DOCUMENT RESUME

ED 171 561 SE 027 752

AUTHOR	Suydam, Marilyn N., Ed.
TITLE	Investigations in Mathematics Education, Vol. 12, No. 2, Spring 1979.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	79
NOTE	72p.
AVAILABLE FROM	Information Reference Center (ERIC/IRC), The Ohio State University, 1200 Chambers Rd., 3rd Floor, Columbus, Ohio 43212 (Subscription $6.00, $1.75 ea.)
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	*Abstracts; *Calculation; Elementary Secondary Education; Higher Education; *Instruction; *Mathematics Education; Open Education; Placement; *Research Reviews (Publications); Student Characteristics; Teacher Education; *Teaching Techniques
IDENTIFIERS	*Calculators

ABSTRACT

Fourteen research reports related to mathematics education are abstracted and analyzed. Five of the reports deal with various student characteristics, four with teaching techniques, two with calculators, and one each with placement, open education, and teacher education. Research related to mathematics education which was reported in RIE and CIJE between January and February 1979 is listed. (MP)

INVESTIGATIONS IN MATHEMATICS EDUCATION

Published quarterly by

The Center for Science and Mathematics Education
The Ohio State University
1945 North High Street
Columbus, Ohio 43210

With the cooperation of the ERIC® Clearinghouse for Science, Mathematics, and Environmental Education

Volume 12, Number 2 - Spring 1979

Subscription Price: $6.00 per year. Single Copy Price: $1.75
Add 50¢ for Canadian mailings and $1.00 for foreign mailings.

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Spring 1979

Becher, Rhoda McShane. THE EFFECTS OF PERCEPTUAL TRANSFORMATION EXPERIENCES AND NUMERICAL OPERATIONAL EXPERIENCES ON NUMERICAL CORRESPONDENCE AND EQUIVALENCE. Journal for Research in Mathematics Education 9: 69-74; January 1978.

Abstract and comments prepared for I.M.E. by JERRY P. BECKER, DeKalb, Illinois.

1. Purpose

The purpose of this study was to determine if there would be a significant difference ($\alpha \leq .05$) in the acquisition of conservation of number or basic mathematical and conservation-related skills by 4- and 5-year-old children of lower socioeconomic status, as shown by their performance scores on an intermediate test or a posttest as a result of (a) treatment conditions, (b) interaction of their level of development and treatment conditions, (c) interaction of the sex of the subjects and treatment conditions, and (d) interaction of level of development and sex and treatment conditions.

2. Rationale

With respect to young children's development of an understanding of numerical correspondence and equivalence, a critical examination of a wide variety of training reports indicates that in a majority of successful investigations, subjects have possessed some aspects of the conservation response (1-1 correspondence or ability to conserve on one but not two items) prior to inclusion in the study. In unsuccessful studies, subjects have been generally classified as "nonconservers" without specification of developmental status. Such non-specific classification suggests to the researcher, firstly, that children at lower stages of development (cannot establish 1-1 correspondence) were included in the samples. Secondly, although training sessions provided for learning to (a) establish 1-1 correspondence, (b) recognize equivalence of matched sets, and (c) maintain equivalence during perceptual transformations, assessment of effectiveness of training has occurred only in terms of maintenance of equivalence during perceptual transformations. Since no examination has been made of effectiveness of training approaches in facilitating learning of the other two aspects (which are conservation subskills) and since no previous studies have dealt with assessment of differential effectiveness of training for preconservation children at various stages of development, the researcher sought to look at these aspects in the present study.

Further, the researcher observes that most introductory or readiness programs in mathematics emphasize noting variance in number brought about by operations (addition and subtraction) on sets, and they do not provide experiences emphasizing invariance in number of perceptually transformed equivalent sets. Since it has not been empirically established whether experiencing a change in quantity through a numerical operation on a set is sufficient to induce the related conclusion that a nonnumerical operation (spatial transformation) means no change in quantity, the need for such an assessment appears critical--for the reason that the most widely used instructional approach for development of numerical understanding is through standard introductory or readiness programs.

3. Research Design and Procedures

Subjects

Subjects were 144 nonconserving 4- and 5-year-old participants of lower socioeconomic status (70 males, 74 females) in Head Start programs, Title I prekindergarten classes, and social welfare-supported preschools and day-care centers in Franklin County, Ohio.

Design-Procedures

Using pretest performance, the researcher classified nonconserving subjects into three levels of development with respect to conservation of number and, within these levels, according to sex. An equal number of subjects were then randomly assigned within blocks to three treatment conditions, each using a pretest/intermediate test/posttest model: Experimental group I (EI): Perceptual transformation approach; Experimental group II (EII): Numerical operational approach; Experimental group III (EIII): Control. Due to skewness of pretest data, 33 subjects were assigned to Level I, 10 to Level II, and 5 to Level

III of each treatment. So, there resulted a total of 48 subjects per treatment, with nearly equal numbers of males and females placed at each level, and a total of 144 subjects.

Treatment Conditions

			EI	EII	EIII
Levels of Development with Respect to Non-conservation of Number	I	M	16	16	16
		F	17	17	17
	II	M	4	4	4
		F	6	6	6
	III	M	3	3	3
		F	2	2	2
			48	48	48

Testing

A four-item modified version of the standard Piagetian conservation-of-number test was individually administered, in a fixed order, to each subject. Items 1 (six objects) and 2 (eight objects) were presented to assess each subject's ability to establish sets equivalent to those modeled by the tester. Items 3 and 4 (eight objects each) were used to evaluate the extent to which the subject was able to maintain equivalence following perceptual transformation (expansion and contraction, respectively). Rothenberg's (1969) two-question, agreement/disagreement-response format was used on each item. An ordinal scoring system for performance scores (0-7) was used, rather than classifying subjects as "conservers" or "non-conservers" in order to represent more specifically the status of all subjects with respect to their ability to (a) set up 1-1 correspondence (0-3), (b) establish equivalence of matched sets (4-5), and (c) maintain equivalence during perceptual transformations (6-7).

Instruction

Each subject in EI and EII participated in 12 group ($n = 4$) sessions 10-15 minutes long. Only one lesson was taught per session, and there were three sessions per week. Lessons and instructional procedures were standardized within each experimental treatment. Both treatments used an interactive instructional strategy (Hough and Duncan, 1970), and both involved subjects as active participants and manipulators of objects. A variety of concrete materials was used in each activity, and each lesson was presented as a new type of "game." Eight trained instructors, blind to the experimental hypothesis and randomly assigned to treatment, administered identical treatments to each of their groups of subjects.

4. Findings

Due to attrition, complete data were collected and analyzed for 109 subjects, with approximately equal numbers at each level and in each treatment (EI, 35; EII, 37; EIII, 37).

- With respect to acquisition of number, six subjects (three in EI and three in EIII) exhibited conservation; neither the perceptual transformation treatment nor the numerical operation treatment was effective in facilitating conservation after 12 instructional sessions.
- There was no significant differential effectiveness exhibited with subjects at different stages of development, nor with each sex. Of six subjects achieving conservation, five were boys and, as mentioned earlier, three were in EIII.
- There were significant differences ($F = 4.10$, 2/91 df, $p = .02$) in acquisition of subskills, but no interaction effects with level of development or sex.
- Dunn's multiple comparison procedure was applied to six a priori established comparisons at the collective .05 level:
  - Subjects in the perceptual transformation treatment exhibited significantly higher ($p < .05$, 91 df) mean scores (4.01) than subjects in the control group (2.78) on the posttest.
  - Subjects in the perceptual transformation treatment exhibited significantly higher ($p < .05$; 91 df) mean scores than subjects in the numerical operations treatment (3.34) on the posttest.
  - There were no significant differences between the numerical operation and control treatments.

- There were no significant differences on the intermediate test among the treatments.

5. Interpretations

The researcher states that after 12 instructional sessions, the perceptual transformation treatment was shown to be most effective in facilitating acquisition of conservation subskills. With respect to facilitating acquisition of conservation of number, the researcher appeals to Piagetian theory to explain the failure, mentioning that subjects may not have been at a level of maturity to permit intellectual shifts prerequisite for conservation; or that experiences may not have been of the type necessary for learning to occur; or both. The researcher also observes that, moreover, the lack of effectiveness of the treatments may be due more to time and number of sessions than a result of activities employed or subject's level of maturity.

In the perceptual treatment, activities placed primary emphasis on development of the concept of "same" with respect to 1-1 correspondence, whereas in the numerical treatment, activities placed equal emphasis on the concepts of "same," "more," and "less." Thus, a unified single-concept emphasis appears to be more effective in the initial establishment of basic mathematical skills and concepts with 4- and 5-year-old subjects of lower socioeconomic status. This may explain the unexpected failure of the numerical treatment to facilitate acquisition of the mathematical and conservation-related skills.

The researcher further discusses an explanation for the greater success of the unified single-concept emphasis of the perceptual transformation treatment by drawing on and integrating the research regarding children's understanding of the relational terms used in questioning procedures with either Piaget's equilibration theory or a learning theory description of the formation of response sets. Results of previous investigations (Beilin, 1965; Pratoomraj and Johnson, 1966; Siegel and Goldstein, 1969) of young children's (ages 4 to 7) understanding of relational terminology indicate that although these children seem to understand the relational terms "more" and "less," the majority do not understand the meaning of "same." Within Piagetian theory, this suggests that in a situation where all three terms receive approximately equal emphasis, the children may tend to focus on the familiar aspects of the situation (i.e., their understanding of "more" and "less" meaning "different"). They assimilate (rather than accommodate) the unfamiliar term "same" (especially when it is used in relation to perceptual transformations) into their existing schemata of "different" without recognition of how the two "differences" were brought about (i.e., operation or transformation). In the perceptual transformation treatment, however, the consistency of emphasis on the unfamiliar concept and terminology ("same") may have prevented assimilation, thus resulting in disequilibrium. In order to resolve this disequilibrium, subjects were forced to accommodate their thinking and thus "learn" the meaning of "same." Similarly, from a learning theory basis, it might be suggested that in the numerical operational treatment there was a greater opportunity to respond to changes, and therefore differences, that can be described by familiar terminology ("more" or "less"). This established a response set that was then generalized to unfamiliar "changes" (transformations) and corresponding terminology ("same"). Again, in the perceptual transformation treatment, the consistent attention to, and emphasis on, the unfamiliar concept of "same," without the interfering familiarity of responses to "more" and "less," may have provided the necessary practice in making the correct responses required for the establishment of an appropriate response set to "same."

