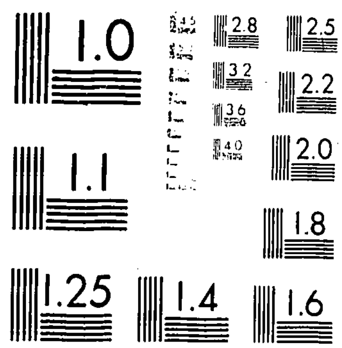


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ABSTRACT

Upper elementary students are the focus of this paper, intended to present a few studies that have been done in the last five years that are related to two aspects of problem solving: strategies used by the problem solver and tasks used in studying problem solving. The review is not intended to be exhaustive, but is designed to present examples of strategies and tasks that have been studied or used in recent research. (MP)

# MATHEMATICAL PROBLEM SOLVING PROJECT

A Project of the
MATHEMATICS EDUCATION DEVELOPMENT CENTER

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## MATHEMATICAL PROBLEM SOLVING PROJECT

FINAL REPORT
MATHEMATICAL PROBLEM SOLVING PROJECT
TECHNICAL REPORT I:
DOCUMENTS RELATED TO A PROBLEM-SOLVING MODEL
PART A: "A Review of the Literature Related to Problem-Solving Tasks and Problem-Solving Strategies Used by Students in Grades 4, 5 and 6"

Report Prepared By

Norman L. Webb

This review was prepared in September, 1974 to identify problem-solving strategies that other researchers have studied or have identified as being used by fourth, fifth, and sixth grade children.

MATHEMATICS EDUCATION DEVELOPMENT CENTER
Indiana University - Bloomington
May, 1977

A Review of the Literature Related to
Problem-Solving Tasks and Problem-Solving
Strategies used by Students in Grades 4,5 and 6

by Norman Webb

The literature on problem solving is vast. Because of the complexity of problem-solving processes and the number of variables associated with problem solving, research in this area has been too diverse to have any real consolidation. This paper focuses on the age level of upper elementary students and is intended to present a few studies that have been done in the past five years that are related to two aspects of problem solving. One aspect is the strategies used by a problem solver. The second aspect is the tasks that have been used in studying problem solving.

What is meant by a problem will first be given. Then a few models will be presented that have been used to describe problem-solving processes. This will be followed by a section reviewing some of the literature related to strategies. Finally, a few tasks that have been used in studying problem solving will be reviewed. This review is by no means exhaustive, but is designed to present examples of strategies and tasks that have been studied or used in recent research.

What is a problem?

A problem will be described in terms of the individual. For an individual, a problem exists if he desires to obtain a goal but the path leading to the attainment of this goal is not immediately known and cannot be found by just using habitual responses. This corresponds to most definitions of a problem given in the literature. Common to the definitions is the existence of a goal, a blockage of the paths used to reach the goal, and motivation to obtain the goal.

A slightly differently worded definition which is more suitable to the psychological literature is given by Davis (1973). For Davis a problem is a stimulus situation for which an organism does not have a ready response. Here the stimulus situation is the goal that is desired to be obtained. Not having a ready response corresponds to not being able to find the solution by just using habitual responses.

Problem-Solving Models:

A review of different theories of problem solving is given by Treffinger (1969). One model reviewed, Klausmeir and Goodwin (1966), lists five problem-solving stages: 1) setting a goal, 2) appraising the situation, 3) trying to attain the goal, 4) confirming or rejecting a solution, and 5) reaching the goal. Guilford (1966) presented problem-solving episodes. First is the input from environmental or somatic sources. Next is an initial filtering process. Following these two stages is a period of cognition and production. Underlying Guilford's model is the continual referring to the memory storage and evaluation.

Polya (1957) gave four stages: understanding the problem, devising a plan, carrying out the plan, and looking back. As Treffinger noted, there is considerable similarity among the models given.

The following model of the problem-solving process is a synthesis of the above models that is felt appropriate in reviewing the existing literature in Mathematics Education. The three main stages in solving a problem are preparation, production, and evaluation.

Preparation - This includes the defining and understanding of the problem. In some cases, the problem is well defined as stated. In other cases, the conditions and constraints for the problem must be given by the problem solver. This is considered preparation for solving the problem. Also included in this stage is the understanding of what is unknown, what is given, and what the problem asks.

Production - This stage includes the search for a path to reach the goal; the recall from memory of principles and facts from previous knowledge; the generation of new principles to be used in solving the problem; and the development of hypotheses and different alternative paths that may lead to the goal. Included in this stage would be Polya's devising and carrying out the plan.

