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ABSTRACT

Factor analyses of two forms of the Graduate Record Examinations (GRE) Aptitude Test were undertaken to better understand the abilities affecting test performance. Results suggest that three global abilities--two verbal and one quantitative--are consistently tapped by the GRE Aptitude Test. Other less prominent dimensions--some of which appear to be specific to test forms--revealed aspects of the test related to item type, speededness, and the content of reading passages. Factor extension analysis was used to estimate the loadings on these operational test factors of new items from eight experimental tests administered with the operational forms. Statistical removal of the extended facto[illegible] from the matrices of tetrachoric interitem correlations of the experimental tests and examination of residual relationships and amount of variance explained suggested that the experimental tests are, in general, adequately explained by the factors in the operational tests. There are, however, several dimensions in the experimental tests distinct from the factors underlying the operational forms. Recommendations based upon these findings are made concerning subtest length, item arrangement, and passage content. The factor analysis method is relevant for supplementing more traditional item classification and analysis techniques, and for the planned restructuring of the GRE Aptitude Test. (Author/ROF)

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A FACTOR ANALYTIC STUDY OF THE GRE APTITUDE TEST

Donald E. Powers
Spencer S. Swinton
Alfred B. Carlson

GRE Board Professional Report GREB No. 75-11P

September 1977

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Abstract

Separate factor analyses of two operational forms of the Graduate Record Examinations (GRE) Aptitude Test were undertaken to gain a better understanding of the abilities that contribute to performance on the examination. Results suggest that three global abilities -- two verbal and one quantitative -- are being consistently tapped by the GRE Aptitude Test. Other less prominent dimensions -- some of which appear to be specific to test forms -- were noted also. These dimensions revealed aspects of the test that are related to item type, speededness, and the content of reading passages.

Factor extension analysis was used to estimate the loadings on these operational test factors of new items from eight experimental tests administered with the operational forms. Statistical removal of the extended factors from the matrices of tetrachoric interitem correlations of the experimental tests and examination of residual relationships and amount of variance explained suggested that the experimental tests are, in general, adequately explained by the factors in the operational tests. There are, however, several dimensions in the experimental tests distinct from the factors underlying the operational forms. Recommendations based on the findings of the analyses are made concerning subtest length, item arrangement, and passage content. The factor analysis method is seen as having relevance in supplementing more traditional item classification and analysis techniques and for the planned restructuring of the GRE Aptitude Test.

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Introduction

Purpose

The investigation reported herein is part of a multiyear plan aimed at revitalizing or renewing the GRE Aptitude Test (Altman et al., 1975). The present study contributes to the renewal effort by (1) determining the factor structure of the current test and (2) determining the structure of several experimental tests through relating each of these tests to the structure of the current test. Since no factor analysis of the GRE Aptitude Test has ever been reported, consideration of the current test's structure was thought to be useful in decisions regarding the addition of certain new tests or item types and the deletion of others. A distinction between global factors (such as verbal and quantitative) and local factors (such as reading comprehension, vocabulary, and verbal reasoning) was thought to be useful also. Local factors were considered to be of greater relevance to decisions regarding the shortening of the current test, while the more global factors might have greater value for deciding which additional modules should be included in any contemplated test revision. Reference to previously reported separate analyses of the SAT verbal and quantitative sections (Coffman, 1966; Pruzek and Coffman, 1966) did not yield conclusions that would apply without reservation to the GRE Aptitude Test, despite similarities between item formats of the two instruments.

To accomplish the first purpose, principal factor solutions, based on interitem tetrachoric correlations, were computed for the responses of two random samples, each of 8,000 examinees, taking the GRE Aptitude Test in October 1975. The second purpose was addressed by extending the current test's factors into each of eight experimental tests, which at each test center were administered in a spiral design in conjunction with the two operational forms (See Table 2). Additional analyses of residuals were planned for any experimental tests not adequately described in terms of the factors found in the operational tests. The factors extracted from the operational forms, however, accounted for sufficient amounts of variance in each experimental test to justify postponing further analyses.

Description of Operational Tests

The operational portion of each GRE Aptitude Test form consists of three separately timed sections containing verbal and quantitative items, and is that portion of the test on which candidates' reported scores are based. Section I, the first of two verbal sections, is made up of questions in three formats: analogies, which test the ability to understand relationships among words and ideas; antonyms, which test vocabulary; and sentence completions, which measure ability to recognize logical and stylistic consistencies among the elements of a sentence. Section II, the second verbal section, contains paragraphs dealing with a range of subjects, each followed by sets of reading comprehension questions. The items in Section III, the quantitative section, require reasoning based on an understanding of arithmetic, algebra, and plane geometry, and the ability to interpret data presented in maps, graphs, charts, or tables (Graduate Record Examinations, 1975). A tabular summary of the operational test appears as Table 1 and examples of selected item types appear in Appendix A. These items have been extracted from the Information Bulletin that is available to all GRE candidates.