Finally, the researcher comments that although it is not possible to explain completely the differences between the numerical and perceptual treatments, the specific findings with respect to lack of effectiveness of the numerical treatment have educational importance. It is concluded that, in view of no success in establishing an understanding of numerical constancy nor in facilitating development of the basic mathematical and conservation subskills of 1-1 correspondence and the concepts of "same" and "more" in relation to number, that a change in the type of basic mathematical readiness experiences provided for 4- and 5-year-old children of lower socioeconomic status appears needed.

Abstractor's Comments

The first reaction I had to the study is that it is tedious and complex. However, after some study and thought, I came to view the study as one which looks at an important aspect of mathematics learning (children's development of understanding of numerical correspondence and equivalence), was well-formulated, and was carried out quite nicely. The rationale seems plausible and seems to provide a sound basis on which the study was conducted, and the design and procedures were clear and straightforward. As such, the research has all the elements of an excellent investigation.

But there are several observations and reactions that seem in order. Not much is reported regarding the actual interaction between instructor and subject. In studies such as this, I want to know more about what is actually said to the subject and how the subject responds. For example, does language (communication) get in the way and provide difficulties for the subject? And how is the response or reaction of the subject to the instructor to be interpreted? I would find more information along these lines helpful and, perhaps, useful in explaining the results. Also, a session lasting just 15 minutes strikes me as a rather short period of time, taking into consideration that a very young subject has to receive a communication, process it, and prepare a response for interpretation by the instructor.

The researcher observed that the lack of effectiveness of the two treatments may be due to time and number of instructional sessions, more so than to the activities employed or the children's level of maturity. I tend to agree and am tempted by the notion that more sessions would yield the results expected in the beginning. This provides the basis for a follow-up study. But related to this, I would also like to know more about the activities themselves--a more thorough description would have been useful.

I, with the researcher, am also tempted by the notion that the numerical treatment is indeed a more complicated treatment from the subject's point of view, since it involved equal emphasis on the concepts of "same," "more," and "less." There is more chance of confusion and more understanding required of numerical treatment subjects than perceptual treatment subjects. This difficulty could be remedied, perhaps easily, in a follow-up study.

I am not sure I'm ready to accept the researcher's last conclusion that a change in the type of basic mathematical readiness experiences provided for 4- and 5-year-old children of lower socio-economic status appears warranted. To begin with, is much known about what the basic mathematical readiness experiences of such children are? And if so, how are these experiences to be modified? It would have been interesting to read the investigator's thoughts on these questions.

Campbell, Patricia F. TEXTBOOK PICTURES AND FIRST-GRADE CHILDREN'S PERCEPTION OF MATHEMATICAL RELATIONSHIPS. Journal for Research in Mathematics Education 9: 368-374; November 1978.

Abstract and comments prepared for I.M.E. by H. LAVERNE THOMAS, State University of New York College at Oneonta.

1. Purpose

This investigation studied the effect of pictures and illustrative mode on the perception of mathematics textbook pictures by first-grade children. The relationship between the ability to tell a story about a picture of a sequence of three pictures and their ability to describe the picture(s) by using a number sentence was also studied.

2. Rationale

The presence in many current primary mathematics texts of pictures intended to convey mathematical concepts to students in a dynamic fashion was taken as a motivation for this study. Reference is made to prior studies regarding features within pictures influencing children's interpretations and forms of pictorial representation that are optimal in conveying concepts. Other research quoted suggests very gradual development among young children of the ability to perceive implied motion in static pictures. Also, ability to infer relationships among illustrated characters is slow to develop. These characteristics are asserted to be necessary to the perception of mathematical relationships from dynamic illustrations.

3. Research Design and Procedures

The illustrative modes studied were (a) stylistic (personified animals or inanimate objects) and (b) realistic (representations of persons). Number sentences were represented in these modes by both a sequence of three illustrations representing a dynamic situation (joining, separating) and a single illustration in the sequences mentioned. Parallel illustrations for five addition and five subtraction number sentences were constructed in each mode. A set of five response cards accompanied each illustrative sequence or single picture, the set containing one card with a correct number sentence and one card for each of four distractors. These cards were used for subjects in a forced-choice response mode. The second response mode required subjects to generate a number sentence to fit the situation illustrated. Each subject responded to ten situations.

A factorial design was employed with number of pictures, illustrative mode, and response type as factors. However, the number-of-pictures factor was further divided by adding two additional treatments: five single illustrations followed by five sequences, and vice versa. Thus 4 x 2 x 2 = 16 treatment groups result as illustrated below:

Presentation		Illustration		Response
10 single				
10 sequence		Stylistic		Forced choice
5 single-5 sequence	X	Realistic	X	Generated
5 sequence-5 single				

From a random sample of 96 first-grade students from two elementary schools, a random assignment was made of six students to each treatment group. Each subject was given the treatment with immediate testing on each item as to two outcome variables: score on a story describing the situation depicted and a number sentence score. Completion of the interview usually occurred in 15 minutes or less.

Data were analyzed by non-parametric methods because of concern for normality. The Kruskal-Wallis (two-sided) and Mann-Whitney U (one-sided) tests of significance were employed. Total scores (story scores, number sentence scores) for each student were employed.

4. Findings

No significant effects were found for illustrative mode or number of pictures (single, sequence, or mixed) on either story response scores or number sentence response scores ($\alpha = .05$). A significant effect on number sentence response scores favoring the forced-choice response mode was observed ($p < .001$).

Post hoc analyses indicated the following:

(a) Subjects responding consistently to at least 7 of the 10 similar items showed a significant association (Chi-square, $p < .025$) between story response scores and number of pictures (sequence or single).

(b) On the last five responses, subjects who had received five sequences scored significantly better on story responses than those whose entire presentation consisted of single pictures (Mann-Whitney U test, $\alpha = .025$, $p = .0103$).

(c) Correlations (Kendall) of number sentence scores with Key Math test scores (given upon entering first grade) and with story response scores revealed correlations of .28 and .21 respectively.

5. Interpretations

(a) The complexity of "stylistic" pictures does not hinder abstraction of mathematical relationships. Thus "stylistic" as well as "realistic" pictures should be employed in primary mathematics textbooks.

(b) The influence that viewing sequenced pictures has on the story response scores on subsequent single picture illustrations implies that the use of sequenced illustrations should precede single pictures in primary textbooks.

(c) Further research regarding the effect of number of pictures (sequential, dynamic, to portray a particular mathematical relationship) is needed.

(d) Investigation of the relationship of the Piagetian conservation tests of cognitive development to picture interpretation skills was suggested since the investigation of relationships with Key Math scores was non-productive. And, also, the analysis of the mathematical content of the pictures employed seems to rest on some understanding of part-whole-relationships.

Abstractor's Comments

The research reported is of such a nature as to have immediate relevance to the construction of primary mathematics textbooks and other printed instructional materials. Although not specifically mentioned, the questions raised are generally significant to the multitudinous "worksheet" or "activity" sheets employed by primary teachers of mathematics. Generally, this study provides some useful information in these directions.

However, a number of questions need to be raised regarding the research reported. Some of these are probably created by the brevity of the report and the abstractor acknowledges that the researcher was operating under considerable restraint in this regard.

It was not clear from the report how the sequencing of addition and subtraction instances was handled. Were there five additions followed by five subtraction instances? In particular, how was this handled for the "mixed" presentations of five single and five sequences of three pictures? With respect to the presentation of the five choices in the forced choice response mode, what is meant by balancing the order of presentation of choices across the 10 sequences? Were they balanced randomly or systematically? Is "10 sequences" the proper referent? Is not "10 items" the proper referent? Also, was the scoring procedure for number sentence responses a right/wrong process (it appears that it was) in both modes? What was the scoring procedure for the story responses?

Perhaps more significantly, the study suffers from the lack of a strong rationale or theoretical base. In this context, the abstractor suggests that a rationale based upon meaning could provide support and add power in analyzing the results of the study. For example, there seemed to be an underlying assumption that "stylistic" illustrations would be less meaningful and hence would be less well reported in terms of both story and number sentence responses. This was not the case. In the abstractor's view this is an expected result since both illustrative modes appear stylistic --the persons portrayed having little if any more "personality" than the personified animals or objects. What is "real" to a child?

Also, from the illustrations seen, the complexity of the "stylistic" illustrations is no greater than that of the "realistic" ones.

On the other hand, the improved story responses on single pictures following viewing sequences would seem to indicate a higher level of meaning for such sequences which was transferred to or projected upon the following single picture illustrations. That this effect did not carry over to number sentence response scores is a little surprising. However, recent work by Greeno, Mayer, and others would suggest that if the task were different, the effect of a different level or type of meaning in the situation would be reflected in differential performance. For example, generating a number sentence to fit an illustration is a higher order activity than selecting a response to fit it. Was there such a differential effect in evidence when contrasting sequential illustrations with single pictures?

Another level where meaning would seem to be involved is the story response requested for each illustration. A high story response score should indicate a high level of meaning which would be reflected in higher number sentence scores. That this did not occur may indicate no effect of meaning, an effect that was "washed out" in the design since everyone gave a story response--this being the critical factor rather than the particular response, or an effect not tested by the number sentence task.

It would be interesting to see a follow-up study which addressed these issues directly in the design; for example, effect of story response required or no story response required on number sentence scores, as well as an interactive effect of sequential vs. single pictures vis-a-vis forced choice vs. generative response number sentence tasks.

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Greeno, J. G. The structure of memory and the process of solving problems (Tech. Rep. 37). Ann Arbor, Mich.: University of Michigan, Human Performance Center, 1972.

Mayer, Richard E. Different rule systems for counting behavior acquired in meaningful and rote contexts of learning. Journal of Educational Psychology, 1977, 69, 537-546.

Coward, P. H. ELECTRONIC CALCULATORS IN FURTHER EDUCATION. Mathematics Teaching 82: 26-28; March 1978.

Abstract and comments prepared for I.M.E. by ZALMAN USISKIN, University of Chicago.

1. Purpose

The study attempted to gather "quantitative evidence of students' performance with calculators." Because attitudes and arithmetic achievement were felt to affect each other, the study involved both of these variables.

2. Rationale

The pocket calculator is perhaps the most significant change in the learning environment in the mathematics classroom in recent years. It affects both the engineer and the consumer. Yet the extent to which calculator use enables a student to pursue his [or her] studies with greater efficacy is not known.

