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ABSTRACT

Several highly motivating lessons particularly pertinent to the study of mathematics in high schools and the first years of college are presented. These lessons fall into four categories: (1) problems that present a challenge and which are interesting for their own sake; (2) problems based on real situations that can be understood by the student; (3) puzzles and mathematical games; and (4) project-oriented mathematics. Included are nctions from graph theory, geometry, number theory, logic, probability, statistics, consumer mathematics, and set theory. (MP)

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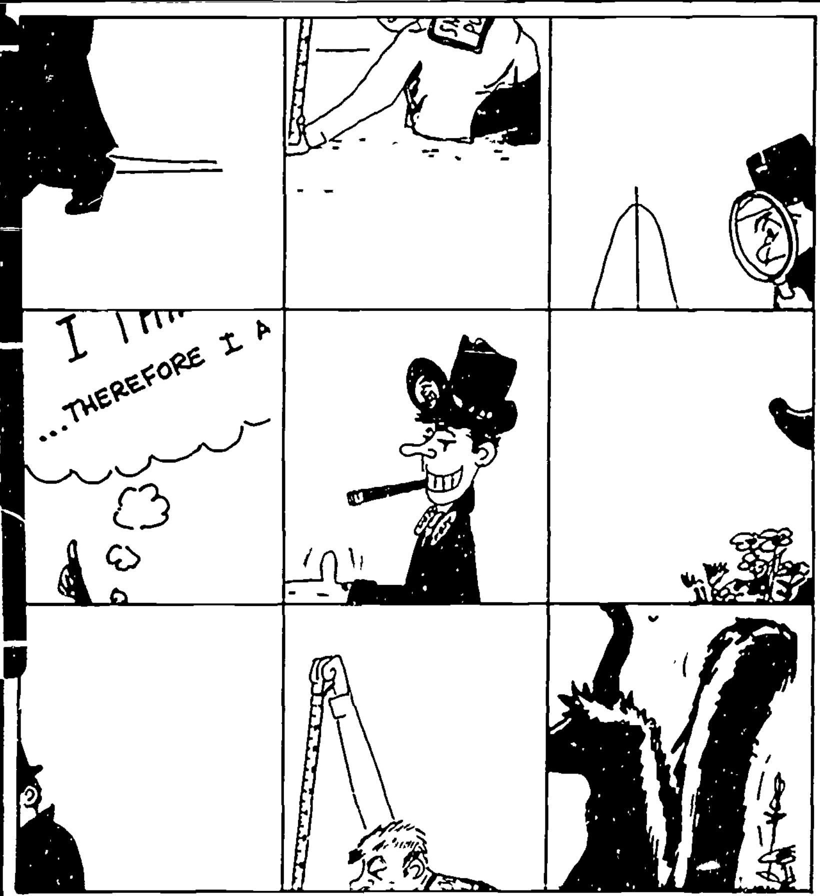
MATHEMATICS LESSONS THAT LIVE



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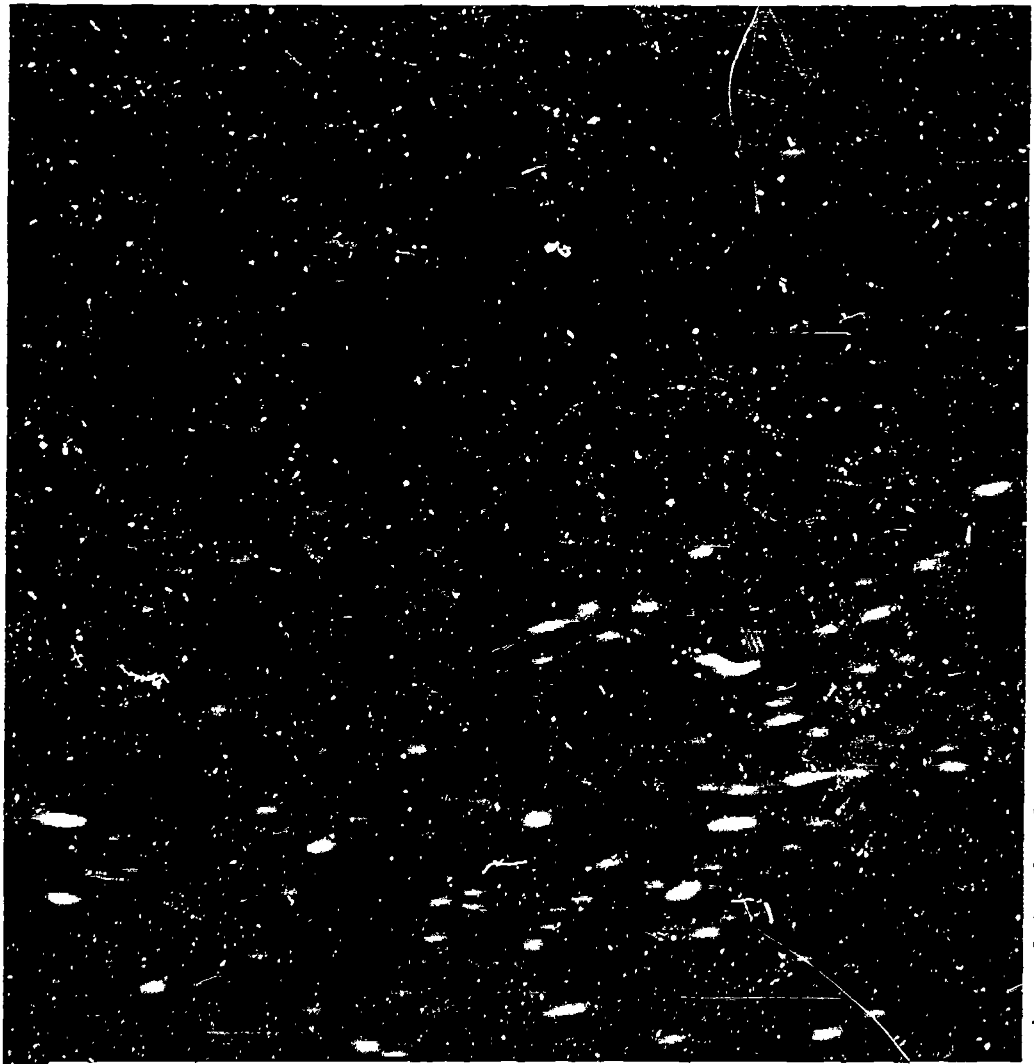

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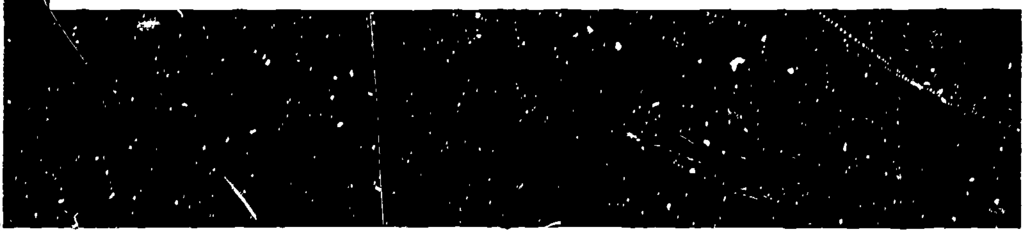

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INTRODUCTION

Ask most mathematics educators to indicate what ingredients they feel are necessary to present stimulating lessons in mathematics and you will hear such things as clear aims, statement of objectives, and motivational devices It has been our experience that of the three characteristics mentioned above, the one that is truly indispensable is motivational devices.

What then is motivation? For our purposes we will simply define it as a way of arousing a student's curiosity. In this presentation, we will attempt to provide a few experiences that have provided motivation in mathematics classes. Generally these experiences fall into four categories:

a) problems that present a challenge and which are interesting for their own sake.
b) problems based on real situations that can be *understood by the student.*
c) puzzles and mathematical games.
d) "project oriented" mathematics.

In the following pages we have attempted to catalogue several highly motivating lessons particularly pertinent to the study of mathematics in high schools and the first years of college. This brief collection of motivational lessons covers a broad range of topics.

Included are notions from Graph Theory, Geometry, Number Theory, Logic, Probability, Statistics, Consumer Mathematics and Set Theory.

It is our hope that the instructor will begin to think about new and exciting ways of presenting mathematics to the not already "turned on" student.

This monograph is only a beginning. Hopefully, it will encourage others to share the ideas that have inspired their students.

Frank J. Avenoso
George M. Miller
Editors-in-chief
The MATYC Journal

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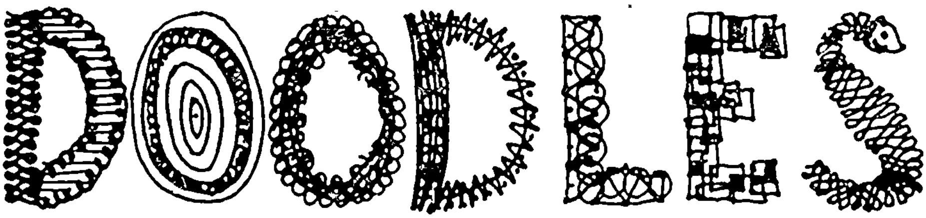
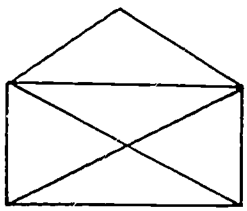


"DOODLES I HAVE DRAWN"
AN INTRODUCTION TO GRAPH THEORY

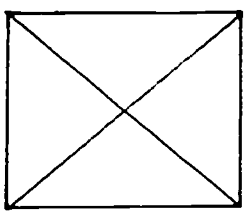
By Frank J. Avenoso

The day will surely arise when one of your students, who is not particularly interested in the lesson at hand, is found drawing a doodle such as.

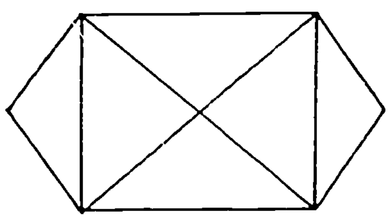


The objective of this "doodle" is to trace out the figure shown without lifting one's pen from the paper and without retracing any line. All doodles must have lines ending at a single point. This doodle, drawn by the student can serve as the basis for a very intriguing and entertaining lesson (based on Graph Theory). The first step in this lesson, appropriate for inclusion in a Survey course, is to show the student that certain doodles can be drawn without lifting the pencil from the paper, while others cannot. Initially, we will present the students with a number of doodles, some of which can be traced, others which can not be traced. Below, are doodles with an indication of whether or not they are traceable ones.

Not Traceable



Traceable



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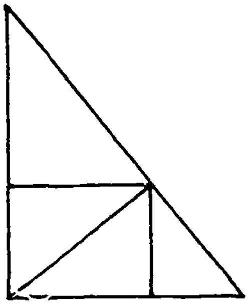
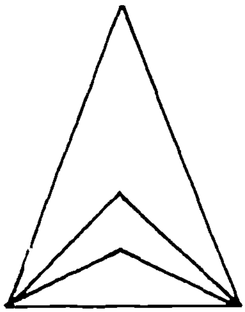
Not Traceable



Traceable



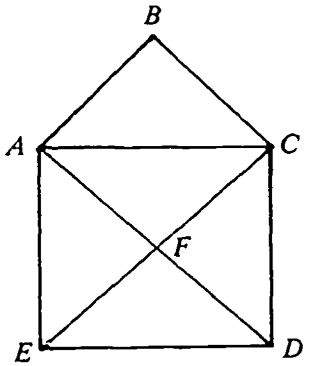
After students have had sufficient time (preferably at home) to attempt to do these doodles, two questions will usually be forthcoming from the class:

(a) Under what conditions can doodles be traced without lifting a pencil from the paper (traceability property)?

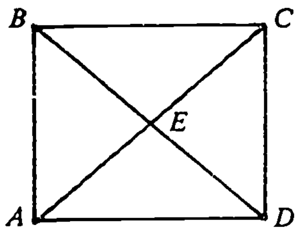
(b) When a doodle has been established as having the traceability property, at what location does one begin to trace it successfully?

This latter consideration is important.

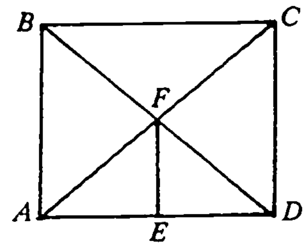
Consider



Beginning from locations *A*, *B*, *C* and *F* the doodle cannot be traced while attempts from *D* or *E* will prove successful. The lessons that follow as a result of these two questions would be best presented only after inquisitive students have been given ample opportunity to pursue the answers to these questions on their own At some future date we would hope that students realize the principle behind the traceability property is based on the "evenness" or "oddness" of vertices. An odd vertex is a vertex where there are an odd number of lines coming together at the vertex. An even number of lines coming together at a vertex will determine an even vertex. Below are several examples of odd and even vertices:



Odd vertices are A, B, C, D
Even vertex at E



All vertices are odd

The observations about traceable doodles we will be seeking are:

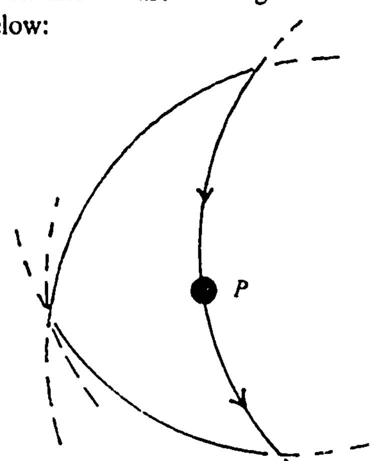
1) Traceable doodles can only have either no odd vertices, or exactly two odd vertices.
2) Traceable doodles can be traced by observing:
   a) if the doodle has no odd vertices then the doodle is traceable beginning at any point on the doodle and proceeding along any line until all lines have been traced.
   b) if the doodle has two odd vertices then the only way the doodle is traceable is by beginning at one of the odd vertices and then along any line until all lines have been traced.

