DOCUMENT RESUME

ED 175 668 SE 028 656

AUTHOR Clark, Ronald J., Ed.
TITLE Supplementary and Enrichment Series: Non-Metric Geometry. Teachers' Commentary. SP-9.
INSTITUTION Stanford Univ., Calif. School Mathematics Study Group.
SPONS AGENCY National Science F[illegible]dation, Washington, D.C.
PUB DATE 64
NOTE 26p.: For related documents, see SE 028 648-675: Contains occasional light and broken type

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Curriculum: *Curriculum Guides: Enrichment: *Geometric Concepts; *Geometry: *Instruction: Mathematics Education: Secondary Education: *Secondary School Mathematics: *Set Theory
IDENTIFIERS *School Mathematics Study Group

ABSTRACT

This is one in a series of manuals for teachers using SMSG high school supplementary materials. The pamphlet includes commentari[illegible] on the sections of the student's booklet, answers to the exercises, [illegible]d sample test questions. Topics covered include points, lines, spa[illegible] planes, names, intersection of sets, intersection of lines and planes, segments, separations, angles, one-to-one correspondence, and simple closed curves. (MP)

SCHOOL
MATHEMATICS
STUDY GROUP

SP-9

SUPPLEMENTARY and ENRICHMENT SERIES

NON-METRIC GEOMETRY

Teachers' Commentary

Edited by Ronald J. Clark

SMSG

Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

PHOTOLITHOPRINTED BY CUSHING - MALLOY, INC.
ANN ARBOR, MICHIGAN, UNITED STATES OF AMERICA

PREFACE

Mathematics is such a vast and rapidly expanding field of study that there are inevitably many important and fascinating aspects of the subject which, though within the grasp of secondary school students, do not find a place in the curriculum simply because of a lack of time.

Many classes and individual students, however, may find time to pursue mathematical topics of special interest to them. This series of pamphlets, whose production is sponsored by the School Mathematics Study Group, is designed to make material for such study readily accessible in classroom quantity.

Some of the pamphlets deal with material found in the regular curriculum but in a more extensive or intensive manner or from a novel point of view. Others deal with topics not usually found at all in the standard curriculum. It is hoped that these pamphlets will find use in classrooms in at least two ways. Some of the pamphlets produced could be used to extend the work done by a class with a regular textbook but others could be used profitably when teachers want to experiment with a treatment of a topic different from the treatment in the regular text of the class. In all cases, the pamphlets are designed to promote the enjoyment of studying mathematics.

CONTENTS

TEACHER'S COMMENTARY AND ANSWERS

NON-METRIC GEOMETRY

The principal objectiv s of this pamphlet are threefold:

(1) To introduce pupils to geometric ideas and ways of thought

(2) To give pupils some familiarity with the terminology and notation of "sets" and geometry, and

(3) To encourage precision of language and thought.

There is an attempt to guide the student to the discovery of unifying concepts as a basis for learning some of the more specific details. This pamphlet attempts to focus attention on ideas which are fundamental but which (while sometimes vaguely taken for granted) are often poorly understood by students.

Traditionally, these ideas have been taught when they were needed for a particular geometric discussion. All too often, however, the teacher has assumed that these properties are obvious or clear without mentioning them. Also, there should be some advantage in considering together this group of closely related analagous properties and observing relations among them. The higher level study of some aspects of non-metric geometry has become a separate mathematical discipline known as projective geometry.

Reading the text. This pamphlet has been written with the intent that it be carefully read by students. We suggest that not only should students be assigned to read the material, but that they also be encouraged to study it. Reading mathematics is not like reading a novel. Students may find it necessary to "get in and dig" for ideas and should be advised to read with a paper and pencil at hand so that they may draw diagrams to assist their understanding.

Precision of language. Pupils should be encouraged to express themselves accurately. Some pupils will be able to do much better than others.

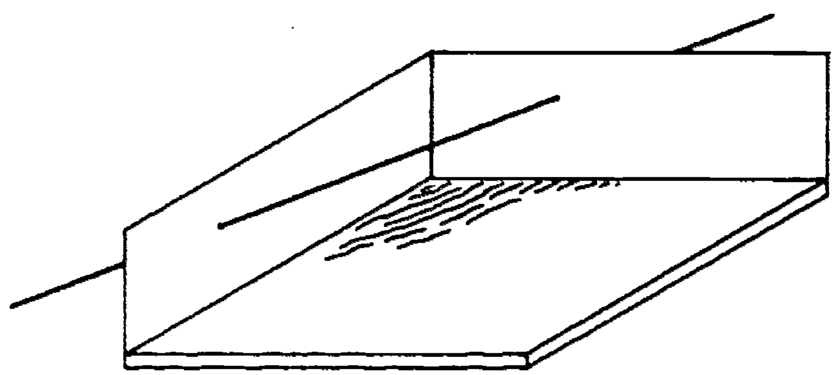
Spatial Perception. It has been our effort here to help boys and girls develop spatial understanding. We do this in part by representations in space. We hope that suggestions given to the students and the notes here may be helpful in selecting other representations appropriate for your class. The pamphlet is intended to provide background which is sometimes assumed in later courses.

