DOCUMENT RESUME

ED 175 703 SE 028 691

TITLE Study Guides In Mathematics: Algebra, Geometry, Number Theory, Probability, and Statistics.
INSTITUTION Stanford Univ., Calif. School Mathematics Study Group.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 62
NOTE 39p.

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Algebra; *Bibliographies; Curriculum; Geometry; Higher Education; *Inservice Education; *Instruction; Mathematics; *Mathematics Education; Probability; Secondary Education; Secondary School Mathematics; Statistics; *Study Guides
IDENTIFIERS *School Mathematics Study Group

ABSTRACT

This SMSG study guide is designed to provide assistance to teachers who wish to improve their professional competence by self-study or by group study. The main purpose of the guide is to list and organize suitable references. Topics covered include: (1) algebra; (2) geometry; (3) number theory; (4) probability; and (5) statistics. (MP)

SCHOOL MATHEMATICS STUDY GROUP

STUDY GUIDES IN MATHEMATICS

ALGEBRA
GEOMETRY
NUMBER THEORY
PROBABILITY and STATISTICS

STUDY GUIDES IN MATHEMATICS

Suggestions for revision of the mathematics curriculum from the kindergarten through the high school, such as those suggested by the Commission on Mathematics of the College Entrance Examination Board, the School Mathematics Study Group, and others, involve materials and points of view that are not part of the training of most elementary and secondary school teachers, or indeed of many college teachers. In order that these revisions may be effected as quickly and as widely as possible, an extensive program for the in-service training of teachers is necessary. Opportunities must be provided for the acquisition of supplementary training that will enable teachers to handle the new concepts and materials with comfort and confidence. The purpose of these Study Guides is to provide assistance to teachers who wish to improve their professional competence by self-study or by group study. Group study might well be undertaken by members of a given mathematics department, by an entire school district, or by a joint effort of neighboring districts. A college or university may wish to use these Guides as the basis of an in-service program.

The Guides offer suggestions on three important aspects of mathematical study:

1. HOW to read mathematics.
2. WHAT basic ideas to seek out.
3. WHERE to find these basic ideas discussed.

It must not be assumed that the use of these Guides is intended to replace formal study of mathematics. Every teacher should avail himself of the opportunities now offered in summer institutes, in year-long institutes, and the like. It is hoped, however, that the Guides will serve as an effective supplement to more formal study.

1. HOW TO READ MATHEMATICS

The student of mathematics must accept the fact that the technique of reading mathematics differs from that of reading a newspaper or a novel. The compact nature of mathematical symbols makes special demands on the reader. For example, in reading a newspaper or a novel, one can often skip superficially over sentences or even paragraphs without losing the thread of the story; in reading mathematics, such a procedure is likely to be disastrous. The result may be complete loss of understanding.

In short, the reader of mathematics must not expect to be relieved of responsibility. He must be prepared to attack his task aggressively. This means reading slowly and carefully, with one's faculties focused on the job in hand. After a first reading to get the general idea, and a second reading to come to grips with details, still further readings may be necessary to gain mastery.

To read mathematics successfully requires equipment—paper and pencil. All calculations and arguments in the text should be checked by the reader to ensure complete understanding.

2. WHAT BASIC IDEAS TO SEEK OUT

In undertaking an organized program of reading and study, it is helpful to have a clear notion of what important concepts are to be mastered. In order to provide well-defined targets, each Study Guide lists, immediately after the title of each topic, the basic ideas that the reading should clarify. These lists also provide a framework for review after the material has been studied.

3. WHERE TO FIND BASIC IDEAS DISCUSSED

The main purpose of the Guides is to list and organize suitable references for study. At the end of each Study Guide, all references are listed alphabetically by authors. Although these lists stem from a rather thorough examination of many books, it is not exhaustive and suggestions for appropriate additions will be welcome.

In addition to the listings mentioned above, the references have been organized with respect to each topic. Each Guide begins with a section entitled COMMENTS where the purposes of the particular Guide are given and discussed. A section entitled ORGANIZATION OF THE GUIDE describes the format and use which should be made of that particular Guide.

A. STUDY GUIDE IN MODERN ALGEBRA

COMMENTS:

This Guide concerns especially the background for a contemporary course in introductory algebra. Consequently, we focus attention on set theory and modern algebra. The topics selected for this part of the Study Guide are those deemed accessible to the teacher and relevant to the teaching of secondary school mathematics.

ORGANIZATION OF THE GUIDE:

In this Guide each reference is classified as a primary reference, a secondary reference, or a supplementary reference. The significance of these classifications is as follows:

(a) A *primary reference* is one in which the presentation is simple, direct, and conveniently organized for the purposes of the Study Guide. Such a reference is recommended as a suitable first approach to the topic to which it is assigned.

(b) A *secondary reference* is one in which the material, although readily readable, is presented in less convenient form.

(c) A *supplementary reference* is one in which the presentation is at a higher mathematical level, or in which the scope is broader than that ordinarily needed for secondary school purposes.

SET THEORY

The use of elementary notions of sets, in one form or another, is included in virtually all suggestions for revision of the secondary school mathematics courses. The reason is that an early introduction of set concepts and notations facilitates greatly the treatment of many topics in mathematics. The guide to set theory is divided into three parts:

I. Concept of set, definitions, and properties of sets.

II. Relations and functions from the point of view of sets.

III. Various ways in which the set concept is used.

I. SETS — CONCEPTS, DEFINITIONS, PROPERTIES

Basic Topics

Concept of a set.
Element, or m nber, of a set.
Notation; hov ets are specified.
Universal set; null, or empty, set.
Subset; proper subsets.
Operations with sets; Intersection, Union, Complement.
Disjoint sets.
Algebra of sets.
Venn diagrams.

Primary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics**, Ch. 5, Sec. 1-7, pp. 103-114

COMMISSION ON MATHEMATICS, **Appendices**, Ch. 9, pp. 94-105

HAAG, **Studies in Mathematics, Vol. III**, Ch. 2, pp. 2.1-2.19

LUCE, **Basic Concepts**, Ch. 1, Sec. 1.1-1.6, pp. 7-31

JOHNSON AND GLENN, **Sets, Sentences and Operations**, pp. 1-34

WOODWARD AND MCLENNAN, **Elementary Concepts of Sets**, pp. 1-25

Secondary References

CHRISTIAN, **Introduction to Logic and Sets**, Part II, Sec. 1-2, pp. 33-41

COMMITTEE ON THE UNDERGRADUATE PROGRAM, **Elem. Math. of Sets**, Ch. 1, Sec. 1-4, pp. 4-22

JOHNSON, **First Course**, Ch. 1, Sec. 1, pp. 1-4

KELLEY, J. L., **Introduction to Modern Algebra**, pp. 36-50

KEMENY, et al., **Finite Mathematics**, Ch. II, Sec. 1-2, pp. 54-63

KERSHNER AND WILCOX, **Anatomy of Mathematics**, Ch. 4, Sec. 4.1-4.9, pp. 28-44

ROSE, **A Modern Introduction to College Mathematics**, Ch. 1, pp. 1-17

STOLL, **Sets, Logic and Axiomatic Theories**, pp. 1-24

Supplementary References

ADLER, **The New Mathematics**, Ch. 2, pp. 35-43

BIRKHOFF AND MCLANE, **Modern Algebra**, Ch. XI, Sec. 1-9, pp. 311-322; Ch. XII, Sec. 1-5, pp. 333-347

BREUER (FEHR), **Theories of Sets**, Ch. 1-6

EVES AND NEWSOM, **Fundamental Concepts**, Ch. 8, Sec. 8.1-8.5, pp. 226-259

MCCOY, **Introduction to Modern Algebra**, pp. 1-3

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, **Modern Mathematics**, Ch. III, pp. 36-64

RICHARDSON, **Fundamentals of Mathematics**, Ch. 7, pp. 176-205

UNIVERSITY OF CHICAGO, **Concepts and Structure**, Ch. 4, pp. 116-150

II. RELATIONS AND FUNCTIONS

Basic Topics

Set of ordered pairs, Cartesian product.
Relation.
Domain and range of a relation.
Variable.
Function.
Notation; how functions are specified.
Domain and range of a function.
Graph of a function.
Inverse of a function.