These properties of doodles can be illustrated for several different examples and with proper planning a proof of why these theorems hold can be developed. The following represents an argument for the theorem. A doodle (called a graph) is traceable if all vertices are even.

The argument which we will present is somewhat wordy but if presented slowly, it is reasonably easy to follow. Its validity is based on an inductive approach.

ARGUMENT

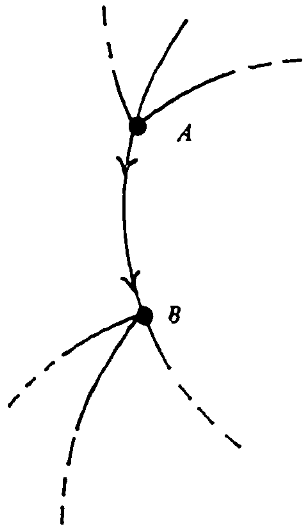
Assume we have a graph consisting of all even vertices. We will trace the graph by beginning at any vertex (notice if we start at any intermediary point along a given line we are treating this intermediary point as "an even vertex") as shown below:



Starting at an intermediary point *P*, treat *P* as an even vertex i.e. one path into *P*, one path out from *P*

13

We will begin our "journey" through the graph below at vertex A. Since A is an even vertex, when we leave A in the direction of another vertex, say B, A becomes an odd vertex: Future "returns" to A will change A to an even vertex.



departure from A to B change vertex A to "odd"—returns to A will change vertex A to "even"

entry to B from A leaves vertex B "odd"—departure from B to any other vertex leaves vertex B "even"

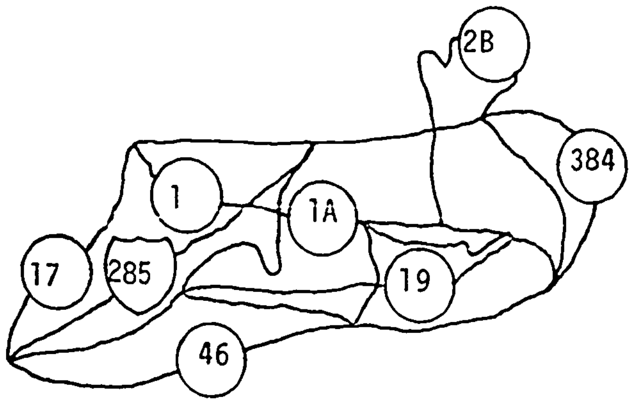
As we enter vertex B, it changes from an even to an odd vertex, then when we leave B along some untraced path we change B to an even vertex. Ultimately, upon returning to B through the last of two untraced paths, the number of paths out of B will be reduced to one and upon leaving B no untraced paths will be left.

Similarly this will happen to all vertices (other than A) whereby we leave the last vertex through the last untraced path and return to vertex A for the last time. Remember, any "returns" to A result in it becoming an even vertex (eventually zero). As a result, there are no untraced paths from all vertices.

The above argument could serve as a model to prove that the only other graph which is traceable is one which contains two odd vertices.

A final statement with regard to the practical application of "doodling" is now appropriate. The use of doodling has implications in the field of transportation. Consider the following problem:

A salesman has the following highway route to cover during the course of a one month period.



Along 'hese various highways shown in the map are several stops that the salesman must make in order to show his products. Realizing that he wishes to cover this route as efficiently as possible, that is, without having to retrace any of his steps, the salesman is essentially faced with a problem of "doodle" traceability. Notice that the map presented to the salesman is a traceable doodle. In fact he can make all his stops without having to repeat going over any of the roads.

Where should he begin?

ALL YOU NEED TO KNOW IS HOW TO TELL TIME

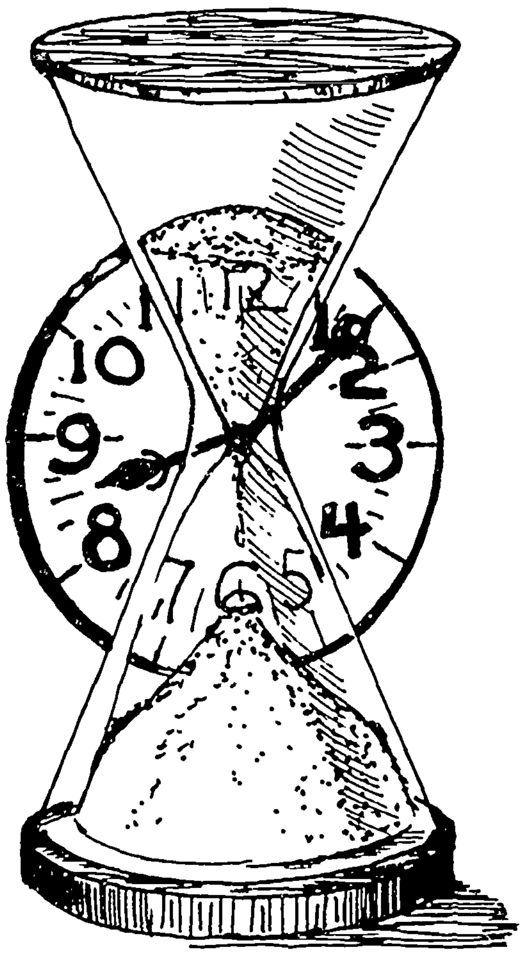
by Frank J. Avenoso

AN INTRODUCTION TO DIOPHANTINE EQUATIONS

One of the considerations of a beginning course in Algebra, that is rarely pursued, is the notion of finding integral solutions to linear equations. From a mathematical standpoint linear equations whose solution set involves only integers are referred to as Diophantine equations (in honor of the Greek Mathematician Diophantus, who first attempted to solve such equations). Euclid was one of the first to examine the concept of how to solve Diophantine equations and several techniques have been developed throughout the history of mathematics. One method that employs a relatively simple approach is based on Modulo Arithmetic.

The following lesson would be appropriate for inclusion at several different levels, e.g., an introductory survey course or an elementary algebra course. The lesson has many variations but one that has been found to work fairly well begins with:

The teacher produces a bag filled with coins and stamps. The coins are all five cent pieces and the stamps are all worth seven cents each. The teacher proclaims that there is an unknown number of coins and stamps contained therein but the total value of the contents is equal to 41 cents.



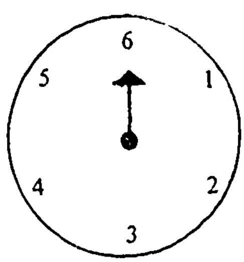
He then challenges the class to tell him exactly how many five cent pieces and seven cent stamps are present. The answer to the question will be forthcoming in not too long a period of time. The solution is three seven cent stamps and four five cent pieces (and is unique). Obviously what the teacher wishes is to have the class gain some insight as to how the solution to this problem was arrived at. The choice of a very easy example to start with, is so that the class as a whole may be allowed to experiment with various combinations. All students will have their own ways of trying to make the combinations come out to the exact amount. The second stage in the lesson would be to give a somewhat more difficult problem.

> As a graduation present Tom received $200 from various relatives and friends. He decided to invest this $200 in two stocks, one which sold at $12 per share and the other at $13 per share. How many shares of each type of stock should he buy in order to exactly spend his $200?

Here again, it is advisable to allow the students the opportunity to find the answer to this problem by trial and error or by whatever means they choose. In this case the answer is not apparent and will take the vast majority of students quite a long time to come up with the correct answer (8 shares of each type of stock). At some point in this development some student should ask "How does one go about a more systematic approach to solving problems such as this?"

If in fact we really look at the problems presented, we see that they amount to finding integral solutions to linear equations. At this point we would begin the introduction to Modulo Arithmetic as a way to solve such equations. The best approach for introducing a modulo system is by "clock arithmetic".

We begin each day at 12 a.m., after 9 hours it would be 9 o'clock, after 11 hours it would have been 11 o'clock, but after 13 hours it would be 1 o'clock; after 26 hours—2 o'clock. After each 12 hours we begin "counting again". The number 12 is critical to the clock. The number 12 in such a case will be referred to as the modulus and the resulting time of day (after a number of hours has elapsed) is called the *residue* for the particular modulus. Let's assume we had a six hour clock (the modulus is 6 or simply "mod 6").



After three hours it would be 3 o'clock, while after 7 hours it would be 1 o'clock. The residues are 3 and 1 respectively Mod. 6. Conventional notation for the several examples presented above would be:

$$9 \bmod 12 = 9$$

$$11 \bmod 12 = 11$$

$$13 \bmod 12 = 1$$

$$26 \bmod 12 = 2$$

$$3 \bmod 6 = 3$$

$$7 \bmod 6 = 1$$

We can see then $A \bmod B = C$ if and only if there is an integer I such that $BI + C = A$. Thus, residues are not unique. $9 \bmod 4 = 1$ or $9 \bmod 4 = 5$ or $9 \bmod 4 = -3$ since $4(2) + 1 = 9$ and $4(1) + 5 = 9$ and $4(3) + (-3) = 9$. To obtain additional residues we can simply add multiples of a Modulus to any one residue obtained.

$$11 \bmod 5 = 1 = 1 + 1(5) = 6 = 1 + 2(5) = 11 = 1 + (-3)(5) = -14.$$

The elementary properties of a modulo system are:

Prop. 1: $(a + b) \bmod c = a \bmod c + b \bmod c$

Prop. 2: $(a \cdot b) \bmod c = (a \bmod c)(b \bmod c)$

Prop. 3: if $x = y$ then $x \bmod c = y \bmod c$.

Now let's begin using Modulo systems to solve Diophantine Equations. Find integral solutions to $2x + 3y = 7$.

We begin by applying a modulus to both sides of this equation (property 3). We will use a modulus of 2 (the choice of 2 will shortly become apparent). Thus we write

$$(2x + 3y) \bmod 2 = 7 \bmod 2.$$

Since the "mod" of a sum is the sum of the "mods" (Property 1) we have:

$$(2x) \bmod 2 + (3y) \bmod 2 = 7 \bmod 2.$$

Then the "mod" of a product is the product of the "mods" (Property 2).
Thus:

$$(2 \bmod 2)(x \bmod 2) + (3 \bmod 2)(y \bmod 2) = 7 \bmod 2$$

or

$$(0)(x \bmod 2) + (1)(y \bmod 2) = 1$$

or

$$y \bmod 2 = 1.$$

This simply means that integral values for y in the equation given, will leave a remainder of one when divided by 2, or y must be of the form:

$$y = 2I + 1 \text{ where } I \text{ is any integer .}$$

Let's go back to the original equation and find out about the value for x.

$$2x + 3(2I + 1) = 7$$

$$2x = 4 - 6I$$

$$x = 2 - 3I.$$

We can now find integral solutions by simply substituting integers for I in both of the expressions for x and y.

	I	0	−1	−3	1	2	4
$x = 2 - 3I$	x	2	5	11	−1	−4	−10
$y = 2I + 1$	y	1	−1	−5	3	5	9

Let's apply our mod technique to the problem involving the stocks, \$200 to be spent on two stocks costing \$12 and \$13 per share. How many of each stock should be bought? We have

$$12x + 13y = 200.$$

It should now be pointed out that a useful modulus will be the smaller of the two coefficients of x and y.

Thus: $12x + 13y = 200$ will be looked at mod 12

Therefore $(12x + 13y) \bmod 12 = 200 \bmod 12$

or $(12x) \bmod 12 + (13y) \bmod 12 = 8$

or $y \bmod 12 = 8$

which means $y = 12I + 8$ where I is an integer.

Returning to the original equation and simplifying, we have:

$$12x + 13(12I + 8) = 200$$

$$12x + 13(12I) = 96$$

$$x + 13I = 8$$

$$x = 8 - 13I$$

The only *positive* integers for both x and y will arise when $I = 0$ Thus for $I = 0$ $x = 8 - 13(0) = 8$ and $y = 12(0) + 8 = 8$ At this point it should be clear that this method produces results with relatively little effort. Students generally have little trouble with the method once the initial considerations of what a Modulo System is and what the basic properties entail are made clear.

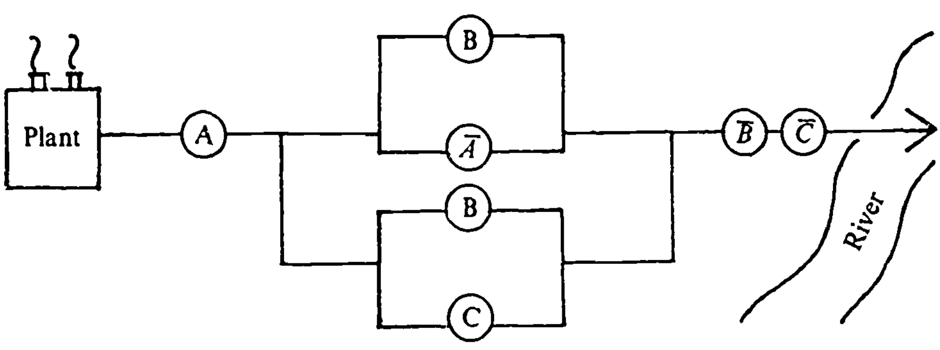
LOGICALLY ECOLOGICAL

by Frank J. Avenoso

This lesson appropriately presented after an introduction to Logic shows an application of "Truth Tables". The motivating problem centers around notions from the field of Environmental Science.

