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ABSTRACT

The paper addresses provisions for a diagnostically based remedial component for children manifesting clear and specific learning disabilities in mathematics. A model, the interactive unit, upon which a sequential program that systematically minimizes learning disabilities on mathematics performance is introduced. It is explained that the underlying strategy is to compensate for deficits in one area (e.g., reading) upon another area (e.g., mathematics) by partialing out the effects of the deficit so that it will not effect performance in the second area. A second factor, error analysis, in the provision of a diagnostically based remedial component is also described. The results of information gathered from students in grades K-12 regarding errors discovered in mathematical computation are reviewed. A sample remedial module written for the purposes of remediating error displayed in the problem "11 + 7 = 9" is offered. (SBH)

MATH EDUCATION FOR THE LEARNING DISABLED - AN ANALYSIS OF ERROR PATTERNS AND TECHNIQUES IN REMEDIATION*

A Paper Presented at the Fifty-Seventh Annual International Convention of The Council for Exceptional Children, Dallas, 1979

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ERRATA SHEET*

page 11 - The student stated that the problem was solved in the following manner.

$$\begin{array}{r} 11 \\ +\ \underline{7} \\ \underline{9} \end{array}$$

page 11 - A sample remedial module written for the purposes of remediating the error displayed in the problem "11 + 7 = _9_"

page 12 - (LEVEL 2) - ...The teacher may incorporate that the sign for addition is " + " and may ask the student to write the " + " next to 11 and seven and thus, 11 + 7 = _18_.

* The underlined answers indicate the correct response.

INTRODUCTION

At present, there is a paucity of comprehensive mathematics curricula developed for the handicapped student. With the exception of Project Math[1] (Cawley, Fitzmaurice, Goodstein, Lepore, Sedlak & Althus, 1976), mathematics materials developed for use by special educators consists primarily of instructional materials, each designed with a singular purpose.

The sole mathematics curricula utilizing a comprehensive design is Project Math (Cawley et.al., 1976). A four level mathematics program that provides a pre-K through secondary school curriculum for mentally handicapped in addition to pre-K through early elementary math content for children with learning disabilities and behavioral disorders, Project Math is currently available on the popular market. As part of an ongoing effort in providing a comprehensive mathematics curricula, Dr. Cawley and his associates are currently involved in the development of a mathematics curriculum[2], in cooperation with the Bureau of Education for the Handicapped, intended for use with learning disabled students at the upper grade levels.

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[1]A Program Project Research and Demonstration Effort in Arithmetic Among the Mentally Handicapped. BEH, U.S.O.E. Grant #OEG-0-70-2250(607), Project #162008, J. F. Cawley, Principal Investigator, University of Connecticut, Storrs, Connecticut, 06268.

[2]"Learning Disabilities in Mathematics: A Curriculum Design for Upper Grades," U.S.O.E. Grant No. G007605223 under the direction of J. F. Cawley, Project Director, University of Connecticut, Storrs, Connecticut, 06268.

LEARNING DISABILITIES DEFINED

Learning Disabilities must be viewed as a two dimensional construct. There is the individual who exhibits a learning disability specific to mathematics. Secondly, there are individuals who exhibit learning disabilities in other areas (e.g., reading) which interfere with mathematics performance.

A curriculum endeavor in mathematics for the learning disabled must focus upon these two different aspects of learning disabilities. As such, a curriculum endeavor must incorporate (1) provisions for a diagnostically-based remedial component for the child who manifests clear and specific learning disabilities in mathematics and (2) a sequential program which systematically minimizes the effects of other specific learning disabilities on performance in mathematics (Cawley, 1976). The focus of this paper will address the first of these two aspects, that is, the provision of a diagnostically-based remedial component.

But first, the reader will be introduced to the model upon which a sequential program that systematically minimizes learning disabilities on performance in mathematics is based. The underlying strategy is to compensate for deficits in one area (e.g., reading) upon another area (e.g., mathematics) by partialing out the effects of the deficit so that it will not effect performance in the second area.

THE INTERACTIVE UNIT (IU)

A means by which greater specificity and variability are provided for in curriculum development, the interactive unit (Cawley & Vitello, 1972) is comprised of four (4) teacher inputs and four (4) learner outputs. When matched against one another the various inputs and outputs result in the formulation of sixteen (16) combinations of teacher-learner combinations. Figure 1 illustrates the Interactive Unit (Cawley, 1976).