3. Research Design and Procedures

Two different samples were used. For the achievement study, the sample consisted of 414 students attending Doncaster College of Technology, including school children and part-time students. It was assumed that these students are generally typical of the student population of the country (England) as a whole. For the attitude study, the sample consisted of twenty staff members and thirty-two students.

For the achievement study, a test of forty arithmetic items taken from engineering, business, and other sources was prepared. "There was a general trend of complexity from simple addition through percentages, powers and Pythagoras to the solution of a quadratic equation. It is emphasized however that no item required anything of the student other than arithmetic manipulation." This test was given twice to each student with a one-week interval between administrations. On the first administration, no calculators were allowed.

Before the second administration, each student was allowed up to one hour's practice with an SR-10 calculator. Twenty-four statements were offered on the attitude questionnaire, with subjects responding by expressing agreement or disagreement on a five-point scale. Written comments could be appended to the questionnaire.

Neither sample was random. Students in the achievement study sample were given the tests in their classes and the researcher "sought to make it as representative as possible by including students in a wide range of courses and disciplines, of both sexes and of all appropriate ages." The students in the attitude study were not volunteers but were selected in twos or threes "from various classes."

In achievement, the percentage of improvement for each student and mean scores for the pre-test and post-test groups were calculated. These scores were analyzed also for common course subgroups of the original sample. Analysis showed weak internal consistency of the items on the questionnaire, so reporting of attitudes was limited to the written comments.

4. Findings

The pretest achievement scores had a mean of 16.28 and a standard deviation of 8.05. For the posttest, the mean was 27.19 and the standard deviation was 6.25. Subgroup mean scores improved from 16.5 percent to 128.6 percent, with many groups showing over 99 percent improvement.

It was noted that many could not complete half of the pretest within the allotted hour, while that time was sufficient for all on the posttest.

Written comments on the questionnaire included (among others) that use of calculators allowed more time to be spent upon principles, gave students more confidence, increased interest, lessened fatigue, forced greater understanding of accuracy, and lessened the use of mental arithmetic.

5. Interpretations

The main conclusions were (1) "the use of a calculator does improve students' arithmetic performance. Further, the improvement is substantial and the statistical significance is very high; (2) the less able students show proportionately about twice the improvement of the more able." The investigator felt that the second conclusion was particularly significant, in that this betterment of performance would help the student psychologically in terms of his [or her] further education.

The increase of interest referred to by many on the questionnaire could be attributed to interest in the calculator itself or to the subject matter. Interest in the calculator itself may simply be due to the powerful visual stimulation caused by the illuminated and changing display.

Whether less use of mental arithmetic was good or bad was felt to be related to fundamental philosophical questions regarding the aims of education.

Abstractor's Comments

The report of this study is sketchy and brief; it could fit on five pages of I.M.E. Missing are many aspects crucial to an intelligent consideration of the results. For example, what were the items on the test? It is difficult to imagine a problem involving the Pythagorean theorem which requires nothing of the student other than arithmetic manipulation. How many students were in each of the subgroups of the sample? Who were the faculty members used for the questionnaire? Was it faculty or student comments which the investigator chose to relate in his report? What were the items on the questionnaire? Who prepared the items on either instrument and why wasn't a standardized instrument used?

Even a more complete report would not have corrected the major shortcomings in the study itself. The first concerns the research design. The investigator noted that a student could not have "memorized forty examples for a week, having worked them once under test conditions, ..." Yet a student could easily have remembered general principles concerning the methods of attack on the problems, thus shortening the amount of time needed for items on the posttest. This could have been corrected by randomly dividing the sample into four groups which took tests under the conditions W-WO (with calculators the first time, without the second), W-W, WO-WO, and WO-W, or by simply changing the test.

A second shortcoming concerns the length of time given to do the problems. Ostensibly the study was one of efficacy, i.e., to what extent does the calculator help a student with arithmetic calculations? Yet it is reported that "many . . . scarcely completed one-half of it [the pretest]." Thus, although the overall mean score improved from 16.28 to 27.19, it is not clear what improvement (if any) there was in the percentage of correct answers among all those that were attempted. So it is possible that speed of calculation, and not the extent to which the calculator allowed the student to concentrate on strategy, was being measured.

Finally, it is not clear that there was suitable rationale for undertaking this study. The year in which this study was undertaken is not given, but we may guess that it was a recent study, done after many comparison studies of performance with and without calculators. The investigator makes no reference to related studies, yet there are presently many such studies, some of which are more carefully done and could have shed light on the general questions about which the investigator was curious.

Doblin, Stephen A. MAPP: A MATHEMATICS PLACEMENT PROGRAM. Educational and Psychological Measurement 38: 831-833; Fall 1978.

Abstract and comments prepared for I.M.E. by JOHN R. KOLB, North Carolina State University.

1. Purpose

The investigator developed a computerized system that utilizes ten weighted scores or characteristics of each entering college freshman at the University of Southern Mississippi (USM) and recommends their placement in a mathematics course.

2. Rationale

Placement of an entering student in a mathematics course at USM had previously been based solely upon the student's American College Testing Program (ACT) examination score and an interview with a faculty advisor. Non-mathematics majors were advised by faculty from the departments in which they were majoring or by admissions counselors. As a result, many students were advised by individuals who had little knowledge of the content of the mathematics courses or the preparation they require. Students were frequently placed in courses for which they were underprepared or in courses that seriously duplicated their existing knowledge. Thus, an effort was made to develop a placement procedure that would correct some of these deficiencies.

3. Research Design and Procedures

The criteria established for a new placement program were: (a) to consider factors other than the ACT score that may in fact be better predictors of success in mathematics; (b) to include a locally-developed algebra/trigonometry placement test; (c) to concentrate solely upon placement in calculus I and lower-level courses; (d) to recommend only one specific course; (e) to invite dialog between the student and a faculty advisor; and (f) to place consistently a student in a mathematics course in which he or she has a reasonable chance of success.

The product that was constructed to meet the criteria above is called the Mathematics Placement Program (MAPP). It consists of two COBOL programs: one that descrambles information from an optical scanner tape and the second that uses biographical data and test scores to recommend a mathematics course. Recommendation of the mathematics course is based upon the following weighted factors:

1. Score on algebra placement test
2. Score on trigonometry placement test
3. Mathematics ACT Score
4. Composite ACT Score
5. Number of years of high school algebra taken
6. Number of years since high school graduation
7. Mathematics grade-point average in high school
8. Registration or lack of registration in a mathematics course during the previous term
9. Identification of high school attended
10. College at USM in which enrolled.

MAPP has been used since the summer of 1975. All freshmen entering USM take the placement test and provide all the required information on a standard optical scanner form. The MAPP computer program grades the placement test and, using the information provided, prints out a recommended mathematics course which is given to the student.

4. Findings

Data are reported only for those students who registered for a mathematics course in the Fall Quarter following the summer in which they participated in MAPP. Successful placement was defined as earning a grade of A, B, or C in the course in which the student was advised to enroll. Excluding incompletes, continuing students, and withdrawals, 242 individuals were monitored over a three-year period of which 88 percent were reported to be successfully placed as defined above.

5. Interpretations

The success rate was felt to be relatively high and the MAPP was described as adequately serving the purposes for which it was intended. Efforts would be continued to monitor results and update the factors, weights, and test questions.

Abstractor's Comments

This is a developmental study in which a specific produce is constructed to specifications to fulfill a local need. The need identified here, that of improved placement, is a common one and so the developmental procedures followed are of interest as well as the effectiveness of the product itself.

No information is given on how MAPP was actually developed. There is no indication given of what pool of factors were originally considered or on what basis factors were selected or rejected to arrive at the final ten. In addition, no description is provided concerning the statistical techniques employed in the computer programs to weigh the factors and combine them to produce a recommended mathematics course. Thus, any person who may need information about the methodology of this study would have to communicate with the investigator, for the report contains no description of the developmental procedures.

A second consideration in a developmental study is the effectiveness of the product. Here data are lacking. We are told that 88 percent of those students who enrolled in the recommended course achieved an A, B, or C. This may represent a good achievement record for those students but it may not be an appropriate measure of the success of the placement program. How many of the students making A's, B's or C's made them because they were placed in a course at a lower level than they should have been? How many of the withdrawals and incompletes (who were left out of the data) were a result of the placement program? All of these would represent placement errors that could lower the 88 percent figure. How does the success rate of MAPP compare to that under the former placement program?

Clearly, this article reports an endeavor that was never intended as research but was a reasoned response to a perplexing problem that resulted in a new product (two computer programs) and a new placement procedure (MAPP). The users are satisfied that it is an improvement over previous practice based upon their experience with it and they are reporting their results as an aid to others who may wish to follow a similar course.

Eshel, Yohaman and Klein, Zev. THE EFFECTS OF INTEGRATION AND OPEN EDUCATION ON MATHEMATICS ACHIEVEMENT IN THE EARLY PRIMARY GRADES IN ISRAEL. American Educational Research Journal 15: 319-323; Spring 1978.

Abstract and comments prepared for I.M.E. by JAMES E. BIERDEN, Rhode Island College.

1. Purpose

The purpose of this study was to examine the effects of both classroom integration and a version of open classroom procedures on the mathematics achievement of first- and second-grade pupils.

2. Rationale

The study was part of a longitudinal investigation of both the effects of integration of lower- and middle-class students and the effects of open classroom practices on the achievement of elementary school children in Jerusalem, Israel. An earlier report of the project had indicated the following:

(a) Lower-class children, who were integrated into classrooms with a majority of middle-class children, achieved at a significantly higher level at the end of first and second grade than similar lower-class children in homogeneous lower-class classrooms.

(b) The effect of an open classroom intervention was most pronounced at the end of the second grade. Lower-class children in integrated classrooms utilizing the special program were similar to their middle-class peers in achievement. Lower-class children in integrated classrooms with no special intervention performed at a lower level.

(c) The achievement of middle-class children in integrated classrooms was not negatively affected.

The present study was an attempt to investigate more specifically both the stability and the components of these effects.

3. Research Design and Procedures

The first- and second-grade classrooms used as the experimental group in this study had been organized in the form of "activity-oriented classes." The defining characteristics of these classrooms were measured by an observation schedule developed by the authors. The characteristics observed were similar to those reported in studies of U.S. and British open classrooms. It was found that the "activity-oriented classes" differed from more traditional classrooms in class-room management characteristics, pupil involvement, and affective dimensions.