PROBLEM:

Engineers employed by a large industrial plant were asked to design a system of waste treatment facilities. Taking into account geographic considerations, daily waste disposal rates, etc., they drew up a series of lock stations (open and closed viaducts) which would apply various treatments to the waste before it was to be deposited in a "pure" state into a nearby river. The nature of the chemical treatments employed necessitated that each time the waste was to be treated "opposition stations", (the three pairs referred to will be A and $\overline{A}$, B and $\overline{B}$, C and $\overline{C}$) could not both accept the waste. In order to accomplish this, opposition stations were designed so that whenever one was open the other was closed and vice versa. The diagram of the treatment scheme looked as follows:



On the trial run of the facilities it was found that instead of the "treated waste" entering the river, the plant received a backup of the waste. Can you explain why?

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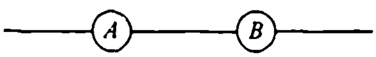
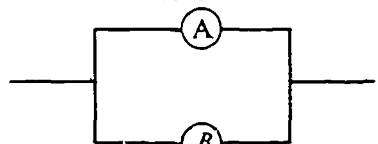
We begin by observing that when locks are in a line we call such an arrangement a series arrangement. In order for waste to proceed through a series arrangement both locks must be open. In the case where either lock is closed waste will not flow from one end to the other. ($\cap$ to denote that the two locks A and B are in series) The next observation is when two locks lie along separate routes we call the scheme a parallel arrangement. We note that waste can flow from one initial point (before the two locks) to a terminal destination (after the two locks) if either or both locks are open. The waste will not flow only in the case where both locks are closed. We will use the following symbol $\cup$ to represent a parallel arrangement.
We can symbolize these conclusions as:

$A \cap B$



$A \cup B$



A	B	$A \cap B$
F	F	F
F	D	D
D	F	D
D	D	D

Flows (F) Does not flow (D)

A	B	$A \cup B$
F	F	F
F	D	F
D	F	F
D	D	D

Students will quickly realize the similarity between the notions developed in symbolic logic i.e., True and False compared to "flow" and "doesn't flow", "and" and "series", "or" and "parallel."

Symbolizing the scheme developed by the engineers' would result in the following:

$$A \cap [(B \cup \bar{A}) \cup (B \cup C)] \cap (\bar{B} \cap \bar{C})$$

Now in order to test whether or not waste would flow through this scheme we could set up a "flow table" similar to previously discussed truth tables. The flow table will be developed in stages as follows:

A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	1 $B \cup \bar{A}$	2 $B \cup C$	3 $\bar{B} \cap \bar{C}$	$1 \cup 2$	4 $A \cap (1 \cup 2)$	$4 \cap 3$
F	F	F	D	D	D	F	F	D	F	F	D
F	F	D	D	D	F	F	F	D	F	F	D
F	D	F	D	F	D	D	F	D	F	F	D
F	D	D	D	F	F	D	D	F	D	D	D
D	F	F	F	D	D	F	F	D	F	D	D
D	F	D	F	D	F	F	F	D	F	D	D
D	D	F	F	F	D	F	F	D	F	D	D
D	D	D	F	F	F	F	D	F	F	D	D

Now the complete scheme (last column) never shows waste passing through it under any state of the locks. This means that there is no possible path that will permit waste to leave the plant, be treated and enter the river.

SEVENTEEN PROBLEMS TO "TURN-ON" STUDENTS

by Philip Cheifetz

Very often, when teaching liberal arts courses to the mathematically reluctant, educators try to help students develop an appreciation for mathematics through its applications. One approach is to cover topics such as statistics, the real number system, algebra and trigonometry and logic and set theory. While this is one way to try to help students appreciate mathematics, some studies have shown that students soon forget even the most basic concepts learned in these courses.

Perhaps we could be more successful if we taught our students the art of mathematical thinking as a prelude to mathematical manipulation. The following thought questions have been culled from many different sources and can be introduced in courses as diverse as logic, statistics, algebra, trigonometry, elementary functions and even differential calculus. While many of the questions can be solved by mathematical means, it should be stressed that all can be solved with a minimum of computation.

Students should be encouraged to think about the solution before attempting to write anything on paper. Hopefully, lively discussions will occur during the course of solving the problems, and the instructor may even discover that he is being drawn into areas which previously were seldom discussed in the course.

Here is a good problem when discussing "common sense" in logic.

1. The brick. A brick weighs six pounds and half a brick. How much does the brick weigh?

This is a natural when doing motion problems in algebra.

2. The track. A racing car driver must circle a track which is 1 mile in circumference twice. His average speed must be 60 m.p.h. On his first lap he averaged 30 m.p.h. How fast must he travel on his second lap in order to qualify?

This problem is best introduced when covering fractions.

3. The horses. A man has 17 horses and wishes to divide them among his three children so that the eldest receives 1/2 of them, the middle child receives 1/3 of them and the youngest receives 1/9 of them. How is it accomplished without killing any horses?

Although a difficult problem, the following can be used when talking about using "extreme" conditions to help solve problems.

4. The beal ·s. Beaker A has a quantity of water and Beaker B has an equal quantity of wine. A teaspoon of A is placed in B and B is mixed thoroughly. Then a teaspoon of the mixture in B is placed in A. Does A have a higher percentage of B in it then B has of A, or vice versa?

Logic is called for to solve this problem.

5. The hats. Persons A, B, & C are placed in a room, so that A can see B & C, B can see C, and C sees no one. They can, however, all hear each other. They are shown six hats, four of which are black and two of which are white. They are then blindfolded and three hats are removed from the room. The remaining hats are placed so that each person has one hat on his head. The blindfolds are removed and the people are asked to determine the color of the hat they have on without looking. A speaks and says, "I see the colors of the hats on B and C, but I don't know my color." B says, "I see C's color, but I don't know my color." C says "I know my color." What color is C wearing and why?

Some background in logic would be helpful for this problem.

6. The marbles. Three cans contain two marbles each. The cans are labeled "Black-Black", "White-White", and "Black-White" to denote their contents, but each can has been mislabeled. You may choose any can, pick one marble from it, and then replace the marble. You may now repeat this process with the other cans, or the same can. How many picks are necessary in order to be sure of relabeling the cans correctly?

While many will try to do the next problem using algebra, it can be done most easily by using arithmetic only.

7. The children. A & B meet on the street and have the following conversation.

A: How old are your three children?

B: The product of their ages is 36.

A: That's not enough information for me to know their ages.

B. The sum of their ages is the number of the house across the street (and then points to the house number).

A: Hum, I almost have it, Tell me more.

B: The oldest child just got a haircut.

A: Now I know the ages.

Assuming the ages to be whole numbers, how old are the children?

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While you can use infinite series to do number 8, logical thinking makes it quite easy.

8. The bird: Trains A and B are 400 miles apart on the same track. A is traveling at 64 m.p.h. towards B and B is traveling at 36 m.p.h. towards A. A bird located on train A began flying towards B when the trains were 200 miles apart and was flying at 120 m.p.h. When it reached B it instantaniously turned and flew back towards A at the same speed. Upon reaching A it turned and headed for B at the same speed, etc. When the trains meet how many miles will the bird have flown?

I like this one in any course that uses an axiomatic approach.

9. The Goobles. If (1) Any two goobles have exactly one bab in common.
(2) Every bab belongs to exactly two distinct goobles.
(3) There are exactly four goobles.
Then (1) How many babs are there?
(2) How many babs does each gooble have?

This problem has no real pigeon hole, but see if you can do it anyway.

10. The coins. You have ten stacks of coins, each consisting of ten half-dollars. One entire stack is counterfeit, but you do not know which stack it is. You do know the weight of a genuine half-dollar, and you are also told that each counterfeit coin weighs one gram more than the genuine mode. You may weigh the coins on a scale. What is the smallest number of weighings necessary to determine which stack is counterfeit?

The next three problems have the same flavor and can be used in logic, algebra or a functions course.

11. The butts. Four cigarette butts can be made into one cigarette. If I have 16 cigarettes, how many smokes can I have?

12. The chickens. A chicken and a half lays an egg and a half in a day and a half. How many eggs do 12 chickens lay in 12 days?

13. The eggs. Mary leaves with a basket of hard boiled eggs to sell. At her first stop she sold half her eggs plus half an egg. At her second stop she sold half her eggs and half an egg. The same thing occurs at her 3rd, 4th and 5th stop. When she finishes, she has no eggs in her basket. How many did she start with?

This is a tough logic problem.

14. The fork. A logician vacationing in the South Seas finds himself on an island inhabited by two proverbial tribes of liars and truthtellers.

Members of one tribe always tell the truth, members of the other always lie. He comes to a fork in the road and has to ask a native bystander which branch he should take to reach a village. He has no way of telling whether the native is a truth-teller or a liar. The logician thinks a moment, then asks one question only. From the reply he knows which road to take. What question does he ask?

This complex logic problem is a classic. You can also use it in a "problem solving" course.

15. The zebra. The facts essential to solving the problem-which can indeed be solved by combining deduction, analysis and sheer persistence—are as follows:

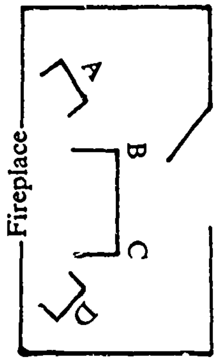
1. There are five houses, each of a different color and inhabited by men of different nationalities, with different pets, drinks and cigarettes.
2. The Englishman lives in the red house.
3. The Spaniard owns the dog.
4. Coffee is drunk in the green house.
5. The Russian drinks tea.
6. The green house is immediately to the right (your right) of the ivory house.
7. The Old Gold smoker owns snails.
8. Kools are smoked in the yellow house.
9. Milk is drunk in the middle house.
10. The Israeli lives in the first house on the left.
11. The man who smokes Chesterfields lives in the house next to the man with the fox.
12. Kools are smoked in the house next to the house where the horse is kept.
13. The Lucky Strike smoker drinks orange juice.
14. The Frenchman smokes Parliaments.
15. The Israeli lives next to the blue house.

Now, who drinks water? And who owns the zebra?

This is also best used in logic but can be used in operations research.

16. The murder. Bilks the bookmaker has just been found dead in the dining room of the club. Poison in his wine. Four men seated as shown on a sofa and two armchairs round the fireplace in the lounge discussing the murder. Their names are Smith, Brown, Jones and Robinson. They are, not necessarily respectively, a General, a Schoolmaster, an Admiral and a Doctor.

1. The waiter has just poured out a glass of whiskey for Jones and of beer for Brown.



2. The General looks up and in the mirror over the fireplace sees the door close behind the waiter. He then turns to Robinson, who is next to him, and starts talking.
3. Neither Smith nor Brown have got any sisters.
4. The Schoolmaster is a teetotaller.
5. Smith, who is sitting in one of the armchairs is the Admiral's brother-in-law. The Schoolmaster is next to him on his left.
6. Suddenly a hand is seen putting something in Jones' whiskey. It is the murderer again. No one has left his seat: no one else is in the room.

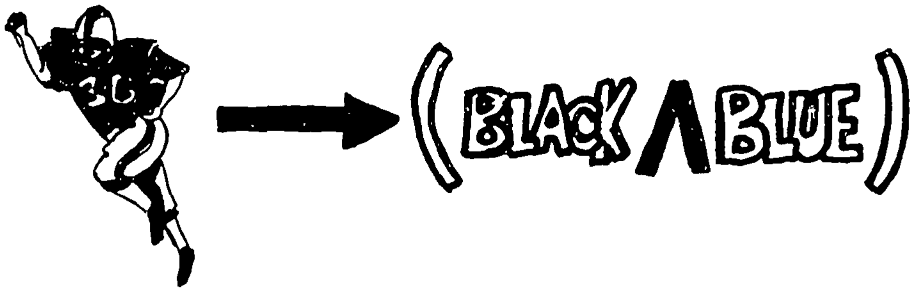
Who is the murderer? What is the profession of each man, and where is he sitting?

This next one is good in a geometry class.

17. The block: A cube consists of a block of white wood, only the outside has been covered with a coat of black paint.
Now imagine that you divided each edge into three equal sections, and cut the block up so that you had 27 small cubes.
How many small cubes would have three painted faces? How many would have two black sides . . . and how many one?
And, finally, how many all white cubes would there be?

ANSWERS:

1. 12
2. Impossible
3. Borrow a horse to make 18.
4. They're equal
5. Black
6. One, pick from the BW container
7. 2, 2, 9
8. 240 miles
9. 6, 3
10. One weighing
11. 21
12. 96
13. 31
14. "If I were to ask you if this road leads to the village, would you say, "yes"?"
15. Israeli, Frenchman
16. Chair A: Dr. Smith
    Chair B: Professor Robinson
    Chair C: General Brown, the murderer
    Chair D: Admiral Jones
17. 8, 12, 6, 1



COLOR PATTERNS AND SYMBOLIC LOGIC

by John Earnest

A. GOAL

To help the student develop the ability to conceptualize and remember eight valid argument forms by describing first the valid argument forms in terms of color patterns, and then to gradually substitute logical connectives and statements for the colors.

B. MOTIVATION

This lesson is presented as a game where the player (student) must be able to identify an incomplete pattern from memory.

