DOCUMENT RESUME

ED 164 157 RC 010 767

TITLE Navajo Area Mathematics Guidelines.
INSTITUTION Bureau of Indian Affairs (Dept. of Interior), Window Rock, Ariz.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 77
GRANT GW-8613; PES-75-01852
NOTE 363p.

EDRS PRICE MF-$0.83 HC-$19.41 Plus Postage.
DESCRIPTORS *American Indians; *Curriculum Guides; Elementary Education; Elementary School Mathematics; Geometric Concepts; Geometry; Illustrations; *Learning Activities; *Mathematical Concepts; *Mathematics; Numbers; Pattern Recognition; Reservations (Indian); *Set Theory
IDENTIFIERS *Navajo (Tribe)

ABSTRACT

Heavy emphasis is placed on development of understanding of arithmetic in this mathematics curriculum prepared specifically for Navajo children and intended for use in reservation schools. Materials are presented so that only when a given concept is understood are skills associated with it and a written format introduced. Learning activities expose students early to ideas of patterning and repatterning. The guidelines are to be used as a mathematical core for elementary school, not as a total curriculum. The more than 140 objectives are categorized according to four strands--comparative (sets, cardinal and sequential relationships, grouping, place value, types of numbers), joining (addition and multiplication operations), separate (subtraction and division operations), and geometry (line, shapes, figures, measurement). Objectives are cross referenced according to strand components, and the guidelines also contain a cross reference with other text books available in the Navajo area. The guidelines are organized according to objective; for each, there are comments, prerequisite objectives, and at least two activities, with materials listed and directions for carrying them out explained. Illustrations of mathematical processes are provided, along with ditto sheets for some activities. The guidelines also contain a sample of the pupil's record sheet and a glossary. (RS)

ED164157

NAVAJO

AREA

MATHEMATICS

GUIDELINES

RC010767

Donald Dodge
Area Director

Assistant Area Director (Education)
Rebecca H. Dotson

Acting Deputy Assistant Area Director (Education)
Ernest L. Magnuson

Chief, Branch of Curriculum and Instruction
James G. Kipp

First Printing, 1975
Second Printing, 1977

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This draft of the Navajo Area Mathematics Guidelines is the result of efforts of many education staff members. Credit goes to school and agency committee members who determined needs upon which these objectives are based, to teachers who developed, piloted and evaluated initial objectives, to the members of the Navajo Area Mathematics Curriculum Committee who provided guidance throughout the development stage and to the following writers who gave so generously of their time, efforts and knowledge in writing the guidelines.

The writing took place in two stages; the working draft (1974) and the final draft (1975). The working draft was prepared by:

Sharron Hunter and Juanita Begay - Chinle Agency

Georgia Robertson and Margaret Sue Fike - Eastern Navajo Agency

Ola Bonner - Fort Defiance Agency

Tom Bryant and Melody Bradshaw - Shiprock Agency

Diane Pettigrew and Fred Ritchhart - Tuba City Agency

Ethyl Yazhe - Navajo Area

with the assistance of

Harry Rosenberg - Fort Lewis College

The final draft was prepared by:

Sharron Hunter - Chinle Agency

Margaret Sue Fike - Eastern Navajo Agency

Ola Bonner - Fort Defiance Agency

Tom Bryant and Melody Bradshaw - Shiprock Agency

Fred Ritchhart - Tuba City Agency

with the assistance of

Dick Gibbs - Fort Lewis College

The test items were prepared by:

Rosalie LeCount - Chinle Agency

Judy Sturm - Fort Defiance Agency

Special thanks goes to: Patricia Hale; Sharrie Holt; Erika Yazzie; Kathy Prouty; Allison Raish; John Ross, Jr.; Peggy Hoyos; Judy Willis; and Janet Carelli for their efforts in the preparation of the guidelines. These guidelines were prepared with the support of National Science Foundation grants GW-8613 and PES-75-01852. The opinions and recommendations expressed herein are those of the authors, and do not necessarily reflect the views of the National Science Foundation.

RATIONALE

These guidelines were prepared with the specific needs of the Navajo child in mind.

The basic purpose of the guidelines is to first develop an understanding of arithmetic and then develop skill in its use. Conceptualization is an integral part of a child's understanding. For our purposes, conceptualization is recognition of some invariant characteristic in a series of concrete experiences. Understanding and skill must develop together. Often, understanding is deemphasized and skill is overemphasized because skill is easier to measure. In the guidelines, however, heavy emphasis is placed on the development of understanding. This does not mean that you should sacrifice the drill necessary to develop skill, only that it should follow understanding.

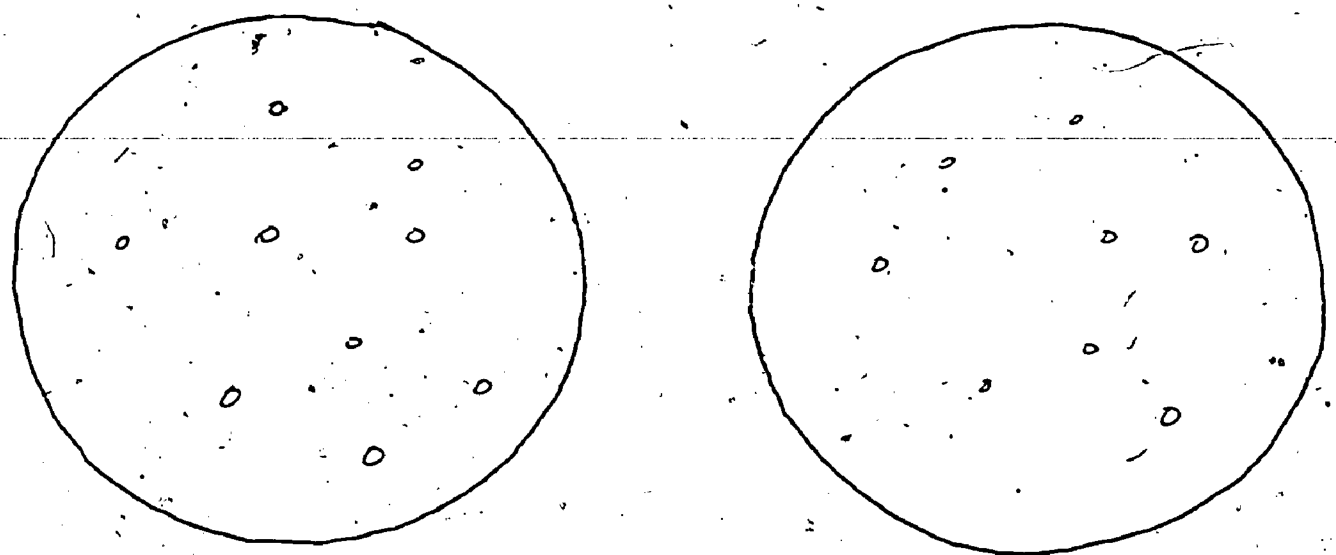
Throughout the guidelines the child is first given experiences related to a concept, then he begins to realize and verbalize the concept involved. Only when this is accomplished should a written format and the skills associated with the concept be introduced. For example, in the guidelines, the child is given experiences with sets, discussing various likes and differences between sets that will, in the end, develop his quantative sense. Only then is the idea of number introduced. Thus number becomes a way of talking about something the child already senses conceptually.

Since arithmetic is a particularly tight conceptual structure, the child is exposed early to the ideas of patterning and repatterning as the basis upon which all arithmetic is developed. Keep in mind that the average person cannot cope with large sets unless they are patterned in some way that is meaningful to him. Thus as a child develops a sense of quantity, he must simultaneously develop a patterning process for recognition's sake. These initial patternings and repatternings will vary from child to child and it is the teacher's task to draw these together, in time, to the standard patterning format (base ten).

Since the patterning-repatterning process is so important it should be carefully developed and is itself an integral part of number recognition. Only after quantity-number and patterning-repatterning are well developed should the arithmetic processes be developed.

All arithmetic operations are first presented with concrete experiences. Only when the child has shown mastery at this level should the written format be developed. When this stage is reached, the more closely the recording process is coupled with the actual experiences the better. If possible, it is best to have materials arranged so that the child can record a particular experience step by step as it is done.

Place value (base ten) was alluded to in the preceding discussion on patterning. However, since it is one of the most important concepts to be developed in the elementary school, it is worth a few additional words. If the method by which quantity is recorded is not meaningful, the child will have no end of trouble in arithmetic. The feeding back of such words as thousands, hundreds, tens, and units does not imply that the child understands what is happening. Teaching place value (base ten) is a very difficult task, if for no other reason than most children cannot discriminate the size of sets much larger than 5 or 6 unless they are patterned or counted. Consider the following two sets. Without patterning or counting, which is larger?



The point is that visual discrimination of set size is very difficult for sets this large. Most people, when looking at large sets, view them as comprised of smaller sets. For example, a set of size 7 may be viewed as a set of size 4 and a set of size 3.

Therefore the child must be introduced to the patterning involved in place value with set sizes he can easily recognize. Once the child grasps how the patterning is being done and what it means, the child can be led to patterning by sets of size "ten". What must be kept in mind when teaching place value is that you are going to use the child's natural tendency to pattern (group) to recognize and discriminate between large sets and, step by step, develop the usual base ten place value system. This is no small task and sufficient time should be taken to move from the simple patterning a child might initially use to the very sophisticated place value patterning the child will use throughout his work in mathematics.

With regard to operations, it is rather common to develop addition and subtraction together as inverses and multiplication and division together on the same basis. Structurally this is sound, but certainly not mandatory. Since this approach forces the child into subtraction, one of the more difficult operations for children, quite early an alternate approach is used in the guideline that will permit us to delay subtraction till the child is at a higher readiness level. Thus the guideline follows addition with multiplication, where multiplication is viewed as repeated (extended) addition. Later when subtraction is developed, it is developed as a separation process and division is developed as repeated subtraction. No attempt is made to avoid informal uses of inverses, but the formal tie is not made till the 7th and 8th grades when structural characteristics are studied.

In all operations, one thing is important for the child to realize. Quantity is <u>conserved</u>, that is, the patterning of a set may change to suit a particular situation, but the quantity involved does not change. Thus in the sentence 12 - 7 = 5, a set of 10 and 2 is being repatterned into a set of 7 and 5. Nothing is lost, thrown away, etc. This is a somewhat different approach regarding subtraction, but one that is sound experientially. In any concrete model of subtraction, the items "removed" or "taken away" are still really there, just set apart for a moment. That is, the set has been repatterned.

When writing mathematical sentences, it is important to be specific. For example, the terms "factor" and "product" are not specific enough for most young children. However, "multiplier" and "multiplicand" suggest specific roles that numbers play. Multiplier tells how many sets, while multiplicand tells the size of each set. Thus in 3 x 4, there are 3 sets of size 4, not just 3 times 4. The more specific the interpretations the better chance of them being used; the more abstract and general the more difficult they are to apply.

The objectives are presented in a particular order which represents the best thinking of the writing team. The various strands are organized creating what may appear to be breaks in the sequence. These "breaks" are necessary in order to continuously move through the various strands Though the order given is strongly recommended, it is not sacred. Individual schools and situations often require unique approaches. Therefore, if a different ordering is needed to meet a particular situation that ordering should be adopted. However, under no circumstances should all of one strand be taught at one time.

Certain special topics such as ratio, proportion, average and percentage do not occur directly as part of the guidelines. Though they are important they do not comprise the "core" of junior high arithmetic. These guidelines are not intended to be a complete curriculum at the upper elementary level -- they present the "core" arithmetic skills needed. You should insert any of these or other topics you feel have been omitted as they are needed, provided they come after any pre-requisite arithmetical skills. Such topics can usually be referenced in any standard arithmetic series.

A special comment about time and money. Both certainly involve the use of numbers, but each is a special skill in its own right and does not require the use of any extensive arithmetic operations. Telling time requires no skill other than number recognition. A person can be readily trained to tell time and use money. In particular, don't use money as a device to teach place value. Once place value is understood, money serves as an excellent example of it.

DIRECTIONS FOR USE

Introduction

These guidelines are to be used as a mathematical core, not as a total curriculum. Any topic that you feel has not been covered should be inserted in the guidelines to complete your mathematics program.

Directions

I. The rationale is a requisite for the understanding of these guidelines. READ CAREFULLY.

II. The pre-tests are to be used as a diagnostic tool for placement of your children.

III. The format of the objectives

A. The objective number is followed by a letter which refers to one of four strands. The strands are:

1)C - Comparative

2)J - Joining

3)S - Separate

4)G - Gecmetry

The Comparative Strand refers to sets, cardinal and sequential relationships, grouping, place value, and types of numbers (fractions, decimals, integers, factors).

The Joining Strand refers to the operations of addition and multiplication.

page viii blank

The Separation Strand refers to the operations of subtraction and division.

The Geometry Strand refers to line, shapes, figures and measurement.

B. The reference indicates the reference area to which the objectives in the guidelines are cross-referenced with most text books available in the Navajo area.

The reference areas are:

1) addition
2) applications and problem solving
3) categories
4) division
5) geometry
6) multiplication
7) number relations
8) sets
9) subtraction
10) types of numbers

C. Some objectives contain comments related to the objective or activities and sometimes provide purpose and definitions.

D. Pre-requisite objectives are those objectives which are related to or reinforce another objective.

E. There are two or more activities for most objectives. Add as many activities as you find necessary.

F. Each objective is self-contained for easy removal and replacement.

LIST OF OBJECTIVES

Page xii blank

OBJECTIVES LISTED BY REFERENCE AREAS

ADDITION

32J	Pattern and repattern a set
33J	Repattern into subsets
34J	Record addition
35C	Word name of ten
36C	Group tens and units
39J	Basic addition facts (0-18)
41J	Basic fact equations plus sets of ten
42J	Addition of two place numbers
43J	Join three or more sets
52J	Addition without models (2 place)
53J	Addition of three place numbers
54J	Addition without models (3 place)
144J	Structural properties of addition

Application and Problem Solving

See individual reference areas and cross-referencing

CATEGORIES

47C	(Odd and Even)	Even and odd numbers (1-90)
48C	(Odd and Even)	Even and odd numbers related to addition

49C (Odd and Even) Unit digit determines odd or even

92C Model sets as rectangles

DIVISION

86S Repeated subtraction

87S Separate sets into two groups

88S Repeated subtraction with recording

89S Partial quotient division algorithm

90S Basic division facts

91S Develop traditional division algorithm

122S One digit divisor

123S Two digit divisor

124S Use traditional division algorithm

125S Three and four digit divisors

126C Check

136S Change remainder to fraction

GEOMETRY

5G Construct lines

6G Straight and curved lines

7G Longest and shortest lines

8G Identify vertex

9G Basic straight line figures and shapes

10G Curved line figures

11G Boundary

12G Inside and Outside of figures

8C	Duplicate a pattern
14C	Continue an established pattern
19C	Create a pattern
55G	Nonstandard linear measurement
56G	Standard linear measurement
57G	Measure perimeter
58G	Line and line segment
59G	Labeling
60G	Perpendicular and parallel lines
61G	Rays and Angles
66G	Weight
67G	Non-standard area measurement
68G	Square units
69G	Formula for area of a square
70G	Formula for area of a rectangle
96G	Formula for area of a right triangle
100G	Volume
101G	Formula for rectangular solids
102G	Cubic measure
103G	Congruent shapes
104G	Similar shapes
105G	Measurement with fractional units
106G	Longest and shortest curved line segment
110G	Parts of a circle
111G	Measure angles

112G	Circumference of a circle
113G	Area of a circle

MULTIPLICATION

62J	Models for multiplication
63J	Times sign
64J	Multiplication facts
65J	Multiplication of one place by two place
71J	Model two place times two place numbers
72J	Two place times two place without models
73J	Two and three place times three place
75C	Model repeated multiplication
95J	Common multiples
114J	Least common multiples
145J	Structual properties of multiplication
146J	Distributive property

NUMBER RELATIONS

20C	Name set size (0 - 9)
22C	Name numbers 0 - 9
23C	Symbols 0 - 9
24C	Numerals 0 - 9
25C	Sequencing
26C	Groups and units
27C	Groups of groups, groups, and units

28C	Concrete representative for a group	
37C	(Place Value)	Names for numbers 10 - 19
38C	(Place Value)	Write numerals 10 - 19
40C	(Place Value)	Name and Write numerals 20 - 99
44C	(Place Value)	Counting by hundreds 100 - 900
45C	(Place Value)	Name and Record hundreds with tens
46C	(Place Value)	Name and Record hundreds, tens, and units
50C	(Place Value)	Counting by thousands 1,000 - 9,000
51C	(Place Value)	Name and Record numerals 1,001 - 9,999
74C	(Place Value)	Name and Write numerals 10,000 - the millions
76C	Exponential notation	
77C	(Place Value)	Exponents and Place Value

SETS

1C	Make a set
2C	Name the members of a set
3C	Subsets
4C	Empty set
15C	Sets by attributes
16C	Compare sets by attributes
17C	"1 - 1" correspondence
18C	More, Less, Same Size
21C	Recognition of set size

SUBTRACTION

78S	Partitioning sets

27

79S	Partitioning sets by given size
80S	Introduce subtraction symbol
81S	Subtraction facts
82S	Two or three place minuends
83S	Minuends containing zero
84S	Subtraction of four and five place numbers
85C	Relate addition to subtraction

TYPES OF NUMBERS

Factors

93C	Factors
94C	Prime Factors

Fractions

29C	Model ½
30C	Model ¼
31C	Model 1/3
97C	Model Fractions
98C	Like denominators
99C	Equivalent fractions
107J	Multiplication
108C	Improper fractions
109J	Multiply mixed numbers
115C	Least common denominator
116C	Simplest form

117J Addition

118J Addition of mixed numbers

119S Subtraction

120S Subtraction of mixed numbers

121C Comparison

134S Division

135S Division of mixed numbers

Decimals

127C "th" sound

128C Denominators - powers of ten

129C Decimal notation

130J Addition

131S Subtraction

132C Money

133J Multiplication

137S Division with decimal in divisor

138S Fractions to decimals

Integers

139C Integers

140J Addition

141S Subtraction

142J Multiplication

143S Division

GENERAL REMARKS This first sequence of objectives is to develop an initial concept of set. One should not hurry through this sequence, but give the children plenty of time to absorb the ideas involved. All activities should be kept in oral format.

1-C	Reference Sets

OBJECTIVE Make a set.

COMMENTS See definition for set.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A.

MATERIALS: odds and ends and set indicators

DIRECTIONS: Teacher will make many models of sets. Start with "I am going to make a set." Make the set and say "This is my set." Repeat with new sets. Ask, "What am I doing?","What am I making?", "What is this called?"

B.

MATERIALS: odds and ends, set indicators

DIRECTIONS: Students make own sets. Ask what they are making or what they have made (a set). "What is this?" "A set."

OBJECTIVE Name the members of a set.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A.

MATERIALS: odds and ends, and set-indicators

DIRECTIONS: Continue making sets as in activity 1-B asking students to name the members of his set.

B. PUPIL CONSTRUCTED SETS

VARIATION 1

MATERIALS: Art supplies, magazines, glue

DIRECTIONS: Teacher asks, "What do we need to make a _____ (see suggestions below)?" Small groups of pupils make a list of what is needed and set out to find pictures and make a collage for the set.

SUGGESTIONS: Classroom, house, carnival, rodeo, book, circus, car, TV programs, ranch, Chapter house meeting, basketball game, sports, dances, library, Trading Post (draw a picture), rug weaving, shearing sheep, pottery, a supermarket.

Ex.: "What do we need to make a classroom?" List--desks, chairs, blackboards, teacher, pupils, pencil, paper, chalk, erasers, etc.

VARIATION 2

MATERIALS: Poster paper, various media (paint, chalk, pencil, charcoal marker)

DIRECTIONS: The pupil will choose a topic for his set. He will cut out and paste pictures to make a collage. Then identify the poster as a set of dogs and name the members either by name, collie, or small spotted dog, or larger brown dog.

Page 2 blank.

VARIATION 3

MATERIALS: Poster paper, colored chalk or paint, crayons.

DIRECTIONS: The pupil will draw his own set. Identify the members of the set by labeling the pictures he has drawn or by telling the teacher what is in the picture.

VARIATION 4

MATERIALS: "Stockpile words" from a lesson in science, social studies, etc.

DIRECTIONS: Identify and make sets by classifying words. Examples of "stockpile words."

cactus	flower	hay	bean
stems	weeds	fruits	corn
seeds	grass	apple	

Stockpile categories in science, geography, and social studies--

food	clothing	house or buildings
climate	animals	transportation

Examples of classifying words--

Kinds of plants	Parts of plants
flower	stems
weeds.	seeds
grass	leaves
cactus	fruits
hay	

32

OBJECTIVE Identify subsets of a given set.

COMMENTS Sets should be separated into as many subsets as possible. See definition of subset.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MAKING SUBSETS

MATERIALS: odds and ends, and set indicator

DIRECTIONS: Place a large set on the floor and use the yarn as a set indicator. Within the large set use yarn to put set indicators around two or three subsets. For example, in a large yellow yarn set, put a spool, pencil, cup, blocks, buttons, comb and pen. Inside the yellow yarn set, put a piece of red yarn to circle a set of the spool, pencil and cup. Also, put a piece of blue yarn to circle the comb and pen. Ask the pupil to name the members of the yellow set, the red set, and blue set.

B. SUBSETS

MATERIALS: odds and ends, set indicator

DIRECTIONS: Put a large group of objects on the floor surrounded by yarn to indicate the set. Give a pupil a small circle of red yarn and say, "Make a red set in the big set." When the set is made, have the pupils name the objects in the red set.

At first have only one set at a time made inside the large set. Later let pupils put two or three sets inside the large set. If the sets overlap, the teacher may need to comment that some objects are in two sets, but this is all right.

If some pupils want to make a set grouping objects by the same size, color, or shape, it might indicate a readiness for starting to build sets by attributes.

4 -C Reference Sets

OBJECTIVE Identify and Model empty set

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. EMPTY SET

MATERIALS: Cups, glasses, jars, cans to use as set indicators; odds and ends to use as set members.

DIRECTIONS: Put several objects in all but one set holder. Ask the pupil to "name the objects in each set." When you get to the set that has no objects in it, disscuss the fact that this set has no objects in it and introduce the term "empty set." Repeat this several times, always leaving at least one set empty.

B. STUDENT CONSTRUCTED EMPTY SET

MATERIALS: Odds and ends, set indicators for each child.

DIRECTIONS: Have children line up in sets (teacher calls names), make a set of children who are absent that day, (teacher again calls names-) this set will be empty.

Give children odds and ends and set indicators. Have them make a set containing objects they do not have.

page 6 blank

GENERAL REMARKS The following sequence of objectives is essentially pre-geometry activities. These ideas and concepts will include line, basic figures, i.e. triangle, square, rectangle, circle, and basic shapes.

5 -G	Reference Geometry

OBJECTIVE Construct lines

COMMENTS A line has no width or depth but length only. Models should be closely related to the idea of line as a continuous set of points. Technically, the pupils will be drawing line <u>segments</u> but the distinction need not be made at this point.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. CONTINUOUS PENCIL DRAWINGS

MATERIALS: $8\frac{1}{2}$ x $11\frac{1}{2}$ sheets of paper and pencils (colored if possible).

DIRECTIONS: Have the children draw designs without lifting the pencil from the paper. They will produce all types of lines. Have a pupil begin at one point (any point on the paper). Tell him to draw any design he wishes, only once he begins, he cannot lift his pencil from the paper.

B. DOT-TO-DOT LINES

MATERIALS: Sheets of paper with dots drawn. Dots may be in a pattern, or drawn randomly.

DIRECTIONS: Have the children connect the dots without lifting the pencil from the paper. The pupils may connect dots in any pattern they wish, as long as they use only one line and never lift the pencil.

VARIATIONS: Let children draw their own dots, and then connect them with one line.

page 8 blank

OBJECTIVE Identify straight and curved lines.

COMMENTS Make sure the pupil makes a conscious distinction between straight and curved lines.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. LINE CLUES

MATERIALS: feel box, pieces of wire bent in curved lines, and in straight lines.

DIRECTIONS: Place the wire in the feel box. The pupil comes up, puts his hand in the feel box and feels the wire and tells what type of line he has found--curved or straight. When several objects are used in the feel box, the pupil should remove the one he has selected, after he has said what it is.

VARIATIONS:
1. Have only one piece of wire in the feel box.
2. Have several pieces in the feel box.
3. Have string glued on a small piece of cardboard in curved and straight lines for the child to feel.
4. Use pipe cleaners.
5. Put several curved and straight line models in the feel box and instruct the child to removed a straight line or a curved line model.

B. LINES--ART PROJECT

MATERIALS: poster paper, starch or flour, glue, string, scissors, pipe cleaners.

DIRECTIONS: Pupils make a poster using string and glue. They glue the string in curved and straight lines.

VARIATION: ~~Find examples in pictures~~ from magazines, label or make straight and curved collages.

page 10 blank

OBJECTIVE Identify longest and shortest straight line.

COMMENTS A comparison of lengths of straight and curved line segments will occur later. Again, technically you are dealing with straight line <u>segments</u> rather than lines but it is not necessary to make the vocabulary distinction at this point.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. LONGEST OR SHORTEST LINE ACTIVITY (LARGE DISPLAY)

MATERIALS: Different colored magic markers, oak tag 24 x 36, colored yarn to match the magic markers.

DIRECTIONS: Divide oak tag into 9" x 6" rectangles using ruler and black magic marker. In each rectangle draw segments of different length and different color. Place the matching pieces of yarn over the line segments.

NOTE: You may want to make the lines equal in one (or several) rectangles. If you can, laminate the sheet after you draw segments. Plastic spray or hair spray can be used.

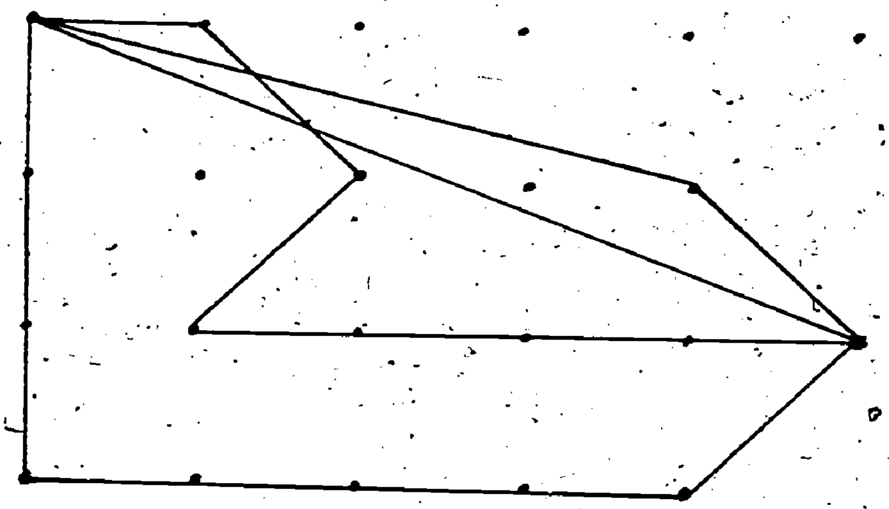
Have the children come up or gather around in a circle; and pointing to each rectangle, have a child guess which segment is longest or shortest. Have them verify this by removing the pieces of yarn and comparing their length.

B. THE SHORTEST PATH

MATERIALS: Peg board (3' x 3'), geoboard, or just any smooth panel or board into which you can drive nails in some pattern.

DIRECTIONS: Place pegs in peg board (drive nails in panel) in some pattern and then designate two of them in some manner. Tie pieces of yarn of different colors around them. Then have the children designate various paths from one of these pegs to the other. Using various colored pieces of yarn, have them indicate what they feel is the shortest path. Ask them if there is one shorter. If not, discuss why not. If yes, have them show it and discuss if it is the shortest possible path.

page 12 blank



NOTE: One could make this more interesting by making miniature trees, houses, lakes, ponds, etc., to set on the board. This activity gives the pupil his first experience that a straight line provides the path of shortest distance.

OBJECTIVE Identify Vertex.

COMMENTS A vertex is the point where two straight lines meet.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MAKING THE VERTEX

MATERIALS: poster paper cut in strips and with holes in the ends, paper fasteners. Paper strips should be as narrow as possible but will depend on size of paper fasteners and hole punch used.

DIRECTIONS: Pupil will make line segments by joining the ends with paper fasteners, pointing out that where the two lines meet (are joined) is the vertex.

B. FINDING THE VERTEX

MATERIALS: none but the classroom itself and what is in it.

DIRECTIONS: Pupils go around the room and find the vertex of lines.

Ex.--where two walls and ceiling meet; where two sides of a desk meet.

OBJECTIVE Identify the basic straight line figures and shapes.

COMMENTS Basic straight line figures and shapes will be the triangle, square, and rectangle. See glossary for distinction between figure and shape.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MAKING GEOMETRIC FIGURES

MATERIALS: wire, yarn, string, or geoboards and rubber bands

DIRECTIONS: Make a figure out of one of the above items. A pupil constructs the same figure and names its parts.

B. FIGURE RECOGNITION

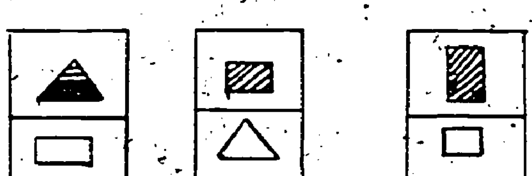
MATERIALS: Cards with a diagram of a square, rectangle, or triangle on each card.

DIRECTIONS: Place all the cards on a table or on the floor, use a variety of sizes, and ask the pupil to make a set of all the squares. Repeat using other figures.

VARIATION: 1. Hunting Squares. Hold up a square and have a pupil find a square in the classroom.
2. Hunting Rectangles
3. Hunting Triangles

C. GEOMETRIC DOMINOES

MATERIALS: Teacher-made cards as follows: On one half of each card, draw a basic straight line figure (△). On the other half of the card, draw a basic straight line shape (▨) (but not the same as the other picture on that card.) For example:



(The number of cards in a deck is up to the teacher and depends on the number of children to be in a group.)

Page 16 blank

DIRECTIONS:

(1) Shuffle the cards, deal 7 cards to each player, lay the remaining cards face down on the table..

(2) Place the top card from the discard pile face up in the center of the table.

(3) Play begins with the player to the dealer's right. Each player in turn must match a figure with the correct shape or vice versa.

(4) If a player cannot make a match he draws one time from the discard pile, and uses that card if possible. (If not, play goes to the next player.)

(5) Winner is the player using all his dominoes first.

OBJECTIVE Identify curved line figures.

COMMENTS Include the circle.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

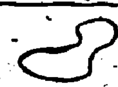
A. DETECTING CURVED LINE FIGURES

MATERIALS: feel box, various curved line figures made of wire or heavy string glued on cards to produce a raised effect. These should include shapes like the following:

oval

circle

general (funny)



DIRECTIONS: Have half a dozen to a dozen different kinds of curved figures, including several circles to put in the box.

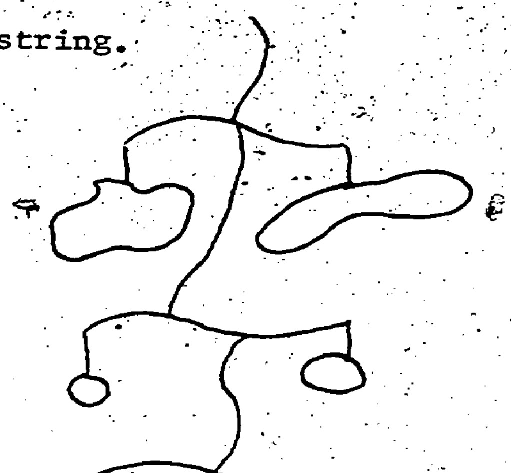
VARIATION 1. Have the children come up one at a time and select a figure to take from the box. Have them tell what it is before they remove it from the box.

VARIATION 2. Remember what you have put in the box and this time direct the child to remove a particular type of figure, that is, a circle, oval, or general (funny).

B.

MATERIALS: Scissors, wire, heavy colored paper, string.

DIRECTIONS: Have students work in pairs or small groups to make curved mobiles. The shapes used must be curved shapes, and the wire must be curved.



OBJECTIVE Identify boundary.

COMMENTS Examples of boundary are limit, territorial boundary, border, walls, fences.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

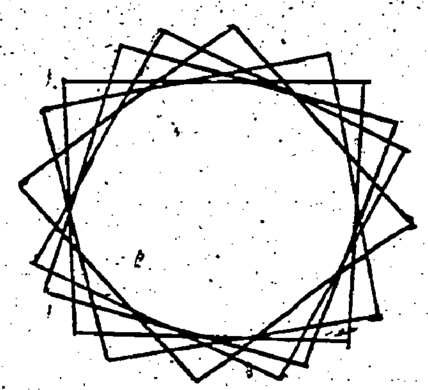
PRE-ACTIVITY: Have children point out boundaries of various shapes (run finger along boundary to develop a sense of its linear nature).

A. MAKING BOUNDARIES

MATERIALS: Magic markers, pencils or crayons (for each pupil), cardboard shapes, eg. , , , , poster paper.

DIRECTIONS: Have pupil place the cardboard shape on the poster paper and draw the boundary with a magic marker, pencil, or crayons and then remove the shape. Then talk with pupils about the figure. For example, "Trace the boundary using your finger," or "Show me the vertices," or "Point out the vertices."

NOTE: At this point, you can use the tracing of boundaries about a geometric shape to make various designs. For example, tack the center of a square to a piece of paper and then draw around the square. Rotate the square a little and then draw around again. Continue this procedure and you will get a design like the following.



page 20 blank

Something like this can be done with any geometric shape and it need not be tacked and rotated about the center. This kind of experience can develop motor skill and at the same time reinforce concretely the idea of boundary.

B. COOKIE

MATERIALS: Cookie dough, cookie cutters, and lifesavers.

DIRECTIONS: Cut dough into different shapes, i.e. ⬡ , △ , ○ . Take these shapes and cut smaller shapes in the center, i.e. ⬡ , △ , ◎ . Place a lifesaver in the center of each cookie before baking. Then bake. The melted lifesaver will show a shape bounded by a geometric figure that is known as a boundary.

NOTE: This activity will build in the pupil the subconscious feeling that math tastes good!

44

OBJECTIVE Identify inside and outside of figures.

COMMENTS Recall that a figure is closed. The figures used in this objective should have only one "inside".

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. OUTSIDE AND INSIDE

MATERIALS: large pieces of yarn--large enough for pupil to stand in, several objects from room.

DIRECTIONS: Spread yarn in a circle on the floor. Place several objects inside the circle and several outside the circle. Lead a discussion on inside and outside--pointing to an object inside the circle and saying, "This is inside the circle." Run hand around the circle of yarn and then point to an object outside and repeat. Do this several times until understood.
Then call on pupils in the group--"Tell me what objects are inside, name ." Or, "Tell me what objects are outside, name ."
After this is done, clear away the objects from inside and outside. Then give directions to pupils " Name , come and stand inside (or outside) the circle." Do this several times.

VARIATION

1. Each pupil has a piece of yarn tied in a circle and placed on the floor in front of him. The teacher or another pupil gives directions such as--"Everyone stand inside." Or "Everyone stand outside." "Boys stand inside and girls stand outside." Or "Boys stand outside and girls stand inside."

2. Use yarn in various shapes--not just in circles.

B. INSIDE AND OUTSIDE

MATERIALS: paper, pencils, rulers, compasses, crayons.

DIRECTIONS: Have pupils draw various triangles, squares, rectangles, circles, and free form figures. Then have the child color the inside of the square one color, the inside of the circles another color and so on.
NOTE: This can be made into an art project and would be a good break for the children.

OBJECTIVE Duplicate a given pattern

COMMENTS : Patterning activities help develop visual perception and will be used later for number recognition. The patterns involved in this objective could embody color, shape, size, etc. Since this is a pre-number objective, number should not be used in the patterns at this stage.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. PATTERNING

MATERIALS: attribute blocks, 1-inch colored cubes, cuisenaire rods, tinker toys, building blocks, or paraquetry blocks.

DIRECTIONS: Build a simple pattern and ask the pupil to repeat it. Colors, shape, and positions should be repeated. As pupils develop skill, the patterns should become more and more difficult.

NOTE: Many different types of materials for your patterns should be used as possible to maintain interest and develop skill. Later, the teacher may draw the models on activity cards for the students to copy. (4 x 6 index cards are good.)

B. MAKING PATTERNS ON THE GEOBOARD

MATERIALS: geoboards and colored rubber bands. If geoboards are not available, pegboards and colored pegs can be used.

DIRECTIONS: Make a pattern on a geoboard and have the pupils copy it on their boards. Have pupils copy patterns from premade geoboards which can be purchased or teacher made. Have pupils transfer their designs from geoboards to dot paper.

NOTE: This is a chance for you and your pupils to work out some patterns together. These could evolve into some interesting art projects.

43

page 24 blank

OBJECTIVE Continue an established repeating pattern.

COMMENTS These activities develop skills which are necessary for later discerning patterns in addition and multiplication tables.

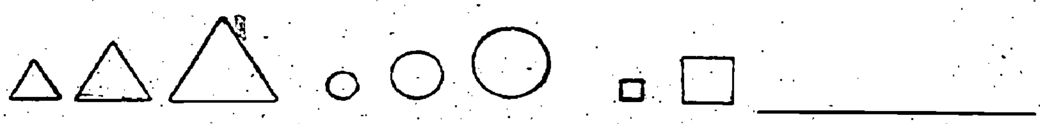
PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. REPEATING PATTERNS

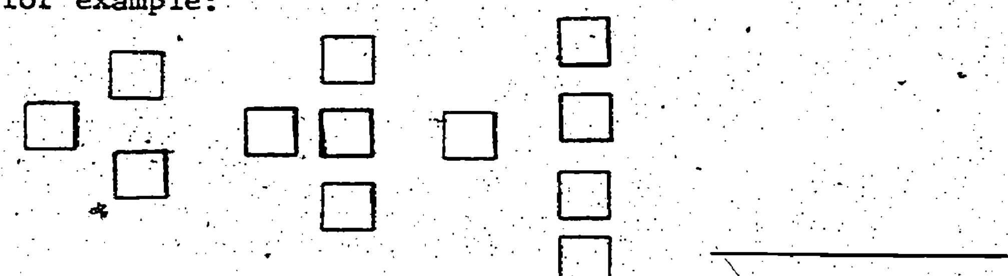
MATERIALS: attribute blocks, 1-inch colored cubes, cuisenaire rods, tinker toys, or building blocks.

DIRECTIONS: Establish a simple repeating pattern with the attribute blocks; for instance.



Ask the pupil, "What block comes next?"

NOTE: Patterns can be made with different lengths, colors, shapes, quantities, for example:



ALTERNATIVE: The teacher can draw the pattern on activity cards for the pupils to copy with the blocks at an activity table.

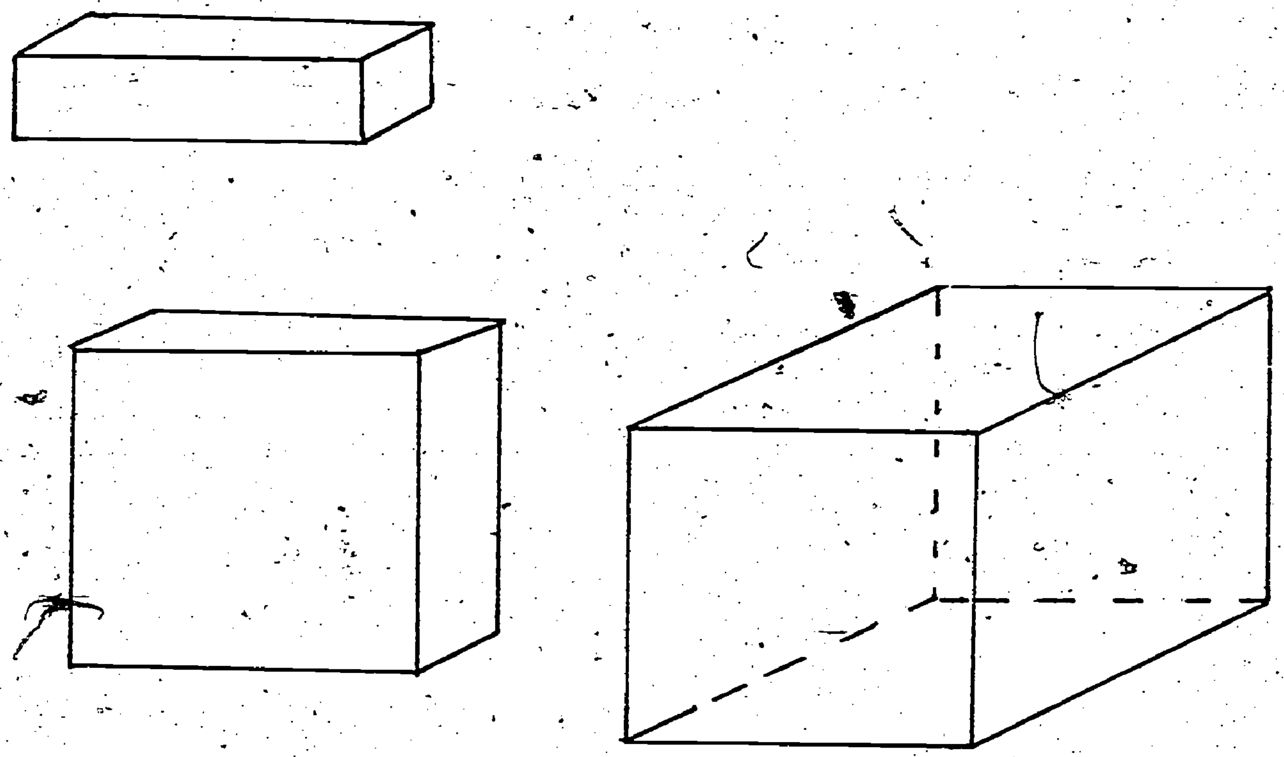
B. REPEATING PATTERNS

MATERIALS: Cuisenaire rods

4

page 26 blank

DIRECTIONS: Take a purple rod, a purple flat, and a purple cube.



Ask a pupil to repeat the pattern with various rods, flats, and cubes.

NOTE: If your school has Cuisenaire rods and activity cards, there are some excellent activities in this area.

48

GENERAL REMARKS The following objectives on attributes develop and sharpen the pupils sense of observation. When recognizing attributes the student is seeing what one member has in common with another. This is a neccessary skill in making comparisons and later becomes important for seeing and understanding relationships in math.

15-C

Reference Sets

OBJECTIVE Make several sets from a given set by using an attribute of the child's choice.

COMMENTS See definition of attributes. At this time be careful to consider only one attribute at a time. The activities should be repeated and expanded with other materials and various attributes.

PRE-REQUISITE OBJECTIVES 10-G, 9-G, 4-C, 3-C, 2-C, 1-C

ACTIVITIES

A. MAKING SETS USING ONLY ONE ATTRIBUTE

MATERIALS: Blocks, paper, popsicle sticks, which are of various colors, shapes, and sizes; yarn.

DIRECTIONS: Using yarn, make a set indicator on the floor or table for the set of objects to be used. Into this set indicator, place objects of various shape, color, and size. Next, using more yarn, make set indicators for several subsets within the large set. Have pupils separate the members of the set into subsets by one attribute of their choice.

NOTE: See definition of attributes.

B. SEPARATING PICTURES BY ATTRIBUTES

MATERIALS: Magazines, butcher paper, glue, yarn.

DIRECTIONS: In the initial activity, have pictures cut out beforehand to insure that only pictures fitting the attributes are used. Separate pictures by attributes such as:

Use	Living things	Foods
Work	Animals	Clothing
Play	Plants	Tools

Create a set of pictures. After discussing the various pictures, have the pupils break into small groups with one assigned attribute. Pupils will get pictures that fit their attribute from the set. The pictures will then be glued onto the paper to create subsets. These can then be discussed by the class.

NOTE: Each of the general attributes can be refined to kinds of work, kinds of animals, kinds of food, etc.

Now, make sets using more than one attribute. Materials are the same. Make a set as previously described. Have pupils separate it into subsets of objects having more than one attribute. Example: "Make a set of red round objects."

NOTE: Include activities involving kind, feel, taste, use, size, shape, color. Example: "Make a set of fat, brown dogs (pictures.)

OBJECTIVE Compare two or more sets by attributes.

COMMENTS The purpose of this objective is to compare attributes of several sets. After the first activity, gradually get away from asking for likenesses or differences and ask the child to make a comparison of the two sets.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. FINDING LIKENESSES AND DIFFERENCES IN SETS

MATERIALS: Attribute blocks or construction paper shapes of various colors and sizes, yarn loops.

DIRECTIONS: Make two different loops of colored yarn as set indicators. Place various objects in each set and have the children find ways in which the sets are alike and ways in which they are different. Plan ahead a little, so that your sets have fairly obvious likenesses and differences. Later, you can let different children make the sets, but in that situation, it may be more difficult to find ways in which the sets are alike or different.

VARIATION 1

MATERIALS: A dozen or so pictures of various kinds.

DIRECTIONS: Repeat the activity using pictures instead of blocks or shapes.

VARIATION 2

MATERIALS: overhead projector, transparencies of sets.

DIRECTIONS: Make 8-12 transparencies with two sets on each transparency. Be sure the sets are alike in some way and different in others. For example, both sets may have red objects, but in one set they are all round and in the other, they are all rectangular. Project each on a screen and discuss ways in which the sets are alike and ways in which they are different.

VARIATION 3

MATERIALS: mimeographed sheets with sets on them

DIRECTIONS: Make several sheets with two sets on each mimeographed sheet. Again have children discuss ways in which they are alike and different.

When they are through, let them color the sets as they would like.

B. MAKING AND COMPARING SETS

MATERIALS: paper, pencil, crayolas

DIRECTIONS: Direct the pupils to draw sets of their choice. The pupils should make two or more sets on separate sheets of paper. Discuss how the sets are alike. Pupils should be able to compare sets by various attributes such as color, kind, etc. This is an individual activity, so the teacher must work with each pupil individually. The pupil should identify how two or more sets are different with respect to some attribute.

C. LANGUAGE ENRICHMENT

Use comparison of sets to introduce vocabulary, such as smaller - smallest
bigger - biggest
heavier - heaviest

5

52

OBJECTIVE Match two sets of the same quantity, using one-to-one correspondence.

COMMENTS The use of one-to-one correspondence is necessary for an understanding of number. Since set size is an attribute, one-to-one correspondence serves as a bridge between the preceding activities on attributes and the following activities on number.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MATCHING SETS TO SETS

MATERIALS: Odds and ends.

DIRECTIONS: 1) Make two sets of the same quantity. Have the pupils match the sets.
Note: Don't draw lines, move the sets side by side.

2) Make two related sets such as cowboys in one set, and horses in the other. Other examples are hook and eye, nuts and bolts, shoes and laces, and hammer and nails.

B. MATCHING DRILLS

MATERIALS: Matching activities for language arts, social studies, science, and mathematics.

EXAMPLE: Social Studies.

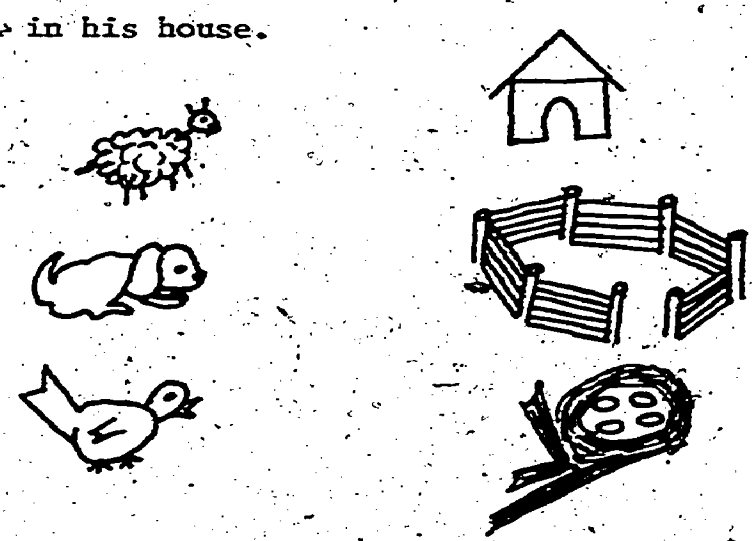
Put the hat on the worker.

COLUMN 1 (pictures of)	COLUMN 2 (pictures of)
fireman's hat	cowboy
nurse's hat	nurse
cowboy's hat	chef
chef's hat	fireman

LANGUAGE ARTS
Put the big letter with the little letter.

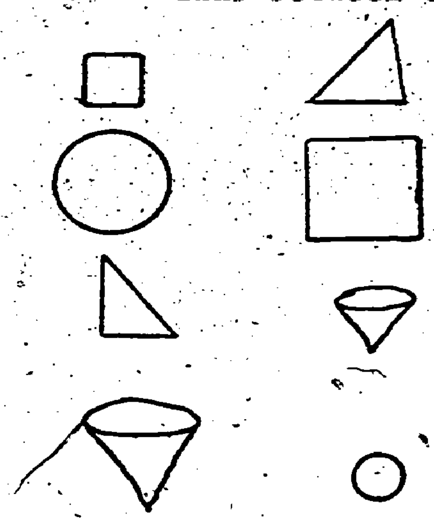
A	a
B	b
C	c
D	d

SCIENCE
Put the animal in his house.



MATH
Shapes

Draw a line between two shapes that are of the same kind.



NOTE: In general do not mix 2 and 3 dimensional figures, particularly where confusion can occur as with circle and sphere.

C. MATCHING SETS

MATERIALS: ditto attached

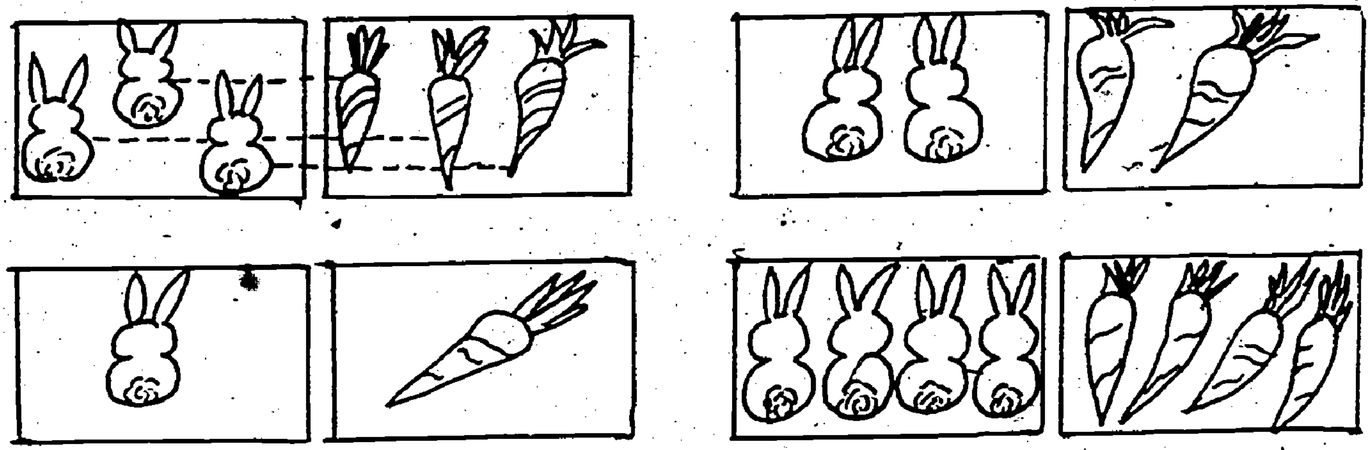
DIRECTIONS: Have pupils match pictures in one set with those in the second set.

NOTE: This ditto is a good transition to Objective 18-C.

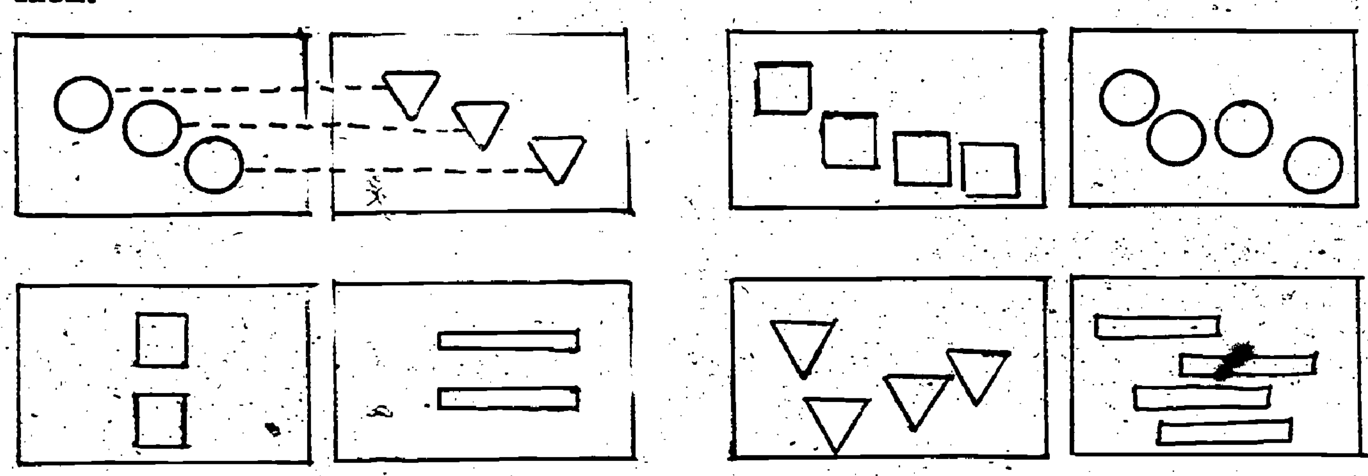
Name ______________________ Date ______________________

MATCHING SETS

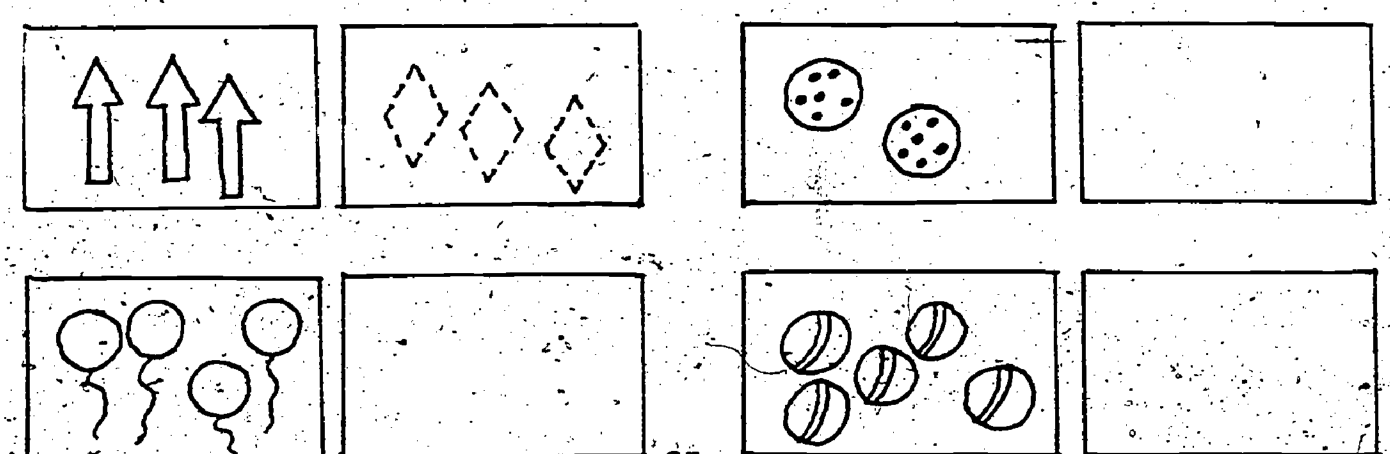
1. Match the rabbits in the first set to the carrots in the second set by drawing lines between them.



2. Match the objects in each pair of sets one-to-one by drawing lines between them.



3. Draw a set that has the same number as the first set. Match the two sets one-to-one.



page 36 blank

OBJECTIVE Compare sets using "1-1" process and tell which has more or less members or if they are the same size.

COMMENTS These activities will be used to develop equivalence and, later, the development of number. Do not use the terms "equal" or "equivalent" at this stage.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. TEACHER-MADE SETS

MATERIALS: Set indicators, odds and ends.

DIRECTIONS: Teacher makes two sets of unequal size and asks one pupil to do "1-1" matching. Ask which set has more (or less). Repeat several times. Then repeat activity using sets of same size.

B. STUDENT-MADE SETS

MATERIALS: Two set indicators for each pupil, odds and ends.

DIRECTIONS: With pupils in small groups (or individually), have each make two sets. The pupil will indicate which has more, which has less, or if they are the same size. Repeat this activity several times.

57

page 38 blank

GENERAL REMARKS Patterning emphasizes the quantity aspect of numbers rather than the sequencing or counting aspects. It enables the child to associate the quantity with its numeral. Throughout the following objectives, quantity is determined by looking at patterns. It is important that the quantity be recognized without counting.

19-C Reference Geometry

OBJECTIVE Create a pattern.

COMMENTS

PRE-REQUISITE OBJECTIVES 14-C, 13-C

ACTIVITIES

A. ART ACTIVITIES

MATERIALS: Construction paper scraps cut into a variety of triangles, rectangles, circles, and squares; glue, paper.

DIRECTIONS: Give each child a pile of shapes and a piece of paper. Ask him to arrange the shapes on his paper to make a pattern. Let him feel free to add pieces or remove them as he desires. When the child is satisfied with his pattern, let him glue it in place.

VARIATIONS: Use corn, beans, macaroni, etc. to create patterns.

B. HEADBANDS

MATERIALS: Strips of construction paper, crayons for each child, scratch paper for each child.

DIRECTIONS: Have each child draw a pattern on his scratch paper to be used on his headband. When he has a definite pattern to use, give him a strip of construction paper to draw his pattern on. When his headband is colored, he may wear it.

ge 40 blank

OBJECTIVE Name orally a given set size from 0 to 9.

COMMENTS This is the childs first encounter with number names. Use patterning for easier recognition of set sizes larger than four. Do not exceed 9 as the largest set size, but do include the empty set (zero).

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. NAMING SET SIZE ORALLY

MATERIALS: Different colored felt pieces large enough to be used as set indicators, odds and ends.

DIRECTIONS: Place 5 objects on a piece of felt and name the objects as 5. Ask pupil to place the same number of objects on another piece of felt and name the set size. Repeat using different set sizes.

B. MAKING SETS OF A "NAMED" SIZE

MATERIALS: Poster paper, magazines, paste, scissors.

DIRECTIONS: Ask the pupils to construct a set of a certain size. Pictures from magazines may be used for the members of the sets. After cutting out the pictures and gluing them to the poster paper, be sure a set indicator is drawn to define the set.

C. SETS--NO MEMBERS (EMPTY SET)

MATERIALS: Two paper sacks, many objects that may be used in making sets (erasers, pop tops, sticks, checkers, dominoes, etc.).

DIRECTIONS: A pupil makes an empty set and another set with one or more members using paper sacks. The other pupils try to guess which sack is the empty set. When a pupil guesses correctly, he looks in the other sack and names its set size. Then it is his turn to fill the sacks. The game continues until all have had the opportunity to guess the empty set.

 page 42 blank

OBJECTIVE Pattern large sets into smaller sets for easier recognition of set size.

COMMENTS Have pupil pattern in the set size he recognizes. This is a verbal pre-division activity, e.g. activity A illustrates 15 divided by 4.

PRE-REQUISITE OBJECTIVES

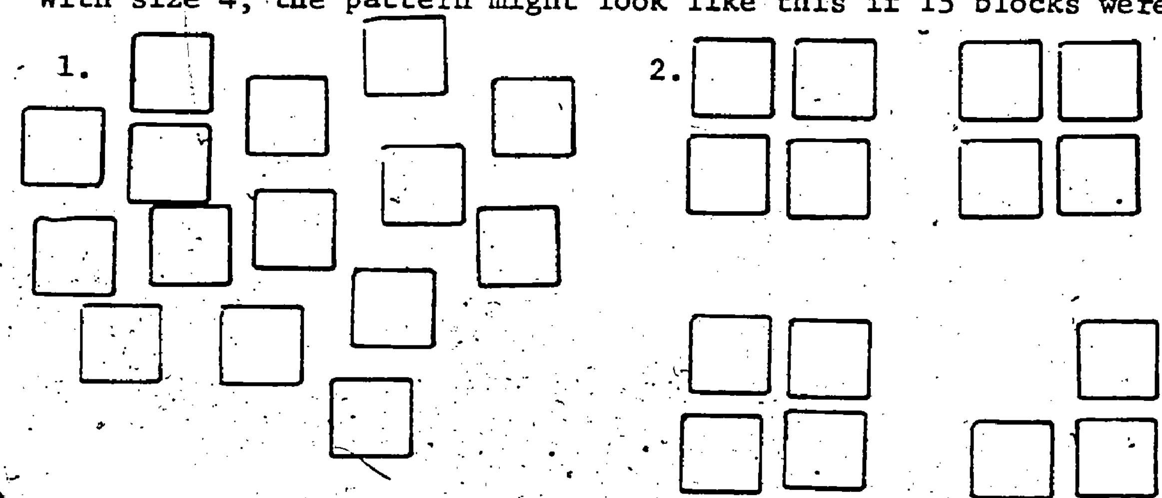
ACTIVITIES

A. PATTERNING OF SETS

MATERIALS: Odds and ends

DIRECTIONS: The teacher makes a set of no more than twenty objects. Ask the child to pattern them into small groups of a given size.

EXAMPLE: With size 4, the pattern might look like this if 15 blocks were used.



page 44 blank

OBJECTIVE Name the numbers from zero to nine after sequencing sets by size.

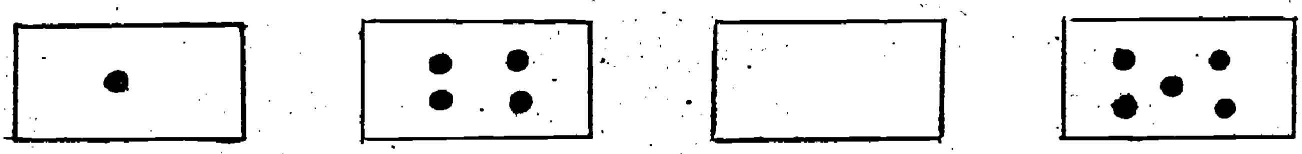
COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. SEQUENCING SETS

MATERIALS: Cards with dots from 0-9 for each pupil. Pattern the dots on each card.



DIRECTIONS: Arrange the cards in the order of least number of dots to greatest number of dots and name number represented 0, 1, 2, 3, 4, etc.

VARIATIONS: Dots could be made of corn, beans, felt, sandpaper, crayons, button, spools, toothpicks.

B. SEQUENCING

MATERIALS: Two sets of cards with dots (or pictures) from 0-9, two teams of 10 children.

DIRECTIONS: Give each team a set of cards, one card to each child. Tell the children not to look at their card until a given signal. At the signal, the children are to look at their card, line their team up in order from 0-9 as quickly as possible. The first team to line up correctly wins.

C. SEQUENCING WITH EGG CARTONS

MATERIALS: Egg cartons torn into sections of 1, 2, 3, 4, 5, 6, 7, 8, and 9, egg holders each for each child.

page 46 blank

DIRECTIONS: Give each child a set of egg carton sections containing 1, 2, 3, 4, 5, 6, 7, 8, and 9. Have each child group the sections from 1-9, discuss 0, and where 0 sections are in the sequence.

OBJECTIVE Associate written symbols with set size from 0-9.

COMMENTS You may want to introduce the written number words at this time.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MATCHING SETS WITH SYMBOLS

MATERIALS: Use materials from preceding objective.

DIRECTIONS: Teacher holds up numeral and pupil points to a set which illustrates symbol.

B. MATCHING--CARDINAL NUMBERS

MATERIALS: crayons, scissors, dittos of cars and garages--garages should be different sizes to contain from 0-9 cars and numbered 0-9, in random order.

DIRECTIONS: Children color cars, cut them out and paste them in garages. For example, 3 cars in garage 3.

VARIATION: Cars and garages are cut out, matched, and lined up in counting order.
---number cars and garages
---color cars and garages the same.

OBJECTIVE Write the numerals from 0 to 9.

COMMENTS Provide ample practice in writing each symbol. The spelling of number words can also be practiced.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. TRACE THE NUMERALS IN REPETITIVE ROWS OF EACH DIGIT.

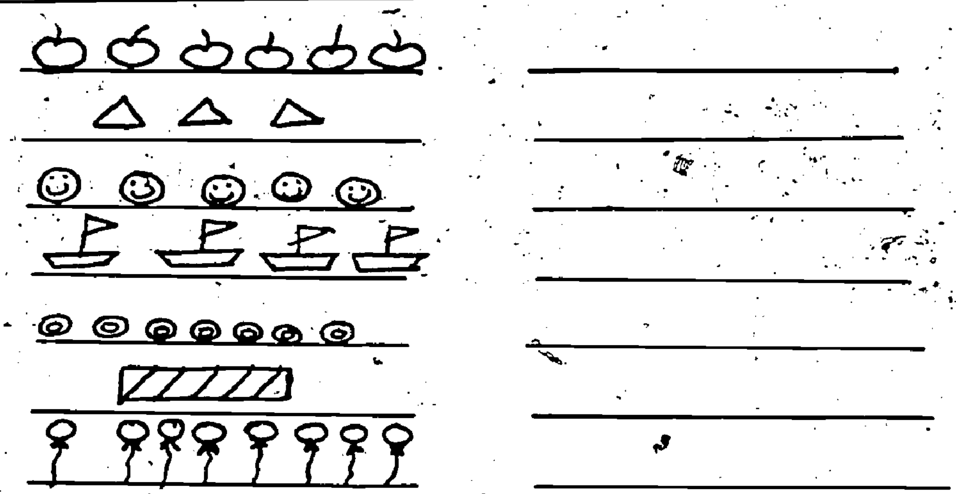
B. ASSOCIATING NUMBER WITH SET SIZE

MATERIALS: Ditto

DIRECTIONS: Count the objects in each set and write the number for that set size.

