DOCUMENT RESUME

ED 165 093 CS 004 565

AUTHOR Sternberg, Robert J.
TITLE Developmental Patterns in the Encoding and Combination of Logical Connectives. Technical Report No. 2.
INSTITUTION Yale Univ., New Haven, Conn. Dept. of Psychology.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE Oct 78
GRANT BNS-76-05311
NOTE 55p.

EDRS PRICE MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS Cognitive Processes; *Comprehension; *Comprehension Development; Elementary Secondary Education; Higher Education; *Language Development; Language Learning Levels; Language Patterns; *Language Research; *Language Skills; Linguistic Competence; Structural Analysis

ABSTRACT

A total of 224 subjects participated in a study to determine how children and adults comprehend logical connectives. Specifically, the study examined the effects of age, content, and practice on the encoding and combination of logical relationships expressed by six types of logical connectives: conjunction, disjunction, conditionality, biconditionality, simple affirmation, and simple negation. The subjects were drawn from grades 2, 4, 6, 8, 11, 12, and college. Half completed a task that required them merely to encode the connectives; the other half were required to combine as well as encode them. The results revealed significant effects of task, age, session, and connective, with comprehension of different connectives developing at different rates. Children as young as second grade encoded the connectives in consistent ways; however, it was not until fourth grade that they showed evidence of consistent combination of connectives. In general, comprehension of the conjunctive connective was the easiest, while comprehension of the conditional and biconditional connectives proved to be the most difficult. (Tables of results are included.) (FL)

Developmental Patterns in the Encoding and Combination of Logical Connectives

Robert J. Sternberg

Yale University

Technical Report No. 2 Cognitive Development Series October 1978

This research was supported in part by Grant BNS76-05311 from the National Science Foundation

Running Head: Logical Connectives

Send proofs to Robert J. Sternberg
Department of Psychology
Yale University
Box 11A Yale Station
New Haven, Connecticut 06520

Abstract

A total of 224 subjects in grades 2, 4, 6, 8, high school, and college solved problems requiring comprehension of the logical connectives and, or, if-then, only if, and if and only if, as well as the terms is and is not. Half the subjects were required merely to encode the connectives; the other half were required to combine as well as encode them. Problems were presented in two replications (over two sessions) via two different content vehicles. Quantitative analyses revealed significant effects of task, age, session, and connective, with comprehension of different connectives developing at different rates. In general, comprehension of the conjunctive connective was easiest; comprehension of the conditional and biconditional connectives was most difficult. Qualitative analyses indicated just how the logical connectives were interpreted at each grade level, and also investigated individual differences within each grade level.

Developmental Patterns in the Encoding and Combination of Logical Connectives

Understanding of logical connectives is essential to comprehension of everyday speech and reading, because these connectives tie together what otherwise would be disconnected strings of ideas. In everyday discourse, we usually assume that children understand logical connectives (such as and, or, if-then, only if, if and only if) in the same way that adults do (to the extent that the children understand them at all). If, however, children of varying ages understand logical connectives in systematic ways, but in ways differing from those of adults, communication between children and adults may be impaired unless the children's understanding of the terms is taken into account. Since there is in fact evidence that children understand logical connectives differently from adults (for example, Neimark & Slotnick, 1970; Paris, 1973; Peel, 1967; Suppes & Feldman, 1971; Taplin, Staudenmayer, & Taddonio, 1974), it is necessary to become aware of developmental patterns in understanding of logical connectives.

Three basic theoretical questions need to be answered in order to become aware of how children and adults understand logical connectives:

1. How is each logical connective encoded (in the context of a single premise or statement) at each age level?

2. How is each logical connective combined (in the context of two statements, a major and a minor premise) at each age level?

3. What effects do age, content, and experience with logical connectives in the test setting have upon encoding and combination of the connectives?

Previous research has dealt with these questions, although not within the context of a single experiment or series of experiments. A brief review of this research will establish the progress that has been made toward answering each of the three questions.

The first question deals with encoding of logical connectives in the context of single premises or statements. A relatively earlier investigation of encoding was performed by Neimark and Slotnick (1970). These authors tested understanding of class inclusion (assertion), exclusion (denial), intersection (conjunction), and union (disjunction) in children in each of grades three through nine and in college. Problems were presented in both verbal and pictorial forms. The authors found that class inclusion and exclusion were understood by a majority of children of all ages, intersection was understood by a majority of all but the third-grade children, but union was understood by a majority of children only at the college level. The exceptional difficulty of the class union relation was confirmed by Neimark (1970), who found that development of the connective <u>or</u> proceeded throughout the high school age range. Neimark attributed errors in understanding of this connective to inappropriate application of concrete operations to a logical connective whose understanding requires application of formal operations.

One of the most comprehensive investigations of logical connectives was performed by Paris (1973), who studied five logical relations as expressed by eight connectives: conjunction (<u>and</u>, <u>but</u>, <u>both</u>-<u>and</u>), conjunctive absence (<u>neither</u>-<u>nor</u>), inclusive disjunction (<u>or</u>, <u>either</u>-<u>or</u>), conditionality (<u>if</u>-<u>then</u>), and biconditionality (<u>if and only if</u>-<u>then</u>). Children in each of grades 2, 5, 8, and 11 had to decide whether verbal descriptions were true or false of pictorial material presented via slides. Paris found that (a) the conjunctive connectives all displayed nearly perfect performance at all grade levels; (b) the conjunctive-absence connective showed a pattern similar to the other connectives for eleventh graders, but was understood only poorly by younger children; (c) the disjunctive connectives showed high error rates at all grade levels--development continued throughout the age range studied, with older subjects more likely to treat disjunction exclusively than younger subjects; (d) the conditional connective was extremely difficult at all levels, and although patterns of errors changed over age, overall level of performance remained almost constant; (e) the biconditional connective became easier to understand with age, with patterns of errors as well as proportions of errors changing over age. On the whole, subjects found conjunctive relationships easiest to understand, followed in order by biconditional, disjunctive, and conditional relationships.

Suppes and Feldman (1971) investigated conjunctive, disjunctive, negative, and exclusive-or relationships as understood by children of preschool and kindergarten age from middle-class and disadvantaged backgrounds. Subjects were commanded to give the experimenter concrete objects, for example, green stars. They found comprehension of positive conjunctive commands highest, followed by commands involving exclusive conjunction, conjunction with negation, positive disjunction, and disjunction with negation.

Peel (1967) investigated three logical relationships--implication, incompatibility (implication with a negated consequent), and disjunction. Subjects were of ages 5, 6, 8, and 11, and played a game with colored beads and counters. Although Peel's primary emphasis in his experiment was methodological, his substantive findings were of some interest. He found that (a) disjunction is generally interpreted in its exclusive sense, (b) the conditional is interpreted primarily as a biconditional (equivalence), and (c) incompatibility, when interpreted reliably, is interpreted in its logical sense. The presence of the additional negation in the consequent thus serves to make subjects more (rather than less) logical.

A number of studies have also been conducted focusing upon cognitive development in conceptual rule learning (for example, Friedman, 1965; King, 1966; Weir, 1964). Since these studies involve learning rather than encoding of logical connectives, they are not reviewed here. Some of these studies are reviewed by Bourne and O'Banion (1971), who also present new data.

The second question deals with combination of logical connectives in the context of major and minor premises presented jointly. Research on this question has dealt almost exclusively with the conditional connective. Taplin, Staudenmayer, and Taddonio (1974) have followed up on the research with college students of Taplin (1971) and Taplin and Staudenmayer (1973) by investigating developmental changes in conditional reasoning. Subjects in each of grades 3, 5, 7, 9, and 11 were required to indicate whether the conclusions of conditional syllogisms such as "If there is a Z, there is an H; there is a Z; there is an H" were "always correct, sometimes correct, or never correct." The authors found that performance improved with age, and specifically, that nine-year olds treat the conditional connective conjunctively or biconditionally, that the conjunctive meaning disappears with increasing age, and that after 13 years of age, the conjunctive meaning has disappeared and been replaced by the true conditional meaning.

Roberge and Paulus (1971) and Kodroff and Roberge (1975) have also investigated the development of conditional reasoning ability. The former authors also compared conditional to class reasoning abilities (where class reasoning problems were presented with the quantifiers some and all). Findings from these experiments were that performance improves with age (from grades 1 to 3 in the latter study and grades 4, 6, 8, and 10 in the latter study, that modus ponens is easier than tollendo tollens (for the first to third graders for whom the contrast was made), and that for grades 4 to 10, class reasoning is easier than conditional reasoning.

Hill (Note 1) tested children of ages 6, 7, and 8 in order to determine whether children could discriminate between a necessary conclusion and its negation. She found that even these very young children performed at a level well above chance in solving problems including conditional and categorical (class) syllogisms. She also found that performance improved with age. O'Brien and Shapiro (1968) followed up on this work, first by replicating Hill, and second by including problems requiring a third category of responses in addition to "yes" and "no." These new problems contained information that was insufficient for the subject to reach a logically valid conclusion, and hence the correct response was that "not enough clues" had beem provided. These authors found, as did Hill, that performance of 6, 7, and 8-year olds is significantly above chance and increases with age when all problems have a logically valid conclusion. They found a considerable amount of below-chance performance, however, when not all problems had a logically valid conclusion, and that growth in the ability to distinguish between problems with and without a valid conclusion was very slow. Shapiro and O'Brien (1970) extended this work to children of ages 6 to 13, finding that there was slow growth in the ability to distinguish the two types of problems in this age range, that there was no evidence of leveling off, and that performance was far from perfect at all age levels.

