DOCUMENT RESUME

ED 173 208 SO 011 766

AUTHOR Tuma, Nancy Brandon; Hannan, Michael T.
TITLE Final Report for Dynamic Models for Causal Analysis of Panel Data. Dynamic Analysis of Event Histories. Part III, Chapter 1.
INSTITUTION Center for Advanced Study in the Behavioral Sciences, Stanford, Calif.; Stanford Univ., Calif.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C.
PUB DATE 7 Jul 78
GRANT 100-89-0004; NIE-G-76-0082; SRS-70-53
NOTE 66p.; For related documents, see SO 011 759-772 ; Some tables may be marginally legible due to small print

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Comparative Analysis; Data Analysis; *Educational Change; Measurement Techniques; Models; *Research Design; *Research Methodology; Research Problems; *Social Change; Social Science Research; Sociometric Techniques; *Validity

ABSTRACT

The document, part of a series of chapters described in SO 011 759, examines sociological research methods for the study of change. The advantages and procedures for dynamic analysis of event-history data (data giving the number, timing, and sequence of changes in a categorical dependent variable) are considered. The authors argue for grounding this analysis in a continuous-time stochastic model. This approach permits the data to be fully utilized and allows a unified treatment of the many approaches that use only part of the information of this data. The report focuses on the familiar continuous-time Markov model, summarizes its properties, reports its implications for various outcomes, describes extensions to deal with population heterogeneity and time-dependence, and outlines a maximum likelihood procedure for estimating the extended model from event-history data. The discussion is illustrated with an empirical analysis of the effects of an income maintenance experiment on change in marital status. Event-history analysis is contrasted with cross-sectional, event-count, and panel analyses. The conclusion is that event-history analysis has substantial advantages over these other approaches. (Author/KC)

Final Report for

DYNAMIC MODELS FOR CAUSAL ANALYSIS OF PANEL DATA

DYNAMIC ANALYSIS OF EVENT HISTORIES[1]

by

Nancy Brandon Tuma
Stanford University

Michael T. Hannan
Center for Advanced Study in the Behavioral Sciences
and
Stanford University

February 28, 1978

Revised July 7, 1978

[1]This paper is coauthored with Lyle P. Groeneveld of SRI International. We gratefully acknowledge the research assistance of Helen Garrison, Beverly Lauwagie, Marlos Viana, and Barbara Warsavage, and the helpful comments of anonymous referees. We appreciate the continuing support and encouragement of Robert G. Spiegelman, Director of the Center for the Study of Welfare Policy, SRI International. Part of the analytic work reported herein was supported by the National Institute of Education grant NIE-G-76-0082. The rest of the analytic work and all empirical research reported herein was performed pursuant to contracts with the states of Washington and Colorado, prime contractors for the Department of Health, Education and Welfare under contract numbers SRS-70-53 and HEW-100-89-0004, respectively. The opinions expressed in the paper are those of the authors and should not be construed as representing the opinions or policies of the states of Washington or Colorado or any agency of the United States Government. This paper will be published in the American Journal of Sociology 84 (Jan. 1979), forthcoming.

TABLE OF CONTENTS

Page

DYNAMIC ANALYSIS OF EVENT HISTORIES

Though most sociologists profess an interest in social processes--how social behavior and social systems change over time--this interest is seldom reflected in sociological research. Usually, sociologists examine relationships among phenomena at only one point in time. Even when temporal data are used, such as in experiments and in panel studies, sociologists seldom study dynamics--the time paths of change. The focus is rather on change from one equilibrium level to another, as measured, for example, by pre- and post-differences in an experiment or by levels at successive waves of a panel.

To some extent professed interests and actual research diverge because data suitable for dynamic analysis are scarce. But opportunities to collect such data are often bypassed because investigators are uncertain how to utilize detailed information on change over time. We are struck particularly by the failure of social scientists to gather and analyze dynamically data on changes in categorical variables. We hope to stimulate interest in the collection and analysis of what we term event histories. An event history records dates of events that occur for some unit of analysis. Examples of events include changes in categorical variables describing individuals--such as marital status, employment status, health status, and membership in voluntary associations--as well as those applying to social collectivities--such as changes in political regimes and outbreaks of strikes, riots, and wars.

This paper has two main goals. We wish to show the value of dynamic analysis of event-history data for the sociological study of change in categorical variables. We also wish to describe in some detail a practical procedure for making full use of event-history data to answer the kinds of questions that interest social scientists.

The exposition has the following structure. Section I begins by noting the wide range of empirical analyses permitted by event-history data--including, of course, application of existing techniques for analyzing cross-sectional and panel data. We argue that the use of continuous-time stochastic models allows a unified treatment of the outcomes in these various other approaches and the full use of the information in event-history data. So we take the position that event-history analysis be linked to an explicit mathematical model of the social process being studied. We focus on a familiar model, the continuous-time Markov model introduced into sociology by Coleman (1964). We summarize properties of the model and its implications for various outcomes. We also describe extensions of this model to deal with population heterogeneity and time-dependence; these extensions make the model much more useful for sociological research. Section I concludes with an outline of a maximum likelihood procedure for using event-history data to estimate and test the extended model.

Section II gives an empirical illustration. This involves an elaboration of the analysis presented in Hannan, Tuma, and Groeneveld (1977) of the effects of an income maintenance experiment on change in marital status. To illustrate the extension dealing with population heterogeneity, we report estimates of the effects of experimental treatments on rates of marriage, marital dissolution and experimental attrition. To illustrate the extension dealing with time-dependence, we report estimates of how treatment effects vary over the experimental period. We conclude by showing that the estimated model not only provides much more information about the process than other procedures more familiar to sociologists but also fits the data well--at least as well as these alternative procedures.

Finally, in Section III we contrast dynamic analysis of event-history data with other kinds of analysis. In particular, we compare it to cross-sectional analysis (assuming the process is in equilibrium), event-count analysis, and panel analysis. We find that when event-history data can be obtained, the strategy and procedures that we advocate have substantial advantages over these three alternatives.

Obviously a paper of this sort builds on a number of intellectual traditions and on contributions of previous authors. Our debt to classical mathematical sociology is obvious in our discussion of this model and its implications. Readers familiar with the causal-modeling (or structural equations) tradition should discern its influence on our approach to modeling causal structure and time-dependence. In a companion paper (Tuma and Hannan, 1978), we have sketched the genealogy of our statistical approach and have briefly reviewed some alternative estimation strategies.

I CHOICE OF A MODEL

Any data set permits a variety of empirical analyses. When we analyze event histories, we have even more options than are usual. This is seen clearly in the various dependent variables that can be selected from event histories. For example, consider how different investigators interested in marital stability might utilize marital histories to study changes in marital status. Some would study the probability of at least one marital dissolution within some fixed period. Others might study the average length of marriage. Others might choose to study the number of marital events during a fixed period. And, some might analyze changes in the proportion of a population that is married.

What are we to make of the richness of the information contained in event histories? We certainly have no objection to different analysts focusing on different aspects of a problem. However, the various outcomes listed above are not independent. Moreover, no one of them conveys all of the information contained in an event history. We believe that much is gained from designing analyses that utilize all of the information in an event history, that is, which use information on the number, sequence and timing of events.

Our solution to this problem--not necessarily the only one[2]--rests upon the recognition that we can infer each of the dependent variables listed earlier if we can describe each person's status at every moment in time. More generally, we can unify the analysis of various outcomes that can be extracted from event histories by building a model that describes the state of the categorical variable at every moment in time. This strategy has the advantage of parsimony--one model replaces several. It also has the advantage of dealing with the interdependence among the various outcomes. Of course, we expect to pay a price for such analytic power. In particular, we must make some restrictive assumptions about the process generating the events. Because of these assumptions, our model may fit sample data less well for any particular outcome at any arbitrary time than a model designed specifically to account for that outcome. After we have described our model and estimation procedure, we return to this issue as part of an extended empirical illustration of our methodology.

A Markov Model of Events

To use event histories fully, we must formulate an explicit model of the process generating the events. Because the model must account for states of categorical variables at every moment in time, and because the events can occur at any time, we choose continuous time models. Exactly what type of model should be used depends on the substantive application; a variety of continuous-time models could be employed with the general methodology we propose. Here we concentrate on one type of model: a finite-state, continuous-time Markov model.[3] We begin by briefly stating the formal assumptions of this model.

Let $Y(t)$ be a random variable denoting the state of the categorical variable occupied by a member of a homogeneous population at time t. The set of all possible values of $Y(t)$ is called the state space of the process; this is assumed to be finite. Let $p_{jk}(u,t)$ represent the probability that someone in state j at time u is in state k at time t, $u \leq t$:

$$p_{jk}(u,t) = \text{Prob}[Y(t) = k|Y(u) = j] \quad , \qquad (1)$$

and let $P(u,t)$ denote the matrix of these transition probabilities, $P(u,t) = \{p_{jk}(u,t)\}$. For example, suppose that $j = k = 2$ = married; then $p_{22}(u,t)$ stands for the probability that those married at time u are also married at time t.

Next we define $r_{jk}(t)$ as the instantaneous rate of transition from state j to state k at time t. The transition rate $r_{jk}(t)$ is the limit, as Δt approaches zero, of the probability of a change from j to k between t and $t+\Delta t$, per unit of time:

$$r_{jk}(t) = \lim_{\Delta t \to 0} \frac{p_{jk}(t,t+\Delta t)}{\Delta t} , \quad j \neq k \qquad (2)$$

The rate of leaving state j at time t, $r_j(t)$, is as follows:

$$r_j(t) = \sum_{\substack{k \\ j \neq k}} r_{jk}(t) \qquad (3)$$

By the Markov assumption, the Chapman-Kolmogorov identity holds:

$$P(v,t) = P(v,u)P(u,t) \qquad v \leq u \leq t \quad . \qquad (4)$$

Given this, and the usual additional assumptions (e.g., continuity, probabilities ranging between 0 and 1, etc.)[4], it can then be shown that

$$\frac{dP(u,t)}{dt} = P(u,t)R(t) , \quad (5)$$

where R is a matrix in which the j-kth off-diagonal element is the transition rate, $r_{jk}(t)$, and the jth diagonal element is the negative of the rate of leaving, $-r_j(t)$. In the time-independent case, i.e., R(t) = R, equation (5) has the solution:

$$P(u,t) = \sum_{m=0}^{\infty} \frac{R^m (t-u)^m}{m!} \triangleq e^{(t-u)R} , \quad (6)$$

where a matrix raised to the zero power is defined to equal the identity matrix I (so $R^0 = I$).