Our Numbers

Count the Pictures — Write the number

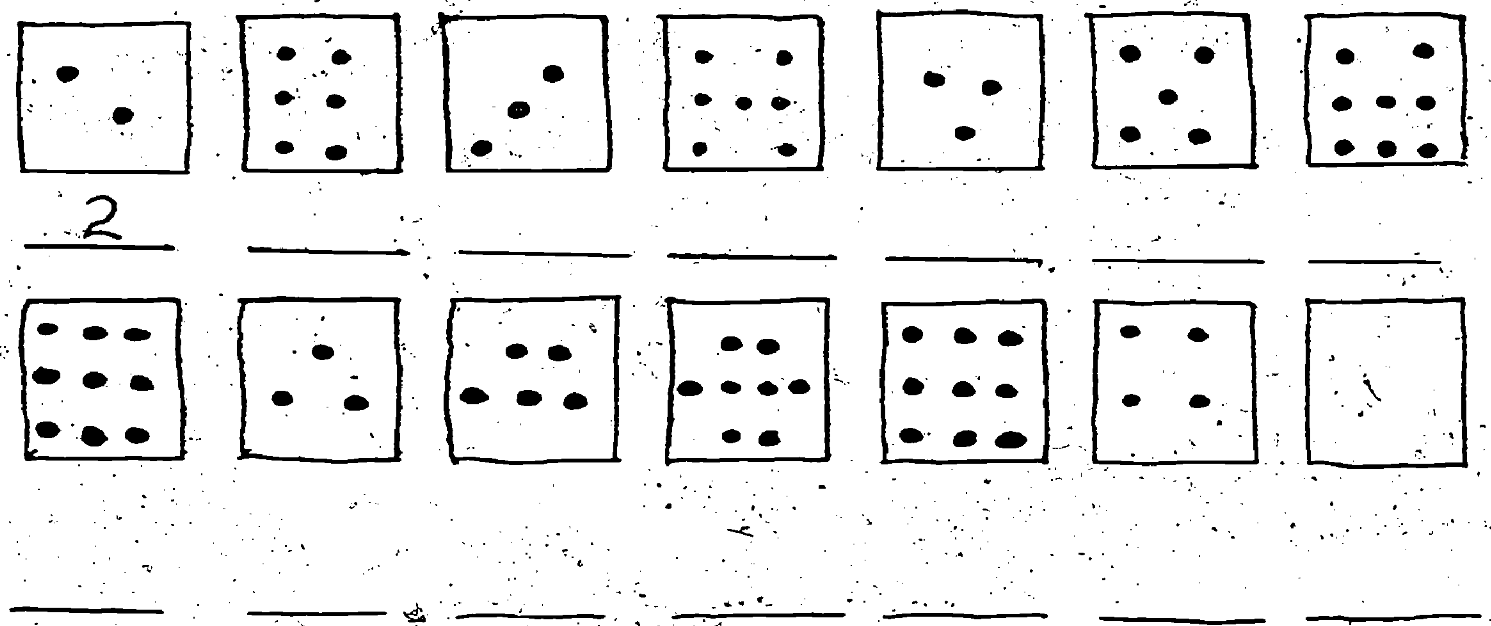


C. DOMINOE DOTS

MATERIALS: Ditto

DIRECTIONS: dots--How many dots in each box? Write the numerals.

page 50 blank


2

OBJECTIVE Name the position an object has in a sequence.

COMMENTS Since this is the only objective dealing with ordinal numbers, adequate time should be allowed for thorough understanding of the concept.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. ORDINAL NUMBERS

MATERIALS: Have several pupils line up at the door. Name their position, first, second, third, etc. Repeat asking, "Who would like to be first, second, third?" Line up several pupils and ask, "Who is first in line?" "Who is second in line?", etc.

VARIATION: Repeat activity above using odds and ends.

B. ORDINAL NUMBERS

MATERIALS: Pictures, either rows of pictures on paper or rows of pictures mounted and placed where the entire can see.

DIRECTIONS: As a group project, use large pictures in rows where everyone can see. Let children take turns pointing out the first, second, fifth... picture, or pick a picture and tell whether it is first, second, etc.

As an individual project, give each child a worksheet with rows of pictures. The teacher may name a row and a picture and have the children point to or put a mark on that picture. For example: In the second row, point to the fourth picture.

NOTICE

SINCE THE DEVELOPMENT OF PLACE-VALUE IS ABOUT THE MOST IMPORTANT CONCEPT IN ELEMENTARY SCHOOL ARITHMETIC, THE NEXT THREE OBJECTIVES ARE AMONG THE MOST IMPORTANT IN THE GUIDELINES. BE SURE TO SPEND PLENTY OF TIME ON THESE OBJECTIVES!

IF A PUPIL HAS DIFFICULTY WITH ADDITION, SUBTRACTION, MULTIPLICATION, OR DIVISION, IT IS USUALLY BECAUSE OF A LACK OF UNDERSTANDING OF PLACE-VALUE. OBJECTIVES 26, 27, AND 28 MODEL THE PLACE-VALUE PROCESS AND IT IS IMPERATIVE THAT THE PUPIL UNDERSTAND THESE OBJECTIVES.

page 54 blank

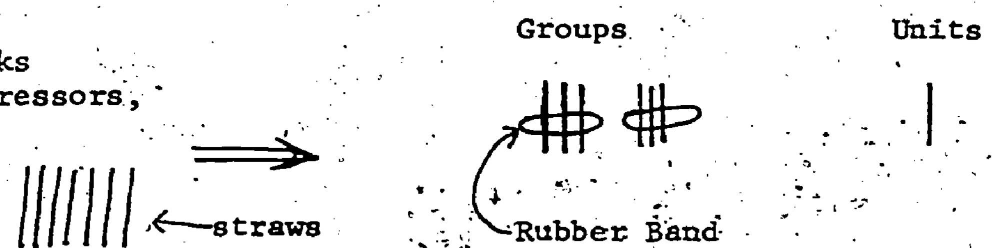
GENERAL REMARKS During the next three objectives, the teacher will be developing models that will be extended to units, tens, and hundreds. In these early objectives the models will be called units, groups, and groups of groups. The teacher should study these objectives and choose a model he and his pupils can use easily. Be sure to spend plenty of time on these objectives.

MODELS FOR GROUPS

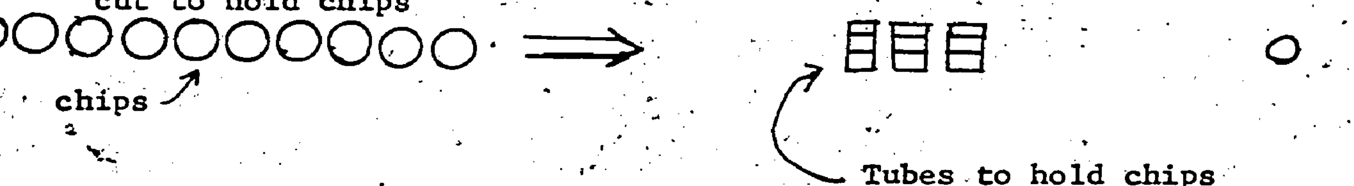
MATERIALS: Drinking straws, popsicle sticks, poker chips, cubes, buttons, safety pins, tongue depressors, rubber bands, toilet tissue tubes.

A group depends on size of grouping. Example: Grouping by size 3.

A. straws or popsicle sticks or tongue depressors, rubber bands

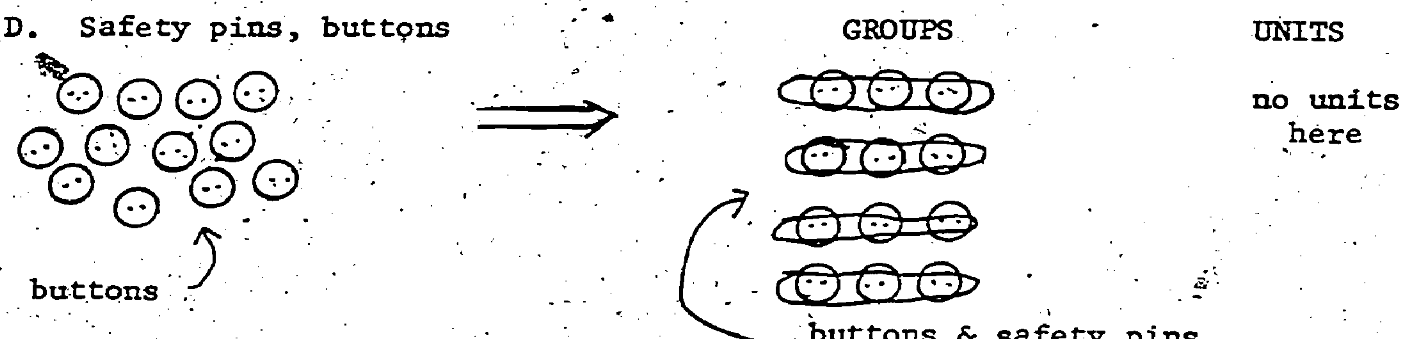


B. poker chips, toilet tissue tubes cut to hold chips



C. cubes

D. Safety pins, buttons



Page 56 blank

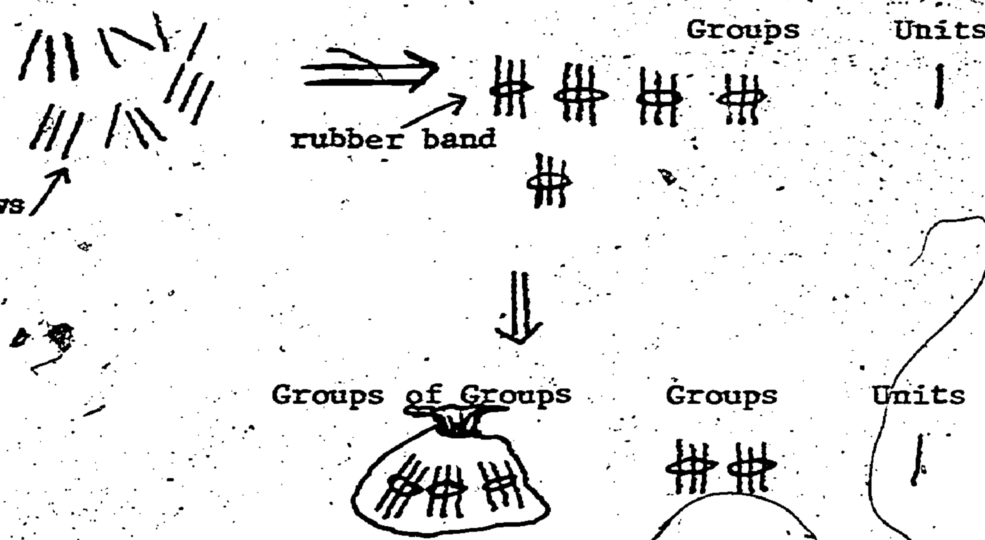
MODELS FOR GROUPS OF GROUPS

MATERIALS: Same as for groups.

Groups of groups depend on grouping size. (Use a different method for modeling the groups of groups that that used for groups.)

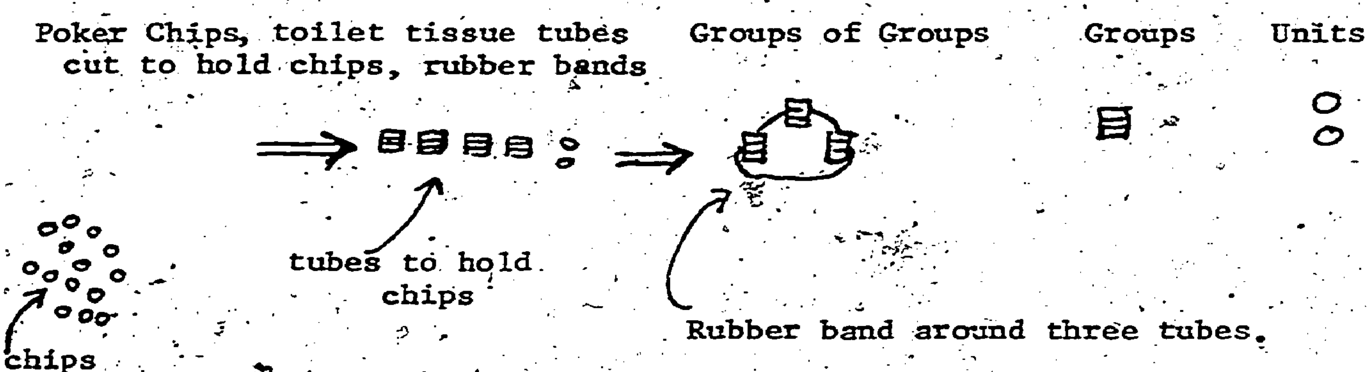
EXAMPLE: Grouping by size 3

A. drinking straws or, popsicle sticks or, tongue depressors, rubber bands, plastic bags or boxes.



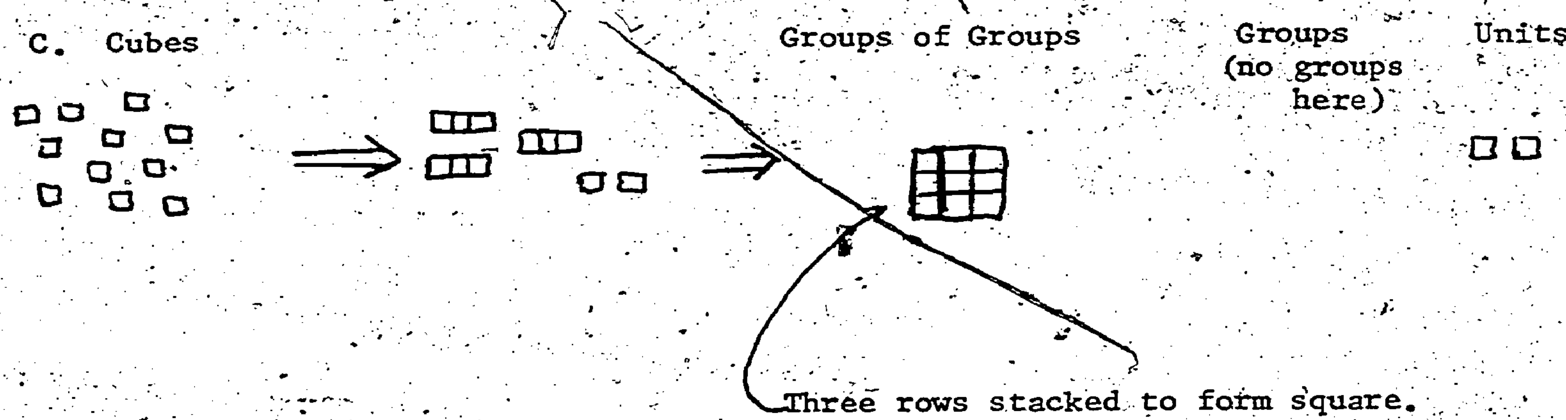
Plastic bag, box, or convenient container holding three groups.

B. Poker Chips, toilet tissue tubes cut to hold chips, rubber bands

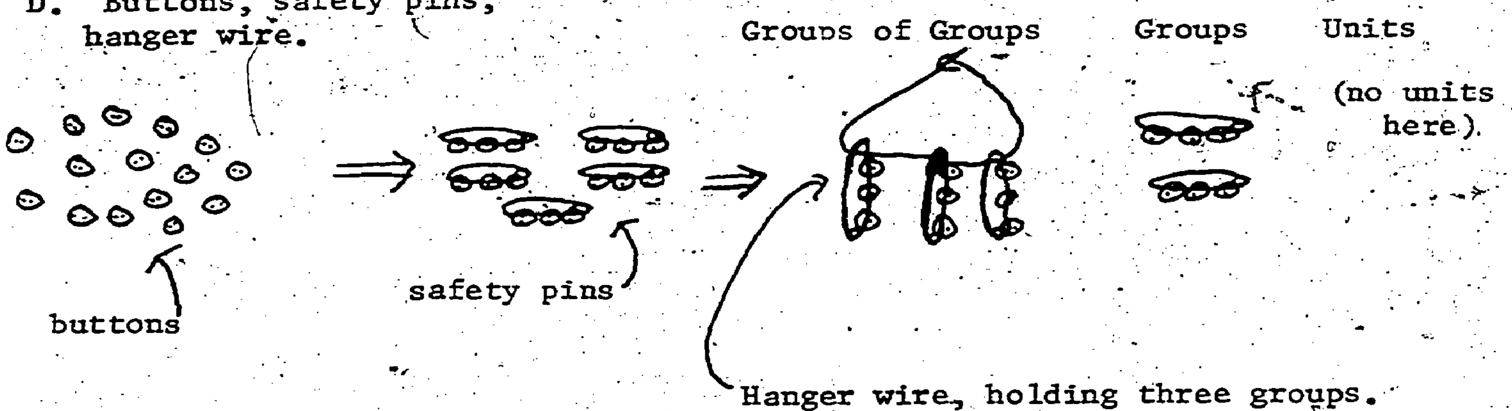


Rubber band around three tubes.

C. Cubes



D. Buttons, safety pins,
hanger wire.



OBJECTIVE Model in groups and units

COMMENTS Each child will need a large set of 20 or more objects. When grouping, the pupils do not count the entire set of objects; they count the groups and units. Be certain you never start with a set so large that the pupil has more than 9 groups and 9 units. Take your time with this objective.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

PRE-ACTIVITY: TEACHER DEMONSTRATION

Ask seven students to come to the front of the room. Name three students to make a group in one corner of the room. Say, "We have one group of size 3. How many groups do we have? (1) Do we have enough students to make another group of size 3?" (yes)

Name three more students to form a group, in the vicinity of the first group. Ask, "How many groups of size 3 do we have now? (2) Do we have enough to make another group of size 3? (no) When we do not have enough to make a group, these left overs are called Units. How many units are left over? (1) We have: how many groups? (2) how many units? (1)."

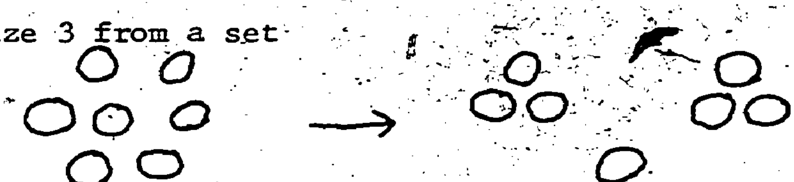
Repeat this activity several times using 4,8, and 10 or 11 students, but keep the set size constant. Because of the difficulty, 0 units should be saved for last. Use 6 or 9 students to show 0 units.

A. GROUPING

MATERIALS: Counters

DIRECTIONS: Establish a set size to be used for grouping (start with groups of size 3 or 4; later use groups of size 5 through 9.) Place a set of counters in front of the pupils. Ask them to group in the set size.

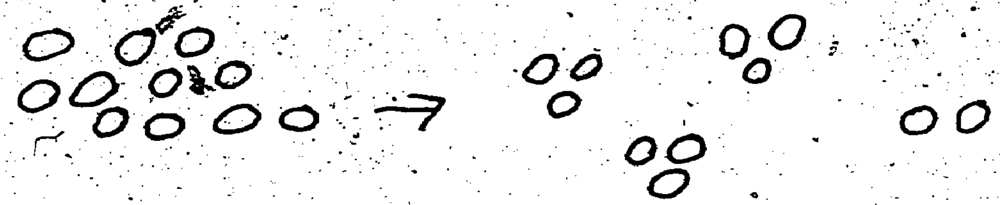
EXAMPLE: Make groups of size 3 from a set



Ask, "How many groups of 3 do we have?" "How many units are left?" Repeat with a new set size.

60 blank.

EXAMPLE:



B. SHOE BOX GROUPING

MATERIALS: counters, shoe boxes

DIRECTIONS: Put sets of counters in boxes on a table or around the room. Label the boxes A,B,C,D,E, etc. Give the pupils an answer sheet labeled to match the boxes. Have pupils group in a set size and record the number of groups and units.

Sample answer sheet:

	Groups	Units
A		
B		
C		
D		

VARIATION: Change the set size and repeat. This variation should be done.

OBJECTIVE Model groups of groups, groups, and units; and record.

COMMENTS See general remarks and models page 57. Be sure to spend adequate time on this objective. Lots of modeling should be done. Do not move to Objective 28-C too soon.

PRE-REQUISITE OBJECTIVES 26-C

ACTIVITIES

A. GROUPS OF GROUPS

MATERIALS: Models from general remarks - page 57.

DIRECTIONS: Have children group by a set size and regroup the groups to make a group of groups. After several examples, begin recording the information as:

Groups of Groups	Groups	Units

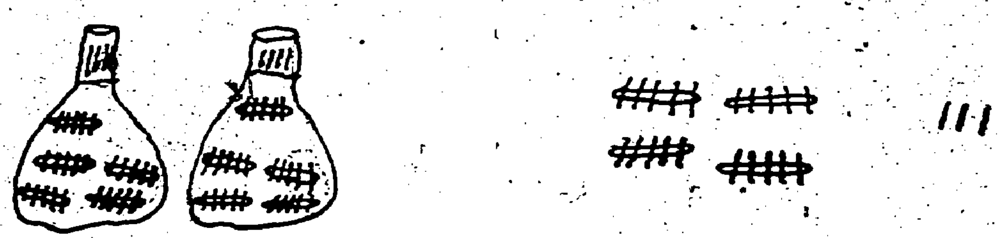
Be certain to include examples where pupils will need zeroes in one or two columns.

B. PICTURES OF MODELS

MATERIALS: Activity cards or dittos with drawings of grouping models; used in activity A.

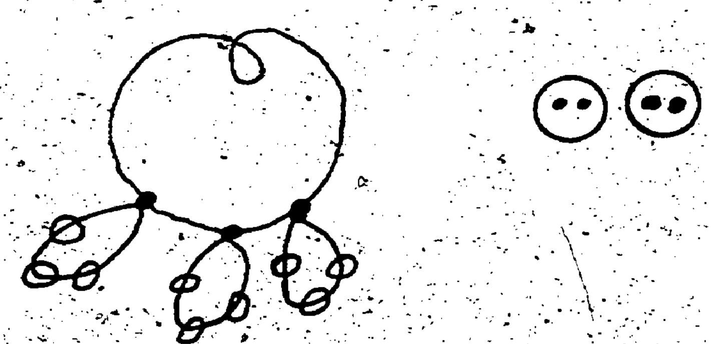
DIRECTIONS: Have children look at the drawings of a model and record the groups of groups, groups and units.

EXAMPLE: a. STRAW MODEL
(using group size five)

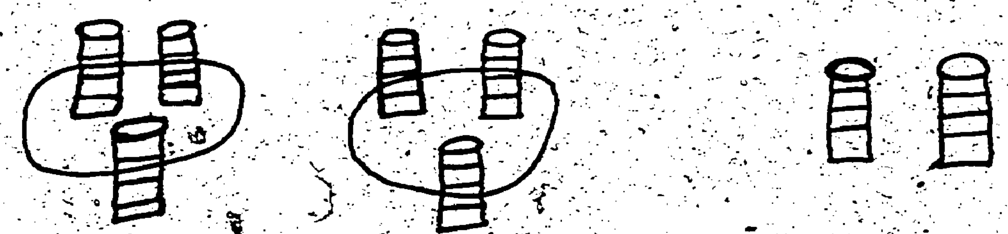


73

b. BUTTON MODEL
(Using group size three)



c. CHIP MODEL
(Using group size four)



ANSWERS

	Groups of Groups	Groups	Units
a.	2	4	3
b.	1	0	2
c.	2	2	0

75

OBJECTIVE Replace a group with a concrete representative for a group. (This refines the model for groups and makes it somewhat more abstract.)

COMMENTS This objective provides the pupil with experiences which are necessary for the understanding of place value notation. Again, be sure to allow plenty of time for these activities.

PRE-REQUISITE OBJECTIVES

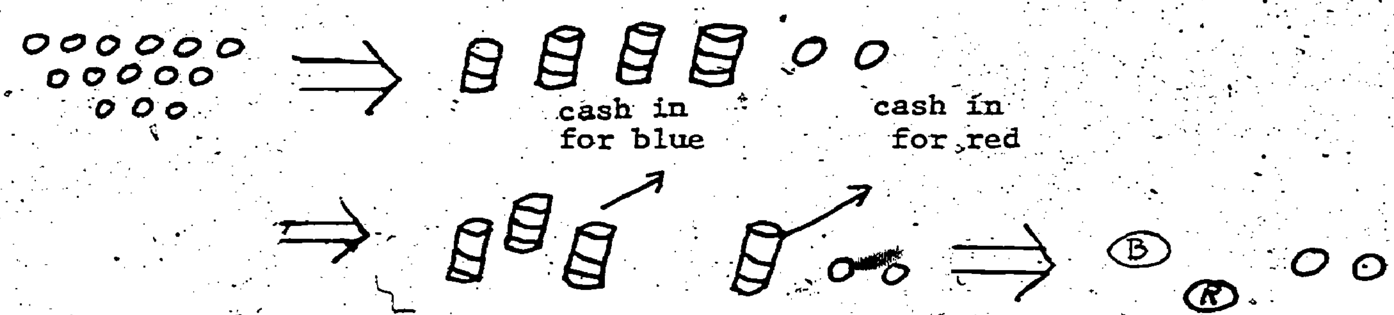
ACTIVITIES

A. TRADING - CASHING IN

MATERIALS: Poker chips, recording charts

DIRECTIONS: Begin with several chips of the same color, (white.) Choose a grouping size and have the children model into units, groups and groups of groups as before. Choose another color, (red) and have the children "trade" or "cash in" each group of white chips for a red chip. Choose another color, (blue) and have the children "trade" or "cash in" each group of red chips (group of groups) for a blue chip. Have the children record the information.

EXAMPLE: Group size 3



BLUE	RED	WHITE
1	1	2

Do several examples with different group sizes.

VARIATION: 1. Use any materials that come in a variety of colors, e.g. construction paper, painted bottle caps, colored popsicle sticks, etc.

VARIATION: 2. Instead of doing all the grouping with white chips before beginning the "cashing in" have the children cash in a group of whites for a red every time a group of whites is formed and cash in a group of reds for a blue every time a group of reds is formed.

VARIATION: 3. If more colors are available you can do further grouping. Eg.
whites are units
a red can be traded for a group of whites
a blue can be traded for a group of reds
a green can be traded for a group of blues
a yellow can be traded for a group of greens
etc.

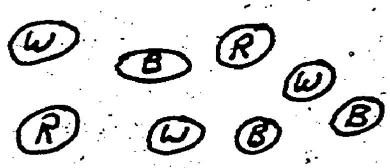
B. POSTERS AND DITTOS

MATERIALS: poster paper, mailers, dittos

DIRECTIONS: Make posters or dittos with drawings of models of units, groups and groups of groups. Where groups are represented by a single object and groups of groups are represented by a different object. Have pupils record answers on answer sheets.

EXAMPLE: white are units, red is worth five whites, blue is worth five reds

B	R	W
3	2	3



GENERAL REMARK The following are pre-fraction activities to familiarize pupils with one-half, one-fourth, and one-third.

29 -C Reference Types of Numbers (Fractions)

OBJECTIVE Identify and model $\frac{1}{2}$ of a unit.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. ONE-HALF

MATERIALS: Paper, crayons, scissors.

DIRECTIONS:

1. Model $\frac{1}{2}$ to the pupils by folding a sheet of paper into two equal parts. Introduce the name one-half. Discuss what $\frac{1}{2}$ is. Have pupils copy model. You might want the pupil to color $\frac{1}{2}$ of the paper to make the concept clearer.

2. Model $\frac{1}{2}$ to the pupils by folding and then cutting a square of paper into two equal parts. Have the pupils cut squares of paper in half as many ways as they can. After doing this, they can use other shapes.

B.

MATERIALS: Use pictures of fractions, flannel-board objects showing fractions, geoboards, or a dittoed worksheet. All of these should show the fraction $\frac{1}{2}$ several ways (rectangles, squares, circles, large figures, small figures, etc).

DIRECTIONS: As a group activity using the flannel board, geoboard, or pictures, have different children point out $\frac{1}{2}$ or point to a fractional part and tell the name $\frac{1}{2}$.

As an individual activity, give each child a worksheet showing $\frac{1}{2}$. Have each child put a ✓ or an X on as many halves as he can find on the page. Go over the page later as a group to make sure everyone marked halves.

30-C Reference Types of Numbers (Fractions)

OBJECTIVE Identify and model $\frac{1}{4}$ of a unit.

COMMENTS

PRE-REQUISITE OBJECTIVE 29-C

ACTIVITIES

A. ONE-FOURTH

MATERIALS: See activity A (Objective 29-C)

DIRECTIONS: Follow directions as in activity A Objective 29-C, except replace $\frac{1}{2}$ by $\frac{1}{4}$ and fold the sheet of paper into two equal parts twice.

B.

MATERIALS: See activity B (Objective 29-C)

DIRECTIONS: Follow directions as in activity B (Objective 29-C), except use pictures, flannel board objects, and dittoed pictures of $\frac{1}{4}$ instead of $\frac{1}{2}$.

page 68 blank.

31-C Reference Types of Numbers (Fractions)

OBJECTIVE Identify and model 1/3 of a unit.

COMMENTS Due to the difficulty a small child might have in folding a paper into three equal parts, it is felt that activities that involve coloring pre-dittoed papers might be best.

PRE-REQUISITE OBJECTIVES 30-C

ACTIVITIES

A. FRACTION DITTO

MATERIALS: Teacher made ditto, crayons, paste, scissors, poster or construction paper.

DIRECTIONS: The ditto should contain only one fraction. Example: a ditto with objects separated in all 1/3's. Have pupil color and cut off and paste on paper.

B. DITTO SHEETS

MATERIALS: Teacher made dittos with objects separated in 1/3's, crayons.

DIRECTIONS: Teacher gives directions on coloring different parts.

79

page 70 blank

OBJECTIVE Pattern and repattern a set of given size as many ways as possible.

COMMENTS This is a non-written activity.

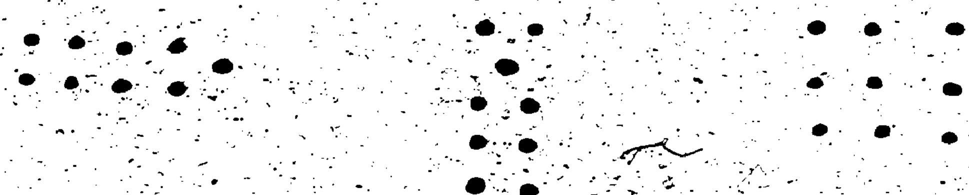
PRE-REQUISITE OBJECTIVES 21-C

ACTIVITIES

A. REPATTERNING

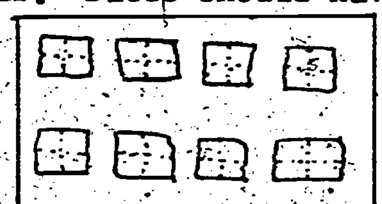
MATERIALS: 9 objects for each pupil.

DIRECTIONS: Use a set size of 2-9, and have pupil make as many patterns as possible. Have the child talk about what he has done. Sample: 3 possible patterns of 9:

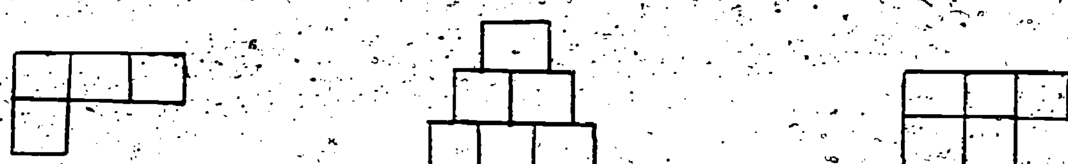


B.

MATERIALS: Ditto sheet with a set of squares patterned in a particular set size, scissors, glue, construction paper. Ditto should have as many patterns (of one size) as possible. Example:



Give a lot of experience with one number before going on with another. Some other patterns might include:



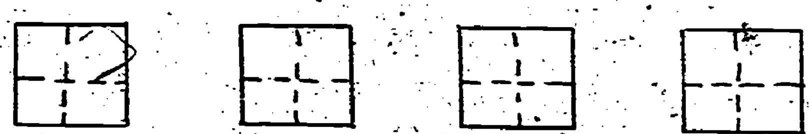
80

page 72 blank.

ACTIVITY B. cont'd.

DIRECTIONS: Have pupil cut outer border of pattern from ditto sheet. Then have him repattern by another set size by cutting some of the patterns and gluing them on the construction paper. Repattern 4's into 3's. Sample:

Shapes from ditto:



Construction paper:

81

33-J Reference Addition

OBJECTIVE Repattern a set of a given size (not to exceed 9) to see how many ways it can be patterned into 2 subsets and tell what combinations have been modeled.

COMMENTS This is a non-written activity.

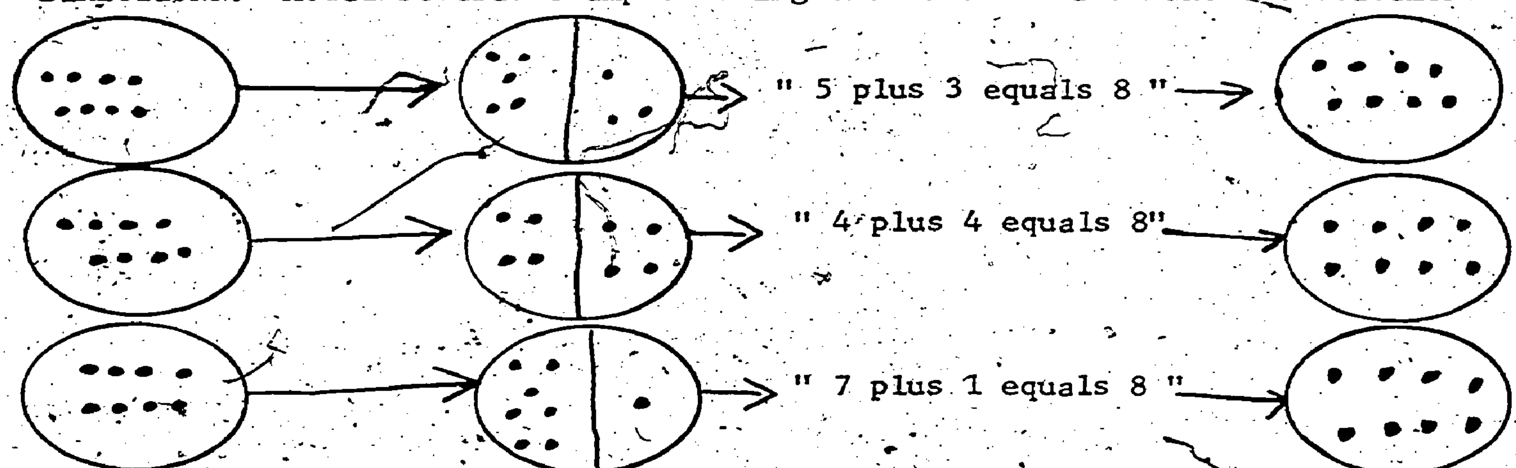
PRE-REQUISITE OBJECTIVES 32-J

ACTIVITIES

A. MODELING ADDITION

MATERIALS: Any set of 9 objects and set indicators.

DIRECTIONS: Model several examples using the same sum and make the statements-



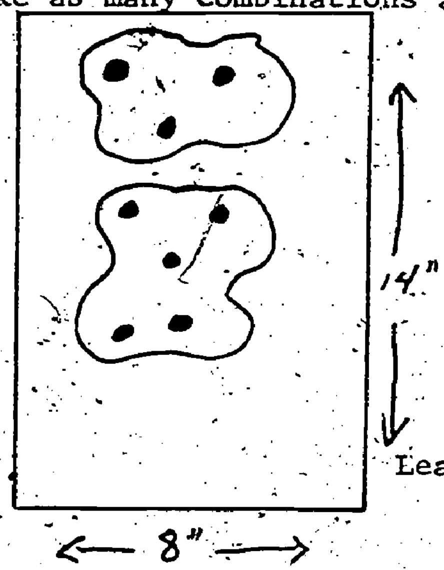
Then give the pupil materials to see how many ways he can model other sets and verbally make the statement about the sums.

Be sure to include zero, but not the beginning example. When all combinations of a given set size have been modeled, change the set size.

B. DROP THE STRING

MATERIALS: Set of large domino cards modeling 2-9, and 2 large circles of yarn.

DIRECTIONS: Instruct student to drop one circle of yarn around some dots. Then encircle the remaining dots with other yarn and verbally give the equation. He should make as many combinations as possible. Example:



Leave space at bottom to indicate zero.

OBJECTIVE Record models of equations.

COMMENTS Discuss the "plus" (+) and "equals" (=) symbols.

PRE-REQUISITE OBJECTIVES 33-J

ACTIVITIES

A.

Use activity A from Objective 33-J. Record the equation. Have the pupil write the equation both ways, i.e. $5 + 3 = 8$ or $8 = 5 + 3$. Use several different set sizes.

B. DUMP TRUCK ADDITION

MATERIALS: Two piles of blocks or other objects, a toy truck.

DIRECTIONS: Instruct pupil to tell how many things are in the first pile. He then picks them up in the toy truck (or pick-up) and proceeds to the second pile. He then tells how many are in that pile and picks them up. He then tells how many there are in the truck. The pupil makes the oral statement "3 plus 4 equals 7", then is shown the written format-- $3 + 4 = 7$.

OBJECTIVE Associate a set of size 10 with its word name of "ten".

COMMENTS This is a non-written objective.

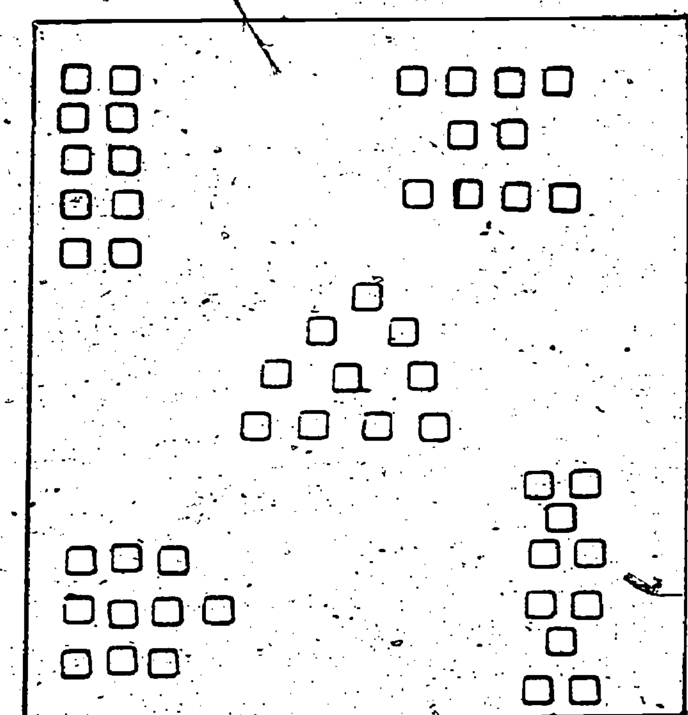
PRE-REQUISITE OBJECTIVES 22-C, 21-C, 20-C, 19-C.

ACTIVITIES

A. PATTERNING TEN

MATERIALS: Counters (Popsicle sticks, blocks, caps, buttons, etc.), oak tag.

DIRECTIONS: Have class get into groups of 4 or 5. Give each pupil a pile of counters. Ask each pupil to quickly pattern the number nine with the counters on their desks or a flat surface. Tell them that ten is 1 more than nine and ask them to make a pattern for ten. Ask each group to make as many patterns of ten as they can.
Later, ask each group to copy their patterns of ten on oak tag posters.



EXAMPLE

1-inch cubes make better patterns than little ones.

B. REFER TO ACTIVITIES USED WHEN LEARNING TO COUNT TO NINE. FOR FURTHER ACTIVITIES SEE PRE-REQUISITE 22-C.

page 78 blank

OBJECTIVE Group sets of size 10-19 in groups of tens and units.

COMMENTS Take your time on this objective. Note in Activity A that the pupil is to record tens and units on a chart. Eg: Record fourteen as

groups	units
1	4

, not as 14.

PRE-REQUISITE OBJECTIVES 26-C

ACTIVITIES

A. GROUPING BY TENS

MATERIALS: Use models as shown with General Remarks on page 57.

DIRECTIONS: Repeat activities for grouping with groups smaller than ten. Use the same recording form. (Objective 26-C). Limit size to 19.

groups	units

B. BEANSTICKS

MATERIALS: Beans, glue, tongue depressors.

DIRECTIONS: Give each child 2 tongue depressors, glue, and some beans (at least 10 but not more than 19). Have the children group their beans in groups of 10, with some units left over. No more than 10 beans may be glued on each tongue depressor, but every one has to have 10 beans before the child can start gluing beans on his second stick. When everyone is finished, record the groups and units in the same form as above.

83

page 80 blank.

OBJECTIVE Learn the names of the numbers from 10-19.

COMMENTS Hereafter, ten is the group size used to write our numbers.

PRE-REQUISITE OBJECTIVES 36-C

ACTIVITIES

A. COUNT 10-19

MATERIALS: Models, chalkboard.

DIRECTIONS: Tell the pupils the numbers larger than ten have special names. Model a group of ten and one unit and tell the puil that we call this number eleven. Continue to record eleven as:

tens	units
1	1

Model a group of ten and two units---call the number twelve and record it as:

tens	units
1	2

Introduce the rest of the numbers up to nineteen as fast as the pupils can remember the names. There will be lots of natural review in future exercises.

B. Repeat activity A from Objective 36-C and associate names of numbers with the quantities. Have the pupils do the modeling.

87

page 82 blank

OBJECTIVE Write the numerals from 10-19.

COMMENTS

PRE-REQUISITE OBJECTIVES 37-C

ACTIVITIES

A. WRITING NUMERALS FROM 10 TO 19.

MATERIALS: Chalkboard, ditto, and models.

DIRECTIONS: Show pupils that we usually write tens and units without the chart.

tens	units	
1	3	= 13
1	8	= 18
1	0	= 10

VARIATIONS:

A) Show the pupils models for numbers from 10-19. Have the pupils write the numerals without the chart. Have them read the numerals.

B) Model-write-read many times. Use posters, answer sheets, dittos, and work book pages.

C) Reverse the order: read a number and have the pupil write the numerals and build or draw a model first at the chalkboard and then at his seat.

page 84 blank

39-J Reference Addition

OBJECTIVE Internalize basic addition facts. (0-18)

COMMENTS See definitions of "internalize" and "basic facts" in glossary.

PRE-REQUISITE OBJECTIVES 34-J

ACTIVITIES

A. BASIC FACTS-ADDITION

MATERIALS: Egg carton with cups numbered 0, 1, 2, 3, 4, 5; two small objects (beads, buttons).

DIRECTIONS: Teacher places objects into egg cartons and the student adds the combination.

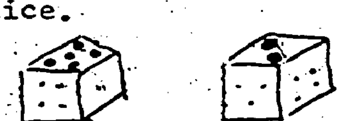
VARIATION: Have the pupil toss the objects into the egg carton one at a time, then add the two numbers.

NOTE: Use other egg cartons with larger numbers to continue through sums up to 18.

B. ADDITION-BASIC FACTS

MATERIALS: Pair of dice (can be made from wood cubes or sponges cut into cubes).

DIRECTIONS: Roll both dice and add the two numbers shown. Have the pupils count the spots on the dice.



5 + 2 =

C. LAY OR LOSE

MATERIALS: One ten by fifteen inch playing board of heavy cardboard, two sets of $1\frac{1}{2}$ by $1\frac{1}{2}$-inch cards to each set and each set of a different color tagboard, answer card.

DIRECTIONS: Draw a line dividing the playing board in half (fig. 1) and then divide each half into 25 spaces 2 by $1\frac{1}{2}$ inches each (five vertical and five horizontal rows).

Make a list of at least 25 pairs of addends and their sums, 11 through 18. Examples: 9 + 9 = 18, 5 + 9 = 14, 6 + 5 = 11, 8 + 3 = 11, 7 + 11 = 18.

page 86 blank

C. Cont'd.

Write the sums, in jumbled order, on each half of the playing board, positioning the numerals so that the player can read his half of the board. (Players face each other; numerals should be right-side-up for each player.) On each set of 25 cards write the pairs of addends, one pair per card.

Make an addition table and mount it on a card. This table may be stored with the game or put near the Math Resource Center for continued reference.

Each player is to place his addend cards face down in a stack adjacent to his end of the playing board. The turns alternate between the two players. The first player will turn up a card in his stack and hold it so both he and his opponent can see it.

The idea is for the player to place the card on its correct sum on his side of the board before his opponent can name the sum.

The player loses the card if his opponent names the sum before the card is played. If the opponent wins the card, he puts it in his bank off the board. Either way....the player lays the card, or loses it....the play goes back to his opponent, who repeats the procedure: picks up a card from his stack, shows it, and tries to place it correctly before it sum is named.

The game continues until each player has played all his cards. The one with the greatest number of cards (those on his half of the board plus those in his bank) wins the game.

Players use the addition table to check their responses.

D. ADDITION CONCENTRATION

MATERIALS: twenty-four two inch cards.

DIRECTIONS: List 12 pairs of addends and their corresponding sums. Write a pair of addends on each of 12 cards; write their sums on the other 12. Put the addends and their sums on the answer card. Place the 24 cards in a manila envelope. The answer card is to be placed in a separate envelope and attached to the back of the manila envelope.

The cards are to be spread, face down, on a table. The first player may turn

D. Cont'd.

up any two cards. If the two cards are a matched pair--a pair of addends (7 + 9) and its sum (16)--the player places the pair in his card bank on the table. If the cards are not a matched pair, the player turns them face down in their original locations and it is the next player's turn.

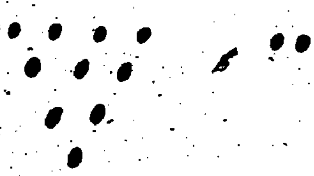
Play continues until all pairs and their sums have been matched, at which time the players use the answer card to check their accuracy. The player with the greatest number of correctly matched pairs wins the game.

E. COMBINATIONS

MATERIALS: Any set of 18 objects.

DIRECTIONS: Using a set size from 10-18, have the pupil group the set in tens and units and record it. He then regroups in 2 sets of size less than ten and records those numbers. The pupil should not exhaust all the combinations of a set size during this first activity.

Example:



Repattern:

12 = 8+4

Note: Give experience with sentences of both types, 12 = 8 + 4 and 8 + 4 = 12

VARIATION:

MATERIALS: Same

E.

VARIATION Cont'd.

DIRECTIONS: Have pupils now record all the combinations of 2 sets of size less than ten that give a given set size between 10 and 18. Be certain to include zero.

Example: combinations of 12
12 = 9 + 3
12 = 6 + 6
12 = 7 + 5
12 = 9 + 3 . . .

figure 1

Lay or Lose				
11	14	17	13	16
18	12	15	11	14
13	17	14	17	15
16	13	15	11	12
12	16	17	14	18
11	14	17	11	16
18	12	15	17	14
13	17	14	11	15
16	13	15	13	12
12	16	17	14	18

OBJECTIVE Learn to name the numbers and write the numerals 20-99.

COMMENTS

PRE-REQUISITE OBJECTIVES 38-C, 37-C

ACTIVITIES

A. NAMING THE TENS

MATERIALS: Chalkboard

DIRECTIONS: Tell the pupils that the groups of ten have special names. Make a chart of tens with no units.

tens	units
1	0
2	0
3	0
4	0
.	.
.	.
.	.

As the ten is written, tell the pupil--"We call one ten; ten, two tens, twenty; three tens, thirty, etc."

Drill counting by tens, naming each decade correctly.

B. EXTEND ACTIVITIES USED IN OBJECTIVES 37-C and 38-C TO INCLUDE NUMBERS 20-99.

93

OBJECTIVE To pattern and record addition equations using basic fact equations plus sets of ten.

COMMENTS This objective emphasizes place value in addition.

PRE-REQUISITE OBJECTIVES 39-J

ACTIVITIES

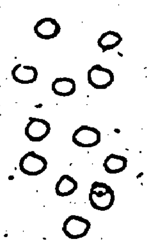
A. PATTERNING AND REPATTERNING

MATERIALS: A set of 30 objects per pupil.

DIRECTIONS: Have the pupil begin with a set of size 18 or less. Pattern into two sets of size less than ten. Record. Add an additional set of size ten to one of the two sets. Record. Repattern and record the sum.

EXAMPLE: The following shows how the recording is done at each step, resulting in chart at step 4.

Step 1.



A	
B	

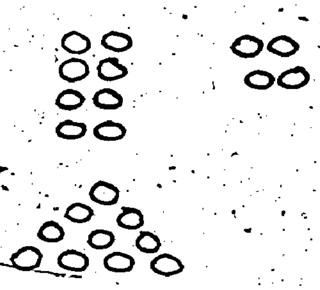
Step 2.

A	8 + 4 = 12
B	

page 92 blank.

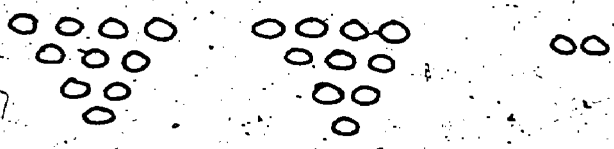
95

Step 3.



A	8 + 4 = 12
B	18 + 4 =

Step 4.



A	8 + 4 = 12
B	18 + 4 = 22

Repeat steps 1-4, this time adding the ten to the four in step 3; and record.

A	8 + 4 = 12
B	8 + 14 = 22

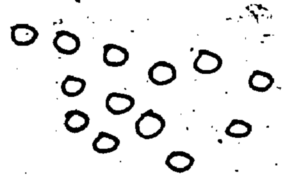
B. CONTINUATION

MATERIALS: A set of 40 objects per pupil.

DIRECTIONS: Same as A except that a set of size twenty is added, either to one of the two sets or ten to each set. See example on next page.

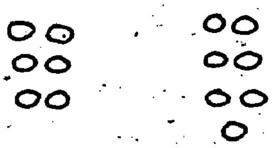
RECORDING

Step 1.



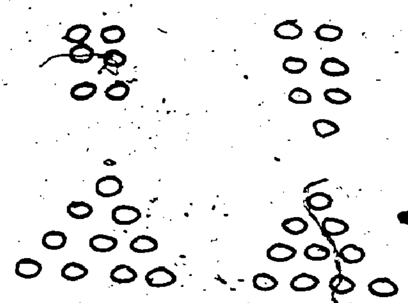
A	
B	

Step 2.



A	6 + 7 = 13
B	

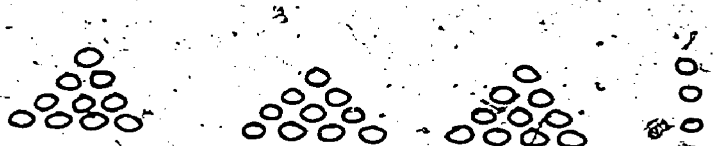
Step 3.



A	6 + 7 = 13
B	16 + 17 =

Step 4.

A	6 + 7 = 13
B	16 + 17 = 33



Repeat steps 1-4 twice, adding all twenty to one set and then the other set in step three, and record.

A	6 + 7 = 13
B	26 + 7 = 33

A	6 + 7 = 13
B	6 + 27 = 33

98

OBJECTIVE Model and record the addition of 2 place numbers whose sum is less than 100.

COMMENTS

PRE-REQUISITE OBJECTIVES 39-J, 36-C

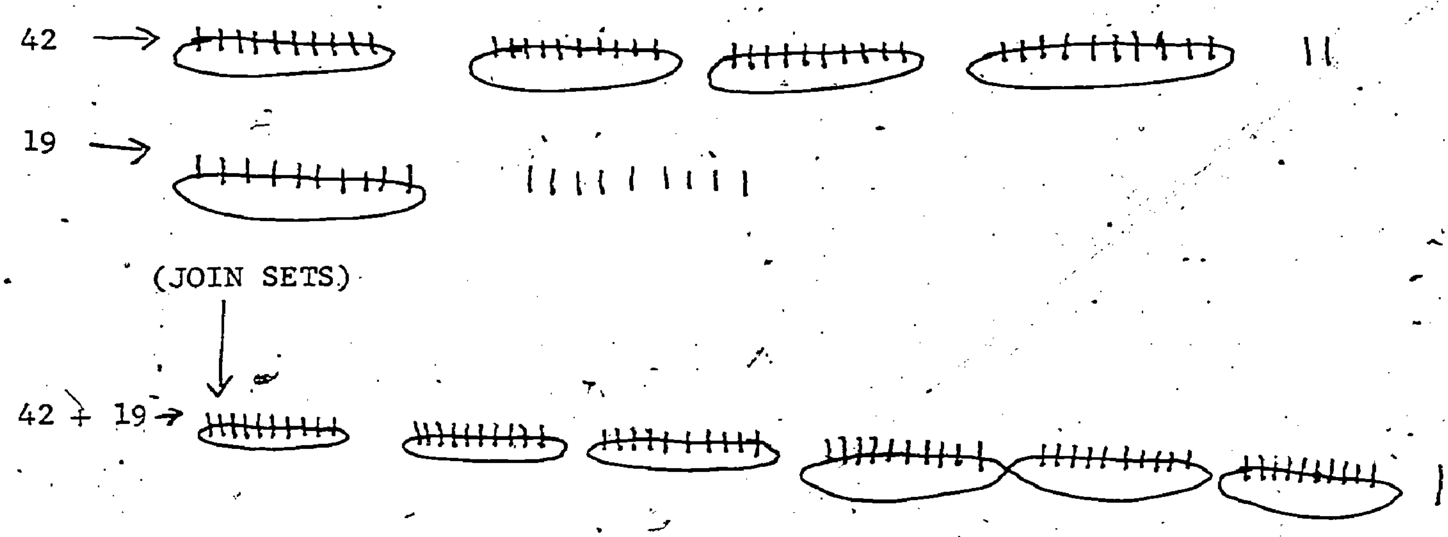
ACTIVITIES

A. ADDITION MODELS

MATERIALS: The models on page 57, chalkboard, activity cards or ditto.

DIRECTIONS: Work with small groups. Each child should have a set of models for tens and units. Review modeling a few numerals--56, 72, 20. Ask the pupils to make a set of 42 and a set of 19. Ask them to join the sets. When they have joined the sets, ask, "What is the sum of 42 plus 19?" Their model indicates the answer.

SAMPLE MODEL: Straws and rubber bands.



Tens	Units	
6	1	= 61

page 96 blank

Example:
$$\begin{array}{r} 42 \\ +\ 19 \\ \hline \end{array}$$

NOTE: Encourage working from <u>right to left</u>. Ask the pupils, "How many do I have if I join the units?" The most likely answer is "11". Then ask, "How many tens is that?" Record the "1" in the tens column. Then ask, "How many units are left?" Record the "1" in the units column.

$$\begin{array}{r} 42 \\ +\ 19 \\ \hline 11 \\ 50 \\ \hline 61 \end{array}$$

Now ask, "How many more tens do we have?" Record 5 tens and ask, "How many altogether?" Record 1 unit and 6 tens.

Some children will already be able to remember the ten, because of the modeling experience. Don't hold these pupils back. Continue with examples:

$$\begin{array}{r} 56 \\ +\ 21 \\ \hline 7 \\ 70 \\ \hline 77 \end{array} \qquad \begin{array}{r} 43 \\ +\ 28 \\ \hline 11 \\ 60 \\ \hline 71 \end{array} \qquad \begin{array}{r} 27 \\ +\ 33 \\ \hline 10 \\ 50 \\ \hline 60 \end{array}$$

Or..

$$\begin{array}{r} 56 \\ +\ 21 \\ \hline 77 \end{array} \qquad \begin{array}{r} 43 \\ +\ 28 \\ \hline 71 \end{array} \qquad \begin{array}{r} 27 \\ +\ 33 \\ \hline 60 \end{array}$$

Provide practice dittos and workbook pages. Encourage pupils to use the models.

OBJECTIVE Join 3 or more sets and repattern the final sum to show tens and units (if the sum is greater than 10).

COMMENTS Include the empty set after the pupils are comfortable with the activity.

PRE-REQUISITE OBJECTIVES 39-J

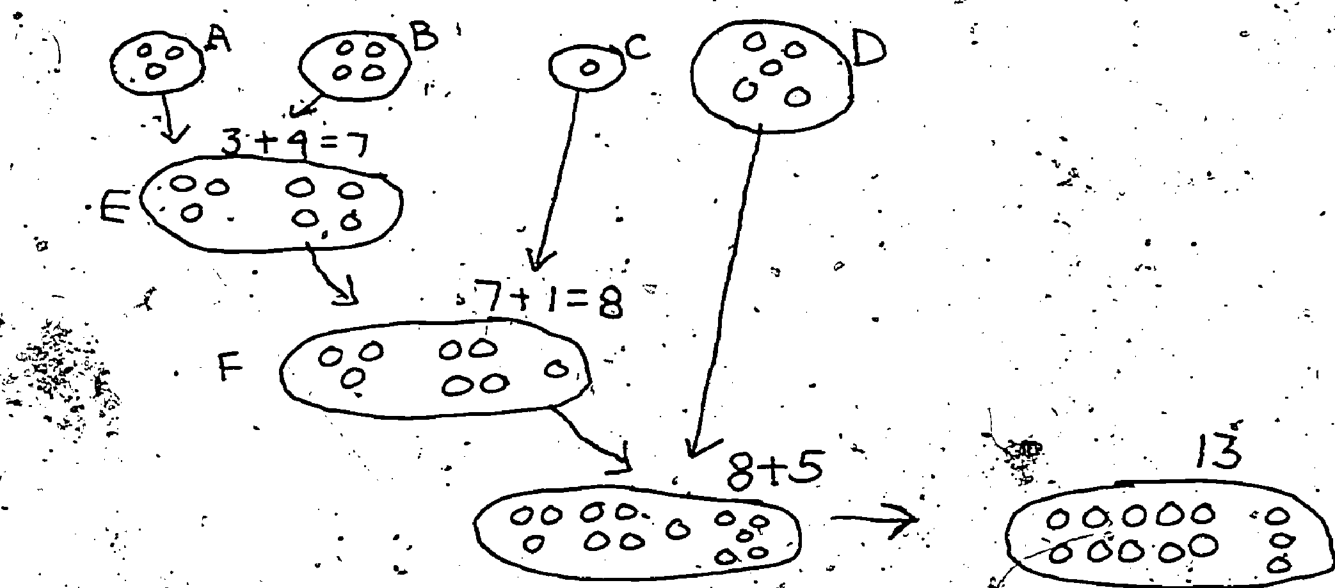
ACTIVITIES

A. 3 AND 4 ADDENDS

MATERIALS: Objects, blocks, cubes, dominoes, bottle caps, etc.; yarn, drawing paper.

DIRECTIONS: Ask the pupil to place a set (not more than 6) of objects in each of 3 or four sets. Have him make a drawing of the sets and record the addition sentence under each set as he moves the objects from one set to the next (moving from left to right). At the last set, have him repattern the objects to show the 10's and units.

SAMPLE: Pupil might fill the sets and make a drawing of it like this.



Using the actual set, he should move the objects from Set A and B to E and record: 3 + 4 = 7. Then he moves the objects from E and C to set F and records 7 + 1 = 8. He continues until he joins set D and records them. Repattern the objects into tens and units.

VARIATION: Use a vertical format.

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline 8 \\ +2 \\ \hline 10 \\ +\ 4 \\ \hline 14 \end{array}$$

or record this way:

$$\begin{array}{r} 3 \\ +5 \\ \hline 8 \\ +2 \\ \hline 10 \\ +4 \\ \hline 14 \end{array}$$

Lead pupils to think the sums like this:

3	
+5	think 8
+2	think 10
+4	think 14
14	

B. PRACTICE ADDING 3 AND 4 ADDENDS.

MATERIALS: Dittos and workbook pages.

NOTICE

THE NEXT SEQUENCE OF OBJECTIVES DEVELOPS THE IDEA OF PLACE VALUE IN BASE TEN. IT SHOULD BE NOTED THAT THE FULL IMPACT OF PLACE VALUE DOES NOT COME ACROSS UNTIL THE NUMERAL HAS THREE DIGITS. WHEN THE PUPIL UNDERSTANDS THE PROCESS OF FORMING GROUPS OF GROUPS (TO FORM HUNDREDS IN BASE TEN) HE IS WELL ON HIS WAY TO COMPREHENDING THE PLACE-VALUE SYSTEM.

OBJECTIVE Rename group of groups as hundreds and learn to name the numbers and record the numerals 100 to 900 by hundreds.

COMMENTS No groups or units should be recorded in this activity.

PRE-REQUISITE OBJECTIVES 28-C, 27-C, 26-C

ACTIVITIES

A. HUNDREDS

MATERIALS: Multibase rods and squares (tens only), paper and pencil for recording.

DIRECTIONS: On paper, pupils may make a grid as follows:

Ten sets of ten (Groups of groups)	Tens (Groups)	Units

Pupils will have a square and ten rods. They will arrange the rods on the square until it is covered. The square will represent one group of groups, ten sets of ten, recorded:

	Ten sets of ten (Groups of groups)	Tens (Groups)	Units
	1	0	0
Two squares would be recorded as	2	0	0
Three squares would be	3	0	0

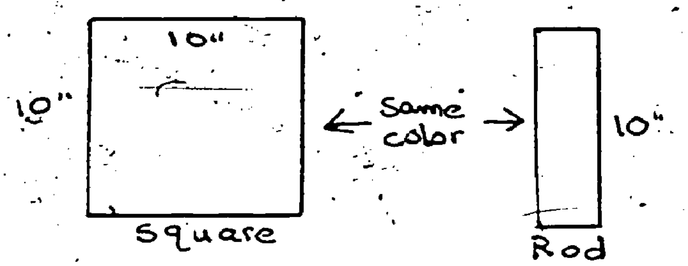
At this point, the term hundred would be substitututed and the above process repeated. "Hundred is a name for ten sets of ten."

Hundreds	Tens	Units
1	0	0
2	0	0

If there is one hundred, no tens, no units, the name of the numeral is 100 (one hundred). Continue the recording and naming process to 900.

page 102 blank

If Multibase materials are not available, the teacher can cut strips and squares from construction paper and use the paper in the same way.



B. REPLACEMENT

MATERIALS: Toilet tissue tubes cut to be no taller than 10 poker chips, colored blue with "tens" printed on the outside; colored red, with "hundreds" printed on the outside; an assortment of poker chips (red, blue, white); paper and pencil for recording.

DIRECTIONS: White poker chips would be used to represent the units. The pupil would fill one blue tube with white poker chips. The teacher could then explain that one blue poker chip could be used to represent the tube full of white chips (the tube should be kept for reference). If the pupil isn't sure, repeat the process until he can see that the blue chip can be a simpler way of showing ten (i.e. 1 tens represent 10 units). Next, have the pupil place 10 blue chips in a red tube and replace the red tube with a red chip. This can be recorded as:

Hundreds	Tens	Units
1	0	0

103

OBJECTIVE Name and record hundreds with tens.

COMMENTS No units should be recorded in this activity. Pupils need to learn to write and say the numbers from 110 to 990 by tens.

PRE-REQUISITE OBJECTIVES 44-C

ACTIVITIES

A. HUNDREDS AND TENS

MATERIALS: Cuisenaire rods and squares, (or multibase blocks); paper and pencil.

DIRECTIONS: On paper, have pupils make a grid as follow:

Ten Sets of Ten (groups of groups)	Tens (Groups)	Units

. Pupils will have a square and more than ten rods. Pupils will arrange rods on the square until it is covered (there will be rods remaining which cannot be placed on the square).

For example, this might occur.

Ten Sets of Ten (Groups of Groups)	Tens (Groups)	Units
1	1	0

After the pupil has done this activity and understands the recording procedure, alter the recording model as follows:

Hundreds	Tens	Units

109

Name and Record as follows:

Name	Hundreds	Tens	Units
"one hundred ten"	1	1	0
"one hundred twenty"	1	2	0
"nine hundred ninety"	9	9	0

B. Modify poker chip activity B from Objective 44-C.

OBJECTIVE Name and record hundreds, tens, and units.

COMMENTS Pupils will name numbers from 101-999 and record the numerals.

PRE-REQUISITE OBJECTIVES 45-C

ACTIVITIES

A. HUNDREDS, TENS, AND UNITS

MATERIALS: Cuisenaire rods (or multibase blocks), squares and units, paper and pencil.

NOTE: Poker chip activity can be used if multibase rods are not available; construction paper may be used to replace square rods and units.

DIRECTIONS: On paper, have pupils make a grid as follows:

Hundreds	Tens	Units

Pupils will have had previous experience modeling and recording Hundreds and Tens and should have little difficulty modeling and recording units.

EXAMPLE: Record and name as follows:

	Hundreds	Tens	Units
(a) →	1	1	1

NAME:

(a) One hundred eleven.

Continue the procedure until the student is able to identify and record numerals to 999 (nine-hundred ninety-nine).

OBJECTIVE Name the even and odd numbers from 1 to 90.

COMMENTS Definitions of even and odd are given in activity A.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. IS THIS NUMBER EVEN?

MATERIALS: Any set of 20 objects.

DIRECTIONS: Step 1. [illegible] the pupil a chart and some objects. Starting with a particular size, a[illegible] [illegible]m if the objects can be repatterned into 2's with no left overs. Record on the chart, yes or no. Use numbers 1 to 10. It is not necessary to take them in order.

Number	Even
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Step 2. Explain that even means n[illegible] left overs when patterning by sets of size two. Those numbers listed on the chart that have no left overs are called even numbers. Have the pupil insert the word "yes" or "no" under the even column.

Step 3. Lead the pupil to understand that when naming the even numbers in order, he is counting by two's.

