DOCUMENT RESUME

ED 171 524 SE 027 561

AUTHOR Pullman, Howard W.
TITLE Cognitive Structure and Performance in Mathematics.
PUB DATE [79]
NOTE 27p.; Contains occasional light and broken type

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Cognitive Development; *Geometry; *Performance; *Prediction; *Research; Secondary Education; *Secondary School Mathematics

ABSTRACT

A selection of 14 tests of five cognitive factors was administered to 95 senior high school geometry students. These cognitive factors were then related to six measures of performance in mathematics and subjected to correlation analysis and stepwise regression. The findings indicated that the selected cognitive factors correlated with all measures of mathematical performance and added a statistically significant amount to intelligence in predicting performance in mathematics. (Author/MP)

Cognitive Structure and Performance in Mathematics

Howard W. Pullman

Youngstown State University

Cognitive Structure and Performance in Mathematics

Abstract

A selection of 14 tests of five cognitive factors were administered to 95 senior high school geometry students. These cognitive factors were then related to six measures of performance in mathematics and subjected to correlation analysis and stepwise regression. The findings indicated that the selected cognitive factors correlated with all measures of mathematical performance and added a statistically significant amount to intelligence in predicting performance in mathematics. Further study into a factor-analytic model of mathematical ability was called for.

Cognitive Structure and Performance in Mathematics

In the study of mathematical ability and performance in mathematics, it is natural to inquire into the nature of thinking itself. Do cognition, in general, and mathematical reasoning, in particular, have a form or structure? Kurt Lewin (1936) posited a general structure of the psychological environment which drew heavily on topological concepts. However, it was Guilford (1947, 1956) who began a systematic investigation into the structure of cognition. Assuming a factorially complex structure of thinking abilities, Guilford proceeded to develop and test a model for a structured intellect. He found that intellectual factors could be divided into two major categories--thinking factors and memory factors. The group of thinking factors he further divided into cognition, or discovery factors, production factors, and evaluation factors. Cognition factors involve becoming aware of constructs. Production factors stress comprehending a situation and producing some end result. These are believed to indicate convergent and divergent thinking. Evaluation factors indicate the "goodness, suitability, or effectiveness of the results of thinking [Guilford, 1956, p. 28]," that is, the validity of the reasoning process. This study assumed Guilford's model of the structured intellect and used several of his factors to investigate correlates of mathematical ability.

In past studies of cognitive structure, researchers have investigated both the internal structure of cognition (Adkins & Lyerly, 1952; Botzum, 1951; Guilford, 1956; Pemberton, 1952; Scott, 1966) and the relationship of cognitive structure to other measures of intellectual functioning (Coombs, 1969; Guilford, 1954; Jones, 1954; Ostrow, 1964; Thurstone, 1951). Vannoy (1965) noted that cognitive structure was probably not generalizeable across cognitive domains. Ostrow (1964) stated that most abilities are factorially complex and called for an investigation into the non-verbal aspects of reasoning.

Although research relating cognitive structure to performance in mathematics is becoming more popular (Behr, 1970; Behr & Eastman, 1975; Eastman & Carry, 1975; Hancock, 1975; Webb & Carry, 1975), most of this research has concentrated on the relationship of a small number of cognitive measures to differential experimental treatment. Gormly (1971) presented a comprehensive review of the literature and concluded that cognitive structure has been found to be related to mathematical performance only if the measures of performance are composed of logically complex problems. This conclusion, however, seems to contradict what one believes intuitively concerning the relationship of cognitive structure to mathematical reasoning. Shuert (1970), on the other hand, concluded that certain elements of cognitive structure predispose one to do well in mathematics, but the elements are so many and diverse that no clear picture can be drawn from the conclusions.

The purpose of this study was to investigate the relationship of five cognitive factors selected for their possible usefulness in explaining mathematical ability with six measures of performance in mathematics. It was hypothesized that each measure of cognitive structure would be positively correlated with each measure of mathematical performance, and that measures of cognitive structure would add significantly to intelligence in predicting performance in mathematics.

Method

The subjects were 95 geometry students, 58 males and 37 females, in five classes in a suburban high school. The five classes were taught by three different teachers who volunteered class time for the rather extensive testing on cognitive factors. Although not all data were available for all 95 students, no subject was dropped from the final data analysis. The mean age of the subjects was 16.09 years and the mean IQ was 117.5. All classes had been grouped by ability and were assigned to one of three levels of instruction. Each of the three ability levels was represented with three classes of middle ability and one each of high and low ability. Data on performance in mathematics were obtained from school records as was the Otis Quick-Scoring Mental Ability Test IQ for each individual. Data on cognitive factors were obtained from 14 written tests administered by the experimenter and the classroom teachers over a period of several weeks. Tests were administered whenever the individual teacher had time in the course of instruction, and no attempt was made to standardize the order of administration across teachers.