The first task in using such a model is specification of the state space, i.e., the exhaustive and mutually exclusive set of discrete values of Y(t). For example, in our analysis of the effects of income maintenance treatments on marital stability (see Section II below), the relevant states of Y(t) are 1 (= not married), 2 (= married), and 3 (= attrited). State 3 includes those who refuse to participate in the study, cannot be located, die or emigrate outside the continental U.S. "Attrited" is an absorbing state; it cannot be left. Thus R has the form:

$$R = \begin{bmatrix} -r_1 & r_{12} & r_{13} \\ r_{21} & -r_2 & r_{23} \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Although the elements of P(u,t) cannot be written as explicit functions of the transition rates for a general matrix R, this can be done when R has the form in (7). In particular, it can be shown that P(u,t) has the elements:[5]

$$p_{jj}(u,t) = \frac{1}{(\lambda_1-\lambda_2)}\left[(r_k+\lambda_1)e^{\lambda_1(t-u)} - (r_k+\lambda_2)e^{\lambda_2(t-u)}\right], \quad (8)$$

$$p_{jk}(u,t) = \frac{r_{jk}}{(\lambda_1-\lambda_2)}\left[e^{\lambda_1(t-u)} - e^{\lambda_2(t-u)}\right] \quad (9)$$

$$p_{j3}(u,t) = 1 + \frac{1}{(\lambda_1-\lambda_2)}\left[(r_{j3}+\lambda_2)e^{\lambda_1(t-u)} - (r_{j3}+\lambda_1)e^{\lambda_2(t-u)}\right], \quad (10)$$

$$p_{31}(u,t) = p_{32}(u,t) = 0 \quad , \quad (11)$$

$$p_{33}(u,t) = 1 \quad (12)$$

where j=1 or 2, k=3-j, $\lambda_1 \neq \lambda_2$, and

$$\lambda_1 = -\left[r_1+r_2+\sqrt{(r_1-r_2)^2+4r_{12}r_{21}}\right]/2 \quad , \quad (13)$$

$$\lambda_2 = -\left[r_1+r_2-\sqrt{(r_1-r_2)^2+4r_{12}r_{21}}\right]/2 \quad (14)$$

The explicit inclusion of attrition as a state is an important feature of our application of a Markov model. All too frequently investigators with temporal data only analyze the fraction of the original sample that has not attrited by the time of some later measurement. This procedure implicitly assumes that attrition has no impact on the assessment of the effects of causal variables. By explicitly including attrition as a state, we avoid this highly dubious assumption. However, it is still necessary to assume that the same model applies both to those who attrite and to those who do not. This assumption could be incorrect, but cannot be tested without a follow-up study of marital status changes of those who attrite.

The inclusion of attrition as a state complicates considerably both the derivation of equations and our discussion. For this reason, parts of our discussion are based on the two-state model in which there is no attrition:

$$R = \begin{bmatrix} -r_1 & r_{12} \\ r_{21} & -r_2 \end{bmatrix}. \tag{15}$$

Then we obtain

$$p_{jk}(u,t) = 1 - p_{jj}(u,t) = \frac{r_{jk}}{r_{12}+r_{21}} \left[1 - e^{-(r_{12}+r_{21})(t-u)}\right] \tag{16}$$

for $j,k = 1, 2$ and $j \neq k$.

Implications of the Model

We claimed earlier that use of such a continuous-time stochastic model lets us derive implications about a variety of observable variables. In this section we show this by discussing a number of well-known results about Markov models.

Time between Transitions. It is widely known that for a Markov model the length of time between transitions has an exponential distribution whose parameter depends on the transition rates (see, e.g., Breiman, 1969, Ch. 7).

In particular, let $F_j(t|u)$ represent the probability of a transition from state j before time t, given state j is occupied at time u. $F_j(t|u)$ is usually called the cumulative probability distribution function, or simply the probability distribution function. We can show that for models in which R(t) = R,

$$F_j(t|u) = 1-e^{-(t-u)r_j} \tag{17}$$

Later we use the definition that

$$G_j(t|u) = 1 - F_j(t|u) \tag{18}$$

$G_j(t|u)$ is often called the survivor function because it gives the probability that a unit in state j at time u remains (or survives) in state j until time t. For models in which R(t) = R,

$$G_j(t|u) = e^{-(t-u)r_j} = e^{-(t-u)\left[\sum_k r_{jk}\right]}$$

This says, for example, that the probability that a marriage existing at time u survives until a later time t declines exponentially as the length of the interval (t-u) increases. This monotonic decline occurs because it becomes increasingly likely as time t increases that either the marriage breaks up or the couple drops from the experiment.

We also use the probability density function:

$$f_j(t|u) \triangleq \frac{dF_j(t|u)}{dt} = \frac{-dG_j(t|u)}{dt} \tag{20}$$

It can be shown that in general

$$f_j(t|u) = r_j(t)\ G_j(t|u). \tag{21}$$

For the particular case in which $R(t) = R$, that is, the transition rates are constant over time,

$$f_j(t|u) = r_j\, e^{-(t-u)r_j} \tag{22}$$

The probability of a change from state j to some other state between t and $(t + dt)$ is approximately equal to $f_j(t|u)dt$. Equation (22) shows that this probability initially (i.e. at $(t-u) = 0$) equals r_j and declines exponentially as the length of the interval $(t-u)$ increases. In other words, the probability of leaving a state varies over time even when transition rates are constant. This is one of the main advantages of modeling social processes in terms of transition rates rather than probabilities of change.

The average duration of state occupancies (e.g., expected duration of marriages and of the intervals between marriages) when $R(t) = R$ is easily shown to be

$$E(t-u) = 1/r_j \quad , \tag{23}$$

where u and t denote the times of entry and exit, respectively, from state j.

Given that a change occurs at time t, r_{jk}/r_j is the conditional probability that k is the destination. Thus, with this model, we can account for both the probability of at least one change within any specified time interval and the conditional probability that a change involves movement into some specified state.

<u>State probabilities</u>: We can also use the model to find the unconditional probability of being in any specified state at any point in time (e.g., the probability of being married at any specified time). Let $p_k(t)$ denote the unconditional probability of being in state k at time t, and $\underline{p}(t)$ represent a row vector giving the probability distribution among the states at time t. It is easy to see that

$$p_k(t) = \sum_j p_j(u)\, p_{jk}(u,t) \quad , \tag{24}$$

or

$$\underline{p}(t) = \underline{p}(u)\, P(u,t) \quad . \tag{25}$$

Usually we are interested in $\underline{p}(t)$ when $\underline{p}(u) = \underline{p}(0)$, the distribution at the start of the process.

For simplicity we begin by considering the two-state model, e.g., for the study of marital stability when there is no attrition. Inserting equation (16) into (24), we obtain:

$$p_1(t) = p_1(0)\, e^{-(r_{12}+r_{21})t} + \frac{r_{21}}{r_{12}+r_{21}}\left[1 - e^{-(r_{12}+r_{21})t}\right] \quad , \tag{26}$$

$$p_2(t) = p_2(0)\, e^{-(r_{12}+r_{21})t} + \frac{r_{12}}{r_{12}+r_{21}}\left[1 - e^{-(r_{12}+r_{21})t}\right] \quad . \tag{27}$$

As time t becomes very large, $p_j(t)$ approaches a steady-state (or equilibrium) value: $p_1(\infty) = r_{21}/(r_{12}+r_{21})$, $p_2(\infty) = r_{12}/(r_{12}+r_{21})$. So $p_j(t)$ is a weighted average of the initial proportion in state j and the steady-state probability in state j. The weight given to $p_j(0)$ declines exponentially over time, while the weight given to the steady-state proportion in state j increases over time until it reaches one.

For the three-state model, e.g., for the study of marital stability when there is attrition, we find (by substituting equations (8) through (12) into (24)) that;

$$p_1(t) = \Big[e^{\lambda_2 t}\{(r_2+\lambda_2)p_1(0) + r_{21}(1-p_1(0))\}$$
$$- e^{\lambda_1 t}\{(r_2+\lambda_1)p_1(0) + r_{21}(1-p_1(0))\}\Big]/(\lambda_2-\lambda_1) \quad , \tag{28}$$

$$p_2(t) = \Big[e^{\lambda_2 t}\{(r_1+\lambda_2)(1-p_1(0)) + r_{12}p_1(0)\}$$
$$- e^{\lambda_1 t}\{(r_1+\lambda_1)(1-p_1(0)) + r_{12}p_1(0)\}\Big]/(\lambda_2-\lambda_1) \quad , \tag{29}$$

$$p_3(t) = 1 + \Big[e^{\lambda_2 t}\{\lambda_1 + r_{13}p_1(0) + r_{23}(1-p_1(0))\}$$
$$- e^{\lambda_1 t}\{\lambda_2 + r_{13}p_1(0) + r_{23}(1-p_1(0))\}\Big]/(\lambda_2-\lambda_1) \quad , \tag{30}$$

where λ_1 and λ_2 are given by (13) and (14), respectively. It is important to emphasize that $r_1 = r_{12} + r_{13}$, $r_2 = r_{21} + r_{23}$ and that λ_1 and λ_2 depend on r_1 and r_2. Thus, each of the above equations depends on the attrition rates, r_{13} and r_{23}.

Expected number of events. Finally, we can derive relationships between the fundamental parameters of the model and the expected number of events that occur in any interval of time. This requires extension of the ordinary Poisson model of the number of events in a time interval, since the Poisson model assumes that only one kind of event can occur. Let $n_{jk}(0,t)$ denote the number of changes to state k between times 0 and t when state j is occupied at time 0, $0 \leq t$. For example, $n_{22}(0,t)$ stands for the number of marriages formed before t for a person who is married at time 0. It can be shown that for the two-state model (when there is no attrition):

$$E[n_{jj}(0,t)] = \frac{r_{12}r_{21}t}{(r_{12}+r_{21})} + \frac{r_{12}r_{21}}{(r_{12}+r_{21})^2}\left[e^{-t(r_{12}+r_{21})} - 1\right] , \qquad (31)$$

$$E[n_{jk}(0,t)] = \frac{r_{12}r_{21}t}{(r_{12}+r_{21})} - \frac{r_{jk}^2}{(r_{12}+r_{21})^2}\left[e^{-t(r_{12}+r_{21})} - 1\right] , \qquad (32)$$

where j,k = 1,2 and j≠k. For the three-state model we find that:

$$E[n_{jj}(0,t)] = \frac{r_{12}r_{21}}{(\lambda_1-\lambda_2)}\left[\frac{(1-e^{\lambda_1 t})}{\lambda_1} - \frac{(1-e^{\lambda_2 t})}{\lambda_2}\right] , \qquad (33)$$

$$E[n_{jk}(0,t)] = \frac{r_{jk}}{(\lambda_1-\lambda_2)}\left[\frac{(r_k+\lambda_2)(1-e^{\lambda_2 t})}{\lambda_2} - \frac{(r_k+\lambda_1)(1-e^{\lambda_1 t})}{\lambda_1}\right] , \qquad (34)$$

$$E[n_{j3}(0,t)] = \frac{1}{(\lambda_1-\lambda_2)}\left[\frac{(r_{jk}r_{k3}+r_{j3}r_k+r_{j3}\lambda_2)(1-e^{\lambda_2 t})}{\lambda_2} - \frac{(r_{jk}r_{k3}+r_{j3}r_k+r_{j3}\lambda_1)(1-e^{\lambda_1 t})}{\lambda_1}\right] , \qquad (35)$$

j,k = 1,2 and j≠k.

The point we wish to emphasize is that these diverse observable measures on event histories are all explicit functions of the rates that define the Markov model. It is not necessary that each of these relationships be estimated separately. Any one of them provides an approach to estimation. Once the rates have been estimated, the remaining quantities can be calculated using these functions. The empirical analysis in Section II below uses these in evaluating the fit of our model.