Table 1

Summary of the GRE Operational Test Forms K and K2*

Section	Item Description Form K	 Form K2	Item Numbers Form K	 Form K2	Time Limit in Minutes
I	Discrete verbal:	Discrete verbal:	1-55	1-55	25
	Analogies	Analogies	1-9	1-9	
	Opposites	Opposites	10-19	10-19	
	Sentence completions	Sentence completions	20-28	20-28	
	Analogies	Analogies	29-37	29-37	
	Opposites	Opposites	38-47	38-47	
	Sentence completions	Sentence completions	48-55	48-55	
II	Reading Comprehension:	Reading Comprehension:	1-40	1-40	50
	Narrative passage	Narrative passage	1-7	1-6	
	Humanities passsge	Scientific passage	8-13	7-13	
	Argumentative passage	Argumentative passage	14-20	14-20	
	Biological science passage	Humanities passage	21-26	21-27	
	Social studies passage	Humanities passage	27-33	28-34	
	Physical science passage	Physical science passage	34-40	35-40	
III	Quantitative:	Quantitative:	1-55	1-55	75
	Algebra, arithmetic, geometry, and miscellaneous	Algebra, arithmetic, geometry, and miscellaneous	1-11	1-22	
	Data interpretation	Data interpretation	12-16	23-30	
	Algebra, arithmetic, geometry	Algebra, arithmetic, geometry	17-21	31-38	
	Data interpretation	Data interpretation	22-26	39-44	
	Algebra, arithmetic, geometry	Algebra, arithmetic, geometry and miscellaneous	27-41	45-55	
	Data interpretation		42-46		
	Algebra, arithmetic, geometry, and miscellaneous		47-55		

---

*Throughout the report the two operational forms will be referred to as K and K2. The actual GRE designation of these forms is, respectively, K-SGR1 and K2-WGR1.

Description of the Experimental Tests

Section IV, the final section in each GRE Aptitude Test book, has been termed an experimental test and designated X_{10} - X_{17}, although the majority of the items contained in these tests are similar to items in the operational form, and were being tried out in preparation for possible inclusion in future operational forms. Two truly experimental tests (X_{15} and X_{16}) contain quantitative comparison items -- an item type not used in the GRE Aptitude Test prior to October 1977. The major experimental aspect of the other tests is the length of passages in the reading tests, the number of questions associated with each, and the grouping of passages by content area -- either scientific or humanities/social studies. A bries description of each experimental test is given in Table 2.

Sample Selection

Samples upon which this investigation is based were drawn from the population of examinees taking the two forms of the GRE Aptitude Test that were given in October 1975. The primary stratification variable was the experimental test form that the examinee received. The two operational forms (K and K2) were administered with the eight experimental tests, the latter being administered in a spiral design as Section IV of the operational forms. That is, approximatel half of the examinees received form K and half received form K2. One of four experimental reading tests (X_{10} - X_{13}) occupied the final section (Section IV) of form K, while one of four experimental quantitative tests (X_{14} - X_{17}) was Section IV of form K2. Approximately one-eighth of the total examinee population was administered each combination (e.g., form K, experimental test X_{10}) of tests. Random samples of 2,000 were drawn for each of these combinations.

Table 2

Description of Experimental Tests

Test Form	General Description	No. of Items	Time Limit in Minutes
X_{10}	Reading Comprehension (Humanities/Social Studies) -- three short passages, each with three associated questions followed by two long passages (same length as operational form) each with eight questions	25	25
X_{11}	Same passages as X_{10}, except each passage has up to two additional questions	30	25
X_{12}	Reading Comprehension (Science) -- three short passages, each with three associated questions, followed by two long passages (same length as operational form) each with eight questions	25	25
X_{13}	Same passages as X_{12} except that two additional short passages and associated questions are added	30	25
X_{14}	Regular quantitative items paralleling those now used in the operational test, but not including data interpretation items	30	25
X_{15}	Quantitative comparison items	40	25
X_{16}	First 23 items -- quantitative comparisons (suggested time 15 minutes), last 12 items -- regular quantitative items (suggested time 10 minutes)	35	25
X_{17}	Data interpretation items paralleling those now used in the operational test	20	25

Selection of Operational Forms

The two operational forms subject to the analysis reported herein were selected solely because their administration coincided with the proposed research. These forms, therefore, were those on which the most recent data were available. The fact that GRE test developers have judged these forms to be representative of a larger number of GRE Aptitude Test forms justifies a certain degree of generality for the results reported herein.

Categorization of Items

Generally, data from three sources were used to interpret the results of the factor analyses of the two operational forms. Naturally, the test forms themselves and the categorization of items in these forms, along with the authors judgments regarding the material contained in the tests, were the major inputs. Secondly, the test content specifications used by GRE examiners to classify items were obtained and scrutinized. Finally, the item statistics (biserial correlations of item with total test score, difficulty indices, and percentage attempting or reaching each item) also served as data useful in suggesting or confirming certain interpretations. Table 3 shows the classification categori associated with each item type.