For the purpose of the study, all children were designated as either lower-class (LC) or middle class (MC). This division was made along the lines of parent education and cultural background. Schools with 40 percent to 60 percent MC children were classified as integrated schools; schools with a lower percentage were classified as homogeneous. Control schools were matched on the following dimensions: percentage of LC children in the classroom, socio-economic status of parents, academic level, school reputation in the community, and religious or non-religious classification of the school. The study sample involved 1,143 children from eight schools, four experimental and four control. The control groups did not include homogeneous, MC schools with open education programs or homogeneous, LC schools without the program. Suitable data from such schools were not available at the time of the study.

The mathematics tests contained items both on arithmetic skills and more abstract basic mathematical operations. Reading comprehension tests were also administered, but a ceiling effect precluded analysis of the variability. The study was conducted in the Spring of 1972 when the project had been under way for three years.

4. Findings

The effects of type of school and grade level were examined separately for LC and MC children. A 3 x 2 analysis of variance indicated the following significant effects for LC children:

type of school	$F(2,347) = 24.59$, $p < .001$
grade	$F(1,347) = 21.64$, $p < .001$
type x grade	$F(2,347) = 7.32$, $p < .001$

The interaction effect is accounted for by the increasing gaps in achievement among students in the three types of schools at the second-grade level.

Analysis of MC scores found a significant effect of type of school: $F(2,733) = 14.92$, $p < .001$. This effect was accounted for by the lower achievement of pupils in the homogeneous traditional schools. The interaction between type of school and grade level was not significant: $F(2,733) = 2.49$.

5. Interpretations

The data suggest that integration had a positive impact on the achievement of LC children, while the open classroom seemed to be differentially effective, depending on classroom social setting. The nature of the interaction remains unclear. The authors caution against generalizing the results because of the specific meanings given to the terms "integration" and "open classroom" in this study.

Abstractor's Comments

The abstractor's reactions to this study are ambivalent at best. On the one hand there is praise for this type of research for both educational and social reasons. On the other hand, the method of reporting the study raises a number of questions which dilute its findings.

There is a definite need for the type of action research described in the study. The authors call it a "natural experiment," because the program studied arose in response to a particular set of educational and socio-political conditions rather than as a vehicle for research. It is often the case that programs which begin in this way are not subjected to the close scrutiny and analysis given by well-designed educational research. In the absence of such research and evaluation, these programs fall back on subjective arguments for their continuation or demise. The abstractor has seen examples of both.

One of the dangers of action research is that all of the pieces of a complete experimental design may not be in place for the investigator. In this study, the lack of sufficient types of schools for the control groups and the imprecision associated with the variables "integration" and "open classroom" were recognized and acknowledged by the two researchers. However, even in these situations educators can use such research to help them understand what is happening in their programs and to give them data for making decisions about future program directions.

The social implications of the study also commend it. The concept of integrated schools has taken on a rather narrow definition in the United States. This study, since it takes place in Israel where there are different sociological factors in the make-up of school populations, serves to expand the issues surrounding the relationships between integration and achievement.

Having praised the study for what it tries to do and for the important issues it addresses, the abstractor still feels the need to criticize the lack of information given by the researchers. The apparent positive impact of integration on the achievement of lower-class children is based on the results of mathematics achievement tests. Since these tests were so crucial to the results of the study, there should have been more discussion of the instruments themselves.

The description of the tests is given in one sentence: "The mathematics tests contained items centering both on arithmetic skills and the more abstract mathematical operations." In the absence of any more information, the following questions can be raised:

1. Since this study involved children from various socio-economic groups, was there any cultural bias in the tests?
2. Did the open classroom effect have any bearing on test performance; i.e., were the testing procedures the same for both experimental and control groups?
3. Would the results have been different if subtests of arithmetic skills and abstract operations had been analyzed separately?

4. Since this was a longitudinal study, to what extent does the test-retest effect enter into the results?

Investigations of relationships among academic achievement, classroom procedures and social groupings continue to be an important facet of educational research. The present study could have contributed more to our understanding of these relationships if more care had been taken in reporting both the design of the study and its results.

Ethelberg-Laursen, J. ELECTRONIC CALCULATORS AND ARITHMETIC: TWO INVESTIGATIONS. AN EXPERIMENT IN DANISH SCHOOLS. Mathematics Teaching 82: 24-25; March 1978.

Abstract and comments prepared for I.M.E. by EDWARD C. BEARDSLEE, Seattle Pacific University, Washington.

1. Purpose

The intent of this study was to determine if calculators could be used with eight-year-old students to teach addition, subtraction, and multiplication.

2. Rationale

The researcher had used calculators with children aged 13 and older, but he was interested in whether the calculator could be used in the primary classes with eight-year-olds where multiplication is introduced. Also, since many teachers had used the calculator with older children, the researcher was interested in what effect the calculator would have on introducing concepts to children.

3. Research Dsign and Procedures

Six different schools were used in the study. Four classes of eight-year-olds were selected for the experimental group, one doing traditional arithmetic and three doing "modern mathematics", and a similar four classes acted as a control group. Six other classes formed a second control group.

All classes were given a test on 100 number facts (addition, subtraction, and multiplication) in June 1975. In the new school year, the four experimental classes began using their calculators. The control classes were asked not to use calculators in their lessons. The teachers of the experimental classes were asked to use the machines as an aid on equal terms with other teaching materials, for checking addition and subtraction, and for solving problems using "everyday" realistic numbers that were otherwise unmanageable. Multiplication was taught as repeated addition; the multiplication key was not used until later.

Each teacher decided how much use to make of the machines. At the end of the year, all classes were tested on addition, subtraction, and multiplication, both mechanically and in problem solving, without using calculators.

4. Findings

The investigator reports the number of mistakes made by ranges (e.g., 0-3 errors, 4-9 errors, 10 or more errors). He also reports the average time in minutes for taking the test and the number of pupils who did not attempt a stated number of questions. No statistical analyses were performed.

5. Interpretations

Based on the raw data, the investigator concluded that the experimental group performed no worse than the control group; hence this is an answer to those critics of calculators who maintain, without evidence, that standards in arithmetic will fail if calculators are used. He also stated: "For me there is no doubt about the place of calculators in teaching."

Abstractor's Comments

This investigation is not an experimental research study; to critique it as such would consume more pages than were used in the original report. As was noted, no statistics were used to analyze data, the experimental treatment was haphazard, and the investigator based his conclusions on observing raw data and his feelings. Hence, this study should not be used in the body of research literature which will be used to support (or refute) the use of calculators in instruction.

However, this investigation appears to have been done by a classroom teacher, using existing tests. More investigations of this type would be useful provided they adhered to accepted educational research practices.

Haylock, Derek W. AN INVESTIGATION INTO THE RELATIONSHIP BETWEEN DIVERGENT THINKING IN NON-MATHEMATICAL AND MATHEMATICAL SITUATIONS. Mathematics in School 7: 25; March 1978.

Abstract and comments prepared for I.M.E. by RICHARD LESH, Northwestern University.

1. Purpose

This study attempted "to relate the notions of fluency, flexibility and originality to pupils' responses in open-ended mathematical situations." In particular, it attempted to find support for the following equation:

GENERAL CREATIVITY	+	MATHEMATICAL SKILL AND KNOWLEDGE	=	MATHEMATICAL CREATIVITY

2. Rationale

General creativity tests, evaluating students for fluency, flexibility, and originality,

> are now well established instruments in the educational psychologist's toolbox. Linked with these tests is the notion that divergent thinking plays a large part in an individual's creativity. Although mathematics is most naturally associated with convergent thinking . . . a number of authors . . . are now finding a place for "creativity" within their objectives for mathematics teaching.

3. Research Design and Procedures

A total of 136 pupils aged 14-15, with a comprehensive ability range excluding those of lowest ability, were given a series of five tests designed to investigate the relationship between divergent production in non-mathematical and mathematical situations.

Tests 1 and 2 were selected from the Minnesota battery of Tests of Creative Thinking. In Test 1, pupils were required to list the most unusual, interesting, and clever ideas they could think of for using tin cans. In Test 2, they had to sketch as many objects as possible which contained a circle as a main part. Both tests were scored for fluency (number of appropriate responses), flexibility (number of different categories of response used), and originality (a score obtained by rewarding a response according to its infrequency).

Test 3 was a multiple-choice mathematics attainment test designed to sort out those pupils who "knew enough mathematics" to make a reasonable attempt at the mathematical creativity tests.

Tests 4 and 5 were two open-ended tests of divergent thinking, one geometrical and the other arithmetical, designed to be scored for fluency, flexibility, and originality in a way similar to Tests 1 and 2. The geometry test required the pupil to make as many statements as possible about a designated line segment in a drawing. The arithmetic test asked the pupil to make the number "8" in as many different and interesting ways as possible (some examples for "9" were given in the question). Marking of this test presented some problems. Clear repetitions of the same idea were discounted even for calculating a fluency score. For flexibility in this context, pupils must dig into their mathematical experiences and select a wide range of different ideas. Many pupils produced only "a mass of complicated sums with the answer '8'". Others made use of number bases, logarithms, zero, negative numbers, fractions, decimals, trigonometry, and exponents; 35 such concepts were used and rewarded for originality.

4. Findings

The two "general creativity" tests gave moderately high correlations between themselves: 0.45 for fluency, 0.44 for flexibility, and 0.49 for originality, all significant at the 1% level. Similarly, the two "mathematical creativity" tests gave moderately high correlations between themselves: 0.50 for fluency, 0.57 for flexibility, and 0.43 for originality.

Scores first for Tests 1 and 2 and then for Tests 3 and 4 were combined to give general creativity scores for fluency, flexibility, and originality. Test 3 was then used to sort out the students who knew enough mathematics and demonstrated sufficient mastery of skills to make a reasonable range of answers possible in the open-ended tests. Fifty-two pupils scored full or nearly full marks on this test, but their scores for mathematical creativity ranged across as much as three standard deviations. For the 52 pupils, the correlations between general and mathematical creativity were near zero: 0.08 for fluency, 0.06 for flexibility, and 0.06 for originality.

5. <u>Interpretations</u>

Haylock noted that the correlations between the tests "were high enough to suggest that there was 'something there' that these two tests were measuring, even if it is somewhat presumptuous to label it 'mathematical creativity.'"

He concluded that

> Creativity within mathematics then would appear to be a specific ability; success in open-ended situations in mathematics, measured by fluency, flexibility and originality scores, cannot be attributed to a simple combination of success in general divergent thinking tasks plus sufficient mathematical skills and knowledge. . . . If we are to make the fostering of creativity one of our goals in mathematics teaching, we need to be clear about what we mean by this term, try to understand better how creativity functions within mathematics, how this is related to other intellectual factors, and seek to develop ways of assessing and rewarding mathematical creativity in our classrooms.