Two Extensions of the Model

Markov models have been used quite often by sociologists in the past twenty years or so. In the simple form discussed above, these models have invariably failed to fit the data. Sociologists' efforts to increase realism and improve fit have concentrated on modification of two assumptions of the simple Markov model:

constant transition rates. Instead, following Coleman, (1964, 1973) and Tuma (1976), we may assume that the same, constant rates only govern the behavior of units with identical values on a set of observable, exogenous variables. In other words, we may establish relationships between observable variables, denoted by X, and the rates of change in categorical variables. The main sociological interest in most analyses of event histories lies in testing hypotheses concerning the effects of exogenous variables on rates. For example, in analyses of marital histories produced by the income maintenance experiments, we concentrate on testing hypotheses about effects of experimental treatments on rates of marital dissolution and remarriage.

To introduce such causal relationships, we must state an explicit dependence of the unobservable rates on the observable variables. In our empirical analysis, we assume a log-linear relationship between each transition rate and $\underline{X}$:[6]

$$\ln r_{jk} = \underline{\theta}_{jk}\underline{X} \quad \text{for all } j \text{ and } k,\ j \neq k \quad , \tag{36}$$

where $\underline{\theta}_{jk}$ represents a vector of parameters to be estimated. The log-linear relationship constrains r_{jk} to be positive for each individual, whatever the value of $\underline{X}$, in accord with equation (2); we find that it usually fits data better than a linear relationship. We also assume that r_{jk} is finite for all j and k and for all individuals.

Time-dependence. We have assumed to this point that the rates are constant over time. Sociologists are accustomed to thinking about population heterogeneity (i.e., causal relationships) but have not devoted much attention to issues involving time-dependence of effects, i.e., the manner in which causal relationships change over time. However, sometimes theory indicates that rates of change are some specific function of time. For example, Sorensen (1975) has argued that rates of leaving a job are exponentially declining functions of labor force experience, while Tuma (1976) has specified a model in which rates of job leaving are second-order polynomials in duration in the job. Such parametric forms of time-dependence may be accommodated in the strategy we discuss without difficulty.

Sometimes we do not have any a priori hypothesis concerning time-dependence. In an experiment, we might expect that transition rates during an initial adjustment period differ from rates later on, but be unsure whether initial rates should be larger or smaller than later rates. In these circumstances we can define a set of time periods and allow the effects of experimental variables to vary freely from period to period while remaining constant within each period. In this situation the time periods may be arbitrary, a disadvantage of this approach. However, in other situations, we may have some idea about points on the time axis when rates may change in some way. For example, rates of collective violence may change when there is a change in the political regime or during periods of warfare.

Mathematically stated, a model in which rates vary from one time-period to another is:

$$\ln r_{jkp} = \underline{\theta}_{jkp}\underline{Z}_p , \quad \tau_{p-1} \leq t < \tau_p , \tag{37}$$

where τ_p is the last moment in period p, $\tau_0 = 0$ is the starting time, $\underline{Z}_p$ is a vector of exogenous variables affecting the rate in period p, and $\underline{\theta}_{jkp}$ is a vector of parameters giving effects on r_{jkp} in period p. We illustrate the use of a model similar to this one below.

Estimation and [illegible]ng

The linear structural equations models with which sociologists have become quite familiar in recent years have made common the view that the estimation equation is identical to the model. But this is not necessarily the case. The equations relating observable outcomes (e.g., average duration, number of events, etc.) to the rates are all possible candidates for estimation equations, but they are not equally promising because of their complexity.

Building upon the work of Bartholomew (1957) and Albert (1962), we form maximum likelihood estimators of rates (and of causal effects on rates) using data on the dates and kind of events. This approach offers a number of advantages in the present circumstances. First, maximum likelihood estimators have good large sample properties under fairly general conditions. Second, maximum likelihood estimators retain their good properties under any monotonic transformation. Thus, one can use maximum likelihood estimators of rates to form maximum likelihood estimators for expected durations and other monotonic functions of rates. Third, maximum likelihood procedures permit a satisfactory solution of what is called the censoring problem. Data in which the observation period is too short to record a change for every case are said to be censored, and errors of inference are likely if appropriate measures for dealing with censoring are not adopted (Tuma and Hannan, 1978).

Assuming independent observations on the N different cases (units) being analyzed, we can write a likelihood function that uses all information in event histories--namely, the number, timing, and sequence of events. This information can be represented by the following kinds of variables: w_{mi} (a dummy variable that equals unity if case i's m^{th} event occurs in the observation period), t_{mi} (the time of case i's m^{th} event if $w_{mi} = 1$ and the time that case i is last observed if $w_{mi} = 0$), and v_{mji} (a dummy variable that is unity if case i's m^{th} event is observed and consists of a change to state j and otherwise equals zero). We define t_{0i} to be the start of the observation period on case i and v_{0ji} to equal unity if state j is occupied by i at t_{0i} and otherwise zero. Note that $\sum_{m=1}^{\infty} w_{mi}$ equals the total <u>number</u> of events observed to occur to case i, while the t_{mi}'s are the <u>times</u> that these events occur and the v_{mji}'s indicate the <u>sequence</u> of events.

For simplicity we begin with the likelihood equation that uses information on the first event only:

$$\mathcal{L} = \prod_{i=1}^{N} \prod_{j=1}^{n} \left\{ \left[G_j(t_1 | t_0, \underline{X}) \right]^{(1-w_1)v_{0j}} \left[f_j(t_1 | t_0, \underline{X}) \right]^{w_1 v_{0j}} \prod_{k=1}^{n} \left[r_{jk}(\underline{X}) / r_j(\underline{X}) \right]^{w_1 v_{0j} v_{1k}} \right\} \qquad (38)$$

where N is the number of cases, n is the number of states, and the subscript i on variables (namely, w_{mi}, t_{mi}, v_{mji} and $\underline{X}_i$) has been suppressed for clarity. The first term [in square brackets], the survivor function, gives the probability that the first event has not occurred by time t_1 (see equation (18)). Since any number raised to the $zero^{th}$ power equals unity, the first term differs from unity only for those cases that have <u>not</u> had a first event ($w_1 = 0$) <u>and</u> are in state j at time t_0 ($v_{0j} = 1$). The second term is the probability density that the first event occurs at time t_1 (see equation (20)).

This term differs from unity only for those cases that have had at least one event ($w_1 = 1$) and are in state j at time t_0 ($v_{0j} = 1$). The third term is the probability that the first event consists of a move to state k, given that state j is left (see equation (36)). This terms differs from unity only for those cases that have had at least one event ($w_1 = 1$), that are in state j at time t_0 ($v_{0j} = 1$), and whose first event consists of a move to state k ($v_{1k} = 1$).

Equation (38) can be written more simply by substituting (2) into (38) and by then collecting terms:

$$\mathcal{L} = \prod_{i=1}^{N} \prod_{j=1}^{n} \left\{ \left[G_j(t_1 \mid t_0, \underline{X}) \right]^{v_{0j}} \cdot \prod_{k=1}^{n} \left[r_{jk}(\underline{X}) \right]^{w_1 v_{0j} v_{1k}} \right\} \tag{39}$$

When $R(t) = R$, so that (19) holds, we can simplify this still more:

$$\mathcal{L} = \prod_{i=1}^{N} \prod_{j=1}^{n} \left\{ \prod_{k=1}^{n} \left[e^{-(t_1 - t_0) r_{jk}(\underline{X})} \right]^{v_{0j}} \right\} \cdot \left\{ \prod_{k=1}^{n} \left[r_{jk}(\underline{X}) \right]^{w_1 v_{0j} v_{1k}} \right\}$$

$$\mathcal{L} = \prod_{j=1}^{n} \prod_{k=1}^{n} \left\{ \prod_{i=1}^{N} \left[e^{-(t_1 - t_0) r_{jk}(\underline{X})} \right]^{v_{0j}} \cdot \left[r_{jk}(\underline{X}) \right]^{w_1 v_{0j} v_{1k}} \right\} \tag{40}$$

Note that the term in curly brackets in (40) above does not depend on $r_{j'k'}$ where $j' \neq j$ and $k' = k$. Therefore, the maximum likelihood estimates of a given r_{jk} -- or of the effects of causal variables on r_{jk} as in (36) -- can be obtained by maximizing the term in curly brackets for that particular j and k. This fact is quite important in practice. It means that we can estimate selected transition rates without having data on all kinds of transitions. To estimate r_{jk}, we need data only on the times of all first events (t_1) for cases originally in state j ($v_{0j} = 1$) and on the outcome of this event (the value of v_{1k}). This means we can concentrate data collection and analysis on units originally in states of particualar theoretical interest and can ignore events occuring to other units -- unless we wish to model or predict the overall evolution of the entire process.

Equations (38) through (40) can be generalized in a straightforward way to the situation in which data on <u>all</u> observed events (and not just the first) are used in the estimation procedure. We give the equations corresponding to (38) and (40) only, leaving the intermediary equation to be supplied by the interested reader:

$$\mathcal{L} = \prod_{i=1}^{N} \prod_{m=1}^{\infty} \prod_{j=1}^{n} \left\{ \left[G_j(t_m \mid t_{m-1}, \underline{X}) \right]^{(1-w_m)v_{m-1,j}} \cdot \left[f_j(t_m \mid t_{m-1}, \underline{X}) \right]^{w_m v_{m-1,j}} \cdot \prod_{k=1}^{n} \left[r_{jk}(\underline{X})/r_j(\underline{X}) \right]^{w_m v_{m-1,j} v_{mk}} \right\} \quad (41)$$

When R(t) = R, this simplifies to:

$$\mathcal{L} = \prod_{j=1}^{n} \prod_{k=1}^{n} \prod_{m=1}^{\infty} \prod_{i=1}^{N} \left\{ \left[e^{-(t_m - t_{m-1}) r_{jk}(\underline{X})} \right]^{v_{m-1,j}} \cdot \left[r_{jk}(\underline{X}) \right]^{w_m v_{m-1,j} v_{mk}} \right\} \quad (42)$$

Maximum likelihood estimators of the parameters are found by maximizing $\mathcal{L}$ (or its logarithm).[7] The optimal asymptotic properties of maximum likelihood estimators (consistency, asymptotic normality) are well-known (e.g., see Dhrymes, 1970). There is also evidence (Tuma and Hannan, 1978) that the properties of the estimators obtained from (40) remain quite good even in small samples and with a high degree of censoring (i.e., the mean of number of events is low).

A likelihood ratio test can be used to test nested models. Let $\mathcal{L}_\Omega$ be the likelihood for a model Ω with q+s estimated parameters and $\mathcal{L}_\omega$ be the likelihood for the nested model ω that has s parameters constrained (usually to equal zero) and q parameters estimated. The likelihood ratio λ is defined to equal $\max(\mathcal{L}_\omega)/\max(\mathcal{L}_\Omega)$. Asymptotically $-2 \ln \lambda$ has a chi-square distribution with q degrees of freedom, permitting us to test the fit of the model ω relative to the model Ω. Furthermore, it is possible to perform tests on the coefficients of individual variables using the estimated covariance matrix of the parameters (the inverse of the matrix of second derivatives of the natural logarithm of $\mathcal{L}$ with respect to the parameters). The square root of the variance of a parameter gives its estimated standard error, which can then be used in a standard fashion to calculate a t-statistic (or F-ratio) that tests whether the parameter differs from its value in the null hypothesis.