Primary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics,** Ch. 6, Sec. 1-5, pp. 124-140

COMMISSION ON MATHEMATICS, **Appendices,** Ch. 2, pp. 8-27

HAAG, **Studies in Mathematics, Vol. III,** Ch. 6, pp. 6.1-6.25

JOHNSON AND GLENN, **Sets, Sentences and Operations,** pp. 40-48

WOODWARD AND MCLENNAN, **Elementary Concepts of Sets,** pp. 30-49

Secondary References

JOHNSON, **First Course,** Ch. 1, Sec. 3, pp. 8-12

KERSHNER AND WILCOX, **Anatomy of Mathematics,** Ch. 5, Sec. 5.1-5.7, pp. 45-62

LUCE, **Basic Concepts,** Ch. 2, Sec. 2.1-2.9, pp. 47-79

ROSE, **A Modern Introduction to College Mathematics,** Ch. 2, pp. 18-38

Supplementary References

KELLEY, J. L., **Introduction to Modern Algebra,** Ch. 3, pp. 85-96

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, **Modern Mathematics,** Ch. 8, pp. 241-272

STOLL, **Sets, Logic, and Axiomatic Theories,** pp. 25-55

UNIVERSITY OF CHICAGO, **Concepts and Structure,** Ch. 6, pp. 187-250; Ch. 7, pp. 251-300

III. USES OF SET CONCEPTS

COMMISSION ON MATHEMATICS, **Appendices,** Ch. 1, pp. 1-7; Ch. 2, pp. 8-27; Ch. 9, pp. 94-105

JOHNSON AND GLENN, **Sets, Sentences, and Operations,** pp. 35-39, pp. 48-57

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, **Modern Mathematics,** Ch. 13, pp. 404-428

ROURKE, **Some Implications,** Pamphlet pp. 74-86

ALGEBRA

A common element in all suggestions for revision of the Algebra course is that there be greater emphasis on "algebraic structure." For secondary school purposes this means that the algebraic properties of the number systems should be stressed. This involves, more than anything else, a change in the attitude of the teacher toward the subject. The objective of this part of the Study Guide is to clarify the meaning of "algebraic structure" so that the teacher will be able to bring out these ideas for the student at the appropriate time. This part of the Study Guide is divided into the following categories:

I. The "discovery" of structure properties of number systems.

II. The formal definition of certain algebraic structures and the study of some of their elementary properties.

III. The construction of one number system out of a simpler one (e.g., rationals out of the integers and complex numbers out of the real numbers).

I. DISCOVERY OF STRUCTURE PROPERTIES

Basic Topics

Number Systems.
- Natural Numbers.
- Integers.
- Rational.
- Real.
- Complex.

Formal properties of number systems.
- Commutative law.
- Associative law.
- Distributive law.
- Closure.
- Identity elements and their special properties.
- Additive and multiplicative inverses.

Order relations, density.

Primary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics**, Ch. 2, Sec. 2.1-2.17, pp. 39-68

HAAG, **Studies in Mathematics**, Vol. III, Ch. 3, pp. 3.1-3.38; Ch. 4, pp. 4.1-4.32; Ch. 5, pp. 5.1-5.19

KELLEY, J. L., **Introduction to Modern Algebra**, pp. 1-35

RICHARDSON, **Fundamentals of Mathematics**, Chs. 3, 4, Sec. 11-26, pp. 41-105

SAWYER, W. W., **Concrete Approach to Abstract Algebra**, pp. 1-25

Secondary References

ADLER, **The New Mathematics**, Ch. 1, pp. 13-34

UNIVERSITY OF CHICAGO, **Concepts and Structure**, Ch. 1, Sec. 1-7, pp. 1-29

Supplementary References

COURANT AND ROBBINS, **What is Mathematics?** Ch. 1, Sec. 1-2, pp. 1-20

MCCOY, **Introduction to Modern Algebra**, Chs. 4-7, pp. 48-125

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, **Modern Mathematics**, Ch. II, pp. 7-35

II. ALGEBRAIC STRUCTURES

Basic Topics

Major aspects of a mathematical structure.
Undefined terms.
Definitions.
Axioms.
Theorems.

Nature of a proof.
(see ANDREE, pp. 12-17).
Groups; properties of, and examples of.
Fields; properties of, and examples of.

Primary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics**, Ch. 3, Sec. 1-5, pp. 69-82; Ch. 4, Sec. 1-10, pp. 83-102

ALLENDOERFER AND OAKLEY, **Fundamentals of Freshman Mathematics**, Ch. 1, pp. 1-19

ANDREE, **Modern Abstract Algebra**, Ch. 1, Sec. 1-4, pp. 1-17; Ch. 4, Sec. 1-4, pp. 78-98; Ch. 8, Sec. 1-2, pp. 180-186

HAAG, **Studies in Mathematics, Vol. III**, Ch. 2, pp. 2.19-2.34

Secondary References

DUBISCH, **The Nature of Number**, Chs. 3, 8 and App.

JOHNSON, **First Course**, Ch. 3, Sec. 8-12, pp. 57-76

KERSHNER AND WILCOX, **Anatomy of Mathematics**, Ch. 7, Sec. 1-5, pp. 83-100; Ch. 20, Sec. 1-6, pp. 348-360

McCOY, **Introduction to Modern Algebra**, Ch. 9, pp. 166-194

ROSE, **Modern Introduction to College Mathematics**, Ch. 2, pp. 38-52; Ch. 4, pp. 70-102

SAWYER, W. W., **Concrete Approach to Abstract Algebra**, Ch. 2, pp. 26-33; Ch. 3, pp. 71-82; Ch. 6, pp. 115-130

Supplementary References

BIRKHOFF AND McLANE, **Modern Algebra**

COMMITTEE ON THE UNDERGRADUATE PROGRAM, **Elementary Mathematics of Sets**, Ch. VII, Sec. 26-29, pp. 137-165

III. CONSTRUCTION OF NUMBER SYSTEMS

Basic Topics

Construction of number systems.
Building the integers.
Building the rationals.
Building the reals.
Building the complexes.

Primary References

ALLENDOERFER AND OAKLEY, **Principles of Mathematics**, Ch. II, Sec. 14, pp. 61-64

COMMISSION ON MATHEMATICS, **Appendices**, Ch. 5, pp. 58-63; Ch. 20, pp. 194-199; Ch. 22, pp. 220-223

JOHNSON, **First Course**, Ch. II, pp. 13-56, Sec. 4-7

RICHARDSON, **Fundamentals of Mathematics**, Ch. 4, Sec. 19-20, pp. 78-87

Secondary References

ADLER, The New Mathematics, Ch. 3, pp. 44-72; Ch. 4, pp. 73-89

KELLEY, J. L., Introduction to Modern Algebra, Ch. 5, pp. 199-227

LEVI, Elements of Algebra

McCOY, Introduction to Modern Algebra, Ch. 5, pp. 86-87; Ch. 7, pp. 112-125

MESERVE, Fundamental Concepts of Algebra, Ch. 1, Sec. 4-9, pp. 15-18

Supplementary References

EVES AND NEWSOM, Fundamental Concepts.

WHITEHEAD, An Introduction to Mathematics.

NOTE: For teachers who wish to gain some additional insight into the system of natural numbers, the following are a few of the many books available for this purpose.

ANDREE, Modern Abstract Algebra.

COURANT AND ROBBINS, What is Mathematics?

DANTZIG, Number, The Language of Science.

ORE, Number Theory.

RADEMACHER AND TOEPLITZ, The Enjoyment of Mathematics.

BIBLIOGRAPHY

ADLER, I. The New Mathematics. New York: The John Day Co., 1958.

ALLENDOERFER, C. B. AND OAKLEY, C. O. Principles of Mathematics. New York: McGraw-Hill Book Co., Inc., 1955.

ALLENDOERFER, C. B. AND OAKLEY, C. O. Fundamentals of Freshman Mathematics. New York: Mc-Graw-Hill Book Co., Inc., 1959.

ANDREE, R. V. Selections from Modern Abstract Algebra. New York: Henry Holt Company, 1958.

BIRKHOFF, G. AND McLANE, S. A Survey of Modern Algebra. New York: The Macmillan Company, 1953.

BREUER, J. (FEHR, H. F.) Introduction to the Theory of Sets. Translated by Fehr, H. F.; Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958.

UNIVERSITY OF CHICAGO COLLEGE MATHEMATICS STAFF. Concepts and Structure of Mathematics. Chicago: University of Chicago Press, 1954.

CHRISTIAN, R. R. Introduction to Logic and Sets, Preliminary Edition. Boston: Ginn and Company, 1958.

COMMISSION ON MATHEMATICS. Appendices to the Report of the Commission on Mathematics. New York: College Entrance Examination Board, 1959.

COMMITTEE ON THE UNDERGRADUATE PROGRAM. Elementary Mathematics of Sets with Applications. Mathematical Association of America, 1958.

COURANT, R. AND ROBBINS, H. What is Mathematics? London and New York: Oxford University Press, 1944.

DANTZIG, T. Number, The Language of Science. New York: Doubleday Anchor Books, 1956.

DUBISCH, R. The Nature of Numbers. New York: The Ronald Press Company, 1952.