Abstractor's Comments

Haylock's study was concisely reported, clearly written, and well designed, and it addressed an interesting issue. Incidentally, it was also based on an M.Ed. dissertation at the Centre for Science Education, Chelsea. Very commendable!

Psychological descriptions of people have for a long time centered around the premise that people have traits, personality traits and cognitive traits, that endure and that are manifest in all situations. This notion is currently under strong attack, particularly in personality theory. Haylock's study furnishes a rather strong argument calling for a reassessment of the kind of content-independent abilities and processes that have dominated research on problem solving and concept formation.

It is common to meet unusually intelligent and highly educated people who claim to lack a "mathematical mind". The presumption is that, regardless of a person's stock of prerequisite knowledge and prior training, there are certain underlying abilities that are unique and specific to mathematics, and that people who lack these abilities (even though they may perform outstandingly in other areas of thought) may be restricted in their ability to reason mathematically. On the other hand, the existence of cognitive abilities that are unique and specific to mathematics and not simply attributable to "lack of knowledge concerning prerequisite facts" is by no means universally accepted among psychologists, mathematicians, or educators.

Although a number of studies have claimed to furnish evidence that ability in mathematics is distinct from general intelligence, the influence of non-ability factors like "motivation and interest", "prior training or practice in specific content areas", and "lack of specific bits of information or specific prerequisite skills" has usually been ignored. For example, Gagné's research (1970) suggests that, after differences in prerequisite knowledge have been factored out, mathematical abilities may be nothing more than specific manifestations of general intelligence.

Several mathematicians (e.g., Poincare, Hadamard, Kolmogorov) and psychologists (e.g., Krutetskii, Binet, Revesz) have argued convincingly for the uniqueness of mathematical abilities. Nonetheless, these individuals have been careful to distinguish between ordinary "school" ability and independent, creative mathematical ability. For example, Kolmogorov (cited in Krutetskii, 1976) stated that "ordinary, average human abilities are quite sufficient for mastering--with good guidance or good books . . .--the mathematics that is taught in secondary school" (p. 4). Krutetskii writes, "Anyone can become an ordinary mathematician; (but) one must be born an outstanding, talented mathematician" (1976, p. 361).

In spite of the above claims, it is quite obvious that people who are good problem solvers or who are creative in one context (or in one type of situation, or in one discipline) may be average or below average in another. Research like Haylock's supports the notion that people may well form severe personae for different situations--family, peer groups, school. The student's personality may be different at different times, in different situations, in different mental and psychological states, as a function of his or her particular agenda at the moment. Cognitive performance may be moved upward and downward by load factors in the individual and in the situations--e.g., noise, emotionality, distraction, confusion, shyness, anxiety. So, students fluctuate in their apparent ability depending upon the time and place. More research is clearly needed in this area, and Haylock's study is a clear first step toward a more sophisticated discussion of mathematical processes and abilities.

Hiebert, James and Tonnessen, Lowell H. DEVELOPMENT OF THE FRACTION CONCEPT IN TWO PHYSICAL CONTEXTS: AN EXPLORATORY INVESTIGATION. Journal for Research in Mathematics Education 9: 374-378; November 1978.

Abstract and comments prepared for I.M.E. by A. EDWARD UPRICHARD, University of South Florida.

1. Purpose

The present study investigated young children's initial understanding of the part-whole fraction concept within physical representations of continuous quantity (length and area) and discrete quantity (set/subset). The purpose of this study was to replicate and extend the work of Piaget, Inhelder, and Szeminska (1960) to determine if Piaget's part-whole interpretation for the continuous cases of length and area apply equally well to the discrete quantity case.

2. Rationale

Piaget et al. suggest that children's understanding of part-whole and part-part relationships (length and area) proceeds through several stages and is guided by an "anticipatory scheme" which provides the child with the ability to recognize beforehand the relationships required in a subdivision problem. The scheme is first operational in situations requiring only one dichotomy (halves), then extended to successive dichotomies (fourths, eighths), and finally transferred to situations involving thirds, fifths, and sixths.

Piaget et al. provide a conceptual analysis of fraction (continuous quantity) to accompany the above described developmental progression of the fraction idea. The seven subconcepts of the analysis (that are suggested to be logically and psychologically interrelated to form a single mental construct) can be defined as the realization that (a) the whole is divisible, potentially composed of separable elements, (b) a fraction implies a determinate number of parts, (c) the subdivision must be exhaustive, (d) there is a fixed relation between the number of parts and the number of divisions, (e) the parts have a nesting or hierarchical character, (f) the whole is conserved under subdivision, and (g) all parts must be equal.

If Piaget's conceptual analysis of continuous quantity tasks (length and area) could be extended to the discrete quantity task (set/subset), its usefulness for instruction would increase.

3. Research Design and Procedures

Nine children, ages 5 years 4 months to 8 years 0 months, were each administered three tasks: an area task, a length task, and a set/subset task. The tasks required the child to divide a quantity equally among a number of stuffed animals seated around a table so that the quantity was completely used up. The area task used a circular clay "pie," the length task used a piece of licorice, and the set/subset task used penny candy (four times as many candies as animals). All tasks for a particular child dealt with the same fractional number, but the fractions between children were varied. Two children received tasks dealing with halves, three children received thirds, three children received fourths, and one child received fifths. The individual interview sessions were videotaped to facilitate a detailed analysis of responses. For purposes of replication and comparison, the task formats and protocols in this study followed closely those employed by Piaget et al.

4. Findings and Interpretations

Within-subject information suggested that the three physical representations of fraction produced tasks of differing difficulty. The set-subset discrete case was easier than the continuous cases of length and area. Complete and immediate solutions to continuous quantity tasks require well-developed anticipatory schemes; children can solve discrete tasks (set/subset) without treating the set as a whole. Also, the set/subset tasks were immediately solvable on the basis of number strategies (counting); length and area tasks were not.

Between-subject information was used to generate results pertaining to the developmental sequence in understanding fractional numbers.

In the area representations, halves and fourths were successfully constructed by some children, but thirds were not. With length representations the level of difficulty corresponded to the number of parts (halves, thirds, fourths, fifths); successive dichotomy was not used to subdivide length. All the set/subset representations were solved with equal success regardless of fractional numbers.

The third set of results concerns the adequacy of Piaget et al.'s conceptual analysis of fraction. The results from this pilot study suggest that the analysis is only appropriate for continuous quantity representations. Thus, the problem of identifying one set of criteria that define a complete part-whole fraction concept across all physical representations remains.

Abstractor's Comments

The authors used a clinical approach (structured interview) to study an interesting problem: children's understanding of the fraction concept (continuous and discrete cases). The investigation was designed to be exploratory in nature rather than to confirm or negate specific hypotheses. In a clinical investigation of this type the significance or worth of the results relate directly to the tasks or protocols used. Hence, the issues I raise with respect to the present investigation focus on the tasks comprising the structured overview.

It's my belief that the number of tasks administered per interview in this investigation was not sufficient. For example, a child received only three tasks per interview with each task involving the same fractional number. I would suggest that at least nine tasks should have been used: three area, three length, and three set/subset. Within each type (i.e., area), the tasks should have focused on halves, fourths, and thirds or fifths. This arrangement (protocol) would have generated more interesting data for the within-subject analysis. Further, a sample of parallel tasks on the pictorial or iconic level could have been included in the interview protocol. It would have been interesting to observe if the counting strategy used by children in the set/subset tasks involving manipulatives was effective on the pictorial level. I suspect that on a pictorial level a child might be forced to think of the set as a whole beforehand (a la Piaget) in order to subdivide it.

In sum, the present investigation focused on a significant problem, and it was carefully implemented and reported; however, the design could have been improved with minor changes. The changes (see above) would have allowed the researchers a more in-depth examination of the construct and may have yielded better insights for future work.

Meyer, Ruth Ann. MATHEMATICAL PROBLEM-SOLVING PERFORMANCE AND INTELLECTUAL ABILITIES OF FOURTH-GRADE CHILDREN. Journal for Research in Mathematics Education 9: 334-348; November 1978.

Abstract and comments prepared for I.M.E. by JAMES H. VANCE, University of Victoria.

1. Purpose

The purpose of the investigation was to identify a structure of intellectual abilities related to mathematical problem-solving performance.

2. Rationale

The investigation grew out of an earlier study of concept attainment abilities (CAA) at the Wisconsin Research and Development Center for Cognitive Learning. In that project, which investigated relationships between cognitive abilities and concept learning in four school subjects, factor analysis was used to identify cognitive abilities underlying several batteries of reference tests.

Factor-analytic procedures have been employed in a variety of other studies of mathematical abilities or problem-solving abilities. The cumulative results of the research suggested to the investigator the existence of a somewhat stable structure of intellectual abilities. This study was designed to explore relationships between intellectual abilities and mathematical problem-solving performance, an aspect which had received little previous attention.

3. Research Design and Procedures

Twenty tests were administered to 179 fourth-grade children from three states. All subjects had been studying from the same mathematics textbook.

Nineteen of the measures were reference tests for intellectual abilities selected from the CAA battery. At least two tests were included for each of seven hypothesized intellectual abilities: Verbal, Induction, Numerical, Word Fluency, Memory, Perceptual Speed, and Simple Visualization.

The other instrument was the Romberg-Wearne mathematical problem solving test which consists of nineteen "superitems" each containing a comprehension question, an application question, and a problem solving question. The test yields separate scores for comprehension, application, and problem solving.

Means, standard deviations, and reliability estimates were obtained for each of the nineteen reference tests and the three parts of the problem solving test. Factor analysis was used on the intercorrelation matrix of the 22 variables.

4. Findings

Interpretation yielded six comparable common factors identified as: Verbal; Induction of classes employing symbolic content; Numerical; Perceptual Speed; Induction employing verbal semantic, pictorial semantic, or figural content; and Mathematics.

Means, standard deviations, Hoyt reliability estimates, and intercorrelations for the three parts of the nineteen-item problem solving test were as follows:

Subtest	Means	Standard Deviation	Reliability	Intercorrelations: Comprehension	Intercorrelations: Application
Comprehension	13.52	2.41	.48		
Application	9.72	3.33	.69	.68	
Problem Solving	3.47	2.40	.59	.48	.60

5. Interpretations

The study suggested that intellectual structures contain a specific Mathematics ability along with Verbal, Induction or Reasoning (two), Numerical, and Perceptual Speed abilities. The six factors resemble the seven hypothesized factors, but there are differences. The General Mathematics factor which emerged was determined by the Comprehension, Application, Problem-Solving, and Mathematics Computation tests, and it resembled factors identified by previous researchers as Arithmetic or Mathematical Reasoning. Further examination of the data suggested that there may be some relationship between problem solving and Numerical ability, and that applications are slightly related to Verbal ability.