Cognitive Factors

Five cognitive factors from a total of 24 orthogonal factors listed in the Manual for Kit of Reference Tests for Cognitive Factors (French, Ekstrom, & Price, 1963) were selected on the basis of their face validity in predicting performance in mathematics. It was believed that the tasks required reasoning similar to that which would produce success in mathematics. The five factors selected are described below.

1. Flexibility of closure (Cf). "The ability to keep one or more definite configurations in mind so as to make identification in spite of perceptual distractions [p. 9]."

2. Speed of closure (Cs). "The ability to unify an apparently disparate perceptual field into a single percept [p. 11]."

3. Induction (I). "Associated abilities involved in the finding of general concepts that will fit sets of data, the forming and trying of hypotheses [p. 19]."

4. Syllogistic reasoning (Rs). "Ability to reason from stated premises to their necessary conclusions [p. 37]."

5. Visualization (Vz). "The ability to manipulate or transform the image of spatial patterns into other visual arrangements [p. 47]."

Each of the factors was measured using tests from the Kit of Reference Tests; three tests were used to measure each of Cf, I, Rs, and Vz, while two tests were used to measure Cs. Split half reliabilities of the fourteen tests ranged from .40 to .89 with a mean of .65. The ranges of all 14 tests were sufficient to allow for variability of the measured factors to appear.

Performance in Mathematics

Performance in mathematics was measured by six different variables--algebra final grade, geometry final grade, School and College Ability Tests (SCAT) quantitative score, Preliminary Scholastic Aptitude Test (PSAT) quantitative score, Scholastic Aptitude Test (SAT) quantitative score (two administrations). Course grades were reported as letter grades and were assigned numerical equivalents of 4, 3, 2, 1, and 0 for A, B, C, D, and F, respectively. Since the students were homogeneous with respect to course, geometry, rather than year in school, the standardized scores were not the result of a single administration of the tests.

The hypotheses of the study were tested using correlation analysis and stepwise regression.

Results

Descriptive statistics for the tests of cognitive factors appear in Table 1. These statistics indicate the appropriateness of the tests for the subjects in this sample. That is, the tests were neither too easy--indicated by a high mean and small standard deviation--nor too difficult--indicated by a low mean and small standard deviation. The scores of the subjects were distributed throughout the range so that discrimination of subjects with respect to each variable was possible.

Insert Table 1 about here

The descriptive statistics for the measures of performance in mathematics appear in Table 2. The mean scores on all standardized measures of mathematical ability were above the national norms, thus indicating that the sample was not typical of the national population in measured ability. The mean Otis IQ was 117.48 with a standard deviation of 10.28. The range of IQ's was from 96 to 143.

Insert Table 2 about here

It was hypothesized that the tests of the five cognitive factors would be positively correlated with all six measures of performance in mathematics. Of the 84 correlation coefficients presented in Table 3, 81 were significant ($p < .05$).

Insert Table 3 about here

It was also hypothesized that the tests of the five cognitive factors would add significantly to intelligence in predicting performance in mathematics. Table 4 presents the results of the stepwise regressions performed on intelligence and the 14 tests of the five cognitive factors to predict each of the six measures of performance in mathematics. In four of the six cases IQ was most highly correlated with the measures of mathematical performance and therefore entered the stepwise regression first. However, individual tests of induction correlated more highly with algebra final grade and SAT-2 than did IQ and in these stepwise regressions entered before IQ. In each of the stepwise regressions, from 11 to 14 of the tests of cognitive factors continued to add a statistically significant amount to the variance common to mathematical performance, intelligence, and the previously entered tests of cognitive factors. Thus, these tests added to the predictability of performance in mathematics.

Insert Table 4 about here

Table 5 presents a summary of the correlations and common variances of the measures of mathematical performance, intelligence, and tests of cognitive factors. The increase in common variance as a result of inclusion of the tests of cognitive factors ranges from 16% in the case of SAT-1 to 35% in the case of SAT-2.

Insert Table 5 about here

Discussion

The fruitfulness of the five factors--flexibility of closure (Cf), speed of closure (Cs), induction (I), syllogistic reasoning (Rs), and visualization (Vz)--as indicators of the type of reasoning necessary in mathematics is evident from the fact that they correlated well with all measures of performance in mathematics. Although the tests of induction and syllogistic reasoning are consistently among the best predictors of each of the mathematical ability variables, all of the cognitive factors can be used to explain mathematical performance.