EVES, H. AND NEWSOM, C. V. **An Introduction to the Foundation and Fundamental Concepts of Mathematics.** New York: Rinehart and Company, 1958.

HAAG, V. **Studies in Mathematics, Vol. III, Structure of Elementary Algebra.** School Mathematics Study Group, 1960. (Revised 1961.)

JOHNSON, D. AND GLENN, W. **Sets, Sentences and Operations.** St. Louis: Webster Publishing Co., 1960.

JOHNSON, R. E. **First Course in Abstract Algebra.** Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958.

KELLEY, J. L. **Introduction to Modern Algebra.** Princeton: D. Van Nostrand Company, 1960.

KEMENY, J. G., SNELL, J. L. AND THOMPSON, G. L. **Introduction to Finite Mathematics.** Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1956.

KERSHNER, R. B. AND WILCOX, L. R. **Anatomy of Mathematics.** New York: The Ronald Press Company, 1950

LEVI, H. **Elements of Algebra.** New York: Chelsea Publishing Company, 1953.

LUCE, R. D. **Studies in Mathematics, Vol. I, Some Basic Mathematical Concepts.** School Mathematics Study Group, 1959.

McCOY, N. **Introduction to Modern Algebra.** Boston: Allyn and Bacon, 1960.

MESERVE, B. E. **Fundamental Concepts of Algebra.** Reading, Mass.: Addison-Wesley Publishing Co., 1953.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. **Insights into Modern Mathematics, 23rd Yearbook.** National Council of Teachers of Mathematics, Washington, D.C., 1957.

ORE, O. **Number Theory and Its History.** New York: McGraw-Hill Book Company, 1948.

RADEMACHER, H. AND TOEPLITZ, O. **The Enjoyment of Mathematics.** Princeton, N. J.: Princeton University Press, 1957.

RICHARDSON, M. **Fundamentals of Mathematics.** New York: The Macmillan Company, 1958.

ROSE, I. **A Modern Introduction to College Mathematics.** New York: John Wiley and Sons, Inc., 1959.

ROURKE, R. E. K. **Some Implications of Twentieth-Century Mathematics for High Schools.** Reprinted from The Mathematics Teacher, Vol. LI, No. 2, February, 1958. National Council of Teachers of Mathematics, Washington, D.C.

SAWYER, W. W. **A Concrete Approach to Abstract Algebra.** San Francisco: W. H. Freeman and Company, 1959.

STOLL, R. R. **Sets, Logic, and Axiomatic Theories.** San Francisco: W. H. Freeman and Company, 1961.

WHITEHEAD, A. N. **An Introduction to Mathematics.** New York and London: Oxford University Press, 1948.

WOODWARD, E. J. AND McLENNAN, R. C. **Elementary Concepts of Sets.** New York: Henry Holt and Company, 1959.

B. STUDY GUIDE IN GEOMETRY

COMMENTS:

This Guide is concerned with the background for teachers of geometry. References are presented under five major headings:

Deduction and Postulates
Geometry of Euclidean Space
Geometries of Other Spaces
Historical Development
Miscellaneous

Teachers undertaking a personal study program and instructors of in-service courses are urged to include at least parts of the references listed under each of the first four major headings. Several reference books will be required, and these must be selected in accord with the mathematical maturity of the reader and the topics to be emphasized.

There does not exist any one or two books covering all of the topics suggested in this Study Guide. An elementary treatment of the topics may be obtained by studying the following references:

ANDERSON, **Concepts of Informal Geometry**

N.C.T.M., **The Growth of Mathematical Ideas (24th Yearbook)**, Ch. 4, pp. 111-181

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics**

Another approach may be made to most of the topics by using the following references:

PRENOWITZ AND JORDAN, **Basic Concepts of Geometry**

STABLER, **An Introduction to Mathematical Thought**

A slightly more advanced approach to most of the topics may be made by using the following reference:

MESERVE, **Fundamental Concepts of Geometry**

In each of these three cases there will be several omitted topics which should be studied using other references.

ORGANIZATION OF THE GUIDE:

Under each heading the Basic Topics are listed. Following this list, each topic is given with the suggested references. In a few cases the references are listed in order of increasing mathematical maturity; in most cases the references are noted as concerned with different aspects of the topic.

The Bibliography at the end of this Guide includes 48 references; 13 of these are triple starred (***) to form a minimal list recommended for all secondary school teachers. The thirteen books provide an introduction to most of the topics considered in this Guide. Many teachers will also find it worthwhile to study some of the more advanced or more specialized books and a selection from the newer secondary school materials.

I. DEDUCTION AND POSTULATES

It is to be hoped that geometry is no longer the first contact a student has with deductive thinking in mathematics. It is essential that the teacher understand the vocabulary and structure of logic in order to discuss the methods of proof used in geometry. The references that follow can supply this need. The references to postulational systems provide background for the variety of contemporary secondary school programs.

Basic Topics

- a) Deduction
- b) The Postulational Method
- c) Some Postulational Systems

a) Deduction

N.C.T.M., **24th Yearbook,** pp. 111-181; Proofs

N.C.T.M., **23rd Yearbook,** pp. 76-93; Logical concepts

STABLER, **Introduction to Logical Thought,** pp. 43-101; Logic

EXNER AND ROSSKOPF, **Logic in Elementary Mathematics,** Full treatment

b) The Postulational Method

C.U.P.M., **Elementary Mathematics of Sets,** pp. 4-22; Background on sets

NEWMAN, **The World of Mathematics,** pp. 1647-1667; Axiomatic method—pp. 1723-1744; Logic and axiomatics

N.C.T.M., **23rd Yearbook,** pp. 273-305; Origins of postulational systems

c) Some Postulational Systems

C.U.P.M., **Elementary Mathematics of Sets,** pp. 110-119; Intro. and a finite geometry

MESERVE, **Fundamental Concepts of Geometry,** pp. 9-20; Two finite geometries

HEATH, **Thirteen Books of Euclid's Elements,** pp. 195-200; Euclid's postulates

STABLER, **Introduction to Mathematical Thought,** pp. 11-19; Description of Euclidean and non-Euclidean Geometries

YOUNG, **Fundamental Concepts of Algebra and Geometry,** pp. 134-154; Hilbert's postulates

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics,** pp. 82-91; Hilbert's postulates

YOUNG, **Monographs on Topics of Modern Mathematics,** pp. 3-51; Veblen's postulates

II. GEOMETRY OF EUCLIDEAN SPACE

These references provide a basis for understanding the various approaches to a deductive study of secondary school geometry. Special attention should be given to the work of Euclid and the newer school programs.

Basic Topics

- a) Euclid's Elements
- b) High School Geometries Using Other Postulates
- c) Coordinates
- d) Length, Area, and Volume

e) Constructions and Constructibility
f) Transformations

a) Euclid's Elements

HEATH, **The Thirteen Books of Euclid's Elements**, Books I-IV; Highly recommended

KLEIN, **Mathematics in Western Culture**, pp. 188-208; Critique of Euclid

KUTUZOV, **Geometry**, pp. 447-472; Survey and Critique of Euclid

b) High School Geometries Using Other Postulates

BIRKHOFF AND BEATLEY, **Basic Geometry**, Birkhoff's postulates

S.M.S.G., **Geometry**, Birkhoff's postulates

CURTIS, DAUS, AND WALKER, **Euclidean Geometry Based on Ruler and Protractor Axioms**, Discussion of structure of SMSG Geomet

BRUMFIEL, EICHOLZ AND SHANKS, **Geometry**, Modification of Hilbert's postulates

U.I.C.S.M., Unit 6; **Geometry**, Betweenness and measure axioms

c) Coordinates

RICHARDSON, **Fundamentals of Mathematics**, pp. 224-273; Elements

KLINE, **Mathematics in Western Culture**, pp. 159-181; General

C.E.E.B., **Appendices**, pp. 120-139; Content for Schools

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics**, pp. 96-101; Coordinate model for Hilbert's postulates

LEVI, **Foundations of Geometry and Trigonometry**, Affine and Euclidean geometry

C.E.E.B., **Appendices**, pp. 176-199; Vectors

N.C.T.M., **23rd Yearbook**, pp. 145-199; Vectors

d) Length, Area, and Volume

ANDERSON, **Concepts of Informal Geometry**

KUTUZOV, **Geometry**, pp. 381-446; Meaning

e) Constructions and Constructibility

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics**, pp. 296-303; Criteria for ruler and compass constructions

KUTUZOV, **Geometry**, pp. 51-133; A fuller treatment

YOUNG, **Monographs on Topics of Modern Mathematics**, pp. 353-386; Constructions, regular polygons

f) Transformations

C.E.E.B., **Appendices**, pp. 159-165; Use in elementary geometry

KUTUZOV, **Geometry**, pp. 135-191; Concept—pp. 192-380; Elementary transformations

III. GEOMETRIES OF OTHER SPACES

The importance to the secondary school teacher of geometries other than Euclidean is in the recognition that such geometries are possible and are entirely consistent. A detailed understanding of the structures of these geometries is less essential. One should distinguish between "not Euclidean" and "non-Euclidean".