In regard to problem-solving performance, the study indicated that prerequisite skills and concepts are related to some of the variance in problem solving, but mastery of these skills and concepts does not guarantee success in problem solving.

Abstractor's Comments

One of two main conclusions drawn by the investigator was that intellectual structures contain a Mathematics ability. Such an ability had not been hypothesized for the study, but given the tests used the result might well have been anticipated. This factor was determined by four tests--the three parts of the Romberg-Wearne test and the Computational Skills test (also constructed by Romberg). One might have expected that these four measures would have high correlations, even more so perhaps since Romberg was the senior author of the textbook used by the subjects. Similarly, the tentative conclusions that problem solving in Mathematics appears to be related to numerical ability and applications to verbal ability are not startling.

Since this was a study on mathematical problem solving--a topic of major importance to classroom teachers as well as to researchers --the instrument used to measure this ability is of particular interest. A sample test item included in the report helps the reader understand how problem solving is defined for the study and to think about possible implications for the classroom. The problem-solving question for one of the superitems is as follows:

> In another parking lot, trucks are parked. Each truck takes the space of 3 cars. There are 12 trucks in the parking lot, and it is completely full. If there were 4 rows in the parking lot, how many cars could be parked in each row?

This highly verbal "problem" requires two steps and two different operations for its solution. The mean for the test was 3.14 (out of 19), confirming that this type of question is very difficult for most fourth graders. The investigator acknowledged this fact and also the low reliability of the test (.59) in discussing limitations of the study.

The other main conclusion of the study had to do with the role of prerequisite skills and concepts in success in problem solving. Again the conclusion that these are important but not sufficient might have been predicted, particularly when one looks at the sample item. The comprehension question was a True-False question, and the application question was a one-step problem using the two numbers in the stem.

In summary, in studies of relationships between mathematical problem-solving performance and intellectual abilities, one might ask:

(1) To what extent will the results depend on the particular tests used?

(2) What new information and implications might be expected from the results regarding furthering problem-solving skills of students?

Nelson, L. D. and Kieren, T. E. CHILDREN'S BEHAVIOR IN SOLVING SPATIAL PROBLEMS. Alberta Journal of Educational Research 23: 22-30; March 1977.

Abstract and comments prepared for I.M.E. by MARY MONTGOMERY LINDQUIST, National College of Education.

1. Purpose

The purpose of this study was to observe the behavior of three- to eight-year-olds while solving spatial problems that involved folding two-dimensional networks into three-dimensional forms.

2. Rationale

Despite the fact that problem solving is a primary aim of mathematics instruction, there is little knowledge about young children's problem-solving behaviors. This study is one of a series of cross-sectional and longitudinal studies (Nelson and Sawada, 1975) that address this concern.

3. Research Design and Procedures

Sixty subjects, 10 from each age three to eight, were selected from 200 volunteers. Each subject was interviewed, video-taped, and then his or her behavior analyzed on the following tasks:

(1) Cube. Each child was shown a Plexiglas-Velcro cube, and was shown how to disassemble it. After the child had mastered disassembling and reassembling the cube, he or she was shown, one at a time, six two-dimensional patterns. The child was asked to predict whether each pattern could be folded into a cube and then permitted to fold. The first four patterns could be folded into a cube and the last two patterns could not be folded. In the case of the two patterns the child was asked to rearrange the squares so the pattern could be folded into a cube.

(2) Tetrahedron. Tasks similar to those of the cube utilizing two different triangular patterns, one foldable and one not foldable, were presented.

(3) Dodecahedron. Each child was given a dodecahedron and asked to disassemble and reassemble it.

4. Findings

(1) Cube. Most children could assemble the cube and had developed a systematic procedure for folding. Older children used more productive procedures. Fifty percent of the children refused to predict; the percentage of correct predictions increased with age. After finding that one pattern was unfoldable, more children were able to predict the next pattern was unfoldable.

(2) Tetrahedron. This was a more difficult task than the cube--more children had difficulty putting one together, very few would make any predictions, and only 20 percent could modify the unfoldable pattern so that it would be foldable.

(3) Dodecahedron. This task was more age-dependent; 8 of the 15 children who could assemble it were eight-year-olds.

5. Interpretations

The following four conclusions were generated from the research, but the research was exploratory in nature and was not designed to test these conclusions:

(a) The geometric problems posed for this sample, especially those involving the cube, seemed to be appropriate for children aged three to eight. The children were able to gain appropriate physical experience and "solve the problem" by constructing the desired model.

(b) The younger children, particularly, did not seem to gain mathematico-logical experience from the problems. They were less able to cope with unfoldable shapes than were older children (eight-year-olds). Although most children were good builders, the younger children tended to use less efficient folding procedures, probably indicating a lesser or different use of analysis.

(c) A majority of the children in this sample, including nearly half of the older children (seven- and eight-year-olds), indicated a lack of operational control over the problems in that they refused to predict the foldability of the various layouts. This willingness and accuracy of prediction changed somewhat with age. The eight-year-olds seemed most nearly in control of difficult situations, e.g., unfoldable cube layouts and the dodecahedron problem.

(d) The noncube problems presented perceptual and conceptual problems to many students. Part of this probably was due to lack of familiarity with the noncube objects. However, inability to work with nonrectilinear lines seemed a major cause of problems.

Besides these four conclusions, several comments about the research were made, including that it is possible to observe and classify problem-solving behaviors, these behaviors seem to be age related, and realistic problems can be created for young children.

Abstractor's Comments

Many important questions were posed in the series of studies to which this belongs. Without examining the series, it is difficult to judge how much this particular study contributed to the whole.

It is good to see exploratory, observational studies being reported. However, if more exploratory work was done in a pilot phase, then a somewhat more systematic observational study could be reported. For example, if the investigators had previously determined that young children could handle the materials and were motivated by the tasks, then they could have examined more systematically the layout presentations. What would have happened if an unfoldable layout was presented earlier in the sequence?

The four conclusions drawn, albeit not proved, follow logically from the observations. One is only surprised that they were not expected in advance. As I read the concluding section, I was struck with the dual interests of the investigators--observing problem-solving behaviors and developing curriculum. Of course, the two interests are not mutually exclusive and both need research. However, it is my opinion that this interaction of interests may have prevented the research from shedding more light on the problem-solving behaviors. At this point they needed to be separated and then, later, brought together.

Prigge, Glenn R. THE DIFFERENTIAL EFFECTS OF THE USE OF MANIPULATIVE AIDS ON THE LEARNING OF GEOMETRIC CONCEPTS BY ELEMENTARY SCHOOL CHILDREN. Journal for Research in Mathematics Education 9: 361-367; November 1978.

Abstract and comments prepared for I.M.E. by STEPHEN S. WILLOUGHBY, New York University.

1. Purpose

The purpose of this study was to determine which of three instructional settings (no manipulative aids, two-dimensional manipulative aids, or three-dimensional manipulative aids) is most effective in teaching geometry to students in the third grade. The question was repeated for high ability and for low ability children as identified by the Iowa Test of Basic Skills (ITBS).

2. Rationale

Although there is considerable evidence supporting the use of manipulative aids in teaching geometry, the question of which aids are most effective is unanswered.

3. Research Design and Procedures

a. Treatments. There were three treatment groups: W using no manipulative aids, M using two-dimensional manipulative aids, and R using three-dimensional manipulative aids. The experimenter prepared special 56-page programmed instructional units for the three groups. The unit lasted for ten days, 60 minutes each day.

b. Sample and Analysis. One hundred forty-six (146) subjects from two suburban schools (66 from one, 80 from the other) were identified as high, middle, or low ability by their ITBS scores and randomly assigned to the three treatment groups. Analysis of variance was used to test for significant differences among means. Bonferroni t-statistics were used to calculate simultaneous confidence intervals for established propositions. The two schools were treated separately.

c. Evaluation Instrument. Parallel forms of a geometry test created by the experimenter were administered on the 11th, 21st, and 31st days.

4. Findings

Twenty-seven (27) propositions were tested in each school. Of these, there were 12 that produced statistically significant results, all favoring R (three-dimensional aids) over W (no aids) or M (two-dimensional aids). There were no statistically significant differences for the high ability levels, but five involving the low ability groups.

5. Interpretations

Since all post-treatment measures favored treatment R (though not all were statistically significant) "the claim that children are better able to learn geometric concepts when geometric solids are included in the presentation is supported." Also, the "research supports the use of solids for low ability students." The research did not support or refute claims for use of two-dimensional aids or for any aids with higher ability students.

Abstractor's Comments

The questions asked in this study are good questions, and there is room for more research that asks and attempts to answer such questions. In particular, the idea that three-dimensional aids might be more useful in teaching concepts about plane geometry than two-dimensional aids is a creative and interesting suggestion. To consider the possibility that different kinds of children might learn more effectively in different kinds of settings is often overlooked in our desire to collect statistical evidence supporting some special method. The author is commended for not doing so.

However, the very small number of subjects in each cell must be remarked upon. The basis of our statistical techniques is that populations are normally distributed and samples are chosen randomly. These assumptions may be violated with relatively little damage if the sample is very large and the population very large with respect to the sample. This was not the case here, and the results are therefore possibly suspect.

Ability level would seem to be a less important variable in determining which children would succeed with each technique than other possible variables. It would be interesting to consider some of the other possibilities. Beyond this, the reader is certainly left to wonder what "high ability" means when told that "because of the variability of the ITBS scores each school had different ranges for high, middle, and low." Obviously, the terms are only relative to the schools in question, and may not pertain to other settings at all.

Using programmed text material to teach the subject is a very good way to avoid the problem of teacher variables, but may limit the generality of the results unless we expect other groups of third graders to be taught in the same way. Incidentally, the number of teachers and teacher aides in the classes was held constant (presumably there was one of each, although that is not specified). Again, there is some question about the generality of the results.

In summary, the questions asked are good, the data appear to support the use of three-dimensional aids, but this reader finds the very sophisticated statistical techniques somewhat superfluous in light of the small cell sizes and recommends care in generalizing the results in light of the special circumstances.