Evaluation - This stage includes the checking of the final solution as well as any intermediate subgoals. After the production of a hypothesis, usually some form of checking is appropriate.

This three-stage model is not a hierarchical model in that preparation always comes before production which always must precede evaluation. This is more a cyclic model. As one proceeds through the problem-solving process, first comes preparation, then production and evaluation. The problem may then be redefined in which case the cycle begins again.

Problem solving is very complex as the three-stage model with its substages represents. Many factors may affect the ability of a student to solve problems at any stage. The research in mathematical problem solving has begun the task of studying various factors relating to solving problems. Some of the reviews of literature include Kilpatrick (1969), Gorman (1967), and Riedesel (1969). A review of the language factors in learning mathematics is given by Aiken (1972). In this he reports on some studies relating language factors to problem solving. The particular concern of this report is the problem-solving strategies used by students in the fourth, fifth, and sixth grades and problem-solving tasks used to measure the ability to solve problems. Peripheral research reports are included if they are felt to add to the relevant knowledge of this report.

Caution must be taken in interpreting any research on problem solving. The criteria used to measure problem-solving ability have varied greatly. Grades in math classes, honor student status, computational tests, standardized mathematics achievement tests, puzzles, and thought problems have all been used as a measure of problem-solving ability. Besides the variance in criteria, the age levels studied have ranged from pre-school to post-graduate. With such variance in age groups and criteria, each study needs to be read with respect to its particular level. In synthesizing across reports, care must be taken to ensure generalizations are not made beyond the context of the setting of the particular study.

## Problem-Solving Strategies

Problem-solving processes can be defined as all behaviors related to the problem-solving procedure performed from the initial step of reading or defining the problem to giving the final solution or the termination of work on the problem. Problem-solving processes are a general category. A strategy or heuristic strategy is a problem-solving process which is a planned action or series of actions performed to aid in the discovery of the solution. Drawing a figure, looking for a pattern, estimation, and recalling a related problem are examples of possible strategies. Each of these actions is performed in hopes of advancing towards a solution. Processes not considered strategies would include reading the problem, selecting the solution on an irrelevant basis, and performing a counting error. These are all behaviors performed as part of the problem-solving procedure, but are not planned and/or do not directly aid in the discovery of a solution.

As one reviews recent dissertations and studies which focus on problem-solving processes, one soon realizes the diverse number of processes being considered without much overlap between studies in regards to the definition of processes and processes being studied. Table 1 lists processes studied or reviewed by ten investigators. Most of the processes were considered by the investigators as strategies, but some could not be classified as strategies under the definition given above. The age level used in most of the studies ranged from third grade to eighth grade.

TABLE 1

Problem-Solving Processes Studied or Reviewed[a]

| Author(s) (year) | Grade/ Age | Problem Solving Criteria | Processes/strategies | N |
|---|---|---|---|---|
| Sanders (1972) | grade 4 | Verbal arithmetic problems from textbook | Logical analysis (use of equation or algorithm)<br>Creative or divergent thinking (unusual way)<br>Insightful thinking<br>Intuitive thinking<br>Blind guessing<br>Trial and error | 455 |
| Foster (1972) | grade 8 | personally constructed problem solving abilities test (PSAT) | Specifying conditions a datum satisfies<br>Selecting a relevant solution<br>Proposing a hypothesis<br>Identifying a pattern<br>Supplying missing information<br>Selecting relevant data (intermediate stage)<br>Using a constructed algorithm<br>Correcting an error in a constructed algorithm<br>Constructing an algorithm | 60 |
| Hollander (1973) | grade 6 | Verbal arithmetic problems<br>3-two step problems<br>3-three step problems | Comprehension of math relationships as expressed through words and symbols within the problems<br>Ability to employ abstract analytical reasoning<br>Ability to reason insightfully<br>Making neither excessive or few references to the text relative to their peers<br>Minimum number of computational steps necessary for solution of the problem | 12 |
| Riedesel (1969) (A review of the literature) | Elem. age group | varied | Make use of mathematical sentences<br>Make use of drawings and diagrams | NA |