The latter term properly refers only to hyperbolic and elliptic geometries; the former to any geometry other than that of Euclidean space.

Basic Topics

a) Non-Euclidean Geometries
b) Projective and Affine Geometries
c) Topology
d) Spaces of Other Dimensions

a) Non-Euclidean Geometries

BELL, **Men of Mathematics**, pp. 294-306; Lobachewski and hyperbolic geometry

RICHARDSON, **Fundamentals of Mathematics**, pp. 437-458; History and a few theorems

COURANT AND ROBBINS, **What Is Mathematics?** pp. 214-227; Brief survey

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics**, pp. 52-85; Thorough intro. with problems

LIEBER AND LIEBER, **Non-Euclidean Geometry**

CARSLAW, **Elements of non-Euclidean Plane Geometry and Trigonometry**, Ch. 1, 2, 8; History and consistency of these geometries

b) Projective and Affine Geometries

EVES AND NEWSOM, **Foundations and Fundamental Concepts of Mathematics**, pp. 102-108

KLINE, **Mathematics in Western Culture**, pp. 144-158; Projective geometry from art

COURANT AND ROBBINS, **What is Mathematics?** pp. 165-214

PRENOWITZ AND JORDAN, **Basic Concepts of Geometry**

MESERVE, **Fundamental Concepts of Geometry**, pp. 25-218; Projective geometry as a basis for Euclidean

c) Topology

COURANT AND ROBBINS, **What is Mathematics?** pp. 235-271; Various aspects

N.C.T.M., **23rd Yearbook**, pp. 306-335; Point set topology with problems

HILBERT AND CONN-VOSSEN, **Geometry and the Imagination**, pp. 289-340; Surfaces

d) Spaces of Other Dimensions

ABBOTT, **Flatland**

MANNING, **Geometry of Four Dimensions**

IV. HISTORICAL DEVELOPMENT

The history of geometry is strikingly different from that of algebra where progress was very gradual. Euclid's *Elements*, a book modern in spirit although some 2000 years old, has had a profound influence on subsequent developments. Consequently, any serious study of geometry must be made in the light of this history.

Basic Topics

a) Comprehensive
b) Early Physical Origins

c) The Greek Era
d) The Renaissance
e) Nineteenth and Twentieth Centuries

a) Comprehensive

COURANT AND ROBBINS, **What is Mathematics?** pp. xv-xix

MESERVE, **Fundamental Concepts of Geometry**, pp. 219-267

b) Early Physical Origins

EVES, **Intro. to the History of Mathematics**, pp. 31-32, 43-44; Babylonian era

c) The Greek Era

EVES, **Intro. to the History of Mathematics**, pp. 51-178; An overview

KLINE, **Mathematics in Western Culture**, pp. 40-59

BELL, **Men of Mathematics**, pp. 19-34; Biographical

NEWMAN, **World of Mathematics**, pp. 79-113; Biographical

DANTZIG, **The Bequest of the Greeks**, pp. 1-45

VAN DER WAERDEN, **Science Awakening**, pp. 82-291; Excellent source-book

HEATH, **Thirteen Books of Euclid's Elements**, Ch. 1-5; Euclid's sources and commentators

NEUGEBAUER, **The Exact Sciences in Antiquity**, pp. 139-146; Commentary on the development of Greek Mathematics

d) The Renaissance

EVES, **Intro. to the History of Mathematics**, pp. 277-286; Analytic geometry—pp. 252-257; Projective geometry

KLINE, **Mathematics in Western Culture**, pp. 126-181

BELL, **Men of Mathematics**, pp. 39-40, 52-55, 59-61, 63-64; Analytic geometry—pp. 76-79, 209-217; Projective geometry—pp. 184-187; Descriptive geometry

e) Nineteenth and Twentieth Centuries

EVES, **Intro. to the History of Mathematics**, pp. 122-126; Non-Euclidean geometries—p. 254; Klein's Erlanger Programm

KLINE, **Mathematics in Western Culture**, pp. 410-431

BELL, **Men of Mathematics**, pp. 263-267; Differential geometry—pp. 267-268, 492; Topology—pp. 299-306, 491-495, 503-509; Non-Euclidean Geometries

WOLFE, **Intro. to Non-Euclidean Geometry**, pp. 17-64; Non-Euclidean Geometries

COURANT AND ROBBINS, **What is Mathematics?** pp. 235-236; Topology

V. MISCELLANEOUS

Teachers are encouraged to select references according to their special interests. The references for informal geometry should be helpful to teachers of elementary school, junior high school, and general mathematics courses in high school.

Basic Topics

a) Geometry in Relation to Algebra and Analysis
b) Geometry in Relation to the Physical Universe
c) Geometry in Relation to Art
d) Induction and the Testing of Hypotheses
e) Informal Geometry for Elementary and High School Teachers of Mathematics

a) Geometry in Relation to Algebra and Analysis

COURANT AND ROBBINS, **What is Mathematics?** pp. 235-271, 312-321; Also topology

HILBERT AND COHN-VOSSEN, **Geometry and the Imagination**, pp. 171-271; Also the non-Euclidean geometries

b) Geometry in Relation to the Physical Universe

BOEHM, **The New World of Math.**

WHITEHEAD, **Introduction to Mathematics**

NEWMAN, **The World of Mathematics**, pp. 671-724; Symmetry—pp. 734-770; Motion—pp. 780-819; Longitude—pp. 871-881; Crystals—pp. 952-957; Size

HOLDEN AND SINGER, **Crystals and Crystal Growing**, pp. 120-212, 225-275; Geometric structure of crystals

KLINE, **Mathematics in Western Culture**, pp. 144-150; Perspective—pp. 150-158; Map projections

DEETZ AND ADAMS, **Elements of Map Projection**

c) Geometry in Relation to Art

NEWMAN, **The World of Mathematics**, pp. 600-622, pp. 622-626; Basic

KLINE, **Mathematics in Western Culture**, pp. 126-143; More comprehensive

IVINS, **Art and Geometry**, Excellent treatment

d) Induction and the Testing of Hypotheses

POLYA, **How to Solve It**

LIEBER AND LIEBER, **The Education of T. C. Mits**, pp. 129-167

e) Informal Geometry for Elementary and High School Teachers of Mathematics

ANDERSON, **Concepts of Informal Geometry**

S.M.S.G., **Mathematics for Junior High School; Mathematics for Elementary School**

RAVIELLI, **An Adventure in Geometry**

ABBOTT, **Flatland**

NEWMAN, **The World of Mathematics**, pp. 2385-2396

STEINHAUS, **Mathematical Snapshots**, pp. 184-190; Maps—pp. 211-213; Surfaces—pp. 214-240; Topology

KASNER AND NEWMAN, **Mathematics and the Imagination**, pp. 112-155, 265-298

OGILVY, **Through the Mathescope**, pp. 49-84, 96-129

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CURTIS, C. W., DAUS, P. H., AND WALKER, R. J. **Euclidean Geometry Based on Ruler and Protractor Axioms; Studies in Mathematics, Vol. II.** School Mathematics Study Group, New Haven, Conn., 1959.

DANTZIG, T. **The Bequest of the Greeks.** London: George Allen and Unwin Ltd., 1955.

DEETZ, C. H. AND ADAMS, O. S. **Elements of Map Projection; U. S. Coast and Geodetic Survey, Special Publication No. 68.** Washington, D. C.: U. S. Government Printing Office, 1934.

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EXNER, R. M. AND ROSSKOPF, M. F. **Logic in Elementary Mathematics.** New York: McGraw-Hill Book Co., 1959.

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***KASNER, E. AND NEWMAN, J. **Mathematics and the Imagination.** New York: Simon and Schuster, 1940.

***KLINE, M. **Mathematics in Western Culture.** New York: Oxford University Press, 1953.

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***NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (N.C.T.M.). **Insights into Modern Mathematics, 23rd Yearbook.** National Council of Teachers of Mathematics, Washington, D. C., 1957.

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***NEWMAN, J. R., ed. **The World of Mathematics, Vol. 1-4.** New York: Simon and Schuster, 1956.

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C. STUDY GUIDE IN NUMBER THEORY

COMMENTS:

Since numbers form the basis for all mathematics, the theory of numbers in the broad sense embraces all elementary mathematics. A knowledge of the structure of number systems is also basic to the study of algebra. For what we might call the algebraic properties of numbers (laws of operation, extensions of the number system, etc.) the reader is referred to the Study Guide in Modern Algebra.