Sherman, Julia and Fennema, Elizabeth. THE STUDY OF MATHEMATICS BY HIGH SCHOOL GIRLS AND BOYS: RELATED VARIABLES. American Educational Research Journal 14: 159-168; Spring 1977.

Abstract and comments prepared for I.M.E. by RUTH ANN MEYER, Western Michigan University.

1. Purpose

The purpose of this study was to determine the relationships between important affective, cognitive, and other variables and the decision to enroll in further mathematics classes.

2. Rationale

Although many studies have investigated sex-related differences in mathematics achievement, little attention has been given in the analyses of the results of these studies to the fact that fewer females than males elect to continue the study of mathematics during their late high school and post high school education. Since time spent in mathematics classes clearly affects performance on both mathematics achievement and aptitude tests, this situation must be considered seriously.

3. Research Design and Procedures

Data were collected from students in grades 9-12 on three cognitive variables (mathematics achievement, verbal ability, and spatial visualization) and eight affective variables (Attitude toward Success in Mathematics, Stereotyping of Mathematics as a Male Domain, Perceived Attitude of Mother, Father, and Teacher towards one as a learner of mathematics, Effective Motivation in Mathematics, Confidence in Learning Mathematics, and Usefulness of Mathematics). In addition, all students were asked to indicate whether or not they intended to enroll in a mathematics course the next shcool year or whether they were undecided. Since intent measures indicated that almost all ninth-grade students were planning to enroll in tenth-grade mathematics and because few girls were studying twelfth-grade mathematics, the ninth and twelfth grades were eliminated from further analysis.

Two mathematics achievement groups (those who scored above the median of the distribution of the mathematics achievement scores and those who scored below the median) and two intent groups (those who intended to take mathematics next year and those who did not or were undecided). The number of males and females in each achievement gtoup who intended/did not intend to take mathematics next year was determined.

Chi squares for each achievement group were calculated from the frequency data. ANOVAs (2 sexes, 2 grades, 2 mathematics achievement levels, 2 intents) were performed on all variables. Analyses of covariance (2 sexes, 2 grades, 2 intents) were performed also on all affective variables using the three cognitive variables as covariates.

4. Findings

The chi squares showed that, in general, more boys than girls intended to continue to study mathematics. The only exception was that there was no significant difference between the number of eleventh-grade boys and girls in the top mathematics level in intent to continue in mathematics.

Students in the top half of their class in mathematics achievement scored significantly higher on all variables. The only sex-related differences found were for verbal ability (girls higher) and Math as a Male Domain (boys more stereotyped). All variables, with the exception of Math as a Male Domain, differentiated between students' planning and not planning to continue their study of mathematics. The grade by level for Confidence, intent by level for Attitude for Success, and a quadruple interaction for Effectance Motivation were the only interactions which were significant. The analyses of covariance confirmed the ANOVA results for the two achievement levels, but found sex-related differences for Effectance Motivation (in favor of girls) and Math as a Male Domain (in favor of boys).

5. Interpretations

The frequency data indicated that more males than females intended to take mathematics the following year. The study also suggested that male and female groups "equated" on intent to study mathematics and mathematics achievement or on the cognitive variables, verbal ability, spatial visualization, and mathematics achievement, differed little in the attitudes relevant to mathematics. The one important exception was Math as Male Domain. The investigators commented that perhaps the girls of this study, who live in a community in which the women's movement receives much publicity, may overtly agree that mathematics is appropriate for women, but when it came to selecting advanced courses they behaved in a sterotypic way. However, when the female and male groups were not "equated", that is, when cognitive factors and intent to study mathematics were not controlled (Fennema and Sherman, 1977), the girls had less confidence in their ability to learn mathematics, less expectation that mathematics would be useful to them, and perceived their fathers and mothers as less positive to them as learners of mathematics than did boys.

Previous research, the findings of this study, and that of Fennema and Sherman (1977) encouraged the investigators to make the following recommendations:

1. Programs designed to increase female participation in mathematics should consider factors such as confidence, usefulness, and perceived expectations of significant others.
2. While studying the mathematics performances of males and females, investigators should either control for mathematics background and attitudes or refrain from drawing conclusions which suggest that differences can be attributed to the sex of the subjects.

Abstracter's Comments

Some of the impact of this study is already evident. The study's results are quoted as rationale for proposed research projects. Many state and national agencies are attempting to replicate the findings of Sherman and Fennema. Variables such as perceptions of the usefulness of mathematics, the influence of significant others, sterotyping of mathematics as a male domain, mathematics anxiety, and self-confidence as a learner of mathematics are being investigated as possible predictors of females' continued enrollment in mathematics classes.

Although this well-organized study has influenced many similar research projects, there may be some question about its generalizability. The subjects were pooled over four high schools each located in a somewhat different socioeconomic community (Fennema and Sherman, 1977). It is doubtful that the same number of girls in each school intended to continue studying mathematics.

Nevertheless, the statistical procedures were appropriate and accurate. The authors noted the hazards of using multiple univariate ANOVAs on somewhat interrelated variables. The analysis of covariance provided necessary confirmation results. This was particularly true for the Effectance Motivation because of its quadruple ANOVA interaction.

In summary, this and other studies by Fennema and Sherman will continue to contribute to mathematics education until factors significantly related to mathematics "drop-out" are found. Identification of these factors that inhibit young women from pursuing mathematics courses will permit educators to plan intervention studies which will attempt to eliminate these barriers.

Reference

Fennema, E. and Sherman, J. Sex-related Differences in Mathematics Achievement. American Educational Research Journal 14: 51-71; Winter 1977.

Trent, John H. NEED FOR IN-SERVICE AND PRE-SERVICE METRIC EDUCATION. School Science and Mathematics 78: 45-52; January 1978.

Abstract and comments prepared for I.M.E. by ROBERT M. TODD, Virginia Polytechnic Institute and State University.

1. Purpose

The purpose of this study was to determine (a) the current (1975) knowledge about the metric system possessed by Nevada teachers and (b) whether or not there was a need for in-service workshops for those teachers.

2. Rationale

The ability of teachers to lead students into a metric world depends in part on their knowledge of the most used measurement system. Thus, a survey to ascertain the level of knowledge or the need for training is appropriate for any state or locality.

3. Research Design and Procedures

The questionnaire. A two-part questionnaire was used:

(1) "Need for Metric Workshop" consisting of nine questions, one of which contained two parts; i.e.,

"7. If a federally funded in-service course in metric education were offered by the University of Nevada, Reno, would you attend it?

A. If it were in your county;

B. If it were offered on the University of Nevada, Reno campus."

(2) "Knowledge of Metric System" consisting of seven questions relating to mass, length, volume, prefixes, temperature and the meaning of 'MKS' and 'SI'.

The sample. The sample included 135 rural elementary school teachers, 145 elementary teachers from large population areas, 103 elementary teachers from medium population areas, 64 junior high school teachers, and 75 high school teachers. An additional sample of preservice teachers included 44 enrolled in an elementary mathematics methods course and 18 enrolled in a secondary mathematics methods course. (These numbers are inferred from the number of responses recorded in each category.)

"In order to obtain the desired information, two questionnaires were sent to a random sample of in-service elementary and secondary teachers in Nevada. These same questionnaires were also administered to both elementary and secondary (pre-service) methods students at the University of Nevada, Reno."

Statistical Treatment. Frequencies and percent responding for each answer choice were summarized for (a) Elementary Teachers (rural, large, medium population); (b) junior high school teachers; (c) high school teachers; (d) elementary and secondary mathematics methods students. Omissions were ignored in calculating the percentage; thus, 100 percent represented 131 rural elementary teachers on one question and 119 on another question.

Comparisons were made across levels (e.g., rural vs. large population) using chi-squared values.

4. Findings

Chi-squared values were significant for 16 of the 17 frequency distributions. The reported significance level (Sign Level = 4) on page 47 is obviously a misprint. For the "Need for Metric Workshop" questionnaire the author reports:

(a) In rural and metropolitan counties (medium and large populations) most elementary teachers had not had a college course in the metric system.
(b) Most of the elementary teachers participating in this survey did not feel qualified to teach an arithmetic or science course in which the metric system was taught or used.
(c) Rural counties indicated students were inadequately prepared in the metric system.
(d) Rural and metropolitan elementary teachers agreed they would attend an in-service metric workshop if offered in their county.

(e) A majority of both rural and metropolitan elementary teachers did not feel adequate guidelines, course outlines, and materials were available to them for teaching the metric system in their classroom.

For the "Knowledge of Metric System" questionnaire the author reports:

(a) Even though teachers from metropolitan areas did somewhat better, there was no significant difference between the rural and metropolitan county elementary teachers in their knowledge and ability on the questions related to meters, kilograms, and liters.

(b) Most of the rural and metropolitan county elementary teachers were unable to respond correctly to the questions related to Celsius temperature and the meaning of MKS and SI.

A comparison between responses made by teachers at different levels indicated:

(a) Most elementary teachers had not taken a college course in the metric system. However, a majority of both the junior high and senior high teachers had taken such a course.

(b) In comparison to junior and secondary teachers, most elementary teachers felt less qualified to teach the metric system.

(c) Elementary teachers were more aware that the Nevada State Textbook Commission had recommended that all textbooks adopted after January 1, 1976, have the metric system as the primary system of measurement.

(d) A majority of teachers on all levels (elementary, junior high and high school) felt that students were inadequately prepared in the metric system.

(e) Approximately 80 percent of the elementary teachers felt there were no adequate guidelines, course outlines, and materials on the metric system available to them to adequately teach their students the metric system, whereas only 40 percent of the junior high and high school teachers believed that there were no sufficient guidelines, course outlines, and materials available to them.

(f) Over 85 percent of Nevada teachers would attend an in-service metric workshop if held in their county. However, only about half of the teachers said they would attend a metric workshop if held on the University of Nevada, Reno campus.

Analysis of data at various levels showed:

(a) Most of the elementary teachers responded correctly to questions related to meters, kilograms and liters. The majority of the secondary and junior high teachers responded correctly to these same questions related to meters, kilograms, and liters.

(b) On the question related to Celsius temperature, 76.4 percent of the elementary teachers responded incorrectly, as compared to 50 percent of the secondary and junior high teachers.

(c) The majority of elementary, junior high, and senior high school teachers responded incorrectly to questions related to SI and MKS. These differences strengthen the conclusion that elementary teachers of Nevada need in-service metric workshops. In addition, they show that there is some need for an in-service metric workshop for junior high and secondary teachers.

