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ABSTRACT

The global objectives and content outlines of mathematics courses for grades 7 through 12 are listed and the overall rationale and goal of the courses are discussed. Several suggestions are made for changes in the curriculum. The contents include: A Curriculum of Choices, Grades 7 and 8, General Mathematics, Introductory Algebra, Algebra I, Geometry, Applied Vocational Mathematics, Algebra II, Consumer Mathematics, Advanced Mathematics, Calculus, and Additional Curriculum Considerations. (MP)

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CURRICULUM GUIDE, GRADES 7—12

DIVISION OF MATHEMATICS / NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION

ERIC

Foreword

The mathematics curriculum in the secondary schools cannot be described as a fixed body of knowledge that is never changing. Instead, the curriculum is forever changing, slowly at times and more rapidly on other occasions. These changes in the curriculum are often the result of societal pressures or technological advances. As the result of such pressures and advances during the late 1950's and early 1960's, a major restructuring of the secondary mathematics curriculum was undertaken.

Another way changes occur in the curriculum is as the result of the efforts of teachers in a particular school. These teachers identify a need for a change in the curriculum. They develop alternatives and experiment with them. Then, if their efforts are successful, the courses they develop are soon adopted by other schools.

The developers of this guide have made an effort to analyze objectively the status of the curriculum that exists in many schools across North Carolina. As a result of this analysis, several suggestions are included in this guide for changes in the curriculum. Hopefully, some of the suggestions included in this publication will provide teachers with the impetus to try them in their schools and share their findings with others.

The high school mathematics curriculum should be broad enough to provide a full sequence of elective courses for non-college-bound students as well as college-bound students. This publication is an attempt to provide guidance to those school systems wishing to expand their course offerings in mathematics to the extent that appropriate courses are available for each student at every grade.

The primary purpose of this guide for secondary school mathematics is to provide an aid for local personnel involved in curriculum planning and experimentation. This guide provides a general orientation and framework upon which a more detailed plan which meets the needs of the community can be built.

Two objectives of this guide are:

To propose a pattern of mathematics offerings that is flexible and diverse enough to accommodate many local innovations and interpretations.

To provide descriptions of different mathematics courses that schools can use in building a curriculum that will insure the availability of a substantive mathematics program for their students, who upon graduation, might enter the work force, technical school or college

A. Craig Phillips
State Superintendent of Public Instruction

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Robert R. Jones

Robert R. Jones, Director
Division of Mathematics

Contents

(1976)

Introduction

The major portion of this publication is devoted to lists of global objectives and content outlines of mathematics courses for grades 7 through 12. Each outline is only one of the many possible. It is hoped that individual school units will develop their own outlines, taking into account the specific needs of the pupils they serve.

Mathematics teachers must consider the interests, aspirations, and abilities of individual students as they develop mathematics programs. It is not enough to teach students the mathematics they must know today; they must, if possible, be given the foundation that will serve them in their world of tomorrow. This suggests that attempts must be made to develop within students the ability to continue their studies under their own directions to enable them to learn the mathematics they will need to survive in a changing world. Provisions must be made for adequate pre-professional and vocational backgrounds, simultaneously emphasizing the role of mathematics as a vital part of general education. The role of mathematics as an independent discipline must be shown, at the same time not neglecting its many applications. The teacher should introduce students to current developments in mathematics so as to make students aware of the fact that mathematics is a subject that is alive and ever changing. This enrichment may get students involved in individual and small interest-group research projects beyond the regular course content of the curriculum.

The mathematics program of the elementary school must be considered along with that of the secondary schools since mathematics of the junior and senior high schools cannot be isolated. A sequential and well-coordinated program should be developed. In planning such a program, representatives from all educational levels should be involved.

The mathematics program described in this publication reflects the view that the basic computational skills introduced in the elementary grades require further development. The extension, refinement, and maintenance of arithmetic processes and concepts are important components to be stressed in the secondary school mathematics program.

A Curriculum of Choices

Shown in Figure 1 is a proposed framework for planning secondary school mathematics programs. Beneath the myriad of choices is the basic change of providing a sequence of mathematics courses that are alternatives to but co-equal in mathematical value with the traditional Algebra I, Geometry, Algebra II, and Advanced Mathematics sequence.

Courses such as Algebra I, Algebra II, Geometry and Advanced Mathematics have a reasonably well-defined core of content that is basically the same from class to class, school to school, and state to state. The teacher has to make provisions for adapting the material to the individual student through well planned teaching strategies. On the other hand, Consumer Mathematics, Applied Vocational Mathematics and General Mathematics do not contain a body of fixed subject matter. Instead the subject matter can be drawn from any part of mathematics and its fringes or related fields. The course content cannot be fully prescribed until the students and their inputs are considered.

It should be emphasized that Consumer Mathematics and Applied Vocational Mathematics are different from each other and from other courses. In particular, these are NOT courses in remedial arithmetic. The only computation taught in these courses is for maintenance and extension of existing skills, not compensatory instruction for lack of skills. Any remedial instruction in arithmetic required by students at this level should be provided in a novel non-traditional way. Students who after eight years have not mastered computational skills often have problems so serious they cannot be solved in the traditional manner. They need the specialized intensive care provided by an individualized diagnostic prescriptive approach in a remedial clinic.

Students who complete two or more years of study in "college preparatory" mathematics beyond Geometry should have an understanding of the non-optional topics contained in the syllabus for Algebra II plus one or more of the modules listed under Advanced Mathematics. If the trigonometry topics are included in Algebra II, this course can be followed by Advanced Mathematics (self-contained or in modules). If, however, the trigonometry topics are not included in Algebra II, then this course should be followed by one or more of the modules from Advanced Mathematics, one of which *must* be Trigonometry.

Several course outlines include suggestions for optional topics. It is hoped that the mathematically talented students will be provided the opportunity to study many of these topics.

For advanced students, a third year beyond Geometry could be offered by combining additional modules from Advanced Mathematics or by teaching Calculus. Schools offering Calculus are encouraged to investigate the feasibility of teaching Advanced Placement Mathematics. This program enables students to complete college-level studies while still in high school. Information on the Advanced Placement Program can be obtained by writing: College Entrance Examination Board, Box 2815, Princeton, New Jersey 08540.

The grade level under which a course is listed indicates where it is ordinarily taught. However, this listing should be used only as a suggestion. With careful planning, any course listed might be taught at a different grade level.

The italicized course titles in Figure 1 are discussed in the section of this guide titled "Additional Curriculum Considerations."

Figure 1

The Mathematics Program Sequences, Grades 7-12

Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
REGULAR SEQUENCE					
Mathematics 7	Mathematics 8	General Mathematics	Applied Vocational Mathematics		Consumer Mathematics
			Algebra I	*Business Mathematics	
		Introductory Algebra (Part I)	Introductory Algebra (Part II)	Geometry	Algebra II
		Algebra I	Geometry	Algebra II	**Advanced Mathematics
ACCELERATED SEQUENCE					
Mathematics 7, Mathematics 8, Algebra I		Geometry	Algebra II	**Advanced Mathematics	Calculus
COMPUTER RELATED SEQUENCE					
Mathematics 7	Mathematics 8	Algebra I	Geometry	*Algebra II With Computer Programming*	*Engineering Concepts Curriculum Project*
POSSIBLE SEQUENCE FOR APPLIED MATHEMATICS					
Mathematics 7	Mathematics 8		*Algebra: An Applied Approach*	*Geometry: An Applied Approach*	*Technical Mathematics*

* *Business Mathematics is described in the curriculum guide For Business Education.*

** *Advanced Mathematics can be taught as a survey course or as a series of topical courses which could include Trigonometry, Analytic Geometry, Advanced Algebra, and Probability and Statistics.*

Grades 7 and 8

The topics of major importance at this level are numbers, operations, algebraic concepts, problem solving, probability and statistics, intuitive geometry, measurement, and graphs and scale drawings. The notion of a set should also be studied, not as an isolated topic but as it is used to unify mathematical concepts.

NUMBERS

One of the major goals of the mathematics program at this level is to provide students with an understanding of the rational number system, its structure and the properties that it possesses. Emphasis should be placed on how these properties can often make computation easier. Once students understand these ideas and principles, they can begin to investigate the integers as another number system. A brief introduction to the irrational numbers can also be made as time permits.

The number line could prove to be a useful model for illustrating the numbers in each number system studied as well as the relationships between them. Models from the real world should be used whenever they are appropriate.

Concepts that should be studied in conjunction with number systems are square roots, multiples, prime numbers, composite numbers, scientific notation, and ratio and proportion.

OPERATIONS

The ability to work with numbers is a necessity if individuals are going to have mathematics serve them in their daily living. It is expected that the computational skills developed in earlier grades with whole numbers and fractional numbers will be maintained and extended here. It is important that students learn the inverse nature of the operations of addition and subtraction; and that division is the operational inverse of multiplication.

The number line is an effective model for illustrating the operations of addition, subtraction, multiplication, and division with the integers. These operations should also be related to real life situations. Efforts should be made to develop an understanding of how operations with the integers differ from operations in other number systems. In particular, students should investigate how the structural properties that were not true for other number systems may be true for the integers.

One of the most useful mathematical ideas is that of ratio and proportion. Students should explore many of the properties of a proportion as well as how to solve problems using the proportion. Since the solution of problems involving proportion requires the use of more than one operation, students should be given a great deal of practice in this area.

ALGEBRAIC CONCEPTS

Many of the concepts of arithmetic can be made much easier by the use of the equation: percentage, for example. The mathematical sentence should be introduced early in a student's study of mathematics. Associated with the mathematical sentence is the concept of a variable (a symbol which represents an unspecified member of a set) and the meaning of equality (identity).

Along with the equation, the following concepts should be introduced: inequalities, formulae, and properties of equations in one and two unknowns. A study of the Cartesian coordinate system and its use in plotting a point when the coordinates are given and in finding the coordinates of a given point should be included. These ideas are to be developed as a basis for the study of graphs as pictures of functions in Algebra 1.

PROBLEM SOLVING

As mathematical skills and concepts are developed, they are fixed by applying them in problem solving situations. Problems relating to sports, transportation, and money management can be used. Ideas and data from current biological, physical, and chemical experiments provide material for other meaningful problems. Puzzle problems may also be used. These problems may be interspersed throughout the course, thus avoiding their presentation as isolated topics.

PROBABILITY AND STATISTICS

Thinking, planning, and decision making are often influenced by one's ability to interpret numerical information. Success or failure in an individual's life can sometimes be affected by her or his knowledge and use of probability and statistics. The increasing importance of these two mathematical ideas in today's world makes it necessary for students to become more knowledgeable of them. In particular, they should know how data are collected, presented (charts, tables, and graphs), read and interpreted, and the meanings of central tendency (mean, median, and mode). They should also know the meaning of a probability ratio and be able to distinguish between theoretical and experimental probability.

INTUITIVE GEOMETRY

Students should be introduced to some geometric concepts at this level. They should know what a simple closed curve is and should be familiar with the various common polygons, circles, and ellipses. They should also become acquainted with certain solids (spheres, cubes, cones, and cylinders).

Following this recognition and identification level, the analytical stage should begin. There should be a discussion of undefined terms (point, line, and plane). The students should understand that diagrams on paper are merely representations of the ideas of point, line, and plane. They should have some concept of the following: a simple closed figure, parallelism, a polygon, angle, surface, face, edge, and vertex. Through exploratory activities, they should be led to discover the idea of congruence and the basic relationships existing between angles formed by intersecting lines and between angles formed by two parallel lines cut by a transversal.

The properties of parallelograms can be developed, and the angle sums of various polygons can be examined. The circle, its radius, diameter, and circumference, can be defined and the measure relationships existing between them can be analyzed.

Basic geometric constructions, including constructions with stipulated conditions using straight edge and compass, can be developed. The construction of the five regular solids should be considered. Some scale and perspective drawings might be introduced. The student should understand the difference between a plane figure and the region which it encloses. Formulae should be developed and used to compute perimeters, areas, and volumes, including the area of a sector of a circle. A study of special triangles should be made (equilateral, isosceles, scalene, and right). Along with applications of the Pythagorean relationship, other concepts in geometry that should receive attention here include the geometry on the coordinate grid, symmetry, similarity, and ratio problems involving similar figures.

MEASUREMENT

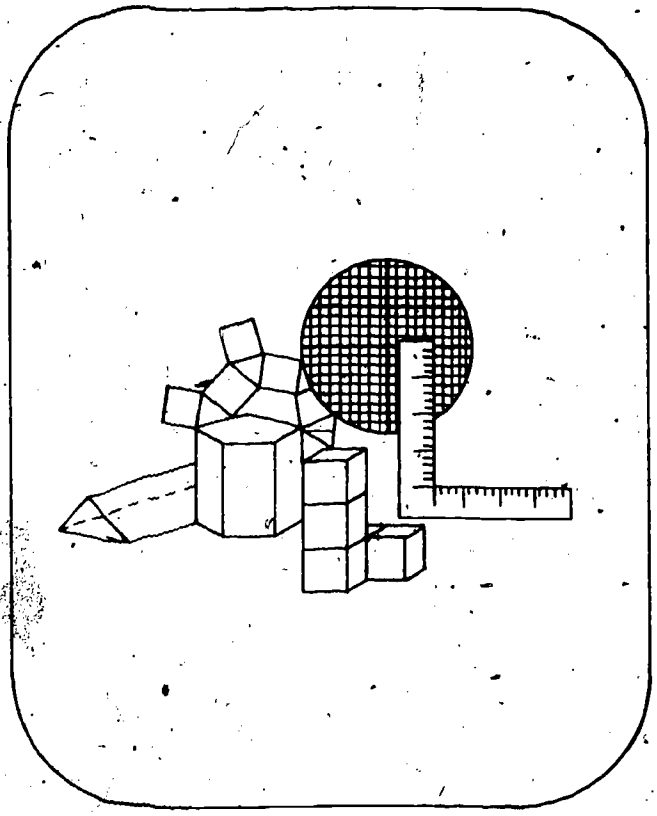
Second to computation, measurement is probably the most frequently used aspect of mathematics. Students should gain an understanding of measurement and the various units of measure (length, area, volume capacity, mass, and temperature) in both the metric system and the U.S. Customary System. Students should also learn to work with angle measure and be provided with opportunities to estimate the measure of objects.

During the teaching of measurement there is no place for conversions between the U.S. Customary System and the metric system. However, comparison of units through estimation is both practical and an example of good pedagogy in mathematics. As measurement applications are being learned, students should be taught how to convert, but not to convert. In specialized cases, it may be necessary to teach students specific conversion techniques which will emphasize the use of tables and calculators.