Both flexibility of closure and speed of closure have highly visual components. That each of the tests of these variables was correlated highly with geometry grades, relative to other measures of mathematical ability, seems consistent with the fact that success in geometry is largely dependent upon the ability to deal with static visual information. Visualization, a factor which is tested using items which require imagined geometric movement, also showed this relationship. Induction and syllogistic reasoning are explicitly indicative of inductive and deductive reasoning. Their relationship to mathematical reasoning and thereby to performance in mathematics is apparent.

In all of the stepwise regressions the tests of cognitive factors added a significant amount of common variance to the correlation of intelligence with performance in mathematics. The results of this study therefore indicate that the five cognitive factors are related to mathematical performance and that this relationship is not entirely attributable to intelligence.

No coherent picture or structural diagram of the tests of cognitive factors was attempted. Continued research should further our understanding of the factor structure of mathematical ability. Those factors chosen represent only five of the 24 cognitive factors listed by French, Ekstrom, and Price (1963).

The relationship of mathematical performance to the remaining factors and to other factors as well should be investigated. One of the goals of research in mathematical education should be the development of a multi-factor model of the abilities required for success in mathematics.

References

Adkins, D. & Lyerly, S. Factor analysis of reasoning tests. Chapel Hill, N.C.: University of North Carolina Press, 1952.

Behr, M. J. Interactions between 'structure-of-intellect' factors and two methods of presenting concepts of modular arithmetic--a summary paper. Journal for Research in Mathematics Education, 1970, 1, 29-42.

Behr, M. J. & Eastman, P. M. Interactions between structure-of-intellect factors and two methods of presenting concepts of modulus seven arithmetic--a follow-up and refinement study. Journal for Research in Mathematics Education, 1975, 6, 150-157.

Botzum, W. A. A factorial study of the reasoning and closure factors. Psychometrika, 1951, 16, 361-386.

Coombs, D. H. Individual cognitive structuring and information assimilation. Dissertation Abstracts, 1969, 29, 4394. (Abstract)

Eastman, P. M. & Carry, L. R. Interaction of spatial visualization and general reasoning abilities with instructional treatment in quadratic inequalities: A follow-up study. Journal for Research in Mathematics Education, 1975, 6, 142-149.

French, J. W., Ekstrom, R. R., & Price, L. A. Manual for kit of reference tests for cognitive factors. Princeton, N.J.: Educational Testing Service, 1963.

Gormly, J. Cognitive structure: Functional unity in verbal and mathematical performance. Journal of Personality, 1971, 39, 70-78.

Guilford, J. P. The discovery of aptitude and achievement variables. Science, 1947, 106, 279-282.

Guilford, J. P. The structure of intellect. Psychological Bulletin, 1956, 53, 267-293.

Guilford, J. P, Christensen, P. R., Kettner, N. W., Green, R. T., & Hertzka, A. F. A factor-analytic study of Navy Reasoning Tests with the Air Force Aircrew Classification Battery. Educational Psychology Measurement, 1954, 14, 301-325.

Hancock, R. R. Cognitive factors and their interaction with instructional mode. Journal for Research in Mathematics Education, 1975, 6, 37-50.

Jones, L. V. Primary abilities in the Stanford-Binet, age 13. Journal of Genetic Psychology, 1954, 84, 125-147.

Lewin, K. Principles of topological psychology. New York: Holt, 1967.

Ostrow, J. A comparative study of certain aspects of intellectual functioning in achieving and low-achieveing high school students. Dissertation Abstracts, 1964, 25, 617. (Abstract)

Pemberton, D. The closure factors related to other cognitive processes. Psychometrika, 1952, 17, 267-288.

Pullman, H. W. The relation of the structure of language to mathematical ability. Unpublished doctoral dissertation, Columbia University, 1972.

Scott, W. A. Brief report: Measures of cognitive structure. Multivariate Behavioral Research, 1966, 1, 391-395.

Shuert, K. L. A study to determine whether a selected type of cognitive style predisposes one to do well in mathematics. Dissertation Abstracts International, 1971, 31, 3352-3353. (Abstract)

Thurstone, L. L. An analysis of mechanical aptitude. Psychometric Laboratory Report Number 62: University of Chicago, 1951.

Vannoy, J. S. Generality of cognitive-simplicity as a personality construct. Journal of Personality and Social Psychology, 1965, 2, 385-396.