Testing. In testing, try to test for grasp of ideas not for mere recall. Students should be encouraged to express ideas in their own words. Because abilities in geometric perception and understanding differ from abilities in arithmetic, you may observe some redistribution of high and low grades among your students.

Materials. Insights into ideas developed in this pamphlet will be greatly enhanced by use of instructional devices. Encourage students to make simple models as a means of helping in basic understandings. Emphasize ideas, not evaluation of models. As in using any instructional material of this kind, seek understanding of ideas without over-dependence upon representations.

Suggested Materials:

String--to represent lines in space
Paper--to represent planes, and folded to represent lines and intersections of planes
Tape or tacks for attaching string to walls, floor, other points in the room
Model (as illustrated)



Suggest making the model as shown above by using a cardboard carton (or, it can be made using heavy paper, oak tag, screen wire). Cut away two sides so that only 2 adjacent sides and bottom of box remain. String, wire, etc., may be used to extend through and beyond "sides", "floor", etc.
Oak tag for making models to be used by both teacher and students
Coat-hanger wire, knitting needles, pickup sticks
Scissors, colored chalk
Light-weight paper (for tracing in exercises)
Yardstick or meter stick with several lengths of string tied to it at different intervals. By fastening stick to wall, lines may be represented by holding the string taut. By gathering together the free ends at one point the plane containing the point and the yardstick may be shown.

Optional:

Long pointer for seeing lines
Tinker-Toys for building models
Toothpicks for student models
Saran wrap, cellophane, and wire frame for representing planes

1. Points, Lines, and Space

1. Understandings:

   (a) A point has no size.

   (b) A line is a certain set of points.

   (c) A line is unlimited in extent.

   (d) Through two points there is one and only one line.

   (e) Space is a set of points.

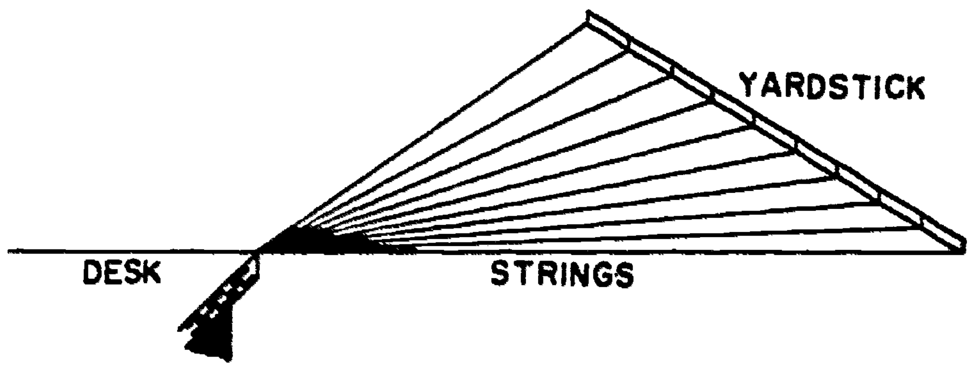
2. Teaching Suggestions

Just as we use representations to develop the concept of the "counting numbers" (2 cars, 2 people, 2 hands, 2 balls, 2 chairs, etc. to develop the concept of twoness) similarly we must select representations for developing the concepts of point, line, plane, and space.

Point. Identify things which suggest the idea of a point keeping in mind that one suggestion by itself is not adequate for developing the idea of a point. One needs to use many illustrations in different situations. Suggestions: tip of pencil, needle, pointer; collection of boxes, putting one inside another and always being able to place one more inside of the last and the point indicated as being in all the boxes; pupil of the eye in intense brightness; progressive closure of shutter of camera; dot of light on some TV screens; particle of dust in the air.

Line. Identify two points using some of the situations as above, such as tips of two pencils, etc. Bring out the idea that given these two points there are many other points of the line that contain them. Some of these are between the two points, some are "beyond" the one, and some are "beyond" the other. Also, through two points there can be only one line. The line has no thickness and no width. It is considered to extend indefinitely. Use string held taut between two points to show representations of lines in positions that are horizontal, vertical, and slanting. Each student may represent lines by using a pencil between his fingertips. With each example talk about thinking of a line as unlimited in extent. Emphasize frequently that we use the word, "line," to mean straight line. Identify other representations of lines such as: edge of tablet (holding tablet in various positions); edge of desk; vapor trails; edges of roof of building; etc. It is important to select illustrations representing lines in space as well as the usual representations made by drawing on chalkboard and paper.

Space. Models will be most helpful here. Using "string on yardstick" and considering some point on a table, desk, or on some object which all students can see, let all the representations of lines from the yardstick pass through the point. Also, use string to show representations of other lines from other points on different walls, the floor, etc., all passing through the point. Use the model as described in drawing under "Suggested Materials". Pass lines (string, wire, thread) through "walls" and "floor" to suggest infinite number of lines and that these lines extend indefinitely.



Bring out the idea that each line is a set of points, and that space is made up of all the points on all such lines.

Answers to Class Discussion Problems

1. Depends on what objects are in the room.
2. The body of the porcupine is like the point. The quills are like lines through this point.