Step 4. Explain that those numbers that have a left over, are called odd. Have the pupil make another chart like one below. Have him name the odd numbers orally.

 e 108 blank

Step 4 cont'd CHART

Odd Numbers	Even Numbers
1	2
3	4
5	6
7	8
9	10

Step 5. Have the pupils continue listing numbers on the chart in Step 4 with or without the patterning of Step 1, as needed. Have them stop at about 30.

Step 6. Spot check each pupil for recognition of even and odd in numbers up to 99. An example is to give him a list of numbers and have him place them under the correct heading as in the example below.

41, 64, 73, 89, 96.

Even	Odd
96	89
64	73
	41

B. ODD OR EVEN

MATERIALS: none

DIRECTIONS: Step 1. Divide the class into 2 evenly matched teams. One team is called "odd" and the other "even".

Step 2. To play the game, one member from each team will come to the front of the room. The two children will stand back to back. Each child will hold up any number of fingers on one hand. Each player must not peek to see how many fingers his opponent has raised.

Step 3. The rest of the class quickly counts how many fingers the two together have held up. If the total is an odd number, the Odd Team scores one point: If it is an even number, the Even Team scores. Instruct the pupil to raise his hand if he sees his team has just won a point. The score is recorded on the board.

Step 4. The next player from each team will come to the front of the room and so on. At the end of the playing time, the team with the highest score wins.

OBJECTIVE Identify oddness and eveness in relation to addition.

COMMENTS This is pre-activity work in preparation for later work with integers.

PRE-REQUISITE OBJECTIVES 47-C

ACTIVITIES

A. EVEN + ODD; ODD + EVEN

MATERIALS: Ditto sheet and pencil (see example below)

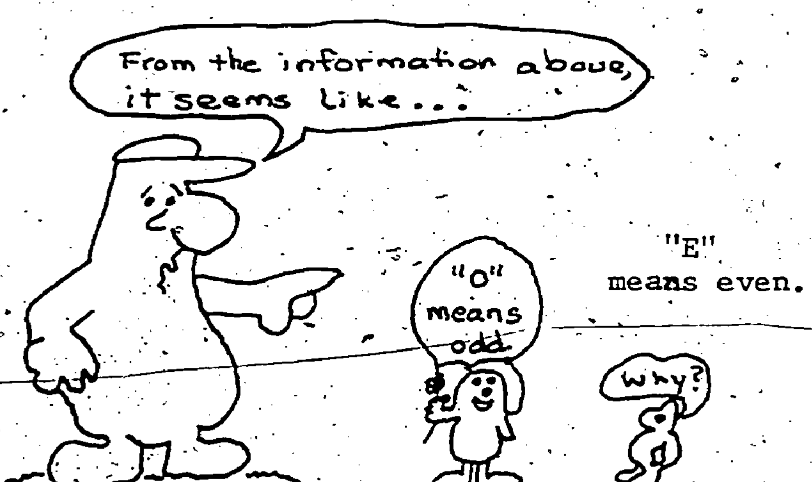
DIRECTIONS: Pupils will complete grids.

+	2	4	6	8
2				
4				
6				
8				

+	1	3	5	7
1				
3				
5				
7				

+	2	4	6	8
1				
3				
5				
7				

Compare the answers in each grid. Ask the pupils, "When we add even numbers to even numbers, are the sums even or odd?"
"When we add odd numbers to odd numbers, are the sums even or odd?"
"When we add odd numbers to even numbers, are the sums even or odd?"



+	O	E
O		
E		

Now Have the Pupils complete this grid

OBJECTIVE To show that in the place-value notation, the unit digit determines whether the number is odd and even.

COMMENTS

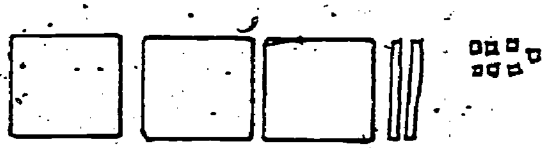
PRE-REQUISITE OBJECTIVES 47-C

ACTIVITIES

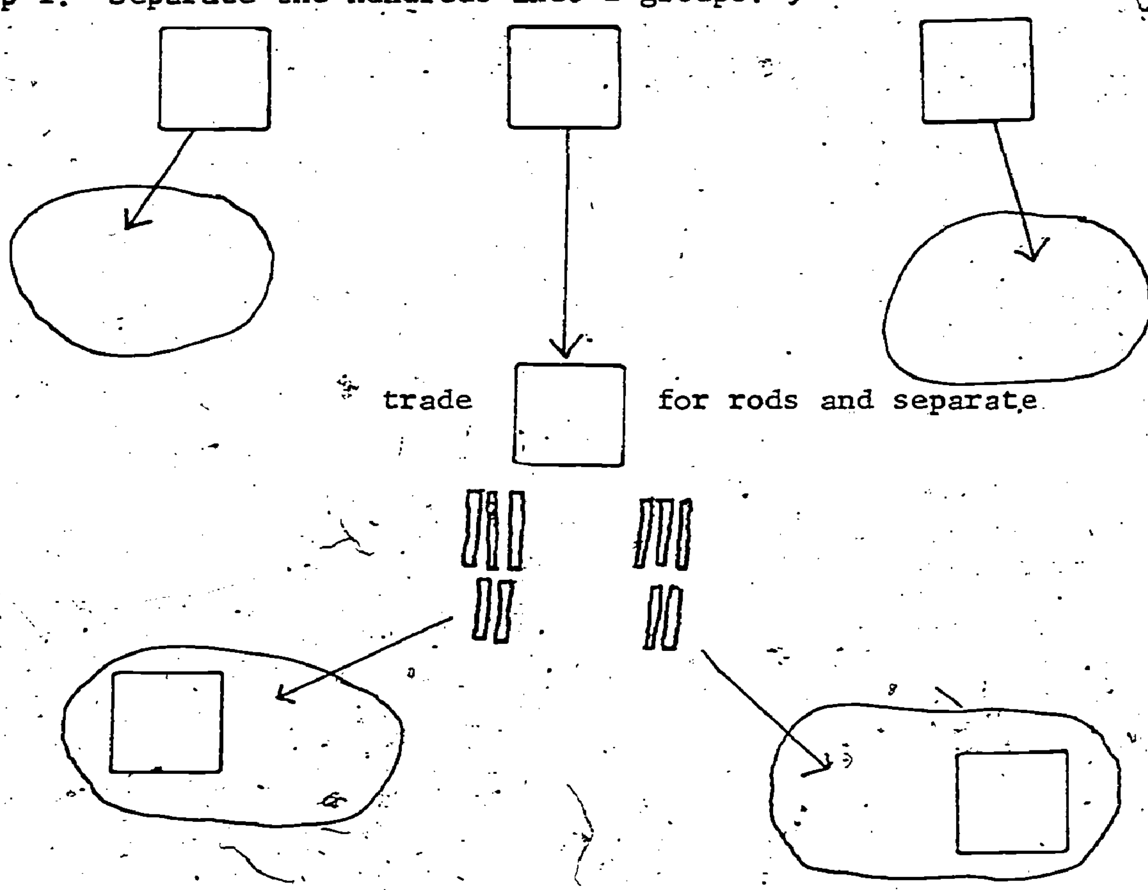
A. DIVIDING LARGE SETS INTO TWO SETS THE SAME SIZE

MATERIALS: Cuisenaire rods (or multibase blocks); or construction paper cut to represent squares (100's), rods (10's), and units.

DIRECTIONS: Model 327.



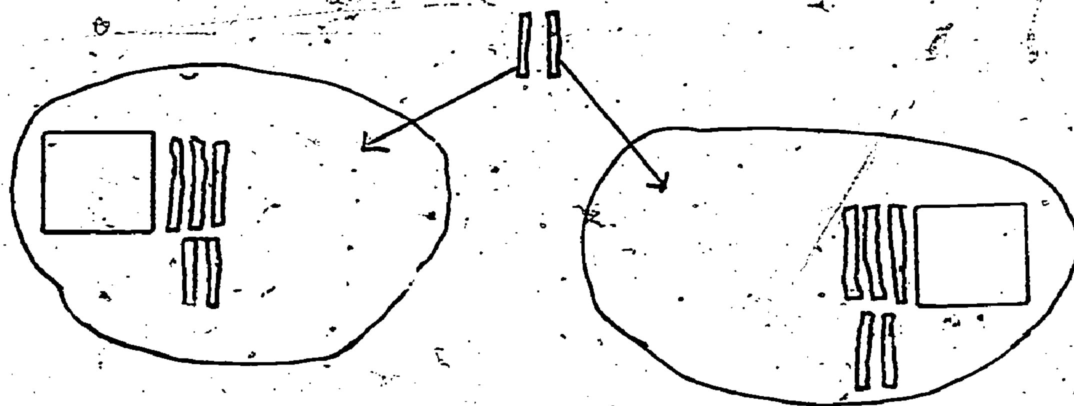
Step 1. Separate the hundreds into 2 groups.



Ask, "Are the hundreds even?" (Yes)
Record the answer on the chart.

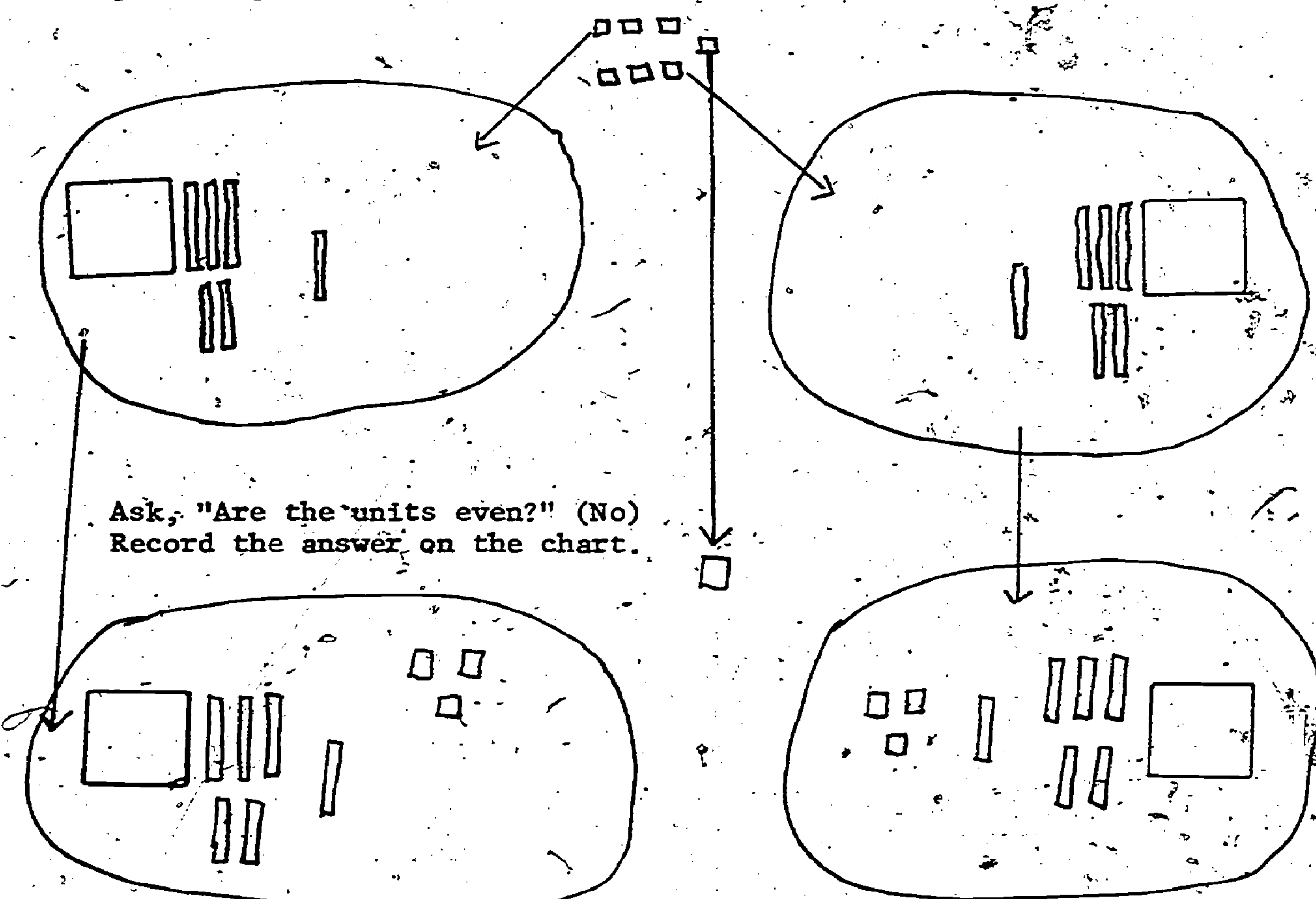
112 blank.

Step 2. Separate the tens into 2 groups



Ask, "Are the tens even?" (Yes)
Record the answer on the chart.

Step 3. Separate the units into 2 groups.



Ask, "Are the units even?" (No)
Record the answer on the chart.

	Are the hundreds even?	Are the tens even?	Are the units even?	Is the whole number even?
327				
232				

Use many examples so that the pupil can realize that the hundreds and tens are always even, and that if the units are even the whole number is even.

This is, of course, not the only way. Experiment with several ways of doing this and use the one you feel most comfortable with. You want to lead the children to the realization that hundreds and tens are even. This may take many examples and quite a bit of time. Don't be afraid to take the necessary time and expend the needed effort, as this is good experience for the children in working with place-value.

B. PLACE VALUE DECIDES EVEN AND ODD

MATERIALS: Clear plastic cookie containers, numbers cut from old calendar, masking tape.

DIRECTIONS: Use masking tape to label each compartment in the cookie tray.

EXAMPLE:

Hundreds	Tens	Units

Get an old calendar that has large numerals on it. Cut all the numerals out and place them in piles in a shoe box. (All the numbers together.) Write a number on the chalkboard or overhead projector and place them in the correct slots in the labeled cookie holders. Now, read a number to the pupils and have them select the numerals to put in the cookie holders.

NOTE TO TEACHER: Drill pupils on this activity until they understand the operation. Have the pupils read the number aloud from the visual aid. Then place them in the slots. Ask, ""What number is in the units place?" When the answer is given, ask, "Is it odd or even?" Then ask, "Is the number odd or even?"

VARIATION: Use a chart like this:

Hundreds	Tens	Units

Follow the same procedure as above.

OBJECTIVE Name the numbers and record the numerals 1,000 to 9,000 by thousands.

COMMENTS

PRE-REQUISITE OBJECTIVES 44-C

ACTIVITIES

A. THOUSANDS

MATERIALS: Cuisenaire cubes, squares (or multibase cubes and squares), or poker chips, tubes, paper and pencil.

DIRECTIONS: Repeat activities described in Objective 44-C using 10 models of 100 to build model for 1,000. (Ten multibase flats form a thousand cube.)

Name and record as follows:

thousands	hundreds	tens	units
1	0	0	0
2	0	0	0
3	0	0	0

Name: one thousand, two thousand, three thousand
Introduce and stress at this point the correct punctuation.
Example: Instead of the recording above, we write 1,000; 2,000; 3,000; etc.

OBJECTIVE Name the numbers and record the numerals 1,001 to 9,999.

COMMENTS Stress at this point that the punctuation begins the naming process.
Ex: 1,268 ...."one thousand two hundred sixty-eight.
2,001 ...."two thousand one"

PRE-REQUISITE OBJECTIVES 44-C

ACTIVITIES

A. THOUSANDS, HUNDREDS, TENS, AND UNITS

MATERIALS: Models developed for 44-C

DIRECTIONS: Modify the Activities using thousands.

Thousands	Hundreds	Tens	Units

e 118 blank.

OBJECTIVE Solve 2-place addition problems without models.

COMMENTS Activity A will be using models for sums over 100 but the end objective is to do these problems without models. Drill is important.

PRE-REQUISITE OBJECTIVES 42-J

ACTIVITIES

A. FROM TENS TO HUNDRED

MATERIALS: Models for hundreds, tens, and units.

DIRECTIONS: Each pupil should have a set of models. Again he models two sets and joins them. This time, he will need to regroup tens into hundreds.

$$\begin{array}{r} 42 \\ +\ 71 \\ \hline \end{array} \qquad \begin{array}{r} 65 \\ +\ 94 \\ \hline \end{array}$$

Again some pupils may automatically exchange 10 tens for a hundred. If not, ask if there are enough tens to make a hundred. Immediately, follow the modeling with recording.

$$\begin{array}{r} 47 \\ +\ 68 \\ \hline 115 \end{array} \quad \text{or} \quad \begin{array}{r} 47 \\ +\ 68 \\ \hline 15 \\ 100 \\ \hline 115 \end{array}$$

Continue providing practice with the models readily available.

B. PRACTICE

MATERIALS: Dittos, drill sheets, workbook pages.

DIRECTIONS: Provide practice in every way possible. The more ways you can give the child experience with adding the better. This is a good place to introduce simple word problems.
If you decide to do this, make it a part of your language arts class having the pupils tell and write about experiences they had had working simple addition. Then introduce problems you have made or found.

118

Page 120 blank.

OBJECTIVE Model and record the addition of 3 place numbers.

COMMENTS Do many examples. Sums should involve both three and four place numbers.

PRE-REQUISITE OBJECTIVES 52-J

ACTIVITIES

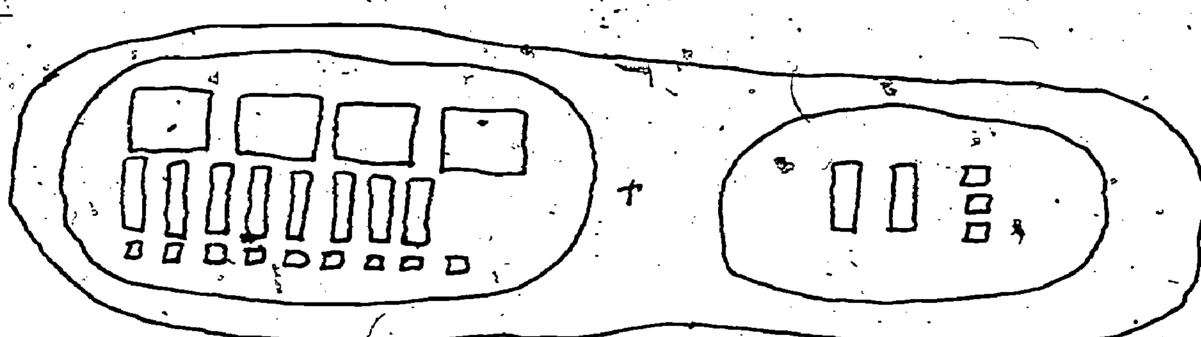
A. ADDITION MODELS

MATERIALS: Models for hundreds, tens, and units.

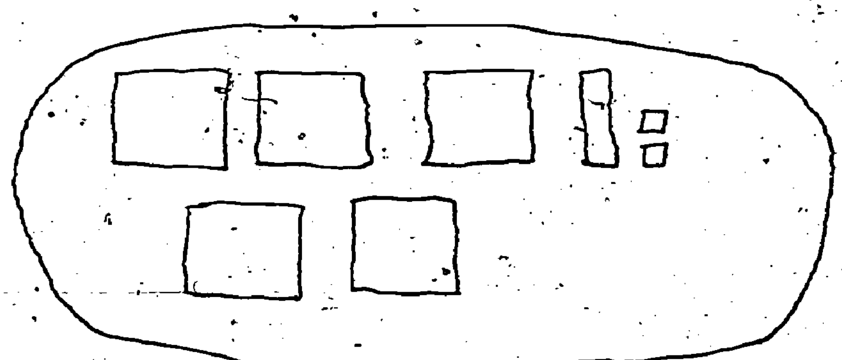
DIRECTIONS: Modify the modeling just done for adding 2 digit numerals to include hundreds. Have pupil first model an addition problem using flats, rods, and units.

For example:

$$\begin{array}{r} 489 \\ +\ 23 \\ \hline \end{array}$$



Regroup the models and replace 10 rods with one flat, 10 units with one rod and record the sum.



$$\begin{array}{r} 489 \\ +\ 23 \\ \hline 512 \end{array}$$

122 blank.

54 -J Reference Addition

OBJECTIVE Solve three place addition problems without models.

COMMENTS Drill is important. Use ditto and workbook pages for drill.

PRE-REQUISITE OBJECTIVES 53-J

ACTIVITIES

A. RECORDING

MATERIALS: Dittos, workbooks.

DIRECTIONS: Repeat activity A, objective 53-J without using models. Encourage adding right to left by recording from right to left.

```
 647        647
+327       +327
  14        974
  60
 900
 974
```

Encourage the shorter process with pupils who are ready for it. Where a "carry" is involved, let them decide how to keep track of it.

ge. 124 blank.

GENERAL REMARKS This next sequence of objectives gives the child experience with, and develops, the idea of linear measure.

55-G Reference Geometry

OBJECTIVE Measure lengths of objects using non-standard units - to recognize a need for a standard unit of measurement.

COMMENTS Use the fact that the pupils arrive at a variety of answers in the activities to establish a need for a standard unit of measurement.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MEASURING LENGTHS

MATERIALS: classroom items

DIRECTIONS: Measure the length of a blackboard, desk, bulletin board, etc. Use pencils, pieces of string, yarn, etc., to show how many pencil lengths it takes to make a length chosen.

B. MEASURING LENGTHS

MATERIALS: parts of body

DIRECTIONS: Have children measure a given object using feet, hands, fingers, arms, etc. Suggestions for objects are windows, doors, desks, tables, halls, rooms, buildings.

C. MEASUREMENT

MATERIALS: see directions

DIRECTIONS: The following are given as suggestions of what one might do to sharpen the idea of measure, but are suggestions only. Some will be used in succeeding activities as we move the pupil toward standard units of measure.

(1) Have the child fill a small cup with beans (anything small, uniform in shape will do) and then ask, "How many beans does it take to fill the cups?" The child has measured the capacity of the cup in beans.

page 126 blank

(2) Have the child count the number of tiles in the room, across the room, the length of the room. He has then measured the area, width, and length in tiles.

(3) Have the child count the numbers of his hand widths it takes to go across the chalkboard. He has then measured the chalkboard in hands. Do the same thing with an eraser.

(4) Have the child count the number of dribbles it takes to get from one end of the gym to the other. He has then measured the gym in dribbles.

D. "WHAT'S MY MEASURE?"

MATERIALS: Various objects

DIRECTIONS: Have the children use their hands to measure their desks or tables. Be sure they understand that they begin at one edge and measure how many hands it takes to get to the other edge, rounding off at the end. Children next measure their desks using a <u>book</u>, then a <u>shoe</u>, a <u>pencil</u>, an <u>arm</u>, other objects, and then a ruler.
Let the children compare their measurements and discover the variety of answers when all measure the same desk. Lead the group into discovering that the ruler gives the same measurement for everyone. The purpose is to understand the need for a standard unit of measure.

E. STRING MEASURE

MATERIALS: String, answer box

DIRECTIONS: Display a piece of string that measures some object in the room such as the width of a chair, length of the chalkboard eraser, height of the vase on teacher's desk, etc. Children take a paper and either write the name, or make a drawing of the object they think the string measures. They drop their guesses in the answer box. At the end of the day, take the string down and have children prove which object was the measurement of the day.

F. LINEAR MEASURE

MATERIALS: See next page.

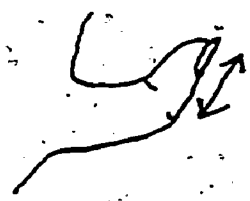
DIRECTIONS: See next page.

A long time ago, before there were no schools or books or teachers, it was very hard to measure the length of almost anything. Inches, feet, meters, miles-- all of these had not been invented yet. Distances were only roughly described. The Indians said that one village was so many "sleeps" distant from another village, or that the length of the summer was so many "moons". Some of the old-fashioned terms for measuring length have survived until today. For, instance, we measure the height of a horse in "hands", like this:

How many hands wide is your desk? ______________ hands

How many hands high is the top of the desk from the floor? __________ hands

The end of your thumb also makes a good measuring tool.



How many "thumbs" long is your math book? ______________ thumbs

How many "thumbs" wide? ______________ thumbs

How many "thumbs" wide is your desk? ______________ thumbs

Is a "thumb" longer or shorter than a "hand"? __________

Compare your answers with someone else.

Here is a line called AB A ______________ Able ______________ B

Take a strip of paper and mark length of it like this:

Is AB longer or shorter than a "thumb"? ____________________

Is AB longer or shorter than a "hand"? ____________________

A ____ B

How many AB's is this piece of paper from top to bottom? ____________ AB's.

Which is longer, AB or FD? ____________

F ____ D

Which is shorter, AB or HO? ____________

H ______________ O

How do AB and MY compare? ____________

M ____ Y

Which is longer, AB or SK? ____________

J ________________________________ K

You might have wondered what else AB can be called. Its full name is AB Line Segment. It is abbreviated like this, $\overline{AB}$.

How would FD Line Segment be abbreviated? ____________

Abbreviate HD, MY, and SK Line Segment. ________, ________, ________.

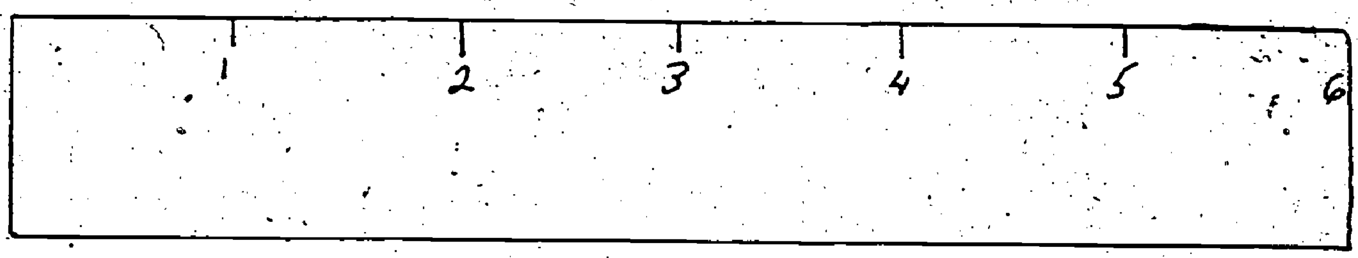
A line segment used on rulers is called an inch.

This is what an inch looks like.

Inch

____

Here is an inch ruler. How many inches are shown on it? __________ inches.



OBJECTIVE Use units to measure length of a line.

COMMENTS Pupils should use both English and metric system. Do not convert, but use both at the same time.

PRE-REQUISITE OBJECTIVES

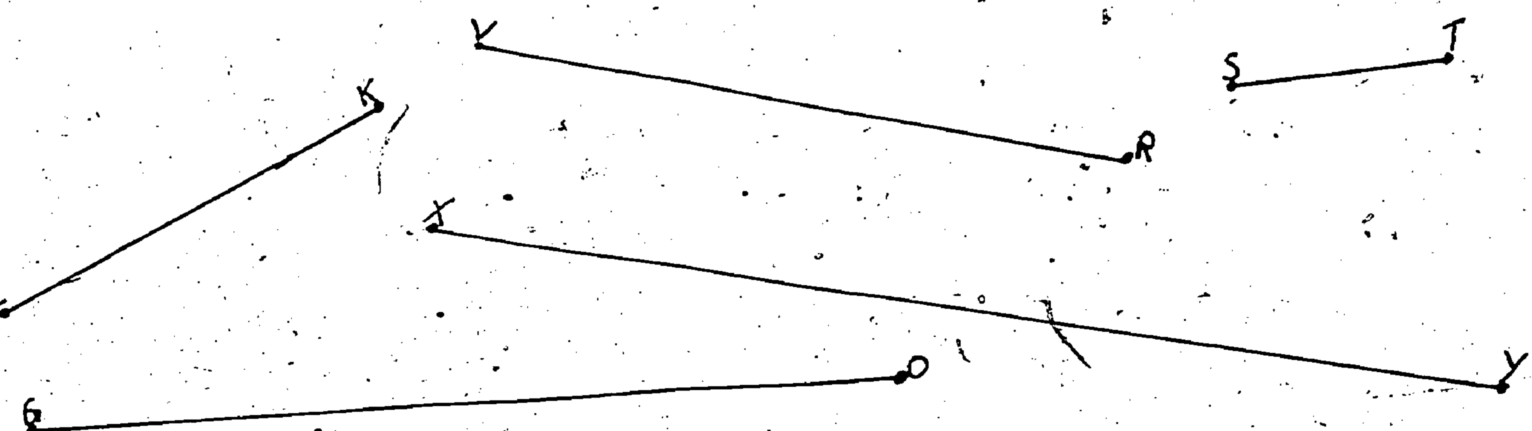
ACTIVITIES

A. MEASURING LINE SEGMENTS

MATERIALS: Dittos, centimenter and inch rulers.

DIRECTIONS: Prepare a ditto somewhat like the following. Be sure measurements are exact and not fractional.

How many inches long is each?



(1) $\overline{LK}$= ______________ inches

(2) $\overline{VR}$= ______________ inches

(3) $\overline{ST}$= ______________ inches

(4) $\overline{XY}$= ______________ inches

(5) $\overline{GO}$= ______________ inches

page 132 blank

A centimeter is this long: ├──┤

Is a centimeter longer or shorter than an inch?

Measure these line segments with your centimeter ruler:

(1) ____________

(2) ____________________

(3) ____

(4) ____________________________________

(5) ____________________

(6) ________________________________

(7) ____

ANSWERS: (1)____________ centimeters

(2)____________ centimeters

(3)______________________

(4)______________________

(5)______________________

(6)______________________

(7)______________________

DID YOU REMEMBER TO WRITE THE WORD CENTIMETER IN BLANKS (3) TO (7)?

OBJECTIVE Use units to measure perimeter of various geometric shapes.

COMMENTS Make shapes in activity B exact measurement, not fractional. Activity B is a pre-multiplication activity.

PRE-REQUISITE OBJECTIVES 56-G

ACTIVITIES

A. MEASURING COMMON THINGS

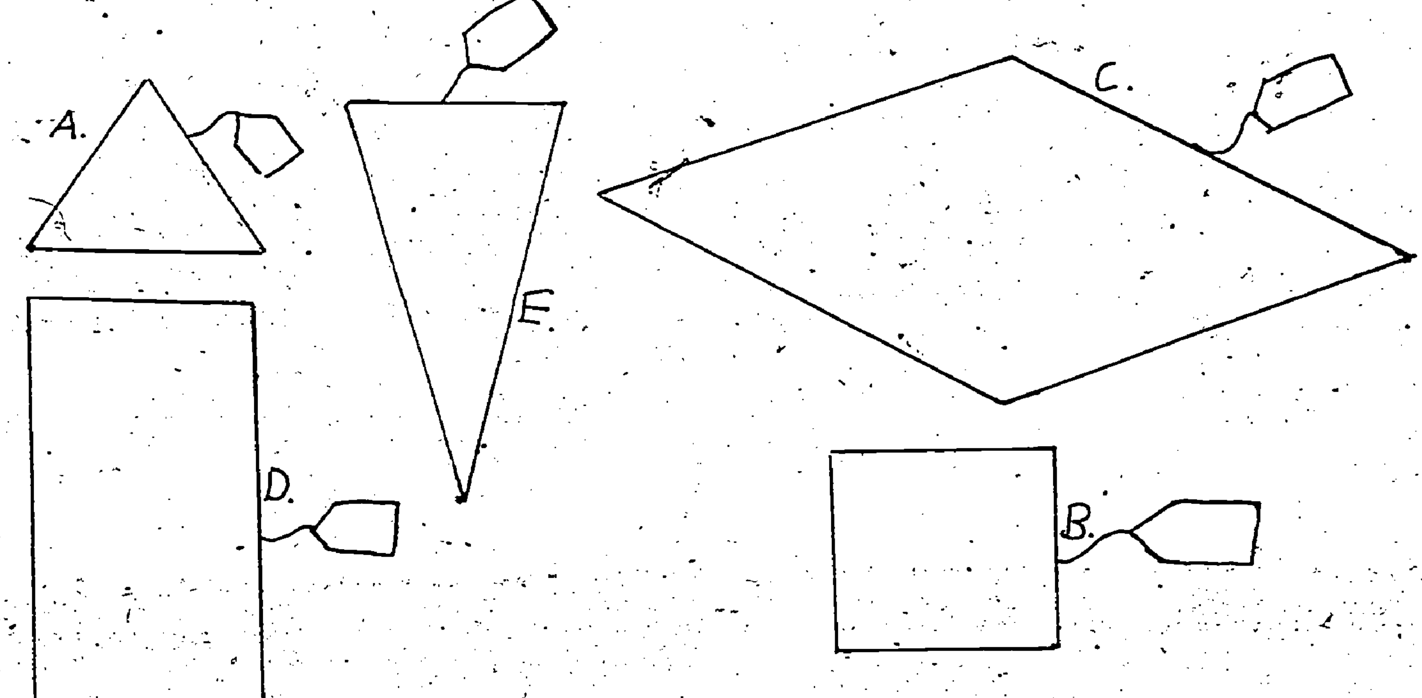
MATERIALS: Sufficient rulers, yardsticks, and meter sticks for the class.

DIRECTIONS: Have pupils measure those common things about them.

B. PERIMETER CENTS

MATERIALS: Dittos, rulers, pencils.

DIRECTIONS: Pupils should measure perimeter of each figure to determine its cost or find the sum of the lengths of all the sides to find the cost in terms of price list. Teacher fixes the cost, for example, 5¢ an inch. How much is each shape worth? Put your answer on the price tag.



GENERAL REMARKS The following three objectives have the purpose of extending the idea of line and defining some relations between lines.

58 -G Reference Geometry

OBJECTIVE Distinguish between <u>line</u> and line <u>segment</u>.

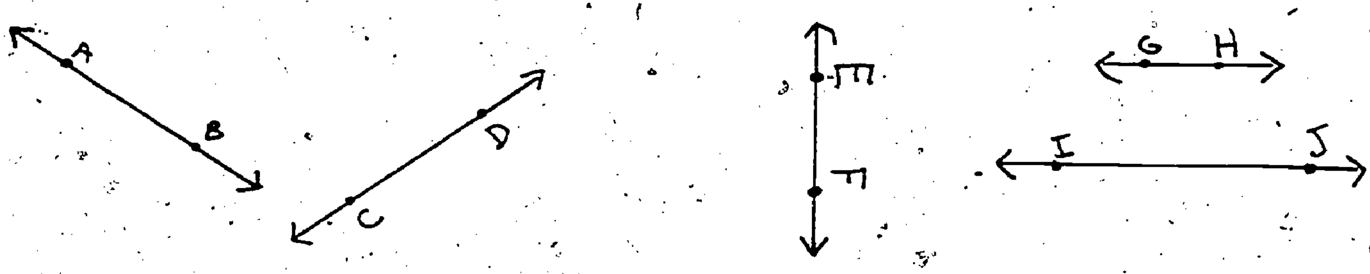
COMMENTS Given two points, a <u>line</u> passes through them and extends infinitely far in both directions. A <u>segment</u> consists of the two points and all the points of the line between them.

PRE-REQUISITE OBJECTIVES 7-G, 5-G

ACTIVITIES

A. LINES

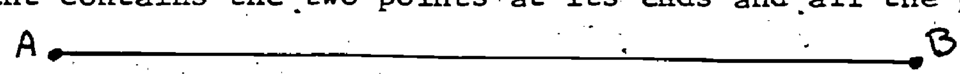
MATERIALS: You may duplicate sheets like this.



DIRECTIONS: Two points on each have been given names so that we have different names to use in talking about the five different lines. Ask, "(a) Which drawing shows line AB? (b) What name will you use for the line in the next drawing? (c) Which drawing of a line is in a vertical position on the page? (d) The drawing of line GH is shorter than the drawing of line IJ, but what shows that both of these lines go on and on and never end?"

Note to the Teacher: Stress that capital letters are used to name points and that two points on a line are used to identify the line.

NOTE: The drawing below shows a part of a line called a <u>line segment</u>. A line segment contains the two points at its ends and all the points in between.



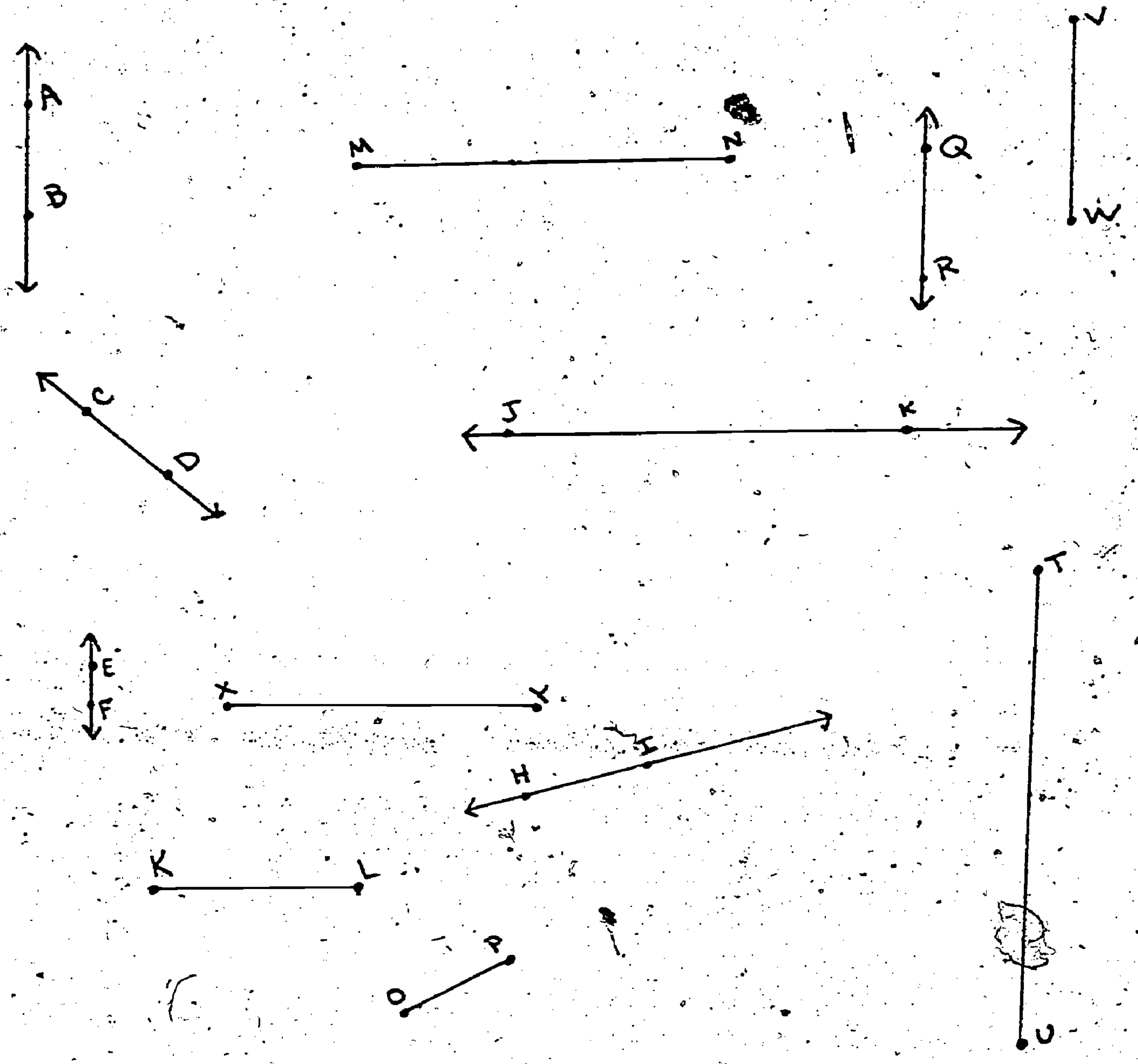
We call the part of a line shown in the drawing above "line segment AB." Points A and B are called the "end points". Since a line segment does not go on and on, we do not draw arrowheads.

 page 136 blank

B. EXERCISES ON LINES

MATERIALS: Ditto sheets with various lines, line segments.

DIRECTIONS: Ask the students to tell which drawings represent lines and which represent line segments. (Note: Duplicate the drawings or draw on the chalkboard.)



OBJECTIVE Learn and use the standard labeling technique to label points, vertices, lines, and line segments.

COMMENTS This method of labeling is a convention adopted by most mathematicians.

PRE-REQUISITE OBJECTIVES

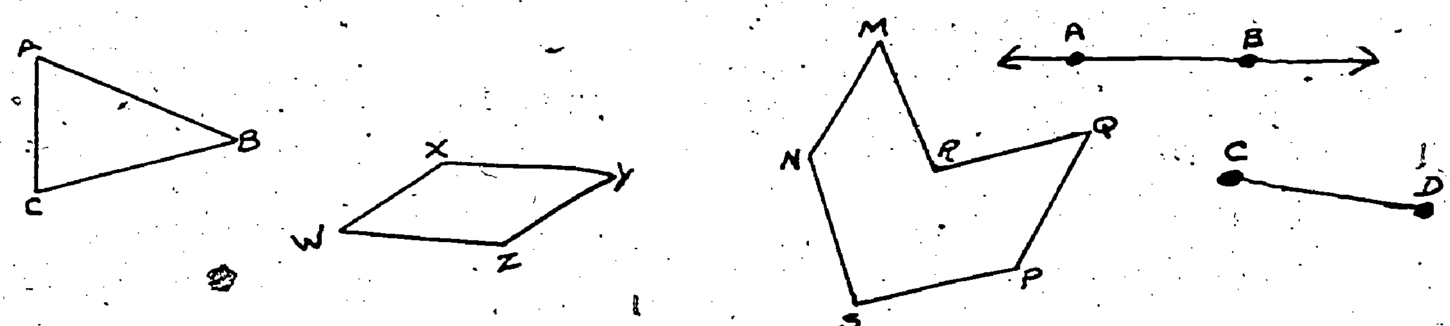
ACTIVITIES

A.

MATERIALS: Chalkboard, geometric figures, practice dittos.

DIRECTIONS:

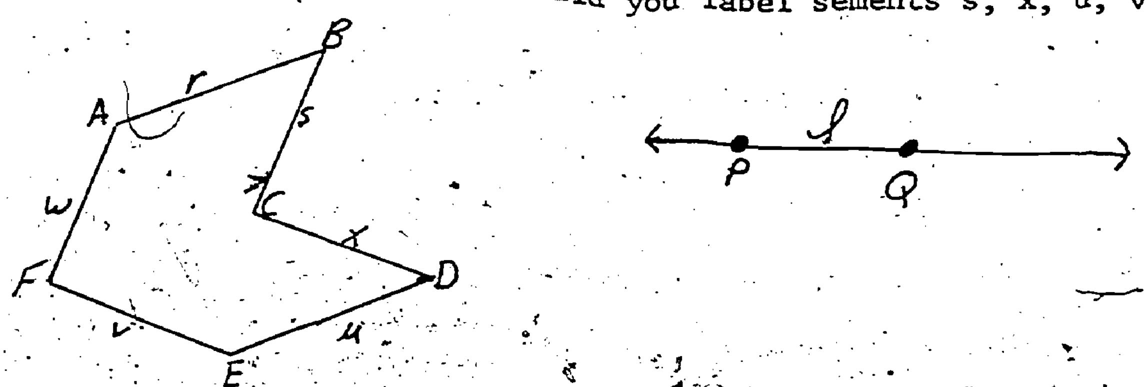
(a) Put several figures on the board. Teach students to label the vertices, points, and endpoints first. Use capital letters, always.



Let students label several figures.

(b) Teach students to label line segments (lines) with small letters or by naming the two end points (points) with a bar (arrow) above the letters.

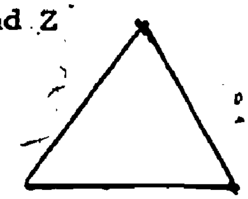
The <u>line</u> l through P and Q will be denoted $\overleftrightarrow{PQ}$. The <u>segment</u> r joining A and B will be denoted $\overline{AB}$ or $\overline{BA}$. How would you label sements s, x, u, v, w ?



129

(c) Have students label vertices and segments on dittos.

Use X, Y, and Z



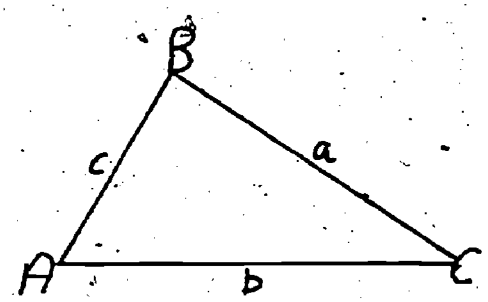
Use a, b, c

Which segment is $\overline{XZ}$? ____________

Which segment is $\overline{YZ}$? ____________

Which segemnt is $\overline{XY}$? ____________

(d) For triangles it is customary that the side opposite angle A gets the letter a, B gets b, and so on.



OBJECTIVE Recognize perpendicular and parallel lines.

COMMENT This is an experiential process, the vocabulary is being developed. Measuring and constructing 90° angles is not necessary.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

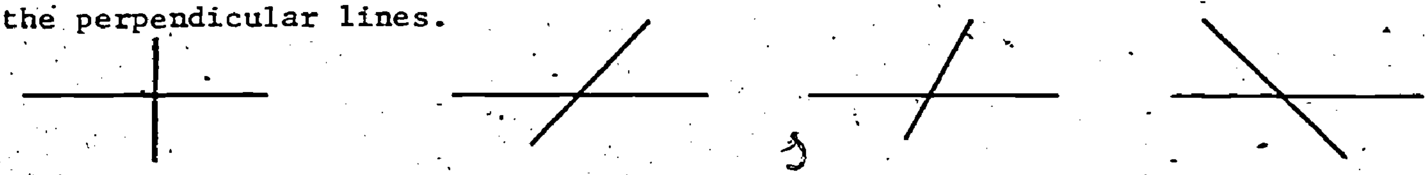
A.

MATERIALS: chalkboard, ruler, geometric figures.

DIRECTIONS:

(a) Find examples of perpendicular lines around the room.

(b) Draw several intersecting lines on the blackboard. Have students point out the perpendicular lines.



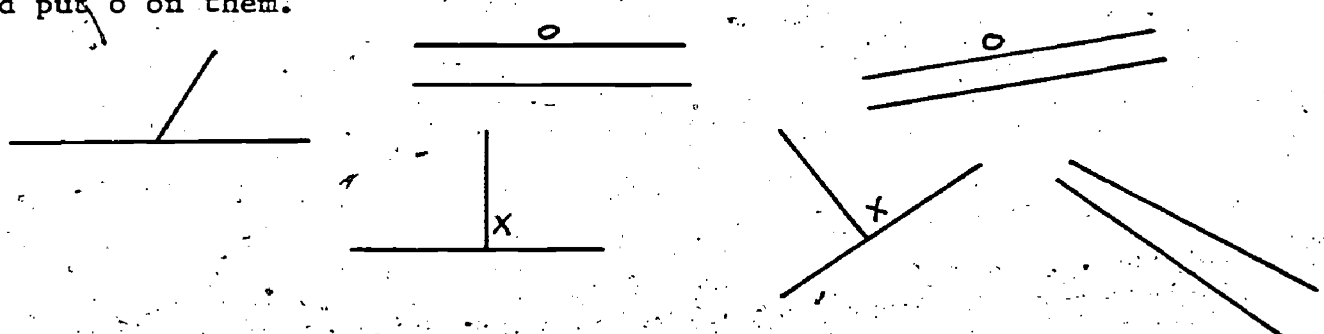
(c) Examine geometric figures. Make a set of those figures with perpendicular lines.

Modify the above to find parallel lines.

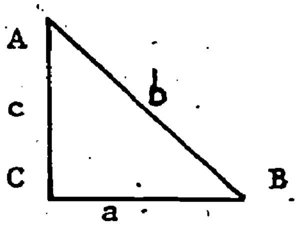
B.

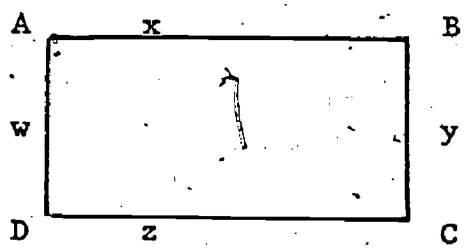
MATERIALS: Teacher-made ditto or activity cards.

DIRECTIONS: Find the perpendicular lines, put an x on them. Find parallel lines and put o on them.



Name the perpendicular and the parallel line segments. Introduce the symbols $\perp$ for perpendicular and $\parallel$ for parallel.





$c \perp a$, $a \perp c$

w $\perp$ x and z ($x \perp w+y$)

y $\perp$ z and x ($z \perp w+y$)

x $\parallel$ z

w $\parallel$ y

132

OBJECTIVE Identify and label rays and angles.

COMMENTS A ray is the part of a line that includes one point (the endpoint) and all points that lie in one direction on the line from that point. An angle is formed by two rays that have a common endpoint (vertex).

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. RAYS AND ANGLES

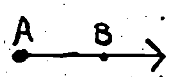
MATERIALS: Paper, rulers, and pencils.

DIRECTIONS: Mark three points D, E, and F.
Example:

.D

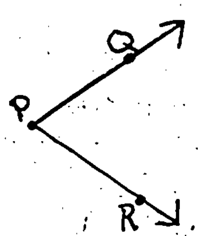
.E

.F

With D as an endpoint, draw ray $\overrightarrow{DE}$. With D as an endpoint, draw ray $\overrightarrow{DF}$. Mark arrowheads to show the directions in which the rays go on and on. Label the angle determined by the two rays.
NOTE: To indicate a ray, list the endpoint first and draw a single arrow above the letters.
EXAMPLE: 

is indicated by $\overrightarrow{AB}$.

To indicate an angle, three letters are required. List the vertex second. Example:



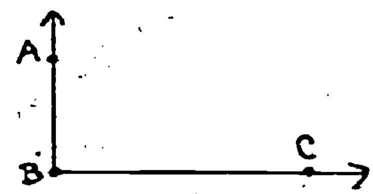
is denoted $\angle$ QPR or $\angle$ RPQ.

B. RIGHT ANGLES

MATERIALS: Same as above, cardboard.

DIRECTIONS: Cut a "square corner" from a sheet of stiff paper or cardboard. Lay the square corner on a piece of paper and use the two straight edges to draw a figure. Draw and label the points A, B, and C. Then mark arrowheads to show that BA and BC are rays.

Label the angle.



Ask, "Does your drawing show an angle? This angle is like a square corner and is called a <u>right angle.</u>" Conclude that when the rays are perpendicular the angle is a right angle.

GENERAL REMARKS The multiplicative process will be modeled as repeated addition in the following objectives.

62-J Reference Multiplication

OBJECTIVE Build models for the multiplication process.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MODELING

MATERIALS: Boxes, cans, and counters.

DIRECTIONS: Teacher models first problem for pupils.

EXAMPLE: Place 3 cans on the table. Into each can place 2 bottle caps. Ask pupils how many caps there are totally. Then give pupils a problem and let them model and explain the problem. Later model and explain 1 and 0 (zero would be an empty can). Due to difficulty, 1 and 0 should not be in the initial lessons.

VARIATIONS: Make activity cards with instructions for making different type models. Let groups of pupils build the models.

EXAMPLE:

MATERIALS: Cans and bottle caps.

Card 1: Take 6 cans. Place 3 caps in each can. How many caps do you have?

Card 2: Take 4 cans. Place 7 caps in each can. How many caps do you have?

EXAMPLE:

MATERIALS: Flannel board figures, flannel board.

Card 1: Put up 3 trees with 2 blue birds in each tree. How many birds?

Card 2: Give 4 rabbits 3 carrots each. How many carrots?

EXAMPLE:

MATERIALS: Cups and beans.

Card 1: Take 7 cups. Place 4 beans in each cup. How many beans do you have?

Card 2: Take 4 cups. Place 2 beans in each cup. How many beans do you have?

B. PICTURE PERFECT

MATERIALS: Counters (bottle caps, poker chips, etc.)

DIRECTIONS:

1) Pass out 8 counters to each pupil.
2) Have pupils arrange their counters in a rectangular array.
3) Ask the following questions.

   a. How many objects in each row?
   b. How many rows?
   c. How many total?

4) Have them rearrange the counters into a different array and repeat 3).
5) Repeat 1) with 12 counters.

VARIATION: Arrange objects in arrays on an overhead. Ask questions.

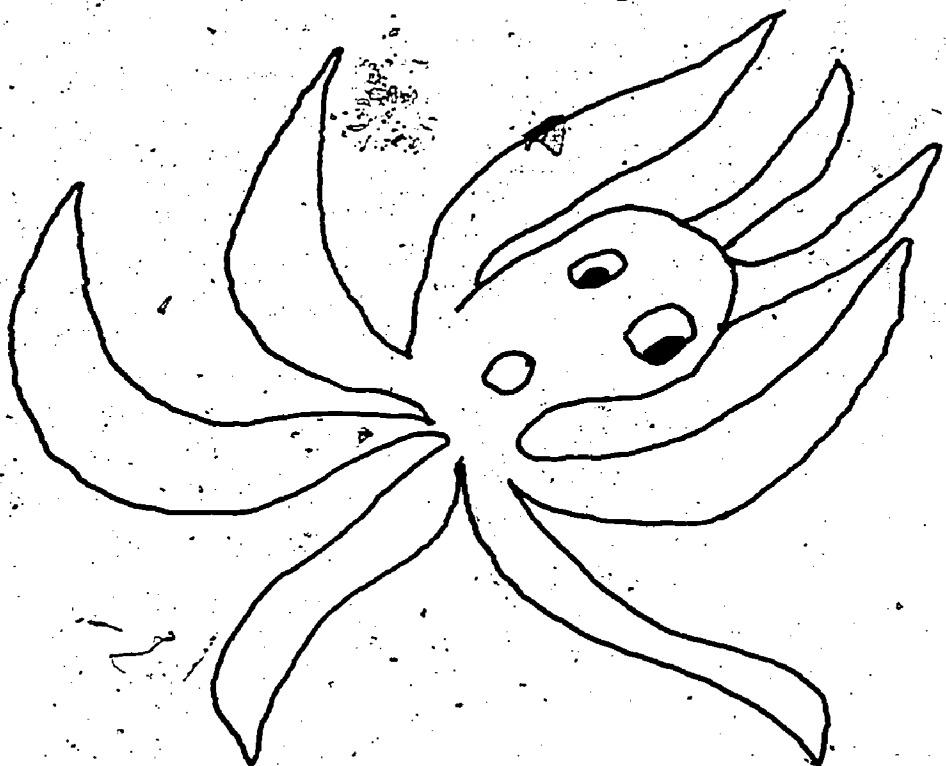
C. STORY PROBLEM POSTERS

MATERIALS: Large sheets of oak tag and colored markers.

DIRECTIONS: Use the following example to design your own posters.

133

Ollie Octopus has 8 legs



He has 3 socks for each foot. How many socks? ____________
He has 5 toes on every foot. How many toes? ____________
He has 2 cowboy boots for every foot. How many boots? ____________
He has 4 tennis shoes for every foot. How many tennis shoes? ____________
He has 1 knee above each foot. How many knees? ____________

OBJECTIVE Use the times sign and record multiplication problems.

COMMENTS In a multiplication equation such as 6 x 4 = 24, the multiplier is 6, the multiplicand is 4 and the product is 24. Do not use zero groups. Use zero as a set size only. Be consistent in using the number of groups as the multiplier, and the size of the groups as the multiplicand. Multiplier times multiplicand--6 x 4. Read: 6 sets of 4 objects.

PRE-REQUISITE OBJECTIVES 62-J

ACTIVITIES

A. RECORDING THE MODELS WITH ADDITION EQUATIONS

MATERIALS: Same as Activity A objective 62-J.

DIRECTIONS: Have the pupils build a model of a multiplication equation.
Example:
6 cans of 2 caps in each.
Ask: How many? 12
How did you get 12? Counted
How did you count? 1, 2, 3, or 2, 4, 6
How many counted by 2's?
When we count by 2s can we write an addition equation?
Record: 2 + 2 + 2 + 2 + 2 + 2 = 12

Do more examples.
Variation: Divide the class into groups. Give the groups the activity cards made for Activity A objective 62-J. Ask them to build the model and write the addition equation for each card on tablet paper.

B. INTRODUCE THE TIMES SIGN (x)

MATERIALS: Same as activity above.

DIRECTIONS: Have the pupil build the model for 6 cans of 2 caps again. Write: 6 cans of 2 caps is 12 caps and 2 + 2 + 2 + 2 + 2 + 2 = 12.

Explain to the pupils there is a shorter way to write this problem. Write: 6 cans x 2 caps = 12. "We read this equation as 6 cans times 2 caps equals 12 caps. It still means 6 cans of 2 caps.

Continue: Write the equation.

Page 148 blank.

B. Cont'd.

Model the equation.
Write the addition equation.
Write the multiplication equation.

Variation: Divide the class into the same groups which did the variation in Activity A above and return their papers and activity cards.

Have them write the multiplication equation that goes with each addition equation.

C. MULTIPLICATION

MATERIALS: Dittos.

DIRECTIONS: Do each problem first by adding, then by multiplying.

1. $\begin{array}{r} 4 \\ 4 \\ +\ 4 \\ \hline \end{array}$ $\begin{array}{r} 4 \\ \times 3 \\ \hline \end{array}$

2. 5 fives

3. 6 twos.

4. 4 nines

5. 2 eights

6. 6 threes

7. 6 zeros

D. PRACTICE

MATERIALS: Form for ditto master on next page, earliest pages on multiplication from workbooks when they apply.

DIRECTIONS: On ditto master.

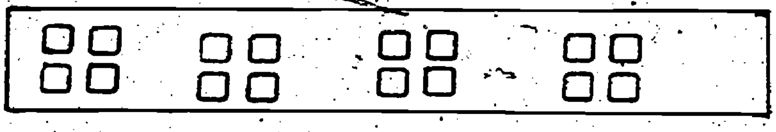
NAME ______________________ DATE ______________________

MULTIPLYING SETS

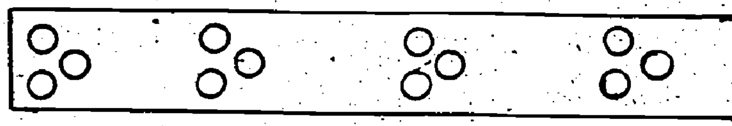
1. Fill in the blanks.

3 sets of 2 are ________

2 sets of 3 are ________



4 sets of 4 are ________



4 sets of 3 are ________

2. Fill in the blanks.

2 + 2 = ________	Two 2's are ________	Two 3's are ________
2 + 2 + 2 = ________	Three 2's are ________	Three 3's are ________
2 + 2 + 2 + 2 = ________	Four 2's are ________	Four 3's are ________
2 + 2 + 2 + 2 + 2 = ________	Five 2's are ________	Five 3's are ________

3. Find the product. (Make sets if you need help.)

2 x 4 = ________	3 x 4 = ________	3 x 2 = ________	5 x 3 = ________
2 x 2 = ________	4 x 4 = ________	2 x 5 = ________	1 x 4 = ________
4 x 3 = ________	5 x 5 = ________	2 x 6 = ________	7 x 2 = ________

4. Fill in the answers. Then match the equations.

2 x 4 = ☐	2 + 2 + 2 = ☐
3 x 2 = ☐	4 + 4 = ☐
5 x 3 = ☐	5 + 5 + 5 = ☐
3 x 5 = ☐	1 + 1 = ☐
4 x 4 = ☐	4 + 4 + 4 + 4 = ☐
2 x 1 = ☐	3 + 3 + 3 + 3 + 3 = ☐

OBJECTIVE Internalize the multiplication facts.

COMMENTS See glossary for definition of basic facts.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. PRESENT THE TABLES

MATERIALS: Models for multiplication, each pupil needs set indicators and counters.

DIRECTIONS:

(1) Have each pupil take 2 set indicators. Put 1 counter in each set so you have 2 sets of 1. Record on the chalkboard. $2 \times 1 = \underline{2}$
Make 2 sets of 2. $2 \times 2 = \underline{4}$
2 sets of 3. $2 \times 3 = \underline{6}$
etc. to
2 sets of 9. $2 \times 9 = \_$

(2) Drill - practice dittos, workbook pages, games, flash cards, tape recorded drills, if you have them or make your own.

(3) Repeat (1) and (2) with a different number of groups.

The rest of these activities are designed to provide drill once the facts are presented.

B. STORY PROBLEM POSTERS

MATERIALS: Story Problem Posters from Objective 62-J, Activity D.

DIRECTIONS: Have pupils write the equations and solve the problems on each poster.

C. "TROUBLED TRIANGLES"

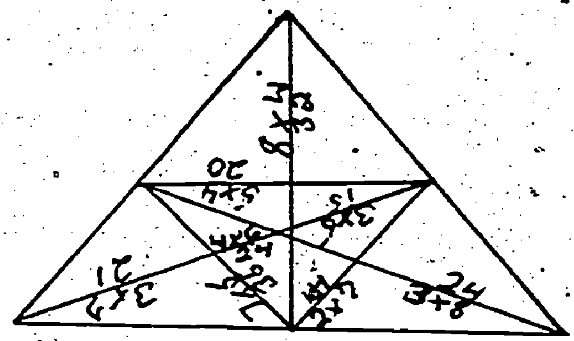
MATERIALS: Poster board, scissors

141

Page 152 blank.

DIRECTIONS:

(1) Work out equations and answers before putting them on the puzzle. One side of each line will have an equation and the opposite side will have the answer to the equation. Pupil matches answers to equations to complete the puzzle.



(2) Copy puzzle on poster board and cut on dark lines.
(3) Mix pieces up.
(4) Fit the triangles together so that all touching edges name the same number.

D. SECRET MESSAGES

MATERIALS: Prepared ditto, see example below.

DIRECTIONS: Have pupils -
(1) Do all the problems given below.
(2) Match their answers to the problems given below with the letters and solve secret messages.

EXAMPLE: The product for number 1 is 16. Looking below the letter M has the value of 16, so put M in the blank above the number 1.

1.	4X4	11.	3X5	A.	14	M.	16
2.	7X2	12.	4X7	B.	9	N.	12
3.	5X7	13.	5X6	C.	28	O.	15
4.	4X5	14.	6X7	D.	8	P.	42
5.	3X8	15.	3X3	E.	0	R.	45
6.	7X3	16.	5X8	F.	18	S.	21
7.	2X9	17.	3X3X3	G.	27	T.	35
8.	8X4	18.	2X2X2	H.	20	U.	32
9.	3X4	19.	3X3X5	I.	24	V.	10
10.	0X8	20.	2X5	L.	30	W.	40

___ ___ ___ ___   ___ ___ ___   ___ ___   ___ ___ ___
1 2 3 4   12 2 9   15 10   7 8 9

___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___ ___   ___ ___ ___
1 2 3 4 10   1 2 3 5 12 6 5 6   3 4 10

(NOTE: Teacher can complete this sentence.)

MATERIALS: Ditto shown below.

DIRECTIONS: Use the chart below to solve the number sentences to see how well pupils know the multiplication facts. Fill the blanks with the letters instead of the products or quotients. Have pupils read completed sentence vertically (top to bottom). Have them try making a similar coded message for a friend to solve.

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t
10	12	14	15	16	18	20	21	24	25	27	28	30	32	35	36	40	42	45	48

u	v	w	x	y	z
49	54	56	64	72	81

6X5 = ______
8X3 = ______
9X5 = ______
4X4 = ______
7X6 = ______
9X8 = ______

6X4 = ______
5X9 = ______

7X2 = ______
5X7 = ______
7X7 = ______
8X4 = ______
6X8 = ______
8X3 = ______
4X8 = ______
5X4 = ______

7X5 = ______
8X4 = ______

8X9 = ______
7X5 = ______
7X7 = ______
6X7 = ______

3X6 = ______
4X6 = ______
8X4 = ______
4X5 = ______
8X2 = ______
7X6 = ______
5X9 = ______

143

OBJECTIVE Model and record multiplication of 1 place numbers times 2 place numbers and 2 place numbers times 1 place numbers.

COMMENTS This is second cycle multiplication. Include zero in this cycle. The pupil will be using the distributive property and the commutative property without naming them.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MODEL UNITS X TENS

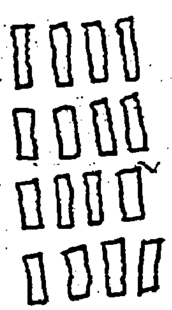
MATERIALS: Models for hundreds, tens, and units used in your class.

DIRECTIONS: Each pupil needs a group of models (lots of tens). Have pupils build models of problems like 2 x 30, 4 x 40, 4 x 50, and then record their answers.

2 x 30 →

→ 60

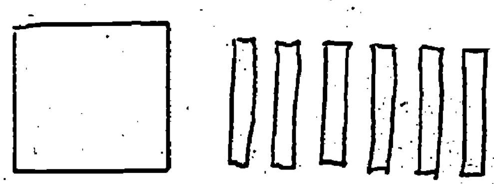
4 x 40 →



Count the tens.

Remind pupils to regroups ten tens as a hundred.

→ 160



B. MODELS

MATERIALS: Counters to represent tens and units (rods and units, poker chips of two different colors, bottle caps, popsicle sticks, etc.)

DIRECTIONS: Give the pupil a problem, such as 4 x 36, and ask him to model it. Example: (0 represents units, X represents ten units.)

page 156 blank

B. Example Cont'd.

Tens	Units
X X X	0 0 0 0 0 0
X X X	0 0 0 0 0 0
X X X	0 0 0 0 0 0
X X X	0 0 0 0 0 0

Student then records:

$$\begin{array}{r} 6 \\ \underline{\times\ 4} \\ 24 \end{array} \qquad \begin{array}{r} 30 \\ \underline{\times\ 4} \\ 120 \end{array} \qquad \begin{array}{r} 36 \\ \underline{\times\ 4} \\ 24 \\ \underline{120} \\ 144 \end{array}$$

(units) (tens)

Repeat with other problems.

Variation: Give students activity cards with problems, have them model to find the answers, and record.

C. 2 DIGITS TIMES 1 DIGIT

MATERIALS: A supply of oral problems, models such as rods, chips, bottle caps, etc.