Utilization of the Interactive Unit as the basis for the development of mathematics curricula provides the instructor with a host of teacher-learner interactions. In this manner, the curriculum is able to partial out the effects of a disability in one area (e.g., reading) on another area (e.g., mathematics). Figure 2 (Cawley, 1976) provides an illustration of the Interactive Unit relative to mathematics curricula. As can be noted, implementation of the Interactive Unit provides a means by which a single mathematics concept can be presented to the learner in up to sixteen different interactive combinations.

Cawley (1976) has outlined three fundamental curriculum qualities that the implementation of the Interactive Unit fosters. These qualities, the minimum in special education curriculum, are:

1. The curriculum must be capable of partialing out or circumventing the effects of one disability upon other areas of development. In this instance, for the child who cannot write, mathematics must be presented in such a way that the effects of the writing disability are partialed out.
2. The curriculum must be capable of interrelating with divergent management strategies and teaching styles in order to facilitate affect. For instance, the combination construct-construct is an ideal method for having students working in close proximity to one another and with the instructor. Utilization of this combination affords the instructor the opportunity to organize instruction such that a withdrawn child operates amidst a group.

Figure 1

The Interactive Unit: Components and Definitions

Instructor	C	P	S	GS
Learner	C	I	S	GS

Construct (C) - Teacher manipulation of the learning environment and pupil constructive or manipulative responses.

Present (P) - Presentation to the learner of fixed non-symbolic visual displays (arrangements of materials, pictures, or pictorial worksheets).

State (S) - Reliance upon oral discourse.

Graphic Symbolic (GS) - Written or drawn symbolic stimulus materials.

Identify (I) - Multiple choice means of responding.

Figure 2

Concept: Division of a whole number by a proper fraction.

Instructor	Learner
Constructs by separating a number of wholes into halves, fourths, etc.	Constructs by separating a number of wholes into halves, fourths, etc.
Constructs by separating a number of wholes into halves, fourths, etc.	Identifies a fixed representation of the same number of wholes divided into the same parts.
Constructs by separating a number of wholes into halves, fourths, etc.	States a description of what the instructor has done, and names the number of pieces resulting.
Constructs by separating a number of wholes into halves, fourths, etc.	Graphically symbolizes by writing the numeral naming the number of pieces resulting.
Presents fixed representations of a number of wholes divided into halves, fourths, etc.	Constructs by separating the same number of wholes into the same number of parts.
Presents fixed representations of a number of wholes divided into halves, fourths, etc.	Identifies a fixed representation of the same number of wholes divided into the same parts.
Presents fixed representations of a number of wholes divided into halves, fourths, etc.	States a description of what the instructor has shown and names the number of pieces.
Presents fixed representations of a number of wholes divided into halves, fourths, etc.	Graphically symbolizes by writing the numeral naming the number of pieces shown in each representation.
States what division by a proper fraction means and gives directions for showing this.	Constructs representations of division by a proper fraction according to the instructor's directions.
States what division by a proper fraction means and gives directions for showing this.	Identifies a representation of division by a proper fraction.
States what division by a proper fraction means and gives directions for showing this.	States what division by a proper fraction means.
States what division by a proper fraction means and gives directions for showing this.	Graphically symbolizes by drawing a picture to represent division by a proper fraction.

Figure 2 (Cont.)

Instructor	Learner
Graphically symbolizes by drawing a picture representing division by a proper fraction and writing the number expression represented.	Constructs a representation of what the instructor has drawn and written.
Graphically symbolizes by drawing a picture representing division by a proper fraction and writing the number expression represented.	Identifies a fixed representation of what the instructor has drawn and written.
Graphically symbolizes by drawing a picture representing division by a proper fraction and writing the number expression represented.	States the meaning of the number expression by describing the picture, and names the number of parts resulting.
Graphically symbolizes by drawing a picture representing division by a proper fraction and writing the number expression represented.	Graphically symbolizes by copying the instructor's picture, writing the number expression, and writing the numeral to name the resulting number of parts.

3. The curriculum must also be capable of transmitting knowledge and information to the learner.

It was stated earlier in this paper that a mathematics curriculum designed for the learning disabled must be comprised of two major components. The first component, the development of a comprehensive, sequential program which systematically minimizes the effects of specific learning disabilities in mathematics, has as its basis the Interactive Unit. It is the second factor, the provision of a diagnostically-based remedial component for the child who manifests a clear and specific learning disability in mathematics, that the remainder of this paper addresses.