| Author(s) (year) | Grade/ Age | Problem Solving Criteria | Processes/Strategies | N |
|---|---|---|---|---|
| Kilpatrick (1967) | grade 8 | twelve insightful word problems | Drawing a figure<br>Using successive approximations<br>Questioning the existence/ uniqueness of the solution<br>Using a deduction process<br>Using an equation<br>Using trial and error<br>Checking the solution | 56 |
| Stern (1967) | grade 3 | Matching exemplars to a model to discover the concept | Multiple hypothesis (focus on two hypotheses associated with correct exemplar)<br>Single hypothesis (remind pupils of concepts possible and which had been tried) | 107 |
| Dienes & Jeeves (1965) | Adults to children | Binary operation card game based on two element group, cyclic group, or klein group.<br>Subject asked to describe how the game worked. Card in window and S played card would determine next card in window. | Operational type (S viewed that his card played operated on the card in window having the power to alter that card)<br><br>Pattern type (S viewed card in window and the one he played at the same level. These hung together in a pattern. Description of game was one large pattern.)<br><br>Memory type (S memorized all the different combinations.) | unknown |
| Crutchfield and Covington (1963) | grades 5 and 6 | Problem-solving and creative tasks similar to those included in the Productive Thinking Program | Value and necessity of properly defining the problem<br>Asking relevant questions<br>Looking carefully at details<br>Observing discrepencies<br>Generating many ideas<br>Being sensitive to all sorts of clues | 3 pairs of classes |
| Jerman (1971) | grade 5 | Arithmetic word problems (some defined as insightful) | Want-given | 261 |

Note [a] - Dissertation abstracts were used for recent dissertations which were unavailable. In these cases, precise definitions of the processes/ strategies were unattainable.

Sanders (1972) used a retrospective technique and asked the students to state how they solved the problems after they had finished working. The strategies used by the most successful students were logical analysis and creative or divergent thinking. Insightful and intuitive thinking were only used by a few of the students. The most unsuccessful strategies were blind guessing and trial and error.

Foster (1972) studied the effects of working with computer programming and flow charts with developing selected problem-solving behavior. Four treatment groups were used over a twelve-week period. One group worked with a computer, one group worked with flow charts, one group used both, and one group used none. The computer group scored significantly higher on the Problem-Solving Abilities Test than the group which used neither the computer nor flow charts. The differences were claimed to be due to the four behaviors of specifying conditions a datum satisfies, selecting a relevant solution, proposing a hypothesis, and constructing an algorithm.

Hollander (1973) had the students describe their processes aloud as they worked the problems. The successful processes identified are given in Table 1. The last two listed would be considered observations of processes but not really strategies.

Riedesel (1969) reviewed the literature on problem-solving ability of elementary children. He reported the use of formal analysis did not produce superior results in problem-solving. Here the use of formal analysis means the student answers such questions as 1) What is given? 2) What is to be found? 3) What operation do you use? 4) What is an estimate of the answer? and 5) What is the answer to the problem? Problem solving was improved if one of these steps was used as a focus for a lesson. For example, the task for the day may be to make an estimate of an answer to the problem. The two strategies identified by Riedesel that do increase problem-solving ability are the use of mathematical sentences and the use of drawings and diagrams.

Kilpatrick (1967) had the students think aloud as they solved twelve word problems. The audio-recorded protocols were coded using a system devised by Kilpatrick. Those processes that were reliably coded are listed in Table 1. The processes which had a positive correlation with the total score on the problem-solving test were successive approximation (r=.44), executive errors (r=.39), and trial and error (r=.30). Only the first two were statistically significant.

Stern (1967) gave third-grade students instruction on two types of strategies. The task given the students was to select from two exemplars the one which matched a given model. The four concepts used to vary the exemplars were number, color, size, and shape. The student picked one of two exemplars and was told yes or no whether it matched the model according to the rule. The student discovered the rule when he was able to select the correct exemplar on each succeeding trial. One group of students was taught a multiple-hypothesis strategy, where the students were asked to focus on two hypotheses associated with the correct exemplar on the first trial and then select one of these two on the basis of the information provided with the second trial. The rest of the students were taught what was called a single-hypothesis strategy where the students were reminded of the possible concepts and which ones had been tried. Students in the single-hypothesis group made larger gains on a pre-posttest than did the multiple-hypothesis group. The multiple-hypothesis group did slightly better on the retention test. Stern concluded that young students were unable to use the sophisticated strategy. However, they could use the simpler strategy wi+' ntimum efficiency.

The three str... ...onal, pattern, and memory, ide... ed by Dienes and Jeev... ...y be useful in defining problem ...g styles or in studying the development of problem-solving ability in individuals. Adults were found to use more strategies than children. The strategies used by adults were evaluated to be of the operational type more often than those used by children. In order of use, the pattern strategy was used the most, followed by the memory type and then the operational type. The most efficient strategy was the operational, followed by the pattern and then the memory.