DIRECTIONS: Have pupils model a couple of oral problems which require a 2 digit numeral times a 1 digit numeral.

Example: There are 24 boys, each has 3 shirts. How many shirts are there altogether?

The equation is $24 \times 3 = \underline{\qquad}$, or $\begin{array}{r} 3 \\ \underline{\times\ 24} \end{array}$

Model: Use white chips for shirts.

C. MODEL cont'd.

```
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
O O O
```

The pupils will soon discover that this is a time-consuming task. (And that is really the point!) Now have them, column by column, replace ten white chips for a red chip so that the model will look like this:

```
R   R   R
R   R   R
W   W   W
W   W   W
W   W   W
W   W   W
```

Point out that this model now looks very much like a model for 3 x 24 (just interchange the rows and columns in this array) so that they can reverse the order of the numerals and replace.

C. cont'd.

24 x 3 or $\begin{array}{r} 3 \\ \underline{\times\ 24} \end{array}$ by 3 x 24 or $\begin{array}{r} 24 \\ \underline{\times\ 3} \end{array}$. The pupils can then record the answer the way they did in B.

NOTE: This will lead to the general rule of always placing the larger numeral on top.

D. MULTIPLICATION

How much money would you have if you had three times each of the amounts below?

DIRECTIONS:
1. Multiply the pennies first.
2. Then multiply the dimes.
3. How many cents would you have?

Example:

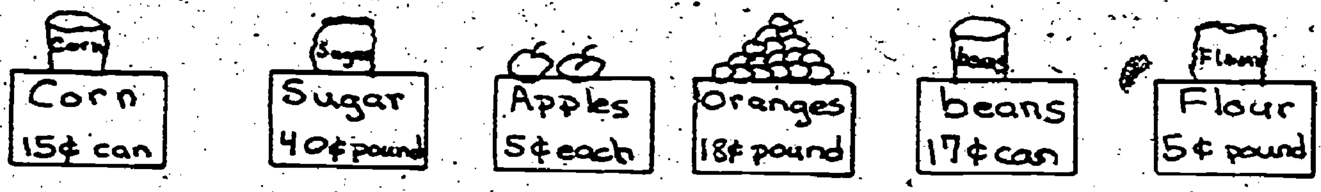
1 dime and 3 pennies
x 3
3 dimes and 9 pennies = 39 pennies

1. 2 dimes and 3 pennies
2. 2 dimes and 2 pennies
3. 3 dimes and 1 penny
4. 3 dimes and 3 pennies
5. 3 dimes and 2 pennies

E. MULTIPLICATION

MATERIALS: See next page.

DIRECTIONS: Fill out the bill for Mrs. Green's groceries:



BROWN'S
GROCERY

Phone 325-4005 23 Baker Street

Mrs. Green

4 pounds of oranges @ 18¢		
10 apples @ 5¢		
6 cans of corn @ 15¢		
8 cans of beans @ 17¢		
7 pounds of sugar @ 40¢		
15 pounds of flour @ 5¢		

OBJECTIVE Identify weight differences by comparison.

COMMENTS This would be a good place to integrate language arts using terms: heavy, heavier, heaviest.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. WEIGHING

MATERIALS: Balance, weighing objects--any two objects which are not close in weight. Example: Pencil and a large lead sinker

DIRECTIONS: Have children weigh and compare any two objects which are not close in weight.

B. WEIGHING

MATERIALS: Same as Activity A, ditto.

PENCIL	
Heavier	Lighter
lead weight	Piece of Paper
Teacher	Staple

DIRECTIONS: Have pupils compare an object with several other objects by weighing. Record results on ditto answer sheet.

C. WEIGHING

MATERIALS: Feel box, several objects with different weights.

DIRECTIONS: Place one object in pupil's hand. Tell the pupil to reach in the feel box and pick an object which is either lighter or heavier than the object in his hand.

D. WEIGHING

MATERIALS: Blindfolds, several objects which are not close in weight.

DIRECTIONS: One child is blindfolded. The teacher place two objects in his hands-one object per hand. The pupil then tells which is heavier.

p. 162 blank

OBJECTIVE Approximate the area of a given surface with non-standard units of area.

COMMENTS Use non-standard measures of area to discover a need for a standard unit of measurement.

PRE-REQUISITE OBJECTIVES 55-G

ACTIVITIES

A. HAND MEASUREMENT

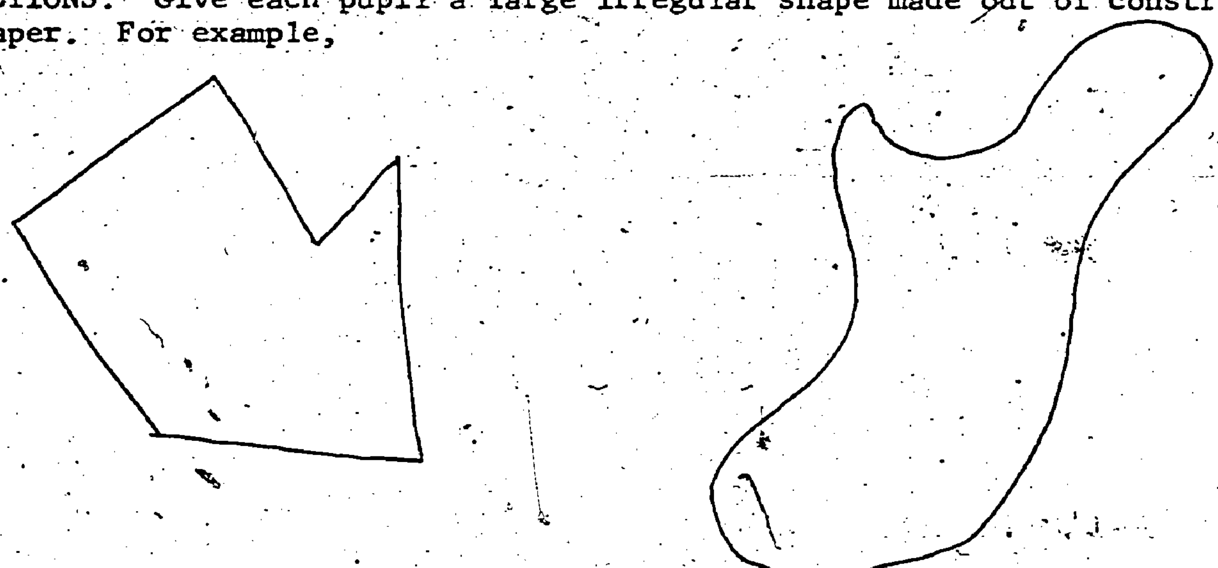
MATERIALS: Table top, two hands for each pupil.

DIRECTIONS: Have the pupils measure a small table or desk top using their flat hand as the unit of measurement. Again, as in Objective 55-G, have them compare their answers and discuss why their answers vary. They should again see that a common unit of measurement is desirable.

B. PAVING AND TILING

MATERIALS: Construction paper; a supply of small geometric shapes (for example,  ,etc.) Some of these could be student made; glue.

DIRECTIONS: Give each pupil a large irregular shape made out of construction paper. For example,



(Continued on next page)

Have the children place the small geometric shapes inside the large shapes you have given them, so that it is entirely covered. You may get overlapping, but at this point that is all right as we are leading to the need for some kind of uniform shape to measure the region with. Have the children glue the small shapes in place. Now have them count the number of shapes they used to cover the large shapes. You should get a number of different answers.

OBJECTIVE Use square units to determine the areas of various shapes.

COMMENTS Grids can be used to determine areas of various shapes.

PRE-REQUISITE OBJECTIVES 67-G

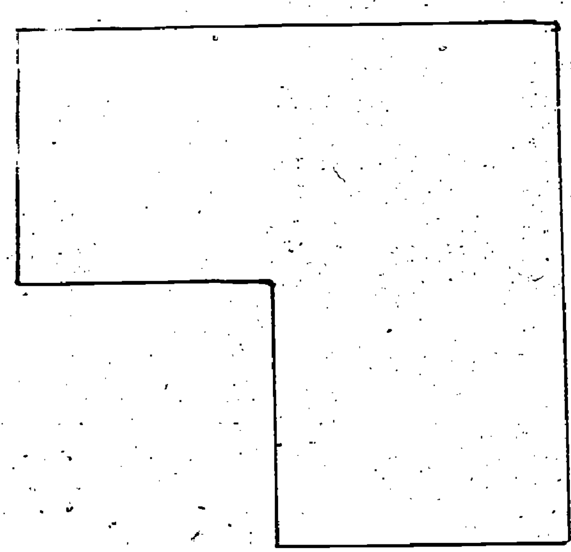
ACTIVITIES

A. AREA

MATERIALS: Ditto shapes such as the one below, construction paper, squares or multibase flats.

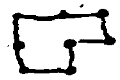
DIRECTIONS: Pupil fits construction paper squares in geometric shape on the on the ditto and records the number of squares used as a mesurement of its area.

Substitute other geometric shapes in the classroom which the student can place on his desk, floor or table and measure with flats to find the area.



B. GEOBOARD ACTIVITIES

MATERIALS: Geoboards or dot paper.

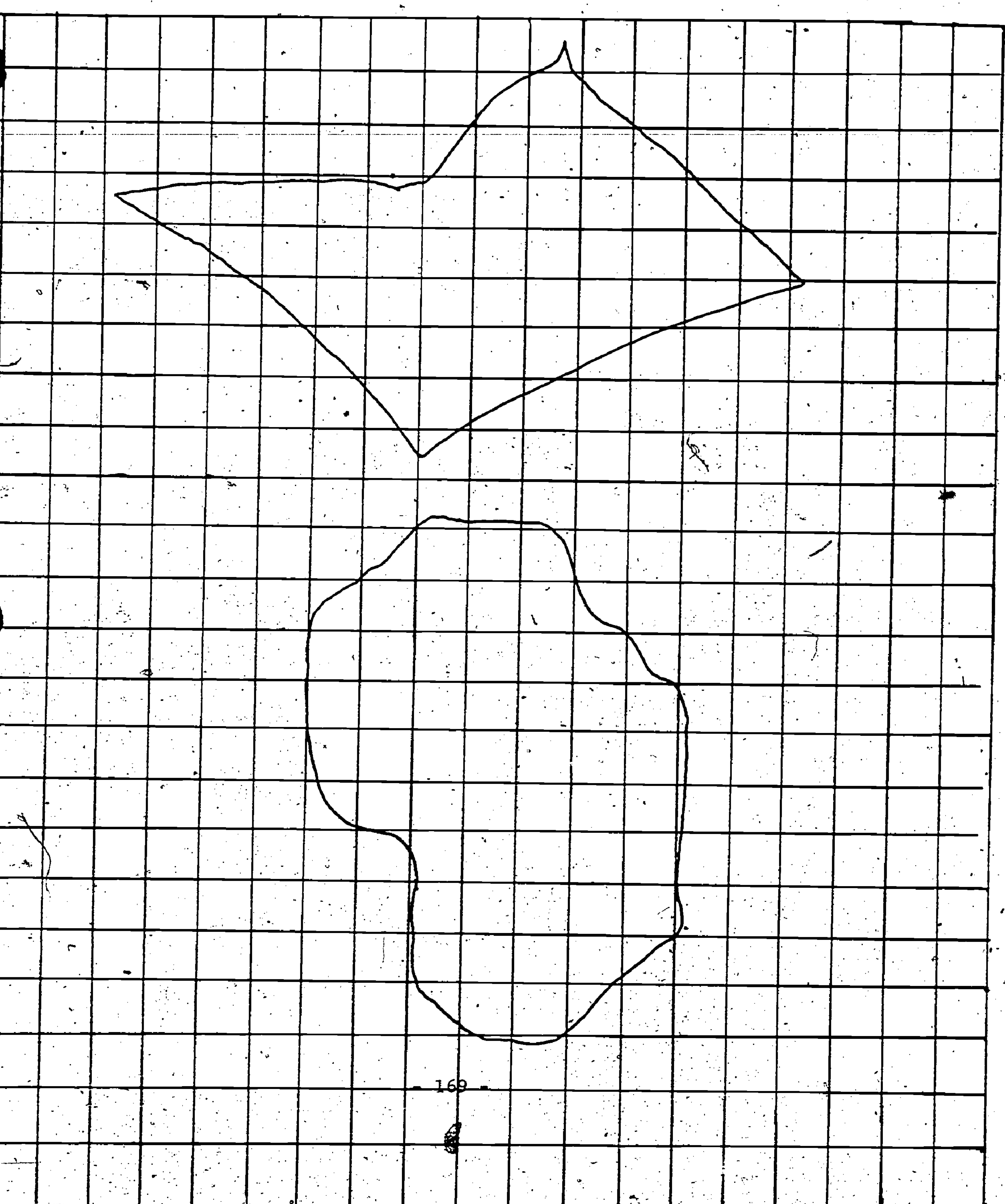
DIRECTIONS: Let the smallest square () be the unit of measurement. Have the pupils construct various figures (ex. ) with horizontal and vertical sides and compute their areas. They may also construct figures which do not have all horizontal and vertical sides and try to guess what their areas
a are. (ex. )

Note: Pegboards can be adapted to this activity using yarn or rubber bands.

C. AREA OF IRREGULAR SHAPES (ENRICHMENT)

MATERIALS: Grid paper, construction paper, squares.

DIRECTIONS: Using grids or squares approximate the area of irregular figures. See example, following page.



GENERAL REMARKS In the next three objectives, pupils move from finding areas of general shapes to finding areas of rectangles, squares, and triangles and developing formulas for these areas.

69-G — Reference Geometry

OBJECTIVE Learn the formula for the area of a square.

COMMENTS Emphasize the fact that length is measured in units and area is measured in square units.

PRE-REQUISITE OBJECTIVES 68-G

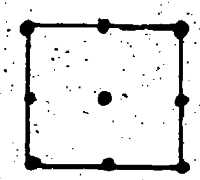
ACTIVITIES

A. GEOBOARD

MATERIALS: Geoboard, rubber bands.

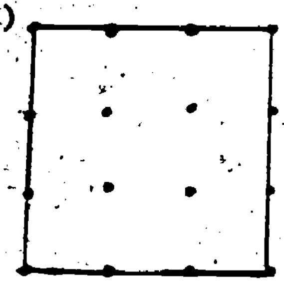
DIRECTIONS: Have the pupils construct squares of various size and using the smallest square on the board, find the number of these squares in the constructed square. Make a chart listing the length of the side and the area. For example:

(1)



	side (units)	area (square units)
(1)	2	4
(2)	3	9
	.	.
	.	.
	.	.

(2)



155

page 170 blank.

B. TAGBOARD SQUARES

MATERIALS: A supply of small tagboard squares.

DIRECTIONS: Have the pupils construct large squares of various size using the small tagboard squares. Find the number of small squares in the large square. Make a chart listing the length of the side and the area.

OBJECTIVE Learn the formula for the area of a rectangle.

COMMENTS

PRE-REQUISITE OBJECTIVES 69-G

ACTIVITIES

A. GEOBOARD

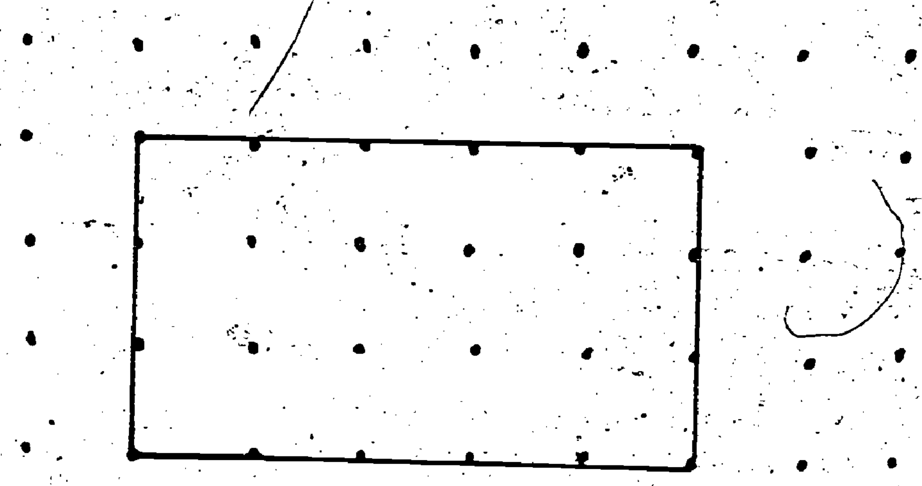
MATERIALS: See Previous Objective - activities A and B; tagboard.

DIRECTIONS: Replace square with rectangle. Using chart, help the student discover that <u>length</u> times <u>width</u> is area.

B. AREA USING GRID

MATERIALS: Square centimeter grid (copy attached), dittos of various rectangular shapes made to be compatible with grid.

DIRECTIONS: You will need a grid for each student and several dittos (2 or3) of various size rectangles. Have the child place the ditto over the grid (the grid lines will show through ditto paper) and indicate the length, width, and area of each rectangle. Have him make a chart of his findings and verify that length x width = area.



length	width	area
5	4	20
.	.	.
.	.	.
.	.	.

Be sure your figures are of even centimeters in length and width.

OBJECTIVE Model and record multiplication of two place numbers times two place numbers.

COMMENTS Continue to use the distributive property without naming it. Do all of the activities for this objective! The distributive property is used especially in activities B, C, and E.

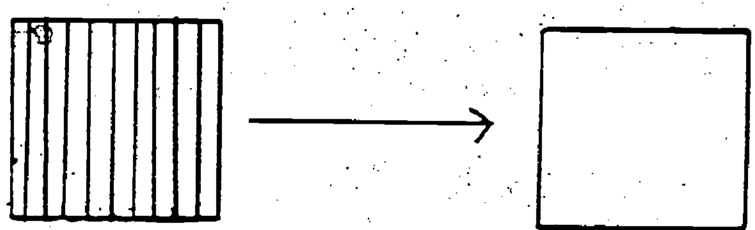
PRE-REQUISITE OBJECTIVE 65-G

ACTIVITIES

A. MODEL TENS X TENS

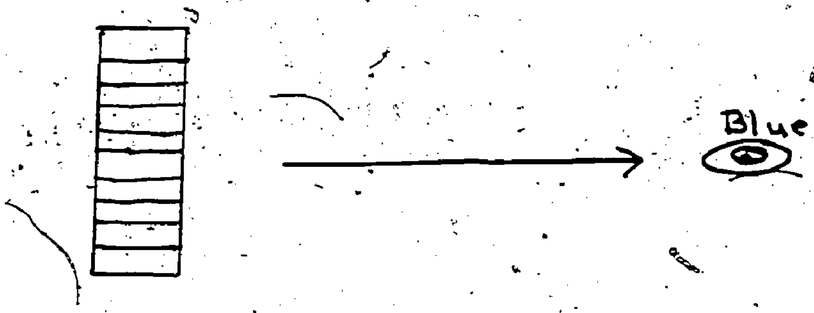
MATERIALS: Models for groups of ten: cuisenaire rods and squares, multibase blocks, poker chips; dittos.

DIRECTIONS: First review the modeling of 10 x 10 as in objective 44-C



Ten rods can be replaced by a square.

or



Ten red chips can be replaced by a blue chip.

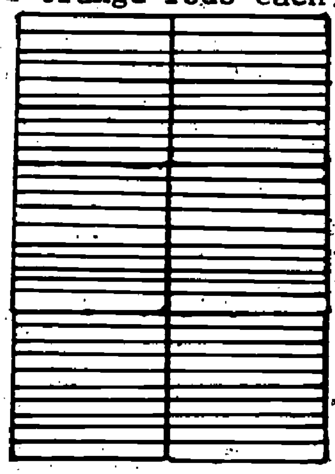
Then build models of 20 x 20, 30 x 10, 30 x 20, 40 x 20, etc. Substitute a hundred model for ten tens.

page 176 blank

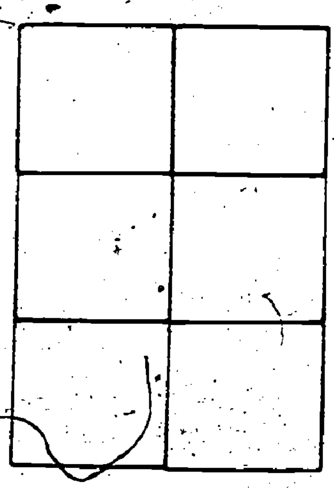
EXAMPLE: 30 X 20

Multibase Model

Model with 30 rows of 2 orange rods each.

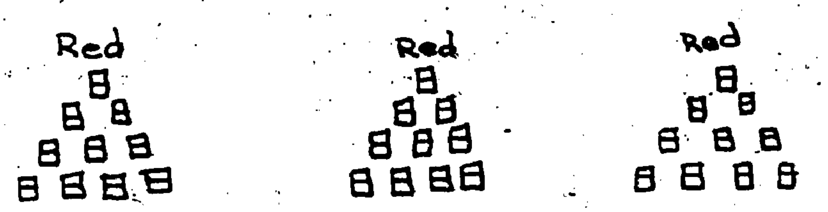


Replace each set of 10 rods with a square and get

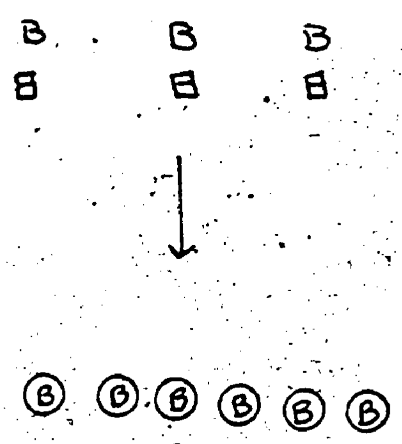


Chip Model

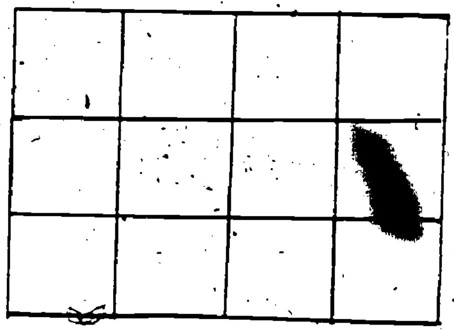
Model 30 stacks of red chips with 2 in each stack.



Replace each group of ten stacks of 2 reds with one stack of two blues and get



Do models with larger numerals, 30 x 40 = 12 hundreds or 1 thousand 2 hundreds.



NOTE: Ten hundreds can be regrouped to make a thousand.

Provide a practice ditto with problems like:

6 x 10 =	40 x 3 =
6 x 1 =	3 x 4 =
60 x 10 =	40 x 30 =

You can mix the order in which these occur.

B. MODEL 2 DIGITS X 2 DIGITS WITH CUISENAIRE RODS OR MULTIBASE BLOCKS

MATERIALS: Orange rods and squares and the units (or multibase blocks) example diagram attached for use.

DIRECTIONS: Teacher: It is important to understand this model before presenting it. Study the diagram carefully.

EXAMPLE 1) 17 x 14 = 17 sets of 14. Build the model with 17 rows of one rod and four cubes each. (See figure 1)
a) Use a square to replace ten of the rods.
b) Use 4 rods to replace the 10 rows of 4 units.
c) Rearrange the 28 cubes so they make 2 rows of ten and 8 units and replace the rows of ten with rods.
d) Rearrange the rods so they are all together (there will be 13). Replace ten rods with a square.

You will then have 2 squares, 3 rods, and 8 units.
The answer 238 is read like place value models.

Figure 1:

17 x 14 (17 Rows of 14 each)

	Cubes
10 x 10 — ten rods replace with a square	10 x 4 — 40 cubes — replace with 4 rods.
7 x 10 — Seven rods	7 x 4 — 28 cubes — replace with 2 rods and 8 cubes.

After replacements indicated above you now have: One square, 13 rods, and 8 cubes. Replace the 13 rods by one square and 3 rods and wind up with 2 squares, 3 rods, and 8 cubes, i.e.

Squares (hundreds)	Rods (tens)	Cubes (units)
2	3	8

or 238.

EXAMPLE 2) 22 x 16. Build model with 22 rows of one rod and six cubes each. (See figure 2)

a) Use two squares to replace the two sets of ten rods.
b) Use 12 rods to replace the 20 rows of 6 units.
c) Rearrange the twelve cubes so they make 1 row of ten and 2 units and replace the row of ten with a rod.
d) Rearrange the rods so they are all together (there will be 15). Replace ten rods with a square.

You will then have 3 squares, 5 rods, and 2 units. The answer 352 is read like place value models.

NOTE: Also model some products where no units are involved, e.g. 40 x 26 or 37 x 20.

Note (Fig. 2) After the replacement indicated above you now have: 2 squares, 15 rods, and 2 cubes. Replace 10 of the fifteen rods with a square and wind up with 3 squares, 5 rods, and 2 cubes...

Squares (Hundreds)	Rods (Tens)	Cubes (Units)	
3	5	2	= 352.

C. MODELING WITH POKER CHIPS

MATERIALS: Colored poker chips.

DIRECTIONS: Model with chips

blue = hundreds = B
red = tens = R
white = units = W

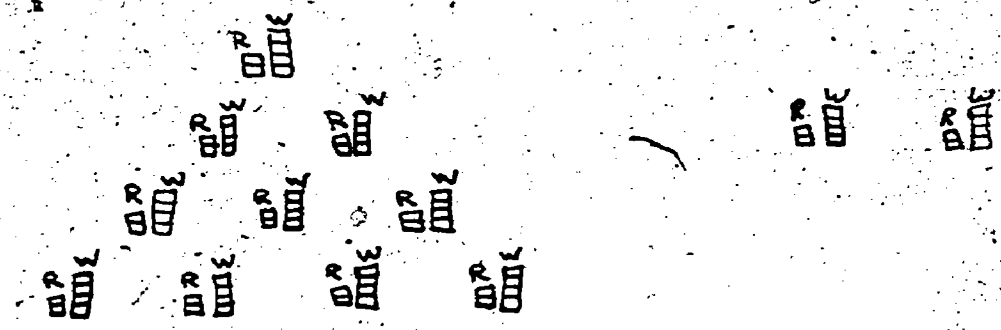
12 x 24 = 12 sets of 24

2 R stacked → 20

4 W stacked → 4

See illustration on next page..

12 Sets of 24

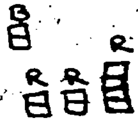


Replace ten of the red stacks  by a stack of two blues .

Replace ten of the white stacks  by a stack of four reds .

You will then have:

two blues: 

eight reds: 

eight whites: 

(unstack them if necessary to see this)

So the product is

Blue (Hundreds)	Red (Tens)	White (Units)	
2	8	8	or 288.

D. FINDING PRODUCTS FROM MODELS

MATERIALS: Models and activity cards.

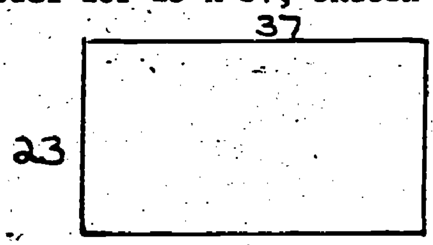
DIRECTIONS: Make activity cards of 2 digit multiplication problems. Use fairly small numbers (eg. 26 x 32, 20 x 13, 45 x 16, 33 x 20). Put the answers on the back of the cards. Have pupils build models to solve the multiplication problems and check their own answers.

E. RECORDING A MODEL

(This activity prepares the pupil for long multiplication without models)

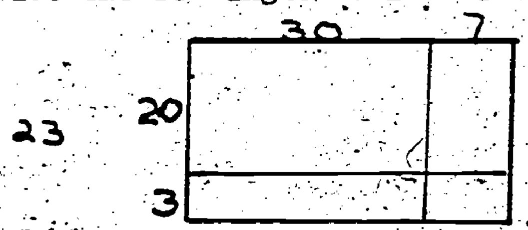
MATERIALS: Unlined paper.

DIRECTIONS: (The pupils will use a drawn, not a concrete, model in this activity). To draw a model for 23 x 37, sketch a rectangle to represent it



(The product will be interpreted as the area of a rectangle--- see Objective 70-G.)

Subdivide the rectangle to indicate units and tens.



Record the areas of the four smaller rectangles.

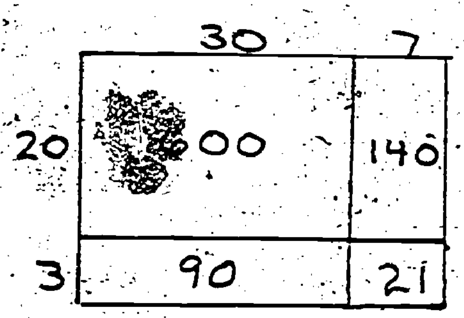
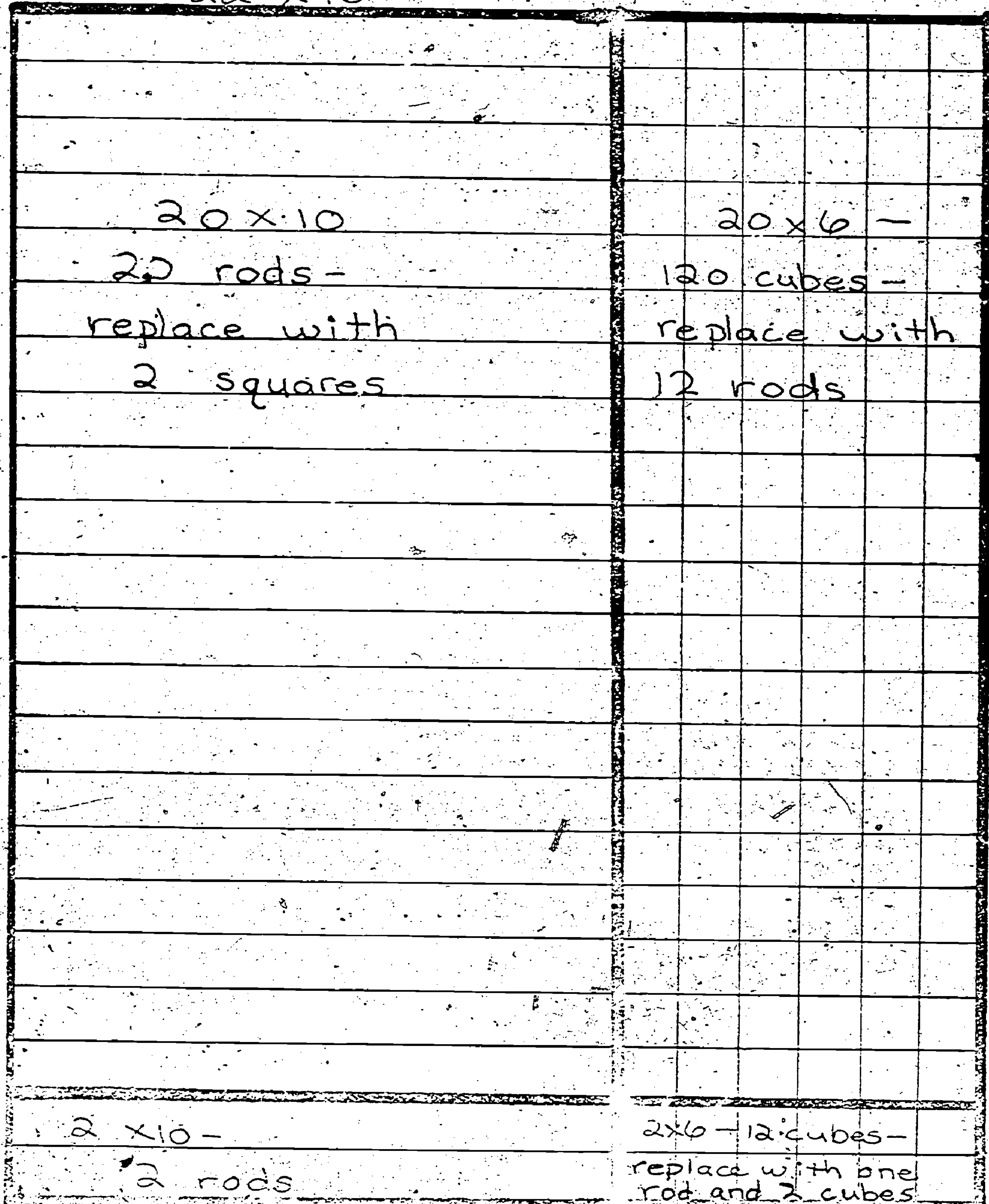


Figure 2
22 x 16



See Note (fig. 2)

Cont'd.

The product 23 x 37 is then the sum of these four numbers.

$$23 \times 37 = 21 + 90 + 140 + 600 = 851.$$

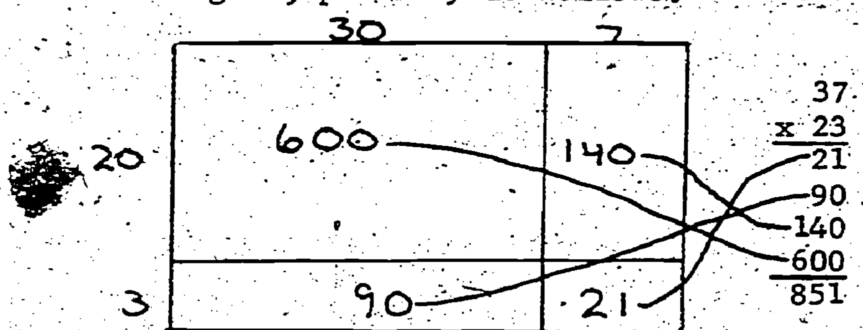
Now set up the standard long multiplication form

$$\begin{array}{r} 37 \\ \underline{\times\ 23} \end{array}$$

and record the partial products:

$$\begin{array}{r} 37 \\ \underline{\times\ 23} \\ 21 \\ 90 \\ 140 \\ \underline{600} \end{array}$$

Indicate the connection between the partial products and the areas of the small rectangles, possibly as follows:

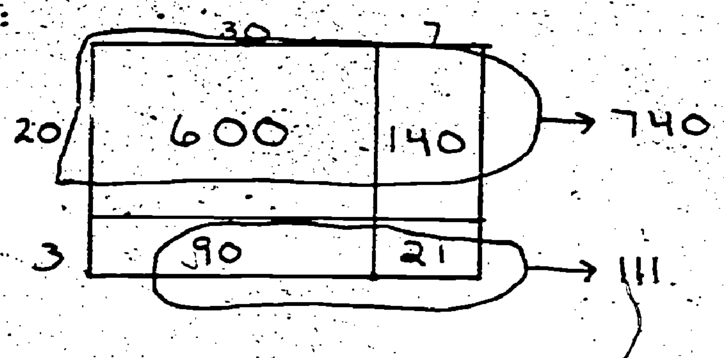


Have the pupils do several examples like this.

NOTE: Later on when you're doing the shortened algorithm:

$$\begin{array}{r} 37 \\ \underline{\times\ 23} \\ 111 \\ \underline{740} \\ 851 \end{array}$$

you can indicate the 111 and the 740 from the drawing as follows:



OBJECTIVE Multiply 2 place numbers x 2 place numbers without models.

COMMENTS

PRE-REQUISITE OBJECTIVES 71-J

ACTIVITIES

A. PRACTICE

MATERIALS: Dittos, workbook pages, chalkboard drill.

DIRECTIONS: Move the students at their own speed from modeling to working a problem with a long algorithm.

```
  24
x 16
  24   (6 x 4)
 120   (6 x 20)
  40   (10 x 4)
 200   (10 x 20)
 384
```

Encourage working from right to left as a convenient method of knowing which partial products have been recorded.

Shorten the algorithm leaving off the parentheses.

```
  24
x 16
  24
 120
  40
 200
 384
```

Some students may further shorten the algorithm to

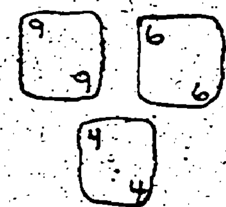
```
  24                    24
x 16                  x 16
 144   (6 x 24)        144
 240   (10 x 24)       240
 384                   384
```


page 186 blank

B. MULTIPLY -- ADD

MATERIALS: Regular playing cards with face cards removed.

DIRECTIONS: Lay three cards face up on the table -- multiply the first two and add the third to the product.



$$\begin{array}{r} 54 \\ +\ 4 \\ \hline 58 \end{array}$$

Do the problems orally.

VARIATION: Race -- 1 person from each team watches as the teacher displays the 3 cards. First person with the correct answer wins a point for his team.

C.

This is a good place for word problems.
Select those suitable at this point. Work them to be sure they can be done with what the pupil knows.

OBJECTIVE Multiply 2 and 3 place numbers times 3 place numbers.

COMMENTS Model the tens times the hundreds since this is the only new process in this problem.

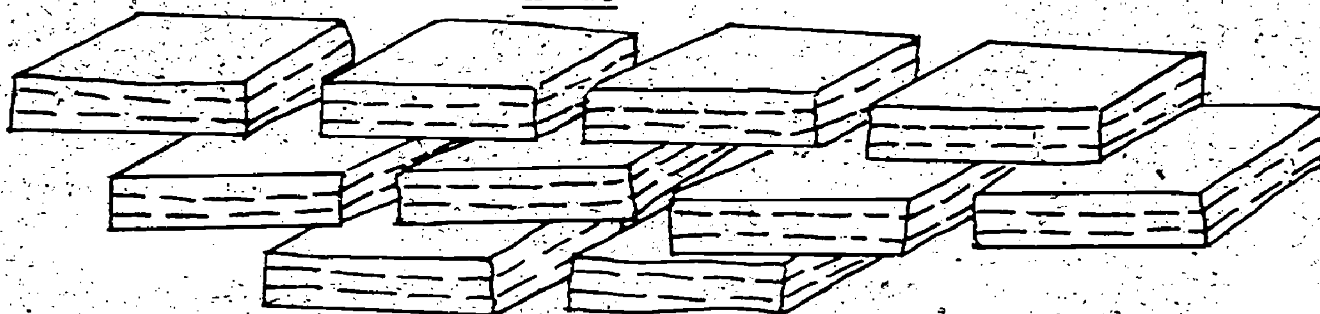
PRE-REQUISITE OBJECTIVES 72-J

ACTIVITIES

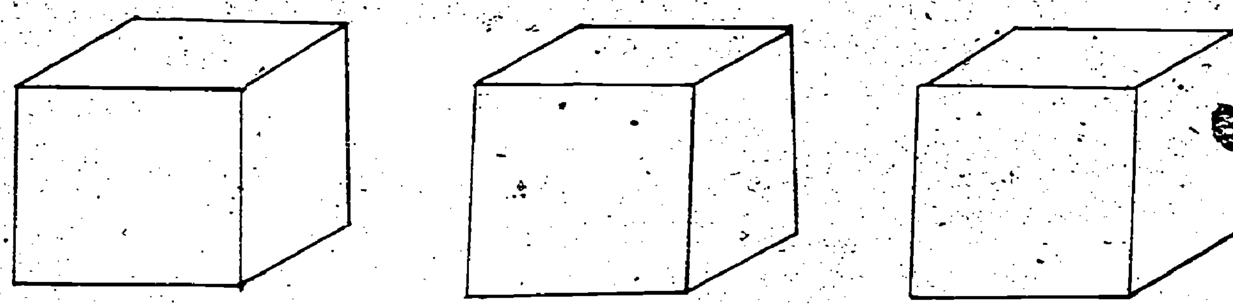
A. TENS X HUNDREDS

MATERIALS: Models for tens, hundreds, and thousands.

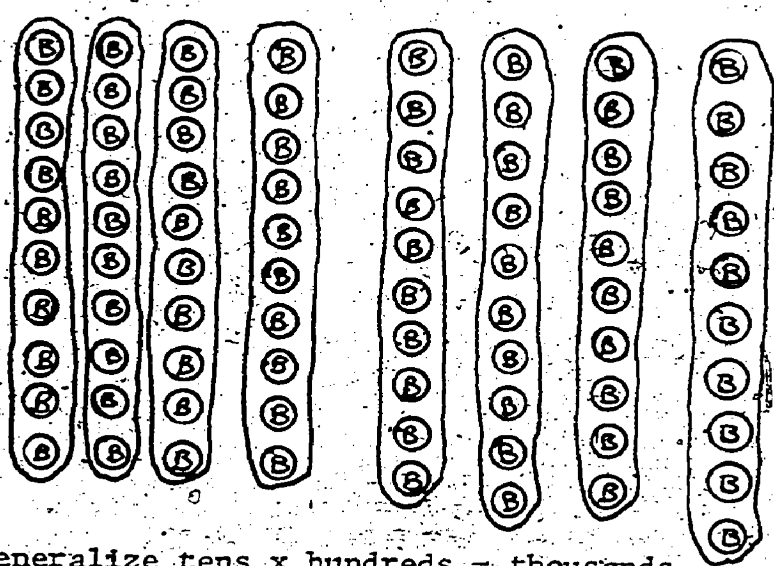
DIRECTIONS: Model problem like $\begin{array}{r} 300 \\ \times\ 10 \\ \hline \end{array}$ (cuisenaire materials or multibase blocks).



Regroup, putting 10 flats on top of each other.



Model 400 (Poker chips) Blue = hundreds = Ⓑ
x 20 Red = thousands = Ⓓ



Replace each set of 10 blue with a red.

Generalize tens x hundreds = thousands.

Note: Usually the thousands have to be regrouped as ten thousands and thousands.

Example:
```
   400
 x  60
24,000
```

B. EXTEND THE ALGORITHM

MATERIALS: Models when needed, practice dittos.

DIRECTIONS:
```
    426
 x   34
     24   (4 x 6)
     80   (4 x 20)
  1,600   (4 x 400)
    180   (30 x 6)
    600   (30 x 20)
 12,000   (30 x 400)
 14,484
```

Some pupils will want to learn the short cut and should be encouraged.

```
    426
 x   34
  1,704
 12,780
 14,484
```

C. ZEROES

MATERIALS: Dittos.

DIRECTIONS: Provide practice and help with zero problems like:

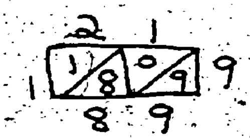
$$\begin{array}{r} 420 \\ \times\ 34 \\ \hline \end{array} \qquad \begin{array}{r} 406 \\ \times\ 34 \\ \hline \end{array} \qquad \begin{array}{r} 420 \\ \times\ 30 \\ \hline \end{array} \qquad \begin{array}{r} 406 \\ \times\ 34 \\ \hline \end{array}$$

D. GALLEY METHOD OF MULTIPLICATION

This is an enrichment activity.

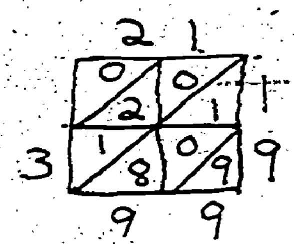
DIRECTIONS: Place the multiplicand along the top of the galley and the multiplier down the right side. Enter the products in the boxes. If there is only one digit, record 0 in the top galley. Add along the diagonal--read answer counter clockwise.

1 digit by 2 digit



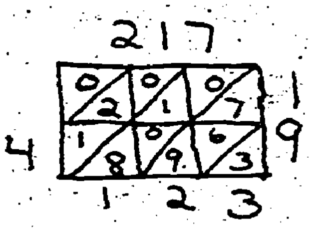
$$\begin{array}{r} 21 \\ \times 9 \\ \hline 189 \end{array}$$

2 digit by 2 digit

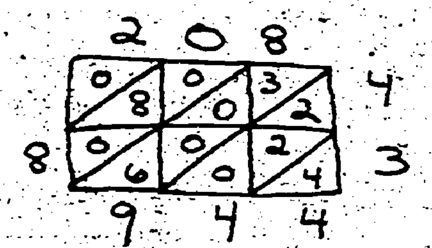


$$\begin{array}{r} 21 \\ \times 19 \\ \hline 189 \\ 21 \\ \hline 399 \end{array}$$

2 digit by 3 digit

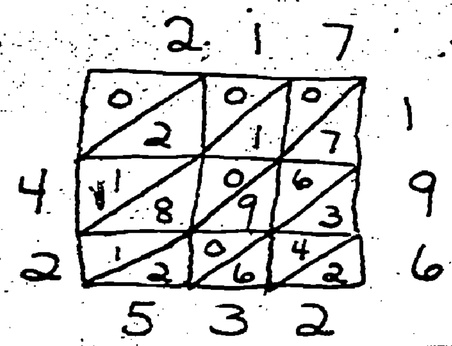


```
  217
 x 19
 1953
 217
4,123
```



```
  208
 x 43
  624
 832
8,994
```

3 digit by 3 digit



```
    217
 x  196
   1302
  1953
 217
 42,532
```

OBJECTIVE Learn to name the numbers and write the numerals from 10,000 to the millions.

COMMENTS Expand place value notation through millions. Use the comma to separate thousands and hundreds.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. NAMING AND WRITING 10,000 to 90,000.

MATERIALS: Models if used, chalkboard.

DIRECTIONS: Count by thousand, write the numerals by thousands. Lead pupils to name 10 thousands and write. Note the new place and add it to the grid.

ten thousands	thousands	hundreds	tens	units

B. NAMING AND WRITING 100,000 to 999,999.

MATERIALS: Same as for previous activity.

DIRECTIONS: Extend previous activity to add hundred thousands to the grid.

C. USE THE COMMA

MATERIALS: Place value grid, chalkboard.

DIRECTIONS: Write the place value grid on the chalkboard.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	units

Ask the pupils what they would add next.

hundred million	ten million

Emphasize the pattern reading right to left. (Ones, tens, hundreds, etc.) Ask pupils where they have learned to use the first comma. Ask where they would put a second comma.

Have pupils read long numbers--stress the ease that commas add to reading.
102 - one hundred two
102,000 - one hundred two thousand
102,000,000 - one hundred two million

Some pupils may know or ask what comes next. It is interesting to talk about billions and trillions.

D. PRACTICE

MATERIALS: Paper and pencil, dittos.

DIRECTIONS: Dictate long numerals to pupils. Ask them to write the numerals with commas.
NOTE: Limit to "millions"

OBJECTIVE Develop a model for repeated multiplication.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. BUILDING CUBES

MATERIALS: Cubes

DIRECTIONS: Have pupils build a large cube -- 2 cubes long, 2 wide, 2 high. Then 3 cubes long, 3 wide, 3 high, and so on with 4, 5, ... Have pupils count the total number of small cubes in each large cube.

B. REPEATED MULTIPLICATION

MATERIALS: Poker chips

DIRECTIONS: Have poker chips (or counting blocks) in a pile (no order). Teacher says, "Make 3". Pupil groups 3 chips -- 000
Teacher says, "Make 3 sets of 3". Children group 3 chips 3 times.

000
000
000

NOTE: The word "of" means multiply (x).

Teacher says, "Make 3 sets of 3 sets of 3." Children group 3 groups with 3 groups of 3.

000 000 000
000 000 000
000 000 000

Teacher asks, "What did you do?" Children respond, "We made 3 sets of 3 sets of 3 and points to the different 3's."

000 000 000
000 000 000
000 000 000

Write on board:

3 of 3 of 3
3 x 3 x 3 = How many (27) count chips.

Repeat with groups of 4 and 5. Compare the answers with those in Activity A.

OBJECTIVE Develop use of exponential notation.

COMMENTS Exponential notation is a convenience, not a necessity. If your pupils have problems in this area, it can be delayed until a later date.

PRE-REQUISITE OBJECTIVES 75-C

ACTIVITIES

A. WRITING EXPONENTS

MATERIALS: See Directions

DIRECTIONS: Repeat Activities from Objective 75-C recording what is done and introducing exponential notation.

EXAMPLE: 3 of 3 of 3 = 27
$3 \times 3 \times 3 = 27$
$3^3 = 27$

The exponent is essentially a counter that tells the number of times a number is used as a factor in a product. It is written to the upper right of the number.

B. TIMES TO EXPONENTS

MATERIALS: See Directions

DIRECTIONS: Teacher-made worksheets drilling on the following:

1. $2 \cdot 2 \cdot 2 \cdot 2 = \underline{2^4}$
2. $4 \cdot 4 \cdot 4 \cdot 4 =$ ____
3. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 =$ ____
4. $6^4 = \underline{6} \cdot \underline{6} \cdot \underline{6} \cdot \underline{6}$
5. $7^8 =$ ___ • ___ • ___ • ___ • ___ • ___ • ___ • ___

178

e 196 blank

OBJECTIVE Relate exponents to place value.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

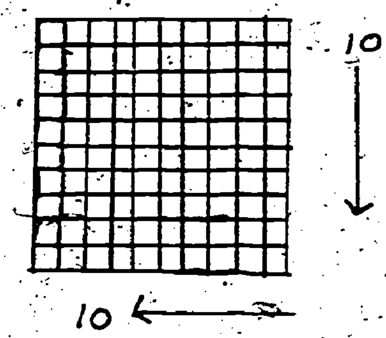
A. MODELING 10, 10^2, 10^3, etc.

MATERIALS: Use place value models.

DIRECTIONS: Model ten. This is noted 10^1 or 10.

EXAMPLE: 

Building on this model, build a model of 10 sets of 10.



This is 10 sets of 10 or 10^2 or 100. Continue making 10 sets of 10 sets of 10. 10 sets of 10 sets of 10 is 10^3 or 1000.

10 sets of 10 sets of 10 sets of 10 is 10^4 or ______________. Ask what 10^4 is. If children are unsure, have them work out problem.

EXAMPLE:
$$\begin{aligned} 10 \times 10 \times 10 \times 10 &= (10 \times 10) \times 10 \times 10 \\ &= (100 \times 10) \times 10 \\ &= 1000 \times 10 \\ &= 10{,}000 \end{aligned}$$

page 198 blank

DISCUSSION: Lead a discussion reviewing what you have done showing (on chalkboard, overhead projector, etc.) that $10^1 = 10$ (tens place); $10^2 = 100$ (hundreds place); $10^3 = 1000$ (thousands place); $10^4 = 10,000$ (ten thousands place); $10^5 = 100,000$ (hundred thousands place). Note: The ex[illegible] determines the number of zeros.

B. EXPONENTS TO PLACE VALUE

MATERIALS: Teacher-made ditto.

Example:
1. $10^1 =$
2. $10^3 =$
3. $10^5 =$
4. $10^2 =$
5. $10^4 =$

DIRECTIONS: Children work out problems.

VARIATION: Children tell place value according to exponent.

Example:
1. 10^1 = tens place.
2. 10^3 = thousands place

C. USE EXPONENTS TO WRITE NUMERALS

MATERIALS: Teacher-made ditto sheets with problems like the following:

$$3\,(10^5) + 2\,(10^4) + 6\,(10^3) + 5\,(10^2) + 4\,(10) + 2$$

DIRECTIONS: Pupils write out numbers using exponents to determine place value.

Example: Problem above is 326,542.

EXTRA WORK FOR FUN: Have child work out the problem (only if children have had background in working equations with parenthesis.)

$3\ (10^5) + 2\ (10^4) + 6\ (10^3) + 5\ (10^2) + 4\ (10) + 2 =$

$3\ (10 \times 10) \times (10 \times 10) \times 10 + 2\ (10 \times 10) \times (10 \times 10) + 6\ (10 \times 10) \times 10$

$+ 5\ (10 \times 10) + 4\ (10) + 2 =$

$3\ (100 \times 100) \times 10 + 2\ (100 \times 100) + 6\ (100 \times 10) + 5\ (100) + 40 + 2 =$

$3\ (10{,}000 \times 10) + 2(10{,}000) + 6(1000) + 500 + 40 + 2 =$

$3(100{,}000) + 20{,}000 + 6000 + 500 + 40 + 2 =$

$(300{,}000 + 20{,}000) + 6000 + 500 + 40 + 2 =$

$(320{,}000 + 6000) + 500 + 40 + 2 =$

$(326{,}000 + 500) + 40 + 2 =$

$(326{,}000 + 40) + 2 =$

$326{,}540 + 2 =$

$326{,}542$

181

OBJECTIVE Partition a set into two parts.

COMMENTS This is a pre-subtraction activity.

PRE-REQUISITE OBJECTIVES 33-J, 32-J, 19-C, 14-C, 13-C

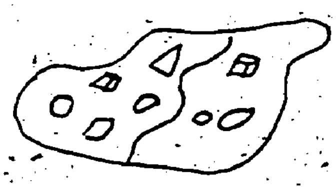
ACTIVITIES

A. REVIEW PRE-REQUISITE ACTIVITIES ON PATTERNING AND REPATTERNING

B. REVIEW PARTITIONING ACTIVITIES ON PATTERNING AND REPATTERNING

MATERIALS: String - one piece per student, tied in a loop, various manipulatives (blocks, bottle caps, buttons, popsicle sticks, etc.).

DIRECTIONS: Students will make a set of any size he wants. The student will then partition the set into 2 parts and tell how many in each part and how many all together.



 Page 202 blank.

OBJECTIVE Partition a set into two groups when one group size is given.

COMMENTS Use term: "What is left" for the unknown set size.

PRE-REQUISITE OBJECTIVES 78-S

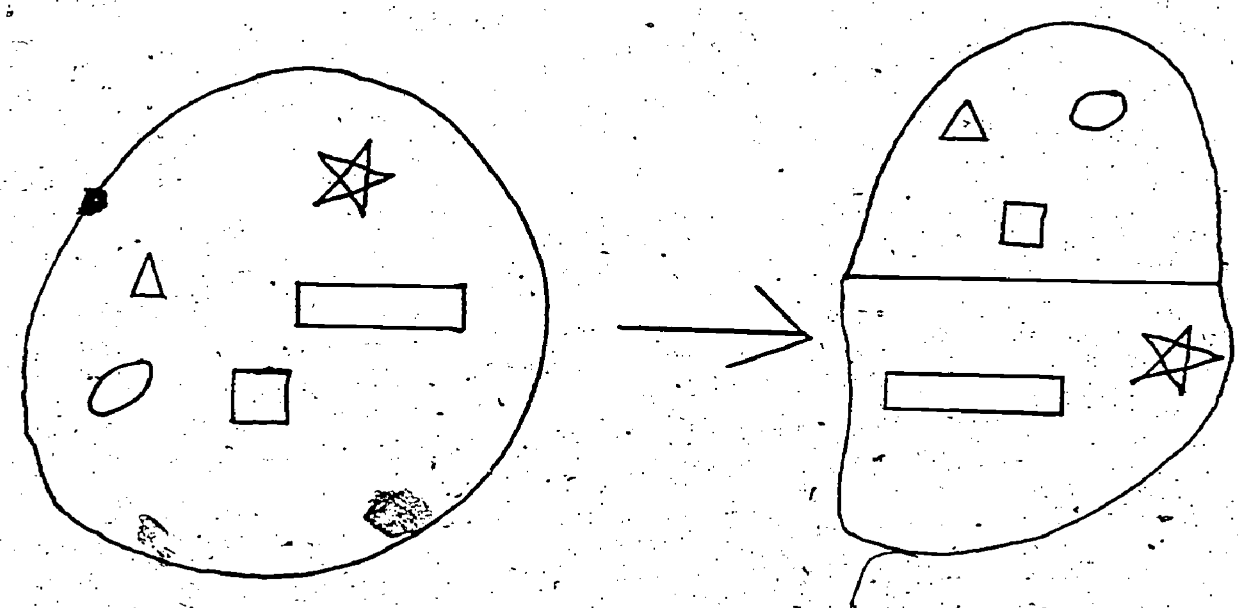
ACTIVITIES

A. PARTITIONING

MATERIALS: Odds and ends, 1 set indicator

DIRECTIONS: Teacher makes a set and asks a student to partition the set into two parts--one part being of size 3, for example. When this is done, teacher asks, "What is left?"

Example:



REPEAT USING DIFFERENT SIZES

page 204 blank

OBJECTIVE Use the separation sign and record partitioning process.

COMMENTS Do not introduce the terms "subtraction" or "minus" at this point. Use partition or separate.

PRE-REQUISITE OBJECTIVES 79-S

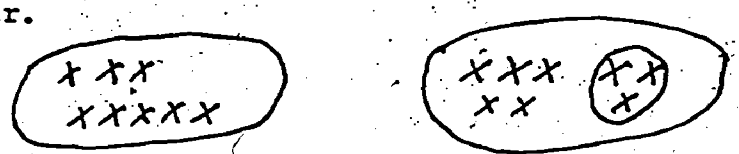
ACTIVITIES

A. PARTITIONING AND RECORDING

MATERIALS: Odds and ends, set indicator, recording sheet.

DIRECTIONS: Use the same procedure as the activities on partitioning in objectives 78-S and 79-S. Have the pupil record what he has done.

EXAMPLE: Make a set. Ask, "How many members are there in this set?" Separate a given set size and circle with yarn. "What do we have left?" Let us write what we did. (A) "How many did we have in the set?" Record the number. "We separated how many?" (B) Record the number. "What's left?" (C) Record the number.



A. Set Size	B. Separate a Given Size	C. What's left
8	3	5

Continue this procedure, modeling many basic facts until the children are competent and comfortable with it. Then introduce the separation sign.

EXAMPLE:

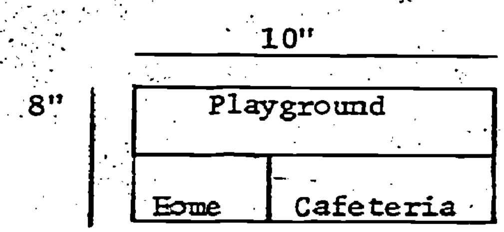
Set Size		Separate a Given Size		What's left
9	-	4	=	5

page 206 blank

B. AN OPERATION OF NUMBERS

MATERIALS: Several 10- by 8-inch playing boards cut from heavy cardboard, several felt cutouts for each pupil.

PREPARATION: Divide the playing board into three sections, the largest measuring 10 by 4 inches and each of the others 5 by 4 inches.



DIRECTIONS: The pupil places his playing board so that the two small sections are nearest him. The teacher or a pupil dictates a story problem. Five boys were playing baseball. (Five cutouts are put in the largest section). Two of the boys went home for lunch. (Two cutouts are moved to the lower-left section). The other boys went to the school cafeteria for lunch. (Three cutouts are moved to the lower right section). How many boys went to the cafeteria?

The pupils can find the number by counting the boys in the lower right section. A discussion of the problem may help the pupils understand that subtraction is an operation on numbers in the problem. Have the students record the equation.

OBJECTIVE To internalize the subtraction facts.

COMMENTS The minuend is the total amount, the subtrahend is the number being separated out. "Separation sign" will now be replaced by "minus sign." The answer is called the difference. Recall that a basic subtraction fact is one where the difference and the subtrahend are single place numbers.

PRE-REQUISITE OBJECTIVES 80-S

ACTIVITIES

A. SUBTRACTION RELAY

MATERIALS: Make 2 sets of cards. Eleven cards in each set. An orange set numbered from 0 to 10 and a yellow set numbered from 0 to 10. Flannel board, flannel numerals and signs to make equations.

DIRECTIONS: Divide class into 2 teams of 10 each. Pass the yellow cards to one team and the orange cards to the other. Teacher makes a partial equation on the flannel board.

$$8 - 5 =$$

The student from each team with the missing numeral "races" to the board. The student reads the equation, and the class can confirm his answer. The student who correctly completes the equation first wins a point for his team. The teacher changes the equation. The team with the most points wins.

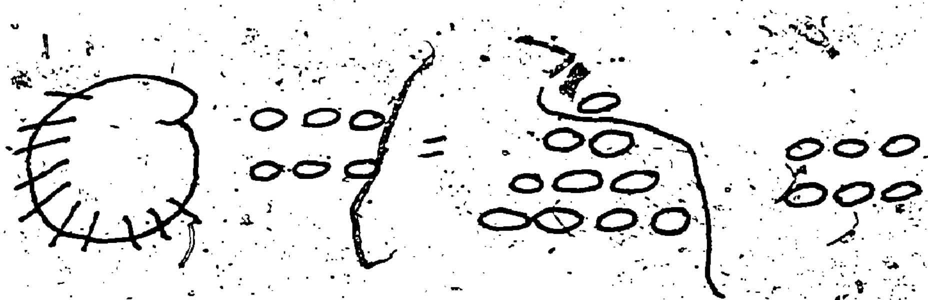
B. SUBTRACTION FACTS 11 - 18 WITH MODELS

MATERIALS: Place value models for tens and 8 units for each pupil. Use a model for the ten that can be easily exchanged for units. 10 buttons on a wire hook, 10 centimenter cubes in a paper sleeve, 10 sticks in a rubberband.

DIRECTIONS: Write a separation problem on the board. 16 - 9 = ______.

Have the pupils model 16 as 10 and 6. To separate 9, they must exchange the ten units (unhook the buttons, take the rubberband off the sticks). Let students regroup objects to model the separation process.

188



Write and model many problems.

$$\begin{array}{r} 14 \\ -8 \\ \hline \end{array} \qquad \begin{array}{r} 13 \\ -5 \\ \hline \end{array} \qquad 17 - 8 = \underline{\hspace{3em}}$$

C. SUBTRACTION CONCENTRATION

MATERIALS: Twenty-four 2-inch cardboard squares, a 4-inch by 6-inch answer card.

PREPARATION: Select 12 subtraction combinations. On each of 12 cards write the answer, on each of the other 12 cards write the problem. Make an answer card.

DIRECTIONS: All cards are to be placed face down on the table. The first player will turn up two cards. If they are a matched pair (as 9 - 6 and 3), the player places them in front of him. If they are not a matched pair, he returns them to their face-down positions on the table. Play continues until all cards have been played. The players then check their responses by the answer card. The player with the greater number of equivalent pairs is the winner.

D. LAY OR LOSE

Modify activity cards in objective /39-J, Activity C for subtraction.

E. CLIMB THE STAIRS

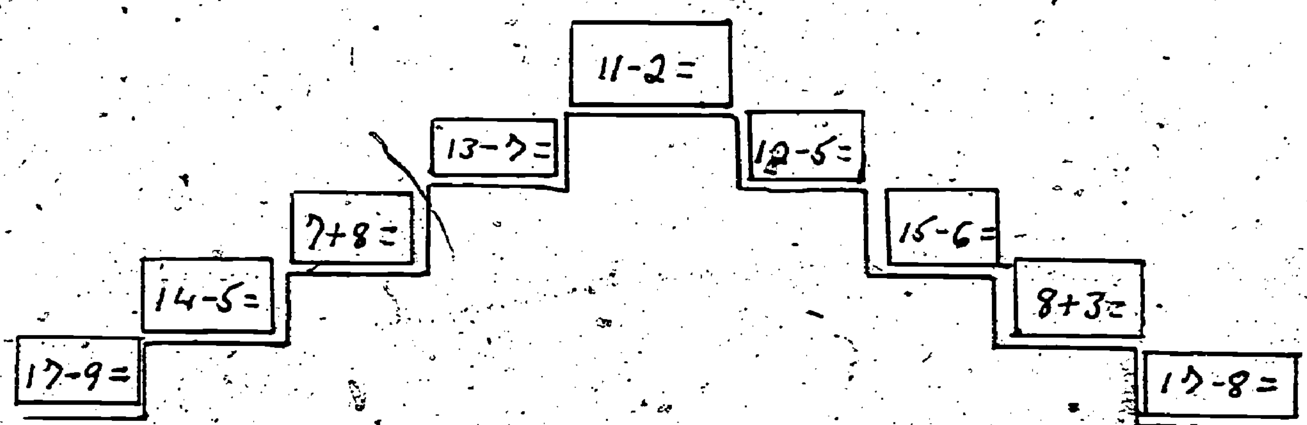
MATERIALS: A playing board, 11 by 8½ inches, made of cardboard, twenty-four 1½-inch by 1-inch tagboard cards, a card pocket.

PREPARATION: Draw on the playing board a broken line to represent stairs. On one set of twelve 1½-inch by 1-inch cards, write pairs of addends, one pair per card; on the reverse side, write the sum of the pair. On each of the other 12 cards, show a sum and known addend; on the reverse side show the missing addend. Glue the card pocket to the back of the playing board and insert the addition and subtraction cards in the pocket.

DIRECTIONS: A pupil, playing alone, places one set of cards on the steps of the "up" stairway. Beginning with the card on the bottom step, he states the sum or missing addend, then turns the card and checks his response. Step by step, he proceeds to the top of the stairs. He then places the cards on the step of the "down" stairway and again states the sum or missing addend and checks his response.

When two pupils play, one player responds to the indicated addition or subtraction pair, and the other checks the responses. The game continues until both players have climbed up and down the stairs.

This activity may be adapted to multiplication and division. The multiplication tables may be used for checking.



OBJECTIVE Subtract from a 2 or 3- place minuend.

COMMENTS No zeros yet in the minuend! The examples illustrate 3-place minuends -- be sure to do several examples with 2-place minuends before moving on to 3-place.

PRE-REQUISITE OBJECTIVES 81-S

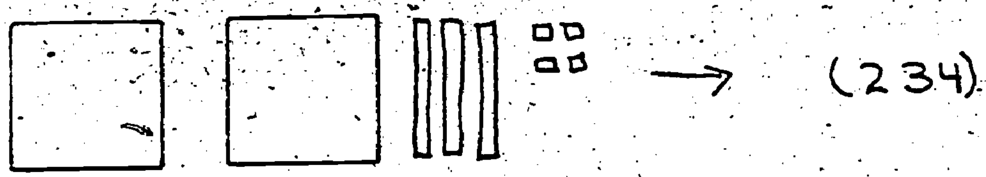
ACTIVITIES

A. SUBTRACTION MODELS

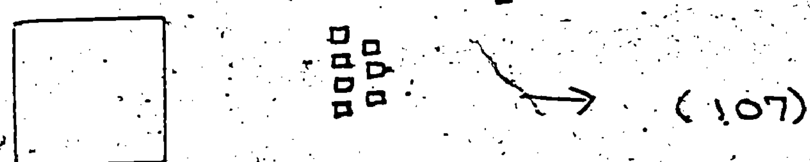
MATERIALS: Place value models for hundreds, tens, and units.