II AN APPLICATION

To illustrate the models and methods discussed above we apply them to data on white women during the first two years of the Seattle and Denver Income Maintenance Experiments (SIME/DIME). We combine data from Seattle and Denver.[8]

We have discussed the experimental design, nature of the sample, definitions of variables, and so forth earlier (Hannan, Tuma, and Groeneveld, 1976, 1977). Here we use similar data that cover a period of 24 months rather than 18. We focus on estimating the effects of the three levels of income support (or guarantee) levels and the length of the treatment (3 or 5 years). Because the design involved stratified random assignment to treatments, we include a number of preexperimental variables in the models: dummy variables representing 6 normal income categories, a site dummy (1=Denver), a dummy variable for previous AFDC experience, number of children, a dummy variable for having any children under six, the woman's age, her wage rate, and her years of schooling.

Though other causal variables, as well as the experimental treatments, may have different effects on rates of events in different time periods, we are primarily interested in the time-dependence of treatment effects. There are a variety of reasons for expecting the effects of experimental treatments to vary over the experimental period. Reasons for treatment effects to be smaller initially than later on include the possibility that subjects do not fully understand the treatments initially and that they may need to search for an opportunity to change state (e.g., to find a marital partner). Reasons for treatments effects to be larger initially than later on include the possibility that the treatments become less salient as the end of the experiment approaches and the possibility that the treatments improve opportunities for a change for some fraction on the verge of changing marital status before the experiment. Indirect treatment effects (e.g., effects of treatments on work behavior, which in turn affect marital status) also can cause treatment effects to vary over time, but it cannot be predicted a priori whether indirect effects enhance or dampen the initial response. Our inability to predict the shape of the pattern of time variation is, of course, the reason for subdividing the observation period and letting rates vary from one period to another.

In the time-dependent models we began by using four time periods with end points of 0.5, 1.0, 1.5, and 2.0 years after the start of the experiment. These represented a compromise between two conflicting goals. First, we wished to have the number of observed events per period be large so that the standard errors of parameters would be comparatively small. Second, we wished to have a large number of time periods so that we could detect the shape of the pattern of time-variation (which according to our reasoning might decrease over time, rise over time, or rise and then decrease, etc.)

To improve efficiency of the estimation of effects still further, we estimated an equation that allowed treatment effects, but not the effects of other causal variables, to vary over time. This model can be represented as follows:

$$\ln r_{jkp} = \underline{\phi}_{jk}\underline{X} + \underline{\Theta}_{jkp}\underline{Z}_p \ , \quad \tau_{p-1} \leq t < \tau_p \ , \qquad (43)$$

where $\underline{X}$ is the vector of other causal variables, $\underline{Z}_p$ is a vector of experimental treatment variables, and p refers to one of P periods.

$\underline{\phi}_{jk}$ and each $\underline{\theta}_{jkp}$ contain a constant term; however, for each transition j to k, only P of these P + 1 constants can be identified. Therefore, to achieve identification we arbitrarily constrain one of them to be zero.[9] The P constants permit the rate of the control group to vary from one time period to another, even though the effects of other causal variables may not.[10]

The method of maximum likelihood can be used to estimate parameters in equation (43). The likelihood equation resembles (39) except that there is a different expression for the survivor function and each transition rate has an additional subscript p to indicate the time period to which it applies. (For details, see the Appendix.) In other respects procedures for estimating and testing the model and effects of individual variables are the same as for a model with transition rates that are constant over time.

Results of the estimation could be reported many different ways. Because the effect of a variable on the rate itself is of more interest than the effect on the log of the rate, we report the antilog of estimates of $\hat{\theta}$s. The antilog indicates the multiplier of the rate for a unit increase in a variable. For dummy variables, which we use to represent experimental treatments, the antilog of the coefficient of the variable is the ratio of the rate for those whose value on the dummy variable is unity to the rate for those in the omitted category. For example, if for some experimental treatment, $e^{\theta} = 2$, where θ is the coefficient of the dummy variable representing the experimental treatment, then the rate for those on the treatment is twice the rate for those in the control group. The percentage change in the rate for an experimental treatment relative to the rate for those in the control group is just $100(e^{\theta}-1)$. Thus, if $e^{\theta} = 2$, the percentage change in the rate for this treatment relative to the rate for the control group is 100%.

Results for Time-independent Models

Table 1 gives the results for time-independent models of rates of marital status change and attrition. We have discussed similar results and their interpretation at length elsewhere (Hannan, Tuma and Groeneveld, 1977; Tuma, Groeneveld and Hannan, 1976).[11] Here we comment briefly, concentrating on the effects of the support levels.

Table 1 about here

All four models significantly (.001 level) improve upon a constant rate model. The set of experimental treatments significantly improves upon a model that includes only the other causal variables in the case of the marital dissolution rate (.001 level), the attrition rate of married women (.10 level) and the attrition rate of unmarried women (.05 level), but not in the case of the remarriage rate.

Women in each support group have higher marital dissolution rates than comparable (in terms of values of other causal variables) women in the control group. Effects of the support levels are large (ranging from a 57 to 129 percent increase in the rate relative to comparable controls) and statistically significant at the .01 level, except for the highest support.

As expected, attrition rates of those with a financial treatment are lower than those of comparable controls, except in the case of married women on the low support. However, there is no clear pattern to the effects of the different support levels on attrition, and none of the individual coefficients is significant.

Results for the Time-dependent Models

We have estimated time-dependent (four-period) models of rates of dissolution, remarriage and attrition. See Tuma, Hannan and Groeneveld (1977) for a more detailed discussion. The results of these analyses indicated that experimental effects on attrition rates did not significantly vary over time or in any patterned way. Experimental effects on the remarriage rate were significant at the .10 level, but there was no particular pattern to these effects, suggesting that they resulted from chance alone. On the other hand, experimental effects on the dissolution rate had a striking pattern. So we focus on these.

Table 2 shows results on the experimental effects on the dissolution rate when the total observation period (24 months) is treated as a single period, four six-month periods, and two periods (the first six months and the remaining eighteen months). In all three models the set of experimental treatments significantly improves upon a model that contains only the other causal variables.

Table 2 about here

Support level effects over the four periods are plotted in Figure 1. The plot shows that all three support levels produce an exceptionally large increase in the marital dissolution rate during the first half-year of the experiment. Except for the $5600 support level in the second half-year, effects of the support levels in periods two through four are positive and show no clear pattern of variation over time. This suggests that (1) the support levels have transitory effects on dissolution rates of white women that subside after six months, but (2) they also have nontransitory positive effects on the dissolution rate. Furthermore, the effects of each support level in the first six-month period relative to its effect in each subsequent period is about the same for all three support levels. Although there is a clear pattern of time-varying effects, the four-period model does not significantly improve upon the one-period model.

Figure 1 about here

With these findings in mind, we estimated a two-period model in which the first half-year is distinguished from the rest of the experimental period. Relative rates across support levels were constrained to be equal in the two periods (see Table 2), but their rate relative to controls was allowed to vary from one period to the other. Treatment effects in this model significantly (.05 level) vary over time, according to both the likelihood ratio test and the F test on a dummy variable for an effect of financial treatments in the first half-year.

The estimates for the two-period model (Table 2) indicate that the effect of each support level is 2.32 times as large in the first half-year as it is thereafter. Based on our prior arguments, this suggests that the treatments immediately changed the opportunities of some respondents of the verge of dissolving their marriage. But the treatments also seem to have changed the long-range opportunity structure. The effects of the support levels during the 0.5 to 2.0 year period are positive and significant (except for the $5600 support), though 14 to 15 percent lower than in the results for the time-independent model. Since the long-range impact of an income maintenance program should not depend on transitory effects on rates during an initial adjustment period, the effects of the support levels in the 0.5 to 2.0 year period should provide more reliable estimates of the ultimate effects of income maintenance than estimates based on the one-period model.

How Well Does the Model Fit?

Markov models have a reputation for fitting data poorly. But our extensions of the Markov model should have helped to improve the ability of the model to fit the data. So far we have used likelihood ratio tests to assess the relative fit of a series of nested models. We have learned that some models do not improve upon others. Here we look at the absolute fit of the model and compare it to a common alternative.

Three main questions are involved in assessing our dynamic model: (1) To what extent do predictions based on our model differ systematically from observed values? (2) If there are systematic differences, are these related to the experimental treatments and so causing us to make erroneous inferences about experimental effects on a particular outcome? (3) As compared to approaches that seek to minimize prediction errors for a single observable variable, how well does our model explain sample variation in outcomes related to marital stability?

In answering these questions we consider three kinds of outcomes: the probability of being in a given state (e.g., single) at any moment, the expected number of marriages and marital dissolutions in any given time interval, and the probability of leaving the original marital status in some time period. Our model implies that each of these is a function of marriage, disssolution, and attrition rates, as indicated in Section I.

Thus we can use a woman's values of treatments and other causal variables and the estimated effects of these variables to predict her rates of marriage, marital dissolution, and attrition, and then her values of the different outcomes listed above. We chose to consider these outcomes at two arbitrary times: one and two years after the start of the experiment. For each woman used in our analysis of transition rates, we retrieved her observed values on the outcomes listed in Table 3. These variables were not directly used in estimating transition rates; of course, they are not independent of those data either, which is the reason for having a single model. We also predicted each woman's values on these outcomes using the estimated effects from the one-period models of remarriage and attrition and the two-period model of marital dissolution (see Tables 1 and 2).

Table 3 about here

To detect systematic differences between predictions and observed values we report the mean residual for each outcome, i.e., the mean difference between observed and predicted variables. We also report the observed mean of each outcome because the relative size of a systematic difference is of some interest too. With predictions from a linear regression model, the mean residual would be zero. This need not be the case with predictions from our model. The results in Table 3 show that the mean residuals for our predictions are usually small both in absolute terms (the largest is .02) and in relative terms. There is little indication of any overall pattern to these differences, except for the last four dummy variables, which have consistently positive mean residuals. It is well-known that no change in status has tended to be underpredicted in sociological applications of Markov models (e.g., see Blumen et al., 1955). Our introduction of population heterogeneity has made this a comparatively small problem, but it has not erased it entirely.

Small mean residuals could hide systematic differences associated with different treatments, which is clearly undesirable if control-financial differences on an outcome are of interest. To answer the second question, we performed one-way ANOVA on the residuals for each outcome. Treatment differences in the residuals never even approached statistical significance. (The smallest prob-value was greater than .50.)