The third question deals with effects of age, content, and practice on encoding and combination of logical connectives.[1] As would be expected, age is a potent variable affecting performance: In all of the studies mentioned above, performance changes with age. Interestingly, though, it does not always change for the better. For example, Paris (1973), scoring disjunction inclusively, found that the tendency to treat disjunction inclusively decreases with age. As children grow older, they are more likely to use the exclusive meaning of <u>or</u>. Taplin et al. (1974) found that although truth functions for conditional relationships change with age, the proportion of subjects having any truth function at all does not change. They concluded that changes in performance are linguistic rather than logical. A number of investigators have found age-related inversions in learning tasks with logical concepts (see Bourne & O'Banion, 1971). In general, though, performance does improve: A pattern of developmental differentiation that seems to capture findings from several laboratories is a "gradual developmental differentiation of a general conjunctive strategy into (a) processing of the independent truth status of elements in disjunctive expressions, and (b) cause and effect interpretation of the relation between elements in conditional and biconditional statements" (Paris, 1973, p. 278).

Investigations of content effects present a mixed picture. Most studies have used only a single type of content, so that no conclusion can be drawn regarding possible differential effects of content. Roberge and Paulus (1971) studied three types of content, condensing the four types used in Wilkins's (1928) classic study of syllogistic reasoning: concrete-familiar, abstract, and suggestive. Concrete-familiar items had conclusions that were facts outside the children's experience. Abstract items were presented via symbols, capital letters, or nonsense words. Suggestive items were ones referring to things within the children's experience, but which contradicted known facts. The authors found a significant effect of content, and a significant interaction between content and type of reasoning. Concrete-familiar items were the easiest of the three types in both class and conditional reasoning problems. In class reasoning, suggestive content was easier than abstract content; but in conditional reasoning, suggestive and abstract content did not differ in difficulty. Kodroff and Roberge (1975) also found a significant effect of content: They presented conditional-reasoning items either verbally or concretely, and found concrete presentation facilitated performance relative to verbal presentation. Neimark and Slotnick (1970, however, found a more complex pattern of results in comparing pictorial to verbal content. They found that pictorial content is easier for younger children, but where any content difference was obtained at all for older children, it favored verbal content. Osherson (1974) studied complex logical reasoning using either toks (little statuettes) or elements of a playground as content vehicles. He found only trivial effects of content in a variety of data analyses.

Practice effects would seem to be important in the study of developing comprehension of logical connectives in children, first, because it often takes children some time to adjust to the demands of complex information-processing tasks of the kinds used in studies of logical connectives, and second, because experience with the logical connectives in a given setting may affect the way in which these connectives are understood. For whatever reasons, practice effects have been virtually ignored in the literature on comprehension of logical connectives. We thus have no basis for assessing the effects of experience with logical connectives in a given setting upon either encoding or combination of those connectives.

The present investigation is intended to address the three questions posed earlier, and which are discussed above. The need for this further research on the comprehension of logical connectives derives from (a) differences across studies in subject populations, experimental procedures, task materials, and experimental designs, which make difficult comparisons across experiments of encoding tasks on the one hand (such as those used by Neimark & Slotnick, 1970, and by Paris, 1973) and combination tasks on the other (such as those used by Roberge & Paulus, 1971, and by Taplin et al., 1974); (b) sole use of the conditional connective (or its class isomorphs) in most combination studies, providing little information about combination of premise information with logical connectives other than the conditional, and making comparison of results from encoding and combination tasks with other connectives almost impossible; (c) failure to study effects of repeated exposure to stimulus materials in previous studies, making it impossible to assess whether performance improves or in any way changes once subjects have become more thoroughly familiarized with the complex tasks required of them; and (d) the possibility of further methodological refinements, which are introduced here. The methodology for inferring logical truth functions builds upon previous work of Taplin et al. (1974), Staudenmayer (1975), and Peel (1967) in studying logical connectives, and upon the work of Levine (1966) in studying concept learning, but includes some novel features that have not been used in previous research.

Method

Subjects

The number of subjects in the main experiment totaled 224, of whom 34 were in grade 2, 38 in grade 4, 38 in grade 6, 40 in grade 8, 38 in high school (grades 11 and 12), and 36 in college (freshmen and sophomores). Mean ages at each grade level were 7½, 9½, 11½, 13½, 17, and 19 years for the respective grades. Elementary and secondary students were from a middle-class suburb of New Haven; college students were Yale undergraduates.[2] High school and college students were enrolled in introductory psychology courses, and received course credit for their participation. Subjects at each grade level were approximately equally divided between sexes. Second-graders were prescreened to exclude grade-repeaters and children classified as non-readers by their teachers; otherwise, all students whose parents returned permission slips were tested at each elementary and secondary grade level. Two second-graders and one fourth-grader were eliminated because of inability or unwillingness to pay attention to the task; otherwise, all subjects who participated contributed analyzable data.

A subsidiary replication experiment was run in which 36 freshmen and sophomores at Yale served as subjects. These subjects were tested 1½ years after the original subjects, by a different experimenter.

<u>Materials</u>

Two different tasks--encoding and combination--were presented in crossed fashion via two different content vehicles--fruits (apple, banana) and shapes (circle, square) over the course of two sessions. Props for the tasks included a box, a towel to cover the box, and two objects (either an artificial apple and banana or a cardboard circle and square covered with silver paper).

In the encoding task, each problem consisted of two sentences describing the contents of the box, namely, a premise and a conclusion drawn from that premise. Premises were constructed on the basis of six logical relationships expressed by seven different logical connectives: conjunction (e.g., there is a circle in the box <u>and</u> there is a square in the box), disjunction (e.g., there is a circle in the box <u>or</u> there is a square in the box), conditionality (e.g., <u>if</u> there is a circle in the box, <u>then</u> there is a square in the box; there is a circle in the box <u>only if</u> there is a square in the box), biconditionality (e.g., there is a circle in the box <u>if and only if</u> there is a square in the box), simple affirmation (e.g., there <u>is</u> a circle in the box), and simple negation (e.g., there <u>is not</u> a circle in the box).[3] Four conclusions were presented (in random order) for each premise, corresponding to the 2^2 possibilities that each of the two items (apple and banana, or circle and square) either was or was not in the box: PQ (e.g., there is a circle in the box and there is a square in the box), $P\overline{Q}$ (e.g., there is only a circle in the box and nothing else), $\overline{P}Q$ (e.g., there is only a square in the box and nothing else), and $\overline{PQ}$ (e.g., there is not a circle in the box and there is not a square in the box). Since there were 7 different premise connectives and 4 different conclusions, there were a total of 28 different encoding problems for each content.

In the combination task, each problem consisted of three sentences describing the contents of the box, namely, a major premise, a minor premise, and a conclusion. Only the first four of the logical relationships described above (conjunction, disjunction, conditionality, biconditionality) were used for the major premise, since the latter two relationships (simple affirmation, negation) apply only to single items and hence do not permit combination of two items. Major premises in the combination task were identical in format to the single premises in the encoding task. There were four possible minor premises, corresponding to the 2^2 possibilities that each of the two objects (apple and banana, or circle and square) was or was not in the box: P (e.g., there is a circle in the box), $\overline{P}$ (e.g., there is not a circle in the box), Q (e.g., there is a square in the box), and $\overline{Q}$ (e.g., there is not a square in the box). There were four possible conclusions, identical in form to the minor premises. However, since the conclusion always referred to the object complementary to the object referred to in the minor premise (for example, square if the object named in the minor premise was circle), there were eight possible pairings of minor premises with conclusions. For each of the four possible minor premises, the corresponding conclusion had to be that the complementary object either was or was not in the box. Since there were 5 different logical connectives in the major premises, and 8 possible pairings between minor premises and conclusions, there were a total of 40 different combination problems for each content.

Design

The dependent variable in this experiment was the subject's response of true, maybe true and maybe false (abbreviated to maybe on answer sheets), or false to each conclusion (cf. Taplin et al., 1974). Independent variables were task (encoding, combination) and grade level (2, 4, 6, 8, high school, college) between subjects,[5] and content (fruits, shapes) and session (1, 2) within subjects. All independent variables were fully crossed. Orders of presentation of logical connectives and content vehicles were counterbalanced so that each subject received each of the logical connectives in a different order in each session, with one content vehicle per session.

Procedure

Subjects were tested in small groups of from two to six members. Each of two sessions began with a pretask designed to familiarize subjects with the experimental procedures, the relevant content vehicle, and the definitions of true, maybe true and maybe false, and false. In the elementary-school groups, the experimenter also made sure that each subject knew what the relevant content objects were, and was able to read each word in the problems. Subjects were given three easy practice problems (one having each of the three possible responses as correct) to insure understanding of the task.