ERROR ANALYSIS

As part of the ongoing effort in the development of comprehensive mathematics curriculum for the learning disabled, attention was focused upon the youngster who exhibits a clear and specific learning disability in mathematics.

During the 1977-1978 academic year, information was gathered from students in grades K-12 in Connecticut regarding errors discovered in mathematical computation. The Bureau of Education for the Handicapped "Learning Disabilities in Mathematics: A Curriculum Design for Upper Grades," U.S.O.E. Grant No. G007605223 sponsored the aforementioned effort. Heavy emphasis was placed upon diagnosing error patterns with whole numbers, decimals and simple fractions in the operations of addition, subtraction, multiplication and division. The actual thought processes that a student utilized were obtained through an oral interview technique. Remedial modules were then written à la Bruner (e.g., symbolic, iconic and enactive modes) in an attempt to remediate these incorrect thought processes. This aforementioned effort is closely aligned with the diagnostic-prescriptive teaching cycle, that is, teaching in which strengths and weaknesses are identified, objectives are formulated for correcting the weaknesses, remedial techniques are applied, and evaluation is continuous (Reisman, 1972; West, 1971; Ysseldyke & Salvia, 1974).

The manner in which the diagnostic-prescriptive backing was performed was twofold:

1. Students who did not attain 60% on the program's screening device (Math Concept Inventory) were administered the Buswell-John Diagnostic Chart for Individual Difficulties: Fundamental Process in Arithmetic.
2. Once error patterns were diagnosed on the Buswell-John, remedial modules were developed to remediate a student's incorrect thought processes.

The Buswell-John was used because it provided the interviewer close scrutiny of a student's errors. Many have had success with this instrument in an oral interview setting (Cox, 1973; Lankford, 1974; Lepore, 1974; Schonell et.al., 1957).

The items on the Buswell-John number 180; however, the instrument takes approximately 1-1 1/2 hours to administer. The items deal with basic concepts in mathematics with whole numbers, fractions, and decimals.

The interviewer presents the students with written examples and the student computes the example. At certain times the interviewer requests that the student "explain aloud" the method used to solve the problem. The student was never told if he or she were correct or not, but <u>each</u> time the student performed an incorrect computation, the student was administered a problem which was conceptually the same and was then asked to compute that problem. After this computation, the student was asked to explain what he or she "thought of" in order to arrive at the answer. For example, a student was administered the following example: $7\overline{)217}$.

The student solved the problem as follows:

$$7\overline{)217}; \qquad \begin{array}{r} 1 \\ 7\overline{)217} \end{array}; \qquad \begin{array}{r} 31 \\ 7\overline{)217} \end{array}$$

The interviewer noticed that the quotient was correct; however, the student had solved the task in reverse. After such an observation, the interviewer wrote: $7\overline{)507}$. The student was asked to solve it. The student solved the problem as follows:

$$7\overline{)507}; \qquad \begin{array}{r} 1 \\ 7\overline{)507} \end{array}; \qquad \begin{array}{l} \;\;\;\;71\text{ r}1 \\ 7\overline{)507} \end{array}$$

Next the student was <u>asked</u> to <u>state</u> what he or she did; he said to solve $7\overline{)217}$, divide 7 into 7, and obtain 1. Next, divide 7 into 21 and obtain 3. Thus, 31 is the answer. To solve $7\overline{)507}$, divide 7 into 7, the answer is 1; next divide 7 into 50, the answer is 7 r1. Thus, 71r1 is the answer.

After such an explanation, it was obvious to the interviewer that the student had some correct concepts concerning division; however, the student had developed an incorrect algorithm for the division of a 3 digit number by a divisor of 1 digit. If the oral interview technique were not utilized, teachers may have thought that this student actually divided problems such as $7\overline{)217}$ correctly.

Remedial modules were originally written in a flow chart format à la Bruner in reverse. Students were started with problems in the symbolic mode; however, if they were unsuccessful they were looped into the iconic mode. If they were unsuccessful in the iconic mode, they were looped into the enactive mode. Once a student met a criterion for success in each mode, they were looped finally back to the symbolic mode. Evaluation of the module was performed each month following the attainment of the criterion of success for the symbolic mode. Most students retained what they had been taught.

Since the writing of these remedial modules was at a very experimental stage, they were consistently rewritten and re-evaluated. Presently the modules begin with the enactive stage, followed by the iconic stage and finally progress to the symbolic stage. Teachers observed the use of the modules and felt it was simpler in the aforementioned manner. The modules were most effective when a "starting point" was located. The modules acted as an alternative method of instructing students by emphasizing the correct aspects of their thought processes in order to eliminate their incorrect thought processes.