In addressing the third question, we focus on a single, common inexpensive alternative--linear regression analysis. We regressed each observed outcome on the prediction from our model; we also regressed each of the fourteen outcomes on initial marital status, treatments, and other causal variables used in our analysis of the transition rates. For both our model and the regression model we report R^2, the square of the correlation between the observed and predicted variables. Since we expected a poorer fit from our model than from one designed to minimize errors, we were surprised to find that for ten of the fourteen outcomes our model explains more of the sample variation than does linear regression analysis. Moreover, the advantage of our model in these ten cases tends to be larger than the advantage of linear regression in the other four.

Though we have considered the predictions of our model for several outcomes at two arbitrary times, we have not yet seen how well it predicts the time path of these outcomes. Computational expense has forced us to examine the time path of only one outcome. We selected the proportion who are unmarried at time t, conditional on having not attrited by time t. We chose this because it is similar to the most important policy outcome, because it should reveal whether experimental effects are confined to an initial, brief adjustment period, and because it depends about equally on our estimates of marriage, dissolution, and attrition rates. This choice provides a severe test of our method because it uses estimates of all four rates of change (r_{12}, r_{13}, r_{21}, r_{23}).

Figure 2 gives observed and predicted curves for this outcome by support level. Points on the observed curve are given by $N_{1j}(t)/(1-N_{3j}(t))$ where $N_{1j}(t)$ and $N_{3j}(t)$ are the number in treatment j at time t who are unmarried and attrited, respectively. Points on the predicted curves are calculated as

$$\frac{1}{N_j} \sum_{i=1}^{N_j} \hat{p}_{1ji}(t)/[1-\hat{p}_{3ji}(t)] \quad , \qquad (44)$$

where $N_j = N_{1j}(0) + N_{2j}(0)$ (the initial number in treatment j), and $\hat{p}_{1ji}(t)$ and $\hat{p}_{3ji}(t)$ are calculated for each woman i enrolled on treatment j using equations (28) and (30), respectively. Predictions are based on the one-period models of remarriage and attrition and on the two-period model of dissolution; they assume each woman has her assigned treatment.

Figure 2 about here

We begin by considering the observed (squiggly) curves. First, note that the proportion of unmarried women at the start of the experiment differs greatly from one treatment to another. These initial differences result from the use of a stratified random design in which marital status was a stratification variable. Unmarried women were more likely to be assigned to treatments with a lower support level. Because of this, a comparision of "post-test" levels is clearly inappropriate.

Next, notice that the observed curve for the control group is relatively flat, suggesting that there are no important "natural" time trends (due to aging, secular change, etc.). On the other hand, the observed curves for the three support levels show noticeable increases in the conditional proportion of unmarried women after two years (+.039 for the low support, +.044 for the medium support, and +.031 for the high support versus +.009 for the control group). Furthermore, the proportion of unmarried women among those on the financial treatments rises fairly steadily throughout the two-year period. This upward trend is quite apparent for the low and medium supports. It is less certain for the high support, which has the fewest subjects and the most extreme fluctuations about any overall time trend. There is little evidence that the proportion of unmarried women among those on financial treatments has reached a plateau within the first two years of the experiment, as we would expect if an equilibrium was reached during this period. This suggests that dynamic analysis is really needed to assess experimental effects accurately. We elaborate on this in Section III.

Now let us consider the fit between the (smooth) curves predicted by our model and the actual (squiggly) curves. We rely on visual inspection to compare the two sets of curves. On the whole, the fit is quite good except for the high support group, for which the actual curve is noticeably below the predicted curve. Because only 240 women are in the high support group, a change in status of a very few women makes a substantial difference in the observed curve. Hence the deviations for this group are less worrisome than they would be for a large group like the controls (N=847).

Our scrutiny of the implications of our model for various outcomes at arbitrary times has revealed no major disadvantages and even some small advantages. The model's primary advantage is, of course, its ability to predict the time path of a variety of interdependent outcomes reasonably well.

III COMPARISON WITH OTHER APPROACHES

We return finally to an issue raised at the outset: the advantages of event-history analysis relative to other approaches. We continue to assume throughout this discussion that events are generated by a Markov process whose transition rates are log-linear functions of exogenous variables (36).

Obviously any comparison of alternative approaches depends on the assumptions about the process generating events. Following Coleman's (1964, 1968) work, nonetheless, we wish to challenge the still widespread view that substantive assumptions ought to be dictated by the form in which the data are collected. That is, we do not believe that sociologists ought to change their assumptions about the underlying process (their model, in our terminology) when they shift from analyzing panel data, say, to analyzing cross-sections or event histories. In our view, a major advantage of formulating problems in terms of dynamic stochastic models is that we can use different data structures to estimates parameters of the same model. This provides a way of unifying a variety of data analytic procedures.

We begin with an extended discussion of cross-sectional analysis because this has been--and will undoubtedly continue to be--the mainstay of sociological research. We then contrast event-history analysis with two other strategies that use temporal data: event-count analysis and panel analysis. Our discussion of each is brief. To the best of our knowledge, event-count analysis has not yet been developed (let alone applied) except for the most elementary kind of Markov model (a Poisson model). So our comments on it are intended to encourage the development of this approach. On the other hand, Singer and Spilerman (1974; 1976) have treated in detail the difficulties of panel analysis with categorical dependent variables. We mainly review these to emphasize that they are either absent or much less serious with event-history analysis.

Cross-sectional Analysis

Cross-sectional data give the state that each member of a sample occupies at a particular time t. Earlier we referred to the unconditional probability of being in a state j at time t as the state probability, $p_j(t)$. Given n possible states, there are only (n-1) unique state probabilities since the n probabilities must sum to unity. But, in general, there are n(n-1) unique transition rates. Because $n(n-1) > (n-1)$ for $n > 1$, it is immediately obvious that cross-sectional analysis does not allow all parameters of a model to be identified unless we can specify $(n-1)^2$ of the transition rates, either on theoretical grounds or from a priori knowledge. With event-history analysis we can estimate all parameters. So event-history analysis is clearly preferable to cross-sectional analysis if we wish to understand the process fully or to predict other outcomes.

Under certain circumstances, however, cross-sectional analysis can supply useful information about the process generating events. It is worthwhile identifying these conditions. We begin by considering the situation in which the process has been operating a comparatively long time so that the distribution of the population across states is in equilibrium. For concreteness we again start with the two-state case.

Since the rates, r_{12} and r_{21}, cannot be negative, the two-state model implies that the probabilities of being in each state eventually reach stable, so-called steady-state, values:

$$p_1(\infty) = \frac{r_{21}}{r_{12}+r_{21}} = \frac{e^{\underline{\theta}_{21}\underline{X}}}{e^{\underline{\theta}_{12}\underline{X}}+e^{\underline{\theta}_{21}\underline{X}}} = 1/(1 + e^{(\underline{\theta}_{12}-\underline{\theta}_{21})\underline{X}}) \quad , \qquad (45)$$

$$p_2(\infty) = \frac{r_{12}}{r_{12}+r_{21}} = \frac{e^{\underline{\theta}_{12}\underline{X}}}{e^{\underline{\theta}_{12}\underline{X}}+e^{\underline{\theta}_{21}\underline{X}}} = 1/(1 + e^{(\underline{\theta}_{21}-\underline{\theta}_{12})\underline{X}}) \quad . \qquad (46)$$

Of course, individuals continue to change from one state to the other. That is, the model implies an equilibrium probability distribution on the aggregate level; it neither assumes nor implies equilibrium on the individual level.

Note that

$$\frac{p_1(\infty)}{p_2(\infty)} = e^{(\underline{\theta}_{21}-\underline{\theta}_{12})\underline{X}} \qquad (47)$$

or

$$\ln \frac{p_1(\infty)}{p_2(\infty)} = \underline{\gamma}_{12}\underline{X} \quad , \qquad (48)$$

where $\underline{\gamma}_{12} = \underline{\theta}_{21}-\underline{\theta}_{12}$. For example, this model implies that in the steady-state, the log of the odds of being unmarried (rather than married) is linear in $\underline{X}$. Equation (48) is the usual form of a binary logit model (Berkson, 1944; Theil, 1969, 1970), and if all members of $\underline{X}$ are dummy variables, then it is just a special case of Goodman's (1972) log-linear model of the odds ratio. Thus, when a population is in equilibrium, logit (or log-linear) analysis of cross-sectional data tells us the difference in the effects of variables on the two rates, r_{12} and r_{21}. Note that "no effect" of a variable in the cross-sectional logit analysis can be due to its equal effects on the two rates. That is, it should not be taken as evidence that a variable is irrevelant to the process, only that it has no net effect on the steady-state distribution.

Is there a similar connection between the general, n-state Markov model and multinomial logit analysis? Unfortunately, the answer is no. This can be proved by a single contrary case. Consider $\ln p_1(\infty)/p_2(\infty)$ for the three-state model with attrition, which we have used in our empirical analysis. Though both $p_1(\infty)$ and $p_2(\infty)$ are zero (because eventually everyone "attrites"), they do have a finite ratio. We find that:

$$\lim_{t\to\infty} \ln\left\{\frac{p_1(t)}{p_2(t)}\right\} = \ln\left\{\frac{r_{21}+p_1(0)[r_{23}+\lambda_2]}{r_{12}+[1-p_1(0)][r_{13}+\lambda_2]}\right\}. \qquad (49)$$

The expression on the right-hand side is quite complicated and substitution of equations like (36) does not produce anything resembling a logit model in general. (If $r_{13}=r_{12}$, i.e., attrition rates for married and unmarried women are identical, then (49) does simplify to (48)).

However, an important class of Markov models--general birth and death models--do have a steady-state distribution that has the form of a multinomial logit model. In these models states can be ordered so that there can occur only transitions between neighboring states (e.g., in a three-state model transitions from 1 to 3 and from 3 to 1 are impossible). Then, for example, in a three-state model with ordered states, equations (47) and (48) continue to apply, and in addition:

$$\frac{p_2(\infty)}{p_3(\infty)} = e^{(\underline{\theta}_{32}-\underline{\theta}_{23})\underline{X}} , \tag{50}$$

$$\ln \frac{p_2(\infty)}{p_3(\infty)} = \underline{\gamma}_{23}\underline{X} , \tag{51}$$

where $\underline{\gamma}_{23} = \underline{\theta}_{32} - \underline{\theta}_{23}$. Furthermore,

$$\frac{p_1(\infty)}{p_3(\infty)} = \frac{r_{32}r_{21}}{r_{12}r_{23}} = e^{(\underline{\theta}_{32}+\underline{\theta}_{21}-\underline{\theta}_{12}-\underline{\theta}_{23})\underline{X}} . \tag{52}$$

The similarity in form of (52) to (47) and (50) means that there are no clues in cross-sectional data to tell us how to order the states. This must be done on theoretical grounds--or else we must have event-history data, which do permit us to observe what kinds of transitions can occur. But, if we do know the order of states in a general birth and death model, cross-sectional multinomial logit analysis does let us make conclusions about the net effect of a variable on transition rates between states.

So far we have considered the situation in which the system is in equilibrium. We have indicated that in the steady-state, the log of the odds of being in one state rather than another has a very simple form if states can be ordered. On the other hand, if the steady-state has not been reached, then the log of the odds of being in one state rather than another is a very complicated function of time, the transition rates, and the initial conditions--even for the very simple two-state case. This can be seen by forming the ratio of the right-hand sides of equations (26) and (27).