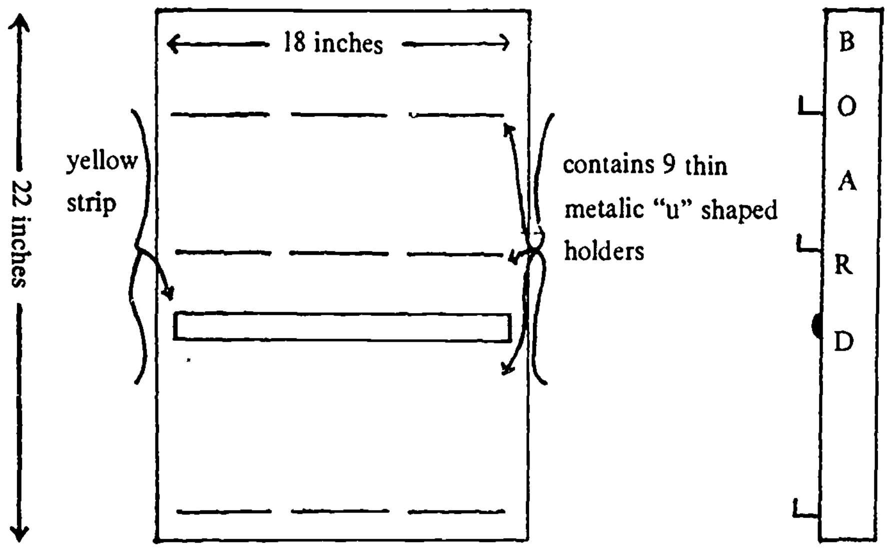
C. GAME OBJECTIVE AND MATERIALS NEEDED

A pattern is shown to be composed of a number of different parts located in positions relative to each other.

1. The number of parts needed to construct a pattern varies from four to nine.
2. The types of parts used consist of logical connectives, parts which are the same (sameness), and parts which are opposites (negations).
3. The parts are located on a 3 by 3 matrix where the last row of the matrix is the conclusion and the above one or two rows are the premises.

This lesson has been presented by using a hanging wooden board designed to hold replaceable cardboard parts.

I. Wooden argument demonstration board:



II. Cardboard parts used:

a. Connectives: dimensions—3 1/2 " × 4 1/2 " by 1/8 " 3 implications, and 3 disjunctions (which are also used for conjunctions).

→	V

b. Colored parts; dimensions same as connectives
6 red cards and 6 negation red cards,

RED	~ RED

3 green cards and 3 negation green cards,

GREEN	~ GREEN

3 yellow cards and 3 negation yellow cards.

YELLOW	~YELLOW

c. Simple logical statements. dimensions same as connectives, color—white, 3 letter "P", 3 Letter "O". 3 Letter "R"

P	Q	R

d. Negation symbols and dimensions—

$$1'' \text{ by } 4\frac{1}{2}'' \text{ by } 1/8''$$

6 symbols are needed and are used with the simple logical statements.

~

e. Compound Logical statements. dimensions—4 1/2 " by 4 1/2 " by 1/8"
2 sets of 3 compound statements, and their negations.

$(P \wedge Q)$	$(P \vee R)$	$(P \rightarrow Q)$

D. SCOPE

This method has been used to demonstrate the following valid argument forms.

1. Modus Ponens
2. Modus Tollens
3. Hypothetical Syllogism
4. Disjunctive Syllogism
5. Disjunctive Simplification
6. Disjunctive Addition
7. Conjunctive Addition
8. Conjunctive Simplication

E. SAMPLE LESSON

The goal of this sample lesson is to give an example of the green diagonal pattern, and how it is used to introduce the patterns of Modus Ponens and Modus Tollens.

1. The green diagonal pattern. In this beginning section the student is introduced to the basic concept of pattern and to the game playing procedure. This initial pattern is not directly related to any valid argument form.
   I. Pattern demonstration. The pattern is displayed on the demonstration board using 6 red cards and 3 green cards.

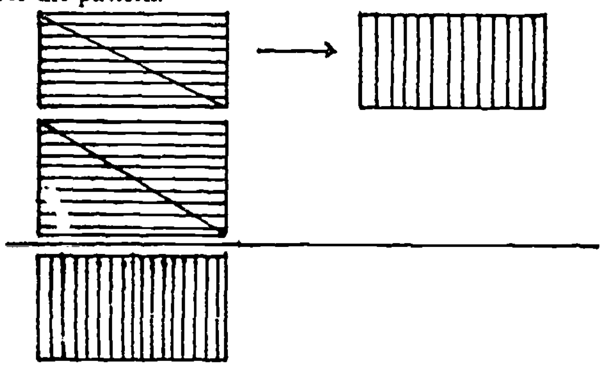
RED	RED	GREEN
RED	GREEN	RE D
GREEN	RED	RED

The concept of pattern is now developed and explained to the class. The students are told to remember the pattern, and then the missing parts game is played.

II. The missing parts game procedure. one or two parts are removed from the demonstration board. The class is asked what parts (color and location) are needed to reconstruct the pattern. When the correct pattern is reconstructed, the students are questioned on how they know the pattern is correct. The students are helped to discover the importance of memorization. This first game should be repeated and other patterns used until the concept and the game procedure are clearly understood. Several patterns using blank spaces should also be practiced.

2. The Modus Ponens pattern:

I. The basic pattern is displayed and discussed. Special attention is given to what parts are the same (sameness), what are the location of the parts, and what type of connective is used. The students are then told to remember the pattern.

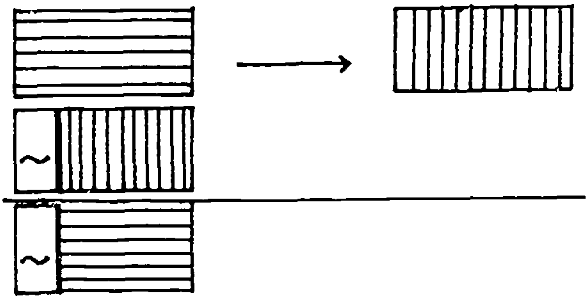


II. The missing parts game. The game is first played with colors until the class demonstrates a proficiency in reconstructing the pattern, each time the game is played different parts of the pattern are removed. Cards with single letters (simple logical statements) are substituted for the colored cards and the game is played until the students again demonstrate pattern recognition proficiency. Cards with compound logical statements are now substituted and the game is played until the students have reconstructed the pattern. The pattern can now be described with proper terminology in the following manner:

antecedent implies consequent	Premise 1
antecedent	Premise 2
Consequent	Conclusion

Naming the pattern: at this time the pattern name of Modus Ponens is given and the class can move onto the next pattern.

3. The Modus Tollens pattern. The basic pattern is displayed and discussed. Special attention is given to what parts are opposites (negations), what their location is, and what type of connective is used.



Variations in the basic pattern are pointed out by showing that the location of the negation may vary, but the basic pattern is still maintained. The missing parts game is played in the same manner as with the Modus Ponens pattern. At this point the pattern can be described with proper terminology and the pattern given the name Modus Tollens.

F. COMPLETE LESSON

The other six valid argument forms can be demonstrated in the same manner.

G. SUMMARY

This method of presentation has been very successful because of its unconventional approach by the use of colors and the concept of pattern to demonstrate the valid argument forms. The concepts first are presented without the use of mathematical terms and this seems to help a large number of students. Because of the use of the demonstration board and the colored cardboard parts, the basic pattern can be quickly demonstrated and then a very easy and natural transition is made to the use of parts that are composed of simple or compound logical statements.

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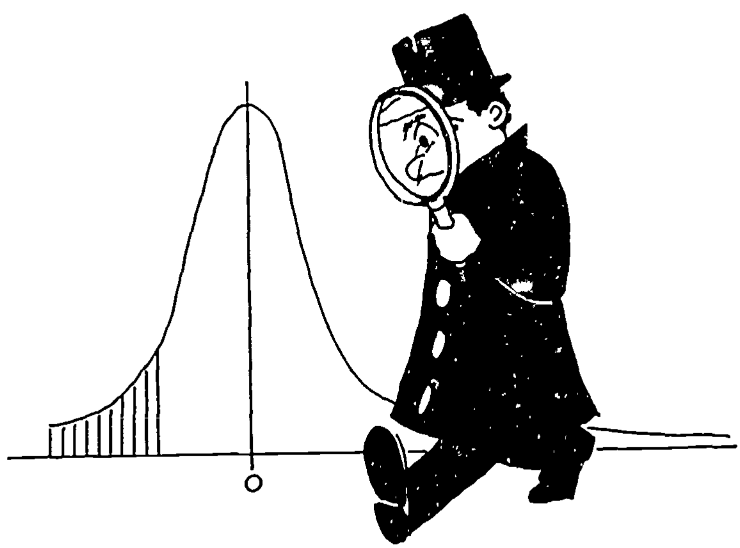
TRACKING DOWN STATISTICS: THE DISCOVERY METHOD

By George M. Miller

Concepts of statistics are generally taught through the use of textbook problems. While an effective tool, this approach often fails to enable students to realize that statistics can have very natural applications which they themselves can put to use. By conducting surveys which the students develop themselves and then relate to statistics taught them in the classroom, students can be made aware of the realities and the dynamics of statistics, and also how statistics can be applied to everyday life.

GENERAL PROCEDURE

Each student will develop his/her own survey in order to determine the attitudes of a particular group concerning a topic which the student has chosen. Both the topic of the opinionnaire as well as the questions developed, although under the guidance of the teacher, are the student's own choices, and so, interest on the part of the student is generated. At the same time, statistical topics are introduced in the classroom, and are subsequently applied by the student to his/her survey data. As the course progresses, the results of the applications of these statistics form a report to be completed by the end of the term.



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SPECIFIC PROCEDURE

A. Topics of interest to the student as well as the population he/she wishes to investigate must be determined. Topics might include Capital Punishment, Premarital Sex, the Cost of Living, Drug Use by High School Students, etc. Examples of the population which the student chooses to investigate may include the residents of a local community, the students at a local high school, the nurses at a nearby hospital, etc.

B. The individual survey form must be developed. Several lectures should be devoted to discussion of criteria for questions to be asked on the survey, which is to be developed in preliminary form and submitted to the teacher for review. The student is responsible for typing the final version on a spirit master for production of 90-100 forms, which are to be handed out to a sample of the population chosen and then collected for analysis of the data obtained.

C. Specific criteria for developing the survey should be followed.
   1. Background of the respondents should be determined. This may include age, sex, income, marital status, education, and so forth.
   2. Questions asked to determine respondents' opinions should be relevant to the objectives of the survey.
   3. Questions should be closed (i.e., respondent may check the appropriate answer) rather than open (i.e., respondent is required to fill in an answer), and should be clearly understood.
   4. Specific information which the respondent may not have, should not be requested.
   5. Questions relating to delicate or personal information should be attempted later in the interview after a rapport has been built up.
   6. Questions should be worded objectively so that respondents will answer how they *honestly* feel, rather than how they think they *should* feel.

D. As the student is developing his/her survey and investigating the sample, the teacher is presenting the various aspects of statistics. The meaning of mean, median, mode, range, standard deviation, etc. are developed to the extent that the student can apply these concepts to the data on his/her survey.

E. The survey is completed when the student has summarized his/her conclusions drawn from analysis of the data which has been obtained. It is also appropriate at this time for the student to criticize his/her survey questions to determine if a more effective approach might have been used Limitations of the survey and of the statistical process should also be considered.

CONCLUSION

Presenting Statistics in this manner has many positive results. The teacher is able to interact with the student on a more personal level than is possible utilizing the standard lecture method. Student interest is generated because the students are able

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to participate throughout the development of the course. Presented in this way, the variety of possible topics and samples to be investigated makes Statistics current and ever-changing. But perhaps most important of all, the students' attitude is generally modified to be able to see mathematics as less abstract, and they can take pride in their new ability to confront raw data without confusion and apply statistical concepts in terms which they can understand.

Student projects can often be useful beyond teaching basic statistics. I briefly site two examples of completed student projects:

Her chi-square analysis showed a strong relationship between the location of the inmate's cell block and attendance at weekly Alcoholics Anonymous meetings.

A student employed at the County Correctional Center (jail) as an alcoholism counselor chose as her student project to study the alcoholism program. She asked inmates the number of drinks they averaged per day (before incarceration), and made population interval estimates. She also included various standard background questions which led to percentages, graphs, modes, etc.

Her chi-square analysis showed a strong relationship between the location of the inmate's cell block and attendance at weekly Alcoholics Anonymous meetings. Further investigation revealed that the complex and annoying security procedures necessary to move a prisoner from some distant locations caused the guards to be less than enthusiastic about a man's attendance. The solution, move all alcoholic prisoners to one area close to where the meetings are held.

Using her newly-calculated information about relationships, percentages, hypothesis tests, etc. she spoke before the school board.

Another student chose to survey parents in her school district who had children with learning disabilities. Her own child had a learning disability and she felt that her public school program was not meeting the needs of these special children. She collected various information. What professional diagnosed your child's problem? How often have you had school conferences concerning your child's disability? Is the child in a special school program?

Using her newly-calculated information about relationships, percentages, hypothesis tests, etc. she spoke before the school board. The board, impressed by her objective and complete consideration of the problem and specifically to learn of the mean amount of money spent by these parents on private help for their children, was moved to give serious consideration to her claim.

Although many of my students choose to survey their fellow students at the college, it is generally interesting when the student chooses instead to survey a segment of the community outside the college. Both the student as well as a part of the community come to know each other better, and often valuable change is brought about as a result.

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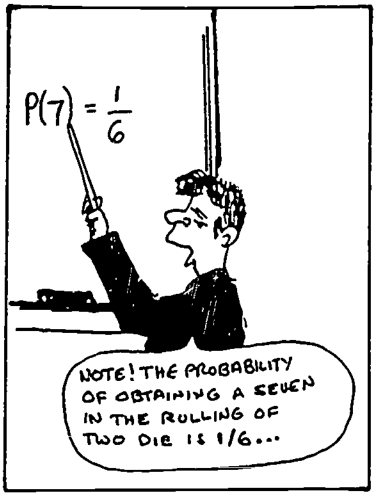

PROBABILITY: THEORY AND PRACTICE

By Robert Rosenfeld

PURPOSE

a. To show how the laws of probability allow us to predict patterns of events but not individual events.
b. To show two interpretations of probability.
c. To introduce the concept of statistical inference.

PREREQUISITES: ARITHMETIC SKILLS.