Crutchfield and Covington (1963) developed the Productive-Thinking Program. This consisted of sixteen booklets in cartoon format that were designed to develop the general problem-solving skills and strategies given in Table 1. Their idea was that such strategies would be useful in solving problems in any content area. They tested their program using three pairs of fifth- and sixth-grade classes. One class of each pair used the Productive-Thinking Program over sixteen days while the other class was given a serial adventure story. The criterion tests were problem-solving and creative-thinking tasks similar to the type of problems in the program. The trained group did better on several of the problems.

Treffinger (1969) studied the effect of the Productive-Thinking Program in teaching skills to solve problems on an Arithmetic Puzzles Test. He used sixteen classes from grades 4, 5, 6 and 7 from which he randomly selected control and experimental groups. The experimental group were given the Productive-Thinking Program for sixteen consecutive school days.

The control group continued with their ordinary classroom instruction. In comparing the difference between pretest and posttest scores, no support was found that the program effectively developed skills or abilities which would transfer to arithmet blem-solving tests of the kind used. There was support for the effectiveness of the program to develop the pupils' attitudes toward creative thinking and problem solving.

Jerman (1971) compared the Productive-Thinking Program with a want-given program. The want-given program instructed the students on considering what is wanted and what is given in a problem before trying to solve the problem. No significant differences at the .05 level were found between the two programs on an arithmetic word problem test. Some differences, at the .10 level, were found on follow-up word problems given after a period of time in favor of the want-given program. Jerman stressed the need to teach problem-solving techniques along with the teaching of content.

The diversity in strategies studied prevents a succinct consolidation of results across studies. The use of equations or mathematical sentences at the elementary level was shown to be successful in some of the studies (Sanders (1972), Riedesel (1969), Hollander (1973)). This as evidence of a successful problem-solving strategy may be in question. The use of equations is probably closely associated with mathematics achievement and may indicate advancement in the content area instead of any special problem-solving strategy. Trial and error as a beneficial strategy appears to be in question. Sanders (1972) identified trial and error as used by the most unsuccessful problem solvers whereas Kilpatrick (1967) found this strategy to be positively correlated with problem-solving ability. The difference may be in the difference in the age groups considered by the two studies and/or the difference in the problems used as the criteria for problem solving. This suggests that when strategies are being considered, they should be studied relative to the criteria problems and the age level.

Ther- oubt that the teaching of general problem-solving strategie m.' isfer to specific content areas such as arithmetic. As suggested by Riedesel, the use of one step in formal analysis or strategy as the focus of a daily math lesson may have more results than global programs such as the Productive-Thinking Program. The study of content along with the study of problem-solving strategies is important.

Sources for strategies exist other than in reports of empirical research. Polya (1957) as a master problem solver has written from his experience his conceptualization of heuristic strategies. Among the possible strategies he gives are using an auxiliary problem, decomposing and recombining the problem, trying to recall a related problem, drawing a figure, examining your guess, forming a generalization of the problem, using a specialization of the problem, and working backwards on the problem. More operational definitions of these strategies based on Polya's work are given by Kilpatrick (1967).

Wickelgren (1974) was influenced greatly by Polya in writing his book, <u>How to Solve Problems</u>. He combines the work of Polya with the work in artificial intelligence and his own experience in psychology in forming his conceptualization of problem solving. For Wickelgren, a formal problem is composed of three types of information, information concerning givens, operations, and goals. A solution of a problem then is the complete specification of the three types of problem information as well as an ordered sequence of problem states from the given state to the goal state.

Wickelgren discussed several general problem-solving methods appropriate for higher level problem solving in mathematics, science, and engineering. In their present form, some of these methods are inappropriate for use by elementary children. A transformation or a simplification of these methods may be beneficial for this young age group. The general problem solving methods are

1) Drawing inferences - Try drawing inferences from the explicit or implicit information similar to inferences drawn from the same type of information in the past and/or draw inferences about the properties in the goal, the givens, or inferences from the goal and the givens.

2) Trial and error - (three types)

   Random - applying allowable operations to the givens with no definite order,

   Systematic - systematizing the sequences of actions,

   Classificatory - organizing sequences of actions into classes that are equivalent with respect to the solution of the problem.

3) Getting out of loops - Make an analysis of what you have been doing, determining the attributes of your approach. After classifying actions, what other general method or action could you use?

4) Incubation - Put the problem aside for a period of time.