Answers to Exercises 1

1. No, the ribbons are usually not straight.
2. There is exactly one line containing the two points on the monuments.
3. The line of the new string passes through the same two points as the line of the old. (Property 1)

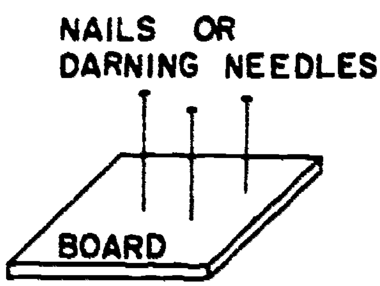
2. Planes

1. Understandings
   (a) A plane is a set of points in space.
   (b) If a line contains two different points of a plane, it lies in the plane.
   (c) Many different planes contain a particular pair of points.
   (d) Three points not exactly in a straight line determine a unique plane.

2. Teaching Suggestions

Identify surfaces in the room which suggest a plane--walls, tops of desks, windows, floor, sheet of paper, piece of cardboard, chalkboard, shadow. Make use of Saran wrap, cellophane, and a wire frame to show further a representation of a plane since this more nearly approaches the mathematician's idea of a plane. With each example bring out the idea that a plane has no boundaries, that it is flat, and extends indefinitely. It is an "ideal" of a situation just as is a line and a point. We try to give this idea by suggesting things that represent a plane. It is important to suggest representations of planes in horizontal, vertical, and slanting positions. Note that if a line contains 2 points of a plane, it lies in the plane and that many planes may be on a particular pair of points as pages of a book, revolving door, etc.

Then using three fingers or sticks of different heights in sets of 3 (not in a straight line) as suggested by the sketch at the right, see what happens when a piece of cardboard is placed on them. Add a fourth finger or a fourth stick and observe what happens. Each student may try this experiment by using three fingers of one hand (and also three fingers using both hands) letting a plane be represented by a book, piece of oak tag, card. Change position of fingers and thumb by bending the wrist (changing sticks in model). Ask class to make a statement about three points not in a straight line (Property 3).



Demonstrate with wires or string the ideas in the last paragraph before asking students to read it or suggest that one or two students be responsible for demonstrating the idea to other members of the class.

The Class Discussion Problems may well be developed as a class activity.

A note. What is a basic motivation for the study of geometry? In our daily living we are forced to deal with many flat surfaces and with things like flat surfaces. It would be foolish not to note similarities of these objects. So, we try to note them. In so doing we try to abstract the notion of flat surface. We try to find properties that all flat surfaces have. Thus, we are led to an abstraction of the flat surface--the geometric plane. We study two aspects of this--(1) What a plane is like, considered by itself (plane geometry), and (2) how various planes (flat surfaces) can be related in space (one aspect of spatial geometry).

Just how do we study the geometric plane? We study it by thinking of what the plane is supposed to represent, namely, a flat surface. However, in trying to understand a plane (or planes), we find it difficult to think just abstractly. Thus, we think of representations of the plane--wall, chalkboard, paper, etc., and we think of these as representations of the abstract idea. The abstract idea enables us to identify characteristics which all flat surfaces have in common.

Answers to Class Discussion Problems

1. Depends on the particular classroom.
2. There are many different sloping positions, illustrating Property 2. Only one, illustrating Property 3.

Answers to Exercises 2

1. All in the same line
2. Not all in the same line
3. All legs must rest on the same plane or the device may rock. Three points not all on the same line are in exactly one plane.
4. (a) Many (unlimited quantity) (b) One
5. (a) Many
   (b) Many
   (c) Exactly one, unless the points are all on the same line.
6. Wing surface, flat surface used on water. Consult dictionary.

*7. (a) Six, as long as no three are on the same line
   (b) Six

*8. Explanation in Section 5

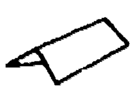
3. Names and Symbols

1. <u>Understandings</u>
   - (a) Students learn to recognize how planes, lines on planes, lines through planes, etc. are represented by drawings.
   - (b) Students learn to name particular points, lines, and planes using letters, etc.
   - (c) Students learn how to interpret and understand perspective drawings.
   - (d) Students learn to develop an awareness of planes and lines suggested by familiar objects.

2. <u>Teaching Suggestions</u>

Bring out the idea that we make agreements as to how to represent certain ideas, i.e. "." for a point, "  or  . " for a line, and the use of letters for naming lines and points. Note that we usually name points by capital letters, lines by lower case letters or pairs of capital letters with bar and arrows above, as, $\overleftrightarrow{AB}$, and that a plane is named by three capital letters or a single capital letter. Also, we sometimes talk about two or more lines, planes, etc. by using subscripts, such as, ℓ_1, ℓ_2, and ℓ_3.

We do not expect students to learn to make drawings showing more than one plane, intersections of planes, etc., but we want them to learn to interpret drawings. However, if some students wish to attempt such drawings, certainly do not discourage them.

Students enjoy a guessing game about abstract figures such as these:  "man looking over fence" or  "boat sinking". They might enjoy a similar game with planar abstractions such as:  "open door" or  "pup tent" or  "butcher knife".