The tasks were presented as games in which the experimenter was secretly placing one, both, or neither of two objects in the cardboard box. The subject's task in the game was to figure out what the experimenter had done. In order to help the subject figure this out, the experimenter provided the subject with certain clues. The single premises in the encoding task and the major premises in the combination task served as such clues. These premises were printed on 9 x 12-inch poster board. The content vehicles were presented as pictures rather than words to facilitate ease of recognition. For the combination task, minor premises served as additional clues (called hints), and were presented on 3 x 5-inch flash cards. A given card either pictured an object in order to show that the object was in the box, or pictured the object with a black X drawn through it to show that the object was not in the box. The experimenter read all problems aloud as well as allowing children to read the problems from the cards.

Problems within each session were blocked by logical connective. Answers to problems were recorded in booklets that had for each problem the words true, maybe, and false printed in them. Subjects responded by circling the appropriate word. Testing sessions lasted 20 to 30 minutes apiece.

Results

Unit of Analysis

The unit of analysis in the data was the response pattern. A response pattern refers to the four (encoding task) or eight (combination task) responses given to the conclusions, and provides the means whereby one can determine how each logical connective was interpreted. For example, if the premise of a problem in the encoding task stated that "the circle is in the box or the square is in the box," the response pattern would refer to the pattern of "trues," "maybes," and "falses" given to the conclusions

"There is only a circle in the box and nothing else," "There is only a square in the box and nothing else," "The circle is in the box and the square is in the box," and "The circle is not in the box and the square is not in the box." Response patterns were each compared to <u>model patterns</u>. A model pattern refers to the pattern of "trues," "maybes," and "falses" that would be predicted if a particular logical connective were interpreted in a particular way. Consider the example above. If <u>or</u> in the premise were interpreted inclusively (either or both objects in the box), one would expect a subject to respond "maybe" to the first three conclusions and "false" to the last conclusion. If, however, <u>or</u> were interpreted exclusively (either but not both of the objects in the box), one would expect a subject to respond "maybe" to the first two conclusions, but "false" to the last two. The third conclusion becomes false in the exclusive interpretation of <u>or</u> because both objects cannot be in the box.

Table 1 shows model patterns associated with 16 possible interpretations of the logical connectives. Each interpretation has a distinct pattern, so that it is possible to distinguish among the different interpretations by the response patterns subjects actually gave. The 16 patterns presented in the table are in fact the only logically consistent patterns that are possible. Thus, if subjects respond in a way that is logically consistent, it will be possible to infer what form this logical consistency takes. The 16 patterns are exhaustive with respect to logically consistent possibilities because in standard logic, each of the four possible states for P and Q—PQ, $P\overline{Q}$, $\overline{P}Q$, $\overline{PQ}$—may be either true or false, yielding 2^4, or 16 possible joint states. In the present experiment, a subset of statements that would be classified as true in standard logic are classified as "maybe true, maybe false," using the kind of defective truth table advocated by Johnson-Laird

(1975) and Wason (1966), and used by Staudenmayer (1975) and Taplin et al. (1974). The kind of defective truth table used in this and other experiments seems better to capture people's intuitions about validity: For example, "if P then Q" would be classified as "maybe true, maybe false" in this experiment whenever P is false, although in standard logic it would be classified as true. This mapping of a subset of trues onto maybes does not change, however, the possibility of there being just 16 logically consistent response (or model) patterns in the solution of the problems used in this experiment.

Insert Table 1 about here

In the table, there are four possible conclusions for the encoding task, as noted earlier, corresponding to the possible combinations of P and Q being true and false. There are four possible conclusions in the combination task, which are paired with the four possible minor premises such that each item referred to in the conclusion is different from the item referred to in the minor premise.

Quantitative Analyses of Response Patterns

The first data analysis dealt with proportions of errors, that is, of response patterns not conforming to the correct model pattern(s) for each logical connective. One logical connective, <u>or</u>, was judged to have two "correct" patterns for the purposes of this experiment. The two patterns corresponded to the logical (inclusive) and everyday (exclusive) interpretations of <u>or</u>. The two interpretations are different, and both are of interest in an experiment designed to understand people's interpretations of logical connectives.

Proportions of response patterns not conforming to the correct model pattern(s) for each logical connective are shown in Table 2, with <u>or</u> scored in the two different ways. In order for a response pattern to be scored as correct, a subject had to respond to all of the four (encoding task) or eight (combination task) conclusions as predicted by the model pattern.[6]

Insert Table 2 about here

The difficulty of the combination task relative to the encoding task was assessed by conducting a paired *t*-test on the overall means for each logical connective, as shown in the last column of Table 2. Only those connectives were used that overlapped between tasks, and the mean for the exclusive rather than the inclusive interpretation of *or* was used since that interpretation seemed better to reflect subjects' use of the term. Since the combination task required all of the operations required for the encoding task, plus additional operations, it was expected that performance would be poorer on this task. This expectation was confirmed, $t(4) = 2.33$, $p < .05$. When the overall proportion of errors in encoding (.59) is compared to the overall proportion of errors in combination (.70), it becomes obvious that most errors made in the interpretation of logical connectives occur in the initial encoding stage rather than the subsequent combination stage. Incremental errors due to combination amount only to .11 (.70-.59).

A four-way analysis of variance was conducted upon means over subjects, with connectives, grade, content, and session as independent variables.[7] The main effect for connective was highly significant in both the encoding task, $F(6,30) = 111.18$, $p < .001$, and the combination task, $F(4,20) = 158.61$, $p < .001$. Examination of the proportions in Table 2 reveals the sources of the main effect. In both tasks, the connectives seem to fall into three groups of differential difficulty. The conjunctive connective stands alone as the easiest of the set. In the second group are exclusive disjunction and, in the encoding task, affirmation and negation. The third group comprises the two conditional connectives and the biconditional. Turning to the individual connectives we examine three patterns of interest. First, it is clear that overall, subjects are far more likely to interpret *or* in its exclusive meaning than in its inclusive meaning. This difference is as would be expected, since the exclusive meaning of *or* is the one used in everyday language. Second, conditionality is better understood when expressed by the *only if* connective than when expressed by the *if-then* connective. Third, encoding of negations was no more difficult than encoding of simple affirmations. The relatively great difficulty of the affirmation task seems to derive from the binary nature of the stimuli, as will be discussed later.

The main effect for grade was also highly significant, $F(5,5) = 95.37$, $p < .001$, in the encoding task, and $F(5,5) = 62.14$, $p < .001$, in the combination task. Visual examination of the last row of data for each of the two tasks reveals a monotone decrease in error rates across grade levels in both tasks, except for a trivial inversion between grade 8 and high school in the encoding task. The rate of decrease in error rate does not seem to show any slackening with increased age. Except for the inexplicably good performance of the eighth graders in the encoding task, the rate of decrease is actually quite steady. Apparently, development in understanding of logical connectives continues even to the college level.

The main effect of session was statistically significant for the encoding task, $F(1,5) = 10.32$, $p < .05$, but not for the combination task, $F(1,5) = 3.82$, $p > .10$. In both tasks, however, the trend was toward improved performance with practice. Not all of this improvement is necessarily task specific, since at least some of the connectives (such as those of conditionality and biconditionality) may have been relatively unfamiliar to some of the subjects (such as the younger elementary-school children).

The main effect of content was trivial in both tasks ($\underline{F} < 1$). Apparently, the logical connectives were abstracted from the content of the sentence without any effect of the content upon the way in which the connectives were abstracted. Although the contents were chosen so that one would be relatively more abstract (shapes) and the other relatively more concrete (fruits), it is of course possible that the failure to obtain a significant effect was due to insufficient variety in the two contents sampled.

Only three interactions were statistically significant in the two tasks. Two of these (one in each task) were for connective by grade, $\underline{F}(30,30) = 5.07$, $\underline{p} < .001$, in the encoding task, and $\underline{F}(20,20) = 7.54$, $\underline{p} < .001$, in the combination task. This interaction signifies different rates of development for the various logical connectives, and is thus of considerable interest. An examination of trends in Table 2 reveals the sources of the interaction. Errors for the conjunctive connective start at a fairly high level in each task, but fall to near zero by grade 8 or high school, depending upon the task (encoding or combination). Errors for the affirmation and negation connectives and for the exclusive interpretation of the disjunctive connective start at very high levels, but fall to relatively low levels by college age. Errors on the conditional connectives (and especially <u>if-then</u>) and the biconditional connective start very high and remain at relatively high levels. Thus, developmental processes proceed at very different rates for the various connectives. The third significant interaction, for connective by session in the combination task, $F(4,20) = 4.04$, $p < .05$, derived from greater improvement in performance for easier than for more difficult connectives. This result would suggest that more of the errors on the easier connectives were task-specific, and were corrected after one session of experience with the task. Errors for the more difficult connectives were due to faulty comprehension of the connectives, and were not alleviated by further task-specific experiences.

Taken as a whole, the quantitative analyses of error rates present a coherent picture, showing that the connectives are differentially difficult at the various ages included in the study, and that understanding of the various connectives develops at different rates. But although the quantitative analyses show how many errors are made at each developmental level, they fail to indicate the kinds of errors that are made. These indications are provided by the qualitative analyses described below.