MATERIALS. Answer sheets prepared for multiple choice tests. A good one would have 20 questions, with 4 responses (a, b, c, d).
STYLE. Informal and Conversational. Highly interactive, demanding class participation.

DEVELOPMENT OF THE LESSON.

1. Present the idea of probability as the ratio of those outcomes favorable to the event to all possible outcomes when all outcomes are equally likely. Present the idea of probability as a long run relative frequency. Show how to go back and forth between these two ideas.

EXAMPLE. The probability that a fair coin comes up heads when tossed is 1/2. This means two things.

1. There are 2 equally likely outcomes, H and T. We are interested in 1 of these (H). Therefore, $P(H)$ = 1 out of 2 or 1/2,

2. In many tosses of a fair coin we expect about 1/2 the results to be heads.
2. Do several examples of this type, perhaps using cards as illustrations, or whatever your favorite examples are. Keep the examples simple and *ask* the class to tell you what the answers are. Do not state the answers first yourself. The idea is to get them used to calling out answers without too much hesitation.
3. Present—*startling counter-example* to underscore the importance of *equally likely* in the definition they have been using. This is a reminder that intuition is sometimes misleading.

EXAMPLE. Social Security numbers are 9 digits. Ask the questions. "What is the probability that the middle digit is odd?" Most will say, $p = 1/2$, which is *wrong*. The fact is that relatively few are odd. It is quite likely that *no one* in the class will have an odd middle digit. This can be dramatically presented by pretending to go along with their guess of 1/2, and then checking it for accuracy by calling on students 1 at a time to state whether their number is even or odd. You might even make a chart on the board. (Figure 1)

Figure 1.

Frequency tally on Social Security Middle Digit

Odd	Even
1	7

Ask the class what went wrong. Why were their guesses in error? Emphasize the basic importance of the phrase "equally likely" in the definition of probability. Just because there are two categories, odd and even, does not mean they are equally likely.

4. If you want to get into the ideas of statistical inference you can point out how the last demonstration can be used to infer that the hypothesis $p = 1/2$ is probably wrong *for the entire U.S. population* even though the evidence is based only on a small sample of citizens. You could then suggest that they test the same hypothesis ($p = 1/2$) on one of the other digits in the social security number. You can discuss how it is easier to use inference to *reject* certain hypothesis than it is to use it to confirm a specific hypothesis.
5. Demonstration of probability at work. This classroom experiment is very successful in confirming the reasonableness of the definitions of probability.
   1. Hand out answer sheets like the ones in (Figure 2)
   2. Explain that you are going to give a test in advanced nuclear physics, so that they can experience the idea of being completely unprepared for a test. Or make up some such story. Tell them not to worry too much since it is multiple choice and they can guess.

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Figure 2

Answer Sheet
Advanced Nuclear Physics

1 a b c d	11 a b c d
2 a b c d	12 a b c d
3	
.	
.	
.	
10 a b c d	20 a b c d

3. When they are ready to begin, fumble around for the questions and fail to find them. Say "Oh, well. It doesn't really matter, since you wouldn't have understood them anyway. Just go ahead and choose one answer for each question." There's a lot of room for joking here, such as "Don't cheat," etc.
   Explain that you *do* have the correct answers, so that we can check the papers. (You should have several sets of "correct" answers because students usually want to repeat the experiment. It's a good idea to use a random number table to get your "correct" answers. For example, you could take: $1 = a, 2 = b, 3 = c, 4 = d$)
4. Analysis of the results.
   a. Ask "On question 1, what is the probability that you got it right?" After ascertaining that the probability is 1/4, ask
   b. "What fraction of the people in the class would you expect got it right?"
   They will say 1/4 or 25%. Read off the correct answer and find out how many got it right. Record it as a raw score, and as a percent. See (Figure 3) for suggested display of results.

Figure 3

NUMBER OF STUDENTS IN CLASS =

QUESTION	NUMBER WHO GOT QUESTION RIGHT	PERCENT OF STUDENTS WHO GOT QUESTION RIGHT
1		
2		
3		
.		
.		
.		
20		

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c. Repeat the above for each of the 20 questions. You should see results very close to theory.

Emphasize that the results for each individual are unpredictable, but that the approximate number of people who get the question right is predictable.

6. Demonstration of the multiplication rule of probability. (Assumes you have talked about this, or are willing to present it quickly.)

   Ask "What is the probability that an individual gets *both* question 1 *and* question 2 right?" The answer will be $1/4 \times 1/4 = 1/16 = .0625$ or about 6%. Again have those who got question 1 right raise their hands, and say "Now all those who got both 1 and 2 right keep your hands up." Most hands will drop. The results should be pretty close to theory. The class usually wants to continue this process until no hands are up. So you can continue by calculating the probability of getting the first 3 questions right, etc. At each stage about 3/4 of the hands drop.

7. Demonstration of the binomial distribution (Assumes you have discussed it.) You can calculate the probability for getting any particular *number* of questions right. Intuition will say that most people will get about 1/4 of the questions right. This would be about 5 questions. The standard deviation is 1.94. So most of the class will get from 3 to 7 right. For a class which is not ready for this level of discussion you can just show them the expected results "according to more advanced theory". It's interesting to list the actual results next to the expected ones. (Figure 4).

Figure 4

NUMBER OF QUESTIONS RIGHT	NUMBER OF STUDENTS WHO GOT THIS MANY RIGHT	PERCENT OF STUDENTS WHO GOT THIS MANY RIGHT	"EXPECTED" PERCENT OF STUDENTS TO GET THIS MANY RIGHT
20			9.09×10^{-11}%
19			5.5×10^{-9}%
18			
.			.
.			.
.			.
0			.32%

SUMMARY AND COMMENTS

You can make this more interesting by wagering on what the results will be before you check any answers. You can say, for example, that 60% is a passing grade. Therefore you need 12 right to pass, 14 for a C, 16 for a B, and 18 for an A. You can announce a prize for various outcomes. You can even announce a prize for "all wrong"—which is very difficult ($.75^{20} = .0032$). You can make an analogy between this experiment and a lottery. If you have to get, say, 7 digits to match the "correct" number, this is like a multiple choice test with 7 questions and 10 choices per question. The class can see how small the chances of winning are. If your class is small, then give each student *several* answer sheets to work on. This increase in sample size will usually make the results closer to theory.

IS THERE LOGIC IN ADVERTISING?

By Aaron Seligman

An introductory lesson about the difficulties we encounter in reasoning from the premises to the conclusion.

An introductory course in logic, or a unit in logic as part of a topics course appropriate for the liberal arts student, is now common place in many mathematics curricula. The first day of class I ask the students to list some of the ways they arrive at conclusions, make decisions and make up their minds about something. Typical responses include observation, experimentation, influence of others (author, elected official, teacher, parent, etc.), prejudice, emotion and past experience. Examples of each are discussed with an emphasis on showing how we sometimes draw the wrong conclusion. I then introduce two particularly important techniques which will be discussed in this course. inductive and deductive reasoning. There is a brief mention that deductive reasoning, unlike any of the other ways we arrive at conclusions, will provide us with a method for being certain whether or not the conclusion follows logically from the information available to us.

After this brief introduction, and before becoming involved in a mathematical approach to logic, I suggest to the students that one area in which each of us is asked each day to draw conclusions is in the field of advertising. Advertising involves the expenditure of billions of dollars annually for the sole purpose of convincing the consumer to draw the conclusion that he or she should purchase the product. The question I pose in class is "Does the decision (conclusion) to purchase the product follow logically from the information (premises) contained in the advertisement?"

One of my favorite examples is a TV commercial from the early 1970's which stated, "Nine out of ten Volvos sold in the U.S. in the past ten years are still on the road." Most of the students are impressed by the claim. They conclude that Volvos last a long time and that they are well built, dependable cars. Most students conclude that, on the basis of the information contained in the advertisement, that the car is worth purchasing. But do the favorable conclusions drawn by many students follow logically from the information contained in the ad? Does the ad state that Volvos last a long time? Is there a 90% chance that the Volvo you purchase will last ten years?

It appears that advertising is so effective because consumers often draw the favorable conclusions advertisers hope they will draw, particularly when the

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company has not made any such claim. The Volvo ad does not claim that Volvos last a long time yet consumers think that is what the ad states. At the time the ad was being used, Volvo had in the past few years experienced tremendous increases in sales. In that ten-year period, Volvo sales had approximately quadrupled so, that, while nine out of ten Volvos sold in the past ten years were still on the road approximately 70% of them had been sold during the past five years. It was not true that 90% of the Volvos sold ten years ago were still on the road, only that 90% of all Volvos sold within the past ten years (including those last year) were still on the road.

Students begin to see that conclusions must be carefully drawn based upon the given information without their reading into the data premises not actually stated. Students also realize that decision making requires all the relevant information and that in advertising data which would be useful in helping consumers make logical decisions is often deliberately withheld. As a result, we are asked to draw conclusions based only on the data the advertiser chooses to supply. Intelligent decision making will require that consumers do some independent research to obtain as much data as possible before drawing conclusions.

The emphasis of the course has now been established we are concerned with how we draw conclusions, that is, if we assume the information we have available to us to be true then must the conclusion we arrive at also be true? Advertising, because it is a concrete, relevant and familiar experience, helps students realize that if they are to draw conclusions in a logical and responsible manner they will have to do some careful thinking.

Although the information contained in the Volvo commercial was entirely truthful, it is my contention that the advertisement was misleading.

DEFINITION. A misleading advertisement is one which makes a claim which is likely to lead the consumer to draw favorable conclusions about the product though there is no evidence in the advertisement to warrant this.

Students quickly learn to be more probing when confronted with advertisements. Each time an advertisement is discussed in class the students list the favorable conclusions consumers might draw and the questions they would like answered before making a decision to purchase the product. (NOTE. as illustrated in the following example the term product includes candidates, causes and charities.)

In the Spring of 1970 a newspaper advertisement appeared in behalf of Paul O'Dwyer, who was opposing Richard Ottinger and Theodore Sorenson, in New York's Democratic Senatorial primary. The ad stated, "Paul O'Dwyer fought in the UN for the establishment of the State of Israel in 1946. Where were the other guys?"

Favorable conclusions that consumers (voters) might draw include. (1) Paul O'Dwyer is very concerned about the State of Israel, (2) Paul O'Dwyer played an important role in helping to establish the State of Israel, and (3) the other candidates did not care enough to fight for the establishment of the State of Israel in 1946 and, perhaps, still do not care enough today.

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Questions which consumers need answered include. (1) How relevant is Paul O'Dwyer's accomplishment in 1946 to an election in 1970? (2) What exactly did he do when he fought for the establishment of the State of Israel? and (3) How old were each of the three candidates in 1946?

Comments. In 1946 Paul O'Dwyer was 39 years old and a member of the American delegation to the UN. In that same year Ottinger and Sorenson were 17 years old. The answer to the extremely clever question "Where were the other guys?" is that they were in high school and certainly were not in a position to fight in the UN for the establishment of the State of Israel.

It really becomes a challenge for the student to select an advertisement which is misleading, to list the unwarranted favorable conclusions which consumers might draw, to list the questions he or she would like answered before deciding whether or not to purchase the product and then to get those questions answered. It is this last matter which provides students the opportunity to do some original research. Students may elect, in lieu of taking the final examination, to research the advertisements which they have selected as possibly being misleading advertisements. On reserve in the college's library is a copy of *The Standard Directory of Advertisers*. This directory contains the names of companies and the advertising agencies for each of the products manufactured by the companies. The student must then call the advertising agency, speak to the account executive and discuss the advertisement with the account executive. Most account executives are pleasant, courteous and willing to speak with the students. Some are more willing than others to respond in detail to the student's questions. Finally the student submits a report based on his conversation with the account executive.

A short list of some of the advertisements students have actually researched is given below followed by some comments taken from the students' reports.

ADVERTISEMENTS

1. "Fleischman's Margarine, selected for use by the U.S. Olympic Team."
2. "How did an American Ford Granada compare in tests of smoothness and quietness of ride with a $20,000 German Mercedes? . . . In three out of four test conditions there were no major differences in smoothness . . . in all tests the Ford Granada consistently rode as quietly as the Mercedes."
3. "Which color TV has the best picture?" People from all over America looked at the six leading big-screen color TVs. They voted Zenith the best picture by more than 2 to 1. "
4. "World wide we've been selling Uniroyal Steel Belted Radial Tires ten years ahead of our American Competitors."
5. "Ivory Snow washes diapers softer as compared to the leading detergent even to the roughest hands."

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COMMENTS

1. The U.S. Olympic Committee must raise millions of dollars to cover the expenses of the American team. The choice of the companies or products entitled to use the phrase "Selected for use . . ." was not related to product performance.