5) Define subgoals - Break the problem into parts or subgoals and solve these.

6) Contradiction - Apply operations to the possible goals in order to get to the givens or produce an impossible state. Four possible types include:

   Indirect proof

   Multiple choice - small space search

   Classificatory contradiction - large space search

   Classificatory contradiction - infinite space search

7) Working backwards - Start with the goal and try to guess a preceding statement or statements that would imply the goal statement.

8) View relations between problems - Two problems can relate to each other in one of five ways:

   a) Unrelated

   b) Equivalent

   c) Similar

   d) Special case

   e) Generalization.

9) Use of mathematical representation - Two possible forms are symbolic or diagrammatic.

Often creativity and problem solving have been linked together. Davis (1973) in his book, Psychology of Problem Solving, reviews some strategies for problem solving or for fostering creativity that have been given by different psychologists. As with the problem-solving methods given by Wickelgren, these strategies in their present form may not be appropriate for elementary children. However, some form of the strategy may be useful in either group or individual problem solving for the elementary age group.

Brainstorming (Osborn, 1963) - Generally used with a group, individuals are asked to throw out ideas as they come into their heads.

Attribute listing (Crawford, 1954, p.96) - Each step is taken by changing an attribute or a quality of something, or by applying that same attribute or quality to some other thing. An example of attributes for physical objects would be size, shape, color, etc. Some area problems can be solved by changing the shape.

Morphological synthesis - This consists of two steps:

1) One identifies two or more attributes or dimensions.
2) One lists ideas for each of these dimensions.

Idea checklists - Examine some kind of list that could suggest solutions to the given problem.

Metaphorical thinking - Two different types:

1) Synectics - apparently unrelated elements are joined together
2) Bionics - biological prototypes are used to suggest the designs of man-made systems. The structure of a bee hive may suggest an appropriate structure for a building.

Davis lists some of the existing projects or programs that are designed to develop general creativity and problem-solving attitudes and techniques. The Wisconsin Project, which Davis has worked on, is intended for use with upper elementary and lower junior high school grades. The students are given material in a cartoon-type format designed to encourage creative and problem-solving behaviors: (1) by instructions and illustrations dealing with creative attitudes and techniques; (2) by student exercises that allow him to find new ideas while practicing the strategies; (3) by example, in that the content of the material was intended to demonstrate flexibility and imagination. Only a small pilot study was reported for this project. Students were reported as having more ideas on three divergent thinking tasks after using the material.

Other projects include the Productive-Thinking Program, Crutchfield and Covington, which has already been discussed; Purdue Creativity Program, Feldhusen, composed of twenty-eight audio tapes accompanied by printed exercises stressing originality, flexibility, fluency, and elaboration; and Ideas Books, Myers and Torrance, six books designed to strengthen thinking abilities and to foster attitudes conducive to imagination.

Problem Types and Tasks

This section is devoted to identifying how people classify problems into types. In addition, a few tasks which have been used in pr ' 'em-solving research will be giver

Polya (1957) dichotomizes mathematical problems into problems t find and problems to prove. The aim of a "problem to find" is to find a certain object, namely the unknown of the problem. The aim of a "problem to prove" is to show by some logical means a stated assertion is true, or else to show it is false.

Davis offers a taxonomy, in the form of questions, of types of problems:

1. Is the problem really a problem?

2. Does the task elicit observable trial-and-error behavior or implicit problem solving and thinking?

3. Does the task require one "correct" solution or many original ones?

4. Is the problem (and its solution) a fairly well-defined, one-shot affair, or is it a creative contribution of substantial magnitude, requiring the creative solving of multiple subproblems? (Davis, 1973, p.21)

Another example of classifying problems is given by Getzels (1964). He listed eight types of problems. In the type 1 problem, the given is known and there is a stardard meth .f solving it known to the problem solver and to others and guaranteeing a solution in a finite number of steps. Getzels referred to this type of problem as a "pseudo-problem" which requires a minimum of innovation and creativeness. The remaining types of problems varied as to whether the problem is known or is to be discovered by the problem solver and if there exists a standard method for solving the problem. For example, in another type of problem, the problem is known to the problem solver, but the method is unknown to him but is known to other people. After discussing some of the prevailing learning theories in regards to problem solving, Getzels concluded, "It is evident that no single set of principles of instruction for creative thinking and problem solving can be drawn from the present theory and research. The theory is too diverse, the research too scant." (Getzels, 1964, p.265)

Problems can be classified in different ways, all of which are beneficial to the study of problem solving. Polya classifies problems according to the types of solutions required. Davis focused on the task or processes demanded by the problem. Getzels considered the conditions under which the problem is stated and the level of the method used to solve it. In selecting problems for problem-solving research, each method of classification should be considered in terms of the population or individuals being used.