Webb, L. F. & Carry, L. R. Interaction of spatial visualization and general reasoning abilities with instructional treatment in quadratic inequalities: A follow-up study. Journal for Research in Mathematics Education, 1975, 6, 132-140.

Table 1

DESCRIPTIVE STATISTICS FOR THE TESTS OF COGNITIVE FACTORS

Test of Cognitive Factor	Statistic				
	Mean	SD	N	Maximum	Minimum
Cf-1	11.53	7.40	82	27.00	-5.75
Cf-2	156.89	45.45	81	295	63
Cf-3	34.82	12.65	82	62	3
Cs-1	15.60	3.80	82	20	3
Cs-2	19.55	4.92	82	31	9
I-1	19.95	4.24	91	27.75	4.75
I-2	9.47	5.43	86	25.75	-3.25
I-3	89.38	42.57	84	184.00	-6.50
Rs-1	7.19	7.36	89	30	-10
Rs-2	30.71	6.11	68	40.00	12.00
Rs-3	11.84	4.00	83	19.00	2.75
Vz-1	100.94	43.26	89	196	19
Vz-2	11.03	4.36	85	20.00	0.75
Vz-3	40.79	13.55	74	58.86	2.23

Table 2

DESCRIPTIVE STATISTICS FOR THE MEASURES OF MATHEMATICAL ABILITY

Mathematical Ability	Statistic				
	Mean	SD	N	Maximum	Minimum
Algebra grade	2.30	1.07	93	4	1
Geometry grade	2.64	.93	91	4	1
SCAT	64.31	25.74	88	99	1
PSAT	52.95	10.89	74	75	28
SAT-1	546.51	113.16	81	780	302
SAT-2	558.25	127.54	73	800	309

Table 3

CORRELATION OF COGNITIVE STRUCTURE
WITH MATHEMATICAL ABILITY

Variable	Mathematical Ability					
	Algebra grade	Geometry grade	SCAT	PSAT	SAT-1	SAT-2
Cf-1	.31* (81)	.36* (81)	.34* (76)	.42* (67)	.40* (72)	.40* (63)
Cf-2	.29* (80)	.41* (79)	.25* (75)	.22* (65)	.32* (70)	.25* (63)
Cf-3	.31* (80)	.41* (80)	.28* (77)	.36* (76)	.40* (72)	.42* (64)
Cs-1	.11 (80)	.25* (80)	.22* (76)	.24* (67)	.26* (71)	.23* (64)
Cs-2	.28* (80)	.45* (80)	.12 (76)	.23* (66)	.28* (71)	.28* (64)
I-1	.39* (90)	.26* (87)	.44* (84)	.35* (71)	.35* (77)	.31* (70)
I-2	.43* (85)	.53* (83)	.51* (80)	.67* (67)	.65* (72)	.62* (64)
I-3	.40* (83)	.47* (82)	.56* (77)	.64* (65)	.61* (72)	.61* (65)
Rs-1	.23* (87)	.33* (85)	.29* (83)	.41* (69)	.44* (75)	.43* (68)
Rs-2	.32* (67)	.31* (67)	.53* (63)	.37* (56)	.42* (59)	.33* (53)
Rs-3	.35* (87)	.53* (85)	.50* (83)	.61* (69)	.59* (75)	.49* (68)

Note. - Numbers in parentheses indicate N for respective correlations.

*$p < .05$.

Table 3, continued

Variable	Mathematical Ability					
	Algebra grade	Geometry grade	SCAT	PSAT	SAT-1	SAT-2
Vz-1	.38* (87)	.50* (86)	.50* (82)	.61* (69)	.58* (76)	.57* (69)
Vz-2	.18* (83)	.39* (82)	.51* (80)	.57* (67)	.52* (72)	.61* (65)
Vz-3	.17 (73)	.29* (73)	.27* (68)	.42* (59)	.37* (64)	.43* (59)
IQ	.44* (92)	.57* (90)	.65* (88)	.75* (74)	.76* (81)	.70* (73)

Note. - Numbers in parentheses indicate N for respective correlations.

*$p < .05$.

Table 4

STEPWISE REGRESSION OF INTELLIGENCE AND COGNITIVE FACTORS ON MATHEMATICAL ABILITY

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
	Algebra Grade (N=52)		
I-2	.5426	.2945	.2945
I-1	.6017	.3620	.0675
IQ	.6190	.3832	.0212
Rs-3	.6389	.4082	.0250
Vz-2	.6604	.4361	.0279
Rs-1	.6659	.4434	.0073
Vz-3	.6695	.4482	.0048
Cf-2	.6720	.4516	.0033
Cs-1	.6744	.4548	.0032
Vz-1	.6760	.4570	.0022
Cf-3	.6778	.4595	.0025
Cs-2	.6787	.4607	.0012

Note. - Level for inclusion is $p < .05$.