<u>Answers to Exercises 3</u>

1. A is to the left, B to the right
   C is nearest the top, D nearest the bottom

2. A table

3. It has turned upside down.

4. (a) Cot
(b) Ping pong table
(c) Football field
(d) Carpet
(e) High jump
(f) Coffee table
(g) Line of laundry
(h) Open door
(i) Chair
(j) Shelf
(k) Ladder

6. Yes (or no). We assume point V is on line $\overleftrightarrow{PQ}$ and not "behind" $\overleftrightarrow{PQ}$.
No
No
One, P.

7. 1, b
2, d
3, c
4, d
5, a
6, a

8. Yes, Yes
Yes, ℓ is the only line through P and Q.
ℓ_2 is the intersection of M_1 and M_2.
No, No, No.

---

4. Intersection of Sets

1. Understandings
   (a) A set contains elements which are collected according to some common property or explicit enumeration.
   (b) The common elements in two or more sets make up the elements of the intersection of two or more sets.

2. Teaching Suggestions

Review the idea of sets by asking students to describe certain sets, as set of names of members of the class, the set of members of the class, set of all students in class whose last name begins with "B", set of even numbers, set of counting numbers between 12 and 70 having a factor 7 (i.e. 14, 21, 28, . . .).

Explain that any 2 sets determine a set which is called their intersection, that is, the set of elements (if any) which are in both sets. Have students give the intersection for the set of odd numbers between 1 and 30. Note the three sets--the two given sets and the intersection of the two sets. Use other illustrations such as set of boys in the class and the set of students with brown eyes. Also, find intersection of three sets--students in class having blonde hair, students in class having birthdays in November, and students in class riding the school bus. In selecting sets, include some in geometry (i.e. the intersection of two lines in the same plane, etc.).

Note that the empty set is the intersection of two sets with no elements in common.

After developing the idea of intersection go back to examples and describe how the idea can be expressed in symbols. It is a code we can use and like many codes it simplifies the expression. For example,

Set A = {3,5,7,9,11,13,15,17,19,21,23,25,27,29}

Set B = {3,6,9,12,15,18,21,24,27}

$A \cap B$ = {3,9,15,21,27}

Answers to Exercises 4

1. (a) {18,19,20,21,22}
   (b) As in your state
   (c) The empty set
2. (a) 1, 3, 5 (a possible answer)
   (b) 5, 10, 15 (a possible answer)
   (c) P, R, T (a possible answer)
3. (a) 9, 10, 11, 12
   (b) As in your class
   (c) The point P
   (d) The line
4. $S \cap T$ = {10,15,23}
5. (a) An edge
   (b) The empty set

6. (a) S = {Maine, N. H., Vt., Conn., Mass., R. I.}
   T = {New Hampshire, New York, New Jersey, New Mexico}
   V = {Texas, New Mexico, Arizona, California}
   (b) New Hampshire
   (c) The empty set
   (d) New Mexico

7. (a) Yes
   (b) Yes. The addition of multiples of 5 gives multiples of 5.

8. Each part is almost obvious from the notions of set and intersection.
   (a) This is by definition since the empty set is a set.
   (b) The set of those elements in X which are also in Y is the set of those elements in Y which are also in X.
   (c) Each "side" of the equality means the same set: namely the set of all elements which are in each of the three sets X, Y, and Z.

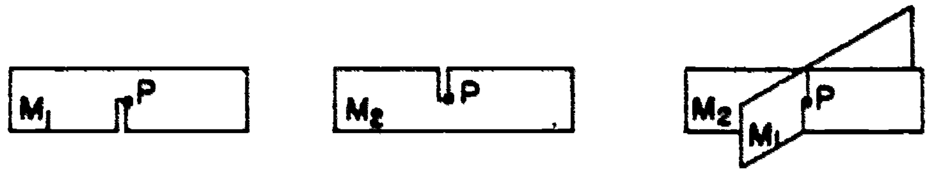
---

5. Intersections of Lines and Planes

1. Understandings
   (a) Two lines may:
      (1) be in the same plane and intersect.
      (2) be in the same plane and not intersect (intersect in the empty set).
      (3) not be in the same plane and not intersect (intersect in the empty set).
   (b) A line and a plane may:
      (1) not intersect (intersect in the empty set).
      (2) intersect in one point.
      (3) intersect in a line.
   (c) Two different planes may:
      (1) intersect and their intersection will be a line.
      (2) not intersect (have an empty intersection).

2. Teaching Suggestions

Use models in order to explore the possible situations for two lines intersecting and not intersecting. (Let each student have materials, too.) Also, use a pencil or some other object to represent a line, and a card to represent a plane. Use two pieces of cardboard each cut to center with the two fitted together to represent the idea of two planes and their intersection, and, from these, state some generalizations that may be made.