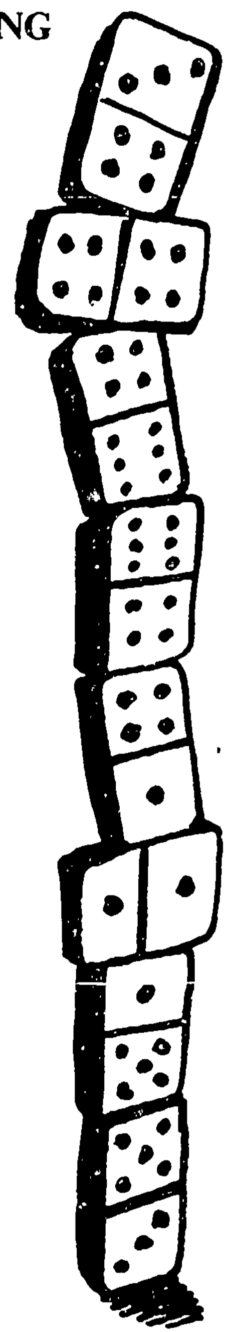
   All companies were solicited in the field of any one product, and the first company to reply and to agree to make the necessary donation was given the use of the phrase.
2. In the smoothness tests there was a major difference in one of the four test conditions, but the account executive would not tell me which one. The cars were both new cars. No tests were done after 10,000 or 20,000 or 50,000 miles—an important consideration. There may be important differences between the 2 cars in engine performance, safety, handling, comfort, etc
3. The phrase " . . . more than 2 to 1" meant that 50.1% of the respondents selected Zenith and 21.1% selected RCA. However, it is also true that 49.9% of the total respondents did not think Zenith had the best picture. By choosing 25" color TV sets for the survey Sony was excluded.
4. The implication here is that since Uniroyal has been making steel belted radial tires longer than its American competitors, consumers should purchase Uniroyal tires. However, if length of time manufacturing the tire is important then, perhaps, consumers should purchase steel belted radial tires made by Michelin, a company which produced them before their competitor, Uniroyal. When I suggested this to the account executive, he said that in their ad they were only talking about their American competitors.
5. I asked if Ivory Snow was ever tested against another soap product rather than a detergent. The account executive told me that this had been done in laboratory tests, but never in a TV commercial. I wonder what the results were. Even if people with rough hands can feel a difference in softness, I wonder if an infant can when the diaper is being worn. Perhaps, more important than how soft the diapers are after being washed is how clean they are after being washed.

   I also called one of the men who appeared in the commercial. He told me that he was not given any special sensitivity to touch tests, but that one day he felt the diapers and then was told to return on another day for the filming of the commercial. It seems that the advertising agency knew before filming the commercial that the individual selected would be able to tell the difference.

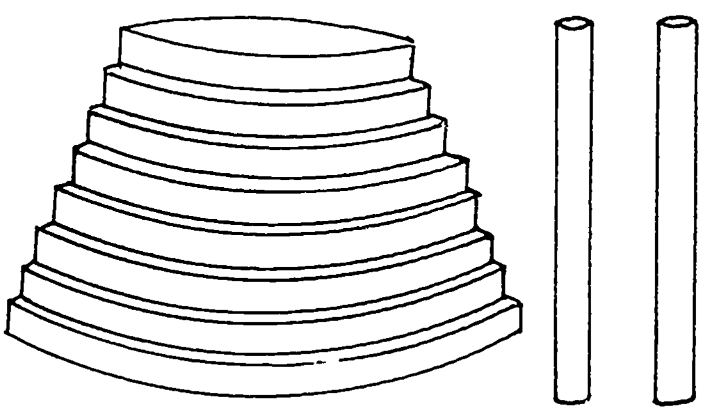
TWO CLASSROOM GAMES: THE TOWER PUZZLE AND POLYOMINOES

A UNIQUE APPROACH TO INTRODUCING THE CONCEPT OF A FUNCTION

by Aaron Seligman



In the 1880's there appeared in toyshops in France a puzzle which had been invented by the French mathematician Edouard Lucas. The problem was to transfer the tower of 8 disks to either of the two vacant pegs in the fewest possible moves, moving one disk at a time and never placing a disk on top of a smaller one.



In class I re-create the game by distributing to each student a set of five paper circles each of a different radius. Circles one, three and five are of one color and circles two and four are of a different color. Students use 3 "points" on their desks to represent the pegs.

The problem is explained, and the students attempt to solve it. The students throughly enjoy the challenge presented by this hands on activity and the change from the typical way in which classes are conducted. Most students struggle with the puzzle, but, after a few minutes, some announce that they have moved the original set of "disks" to one of the empty "pegs". When I ask them to try it one more time students realize that it is not so easy to repeat the solution. The first question the students ask is what is the minimum number of moves in which the puzzle can be solved.

I suggest that in order to answer that question we try to find a way to simplify the problem. Students in turn, suggest that we try the puzzle with fewer circles. This leads us to set up a table as follows:

NUMBER OF CIRCLES	MINIMUM NUMBER OF MOVES
1	1
2	3
3	7
4	15
5	31

The pattern students most often suggest is that the number of moves is twice the previous number of moves plus one.

I pose the problem of trying to find a formula, a rule, so that for any number of circles we can find the minimum number of moves by substitution into the formula.

If we let x represent the number of circles, then the minimum number of moves, $f(x)$, will be $2^x - 1$, i.e., $f(x) = 2^x - 1$.

DEFINITION: A function from set A to set B is a rule which assigns to every element in A exactly one element in B.

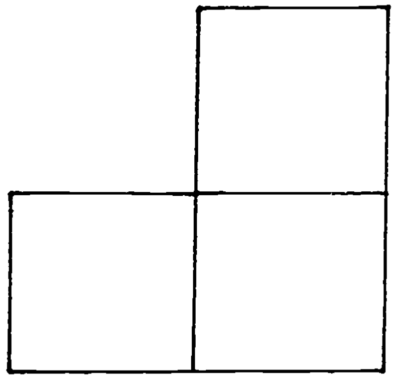
We also introduce and discuss domain, range and functional notation. I make sure the students understand that we have not proved that $f(x) = 2^x - 1$ is the correct solution to the problem, rather, we have drawn the conclusion inductively.

The final discussion concerns whether or not we can find a pattern to the movement of the circles so that we can actually do the puzzle in 31 moves. The two colors used in the set of circles help students discover that the odd number circles always go on top of the even number circles and vice versa. Students also discover that if the 3 "pegs" are positioned as. ABC then the smallest circle moves "clockwise" (or counter-clockwise) on alternate moves.

After spending an entire lesson working with circles I inform the students that their assignment will now involve working with squares.

DEFINITION. A *polyomino* is a set of squares joined along their edges.

For example. a polyomino consisting of three squares, called a tromino is pictured below.



The problem for the students is to find the number of (non-congruent) pentominoes (polyominoes with five squares). I distribute a ditto which has been filled with 1" × 1" squares. The students are to draw their pentominoes on the worksheet, cut them out, make sure no two are congruent and bring the cut-outs to the next class. Our main objective is to find a function to predict the number of pentominoes.

NUMBER OF SQUARES	NUMBER OF POLYOMINOES
1	1
2	1
3	2
4	5
5	12

A function which describes the above table is

$$f(x) = 2^{x-1} - (x - 1).$$

Based on this rule students predict that $f(6) = 27$. Again we have used inductive reasoning to conclude that 27 is the correct answer. Unfortunately, that is not the right answer. This comes as a considerable surprise to the class, but, illustrates very effectively the limitations of inductive reasoning. No function has yet been found which gives the number of polyominoes for n squares.

Additional ideas related to these two puzzles can be obtained from *The Scientific American Book of Mathematical Puzzles & Diversions* by Martin Gardner, 1959.

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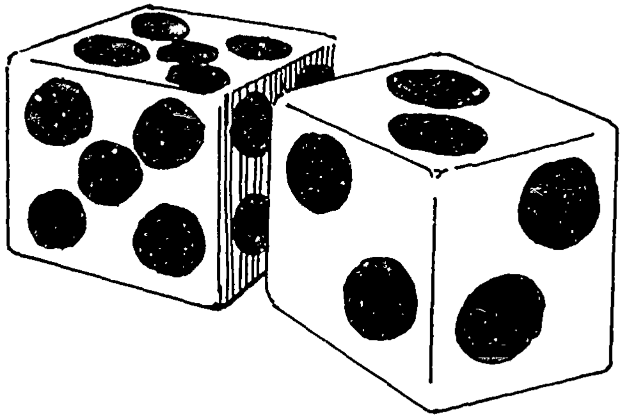

LAS VEGAS ALWAYS WINS

by Thomas Timchek
and
Michael Totoro

It is very likely that students in an Elementary Course in Probability and Statistics have asked themselves the question. "What is my chance of winning a certain contest, game, lottery, etc.? An informal analysis of elementary probability can lead him to the answer. For our purposes we will define probability as a ratio; the number of experimental results that would produce an event, to the total number of results considered possible (provided all events have an equal chance of occurring). By examining some of the games played in an organized gaming establishment it is possible to generate an interest, as well as, an understanding of the mathematics used to produce advantageous conditions for the casino. First let us consider the American Roulette Wheel. On this wheel (see figure 1) there are 38

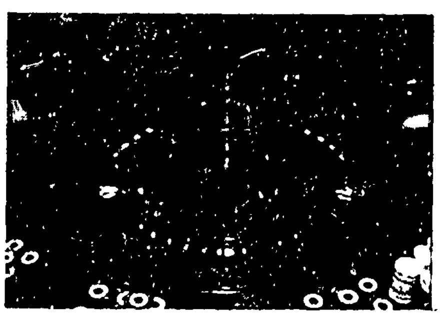

Figure 1. Roulette Wheel

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numbers, the numbers 1 through 36, 0 and 00. Of the 38 numbers 18 are considered even, 18 odd, and 0 and 00 are neither even nor odd. The probability of spinning the roulette wheel and obtaining an even number is given by the ratio 18/38.

While referring to the roulette game, one might ask such questions as. What is the probability of obtaining a result that is a red number?, a black number?, a five?, etc. After having discussed the probability for the various outcomes in this game, a natural follow up is the question. "How does a casino make a profit on this game?" To formalize the understanding of how a casino makes a profit a discussion of odds must follow, emphasizing the differences between the actual betting odds or payoffs given by the casino and the true mathematical odds. In roulette, the payoff on wages placed on "even", "odd", "red," or "black" numbers is even money, 1 to 1. On the other hand, the mathematical odds against obtaining any of these events are 10 to 9 It must be pointed out that the casino makes its money by offering odds (payoffs) that are less than the true mathematical odds. By true mathematical odds, we mean that, in the long run, neither party involved in the wager will win nor lose but will come out even. To determine the casino's rate of return we demonstrate that in the long run (a theoretical infinite number of spins or trials) the player wins 18 out of 38 times and loses 20 out of 38 trials. With a payoff of 1 chip per win and a loss of 1 chip per loss the player wins 18 chips and loses 20 chips for a net loss of 2 chips out of a total of 38 wagered. This rate of loss to the player is therefore defined as 2/38 or .0526. That is to say that the casino expects to profit 5.26% of all the money wagered on these even money payoff wagers. It can be easily shown that the casino's percentage of profit is 5.26% for most of the other wagers in this game. We can now define the Casino or House Percentage as a number which expresses how much the casino expects to win for every dollar that is wagered. To continue with the development of the concepts of probability we ask, "Given that an even number was the result on the first spin, what is the probability of an even number on the second spin?" We now define the concept of an independent event. To motivate its meaning we introduce another device, a pair of dice and the Casino Crap Game.

Figure 2. Crap Table

By independence we mean the occurrence of one event is not influenced by the occurrence of another event. We demonstrate that on successive rolls of the dice each ensuing total is independent of or not influenced by the previous result.

Moreover in using this device to enchance the understanding of the various concepts of probability one must have the class determine the space of all possible outcomes on tossing the pair of dice. Unlike the roulette game, where the space of all possible outcomes are spelled out on the wheel, here we must determine the set of all possibilities by either listing them or introducing a counting technique For a clearer understanding we do both. Table 1 contains the list of all possible ways the dice may fall. The first entry is the face up result of the first die, the second is the face up entry of the second die and the third is the total of the face up entries

Table 1

1, 1 = 2	2, 1 = 3	3, 1 = 4	4, 1 = 5	5, 1 = 6	6, 1 = 7
1, 2 = 3	2, 2 = 4	3, 2 = 5	4, 2 = 6	5, 2 = 7	6, 2 = 8
1, 3 = 4	2, 3 = 5	3, 3 = 6	4, 3 = 7	5, 3 = 8	6, 3 = 9
1, 4 = 5	2, 4 = 6	3, 4 = 7	4, 4 = 8	5, 4 = 9	6, 4 = 10
1, 5 = 6	2, 5 = 7	3, 5 = 8	4, 5 = 9	5, 5 = 10	6, 5 = 11
1, 6 = 7	2, 6 = 8	3, 6 = 9	4, 6 = 10	5, 6 = 11	6, 6 = 12

By counting the events listed it is clear that there are 36 possible events. We then demonstrate how this total is obtained by examining the following general principle:

With M elements in Group 1, $X_1, X_2, \ldots, X_m$ and N elements in Group 2, $Y_1, Y_2, \ldots Y_n$ it is possible to form MN pairs containing one element from each group. We will call this the "MN" rule.

In our example, there are 6 elements (numbers) in group 1 (die 1) and 6 elements (numbers) in group 2 (die 2). Hence, $6 \times 6 = 36$ pairs or possibilities. Putting these ideas together we now ask the question. "Given a total of seven on the first toss, what is the probability of a seven on the second toss?" Noting the independence of each toss, the student realizes that the probability is the same on any single toss. By further examining table 1 we see there are six events out of the possible 36 which have a total of 7.

$$\text{Therefore, } P(\text{total of } 7) = \frac{6}{36} = \frac{1}{6}.$$

Using this game we next explore the concept "mutually exclusive" events, that is, two or more events which cannot occur at the same time. We pose the question, "On a single toss of the dice, what is the probability of producing a value less than three on one die and a total of 11? Since this is impossible the probability is 0. Our next question is, "What is the probability of tossing a pair of dice where the total is not 7?" Allowing the students a few moments to find the solution we see tha the answer is 30/36 or 5/6.

As a result of these questions and solutions, complementary, independent, and mutually exclusive events are explored.

Using the dice or roulette wheel, it is then demonstrated that $\sum_S P(E) = 1$ where S is the space of all simple events in our experiment.