GRAPHS AND SCALE DRAWINGS

Graphs are used extensively to display data. They are an effective way to communicate information quickly and clearly. Attention should be given to the bar, circle, and broken-line graphs. Some time should also be devoted to the graph of points on a number line, and points on a rectangular coordinate system. Not only do students need to learn how to display data in a graph, they also need to learn how to interpret data presented in a graph and to draw inferences from the data presented there.

The use of scale drawings is almost commonplace in today's world. Students should be taught to interpret and construct scale drawings. Maps, charts, and house plans are some examples of scale drawings that should be studied. The construction of scale drawings allows students to make use of some of the knowledge they have gained in the study of geometry and measurement. For example, to make a scale drawing of some object of shape, the student has to draw angles the same size as the original and either increase or decrease the lengths of the corresponding sides in a fixed ratio.



Mathematics 7

OBJECTIVES

1. Review or develop computational skills involving the whole, fractional, and decimal numbers.
2. Explore some of the basic concepts of the geometry of two and three dimensions.
3. Use some of the properties of number theory as a means of developing an understanding of the algorithms involving whole, fractional, and decimal numbers.
4. Provide opportunities for students to learn to estimate the answers to mathematical problems.
5. Examine the concept of measurement and then apply this to situations both real and contrived. Both the U.S. Customary and metric systems should be examined; however, emphasis should be on the metric system.
6. Investigate problem solving techniques and then make use of these throughout the year.
7. Explore some of the elementary notions of probability and statistics.
8. Provide students experiences in reading and constructing graphs, gathering data and recording this graphically.

CONTENT

I. NUMBER CONCEPTS
   - A. Reading and writing numerals
   - B. Historic numeration systems
   - C. Place value systems
   - D. Rules for rounding numbers
   - E. Powers and roots

II. MATHEMATICAL SENTENCES
- A. True sentences, false sentences, and open sentences
- B. Variables
- C. Number-line representations of solution sets

III. NATURAL NUMBERS AND ZERO
- A. Basic principles of addition and multiplication
- B. Algorithms of addition and multiplication
- C. Inverse operations
- D. Addition, subtraction, multiplication, and division algorithms
- E. Estimation and approximation

IV. FACTORS, PRIME AND COMPOSITE NUMBERS
- A. Tests for divisibility
- B. Review and definitions
- C. Greatest common factor and least common multiple

V. NON-NEGATIVE RATIONAL NUMBERS
(Fractions)
- A. Changing to higher terms
- B. Changing to lower terms and simplifying
- C. Finding the least common denominator
- D. Addition, subtraction, multiplication, and division (vertical and horizontal notation)
- E. Finding the multiplicative inverse (reciprocal)
- F. Comparing fractions (equal or not equal, greater than or less than)
- G. Ratio and proportion
- H. Percent
- I. Using fractions to solve problems
- J. Estimation and approximation
- K. Properties of operations on fractional numbers

(Decimals)
- A. Reading and writing (to four decimal places)
- B. Adding, subtracting, multiplying, and dividing (vertical and horizontal notation; expanded notation)
- C. Changing fractions to decimals and decimals to fractions (terminating and repeating decimals)
- D. Rounding numbers (nearest thousandth, hundredth, tenth, one, ten, hundred, etc.)
- E. Multiplying and dividing by powers of ten
- F. Using decimals to solve problems
- G. Estimating and approximating

VI. GEOMETRY
- A. Sets of points in space
- B. Measurement of segments and angles
- C. Sketchs of geometric figures
- D. Constructions
- E. Properties of polygons

VII. MEASUREMENT
- A. Perimeters, areas, and volumes
- B. Formulas in measurement
- C. Standard units of measure emphasizing the metric system
- D. Appropriate units of measure
- E. Estimation

VIII. INTEGERS (Optional)
- A. Negative integers
- B. Addition of integers
- C. Properties of addition
- D. Multiplication of integers
- E. Properties of multiplication
- F. Subtraction and division of integers
- G. Number sentences and pictorial representations
- H. Estimation and approximation

IX. MATHEMATICAL SYSTEMS (Optional)
- A. Properties of binary operations
- B. Systems with rational numbers for elements
- C. Mathematical systems from physical models
- D. Abstract systems

Mathematics 8

OBJECTIVES

1. Develop proficiency with the operations involving integers.
2. Maintain previously learned computational skills with whole, fractional, and decimal numbers.
3. Explore some of the structural properties of the whole, fractional, and decimal number systems. Particular emphasis should be placed on the usefulness of these properties in simplifying computations.
4. Extend previously gained knowledge about geometry to coordinate geometry.
5. Investigate problem solving techniques and make use of these in solving mathematical problems.
6. Provide experiences which will enable students to estimate answers with a reasonable degree of accuracy.
7. Develop techniques for solving linear equations and inequalities with one variable.
8. Study, on an informal basis, some of the ideas of probability and statistics.
9. Provide students experiences in reading graphs, gathering data and recording this graphically.

CONTENT

I. NUMBER SYSTEMS
- A. Natural Numbers
  1. Prime and composite numbers
  2. Place value
  3. Order of numbers
  4. Scientific notation
  5. Greatest common factor
  6. Least common multiple
- B. Integers
  1. Negative integers
  2. Addition of integers
  3. Properties of addition
  4. Multiplication of integers
  5. Properties of multiplication
  6. Subtraction and division of integers
  7. Number sentences and pictorial representations
- C. Rational Numbers
  1. Fractional numbers
  2. Decimal numbers

3. Equivalent and non-equivalent
4. Order relationship
5. Terminating and non-terminating
6. Estimation and approximation

D. Real Numbers
1. Rational numbers
2. Irrational numbers

E. Structural Properties of Various Number Systems
1. Closure
2. Commutativity
3. Associativity
4. Distributivity
5. Identity elements: 0 for addition and 1 for multiplication

II. OPERATIONS AND NUMBER SENTENCES
A. Addition, subtraction, multiplication, and division of whole numbers, rational numbers, and integers
B. Linear equations with one variable
C. Percent
D. Ratio and proportion
E. Powers and roots
F. Estimation and approximation

III. GEOMETRY
A. Language of one, two, and three dimensions
B. Informal experiences in one, two and three dimensions
C. Coordinate geometry
D. Pythagorean theorem

IV. MEASUREMENT
A. Standard units emphasizing the metric system
B. Approximation and estimation
C. Direct and indirect
D. Experiences with length, area, volume, and temperature

V. GRAPHS AND SCALE DRAWINGS
A. Bar, circle, and broken-line graphs
B. Scale drawings

VI. ALGEBRA OF POLYNOMIALS (Optional)
A. Polynomial expressions
B. Operations and polynomial elements
C. Equalities and inequalities

VII. INFORMAL PROBABILITY AND STATISTICS
A. Mean, median, and mode
B. Certain and uncertain events (optional)
C. Meaning of a probability ratio and odds
D. Random and biased sample (optional)
E. Equally likely events (optional)
F. Frequency distribution
G. Histogram
H. Theoretical versus experimental probability (optional)

VIII. TRIGONOMETRY (Optional)
A. Trigonometric ratios - sine, cosine, and tangent
B. Using the tables for trigonometric ratios

General Mathematics

COURSE NUMBER 2030

The purposes of this course are (1) to teach or reteach computational skills involving whole, fractional, and decimal numbers; (2) to survey some of the other areas of mathematics like geometry, measurement, probability, statistics, and elementary algebra; and (3) to examine some of the applications of mathematics. The approach for teaching computational skills needs to be novel, as it is unlikely that a person who has previously been unsuccessful in learning computation will suddenly learn it in another year of similar content and methodology. Games, contests, puzzles, laboratory activities, outdoor mathematics and the hand-held or desk calculator could be used to motivate students to want to learn how to do computations.

One way of organizing this course is around topics. Efforts should be made to include experiments, games, surveys, and computing devices in the study of as many topics as feasible. The major concern should be that students discover mathematical ideas. The activities in the course should teach the students how to learn and add meaning to their understanding of mathematical ideas. In the process students would perform computations as needed. The course could be developed around various combinations of the following topics: elementary statistics; patterns; flow charts; calculating devices; mathematical recreations; rational numbers; measurement; experimental probability; equations and relations; ratio, proportion, and percent; graphs, formulas, and patterns; mathematical reasoning; metric and motion geometry; and creative geometric constructions.

If General Mathematics is to be limited to remedial arithmetic, then major emphasis should be placed on the first three headings in the content outline.

OBJECTIVES

1. Provide experiences which will enable students to become proficient in the basic computational skills involving whole numbers, fractions, and decimals.
2. Explore some of the basic concepts of the geometry of two and three dimensions.
3. Make use of some basic concepts of number theory as a means of developing an understanding of the important properties of the whole numbers.
4. Provide experiences which will enable students to become proficient in estimating answers and doing mental arithmetic.
5. Present experiences in ratio and proportion and then use these in solving problems involving percentage, similar triangles, and numerical trigonometry.
6. Explore some of the elementary principles of probability and statistics.
7. Extend the students' knowledge of numbers to the integers. Then explore the addition, subtraction, multiplication, and division of integers.
8. Develop the ability to solve linear equations and inequalities.
9. Present problems, both real and contrived, which can be solved by means of equation solving.

10. Emphasize problem solving techniques.
11. Present arithmetic concepts in a non-traditional manner and on a level in keeping with the maturity of these students.
12. Examine some of the applications of mathematics which make use of the mathematics explored in this course.

CONTENT

I. WHOLE NUMBERS
- A. Addition
- B. Subtraction
- C. Multiplication
- D. Division
- E. Estimation and approximation
- F. Raising to a power
- G. Finding the root of a number
- H. Applications involving whole numbers

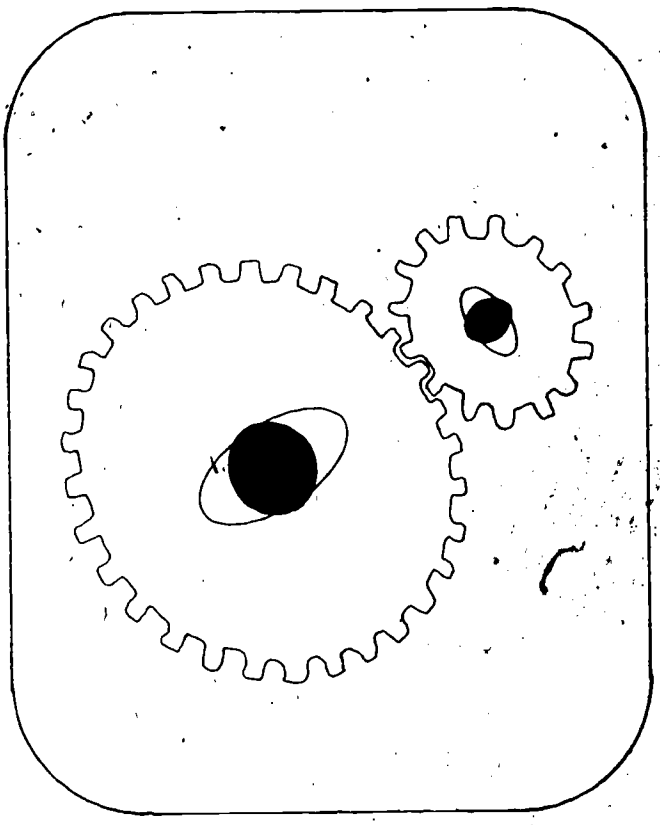
II. FRACTIONAL NUMBERS
- A. Addition
- B. Subtraction
- C. Multiplication
- D. Division
- E. Estimation and approximation
- F. Basic operations involving mixed numerals
- G. Applications involving fractional numbers

III. DECIMALS
- A. Addition
- B. Subtraction
- C. Multiplication
- D. Division
- E. Estimation and approximation
- F. Applications involving decimals

IV. INTEGERS
- A. Meaning and representation of integers on a number line
- B. Addition
- C. Subtraction
- D. Multiplication
- E. Division
- F. Applications involving integers

V. INFORMAL GEOMETRY
- A. Geometry of two dimensions
- B. Geometry of three dimensions
- C. Applications of geometry

VI. LINEAR EQUATIONS AND INEQUALITIES
- A. Solving equations by addition
- B. Solving equations by subtraction
- C. Solving equations by multiplication
- D. Solving equations by division
- E. Solving equations involving more than one operation
- F. Problem solving involving equations
- G. Solving linear inequalities



VII. MEASUREMENT
- A. Linear
- B. Area
- C. Volume
- D. Temperature
- E. Time
- F. Indirect

VIII. PROBABILITY AND STATISTICS
- A. Gathering data including sampling
- B. Analyzing data
- C. Reporting data including histograms and graphs
- D. Probability of an event

IX RATIO AND PROPORTION
- A. Using ratios and proportions to solve problems
- B. Scale drawings
- C. Percent as a ratio

X. NUMERICAL TRIGONOMETRY
- A. Sine, cosine, and tangent ratios
- B. Using a table of trigonometric ratios to solve problems

Introductory Algebra

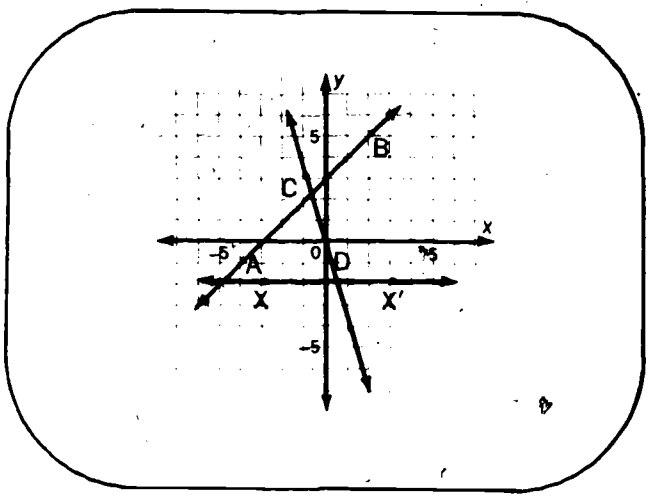
COURSE NUMBERS 2101 and 2102

Essentially, this is an Algebra I course taught over a two-year period. In addition to being taught at a slower pace, the presentation of this content should be of a less abstract nature with less stress being put on precision of language and should include many opportunities for reinforcement.