DIRECTIONS: Directions a-g are steps that should be taken in order. Work with a group of 8 to 10 students.

a) Demonstrate first. Build a model of a number in the hundreds. For example



Separate a set of 127. After regrouping, the following set will remain:



Do several examples. Subtract some one and two place numbers from the minuends.

b) Demonstrate again; this time write the problem on the board. Diagramming the problem the following way may be helpful when regrouping is required. Arrange in columns of hundreds (H), tens (T) and units (U) and then regroup till all columns can be subtracted. For example:

$$\begin{array}{r} 342 \\ -\ 187 \\ \hline \end{array}$$

Regroup →

H	T	U
3	3	12
1	8	7

Regroup →

H	T	U
2	13	12
1	8	7
1	5	5

c) Provide pupils with practice dittos. Include examples with regrouping for units and with no regrouping. Do just those problems which require regrouping at the units and tens only. You can have pupils show work as in b) above.

page 212 blank

d) Demonstrate again; this time let the tens digit of the subtrahend be larger than the minuend. Example:

$$\begin{array}{r} 368 \\ -\ 174 \\ \hline \end{array}$$

H	T	U
3	6	8
1	7	4

Subtract →

	H	T	U
	3	6	8
−	1	7	4
			4

Regroup and subtract →

	H	T	U
	2	16	8
−	1	7	4
	1	9	4

e) Provide practice dittos; include problems with no regrouping for the units or tens, but only one regrouping in each problem.

$$\begin{array}{r} 368 \\ -\ 245 \\ \hline \end{array} \qquad \begin{array}{r} 327 \\ -\ 182 \\ \hline \end{array} \qquad \begin{array}{r} 362 \\ -\ 147 \\ \hline \end{array}$$

f) Demonstrate again: This time let both the units digit and the tens digit of the subtrahend be larger than the minuend.

g) Model and record again.

Provide lots of practice --- dittos --- work book pages --- games.

OBJECTIVE Subtract from a minuend containing zero.

COMMENTS

PRE-REQUISITE OBJECTIVES 82-S

ACTIVITIES

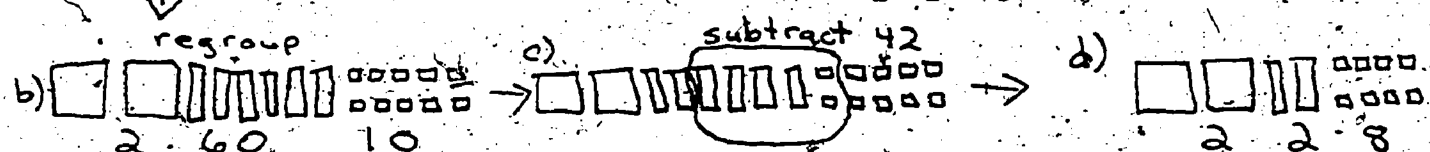
A. SUBTRACTING FROM ZERO IN THE UNITS PLACE

MATERIALS: Place value models

DIRECTIONS: Extend the modeling for three digit subtraction. Model numbers with zero in the units place -- record problems from the model. Use place value models on page 57.

FOR EXAMPLE:

270	Regroup	H	T	U
-42		2	6	10
			4	2
		2	2	8



This shouldn't be too difficult as there is nothing new. But some pupils have trouble just because there is a zero.

B. SUBTRACTING FROM ZEROS IN THE TENS OR UNITS PLACE

MATERIALS: Place value models for hundreds, tens and units.

DIRECTIONS: (a) Review place value, particularly numerals with zero in the ones, tens or both places. Have the pupils build or draw models for numerals, 104, 106, 240, and 300, for example. Build models or pictures of models and have children write the numeral from the model. Be certain all pupils can write a numeral that contains a zero as you dictate it. Any pupil who can not probably needs help with place value.

C. PROVIDE MANY PRACTICE MODELS

OBJECTIVE Subtract 4 and 5 place numbers from 4 and 5 place numbers.

COMMENTS

PRE-REQUISITE OBJECTIVES 83-S

ACTIVITIES

A. ADD A DIGIT

MATERIALS: Chalkboard

DIRECTIONS: Put a three digit subtraction problem on the chalkboard. Have the children give you directions on how to do the problem.

$$\begin{array}{r} 642 \\ -389 \\ \hline 253 \end{array}$$

Add a digit to either end of both the subtrahend and minuend.

$$\begin{array}{r} 3642 \\ -1389 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{r} 6425 \\ -3896 \\ \hline \end{array}$$

B. CHALKBOARD DRILL

MATERIALS: Chalkboard, chalk

DIRECTIONS: Send 2 or 3 pupils to the chalkboard. Have them work several problems. Correct mistakes for the class.

C. PRACTICE

MATERIALS: Dittos and workbook pages of 4 and 5 place subtraction problems.

DIRECTIONS: Give lots of practice.

D. TRICKY ZEROS

MATERIALS: Chalkboard, problems with zeros in the minuend.

192

page 216 blank

DIRECTIONS: Put a problem like 7000 on the board. Ask pupils how they would
-2648

work it. Accept all methods that will consistently get the correct answer.

CONTINUE:	6002	7026	60000
	-2340	-2348	- 2486

E. PRACTICE

MATERIALS: Dittos, workbook pages, games, chalkboard contests.

DIRECTIONS: Provide practice with zeros.

F. MAGIC NUMBER 6174

MATERIALS:

DIRECTIONS:
(a) write any 4 place number with different digits: 3412
(b) Rearrange the digits to form the largest possible number: 4321
(c) Rearrange to form the smallest number: 1234
(d) Subtract: 3087

Repeat b,c,d for the answer
8730
-0378
8352

Continue until the "magic" answer 6174 is obtained:
8532
-2358
6174

(The pupil will never do more than 7 subtractions before the "magic" answer appears.)

OBJECTIVE Relate addition to subtraction as a checking process.

COMMENTS Where addition is a joining process, subtraction is a separating process. This objective shows the pupil that what is separated can be re-joined.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. ADDITION AND SUBTRACTION

Modify the activities from Objective 33-J by asking for subtraction combinations as well as addition combinations.

B. "SUBTRACTION"

MATERIALS: Prepared ditto.

DIRECTIONS:

EXAMPLE:

SUBTRACT		CHECK	
Jane had	56¢	Jane had left	32¢
Jane spent	-24¢	Jane spent	+24¢
Jane had left	32¢	same	56¢

NOTE: Make up your own stories or use worksheets. Vary the number of digits in the problems.

19

GENERAL REMARKS The next sequence of objectives models repeated subtraction as a means of developing algorithms for division.

86 - S Reference Division

OBJECTIVE Separate sets and record as repeated subtraction.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. PARTITIONING AND RECORDING

MATERIALS: Counters

DIRECTIONS: Direct the student to separate the original set into several sets and record as repeated subtraction. The sets should not be of equal size.

EXAMPLE: Set of 15

```
x x x x x
x x x x x
x x x x x
```

The pupil separates thus:

```
x x x x x
x x x x x
x x x x x
```

Recording would be:

```
  15
 - 4
  11
 - 6
   5
```

VARIATION: Place a set of X's on ditto and have child indicate the set to be subtracted by circling them. Record next to drawing.

B. PRACTICE

MATERIALS: Dittos and workbook pages.

DIRECTIONS: Practice

195

page 220 blank.

OBJECTIVE Separate sets into 2 or more equal parts of a given size.

COMMENTS

PRE-REQUISITE OBJECTIVES - 86-S

ACTIVITIES

A. SEPARATING SETS

MATERIALS: Counters, set indicators.

DIRECTIONS: Choose the set size you want to group by, then make a large set to be separated into sets of the chosen size. In this first experience, it would be best to have the grouping come out evenly. For example, if you want to group by sets of size 3, then your large set should have 6, 9, 12, 15, etc. objects. After you have formed the large set, have a pupil group it into sets of the chosen size. Ask questions about what was done and set sizes involved. Do not record yet.

B. DIVISION--SEPARATING SETS

MATERIALS: Same as above.

DIRECTIONS: In this activity we will not be concerned if the chosen set size will evenly separate the larger set. Choose a large set of objects and let the children choose a set size to separate it by. When the pupil has separated as many sets as possible there will usually be some elements left over. This should be noted, by asking, "How many left-over?". For example, if we have a set of size 24 and decide to separate it into sets of size 5, we will have 4 sets of 5 and 4 left over.

page 222 blank

OBJECTIVE Repeated subtraction of sets of multiples of a given size with recording.

COMMENTS This objective provides experiences leading to the partial quotient algorithm for division.

PRE-REQUISITE OBJECTIVES 87-S

ACTIVITIES

A. SEPARATING SETS WITH LEFT-OVERS

MATERIALS: Counters, set indicators.

DIRECTIONS: Repeat activity B of Objective 87-S, using the variation example on the next page.

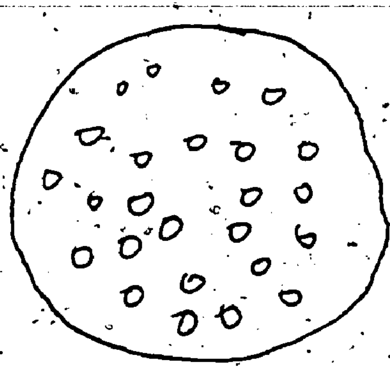
Have the pupils do several examples, varying original set size and the separating set size. In the first example you may tell the pupil how many sets to separate (In the example: First separate 4 threes, then two threes, etc.). Afterwards have the pupil do the separating according to his own choice.

NOTE: It may be helpful to fix the original set size and have the pupil do two or three examples with various separating set sizes.

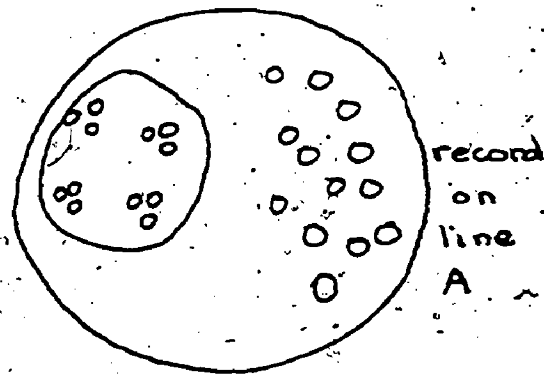
197

page 224 blank.

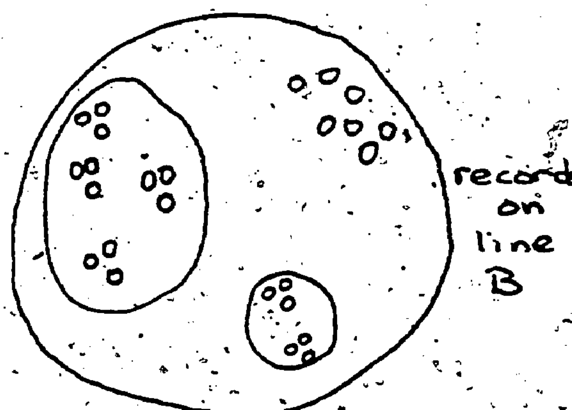
Original set size: 25
Separate by sets of size: 3

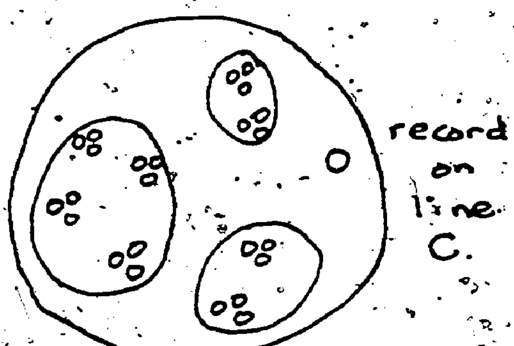


		Separated
	25	
A	−12	4 threes
	13	
B	−6	2 threes
	7	
C	−6	2 threes
D	1	8 threes with 1 left-over.



As in activity B of objective 87-S have the pupil note the total number of threes and left-overs, and then do a final recording on line D.





OBJECTIVE Develop the partial quotient division algorithm using single digit divisors and single or double digit dividends.

COMMENTS The dividend is the number being divided, the divisor is the number doing the dividing, the quotient is the answer.

PRE-REQUISITE OBJECTIVES 88-S

ACTIVITIES

A. TRANSFER REPEATED SUBTRACTION TO THE DIVISION ALGORITHM

MATERIALS: Chalkboard

DIRECTIONS: Do the written part of Activity A objective 88-S. Modeling should not be needed now.

EXAMPLE: How many fives in 37?

```
 37
-25   (5 fives)
 12
-10   (2 fives)
  2
        7 fives with 2 left over
```

Transfer the problem to the division algorithm.

```
5 | 37 |      Read, "How many fives in 37?"
   -25 | 5
    12 | 2
   -10 | -
     2 | 7 with 2 left over
```

B. PRACTICE

MATERIALS: Dittos, work book pages.

DIRECTIONS: Begin with dittos that transfer the repeated subtraction to the division algorithm.

199

EXAMPLE:

How many 7's in 48?

Subtract		Divide	
48		7 ⟌ 48	
-21	3 groups of 7	21	3
27		27	
-21	3 groups of 7	21	3
6	6 groups of 7 with 6 left over	6	6 groups with 6 left over.

Have the pupils do several examples.

6 ⟌ 84   3 ⟌ 78   6 ⟌ 19

OBJECTIVE Internalize the basic division facts and notation.

COMMENTS Recall that basic division facts involve a one digit divisor, a one digit quotient and no left overs. Two different models of division are presented here. Activity A uses measurement to see how many sets of a given size are contained in a given set (the large set is "measured" in terms of the small set). Activity B uses partitioning to separate a set into a given number of subsets of equal size to see how large each subset is (this is sometimes known as "divvy-up" division). Be sure to provide lots of practice with each model. One or the other of these models will apply in division story problems, so the pupil must be familiar with both of them.

PRE-REQUISITE OBJECTIVES 87-S

ACTIVITIES

A. SEPARATING SETS WITH RECORDING

MATERIALS: Counters, set indicators. Repeat Activity A of Objective 87-S with recording.

EXAMPLE:

Original set size 24
Separate by sets of size 6

Original Set Size	Separate By Sets of Size	Number of Sets
24	6	4

Record several examples using a variety of sizes.

Original Set Size	Separate By Sets of Size	Number of Sets
24	6	4
35	7	5
28	4	7

B. MODEL VARIATION

Ask the pupil to divide 18 counters into 6 sets to see how many counters are in each set. The pupil models each problem and records the answer on the chart.

Original Set Size	Number of Sets	Size of Each Set
18	6	
24	6	
12	6	

C.

Use the information from the charts in activities A and B to record. Use these formats. Make a special note of ÷ sign.

$$6\overline{)24}^{\,4} \qquad 24 \div 6 = 4$$

D.

MATERIALS: Dittos, worksheets, flashcards.

DIRECTIONS: Provide lots of practice.

202

OBJECTIVE Develop the traditional division algoithm using one digit divisors with one or two digit dividends.

COMMENTS The traditional algorithm uses the largest possible multiple. In Activity A, the pupil determines the largest multiple, a skill necessary for the traditional algorithm presented in Activity B. These problems involve 1 digit quotients. In Objective 121-S, problems with more than 1 digit quotients will be considered.

PRE-REQUISITE OBJECTIVES 90-S

ACTIVITIES

A. TRIAL MULTIPLE

DIRECTIONS: Start by asking, "How many 5's in 27?" Ask what is the largest number of 5's in 27. It is 5.
How many 6's in 47? What is the largest multiple of 6 in 47? (If a child answers 36, ask how many left overs? 11) Is there a larger multiple? Yes.

Record the information on the chart.

Given Set	Grouping Set	Groups	Trial Multiple	Left Over
27	5	5	25	2
47	6	6 7	36 42	11 5
78	8	5 7 8 9	40 56 64 72	38 22 12 6
29	3	7 9	21 27	8 2
24	4	6	24	0

B. TRADITIONAL ALGORITHM

ACTIVITIES: Use the new format with problems on the chart in A and explain each part.

```
                    5   groups
grouping set 5 | 27   given set
                   25   trial multiple
                    2   left overs
```

C. PRACTICE

Use the dittos and workbook pages.

OBJECTIVE Determine how a set of a given size can be modeled as a rectangle.

COMMENTS The pupil should be aware that a square is a rectangle but not all rectangles are squares.

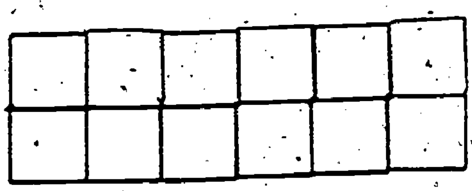
PRE-REQUISITE OBJECTIVES 70-G

ACTIVITIES

A. BUILDING RECTANGLES

MATERIALS: Use objects of uniform width and height.

DIRECTIONS: Group different size sets in rectangular arrangements and record. See sample arrangement of cubes below.



6 x 2 = 12

B. PROPERTY CHART

MATERIALS: Objects of a uniform width and height. Teacher prepared ditto with chart as shown in sample (children to fill in yes and no in blanks).

DIRECTIONS:

Step (1) Beginning with a set size of 5, the child determines if he can arrange it into a rectangular pattern of 1 x 5, then determines if 5 can be arranged into a rectangular shape other than 1 x 5 and last if it can be arranged into a square. At each step, he records yes or no in the appropriate place on the chart. Then proceed to the next number. Do not use the number 1 as a model.

N	Rectangular Arrangement of 1 x N	Rectangular Arrangement other than 1 x N or N x 1	Square Arrangement
5	yes	no	no
6	yes	yes	no
7	yes	no	no
8	yes	yes	no
9	yes	yes	yes
4	yes	yes	yes
3	yes	no	no
2	yes	yes	no
10	yes	yes	no
11	yes	no	no
12	yes	yes	no
13	yes	no	no
14	yes	yes	no
15	yes	yes	no
16	yes	yes	yes
17	yes	no	no

Step (2) Tell pupils that those numbers which have a square arrangement are called square numbers. Using the chart he should make a list of square numbers. Tell the pupils that those numbers that have a rectangualar shape are rectangular numbers. Then make a list of those numbers. He should begin to realize that all numbers have a rectangular arrangement of 1 x N and N x 1. Tell pupil that numbers which cannot be arranged into any rectangular arrangement other than 1 x N and N x 1 are called <u>prime numbers</u>. Then have him make a list of those numbers.

Step (3) Have the pupil continue listing the first 20 prime numbers. Lead him to discover that prime numbers are numbers that are not the product of any multiplication sentences (except for 1 x N and N x 1). As the pupil lists primes such as 19, 23, 29, he can confirm them by seeing that the objects cannot be patterned into rectangles other than 1 x N and N x 1, but it may become more impractical to model as the primes get larger.

Step (4) Tell the pupil that all numbers which are not primes are called composites. List composites from the chart.

OBJECTIVE Pattern objects into as many rectangular arrangements as possible and record factors.

COMMENTS Introduce the term "factors" as a general term to replace both multiplier and multiplicand.

PRE-REQUISITE OBJECTIVES 92-C

ACTIVITIES

A. MODELING RECTANGUALR ARRANGEMENTS

MATERIALS: Objects of uniform width and height.

DIRECTIONS: Pupil takes object and determines if they can be arranged in a rectangle of 1 x ______, then 2 x ______, 3 x __________, etc. and records the rectangular arrangements.

EXAMPLE:

12 = 1 x 12
2 x 6
3 x 4
4 x 3
6 x 2
12 x 1

Tell pupils that the name given those numbers in the chart are called factors. On a separate sheet, keep tabs of the factors of numbers 2 - 20. Circle those numbers that are prime numbers.

207

The factors of

2 - 1, 2
3 - 1, 3
4 - 1, 2, 4
5 - 1, 5
6 - 1, 2, 3, 6
7 - 1, 7
8 - 1, 2, 4, 8
9 - 1, 3, 9
10 - 1, 2, 5, 10
11 - 1, 11
12 - 1, 2, 3, 4, 6, 12
13 - 1, 13
14 - 1, 2, 7, 14
15 - 1, 3, 5, 15
16 - 1, 2, 4, 8, 16
17 - 1, 17
18 - 1, 2, 3, 6, 9, 18
19 - 1, 19
20 - 1, 2, 4, 5, 10, 20

Find factors for other numbers without using models. You can use this form:

36
4 × 9

24
3 × 8

Do not continue factoring the 3 and 8. That will be done in the next objectiv

OBJECTIVE Factor a number into prime factors.

COMMENTS This objective develops a factor tree. Do several different examples.

PRE-REQUISITE OBJECTIVES 93-C

ACTIVITIES

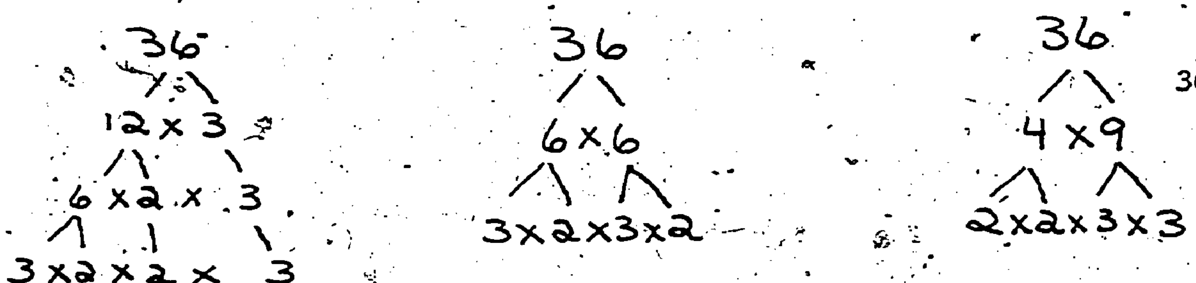
A. PRIME FACTORS

MATERIALS: Chalkboard

DIRECTIONS: Choose a number and find several pairs of factors in the number. Have the students refactor any factor they can. When the students write the product, have them list the primes from smallest to largest.

NOTE: No matter which pair of initial factors were used, the prime factors are the same.

24 = 2·2·2·3



36 = 2·2·3·3

70 = 2·5·7

OBJECTIVE Identify and record common multiples.

COMMENTS

PRE-REQUISITE OBJECTIVES

PRE-ACTIVITY

MULTIPLES

EXAMPLE: Multiples of 4: 4, 8, 12, 16, 20...
Multiples of 12: 12, 24, 36, 48, 60....

ACTIVITIES

A. TEACHER-MADE DITTO

MATERIALS: Colored pencils or crayons, pencils.

Teacher-prepared ditto as follows.
Example:

6	36	12	42	18	48	30
9	18	81	36	63	27	45

18, 36

5										
3										
10										

3	9	18	30	27	12	45
10	40	60	20	100	80	30

30

5	15	45	30	50	100	75
6	18	36	42	30	48	72

30

DIRECTIONS: Ask the pupils to shade, color, or circle common multiples of the first two numbers in each double row; then record those at end. For three rows, ask the pupils to record the multiples and circle the common multiples. This may be done with three or more numbers in first column and as many multiples as teacher wants.

Page 238 blank.

B. FRACTION STICKS

MATERIALS: Popsicle sticks, tongue depressors, or similar items for each child.

DIRECTIONS: Have each child write multiplication facts on his own set of "fraction sticks" as shown below:

1	2	3	4	5	6	7	8	9	...
2	4	6	8	10	12	14	16	18	...
3	6	9	12	15	18	21	24	27	...
4	8	12	16	20	24	28	32	36	...

Make sticks for multiples of all numbers 1-12. These sticks can be used for number multiples, equivalent fractions, common denominators, ordering of fractions, and addition and subtraction of fractions. For example:

The "2" stick gives the first multiples of 2 (even numbers).
The "7" stick gives the first multiples of 7.
The Least Common Multiple (LCM) of 2 and 3 is 6.
Using the "2" stick above the "3" stick gives the first equivalent forms of the fraction 2/3. (i.e. 4/6, 6/9, 8/12,...)
To order the fractions 2/3 and 5/7, place the "2" above the "3" and the "5" above the "7", find the LCM (35) and see which numerator is largest.

To add 2/3 and 4/5 (avoid using the same digit more than once) use the "2" above the "3" and the "4" above the "5", shift one pair until the LCM or another multiple, "line up" vertically, then add the numerators. Emphasize common lower multiple.

2	4	6	8	10	...
3	6	9	12	...	
		4	8	12	...
		5	10	15	...

43

211

OBJECTIVE Learn the formula for the area of a right triangle.

COMMENTS The formula for the area of an arbitrary triangle can be developed later. In going from Activity A to Activity B introduce the term "base" for "length". When the formula for area is developed, emphasize that it is the height times base divided by 2 rather than ½ the height times base (they haven't multipled by fractions yet!)

PRE-REQUISITE OBJECTIVES 70-G

ACTIVITIES

A. GEOBOARD ACTIVITIES

MATERIALS: Geoboards, rubber bands, dot paper and pencil.

DIRECTIONS: Have the pupils record heights, lengths, and areas of all figures in this activity. By the end of step (10) the pupils should realize that the area of a right triangle is the area of its corresponding rectangle divided by 2.

(1) Have the children make a square of one unit on their Geo-Boards and stretch a rubber band from corner to corner of the square (diagonally) as shown in Figure 1. Have them draw a one-unit square on 5 x 5 dot paper, and then draw a line diagonally to divide the square. They should then cut out the square and fold it on the diagonal to make two triangles. Have the children compare the area of the square with the area of each triangle.

Figure 1

(2) Have the children make a two-unit rectangle and divide it into two equal triangles by drawing a rubber band diagonally from corner to corner (See Figure 2). Ask them how the area of each triangle compares with the area of the rectangle.

Figure 2

(3) Have the children make a four-unit rectangle or square and have them make two equal triangles by stretching a rubber band along a diagonal as shown in Figure 3. How does each triangle compare in area with the area of the rectangle?

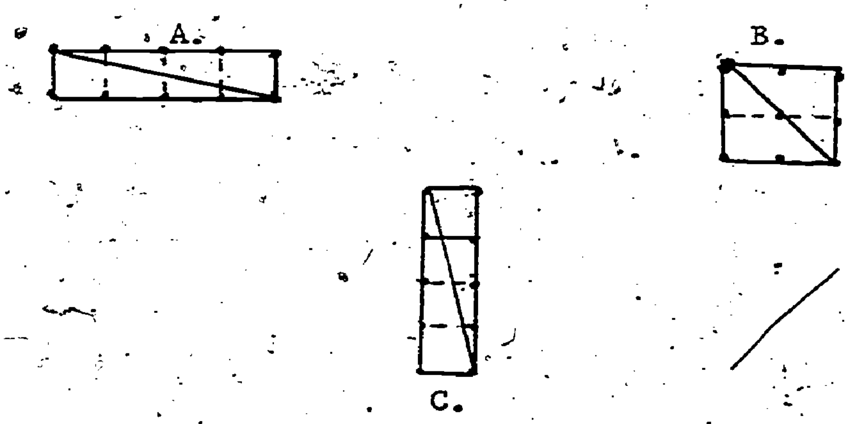


Figure 3

(4) Have the children copy on their Geo-Boards or on dot paper the pairs of triangles in Figure 4. Each small triangle equals one half a square unit. Each large triangle equals one unit. Have the children compare the height and length of each pair of triangles. Conclude that if the height and length are interchanged the area remains the same.

213

Figure 4

(5) Have the children copy the two shapes in Figure 5 on their Geo-Boards or on dot paper. The area of each of these figures is one square unit. Have the children compare the heights and lengths of the square and triangle.

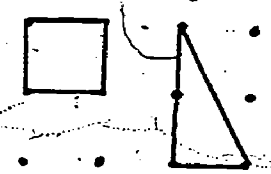


Figure 5

(6) Have the children make a rectangle on their Geo-Boards or on dot paper. It should be two units wide and four units long, divided by a diagonal. (See Figure 6). Ask them to find the area of each triangle and the area of the rectangle.

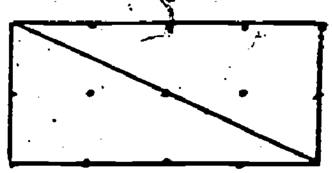


Figure 6

(7) Have the children make a triangle that has a height of four units and a length of one unit. Have them make another triangle with a height of two units and a length of two units, as shown in Figure 7. Ask them to compare the areas of the two triangles.

A. Figure 7 B.

(8) Have the children draw on dot paper a triangle with a height of three units and length of two units. Have them make another triangle with a height of six units and one unit in length. (See Figure 8). Let the children compare the areas of the triangles.

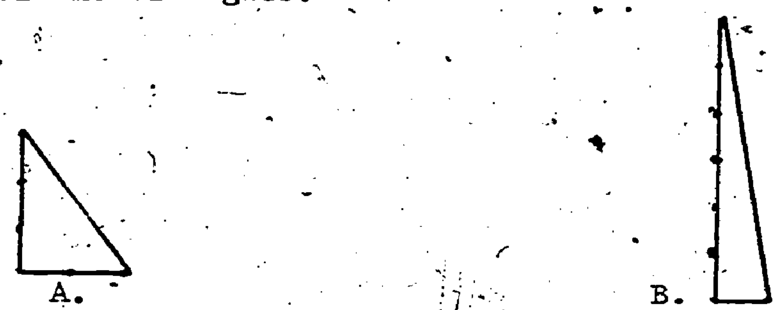


Figure 8

(9) Have the children draw on dot paper the two triangles shown in Figure 9. Triangle A has a height of four units and is three units in length. Triangle B has a height of five units and is two units in length. Ask which triangle has a larger area.

Figure 9

(10) On the Geo-Board or on dot paper, have the children make a triangle that has an area of one square unit and a rectangle that has an area of one square unit. (See Figure 10)

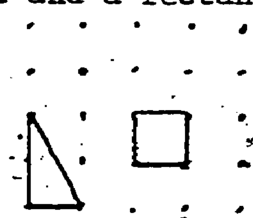


Figure 10

215

B. FINDING AREA OF TRIANGLE

MATERIALS: 3 x 5 cards or construction paper, scissors, ruler, pencil

DIRECTIONS: Make any 2 right triangles of the same shape and size. Put them together to form a rectangle. Recall that the area of the rectangle is the base times the height. Therefore, the area of the right triangle is the base times the height divided by 2.

GENERAL REMARKS This sequence of objectives is designed to review and extend some of the concepts involving fractions.

97-C | Reference Fractions

OBJECTIVE Name a fraction from a model or make a model of a fraction.

COMMENTS The pupil should know that the denominator tells the number of equivalent parts; and the numerator tells how many parts are considered. They do not have to know the words denominator and numerator necessarily. Be sure model fractions greater than 1.

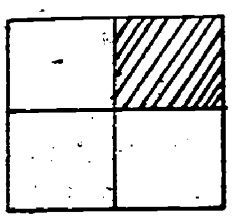
PRE-REQUISITE OBJECTIVES 31-C, 30-C, 29-C

ACTIVITIES

A. PAPER FRACTIONS

MATERIALS: Construction paper circles, squares, rectangles, triangles, scissors. (Teacher would be wise to make some guidelines for cutting into thirds, fifths, and nineths.)

DIRECTIONS: Hold up a figure, cut it into equivalent parts. Ask the child how many parts were cut. Write the number of parts. Hold up one of the parts. Write one and the fraction line over the number of parts.

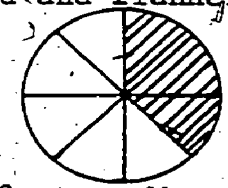


¼ 1 of 4 parts is one fourth.

Continue cutting figures into equivalent parts, counting the parts and then naming a part of the figure.

VARIATION: Flannel board and flannel may be used instead of paper.

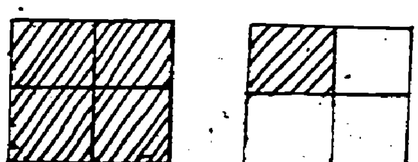
1/2



3/8

Also illustrate 3/2, 11/8 etc. More than one figure will, of course, be needed.

EXAMPLE:

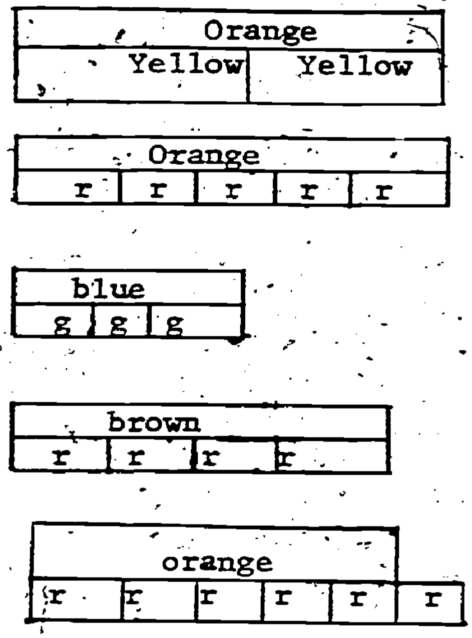


5/4

B. CUISENAIRE FRACTIONS

MATERIALS: Cuisenaire rods, chalkboard

DIRECTIONS: Use the rods to model fractions.



Ask, "How many yellow make the orange?" Write 2 on the board. Move 1 yellow and show the pupils that it is 1 of 2 equal parts. Write the fraction as ½ - one half.

Ask, "How many red make the orange?" Write the 5. Move two. Ask the pupils how they would name the two parts. Continue with each model. Make new models.

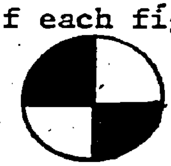
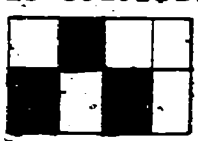
Emphasize that a fraction tells the number of equivalent parts. 5/8 is 5 of 8 equal parts.

Also model fractions larger than 1, e.g. 6 red rods is 6/5 of an orange rod.

C. PRACTICE

MATERIALS: dittos

DIRECTIONS: What part of each figure is colored?

 2/4  3/8

NOTE: If you are lucky, you'll find pages like this in a workbook. Be sure to include some fractions not modeled in class (they serve as a simple test).

D. NAME EACH FRACTION

MATERIALS: ditto sheet

DIRECTIONS: Have pupils complete chart.

<table>
<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{1}{2}$</td></tr>
<tr><td colspan="4"></td><td colspan="4"></td><td colspan="4"></td><td colspan="4"></td></tr>
<tr><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Answer to Activity D Chart

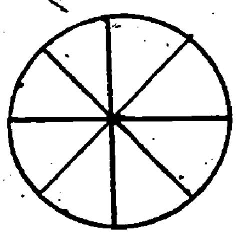
<table>
<tr><td colspan="8"></td><td colspan="8"></td></tr>
<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{1}{2}$</td></tr>
<tr><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td><td colspan="4">$\frac{1}{4}$</td></tr>
<tr><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td><td colspan="2">$\frac{1}{8}$</td></tr>
<tr><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td><td>$\frac{1}{16}$</td></tr>
</table>

E. PRACTICE

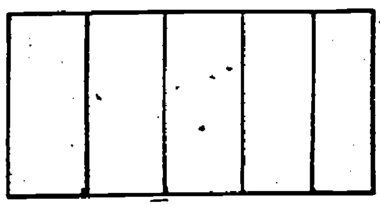
MATERIALS: Dittos and workbook pages

DIRECTIONS:

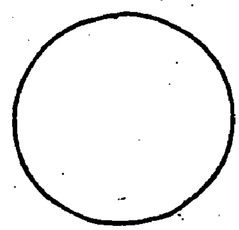
Color 3/8 of the circle.



Color 2/5 of the rectangle.

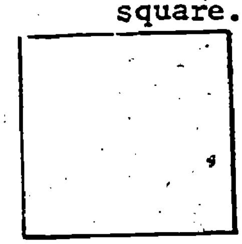


Color 1/4 of the circle.



Can you color 5/4 of the circle?

Color 4/9 of the square.



Can you name 13/9 of the square?

OBJECTIVE Arrange fractions with like denominators according to size.

COMMENTS You will want to introduce the symbols "<" and ">" for "less than" and "greater than" respectively.

PRE-REQUISITE OBJECTIVES 97-C

ACTIVITIES

A. COMPARING FRACTIONS

MATERIALS: Models of fractions from objective 97-C.

DIRECTIONS:

1. Choose a set of related fractions -- $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, . . ., $\frac{8}{8}$.

Have the pupils name the fractions.

Repeat with other related fractions.

2. Choose 3 or 4 related fractions -- $\frac{1}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, and $\frac{7}{8}$. Have the pupils name the fractions and sequence them from smallest to largest.

3. Write several related fractions in scrambled order on the chalkboard or on note cards. Ask the pupils to put the fractions in order from smallest to largest.

B. MODEL BIG AND LITTLE FRACTIONS

MATERIALS: Models of fractions.

DIRECTIONS: Have the students make a model for some fraction.

Fifths for example:

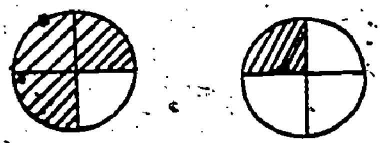
orange				
r	r	r	r	r

Ask the students, "Which is greater, $\frac{1}{5}$ or $\frac{3}{5}$?" Make them show why. "Which is greater, $\frac{2}{5}$ or $\frac{1}{5}$?" Continue with other models.

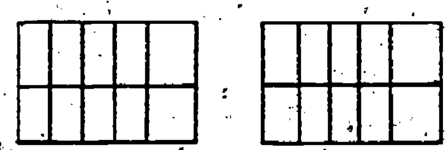
C. PRACTICE

MATERIALS: Teacher-made dittos and workbook pages.

DIRECTIONS: Use > < to compare these fractions.



$\frac{3}{4}$ $\square$ $\frac{1}{4}$



$\frac{4}{10}$ $\square$ $\frac{6}{10}$

$\frac{2}{4}$ $\square$ $\frac{3}{4}$ $\frac{5}{8}$ $\square$ $\frac{4}{8}$ $\frac{6}{7}$ $\square$ $\frac{2}{7}$

OBJECTIVE Compare and name equivalent fractions.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A.

MATERIALS: Ditto shown below. Directions following chart.

x		1	2	3	4	5	6	7	8	9
0		0	0	0	0	0	0	0	0	0
1		1	2	3	4	5	6	7	8	9
2		2	4	6	8	10	12	14	16	18
3		3	6	9	12	15	18	21	24	27
4		4	8	12	16	20	24	28	32	36
5		5	10	15	20	25	30	35	40	45
6		6	12	18	24	30	36	42	48	54
7		7	14	21	28	35	42	49	56	63
8		8	16	24	32	40	48	56	64	72
9		9	18	27	36	45	54	63	72	81

Directions: Use a multiplication table to show equivalent fractions. Cover spaces between numerator and denominator. For example: To show 1/3 cover the space between 1 and 3.

Example:

covered with paper →

1		1	2	3	4	5	6	7	8	9
3		3	6	9	12	15	18	21	24	27

223

1								
1/3			1/3			1/3		
1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9	1/9

1							
1/2				1/2			
1/4		1/4		1/4		1/4	
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

1									
1/2					1/2				
1/5		1/5		1/5		1/5		1/5	
1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

D. PRACTICE DITTOS FOR EQUIVALENT FRACTIONS

MATERIALS: Dittos similar to the two below; crayons.

DIRECTIONS: Have the pupils fill in the blanks with equivalent fractions.

1.

1							
1/2				1/2			
1/4		1/4		1/4		1/4	
1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

a) 2/4 = ____ = ____ b) 4/8 = ____ = ____

c) 1/2 = ____ = ____ d) 3/4 = ____

Variation:

MATERIALS: Popsicle sticks.

DIRECTIONS: Write the multiplication tables on popsicle sticks. Place the two sticks together to form the fraction line.
Example--Equivalent Fractions for 4/6

4		4	8	12	16	20	24	28	32	36
6		6	12	18	24	30	36	42	48	54

B.

MATERIALS: Flannel board and geometric shapes cut from flannel. You'll need circles (all the same size) of different colors cut into halves, thirds, fourths, fifths, sixths, eighths, ninths, and tenths of squares (all the same size) cut into halves, thirds, fourths, fifths, sixths, eighths, ninths, and tenths.

DIRECTIONS:

a) Let the pupils experiment to see which pieces can be stacked exactly on top of each other.

b) Start with ½ and ask pupils how many ways they can make ½. Record the answers on the chalkboard.

Example: 1/2 = 3/6 = 4/8 = 2/4 = 5/10
They might also make 1/3 + 1/6 and 2/5 + 1/10; or other combinations.
Continue with 1/3 = 3/9 = 2/6; 1/4 = 2/8; 1/5 = 2/10; etc.
Continue with 2/5 = ____, 4/8 = ____, 2/3 = ____ and other combinations.
Don't forget to ask which fractions equal 1. (2/2, 3/3, 6/6, etc.)

c) Make equations on index cards. Have pupils see if they are equal or not equal. Put the answers (equal or not equal) on the back of the cards.

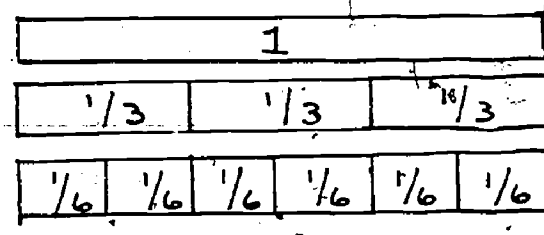
C. EQUIVALENT ROD FRACTIONS

MATERIALS: Cuisenaire rods or multibase blocks, chalkboard.

DIRECTIONS: Experiment with the blocks to find equivalent fractions.

2. a) 1/3 = ____ b) 2/3 = ____

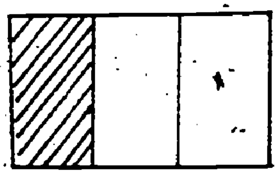
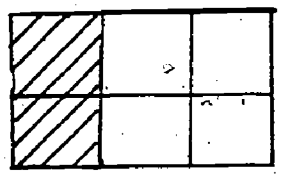
c) 2/6 = ____ d) 4/6 = ____

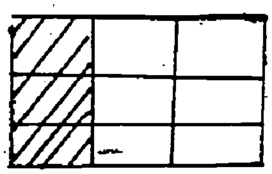
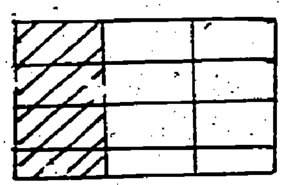


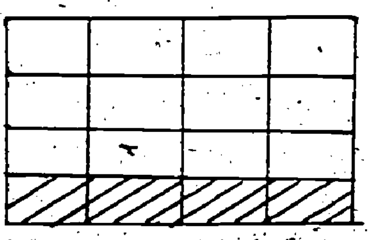
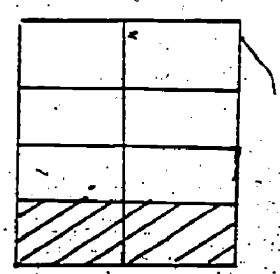
E. PRACTICE DITTOS FOR EQUIVALENT FRACTIONS

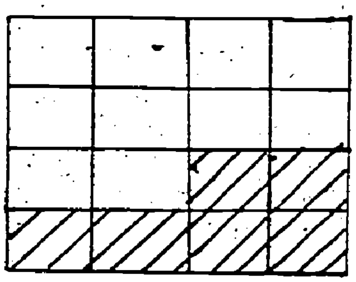
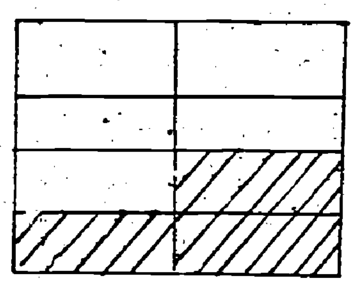
MATERIALS: Dittos as shown below.

DIRECTIONS: Have pupils fill in the blanks.

1.  a) ________  b) ________

 c) ________  d) ________

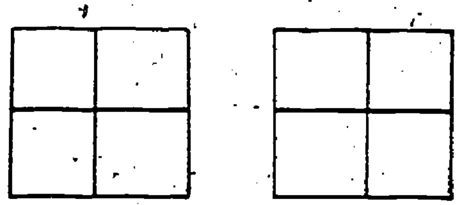
2.  a) ________  b) ________

3.  a) ________  b) ________

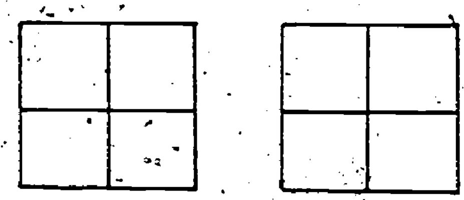
F. INEQUALITIES

MATERIALS: Dittos as shown below.

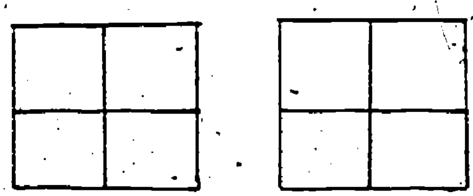
DIRECTIONS: Have pupils color in the fractional part and then fill in the blanks with > or < .



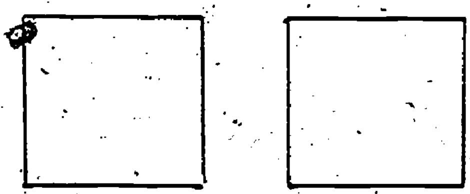
a) 3/4 ___ 2/4



b) 2/4 ___ 3/4



c) 2/4 ___ 1/4



d) 1/4 ___ 1/2

G. FRACTION PATTERNS

MATERIALS: Chalkboard, activity cards if you want; models for fractions.

DIRECTIONS: Some pupils enjoy finding patterns with equivalent fractions. Start by writing part of a fraction on the chalkboard.

1/2 = 2/? = ?/6 = ?/8 = 6/12 = 7/? = ?/16

Ask the pupils to finish the pattern. They may use the models. Start the next pattern on the chalkboard.

1/3 = 2/6 = 3/? = ?/12 = ?/? = ?/? = ?/?

Continue starting patterns on the chalkboard, giving fewer clues each time. Or have the patterns started on activity cards. Let the pupils work alone.

Include 1 = ? /? = ?/? = ?/ ?

GENERAL REMARKS Volume measure pertains to three-dimensional measurement of solids.

100-G

Reference Geometry

OBJECTIVE Discuss and define a unit of measure for volume.

COMMENTS This is a pre-activity for developing the concept of volume. Try various kinds of material to measure volume and show difficulties. Lead to cube; show use of standard cube, even when measuring irregular volume, e.g., cylinders.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. MEASUREMENT OF VOLUME

MATERIALS: Different size and shape containers; water, sand, marbles, etc. (NOTE: Containers should have different shape but same volume or, same shape but different volume, or may be different in shape and volume.

DIRECTIONS: Place two containers (having different shape but same volume). Example: ▭ - Pint - ▯ Fill one of the containers with one of the above; ask, "Will the sand fit in the other container?" Ask child to come up and fill the other container with the sand from the first container. Ask, "Are the containers equal in volume? How can you tell?" Repeat, using containers which are not equal in volume.

B. MEASURING WITH BEANS AND B.B.'S

MATERIALS: Jars, plastic containers, beans, b.b.'s, etc.

DIRECTIONS: Take a container (any container you wish to use -- a square based container may work best). Put beans (or whatever you choose to use) into the container until it is filled. Then count the number of beans used. Repeat with different containers and different fillers.

C. 3 DIMENSIONAL FIGURE

MATERIALS: Shoe boxes, milk cartons, cuisenaire rods or multibase blocks (if available), plastic freezer containers, cubes, ABC blocks.

258 blank.

DIRECTIONS: Take a large cube, have pupil fit smaller cubes inside. When one of the containers listed above is filled, remove the small cubes and count them.

NOTE: Number of small cubes counted is the volume.

VARIATIONS: Do other models using materials listed.

101-G Reference Geometry

OBJECTIVE Develop formula for measuring rectangular solids.

COMMENTS

PRE-REQUISITE OBJECTIVES 100-G

ACTIVITIES

A. SMALL GROUP

MATERIALS: Multibase blocks or wooden cubes, ice cubes.

DIRECTIONS: Tell children that they are going to build a box. Teacher says, "Our box is this long." Set out blocks.  Then say, "Our box is this long (point to blocks) and this wide." Set out blocks.

"Our box is this long, this wide, and this high (stack at one corner)." Have student build a box. After this is done, ask, "How many blocks does our box hold?" Child counts. After this has been done several times using different lenghts, widths, and heights, review what has been done. Ask, "What did we do?" (built a box) "What did we need to find that?" (We need length, width and height).

Go back to one of the boxes modeled before. Ask, "How long is our box?" (say 4) "How wide is our box?" (say 2) "How high is our box?" (say 3) Write these numbers down on the chalkboard. 4 2 3 Ask, "How many blocks could the box hold?" (24) Write on chalkboard. 4 2 3 24. From the chart you have made of the various volumes, see if the children can guess the formula for volume. If not, point out the product. For example: $4 \times 2 \times 3 = 24$

Say, "We are looking for the volume of our box, how much the box holds or how much space is inside the box. We found that we can multiply the length times the width times the height to find the volume. We can write this a short way in a formula which we can use to find the volume of any rectangular shape. The formula is volume is equal to length times width, times height, or

$$v = l \cdot w \cdot h$$

230

OBJECTIVE Relate cubic measure of volume to other units of volume measure.

COMMENTS Gallons, quarts, and liters and cubic measures all measure volume.

PRE-REQUISITE OBJECTIVES 101 -G

ACTIVITIES

A. CUBES IN GALLONS

MATERIALS: Gallon, quart, pint, and liter containers, be sure to include some rectangular containers like milk cartons and a supply of cubes.

DIRECTIONS: Ask students how many cubes in a gallon. Let them experiment to find out. Be certain they use the round containers also. Experiment.

Ask students -- "Do we use cubes to measure gasoline?" "to measure water?" "to measure milk?"

Make a chart on the board -- ask would I measure in cubes or gallons?

Volume of	cubes	gallons
milk		
a box		
air in a room		
gas		
kool-aid		
an elevator		

B. CUBIC CENTIMETERS AND LITERS

(Note this relationship is deliberate: 1000 cubic centimeters = 1 liter).

MATERIALS: liters, centimeters cubes

DIRECTIONS: Have students experiment to find how many cubic centimeters in a liter. Build a cube 10cm. long, wide, and high. Have the students find the volume. Ask if they could give the volume 1000 centimeters another name. Usually 1000 cm. = 1 liter.

231

page 262 blank

OBJECTIVE Identify geometric shapes which are congruent.

COMMENTS

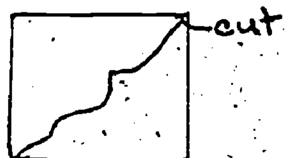
PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. NEGATIVE PICTURES

MATERIALS: Paste, paper (poster), white and black construction paper, scissors.

DIRECTIONS: Fit one sheet of white construction paper over a piece of black construction paper (or vice versa) so that they are even on all sides. Cut paper in some design. Example:



Paste on poster paper the top half of the white and bottom half of the black together and the top half of the black and the bottom half of the white together making two congruent pictures.

B. CONGRUENCE

MATERIALS: See the list below.

DIRECTIONS: Which of the following figures would always be congruent? Let the students experiment with paper models to prove their answers.

a) squares
b) right angles
c) rectangles
d) circles
e) squares with the same area
f) right triangles with the same area
g) rectangles with the same area

232

page 264 blank

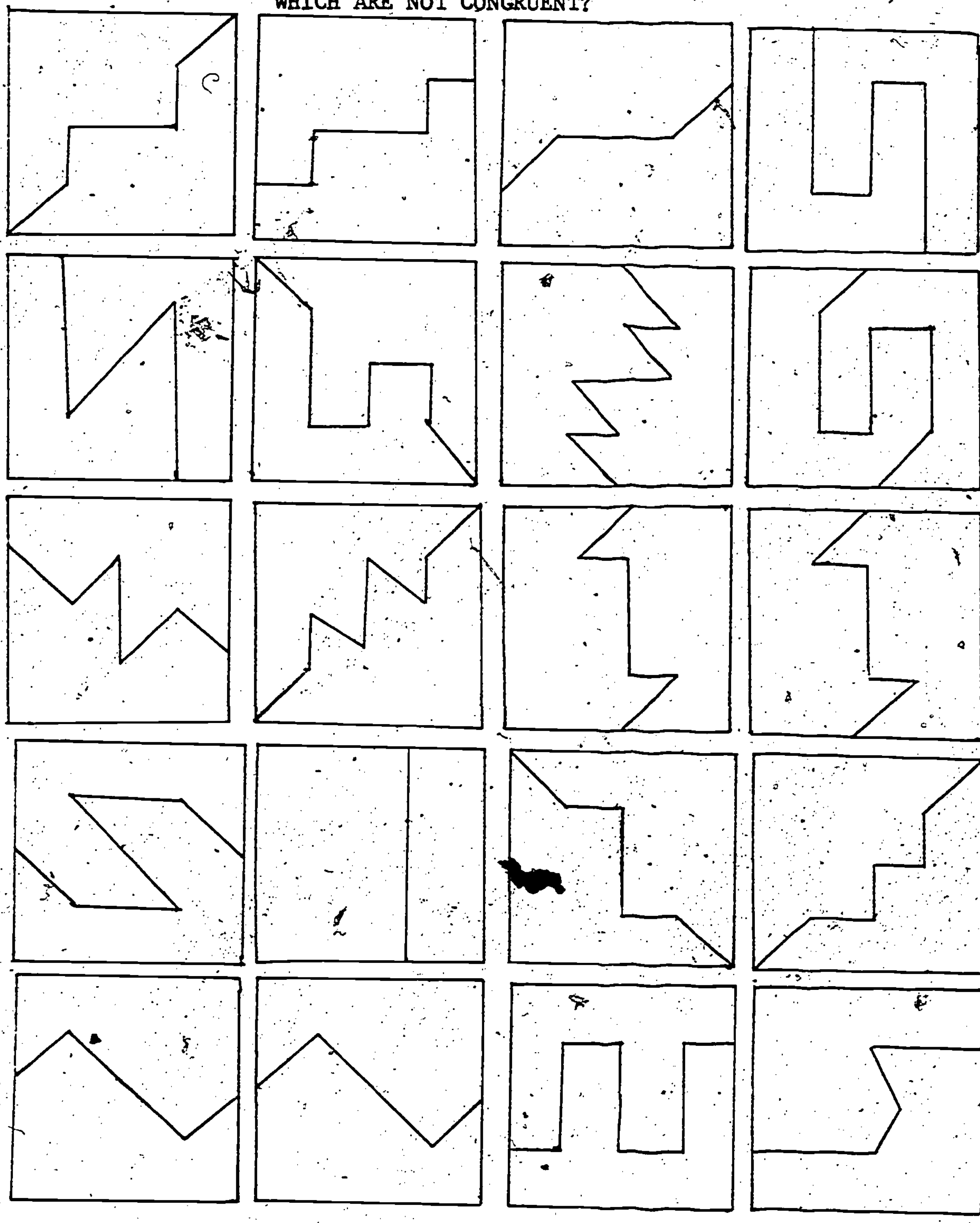
C. WHICH ARE NOT CONGRUENT

MATERIALS: Ditto (see copy on next page)

DIRECTIONS: Student is given two copies of the ditto. Student cuts out square and then cuts out along line, fits the two parts together, one on top of the other. If they fit evenly, they are congruent. Put an X on those figures for which the parts are not congruent.

233

WHICH ARE NOT CONGRUENT?



OBJECTIVE Identify geometric shapes which are similar.

COMMENTS Similar figures "look alike". Technically their corresponding angles are the same size and their sides are proportional. At this stage, however, measure of angles and proportion have not been studied.

PRE-REQUISITE OBJECTIVES 103-G

ACTIVITIES

A. SIMILAR SHAPES

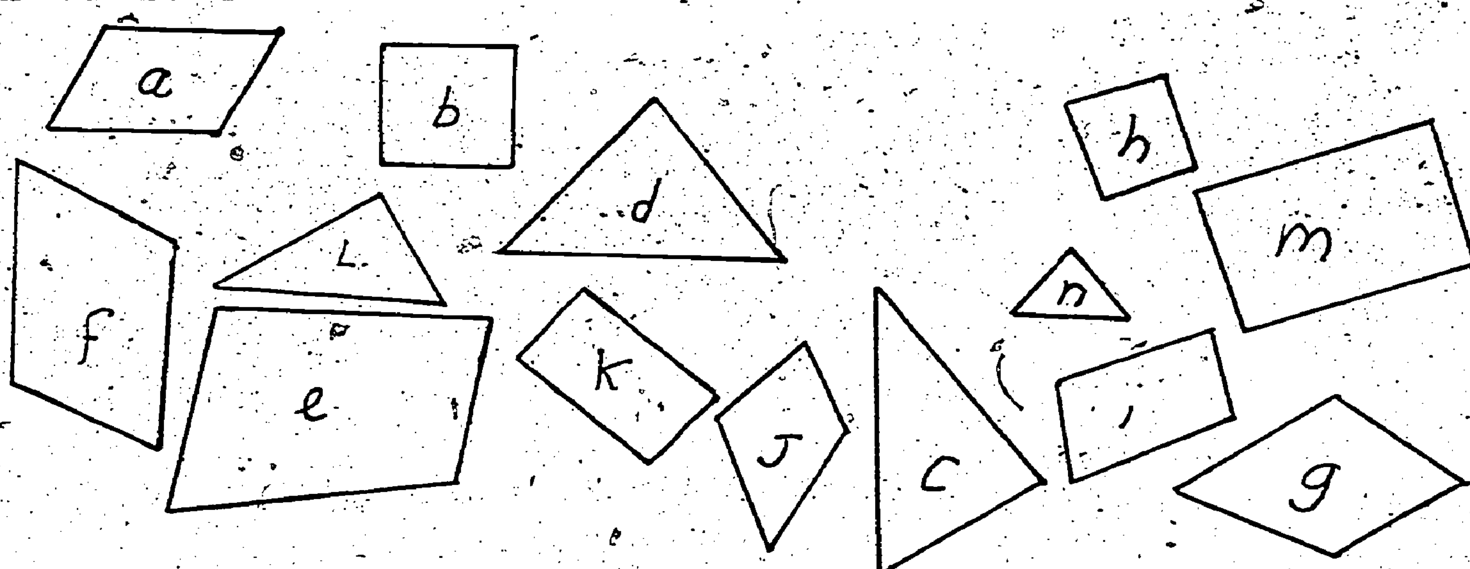
MATERIALS: Various geometrical shapes.

DIRECTIONS: Discuss how shapes are alike.

EXAMPLE: Take 2 squares. Ask, "How are these 2 squares alike?" Develop the idea of similar figures, those that look alike.

B. FIND THE SIMILAR FIGURES

MATERIALS: Teacher made ditto. Example:



(a) is similar to (f)
(b) " " " (h)
(c) " " " (l)
(d) " " " (n)
(e) " " " (i)
(g) " " " (j)
(k) " " " (m)

DIRECTIONS: Match similar figures on dittos. Prepare answer sheet with some spaces left blank.

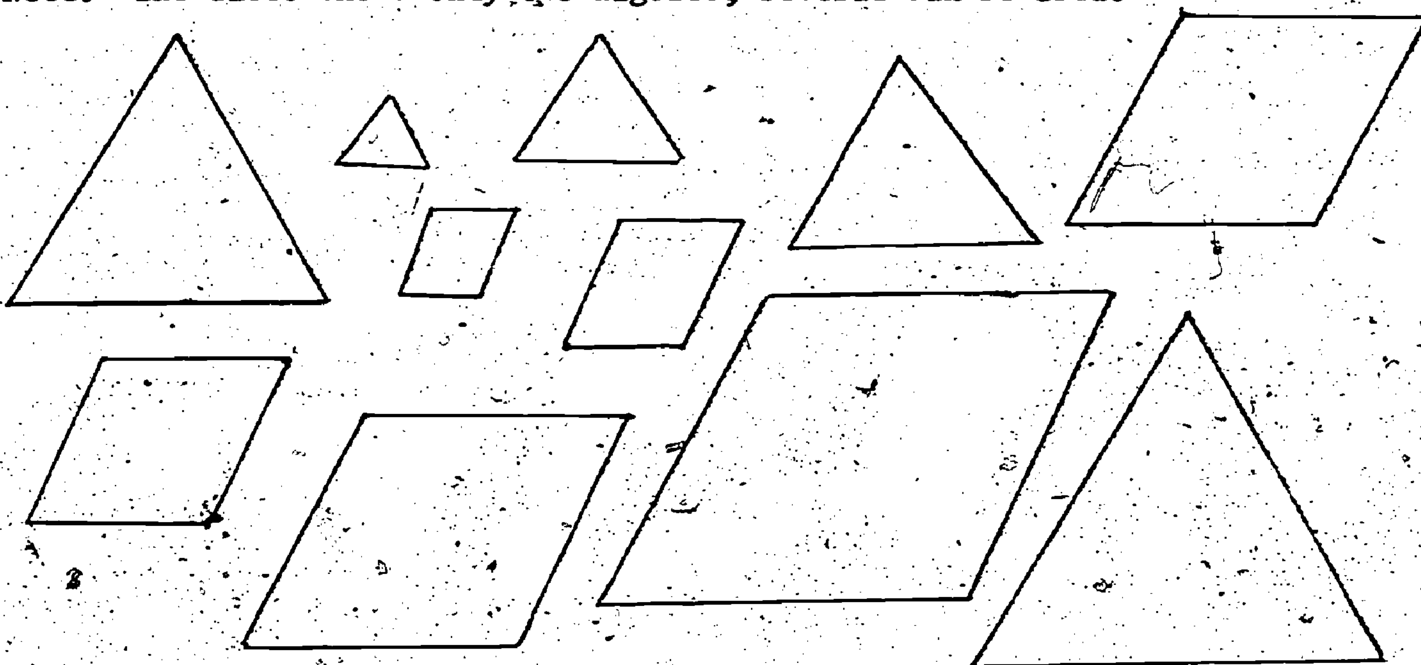
Page 268 blank

C. SIMILARITY

MATERIALS: Teacher made ditto (see example), crayons, scissors.

DIRECTIONS: Color the similar figures different colors, cut them out, and stack them.

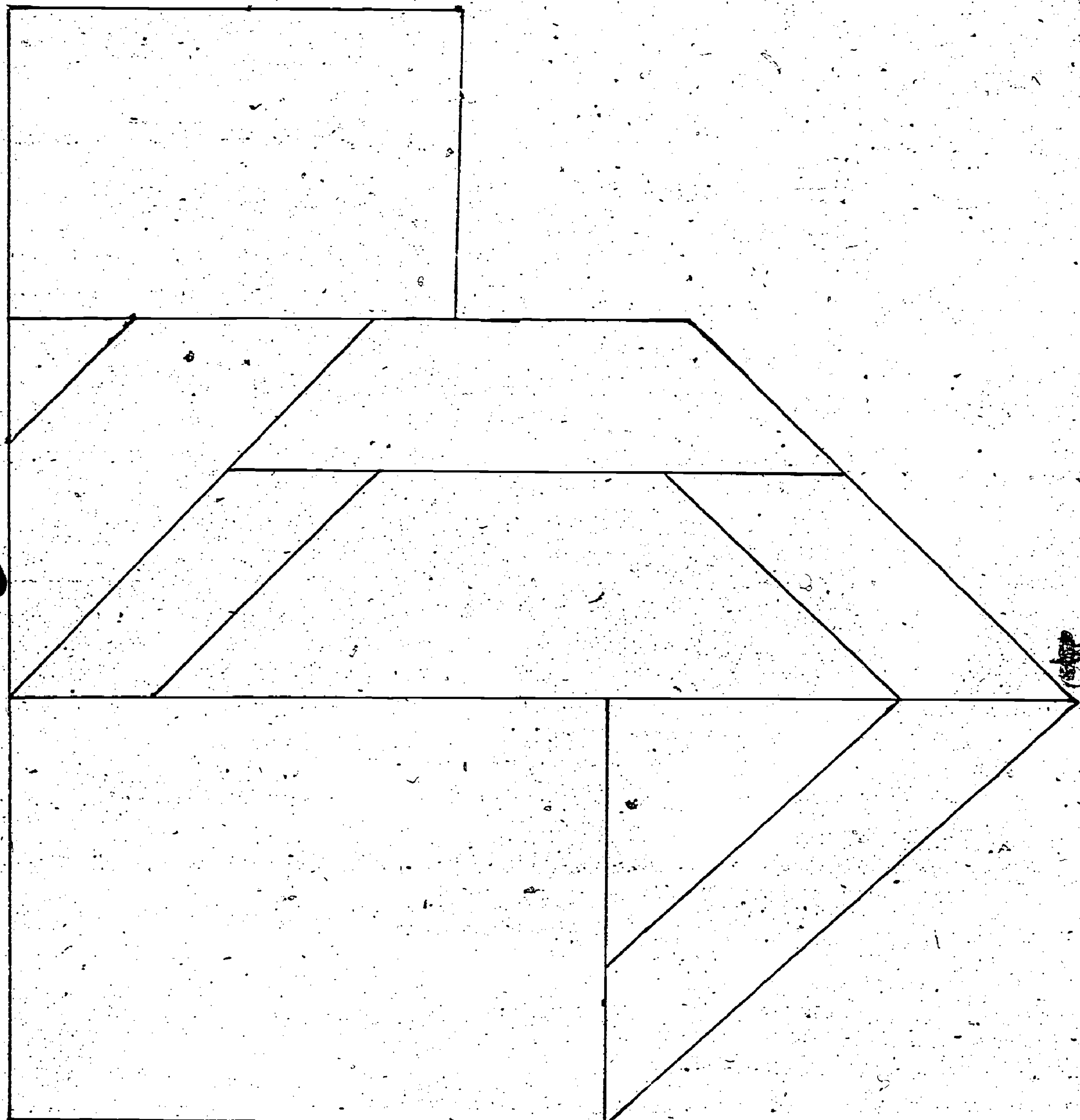
Note: The ditto shows only two figures, several can be used.