5. Interpretations

The author drew the following inferences:

(a) "There is a need for in-service workshops on the metric system by both rural and metropolitan (medium and large population) elementary teachers of Nevada, as almost all teachers indicated they would attend an in-service metric workshop if offered in their home county" (p. 48).

(b) "There is a need for in-service workshops for Nevada elementary teachers from both rural and metropolitan counties, even though the teachers from the metropolitan areas showed a somewhat greater knowledge of the metric system" (p. 50).

(c) "While there is definitely a need for in-service metric workshops for elementary teachers in Nevada, the need is not nearly as great at the junior and senior high school levels" (p. 51).

(d) "Each sample group was unable to respond to the questions on SI and MKS and were therefore unable to think in "metric terms."

(e) "There is a need for metric workshops for secondary math methods students, and for junior high and secondary teachers of Nevada; however, this need is probably not as great as the need for in-service metric workshops for the elementary teachers of Nevada and elementary math methods students..."

Abstractor's Comments

This is an example of data collection that establishes a "need" for something, in this case in-service metric workshops. No hypotheses were stated.

Two distinct populations were used: (1) University of Nevada students enrolled in elementary or secondary mathematics methods courses and (2) a random sample of in-service teachers in Nevada. The reader may reasonably assume 100 percent response from the first group, but there is no indication of the number of teachers randomly selected nor of the randomness of the sample which actually responded. To generalize from the respondents to all the teachers in the random sample or to all teachers of Nevada may not be valid. If the number selected and the number responding (and their distributions) had been provided, a reader could make his or her own decision about generalizing.

Any test of metric knowledge involves choosing items to sample that knowledge. I think that using two of seven items to test "metric terms" is inappropriate, especially when one is "What does MKS stand for?" and the other is "What does SI stand for?" One might have learned the metric system's base units, prefixes, and practical applications without ever needing the symbols SI and MKS. Those two questions, for me, are about as important as whether to spell metre with an 're' or 'er'.

The author reported chi-square values without noting his expected frequencies. His conclusions suggest he made more than one comparison (rural versus metropolitan and also elementary versus secondary teachers), but his single chi-squared values do not indicate which comparisons were significant.

The comments I have made do not mean the conclusions of the study are incorrect. More than 400 teachers answered the questionnaires and the knowledge results are similar to a non-random, voluntary testing conducted in Virginia at about the same time (Todd and Weber, 1977).

The fact that teachers are willing to attend a metric workshop in their own county but less willing to travel long distances for a metric workshop is no surprise: many teachers feel they should learn more about teaching the metric system, but they have not decided it is of overwhelming importance.

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Uprichard, Edward A. and Phillips, E. Ray. AN INTRACONCEPT ANALYSIS OF RATIONAL NUMBER ADDITION: A VALIDATION STUDY. Journal for Research in Mathematics Education 8: 7-16; January 1977.

Abstract and comments prepared for I.M.E. by THOMAS E. KIEREN, University of Alberta.

1. Purpose

"The purposes of the study were (1) to develop a learning hierarchy for rational numbers using the intraconcept analysis technique and (2) to test the validity of the hypothesized hierarchy using the Walbesser method and pattern analysis" (p. 7).

2. Rationale

The study is one of many in the field of hierarchy development and analysis. It is related to the work of Gagné and Paradise (1971), Callahan and Robinson (1973), Walbesser (1978), and Rimoldi and Grib (1960), among many others. The study has an epistemological facet in that it attempts to use a conceptual structure of addition of fractions to develop a hierarchy. It has a related measurement-empirical facet using two methods for validating the hierarchy to generate instructional hypotheses.

3. Research Design and Procedures

a. Epistemological

"The addition of rationals in fraction form was analyzed using an intraconcept analysis technique" (p. 8). Addition of rationals is treated completely from a formal symbolic algorithm viewpoint. Using a traditional concept of rational number addition, two levels of tasks were used, like and unlike denominators. Within levels, classes based on the prime, composite relationships between denominators were used. For example, composite denominators were assumed to be easier than prime denominators within the like denominator level. The most difficult setting was hypothesized to be unlike denominators, both composite but not multiples of one another.

Hence, in the initial hypothesized item hierarchy, 3/4 + 3/6 = _____ was hypothesized as the most difficult item.

Within each denominator class, a hypothesized order of difficulty was developed based not on the addends, but on the character of the sum. These final "sum levels" ranged from "proper fractions in simplest form without renaming" to "improper fraction renamed to mixed numeral in simplest form, two renamings." These levels also give rise to the item cited above as being most difficult. For each sum category within a class, two mixed numeral tasks were generated.

The intraconcept procedure yielded 89 specific tasks. For testing purposes these were reduced to 45 tasks including all existing sum categories within classes, with a sampling of mixed numeral items within classes.

b. Empirical

A 45-item test was administered to 251 subjects in grades 4 through 8, with the bulk in grades 5 and 6. Eliminating subjects who passed or failed all items left the results from 200 subjects for hierarchy analysis.

Responses for each transfer in the hierarchy (pair-wise a vs b where $a < b$) were subjected to the Wallbesser and pattern analyses. The former is based on the study of 2 x 2 contingency tables and three ratios are computed for the pair-wise item contingencies noted above:

$$\text{consistency: } \frac{N\text{ (pass, pass)}}{N\text{ (pass, pass)} + N\text{ (pass, fail)}}$$

$$\text{adequacy: } \frac{N\text{ (pass, pass)}}{N\text{ (pass, pass)} + N\text{ (fail, pass)}}$$

$$\text{completeness: } \frac{N\text{ (pass, pass)}}{N\text{ (pass, pass)} + N\text{ (fail, fail)}}$$

Values of .85, .70, and .50 were used to test each hierarchical transfer.

Under patterns analysis, an index of agreement was computed between each subject's performance and the hypothesized sequence, using the rationale that if tasks were truly hierarchical, then once a learner failed at a level, failure would occur at all succeeding levels.

The various analyses were carried out using the total test and a sub-test not containing mixed numeral items.

4. Findings

a. Reliability

The test as a whole proved reliable (KR-20 ≈ .98)

b. Wallbesser Analysis

The consistency ratio met criterion on 35 of 44 comparisons, the adequacy ratio on 43 of 44, and the completeness ratio on 27 of 44, with the former two being more critical for hierarchy testing. A considerable improvement was noted with the elimination of mixed numeral items.

c. Pattern Analysis

The final hypothesized ordering of subordinate tasks yielded an index of agreement of .70 for which no statistical test of significance is available. An improvement of this index to .75 was noted with the elimination of mixed numeral items.

An item difficulty sequence proved somewhat different from the hypothesized sequence with a preponderance of mixed numeral items proving to be relatively difficult.

5. Interpretations

The interpretations of the data were mainly in the form of instructional hypotheses. It was noted that these hypotheses could not have arisen from logical-epistemological analysis alone. Some of these hypotheses are:

a. Like denominator tasks should be taught before unlike tasks.
b. The sum category hierarchy should obtain in instruction within like denominators with the unusual reversal that a mixed numeral in simplest form with one renaming precedes a proper fraction in simplest form in one renaming.

c. Sequencing unlike denominator tasks is difficult and not settled by the empirical results of this study, but a possible pattern emerged.

d. Tasks involving mixed numerals should not be introduced early in the development of rational number addition skills.

Abstractor's Comments

The study, particularly the analysis and its results, were clearly presented with very useful tables. The development of the index of agreement in pattern analysis is an exception to this clarity. The study also was carefully tied to related literature.

The sample is not adequately described. While this may not be critical to the Walbesser analysis, it would seem that the pattern analysis might be greatly affected by choice of sample and its experience. This is particularly true since the concept was construed to have a specific formal algorithmic meaning.

Given the particular epistemological position, the analysis used appeared appropriate. It is unfortunate that there was no further probing into the meaning of the concept for the subjects involved. It is also interesting that a mathematical processing model of the behaviour was not considered.

The derived instructional hypotheses are a reflection of the particular epistemological view. One very interesting finding is the difficulty caused by both mixed numerals for the subjects and by unlike denominator tasks for the researcher. Both findings suggest that a further analysis of the meaning of the task and the reasons for a particular response to a task by a subject are in order.

The study suggests that epistemological analysis is useful in developing teaching sequences in mathematics. Yet the authors did very little of this. Their analysis assumed that rational numbers (at least under addition) are symbols to be manipulated according to formal algorithms. Within this assumption their analysis makes sense. Yet, if addition means putting two quantities together to yield a new quantity then their analysis does not obtain. For example, a child who thinks of 1 1/2 and 3/4 as measures can easily conceptualize the sum as 2 1/4. This reviewer interviewed an eight-year-old who had no instruction in fractions who said, "Well, 1/2 takes you up to 2 and then there is a quarter more." This same child found 3/4 + 3/6 impossible because he did not know what 3/6 meant. Once he was shown a picture of 3/6 he said, "Oh, I see, 3/6 is like 1/2," and went on to give a correct result for 3/4 + 3/6.

The digression above is central to the criticism of this research from an epistemological point of view. Addition of rationals has two meanings. One is related to vector addition, and performance is closely related to the particular knowledge of the concept of rational number of the child. Thus a child who sees a rational number as a quantity is not confounded by mixed numerals and can add 2 1/4 + 3 1/2 in the same sense as 3 + 2. It is certainly true that addition is related to equivalence. While equivalence underlies some of the hierarchy decisions in this study, the researchers do not seem to take into account different kinds of knowledge of equivalence. A child who sees 1/2 as 1/4 + 1/4 and hence easily adds 3/4 + 1/2 (and 3/4 + 3/6 once he sees the connection) is using a very different notion of equivalence than a child who responds 3/4 + 1/2 = 6/8 + 4/8 = 10/8 = 1 2/8 = 1 1/4 or 3/4 + 1/2 = 3/4 + 2/4 = 5/4 = 1 1/4.

Because the researchers chose a very limited view of rational numbers (an unfortunately popular view which might be well reflected in their results here), their theoretical hierarchy is easily left open to justified criticism. The hierarchy is based on symbols and algorithms rather than mathematical and cognitive structures.

Studies such as this need to have a sound epistemological basis from which to work. Otherwise the concluding instructional hypotheses will be founded on a weak empirical base. Further, such studies need to make use of clinical evidence to help interpret the statistical analyses. A different epistemological stance and an added clinical dimension would hopefully generate more valid instructional hypotheses based on a sounder view of rational numbers as numbers (and not symbols) and of the role the developing concept of equivalence plays in addition.

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