In addition to algebraic topics, some attention should be devoted to arithmetic, in particular in the sense that some algebraic concepts can be thought of as being generalized arithmetic. It is hoped that students will improve and extend their computational skills as the result of experiences in this two-part course.

Whenever appropriate, applications of mathematics should be presented. Many students have a need to see the usefulness of algebra in order for them to become motivated to study it. In particular, they want to know how the mathematics they study applies to the world in which they live.

Some of the elementary concepts of geometry should be included so as to enable students to acquire an understanding of the interrelatedness of geometric and algebraic concepts.



OBJECTIVES

1. To use algebraic ideas as a vehicle for improving computational skills in arithmetic.
2. To examine the structure of algebra at an informal level.
3. To utilize some of the structural aspects of various number systems in explaining many of the algorithms of algebra.
4. To develop facility in algebraic computations.
5. To examine several types of applied problems which require algebra ideas for their solution.
6. To explore some of the elementary aspects of geometry with particular emphasis on the interrelatedness of geometric and algebraic concepts.

Part I

CONTENT

I. LANGUAGE OF ALGEBRA
- A. Variables
- B. Order of operations
- C. Using punctuation marks in numerical phrases
- D. Exponents

II. PROPERTIES OF NUMBER SYSTEMS
- A. Closure
- B. Order (commutative) property
- C. Grouping (associative) property
- D. Inverse elements
- E. Identity elements

III. WHOLE NUMBER OPERATIONS
- A. Addition, subtraction, multiplication, division
- B. Factors
- C. Multiples
- D. Prime factors
- E. Divisibility
- F. Tests of divisibility
- G. Least common multiple (LCM)
- H. Least common divisor (LCD)

IV. OPERATIONS WITH FRACTIONS AND DECIMALS
- A. Addition, subtraction, multiplication, division
- B. Ratio
- C. Proportion
- D. Percent
- E. Interest
- F. Discount

V. OPEN SENTENCES
- A. Addition property of equality
- B. Subtraction property of equality
- C. Multiplication property of equality
- D. Division property of equality

VI. MEASUREMENT INVOLVING U.S. CUSTOMARY AND METRIC UNITS WITH EMPHASIS ON THE METRIC SYSTEM
- A. Length
- B. Area
- C. Volume
- D. Mass (weight)
- E. Temperature
- F. Indirect measurement

VII. INTEGERS
- A. Number line
- B. Opposites (additive inverse)
- C. Comparison
- D. Number line addition
- E. Number line subtraction
- F. Addition
- G. Subtraction

H. Multiplication
I. Division

VIII. EQUATIONS AND PROBLEM SOLVING
A. Addition property of equality
B. Multiplication property of equality
C. Using equations to solve problems

IX. INEQUALITIES OF ONE VARIABLE
A. Using addition to solve inequalities
B. Using multiplication to solve inequalities
C. Using more than one operation to solve inequalities

X. GEOMETRIC FIGURES
A. Points, lines, and plane figures
B. Curves
C. Angles
D. Triangles
E. Pythagorean Theorem
F. Polygons

Part II

CONTENT

I. REAL NUMBERS
A. Locating points on the number line
B. Order relationships
C. Addition
D. Subtraction
E. Multiplication
F. Division

II. SOLVING EQUATIONS AND INEQUALITIES OF ONE VARIABLE
A. Graphing solution sets on the number line
B. Using the addition property of equality to solve equations and inequalities
C. Using the multiplication property of equality to solve equations and inequalities
D. Using more than one property to solve equations and inequalities

III. PRODUCTS AND FACTORS
A. Exponents
B. Factors
C. Prime factors
D. Greatest common factor

IV. POLYNOMIALS
A. Adding polynomials
B. Subtracting polynomials
C. Multiplying polynomials
D. Raising polynomials to a power
E. Dividing polynomials
F. Removing a common monomial factor
G. Computing the products of binomials
H. Squaring a binomial
I. Factoring a perfect square trinomial

J. Determining the product of sum and difference of two binomials
K. Factoring the difference of two squares
L. Factoring a trinomial
M. Doing complete factoring

V. ALGEBRAIC FRACTIONS
A. Simplifying fractions
B. Adding fractions
C. Subtracting fractions
D. Multiplying fractions
E. Dividing fractions

VI. SOLVING EQUATIONS WITH FRACTIONS
A. Equations with fractions
B. Algebraic equations
C. Using algebraic equations in problem solving

VII. SYSTEMS OF EQUATIONS
A. Equations with two variables
B. Graphing on the coordinate plane
C. Graphing equations on the coordinate plane
D. Slope of a line
E. Slope-intercept form of a line
F. Solving systems of equations by graphing
G. Solving systems of equations by addition
H. Solving systems of equations by substitution
I. Using multiplication in solving systems of equations
J. Using systems of equations to solve problems

VIII. EXPONENTS AND ROOTS
A. Powers of numbers
B. Irrational numbers
C. Square roots of numbers
D. Roots other than square roots of numbers
E. Simplifying radicals
F. Multiplication and division of radicals
G. Addition and subtraction of radicals
H. Approximations of the values of real numbers

IX. LINEAR AND QUADRATIC FUNCTIONS
A. Meaning of a relation
B. Meaning of a function
C. Graphing linear functions
D. Graphing quadratic functions
E. Graphing parabolas (optional)
F. Graphing hyperbolas (optional)
G. Graphing circles (optional)
H. Quadratic equations
I. Solving quadratic equations
J. Quadratic formula
K. Solving problems involving quadratics

X. TRIGONOMETRY OF THE RIGHT TRIANGLE
A. Similar figures
B. Sine ratio
C. Cosine ratio
D. Tangent ratio
E. Solving problems using trigonometric ratios

Algebra I

COURSE NUMBER 2100

The major goal of Algebra I, normally studied in the ninth grade, is to develop within each student, skill in certain algebraic techniques and some understanding of algebra as a logical structure.

The course might start with a study of some elementary concepts and notions concerning sets and sentences, such as equality, subsets, mappings, the empty set, variables, and open sentences. This study would be for the purpose of supplying the student with concepts and language useful for clarifying fundamental ideas in mathematics, such as operation, function, and graphs. The amount of time needed for this aspect of the course would depend upon the nature of the students' mathematical training in previous courses. For some students, this part of the course (as well as some of the topics mentioned later) would be a review, and could be covered rapidly.

Algebra I should include a careful treatment of the real number system and its various subsystems: the natural numbers, integers, rational numbers and irrational numbers. Operations of addition and multiplication and the importance of the commutative, associative, and distributive properties governing these operations should be stressed. Justification by means of these properties of manipulative maneuvers of algebra should be made where feasible. The unique roles played by 0 and 1, the identity elements, should be pointed out, as should be the meanings of inverse elements and inverse operations.

The number line should be employed to illuminate such processes as operations and the extension of a number system to a larger one. Set terminology should be used where beneficial in clarifying concepts, but should not be studied as a separate entity.

Solution of simple equations should be carried out by logical steps based on explicitly stated axioms, with attention paid to the reversibility of the steps.

The order properties of the rational and real number systems and the solution of inequalities by application of these properties should be treated thoroughly.

To increase interest, historical developments and biographical sketches of mathematicians should be used where feasible.

Without being too formal, an attempt should be made to exhibit structure in algebra by showing that the subject is based upon a few simple principles and is developed by logical reasoning from these basic assumptions.

In treating symbols of grouping, the roles of the distributive, commutative, and associative properties in simplifying algebraic expressions should be stressed.

The properties of polynomials and rational operations with polynomials should be taught. Special products and factoring should be treated by means of the distributive property. Attention should be called to the analogy between the system of polynomials with coefficients in some number field and the system of integers concerning such matters as factorization into primes and the division algorithm. Insistence upon identification of the number system from which coefficients are taken is important, especially in factorization of polynomials.

Rational expressions — quotients of polynomials — may be treated as analogous to

rational numbers. Students need considerable practice in performing operations with algebraic functions.

The concept of a function should be defined both as a mapping from one set to a second set and as a collection of ordered parts in which no two pairs have the same first number. The equivalence of the two definitions should be made clear. Relations as generalizations of functions may be included. The terms, domain and range of a function, should be introduced and taught using many examples. Special types of functions — linear, quadratic, and polynomial — should be studied. Modern notation, such as the use of a single letter to denote a function, should be used.

Graphs may be introduced as pictures of functions and applied to solving such problems involving functions and relations as determining zeros or maximum-minimum values.

Solution of systems of linear equations by various methods including graphs should be presented.

An introduction to quadratic equations and quadratic inequalities should be included in the course.

Correct mathematical language and usage should be introduced at appropriate places and employed thereafter. Such ideas as equations and inequalities being sentences, the practice of denoting numbers by letters, the meaning of integral exponents, and symbols of grouping are examples of mathematical language.

Proofs should be given where feasible, with the purpose of introducing to students the deductive aspect of mathematics, but excessive formality should be avoided. With each topic, good realistic applications — word problems — should be given where feasible.

OBJECTIVES

1. To develop a foundation in algebra which will prepare the student for further mathematics courses.
2. To extend the student's understanding of the real number system.
3. To further examine the elementary notions of relations and functions.
4. To study in-depth the procedures for solving equations and inequalities.
5. To consider some of the principles used to interpret equations and inequalities geometrically.
6. To examine some of the fundamental ideas of trigonometry.
7. To emphasize the necessity for using precise mathematical language and symbolism.
8. To introduce students to some of the beginning notions of mathematical proof.
9. To emphasize the use of algebraic techniques to solve problems.

SUGGESTED CONTENT

I. THE LANGUAGE AND STRUCTURE OF ALGEBRA
   - A. A finite number system
   - B. Properties of operations

II. THE REAL NUMBER SYSTEM
   - A. Natural numbers
   - B. Integers
   - C. Rational numbers
   - D. Irrational numbers

III. FIRST DEGREE EQUATIONS AND INEQUALITIES IN ONE VARIABLE
   - A. First degree equations in one variable
   - B. First degree inequalities in one variable

IV. FIRST DEGREE EQUATIONS AND INEQUALITIES IN TWO VARIABLES
   - A. The Cartesian plane
   - B. Lines in the coordinate plane
   - C. First degree equations in two variables
   - D. Mathematical relations

V. ALGEBRAIC EXPRESSIONS
- A. Expressions and identities
- B. Addition and subtraction
- C. Multiplication
- D. Exponents
- E. Factoring
- F. Division
- G. Rational expressions

VI. POLYNOMIALS
- A. System of polynomials
- B. Rational functions

VII. RATIONAL SENTENCES AND FUNCTIONS
- A. Fractional equations and inequalities
- B. Rational functions

VIII. SYSTEMS OF LINEAR EQUATIONS AND INEQUALITIES
- A. Graphical solutions of systems of linear equations
- B. Algebraic solutions of systems of equations
- C. Graphical solutions of systems of linear inequalities
- D. Linear programming (optional)

IX EXPONENTS AND RADICALS
- A. Rational exponents
- B. Radicals

X QUADRATIC FUNCTIONS AND QUADRATIC EQUATIONS
- A. Graphs and quadratic functions
- B. Quadratic equations and inequalities (optional)
- C. Systems of equations and inequalities (optional)

XI. TRIGONOMETRY (Optional)
- A. Geometric concepts
- B. Trigonometric functions
- C. Tables of trigonometric functions
- D. Applications of trigonometry

XII. STATISTICS AND PROBABILITY (Optional)
- A. Representing data graphically
- B. Describing data
- C. Statistical averages
- D. Probability of an event
- E. Probability experiments

Geometry

COURSE NUMBER 2200

In planning for a well-coordinated and sequential program of mathematics, a single course cannot be considered out of the context of the total program. Mathematics development continues to break down the barriers between the traditional compartments of mathematical thought. No longer is a definitive distinction made between algebra and geometry. Neither is there a need for a separation of the study of plane geometry and solid geometry.

Since a sequential study of mathematics in grades 7-12 is planned, this guideline for geometry assumes but does not assure that students have dealt with the following areas of study.

1. Vocabulary of sets
2. Algebraic development of the basic properties of the real numbers including the order relationships and elementary simplification of radicals
3. Definitions and descriptions of elementary geometric figures such as triangles, quadrilaterals and circles
4. Exploration of elementary properties of physical lines and points
5. Basic constructions using compass and straight edge
6. Basic graphing of straight lines

Any program must be adapted by the individual system and teacher to meet the needs and capabilities of the students. There are concepts that should permeate the entire course and should not necessarily be taught as separate units. There are basic topics that every geometry course should include; there are topics for the accelerated groups. This geometry guideline is developed according to these categories. The sequence in which the materials is to be taught is to be determined by the individual system or teacher.

CONCEPTS TO PERMEATE THE STUDY OF GEOMETRY

Geometry should be presented as much as possible as a discovery topic to develop insight into the basic structure of a mathematical system consisting of undefined terms, assumptions, definitions, and theorems. It is not important at this level to prove all statements. However, what is assumed and what is a theorem should be stressed. That there are different developments of geometry, what is a theorem in one might be an assumption in the other and vice versa, should be noted. The study of geometry as a formal, rigid body of materials to be memorized should be avoided, with creativity and individual thinking encouraged.

The course should be thoroughly integrated, not plane geometry and solid geometry taught as separate courses in the same year. Algebra and geometry should also be integrated wherever possible. However, geometry should not be sacrificed for the teaching of algebra.

Equality, as used mathematicly, should be discussed and used. Equality means identity in mathematics.

An introduction to the elementary ideas of logic, especially those pertaining to deductive reasoning, is advisable early in the course. These ideas should be extended and new principles introduced whenever it is advantageous. A study of logic, per se, is not advocated.

The history of mathematics should be utilized throughout geometry to give the student a better appreciation of mathematics. If students are aware of the processes of development, they will better appreciate mathematical thought and structure. History is especially useful in the topics in which the geometry now amends or deviates from Euclid.

TOPICS BASIC TO A GEOMETRY COURSE

A study of geometry should use the properties of the real numbers, including order. The concept of betweenness can be used to define segment and ray. In turn, segment and ray can be used in studying angles and triangles, separation of a plane by a line, and separation of space by a plane. A distinction should be made between interior of, exterior of, and the geometric figure itself. The correspondence of a real number to an angle gives a means of angle measurement. Perpendicular lines, dihedral angles, and perpendicular planes are defined in the spiraling process of definitions and development of concepts.

Parallel lines are defined and a statement of a parallel postulate is given. The history of this idea is very interesting and informative, as is an introduction to the ideas of non-Euclidean geometries. Basic properties of parallel lines and planes can then be developed.

