MATHEMATICS STUDY GROUP, **Supplementary and Enrichment Series: Circular Functions**

This reference contains essentially the same material as Chapter 5 of School Mathematics Study Group's *Elementary Functions.*

Advanced References

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry,** Ch. 4, pp. 107-198; Ch. 10, pp. 365-391

This is an excellent reference to plane trigonometry. The presentation is developed at a level which will not overwhelm readers who are studying the subject for the first time, nor bore those who already have an acquaintance with the subject. In Chapter 4 the trigonometric functions of real numbers are introduced. Later on the trigonometric functions of angles are considered. Chapter 10 deals with inverse trigonometric functions and trigonometric equations.

GLICKSMAN AND RUDERMAN, **Fundamentals for Advanced Mathematics,** Chs. 13-14, pp. 340-377

This book provides a treatment of trigonometry which is rather formal but nonetheless an interesting variation of the treatment in *Intermediate Mathematics.*

MOORE, **Modern Algebra with Trigonometry,** Chs. 6-7, pp. 122-207

Chapter 6 contains an elementary development of the trigonometric functions of angles. Chapter 7 provides an advanced level treatment of analytic trigonometry.

TRIMBLE AND LOTT, **Elementary Analysis: A Modern Approach,** Ch. 4, pp. 148-171; Ch. 9, pp. 407-496

In this pre-calculus text, the essentials of plane trigonometry are presented from a modern point of view. A modest introduction to polar coordinates, complex numbers as vectors, and the polar form of complex numbers is included.

Chapter 11 — THE SYSTEM OF VECTORS

In this chapter vectors in the plane and the operations of addition and multiplication by a scalar are introduced in terms of directed line segments. The introduction of a coordinate system leads to the notion of components of a vector and the representations of vectors and vector operations in terms of ordered pairs of real numbers. The "geometric" and "algebraic" representations of the system of plane vectors are related by way of the notion of a basis. The inner product of two vectors is defined both "geometrically" and "algebraically." The discussion of plane vectors is extended, by analogy, to vectors in space.

Applications to the proof of geometric theorems and to physical problems involving forces and velocities are discussed. There is a brief discussion of the abstract notion of a vector space.

Elementary References

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra, Second Course**, Ch. 14, pp. 540-568

The discourse is comparable to that of Chapter 11 of *Intermediate Mathematics*, except that there are no physical applications given and abstract vector spaces are not discussed.

SCHOOL MATHEMATICS STUDY GROUP, **Analytic Geometry**, Ch. 3, pp. 91-138

The discussion in this book parallels that of Chapter 11 of *Intermediate Mathematics*. No physical applications appear and the notion of an abstract vector space is not introduced. Considerably more is done with geometric applications than in *Intermediate Mathematics*.

SCHUSTER, **Elementary Vector Geometry**

The discussion parallels that of Chapter 11 of *Intermediate Mathematics* but is at a slightly more advanced level. Although there is no mention made of abstract vector spaces, many topics of interest are included that are not found in *Intermediate Mathematics*.

Advanced References

MACBEATH, **Elementary Vector Algebra**, pp. 11-58

The discussion in this book concerns vectors in space from the outset and treats plane vectors as a special case. Abstract vector spaces are not introduced, but many other topics of interest are included.

McCOY, **Introduction to Modern Algebra**, Ch. 10, pp. 195-220

A largely self-contained and reasonably well-motivated discussion of abstract vector spaces in the context of a general discussion of modern algebra.

POLYA, **Mathematical Methods in Science**, Ch. 2-3, pp. 53-172

Addressed specifically to secondary school mathematics teachers. Chapter 2 presents a particularly well-motivated discussion of most of the physical applications considered in the text, and additional physical applications are discussed in Chapter 3.

Chapter 12 — POLAR FORM OF COMPLEX NUMBERS

In this chapter the properties of complex number system which require trigonometry for their exposition are discussed. The polar form is used in multiplication, division, raising to a power, and finding roots of complex numbers. The method of solution of quadratic equations with complex coefficients is given.

Elementary References

HART, **Algebra, Elementary Functions, and Probability**, Ch. 7, pp. 190-197
This reference gives a standard treatment of the trigonometric form and its use in computation. There is an ample supply of drill-type exercises.

MCCOY, **Introduction to Modern Algebra**, Ch. 7, pp. 117-125
A concise treatment of the trigonometric form of representation of a complex number and its use in multiplication, division, raising to a power, and finding roots of complex numbers.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics, Algebra Two and Trigonometry**, Ch. 14, pp. 437-450
The polar form of a complex number is developed in a clear fashion. De Moivre's Theorem, given without proof, is used to find powers and roots of complex numbers. Principal roots are introduced.

Advanced References

DEAUX, **Introduction to the Geometry of Complex Numbers**, pp. 16; 19-21
The level, both of content and exposition, is considerably above that of Chapter 12 of *Intermediate Mathematics.*

KNOPP, **Elements of the Theory of Functions**, Ch. 2, pp. 27-31
A very concise treatment of the polar form of a complex number.

KNOPP, **Theory of Functions, Part II**, Ch. 4, pp. 93-106
The subject of Riemann surfaces in the appendix to Chapter 12 of *Intermediate Mathematics* is a very advanced topic. Knopp's book presents the simplest and clearest possible discussion.