On the other hand, the theory of numbers is usually understood to refer to the so-called arithmetic properties of integers. A great many of these are concerned with divisibility, and all of them emphasize the discrete aspect of the integers as opposed to the continuous properties of the reals. The applications of number theory usually depend on the fact that many physical phenomena are discrete rath. than continuous.

Very early in his mathematical experience, the student meets and uses the concepts of factor, divisor, prime, greatest common divisor, and least common multiple. The similar properties of polynomials, when they occur later on in algebra, appear as natural generalizations of these ideas from number theory.

Moreover in this field one encounters solutions to many of the puzzle problems which through the years have held the attention of layman and mathematician alike. In this subject many questions can easily be asked which are understandable to anyone with a minimum knowledge of mathematics. To some of these questions there are elegantly simple answers which can be discovered by the clever student, others have yielded only to intricate mathematics of the deepest kind, and still others remain unanswered to this day.

A basic preparation in number theory then, in addition to furnishing the teacher with a storehouse of new and interesting topics for the bright student, should help him to deal more confidently and competently with many of the concepts encountered in mathematics today.

ORGANIZATION OF THE GUIDE:

The Guide to Number Theory lists six major areas for study. After each heading, essential ideas and important results are indicated. The topics selected are those deemed accessible to the teacher and relevant to his teaching. Several developments which are important, but not absolutely essential in a brief study, are starred (*).

In this Guide at least one primary reference and frequently secondary references are given after each topic in the outline. The significance of these classifications is as follows:

a) A primary reference is one in which the presentation is simple, direct, and conveniently organized for the purposes of this Study Guide. Such a reference is recommended as a suitable first approach to the topic to which it is assigned.

b) A secondary reference is one in which the material is presented in less convenient form, usually at a higher level of mathematical sophistication. The treatment may be more extensive and have greater depth than ordinarily would be needed.

A list of Supplementary Topics is given rather than lists of Supplementary References. These supplementary topics include many interesting sperial topics which are fascinating in their own right, but which are not central in the theory of numbers. It is hoped that the users of this Guide, having tasted the unusual flavors of this branch of mathematics, will want to investigate some of these interesting supplementary fields.

I. NUMERAL SYSTEMS

Basic Topics

a) History and development of the notation for the integers
b) Positional number systems
c) Number bases

Primary References

DANTZIG, **Number, the Language of Science,** Ch. 2, pp. 20-36
DUBISCH, **The Nature of Number,** Ch. 1 & 2, pp. 3-24

For (a) above:

ORE, **Number Theory and Its History,** Ch. 1, Sec. 1-8, pp. 1-15

For (b) above:

ORE, **Number Theory and Its History,** Ch. 1, Sec. 9, pp. 16-24

For (c) above:

DANTZIG, **Number, The Language of Science,** Ch. 1, Sec. 9-11, pp. 12-17
GRIFFIN, **Elementary Theory of Numbers,** Ch. 3, Sec. 10, pp. 37-40
JONES, **The Theory of Numbers,** Ch. 1, Sec. 9; pp. 23-26
MERRILL, **Mathematical Excursions,** Ch. 2, pp. 13-22
ORE, **Number Theory and Its History,** Ch. 2, Sec. 4, 5, pp. 34-39
STEWART, **Theory of Numbers,** Ch. 4, pp. 21-26
USPENSKY AND HEASLET, **Elementary Number Theory,** Ch. 1, Sec. 6, pp. 13-15

Secondary References

For (c) above:

HARDY AND WRIGHT, **An Introduction to the Theory of Numbers,** Ch. 9, Sec. 1-7, pp. 107-117

II. DIVISIBILITY PROPERTIES OF THE INTEGERS

Basic Topics

a) Fundamental concepts: factor, divisor, prime, composite, greatest common divisor, least common multiple
b) The Euclidean algorithm
c) The Fundamental Theorem of Arithmetic
*d) $\tau(n)$, $\sigma(n)$ and perfect numbers, $\phi(n)$

Primary References

For (a) above:

GRIFFIN, **Elementary Theory of Numbers,** Ch. 1, Sec. 3, pp. 6-9; Ch. 3, Sec. 7, p. 33
JONES, **The Theory of Numbers,** Ch. 1, Sec. 8, pp. 14-19

For (b) above:

COURANT AND ROBBINS, **What Is Mathematics?** Ch. 1 Supp., Sec. 4, pp. 42-46

GRIFFIN, **Elementary Theory of Numbers,** Ch. 3, Sec. 6, pp. 31-32

JONES, **The Theory of Numbers,** Ch. 1, Sec. 8, pp. 16-17

ORE, **Number Theory and Its History,** Ch. 3, pp. 41-49

SMSG, **Essays on Number Theory,** Vol. 2, Sec. 4, pp. 19-23

STEWART, **Theory of Numbers,** Ch. 5, pp. 27-31

For (c) above:

COURANT AND ROBBINS, **What Is Mathematics?** Ch. 1 Supp., Sec. 2, pp. 46-48

DAVENPORT, **The Higher Arithmetic,** pp. 17-22

GRIFFIN, **Elementary Theory of Numbers,** Ch. 3, Sec. 4, pp. 28-30

JONES, **The Theory of Numbers,** Ch. 1, Sec. 8, pp. 18-19

ORE, **Number Theory and Its History,** Ch. 4, Sec. 1, pp. 50-52

SMSG, **Essays on Number Theory,** Vol. 1, Sec. 3, pp. 19-27

STEWART, **Theory of Numbers,** Ch. 6, Sec. 1-3, pp. 32-35

For (d) above:

DANTZIG, **Number, The Language of Science,** Ch. 3, Sec. 9, pp. 45-46

GRIFFIN, **Elementary Theory of Numbers,** Ch. 3, Sec. 8, 9, pp. 34-37; Ch. 4, Sec. 4, pp. 58-60

JONES, **The Theory of Numbers,** Ch. 2, Sec. 6, 9, pp. 48-50, 56-58

ORE, **Number Theory and Its History,** Ch. 5, Sec. 1, 2, pp. 86-96.

SMSG, **Essays on Number Theory,** Vol. 2, Sec. 1, 2, pp. 1-11

STEWART, **Theory of Numbers,** Ch. 8, Sec. 1-3, pp. 45-48; Ch. 16, pp. 102-110

Secondary References

For (a) above:

NAGELL, **Introduction to Number Theory,** Ch. 1, Sec. 1, 3, 5, pp. 1-19

For (c) above:

NAGELL, **Introduction to Number Theory,** Ch. 1, Sec. 4, pp. 14-15

RADEMACHER AND TOEPLITZ, **The Enjoyment of Mathematics,** Ch. 11, pp. 66-73

USPENSKY AND HEASLET, **Elementary Number Theory,** Ch. 4, Sec. 4, pp. 71-73

For (d) above:

NAGELL, **Introduction to Number Theory,** Ch. 1, Sec. 8, 9, pp. 23-29

NIVEN AND ZUCKERMAN, **An Introduction to the Theory of Numbers,** Ch. 2, pp. 34-38

RADEMACHER AND TOEPLITZ, **The Enjoyment of Mathematics,** Ch. 19, pp. 129-135

USPENSKY AND HEASLET, **Elementary Number Theory,** Ch. 4, Sec. 7, 10, pp. 76-77, 80-82

III. PRIME NUMBERS

Basic Topics

a) Euclid's proof of the infinitude of primes
b) The sieve of Eratosthenes
c) The distribution of primes and the prime number theorem

Primary References

DAVENPORT, The Higher Arithmetic, Ch. 1, pp. 9-40

For (a) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 3, Sec. 3, 4, pp. 26-27
JONES, The Theory of Numbers, Ch. 1, Sec. 8, p. 19
ORE, Number Theory and Its History, Ch. 4, Sec. 6, p. 65
SMSG, Essays on Number Theory, Vol. 1, Sec. 1, pp. 2-3
STEWART, Theory of Numbers, Ch. 7, Sec. 3, pp. 40-42

For (b) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 3, Sec. 1, p. 25
ORE, Number Theory and Its History, Ch. 4, Sec. 6, pp. 64-69
STEWART, Theory of Numbers, Ch. 7, Sec. 2, pp. 37-40
USPENSKY AND HEASLET, Elementary Number Theory, Ch. 4, Sec. 3, pp. 69-70

For (c) above:

COURANT AND ROBBINS, What Is Mathematics? Ch. 1, Sec. 2, pp. 25-31
ORE, Number Theory and Its History, Ch. 4, Sec. 8, pp. 75-85
SMSG, Essays on Number Theory, Vol. I, Sec. 1, pp. 4-7; Vol. 2, Sec. 3, pp. 13-17
STEWART, Theory of Numbers, Ch. 7, Sec. 4, pp. 42-43

Secondary References

For (a) above:

USPENSKY AND HEASLET, Elementary Number Theory, Ch. 4, Sec. 11, pp. 85-86

For (c) above:

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 1, pp. 1-11
NAGELL, Introduction to Number Theory, Ch. 2, pp. 47-67; Ch. 8, pp. 275-299

IV. CONGRUENCES

Basic Topics

a) Elementary properties
b) The solution of congruences
c) Tests for divisibility
*d) Fermat's and Euler's theorems
*e) Wilson's theorem

Primary References

For (a) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 4, Sec. 1, 2, pp. 53-55

JONES, The Theory of Numbers, Ch. 2, Sec. 3, 4, pp 41-44

ORE, Number Theory and Its History, Ch. 9, Sec. 1, 2, pp. 209-213, Sec. 4, pp. 216-224

SMSG, Essays on Number Theory, Vol. 1, Sec. 2, pp. 9-13

YOUNG, Monographs in Modern Mathematics, Ch. 7, Sec. 4, pp. 320-325

For (b) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 5, pp. 66-87

JONES, The Theory of Numbers, Ch. 2, Sec. 5, pp. 45-47

ORE, Number Theory and Its History, Ch. 10, pp. 234-257

For (c) above:

DANTZIG, Number, the Language of Science, Part II, Sec. A, pp. 268-270

MERRILL, Mathematical Excursions, Ch. 1, pp. 1-7

ORE, Number Theory and Its History, Ch. 9, Sec. 5, pp. 225-233

SMSG, Essays on Number Theory, Vol. 1, Sec. 2, pp. 11-12

STEWART, Theory of Numbers, Ch. 17, Sec. 4, pp. 115-117

YOUNG, Monographs in Modern Mathematics, Ch. 7, Sec. 4, pp. 326-327

For *(d) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 6, Sec. 1, 2, pp. 88-89

ORE, Number Theory and Its History, Ch. 12, Sec. 1, 2, pp. 272-279

STEWART, Theory of Numbers, Ch. 18, pp. 119-125

USPENSKY AND HEASLET, Elementary Number Theory, Ch. 6, Sec. 9, pp. 146-147

For *(e) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 6, Sec. 3, pp. 92-93

JONES, The Theory of Numbers, Ch. 2, Sec. 11, pp. 55-56

STEWART, Theory of Numbers, Ch. 20, Sec. 7, p. 144

USPENSKY AND HEASLET, Elementary Number Theory, Ch. 6, Sec. 12, pp. 153-157

Secondary References

LEVEQUE, Topics in Number Theory, Ch. 3, pp. 24-47

NAGELL, Introduction to Number Theory, Ch. 3, Sec. 19-23, pp. 68-78

NIVEN AND ZUCKERMAN, An Introduction to the Theory of Numbers, Ch. 2, pp. 20-34

For (a) above:

USPENSKY AND HEASLET, Elementary Number Theory, Ch. 6, Sec. 1-3, pp. 126-134

For *(d) above:

LEVEQUE, Topics in Number Theory, Ch. 3, Sec. 7, pp. 42-47

NAGELL, Introduction to Number Theory, Ch. 3, Sec. 21, pp. 71-73

For *(e) above:

NAGELL, Introduction to Number Theory, Ch. 3, Sec. 30, pp. 99-101

ORE, Number Theory and Its History, Ch. 11, Sec. 1, 2, pp. 259-267

V. DIOPHANTINE PROBLEMS

Basic Topics

a) Linear Diophantine equations

b) Some quadratic equations; e.g. Pythagorean triples

Primary References

For (a) above:

GRIFFIN, **Elementary Theory of Numbers**, Ch. 2, pp. 14-24

JONES, **The Theory of Numbers**, Ch. 3, Sec. 1-3, pp. 29-32

MERRILL, **Mathematical Excursions**, Ch. 12, pp. 119-127

ORE, **Number Theory and Its History**, Ch. 6, 7, pp. 116-157

SMSG, **Essays on Number Theory**, Vol. 2, Sec. 4, pp. 19-26

STEWART, **Theory of Numbers**, Ch. 12, 13, pp. 72-85

For (b) above:

COURANT AND ROBBINS, **What Is Mathematics?** Ch. 1 Supp., Sec. 3, pp. 40-42

JONES, **The Theory of Numbers**, Ch. 3, Sec. 4, 5, pp. 69-73

ORE, **Number Theory and Its History**, Ch. 8, Sec. 1, pp. 165-170, Sec. 3-6, pp. 179-203

STEWART, **Theory of Numbers**, Ch. 14, pp. 86-93

Secondary References

NIVEN AND ZUCKERMAN, **An Introduction to the Theory of Numbers**, Ch. 5, pp. 94-110

For (a) above:

NAGELL, **Introduction to Number Theory**, Ch. 1, Sec. 10, pp. 29-32

For (b) above:

DAVENPORT, **The Higher Arithmetic**, Ch. 7, pp. 152-168

HARDY AND WRIGHT, **An Introduction to the Theory of Numbers**, Ch. 13, Sec. 1, 2, pp. 190-191

NAGELL, **Introduction to Number Theory**, Ch. 6, pp. 188-226

RADEMACHER AND TOEPLITZ, **The Enjoyment of Mathematics**, Ch. 14, pp. 88-95

VI. FAMOUS UNSOLVED PROBLEMS

Basic Topics

a) Formulas for primes

b) Goldbach's Conjecture

c) Waring's problem and the sums of squares

d) Fermat's Last Theorem

Primary References

For (a) above:

COURANT AND ROBBINS, **What Is Mathematics?** Ch. 1 Supp., Sec. 2, pp. 25-27

DANTZIG, **Number, The Language of Science**, Part II, Sec. B-7, pp. 287-290

ORE, **Number Theory and Its History**, Ch. 4, Sec. 7, pp. 69-75

For (b) above:

COURANT AND ROBBINS, What Is Mathematics? Ch. 1 Supp., Sec. 1, pp. 30-31

ORE, Number Theory and Its History, Ch. 4, Sec. 8, pp. 81-85

For (c) above:

GRIFFIN, Elementary Theory of Numbers, Ch. 10, Sec. 1, pp. 158-166

JONES, The Theory of Numbers, Ch. 6, Sec. 6, pp. 133-135

ORE, Number Theory and Its History, Ch. 11, Sec. 3, pp. 267-271

STEWART, Theory of Numbers, Ch. 25, 26, pp. 175-186

For (d) above:

DANTZIG, Number, The Language of Science, Ch. 1, Sec. 14, 15, pp. 53-56

GRIFFIN, Elementary Theory of Numbers, Ch. 10, Sec. 3, p. 168

ORE, Number Theory and Its History, Ch. 8, Sec. 7, pp. 203-207

STEWART, Theory of Numbers, Ch. 15, Sec. 1, pp. 94-95

Secondary References

For (a) above:

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 1, Sec. 5, pp. 5-6; Ch. 2, Sec. 7, pp. 17-18

For (c) above:

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 20-21, pp. 297-339

RADEMACHER AND TOEPLITZ, The Enjoyment of Mathematics, Ch. 9, pp. 52-61

LEVEQUE, Topics in Number Theory, Ch. 7, pp. 125-136

For (d) above:

RADEMACHER AND TOEPLITZ, The Enjoyment of Mathematics, Ch. 14, pp. 92-95

USPENSKY AND HEASLET, Elementary Number Theory, Ch. 12, Sec. 7, pp. 407-412

SUPPLEMENTARY TOPICS

A. Partitions

Primary References

GRIFFIN, Elementary Theory of Numbers, Ch. 12, pp. 190-195

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 19, pp. 273-296

NIVEN, An Introduction to the Theory of Numbers, Ch. 10, pp. 200-206

B. Continued Fractions

Primary References

COURANT AND ROBBINS, What Is Mathematics? Ch. 1 Supp., Sec. 4, pp. 49-51; Ch. 6, Sec. 5, pp. 301-303