<u>Qualitative Analyses of Response Patterns</u>

The second data analysis dealt with fit between response patterns and model patterns for each logical connective. In this analysis, response patterns for each logical connective in each task were compared to the 16 model patterns shown in Table 1. In order to permit a quantified comparison, responses of "true" were coded as 1, responses of "maybe" as 2, and responses of "false" as 3.[8] This quantification procedure permitted fit to be assessed in terms of two indices, the root-mean-square deviation (RMSD) and the squared correlation (r^2) between a given response pattern and each possible model pattern. The first measure, RMSD, provides an index of badness-of-fit in terms of the original unit of measurement. Thus, an RMSD of 0 would indicate perfect fit between a response pattern and a particular model pattern; an RMSD of 2 (the difference between 1, for a "true" response, and 3, for a "false" response) would indicate the poorest possible fit between a response pattern and a particular model pattern. The second measure, r^2, provides an index of goodness of fit between a given response pattern and each possible model pattern in invariant units (-1 to 1) that standardize the variances of the patterns being correlated. The two measures provide complementary information in assessing model fits, since both absolute fit (RMSD) and relative fit (r^2) are of interest in determining whether a given model adequately describes a set of data.

In order for the qualitative analysis to be informative, it is necessary that the data be internally consistent across subjects. Otherwise, one runs the risk of discovering group patterns that do not represent the majority of individual response patterns, or even any individual response patterns, but rather some artifact of averaging across subjects. With all of the logical connectives considered together, alpha (internal-consistency) reliabilities across subjects in the encoding task were .92, .96, .98, .99, .99, and .99 for grade 2, grade 4, grade 6, grade 8, high school, and college students respectively; reliabilities in the combination task were .40, .95, .98, .99, .99, and .99 for the respective grade levels. Reliabilities were also computed for logical connectives individually: In the encoding task, all reliabilities were .90 or above except for negation at the grade 2 level (.60); in the combination task, all reliabilities for grade 4 and above were over .90. The large majority of reliabilities for the individual connectives were in the high .90's. These data suggest that the data were all highly reliable except for combination at the grade 2 level and encoding of negations at the grade 2 level. Because of its unreliability, the combination task at the grade 2 level was not further analyzed.

Table 3 shows fits at each grade level between response patterns and selected model patterns for each of the logical connectives. The selected model patterns are the logically correct model pattern, and the model patterns with the lowest (first-ranked), second-lowest (second-ranked), and third-lowest (third-ranked) group RMSD's. The RMSD's are displayed in the table, as are the proportions of individual subjects whose response patterns were best fit by each of the three top-ranked models (ignoring the others). Generally, one can expect the group data to represent the individual data accurately. This is not always the case, however, especially when none of the models fit the group data particularly well. These data make it possible to determine (a) how well the model pattern for the logically correct interpretation of each connective fit the group data, (b) whether the first-ranked model pattern was the same as the logically correct model pattern for the group data (and if not, what the first-ranked model pattern was), (c) how well the first-ranked model pattern fit the group response pattern, (d) how well the first-ranked model pattern fit the group response pattern compared to the two strongest competitor model patterns, and (e) how well the group data corresponded to the individual data.

Insert Table 3 about here

Conjunction. The results in the encoding task for the conjunctive connective, and, were simple and straightforward: The conjunctive interpretation of and provided the best fit to the data at each grade level. In the combination task, response patterns for the conjunctive and biconditional interpretations were indistinguishable because, as mentioned earlier, certain (contradictory) conclusions were omitted in this task. These conclusions would have been needed, however, and were available for other tasks, in order to provide unique model patterns for each possible interpretation of the logical connectives. In view of the encoding results, however, it seems reasonable to conclude that the conjunctive interpretation was in fact the one subjects used. The absolute levels of fit for the conjunctive interpretation were extremely good at all except the second-grade level, where about 30% of the subjects in the encoding task appear to have used an inclusive disjunctive interpretation. All values of r^2 between model and group response patterns for the conjunctive interpretation were .99+, whereas competitor models did much worse.[9]

Disjunction. Disjunction, it will be recalled, was scored in two ways--for the inclusive interpretation and for the exclusive interpretation--and therefore two correct patterns are shown in Table 3. The data show an interesting interaction between age and interpretation of or. In the encoding task, children at the lowest grade level (2) use the inclusive interpretation of or in preference to the exclusive interpretation. This pattern continues in diluted form at the grade-four level, where individual differences in interpretations become more pronounced. At the higher grade levels, children show a strong tendency to use the exclusive interpretation in preference to the inclusive interpretation. The combination task shows a similar although somewhat less clearcut pattern, with the transition apparently occurring slightly later, perhaps because the combination task is so difficult that even sixth graders are not yet ready to apply to it their newly developing meaning of the word or. The replacement of the inclusive interpretation of or by its exclusive interpretation appears to continue even through the college level: The discrepancy between model fits for the two interpretations continues to increase even to college age. Correlational results show the same pattern as RMSD's. In the encoding task, for example, the highest r^2 among the model patterns is for the inclusive interpretation of or (.72), whereas r^2 for the exclusive interpretation is a mere .01 at the second-grade level. At the adult level, however, r^2 for the exclusive interpretation is .99, whereas r^2 for the inclusive interpretation is a mere .44.

<u>Conditionality</u>. The conditional was shown in the quantitative analyses to be among the most difficult of the relations. An examination of Table 3 shows the sources of errors. Consider first the connective, <u>if-then</u>. At the second-grade level, none of the model patterns fit the data very well, with the plurality of subjects appearing to use a conjunctive interpretation. In the encoding task, the conditional is interpreted biconditionally by at least a plurality of subjects at the next three grade levels, with strong competition from the correct conditional interpretation first appearing at grade 8. Most subjects at the high school and college levels use the conditional interpretation, although the biconditional provides strong competition as late as high school. In the combination task, <u>if-then</u> is interpreted as a reverse conditional or a biconditional by substantial numbers of fourth graders. At the sixth and eighth grade levels, the biconditional interpretation clearly predominates. At the high school level, the conditional interpretation comes close to taking over, and by the college level, it has indeed taken over. Again, performance on the combination task seems to lag one level behind performance on the encoding task, perhaps because of the combination task's greater difficulty (but possibly because of differences in the subject samples). The high school level appears to be transitional in both tasks, with substantial numbers of subjects using both the biconditional and the developing conditional interpretation. Only at the college level does the conditional interpretation enjoy a clear advantage over the biconditional one. It is noteworthy that even at this level, RMSD is relatively higher than it was for the two connectives con-

sidered previously, presumably because of the presence of greater individual differences in interpretation. The RMSD of .19 in the encoding task corresponds to an r^2 of .86 at the college level, and the RMSD of .34 in the combination task corresponds to an r^2 of .77.

The <u>only if</u> connective showed relatively faster development than the <u>if-then</u> connective. It, too, was interpreted variably by the youngest subjects, then biconditionally, and then conditionally. But the transition to the conditional interpretation occurred at grade 8 in both tasks, earlier than for the <u>if-then</u> connective. Wherever the conditional interpretation is the first-ranked model, the biconditional is the second-ranked model, and the discrepancy between the performance of the two models increases with increasing age, as more subjects adopt the conditional meaning. Good distinguishability between the two model patterns as applied to the group data is not achieved until high school (as contrasted with college for the <u>if-then</u> connective).

<u>Biconditionality</u>. The biconditional relation was also among the most difficult for subjects. The data in Table 3 suggest substantial individual differences in the interpretation of the biconditional. From grade 4 on, substantial numbers of subjects divide between the conditional and biconditional interpretations. If anything, there is a tendency for the (incorrect) conditional interpretation to become more predominant with increasing age.

<u>Affirmation</u>. Affirmation was tested only in the encoding task, since it does not involve combination of information. The error rate for simple affirmation appeared rather higher in Table 2 than one might have expected, and the pattern of results in Table 2 shows why. The difficulty of simple affirmation for some subjects appears to have derived largely from the binary nature of the task. At the second-grade level, being told that an object,

say, the circle, was in the box, was interpreted by some (although not a majority) of subjects as tantamount to being told that either the circle was in the box or the square was in the box; and indeed, it was possible that the square was in the box. Some fourth graders went to the opposite extreme, interpreting the statement that the circle was in the box as implying that the other object, the square, was not in the box. Although it was emphasized to subjects in the initial instructions that a statement about one object supplied information only about that object, some second- and fourth-graders incorrectly inferred a connection between the object about which a statement was made and the object about which no statement was made. It should be noted, however, that substantial numbers of subjects interpreted affirmation correctly at both grade levels. By grade 6, the correct meaning of affirmation clearly predominated, and by grade 8, it was firmly entrenched.[10]

Negation. Second-graders appear not to have interpreted not in its logical sense; but at each of the other levels, the first-ranked model pattern was that of the logical interpretation of not, and the performance of this model was far superior to the performance of alternative models.

Discussion

Summary of results. Children as young as second grade encode logical connectives in consistent ways, although it was not until the fourth grade that children showed evidence of consistent combination of logical connectives. The failure of second-graders to combine in a consistent fashion may, of course, have been due to their failure to understand the directions for this particular task as presented in this particular experiment. Moreover, the second-graders' nonstandard interpretations of even the simplest connectives (such as not) necessitates interpreting all results for these children with caution.