The concept of congruence is used to define congruent segments and congruent polygons. The difference between congruence and equality should be emphasized and used. Congruence theorems in two and three dimensions should be studied.

The algebraic idea of ratio should be applied to the measures of segments. Similarity may be developed as a generalization of congruence, with similarity of triangles and polygons in general defined, followed by proofs of similarity theorems in two and three dimensions. The study of the trigonometric ratios (sine, cosine, and tangent) as developed from the similarity of two right triangles could also be included.

Proof of the Pythagorean theorem using properties of similarity follows. The application to special right triangles such as the 30°- 60°- 90°, the 45°-45°- 90°, and the 3-4-5, is studied.

The study of geometry extends to polygons and polyhedrons. The special polygons and polyhedrons are defined and the history of the regular polyhedrons studied. The properties of polygons and polyhedrons should be discussed and selected properties proved.

A circle and a sphere should be defined, using the idea of a set of points, and their basic properties developed. Angles connected with circles and spheres should be studied, along with tangent lines and tangent planes. The relationships between polygons and circles should lead to an introduction of the idea of a limit.

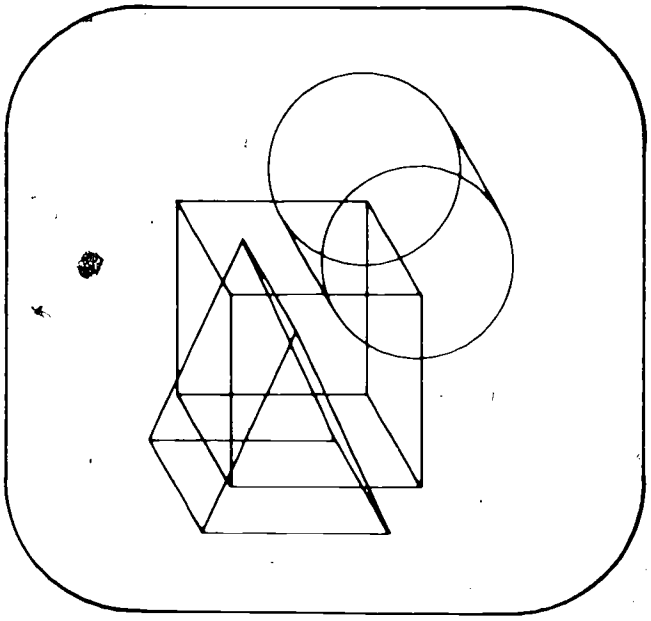
Measurement should not be thought of as a separate unit of study but should be integrated throughout the course, the measure of segments, angles, and arcs being defined and extended to include the measurements of perimeters, circumferences, and calculations of areas and perimeters. Applications and basic theorems should be developed.

A separate study could be made on constructions, with special emphasis on the logical properties or proofs of the constructions.

A unit that possibly would be considered as basic is the study of coordinate geometry and loci. This would provide another method of proof for use in the remainder of the course.

A study of finite geometries is a good enrichment topic for a group to foster creativity and mathematical insight.

The history mentioned throughout the course could be extended to a complete unit on the history of mathematics, as could the logic permeating the course. This unit would include some symbolic logic; however, topics such as "truth tables" should not be overemphasized.



Other topics for consideration are: geometric interpretation of vectors, geometric transformations, and non-Euclidean geometries.

This course in geometry emphasizes ideas developed in previous courses such as sets, real numbers, and deductive reasoning. The role of undefined terms, definitions, axioms, theorems, and the nature of proof is made clear. Thus the value inherent in Euclidean geometry is retained and strengthened. Concepts of three dimensions are introduced along with analogous two dimensional concepts.

OBJECTIVES

1. To show the inadequacy of common sense reasoning in establishing mathematical proof.
2. To teach the process of deductive reasoning in mathematical and nonmathematical situations.
3. To extend the concepts of sets to include sets of points as well as sets of numbers, and to use sets for purposes of clarity and convenience throughout the course.
4. To use the properties of the real numbers, including order, in the study of various geometric figures.
5. To develop and to use the concepts of betweenness and separation.
6. To study relationships between lines in a plane and in space, between planes, and between lines and planes
7. To teach the idea of congruences as correspondences and to study congruences of a variety of geometric figures.
8. To distinguish between the concepts of existence and uniqueness, and to prove some theorems involving these concepts.
9. To teach the principles of ratio and proportion, and to use these principles in proving similarity theorems and in solving problems.
10. To study regions whose boundaries are rectilinear and to derive formulas for finding the area of these regions.
11. To consider the incidence properties of lines and circles and the analogous properties of planes and spheres.
12. To develop intuitively formulas for the circumference and the area of a circle, for the length of an arc, for the area of a sector, and for the surface areas and volumes of familiar three-dimensional geometric figures.

CONTENT

I. EUCLIDEAN GEOMETRY AS A MATHEMATICAL SYSTEM
   - A. Undefined terms: point, line, and plane
   - B. Relation: betweenness
   - C. Postulates
   - D. Definitions
   - E. Theorems

II. CONGRUENCE
   - A. Congruence of segments and angles
   - B. Congruence of triangles

III. INEQUALITIES
   - A. Inequalities of measures of segments and angles
   - B. Inequalities of lengths of sizes of angles in a triangle, and also in pairs of triangles

IV. PERPENDICULAR AND PARALLEL LINES AND PLANES
   - A. Perpendicular lines in a plane and lines perpendicular to a plane
   - B. Perpendicular and parallel relations
   - C. Dihedral angles and perpendicular planes

V. SIMILARITY OF TRIANGLES
   - A. Line segments whose measures are proportional
   - B. Similar triangles

VI. COORDINATE GEOMETRY
- A. One-to-one correspondence between points of a plane and ordered pairs of real numbers
- B. Midpoint and distance formulas
- C. The slope of a line
- D. Theorems pertaining to the slopes of parallel lines and the slopes of perpendicular lines
- E. The equation of a line
- F. Simple analytic proofs

VII. POLYGONS AND POLYHEDRONS
- A. Definition of polygon and names of common polygons
- B. A detailed study of quadrilaterals
- C. Regular polygons
- D. Sum of measures of angles of a polygon
- E. Ratios of measures of corresponding line segments in polygons
- F. Polyhedra
- G. Polyhedral angles
- H. Coordinate systems in space

VIII. CIRCLES, SPHERES, CYLINDERS, AND CONES
- A. Geometry of the circle and the sphere
- B. Cylinders
- C. Cones

IX. LOCI AND CONSTRUCTIONS
- A. Two and three-dimensional loci
- B. Constructions with compasses and straightedge
- C. Constructions utilizing the compass alone (Optional)
- D. Constructions using the straightedge and a fixed circle (Optional)

X. AREAS AND VOLUMES
- A. Area of polygonal regions
- B. Polyhedrons — prisms, pyramids, frustums of pyramids and prisms
- C. Circumferences of circles and lengths of arcs
- D. Areas of circular regions and sectors of circles
- E. Surface area and volume of a sphere, cylinder, cone, and frustum of a cone

XI. A GEOMETRIC INTERPRETATION OF VECTORS (Optional)
- A. Directed segments
- B. Vectors

XII. GEOMETRIC TRANSFORMATIONS (Optional)
- A. Transformations of Euclidean Geometry
- B. Introduction to Projective Geometry

XIII. NON-EUCLIDEAN GEOMETRIES (Optional)
- A. Historical background of non-Euclidean Geometries
- B. Hyperbolic Geometry
- C. Elliptic Geometry

Applied Vocational Mathematics

COURSE NUMBER 2250

Many students tend to reject studying any course for which they fail to see a practical application. The purpose of Applied Vocational Mathematics is to relate mathematics to a wide range of vocational applications. Students should be provided with the opportunity to see how mathematics is used in several careers.

At times it might be advantageous to teach a mathematical concept or skill to a large group of students or to the entire class and then allow the individuals within the group to apply this knowledge to an application related to their own career goals. For example, several students might be taught how to find what percent one number is of another, then they could apply this skill to several areas of interest like the masonry trades, electrical trades, or health occupations.

Mathematics is a common language among many vocations. Very often facility with the basic computational skills is a job entry requirement. Thus, the teacher will want to work with students on improving their ability to compute with whole, fractional, and decimal numbers. Attention should also be given to proficiency with operations involving percents. However, care should be taken not to overemphasize computational skill to the extent that students never get a chance to see the applications of mathematics.

Students need to learn how to work with some of the formulas used in the vocations. Techniques for solving formulas related to a specific vocational area should be studied in more detail by those students with an interest in this area.

Since geometric shapes are used in every vocational area, students should become familiar with both two and three-dimensional geometric figures. Some of the special properties of geometric figures should be examined. For example, because the triangle is a rigid figure, it is widely used in the construction industries. Time should be taken to review or teach students the techniques for constructing some of the geometric figures. Many of the ideas studied concerning geometric constructions will prove useful when students work with technical drawings.

The use of technical drawings is widespread in many vocations. Students should be provided with the opportunity to solve mathematical problems, which arise from the use of these drawings. Efforts should be made to ensure that the metric system of measurement is included on the technical drawings as the students taking this course will find an increased usage of this system of measurement by the time they enter the labor force.

One of the most common uses of mathematics is in measurement. Attention should be given to the study of length, area, volume, temperature, and mass (weight). Both the U.S. Customary and the metric system of measurement should be studied. Some aspects of the study of measurement will allow students to apply knowledge they have gained earlier in the course. For example, if they are to find the displacement of a motorcycle engine, they will need to work out a formula which will involve several mathematical operations.

Students should gain a good understanding of ratio and proportion and be able to apply this useful mathematical tool in the solution of different types of real life settings. Work with scale drawings can provide students with additional experiences with ratio and proportion. Provisions should be made to provide students with hands-on experiences in working with scale drawings.

GENERAL OBJECTIVES

1. To use applications from the vocations as a vehicle for developing and reinforcing students' backgrounds.
2. To present mathematical problems as they exist in a real-life setting with which students can relate.
3. To familiarize students with some of the technical vocabulary used in various vocations.
4. To acquaint students with some of the mathematical concepts and skills needed for selected vocations.

SPECIFIC OBJECTIVES

1. Review and reinforce computational skills involving whole numbers, fractions, and decimals.
2. Extend computations involving fractions and decimals to include percents.
3. Develop equation and formula solving facility.
4. Provide opportunities for measurements with the U.S. Customary and metric systems using basic formulas to compute linear, area, and volume measures of various geometric figures.
5. Develop an understanding of some of the elementary concepts of geometry. Use these concepts to help solve problems related to layout, sketching, and scale drawings.
6. Read and interpret bar, line, and circle graphs.
7. Construct bar, line, and circle graphs as a means of communicating information.
8. Apply the ideas of ratio and proportion to help solve real or contrived problems.
9. Use the Pythagorean Theorem to solve problems.
10. Read and use mathematical tables.

NOTE: As mathematical ideas are presented, opportunities should be provided to apply these to problems from various vocations.

CONTENT

I. WHOLE NUMBERS
   - A. Applications involving whole number operations as used in various trades and industries
   - B. Estimating answers

II. FRACTIONS
   - A. Applications from various trades and industries
   - B. Fractions as percents
   - C. Estimating answers

III. DECIMALS
   - A. Applications from various trades and industries
   - B. Decimals as percents
   - C. Estimating answers

IV. EQUATIONS
   - A. Using informal techniques to solve equations
   - B. Using addition, subtraction, multiplication, and division to solve equations
   - C. Applications from various trades and industries

V. FORMULAS
   - A. Working with formulas
   - B. Using formulas from various vocations

VI. GEOMETRY
   - A. Angles
   - B. Plane figures
   - C. Three-dimensional figures
   - D. Constructions
   - E. Applications

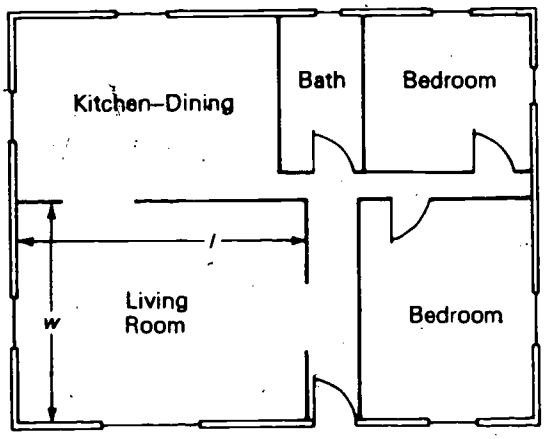
VII. MEASUREMENT INVOLVING BOTH THE U.S. CUSTOMARY AND METRIC UNITS
- A. Length
- B. Angle
- C. Area
- D. Volume
- E. Temperature
- F. Mass (Weight)
- G. Applications

VIII. GRAPHS
- A. Bar
- B. Line
- C. Circle
- D. Interpreting and drawing inferences from graphs
- E. Constructing graphs
- F. Applications

IX. SCALE DRAWINGS
- A. Ratio
- B. Proportion
- C. Similarity
- D. Applications

X. APPLICATIONS OF THE PYTHAGOREAN THEOREM

XI. TRIGONOMETRY OF THE RIGHT TRIANGLE
- A. Sine, cosine and tangent ratios
- B. Using trigonometric tables
- C. Using sines, cosines, and tangents in problem solving



EXAMPLE An architect is designing a house which will be 10 meters long and 8 meters wide. A scale drawing of the floor plan is 30 cm by 24 cm. In the drawing, find the dimensions representing a 4.2 m × 5.5 m living room.

SOLUTION 1 m = 100 cm, so 10 m = 1000 cm
Using the proportion

$$\frac{\text{actual size}}{\text{drawing size}} = \frac{1000}{30}$$

we have

$$\frac{420}{w} = \frac{1000}{30} \quad \text{and} \quad \frac{550}{l} = \frac{1000}{30}$$

Solving

$$w = \frac{420 \cdot 30}{1000} = 12.6 \text{ cm}$$

$$l = \frac{550 \cdot 30}{1000} = 16.5 \text{ cm}$$

Can you find the dimensions that represent the other rooms in the drawing?

Algebra II

COURSE NUMBER 2300

The study of the real number system and functions begun in Algebra I should be continued in Algebra II. The field and order axioms of the real number system should be reviewed and applied to equations and inequalities. Throughout the course, emphasis should be placed upon understanding the structure of algebra as well as developing skill in algebraic techniques.