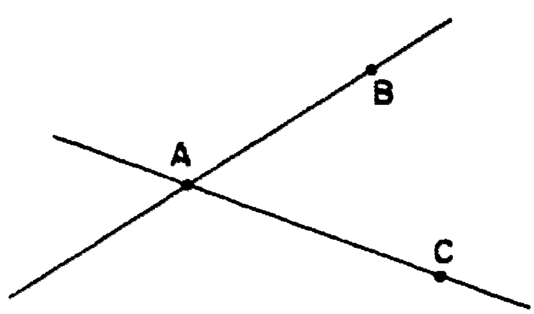
Also identify situations in the room which are representations of different cases of intersections of lines and planes. Some may wish to express the ideas in symbols. For example, name the line of one of the front vertical edges ℓ, and line along the top front of desk, k. Then $\ell \cap k$ is a point.

Subscripts also may be used so that we talk about lines ℓ_1 and ℓ_2. The use of a few subscripts should be encouraged.

Answers to Exercises 5

1. (a) Intersection of line and plane, empty set. Line and plane are parallel. No point.
   (b) Intersection of line and plane, one point line pierces plane.
   (c) Intersection of line and plane more than one point. Line lies in plane.

2. Intersection of floor and front wall, intersection of ceiling and side wall, etc.

3. 

   $AB \cap AC$ = Point A

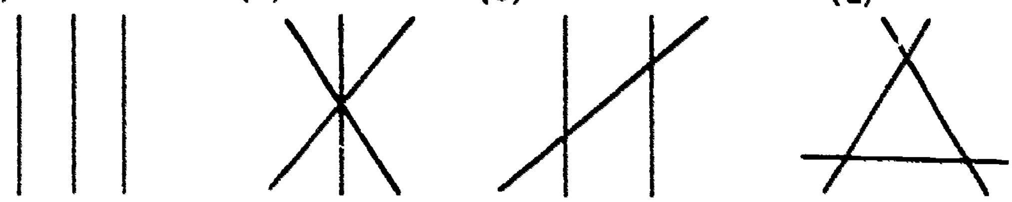
4. Yes, yes, no (no point in all planes), the empty set, no.

5. Yes, yes, one point

6. Yes, yes, a line (the fold)

7. (4) The empty set
   (5) One point
   (6) The line

8. (a) (b) (c) (d)



0 points 1 point 2 points 3 points

Using Only Lettered Points

9. (a) HGD and ABC (one other possibility)
   (b) HGB and GBC (many other possibilities)
   (c) HGB, BGD, FGD (many other possibilities)
   (d) HGD, FGD, BGD (some other possibilities)
   (e) ABC, FE (many other possibilities)
   (f) FE, GD (many other possibilities)
   (g) ED, AB (many other possibilities)
   (h) ED, GD, CD (many other possibilities)
   (i) FGB, FGD, HGD, BGD

---

6. Segments

1. Understandings
   (a) If A, B, and C are three points on a line, our intuition tells us which point is between the other two.
   (b) A line segment is determined by any two points and is on the line containing those points.
   (c) The two points which determine a segment are called endpoints of the segment.
   (d) A line segment is a set of points which consists of its endpoints and all points between them.
   (e) The union of two sets consists of all the elements of the two sets.

2. Teaching Suggestions

Bring out the idea that when we draw a sketch or a picture of a line, we draw a picture of one part of the line, and that this is, properly, a line segment. However, we often represent a line by a part of a line (since we cannot do anything else). One should be careful to say that the sketch represents a line or line segment as is appropriate. Draw a representation of a line on the chalkboard and name two points of the line, A and B. Note that $\overline{AB}$ means points A and B and all points between them. Name other points on the line and various line segments.

Review the idea of intersection of two sets. Then develop the idea of union of two sets using line segments much as is suggested by student text. Extend this idea by selecting other illustrations, as, set of girls in class and set of boys in class and that the union of the two sets is the set obtained by combining the two sets. Each member of the union is then either a boy or a girl in the class. Have students suggest other sets, noting what can be said about the elements of the union of the two or more sets. Take illustrations from geometry too, such as, the union of two lines, line and plane, etc.

Some students prefer expressing these ideas in symbols. If so, you may wish to express one of illustrations in symbols, as,

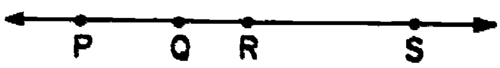
$$\text{Set } A = \{4,8,12,16,20,24,28\}$$
$$\text{Set } B = \{6,12,18,24\}$$
$$A \cup B = \{4,6,8,12,16,18,20,24,28\}$$

Again note that if an element is a member of both sets, it appears only once in their union.

<u>Answers to Exercises 6</u>

In text $\overline{BC}$

1.



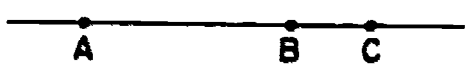
(a) $\overline{PR}$, $\overline{QS}$ or $\overline{PR}$, $\overline{QR}$ etc.

(b) $\overline{PQ}$, $\overline{QR}$ etc.

(c) $\overline{PQ}$, $\overline{RS}$

(d) $\overline{PQ}$, $\overline{RS}$

2.