There are large differences in the difficulties of the various logical connectives, with conjunction clearly the easiest, exclusive disjunction (and in the encoding task, affirmation and negation) of intermediate difficulty, and conditionality and biconditionality the most difficult. As would be expected, children improve in performance with increasing age, except for the biconditional, where there is some tendency to turn successively more toward the conditional interpretation with increasing age. Comprehension of the various connectives changes at different rates: Comprehension of conjunctions starts relatively high and stays high; comprehension of conditionals and biconditionals starts low and never becomes very high; comprehension of affirmation, negation, and disjunction shows substantial improvement with age. It is more difficult to combine the connectives than to encode them, as would be expected, since combination requires encoding as a prerequisite. Effects of content were trivial in this experiment, perhaps because the difference between the more abstract content (shapes) and the more concrete content (fruits) was not large enough to result in differential strategy. The data suggest that subjects encoded and combined the logical connectives without regard to the content vehicle through which they were presented. Effects of practice with the connectives in the experimental situation were significant in the encoding task, suggesting that single-trial experiments may fail to give a full picture of subjects' capabilities in comprehending the logical connectives. Improvement was greater for easier than for harder connectives, implying that errors on these connectives in the early trials were more likely to be task-specific.

The connective, <u>and</u>, was interpreted conjunctively by the majority of children at all ages represented in the experiment. <u>Or</u> was interpreted first inclusively and then exclusively. <u>If-then</u> and <u>only if</u> seem to have been interpreted as inclusive disjunctions or as conjunctions by many children at the second-grade level; they acquired their potentially causal meanings only at the fourth-grade level and above. The biconditional meaning first dominated the conditional one, but was gradually replaced by the conditional meaning at the higher grade levels. Development of only if proceeded at a more rapid rate than development of if-then. If and only if was interpreted either conditionally or biconditionally by children as young as grade 4 and as old as college age. The proportion of subjects using the logically incorrect conditional interpretation actually increased with age. Simple affirmation presented difficulty for some young children in a binary situation, because of incorrectly inferred constraints upon the object not mentioned. Nevertheless, many children interpreted affirmation correctly even at the second-grade level. Negation was understood in its logical sense by all but the second-graders.

Correspondence between the present results and previous ones. Where the present data overlap in scope with previous data, they show good agreement with those previous data. The present results for the conjunctive connective replicated Paris's (1973) finding of nearly perfect performance at all age levels, and his finding that the conjunctive connective was the easiest of all; Suppes and Feldman (1971), too, found conjunction to be the easiest of the logical relationships they studied. The present study also replicated Neimark's (1970) finding that development in comprehension of the connective, or, proceeds throughout high school (and even into college). As was the case in the experiments of Paris and of Suppes and Feldman, disjunction was more difficult than conjunction. And it was found, as in the studies of Paris and of Peel (1967), that disjunction is generally interpreted in its exclusive sense by older subjects. The present study also replicated Taplin et al.'s (1971) finding that the conditional tends to be interpreted conjunctively or biconditionally by very young children, that the conjunctive meaning disappears with increasing age, and that the true conditional meaning eventually replaces the biconditional one. The biconditional meaning was predominant in approximately the same age range as in Peel's (1967) experiment. And as was the case in the Taplin et al., Roberge and Paulus (1971), and Kodroff and Roberge (1975) experiments, performance on the conditional improved with age.

An apparent difference between the presents results and some previous ones disappears when the data are examined carefully. Paris (1973) found the biconditional to be considerably easier than it was found to be in the present experiment. The difference in difficulty appears to result from the difference in scoring methods between experiments, not from differences in subjects' performance. Paris scored his data by individual item rather than by response pattern. When the present data were scored in this way, the biconditional appeared to be quite a bit easier. The problem with this scoring method, however, is that subjects may respond to individual items correctly for the wrong reasons, simply because response patterns make overlapping predictions. Thus, a subject using a model pattern other than that of the biconditional will receive credit for some correct answers solely because the model pattern he or she is using overlaps in some of its responses with the biconditional model pattern.

Findings regarding content have been mixed, and apparently the specific contents used in an experiment make a considerable difference in whether content will yield nontrivial effects. Roberge and Paulus (1971), for example, used suggestive content that was deliberately misleading. They found that subjects performed relatively poorly on this content type. The present experiment was more like Osherson's (1974) experiments in using contents that were intended to be quite different, but not misleading. Neither Osherson's data nor the present data showed any content effects.

Conclusions. The present experiment was intended to go beyond previous experiments in its (a) simultaneous investigation of encoding and combination, (b) use of all connectives, rather than just the conditional, in the combination task, (c) study of practice effects, and (d) methodological refinements allowing the experimenter to infer both individual and group interpretations of the logical connectives. In its juxtaposition of a task (combination) and subtask (encoding) required for performance of the full task, and in its techniques of mathematical modeling for eliciting cognitive contents, the experiment may be viewed as following a modified form of componential analysis (Sternberg, 1977, 1978; Sternberg & Rifkin, in press). The methodology seemed to be reasonably successful in shedding light upon developmental patterns in the encoding and combination of logical connectives, and might be suitable for studying the development of other types of comprehension as well. If the study has shown anything, it is the richness of the cognitive contents of children as young as grade 2 and as old as college age.

Appendix

The table presented here shows mean response patterns associated with each logical connective. These group data formed the basis for the group analyses performed in the section of the article on qualitative analyses of response patterns.

Insert Table A about here

Reference Note

1. Hill, S. A. A study of the logical abilities of children. Doctoral dissertation, Stanford University. Ann Arbor, Michigan: University Microfilms, 1961. No. 61-1229.

References

Bourne, L. E., Jr., & O'Banion, K. Conceptual rule learning and chronological age. Developmental Psychology, 1971, 5, 525-534.

Friedman, S. R. Developmental level and concept learning: Confirmation of an inverse relationship. Psychonomic Science, 1965, 2, 3-4.

Johnson-Laird, P. N. Models of deduction. In R. Falmagne (Ed.), Reasoning: Representation and process. Hillsdale, N.J.: Erlbaum, 1975.

King, W. A developmental study of rule learning. Journal of Experimental Child Psychology, 1966, 4, 217-231.

Kodroff, J. K., & Roberge, J. J. Developmental analysis of the conditional reasoning abilities of primary-grade children. Developmental Psychology, 1975, 11, 21-28.

Levine, M. Hypothesis behavior by humans during discrimination learning. Journal of Experimental Psychology, 1966, 71, 331-338.

Neimark, E. D. Development of comprehension of logical connectives: Understanding of "or." Psychonomic Science, 1970, 21, 217-219.

Neimark, E. D., & Slotnick, N. S. Development of the understanding of logical connectives. Journal of Educational Psychology, 1970, 61, 451-460.

O'Brien, T. C., & Shapiro, B. J. The development of logical thinking in children. American Educational Research Journal, 1968, 5, 531-542.

Osherson, D. N. Logical abilities in children (Vol. 2): Logical inference: Underlying operations. Hillsdale, N.J.: Erlbaum, 1974.

Paris, S. G. Comprehension of language connectives and propositional logical relationships. Journal of Experimental Child Psychology, 1973, 16, 278-291.

Peel, E. A. A method for investigating children's understanding of certain logical connectives used in binary propositional thinking. British Journal of Mathematical and Statistical Psychology, 1967, 20, 81-92.

Roberge, J. J., & Paulus, D. H. Developmental patterns for children's class and conditional reasoning abilities. Developmental Psychology, 1971, 4, 191-200.

Shapiro, B. J., O'Brien, T. C. Logical thinking in children ages six through thirteen. Child Development, 1970, 41, 823-829.

Staudenmayer, H. Understanding conditional reasoning with meaningful propositions. In R. Falmagne (Ed.), Reasoning: Representation and process. Hillsdale, N.J.: Erlbaum, 1975.

Sternberg, R. J. Intelligence, information processing, and analogical reasoning: The componential analysis of human abilities. Hillsdale, N.J.: Erlbaum, 1977.

Sternberg, R. J. Componential investigations of human intelligence. In A. Lesgold, J. Pellegrino, S. Fokkema, & R. Glaser (Eds.), Cognitive psychology and instruction. New York: Plenum, 1978.

Sternberg, R. J., & Rifkin, B. The development of analogical reasoning processes. Journal of Experimental Child Psychology, in press.

Suppes, P., & Feldman, S. Young children's comprehension of logical connectives. Journal of Experimental Child Psychology, 1971, 12, 304-317.

Taplin, J. E. Reasoning with conditional sentences. Journal of Verbal Learning and Verbal Behavior, 1971, 10, 219-225.

Taplin, J. E., & Staundemayer, H. Interpretation of abstract conditional sentences in deductive reasoning. Journal of Verbal Learning and Verbal Behavior, 1973, 12, 530-542.

Taplin, J. E., Staudenmayer, H., & Taddonio, J. L. Developmental changes in conditional reasoning: Linguistic or logical? Journal of Experimental Child Psychology, 1974, 17, 360-373.

Wason, P. C. Reasoning. In B. Foss (Ed.), New horizons in psychology. Harmondsworth, England: Penguin, 1966.

Weir, M. W. Developmental changes in problem-solving strategies. Psychological Review, 1964, 71, 473-490.

Wilkins, M. C. The effect of changed material on ability to do formal syllogistic reasoning. Archives of Psychology, 1923, No. 102.