The function concept should be presented both as a mapping and as a special set of ordered pairs, with stress on correct terminology and notation. This concept should be used extensively throughout the course. Examples should be given of various types of functions.

A thorough study should be made of linear and quadratic functions, including their zeros. An introduction to parabolas can be given by means of the graphs of quadratic functions.

The process of determining the zeros of quadratic functions (solutions of quadratic equations) indicates the need for extending the real number system to the complex number system. Properties of the latter should be clearly stated and compared with those of the real number system. The relations of the various number systems to each other should be made clear.

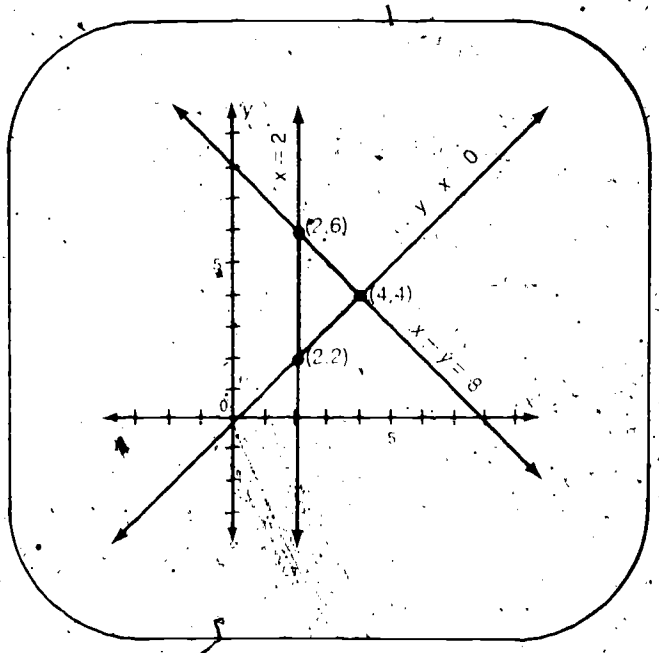
After complex numbers have been introduced, the problem of solving quadratics in general may be approached by completing the square, with the quadratic formula derived.

Equations involving rational algebraic fractions and square roots of simple algebraic expressions should be solved. It is important to provide adequate review and practice material for operations with algebraic functions.

Systems of linear equations and systems of linear inequalities should be studied, with graphs and set terminology used freely to clarify the meaning of solutions.

Integral exponents should be reviewed and rational fractional exponents introduced. Rules of operations with radicals, along with their limitations should be established and ample practice provided.

The elementary properties of logarithmic and exponential functions, including their graphs, should be studied. Logarithms and their use in computation should be included. Applications of exponents and logarithms to realistic problems in the natural and social sciences is desirable.



The more elementary aspects of trigonometry should be treated in this course. The trigonometric functions should be defined for general angles. Measurement of angles, special angles, simple identities, solution of right triangles, and the Laws of Sines and Cosines should be included. In connection with the definitions of the functions, polar coordinates and their relations to rectangular coordinates might be pointed out. An introduction to vectors provides a good application of elementary trigonometry.

A treatment should be given of sequences, including arithmetic and geometric progressions.

Throughout the course, where possible, meaningful applications drawn from the biological, physical, and social sciences should be presented. Every suitable occasion should be utilized to develop deductive reasoning. Students should not be left with the impression that algebra is a subject without form or pattern.

Optional topics to be covered if time permits or used as supplementary or enrichment topics include matrices and determinants, linear programming, algebra of functions, logarithms of bases other than 10, logarithmic interpolation, rational roots of rational polynomials, irrationality of roots, factoring rational polynomials, complex roots, trigonometric equations, graphs of trigonometric functions, amplitude and period, inverse trigonometric functions, polar coordinates, trigonometric functions of complex numbers, vectors in physics and vector components, and finite probability.

OBJECTIVES

1. To provide a review of previously introduced algebraic topics as they relate to the structure of the real number system.
2. To consider the role of deductive reasoning in algebra and to apply this in the manipulative techniques used in algebra.
3. To use the basic ideas of sets to define and to explain algebraic concepts.
4. To review and to extend the concept of open mathematical sentences, and systems of sentences, involving inequality as well as equality.
5. To use one, two, and three-dimensional coordinate systems to represent graphically various mathematical relationships.
6. To strengthen understanding of relations and functions and to use these ideas throughout the course.
7. To teach in-depth linear and quadratic functions and relations.
8. To extend the laws of exponents to include rational and irrational exponents.
9. To develop logarithms from exponents and to develop skill in using logarithmic and exponential functions.
10. To teach the fundamental ideas of numerical trigonometry of the general angle.

CONTENT

I. SETS OF NUMBERS
   - A. Natural numbers
   - B. Integers
   - C. Rational numbers
   - D. Irrational numbers
   - E. Complex numbers

II. RELATIONS
   - A. Definitions and notations
   - B. Linear and quadratic functions
   - C. Inverse of a relation
   - D. Direct, inverse, and joint variation

III. POLYNOMIALS AND RATIONAL FRACTIONAL EXPRESSIONS

IV. ALGEBRAIC EQUATIONS AND INEQUALITIES INVOLVING ONE VARIABLE
- A. Solution of quadratic equations
- B. Solution of special higher degree polynomial equations
- C. Solution of special irrational equations
- D. Solution of quadratic inequalities

V. SYSTEMS OF EQUATIONS AND INEQUALITIES
- A. Solutions of systems of linear equations and inequalities by linear combinations, substitution and graphs
- B. Determinants
- C. Solutions of systems of linear equations using determinants
- D. Solutions of systems involving linear and quadratic equations in two variables

VI. EXPONENTIAL AND LOGARITHMIC FUNCTIONS
- A. A review of integral and rational exponents and the extension of the concept to include any real exponents
- B. Exponential functions and equations
- C. The logarithm of a number to a base "a" as the inverse of an exponential function
- D. Properties of logarithms and computation using logarithms
- E. Solutions of exponential and logarithmic equations and inequalities

VII. COMPLEX NUMBERS
- A. Sums, products, differences, quotients, powers and roots of complex numbers in rectangular form
- B. Fundamental operations with complex numbers in exponential form

VIII. TRIGONOMETRIC FUNCTIONS
- A. Radian measure
- B. Definition of certain quotient functions
- C. Reference angle
- D. Periodicity
- E. Complementary angles
- F. Complementary functions
- G. Use of trigonometric tables
- H. Graphs of the basic trigonometric functions (cosine, sine, tangent, secant, cosecant, cotangent) through one complete period
- I. Use of trigonometric functions to determine components of vector quantities
- J. Pythagorean identities
- K. Law of Cosines
- L. Law of Sines
- M. Formulas for sums and differences of angles
- N. Solutions of trigonometric equations and proof of trigonometric identities
- O. Solutions of triangles
- P. Inverse trigonometric relations and functions
- Q. Algorithms with complex numbers using polar form
- R. DeMoivre's Theorem (proof optional)

IX. SEQUENCES AND SERIES
- A. Sequences and Series
- B. Progressions and sums of progressions

X. MATHEMATICAL INDUCTION (Optional)

XI. PROBABILITY (Optional)
- A. Permutations and combinations
- B. Binomial Theorem and Binomial Series
- C. Probability

XII. MATRICES (Optional)

A. Symbolism used with matrices
B. Special kinds of matrices
C. Definition of the sum of certain kinds of matrices
D. Definition of the product of certain kinds of matrices
E. Properties of addition and multiplication of matrices
F. Applications

Consumer Mathematics

COURSE NUMBER 2410

The goal of the course in consumer mathematics is to provide students with basic mathematical skills and competencies necessary for economic survival in the highly competitive society of today. The skills and competencies in mathematics of the students enrolling in this program should be carefully assessed. Instruction should then be based on the findings of such an assessment. If the student is lacking in basic computational skills, remedial work should be provided at a level appropriate for the individual student.

The ability to read and interpret intelligently current data and statistical information is needed in many activities of the consumer. An understanding of appropriate methods of selecting data for given purposes, desirable sizes of samples, the representativeness of given samples, the comparability of data given, and the relevance of data should be striven for. Methods of presentation of data (various types of tables and graphs) and interpretation of data should be investigated. The above-named topics in elementary descriptive statistics should be developed as tools for later topics in the course, and should be used in discussions of later work as much as possible. Introductory experiences should involve as much concrete material for illustration purposes as possible. It is not the purpose of this unit to provide an in-depth study of statistics. Some classes may, however, have students with the interests and ability to do further work in statistics. For these students, a beginning study of measures of central tendency (mean, median, and mode) and dispersion (range, mean deviation, average deviation, and standard deviation) may be appropriate.

Almost all — if not all — adults have to be concerned with the earning of a living and the spending of their money. Many problems in adult life can be traced to the inability to control the spending of money. To help in learning how to curb unnecessary spending and to find better ways of saving and spending, budgeting should be studied. The reasons for and the purposes of budgets can be developed by students in class discussion. Income and outgo of money from and to all sources need to be considered, along with the types of records possible and the problems that can exist.

Our American economy has often been described as one built upon credit. Being an integral part of our modern life, credit should be understood as actually being a way of borrowing money rather than of borrowing time to pay. The dangers and advantages of the various plans should be known by all who expect to use them. Short-term charge accounts and long-term installment buying should be compared and contrasted, along with the various plans of installment buying. The paying of cash by means of borrowing money should be compared with these other more usual credit plans.

The members of our society have come to expect local, state, and federal governments to provide many comforts, conveniences, and services. All three forms of government are financed by taxes, usually property taxes at the local level, income and sales taxes at the state level, and income taxes at the national level. In the study of income taxes, concern must be given to income deductions, exemptions, and computation of taxes. The difference between real and personal property must be considered with property taxes, as must such concepts as assessed value and tax rates. Many other taxes

also exist that are a normal part of our expenses: excise taxes, licenses, gasoline taxes, inheritance taxes, and import duties.

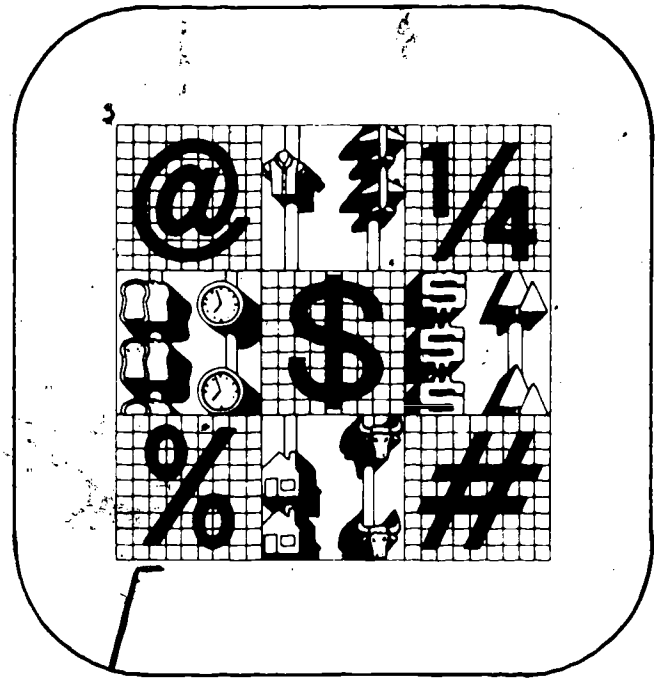
What is insurance and why is it desirable or necessary? Is insurance an investment or is it an operating expense? In the study of personal and property insurances, these are just two of the many possible questions. The various types of life insurance policies should be discussed. Health, accident, and hospitalization insurances are often provided in many employment situations. The worker should know the features of these policies. In North Carolina, automobile owners must by law have automobile liability insurance. Many abuses have occurred in financing this type of insurance because of a lack of understanding. Other property insurances — fire, theft, damage — are often needed by the owner. Personal liability insurance can often prove of value in protecting a person from financial disaster. Insurance can also provide funds for future retirement. Social security and other forms of annuities are appropriate topics for discussion.

If persons making living wages can manage their money well, they should have money beyond that needed for ordinary living expenses, money which perhaps should be considered for investment. The characteristics of a good investment — safety, salability, and income — are points to study. Various kinds of investments should be compared and contrasted: life insurance, savings accounts, social security, real estate, and stocks and bonds.

Although many of the previous topics will of necessity have to be considered as isolated topics, wherever possible, broad units that will involve many of these topics should be developed. For example, a study of the problem of home ownership versus rental would entail many of these separate areas. The problem of providing adequately for retirement would be another such broad unit. The teacher should develop this type of unit where possible.

The teacher should involve the type of student taking this course as much as possible in the actual discussion and development of content. Much work of a concrete level will be necessary, as will be many specific illustrations and examples from everyday sources. The teacher should not attempt to go excessively into the theories behind these areas of concern with the majority of students, but should attempt to provide the basic survival skills needed by average citizens.

This is a mathematics course, not an economics course, and should be taught with this in mind. Although primary emphasis is upon the applications of mathematics to social situations, mathematical theory should not be completely neglected. At all times, whenever computational algorithms are used, efforts should be made to continue the development of the understanding of the mathematical principles behind them.



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OBJECTIVES

1. Investigate the functions of a bank and the services a bank provides consumers.
2. Perform the following tasks: (a) open an account; (b) write a check; (c) determine a balance; (d) make out a deposit ticket; and (e) reconcile a bank statement.
3. Investigate the interrelationships of credit and the nation's economy.
4. Perform the following tasks: (a) determine the amount of interest on credit; (b) compute the rate of interest; (c) find the difference between the costs of various credit plans; and (d) compute gross, net or disposable income, and discretionary income.
5. Identify variations in housing needs and the factors influencing them.

6. Perform the following tasks: (a) find the difference between the purchase price of a home and the cost of owning and maintaining a home; (b) use interest tables to derive the purchase price of a house under different mortgage plans; (c) find variations in mortgage costs as determined by the number of payments, interest rate, and amount of the loan; (d) make a scale drawing of a floor plan; and (e) estimate the costs of simple home repairs, renovations, and utilities.
7. Identify various forms of insurance along with the purposes, advantages, and disadvantages of each.
8. Identify factors which are used in determining premium rates of insurance policies.
9. Use tables to determine: (a) the costs of various types of insurance, and (b) the amount of an employee's current FICA tax contribution.
10. Investigate the elementary concepts of probability and statistics that affect the typical consumer, in particular, how sampling techniques and graphs can be used to present a biased picture.
11. Perform activities in probability and statistics which will make the student better aware of these important branches of mathematics.
12. Identify various forms of taxation as well as the derived benefits from tax dollars.
13. Perform the following tasks: (a) complete state and federal tax forms; (b) compute direct and indirect taxes on specified items; and (c) keep records for making tax reporting easier.
14. Investigate the various modes of transportation and the advantages and disadvantages of each.
15. Identify the costs other than purchase cost relative to automobile ownership, operation, and maintenance, and determine the aggregate cost.
16. Investigate various schemes used to defraud the public and the agencies which can be called upon to assist a victim of fraudulent practices.