Suppose we perform cross-sectional analysis at two successive time points and determine that variables have very different effects. We cannot be certain what to conclude. Has the underlying process changed--that is, has the relationship between variables and transition rates altered? Or has the system just moved closer to its ultimate steady-state value? Without some form of temporal analysis we cannot answer these questions.

Social scientists have been so wedded to cross-sectional analysis that they seldom seem to reflect on the likelihood that equilibrium exists or on the length of time required for a system to reach a new equilibrium following some intervention or structural upheaval. We suspect that in most cases inertia greatly slows the speed with which social systems reach equilibria. We note that the equilibrium assumption implicit in sociological theories prominent a few decades ago (e.g., functionalism) began to be attacked more than a decade ago. But these criticisms have barely begun to penetrate sociological methodology.

We will make our discussion of this issue more concrete by referring to our empirical illustration. Many analysts faced with data like ours might conduct some sort of logit analysis that assumes that equilibrium is reached within the experimental period. But can we expect the steady-state probability distribution of marital status to be approached during the 3 or 5 years of the experimental period? As social experiments go, SIME/DIME is long, so it might seem that this would happen. However, according to the models we have discussed, how long it takes to approach a new steady-state (say, to within .01 of its ultimate value) depends on both preexperimental and experimental rates of marital status change. Our results imply that SIME/DIME is much too short for the steady-state to be approached during the experimental period.

Marriage and dissolution rates are approximately equal to annual probabilities of forming and dissolving a marriage, respectively.[12] Among SIME/DIME's participants, who have incomes below the U.S. median, both rates are somewhat higher than in the overall U.S. population. In the environment facing the control group, $r_{12} = .10$ and $r_{21} = .05$ are fairly typical.[13] If an aggregate-level equilibrium exists at the beginning of the experiment, the initial probability of being unmarried is $.05/(.10 + .05) = .333$. This is close to the observed proportion of unmarried women among the control group at the beginning of SIME/DIME (see Figure 2).

Our analyses reported in Section II indicate that for whites SIME/DIME has a negligible effect on marriage rates, but roughly doubles the dissolution rate of those on most treatments. If the dissolution rate under income maintenance is twice that of the controls, i.e., $r_{21} = .10$, then according to the two-state model the equilibrium probability of being unmarried under income maintenance is $.10/(.10 + .10) = .50$. Thus, the model predicts that under such conditions, the proportion of unmarried women in the population would eventually increase about 50% above its pre-income maintenance value (from .333 to .500). However, the proportion of unmarried women would only increase by about .04, or about 13%, in the first two years (see Figure 2), and by about .09, or about 26%, in the first five years. It would take nearly 19 years to be within .01 of the steady-state proportion. If data on marital status of participants at any point during the 3 to 5 years of the experiment are analyzed cross-sectionally, the ultimate effect of income maintenance will be greatly underestimated. As we mentioned earlier, the observed curves for support levels in Figure 2 do not suggest that a plateau or equilibrium has been reached within the first two years.

The only way to decide whether a system is in equilibrium is to collect data over time and to analyze it dynamically--that is, in a way that lets us study the time path of change in the phenomenon.

Event-count Analysis

There have been a number of sociological studies that analyze the number of times a particular event occurs in some time period. For example, Spilerman (1970) analyzed the number of riots per city during the mid-sixties. We refer to this type of analysis as event-count analysis. We suspect that a number of surveys have asked such questions as "How many times have you been married?" and "How many times have you been divorced?" However, we are not aware of any event-count analyses in sociology where counts of more than one type of event are analyzed within a single model.

Given the assumptions of a Markov model we can derive expressions for the expected number of different types of events in some time interval. For example, for the two- and three-state models discussed in this paper, we derived the expressions given in equations (31) through (35). These equations, combined with observed data on the counts of different types of events, permit transition rates to be estimated by a nonlinear regression approach. That is, we can estimate rates--or the causal effects of variables on rates--by minimizing the sum over all units of the squared deviation between the observed count of events for each unit and that predicted by the model. This approach, which we have not yet used, has one inherent limitation: we know of no theorem (comparable to the Gauss-Markov theorem in linear regression analysis) that estimators obtained in this way will have optimal statistical properties--even in an infinite sample.

Maximum likelihood estimators typically have optimal asymptotic properties. But to perform maximum likelihood estimation we must know the probability mass function for the count of events. To the best of our knowledge, the expression for this function has not yet been derived for a general n-state Markov model. In fact, it is not even clear that such an expression can be written in closed form for a general n-state model. The probability mass function for the number of events can, of course, be written explicitly for certain special cases (e.g., a Poisson model), but it is mathematically intractable even for the two- and three-state models used in the illustrations in this paper.

Given these difficulties, it seems obvious that event-history analysis is preferable to event-count analysis. Nevertheless, event-count analysis deserves further study. Under some circumstances, event counts either already exist or are feasible to collect, while event histories or panel data cannot be obtained.

Panel Analysis

Panel data, which record the states occupied by members of a sample at a series of discrete points in time, are the temporal data most commonly available to sociologists. Singer and Spilerman (1974; 1976) have identified the following problems regarding estimation of transition rates in a general n-state Markov model from panel data.[14]

First, sometimes panel data on categorical variabl cannot be embedded in--that is, described by--a Markov process. Moreover, sampling variability and measurement error can cause panel data to be unembeddable even though they are truly generated by a Markov process. Second, even though the panel data may be describable by a Markov process, there may not be a unique matrix of transition rates that describe the data. Furthermore, the different matrices obtained in the nonunique cases may suggest substantially different qualitative conclusions. (See, for example, Singer and Spilerman, 1976, p. 31). Neither embeddability nor uniqueness is a problem in event-history analysis because maximum likelihood estimators based on such data give unique estimates of rates--or of causal effects on rates.

Third, Singer and Spilerman (1976, pp. 44-48) note that small changes in an observed matrix of transition probabilities (due to sampling variability or measurement error) can sometimes lead to very marked changes in estimates of transition rates. This is clearly undesirable. On the other hand, in our experience in analyzing event histories, given a moderately large sample, fairly substantial errors in records on the occurence or timing of events do not qualitatively alter estimated patterns of causal effects of variables on transition rates.[15] Insensitivity to sampling and measurement error is, we believe, an important advantage of event-history analysis. Because such errors are unavoidable, the sensitivity issue clearly deserves further study--in both panel and event-history analysis.

Fourth, estimation of transition rates from panel data is also sensitive to the length of the time interval between waves of the panel. When the time interval is large, each row of the matrix of transition probabilities approaches the steady-state probability distribution. (See, for example, equations (8) through (12)). In this situation there are only (n-1) unique transition probabilities, rather than n(n-1). This means that the data contain no more information than cross-sectional data. On the other hand, if the time interval between waves of the panel is very short, almost all members of the sample will be in their original state. This is not very informative, either. With event-history analysis the length of the observation period cannot be too long. It can be too short--if no event has occurred. However, Tuma and Hannan (1978) have shown that with samples that are moderate in size, rates can be estimated well if as few as ten percent of the sample have had an event. We have not seen comparable results based on panel analysis, but we suspect that it does not perform as well.

Fifth, we know of no way of estimating parameters in a general n-state Markov model from panel data when transition rates are functions of exogenous variables, as is almost always the case in problems of interest to sociologists. Singer and Spilerman (1974) have reported some work on estimating parameters from panel data generated by a mixture of Markov processes, when the mixture is described by some specified probability distribution. This work is helpful, but it still does not permit causal inferences to be made. As we have shown, causal relationships are easily studied with event-history analysis.

A sixth problem with panel analysis concerns the ability to study and detect time-dependence in the process generating events. This appears to present a very difficult problem for panel analysis. As we have indicated, various kinds of time-dependence can readily be investigated through event-history analysis.

Conclusions

Our conclusions are very simple. Event histories provide rich opportunities for answering fundamental sociological questions. We have shown how to analyze event histories when data are generated by a well-behaved stochastic process. The procedures we have outlined permit analysis of causal effects on the rates at which events occur and of time-dependence in such rates. These procedures are simple to implement, and in our empirical application they yield good predictions about a variety of observable variables.

Event-history analysis offers substantial advantages over other common approaches to the study of causal effects on changes in qualitative variables. Since in many situations it is no more difficult to obtain information on the timing of events than the count of events, we urge that sociologists begin to collect and analyze such data.

FOOTNOTES

[1]We gratefully acknowledge the research assistance of Helen Garrison, Beverly Lauwagie, Marlos Viana, and Barbara Warsavage, and the helpful comments of anonymous referees. We appreciate the continuing support and encouragement of Robert G. Spiegelman, Director of the Center for the Study of Welfare Policy, SRI International.

Part of the analytic work reported herein was supported by the National Institute of Education Grant #NIE-G-76-0082. The rest of the analytic work and all empirical research reported herein was performed pursuant to contracts with the states of Washington and Colorado, prime contractors for the Department of Health, Education, and Welfare under contract numbers SRS-70-53 and HEW-100-89-0004, respectively. The opinions expressed in the paper are those of the authors and should not be construed as representing the opinions or policies of the states of Washington or Colorado or any agency of the United States Government.

[2]A simultaneous linear equations model might appear to be one alternative. However, the same exogenous variables appear in every equation, leading to underidentification of the structural parameters.

[3]For a more thorough discussion, see Cox and Miller, 1965; Feller, 1968; Karlin and Taylor, 1975; or any other standard text on stochastic processes.

[4]Mathematically these assumptions can be expressed as:

$$0 \leq p_{jk}(u,t) \leq 1 \quad ;$$

$$\sum_k p_{jk}(u,t) = 1 \quad ;$$

$$P(t,t) = I \quad .$$

[5]Equations (8) through (12) do not apply to a general three-state model, i.e., one in which r_{31} and r_{32} are greater than zero.

[6]As mentioned by an anonymous referee, we could have chosen to model r_j and m_{jk} ($\triangleq r_{jk}/r_j$) rather than r_{jk} as functions of exogenous variables. The choice involves a decision about the substantive nature of the process; it is not just a methodological issue. The approach suggested by the referee, which has also been advocated by Tuma (1976) and by Singer and Spilerman (1974), is appropriate when the decision to leave the current state and the choice of a destination are separate or sequential. In the example used in this paper, it seems reasonable to think that the decision to leave a state and the choice of a destination are <u>not</u> separate; therefore, we have not adopted this other approach. For other social processes it may, indeed, be preferable to model r_j and m_{jk} separately. The method of estimation and accompanying soft-ware described below can readily be used to estimate $r_j = e^{\underline{\alpha X}}$ from data on the dates of entering and leaving state j.

[7]A FORTRAN computer program called RATE has been developed to find the maximum likelihood estimates of parameters in Equations (36), (37), and (43), among others. Written documentation, test data and test output are also available. For information, write the first author.

[8]For our definition of marriage and marital dissolution, see Hannan, Tuma, and Groeneveld, 1977.

[9]The same problem arises in regression analysis when dummy variables are used to represent a categorical variable. One category must be omitted to achieve identification; the one chosen affects the interpretation of coefficients of the included dummy variables but does not affect predictions of the dependent variable.