Footnotes

This research was supported in part by grant BNS76-05311 from the National Science Foundation to Robert J. Sternberg. I am grateful to Margaret E. Turner for invaluable assistance in all phases of the experiment, to Barbara Conway and Elizabeth Charles for assistance in data analysis, and to Joseph J. Cirasuolo, Director of Learning Services for the Hamden, Connecticut, Public Schools, for permitting access to the Hamden Public Schools. I also thank the children, teachers, and principals of the Hamden schools who made the execution of this experiment possible. Requests for reprints should be sent to Robert J. Sternberg, Department of Psychology, Yale University, Box 11A Yale Station, New Haven, Connecticut 06520.

[1] A variable of potential interest would be subjects' sex. However, previous research investigating sex differences has failed to find any (for example, Neimark, 1970; O'Brien & Shapiro, 1968; Roberge & Paulus, 1971; Suppes & Feldman, 1971; Hill, Note 1).

[2] The college students unfortunately do not come from the same school population as the elementary and secondary students. This is a common and probably insoluble problem in much developmental research, however, caused by the failure of community school populations to remain intact after high school.

[3] Strictly speaking, <u>is</u> and <u>is not</u> are not logical connectives. They were included in the study, however, because of their obvious importance in understanding communications (including the ones in this experiment) in which the conventional logical connectives are used.

[4] Sex was originally used as an additional independent variable, but because of the absence of either main effects or interactions with this variable,

and because of the variable's insignificance in past experiments, it was dropped in order to increase residual degrees of freedom.

[5]For the conjunctive connective, certain minor premises resulted in contradictions when paired with certain major premises. For example, if a major premise stated that there is a circle and a square in the box, a minor premise stating that there is not a circle in the box would be contradictory. No contradictory items were presented to subjects.

[6]This criterion for correctness is obviously a strict one, and to the extent that nonsystematic factors result in responses straying from the model pattern, the criterion results in an inflation of the error rate. An alternative scoring procedure is simply to score individual responses, rather than whole response patterns, as the unit of analysis. There is reason to believe that this procedure (which has been used in the past) results in serious distortion of the data, however, for reasons presented in the Discussion section.

[7]Analyses were done separately using both the original and replication groups of adults. The replication group was added to the experiment because all of the original groups were tested by a single experimenter, and it seemed wise to get at least some idea of the effect upon results of using a different experimenter (who tested the replication group only). Since correlations between adult data in the original and replication groups were all over .98, and since the same patterns of results were obtained for both groups, analyses for the replication group are not presented here.

[8]This quantification procedure assumes that the three responses are equidistant on a continuous interval scale. Although this strong assumption is probably not wholly justified, it seems to work well in practice: First, the results of the qualitative analyses to be described are consistent with and amplify upon the results of the quantitative analyses, where this assump-

tion was not made. Contradictions, which were not found, would have provided evidence for failure of the data to conform to the assumption. Second, where overlap exists between the present qualitative results and those from previous research where the assumption was not made, there is good agreement between results, suggesting that the assumption did not alter the pattern of results. Third, the data upon which operations were performed requiring the assumption of an interval scale (such as averaging) were highly reliable across subjects (with one exception to be noted later), according to several indices of internal-consistency reliability. Thus, there is no reason to believe that the operations (such as averaging) would result in group patterns unrepresentative of individual data. Fourth, the subassumption of equal intervals between responses is not critical, since it is well-known that the correlation coefficient is remarkably stable under monotone transformations of a set of data.

[9]The comprehension of <u>and</u> according to its logical meaning at all grade levels was fortunate, since the conclusions about which the subjects had to make judgments required understanding of the logical meaning of <u>and</u>.

[10]It should be noted that simple affirmation as a premise was not the same as affirmative statements in the conclusions. In the conclusions, <u>is</u> was qualified by <u>only</u>, so that it was clear that the conclusion referred only to a single object being in the box. In the simple affirmative premises, <u>is</u> was unqualified and thus open to multiple interpretations, as the data show.

Table 1

Model Patterns Associated with Sixteen Possible Interpretations of the Logical Connectives

Encoding Task

Conclusion	P	Q	$\bar{P}$	$\bar{Q}$	$P \vee_{in} Q$	$P \vee_{ex} Q$	$\bar{P} \vee_{in} \bar{Q}$	$P \wedge Q$	$\bar{P} \wedge Q$	$P \wedge \bar{Q}$	$\bar{P} \wedge \bar{Q}$	$P \rightarrow Q$	$Q \rightarrow P$	$P \leftrightarrow Q$	U	$\bar{U}$
PQ	M	M	F	F	M	F	F	T	F	F	F	M	M	M	M	F
$P\bar{Q}$	M	F	F	M	M	M	M	F	F	T	F	F	M	F	M	F
$\bar{P}Q$	F	M	M	F	M	M	M	F	T	F	F	M	F	F	M	F
$\overline{PQ}$	F	F	M	M	F	F	M	F	F	F	T	M	M	M	M	F

Combination Task

Minor Premise	Conclusion	P	Q	$\bar{P}$	$\bar{Q}$	$P \vee_{in} Q$	$P \vee_{ex} Q$	$\bar{P} \vee_{in} \bar{Q}$	$P \wedge Q$	$\bar{P} \wedge Q$	$P \wedge \bar{Q}$	$\bar{P} \wedge \bar{Q}$	$P \rightarrow Q$	$Q \rightarrow P$	$P \leftrightarrow Q$	U	$\bar{U}$
P	Q	M	T	F	F	M	F	F	T	F	F	F	T	M	T	M	F
P	$\bar{Q}$	M	F	F	T	M	T	T	F	F	T	F	F	M	F	M	F
$\bar{P}$	Q	F	T	M	F	T	T	M	F	T	F	F	M	F	F	M	F
$\bar{P}$	$\bar{Q}$	F	F	M	T	F	F	M	F	F	F	T	M	T	T	M	F
Q	P	T	M	F	F	M	F	F	T	F	F	F	M	T	T	M	F
Q	$\bar{P}$	F	M	T	F	M	T	T	F	T	F	F	M	F	F	M	F
$\bar{Q}$	P	T	F	F	M	T	T	M	F	F	T	F	F	M	F	M	F
$\bar{Q}$	$\bar{P}$	F	F	T	M	F	F	M	F	F	F	T	T	M	T	M	F

Note: T=True, F=False, M=Maybe, V_{in}=Inclusive Or, V_{ex}=Exclusive Or, $\wedge$=And, $\rightarrow$=Implies (Conditionality, $\leftrightarrow$=Implies and is implied by (Biconditionality); U=Universe (Anything possible)

Table 2

Proportions of Response Patterns not Conforming to Correct Model Pattern for Each Logical Connective

Relationship	Grade 2	Grade 4	Grade 6	Grade 8	High School	College	Overall
			Encoding Task				
Conjunction							
AND	.62	.29	.18	.05	.03	.06	.20
Disjunction							
Inclusive OR	.94	.84	.92	.82	.81	.86	.86
Exclusive OR	.88	.82	.55	.45	.47	.25	.57
Conditionality							
IF-THEN	1.00	.95	.92	.71	.67	.44	.78
ONLY IF	1.00	.97	.84	.63	.61	.39	.74
Biconditionality							
IF AND ONLY IF	.94	.79	.87	.74	.86	.79	.83
Affirmation							
IS	.97	.76	.63	.16	.31	.22	.51
Negation							
IS NOT	.91	.74	.68	.32	.36	.14	.52
Overall	.90	.76	.67	.44	.47	.33	.59
			Combination Task				
Conjunction							
AND	.79	.50	.24	.21	.05	.08	.31
Disjunction							
Inclusive OR	1.00	1.00	.97	.86	.93	.97	.94
Exclusive OR	.88	.79	.84	.71	.48	.19	.65
Conditionality							
IF-THEN	1.00	1.00	.97	1.00	.83	.81	.94
ONLY IF	1.00	1.00	.92	.81	.62	.39	.79
Biconditionality							
IF AND ONLY IF	.94	.84	.84	.71	.80	.69	.80
Overall	.92	.83	.76	.69	.56	.43	.70

Table 3

Fit between Response Patterns and Selected Model Patterns for Each Logical Connective

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
Conjunction: AND				
Encoding Task				
2	$P \wedge Q$ .44	$P \wedge Q$ .44 .51	Q .52 .19	$P \vee_{in} Q$ .53 .30
4	$P \wedge Q$ .18	$P \wedge Q$ .18 .82	$P \rightarrow Q$ .58 .11	P .61 .08
6	$P \wedge Q$ .06	$P \wedge Q$ .06 .97	P .68 .03	Q .70 .00
8	$P \wedge Q$ .03	$P \wedge Q$ .03 1.00	P $.70_t$ .00	Q $.70_t$ .00
High School	$P \wedge Q$ .01	$P \wedge Q$ .01 1.00	P $.70_t$ .00	Q $.70_t$ .00
College	$P \wedge Q$ .03	$P \wedge Q$ .03 .96	P $.69_t$ .02	Q $.69_t$ .02
Combination Task				
4	$P \wedge Q$ .33	$P \wedge Q$ $.33_t$ .64	$P \rightarrow Q$ $.33_t$ .64	$P \rightarrow Q$ .43 .37
6	$P \wedge Q$ .14	$P \wedge Q$ $.14_t$ .90	$P \rightarrow Q$ $.14_t$ .90	$Q \rightarrow P$ .60 .11
8	$P \wedge Q$ .12	$P \wedge Q$ $.12_t$ .88	$P \rightarrow Q$ $.12_t$ .88	$Q \rightarrow P$ .61 .12
High School	$P \wedge Q$ .04	$P \wedge Q$ $.04_t$ 1.00	$P \rightarrow Q$ $.04_t$ 1.00	$Q \rightarrow P$ .69 .00
College	$P \wedge Q$ .08	$P \wedge Q$ $.08_t$ .94	$P \rightarrow Q$ $.08_t$ .94	$Q \rightarrow P$ .65 .06