CONTENT

I. BANKS AND THEIR SERVICES
   A. Types of banks
   B. Bank services

II. CONSUMER CREDIT
   A. Using credit
   B. Kinds of credit plans
   C. Cost of consumer credit
   D. The credit contract
   E. Legal aspects

III. HOUSING
   A. Buying a house, condominium, and mobile home
   B. Renting a house, apartment, and mobile home

IV. INSURANCE
   A. Basic kinds of insurance
   B. Buying insurance

V. PROBABILITY AND STATISTICS
   A. Probability
   B. Statistical data
      1. Collection
      2. Analysis

VI. MONEY MANAGEMENT
   A. Income
   B. Budgets
   C. Buying practices
   D. Consumer protection

VII. SAVINGS AND INVESTMENTS
- A. Savings
- B. Investments

VIII. TAXES
- A. Federal income tax
- B. State income tax
- C. Property taxes
- D. Other taxes: sales, excise, inheritance, gift, use, and intangible

IX. SWINDLES AND GYPS
- A. Deceptive advertising
- B. Deceptive pricing
- C. Business opportunity schemes
- D. Home improvement schemes
- E. Referral schemes
- F. Vacation schemes
- G. Games of chance
- H. Charity collections
- I. Self-improvement schemes
- J. Flim flam
- K. Worthless stocks and bonds
- L. Land sales
- M. Family swindles

X. TRANSPORTATION
- A. Personal transportation
  1. Purchasing, leasing, and renting a vehicle
  2. Continuing costs
  3. Insurance
- B. Trip planning
- C. Specialized recreational vehicles

Advanced Mathematics

COURSE NUMBER 2400

This course, usually taken in the twelfth grade, is designed primarily for those students who intend to continue their study of mathematics beyond the high school level. Completion should prepare the students to begin the study of calculus and analytic geometry in college.

With this objective in view, any topics not covered in previous courses should be studied before proceeding with the new material of this course. This may include sequences, permutations, probability, and the binomial theorem. An introduction to infinite sequences and series should be made, and summation notation ($\sum$) used.

Mathematical induction should be studied and used to prove a variety of properties, including formulas for progressions and the binomial theorem.

The algebra of functions — addition, subtraction, multiplication, division, and composition of functions — should be given careful treatment. This work will provide a good review of the concept of a function, considered both as a mapping and as a special set of ordered pairs. Particular attention should be paid to the domains and ranges of new functions constructed by combining two functions. The concept of the inverse of a function should be presented.

Polynomials and polynomial functions should be studied intensively. This study should include the factor and remainder theorems, synthetic division, theorems about roots, and methods of calculating roots.

Some further study of the exponential and logarithmic functions may be included. The slope function may be introduced as a preparation for calculus. Trigonometric (circular) functions should be defined as functions of real numbers and their relations to functions of angles made clear. Work in trigonometry should include a study of the periodicity of the trigonometric functions, graphs, applications to uniform circular motion and waves, identities, equations, addition and related formulas, and inverse trigonometric functions. Careful attention should be given to the domain and range of all the functions studied.

Applications of trigonometry may be made to vectors and the polar form of complex numbers, using DeMoivre's theorem, to calculation of roots, and to complex numbers.

The remainder of the course should be devoted to analytic geometry. Lines and linear equations, systems of lines and systems of linear equations, and conics and second degree equations constitute the material to be studied.

As many properties of circles, parabolas, ellipses and hyperbolas should be investigated as time permits.

Optional additional topics include matrices and determinants.

OBJECTIVES

1. To extend previously studied principles of analytic geometry.
2. To emphasize the role of logic in deductive systems of mathematics.
3. To emphasize the circular functions of real numbers and the analytical aspects of geometry
4. To consider some of the principles and applications of permutations, combinations, and probability.

5. To develop understanding and skill in expanding binomial expressions.
6. To introduce mathematical induction as a method of proof and to use this method to prove a variety of properties, including formulas for progressions, the binomial theorem and DeMoivre's theorem.
7. To study in depth the properties of polynomial functions and methods for solving polynomial equations.
8. To further examine the exponential and logaritr ic functions.
9. To give the student experience with an algebraic system different from the real and complex number systems by introducing the algebra of matrices.
10. To use matrices and determinants to solve systems of linear equations.

CONTENT

I. FUNCTIONS
   A. The function concept
   B. Classificat of functions
   C. Algebra of functions
   D. Graphs of functions
   E. Inverse of a function

II. POLYNOMIALS
   A. Factor and remainder theorem
   B. Synthetic division
   C. Fundamental Theorem of Algebra
   D. Other theorems concerning existence and number of roots
   E. Methods of computing roots

III. EXPONENTIAL AND LOGARITHMIC FUNCTIONS
   A. Review of rational exponents
   B. Exponential functions
   C. Continuity of exponential functions
   D. Inverse of exponential functions
   E. Linear interpolation
   F. Combination of logarithmic functions and other functions

IV. CIRCULAR (TRIGONOMETRIC) FUNCTIONS
   A. Functions of real numbers
   B. Periodicity
   C. Graphs
   D. Applications to circular motion and waves
   E. Identities
   F. Equations
   G. Addition formulas and similar formulas
   H. Inverse trigonometric functions
   I. Applications to vectors and complex numbers

V. ANALYTIC GEOMETRY
   A. Lines
   B. Systems of lines
   C. Circles
   D. Parabolas
   E. Ellipses
   F. Hyperbolas
   G. The Slope Function
   H. Lines and planes in three space

VI. MATHEMATICAL INDUCTION

VII. SEQUENCES, SERIES, AND LIMITS
- A. Arithmetic and geometric sequences
- B. Sums of arithmetic and geometric sequences
- C. Limits of infinite sequences
- D. Sums of infinite series
- E. Sigma notation

VIII. LIMITS
- A. Intervals on the real number line (Neighborhoods)
- B. Limit of a sequence
- C. Definition of limit
- D. Convergence of a sequence
- E. Divergence of a sequence

IX. MATRICES AND DETERMINANTS
- A. Matrices and determinants of order 2
- B. Matrices and determinants of order 3
- C. Properties of determinants
- D. Cramer's Rule
- E. The sums and products of matrices
- F. Multiplicative inverse of a matrix
- G. Matrix solution of a system of equations

X. PROBABILITY AND STATISTICS
- A. Basic concepts
- B. Counting principles
  1. Permutations
  2. Combinations
  3. Binomial Theorem
- C. Probability of equally likely terms
- D. Organizing and reporting data
- E. Random sampling
- F. Theoretical distributions
- G. Inferential statistics
- H. Game theory

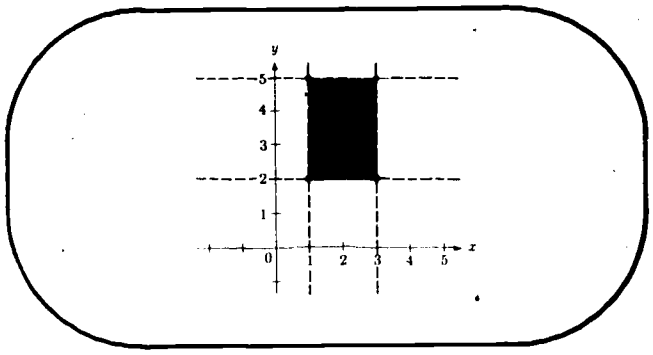
XI. GROUPS, RINGS, AND FIELDS (Optional)

If a school is operating on the quarter or semester system, it might be more convenient to teach the course content from Advanced Mathematics as self-contained topical courses. Three separate courses which could make up the course content of Advanced Mathematics are Trigonometry, Analytic Geometry, and Advanced Algebra. A brief discussion of each course and a course description of each of these follows.

Advanced Algebra

Course Number 2402

This course is designed to provide an in-depth look at some of the important algebraic ideas that often receive only a brief treatment in Algebra II or in some cases, they are not discussed at all. Some of the topics included in this course are of the nature that they will be used as "stepping stones" to more complex mathematical topics. Yet, others are of the nature that they can be used in some of the applications of mathematics.



OBJECTIVES

1. To investigate the nature of polynomial functions of degree n, $n \geq 2$, and methods for finding their zeros whenever they exist.
2. To study systems of equations which contain two or more variables and the various methods for determining common solutions.
3. To examine the nature of combinations and permutations and to solve problems involving them.
4. To explore the properties of matrices and determinants and how they can be used to solve systems of equations.
5. To examine the method of mathematical induction and to use this to prove various mathematical statements.
6. To study the characteristics of arithmetic and geometric series along with techniques for representing series.
7. To explore the notion of a limit and some of the theorems about limits.

CONTENT

I. POLYNOMIAL FUNCTIONS OF DEGREE n, $n \geq 2$
   - A. Product of two binomials
   - B. Product of two trinomials
   - C. Factoring binomials and trinomials
   - D. Remainder Theorem
   - E. Factor Theorem and its converse
   - F. Synthetic division
   - G. Graph of a polynomial
   - H. Locating the roots
   - I. Number of roots
   - J. Bounds of the real roots
   - K. Descartes' Rule of Signs
   - L. Imaginary roots
   - M. Approximation of irrational roots

II. SYSTEMS OF EQUATIONS IN TWO OR MORE VARIABLES
   - A. Solution by graphical methods
   - B. Consistent, inconsistent, and dependent equations
   - C. Solution by algebraic methods
   - D. Linear programming

III. PERMUTATIONS AND COMBINATIONS
- A. The Fundamental Counting Principle
- B. Permutations of n things taken r at a time
- C. Permutations of n things not all different
- D. Cyclic permutations
- E. Combinations
- F. The sum of certain combinations

VI. MATRICES AND DETERMINANTS
- A. Matrices and determinants of order 2
- B. Matrices and determinants of order 3
- C. Properties of determinants
- D. Cramer's Rule
- E. The sums and products of matrices
- F. Multiplicative inverse of a matrix
- G. Matrix solution of a system of equations

V. MATHEMATICAL INDUCTION

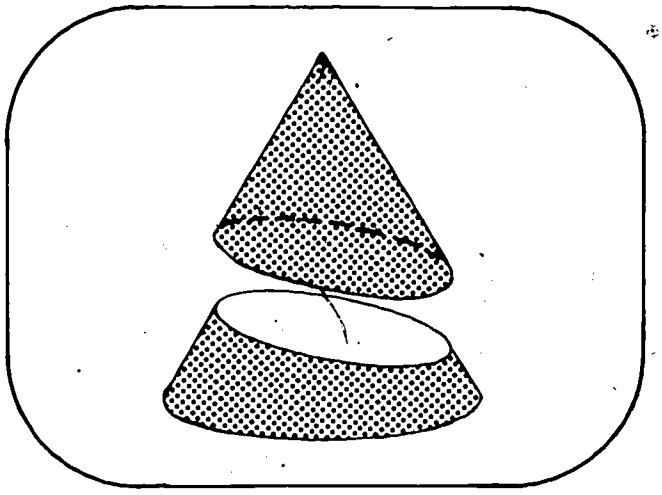
VI. SEQUENCES, SERIES AND LIMITS
- A. Arithmetic and geometric sequences
- B. Sums of arithmetic and geometric sequences
- C. Limits of infinite sequences
- D. Sums of infinite series
- E. Sigma notation

Analytic Geometry

Course Number 2406

Analytic Geometry serves as a bridge between algebra and geometry. It should help the student to see how one area of mathematics can help clarify and extend ideas from another area. The examination of coordinate systems of one and two spaces should enable the student to begin to generalize coordinate systems beyond two dimensions.

Considerable attention should be given to the study of the properties of lines, linear equations, systems of linear equations, conics and second degree equations. This should be followed by an examination of the properties of circles, parabolas, ellipses, and hyperbolas.



OBJECTIVES

1. To examine the use of the rectangular coordinate system as a means of illustrating algebraic expressions and the relationships that exist between many of them.
2. To investigate the equations of lines with special characteristics.
3. To stress techniques for sketching the graphs of any equation of the form $AX^2 + BY^2 + CX + DY + E = 0$.

4. To examine how to graph some algebraic equations on a polar coordinate system.
5. To introduce students to the use of space coordinates in graphing.
6. To study how analytical methods can be applied to the proofs of geometric theorems.