[10]The rate of the control group may vary over time because of aging, secular trends, etc.

[11]Although these papers do not explicity mention attrition as a third state, the actual estimation procedures were the same as those reported here.

[12]If there is no attrition, the probability of a change in marital status before t is $1 - \exp[-r_{jk}t]$. (See equation (17)). By a Taylor expansion this is approximately $1 - [1 - r_{jk}t] = r_{jk}t$. So when $t = 1$, the probability of a change is approximately r_{jk}.

[13]These numbers are obtained by rounding off the crude proportion of controls who marry (if initially unmarried) or end their marriage (if initially married) in the first year of the experiment.

[14]Some of these problems do not arise in certain special cases. For example, the first, second and fifth problems mentioned below do not occur for the two-state model.

[15] We used this feature in Hannan, Tuma, and Groeneveld (1976) to eliminate the possibility that effects of income maintenance treatment on the rate marital dissolution were due to attrition bias. Our unpublished Monte Carlo studies show that random error in the timing of events has surprisingly small effects on the quality of maximum likelihood estimators of rates obtained from event history analysis.

Table I. Effects of Variables on Rates of Marital Status Change and Attrition of White Women: Time-Independent Model.†

Variables	Marital Dissolution	Attrition of Married Women	Marital Formation	Attrition of Unmarried Women
$3800 Support Level	2.29***	1.24	1.27	.86
$4800 Support Level	2.06***	.96	1.08	.40**
$5600 Support Level	1.57	.53	.80	.64
Three-year program	.86	.79	1.01	.91
Normal Income Level:				
$0-999	3.80***	2.52*	.31**	.82
$1,000-2,999	2.71***	1.57	.48	.64
$3,000-4,999	2.21***	.89	.53	.51
$5,000-6,999	1.88**	1.09	.54	.60
$7,000-8,999	1.21	1.06	.62	.65
Unclassified	4.13***	1.26	.64	1.12
One if on AFDC before enrollment	1.54**	.85	1.09	.81
One if any children under 6 years	.97	1.10	.83	.62*
Number of children	.88**	.93	1.20**	1.01
Woman's age in years	.97***	1.00	.91***	.95***
Woman's education in years	.97	.95	1.02	.98
Woman's wage in $/hr.	1.33**	1.41**	1.05	1.00
One if Denver	.90	1.85***	.92	1.79**
Constant	.10***	.04***	2.60	.73
N	1376	1376	914	914
Likelihood ratio test for model (17 d.f.)	107.55***	26.52*	97.87***	32.16**
Likelihood ratio test for experimental effects (4 d.f.)	18.78***	8.03*	2.91	9.54**

† Coefficients are the multipliers of the rate for a unit change in a variable i.e., exp ($\hat{\theta}$). A coefficient of 1.0 means the variable has no effect.

Table 2. Time-Dependence of Effects of Treatments on Marital Dissolution Rates of White Women† (N=1376)

	$3800 Support	$4800 Support	$5600 Support	Three-year Treatment	Financial Treatment	Likelihood Ratio Test (χ^2) for Effects of Experimental Treatments	Degrees of Freedom	Likelihood Ratio Test (χ^2) for Time-Dependent Effects of Experimental Treatments	Degrees of Freedom
ONE-PERIOD MODEL									
(0-2.0 years)	2.29***	2.06**	1.57	.86	--	18.78***	4	--	--
FOUR-PERIOD MODEL									
First Period (0-0.5 year)	5.69***	3.37**	4.27**	.74	--	33.71***	16	14.80	12
Second Period (0.5-1.0 year)	1.83	1.55	.71	.76	--				
Third Period (1.0-1.5 years)	2.07*	1.75*	1.34	.68	--				
Fourth Period (1.5-2.0 years)	1.53	2.23*	1.50	1.47	--				
TWO-PERIOD MODEL									
First Period (0-0.5 year)	1.96***	1.76**	1.35	.86	2.32**	23.33***	5	4.40**	1
Second Period (0.5-2.0 years)	1.96***	1.76**	1.35	.86	--				

† All equations contain the other causal variables given in Table 1. Coefficients are exp(θ) and indicate the multipliers of the rate. A coefficient of 1.0 means the variable has no effect.

* $0.10 \geq p \geq 0.5$; **$0.05 \geq p > 0.01$; ***$0.01 \geq p$.

Table 3. Observed and Predicted Values of Arbitrary Outcomes.

Outcome†	Sample§ Size	Obsvd. Mean	Our Model Mean Residual	Our Model R^2	Linear Regression Model, R^2	Diff. in R^2
Single at t=1	1917	.351	-.005	.675	.664	.011
Single at t=2	1917	.350	.002	.500	.481	.019
Married at t=1	1917	.598	.011	.631	.627	.003
Married at t=2	1917	.540	-.004	.439	.428	.011
Attrited at t=1	1917	.050	-.006	.008	.010	-.002
Attrited at t=2	1917	.111	.002	.025	.023	.002
No. Marital Dissolutions, t=1	1917	.050	-.002	.050	.043	.007
No. Marital Dissolutions, t=2	1917	.109	-.001	.083	.068	.015
No. Marriages, t=1	1917	.046	.002	.111	.087	.024
No. Marriages, t=2	1917	.086	-.006	.138	.109	.029
Continuously Single to t=1	705	.848	.004	.069	.072	-.003
Continuously Single to t=2	705	.740	.020	.092	.089	.003
Continuously Married to t=1	1212	.885	.016	.022	.029	-.007
Continuously Married to t=2	1212	.768	.017	.044	.045	-.001

† Except for the variables on number of marital events, the observed variables are dummy (0-1) variables.

§ There were 705 initially single white women and 1212 initially married white women, giving a total of 1917 white women.

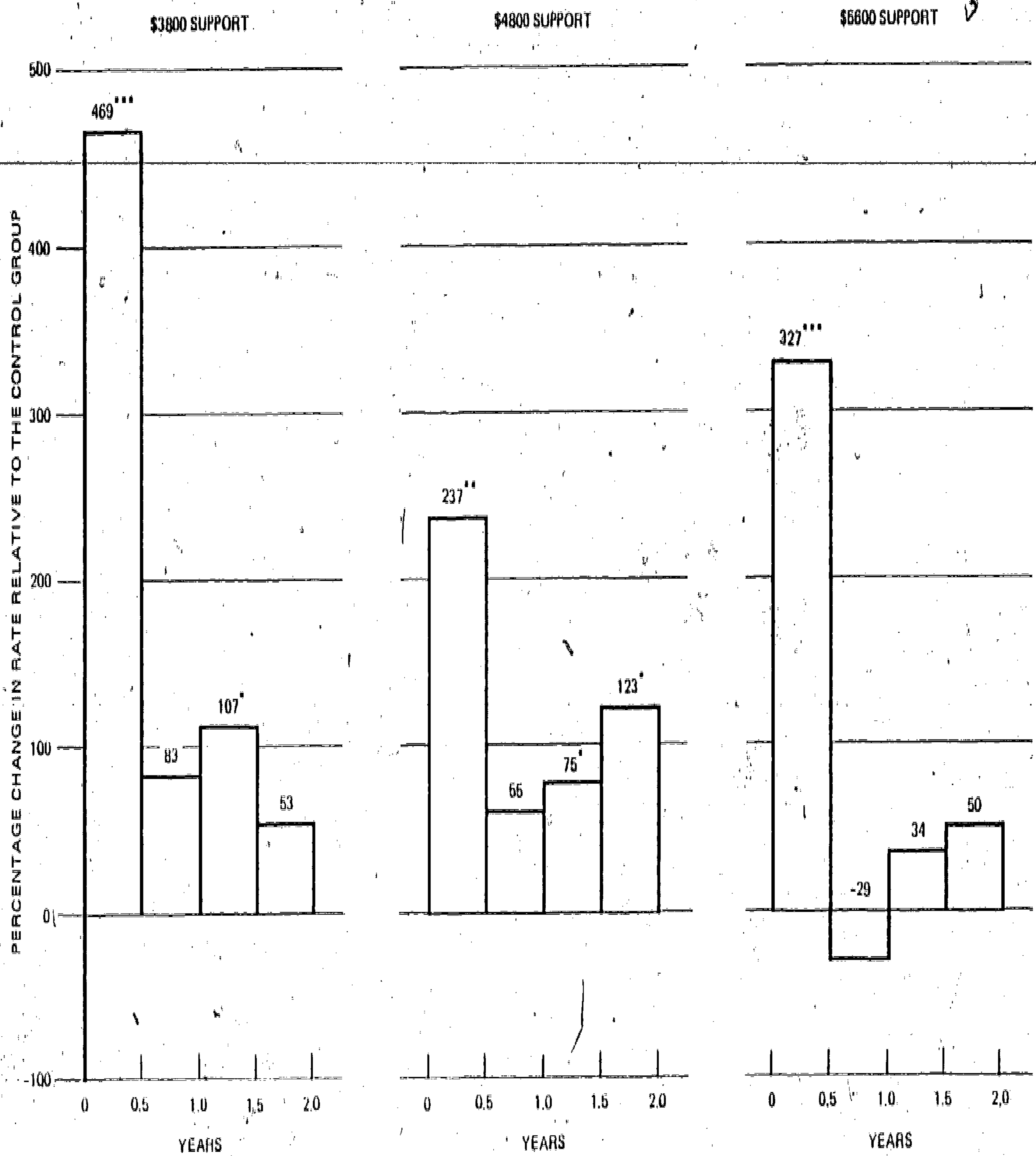

FIGURE 1 RELATIONSHIP OF INCOME MAINTENANCE SUPPORT LEVELS TO MARITAL DISSOLUTION RATES OF WHITE WOMEN OVER TIME

*0.10 > p > 0.05; **0.05 > p > 0.01; ***0.01 > p.