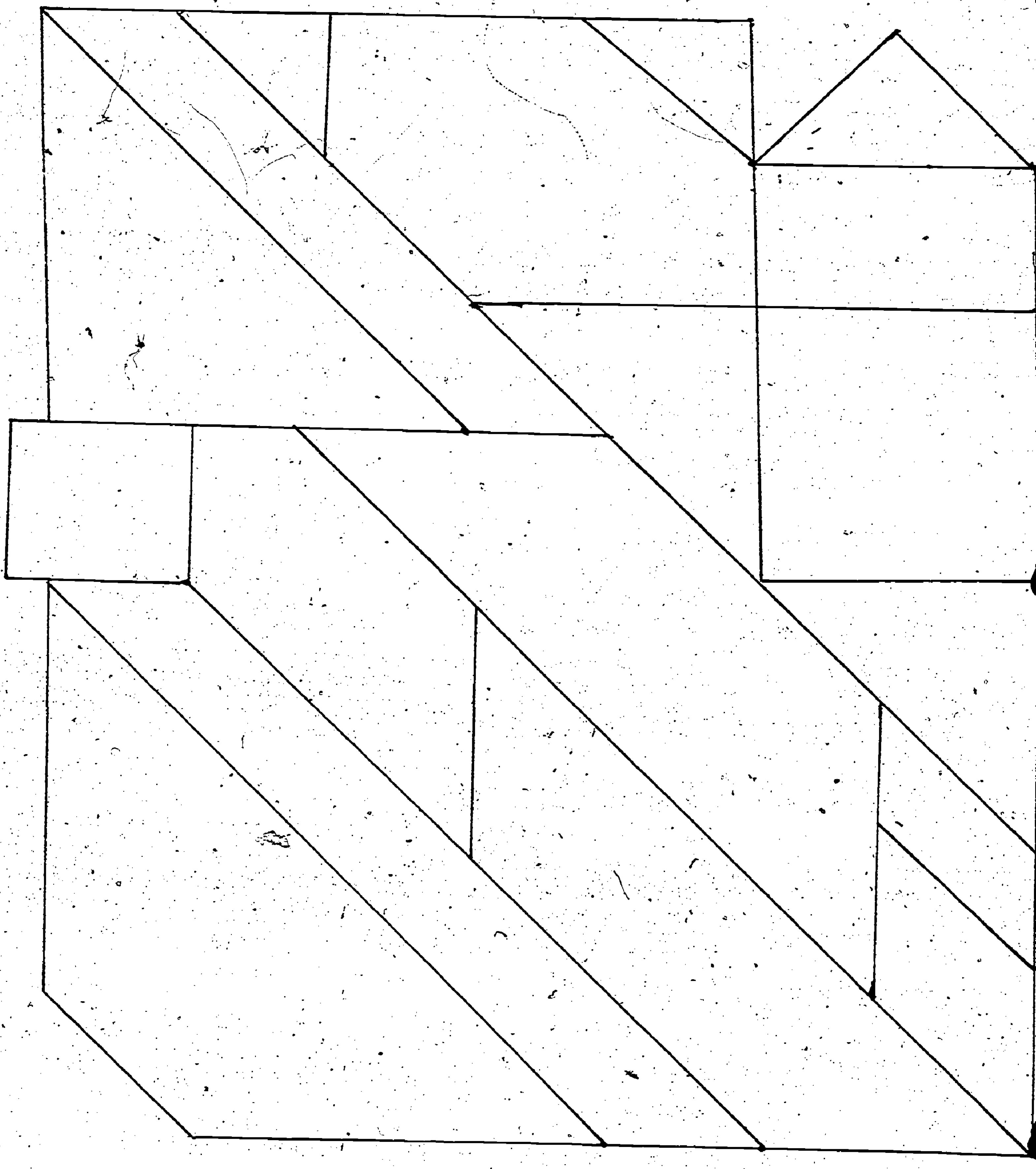


D. ART PROJECT

MATERIALS: Dittos on next two pages, scissors, paste, crayons, poster paper.

DIRECTIONS: Give pupils ditto copies of the next two pages. Have them cut out the figures, collect similar figures, color them, and make a poster.





OBJECTIVE Measure with fractional units.

COMMENTS This is linear measure. Earlier, the measurements were in exact units, now fractional measurements will be made.

PRE-REQUISITE OBJECTIVES 97-C, 56-G

ACTIVITIES

A. MEASURING

MATERIALS Ditto like the example below.

DIRECTIONS Try to find the length of $\overline{WZ}$.

W ______________________________________ Z

Can you say WZ = 3 inches? __________ Can you say WZ = 4 inches? __________

Since $\overline{WZ}$ ends halfway between 3 inches and 4 inches, we say $\overline{WZ}$ = 3½ inches.

How long are the following line segments?

C ____________________________ D $\overline{CD}$ = __________ inches

E ________________ F $\overline{EF}$ = __________ inches

G ____ H $\overline{GH}$ = __________

I ____ J $\overline{IJ}$ = __________

Did you remember to write the word "inches" at the end of the last two blanks?

How long is $\overline{QV}$? Q ____________________________ V

This is ¼ inch (_____), this is 2/4 or ½ inch (________), and this is 3/4 inch (__________). This means that QV = 2 3/4 inches.

239

Measure each of these line segments.

(1) ____________

(2) ____________________________________

(3) __________________________

(4) ____

(5) ____

(6) _______

(7) ______________________

(8) _________________________________

(9) _________________

(10) ____

Answers:

(1) __________ inches (6) __________

(2) __________ inches (7) __________

(3) __________ (8) __________

(4) __________ (9) __________

(5) __________ (10) __________

Did you remember to write the word "inches" in blanks (3) to (10)?
Which line segment is just as long as $\overline{QV}$?

OBJECTIVE Determine the longest and shortest curved line segments.

COMMENTS In objective 7-G straight line segments were compared. The abilty to determine longest curved length is more difficult because first the principle of conservation of length must be developed.

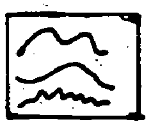
PRE-REQUISITE OBJECTIVES 7-G

ACTIVITIES

A. COMPARISON

MATERIALS: Same as activity A, objective 7-G with curved instead of straight segments on some of the rectangles.

DIRECTIONS: Compare the lengths of yarn as in objective 7-G. The pupils should guess the longest and shortest before they compare.



B. ALPHABET MEASUREMENT

MATERIALS: Set of block letters of the alphabet.

DIRECTIONS: Have pupils compare letters of the alphabet for total length. They can measure using pieces of yarn, put end to end if need be. e.g. K will need 3 pieces.

OBJECTIVE Multiply fractions less than 1.

COMMENTS Recall that "of" means "times". Do many examples in each activity. At the conclusion of the activities the pupils should see that the numerator and denominator of the product are the products of the numerators and denominators of the factors, respectively. This is the rule for multiplication.

PRE-REQUISITE OBJECTIVES 97-C

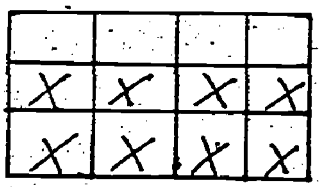
ACTIVITIES

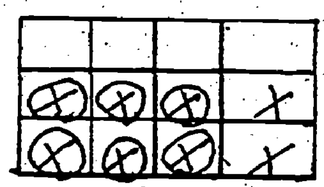
A. DOT TO DOT FRACTIONS

MATERIALS: Dot paper and pencil.

DIRECTIONS: Teacher gives pupil a problem and has him model it on the dot paper by drawing in squares to represent the denominators of the fractions.

3/4 x 2/3 would be a rectangle 4 columns by 3 rows. Pupils would mark, with an "x", 2 of the 3 rows. This illustrates 2/3. Then circle 3/4 of the columns. The circled x's (there are 6 of them) represent the numerator, the total number of squares (12) represents the denominator. The answer is 6/12.





Do several examples of this modeling, using fractions less than 1 for both factors. (Factors greater than 1 will be modeled later.) Have the pupils record the results of each example as follows:

Numerator of Multiplier	Numerator of Multiplicand	Numerator of Product
3	2	6
.	.	.
.	.	.

Have the pupil try to guess the rule for determining the numerator and denominator of the product.

Denominator of Multiplier	Denominator of Multiplicand	Numerator of Product
4	3	12
.	.	.
.	.	.

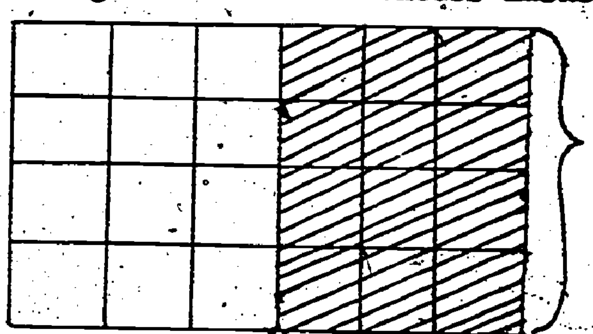
Page 276 blank.

B. CARTON FRACTIONS

MATERIALS: Egg cartons, markers (poker chips, bottle caps, pieces of paper), paper and pencil for recording.

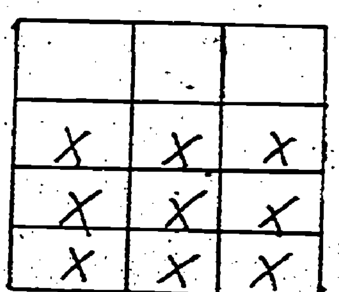
DIRECTIONS: Teacher gives pupil a problem and asks him to model it using egg cartons and markers. The denominators of the fractions being multiplied indicate the number of rows and columns needed.

EXAMPLE: 2/3 x 3/4 The model will be 3 columns by 4 rows. This is 2 egg cartons taped together with the excess masked off. This will be the unit.

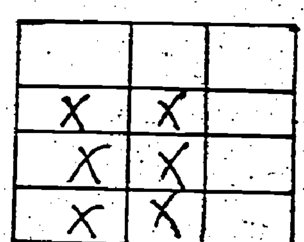


Portion masked off.

(Note that the number of squares, 12, shows a common denominator). The pupil will model 3/4 of the unit and then model 2/3 of 3/4 of the unit. i.e. 2/3 x 3/4 Taking markers, the pupil fills in rows to show 3/4, i.e., he fills in three of the four rows.



Now, looking at the columns, the pupil removes markers so that only 2 columns have markers, i.e. 2/3 of the columns have markers.



This model illustrates the answer, 6/12, 6 of 12 parts of the original unit. Again, as in A, have the pupil record the results of the examples.

NOTE: You should not proceed to the next activity until the pupil sees the rule for multiplying.

C. EXTENDING MULTIPLICATION

DIRECTIONS: Ask the pupil how to calculate, for example, $2/3 \cdot 5/6 \cdot 3/7$. Point out that $5/6 \cdot 3/7 = 15/42$ so that $2/3 \cdot 5/6 \cdot 3/7 = 2/3 \cdot 15/42 = 30/126$. Do enough examples so that the pupil sees the multiplication rule; multiply numerators and multiply denominators.

D. PRACTICE

MATERIALS: Teacher prepared dittos, paper and pencil.

DIRECTIONS: Prepare dittos of multiplication of fractions such as $1/2 \cdot 2/3 = 1 \cdot 2/2 \cdot 3 = 2/6$.

$1/4 \cdot 4/6 =$ _____ $=$ _______

$2/3 \cdot 5/6 =$ _____ $=$ _______

$3/4 \cdot 1/2 \cdot 2/3 =$ _____ $=$ _____

Eventually the pupils will omit the middle step. E.g. $2/3 \cdot 5/6 = 10/18$, $1/4 \cdot 2/3 \cdot 4/7 = 8/84$.

244

OBJECTIVE Change improper fractions into mixed numbers and vice versa.

COMMENTS A fraction with a numerator larger than the denominator is improper. Introduce the notation for mixed number, e.g., 1 + 3/4 is expressed as 1 3/4.

PRE-REQUISITE OBJECTIVES 97-C

ACTIVITIES

A. CHANGING

MATERIALS Fractional models, practice ditto.

DIRECTIONS Put a group of equivalent fractional parts on the table enough to make a whole number with some left (an improper fraction).
Example: Put seven fifths on the table. Have students group them. They'll get one unit with 2/5 of a unit left.

$\frac{7}{5} = 1 + \frac{2}{5} = 1\ 2/5$

After doing several models, have pupils work several examples from the chalkboard.

$\frac{12}{9} = 1 + \frac{3}{9} = 1 + \frac{1}{3} = 1\ 1/3$ $\qquad \frac{7}{4} = 1 + \frac{3}{4} = 1\ 3/4$ $\qquad \frac{6}{2} = 3$

B.

DIRECTIONS Have the pupils also change mixed numbers into improper fractions.

$4\frac{1}{9} = \frac{?}{9}$

Use models as needed by your class. Ask, "4 is how many 9ths?" (It is $\frac{36}{9}$).
Thus,

$4\frac{1}{9} = 4 + \frac{1}{9} = \frac{36}{9} + \frac{1}{9} = \frac{37}{9}$

$2\frac{4}{5} = \frac{?}{5}$ $\qquad$ 2 is $\frac{10}{5}$, thus $2\frac{4}{5} = \frac{10}{5} + \frac{4}{5} = \frac{14}{5}$

245

page 280 blank

OBJECTIVE Multiply mixed numbers

COMMENTS

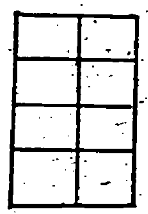
PRE-REQUISITE OBJECTIVES 108-C, 107-J

ACTIVITIES

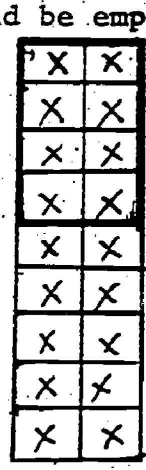
A. MODELS

MATERIALS Dot paper, pencil.

DIRECTIONS Do several examples like the following:
$1\frac{1}{2} \cdot 2\frac{1}{4}$ is the same as $3/2 \cdot 9/4$.
The model for the unit will be, as in objective 107-J, a rectangle with 2 columns and 4 rows.

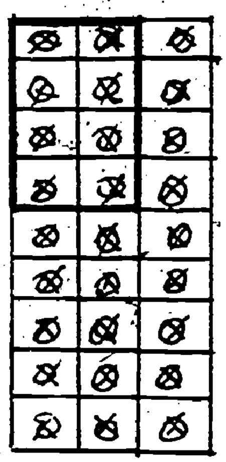


Have the pupil model 9/4 of the unit: each row is $\frac{1}{4}$, so he will mark x's in 9 rows. The unit should be emphasized with special outlining.



page 282 blank.

The pupil then models 3/2 of this: each column is $\frac{1}{2}$, so he will mark 0's in 3 columns (the third column, since he need $1\frac{1}{2}$ times the 9/4, will also have to be x'd). The final model will be



and the number of squares with x's and 0's is 27, each of which is 1/8 of the unit. The answer is therefore, 27/8. As in Objective 107-J, the pupil should record the results of several examples and see that the rule for multiplying applies for improper fractions as well (3/2 · 9/4 = 27/8).

B. WORKSHEET

MATERIALS Paper and pencil.

DIRECTIONS Give a problem, the teacher directs the pupil to change the mixed numerals to fractional numbers.

$$2\frac{1}{2} \cdot 3\frac{1}{3} \longrightarrow \frac{5}{2} \cdot \frac{10}{3} = \frac{50}{5}$$

Give a number of problems for practice.

Alternate: If the class understands the commutative property--and the teacher wishes--this short cut can be used. Since the commutative property can be applied to multiplication, the fractions can be changed so that they are easier for the children to handle.

$\frac{5}{2} \times \frac{10}{3}$ Ten is a multiple of two, so the numerators can be commuted.

$\frac{10}{2} \times \frac{5}{3} \longrightarrow \frac{10}{2}$ can be changed to its simplest form $\frac{5}{1} \times \frac{5}{3}$ and the pupil can then multiply $\frac{5}{1} \times \frac{5}{3} = \frac{25}{3} = 8\frac{1}{3}$

OBJECTIVE Identify the parts of a circle.

COMMENTS

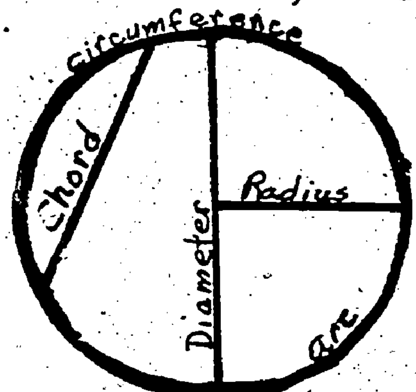
PRE-REQUISITE OBJECTIVES

ACTIVITIES

A.

MATERIALS: Paper plates, popsicle sticks, magic markers.

DIRECTIONS: Have each pupil mark the circumference of the paper plate with red magic marker. Draw in the radius and color blue; the diamter green; the chord brown; and the arc yellow--label.

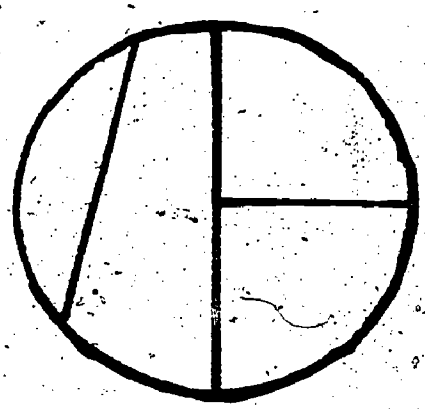


Colored popsicle sticks can be pasted over the diameter, chord, and radius.

Variation: The same can be done with plastic coffee can lids.

B.

Make a ditto of a circle with the various parts drawn in. Have the pupils label the parts.



OBJECTIVE Measure angles.

COMMENTS When two radii are drawn, the smaller arc determined by them is called the <u>intercepted arc</u>. You may want to introduce terms acute (less than 90°), right (90°), obtuse (greater than 90°), and straight angle (180°).

PRE-REQUISITE OBJECTIVES 61-G

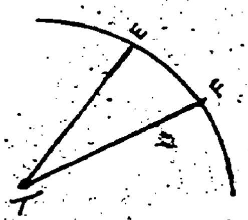
ACTIVITIES

A. MEASURING

NOTE: Always label angles with capital letters.

MATERIALS: Chalkboard.

DIRECTIONS: A unit of 1 degree (1°) was determined as follows: A circle was divided into 360 arcs of equal size. Two radii were drawn to intercept one of these arcs. The angle formed by these two radii has degree-- measure 1.



T is the center of the circle. Arc EF is 1/360 of the way around the circle. $\angle$ ETF has degree measure 1.

Explain how angles of measure 10°, 20°, 45°, etc. are determined.

B. PRACTICE

MATERIALS: Dittos of angles or cardboard angles to measure, protractors.

DIRECTIONS: Students use protractors to measure angles.

249

page 286 blank

OBJECTIVE Find the circumference of a circle.

COMMENTS The perimeter of a circle is called the circumference. The formula is $C = \pi 2 \cdot r$ or $C = \pi \cdot d$. The difficulty involves π (pi). No fraction exactly equals π. A good fractional approximation for is 22/7.

PRE-REQUISITE OBJECTIVES 110-G, 108-C, 107-J, 105-G, 57-G

ACTIVITIES

A. FIND THE RELATION BETWEEN DIAMETER OF A CIRCLE AND CIRCUMFERENCE.

MATERIALS: Circles of various sizes cut out of heavy cardboard, rulers.

DIRECTIONS: The children measure the diameter and circumference of each circle. (If you have flexible rulers you can measure the circumference with these.) Mark a starting point on the circle and then (starting with the mark down) mark a piece of paper and roll the circle, till the mark is down again and mark that on the paper. The length of the line is the circumference. The child can also roll the circle along the ruler. Record this information on a chart. Pupils answers will vary according to the accuracy of their measurement.

A typical chart may look like this:

d	C
3"	9 3/8"
2"	6"
4"	12 ½ "

This is where you will have to help the children guess about how many diameters make the circumference. It is about 3 1/7. Ultimately you want them to see that C is approximately $\frac{22}{7} \cdot d$.

B. FINDING CIRCUMFERENCE OF CIRCLES

MATERIALS: A variety of circular objects.

DIRECTIONS: Children measure the diameter, using the formula $C = \pi \cdot d$ or $C = \pi 2r$. Find circumference by multiplying. Have them check by measuring circumference. See example on next page.

250

page 288 blank

EXAMPLE:

Using Diameter

$d = 3''$
$C = \quad d$
$C = 22/7 \cdot 3/1''$
$C = 66/7''$
$C = 9\ 3/7''$

Using Radius

$r = 3''$
$C = \pi\, 2r$
$C = 22/7 \cdot 2/1 \cdot 3/1''$
$C = 132/7''$
$C = 18\ 6/7''$

251

OBJECTIVE Find the area of a circle.

COMMENTS The area is given by the formula $A = \pi \cdot r^2$. Emphasize that the area is measured in square units of measure.

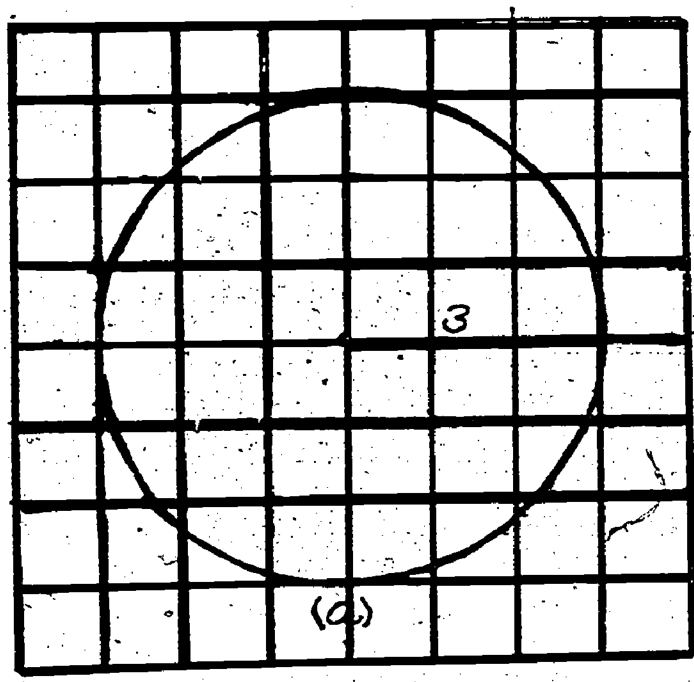
PRE-REQUISITE OBJECTIVES 112-G

ACTIVITIES

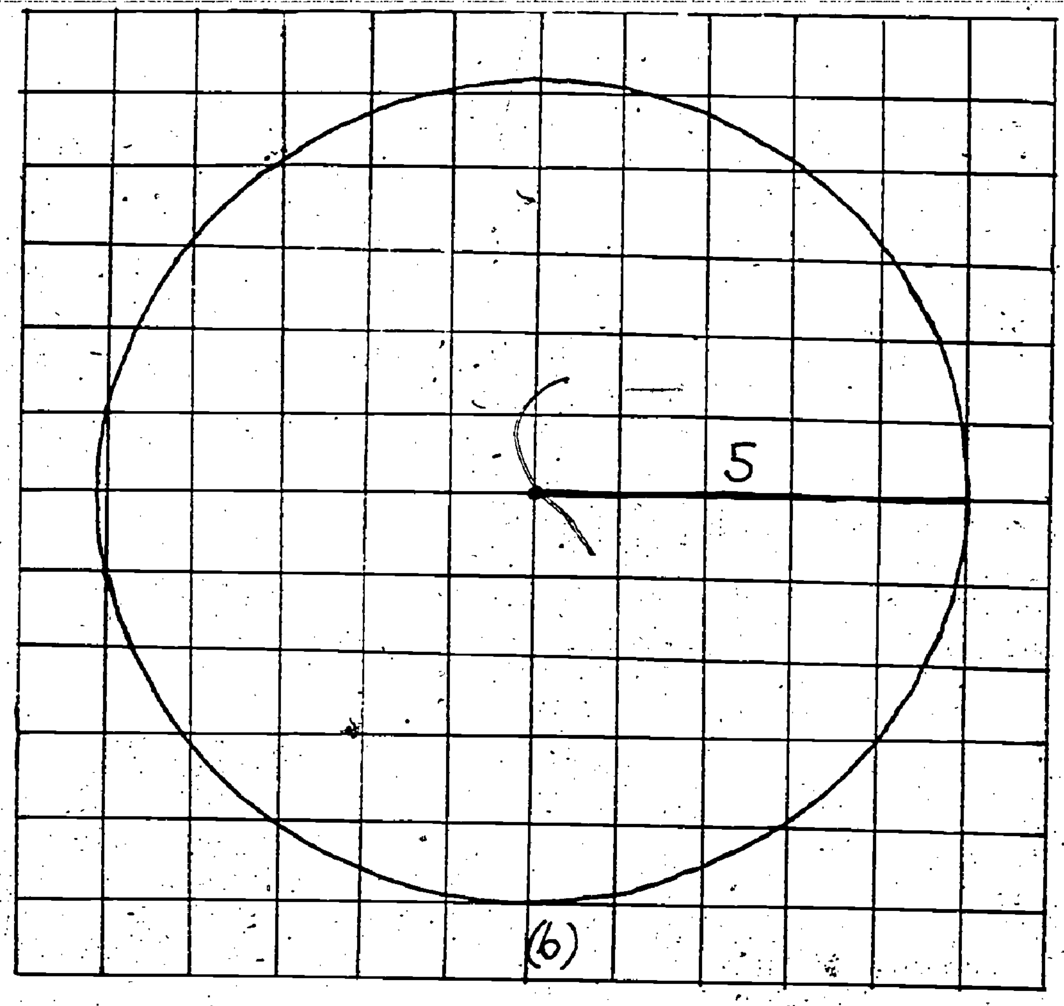
A. RELATING AREA TO RADIUS

MATERIALS: Circles of various size dittoed on graph paper. Take care with this to make your centers coincide with grid intersections and your radii to even multiples of your grid gauge.

Example:



252



DIRECTIONS: Have the children measure the radius in terms of the grid gauge. For example, in (a), it would be 3, and in (b), it would be 5. Then have the children estimate the corresponding area, using the squares in the graph paper as a unit of area. For example, in (a) you might get 28. Make a chart.

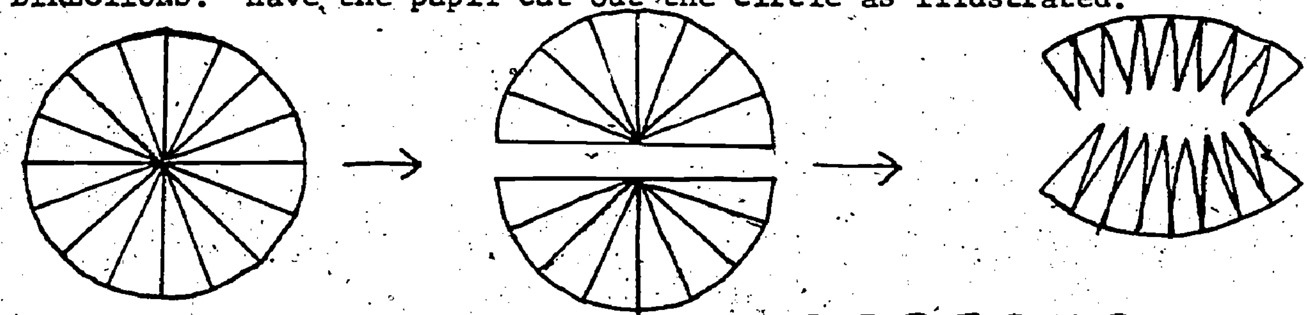
	r	r^2	A
(a)	3	9	28
(b)	5	25	78

This is where you can help the children guess that A is approximately $3\ 1/7 \cdot r^2$. Involve the students in a discovery dialogue. Lead to A is $\pi \cdot r^2$ where π is approximately 3 1/7 or 22/7.

B. AN AREA MODEL

MATERIALS: Ditto of a circle with a 3" radius--the circle should be divided into 16 parts. Label the radius r.

DIRECTIONS: Have the pupil cut out the circle as illustrated:



Then fit the two pieces together:

This shape resembles a rectangle. "What is its height?" (r). "What is its base?" (half of the circumference, πr). "So, what is its area?" ($r \times \pi r$ or πr^2).

C. FIND AREA OF CIRCLES

MATERIALS: A variety of circular objects.

DIRECTIONS: Have the pupils measure the radius of each circular object and then find the area of each using the formula $A = \pi r^2$. Check using a grid or graph paper tracing of object.

254

OBJECTIVE Identify and record least common multiple of two or more numbers.

COMMENTS "Least common multiple" is abbreviated LCM.

PRE-REQUISITE OBJECTIVES 95-J, 94-C

ACTIVITIES

A. MULTIPLES

MATERIALS: Paper and pencil.

DIRECTIONS: Teacher will ask children to list the multiples of two numbers between ______ and ______. Example: "List the multiples of 6 between 6 and 54 and the multiples of 9 between 9 and 72."

Children should do the following:

6, 12, 18, 24, 30, 36, 42, 48
9, 18, 27, 36, 45, 54, 63

Teacher will ask pupils to circle common multiples (common may have to be called "the same".)

6, 12, 18, 24, 30, 36, 42, 48
9, 18, 27, 36, 45, 54, 63

Teacher will ask, "Which is the least common multiple?" (Note: again least and common may require some explanation.)

Children should recognize 18 as the LCM. Repeat the procedure with many combinations until children identify LCM.

B. USE PRIME FACTORS TO FIND LEAST COMMON MULTIPLES

MATERIALS: Chalkboard.

DIRECTIONS: Have pupils compare the prime factors of pairs of numbers and find their least common multiple.

See example next page.

255

Page 294 blank.

Example: LCM

$\begin{cases} 6 = 2\cdot3 \\ 9 = 3\cdot3 \end{cases}$ $18 = 2\cdot3\cdot3$

$\begin{cases} 8 = 2\cdot2\cdot2 \\ 12 = 2\cdot2\cdot3 \end{cases}$ $24 = 2\cdot2\cdot2\cdot3$

$\begin{cases} 6 = 2\cdot3 \\ 10 = 2\cdot5 \end{cases}$ $30 = 2\cdot3\cdot5$

$\begin{cases} 10 = 2\cdot5 \\ 8 = 2\cdot2\cdot2 \end{cases}$ $40 = 2\cdot2\cdot2\cdot5$

$\begin{cases} 21 = 3\cdot7 \\ 5 = 5 \end{cases}$ $105 = 3\cdot5\cdot7$

Note: Prime factors that occur in both numbers occur only once for each pair in the least common multiple.

Example: $54 = 3\cdot3\cdot3\cdot2$ LCM
$81 = 3\cdot3\cdot3\cdot3$ $162 = 3\cdot3\cdot3\cdot3\cdot2$

25

OBJECTIVE Find the least common denominator of two fractions, and compare.

COMMENTS

PRE-REQUISITE OBJECTIVES 114-J, 99-C

ACTIVITIES

A. COMPARE LCD TO LCM

MATERIALS: Chalkboard.

DIRECTIONS: Find an equivalent fraction for 1/6 with the same denominator as an equivalent fraction for 2/9.

$$\frac{1}{6} = \frac{2}{12} = \frac{3}{18}$$

$$\frac{2}{9} = \frac{4}{18}$$

Have children do this and circle those fractions with common denominators. Note that the least common denominator is also the least common multiple of the two denominators. Do several examples.

B. USE LCD TO COMPARE UNLIKE FRACTIONS

MATERIALS: Models of fractions, ditto.

DIRECTIONS: Ask pupils to compare fractions with unlike denominators using the models. Which is bigger, 1/6 or 2/9?

Have pupils change fractions to equivalent fractions with least common denominator and compare.

Use the least common multiple to find the denominator.

$6 = 2 \cdot 3$

$9 = 3 \cdot 3$

LCM, LCD is

$2 \cdot 3 \cdot 3 = 18$

$$\frac{1}{6} = \frac{3}{18}$$

$$\frac{2}{9} = \frac{4}{18}$$

Which is larger? $\left(\frac{2}{9}\right)$.

257

Compare many fractions on the chalkboard using LCD.

Example:

$$\begin{array}{ccc} \frac{2}{3} & & \frac{3}{4} \\ \downarrow & \uparrow & \downarrow \\ \frac{8}{12} & < & \frac{9}{12} \end{array} \qquad \begin{array}{ccc} \frac{5}{6} & & \frac{1}{3} \\ \downarrow & \uparrow & \downarrow \\ \frac{5}{6} & > & \frac{2}{6} \end{array} \qquad \begin{array}{ccc} \frac{1}{2} & & \frac{4}{8} \\ \downarrow & \uparrow & \downarrow \\ \frac{4}{8} & = & \frac{4}{8} \end{array}$$

Assign a ditto with problems like these. Use <, >, or = to compare.

$\frac{5}{10} \bigcirc \frac{7}{14}$, $\frac{5}{8} \bigcirc \frac{10}{16}$, $\frac{3}{4} \bigcirc \frac{2}{3}$, $\frac{3}{12} \bigcirc \frac{6}{15}$.

OBJECTIVE Name fractions in the simplest form.

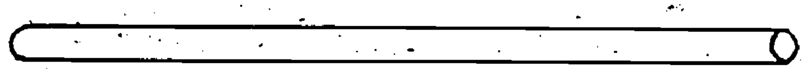
COMMENTS The <u>simplest form</u> of a fraction is the equivalent fraction with the smallest possible numerator and denominator. An understanding of equivalent fractions is necessary if pupils are to express fractions in simplest form (lowest terms). Once pupils understand this concept, they should experience no difficulty later; when addition and subtraction of unlike fractions are presented.

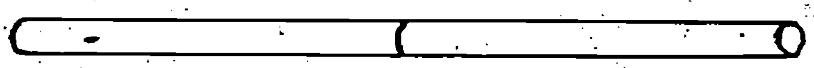
PRE-REQUISITE OBJECTIVES 99-C, 97-C

ACTIVITIES

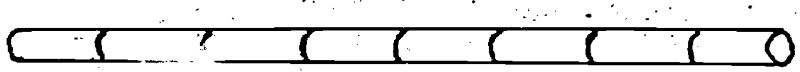
A. EQUIVALENT AND SIMPLE FRACTIONS: RODS

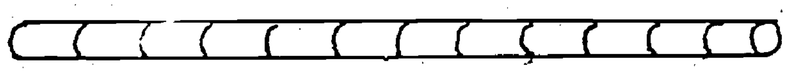
MATERIALS: Rods or drawings of rods divided into into fractions. Examples below (soda straws can be used for rods).

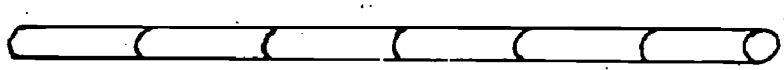
a.


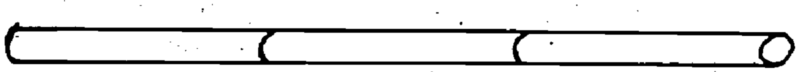
b.


c.


d.


e.


f.


g.


DIRECTIONS: 1. Use the rods or drawings above to help you replace each frame with a figure that will make the fraction equi alent.

a. $\frac{1}{4} = \frac{\ }{8}$ b. $\frac{2}{4} = \frac{\ }{8}$ c. $\frac{3}{4} = \frac{\ }{8}$

d. $\frac{1}{2} = \frac{\ }{4}$ e. $\frac{1}{2} = \frac{\ }{8}$ f. $\frac{1}{2} = \frac{\ }{4} = \frac{\ }{8}$

2. Find the simplest fractions using rods.

a. $\frac{2}{12} = \frac{\ }{6}$ b. $\frac{6}{12} = \frac{\ }{6}$ c. $\frac{4}{12} = \frac{\ }{6} = \frac{\ }{3}$

d. $\frac{8}{12} = \frac{\ }{6} = \frac{\ }{3}$

How many twelfths are equal to $\frac{1}{3}$ of rod C? $\frac{2}{3}$ of rod G? Are $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{4}{12}$ equivalent fractions? Are $\frac{8}{12}$, $\frac{4}{6}$, and $\frac{2}{3}$ equivalent fractions?

B. EQUIVALENT FRACTIONS

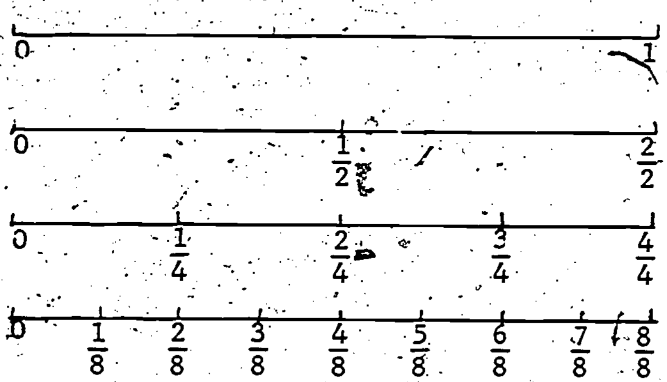
MATERIALS: Paper for folding.

DIRECTIONS: Have each pupil fold a piece of paper into halves and color one half of it. Then have them fold it into fourths and discover that ½ of the paper is also 2/4 of the paper. Have them fold the paper into eighths. Then they will discover that ½ of the paper is 4/8 of the paper.

C. EQUIVALENT FRACTIONS USING NUMBER LINE

MATERIALS: Masking tape may be used on the chalkboards marked off into twelfths, fourths, thirds, sixths, etc. You may also us. masking tape for the number lines measured off into wholes, halves, fourths, eighths, sixths, twelfths, thirds, etc. See Example next page.

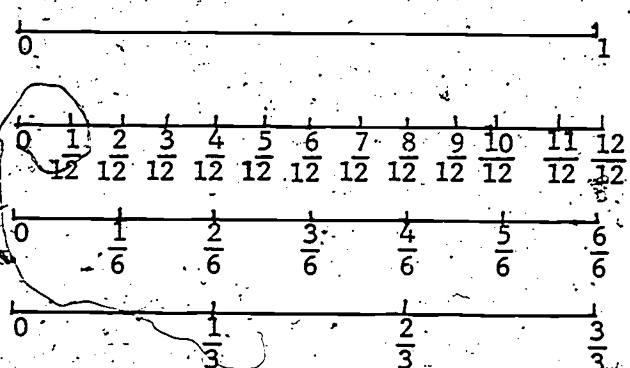
Example:



DIRECTIONS: This activity will give the entire class additional practice with fractions and equivalent fractions.

1. Have a pupil place 1/2 and 2/2 on a chalkboard model of the number line. Then have another pupil point out equivalent fractions while placing 1/4, 2/4, 3/4, and 4/4, in their correct positions. Use the eighths similarly.

2. Repeat step 1. but modify for thirds.



D. PRIME FACTORS TO SIMPLEST FRACTIONS

MATERIALS: Model two equivalent fractions like $\frac{7}{14}$ and $\frac{1}{2}$.

Write the prime factors for both numerator and denominator.

$$\frac{7}{14} = \frac{1 \cdot 7}{2 \cdot 7}$$

Note that both the numerator and the denominator can be grouped by 7. And 1 group of 7 out of 2 groups of 7 gives $\frac{1}{2}$.

Repeat:

$$\frac{6}{12} = \frac{1 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3} = \frac{1}{2}$$

Note: Both the numerator and denominator can be grouped by 2 and by 3. This leaves 1 part of 2.

$$\frac{9}{12} = \frac{3 \cdot 3}{2 \cdot 2 \cdot 3} = \frac{3}{4}$$

Note: Both the numerator and denominator can be grouped by 3. This leaves 3 of 4 parts.

Generalize: Common prime factors drop out.

262

OBJECTIVE Add fractions.

COMMENTS Record answers in simplest form. Stress that, in order to add, the denominators must be the same. Then the numerator of the sum is just the sum of the numerators.

PRE-REQUISITE 116-C, 115-C

ACTIVITIES

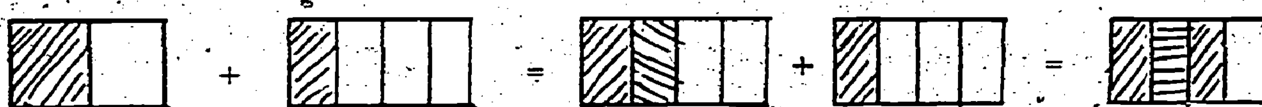
A. ADDING FRACTIONS

MATERIALS Paper, pencil, teacher-prepared ditto.

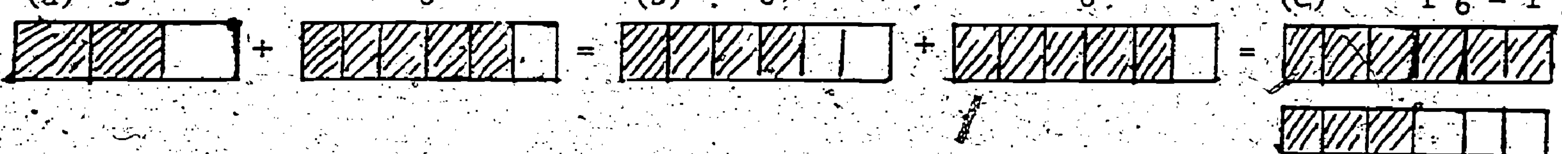
DIRECTIONS Example: $\frac{1}{2} + \frac{1}{4}$ $\frac{2}{3} + \frac{5}{6}$

Note to teacher: This addition activity involves addition of fractions where one denominator is a factor of the other.

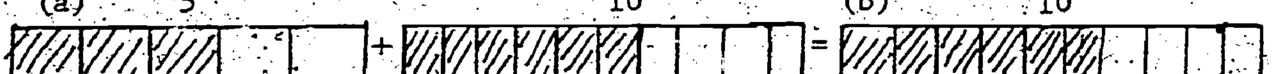
(a) $\frac{1}{2}$ + $\frac{1}{4}$ (b) $\frac{2}{4}$ + $\frac{1}{4}$ (c) $\frac{3}{4}$



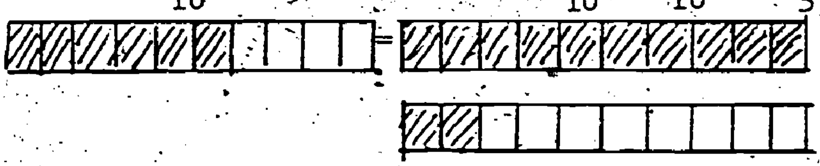
(a) $\frac{2}{3}$ + $\frac{5}{6}$ = (b) $\frac{4}{6}$ + $\frac{5}{6}$ = (c) $1\frac{3}{6} = 1\frac{1}{2}$



(a) $\frac{3}{5}$ + $\frac{6}{10}$ = (b) $\frac{6}{10}$ +



$\frac{6}{10}$ = $\frac{12}{10} = 1\frac{2}{10} = 1\frac{1}{5}$



(a) Teacher will ask children to shade the models to match written fractions.

(b) Teacher will ask children to make the fractions equivalent in size.

(c) Teacher will ask children to combine (add) shaded portions and record.

Note: The conclusion that <u>must</u> be reached here (before proceeding!) is that as long as the denominators are the same the numerator of the sum is found by adding the numerators.

B. ADDING FRACTIONS

MATERIALS Paper and pencil, teacher-prepared ditto.

DIRECTIONS: Pupil has had previous experience with common multiples, but review may be required at this time. Teacher prepares ditto of addition problems written both horizontally and vertically.

Example 1: $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}$

$$\begin{array}{rl} & \frac{3}{4} = \frac{9}{12} \\ + & \frac{5}{6} = \frac{10}{12} \\ \hline & \quad\;\; \frac{19}{12} \end{array}$$

Teacher will ask pupils to find LCM and record as denominator with corresponding equivalent numerator. Pupil will list multiples of 4 and 6 and circle the least (or smallest) or use prime factors.

Multiples: 4, 8, 12, 16, 20, 24
6, 12, 18

Factoring: $\left.\begin{array}{l} 4 = 2 \cdot 2 \\ 6 = 2 \cdot 3 \end{array}\right\}$ LCM $2 \cdot 2 \cdot 3 = 12$

Example 2: $\frac{3}{20} + \frac{4}{15} = \frac{9}{60} + \frac{16}{60} = \frac{25}{60} = \frac{5}{12}$

$$\begin{array}{rl} & \frac{3}{20} = \frac{9}{60} \\ + & \frac{4}{15} = \frac{16}{60} \\ \hline & \quad\;\; \frac{25}{60} = \frac{5}{12} \end{array}$$

284

Multiples: 20, 40, 60, 80, 100
15, 30, 60, 75

Factoring: $20 = 2 \cdot 2 \cdot 5$
$15 = 3 \cdot 5$

LCM
$2 \cdot 2 \cdot 3 \cdot 5 = 60$

Example 3: $\frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{6}{18} + \frac{3}{18} + \frac{2}{18} = \frac{11}{18}$

$$\begin{array}{rl} & \frac{1}{3} = \frac{6}{18} \\ & \frac{1}{6} = \frac{3}{18} \\ + & \frac{1}{9} = \frac{2}{18} \\ \hline & \frac{11}{18} \end{array}$$

Multiples: 9, 18, 27, 36
6, 12, 18, 24
3, 6, 9, 12, 18

OBJECTIVE Add mixed numbers, regroup and record in simplest form.

COMMENTS

PRE-REQUISITE OBJECTIVES 117-J

ACTIVITIES

A. WORKSHEET

MATERIALS: Paper and pencil, teacher-prepared ditto.

DIRECTIONS: Teacher prepares ditto of addition problems written both horizonally and vertically.

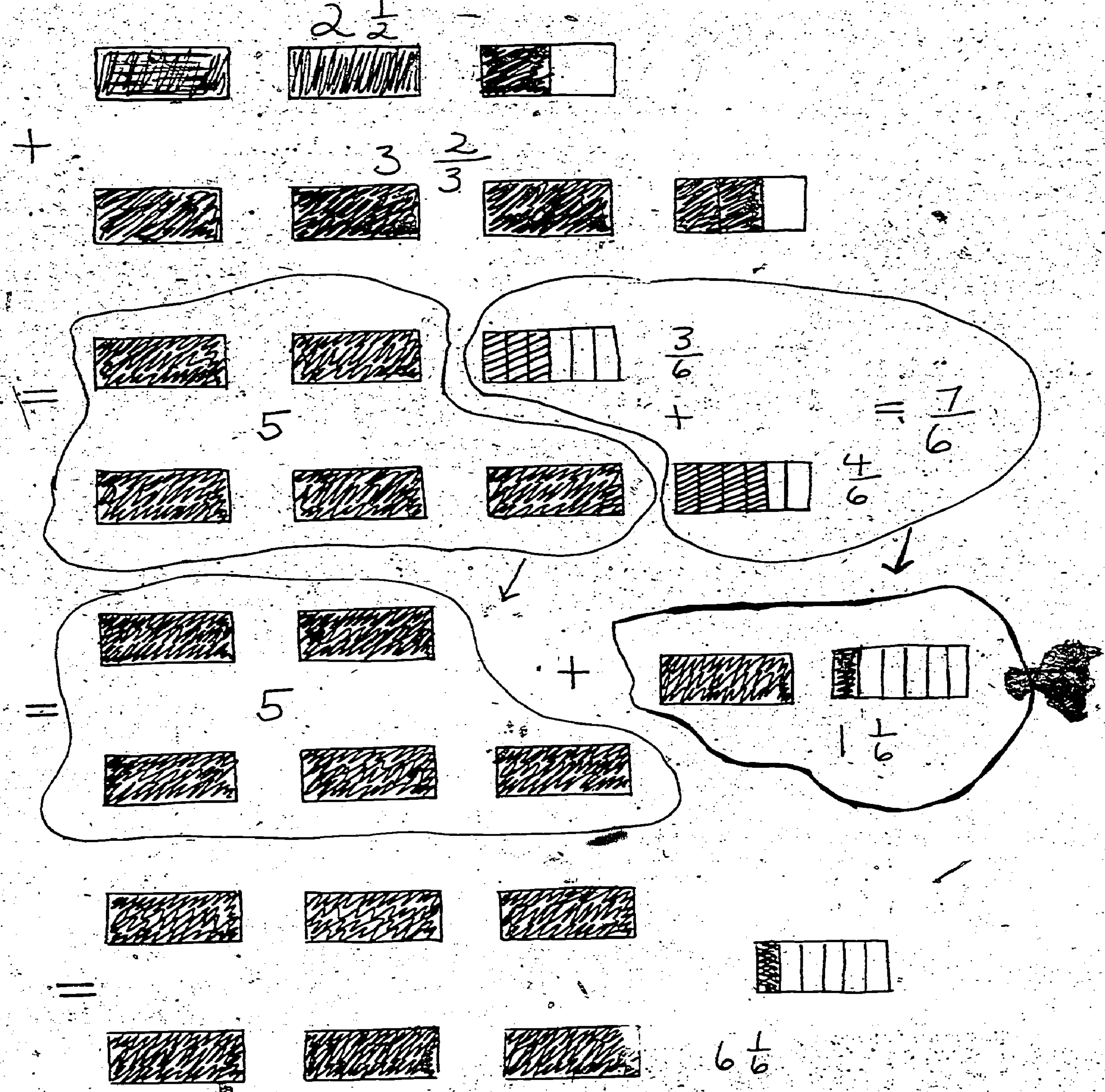
EXAMPLE:

$$2\frac{1}{2} + 3\frac{2}{3} = 2\frac{3}{6} + 3\frac{4}{6} = 5 + \frac{7}{6} = 5 + \frac{6}{6} + \frac{1}{6} = 5 + 1 + \frac{1}{6} = 6 + \frac{1}{6} = 6\frac{1}{6}$$

$$\begin{array}{rl} 2\frac{1}{2} = & 2\frac{3}{6} \\ + \ 3\frac{2}{3} = & 3\frac{4}{6} \\ \hline & 5\frac{7}{6} = 5 + \frac{6}{6} + \frac{1}{6} = 5 + 1 + \frac{1}{6} = 6\frac{1}{6} \end{array}$$

NOTE: This process should be modeled if pupils have difficulty.
(See next page.)

page 306 blank


2 1/2
+
3 2/3
=
5
3/6
+
4/6
= 7/6
=
5
+
1 1/6
=
6 1/6

OBJECTIVE Subtract fractions and record results in simplest form.

COMMENTS Stress that, as in addition, subtraction is only possible when the denominators are the same and only the numerators are used in the subtraction operation. Determining LCM should not be a problem at this point, but review may be required.

PRE-REQUISITE OBJECTIVES 116-C, 115-C

ACTIVITIES

A.

MATERIALS: Teacher-prepared ditto, paper and pencil.

DIRECTIONS: Teacher prepares ditto of subtraction problems written both horizonally and vertically.

EXAMPLE 1: $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

$$\begin{array}{rl} & \frac{5}{6} = \frac{10}{12} \\ - & \frac{3}{4} = \frac{9}{12} \\ \hline & \quad\;\; \frac{1}{12} \end{array}$$

Teacher will ask pupil to find LCM and record as denominator and record correct numerators.

EXAMPLE 2: $\frac{4}{15} - \frac{3}{20} = \frac{16}{60} - \frac{9}{60} = \frac{7}{60}$

$$\begin{array}{rl} & \frac{4}{15} = \frac{16}{60} \\ - & \frac{3}{20} = \frac{9}{60} \\ \hline & \quad\;\; \frac{7}{60} \end{array}$$

NOTE: Modeling may be required.

268

OBJECTIVE Subtract mixed numbers where regrouping is required.

COMMENTS Often fractions have to be renamed in order for the subtraction to be possible.

PRE-REQUISITE OBJECTIVES 119-S

ACTIVITIES

A.

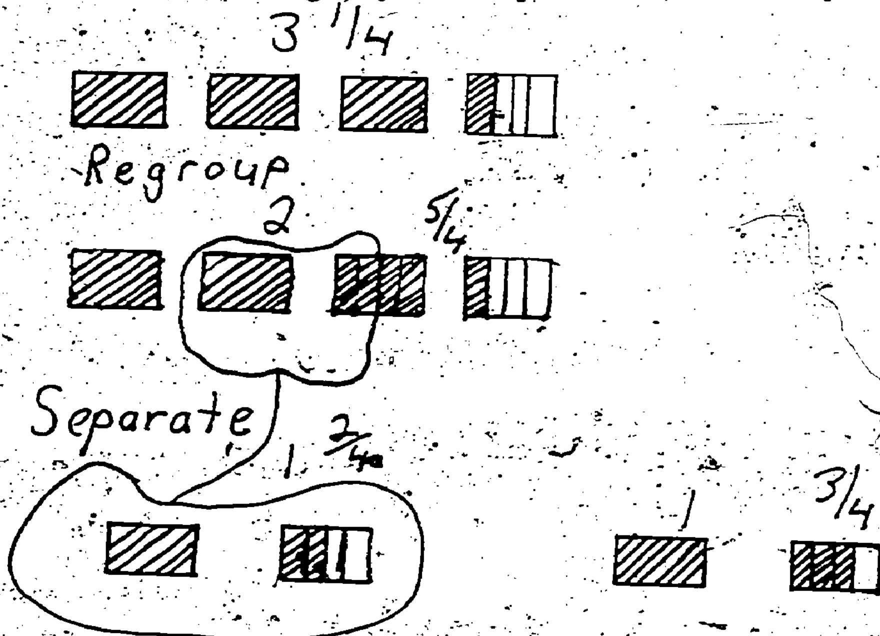
MATERIALS: Ditto prepared by teacher, paper and pencil.

DIRECTIONS: Teacher prepares ditto of subtraction problems written both horizontally and vertically.

EXAMPLE: $3\frac{1}{4} - 1\frac{1}{2} = 3\frac{1}{4} - 1\frac{2}{4} = 2 + (\frac{4}{4} + \frac{1}{4}) - 1\frac{2}{4} = 2\frac{5}{4} - 1\frac{2}{4} = 1\frac{3}{4}$

$$\begin{array}{rcrcrcr} 3\frac{1}{4} & = & 2 + \frac{4}{4} + \frac{1}{4} & = & 2\ \frac{5}{4} & = & 2\ \frac{5}{4} \\ -\ 1\frac{1}{2} & = & -\ 1\ \frac{2}{4} & = & -\ 1\ \frac{2}{4} & = & -\ 1\ \frac{2}{4} \\ \hline & & & & & & 1\ \frac{3}{4} \end{array}$$

NOTE: Modeling may be required. For example:



If Cuisenaire rods are available, they ~~can~~ be used very easily to model addition and subtraction of fractions.

Construction paper can be used also for making models.

260

Page 310 blank.

OBJECTIVE Compare more than 2 fractions with unlike denominators and like numerators.

COMMENTS Given equal numerators: The larger the denominator, the smaller the fraction; the smaller the denominator, the larger the fraction.

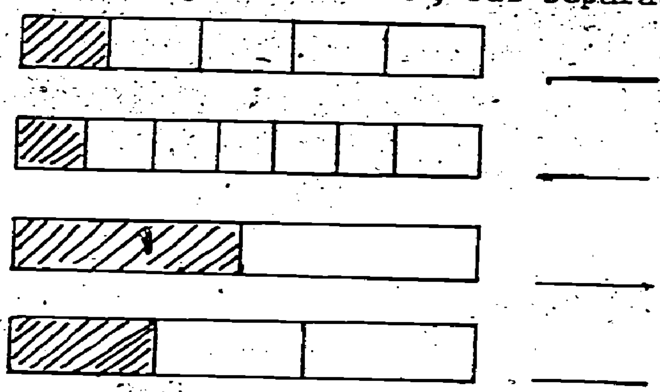
PRE-REQUISITE OBJECTIVES

ACTIVITIES

A.

MATERIALS: Teacher-prepared ditto, pencil.

DIRECTIONS: Teacher will need to prepare a ditto containing rectangles of the same length and width, but separated to represent various fractions.



The pupil must name the denominator for each rectangle. Rectangles will be shaded in to show the numerator. Pupils will record these.

Pupils can then compare the models and order the fractions from smallest to largest by looking at the size of the shaded areas.

Repeat with examples where each numerator is 2, then 3, etc. The pupils should conclude that when the numerators are equal, the largest fraction has the smallest denominator and the smallest fraction has the largest denominator.

270

e 312 blank.

3.

MATERIALS: Teacher-prepared ditto containing lists of fractions.

Example: $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{3}{8}$, $\frac{3}{5}$, $\frac{3}{9}$, $\frac{3}{10}$, $\frac{3}{4}$, $\frac{3}{6}$,

DIRECTIONS: Pupils are to list fractions by size-- from smallest to largest.

OBJECTIVE Divide by 1 digit divisors.

COMMENT Recall that in Objective 91-S the quotient had only one digit. In this objective the quotient should have two or more digits. Begin with two digit dividends. The examples illustrate problems with three digit dividends.

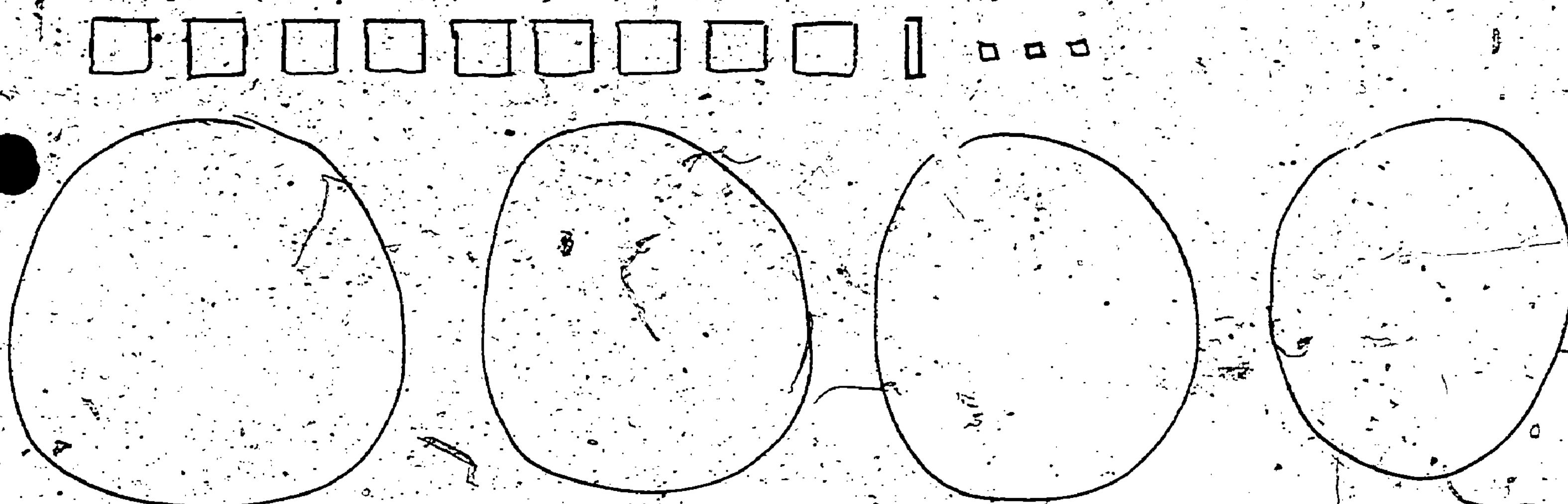
ACTIVITIES

A. PARTITIONING MODEL

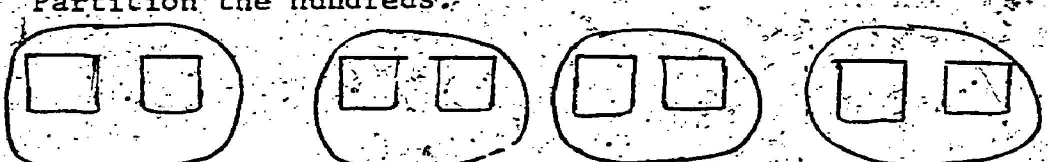
MATERIALS Place value models and set indicators.

DIRECTIONS When dividing by 4, say, "We will partition the dividend into 4 sets and the quotient will be the size of each of the sets." Note how nicely the recording of this process leads to the traditional division algorithm.

Example: $4\overline{)913}$



Partition the hundreds.

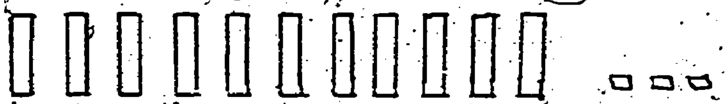


How many in each set? Two. Record $4\overline{)913}$

How many hundreds accounted for? Eight. Record

$$\begin{array}{r} 2 \\ 4\overline{)913} \\ 8 \\ \hline 1 \end{array}$$

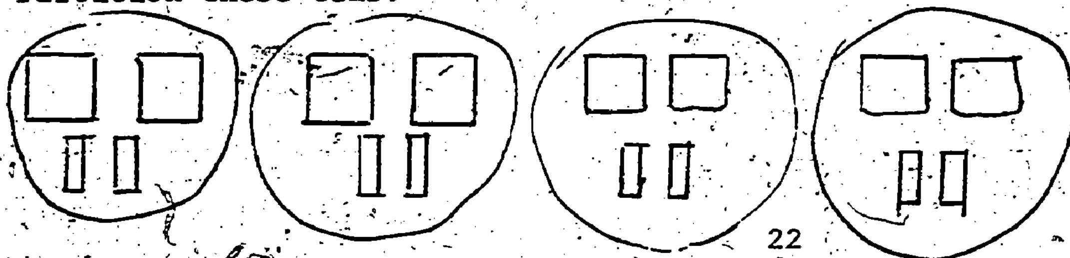
Cash the remaining hundred in for tens.



How many tens altogether? Eleven. Record $\begin{array}{r} 2 \\ 4\overline{)913} \\ \underline{8} \\ 11 \end{array}$

(This is why the "subtract and bring down the next digit" step makes sense!)

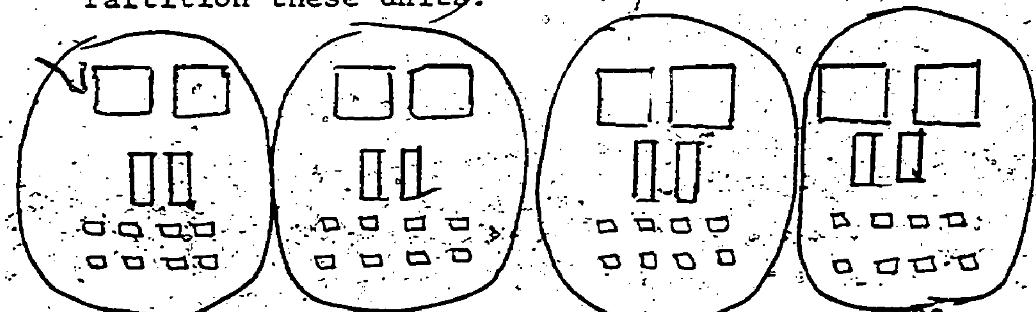
Partition these tens.



How many in each set? Two. Record $\begin{array}{r} 22 \\ 4\overline{)913} \\ \underline{8} \\ 11 \end{array}$

How many accounted for? Eight. Record $\begin{array}{r} 22 \\ 4\overline{)913} \\ \underline{8} \\ 11 \\ \underline{8} \\ 33 \end{array}$

Partition these units.



How many in each set? Eight. Record $\begin{array}{r} 228 \\ 4\overline{)913} \\ \underline{8} \\ 11 \\ \underline{8} \\ 33 \end{array}$

How many accounted for? Thirty-two. Record $\begin{array}{r} 228 \\ 4\overline{)913} \\ \underline{8} \\ 11 \\ \underline{8} \\ 33 \\ \underline{32} \end{array}$

How many left over? One. Record and indicate

```
   228 R1
4 /913
   8
   11
    8
    33
    32
     1
```

So the quotient is 228 with remainder 1.
Have the pupil do as many of these examples as is necessary. Take enough time.
Use 2, 3, 4 digit dividends.

B. REPEATED SUBTRACTION (PARTIAL QUOTIENT)

MATERIALS Chalkboard, pencil and paper.

DIRECTIONS This method is difficult to model concretely but by this time the pupil has had (hopefully) sufficient experience with multiplication to be able to estimate products.

Let's consider the same example as in activity A: 4 /913
First, have the pupil estimate whether 4 x various powers of ten will exceed 913. Is 4 x 1 too big? No. Is 4 x 10 too big? No. Is 4 x 100 too big? No. Is 4 x 1000 too big? Yes. So we know the quotient is somewhere between 100 and 1000. Try to bracket the quotient between consecutive multiples of 100. Is 4 x 100 too big? No. Is 4 x 200 too big? No. Is 4 x 300 too big? Yes. So we know the quotient is between 200 and 300. This is where the first recording occurs

```
    200
4 /913
   800
   113
```

We now must do the same thing with 113 as we did with 913 but with <u>one important difference</u>. The previous steps have established 1) that the quotient is less than 1000 and, then, 2) that the quotient is between 200 and 300. So we know that when we have subtracted 200 fours there are <u>less than 100 fours</u> left to be subtracted. That is, we only have to concentrate on tens now. The next step then is to locate the rest of the quotient between consecutive multiples of 10. Is 4 x 10 too big? No. Is 4 x 20 too big? No. Is 4 x 30 too big? Yes. So we know that the rest of the quotient is between 20 and 30. Record the 20 and subtract 4 x 20

```
     20
    200
4 /913
   800
   113
    80
    33
```

We are now down to the units part of the quotient and it is easy to see that there are 8 fours in 33 with a remainder of 1. Record as

$$
\begin{array}{r|l}
\multicolumn{2}{r}{\left.\begin{array}{r} 8 \\ 20 \\ 200 \end{array}\right\} \; 228 \text{ R } 1} \\
\cline{2-2}
4 & 913 \\
 & \underline{800} \\
 & 113 \\
 & \underline{80} \\
 & 33 \\
 & \underline{32} \\
 & 1
\end{array}
$$

NOTE: This is a drawn-out explanation of the process. In actual practice it will go quickly. The idea is to "generate" the quotient one digit at a time starting with the largest place value, in our case, first the hundreds, then the tens, then the units. All that is required of the pupil is to know 1) his multiplication facts and 2) that multiplying by ten or one hundred, etc., means that one zero or two zeros, etc. are appended to the basic product.

OBJECTIVE Divide by 2 digit divisors.

COMMENTS As the partitioning model is impractical with a 2-digit divisor, the repeated subtraction method will be used here.