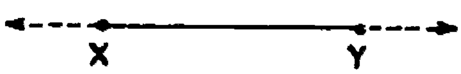
(a) The point B

(b) $\overline{BC}$

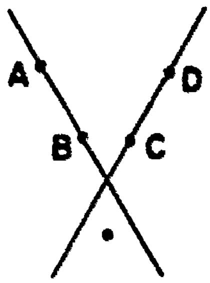
(c) $\overline{BC}$

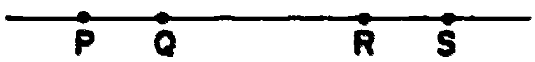
(d) $\overline{AC}$

3. No;

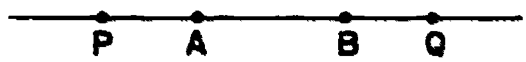


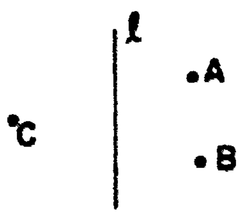
yes

4. 

5. 

6. No.



7. 

Yes; no.

8. We would have more than one "straight line" through two points. See question 6.

9. (a) Definition
   (b) The set X ⋃ Y contains exactly the same members as Y ⋃ X. The sets, therefore, are equal.
   (c) (X ⋃ Y) ⋃ Z is made up of the same elements as X ⋃ (Y ⋃ Z).

10. X ⋃ X will contain <u>one</u> set of elements, exactly the same set as X.

---

7. Separations

1. <u>Understandings</u>
   (a) A plane separates space into two half-spaces.
   (b) A line separates a plane into two half-planes.
   (c) A point separates a line into two half-lines.
   (d) A ray is the union of a half-line and the point which determines the half-line.

2. Teaching Suggestions

Use cardboard models to develop understanding of these ideas. Note how one side of the model pictured in the materials section of this Commentary separates space and that we call the two sides half-spaces. Ask how we could separate a plane into two half-planes and a line into two half-lines before the students have read these paragraphs.

This section gives an unusually good opportunity to emphasize relations among point, line, plane, and space. You can expect seventh grade students particularly to enjoy this section. It gives a certain structure to geometry.

Draw a number of lines on the chalkboard. Mark points on them and talk about half-lines, rays, and endpoints. Talk about the intersection of 2 rays, two half-lines, and ray and half-line. If students are worried as to whether a half-line has an endpoint, see explanation in student text for saying "an angle of a triangle."

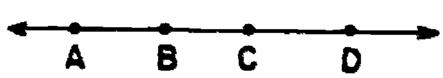
Also identify representations of half-spaces, produced by room dividers, walls in buildings; of half-planes, by lines on paper, lines on wall, etc.; and of half-line by naming a particular point along the edge of a ruler.

Answers to Exercises 7

1. Yes; no; no; no; yes; yes.

2. $\overline{AB}$ (or $\overline{BA}$); a half line to the left of A.

3.



(a) $\overrightarrow{BA}$, $\overrightarrow{BC}$

(b) $\overrightarrow{BA}$, $\overrightarrow{CD}$

(c) $\overrightarrow{BC}$, $\overrightarrow{CD}$

(d) $\overrightarrow{BA}$, $\overrightarrow{BC}$

(e) $\overrightarrow{BA}$, $\overrightarrow{CD}$

4. No, no

5. Yes, yes

6. Yes

* 7. Not if they are half-planes from the same line, but yes if you use parallel lines and overlapping half-planes

* 8. No; yes

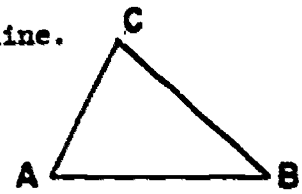
8. Angles and Triangles

1. Understandings

(a) An angle is a set of points consisting of 2 rays not both on the same straight line and having an endpoint in common.

(b) An angle separates the plane containing it.

(c) A triangle is the union of three sets, $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ where A, B, and C are any points not on the same line.



(d) A triangle determines its angles but does not contain its angles.

2. Teaching Suggestions

Illustrate the idea of angle as two rays with the same endpoint. Use colored chalk to show interior and exterior. Also note how we name an angle. (This is not the only way to consider an angle. This definition, however, is identical with that which is very likely to be used in the 10th grade. Later, the measure of an angle and angle as a rotation will be considered.

In developing the idea of triangle, put three points on board and note them as endpoints of 3 line segments, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$. Note the set of points in each segment and that a triangle is the union of these three sets. Use colored chalk to show interior and exterior. Emphasize the set of points of the interior, the exterior, and that of the triangle.

Again students may be interested in drawing angles, triangles, shading, etc. (This is not perspective drawing.) This is good material to develop imagination and to encourage drawing etc. How many triangles can be drawn with just four lines? How many with four lines two of which are parallel? Etc.

In discussing the angles of the triangle bring out the idea that although people often talk about angles of a triangle, it is a short way of saying that they are the angles determined by the triangle. Use such analogies as the school determines its graduates but graduates are not in school; a city "has" suburbs, but the suburbs are not part of the city, etc.