Table 3 Continued

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
	Disjunction: OR			
	Encoding Task			
2	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .26 .69	$P\ V_{in}\ Q$ .26 $\underline{.43}$	U .36 $\underline{.34}$	Q .50 $\underline{.23}$
4	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .31 .39	$P\ V_{in}\ Q$ .31 $\underline{.28}$	U .33 $\underline{.39}$	$P\ V_{ex}\ Q$ .39 $\underline{.33}$
6	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .41 .22	$P\ V_{ex}\ Q$ .22 $\underline{.68}$	$P\ V_{in}\ Q$ .41 $\underline{.18}$	$\bar{P}\ V_{in}\ \bar{Q}$ .41 $\underline{.15}$
8	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .35 .17	$P\ V_{ex}\ Q$ .17 $\underline{.68}$	$P\ V_{in}\ Q$ .35 $\underline{.28}$	$\bar{P}\ V_{in}\ \bar{Q}$ .49 $\underline{.04}$
High School	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .36 .18	$P\ V_{ex}\ Q$ .18 $\underline{.65}$	$P\ V_{in}\ Q$ .36 $\underline{.26}$	$\bar{P}\ V_{in}\ \bar{Q}$ .46 $\underline{.09}$
College	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .42 .10	$P\ V_{ex}\ Q$ .10 $\underline{.82}$	$P\ V_{in}\ Q$ .42 $\underline{.16}$	$\bar{P}\ V_{in}\ \bar{Q}$ .50 $\underline{.02}$
	Combination Task			
4	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .40 .54	$P\ V_{in}\ Q$ .40 $\underline{.29}$	U .49 $\underline{.29}$	$P\ V_{ex}\ Q$ .54 $\underline{.42}$
6	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .43 .65	U .39 $\underline{.34}$	$P\ V_{in}\ Q$ .43 $\underline{.45}$	$\bar{P}\ V_{in}\ \bar{Q}$ .63 $\underline{.21}$
8	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .40 .38	$P\ V_{ex}\ Q$ .38 $\underline{.48}$	$P\ V_{in}\ Q$ .40 $\underline{.40}$	$\bar{P}\ V_{in}\ \bar{Q}$ .67 $\underline{.12}$
High School	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .54 .20	$P\ V_{ex}\ Q$ .20 $\underline{.80}$	$P\ V_{in}\ Q$ .54 $\underline{.20}$	$\bar{P}\ V_{in}\ \bar{Q}$ .66 $\underline{.00}$
College	$P\ V_{in}\ Q$, $P\ V_{ex}\ Q$ .64 .09	$P\ V_{ex}\ Q$ .09 $\underline{.94}$	$P\ V_{in}\ Q$ .64 $\underline{.06}$	$\bar{P}\ V_{in}\ \bar{Q}$ .68 $\underline{.00}$

Table 3 Continued

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
Conditionality: IF-THEN				
Encoding Task				
2	$P \rightarrow Q$ .55	$P \vee_{in} Q$ .50 .20	$P \wedge Q$ $.52_t$ .43	U $.52_t$ .37
4	$P \rightarrow Q$ .50	$P \leftrightarrow Q$ .34 .57	$Q \rightarrow P$ .46 .23	$P \rightarrow Q$ .50 .20
6	$P \rightarrow Q$ .50	$P \leftrightarrow Q$ .38 .43	$P \wedge Q$ .50 .35	$P \rightarrow Q$ .50 .22
8	$P \rightarrow Q$ .37	$P \leftrightarrow Q$ .31 .49	$P \nrightarrow Q$ .37 .41	$Q \rightarrow P$ .53 .10
High School	$P \rightarrow Q$ .38	$P \rightarrow Q$ .38 .57	$P \leftrightarrow Q$ .45 .38	U .60 .05
College	$P \rightarrow Q$ .19	$P \rightarrow Q$ .19 .78	$P \leftrightarrow Q$ .45 .14	U .48 .08
Combination Task				
4	$P \rightarrow Q$ .64	$Q \rightarrow P$ .52 .40	$P \leftrightarrow$ [illegible] .58 .35	U .58 .25
6	$P \rightarrow Q$ .57	$P \leftrightarrow Q$ .41 .58	$Q \rightarrow P$ .56 .24	$P \rightarrow Q$ .57 .18
8	$P \rightarrow Q$ .54	$P \leftrightarrow Q$ .36 .64	$P \nrightarrow Q$ .54 .24	$Q \rightarrow P$ .59 .12
High School	$P \rightarrow Q$ .43	$P \leftrightarrow Q$ .39 .52	$P \rightarrow Q$ .43 .45	$Q \rightarrow P$ .67 .02
College	$P \rightarrow Q$ .34	$P \rightarrow Q$ .34 .72	$P \leftrightarrow Q$ .56 .19	U .58 .08

Table 3 Continued

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
Conditionality: ONLY IF				
Encoding Task				
2	$P \rightarrow Q$ .57	$P\ V_{in}\ Q$ .48 .30	U .50 .33	$P \wedge Q$ .55 .36
4	$P \rightarrow Q$ .48	$P \leftrightarrow Q$ .39 .40	$Q \rightarrow P$ .42 .27	$P \rightarrow Q$ .48 .32
6	$P \rightarrow Q$ .42	$P \leftrightarrow Q$ .38 .41	$P \rightarrow Q$ .42 .33	$P \wedge Q$ .55 .25
8	$P \rightarrow Q$ .24	$P \rightarrow Q$ .24 .54	$P \leftrightarrow Q$ .35 .41	U .54 .04
High School	$P \rightarrow Q$ .28	$P \rightarrow Q$ .28 .75	$P \leftrightarrow Q$ .52 .25	U .57 .00
College	$P \rightarrow Q$ .13	$P \rightarrow Q$ .13 .94	$P \leftrightarrow Q$ .50 .06	U .52 .00
Combination Task				
4	$P \rightarrow Q$ .51	$P \rightarrow Q$ .51 .34	U .56 .42	$P \leftrightarrow Q$ .61 .24
6	$P \rightarrow Q$ .44	$P \leftrightarrow Q$ .35 .62	$P \rightarrow Q$ .44 .28	$Q \rightarrow P$ .63 .10
8	$P \rightarrow Q$ .38	$P \rightarrow Q$ .38 .62	$P \leftrightarrow Q$ .41 .36	U .73 .02
High School	$P \rightarrow Q$ .23	$P \rightarrow Q$ .23 .78	$P \leftrightarrow Q$ .53 .22	U .69 .00
College	$P \rightarrow Q$ .06	$P \rightarrow Q$ .06 .97	$P \leftrightarrow Q$ .66 .03	U .68 .00

Table 3 Continued

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
Biconditionality: IF AND ONLY IF				
Encoding Task				
2	$P \leftrightarrow Q$ .58	U .42 .32	$P \vee_{in} Q$ .43 .35	$P \rightarrow Q$ .45 .32
4	$P \leftrightarrow Q$ .38	$P \rightarrow Q$ .36 .20	$P \leftrightarrow Q$ .38 .54	U .46 .25
6	$P \leftrightarrow Q$ .38	$P \leftrightarrow Q$ .38 .43	$P \rightarrow Q$ .46 .27	$P \wedge Q$ .54 .30
8	$P \leftrightarrow Q$ .31	$P \rightarrow Q$ .29 .44	$P \leftrightarrow Q$ .31 .52	U .53 .04
High School	$P \leftrightarrow Q$ .47	$P \rightarrow Q$ .33 .64	$P \leftrightarrow Q$ .47 .36	U .59 .00
College	$P \leftrightarrow Q$ .39	$P \rightarrow Q$ .15 .75	$P \leftrightarrow Q$ .39 .25	U .52 .00
Combination Task				
4	$P \leftrightarrow Q$ .55	$P \rightarrow Q$ .46 .36	$P \leftrightarrow Q$ .55 .39	U .56 .25
6	$P \leftrightarrow Q$ .46	$P \rightarrow Q$ .37 .39	$P \leftrightarrow Q$ .46 .50	U .64 .11
8	$P \leftrightarrow Q$ .38	$P \rightarrow Q$ .37 .36	$P \leftrightarrow Q$ .38 .60	U .72 .05
High School	$P \leftrightarrow Q$ .39	$P \rightarrow Q$ .36 .48	$P \leftrightarrow Q$ .39 .52	U .74 .00
College	$P \leftrightarrow Q$ .39	$P \rightarrow Q$ .32 .61	$P \leftrightarrow Q$ .39 .39	U .74 .00