Furthermore, the probability of the complement of A, denoted $P(\overline{A})$, is given by

$$P(\bar{A}) = 1 - P(A) \text{ or } P(A) + P(\bar{A}) = 1$$

Using the crap table layout (see figure 2) the meaning of the field wager is explained. On any single toss of the dice if a total of 2, 3, 4, 9, 10, 11, or 12 occurs the wager on the field wins and loses otherwise. By examining table 1 it can be seen that there are 16 ways of winning this wager and 20 ways of losing. Hence, P (winning on Field) = 16/36. In addition, we examine the casino's profit rate or casino winning percentage by inquiring about the casino payoff for a winning roll. If the total is 3, 4, 9, 10, or 11 the payoff is even money and if it is 2, or 12 the payoff is 2 to 1.

The winning totals 3, 4, 9, 10, and 11 occur 14 out of 36 times with a payoff of 1 chip per win for each chip wagered. In addition, the winning totals of 2 and 12 occur 2 out of 36 times with a payoff of 2 chips per win for each chip wagered. At this point there is a net gain of 18 chips. However, the losing totals of 5, 6, 7, and 8 occur 20 out of 36 times. Altogether the net loss for the 36 outcomes is 2 chips. Therefore, the casino percentage for the field wager is 2/36 or $.05\bar{5}$. Consequently, the casino expects to profit approximately 5.56% of the money wagered on the field.

In order to understand the probability of winning when a player wagers on the pass line as well as many other wagers on the crap table one must discuss conditional probability. If two events are related in such a way that the probability of occurrence of one depends upon whether or not the other has occurred we say the two events are dependent. Let A be the event "observe snow" and B be the event "observe a cloudy sky". The probability of snow is the fraction of the entire population of observations which result in snow. This probability is not the same as the probability of snow given prior information that the day is overcast. If we examine the subpopulation of observations resulting in B, a cloudy day, and the fraction of these which result in A, snow, we expect the chance of snow, given the day is cloudy to be larger. The conditional probability of A, given that B has occurred, is denoted as

$$P(A/B)$$

where the slant is read "given" and the event appearing to the right of the slant is the event that has occurred.

To study conditional probability as well as the probability of the simultaneous occurrence of two events, that is, the Probability of A and B, denoted $P(AB)$, we will use the Venn diagram in figure 3.

From this diagram we can show the following

1. $P(AB) = \dfrac{\text{Area } AB}{\text{Area } C}$
2. Multiplying and dividing the right side by area B (or area A) we can produce

$$P(AB) = \frac{\text{Area } AB}{\text{Area } B} \times \frac{\text{Area } B}{\text{Area } C}$$

or

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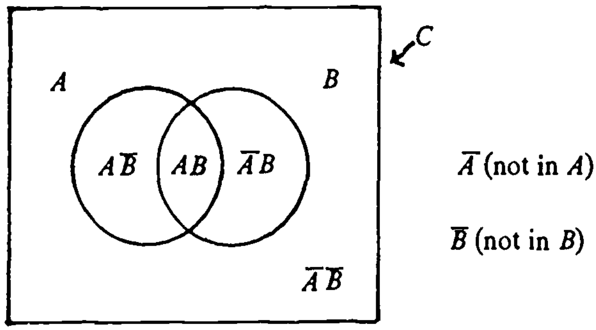


Figure 3

$$P(AB) = \frac{\text{Area } AB}{\text{Area } A} \times \frac{\text{Area } A}{\text{Area } C}$$

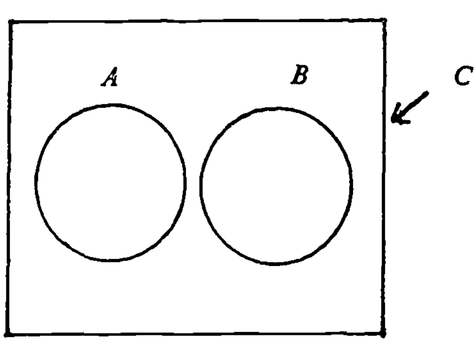
3. The ratio (area AB/area B) is what was described in the illustration where B was cloudy weather and A was snow as the conditional probability of A given B. Similarly, (area AB/area A) is the conditional probability of B, given A.
4. Hence, the results in step 2 become

$$P(AB) = P(A/B) \cdot P(B)$$

or

$$P(AB) = P(B/A) \cdot P(A)$$

5. In addition, if A and B are independent, then $P(A/B)$ is $P(A)$. Similarly, $P(B/A) = P(B)$.



6. Therefore, if A and B are independent

$$P(AB) = P(A) \cdot P(B) = P(B) \cdot P(A)$$

Finally, using Figure 3 the probability of A or B, denoted

$P(A \cup B)$, is given by

$$P(A \cup B) = P(A) + P(B) - P(AB).$$

We are now ready to analyze the probability of winning for a wager on the "Pass Line". A wager on this line wins if on the first toss of the dice a total of 7 or 11 appears. If a total of 2, 3, or 12, commonly called craps, appears the wager is lost. If a total of 4, 5, 6, 8, 9, or 10 appears the wagerer has not won nor lost at this point, but rather this total becomes the POINT of the game. This means that the person throwing the dice must roll this total again before a *seven* appears. If a seven appears before this point the wager is lost. Any other total rolled before the point or seven is irrelevant in deciding the outcome.

The first step in calculating the probability of a win on the pass line is to determine the probability of rolling a seven or eleven in a single toss. By examining table 1 one can see

$$P(7 \text{ or } 11) = \frac{8}{36}$$

Next, analyzing the compound probabilities produce

$$P(4 \text{ and } 4 \text{ before a } 7) = \frac{3}{36} \cdot \frac{3}{9} = \frac{1}{36}$$

$$P(5 \text{ and } 5 \text{ before a } 7) = \frac{4}{36} \cdot \frac{4}{10} = \frac{2}{45}$$

$$P(6 \text{ and } 6 \text{ before a } 7) = \frac{5}{36} \cdot \frac{5}{11} = \frac{25}{396}$$

$$P(8 \text{ and } 8 \text{ before a } 7) = \frac{5}{36} \cdot \frac{5}{11} = \frac{25}{396}$$

$$P(9 \text{ and } 9 \text{ before a } 7) = \frac{4}{36} \cdot \frac{4}{10} = \frac{2}{45}$$

$$P(10 \text{ and } 10 \text{ before a } 7) = \frac{3}{36} \cdot \frac{3}{9} = \frac{1}{36}.$$

Therefore, the probability of a win on the pass line is equal to the sum of the above probabilities.

$$P(\text{win on pass line}) = \frac{976}{1980}$$

In addition,

$$P(\text{loss on pass line}) = 1 - P(\text{win}) = \frac{1004}{1980}$$

Since the pass line is an even money payoff bet, in the long run you win \$976 on every \$1980 wagered and lose \$1004. The net loss in \$28 out of a total of \$1980 wagered.

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The casino percentage is 28/1980 = .01414 or 1.414%
To further illustrate the calculation of conditional probabilities the blackjack game is introduced.

Figure 4. Blackjack Table

After a brief description of the basic rules of blackjack, we ask the question· "If an ace is valued 11, face cards valued 10, and the number cards are valued according to their given numeral, then what is the probability of receiving a total of 21 in two cards?"

The solution is found by determining the probability of an ace on the first card and a 10 on the second card given an ace was selected first or by determining the probability of a 10 on the first card and an ace on the second card given a 10 was selected first. Designating these probabilities as $P(A)$ and $P(B)$ respectively we have,

$$P(A) = P(\text{Ace and } 10) = P(\text{Ace})P(10/\text{Ace}) = \frac{1}{13} \cdot \frac{16}{51} = \frac{16}{663}$$

$$P(B) = P(10 \text{ and Ace}) = P(10)P(\text{Ace}/10) = \frac{4}{13} \cdot \frac{4}{51} = \frac{16}{663}$$

Hence, the probability of a total of 21 is given by $P(A \text{ or } B)$

$$P(21) = P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Since A and B are mutually exclusive, $P(A \text{ and } B) = 0$

$$P(21) = P(A) + P(B) = \frac{16}{663} + \frac{16}{663} - 0 = \frac{32}{663}.$$

MEASURING POLITICAL POWER

by Thomas Timchek

Traditional mathematics courses for non-science students usually include a section on set theory. This lesson deals with a practical application of Set Theory and arithmetic to "measuring political power."

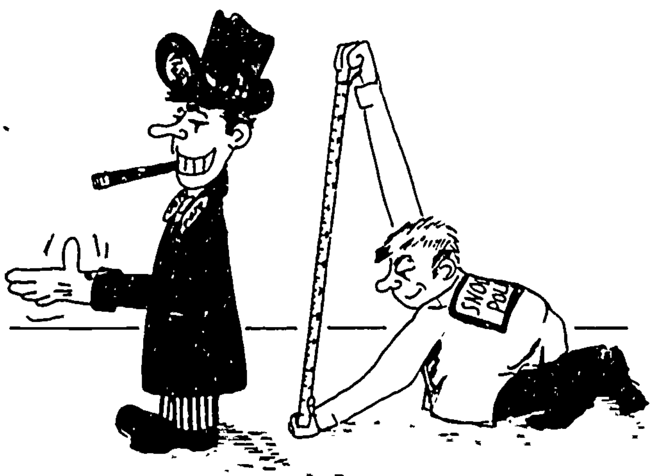
Before introducing the notion of sets I ask a series of questions about the government in our community.

1. What type of legislative body do we have in our county?
2. Who are the members of this legislative body and what communities do they represent?
3. What are the procedures for voting on legislative issues in this county?
4. Who are the most powerful politicians in the community?

After listening to the responses I point out that the answers to these and other similar questions will be researched and presented in a mathematical project as part of the requirements for completing this course.

To begin laying the foundation for the project the basic definitions of sets are discussed and the meaning of the power set of a given set is stressed. The power set of A is the set of all subsets of A. After giving a few examples in determining the power set, I introduce the concept of a weighted voting system. The following example is used to demonstrate the above principles:

Given a set A with 3 players (voters), $A = \{a, b, c\}$, "a" has 20 votes, "b" has 15 votes, and "c" has 5 votes. Any subset of A is called a *coalition*. A subset is a winning coalition if it contains enough votes to pass a law or win, (by simple majority) otherwise it is a losing coalition. For our example, 21 votes are needed for a win or passage. To illustrate, the subset $\{a, b\}$ means "a" and "b" are voting affirmatively and "c" is voting negatively. Hence, the subset $\{a, b\}$ contains 35 votes and is a winning coalition. To determine the *political power* of a member in a weighted voting system we analyze whether or not a voter is *marginal* or *pivotal* in a particular winning coalition. Using the above example in which "a" and "b"



voted yes and "c" voted no if only "a" changed his vote (defected) the result would change from a winning coalition to a losing one i.e. {b} is a losing coalition. Hence, we say "a" is *marginal*. Likewise, if "b" defected and "a" and "c" do not change their votes we would have a change from a winning to a losing coalition i.e {a} is a losing coalition. Whenever a defection causes a winning coalition to become a losing coalition we call it a critical defection. In general, we will determine the [illegible] *power* of a member to be. the number of winning coalitions in which his defection would render it losing divided by the total number of critical defections for all members. John Banzhaf introduced this concept in a series of articles he wrote for the Rutgers Law Review in 1965 and for the Yale Law Journal in 1966. In these articles entitled, "Weighted Voting Doesn't Work. A Mathematical Analysis", Banzhaf calls such winning coalitions, wherein the subtraction of one member would change its status from winning to losing, minimal winning coalitions. Symbolically, the voting power of player i is given by

$$B_i = b_i \Big/ \sum_{j=1}^{n} b_j$$

where b_i is the number of defections by player i which render a minimal winning coalition, a losing coalition.

The chart below computes the frequency for which each player in the given example has a marginal vote. An x under the player indicates that he/she is marginal.

MINIMAL WINNING COALITIONS	PLAYER'S VOTES 20 a	15 b	5 c	MARGINAL PLAYER a	b	c
{a, b, c}	Yes	Yes	Yes	x	–	–
{a, b}	Yes	Yes	No	x	x	–
{a, c}	Yes	No	Yes	x	–	x

Using the above chart we compute the Banzhaf index for each player.

$$B_1 = \frac{3}{5}$$

$$B_2 = B_3 = \frac{1}{5}.$$

We will say that the structure [21, 20, 15, 5] has the power vector (3, 1, 1)/5 (Remember 21 is the number of votes needed to pass). We see an interesting consequence from this example. Although "b" has three times as many votes as "c" they have the same power. However, "a" having only a few more votes than "b" has three times the power.

As citizens of Nassau County, one of the largest counties in New York State, students are given the task of researching the political make up and governmental structure of their community. The information they gather is then evaluated to determine the political power of each member of the Nassau County legislative board, (Board of Supervisors).

ANALYSIS OF THE WEIGHTED VOTING STRUCTURE OF THE BOARD OF SUPERVISORS OF NASSAU COUNTY, NEW YORK

I. *Political Background*

In reviewing the political legislative system in Nassau County, New York, the 1970 weighted voting system for the Board of Supervisors was [63. 31, 31, 28, 21, 2, 2.]

The Board of Supervisors is made up of four Republicans and two Democrats. Due to the Republican dominance a suit to change the legislative system was initiated by the Democrats. As a result, the courts ordered Nassau County to come up with a new system. A referendum was placed on the November, 1974 ballot to retain the present legislative system. This referendum was defeated.

Presiding Supervisor, Frank Purcell, said that a special election on a new plan could be held in June, 1975, with a referendum in November, 1975. In November, 1975 two referendums were placed on the ballot. The first called for a county legislature similar to that of Suffolk County, New York, that is, a system made up of representatives from equal size districts. At the present time the legislative body in Suffolk County is made up of 18 representatives. The second called for the continuation of the existing system. The voters defeated the first referendum and voted for the second. The table below evaluates the power structure of the present weighted voting system of the Board of Supervisors, Nassau County, New York.