CONTENT

I. LINES
   A. Coordinate systems
   B. Slope of a line
   C. Forms of first degree equations
   D. Distances

II. SYSTEMS OF LINES
   A. Parallel lines
   B. Concurrent lines
   C. Intersecting lines

III. PLANES
   A. Parallel planes
   B. Intersecting planes

IV. CONIC SECTIONS
   A. General equations of a conic
   B. Circle
   C. Parabola
   D. Ellipse
   E. Hyperbola

V. CURVE TRACING
   A. Intercepts
   B. Asymptotes
   C. Symmetry
   D. Periodicity
   E. Excluded values

VI. POLAR COORDINATES
   A. Graphs of polar coordinate equations
   B. Relations between polar and rectangular coordinates
   C. Polar equations of lines and conics

VII. THREE DIMENSIONAL ANALYTIC GEOMETRY
   A. Space coordinates
   B. Cylindrical surfaces
   C. Figures of revolution

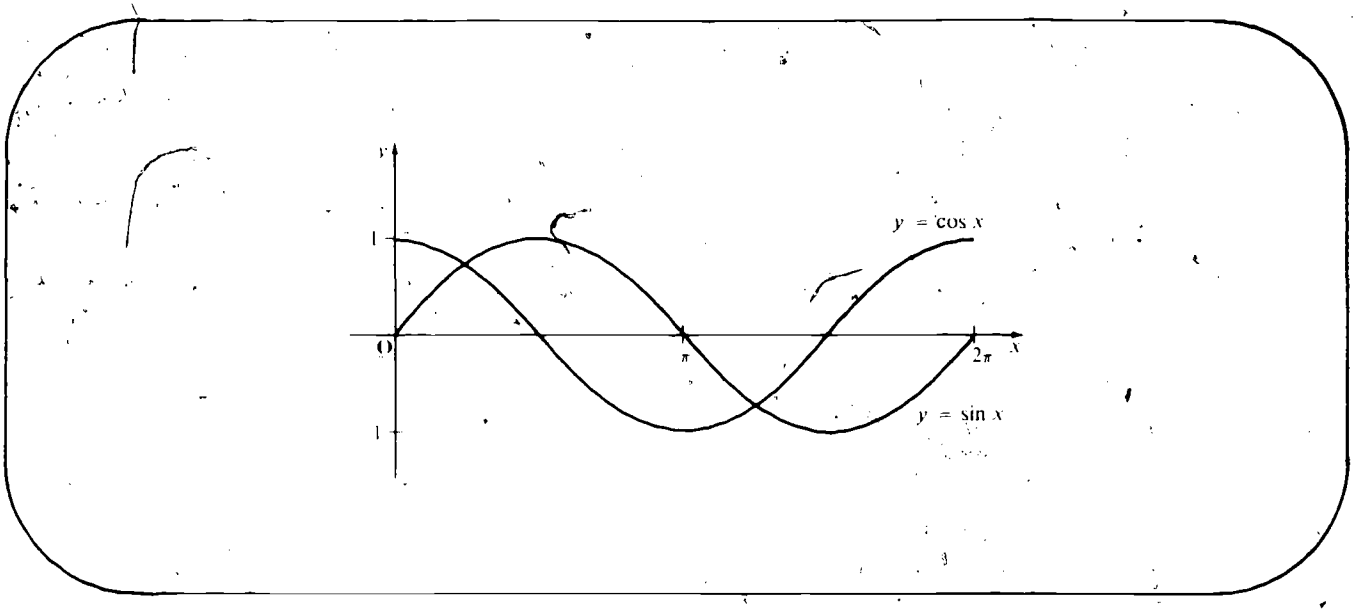
Trigonometry

Course Number 2401

Efforts should be made to show how trigonometry relates to other branches of mathematics. The role of functions, both circular and trigonometric, should be emphasized in developing trigonometric concepts. This emphasis should help the student gain an understanding of the ideas associated with angles, triangles, and vectors. In addition, this course should provide the student with opportunities to explore some of the applications of trigonometric concepts.

OBJECTIVES

1. To explore the definitions of the six trigonometric functions and to use these in the derivation of other trigonometric relations.
2. To emphasize the graphs of trigonometric functions as a means of picturing how the domain, range, period, and amplitude of an equation influence the graph of the equation.
3. To examine the definitions and graphs of the inverse trigonometric functions.
4. To investigate the solutions of equations involving trigonometric and inverse trigonometric functions.
5. To study many of the applications of trigonometry including those which involve the Law of Sines, Law of Cosines, and right triangle relations.
6. To explore the polar coordinate plane and use this as another method of exploring complex numbers.
7. To re-examine logarithmic functions and extend these to include logarithms of some of the trigonometric functions.



CONTENT

I. INTRODUCTION
   A. Review pertinent algebraic and geometric concepts: relations, functions, mappings, domain, range, axioms for real numbers, exponents and radicals, absolute value, and inequality statements; Pythagorean Theorem, and the distance formula.

II. THE WRAPPING FUNCTION AND/OR CIRCULAR FUNCTIONS
   A. Definition of the six trigonometric functions
   B. Cofunctions
   C. Reciprocal functions
   D. Special angles
   E. Radians to degrees and degrees to radians

III. GRAPHING TRIGONOMETRIC FUNCTIONS
   A. Amplitude
   B. Period
   C. Translation
   D. Converses and inverses

IV. EQUATIONS AND IDENTITIES
   A. The fundamental identities
   B. Equivalent trigonometric expressions

C. Solutions of trigonometric equations
D. Proving identities
E. Using identities to solve equations
F. Sine, cosine, and tangent of the sum and difference of two angles
G. Functions of twice an angle
H. The product and sum formulas

V. INVERSE TRIGONOMETRIC FUNCTIONS
A. Inverse of a function
B. Inverse trigonometric functions
C. Inverse trigonometric equations

VI. POLAR FORM OF COMPLEX NUMBERS
A. Polar coordinates
B. Complex numbers: introduction, operations, and trigonometric representation
C. DeMoivre's Theorem

VII. SOLVING TRIANGLES
A. Right triangles
B. The Law of Sines
C. The Law of Cosines

VIII. VECTORS
A. Addition and subtraction
B. Scalar multiplication and dot product
C. Vectors in physics
D. Vector components

IX. LOGARITHMS
A. Exponential and logarithmic functions and their graphs
B. Laws of logarithms
C. Common logarithms, operations, and applications
D. Computation
E. Interpolation
F. Logarithms of other number bases

Probability and Statistics

Course Number 2408

The study of probability includes the study of experiments involving chance events, the outcomes of such experiments, and the likelihood that particular outcomes will occur. The probability of an outcome for a particular experiment is a numerical measure of the likelihood that the outcome will occur. The theory of probability can then be considered to be the methods for assigning probabilities to outcomes for experiments and the study of the relationships among them.

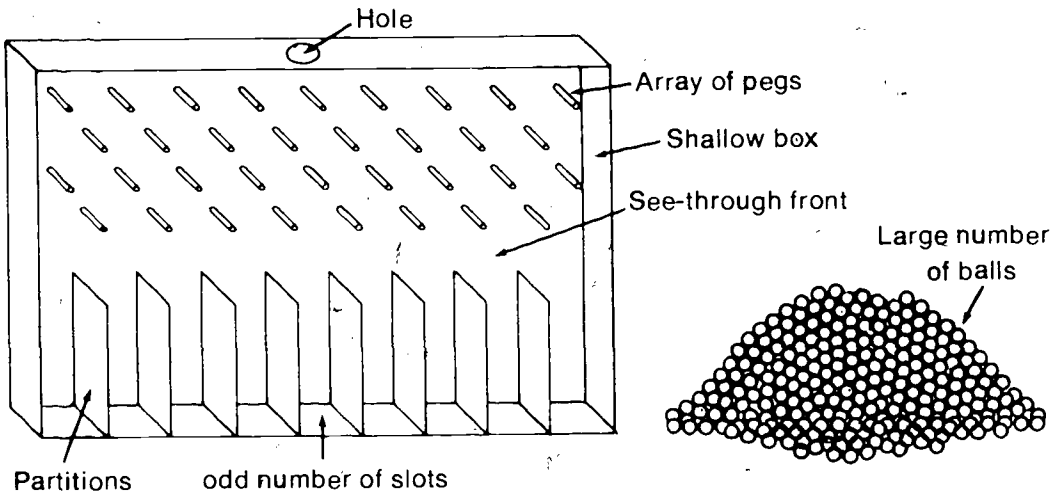
Statistics refers to the ways of collecting, organizing, analyzing, interpreting, and summarizing numerical data of all kinds.

Recent applications of mathematics to the social, biological, and physical sciences have increasingly involved probability and statistics. Understanding the predictions of voting patterns, the validity of opinion polls, and the merchants' application of opinions of questionnaires to decision making requires an understanding of probability and statistics. Such an

understanding will better enable students to appreciate the extent of certainty of events in their world.

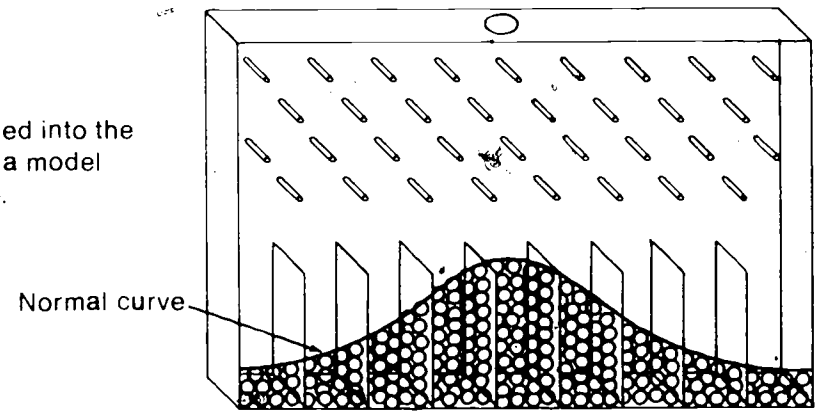
It is suggested that many students be encouraged to take this course. Strong emphasis should be put upon laboratory exercises and individual or group projects that will allow students to investigate probability and statistics at the various levels of their mathematical abilities.

The Galton Board



Build a model of the Galton Board.
(Suggested materials: wood, plastic, heavy cardboard)

If the balls are poured into the hole, they will form a model of the normal curve.



The board was invented by a British mathematician, Sir Francis Galton (1822-1911).

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OBJECTIVES

1. Interpret data presented in tabular and graphical form.
2. Organize and present data in tabular and graphical form.
3. Summarize and analyze selected data by calculating the measures of central tendency (mean, median, and mode); discuss their relative merits.
4. Calculate measures of dispersion: range, quartile deviation, variance, and standard deviation.
5. Compute the mean and standard deviation for large numbers of measurements by grouping techniques.
6. Determine whether outcomes of an experiment are equally likely.
7. Determine whether outcomes of an experiment are mutually exclusive.
8. Determine the sample spaces of various experiments.
9. Express events as subsets of a sample space.
10. Determine whether two events are complementary.
11. Determine whether events are exhaustive.
12. Develop and apply the formulas for calculating the number of permutations and combinations of n objects taken r at a time, and determine whether the problem involves permutations or combinations.
13. Apply correctly the two basic counting principles to determine the number of outcomes in event "A or B" and event "A and B."
14. Determine the probability of an event in a finite sample space using the classical definition of theoretical probability.
15. Identify binomial experiments and apply the laws of chance to the binomial distribution.
16. Compute the conditional probability of event A given B.
17. Apply the appropriate additive and multiplicative theorems to determine the probability of multiple events.
18. Distinguish between samples and populations.
19. Use elementary sampling theory to perform a random sampling.
20. Apply the theory of probability to acceptance sampling.
21. Apply the theory of probability to test statistical hypotheses involving normal and binomial distributions
22. Use samples to make estimates of population measures.

CONTENT

I. BASIC CONCEPTS

II. COUNTING PRINCIPLES
   - A. Permutations
   - B. Combinations
   - C. Binomial Theorem

III. PROBABILITY OF EQUALLY LIKE OUTCOMES

IV. ORGANIZING AND REPORTING DATA

V. RANDOM SAMPLING

VI. THEORETICAL DISTRIBUTIONS

VII. INFERENTIAL STATISTICS

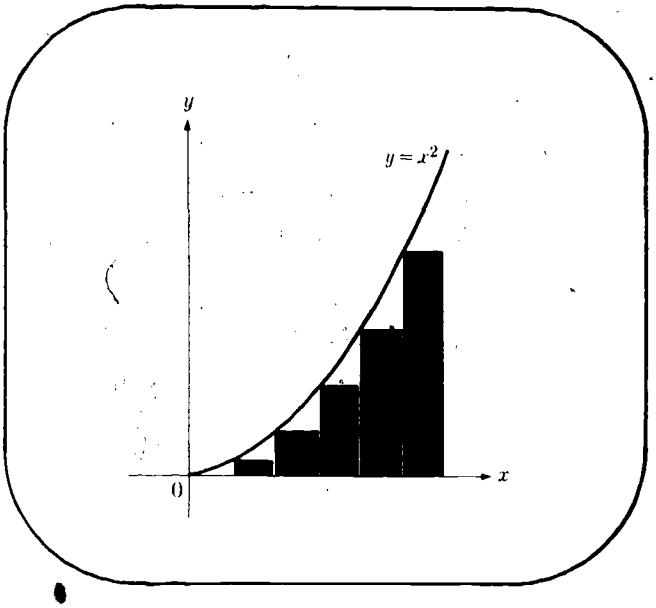
VIII. GAME THEORY

Calculus

COURSE NUMBER 2420

A first course in calculus at the high school level could serve as the culmination of a mathematically talented student's high school program. It should serve as a course which takes the mathematics a student has previously learned and applies this in a new setting. The content should be presented from a point of view that maintains a balance between the theory of calculus and its many applications.

The students in this course will be at many levels of intellectual maturity and will have varying amounts of mathematical motivation. Some students will be able to understand the methods of proof while others may only be able to grasp an intuitive understanding of the notion being proved. The teachers will have to determine the degree or rigor and abstraction to be maintained in the course.



OBJECTIVES

1. Define functions, and state the domain and range of a function. Illustrate various methods of combining functions.
2. Analyze the properties of the conic sections, write the general equations of the conic sections using the definitions or standard forms, and write basic geometric proofs using analytic geometry.
3. Write the equation of a line using the two-point form, point-slope form, or slope-intercept form; and express in the standard form, $AX + BY + C = 0$.
4. Graph functions and relations using the concepts of symmetry, asymptotes, domain, range, translation of axes, and rotation of axes.
5. Find the angle between two intersecting lines, and find the distance from a point to a line.
6. Find the limit at a finite point and at infinity, if either or both exist, of selected algebraic functions.
7. Use L'Hospital's rule, if applicable, to find the limit, if it exists, of a function which is discontinuous at a point.
8. Apply theorems on limits to find the limit at a finite point or at infinity, if either or both exist, of selected functions.
9. Prove: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$
10. Use the definition of a derivative to find the derivative of selected algebraic functions.

11. Use the sum, difference, product, quotient, and chain rule to find the derivative of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
12. Use the derivatives of f(x) = sin x and g(x) = cos x to derive the derivatives of sec x, csc x, tan x, and cot x.
13. Use the derivative to find the maximum, minimum, and points of inflection of the graph of a function.
14. Use the derivative to find the equation of the tangent and normal line to a curve at a point on the curve.
15. Use the derivative to find the equation of the tangent lines from an external point to a curve.
16. Use the derivative to find the velocity and acceleration, and solve related rate problems.
17. Use the derivative to approximate roots of an equation.
18. Use the derivative to find the angle between the graphs of two curves.
19. Find and apply the derivative of implicit functions and equations written in parametric form.
20. Use the methods of substitution, partial fractions, and integration by parts to find the integral of algebraic, trigonometric, inverse trigonometric, exponential and logarithmic functions.
21. Use the definite integral to find the area under a curve and between two curves, length of an arc, surfaces of revolution, and centroids.
22. Use the definite integral to find volumes of solids of revolution, find volume by the cylindrical shell and disk methods, and find volume by the method of slicing.
23. Apply the integral to selected problems of physics such as hydrostatic force, work, acceleration, and velocity.
24. Use the trapezoidal rule to approximate selected definite integrals.
25. Evaluate integrals which are infinite or discontinuous.
26. State and illustrate the basic theorems such as the fundamental theorems of differential and integral calculus, mean value theorems of differential and integral calculus, and theorems of continuous functions.