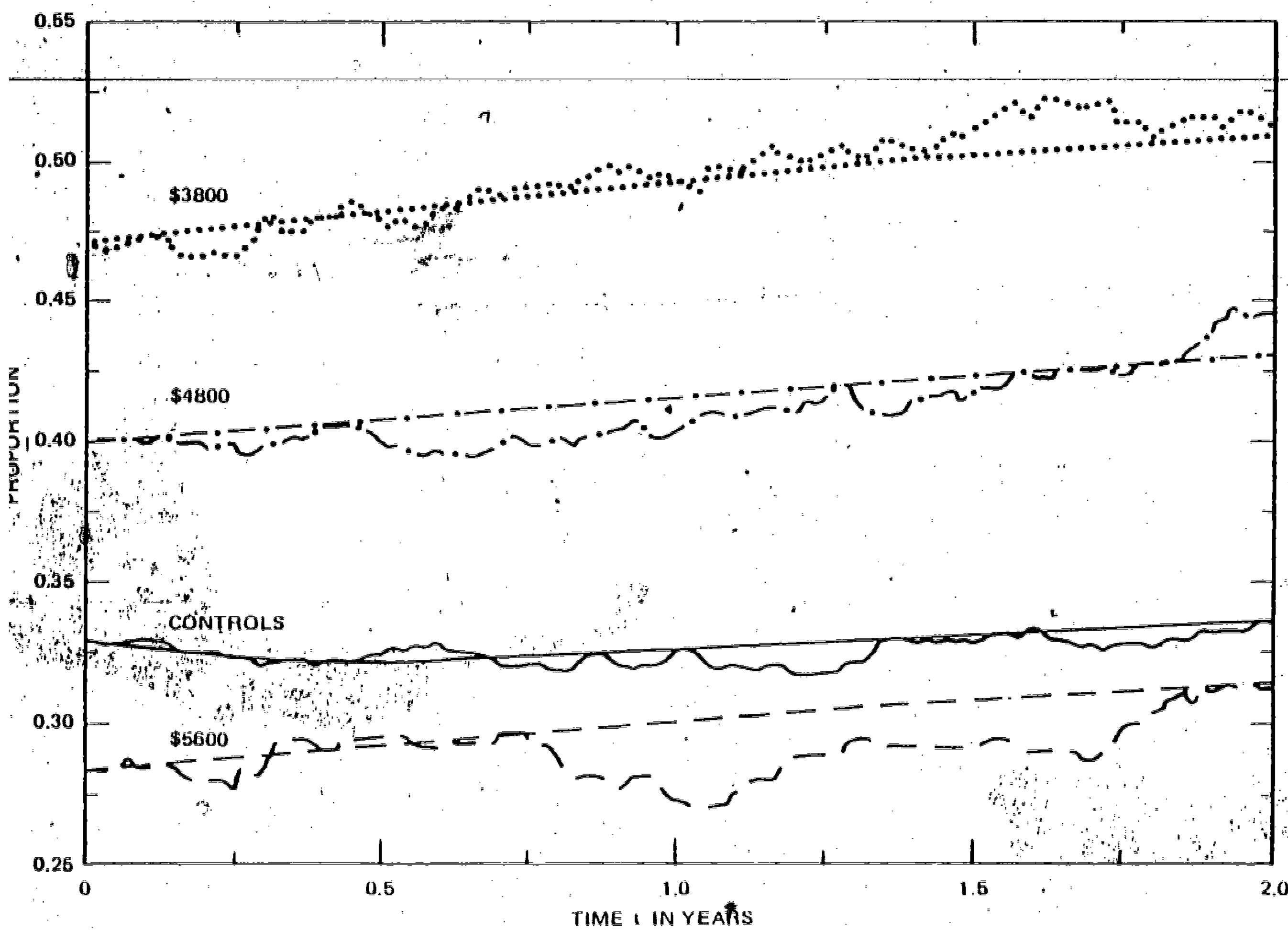

FIGURE 2 OBSERVED AND PREDICTED PROPORTION OF WHITE WOMEN WHO ARE SINGLE AT TIME t, CONDITIONAL ON NOT HAVING ATTRITED BY TIME t, BY SUPPORT LEVEL

APPENDIX

To write a likelihood equation like (39) for the model with transition rates that vary from one period p to another (43), we need the survivor function for state j, $G_j(t|u,\underline{X},\underline{Z}_p)$, where $u \overset{\Delta}{=} t_{m-1}$ (the time of the $(m-1)^{th}$ event) and t is some later time, $u \leq t$. The survivor function is obtained by solving the analogue of (21):

$$f_j(t|u,\underline{X},\underline{Z}_p) = r_{jp}\, G_j(t|u,\underline{X},\underline{Z}_p) \qquad (A1)$$

where $r_{jp} \overset{\Delta}{=} \sum_k r_{jkp}$. Since $f_j(t|u,\underline{X},\underline{Z}_p) = - dG_j(t|u,\underline{X},\underline{Z}_p)/dt$, this implies that

$$\frac{dG_j(t|u,\underline{X},\underline{Z}_p)}{G_j(t|u,\underline{X},\underline{Z}_p)} = - r_{jp}\, dt \qquad (A2)$$

Integrating both sides from u to t, we obtain

$$\ln G_j(t|u,\underline{X},\underline{Z}_p) = - \int_u^t r_{jp}\, dt \qquad (A3)$$

$$G_j(t|u,\underline{X},\underline{Z}_p) = e^{-\int_u^t r_{jp}\, dt} \qquad (A4)$$

To eliminate the integral on the right-hand side of the expression above, define u'_p and t'_p as follows:

$u'_p = 0$ if $\tau_p < u$ or $t < \tau_{p-1}$ (state j is entered after period p ends or t occurs before period p begins)

$= u$ if $\tau_{p-1} \leq u < \tau_p$ (state j is entered in period p)

$= \tau_{p-1}$ if $u < \tau_{p-1} < t$ (state j is entered before period p begins and t occurs after period p begins)

$t'_p = 0$ if $\tau_p < u$ or $t \leq \tau_{p-1}$ (state j is entered after period p ends or it occurs before period p begins)

$= \tau_p$ if $u < \tau_p < t$ (state j is entered before period p ends and t occurs after period p ends)

$= t$ if $\tau_{p-1} < t \leq \tau_p$ (t occurs in period p),

where τ_p is the end of period p, as defined in the text. Using these definitions,

$$G_j(t|u,\underline{X},\underline{Z}_p) = e^{-\sum_{p=1}^{P}(t'_p - u'_p)r_{jp}(\underline{X},\underline{Z}_p)} \qquad \text{(A5)}$$

So, analogous to (39), we can write the likelihood equation for the model given by (43) as:

$$\mathcal{L} = \prod_{j=1}^{n} \prod_{i=1}^{N} \prod_{m=1}^{\infty} \left\{ \left[G_j(t_m|t_{m-1},\underline{X},\underline{Z}_p) \right]^{v_{m-1,j}} \cdot \prod_{k=1}^{n} \left[r_{jkp}(\underline{X},\underline{Z}_p) \right]^{w_m v_{m-1,j} v_{mk}} \right\} \qquad \text{(A6)}$$

Albert, A. 1962. "Estimating the Infinitesimal Generator of a Continuous Time, Finite State Markov Process." Annals of Mathematical Statistics 33 (June): 727-753.

Bartholomew, David J. 1957. "A Problem in Life Testing." Journal of the American Statistical Association 52 (September): 350-355.

__________. 1973. Stochastic Models for Social Processes, 2nd edition. New York: John Wiley and Sons.

Berkson, J. 1944. "Application of the Logistic Function to Bioassay." Journal of the American Statistical Association 39 (September): 357-365.

Blumen, Isadore, Marvin Kogan, and Philip J. McCarthy. 1955. "The Industrial Mobility of Labor As a Probability Process." Cornell Studies in Industrial and Labor Relations 6. Ithaca: Cornell University Press.

Breiman, L. 1969. Probability and Stochastic Processes. Boston: Houghton-Mifflin.

Coleman, James S. 1964. Introduction to Mathematical Sociology. Glencoe: Free Press.

__________. 1968. "The Mathematical Study of Change." In H. Blalock and A. Blalock (eds.), Methodology in Social Research. New York: McGraw Hill.

__________. 1973. The Mathematics of Collective Action. Chicago: Aldine Publishing Co.

Cox, D. R. and H. D. Miller. 1965. The Theory of Stochastic Processes. London: Methuen & Co., Ltd.

Dhrymes, Pheobus. 1970. Econometrics: Statistical Foundations and Application. New York: Harper & Row.

Feller, William. 1968. An Introduction to Probability Theory and its Applications, Vol. 1, 3rd edition. New York: John Wiley & Sons, Inc.

Goodman, Leo A. 1972. "A Modified Multiple Regression Approach to the Analysis of Dichotomous Variables." American Sociological Review 37 (February): 28-45.

Hannan, Michael T., Nancy Brandon Tuma, and Lyle P. Groeneveld. 1976. "The Impact of Income Maintenance on the Making and Breaking of Marital Unions: Interim Report." Research Memorandum 28. Menlo Park, California. Stanford Research Institute, Center for the Study of Welfare Policy.

Hannan, Michael T., Nancy Brandon Tuma, and Lyle P. Groeneveld. 1977. "Income and Marital Events: Evidence from an Income-Maintenance Experiment." American Journal of Sociology 82 (May): 1186-1211.

Karlin, Samuel and Howard Taylor. 1975. A First Course in Stochastic Processes. New York: Academic Press.

Land, Kenneth C. 1971. "Some Exhaustible Poisson Process Models of Divorce by Marriage Cohort." Journal of Mathematical Sociology 1 (2): 213-232.

Mayer, Thomas F. 1972. "Models of Intragenerational Mobility." Pp. 308-357 in J. Berger, M. Zelditch, Jr., and B. Anderson (eds.), Sociological Theories in Progress, Vol. 2, Boston: Houghton-Mifflin Company.

McFarland, David D. 1970. "Intragenerational Social Mobility as a Markov Process: Including a Time-Stationary Markovian Model That Explains Observed Declines in Mobility Rates." American Sociological Review 35 (June): 463-76.

Singer, Burton and Seymour Spilerman. 1974. "Social Mobility Models for Heterogeneous Populations." Pp. 256-401 in Herbert L. Costner (ed.), Sociological Methodology 1973-1974. San Francisco: Jossey-Bass.

Singer, Burton and Seymour Spilerman. 1976. "The Representation of Social Processes by Markov Models." American Journal of Sociology 82 (July): 1-54.

Sorensen, Aage B. 1975. "The Structure of Intragenerational Mobility." American Sociological Review 40 (August): 456-471.

Spilerman, Seymour. 1970. "The Causes of Racial Disturbances: A Comparison of Alternative Explanations." American Sociological Review 35 (August): 627-649.

______. 1972b. "The Analysis of Mobility Processes by the Introduction of Independent Variables into a Markov Chain." American Sociological Review 37 (June): 277-294.

______. 1976. "Structural Characteristics of Cities and the Severity of Racial Disorders." American Sociological Review 41 (October): 771-793.

Theil, H. 1969. "A Multinomial Extension of the Linear Logit Model." International Economic Review 10 (October): 251-259.

______. 1970. "On the Estimation of Relationships Involving Qualitative Variables." American Journal of Sociology 76 (July): 103-154.

Tuma, Nancy Brandon. 1976. "Rewards, Resources, and the Rate of Mobility: A Non-Stationary Multivariate Stochastic Model." American Sociological Review 41 (April): 338-360.

Tuma, Nancy Brandon and Michael T. Hannan. Forthcoming, 1978. "Approaches to the Censoring Problem in Analysis of Event Histories." In Karl Schuessler (ed.), Sociological Methodology 1979. San Francisco: Jossey-Boss.

Tuma, Nancy Brandon, Lyle P. Groeneveld and Michael T. Hannan. 1976. "First Dissolutions and Marriages: Impacts in 24 Months of SIME/DIME." Research Memorandum 35. Menlo Park, California. Stanford Research Institute, Center for the Study of Welfare Policy.

Tuma, Nancy Brandon, Michael T. Hannan and Lyle P. Groeneveld. 1977. "Variation Over Time in the Impact of the Seattle and Denver Income Maintenance Experiments on the Making and Breaking of Marriages." Research Memorandum 43. Menlo Park, California: Stanford Research Institute, Center for the Study of Welfare Policy.