Answers to Exercises 8

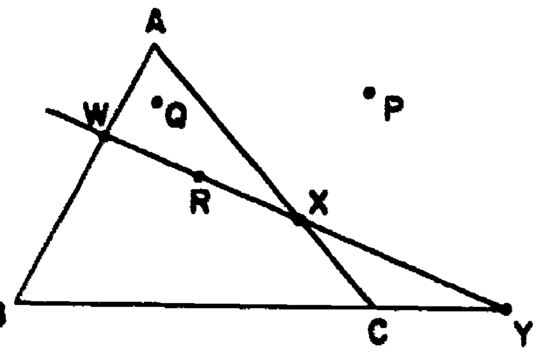
1. (a) The interior of $\angle ABC$
   (b) The interior of $\triangle ABC$

2. (a) Yes, they have different vertices
   (b) Yes
   (c) The lines containing the rays determine a plane.

3. (a) The point A
   (b) No
   (c) $\overline{AB}$
   (d) $\overline{AB}$

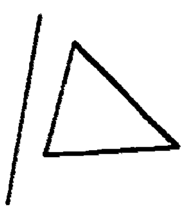
4. (a) The two points X and W
   (b) $\triangle AWX$, $\triangle ABC$, $\triangle BWY$, $\triangle XCY$
   (c) None
   (d) A, B, C, W and Y
   (e) B, A

5. (a)
   (b)
   (c)

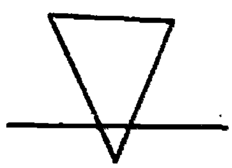


R can be on $\overline{WX}$

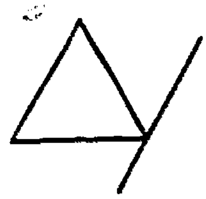
6. (a)



(b)

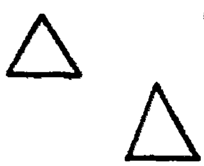


(c)

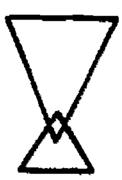


   (d) Not possible

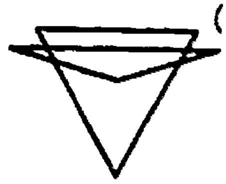
7. (a)



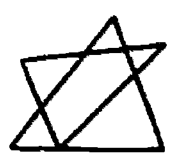
(b)



(c)



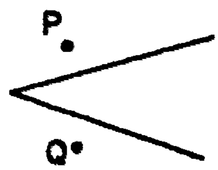
(d)



8. (a) The points A and C
   (b) $\overleftrightarrow{AB}$
   (c) The points A and B
   (d) The point B
   (e) $\angle ACB$
   (f) $\overline{BC}$
   (g) $\overleftrightarrow{BC}$
   (h) The union of $\overline{AB}$ and $\overline{BC}$

*9. No; yes.

*10. (a) Yes
   (b) It may or may not, depending on choice of P and Q.
   (c) Yes
   (d) No



---

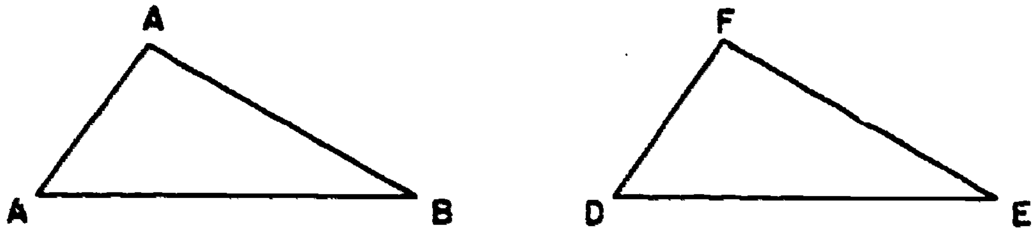
9. One-to-one Correspondence

1. Understandings
   (a) The idea of one-to-one correspondence is fundamental in counting.
   (b) One-to-one correspondences in geometry can be established:
      (1) Between a certain set of lines and a certain set of points.
      (2) Between the set of points of one segment and the set of points of another segment.
      (3) Between certain other geometric sets in pairs.

2. Teaching Suggestions

Review the idea of one-to-one correspondence and the necessary condition that for each element in set A there corresponds an element in set B. For example, if there are 5 chairs and 5 people, for each chair there is a person and for each person there is a chair.

This idea, while elementary, is sometimes hard to grasp. One-to-one correspondences between finite sets (sets having a specific number of elements as in the illustration above) are easy to observe if they exist. If sets A and B are finite, then if A and B have the same number of elements, there exists a one-to-one correspondence between them. Such follows simply by counting. But we are sometimes interested in a particular one of the possible one-to-one correspondences. For the congruent triangles below we are interested in matching A with D, B with E and C with



F. It is on such basis that we get the congruence. If we were to match A with F, B with D, and C with E we would not be noting the congruence.

For infinite sets H and K, the problem is much more complex. To establish a one-to-one correspondence we need (1) a complete matching scheme, and (2) in this particular device it must be true that for any element of either set there corresponds a unique element of the other set. It is implied by what we say that if <u>a</u> corresponds to <u>b</u>, then <u>b</u> corresponds to <u>a</u>.

In effect, to establish a one-to-one correspondence we must have a way of "tying" each element of either set to a particular element of the other. And the "string" we use for tying <u>a</u> to <u>b</u> also ties <u>b</u> to <u>a</u>.