PRE-REQUISITE 122-S

ACTIVITY

A. REPEATED SUBTRACTION

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS: This is just a modification of Objective 122-S Activity B. A pre-requisite skill necessary for this, or any other, method for division by 2 digit divisors is the ability to estimate multiples (up to 9) of a number. For a single number these are just the multiplication facts, but for a number with 2 or more digits this ability takes time and practice to develop.

Example: 24 / 6313

As before, in Activity B of the previous objective, we ask: Is 24 x 1 too big? No. Is 24 x 10 too big? No. Is 24 x 100 to big? No. Is 24 x 1000 too big? Yes. So we know the quotient is between 100 and 1000. Then we ask: Is 24 x 100 too big? No. Is 24 x 200 too big? No. Is 24 x 300 too big? Yes. So the quotient is between 200 and 300. Record and subtract.

```
       200
  24 / 6313
       4800
       1513
```

We now find the tens place of the quotient: Is 24 x 10 too big? No. Is 24 x 20 too big? No. Is 24 x 40 too big? No. Is 24 x 50 too big? No. Is 24 x 70 too big? Yes. So the quotient h[illegible] Record and subtract.

```
        60
       200
  24 / 6313
       4800
       1513
       1440
         73
```

278

We now find the units place of the quotient. Is 24 x 1 too big? No. Is 24 x 2 too big? No. Is 24 x 3 too big? No. Is 24 x 4 too big? Yes. So the quotient has 3 units. Record and subtract.

```
          3
         60
        200   263  R1
  24 | 6313
       4800
       1513
       1440
         73
         72
          1
```

VARIATION: The previous example requires that the pupil have a pretty good grasp of the multiples of 24 up to 24 x 9. If the pupil cannot compute these in his head, a table could be made ahead of time. This, however, is quite tedious and a simpler, but less precise, variation could be used. The pupil could mentally "round-up" 24 to 30 and "round-down" 6313 to 6000 and use 30 and 6000 for the partial quotient estimations for the hundreds place. This will give an estimate of 200 which is correct.

```
        200
  24 | 6313
       4800
       1513
```

At the next step "round-up" 24 to 30 and "round-down" 1513 to 1500. This will give an estimate of 50 for the tens part of the quotient.

```
         50
        200
  24 | 6313
       4800
       1513
       1200
        313
```

Then "round-up" 24 to 30 and "round-down" 313 to 300. This will give an estimate of 10.

```
        10
        50
       200
   ________
24 | 6313
     4800
     ----
     1513
     1200
     ----
      313
      240
      ---
       73
```

At this stage "rounding" probably will not need to be done because the pupil will know that 3 is correct. Record and the problem is complete.

```
         3
        50
       200
   ________
24 | 6313
     4800
     ----
     1513
     1200
     ----
      313
      240
      ---
       73
       72
       --
        1
```

OBJECTIVE Use the traditional algorithm with one and two digit divisors.

COMMENTS This algorithm should only be used when the pupil thoroughly understands place value and has built up a skill at estimating.

PRE-REQUISITE OBJECTIVES 123-S, 122-S

ACTIVITIES

A. TRADITIONAL ALGORITHM

In the example given in Objective 123-S, the student would record

```
       2
24 ) 6313
     48
     15
```

then

```
       26
24 ) 6313
     48
     151
     144
      73
```

then

```
       263 R.1
24 ) 6313
     48
     151
     144
      73
      72
       1
```

279

Page 322 blank.

This is simply a shortened version of the repeated subtraction method. Note that 1). zeros are not recorded so that an understanding of place-value is necessary for an understanding of the procedure

2). accurate estimating is necessary or else there will be a lot of erasing.

Lots of practice is needed to do well with this algorithm.

OBJECTIVE Divide with 3 and 4 digit divisors.

COMMENT This is an extention of already known skills. Skillful pupils should be encouraged to experiment and develop their own techniques. The teacher can provide problems, motivation, encouragement and help. Unskillful pupils need not tread such treacherous waters!

PRE-REQUISITE 124-S, 123-S

ACTIVITIES

A. REPEATED SUBTRACTION METHOD (PARTIAL QUOTIENT)

MATERIALS Chalkboard, pencil and paper, small eraser.

DIRECTIONS Modify Activity A, Objective 123-S using 3 and 4 digit divisors. Much more skill in estimating will be needed here.

B. TRADITIONAL ALGORITHM

MATERIALS Chalkboard, pencil and paper, large eraser.

DIRECTIONS Modify Activity A, Objective 123-S using 3 and 4 digit divisors. Much, much more skill in estimating will be needed here. Otherwise, a lot of erasing will be done!

OBJECTIVE Check division by multiplying.

COMMENT This may have been done before, but due to the level of abstraction at this point, it may be best to go over it again.

PRE-REQUISITE

ACTIVITIES

A. CHECKING

MATERIALS Chalkboard, teacher-made ditto, etc.

DIRECTIONS Teacher should discuss with class such problems as 3 x 4 = 12 and 12 ÷ 4 = 3.

Teacher-made worksheet would be for practicing.

Example: <u>If</u>	7 x 4 = 28	<u>Then</u>	28 ÷ 7 = ______
	6 x 8 = 48		48 ÷ ______ = 8
	521 x 63 = 32823		32823 ÷ ______ = 63
	45 ÷ 9 = 5		9 x ______ = 45
	49 ÷ 7 = 7		7 x ______ = 49
	1431 ÷ 27 = 53		53 x ______ = 1431

Here we want the pupil to come away with the idea that one way to find an answer to a division problem is to find a number which when multiplied by the divisor will give the dividend.

Example: The answer to 12 ÷ 2 is 6 since 2 x 6 = 12.

B. EXAMPLES WITH REMAINDERS

MATERIALS Chalkboard, dittos.

DIRECTIONS Practice checking division problems with a remainder. E.g., if 1102 ÷ 26 = 42 with remainder 10, then (26 x 42) + 10 = 1102. Do several examples.

282

Page 326 blank.

GENERAL REMARKS The next sequence of objectives deals with decimal fractions and operations with these in combination with whole numbers.

127-C Reference Types of Numbers (Decimals)

OBJECTIVE Recognize "th" sound when naming fractions.

COMMENTS Many Navajo pupils have trouble hearing the "th" on the end of a number. Ten and tenth sound the same.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. LISTENING AND PRONOUNCING

MATERIALS: Chalkboard, pencils and paper.

DIRECTIONS:

a) Send several pupils to the chalkboard. As you dictate, pupils write the numerals on the board-- 1/8, eight, 1/10, ten, 100, 1/100. Repeat the dictation to encourage pupils to listen for the "th".

b) Write the words on the chalkboard. Compare the spellings. Dictate, have pupils spell, self-check to see if they hear the difference.

B. DICTATE AND MODEL

MATERIALS: Paper and pencils.

DIRECTIONS: Dictate 3/10 and 3 tens. Have pupils do the models on the chalkboard to show each. Dictate 400 and 4/100; 5000 and 5/1000; etc.

283

328 blank

OBJECTIVE Change common fractions to equivalent fractions with denominator of a power of ten.

COMMENTS This should be done with halves, fourths, fifths, eighths--avoid thirds, sevenths, ninths. Denominators must have factors of 2 and/or 5 since only these fractions will be equivalent to fractions with a power of ten in the denominator.

PRE-REQUISITE OBJECTIVES 99-C

ACTIVITIES

A. COMMON FRACTIONS TO EQUIVALENT FRACTIONS

MATERIALS: Chalkboard, teacher prepared dittos.

DIRECTIONS: Make ditto or write on chalkboard the following fractions and have the pupils change the fractions to equivalent fractions using a denominator that is a power of ten:

$\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$ $\qquad$ $\frac{3}{4} = \_\_ = \frac{\quad}{100}$

$\frac{1}{4} = \_\_ = \frac{\quad}{100}$ $\qquad$ $\frac{1}{5} = \frac{\quad}{10} = \frac{\quad}{100}$

$\frac{4}{5} = \_\_ = \frac{\quad}{1000}$ $\qquad$ $\frac{3}{20} = \frac{\quad}{100}$

$\frac{7}{40} = \_\_$ $\qquad$ $\frac{17}{50} = \_\_$

28

330 blank.

OBJECTIVE Convert fractions that have denominators of powers of ten to decimal notation.

COMMENT The decimal point (.) may be read as, or called, "and" or "point". Since decimal numbers are just different names for fractions, we use already established properties of numbers expressed in fraction form to derive operational procedures for numbers expressed in decimal form.

PRE-REQUISITE 128-C

ACTIVITIES

A. DECIMAL POINT

MATERIALS Chalkboard

DIRECTIONS Write some fractions with denominators of thousand, hundred and ten. Tell the pupils they can rewrite them as decimals. Illustrate.

$\frac{31}{100} = .31$ read "31 hundredths" or "point 31"

$\frac{28}{100} = .28$ $\qquad \frac{8}{10} = .8$ $\qquad \frac{642}{1000} = .642$

Send students to the chalkboard. Have them write decimal fractions and say the names.

B. FRACTION DOMINOES

MATERIALS Cut "dominoes" from tagboard, using pieces 2" by 4". On one side write a common fraction, and on the other side a decimal fraction. The items on the two sides of a single domino will not be equal. Make four copies of each domino.

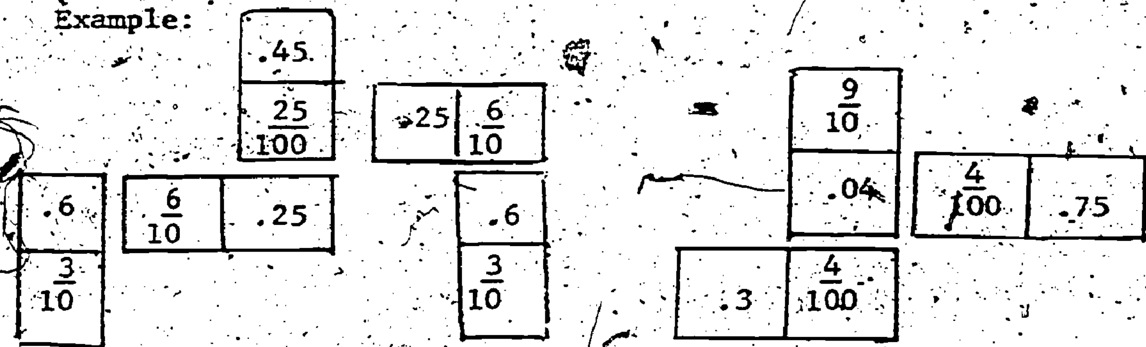
Example:

$\frac{3}{10}$	.6
$\frac{6}{10}$	.5
$\frac{7}{10}$	.8
$\frac{8}{10}$	.30
$\frac{25}{100}$	.45
$\frac{45}{100}$	.75

Page 332 blank.

DIRECTIONS Here is a fraction game that is played like regular dominoes. Place all the dominoes face down in the center of the table. Four players may draw four cards each. The first player may place any one of his dominoes in the center, and succeeding players may match the decimal fraction or common fraction with its equivalent (demonstrate).

Example:



If a person cannot play, he draws another card from the center, and play passes on. The first player to use all his dominoes is the winner.

C. CONVERTING FRACTIONS TO DECIMALS

MATERIALS Chalkboard or teacher-made dittos.

DIRECTIONS Make ditto or write on the chalkboard the following fractions and have the pupils change them into decimal numerals:

$\frac{47}{100}$ (.47) ; $5\frac{1}{2}$ (5.2) ; $17\frac{3}{4}$ (17.75) ; $\frac{9}{1000}$ (.009)

$43\frac{3}{8}$ (43.375) ; $61\frac{15}{40}$ (61.375) ; $\frac{17}{10000}$ (.0017)

$29\frac{13}{20}$ (29.65) ; $509\frac{21}{80}$ (509.2625)

D. DECIMAL PLACES EXPRESSED BY EXPONENTS

MATERIALS Chalkboard

Note: The digits to the right of the decimal point name the numerator of the fraction, and the number of such digits names the denominator in powers of ten.

Example: .217 denotes numerator 217 and denominator 10^3 or 1000, i.e., $\frac{217}{1000}$.

DIRECTIONS Oral exercise -- Powers of ten denoted in the denominator of the following decimals.
Ask "What power of ten is denoted in the denominator of each decimal below?"

.27 (10^2)	.490 (10^3)	.0576 (10^4)
.0037 (10^4)	.57001 (10^5)	.236005 (10^6)

E. USING "ZERO" IN DECIMALS

MATERIALS Chalkboard

DIRECTIONS Look at decimals with zeroes.

$$.0037 = \frac{37}{10000}$$

Note: The zeroes are necessary to keep the places in the denominator straight. Zeroes to the right don't change the value of the decimal.

$$7.0 = 7\frac{0}{10} \text{ or } 7.00 = 7\frac{0}{100}$$

Give lots of practice in this area.

287

OBJECTIVE Add decimal numerals.

COMMENTS Be sure that, when adding, the numerals line up so that the place-value in each column is the same. This can be done by aligning the decimal points.

PRE-REQUISITE OBJECTIVES 129-C

ACTIVITIES

A.

MATERIALS: Chalkboard and ditto sheets.

DIRECTIONS: Add:

a.	b.	c.	d.
.34	1.237	.436	2.36
+.45	+2.051	+.023	+24.5

NOTE: We may place 0's in decimal numerals so that they have the same number of decimal places.

EXAMPLE:

3.42 + .234 + 13.1	3.42	or	3.420
	.234		.234
	13.1		13.100
	16.754		16.754

B. EXERCISES

MATERIALS: Ditto sheets

DIRECTIONS: Add:

a.	b.	c.
4.3	17.346	30.489
+2.91	+11.86	+1.346

.3	.24	.304	.23	26.3	15.03	2.14
+.5	.13	.12	.408	1.24	2.114	.037

.372	.146	14.32	19.48	9.11
13.4	.23	.037	.035	.037
	2.1	1.7	1.7	24.87

.52 + .304 + 2.78 + 24.1 =

6.2 + 5.1 + 123 + .426 =

50 + 2.314 + .0062 + 5.34 =

.246 + .1032 + 15.1 + 256.4 + 1.0003 =

page 336 blank.

OBJECTIVE Subtract with decimal numerals.

COMMENTS As with addition, the important thing to remember in subtraction using decimal numerals is to align the decimal points.

PRE-REQUISITE OBJECTIVES 130-J, 129-C

ACTIVITIES

A. "REGROUPING"

MATERIALS: Chalkboard.

DIRECTIONS: Place on the chalkboard such examples as

$$\begin{array}{r} 39.62 \\ -\ 26.896 \\ \hline \end{array}$$

Emphasize the need to put in a zero either actually or mentally, depending on the abilities of your class.

Example:
Pat wanted to subtract 2.368 from 28.42.
a. First she lined up the decimal point.
b. Then she placed a 0 like this.

$$\begin{array}{r} 28.42 \\ -\ 2.368 \\ \hline \end{array} \qquad \begin{array}{r} 28.420 \\ -\ 2.368 \\ \hline \end{array}$$

Do the subtraction.

B. "SUBTRACT AND CHECK"

MATERIALS: Teacher prepared dittos.

a. 14.324 - 2.103 =
b. 5.72 - .68 =
c. 2.5 - 1.462 =
d. 3.024 - 1.83 =
e. $\begin{array}{r} 1.58 \\ -1.104 \\ \hline \end{array}$
f. $\begin{array}{r} 13.25 \\ -10.5 \\ \hline \end{array}$
g. $\begin{array}{r} 3.617 \\ -.0048 \\ \hline \end{array}$
h. $\begin{array}{r} .48 \\ -.23 \\ \hline \end{array}$
i. 1.46 - .23 =
j. .12 - .000016 =

289

page 338 blank.

OBJECTIVE Use the dollar sign and decimal point.

COMMENTS

PRE-REQUISITE OBJECTIVES 131-S, 130-J

ACTIVITIES

A. CHALKBOARD ACTIVITY

MATERIALS: Chalkboard

DIRECTIONS: First demonstrate writing dollars and cents. Have pupils tell which numeral is in each place--hundreds, tens, units, tenths, and hundredths. Second, send several pupils at a time to the chalkboard. Have them write dollars and cents from dictation.

B. DRILL ACTIVITY

MATERIALS: Problem sheets.

DIRECTIONS: Write application problems. Adding money, subtracting--have pupils solve. From your examples, have pupils write and solve their own problems.

ge 340 blank.

OBJECTIVE Multiply with decimals.

COMMENTS

PRE-REQUISITE OBJECTIVES 129-C, 107-C

ACTIVITIES

A. TENTHS TIMES A WHOLE NUMBER

MATERIALS: Chalkboard.

DIRECTIONS: Write 2 x .3 = ____ on the chalkboard.

2 x .3 = 2 x 3/10 = 6/10 = .6

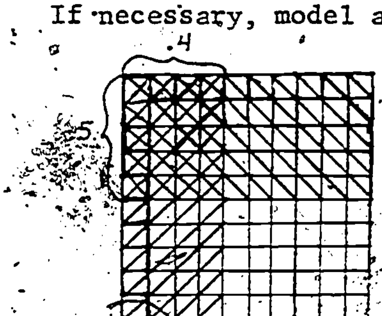
3 x .4 = 3 x 4/10 = 12/10 = 1.2

B. TENTHS TIMES TENTHS

.2 x .3 = 2/10 = 3/10 = 6/100 = .06

.4 x .5 = 4/10 x 5/10 = 20/100 = .20

If necessary, model as in Objective 107-J.



The cross-hatched area is

$\frac{.20}{100}$ = .20

291

page 342 blank.

Note: The sum of the number of places after the decimal in the multiplier and the multiplicand is equal to the number of places in the product. Pupils are to work many problems vertically and horizontally.

Example: .5 x .6 = .30
.20 x .15 =
.125 x .10 =

$$\begin{array}{r} .8 \\ \underline{x\ .5} \\ .40 \end{array} \qquad \begin{array}{r} .30 \\ \underline{x .15} \end{array} \qquad \begin{array}{r} .215 \\ \underline{x\ \ .12} \end{array}$$

C. EXTEND UNDERSTANDING OF DECIMAL MULTIPLICATION

MATERIALS: Chalkboard.

DIRECTIONS: Copy these charts on the chalkboard.

.4 x 1 = .4	tenths x units = tenths
.4 x .2 = .08	tenths x tenths = hundredths
.04 x 1 = ____	hundredths x units = ____________
.04 x .2 = ____	hundredths x tenths = ____________
.04 x .04 = ____	hundredths x hundredths = ____________

Have the pupils look the chart over. Beside the chart write this chart which pupils should know well and be able to help write.

tens x units = tens
tens x tens = hundreds
hundreds x units = hundreds
hundreds x tens = thousands
hundreds x hundreds = ten thousands

Ask pupils if they see any patterns. Discuss answers. Finish the first two charts. Look at the first chart. Ask if pupils see any patterns in the chart. Lead them to see a pattern in the number of places after the decimal. Compare sides of the equation. Supply practice dittos and workbook pages.

292

OBJECTIVE Divide fractions.

COMMENT

PRE-REQUISITE 115-C, 90-S

PRE-ACTIVITY

Review A ÷ B as "How many B's in A?" This is the measurement model for division. 12 ÷ 4: "How many 4's in 12?" Give many examples using whole numbers.

Examples:		How many 4's in 12	3
		2's in 10	5
		2's in 8	4
		10's in 20	2
etc.	Extend to (5 ÷ 2)	2's in 5	5/2
	(2 ÷ 3)	3's in 2	2/3
	(7 ÷ 5)	5's in 7	5/7

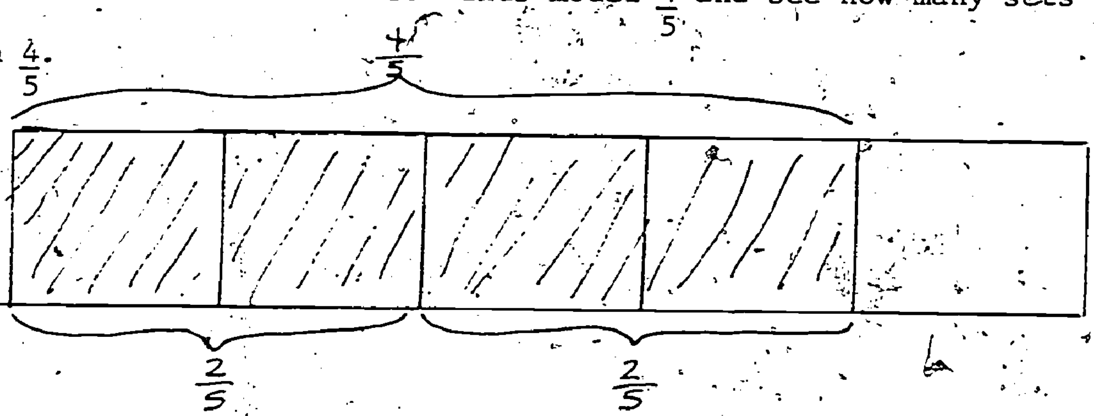
ACTIVITIES

A. MODELING DIVISION

MATERIALS Cuisenaire rods, egg cartons, construction paper or any material you have used to model fractions; paper and pencil.

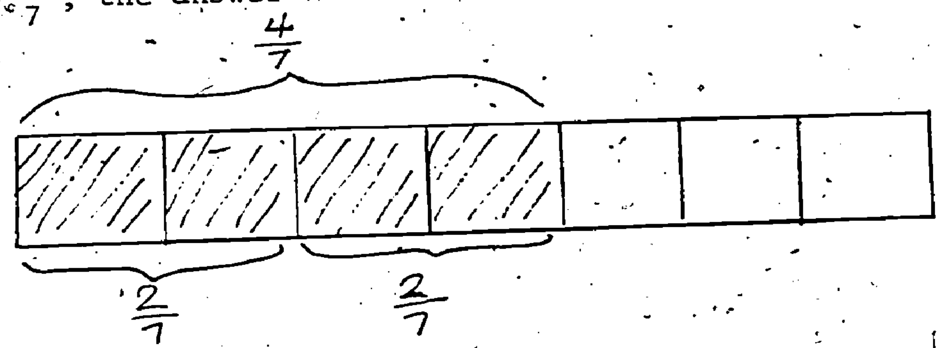
DIRECTIONS Review the division sign, ÷, as used in Activity C, Objective 90-S.

1. To start division of fractions, choose fractions that will have a whole number answer. For example, $\frac{4}{5} \div \frac{2}{5}$. This can be easily modeled and will lead to a nice technique for dividing fractions.
   First remind the children that the denominator of a fraction indicates its type only. Thus we are asking in $\frac{4}{5} \div \frac{2}{5}$, how many two-fifths are contained in four-fifths. Thus model $\frac{4}{5}$ and see how many sets of $\frac{2}{5}$ are in $\frac{4}{5}$.



There are 2 sets of $\frac{2}{5}$ in $\frac{4}{5}$. Point out that as long as the types (denominators) are the same, it is nothing more than asking how many sets of 2 are in 4. Thus $\frac{4}{5} \div \frac{2}{5} = 2$.

Now, if we use $\frac{4}{7} \div \frac{2}{7}$, the answer is still 2 and relates basically to how many sets of 2 in 4.



Do a number of examples of this kind, ultimately, leading to the idea that as long as the denominators (types) are alike, then all we have to do is divide the last numerator into the first as we have done with whole numbers.

2. Now consider problems where the denominators are not alike. Our first step will be to find the LCD and write the problem in this form and then do what we did in (1) above.

Example: $\frac{2}{3} \div \frac{4}{5}$ $\qquad$ $\frac{2}{3} \div \frac{4}{5} = \frac{10}{15} \div \frac{12}{15} = 10 \div 12 = \frac{10}{12} = \frac{5}{6}$

This example did not come out even, in the sense that the division gave a fraction as an answer. If the children have difficulty with this, go back and select problems where the answers will be whole numbers.

Example:

$\frac{2}{7} \div \frac{2}{21}$ $\qquad$ $\frac{2}{7} \div \frac{2}{21} = \frac{6}{21} \div \frac{2}{21} = 6 \div 2 = 3$

Continue to model as needed, but push (gently) toward the idea that if fractions have a common denominator, one need only divide the numerator of dividend by numerator of divisor in linear format.

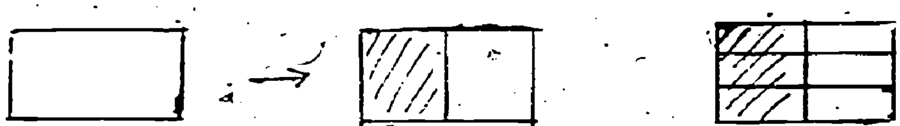
B. ALTERNATE APPROACH

MATERIALS Same as Activity A in this objective.

DIRECTIONS In Activity A, division was presented from the subtractive or measuring point of view. However, "divide" can be a command to partition a set into a number of parts. For example, dividing by 3, as we have used it, means to find out how many sets of size 3 are contained in a given set. It could mean, however, partition the set into 3 parts and find out how many in each part. This partitioning method works well when the divisor is a whole number (and it doesn't work at all when the divisor isn't a whole number!) Consider the following example:

$$\frac{1}{2} \div 3$$

Model $\frac{1}{2}$ and partition it into 3 parts. Ask, "How much in each part?" The answer is $\frac{1}{6}$.



unit $\qquad$ $\frac{1}{2}$ $\qquad$ $\frac{1}{6}$ of the original unit in each part.

OBJECTIVE Divide mixed numbers and record answers in simplest form.

COMMENTS

PRE-REQUISITE 134-S

ACTIVITIES

A. WORKSHEET

MATERIALS Paper and pencil.

DIRECTIONS (Note to teacher: Stress that problems such as $2\frac{1}{2} \div 3\frac{2}{3}$ will be read as "How many $3\frac{2}{3}$'s in $2\frac{1}{2}$?" Modify Activity A, Objective 134-S.

Present a problem such as $2\frac{1}{2} \div 3\frac{2}{3}$.

Pupils will rename as $\frac{5}{2} \div \frac{11}{3}$ and read as "How many $\frac{11}{3}$ in $\frac{5}{2}$?"

Pupils will find a common denominator and rewrite the problem as $\frac{22}{6}$'s in $\frac{15}{6}$?", which is the same as "How many 22's in 15?" Answer: $\frac{15}{22}$.

Pupils should have experience with many problems of this type and identifying. $A \div B$ as "How many B's in A?"

Example: $4\frac{1}{8} \div 1\frac{1}{3}$ How many $1\frac{1}{3}$'s in $4\frac{1}{8}$?

Rename: How many $\frac{4}{3}$'s in $\frac{33}{8}$?

Common denominator -- How many $\frac{32}{24}$ in $\frac{99}{24}$? How many 32's in 99?

Answer: $\frac{99}{32}$ or $3\frac{3}{32}$

29

e 348 blank

OBJECTIVE Change leftovers (remainders) in division problems to a fraction.

COMMENTS

PRE-REQUISITE OBJECTIVES 123-S

ACTIVITIES

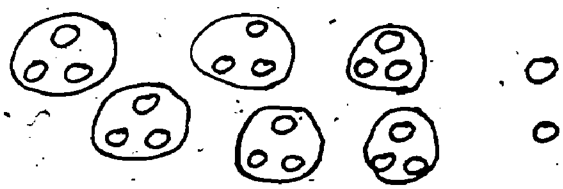
A. MODELING

MATERIALS: Counters

DIRECTIONS: Work simple division problems and work with models.

How many 3's in 20? 6 with 2 units left. How many 3's in 2? Only 2/3 of 3.

$$\begin{array}{r} 6 \\ 3\overline{)20} \\ \underline{18} \\ 2 \end{array}$$



So there are 6 and 2/3 threes in 20 or 20÷3 = 6 2/3.

Do more examples.

B. PRACTICE

MATERIALS: Chalkboard, dittos, work pages.

DIRECTIONS: Expand to larger division problems.

$$\begin{array}{r} 65\overline{)264} \\ \underline{260} \\ 4 \end{array} \quad 4\tfrac{4}{65} \quad 4/65$$

$$\left.\begin{array}{r} 9 \\ 30 \\ 200 \end{array}\right\} 239\ 5/33 \qquad \begin{array}{r} 33\overline{)7892} \\ \underline{6600} \\ 1292 \\ \underline{990} \\ 302 \\ \underline{297} \\ 5 \end{array} /33$$

$$\left.\begin{array}{r} 4 \\ 20 \end{array}\right\} 24\tfrac{18}{32} \qquad \begin{array}{r} 32\overline{)786} \\ \underline{640} \\ 146 \\ \underline{128} \\ 18 \end{array} /32$$

pg 350 blank

If the simplest form of the fraction is desired, instruct pupils to reduce

$\frac{18}{32}$ to $\frac{9}{16}$, etc.

Provide practice with dittos and workbook pages.

OBJECTIVE Divide with decimal in the divisor.

COMMENTS

PRE-REQUISITE OBJECTIVES 134-S

ACTIVITIES

A. REPLACE DECIMALS BY WHOLE NUMBERS TO DIVIDE

MATERIALS: See objective 134-S for models.

DIRECTIONS: (Note: This is a review, basically of the work done in Objective 134-S.)

$$.6 \div .2 = \frac{6}{10} \div \frac{2}{10}$$

How many $\frac{2}{10}$ in $\frac{6}{10}$?

How many 2's in 6 = $\frac{3}{1}$ = 3.

$$2 \div .4 = 2 \div \frac{4}{10} = \frac{20}{10} \div \frac{4}{10}$$

Ask how many $\frac{4}{10}$ in $\frac{20}{10}$?

How many 4's in 20 = $\frac{5}{1}$ = 5.

$$3 \div .7 = 3 \div \frac{7}{10} = \frac{30}{10} \div \frac{7}{10}$$

Ask how many $\frac{7}{10}$'s in $\frac{30}{10}$?

How many 7's in 30 = $\frac{30}{7}$ = 4 $\frac{2}{7}$.

Once the children have the idea provide them with problems to work. These can be on ditto, dictated, or from the blackboard.

B. REPLACE DECIMALS WITH WHOLE NUMBERS TO DIVIDE

MATERIALS: Chalkboard and dittos.

DIRECTIONS:

(1) First the pupil needs to learn that if the divisor and dividend of a division problem are multiplied by the same number, then the quotient of each problem is the same.

For example: $6 \div 2 = 3$. Now multiply 6 and 2 by 3.
Then $18 \div 6 = 3$. Now multiply 6 and 2 by 6.
Then $36 \div 12 = 3$.

Do a number of these examples to get the idea across that one division problem can be changed to another by this technique and the answers will be the same.

(2) Once the children have a feeling for part (1) above, go back to decimals and point out that they can multiply both divisor and dividend by the same power of 10 (i.e. 10, 100, 1000...) to replace the problem with one that is a division of whole numbers, which they can do.

For example: $.52 \div .13$. This can be replaced by $52 \div 13$ by multiplying .52 by 100 and .13 by 100.
Thus $.52 \div .13 = 52 \div 13 = 4$

Do a number of problems on the chalkboard with the children until they understand the idea.

OBJECTIVE Convert proper fractions to decimals by division.

COMMENTS To convert a proper fraction to a decimal the pupil will divide the numerator by the denominator using the division algorithm. Prior to this we have divided large numbers by smaller numbers and obtained whole number quotients (units, tens, hundreds, etc.). In this objective, we will divide smaller numbers by larger numbers and obtain decimal quotients (tenths, hundredths, thousandths, etc.). It is important that the pupil understand this difference.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

PRE-ACTIVITY

Review division emphasizing place-value and "cashing - in" by drawing columns.

Example: 300 ÷ 4

```
           Tens
  Hundreds  |  Ones
         \  ↓ /
          | 7 | 5
   4 ) 3  | 0 | 0
      -3  | 0 |
          |   |
        3 | 0 |      (cash hundreds in for tens)
        2 | 8 |
          | 2 | 0    (cash tens in for units)
          | 2 | 0
          |   | 0
```

A. CONVERTING FRACTIONS TO DECIMALS

MATERIALS: Chalkboard, paper and pencil.

DIRECTIONS: Emphasize place-value and "cashing - in" by drawing columns.

Example: $\frac{3}{4}$ $3 \div 4$ $4\overline{)3}$

Tenths

Units Hundredths

```
   .75
4) 3.00
   3 0      (cash in units for tenths)
   2 8      (cash in tenths for hundredths)
     20
     20
```

$\frac{7}{8}$ $7 \div 8$ $8\overline{)7}$

Units Tenths Hundred ths Thousandths

```
   .875
8) 7.000
   7 0      (cash in units for tenths)
   6 4
     60     (cash in tenths for hundredths)
     56
      40    (cash in hundredths for thousandths)
      40
```

B. CONVERTING FRACTIONS TO DECIMALS BY DIVISION

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS: Write the algorithm in standard decimal format instead of place-value columns as in Activity A.

$\frac{3}{4}$

```
   .75
4)3.00
  2 8
   20
   20
```

302

$\frac{7}{8}$

```
   .875
8)7.000
  6 4
   60
   56
    40
    40
```

$\frac{5}{6}$

```
   .833
6)5.000
  4 8
   20
   18
    20
    18
     2
```

NOTE: When the denominator of the proper fraction (in simplest terms) has prime factors other than 2 or 5, the decimal conversion will not terminate. Instead, a repeating pattern will occur. It is common to indicate the repeating part by putting a bar over it. For example: $\frac{5}{6}$ = .8333... = .83

303

GENERAL REMARKS: The set of integers consists of positive and negative numbers and zero.

139-C Reference Types of Numbers (Integers)

OBJECTIVE Identify positive and negative numbers as opposites.

COMMENTS Pupil's activities provide the first experiential background for positive and negative numbers.

PRE-REQUISITE

ACTIVITIES

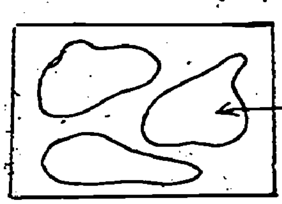
A. DISCUSSION

MATERIALS None

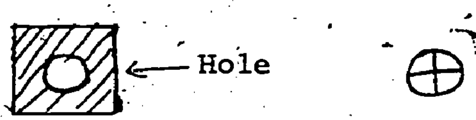
DIRECTIONS Teacher or pupil lead discussion on opposites (right-left, up-down, backward-forward, north-south, hot-cold). Work toward things that are truly opposites in the sense of measurement. E.g., 5 feet to the right is the opposite of 5 feet to the left, 3 miles north is the opposite of 3 miles south, heating up 25° is the opposite of cooling down 25°, etc.

B. INTRODUCING HOLES AND PLUGS

MATERIALS Felt, a large piece with holes cut out and put back so they cannot be seen (so the felt looks uncut).



Another good material for models is scraps of carpet cut into 4 inch squares with a hole cut in the center.



Carpet scraps are easier to use than felt when modeling the operations.

page 358 blank

DIRECTIONS Place felt on the floor or table (holes cannot be seen). Teacher tells class we are going to make a set using the piece of felt as a set indicator. Run hand over felt (but do not disturb the felt) to show there is nothing in the set. Drill until it is clear that there is nothing in the set. This can be done by putting objects in the set and taking them out again.

Take out one plug. Hold it up and ask how many are in this set (one). Point to the hole and ask how many in this set (one). Replace the hole and smooth out felt and ask how many in this set (zero). Repeat until this is clearly understood.

Note: The teacher explains at this point that they are dealing with a special kind of number that acts differently than other numbers.

Plugs are models for positive numbers that act like numbers always have acted.
Holes are models for numbers which act differently -- negative numbers.
Do simple addition problems with the holes and plugs, without writing the problems or trying to generalize any solid rules yet.
Example: Put 3 holes in the set and add 2 plugs. The 2 plugs fill up 2 holes and you are left with one hole.
Put 4 plugs in the set and add 3 holes. 3 plugs fill up 3 holes and you are left with 1 plug.

Note: A positive number pairs with a negative number and they cancel each other if they are in the same set, just as a plug always goes inside a hole, if there is one, and a hole goes around a plug, if they are in the same set, and cancel one another in pairs. The holes or plugs left show the answer.

Problems can be written on the chalkboard. Abbreviate Plug - P and Hole - H.

$P_6 + H_4 = P_2$ $\qquad$ $H_7 + P_8 = P_1$ $\quad$ etc.

As they work the examples pupils must note that the plug and hole numbers are opposites. There will be much confusion here. Be patient and keep giving examples. A good example is digging a hole in the ground. You end up with one pile of dirt and one hole. Put the dirt back and you have neither a pile of dirt or a hole. Keep in mind that plug (P) numbers are nothing more than a concrete way of developing positive and negative numbers (integers). Thus you will want to work toward this. When you make the shift from the model to the integers will have to be determined by the individual teacher. However, it is best to wait till the child has a good feel for the "oppositing" model you are using.

From the fact that $H_3 + P_3 = 0$ or $P_3 + H_3 = 0$ you can eventually reach the fact that $(+3) + (-3) = 0$. The sum of a number with its <u>opposite</u> type is always 0.

305

OBJECTIVE. Model and record addition of integers.

COMMENTS

PRE-REQUISITE OBJECTIVES 139-C

ACTIVITIES

A. INTRODUCE THE SIGNS "+" AND "-"

MATERIALS: Models for holes and plugs -- a number line either blank or with positive and negative numbers, large enough to lay on the floor and walk on.

DIRECTIONS: Write a few simple addition problems on the chalkboard and model.

PROBLEM: Positive 6 + Negative 3 = Positive 3.

MODEL: $P_6 + H_3 = P_3$

PROBLEM: Negative 4 + Positive 2 = Negative 2.

MODEL: $H_4 + P_2 = H_2$

Note the meaning of the plus sign, it means to combine sets.

Tell students, mathematicians also use the "+" to mean positive numbers and they use "-" to mean negative numbers. Write the mathematical equations with the problems on the board.
For Example: $(+6) + (-3) = (+3)$
Use the equations and read signs inside the parenthesis as positive or negative never as plus or minus.

Have pupils practice reading simple equations and modeling to find answers. Rewrite problems initially given with "+" and "-" in place of positive and negative.

Introduce the number line, positive numbers are to the right of the zero, negative numbers to the left.

$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad +4$$

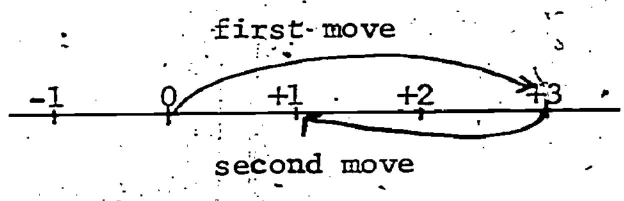
Be sure to read (-3) as negative 3 and (+3) as positive 3.

30?

B. ADDING ON THE NUMBER LINE

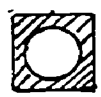
MATERIALS: Number line on the floor.

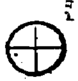
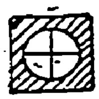
DIRECTIONS: Given a problem, for example, (+3) + (-2), start at zero. Then walk through the problem in the following way. "Positive" means face right; "Negative" means face left and "plus" means walk ahead. In our problem (+3) means face right and walk forward. Now we look at (-2). This means face left and since we are adding walk forward 2 paces. This brings us to (+1).



Do a number of problems like this. Remember you must always start at zero and <u>add</u> the first number, whatever it is, so you will walk forward, whether facing left or right.

C. ADDING WITH THE HOLES AND PLUGS

MATERIALS: Carpet holes and plugs: In examples below  = holes,

 = plugs,  = a plug in a hole.

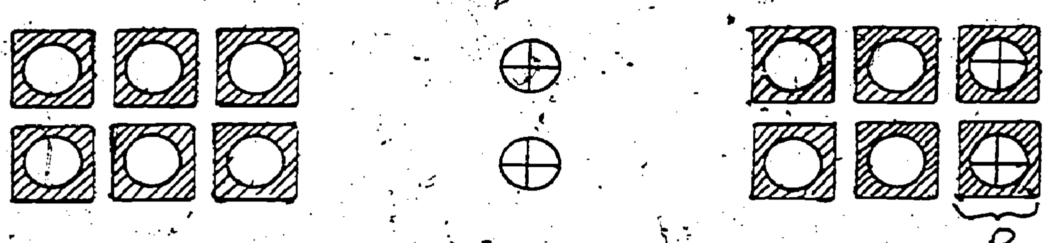
DIRECTIONS:

(1) Start with an empty set. (To keep later models with subtraction and multiplication consistent with the addition model begin with a quantity of carpet blanks (plugs in the holes)).

(2) <u>Put holes in the set to represent negative numbers. Put plugs in the set to represent positive numbers.</u> A plug and a hole must go together to become zero if in the same set.

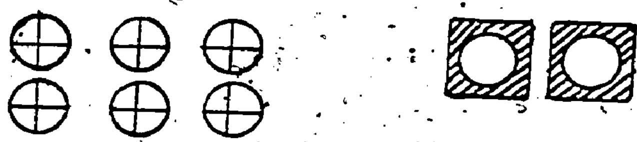
(-6) + (+2) = (-4)

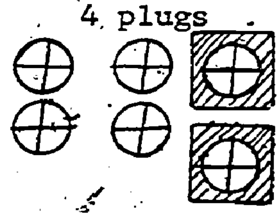
MODEL: 6 holes + 2 plugs = 4 holes



(+6) + (-2) = (+4)

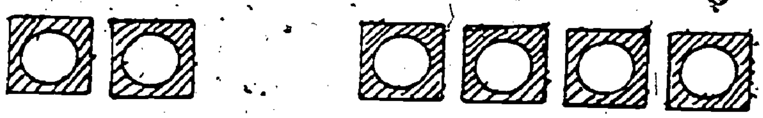
MODEL: 6 plugs + 2 holes = 4 plugs

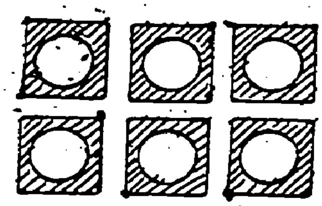




(-2) + (-4) = (-6)

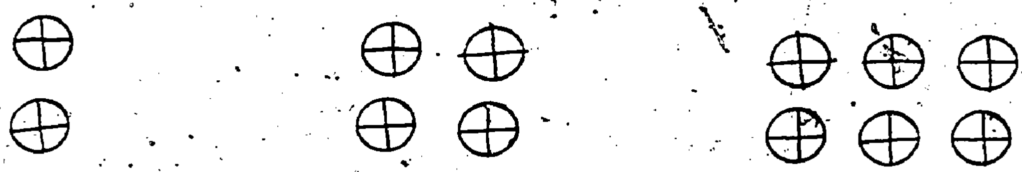
MODEL: 2 holes + 4 holes = 6 holes





(+2) + (+4) = (+6)

MODEL: 2 plugs + 4 plugs = 6 plugs



NOTE: The answers always have the sign of the larger set.

Simplify the rule: When a positive and negative number are added, the smaller number cancels that much of the larger set and the answer has the sign of the larger set.

D. ANALYZING ADDITION MODELS TO INTERNALIZE PROCESS

MATERIALS: Models.

DIRECTIONS: Review the rule that the sum of a number and its opposite number is zero.

(-3) + (+3) = 0
(+3) + (-3) = 0

Analyze enough problems adding numerals with like signs to realize that if you add like numbers you get the same numbers.

(+3) + (+3) = (+6)
(-6) + (-6) = (-12)

Write an addition problem, such as (-6) + (+2) = (-4).
Ask "What did we do to (-6) to make it (-4)?"
Answer will be something like "I put 2 plugs in 2 of the holes."
In other words in (-6) + (+2) the -6 becomes (-4 and -2) and the (-2) combines with the (+2) to make zero.

Another problem: (+6) + (-2) = (+4).
Ask "What did we do to (+6) to make it (+4)?"
Answer will be something like "Take out 2." "Or cover 2 plugs with 2 holes." (The second answer is more exact.)

$$(+6) + (-2) = (+4)$$
$$(+4+2) + (-2) = (+4) + \overbrace{[(+2) - (-2)]}^{0} = (+4)$$
$$(-2) + (+6) = (+4)$$

Ask what happens. Answer: "2 of the six plugs went inside the 2 holes." So $(-2) + [(+2) + (+4)] = (+4)$

$$(+2) + [(-2) + (-4)] = (-4)$$

NOTE: 2 of 6 holes surround the 2 plugs and leave 4 holes.

$$(+2) + [(-2) + (-4)] = (-4)$$

Do examples until pupils have had sufficient practice to realize that they are really subtracting the smaller set and the answer is part of the larger set (has the sign of the larger set).

$$\text{So } (+64) + (-14)$$
$$(50 + 14) + (-14) = (+50)$$

Pupils may have to be reminded to subtract (64 - 14) to find that 50 + 14 = 64.

$$\text{or } (-64) + (+14)$$
$$[(-50) + (-14)] + (+14) = (-50)$$

$$\text{or } (+14) + (-64)$$
$$(+14) + [(-14) + (-50)] = (-50)$$

309

OBJECTIVE Model and record subtraction of integers.

COMMENTS.

PRE-REQUISITE 140-J, 139-C

ACTIVITIES

A. SUBTRACTING WITH HOLES AND PLUGS

MATERIALS: Carpet models

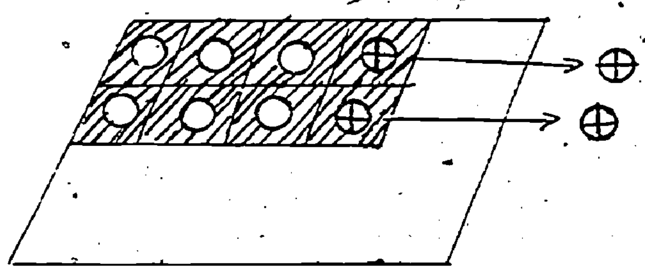
DIRECTIONS: (+7) - (+2) = (+5)

The minus means to remove. (Note to teacher: For modeling integers, we almost have to mean "remove" a change of connotation from earlier objectives.)

Build an empty set of carpet blanks (plugs inserted in holes) on the floor. Keep a stock pile of holes and plugs.

1. $H_6 - P_2 = H_8$

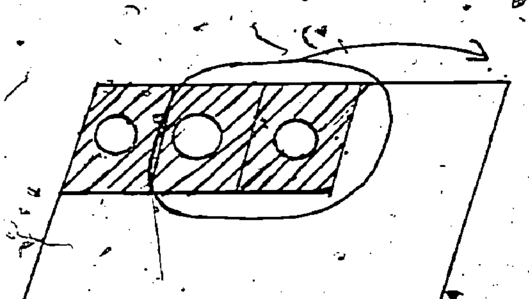
Start by putting 6 holes into the set.



Separate 2 plugs from the blanks--leaving 8 holes. In integer form (-6) - (+2) = (-8).

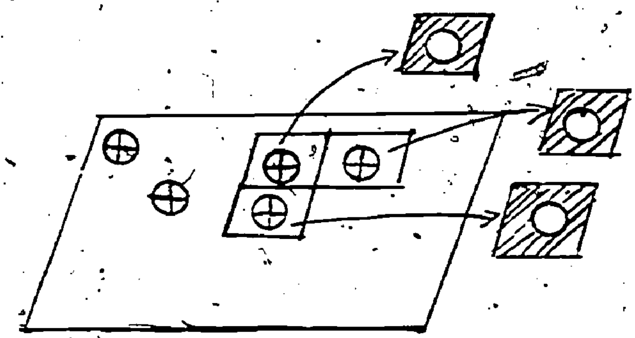
2. $H_3 - H_2 = H_1$

Start by putting 3 holes in the set. (-3) - (-2) = (-1)



Separate 2 holes leaving 1 hole.

3. $P_2 - H_3 = P_5$
Start by putting 2 plugs into the set.



Separate 3 holes leaving 5 plugs. (+2) - (-3) = (+5)

B. SUBTRACTING WITH THE NUMBER LINE

MATERIALS: Number line on the floor.

DIRECTIONS: Always start from zero. Walk through the problems. Positive still means face right, negative still means face left plus still means walk forward, but minus means walk backward.
Example: (+5) - (-3) would be acted out as follows: face right (because the 5 is positive), take five steps forward then face left (because the 3 is negative) and take three steps backward (because you're subtracting). You will end up 8 steps to the right (+8) and so (+5) - (-3) = (+8)

C. ANALYZING SUBTRACTION MODELS TO INTERNALIZE THE PROCESS

MATERIALS: Holes and plugs, number line.

DIRECTIONS: Do as many examples as it takes for pupils to realize that subtracting an integer is the same as adding the opposite. E.g., removing holes is the same as contributing plugs, facing left and walking backwards is the same as facing right and walking forward, etc.

(-6) - (+2) = (-6) + (-2) = (-8)
(+6) - (+2) = (+6) + (-2) = (+4)
Therefore,
(-64) - (-14) = (-64) + (+14) = (-50)
(-64) - (+14) = (-64) + (-14) = (-78)

311

OBJECTIVE Model and record multiplication of integers.

COMMENTS

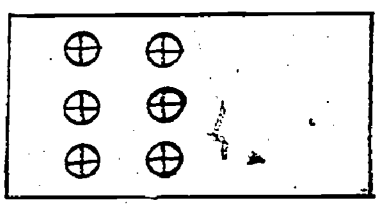
PRE-REQUISITE 140-S, 139-C

ACTIVITIES

A. MODELING WITH PLUGS AND HOLES

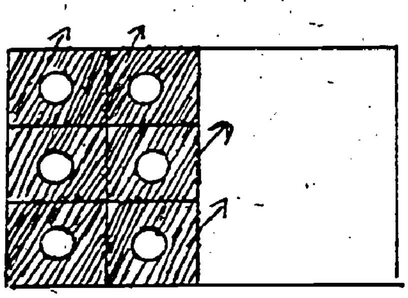
MATERIALS: Models of plugs and holes.

DIRECTIONS: Spread a set of blanks on the floor as for subtraction. Recall 6 x 2 is 6 sets of 2. Let "+" mean "put in" and "-" mean "take out" for the multiplier. For the multiplicand "+" still means plugs and "-" still means holes. Then (-6) x (+2) means take 6 sets of 2 plugs from the set.
So: (+3) x (+2) means put 3 sets of 2 plugs into the set.



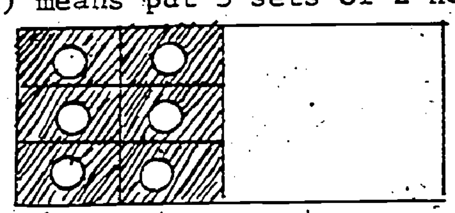
Thus (+3) x (+2) = (+6)

(-3) x (+2) means take 3 sets of 2 plugs out of the set, leaving 6 holes in the set.



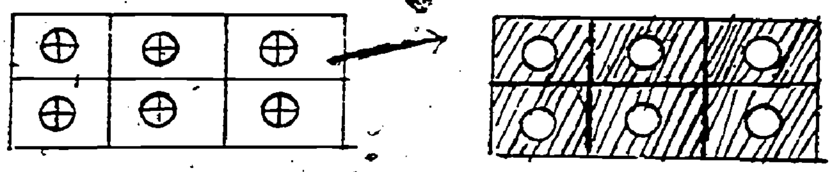
Thus (-3) x (-2) = (-6)

(+3) x (-2) means put 3 sets of 2 holes into the set giving 6 holes.



Thus (+3) x (-2) = (-6)

(-3) x (-2) means take 3 sets of 2 holes out of the set leaving 6 plugs.



Thus (-3) x (-2) = (+6)

B. NUMBER LINE MODEL

MATERIALS: Chalkboard

DIRECTIONS: The forward-backward, right-left approach works real well here. Study the example and make sure it make sense to you.
The sign on the multiplier indicates whether you are going to walk forward (+) or backward (-), the number indicates how many steps. The sign on the <u>multiplicand</u> indicates whether you are going to face right (+) or left (-) and the number indicates the size of each step you take.
For example: (+3) x (-2) would be modeled as: Face left and take 3 steps forward of size 2. You will end up 6 units to the left (-6) so (+3) x (-2) = (-6).
(-3) x (-4) would be modeled as: Face left and take 3 steps backward of size 4. You will end up 12 units to the right (+12) so (-3) x (-4) = (+12).

C. ANALYZE THE MODELS TO INTERNALIZE THE RULES

MATERIALS: Models

DIRECTIONS: Do enough models for students to realize that:
Positive numbers x positive numbers = positive numbers
Positive numbers x negative numbers = negative numbers
Negative numbers x positive numbers = negative numbers
Negative numbers x negative numbers = positive numbers

OBJECTIVE Division of integers.

COMMENTS Relate multiplication of integers with division of integers.

PRE-REQUISITE OBJECTIVES 126-S

ACTIVITIES

A. DIVIDING INTEGERS

MATERIALS: Chalkboard, paper and pencils, teacher prepared dittos.

DIRECTIONS: The teacher would demonstrate that in a problem such as
$(+12) \div (-3) =$ ___?___ is the same as
$(?) \times (-3) = (+12)$.
Since $(-4) \times (-3) = (+12)$, we know that $(+12) \div (-3) = (-4)$

Do as many of these problems on the board as needed to get the idea across. Give lots of practice.

The pupils should see that

Positive ÷ Positive = Positive
Positive ÷ Negative = Negative
Negative ÷ Positive = Negative
Negative ÷ Negative = Positive

OBJECTIVE Identify the structural properties of addition.

COMMENTS The pupil will have encountered these concepts previously but not in a formal setting. The structural properties of addition are commutativity, idendity, associativity, and inverses.

ACTIVITIES

A. COMMUTATIVITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:

1. Using specific examples let the pupil observe that $3 + 4 = 4 + 3$; $8 + 7 = 7 + 8$.

2. Using a single variable have the pupil observe that for <u>any</u> replacement of the variable the resulting sentence is true. Examples:

$$A + 3 = 3 + A$$
$$4 + \square = \square + 4$$

3. Through a sequence such as

$$A + 1 = 1 + A$$
$$A + 2 = 2 + A$$
$$A + 3 = 3 + A$$

lead to the final result that

$$A + B = B + A$$

is true for any replacement of A and any replacement of B. The fact that $A + B = B + A$ for all A and B is known as the <u>commutative</u> property of addition. We say that <u>addition is commutative</u>.

4. A good way to emphasize the presence of a mathematical property is to illustrate examples where it is not present. Subtraction and division are <u>not</u> commutative. Have the pupils test their own examples.

$$8 - 3 \neq 3 - 8$$
$$12 \div 4 \neq 4 \div 12$$

 page 370 blank.

B. ASSOCIATIVITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS: 1. Since addition is a process involving only two quantities, a symbol such as

$$3 + 8 + 2$$

is meaningless unless we adopt some convention as to how to interpret it. This causes no real problem, however, since the two natural interpretations

$$(3 + 8) + 2$$

and

$$3 + (8 + 2)$$

both give the same result, 13.

So we write $3 + 8 + 2 = 13$.

Have the pupils consider several of their own examples to convince themselves that it does not matter whether the middle number is "associated" with the left-hand number or the right-hand number.

$$(8 + 7) + 5 = 20 = 8 + (7 + 5)$$

$$(2 + 9) + 6 = 17 = 2 + (9 + 6)$$

2. Using one, two and finally three variables, lead the pupil through sentences such as

$$(A + 1) + 4 = A + (1 + 4)$$

$$(3 + A) + 6 = 3 + (A + 6)$$

$$(7 + 4) + A = 7 + (4 + A)$$

and then

$$(A + B) + 4 = A + (B + 4)$$

$$(A + 6) + B = A + (6 + B)$$

$$(7 + A) + B = 7 + (A + B)$$

1) as a number sentence with variable(s) which is true for any replacement of the variable(s) and 2) as the number zero when speaking of addition.

This unfortunate double use of a word should be pointed out.

D. INVERSES

MATERIALS: Holes and plugs, chalkboard, pencil and paper.

DIRECTIONS:

1. Holes and plugs give an excellent model for the concept of <u>opposite</u> in addition. It is this notion of opposites that is fundamental to an understanding of inverses with respect to addition.

   If a certain amount of one type of object is contributed to a set (which may or may not be empty to begin with) and then an equal amount of the opposite is contributed, the set ends up as it began. Thus, a quantity of a given type and the same quantity of the opposite type are known as the <u>additive inverses</u> of one another because of their successive contribution to a set leaves the set as it originally began.

   Examples: $5^H + \underbrace{3^P + 3^H}_{4^H + 4^P = 0} = 5^H$

   $2^P + 7^H + 7^P = 2^P$

   $7 + 3 + (-3) = 7$

   $-4 + (-5) + 5 = -4$

   $(6 + -6) = 0$

2. It should be observed that if the set was initially empty, then successive contribution of a quantity of one type and the same quantity of the opposite type results again in the "zero-state". That is, adding the <u>inverse</u> of a number to itself results in the <u>identity.</u>

and finally to

$$(A + B) + C = A + (B + C)$$

The fact that $(A + B) + C = A + (B + C)$ is true for all replacements of A, B and C is known as the associative property of addition. We say that addition is associative.

3. As with commutativity, associativity can be emphasized by illustrating cases where it does not occur. Subtraction and division are not associative. Have the pupils test their own examples.

$$1 = (7 - 5) - 1 \neq 7 - (5 - 1) = 3$$

$$1 = (8 \div 4) \div 2 = 8 \div (4 \div 2) = 4$$

Note: Point out to the pupils that commutativity involves a change in the order of the operation while associativity, leaving the order of the symbols unchanged, involves a change in the punctuation.

C. IDENTITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS: 1. Note that when 0 is added to any number, we get identically the same number. The pupil will have noted that

$3 + 0 = 3$

$0 + 4 = 4$

etc.

They should finally observe that

$A + 0 = A$

$0 + B = B$

are true for all replacements for A and B. Since identically the same number is obtained when adding zero, zero is called the additive identity or the identity number for addition.

Note: Possible confusion may result in the dual use of "identity":

OBJECTIVE Identify the structural properties of multiplication.

COMMENTS The structural properties of multiplication are the same as addition. As with addition, the pupil will have encountered these concepts previously but in an informal manner.

PRE-REQUISITE OBJECTIVES

ACTIVITIES

A. COMMUTATIVITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:

(1) Using specific examples, let the pupil observe that (three groups of 4) $3 \times 4 = 4 \times 3$ (four groups of 3).

(2) Using a single variable, the pupil should note that for any replacement of the variable, the resulting sentence is true.

$A \cdot 3 = 3 \cdot A$

$5 \cdot \square = \square \cdot 5$

(3) Through a sequence such as

$A \cdot 1 = 1 \cdot A$

$A \cdot 2 = 2 \cdot A$

$A \cdot 3 = 3 \cdot A$

lead to the final result that

$A \cdot B = B \cdot A$

is true for all replacements of A and B. This result is known as the <u>commutative</u> property of multiplication and we say that <u>multiplication</u> is <u>commutative</u>.

(4) Recall that subtraction and division are not commutative.

B. ASSOCIATIVITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:

(1) As with addition, multiplication is a process that is performed on only two numbers and so a symbol such as $4 \times 7 \times 3$ is meaningless unless we understand how to interpret it. Fortunately, the two natural interpretations, $(4 \times 7) \times 3$ and $4 \times (7 \times 3)$ both give the same result, 84. Therefore we write $4 \times 7 \times 3 = 84$.

The pupils should consider several examples to convince themselves that the same result is obtained whether the middle number is "associated" with the left-hand number or the right-hand number.

$$(5 \times 3) \times 4 = 60 = 5 \times (3 \times 4)$$

$$(2 \times 6) \times 3 = 36 = 2 \times (6 \times 3)$$

(2) Using one, two, and finally three variables, lead the pupils through sentences such as $(A \times 2) \times 3 = A \times (2 \times 3)$

$$(4 \times A) \times 5 = 4 \times (A \times 5)$$
$$(4 \times 3) \times A = 4 \times (3 \times A)$$

to

$$(A \times B) \times 2 = A \times (B \times 2)$$

$$(A \times 5) \times B = A \times (5 \times B)$$

$$(6 \times A) \times B = 6 \times (A \times B)$$

and finally to

$$(A \times B) \times C = A \times (B \times C)$$

The fact that this result is always true for all replacements of A, B, and C is known as the <u>associative</u> property of multiplication. We say that <u>multiplication is associative.</u>

(3) Recall that subtraction and division are not associative.

C. IDENTITY

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:

(1) The pupil will have observed that

$4 \cdot 1 = 4$

$1 \cdot 7\frac{1}{2} = 7\frac{1}{2}$

etc.

He should observe that

$A \cdot 1 = A$
and
$1 \cdot A = A$

are true for all replacements for A. That is, multiplication of a number by 1 or multiplication of 1 by a number results in identically the same number. We say that 1 is the identity number for multiplication, or that 1 is the multiplicative identity.

D. INVERSE

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:

(1) Just as the additive inverse of a number A is that number which when added to A gives the additive identity zero, the multiplicative inverse of a number A is that number which when multiplied by A gives the multiplicative identity, one.

The pupil will have previously observed that

$\frac{1}{3}$ of $3 = 1$

4 of $\frac{1}{4} = 1$

$\frac{2}{3}$ of $\frac{3}{2} = 1$, etc.

The two numbers on the left of each equation are multiplicative inverses of each other. (The term reciprocal is also used, $\frac{1}{3}$ is the reciprocal of 3, $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$, etc.)

Have the pupil solve several exercises such as

$\square \cdot \frac{2}{5} = 1$

$\frac{4}{3} \cdot A = 1$

until he is sure that the reciprocal will always give the answer. Ask the pupil whether zero has a reciprocal. That is, can $A \cdot 0 = 1$ be solved?

Since any quanity of empty sets will together be empty, a set of size one will never result. That is, zero has no multiplicative inverse. This unusual property of the additive identity should be made clear. The pupil should feel comfortable with the fact that although most members have inverses, not all of them must.

OBJECTIVE Identify the distributive property of multiplication over addition.

COMMENTS

PRE-REQUISITE 71-J, 65-J

ACTIVITIES

A.

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS: 1. Ask the pupil, "Can you multiply by a sum?" "Can you add to a product?" Ask him to give examples illustrating each question, such as

$(4 + 3)\cdot 8$ (multiply by a sum)
$7 \cdot (5 + 2)$
and
$3 + (5\cdot 8)$ (add to a product)
$(4 \cdot 7) + 6$

By the convention of the parentheses (punctuation) in the first two examples you add first and then multiply while in the last two examples you multiply first and then add. The pupil should then search for alternative ways to combine the three given numbers using addition and multiplication to get the same result. Someone should onserve (with directed questions if necessary) that

$(4 + 3)\cdot 8$ is the same as $(4 \cdot 8) + (3 \cdot 8)$

and that

$7 \cdot (5 + 2)$ is the same as $(7 \cdot 5) + (7 \cdot 2)$.

Note: This is a good time to stress the language arts aspect. That is, 7 eights is 4 eights and 3 eights, and 7 sevens is 7 fives and 7 twos.

It should be pointed out that a similar result does not in general hold for the last two examples. That

$3 + (5 \cdot 8)$ is not $(3 + 5) \cdot (3 + 8)$
and
$(4 \cdot 7) + 6$ is not $(4 + 6) \cdot (7 + 6)$

Note: In special cases it is possible to get the result to work. If the first number is zero, or if all three numbers have the sum of 1 then it will work.

E.g.: $0 + (4 \cdot 6) = (0 + 6)$
$2 + [(-4) \cdot 3) = (2 + (-4)] \cdot (2 + 3)$.

This is a good time to point out that, while something may not in general be true, this does not mean that it cannot be true in special cases.

2. The fact that $(A + B) \cdot C = (A \cdot C) + (B \cdot C)$
and
$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

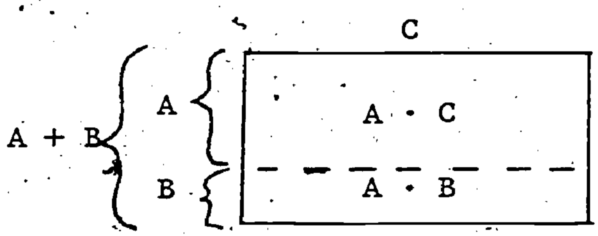
is known as the <u>distributive</u> property. More specifically we say that <u>multiplication distributes over addition</u>. To stress this relationship of addition and multiplication, ask the pupil if addition distributes over multiplication. In general (see note above) $A + (B \cdot C)$ is not the same as $(A + B) \cdot (A + C)$. (The exceptions are $A = 0$ or $A + B + C = 1$.)

3. The term "distribute" can be remembered if you think of the multiplication symbol and numeral being "distributed" to each of the addends in the sum. E.g., distribute this

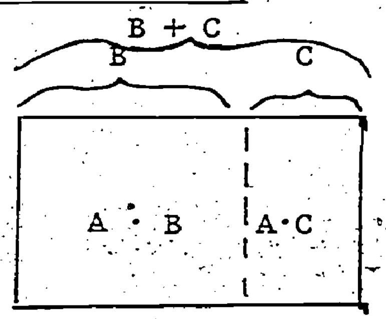
$$(4 + 6) \cdot 5$$
$$(4 \cdot 5) + (6 \cdot 5)$$

Note that we have gone from a situation where we added first and then multiplied to one where we do 2 multiplications first and then add.

4. The distributive property can be illustrated as follows (see Activity E, Objective 71-J).



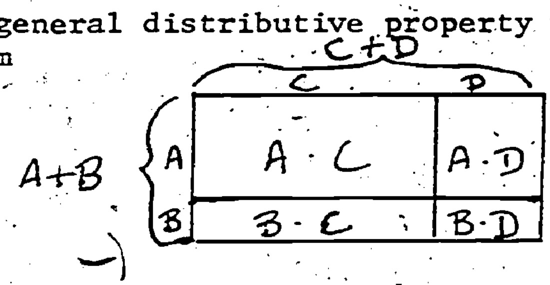
$(A + B) \cdot C = (A \cdot C) + (B \cdot C)$



$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

The pupil should do several of these with numbers to convince himself that this works.