LEDERMANN, **Complex Numbers**, Ch. 1, pp. 27-41
See Ledermann, *Complex Numbers*, in the references for Chapter 5 of *Intermediate Mathematics.*

Chapter 13 — SEQUENCES AND SERIES

In this chapter a sequence of numbers

$$a_1, a_2, a_3 \ldots, a_n, \ldots,$$

is defined as a function whose domain is the set of all positive integers. The indicated sum $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ is called an infinite series. Series, both finite and infinite, are denoted by means of summation (sigma) notation. The sums of finite arithmetic and finite geometric series are obtained without the use of mathematical induction. The notion of the limit of a sequence is introduced and the sum of an infinite series (if it has a sum) is defined as the limit of the sequence of partial sums.

Elementary References

BECKENBACH, DROOYAN AND WOOTEN, **College Algebra,** Ch. 12, pp. 340-375

This chapter contains quite an extensive treatment of mathematical induction, finite sequences (including arithmetic progressions and geometric progressions), finite series, the limit of a sequence, and infinite geometric series.

BRUMFIEL, EICHOLZ AND SHANKS, **Algebra II,** Ch. 12, pp. 353-358

This short discussion deals chiefly with arithmetic and geometric progressions (series), using mathematical induction to prove results.

DOLCIANI, BERMAN AND WOOTEN, **Modern Algebra and Trigonometry, Book 2,** Ch. 13, pp. 487-509

A treatment of arithmetic and geometric series in which summation notation is employed. Infinite geometric series are included.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra, Second Course,** Ch. 10, pp. 357-373

Mathematical induction is used to derive formulas for the sums of finite arithmetic and finite geometric series.

ROSSKOPF, WILLOUGHBY AND VOGELI, **Modern Mathematics, Algebra II and Trigonometry,** Ch. 15, pp. 455-468

This easily read discussion employs the discovery approach.

Advanced References

APOSTOL, **Calculus, Volume I, Introduction; With Vectors and Analytic Geometry,** Ch. 9, pp. 410-429

This brief introduction to infinite sequences and infinite series contains an interesting opening section on Zeno's paradox.

BELLMAN AND BECKENBACH, **An Introduction to Inequalities,** pp. 48-62

An intermediate level discussion of arithmetic and geometric means.

FISHER AND ZIEBUR, **Integrated Algebra and Trigonometry,** Ch. 9, pp. 337-359

In this reference the induction axiom is stated in terms of a sequence of numbers. The relationship of arithmetic series to linear functions and of geometric series to exponential functions is discussed. Infinite geometric series are included.

LANG, **A First Course in Calculus,** Ch. XV, pp. 211-218

A brief introduction on the calculus level to infinite series. A few simple tests of convergence are given.

Chapter 14 — PERMUTATIONS, COMBINATION, AND THE BINOMIAL THEOREM

This chapter is devoted to counting problems, with emphasis upon set terminology and the avoidance of ambiguous terms such as "choice," "act," "event," "operation," "occur," "perform," "happen," and so on. The fundamental ideas underlying the counting procedures developed are precisely those at the very core of the definitions of equality, sum, and product for natural numbers.

Applications of the multiplication principle (I_3) are repeatedly made by counting the number of items in a rectangular array. This leads to a straight forward, almost mechanical, procedure which enables the student to apply the most naive of numerical ideas to the solution of some relatively sophisticated problems of enumeration.

The other matters discussed in the chapter, including the binomial theorem, arrangements, and selections with repetition, are handled in the orthodox ways using the results previously obtained in the studies of n-tuples and permutations.

Elementary References

GLICKSMAN AND RUDERMAN, **Fundamentals of Advanced Mathematics,** Ch. 9, pp. 241-259

A brief treatment similar to that of *Intermediate Mathematics*. Mathematical induction is used in proofs of some of the theorems.

HAYDEN AND FISCHER, **Algebra Two,** Ch. 11

The discussion of permutations and combinations is preceded by a careful development of inductive sets and their use to define the addition and multiplication operations for natural numbers. Linear arrangements are then carried out in terms of mappings, with combinations then defined in terms of permutations.

JOHNSON, LENDSEY, SLESNICK AND BATES, **Modern Algebra, Second Course,** Ch. 11, pp. 374-413

A more traditional approach to permutations and combinations than in *Intermediate Mathematics*, necessitating the use of such terms as "choice," "action," "results," and so on.

Advanced References

BECKENBACH, DROOYAN AND WOOTON, **College Algebra,** Ch. 13, pp. 376-399

A brief treatment of permutations and combinations which employ the counting properties of the cardinality of finite sets. There is an introduction to sample spaces and events.

KEMENY, SNELL AND THOMPSON, **Introduction to Finite Mathematics,** Ch. III, pp. 79-108

Counting problems are approached in terms of partitions of a set. Set terminology and the algebra of statements developed earlier in the book are used in this chapter.

Chapter 15 — ALGEBRAIC STRUCTURES

The algebraic structures of groups, fields and subfields are developed in this chapter in relation to earlier content of the text.

Elementary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** (Revised Edition), Ch. 4, pp. 106-123

A readable treatment of group structure with a number of examples and proofs of elementary properties. A discussion of subgroups is included.

MESERVE AND SOBEL, Mathematics for Secondary School Teachers, Ch. 4
A brief treatment of mathematical systems.

SAWYER, Prelude to Mathematics, pp. 97-101
A popular treatment for laymen of groups of movements in relation to symmetries of the rectangle and equilateral triangle.

Advanced References

ANDREE, Selections from Modern Abstract Algebra, Ch. 4, pp. 78-103
A well-motivated treatment of groups with a number of examples and proofs of elementary theorems.

BIRKHOFF AND MACLANE, A Survey of Modern Algebra (Revised Edition), Ch. 6
A treatment of groups which goes far beyond the content of *Intermediate Mathematics*.

McCOY, Introduction to Modern Algebra, Ch 9, pp. 166-180
This treatment of groups is on a much wider scope than that of *Intermediate Mathematics*.

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