Table 3 Continued

Grade Level	Correct Pattern	First-Ranked Pattern	Second-Ranked Pattern	Third-Ranked Pattern
Affirmation: IS				
Encoding Task				
2	P .46	$P \vee_{in} Q$ .37 .22	U .39 .33	P .46 .45
4	P .45	$P \wedge \bar{Q}$ .34 .50	P .45 .34	$\bar{Q}$ .55 .16
6	P .33	P .33 .44	$P \wedge \bar{Q}$ .42 .41	$Q \rightarrow P$ .53 .15
8	P .07	P .07 .89	$Q \rightarrow P$ .44 .11	$P \vee_{in} Q$ .51 .00
High School	P .08	P .08 .85	$Q \rightarrow P$ $.45_t$ .13	$P \vee_{in} Q$ $.45_t$ .02
College	P .06	P .06 .86	$Q \rightarrow P$ .46 .08	$P \vee_{in} Q$ .48 .06
Negation: IS NOT				
Encoding Task				
2	$\bar{P}$ .56	U .28 .43	$P \vee_{in} Q$ .35 .25	$P \rightarrow Q$ .41 .31
4	$\bar{P}$ .29	$\bar{P}$ .29 .61	$\bar{P} \vee_{in} \bar{Q}$ .42 .22	$P \rightarrow Q$ .47 .17
6	$\bar{P}$ .22	$\bar{P}$ .22 .66	$P \rightarrow Q$ $.44_t$ .21	$\bar{P} \vee_{in} \bar{Q}$ $.44_t$ .13
8	$\bar{P}$ .14	$\bar{P}$ .14 .83	$P \rightarrow Q$ .44 .10	$\bar{P} \vee_{in} \bar{Q}$ .47 .07
High School	$\bar{P}$ .15	$\bar{P}$ .15 .74	$P \rightarrow Q$ .40 .19	$\bar{P} \vee_{in} \bar{Q}$ .47 .07
College	$\bar{P}$ .06	$\bar{P}$ .06 .89	$P \rightarrow Q$ $.48_t$ .06	$\bar{P} \vee_{in} \bar{Q}$ $.48_t$ .06

Table 3 Continued

Note: Fit in this table is measured by root-mean-square deviation (RMSD). The numeral in the center of each cell is the overall RMSD (badness of fit) for a given interpretation of the logical connective at a particular grade level. This measure is computed by taking the mean of the squares of the differences between predicted and observed values for the group data, and finding the square root of this mean. In the present application, a value of 0 would indicate perfect fit of a response pattern to an observed pattern, whereas a value of 2 would indicate maximally discrepant fit. The italicized numeral at the right of each cell is the proportion of individual subjects for whom a given interpretation of the logical connective at a particular grade level was best of the top three models. For example, in the encoding task, the data of .51 of the second-graders were best fit by the conjunctive interpretation of <u>and</u>; the data of .19 were best fit by an affirmation (Q); and the data of .30 were best fit by the inclusive disjunction. In this table, logical symbols are the same as in Table 1. The letter <u>t</u> is used as a sbuscript to denote a tied RMSD between two entries in a single row. In such cases, the order of entries is arbitrary. All fits were also assessed by squared correlations between predicted (model) and observed (response) patterns, with similar results.

[a]Since combination data for the second graders were unreliable, they are not presented in the table. Note that in the combination task for <u>and</u>, the conjunctive and biconditional interpretations of <u>and</u> were completely confounded (see text). Proportions were therefore not split between entries, and instead the full proportion of subjects for whom the relevant model pattern provided the best fit is shown.

Table A

Mean Response Patterns Associated with Each Logical Connective

Conclusion	2	4	6	8	HS	COL	2	4	6	8	HS	COL
Encoding Task												
	AND						OR					
PQ	1.24	1.16	1.03	1.00	1.03	1.06	1.68	2.40	2.74	2.68	2.69	2.83
P$\bar{Q}$	2.53	2.84	2.95	2.97	3.00	3.00	2.15	1.90	1.87	1.92	1.89	1.92
$\bar{P}$Q	2.38	2.84	3.00	2.97	3.00	3.00	2.06	2.03	1.82	1.95	1.92	1.92
$\overline{PQ}$	2.65	2.76	2.90	2.95	3.00	2.97	2.62	2.53	2.74	2.92	2.86	2.97
	IF-THEN						ONLY IF					
PQ	1.29	1.47	1.34	1.58	1.36	1.69	1.29	1.55	1.40	1.74	1.47	1.75
P$\bar{Q}$	2.44	2.68	2.79	2.87	2.94	2.89	2.35	2.55	2.87	2.95	3.00	3.00
$\bar{P}$Q	2.32	2.76	2.71	2.58	2.39	2.17	2.29	2.66	2.55	2.40	2.11	2.03
$\overline{PQ}$	2.53	2.18	2.13	2.03	1.89	1.92	2.53	2.29	2.08	1.87	1.86	2.00
	IF AND ONLY IF											
PQ	1.53	1.58	1.37	1.74	1.42	1.83						
P$\bar{Q}$	2.44	2.66	2.82	2.90	2.97	3.00						
$\bar{P}$Q	2.24	2.47	2.63	2.50	2.28	2.25						
$\overline{PQ}$	2.47	2.08	2.08	2.08	1.89	1.94						
	IS						IS NOT					
PQ	2.15	2.53	2.32	1.95	2.00	2.00	2.21	2.76	2.79	2.87	2.78	2.94
P$\bar{Q}$	1.59	1.34	1.45	1.98	1.94	1.92	2.32	2.68	2.79	2.92	2.89	2.94
$\bar{P}$Q	2.35	2.82	2.97	3.00	2.89	2.94	1.91	1.63	1.68	1.82	1.83	1.92
$\overline{PQ}$	2.53	2.71	2.84	2.90	2.89	2.92	2.41	2.24	1.90	2.00	2.03	1.94

Table A Continued

		2	4	6	8	HS	COL	2	4	6	8	HS	COL
Combination Task													
		AND						OR					
Minor Premise	Conclusion												
P	Q	1.68	1.18	1.05	1.12	1.00	1.00	1.91	2.29	2.29	2.48	2.75	2.92
P	$\bar{Q}$	2.12	2.79	2.74	2.83	2.95	2.83	1.79	1.60	1.71	1.40	1.15	1.06
$\bar{P}$	Q	----	----	----	----	----	----	1.59	1.45	1.47	1.14	1.00	1.03
$\bar{P}$	$\bar{Q}$	----	----	----	----	----	----	2.29	2.63	2.34	2.67	2.82	2.97
Q	P	1.79	1.42	1.03	1.12	1.00	1.00	1.85	2.37	2.24	2.64	2.72	2.81
Q	$\bar{P}$	2.21	2.58	2.92	2.93	2.95	3.00	2.09	1.66	1.76	1.64	1.30	1.06
$\bar{Q}$	P	----	----	----	----	----	----	1.65	1.32	1.34	1.17	1.08	1.08
$\bar{Q}$	$\bar{P}$	----	----	----	----	----	----	2.38	2.53	2.37	2.83	2.85	2.94
		IF-THEN						ONLY IF					
P	Q	1.65	1.13	1.05	1.02	1.02	1.00	1.59	1.40	1.84	1.00	1.10	1.03
P	$\bar{Q}$	2.26	2.68	2.82	2.93	3.00	2.94	1.97	2.79	2.76	2.86	2.80	2.94
$\bar{P}$	Q	1.71	2.37	2.84	2.71	2.50	2.31	1.76	2.03	2.47	2.19	2.20	2.08
$\bar{P}$	$\bar{Q}$	1.97	1.63	1.40	1.31	1.52	1.72	2.03	2.03	1.47	1.60	1.88	1.97
Q	P	1.85	1.16	1.29	1.24	1.40	1.75	1.85	1.21	1.29	1.29	1.52	1.89
Q	$\bar{P}$	1.94	2.71	2.47	2.50	2.55	2.36	1.88	2.74	2.58	2.60	2.28	2.03
$\bar{Q}$	P	1.85	2.03	2.55	2.48	2.60	2.47	1.85	2.47	2.90	2.95	3.00	3.00
$\bar{Q}$	$\bar{P}$	1.88	1.84	1.76	1.52	1.40	1.53	1.97	1.63	1.37	1.29	1.00	1.08

Table A Continued

Minor Premise	Conclusion	2	4	6	8	HS	COL
				IF AND ONLY IF			
P	Q	1.97	1.34	1.16	1.05	1.02	1.00
P	$\bar{Q}$	2.21	2.63	2.82	2.93	2.88	2.94
$\bar{P}$	Q	1.68	2.18	2.34	2.33	2.30	2.42
$\bar{P}$	$\bar{Q}$	2.15	1.92	1.79	1.60	1.55	1.56
Q	P	1.88	1.18	1.29	1.31	1.28	1.50
Q	$\bar{P}$	2.06	2.58	2.45	2.55	2.42	2.44
$\bar{Q}$	P	1.56	2.60	2.71	2.88	2.92	2.92
$\bar{Q}$	$\bar{P}$	1.97	1.50	1.26	1.24	1.10	1.08

Note: Numerical entries in table are means computed such that 1=Definitely True; 2=Maybe True, Maybe False; 3=Definitely False.

Technical Reports Presently in this Series

NSF Grant 76-05311

#1. Sternberg, R. J., & Rifkin, B. The development of analogical reasoning processes. June, 1978.

#2. Sternberg, R. J. Developmental patterns in the encoding and combination of logical connectives. October, 1978.