In the text we describe a one-to-one correspondence between a certain set of lines and a certain set of points.

Answers to Exercises 9

1. That depends on the class and classroom.

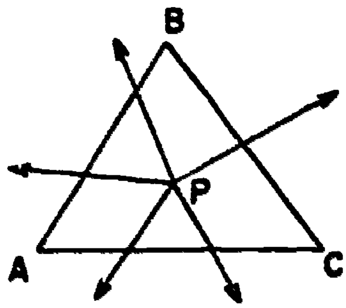
2. No. There are more states than there are cities of over 1,000,000 in population.

3. Yes, one nose per person; one person per number.

4. 1 3 5 7 . . . .
   ↕ ↕ ↕ ↕   each odd to next lower even.
   0 2 4 6 . . . .

5. Yes, match elements of R with elements of T by means of element in S.

6. Each vertex to the side opposite. Yes

7. 

Yes, you should observe it.

Yes; yes.

8. Yes; yes; yes.

* 9. Similar to rays in space.

* 10. The set of intersections of the elements of K with a plane form the set H.

* 11. (a) Yes

(b) Yes

(c) Yes, see Problem 7.

* 12. Each whole number is matched with the whole number which is twice its value. Thus

0	2	4	6	8
↕	↕	↕	↕	↕
0	1	2	3	4

---

10. Simple Closed Curves

1. <u>Understandings</u>

(a) Broken-line figures such as those we see in statistical graphs, triangles, rectangles, as well as circles, and figure eights are curves.

(b) A simple closed curve in the plane separates the plane into 2 sets--the points in the interior of the curve and the points on the exterior of the curve. The curve itself is contained in neither set.

(c) The curve is called the boundary of the interior (or the exterior).

(d) If a point A is in the interior of a curve and a point B is in the exterior of the curve, then the intersection of $\overline{AB}$ and the curve contains at least one element.

2. <u>Teaching Suggestions</u>

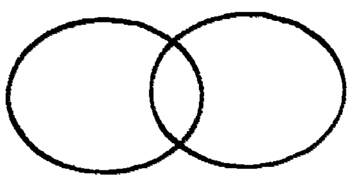
Draw some curves on the chalkboard, bringing out the idea that we call them "curves" and that a segment is just one kind of curve. We use the word "curve" in a special way in mathematics.

Note that a simple closed curve separates a plane into two sets and that the curve itself is the boundary of the two sets. Also, that any quadrilateral, parallelogram or rectangle is a simple closed curve. Identify some of the many curves which are suggested in the room, such as boundary of chalkboard, total boundary of floor surface, etc.

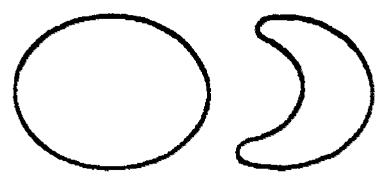
Students enjoy drawing elaborate curves which may still be classified as simple closed curves. Encourage their drawing a few simple closed curves for a bulletin board exhibit.

<u>Answers to Exercises 10</u>

1.

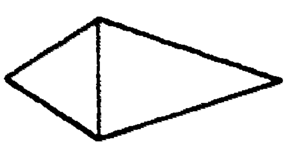


or maybe

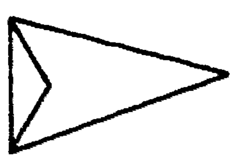


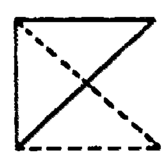
2. The intersection of the exterior of J_1 and the interior of J_2.

3.



OR



[

This is ruled out because of the extra point of intersection.]

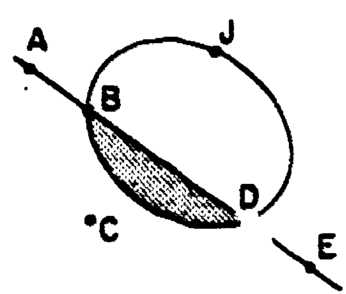
Yes, 3

4. No, it contains two.

5. Simple closed curve, line, angle

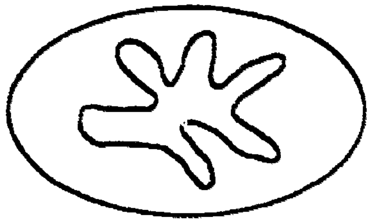
6. (a) B and D

(b)

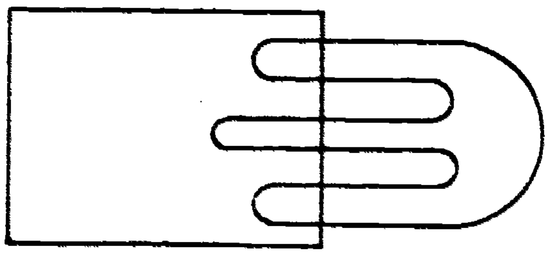


(c) $\overline{BA}$ and $\overline{DE}$

*7.



*8.



9. Because the curve would have to cross an even number of times. If the curve crosses three times the pencil is then not on the same side of the line as where it started.