DANTZIG, Number, The Language of Science, Ch. 8, Sec. 13, 14, pp. 157-159

DAVENPORT, The Higher Arithmetic, Ch. 4, pp. 79-114

JONES, The Theory of Numbers, Ch. 4, Sec. 3-6, pp. 82-91

LEVEQUE, Elementary Theory of Numbers, Ch. 5, pp. 73-95

STEWART, Theory of Numbers, Ch. 32, pp. 238-247

WRIGHT, First Course in the Theory of Numbers, Ch. 2, pp. 15-42

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 10, pp. 129-153

NIVEN, An Introduction to the Theory of Numbers, Ch. 7, pp. 134-148

C. The equation $x^2 - Dy^2 = 1$

Primary References

GRIFFIN, Elementary Theory of Numbers, Ch. 10, Sec. 6, pp. 172-176

JONES, The Theory of Numbers, Ch. 4, Sec. 8-10, pp. 96-104

Secondary References

LEVEQUE, Topics in Number Theory, Ch. 8, Sec. 1-3, pp. 137-148

D. Fibonacci Numbers

Primary References

JONES, The Theory of Numbers, Ch. 4, Sec. 1, 2, pp. 76-82

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 10, Sec. 14, pp. 148-150

E. Classical Construction Problems

Primary References

COURANT AND ROBBINS, What Is Mathematics? Ch. 3, Part 1, pp. 117-140

ORE, Number Theory and Its History, Ch. 15, pp. 340-358

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 5, Sec. 8, pp. 57-62

RADEMACHER AND TOEPLITZ, The Enjoyment of Mathematics, Ch. 13, pp. 82-88

YOUNG, Monographs in Modern Mathematics, Ch. 8, pp. 353-386

F. Farey Series

Primary References

JONES, The Theory of Numbers, Ch. 4, p. 92

NAGELL, Introduction to Number Theory, Ch. 1, p. 46

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 3, pp. 23-36

LEVEQUE, Topics in Number Theory, Ch. 8, Sec. 6, pp. 154-158

NIVEN, An Introduction to the Theory of Numbers, Ch. 6, pp. 128-133

G. The Period Of A Repeating Decimal

Primary References

JONES, The Theory of Numbers, Ch. 2, pp. 38-41, p. 51

MERRILL, Mathematical Excursions, Ch. 5, pp. 44-58

ORE, Number Theory and Its History, Ch. 13, pp. 311-325

STEWART, Theory of Numbers, Ch. 31, Sec. 4, pp. 228-233

Secondary References

HARDY AND WRIGHT, An Introduction to the Theory of Numbers, Ch. 9, pp. 107-128

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I. Magic Squares

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K. A Property of 30

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D. STUDY GUIDE IN PROBABILITY AND STATISTICS

COMMENTS:

This Guide concerns especially the background for a contemporary course in elementary probability and statistics, such as the course recommended as an option in Grade 12 by the Commission on Mathematics of the College Entrance Examination Board.

Descriptive statistics, which is concerned with collecting, organizing, summarizing, and presenting data, has, to a varying extent, often found a place in school mathematics programs; this aspect of statistics is mathematically almost trivial and places no special demands on the teacher. To the statistician, however, as to most users of statistics, the accumulation of data is only the beginning; the data are the known facts from which he seeks to make valid and useful inferences about the unknown. The investor studying a financial report is interested not so much in past performance as in future profit; the inspector studying a sample of mass-produced items must make a decision about the entire lot from which the sample is drawn. This problem of inference is the main issue in modern statistics, so much so that the subject has been aptly defined as the science of making wise decisions in the face of uncertainty. Probability not only furnished the mathematical foundation on which statistics is built but is, in its own right, an important, interesting, and fast-growing branch of mathematics.

The backgrounds of many teachers are likely to include a smattering of classical probability, usually taken as part of a course in Advanced Algebra. This may be more deceptive than helpful. Probability, in the classical treatment, consists largely of a collection of techniques for counting "equally likely" alternatives. The subject in its contemporary form makes extensive use of truly modern mathematical concepts, and a study of probability can therefore serve to illuminate and extend many important mathematical notions which are now beginning to find their way into the high-school program; it can furnish significant answers to the question "What and why is modern mathematics?" It sheds, for example, new light on the concepts of set and of function, in a way which is neither trivial nor contrived. The insight gained through a study of probability can therefore be useful to a teacher, even though he may not be teaching a course in the subject. He must recognize, however, that these intellectual gains (like most) are not to be achieved without effort. Probability is not a subject in which the beginner can fruitfully browse; it takes work, but rewards are in proportion.

ORGANIZATION OF THE GUIDE:

The subjects of probability and statistics have to do with the analysis of processes (experiments) whose outcomes are uncertain and therefore depend on chance. It is mathematically convenient to distinguish two basic types of experiments. The first, which is called the *discrete* type, is one in which the possible outcomes can be numbered, using positive integers. Examples of this type are the toss of a coin, in which there are two possible outcomes, or the roll of a dice, in which there are six, or the combination of hereditary factors in reproduction, where the number of possible outcomes may be very large indeed. The second, called the *continuous* type, is an experiment in which any one in a continuous interval of numbers may occur. Some examples of this type are the height of a man selected at random, or the duration of a telephone call, or the selection of a real number between 0 and 1.

The basic concepts of probability and statistics can be presented in terms of discrete experiments. When this is done, the only mathematics used is elementary algebra and set theory. The introduction of continuous experiments carries with it the introduction of more advanced mathematical concepts. In particular a working knowledge of the calculus is required.

The thorough knowledge of the basic concepts of probability and statistics for discrete experiments should serve as a sufficient background to teach an elementary course in probability and statistical inference. Part I of this outline presents the topics and references for this background.

The study of any branch of mathematics proceeds by successive levels of difficulty. The understanding of the subject at one level is generally very much increased by the study of the next level of difficulty. For this reason, Part II includes references for some of the more advanced topics in probability and statistics. The teacher, after mastering the material in Part I, may wish to study one or more of the topics in Part II. Similarly, a greater appreciation of the importance of probability and statistics comes from the study of applications. The references in Part I include many applications. However, the treatment is often adapted to illustrate a mathematical point rather than to contribute to a subject area. Therefore, Part III includes certain references where the emphasis is on the contribution of probability and statistics to subject areas outside of mathematics. It would be expected that the material in Part II and Part III is primarily for the background of the teacher. However, some of the material would serve for special projects and reading for advanced students.

For the basic Part I, the references have been organized with respect to each topic, and each reference is classified as a primary reference, a secondary reference, or a supplementary reference. The significance of these classifications is as follows:

a) A *primary reference* is one in which the presentation is simple, direct, and conveniently organized for the purpose of this Study Guide. Such a reference is recommended as a suitable first approach to the topic to which it is assigned.

b) A *secondary reference* is one in which the material, although readily readable, is presented in less convenient form.

c) A *supplementary reference* is one in which the presentation is at a higher mathematical level, or in which the scope is broader than that ordinarily needed for high school purposes.

For Part I of this Study Guide primary and secondary references are those which develop the discrete theory and which do not require mathematics other than elementary algebra. (The set theory used is developed in the probability references.) Since the statistical concepts are based on a knowledge of probability, the latter should be studied first.

I. BASIC CONCEPTS

Probability

a) Sample space
b) Events
c) Combinatorial problems
d) Conditional probability (events)
e) Binomial distribution
f) Random variable
g) Mean and variance of a random variable

Primary References

COMMISSION ON MATHEMATICS, **Introductory Probability**, Chs. 4, 5, 6, 8

This is an experimental text for use in high school courses in probability and statistics. It covers topics (a) through (e), and (g). There is an appendix on elementary set theory. A Teachers' Notes and Answer Guide is available.

GOLDBERG, **Probability**, Entire book

The content of this book is essentially the above topics treated for the discrete case. The prerequisite set theory is developed in the book.

MOSTELLER et al., **Probability with Statistical Applications.**

This book treats all of the above topics. The treatment is elementary but careful. The discussion of the binomial distribution is especially complete including a detailed discussion of the normal distribution as an approximation to the binomial. No calculus is assumed. (Note that it is also a primary reference for statistics.)

Secondary References

KEMENY et al., **Introduction to Finite Mathematics**, Chs. 2, 3, 4

This book covers topics (a) through (e) for the discrete case. It has the prerequisite material on set theory and also makes use of some elementary ideas from logic developed in Chapter I.

KEMENY et al., **Finite Mathematical Structures**, Chs. 2, 3, 7

This book covers all the above topics for both the discrete and continuous case. The discrete case may be studied separately. (Chs. 2, 3.) As in the case of FINITE MATHEMATICS, it makes use of some elementary logic. The topics treated here include those in FINITE MATHEMATICS, but from a somewhat more advanced viewpoint.

BRUNK, **Mathematical Statistics**, Chs. 1-5

This book covers all of the above topics. It develops these topics for both the discrete and continuous experiments, but in such a manner that the continuous cases can be omitted if desired. (Note that this book is also a secondary reference for the basic statistics.)

BIZLEY, **Probability**, Entire book

This book covers topics (a) through (e). It has both the discrete case and the continuous case, but the discrete case is treated first (Chs. 1-6), and may be considered separately. The topics of combinatorial problems and theorems on events are pursued further than is usual in an elementary book. There are a large number of non-routine problems.