On page 4, some of the criteria used to measure problem-solving ability have already been listed. Considering the definition of a problem given at the beginning of this report, many of these criteria would not be measures of problem-solving ability but of some other form of learning or mental abilities. In the remainder of this section, a few tasks used in the problem-solving research will be presented that could be considered as problems and appropriate for elementary age students.

Lester (1972) had students at different ages construct proofs in a simple mathematical system. These would be problems to prove under Polya's dichotomous classification. An integral part of this study was that computers were used to present tasks and to record vital data such as responses, response times and errors, and number of trials. Younger age students, grades 1 to 6, were found to be able to construct some proofs within the mathematical systems used.

Rimoldi, Aghi, and Burger (1968) studied the effects of logical structure, language, and age in problem solving in children of ages 7, 9, 11, and 13. 120 students were individually administered tasks for which the students were instructed to ask as many questions as they wanted, but that they would do better if they did not ask all the questions. The sequence of questions asked, called a tactic, was analyzed by (a) the number of questions asked, (b) the schema pulling out method, and (c) analysis of right and wrong answers. An example of a concrete task used included a square divided into four quadrants. The left half of the square was blue and the right half was red. The two top quadrants had circles and the two bottom quadrants had squares (figure 1).

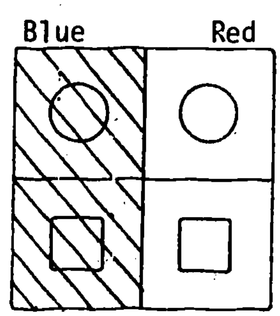


Figure 1

Concrete task used by
Rimoldi, Aghi, and Burger (1968)

Each student was asked to find out how many objects were inside the red-circle portion of the figure without asking that question directly. A minimum sequence of questions would be: How many circles are there? How many blue circles are there?

A verbal problem used, presented to the students on a 3x5 card, asked "We are at the race track. There are two types of cars in the race: station wagons and convertibles. Some of the station wagons are big and the rest of the station wagons are small. Also, some of the convertibles are big and the rest of the convertibles are small. You have to figure out how many of the cars in the race are big station wagons." The students were given eight separate cards on which questions the students could ask were written, and on the reverse side, the corresponding answers were provided.

The problems used by Rimoldi, Aghi, and Burger each could have their logical structure depicted by tree diagrams. In this way, the "thinking processes" of the students could be evaluated. What is lost by such a structure is free thinking and implementation of approaches to the problems which cannot be depicted just as a series of questions.

Houtz (1973) looked at four forms of problem-solving tests in comparing advantaged and disadvantaged second and fourth graders. The four forms varied from abstract to three more and more concrete forms. The abstract form contained written statements of the problems. The picture book form had drawings representing the problem placed in the test booklets above the response alternatives. One form had slides made of the drawings and used these in depicting the problem. The model form was made up of threedimensional full-color models of the drawings. The three concrete forms resulted in a higher performance than the abstract form for both the advantaged and disadvantaged children. The models did not result in the highest performance and appeared to result in a decrease in the performance of non-white children.

Another type of task associated with some problem-solving research is illustrated by work by Stern (1967) and Dienes and Jeeves (1965). Both of these have already been reported. Stern tested the use of two levels of a strategy using the task of matching an exemplar to a model and trying to find the rule. Dienes and Jeeves had their subjects try to find a rule of a game based on a group structure such as a Klein group or a cyclic group. These tasks are similar to ones used by Bruner, Goodnow, and Austin (1956) in their work on concept learning.

Problems used by psychologists in the study of problem solving include such tasks as anagrams, match stick problems, light switch problems, and the Tower of Hanoi problem. Luchins (1942) used water ja problems in studying fixation and rigidity. Maier (1970) reports on studies using the hatrack problem and the string problem. Duncker (1945) u the X-ray problem, among other problems, in his research on problem solving.

The tasks used in problem-solving research can vary greatly. What is important is that problems or tasks are really problems for the participants used in the study. The problem solvers must exert some thought in solving the problems beyond a simple application of an algorithm or series of algorithms. In designing studies on the teaching or development of a general problem-solving ability, several different types of tasks or problems should be used as measures of problem-solving ability. This would help check the possibility that one is not just teaching how to solve a particular type of problem.

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