This diagramming leads to a more general distributive property which should be observed. Consider the diagram



The large rectangle has area $(A + B) \cdot (C + D)$
which is equal to the sum of the four smaller rectangles
$(A \cdot C) + (A \cdot D) + (B \cdot C) + (B \cdot D)$

Thus $(A + B)(C + D) = (A \cdot C) + (A \cdot D) + (B \cdot C) + (B \cdot D)$

Note that here each term in the left-hand parentheses is distributed multiplicatively to each term in the right-hand parentheses. This is the case in general. Consider

$D + E + F + G$

$A + B + C$	D	E	F	G
A	$A \cdot D$	$A \cdot E$	$A \cdot F$	$A \cdot G$
B	$B \cdot D$	$B \cdot E$	$B \cdot F$	$B \cdot G$
C	$C \cdot D$	$C \cdot E$	$C \cdot F$	$C \cdot G$

This model serves to illustrate that

$$(A + B + C) \cdot (D + E + F + G) =$$
$$(A \cdot D) + (A \cdot E) + (A \cdot F) + (A \cdot G) +$$
$$(B \cdot D) + (B \cdot E) + (B \cdot F) + (B \cdot G) +$$
$$(C \cdot D) + (C \cdot E) + (C \cdot F) + (C \cdot G).$$

Note again that <u>each</u> term in the left-hand parentheses multiplies <u>each</u> term in the right-hand parentheses.

OBJECTIVE Identify a field as a mathematical structure.

COMMENTS

PRE-REQUISITE OBJECTIVES

ACTIVITIES

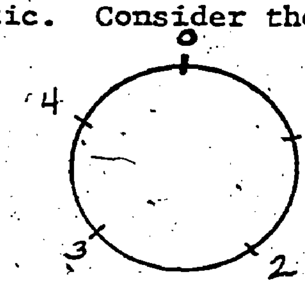
A. FIELDS

MATERIALS: Chalkboard, pencil and paper.

DIRECTIONS:
(1) Point out that a <u>field</u> is basically any set in which you can add, subtract, multiply and divide. This general discription must be qualified some, however. Specifically, the addition must be commutative, associative, with an identity and minuses for <u>all</u> elements. The same is true for multiplication except that the additive identity (zero) can not have a multiplicative inverse. (Since division is multiplication by the multiplicative inverse, what we're saying is that you can't divide by zero.) Finally, multiplication must distribute over addition.

(2) Ask the pupils:
(a) Is the set of counting numbers a field? (No, don't have additive or multiplicative inverses.)
(b) Is the set of integers a field? (No, don't have multiplicative inverses.)
(c) Is the set of rational numbers a field? (Yes, this is the pupil's first example of a field.)

(3) There are fields other than the rational numbers. An excellent example is provided by "clock" arithmetic. Consider the five digit clock



To perform arithmetic on the clock we always begin at zero. All moves are clockwise. A + B is found by moving B spaces A times. The answer is always the location where the moving ends. Have the pupil complete the addition and multiplication tables.

 ge 382 blank

147 cont 'd.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

•	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

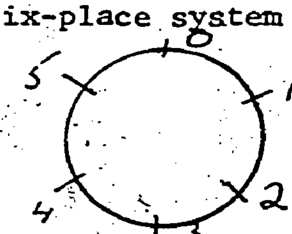
Note that 0 is the additive identity and that 1 is the multiplicative identity and that both addition and multiplication are commutative. (Commutativity is easy to verify because the tables are symmetric about the upper-left to lower-right diagonals.) Observe that each element can be added to some other element to get 0 and that each element except zero can be multiplied by some element to get 1. Hence, inverses exist. Associativity and distributivity are harder to verify but the pupil should do several examples to convince himself that the operations are associative and that multiplication does distribute over addition.

e.g.

$$(3 + 4) + 2 = 2 + 2 = 4$$
$$3 + (4 + 2) = 3 + 1 = 4$$
$$(4 \cdot 2) \cdot 3 = 3 \cdot 3 = 4$$
$$4 \cdot (2 \cdot 3) = 4 \cdot 1 = 4$$
$$3 \cdot (4 + 2) = 3 \cdot 1 = 3$$
$$(3 \cdot 4) + (3 \cdot 2) = 2 + 1 = 3$$

Thus the five-place clock arithmetic yields a field.

(4) As a final example, the pupil should be shown a clock-arithmetic system that is not a field. The six-place system will do.



The reason we do not get a field is that certain non-zero elements have no multiplicative inverses, namely 2 and 3.
In the multiplication table the rows for 2 and 3 are

	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3

Note that neither can be multiplied by a number to get the mulitiplicative identity 1.

(If the number of places on the clock is a prime, a field will result. If the number of places on the clock is not a prime, then a field will never result because there will always be non-zero elements with no multiplicative inverses.)

METRIC SYSTEM

GENERAL REMARKS These activities give the pupils a good knowledge of the metric system of measurement. This may help eliminate some difficulties with operational process in decimal fractions or numerals. Use these activities throughout the guidelines when working with measurement. The standard units of measure in the metric system for length, capacity, volume, and weight are the meter, liter, and grams.

ACTIVITIES

A. TERMINOLOGY, ABBREVIATIONS, AND CONNECTION WITH BASE TEN

MATERIALS: Meter stick, liter container, gram weight, and chalkboard.

DIRECTIONS: Explain to the pupils that the standard units of measurement in the metric system are subdivided into tenths, hundredths, thousandths, etc., and also considered in multiples of ten, hundred, thousand, etc. There are prefixes to attach to the standard unit words to indicate these subdivisions or multiples. They are:

Prefix	Multiple	Abbreviation
kilo	1,000	k
hecto	100	h
deka	10	dk
deci	1/10	d
centi	1/100	c
milli	1/1000	m

The abbreviations for meter, liter and gram are m, l, g. Thus, a kilometer (km) is 1000 meters, a dekagram (dkg) is 10 grams, a milliliter (ml) is 1/1000 of a liter, etc. Have the pupils practice these names and abbreviations.

Point out that a liter is a cubic decimeter and have the pupils determine that there are 1000 cubic centimeters in a liter. That is, a cubic centimeter (cc) is the same as a milliliter (ml). Often the size (volume) of an engine on a car or motor cycle is given in cubic centimeters (cc). Have the pupils check this out.

B. ACTIVITIES

MATERIALS: Dittos similar to the following.

Page 386 blank.

DIRECTIONS:

(1) THE GRAM
is the basic unit of weight.
Five wooden kitchen matches weigh one gram.
Two paper clips also weigh a gram.
a. Do you think a gram is heavy?
b. How many grams do you think you weigh?
c. How many grams do you think a penny weighs?
d. How many grams do you think a postcard weighs?

THE METER
is the basic unit of length.
A yardstick is almost a meter long.
a. Is a baseball bat longer than a meter?
b. Are you taller than a meter?
c. Is a door wider than a meter?
d. Is a room higher than 3 meters?

THE LITER
is the basic unit of capacity.
It is used in measuring liquids.
A quart container would hold about a liter.
a. How many liters of milk do you drink each day?
b. Do you think you can drink two liters of pop?
c. Could gasoline be measured in liters?
d. Have you seen any liquids marked in liters?

(2) Group metric words by roots and prefixes. Fill in the ?'s.

Length	Weight	Capacity
kilometer	?	kiloliter
hectometer	hectogram	?
?	dekagram	?
meter	gram	liter
?	?	deciliter
centimeter	?	centiliter
millimeter	milligram	milliliter

HOW MANY

1. millimeters in a meter?
2. centimeters in a meter?
3. meters in a kilometer?

4. centiliters in a liter?
5. grams in a dekagram?
6. meters in 3.86 centimeters?
7. milligrams in 42.7 grams?

Note: Have the pupils note that the answer to all these conversion problems involves only a simple movement of the decimal point.

(3) Copy and complete the following table. Compare the first unit of measure with the second. Write > or < . Then tell if you have compared units of length or weight or capacity.

First Unit of Measure	> or <	Second Unit of Measure	Weight, Length or Capacity
1. meter	a	kilometer	b
2. deciliter	a	liter	b
3. gram	a	milligram	b
4. centimeter	a	meter	b
5. liter	a	milliliter	b
6. millimeter	a	meter	b
7. kilogram	a	gram	b
8. centimeter	a	millimeter	b
9. milliliter	a	deciliter	b
10. kilogram	a	dekagram	b
11. kilometer	a	millimeter	b
12. gram	a	dekagram	b

Which is the largest unit measure of length on this page?
Which is the smallest?

Which is the largest unit of weight on this page?
Which is the smallest?

Which is the largest unit of cpapcity on this page?
Which is the smallest?

What unit of metric measure would you use to describe each of these items?

a. A bag of potatoes
b. The weight of a postcard
c. The capacity of a milk bottle
d. A bottle of apple juice
e. The distance to Mexico
f. The capacity of a medicine dropper
g. A bottle of castor oil
h. The length of a kite string
i. The length of a sewing needle
j. The weight of a loaf of bread
k. A can of paint
l. The weight of a postage stamp

C. ACTIVITIES

MATERIALS: Meter sticks, balance and gram weights, graduated liter containers, objects for weighing and containers to fill.

DIRECTIONS: Have the pupils perform the following measurement activities:

1. How many cm. can you jump?
2. How many meters around your waist?
3. How many cm. tall are you?
4. How many grams does a potato weigh?
5. How many containers (whatever you have) does it take to fill a liter?
6. How many liters does it take to fill the container?
7. How many meters wide is the room?

Make up more activities of your own.

332

IN REPLY REFER TO:
Area Education

UNITED STATES
DEPARTMENT OF THE INTERIOR
BUREAU OF INDIAN AFFAIRS
Navajo Area Office
Window Rock, Arizona 86515

JAN 3 1979

Betty Rose D. Rios
Clearinghouse on Rural Education
and Small Schools
Box 3AP
Las Cruces, New Mexico 88003

Dear Mrs. Rios:

Your letter expressing concern over the absence of certain pages in the Navajo Area Mathematics Guidelines has been received. In the pilot copy, we had a test for the teachers to use with their children. In the revised edition, the one you received, the test pages were removed for further study and refinement. So your edition is a completed edition.

Thank you for your concern.

Sincerely yours,

Mary E. Ross

Education Specialist

GLOSSARY

Since several words that are used in the guidelines might have more than one meaning, the following definitions are given.

1. ARRAY - A rectangular arrangement of objects in rows and columns.

2. ATTRIBUTE - A particular characteristic or quality that some or all of the members of a given set share. For example: Color, size, shape, texture, taste, material, use, smell, sound.

3. BASIC FACTS -
   - (a) Addition - All additions where the addends are single place numbers.
   - (b) Multiplication - All multiplications where the factors are single place numbers.
   - (c) Subtraction - All subtractions where the subtrahend and the difference are single place numbers.
   - (d) Division - All divisions where the divisor and the quotient are single place numbers and there is no remainder.

4. COUNTERS - Objects which have the same size and shape. Examples: Poker chips, wooden blocks, drinking straws, bottle caps, popsicle sticks, beans, tongue depressors. (Elongated counters do not pattern well.)

5. EQUATION - A mathematical sentence stating equality.

6. FIGURE - The outer edge (or boundary) of a shape. Examples: 

7. GROUP - (as a verb)- To form sets (or subsets) of a given size from some collection of objects. Example: "Group this set by size 4." (as a noun)- A set of an established size. Example: There are 4 groups and 2 left-overs.

8. INTERNALIZE - Understanding coupled with practice (drill, memorization) such that a response is automatic.

9. JOIN - The uniting of two separate sets (or subsets) to make one set.

10. LINE - A continuous set of points of infinite extent in both directions. A line can be straight or curved.

11. LINEAR MEASURE - Measurement along a straight or curved line.

12. MEMBER (ELEMENT) - Anything in a given set.

13. MODEL - A concrete example used to illustrate a concept.

14. ODDS AND ENDS - Anything that can be used in making sets. Example: Buttons, thread spools, pencils, jacks, spiders, ice cubes, etc.

pages 391 - 404 missing

15. OPERATION - Any of the arithmetical processes: addition, multiplication, subtraction, division.

16. PATTERN - An arrangement of members of a set designed to make an attribute of the set apparent.

17. SET - Any definite collection of objects.

18. SET INDICATOR - Any object used to indicate a set such as yarn, shoe boxes, coffee cans, felt squares, drawn boundaries, etc. It is important to note that the boundary is not a part of the set.

19. SHAPE - The inside of a figure. Examples: 

20. SUBSET - A set which is part of another set. It should be noted that a set is a subset of itself.

335

ADDITION

Addison Wesley 1968	American Book Co. 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Harcourt Brace 1971
1: 15, 16, 59-72, 93-114, 121, 125-130, 139-144, 152, 199-208, 214-219, 222-260 2: 21-38, 51-74, 77-100, 109-130, 142-164, 174-176, 200-204, 213-220, 236, 269 3: 24-26, 32-35, 41, 52-58, 68-69, 269, 298-300, 303, 308 4: 22-25, 34-41, 158, 159, 322, 324 5: 102, 103	1: 15, 16, 30-32, 40, 42, 51-52, 55-58, 63-65, 70-73, 80-87, 92, 98-99, 102, 104-105, 122, 127, 134, 139, 140, 151, 158-163 2: 2, 4-6, 9, 10, 12, 15, 17, 19-20, 23, 26-27, 36, 38, 41, 60, 63, 72, 84, 86-87, 95, 105, 110-112, 133, 136, 139-140, 159, 161, 177 3: 1, 2, 5, 18, 19, 21, 30-32, 34, 38, 109-111, 114-115, 118, 147, 161, 176, 203, 208, 248-251, 258, 262, 294-296	4: ~~73~~, 114, 115, 118, 119, 121, 247, 248, 251, 273, 30-32, 37-43 5: 69-74, 85 6: 35, 36, 43, 44	3: 9-11, 18-22, 26, 36, 40-41, 52, 75-76, 79, 80, 83, 88, 44-45, 53-57, 159, 184, 185, 102-104, 109-114, 123, 169, 184-187, 168-170, 173 4: 10-12, 14, 17, 258, 320, 322-323 5: 20, 21	B: 17-28, 56-80 1: 53-69, 85-95 2: 16-25, 36-50, 61-70, 71-80 3: 32-45, 5, 7, 68-71, 75, 122 4: 24-29, 40-44, 148-149, 226, 260-266, 358-359 5: 26-33, 40-41, 152-158, 162-163, 176-177, 257-261 6: 24-34, 43, 142-143, 146, 194-195, 236, 346-347 7: 16, 20, 30-31, 54, 59-61, 130-138, 276, 285	8: 2-4, 8, 9, 20, 50-51, 74-75, 92-93, 218-219, 226-229, 280

ADDITION

Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970	Houghton Mifflin 1967-1970	Houghton Mifflin 1972	Houghton Mifflin 1972	Laidlaw 1968
1: 38-41, 57, 96, 105-108, 132, 133, 137, 140, 145, 160-162, 165, 168-171, 177, 178, 192-197, 202-218 2: 14-18; 30-49, 50-61, 68, 74, 75, 118, 120, 123-125, 129-134,1 150-155, 159-165, 190-194 3: 16-20, 37-38, 53, 56-67, 74-79, 90, 108, 109, 114-118, 122-126, 130, 140, 141, 150, 363, 365, 368-371 4: 25, 48-49, 56-57, 62-67, 256 5: 42-50 6: 20-22	1: 61-70, 81, 86-87, 91-98, 101, 105, 107, 108, 120, 131, 132, 157-160, 162-166, 169-179, 182, 195, 196, 223, 225-230, 233, 237, 239, 241-244, 249-250, 277-290, 293, 295, 296, 302-308, 317, 318 2: 15-18, 23, 25, 30, 33-47, 53, 54, 57, 62-64, 93, 103-110, 113, 115-118; 127-133, 136-146, 153, 154, 157-167, 177-181, 199-211, 267, 268, 271, 272, 277 3: 34-39, 40-45, 48, 59, 94-95, 98-99, 102-	3: cont'd. 102-111, 114-117, 180-185, 320 4: 30, 44-49, 52, 55, 60, 320-321 5: 47, 48 6: 32-35	1: 51-56, 61-64, 75-76, 83-84, 111-112, 137, 138, 141, 148-174, 178-187, 223, 224, 233-238, 259-272, 278-300, 311 2: 15-27, 33-46, 51, 55-56, 75-76, 103-118, 124-167, 172, 195-206, 310 3: 27-35, 38-45, 49-56, 69, 75, 83, 160, 188, 221 4: 8, 30-33, 36, 38-39, 40-53, 58, 78, 228, 229, 251 5: 30-35, 38, 39, 40-47, 53, 57-58, 202, 222-223, 237, 321, 322, 335	6: 29-33, 36-37, 39-41, 175, 193, 197, 202-203	1: 43-53, 56, 60-78, 129-132, 134, 135, 139-149, 156-161, 164-168, 173-174, 187, 189, 191 2: 48, 59, 60, 63-67, 72-83, 117-122, 125-127, 88-92 3: 11-17, 23, 28, 36-38, 67, 71-73, 82-87, 90-93, 125, 129, 135, 137, 289, 301, 306, 307, 318, 319 4: 33, 35, 41-49, 53, 94-101, 305, 306, 311 5: 38, 40, 47

ADDITION

Scott Foresman 1966-68	Silver Burdett 1970	Silver Burdett 1970	SRA 1974	Sterns 1965	
1: 99-104, 110, 111, 114-131, 147-159 2: 7-20, 29-31, 114-120, 124, 129, 134-136, 183, 186-203, 215 3: 21-28, 33, 59, 107-120, 123-125, 127, 128, 130, 132-142, 144-149, 151, 159, 165, 232 4: 22, 30, 54-60, 70, 71, 359-363, 367-369	1: 23-30, 33, 41-45, 75-78, 80, 111-115, 120, 127-130 -139, 141-145, 161-165, 175-178, 191, 194-197, 225-234, 237-241, 250, 154-156, 166, 167, 171, 187, 205, 218, 220, 222, 107, 108, 150, 151, 153, 180 2: 15-20, 37, 51-52, 57-58, 61-68, 75-76, 83, 88, 106, 110-112, 117, 120, 127-129,131, 132, 135, 141-143, 147, 149-151, 153, 158, 167, 179, 203, 231, 234, 238-244, 3: 6-7, 26-30, 33-35,	3: cont'd. 37,47, 54-59, 64, 74-78, 84, 91, 108, 117-123, 129, 132-135, 146-148, 151, 172, 178, 186, 232, 239 4: 5-7, 30, 46-47, 50-51, 64 5: 13-20 6: 16-21	1: 10-18, 65-128, 194-218 2: 163-179 3: 3, 7, 9-23, 33-35, 47, 60-71, 153, 155, 157, 159-168, 174-176, 287, 288, 293, 294 4: 22-31, 39, 41, 42, 47, 51-53, 101, 102, 163-170, 314-319, 336 5: 16-17, 296-299 6: 2-11, 13, 15, 26, 150, 315	1: 8, 9, 26-46, 75-94, 112-126 2: 1-18, 73-92, 120-124 3: 8-16, 18-28, 30-39, 47, 57-65, 69, 73, 74, 76, 81, 82, 84, 85, 88, 131, 171-175, 187-189	

APPLICATION AND PROBLEM SOLVING

Addison Wesley 1968	Addison Wesley 1968	American Book Co. 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971
1: 57-62, 73-79, 80-109, 115-119, 130-134, 201, 211, 223-226, 243-254, 321-334 2: 33-52, 63-69, 81-129, 142-154, 161-175, 217-219, 230-235, 247-259, 265-266, 286 3: 20-39, 59, 70, 79-105, 116, 119, 126-142, 160-163, 214-217, 223-238, 275-279 4: 23-36, 72-83, 91-96, 156-168, 182, 200, 330 5: 66-72, 122, 133, 135, 141 6: 9, 29, 74, 200, 201, 236	7: 55, 57, 76, 78, 100-109, 207, 221, 245, 303, 308, 309, 321, 336, 356, 359, 368 8: 22, 41, 69, 140, 150, 264, 273, 283, 285, 230-238, 334	1: 15-19, 30, 40, 52, 55-59, 64, 65, 71-81, 90, 98-100, 142-156 2: 2-8, 12, 27, 36, 74-75, 86-87, 32-49, 99-119, 128, 129, 170-173 3: 1-4, 8, 10, 15-18, 27-29, 32-37, 45, 49, 73-89, 119, 172, 184-196, 201-207, 214-222, 260, 265, 282-283, 290-292, 304 4: 30-57, 108-113, 123-132, 141-155, 183, 184, 198, 199, 206, 209, 230, 233, 236, 250-253, 266-267, 273, 283, 290, 303, 305	5: 8, 69, 72, 86-88, 92-95, 112, 113, 138-139, 155, 158 6: 47, 57, 69, 139-143, 207-210, 231-236, 242-249, 254	3: 9, 20-22, 40-41, 66-67, 109, 159-161, 179, 182, 188, 219-220, 228, 250, 251, 272, 278, 300-301, 316 4: 8, 9, 16, 27, 28, 49, 50, 65, 69, 90, 112, 128, 138, 144, 147, 150, 155, 169, 196, 214, 256, 290, 278, 285, 311 5: 16, 18, 52, 53, 65, 28-29, 75-77, 147, 160, 211, 222-223, 210-214, 257 6: 34-35, 54-57, 65, 66, 77-80, 109, 165, 170, 199, 207, 250, 292-295	4: 19, 52, 80-81, 90, 123, 150-151, 168-169, 187, 228, 229, 245, 270-271, 296, 308, 309, 339, 362 5: 21, 42, 72, 88, 91, 116, 134, 151, 182, 206, 229, 255, 277, 307, 325, 342, 364 6: 12-13, 43, 54, 64, 90, 127, 163, 189, 210, 213, 262, 263, 277, 302, 316, 317, 333, 363 7: 97, 168-169, 222, 228-229, 238, 245, 252, 260-261

APPLICATION AND PROBLEM SOLVING

Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970	Houghton Mifflin 1972	Laidlaw 1968	Laidlaw 1968	Scott Foresman 1966-68
1: 35-58, 76-79, 84, 94-107, 111, 114, 131-142, 193-199, 206-217 2: 12-17, 30-40, 50-60, 120-133, 148-163, 169, 201-214, [illegible]77, 233 3: 14-18, 37-46, 49-54, 66-73, 81, 110-113, 131-139, 149-156, 169-176, 182-198, 242-243, 294-295, 259, 268, 305, 326 4: 42-61, 71-74, 83-95, 102-104, 116-129, 132, 174-182, 200-201, 158, 386-387 5: 225, 231 6: 79, 111, 242, 270-279	1; 57-68, 70-85, 91-107, 131-134, 150-168, 175-179, 197-198, 215-248, 278-298, 307-310 2: 15-18, 23, 29, 39-59, 104-123, 129-144, 149-155, 163-171, 189-191, 249-256, 283-296, 301-303, 309-312 3: 26-51, 86-99, 101, 126-142, 182-196, 252-255, 262-266, 269-275, 280, 313-317, 339	3: 269 4: 82, 83, 88, 89, 140-141, 167, 172 5: 38, 136, 192-198 6: 47, 109, 118, 259, 290	1: 43, 46-54, 56, 63, 65, 67, 70, 74, 75, 110, 114, 129, 132, 133, 134-138, 140, 142-147, 150-152, 171, 172, 188, 194 2: 48, 50, 54-67, 86, 87, 99, 128, 132-136, 154-162, 165, 166, 169, 170 3: 12-16, 19-23, 17, 28-33, 36, 38, 43, 71-81, 96, 99-103, 106, 110-113, 116-119, 145-155, 159-161, 186, 194, 197, 207, 211, 128, 251, 274, 281, 299, 300, 305, 308-311,	3: cont'd. 315, 316, 120-122, 201 4: 3, 20, 21, 33-43, 45, 49, 56, 59-70, 76-79, 84, 93, 94, 104-106, 110, 118-120, 134, 137, 144, 145, 152, 153, 158, 304, 305, 307, 319 5: 44, 45, 58-62, 64, 70, 82, 83, 96, 97, 166, 186, 187, 322 6: 52, 58, 62, 72-77, 84, 88, 128, 150, 191, 192, 353	1: 99-108, 110-121, 123-128, 130-133, 147-158 2: 22-26, 28-49, 52, 53, 69-78, 108, 118, 119, 123, 124, 129, 134-161, 183, 186-189, 192, 193, 196, 198, 199, 215 3: 16-17, 18, 22-31, 46-50, 57, 59-71, 107-111, 113-115, 118-119, 121, 122, 124, 125, 127, 133-138, 150, 167, 168, 177, 181-192, 194-199, 202-204, 214, 215, 218, 219, 239-245, 293, 328-331 (continued on the next page)

APPLICATION AND PROBLEM SOLVING

Scott Foresman 1966-68	Silver Burdett 1970	Silver Burdett 1970	Silver Burdett 1970	SRA 1974	SRA 1974
4: 20, 22-25, 27-32, 36, 51, 69, 74, 79, 80, 82, 84-96, 98, 101, 103, 104, 107, 127, 131, 132, 144, 146, 152, 154-156, 169, 190, 191, 197, 201, 204, 207, 209, 210, 213, 214, 224, 227, 230-240 5: 35, 82, 86, 87, 91-99, 200-207, 269-280, 213 6: 53-56, 138, 143, 199-201, 152, 212, 265	1: 27-30, 41-44, 54-56, 63-68, 75-78, 89-92, 100-102, 105-108, 111, 112, 120, 121, 127, 129, 135, 141-143, 145, 147, 148, 161-164, 175-178, 181, 191, 192, 197, 218, 221-223, 227, 228, 235-237, 282, 283, 284, 288, 291 2: 15-20, 37, 38, 57-61, 66, 87, 89, 105-107, 110-113, 127-129, 135-137, 149-152, 163-167, 179, 182, 191, 206-208, 210, 218, 219 3: 6, 10, 14-16, 18, 26-37, 46, 49, 57, 61, 62,	3: cont'd. 130, 131, 134, 135, 152, 153, 155-164, 166, 167, 230, 232-235, 251-253, 257, 260, 270-279, 310-312, 316, 338, 339 4: 5-17, 36, 37, 42, 54, 60-62, 64, 76-78, 80-82, 87, 88, 91, 92, 116, 119, 122, 123, 126, 127, 154, 171, 190, 203, 208, 241-245, 266, 277, 278, 281, 231, 307-309, 312, 313, 329, 330, 342, 343 5: 56, 57, 173-174, 196, 261, 262, 361-367, 370, 371, 382	6: 217, 240-254, 270, 330	3: 7, 10, 11, 13, 14, 19, 32, 34, 35, 49, 50, 62, 64, 69, 70, 83, 89, 93, 96, 97, 101, 107, 111, 112, 118, 120, 129, 141, 143, 159, 162, 163, 175, 181, 182, 188-194, 203, 215, 218, 219, 227, 230, 231, 235-240, 242, 245, 247-263, 274, 276, 277, 283, 284, 298, 299, 303, 304 4: 22-25, 29, 30, 34, 37, 38, 40-42, 44, 48, 49, 51, 56, 57, 61, 66, 68, 70, 74, 75, 78-81, 83, 85, 87, 88, 91-93, 98, 99, 103, 104, 106, 109, 110,	4: cont'd. 115-119, 122, 123, 125, 131, 135-137, 139, 141, 147-153, 165-166, 172-175, 178, 181, 184, 195, 198, 200, 209-213, 206, 221, 224, 226, 230-234, 237, 240, 250, 264, 265-277, 279, 283, 288, 313, 316, 336 5: 11, 21, 28, 41, 49, 50, 74, 97, 105, 107-109, 111, 113, 115, 131-133, 135-143, 149, 150, 152, 164, 168, 172, 174, 177-180, 185-188, 191-194, 199-120, 225, 228, 236-238, 241, 242, 267, 292, 293, 297, 301, 315, 320 6: 29, 32, 34, 36, 42, 50, 105, 124, 125, 127, 128, 138, 140, 150, 155-163, 168, 188, 189, 191, 213, 270, 271, 276, 300-309, 330

CATEGORIES: EVEN AND ODD

Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968	Houghton Mifflin. 1967-1970
3: 198-205, 208 4: 45, 232-235, 238-241 5: 160-165, 168- 78, -200, 23 -25 , 310, 3 6: 10-12, 34, 35, 20- 2, 85, 87-89, 142, 143, 152, 162, 163, 290-297 7: 169, 176 8: 95	3: 39-41, 181, 182, 274 4: 5: 108, 136, 138, 251-252 6: 10-12, 85, 146, 147, 162, 163, 177-179	3: 30, 31, 34, 35, 206, 221, 224, 239 4: 29, 46, 47, 56, 67, 76-78, 98, 132, 235, 224, 226, 230, 280, 287 5: 48, 55, 56, 110, 185, 188-189, 193-197, 245-287 6: 66-69, 97-98, 138, 207, 234	1: 109-118 2: 1-15 3: 16, 330, 331 4: 18, 238, 239 5: 2, 3, 97 6: 132-133 7: 5, 106, 107 8: 34, 35	4: 44, 120, 224, 225, 225, 264, 394-395 5: 83-90, 116, 140-142, 218, 250-255 6: 33, 90, 138, 139, 144, 145, 204, 207	3: 226-228, 230-234 4: 200-203, 210-211 5: 44, 164-166, 170-178 6: 164-168, 170-172, 174, 175

CATEGORIES: EVEN AND ODD

Laidlaw 1968	Scott Foresman 1966-68	Silver Burdett 1970	SRA *1974	Sterns 1965	
3: 46, 59, 104, 105, 108, 110 4: 62, 68, 156, 157, 190, 191, 204-208 5: 83, 103, 108, 109, 112-122, 150-155 6: 12, 81-83, 116, 138-142, 144, 145, 204	3: 356 4: 33, 215, 216, 217, 274 5: 49, 51-56, 60, 135-137, 140-142, 166 6: 11-16, 70-71, 88	3: 9, 10, 180-188, 288 4: 4, 41, 63, 215, 217, 267, 284, 285 5: 85-87, 112-116, 344, 345, 377, 378, 383 6: 52, 55, 62, 82, 92, 93, 101, 102, 343-349, 356	3: 128, 172, 204, 287-291 4: 43, 66, 71, 134, 171, 176, 187 5: 26, 27, 40, 326-328 6: 171-173, 176, 206, 208, 209	1: 24, 106-108 2: 52, 81, 96, 103, 104, 127 3: 10-13, 20, 97-100, 110	

343

DIVISION

Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970
3: 140-142, 144, 145, 148, 149, 152, 156, 158, 162, 254-258, 272-274, 278-281, 136-138, 264-267, 313, 317 4: 92, 93, 95, 110, 96, 98, 202, 211, 221, 222, 225, 227, 230, 329, 335 5: 121 6: 56, 57 7: 64, 74, 109, 304, 309, 319 8: 109, 271	3: 74, 75, 78, 79, 83, 123, 125, 128, 164-168, 173, 231, 197-199, 210, 231-233, 244, 255, 43-45, 69-71, 299 4: 54, 55, 57, 58, 134-139, 323, 262, 263, 268-271, 278-282 5: 169 6: 118, 140	3: 280-288, 290-296, 304-310, 322-328, 330-340 4: 104-106, 110-132, 140, 162, 167, 171, 76-79, 176-178, 183, 330 5: 50	2: 16-25 3: 188, 146, 147, 142, 143, 156-161, 204, 290, 342, 316-319, 306-312, 346 4: 88-89, 92-97, 104-109, 156-161, 172-173, 196-198 5: 112-114, 200, 201, 294-297, 188-197, 352-360 6: 60-67, 80, 89, 172, 202-219, 248-262, 358-359 7: 6, 24, 90-96, 88, 112-113, 140-146, 216, 210-211 8: 28, 29, 82-86, 220, 221, 240, 241, 276	3: 160-165, 67, 168, 181-183, 173, 174, 244-250, 186-188, 233, 234, 311-314, 307-310, 239-241, 261-266, 257, 258 4: 88, 89, 91-93, 101, 104, 109, 118-121, 160-166, 174-180, 169-172, 186-190, 195-197 5: 113, 114, 128-132, 135, 136 6: 113-116	3: 130, 134, 138, 143, 157, 270, 271, 274-276, 196-200, 327-333, 336, 337, 338, 326-332 4: 106, 160, 180-185, 189, 230-236, 237, 250-253 5: 118, 122, 123

DIVISION

Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1966-68	Silver Burdett 1970	SRA 1974	Sterns 1965
1: 305, 305 2: 287-307, 313-316 3: 125-138, 142, 144, 164, 165, 271, 273, 307, 325, 191, 268, 272-276, 310-313 4: 61-63, 70-77, 210-216, 189, 230-232 5: 61-87, 130-133, 142-147, 180-184, 253 6: 40, 49-53, 66-70, 79-89, 317	3: 146-166, 202-214, 323, 324 4: 72-75, 82, 83, 120-127, 130-133, 146-151 5: 6: 69	2: 140-164, 204-[illegible] 3: 29, 30, 32, [illegible] [illegible]7, [illegible]7-225, [illegible]-3[illegible] 202 4: 98-[illegible]03, 133-136 142-146, 199-200, 173-188, 364-366	3: 158-167, 184-185, 230, 231, 234-237, 240-248, 250, 251, 254, 257-259, 296-304, 271-276, 283, 313-315, 330, 331 4: 78-81, 85, 86, 89, 90, 93-96, 107-109, 113-120, 123, 141-143, 160-166, 172, 192-200, 204, 213, 271-276, 283 5: 58, 60, 63-67, 78	3: 218, 219, 222-231, 301-308 4: 58, 68, 190-215, 324, 326 5: 24, 39-46, 48-50, 100-121, 306-308 6: 45-48, 328, 329	3: 141-151, 153-161, 164-170, 176-181

GEOMETRY

Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970
3: 48, 49, 128, 129, 132, 135, 164-171, 178-181, 190-192 4: 64-71, 112, 118-127, 174, 185 5: 82, 96-99 276, 277, 282, 283, 286 6: 107-109, 113-117, 253, 262-265 7: 117, 120, 124, 129, 259, 264, 267, 270, 272, 275, 278, 282, 377, 380, 381, 383, 389, 393, 405, 409, 412, 415 8: 171, 175, 177, 180-189, 191, 193, 291, 296, 298, 299, 349, 395, 413, 417	2: 47-50 3: 54, 57-61 63, 80-83, 134, 135, 139, 242 4: 86-89, 220-226, 228, 229, 231-235 5: 7, 26-31 6: 88-96, 102, 105	3: 64,65, 116,117 4: 64,65, 116,117, 268,269 5: 90-101 6: 129-130, 150-157, 174-176, 294,295, 310,311	8: 1-16, 46-55 1: 1-15 2: 51-60 3: 80-87, 89-91, 224-231 4: 114-117, 121,204-213,302-305,310-317 5: 48-65, 178-180, 212-219, 222-225, 272-275, 286-289, 350,351 6: 98-107, 110,111, 272-275, 280-289 7: 74-76 8: 102-111, 130,131	1: 118-126, 2: 138-147 3: 204-211, 214-219, 222, 223, 225-227, 331-337 4: 299, 301, 314, 315, 317-322, 326-331, 333-345, 360-362 5: 365-384 6: 189-195, 352-359, 361-368, 372-374, 382-394	3: 68-72, 88-89, 154, 155, 174, 236-237, 291, 298-301, 306 4: 88-90, 267-270 5: 86, 114, 115, 261, 273 6: 78, 79, 142, 143, 269, 270, 271, 273, 275

GEOMETRY

Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1966-68	Silver Burdett 1970	SRA 1974	
1: 119-122, 199-212 2: 61-90, 217-251 3: 93-101, 114-185, 214-295 4: 93-124, 197, 285-325 5: 21-23, 54, 55, 93-124, 286-315 6: 20, 21, 46, 93-120, 230, 289-316	1: 2, 3, 89, 91-94, 180-182, 190 2: 104-115 3: 68, 72, 221, 223-226, 228, 230, 235, 278-291 4: 217-219, 221-226, 228-235, 264-273, 278, 279 5: 241-243, 248-254, 258-260 6: 269-281, 288, 299	1: 3-11, 134 2: 3-9 3: 34-39, 42, 43, 45, 281-283, 287-291 4: 110-113, 115, 119, 120, 243, 245, 284-290 5: 215-220, 281-290 6: 272-278, 330-336	1: 125, 126, 243, 244 2: 169, 170, 228, 229 3: 213-221, 340-343, 299, 144 4: 20, 21, 216, 227 5: 289-291, 294, 299, 307-309 6: 272-278, 330-336	2: 93-101 3: 75-83, 110-118 4: 146-155, 157-159, 291-305 5: 54-71, 271-287, 330-333 6: 55-71, 218-237, 270-287	

MULTIPLICATION

Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970
2: 247-260 3: 84, 85, 92, 96, 97, 106-115, 120, 121, 126, 127, 212, 213, 220-226, 230-234, 241, 256, 312, 314-316 4: 72, 82-86, 136, 137, 168-173, 176-178, 182, 183, 329, 331-333 5: 30, 31, 50, 51, 54, 55 6: 36, 46-49, 230-232	2: 117-119, 122, 123 3: 42, 43, 69-73, 78, 82, 124-127, 164, 172, 196-200, 209, 210, 232-235, 240, 241, 245-247, 253, 254, 278-281 4: 50-52, 124-128, 258-260, 264, 265, 335-339 5: 95-96 6: 54, 55, 124	3: 206-209, 221-226, 239-242, 248-249, 258-260, 266-269, 318-320 325, 326 4: 46, 47, 97-104, 132, 135, 160, 162-164, 171, 172, 234-239, 334-339 5: 58-59 6: 60, 234	3: 134, 135, 138, 181-201, 208-213, 216, 217, 252-254, 257-264, 296, 336, 337, 354, 355, 358, 359, 362, 363 4: 58-70, 156-167, 226, 227 5: 78-86, 89, 92-95, 176, 177, 312-321, 323, 326, 327 6: 10, 11, 48-53, 62, 63, 132, 133, 172, 180-188, 194, 195, 240-243, 251, 254-257 7: 32, 33, 78, 79, 82, 83, 91, 120, 121, 140-145, 150, 151, 204-207, 285	3: 151-154, 156, 157, 162, 163, 167, 168, 171-176, 178-181, 185, 186, 190, 191, 232, 233, 235-238, 242-245, 247-249, 256, 257, 259, 260, 296-304, 325 4: 86-88, 90-96, 100-105, 109-114, 128-141, 148-150, 153-157 5: 71-73, 101, 102 6: 93, 94, 104	2: 283-290, 293-299, 301-306, 309-318 3: 126-129, 132-139, 141-143, 145, 157, 194, 195, 198, 200-204, 209-213, 220, 223, 256-261, 264, 265, 283, 316, 322 4: 55, 94-97, 104-108, 116-121, 162-164, 168-174, 224-229, 255

MULTIPLICATION

Houghton Mifflin 1972	Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1966-68	Silver Burdett 1970	SRA 1974
1: 305, 306 2: 131, 287, 288, 291-294, 297-300, 303-307, 310-316 3: 125-138, 140, 142, 144, 147, 164, 165, 191, 194, 196, 197, 213, 233, 253-257, 260, 261, 266, 268, 270, 271, 273-275, 280, 306-309, 313, 325 4: 61-78, 82-86, 91, 92, 189, 191-196, 198-204, 207, 210, 222-229, 246, 247, 258, 317 5: 61-67, 72-77, 82-84, 130-133, 146, 147, 222, 223, 253-261, 270	6: 40-45, 48-53, 61-67, 70, 79-86, 89, 230, 317, 332	2: 158-168 3: 99, 100, 104, 106-111, 114-117, 189-191, 194-196, 216, 217, 308, 310, 311, 321, 324 4: 69-71, 102, 103, 109-115, 137, 139, 140, 141, 307-309, 312, 314 5: 26, 27 6: 64, 65	2: 140-142, 146, 149-162, 166, 167, 204-214 3: 29-31, 62, 68, 114, 118, 121, 124, 127, 202, 205-207, 210-212, 216, 238, 242, 245, 314-316, 323, 328-341 4: 24, 27-29, 96-104, 129, 140, 141, 146, 154-168, 364, 366 5: 6: 343, 344	2: 181-186, 205-211, 251, 253-256 3: 152-157, 183, 185, 230-237, 240-253, 259, 269-277, 291, 292, 317, 318, 320, 325 4: 76, 77, 82, 89-91, 111, 113-117, 120, 122, 123, 127, 129, 142, 145, 153-157, 169, 170, 187-191, 206, 212, 231, 232, 258-265, 280, 281 5: 44-49, 52, 55	3: 38, 39, 41-50, 120-127, 129-141, 143, 223-230, 301, 302, 305-308 4: 48-61, 63, 64, 70, 98-119, 178, 181, 184, 192, 193, 212-215, 320, 321 5: 24, 25, 28, 30-38, 51, 296, 301, 303, 304 6: 38-41, 49-51, 117, 119, 155, 306, 321, 323-325, 328, 329

NUMBER RELATIONS

Addison Wesley 1968	American Book Co. 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968
K: 21-64, 66-74, 76, 78-80 1: 1-32, 35-43, 45-48, 51-56, 157-175, 177, 178, 181-184, 187-191, 239-242, 271-272 2: 1-30, 179-180, 184-191, 195, 196, 199, 202, 223, 224, 239, 242-245 3: 2-14, 18, 19, 26, 27, 30-37, 62, 66, 67, 72, 81, 84-91, 98-104, 144, 145, 152, 180, 220 4: 3-13, 30-34, 46, 76, 79, 80, 92, 108, 221, 222, 225, 136, 168-170, 227, 230, 335	1: 1-5, 11-14, 18, 22-29, 34, 44-46, 86, 89, 109, 111-113, 124-126, 138, 143, 144, 67-69, 90 2: 28, 29, 35, 37, 107, 131, 132, 146, 150, 154, 158, 183, 185, 186 3: 6-27, 42-45, 69, 71, 73, 74, 85, 119, 120, 121, 123, 160, 167, 168, 209, 234, 235, 246, 272, 281, 285 4: 3-9, 12, 13, 17-21, 26-29, 38, 45-49, 54-59, 134, 171, 172, 200, 201, 262, 263, 268, 271, 276-282	5: 68-72, 78-80, 86, 88, 90-94, 205, 206 6: 4, 13-18, 37-39, 48, 50-52, 56, 111-113, 118, 120, 146-147, 178, 315-317, 320-321	3: 15-18, 40, 41, 70, 71, 76, 86-88, 119, 123, 206, 221, 224, 239, 283, 284, 322 4: 8, 9, 19, 46-55, 59-66, 94-98, 132, 133, 158 5: 4, 11, 15, 18, 28, 48, 49, 52-55, 60, 158 6: 28-30, 54-56	B: 1-16 1: 1-15 2: 1-15 3: 10, 16, 26-27, 144 4: 32-34, 36, 37, 136, 137 5: 2, 3, 8-11, 242, 243, 250, 251 6: 2, 3, 6, 232, 233, 330, 331 7: 45, 48-51, 200, 202 8: 62, 64, 69, 146, 148, 149, 264	1: 2-34, 62-75, 80-85, 90, 148-156, 178, 188-190 2: 4-12, 19-24, 28, 32, 63, 64, 78-88, 108, 189, 197-199 3: 3-5, 11, 12, 18-23, 47-49, 55, 68, 83, 84, 87-89, 91-95, 107, 155, 160-169, 171, 193-197, 232, 318, 320-324, 369 4: 1-5, 14, 17-19, 24-26, 28, 31-34, 57, 71, 93, 102-107, 114, 126-128, 132, 133, 174, 182, 195, 197, 199, 387 5: 11-13, 23, 45, 58, 59, 61, 74, 75, 79, 94, 101, 114, 115 6: 7, 8, 24-27, 59-62

350

NUMBER RELATIONS

Houghton Mifflin 1967-1970	Houghton Mifflin 1967-1970	Houghton Mifflin 1972	Houghton Mifflin 1972	Laidlaw 1968	
K: 33-40, 43-49, 69-77, 82m 84 1: 5-54, 122-135, 141-147, 150, 213, 241-248, 251, 252, 254-262 2: 2-16, 30, 93-100, 263, 265, 266 3: 2-9, 12, 18, 20, 21, 25-27, 30-35, 40-42, 134-138, 141-145, 190-193, 211, 223, 240-243, 255, 268-271, 312-316, 322 4: 2, 11, 12, 14, 17-19, 28, 29 32-37,94-99,106, 106, 108, 115, 117, 162, 163, 150-253, 321-324	5: 98, 102, 320, 322 6: 32, 42-44, 54, 112, 113, 130, 131	1: 25-28, 43-46, 57-60,77, 78, 91-101, 125-132, 153, 154, 221-231, 240-245, 250, 251, 259, 265, 277, 285, 287, 290, 291, 295-299, 307, 308 2: 7-12, 25-29, 52, 75, 93-98, 163-166, 175-178, 193, 199, 200, 257, 258, 263, 269, 275, 279, 310 3: 4-9, 14-16, 18, 21, 23, 29, 32, 33, 36, 50, 89, 113, 141, 242, 243, 244 4: 6-9, 16, 16, 17, 20-23, 27, 42, 46, 47, 58, 125, 136-	141, 145, 146, 155, 174-175. 5: 5, 12, 13, 24, 40-46, 71, 86, 150, 151, 239, 280, 321, 322. 6: 12, 16, 24, 29, 34, 36, 130-139, 189, 283.	K: 5, 10-17, 19, 21-25, 31, 33, 35, 37, 41, 43, 61, 63, 69, 77-81, 85, 89, 97-101, 105, 109, 115-125. 1: 6-9, 14-19, 23-35, 39-41, 102, 103, 106, 107, 110-124, 126-128, 183, 186. 2: 3-6, 7-9, 10, 11, 12, 13, 20-46, 62. 3: 7, 15, 16, 20-22, 27, 35-42, 50-67, 76-79. 99-103, 106, 107, 110, 112, 113, 116, 120-122, 140-142, 145-150, 152, 153, 194, 198.	4: 8, 9, 15-21, 23, 25, 34-42, 54, 59, 60, 74, 76, 78. 79, 93, 94, 100, 101, 110, 112, 114, 116, 118, 137-151. 5: 31-35, 41-45, 54, 57-62, 64-66, 70, 82, 85, 96, 97, 166. 6: 29, 30, 32-43, 52, 54, 55, 58, 60, 62, 66, 67, 72-77, 84, 128.

351

NUMBER RELATIONS

Scott Foresman 1966-68	Silver Burdett 1970	Silver Burdett 1970	SRA 1974	Sterns 1965
1: 14-53, 58-63, 67-87, 98, 122, 134, 145, 146. 2: 16-20, 27, 35, 54, 81-96, 109-111, 184. 3: 10, 11, 29, 30, 47-50, 54, 57, 75-82, 84-91, 112, 117, 126, 127, 129, 149, 151, 200-206, 222-224, 226, 230, 233, 234, 239, 246, 274. 4: 14-19, 23-26, 37-43, 46, 48, 50, 56, 69, 73, 90, 98-102, 182-189, 228, 340-349, 355, 356. 5: 10-14, 16-22, 28, 46, 96. 6: 23-25, 31-33, 342.	K: 9, 17-29, 31, 57, 59, 60-75. 1: 2-5, 7, 11, 13-18, 24, 31, 36, 62, 71-74, 84, 85, 95, 96, 101, 102, 145, 146, 157-159, 162, 169, 172, 182, 185, 186, 191, 192, 203, 204, 211, 212, 236, 253, 256. 2: 2-5, 7, 8, 10-14, 18, 22-24, 31, 36, 55, 56, 61, 62, 67, 69, 70, 81-93, 98, 101-111, 119, 123, 145, 146, 162, 177, 178, 235, 238, 245, 246. 3: 1, 2, 6-8, 10-13, 21, 22, 24, 25, 38, 39, 43, 45, 54, 55, 62, 75-77.	79, 81, 82, 85-89, 91, 112-119, 129, 149, 151, 160-162, 166, 167, 188, 235, 246, 252, 270, 276-279. 4: 3, 5-7, 9-11, 17, 20, 42, 64, 65, 85, 86, 98-101, 104, 122, 161, 189, 231, 232, 271, 273, 276, 283. 5: 12, 14, 41, 43, 59, 155. 6: 16-19, 26, 30-38, 42, 43, 231, 232.	3: 2-36. 4: 2-20, 22-45. 5: 2-22. 6: 2-27, 29-52.	1: 1-6, 48, 127, 128. 2: 17, 18, 27-43, 56-61. 3: 1-9, 41-44, 70-72, 84, 92, 191.

SETS

Addison Wesley 1968	Addison Wesley 1968	American Book Co. 1968	Harcourt Brace 1971	Holt-Rinehart 1966-1970	Houghton Mifflin 1967-1970
K: 21-54, 56-64, 66-74, 76, 78-80. 1: 1-32, 39-43, 45-48, 51, 52, 55, 56, 153, 154, 157-175, 177, 178, 181-184, 187-191, 239-242, 271, 272. 2: 1-11, 13-16, 18-30, 178-189, 191, 195, 196, 199, 202, 223, 224, 239, 242-246. 3: 2, 3, 6-14, 18, 19, 66, 67, 72, 80, 81, 243-251. 4: 2-9, 12, 46, 108, 244-251. 5: 38, 304-311, 314. 6: 304, 305, 310, 311, 314.	7: 1, 3, 35, 43, 95, 118, 144, 169, 179, 189, 193, 201, 343. 8: 1, 25, 27, 29, 31, 47, 56, 68, 89, 92, 121, 152, 171, 207, 214, 221, 321, 339, 371.	1: 1-5, 11-14, 18, 22-29, 34, 44-46, 62, 67-69, 86, 89, 109, 111-115, 124-126, 138, 143, 144. 2: 24, 26, 28, 29, 31, 35, 37, 107, 131, 132, 146, 150, 154, 158, 183, 185, 186. 3: 6-12, 24, 25, 69, 85, 120, 121, 160, 234, 235, 272, 285 4: 3-9, 12, 13, 21, 26, 27, 29, 59, 62, 156, 262, 263, 268-271, 276-283, 288 5: 6: 40-42, 113, 345	B: 17-45 1: 1-69 2: 16-25 3: 1-10 4: 1-17, 32, 33, 114-120 5: 1-5, 18, 19, 48, 49 6: 15-27, 96, 97 7: 4-13 8: 12-19, 97	1: 2-34, 62-65, 68-75, 80-85, 88-90, 148-156, 178 2: 4-28, 32, 63, 64, 78-88, 108, 189, 197-199 3: 3-5, 11, 12, 18-23, 83, 84, 87-95, 107, 318-324 4: 3-5, 12, 14, 17-19, 24, 29, 33, 34, 195-197	K: 33-59, 69-77, 82, 84 1: 5-54, 122-135, 141-150, 213, 251-262, 276 2: 2-16, 30, 93-100, 263-266 3: 7-12, 18-21, 25-27, 30, 53, 208, 209, 223, 240-243, 298-301, 306 4: 11-14, 17-19, 35, 84-87, 112, 238, 272-277, 250-253 5: 38, 88-91, 104, 140, 280-282 6: 34, 39, 46, 52, 64, 94, 95, 140-141, 144, 192

SETS

Houghton Mifflin 1972	Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1966-68	Silver Burdett 1970	Silver Burdett 1970
<u>K</u>: 2-8, 17-56, 63-92 <u>1</u>: 1-21, 42, 47-50, 65-74, 96-98, 135-136, 161-173, 179, 182, 259, 265, 277, 285 <u>2</u>: 1-10, 15-25, 29-34, 39, 40, 53, 54, 107, 108, 115, 116, 126-131, 134-138, 161-164 <u>3</u>: 1-5, 27, 39-43, 46-52, 55, 61, 157, 166, 190, 192 <u>4</u>: 1-5, 12, 23, 27, 31, 32, 46-48, 55, 61, 64-67, 125, 157, 187, 210-211, 317	<u>5</u>: 1-5, 10, 11, 29, 31, 38, 61, 157 <u>6</u>: 1-8, 322-325, 331	<u>K</u>: 10, 12-[illegible] 25, 31-4[illegible], 49-55, 61, 63, 67, 69, 77-81, 85, 89, 97-101, 105, 109, 115-125 <u>1</u>: 6-11, 27-29, 36-38, 102, 103, 106, 107, 110-128, 137, 183, 186 <u>2</u>: 2-46, 62, 173 <u>3</u>: 7, 40, 41, 50-67, 171, 198, 297, 302, 303 <u>4</u>: 9, 15, 23, 25, 100, 101, 146-151, 290-294 <u>5</u>: 270-272	<u>1</u>: 14-63, 72-87, 98, 122, 134, 135, 145, 146 <u>2</u>: 16-20, 27, 35, 54-56, 81-96, 184 <u>3</u>: 10, 11, 54-57, 74-91, 112, 117, 222-230, 126, 129, 149, 151 <u>4</u>: 16-18, 37-43, 182-188, 219, 223, 340-349	<u>K</u>: 9-11, 17-29, 31, 57-68 <u>1</u>: 7-18, 20, 31, 62, 71-74, 79, 84, 85, 93-96, 102, 157-159, 169, 172, 182, 185, 186, 191, 192, 203, 211, 212, 221, 236, 253-256 <u>2</u>: 2-14, 22-24, 35, 36, 55, 56, 61, 62, 67-70, 81, 82, 98, 101-104, 119, 123-126, 145, 146, 162, 177, 178, 199, 200, 237, 238, 245, 246 <u>3</u>: 1, 2, 21-25, 38, 39, 42-45, 73, 79-81, 102-105, 186, 188, 346, 347	<u>4</u>: 3, 11, 19, 20, 98-101, 271-276, 283, 336-338 <u>5</u>: 268-269, 285-287 <u>6</u>: 357-359, 387

SUBTRACTION

Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968	Houghton Mifflin 1967-1970
1: 73-96, 103-107, 115, 120, 124, 131-138, 145-152, 209-220, 261-266. 2: 39-50, 63, 112, 114, 120, 165, 166, 169, 170-174, 205, 206, 229, 232. 3: 26-28, 36,37, 60-63, 71-73, 297, 301, 305-307. 4: 24, 25, 48-53, 57-63. 5: 26, 27. 6: 24.	1: 33, 38, 48, 49, 53-55, 59-63, 70, 71, 80, 84, 85, 87, 93, 103, 127, 131, 139, 148, 158, 159-163, 165. 2: 3, 11, 15, 18, 29, 31, 40, 45, 60-66, 73, 84, 95-98, 111, 112, 133, 143, 159. 3: 3-5, 14-16, 28, 31, 33, 38, 48, 81, 112, 113, 116-119, 162, 163, 178, 179, 204, 248, 250, 259, 263. 4: 38-41, 51, 107-109, 116-117, 120, 121, 133, 249-251, 272.	3: 12-18, 20-22, 37, 38, 58-63, 68-72, 80, 114, 122. 4: 6, 7, 12, 13, 296, 319, 320.	B: 17-28. 1: 53-69. 2: 16-25. 3: 8-9, 11, 32, 33, 42, 43, 65, 112, 113, 116, 117, 120-125. 4: 26, 28, 40, 41, 48, 49, 148, 149, 262, 263, 360, 361. 5: 26-35, 40, 41, 152-158, 164-165, 176, 177, 258-261. 6: 32-35, 142, 147, 236, 348, 349. 7: 18, 20, 64-67, 90, 130-147. 8: 22, 54, 55, 74-77, 92, 93, 219, 230, 231.	1: 42-55, 58, 64, 77-79, 95-98, 108-111, 114, 116, 131-144, 159, 163, 166-168, 172-174, 177, 178, 192-199, 202, 206, 207, 210, 211, 216, 217. 2: 15-17, 30-49, 54-59, 69-73, 76, 77, 116-118, 120-133, 148-155, 169-175, 191, 193, 196. 3: 16-20, 37, 38, 49, 50, 53, 58-64, 69, 74-91, 116-122, 127-130, 143-145. 4: 44, 45, 50-53, 56, 72-81.	1: 81, 97, 105, 120, 158, 164, 170, 172, 182, 195, 196, 229, 230, 241-244, 249, 250, 263, 264, 268-272, 288, 289, 292-294, 298-300, 304, 306, 317, 318. 2: 30, 40, 61, 62, 117, 130, 142, 147-151, 169-176, 199, 200, 249-261, 269, 270, 273, 274, 276, 277. 3: 35, 50, 96, 100-113, 118-121, 124, 125, 158, 179, 279, 281, 319. 4: 36, 37, 46, 47, 50, 51, 53.

SUBTRACTION

Houghton Mifflin 1972	Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1970	Silver Burdett 1970	SRA 1974
1: 55, 56, 61-64, 69-76, 79-85, 110, 113, 135-160, 170-187, 201, 202, 223-254. 2: 23, 24, 31-46, 55, 56, 75, 76, 92, 103-118, 124-146, 150, 156, 172-182, 188, 193, 194, 197, 223, 224. 3: 13, 27-35, 38-48, 51-57, 63-66, 69-75, 79, 83, 113, 141, 221, 226, 230, 236, 281, 300, 333. 4: 8, 27-53, 58, 78, 228, 229, 251. 5: 31-47, 56-58,	202, 222, 223, 237, 321, 322. 6: 29, 31-39, 175, 193, 197, 202, 203.	1: 46-49, 52-56, 60-65, 68, 70, 74, 77, 78, 129, 133, 150-157, 162-165, 169, 170-174. 2: 50-54, 58, 68-79, 84, 85, 93-97, 123, 124, 129, 130. 3: 19-23, 29-33, 74-83, 88, 89, 94, 95, 126, 127, 130-133, 136-141, 177, 180-185, 299, 305, 312-314, 318-320. 4: 36, 39, 46, 47, 50, 51, 54, 55, 98, 99, 102, 103, 302, 306, 311.	1: 115, 118, 121, 122, 125, 128, 146, 149, 153, 158, 159. 2: 30-35, 38, 42, 47, 51, 54, 60, 66, 68, 73, 78-80, 108, 118, 119, 123, 124, 129, 134-136, 183, 192, 193, 201-203. 3: 21-28, 50, 113, 118, 124, 130, 153, 164, 232. 4: 6, 26, 61-68, 176, 363.	1: 109, 110, 113, 114, 121, 150, 152, 153, 168, 180, 206, 223, 225, 235, 251. 2: 65-67, 76, 83, 84, 120, 133, 134, 138, 140-144, 148, 153, 204, 234, 246-250. 3: 12-14, 17, 26-30, 33-35, 47, 48, 51, 52, 57-59, 75, 86, 124-130, 181, 192-199, 200-204, 225. 4: 7, 8, 15, 22, 23, 48, 49, 52, 53, 71. 5: 28-32.	1: 161-162, 169-186, 225, 226, 230-235, 238-245. 2: 26, 29-37, 40, 41, 87-90, 108-110, 145-163, 193-216. 3: 13-16, 24-27, 32-35, 47, 86-89, 91-98, 101, 106, 107, 152-158, 161-164, 170-176, 289, 290, 295. 4: 22-44, 62, 63, 164, 172-175, 310-313, 336. 5: 13-16, 21, 300. 6: 17-24, 26, 27, 327, 329.

TYPES OF NUMBERS: FRACTIONS, DECIMALS

Addison Wesley 1968	Addison Wesley 1968	American Book Co. 1968	Ginn 1966	Harcourt Brace 1971	Holt-Rinehart 1966-1968
1: 277, 278 2: 275-280 3: 252, 253, 286-293, 352 4: 45, 250-265, 290-297, 308-311 5: 176-183, 226-241, 250-251, 261-265, 290-297, 300, 310-311 6: 138-145, 151, 153, 166, 167, 170-177, 185-187, 191, 200-204, 222, 237-240, 244, 290, 294-298, 302, 303	7: 6, 19, 213, 228, 234, 241, 295-297, 300-321, 328, 331, 333 8: 6, 123, 130, 133, 136, 142, 144, 153, 250-264, 273, 277, 278	1: 116, 117, 124 2: 124-127 3: 90-101, 105, 155, 182, 352 4: 159-177, 180-183, 192-193, 202, 204, 211, 212, 213, 246, 261, 288 5: 112, 113, 126, 127, 130, 131, 134-135, 140, 141, 144-147, 212-214, 221, 222, 252-259, 264-273, 280 6: 62-64, 156-158, 172-176, 180-183, 193, 197, 200-204, 240-243, 248, 249, 333-337	1: 42, 61, 68, 87 2: 42, 61, 68, 87 3: 140-143, 310-312, 230-235, 324 4: 29, 42, 43, 56, 58, 114-117, 222, 223, 235, 243, 280-283, 298-303 5: 8-11, 111, 188-195, 203, 204, 208, 213, 215, 224, 245, 247, 254, 308, 309, 312-315 6: 94, 103-106, 114, 162-166, 188, 189, 196, 197, 203-205, 214-216, 228-230, 236, 238, 240, 243, 248-249, 312	2: 81-94 3: 26-27, 94-97, 166-169, 322-325 4: 6-10, 138-149, 258-262, 348, 358-365 5: 140-145, 148-149, 154, 155, 172, 173, 246-261, 341, 352-363 6: 1-5, 114-119, 136-147, 180-188, 202-219, 232, 233, 229, 234-237, 248-262 7: 44, 119-139, 202-221 8: 48-56, 62-69, 78-85, 114, 115, 134-143, 243	1: 180-185 2: 91-97, 219 3: 270-277, 285, 286, 289-290 4: 204-208, 212-219, 246-257, 281-285, 288-292, 404-405 5: 204-205, 236-239, 245, 248-252, 258-259, 263-265, 288, 289, 299-306, 325-329, 332-340, 344-347 6: 168, 169, 199-202, 205-211, 221-223, 249-252, 289-300, 308-313, 319, 320

TYPES OF NUMBERS: FRACTIONS, DECIMALS

Houghton Mifflin 1967-1970	Houghton Mifflin 1972	Laidlaw 1968	Scott Foresman 1966-1968	Silver Burdett 1970	SRA 1974
1: 113, 114, 199, 200-204. 2: 79-81, 229-232, 235-238, 243-248. 3: 160-169, 170, 171, 295, 300, 301. 4: 22, 130, 134, 136, 138-142, 292-297, 304, 307, 308. 5: 22, 174-178, 232-239, 249, 335-339. 6: 22, 172, 174, 175, 207-209, 214-219, 246, 260, 261, 302-310, 332-335.	2: 81-82. 3: 163, 166, 171, 176-181. 4: 130-141, 144-149, 155, 253-259, 260-266, 270-277, 282, 283, 320-323, 327, 331. 5: 126-137, 140, 141, 149-151, 189-202, 208, 210-214, 218, 253-265, 270-280, 318-320, 323-325, 329-331. 6: 125-137, 144-149, 190-195, 200-213, [illegible]-264, 268-269, [illegible]-277, 281, 284, 285, 318, 321-323.	1: 79-85. 2: 141-149, 152. 3: 239-244, 246-252. 4: 161-163, 166, 167, 170-176, 179, 188, 201-204, 207-209, 244, 245, 317-319, 323. 5: 129, 142, 143, 150, 154, 174, 176, 177, 180-183, 232-3. 6: 74, 122, 128-130, 146, 147, 182, 183.	3: 71, 210-211, 267, 282, 283, 286-299, 301-306. 4: 32, 33, 130-134, 255-259, 263-265, 300, 301, 312, 320-322, 328-333. 5: 154-164, 174-177, 255-257, 306, 307, 311, 320-323. 6: 10, 15, 74-86, 92-97, 117-119, 155-159, 162-163, 175-179, 182-185, 191, 308-316.	1: 87, 88, 201, 202. 2: 45, 116, 171, 172, 223, 224. 3: 68-71, 209-211, 263-268, 348-351. 4: 130-134, 284-290, 294, 295, 298, 320, 321, 347-350. 5: 85, 86, 112, 129-133, 139-141, 210-212, 220-225, 326-330, 346, 374-376, 383. 6: 52, 53, 73, 74, 92-94, 101, 102, 108, 112-115, 120-127, 130-132, 141, 142, 192-194, 218-223, 227, 343-346.	1: 218-224, 255. 2: 216-229. 3: 200-215. 4: 122-144, 237-257, 327-332. 5: 74-97, 146-155, 158-174, 177-189, 218-238, 3[illegible], 316, 320, 321. 6: 74-83, 87, 102-106, 109-138, 167-169, 174-190, 240-267, 290-303, 306, 320, 325, 327, 331.

358

Individual Record

An individual record consisting of a list of objectives with spaces for recording evaluation has been provided. A mimeograph stencil should be made and a copy of the individual record run for every pupil.

After the pupil completes each objective mastery should be noted on his individual record. The extra spaces can be used to indicate when an objective is reviewed or if a teacher finds evidence that any objective should be reviewed.

This individual record should follow the pupil whenever he transfers.

page 430 blank

Name ____________

Objective Number				Mastery		Objective Number				Mastery
1C						20C				
2C						21C				
3C						22C				
4C						23C				
5G						24C				
6G						25C				
7G						26C				
8G						27C				
9G						28C				
10G						29C				
11G						30C				
12G						31C				
13C						32J				
14C						33J				
15C						34J				
16C						35C				
17C						36C				
18C						37C				
19C						38C				

Name________________

Objective Number				Mastery		Objective Number				Mastery
39J						58G				
40C						59G				
41J						60G				
42J						61G				
43J						62J				
44C						63J				
45C						64J				
46C						65J				
47C						66G				
48C						67G				
49C						68G				
50C						69G				
51C						70G				
52J						71J				
53J						72J				
54J						73J				
55G						74C				
56G						75C				
57G						76C				

Name ______________________

Objective Number				Mastery		Objective Number				Mastery
77C						96G				
78S						97C				
79S						98C				
80S						99C				
81S						100G				
82S						101G				
83S						102G				
84S						103G				
85C						104G				
86S						105G				
87S						106G				
88S						107J				
89S						108C				
90S						109J				
91S						110G				
92C						111G				
93C						112G				
94C						113G				
95J						114J				

Name ____________________

Objective Number				Mastery		Objective Number				Mastery
115C						134S				
116C						135S				
117J						136S				
118J						137S				
119S						138S				
120S						139C				
121C						140J				
122S						141S				
123S						142J				
124S						143S				
125S						144J				
126C						145J				
127C						146J				
128C						147				
129C										
130J										
131S										
132C										
133J										