NEYMAN, **Probability and Statistics**, Ch. 2; Ch. 4, Sec. 1, 2

This book treats essentially all of the above topics. The treatment is restricted to the discrete case. The going is somewhat slow because the author has given more than the usual amount of attention to the nature of a probability model, and because of the introduction of some quite complicated but very interesting examples. (Note that this book is a primary reference for statistics.)

Supplementary References

FELLER, **Probability Theory and Its Applications**, Chs. 1-9

This book restricts itself to the discrete case. It is simultaneously a text book and a basic reference. A wealth of applications, both elementary and advanced, are given. This book is generally acknowledged to be the most stimulating book yet written on the subject of probability theory. The work is often hard, but the rewards are correspondingly high.

PARZEN, **Modern Probability Theory**, Chs. 1-8

This book treats all of the above topics in a very thorough manner, covering both the discrete and continuous case. A variety of applications of probability are given during the development of the theory.

CRAMER, **The Elements of Probability Theory**, Chs. 1-6

MOOD, **Theory of Statistics**, Chs. 1-5

These two books treat the above topics for both discrete and continuous cases. They require a good knowledge of calculus topics. (Note that they serve also as supplementary references for the basic statistics topics.)

Statistics

a) Descriptive statistics
b) Elements of sampling
c) Hypothesis testing
d) Estimation (introduction)

Primary References

COMMISSION ON MATHEMATICS, **Introductory Probability**, Chs. 1, 2, 3, 7

Topics (a), (b), and (c) are discussed mainly in terms of examples. The relation of probability theory to these topics is stressed.

MCCARTHY, **Introduction to Statistical Reasoning**, Chs. 1-8

By restricting the discussion to discrete experiments, the author is able to give a quite thorough discussion of the above topics. The concepts of statistics are illustrated primarily in terms of the binomial model.

NEYMAN, **Probability and Statistics**, Chs. 1, 5

This book contains a very fine discussion of hypothesis testing in terms of discrete models. The author is one of the founders of this subject.

WALLIS AND ROBERTS, **Statistics**, Parts I, II, III

This is a particularly good reference for topics (a) and (b). There is a wealth of examples of the analysis of data from a great variety of fields. There is a lively discussion of some of the misuses of statistics. There are also a large number of interesting exercises. The treatment of topics

(c) and (d) is somewhat superficial because the authors do not assume a knowledge of probability theory.

MOSTELLER et al., **Probability with Statistical Applications**

This book treats all of the above topics with the development based on probability theory. (Note that it is also a primary reference for probability theory.)

Secondary References

BRUNK, **Mathematical Statistics**, Chs. 6-10

This book considers all of the above topics. Some probability topics beyond those of Part I are introduced to give a more thorough treatment. The book does a very good job of integrating the probability theory and the statistics. Calculus is used, but moderately.

Supplementary References

CRAMER, **The Elements of Probability Theory**, Chs. 11-14

MOOD, **Theory of Statistics**, Chs. 7-12

These are two standard books which cover all of the above topics for both the discrete and continuous case. A working knowledge of calculus is therefore required. The treatment in both cases is very complete.

NOTE: In addition to the above books there have been interesting articles on topics in mathematics and statistics in the yearbooks of the National Council of Teachers of Mathematics. The TWENTY THIRD YEARBOOK, Ch. 11 is a fine survey article on basic probability theory written by H. Robbins. The article makes use of calculus concepts. A more elementary approach is given in the TWENTY FOURTH YEARBOOK, Chs. 6, and 7. Chapter 6 is an article on probability by David A. Page. Chapter 7, by Richard S. Pieters and John Kinsella, is on statistics and draws on the ideas of probability introduced in Chapter 6.

II. MORE ADVANCED TOPICS

Probability

a) Basic distributions and their applications (Poisson, normal, exponential, etc.)

CRAMER, **The Elements of Probability Theory**, Chs. 5-8

PARZEN, **Modern Probability Theory**, Ch. 6

Both of these books have quite complete discussions of distributions. Calculus is used. Parzen's book has a variety of applications.

NEYMAN, **Probability and Statistics**, Ch. 4

FELLER, **Probability Theory and Its Applications**, Chs. 6, 7

These authors restrict themselves to distributions which arise from discrete experiments and to a discussion of continuous distributions as they arise as limiting cases of discrete experiments. Both have numerous interesting applications.

b) Limit theorems (Poisson approximation to binomial, central limit theorem, law of large numbers).

CRAMER, The Elements of Probability Theory, Chs. 6, 7

BRUNK, Mathematical Statistics, Chs. 7, 9

These books give careful statements of the basic limit theorems for both discrete and continuous experiments. Proofs are given only for certain discrete cases.

KEMENY et al., Finite Mathematical Structures, Ch. 3

NEYMAN, Probability and Statistics, Ch. 4

Both books contain a discussion of the limit theorems for the discrete case only. Neyman has a quite complete discussion of the central limit theorem using only elementary mathematics.

FELLER, Probability Theory and Its Applications, Chs. 7-10

The discussion is restricted to discrete experiments but is very thorough. A good familiarity with limit techniques is necessary to follow this treatment.

c) Stochastic processes (Markov chains, random walks, and Poisson processes).

KEMENY et al., Finite Mathematical Structures, Ch. 6

This book treats finite Markov chains and random walks. The only advanced mathematics used is elementary matrix theory.

KEMENY AND SNELL, Finite Markov Chains, Entire book

This is a quite complete treatment of finite Markov chains based on matrix methods. A number of applications are discussed.

FELLER, Probability Theory and Its Applications, Chs. 14-17

This book goes quite far into these topics and gives a number of significant applications. The method used requires a good knowledge of topics in elementary analysis.

Statistics

a) Theory of estimation (Properties of estimators and types of estimators)

CRAMER, The Elements of Probability Theory, Ch. 14

MOOD, Theory of Statistics, Ch. 8

BRUNK, Mathematical Statistics, Ch. 8

All of these books have quite complete discussions of these topics. The treatments rely on a good knowledge of calculus, including ideas in calculus of more than one variable.

b) Decision theory (strategies, decision criteria)

CHERNOFF AND MOSES, Elementary Decision Theory, Chs. 4-8

A very complete discussion of decision theory. The restriction primarily to discrete experiments enables the authors to give a careful treatment of the basic ideas of the subject, using only elementary mathematics.

LUCE AND RAIFFA, Games and Decisions, Chs. 4, 13

An introduction to the basic ideas of decision theory with particular emphasis on the arguments which have been presented pro and con for the use of decision theory.

BRUNK, **Mathematical Statistics**, Ch. 11

A good introduction to the basic ideas of decision theory. The treatment is brief and uses calculus.

DERMAN AND KLEIN, **Probability and Statistical Inference**, Ch. 3

A quite elementary account of decision theory, with special attention to applications in Engineering.

SCHLAIFER, **Probability and Statistics for Business Decisions**, Chs. 1-4

A very complete treatment with special emphasis on business problems. The book uses only elementary mathematics.

III. APPLICATIONS

Probability

Genetics: Applications to genetics involving only elementary probability theory can be found in Goldberg (11) and Neyman (20). Applications which make use of Markov chain theory can be found in Kemeny (13) and (14) and Feller (8).

Physics: Applications of random walk theory to simple gas models may be found in Kemeny (12), (13) and Feller (8). A discussion of the role of special distributions in statistical mechanics may be found in Fry (10).

Engineering: Applications of the Poisson process to problems in telephone exchanges and other engineering problems can be found in Fry (7). A discussion of the relation between random walk theory and electric circuits may be found in Kemeny (12). For applications of probability theory in communication theory see Shannon (23).

Psychology: Certain of the modern theories of learning are based on probability models. For an introduction to one such theory see Kemeny (14). A more complete discussion may be found in Bush and Mosteller (2) and Luce (15). The latter book contains other applications of probability theory to psychology.

Sociology: For an application of Markov chain theory to social mobility see Kemeny (13).

Statistics

Applications of statistics consist chiefly of developing special tests for certain probability models. The tests are usually described in the discussion of the following topics:

a) Tests for mean and variance (t-test, F-test)
b) Fitting a straight line to data (correlation)
c) Contingency tables
d) Quality control
e) Non-parametric tests

For an elementary discussion of the way in which these tests are applied see either Wilks (25), Wallis and Roberts (24), Freund (9) or Dixon and Massey (7). There is a very good elementary treatment of (b) in Mosteller (19). Applications in the social sciences make particular use of non-parametric tests. One such test, the sign test, is discussed in Mosteller (19). These topics are discussed on a more theoretical level in Mood (18), Cramer (5), and Brunk (3.)

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