Table 4, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
	Geometry Grade (N=52)		
IQ	.5820	.3388	.3388
Cs-2	.6373	.4062	.0674
Rs-1	.6809	.4637	.0575
I-2	.7228	.5224	.0587
I-3	.7274	.5291	.0068
Vz-1	.7340	.5387	.0096
Vz-3	.7374	.5438	.0051
Cs-1	.7403	.5481	.0042
I-1	.7422	.5508	.0028
Vz-2	.7432	.5524	.0015
Cf-3	.7440	.5535	.0011
Rs-2	.7449	.5548	.0014

Note. - Level for inclusion is $p < .05$.

Table 4, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
		SCAT (N=50)	
IQ	.4787	.2287	.2287
Vz-2	.5572	.3104	.0818
Cf-2	.6175	.3813	.0708
I-1	.6584	.4335	.0522
Rs-2	.6805	.4631	.0296
I-2	.6936	.4811	.0180
Cf-1	.7044	.4961	.0150
I-3	.7104	.5046	.0085
Cf-3	.7167	.5137	.0090
Rs-3	.7248	.5254	.0117
Cs-1	.7270	.5285	.0031
Vz-1	.7292	.5318	.0033
Rs-1	.7315	.5352	.0034
Cs-2	.7320	.5359	.0007

Note. - Level for inclusion is $p < .05$.

Table 4, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
		PSAT (N=43)	
IQ	.7659	.5866	.5866
I-2	.8265	.6832	.0966
Cf-2	.8440	.7123	.0292
Vz-1	.8518	.7256	.0133
Rs-2	.8578	.7358	.0102
I-1	.8640	.7465	.0107
Cf-1	.8693	.7557	.0092
I-3	.8744	.7646	.0089
Rs-3	.8780	.7708	.0062
Cf-3	.8813	.7766	.0058
Vz-3	.8836	.7808	.0042
Cs-1	.8873	.7873	.0065
Cs-2	.8876	.7878	.0005
Rs-1	.8879	.7884	.0007

Note. - Level for inclusion is $p < .05$.

Table 4, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
		SAT-1 (N=45)	
IQ	.7552	.5704	.5704
I-2	.8035	.6457	.0753
Vz-3	.8115	.6585	.0129
Cf-1	.8230	.6773	.0188
I-3	.8290	.6872	.0099
Cf-3	.8341	.6957	.0085
Cf-2	.8414	.7080	.0123
Rs-3	.8468	.7171	.0091
I-1	.8495	.7216	.0045
Rs-2	.8524	.7265	.0050
Rs-1	.8541	.7295	.0029
Vz-1	.8546	.7304	.0010
Cs-2	.8553	.7316	.0012

Note. - Level for inclusion is $p < .05$.

Table 4, continued

Independent Variable	Mathematical Ability		
	Multiple R	R squared	Increase in R squared
		SAT-2 (N=41)	
I-2	.6571	.4318	.4318
IQ	.7116	.5064	.0745
Cf-2	.7350	.5402	.0338
Cf-3	.7698	.5926	.0524
Vz-3	.7923	.6278	.0352
Rs-1	.8098	.6557	.0279
I-1	.8139	.6624	.0066
Cf-1	.8172	.6678	.0055
Cs-1	.8237	.6785	.0107
Rs-2	.8271	.6841	.0055
Vz-1	.8285	.6865	.0024
I-3	.8296	.6882	.0017
Rs-3	.8305	.6898	.0015
Cs-2	.8311	.6906	.0009
Vz-2	.8315	.6914	.0008

Note. - Level for inclusion is $p < .05$.

Table 5

CORRELATIONS AND COMMON VARIANCE OF MATHEMATICAL ABILITY WITH INTELLIGENCE AND COGNITIVE FACTORS

Mathematical Ability	Intelligence Alone		Intelligence and Cognitive Factors	
	r	r squared	R	R squared
Albegra grade	.48	23%	.68	46%
Geometry grade	.58	34%	.74	55%
SCAT	.48	23%	.73	54%
PSAT	.77	59%	.89	79%
SAT-1	.75	57%	.86	73%
SAT-2	.58	34%	.83	69%