Political Power Board Of Supervisors, Nassau County, New York

Municipality	Supervisor	Number of Votes in 1970	Pop. (1976)	Number of Votes 1976
1. Hempstead No. 1	Frank Purcell	31	816,251	35
2. Hempstead No. 2	Alfonse DAmato	31		35
3. Oyster Bay	Joseph Colby	28	343,203	32
4. North Hempstead	Mike Tully	21	238,974	23
5. Long Beach	Hannah Komonoff	2	34,902	3
6. Glen Cove	Vincent Suozzi	2	27,091	2

A majority of 71 votes is needed to carry. Hence, in 1976 we have the structure

$[71; 35, 35, 32, 23, 3, 2]$ as compared to that in 1970 of

$[63; 31, 31, 28, 21, 2, 2]$.

The following represents the analysis of the 1976 voting structure

Banzhaf Index

	PLAYERS						MARGINAL					
	35	35	32	23	3	2	1	2	3	4	5	6
	1	2	3	4	5	6						
1.	Y	Y	N	Y	Y	Y	X	X				
2.	Y	N	Y	Y	Y	Y	X		X			
3.	N	Y	Y	Y	Y	Y		X	X			
4.	Y	Y	Y	N	Y	N	X	X				
5.	Y	Y	N	Y	Y	N	X	X				
6.	Y	N	Y	Y	Y	N	X		X	X		
7.	N	Y	Y	Y	Y	N		X	X	X		
8.	Y	Y	Y	N	N	Y	X	X				
9.	Y	Y	N	Y	N	Y	X	X				
10.	Y	N	Y	Y	N	Y	X		X	X		
11.	N	Y	Y	Y	N	Y		X	X	X		
12.	Y	Y	N	N	Y	Y	X	X				
13.	Y	N	Y	N	Y	Y	X		X		X	X
14.	N	Y	Y	N	Y	Y		X	X		X	X
15.	Y	Y	Y	N	N	N	X	X	X			
16.	Y	Y	N	Y	N	N	X	X		X		
17.	Y	N	Y	Y	N	N	X		X	X		
18.	N	Y	Y	Y	N	N		X	X	X		
19.	Y	Y	N	N	Y	N	X	X			X	
20.	Y	Y	N	N	N	Y	X	X				X

$$B_1 = \frac{15}{54}$$

$$B_2 = \frac{15}{54}$$

$$B_3 = \frac{11}{54}$$

$$B_4 = \frac{7}{54}$$

$$B_5 = \frac{3}{54} = B_6$$

The power vector is:

$$B = (15, 15, 11, 7, 3, 3)/54$$

Interestingly, if one also computed the power vector for the 1970 voting game, it would be the same as the above result. In this instance a change in the number of votes for each player produced no effect in the power structure.

For those who wish to pursue this idea in other situations, one interesting albeit ambitious project would be to calculate the power of the states in the United States Electoral College.

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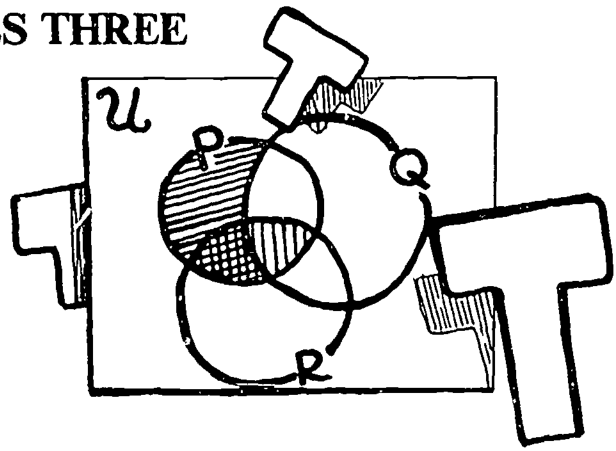
VALIDITY TIMES THREE



by Abraham Weinstein

Many "topics" courses include a section on logic. In this lesson we will show three different ways of dealing with argument forms. Before we begin, let's briefly reiterate the usual truth table ideas.

p	q	$\sim p$	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	T	T
T	F	F	T	F	F	F
F	T	T	T	F	T	F
F	F	T	F	F	T	T

Consider the following three statements that comprise an argument:

Statement 1. He is not truly concerned about the welfare of his country or he is involved in politics.

Statement 2: He is not involved in politics.

Statement 3: Therefore, he is not truly concerned about the welfare of his country.

"If we agree with the first two statements, must we then agree with the third statement or conclusion?" That is, if we consider the first two statements to be true statements, are we then forced to conclude that the third statement or conclusion is also true? If we are, we say the argument is logical or valid.

To answer these questions, we shall develop a method of studying arguments to see if they are "logical" but first, we must define what we mean by an argument

DEFINITION I

Argument: An argument consists of a series of non-contradictory statements called premises, and a final statement called the conclusion. The conclusion is said to follow from the premises.

In our example, the first two statements are the premises and the last statement is the conclusion. In examining this and all following arguments, the term "logical" will not be used. Instead, we shall determine whether the argument is valid or invalid. We use the word "valid" instead of "logical" in order to avoid students' preconceived definitions of the word logical.

DEFINITION II

Valid Argument. An argument is valid if, whenever we assume the premises are true, we can then prove the conclusion is also true.

The word "whenever" is extremely important. This means that each time premises are true, the conclusion must be true in order for the argument to be valid. In some arguments, true premises lead to a conclusion that is sometimes true and sometimes false. In this case, the argument is invalid.

Test of Validity

Since by definition a valid argument is one in which true premises always lead to a true conclusion, we shall test an argument's validity by determining what the truth value of the conclusion would be if we assume that the premises are true.

Test of Validity by Truth Tables

Construct a truth table that contains the truth values of the individual premises and the truth values of the conclusion. Then examine the truth table to determine the truth value of the conclusion when the truth values of the premises are all true.

Symbolizing the original argument:

$$\sim p \vee q$$
$$\sim q$$
$$\therefore \sim p$$

		PREMISES		CONCLUSION
p	q	$\sim p \vee q$	$\sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

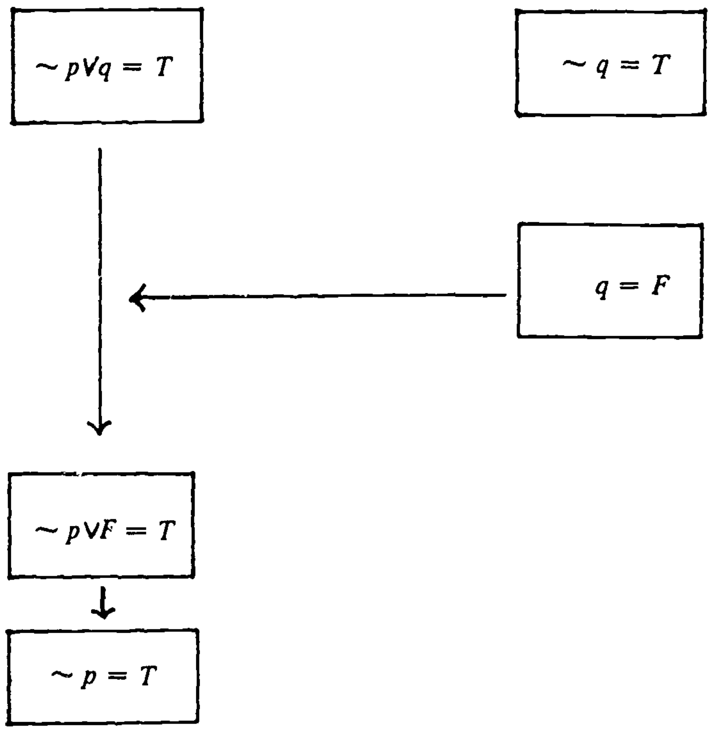
In the fourth row of the truth table, both the premises and the conclusion are true. Therefore, this argument is valid.

Test of Validity by Flow Charting

An alternate method of testing the validity of an argument is flow charting. In this method we directly apply the definition of a valid argument by determining whether or not we obtain a true conclusion when the premises are true.

To use the flow charting method we set the premises of the argument to true and then use this information to find the truth value of the conclusion. If we show the truth value of the conclusion to be true, then the argument is valid. If the conclusion's truth value is false or if it cannot be determined, then the argument is invalid. Consider the illustrative argument:

$$\begin{array}{l} \sim p \vee q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$



1. Set the premises equal to true.
2. Since $\sim q = T$, we have by negation $q = F$.
3. Substituting $q = F$ in the statement $\sim p \vee q = T$, we have $\sim p \vee F = T$
4. By the definition of disjunction $\sim p \vee F = T$ only if $\sim p = T$.

This shows the argument to be valid since true premises led to a true conclusion.

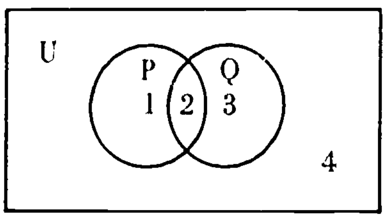
DEFINITION III

Truth Set. The truth set of a compound statement made up of component statements $p, q, r, \ldots$ is a subset of the universal set $U(p, q, r, \ldots)$, all of whose elements make the compound statement true. Elements from any other set of U make the compound statement false.

To illustrate this definition we shall examine the truth sets for negation and the four basic logical connectives. We first construct the truth tables for the connectives and also include the truth sets for the four disjoint subsets.

TABLE 1.

p	q	AREA	TRUTH SETS	$\sim p$	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	1	$P \wedge Q$	F	T	T	T	T
T	F	2	$P \wedge Q'$	F	T	F	F	F
F	T	3	$P' \wedge Q$	T	T	F	T	F
F	F	4	$P' \wedge Q'$	T	F	F	T	T



Note. Since $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$, the simplest form of the truth set for $p \rightarrow q$ is $P' \cup Q$.

Using these relationships we can now write the truth set for any given compound statement by converting negation ($\sim$) to complement ($'$), disjunction ($\vee$) to union ($\cup$), conjunction ($\wedge$) to intersection ($\cap$), and implication ($\rightarrow$) to the union of the complement of the antecedent with the consequent.

Since we can write truth sets for compound statements, it is now possible to use truth sets to test the validity of an argument.

An argument is said to be valid if, whenever the premises are true, the conclusion is also true. To apply truth sets to this definition, we find the truth set whose elements make the premises all true. Once we have found this truth set of the premises, we then determine whether all of its elements also make the conclusion true. If so, the argument is valid. If any of the elements of the truth set of the premises yield a false conclusion, the argument is invalid. In other words, we determine whether or not the truth set for which the premises are all true is a subset of the truth set of the conclusion. When it is, we have true premises always yielding a true conclusion which is a valid argument.

Therefore, we have the following:

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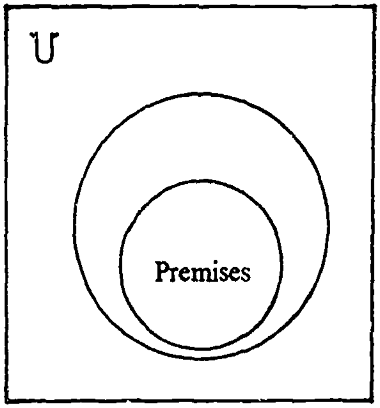
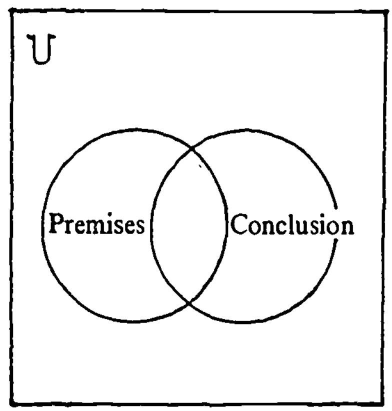


Valid argument: (truth set of the premises) $\subset$ (truth set of the conclusion).



Invalid argument: (truth set of the premises) $\not\subset$ (truth set of the conclusion).

DEFINITION IV

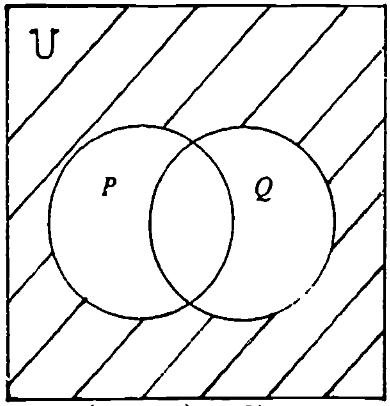
Valid Argument. An argument is valid if the truth set of the conjunction of the premises is a subset of the truth set of the conclusion.

Testing the validity of the illustrative argument:

$$\begin{array}{l} \sim p \lor q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

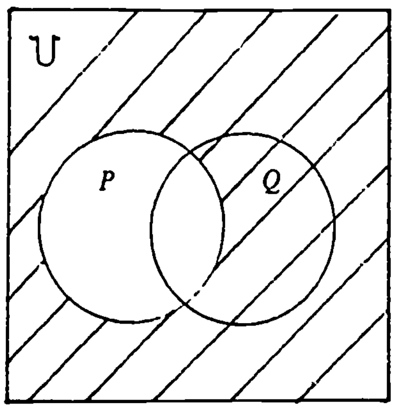
$$\begin{array}{l} P' \cup Q \\ \underline{Q'} \\ \therefore P' \end{array}$$

Premises



$(P' \cup Q) \cap Q'$

CONCLUSION



P'

Since the conjunction of the premises is a subset of the conclusion, the argument is valid.

In conclusion, the student can be shown that a close relationship exists between the rules governing symbolic logic and laws of set theory in order to test the validity of arguments.