CONCLUSION

Basic to the provision of instruction is diagnosis. Without efficient diagnostic procedures available to the instructor, efforts at remediation will consistently fall short of their mark.

Specific to the area of mathematics, diagnosis should afford the instructor more than the opportunity to <u>observe</u> the student in the process of problem solving. What is needed is a process through which the instructor not only <u>observes</u>, but also discovers <u>how</u> the student is performing. This is the foundation upon which effective remediation occurs. In the authors' opinion, with the exception of Cawley (1976), neither general nor special educators are attacking the question of diagnosis and remediation in mathematics that is of any substantial help to the instructor.

A proposed method of establishing a link between diagnosis and remediation has been discussed in the aforementioned section. Utilization of the oral interview method opens the door to a clearer understanding of a student's actual thought processes. Eliminated is all guess work on the part of the teacher. Take, for instance, Ms. Jones and Mr. Smith. When asked to solve the following problem, student A arrived at the following answer.

Student A
$$\begin{array}{r} 12 \\ +\ 5 \\ \hline 8 \end{array}$$

Ms. Jones assumed that student A did not understand the process of addition and subsequently geared her teaching to meet the student's apparent need.

Using the same student as an example, let us see how Mr. Smith deals with the situation. Given the problem "12 + 5," student A arrives at the answer "8." Utilizing the oral interview technique, Mr. Smith requests the student to solve a problem which is conceptually the same, "11 + 7." The student writes "9" as the answer. Mr. Smith now asks that the child <u>tell</u> him exactly how the problem was solved. The student stated that the problem was solved in the following manner.

$$\begin{array}{r} 11 \\ +\ 7 \\ \hline 8 \end{array}$$

7 + 1 = 8, and 8 + 1 = 9

(In other words "7 + 1 + 1 = 9")

Mr. Smith discovered that the algorithm rule employed by the student was: add together all of the digits in order to obtain the answer. Place value <u>was</u> ignored.

A sample remedial module written for the purposes of remediating the error displayed in the problem "11 + 7 = 8" is as follows:

Sample Remedial Module for Addition
of a Two Digit and One Digit
Problem with No Renaming

<u>LEVEL 1</u>

The student would work with objects, namely, the student would count 11 objects aloud, then count 7 objects aloud.

Next, the student would be told to place all objects together and then to count and state the total. (The teacher may or may not request that students write the numeral each time he or she counts and states it.) The student would work at this level until the teacher felt the child could proceed to the next level. [If the teacher has the student write the numeral after he or she counts and states it; the symbolic (writing of the numeral) and the enactive (manipulations) levels will be integrated.]

LEVEL 2

The student would work on pictorial displays of "11 + 7." For example, the student may be told to circle 11 objects on the paper. (The teacher may or may not ask the student to now write the numeral 11.) Next the student would be asked to circle 7 objects on the paper. (Perhaps the teacher may request that the student write the numeral 7.) The student would then be told to count the total amount in both circles and state the answer. The teacher may incorporate that the sign for addition is "+" and may ask the student to write the "+" next to 11 and 7 and thus, 11 + 7 = 8. [Note: In this alternative, the iconic (pictorial) and symbolic (writing the numerals) were integrated.] If a student performs a number of these problems correctly at this level, the teacher would have the student proceed to the next level. If the teacher is not satisfied, the student would be placed back in Level 1.

LEVEL 3

The student would perform problems strictly on the symbolic level. Worksheets with problems which were conceptually the same would be supplied. For example,

$$\begin{array}{r}11\\+\underline{\ 7}\end{array}\qquad\begin{array}{r}18\\+\underline{\ 1}\end{array}\qquad\begin{array}{r}25\\+\underline{\ 3}\end{array}\qquad\begin{array}{r}35\\+\underline{\ 2}\end{array}\qquad\text{(no renaming)}$$

(Since the modules are being field-tested and, therefore, re-evaluated, the sample remedial module is subject to change. For final versions of the modules, write to Dr. John F. Cawley, University of Connecticut, Storrs, CT., 06268. Please state that Schenck and Pelosi recommended your request.)

The major purpose of this paper was to present to the reader a method through which error analysis can be achieved by the classroom instructor. Remediation which meets diagnosed needs can then more readily be enacted. Through the establishment of such a system the mathematics education of learning disabled youngsters will be upgraded.

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