CONTENT

I. FUNCTIONS
   - A. Relations
   - B. Definition of a function
   - C. Domain and range of a function
   - D. Combining functions
   - E. Intercepts, symmetry, and asymptotes
   - F. Composite function
   - F. Composite function
   - G. Continuous and discontinuous functions
   - H. Power functions
   - I. Inverse functions

II. ELEMENTS OF ANALYTIC GEOMETRY
   - A. The rectangular coordinate plane
   - B. Distance formula
   - C. Midpoint formula
   - D. Slope of a line
   - E. Equation of a line
   - F. Parallel and perpendicular lines
   - G. Angles between two lines
   - H. The circle
   - I. The parabola
   - J. The ellipse
   - K. The hyperbola
   - L. Translation and rotation of axes
   - M. Conic sections

III. LIMITS
- A. Intuitive view of limits
- B. Definition of limits
- C. Theorems on limits
- D. Limits at infinity
- E. Limits of sequences

IV. DIFFERENTIATION OF ALGEBRAIC FUNCTIONS
- A. Polynomial functions and their derivatives
- B. Inverse functions and their derivatives
- C. The chain rule
- D. The differentials dx and dy
- E. Higher order differentiation

V. APPLICATIONS OF DIFFERENTIATION
- A. Curve plotting
- B. Rolle's theorem
- C. The mean value theorem
- D. Maxima and minima
- E. Points of inflection

VI. INTEGRATION
- A. The indefinite integral
- B. Area under a curve
- C. Area between curves

VII. THE DEFINITE INTEGRAL
- A. The definite integral
- B. The fundamental theorem of calculus
- C. Area between curves
- D. Distance
- E. Volume
- F. Area of a surface of revolution
- G. Work

VIII. TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS
- A. Trigonometric functions
- B. Differentiation of trigonometric functions
- C. Integration of trigonometric functions
- D. Inverse trigonometric functions
- E. Logarithmic functions
- F. Exponential functions
- G. Differentiation of logarithmic and exponential functions

Additional Curriculum Considerations

Some of the courses (Engineering Concepts Curriculum Project and Algebra II with Computer Programming) discussed in this section have already been successfully taught in a limited number of schools in North Carolina. The others are courses which a school system might develop locally on an experimental basis. If the result of such an effort is satisfactory, the local school system should share this with the Mathematics Division so that other schools might be encouraged to include the course in their curriculum.

The Role Of The Computer In Education

Many recent changes in our society can be traced to the electronic computer. Automation is often controlled by the computer. Data processing by the computer is changing our accounting and record-keeping procedures. Scientific research can be done much more quickly than in the past due to the capability of the computer to provide an almost instantaneous analysis of data. The computer is used as an aid in decision making in government, education, and business. Medical doctors are using information retrieval by the computer for diagnostic purposes when working with their patients. Space travel and commercial airline scheduling are dependent on the computer. Even historians, linguists, and psychologists are using the computer for research in their fields.

Some schools have access to computer facilities through the use of computer terminals. These terminals make it possible for students to use a computer for the solution of a variety of mathematical problems. It is increasingly evident that more schools must explore ways of providing computer access to all students to the extent that they become acquainted with the nature of the computer.

The next two courses described in this publication were designed for use with the computer. They have been successfully taught in several schools across the State. It is hoped that more schools will be able to offer these two courses and other computer related mathematics courses in the future.

Algebra II With Computer Programming

Course Number 2301

One of the objectives of this course is to use computer methods to stimulate interest in mathematics. Most of the topics of Algebra II as described on page 00 are found in this course although the emphasis placed on various topics may be different from that normally given. Computer techniques are used to support and reinforce the presentation of the mathematics.

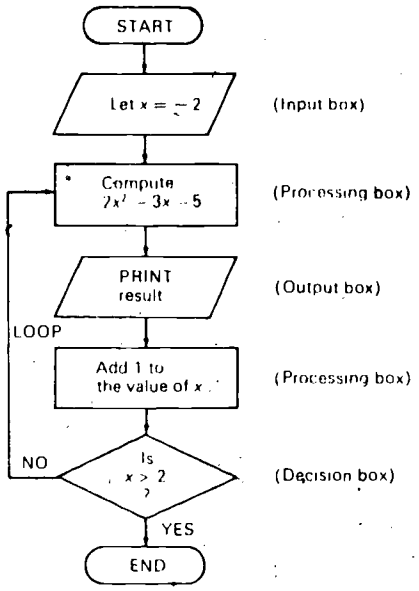
New learning experiences and techniques enhance the student's understanding of mathematics. These include flowcharting, the development of algorithms, modeling of mathematical concepts, and student programming.

Experience with the computer in schools is demonstrating that one of the finest ways for a student to reinforce his understanding of a topic is to teach the computer — that is, write a program that requires the computer to handle that particular topic.

BASIC language is used throughout. BASIC is easy to learn and at the same time its capabilities are sufficient for the problems that will be confronted in the course. In addition, BASIC is a language available to most schools having access to computing.

```
      10  LET X=-2
   ┌  20  PRINT 2*X↑2-3*X-5
loop  30  LET X=X+1
   │  40  IF X>2 THEN 60
   └  50  GOTO 20
      60  END
```

This program can be represented by the following flow chart (notice the shapes of the boxes)



There is no attempt in this course to develop exceptionally proficient programmers, although this may happen, but rather to introduce computer concepts and techniques that will strengthen the understanding of mathematics. This is a mathematics course rather than a computer science course and would normally be offered over two semesters.

Engineering Concepts and Curriculum Project-(ECCP)

Course Number 2404

The development of this course grew out of a concern over the fact that fewer and fewer students were electing to take mathematics and physical science courses at a time when the United States was entering "the age of technology." Survival of our civilization depends on our ability to adapt to changes which would follow technological developments and to control these changes. In a democratic society, this control can only result from an informed public. The widespread ignorance regarding almost anything which might be classified as "modern" in the field of technology justifies the development of this course.

The course is multi-disciplinary and, if taught as it is intended, the social studies aspects would be very evident as the mathematical and scientific techniques are presented. The interplay between technology and social, economic, political, and psychological forces is a predominant theme.

The "Engineering Concepts Curriculum Project" is intended for students with a background of at least one year of algebra who are college bound.

The course deals with that section of technology which is known as information systems to scientists and engineers. The concern then is with information, how it is communicated, how it is used to control systems, and how men and machines interact in a system. The computer seems to be the single most significant development in modern technology and, as an infor-

mation processor, this plays an important part in the course.

The systems approach to decision making and problem solving is the principal theme. Optimization (linear programming), modeling of real world problems on the analog computer (docking of a boat, landing a LEM on the moon, etc.), pollution (noise, air, water), communication (speech, music), feedback (disturbance control, self-regulation), stability (supply and demand, epidemics), man-machine interface (bio-medical engineering) are topics covered in the course which are understandable to the high school student. Students also study the history of the digital computer along with some of its uses.

Discussion of communication with the computer leads into an appreciation of the organization of the computer, of machine code, compilers and higher level languages.

The course emphasizes discussion and is a laboratory course which uses specially designed equipment. The cost of such equipment, in order to do a satisfactory job, would be between approximately $500 and $5,000, ranging from demonstrations to a complete set for each pair of students in a class of 24.

The mathematical content of the course touches on many topics (normally not included to any extent in high school) although vital to the world in which we live. These include queuing, probability, development of algorithms, game theory, exponential growth and decay, linear programming, and sinusoidal motion.

Algebra: An Applied Approach

The traditional course in Algebra I is not suitable for many students. Yet, there are some students who need and can profit from the study of algebraic ideas and problem-solving in practical situations. ALGEBRA: AN APPLIED APPROACH is a course to meet these needs. It is built around the concept of motivating the study of algebra. Students will not have to ask "what good is this?" or "what is it used for?" The uses of algebra will be as constantly before the student as the mechanics of algebra. Thinking in a concrete way will be more emphasized than the abstract, deductive mathematics of Algebra I.

It is intended that this course would contain enough mastery of algebraic content to serve the needs of students who must have algebra for college admission or job qualifications. Students who are more mature in their development and need little motivation other than intellectual stimulation should take Algebra I.

The content of this course would consist of four main strands: (1) the mechanics of manipulating algebraic entities; (2) the direct application of each newly developed technique to show its usefulness; (3) the development of formula reasoning and functional dependence; and (4) the formulation of algebraic models to solve problems.

The first strand contains the basic algebraic facts which will be presented without proofs. The algebraic content is presented as generalized arithmetic rather than as an abstract deductive system. Topics to be covered might include:

- review of integers
- using variables to write mathematical expressions
- solving simple equations
- fundamental operations with polynomials
- special products and factoring
- fundamental operations with rational expressions
- solving fractional equations
- fundamental operations with powers and radicals
- solving radical equations
- solving quadratic equations

The second strand would emphasize or highlight the use of each of the manipulative skills listed above. Situations and problems already formulated would be used to motivate the need for studying a topic. That is, the students would be shown what they will be able to do as a result of mastering each new aspect of the topic. After the topic is developed, it will be practiced in applied situations where its use can be shown. The applications of strand 2 are much more low-level than those of strand 4. In strand 2, students substitute into already derived formulas or relationships and interpret the results within the setting of the problem.

Some of the word problems in present textbooks dealing with physical situations that only call for direct translation with little real formulation would fall in this category.

Strand 3 emphasizes algebra as a subject that extends our power to reason. In this phase, solving, manipulating, and, more importantly, interpreting literal equations would be stressed. Functional relationships would be stressed as well as the ideas of variation.

The fourth strand involves identifying relationships in problems that can be presented algebraically or graphically. The stress in this strand is the formulation of a problem into mathematical terms. Students would generate formulas or graphical means to represent relationships in sets of ordered pairs, relationships in word problems, physical principles or relationships shown in geometric situations.

ALGEBRA: AN APPLIED APPROACH would not cover as broad a range of topics in algebra as Algebra I nor to the same depth. Rather it would concentrate on the mastery of a narrow band of essential content. The remaining time would be spent on motivation of the study of algebra, applications of the subject, and how it is used to model and solve realistic, worthwhile problems.

Geometry: An Applied Approach

GEOMETRY: AN APPLIED APPROACH is a course built around the concept of showing the uses and usefulness of geometry and geometric thinking. The course itself would be organized around four main strands: (1) geometric facts; (2) meaningful applications of geometric facts in practical problems; (3) logical reasoning and deduction; and (4) problem-solving using geometric modeling.

It is intended that this course would contain enough geometric content to serve the needs of students who must have geometry for college admission or job qualifications. Students who are more mature in their development of abstract thinking should take the regular geometry course which is less oriented toward concrete thinking.

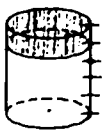
The content of the regular geometry course would be transmitted as basic facts without formal proofs. Theorems would be presented as statements concerning relationships in the physical world rather than propositions that can be logically derived from a set of assumptions called axioms. Instead of proofs, there would be verifications such as measuring to justify that an angle inscribed in a circle is one-half its intercepted arc. Or, pouring sand to show the volume of a right circular cone is one-third the volume of a right circular cylinder with the same base and height.

As theorems are proved, applications of them to the direct solutions of problems would be given. Applications would include discipline drawings, the science of perspective and other draftsmen and trades orientations of geometry. Drawings with ruler and protractor would be utilized as well as constructions with compass and straightedge.

Logical reasoning would be introduced through the use of flowcharts. Students would learn to analyze problems and reduce them to a step-by-step algorithm procedure. From this background, deduction would be introduced as well as some geometric proofs in flowchart form.

The entire emphasis of the logical reasoning strand would not be upon proofs of geometric theorems, but teaching for understanding of and transfer of deductive thinking and logical reasoning to other areas and activities.

A water tank is in the shape of a large cylinder. Its base has an area of 86π square meters, and its altitude measures 5 meters. Find the volume.



$$\text{Volume} = \frac{\text{no. of cubes}}{\text{per layer}} \times \text{no. of layers}$$

$$\text{Volume} = \text{base area} \times \text{length of altitude}$$

$$V = 86\pi \times 5$$

$$= 430\pi$$

Thus, the volume is 430π cubic meters.

The fourth strand would emphasize solving problems using geometric content as mathematical models. The stress would be upon problem formulation, i.e., abstracting from physical situations relationships that can be represented as relationships among points, lines, circles, triangles and angles. The students would then solve the mathematical formulation and translate the mathematical solution back to the physical settings. Problems would include construction problems and locus problems. Another example would be drawing the pattern for a Christmas ball where 5 congruent lines are to cover a ball with a 5-inch diameter. Another example would be to find the inside diameter of a pipe that must slip over a triangular rod.

A part of the course would have the spirit of a geometry drawing course for a draftsman or a skilled machinist. Included also would be the notion of proof and deduction in a broader sense and problem formulation skills associated with mathematical modeling.

Technical Mathematics

Technical mathematics provides the needed background for students who wish to enter skilled crafts and trades. These students need the mathematics that will enable them to pass employment and apprenticeship tests or to enter a technical or vocational curriculum for technicians and skilled workers. Algebra, geometry and trigonometry provide the needed background for students who wish to become scientists, engineers, and mathematicians. Business mathematics provides the necessary background for students who will enter the business world or business colleges.

A shortage of craftsmen, technicians, and skilled workers continues in our society at a time when we have an oversupply of both unskilled labor and college-trained specialists. The high school curriculum is in a unique position to offer a program that will both meet the interests of many students and also prepare these students for entry into the skilled trade and service areas. Mathematics courses that have been offered in most schools have not provided students with the mathematics they need to enter skilled areas immediately after they leave high school. Technical mathematics is an effort to rectify this situation.

The fixed portion of the content of Technical Mathematics should draw upon algebra, geometry, graphing, measurement, ratio and proportion, and numerical trigonometry. The first step in identifying appropriate mathematical applications should be to survey local industries so as to determine the skilled trades of the area and the mathematical requirements. A local technical or trade school could be helpful in this endeavor as they have records about trade and industrial needs in the area. Sample employment and apprenticeship tests should be used which simulate the type of situation that a prospective employee might experience. The sample should not be confined to just shop or industrial vocations. Vocations should be included that have been traditionally female as well as those which have been traditionally male.