Table 3

GRE Aptitude Test Item
Classification Scheme

	Item Type	Classification Categories
Discrete Verbal	Analogies	(1) Level (concrete, mixed, or abstract) (2) Structure (independent or overlapping) (3) Content area*
	Opposites	(1) Specificity (general definition or fine distinction) (2) Length (single words or phrases) (3) Part of speech (4) Content area*
	Sentence Completions	(1) Number of blanks (one or two) (2) Content area*
Reading Comprehension		(1) Passage content (narrative, biological science, physical science, synthesis, argumentative, social studies, or miscellaneous) (2) Item type (main ideas, supporting ideas, intended inferences, application, evaluation of logic, or style and tone)
Quantitative		(1) Content (algebra, arithmetic, geometry, data interpretation, or miscellaneous)

*Aesthetic/philosophical, world of practical affairs, science, human relationships, or general.

Results

Form K - Internal Structure

Estimated communalities, obtained by Tucker's method (Appendix B), were employed, leading to a total communality estimate of nearly 60 or about 40% of th total normalized test variance of 150. The first 12 principal factors, derived from the interitem tetrachoric correlation matrix, were sufficient to account for virtually all of this estimated common variance. Inspection of the roots and trial rotation of 6, 8, 9, and 10 factors resulted in the retention of eight factors, with total estimated communality of 56.3, or 37.5% of the total test variance (94% of the estimated common variance). (See Tables C-7 and C-8 in Appendix C.)

While factoring of the common variance allows the parsimonious description of the relationships among test items in a space of lower dimensionality, psychological interpretation of the resulting latent dimensions usually requires a rotation of axes to some criterion of simple structure. Since projection of the experimental tests into the factor space determined by the operational tests was planned, it seemed prudent to work with uncorrelated (orthogonal) factors..

The most common analytic method for approximating orthogonal simple structure uses Kaiser's (1958) varimax criterion, in which the variance of the column of squares of item loadings on each factor is maximized. This procedure leads to item loadings near one or near zero on each factor and tends to result in relatively easily characterized factors with "clean" patterns of loadings. Although the patterns resulting from oblique rotations, such as Yates' (1974) geomin, are likely to be even more interpretable, the fact that the resulting factor structure is not orthogonal could have led to difficulties in interpreting the results of the extension of the factors to the experimental tests. Upon vari max rotation, the first three rotated factors were found to account for 76.5% of the common variance, while the first five accounted for 90.1% of the common variance.

The factors showed the influence of item type and differential speededness, with little evidence of covariance due to content classifications (See Table 3) among the verbal items, but some indication of a content structure among the quantitative items, for which content and type appear to be more closely related.

The first three factors characterize the major dimensions of item covariance and reflect the global structure of the skills assessed by the Aptitude Test. It would appear that three relatively distinct skills, one quantitative and two verbal, are being tapped. Although the two verbal factors are not completely independent, with all but two verbal items displaying positive loadings on both factors, a clear tendency is evident for completion items from Section I to relate more to the reading passage items of Section II than to the opposites and analogies items forming the bulk of Section I. The remaining factors account for relatively smaller proportions of test variance, revealing aspects of the structure of the quantitative test (Section III) and differential effects of speededness on the three sections. These latter factors identify additional dimensions of covariation within algebra, data interpretation, and applications ("word problems") items, and separate speed factors for each of the two verbal sections. A discussion of each factor identified in form K follows below. (Also see Table 12.) Although the discussion fairly bristles with item numbers, item position is central to a major point, and we request the reader's forbearance.

Factor I, accounting for 28.3% of the common variance, underlies the common variance of the quantitative items, since most (43 of 55) of the quantitative items, but none of the verbal items, load highest on this *general quantitative factor*. Nearly all (53) of the quantitative items display loadings of .2 or greater on Factor I, while only 12 of the 95 verbal items have loadings this large. Items at the end of the quantitative section (#48, 51, 52, 53, 54, and 55) display the highest loadings on this factor, suggesting a strong relationship between speed and ability in this domain. While carrying the analysis out to 10 factors resulted in the splitting off of a speed factor tapping only these final items, this factor accounts for less than one-tenth as much variance as was contributed by the quantitative factor, and the final items continue to represent the highest loadings on Factor I. It was concluded that the apparent speededness component among the quantitative items is not an artifact, but that alacrity is linked to the ability measured by this factor. (See Swinton and Powers, 1976.)

Factor II, accounting for 26.7% of the common variance, is a verbal factor--*reading comprehension: connected discourse*. Thirty-eight of 40 reading passage items and all but one of the eight items in the first sentence-completion section exhibit their highest loadings on this factor, with physical-science-related reading passages appearing less strongly related to this comprehension dimension than passages based on literary or social-science content. The second completion section, appearing at the end of Section I, shows substantial loadings (greater than .3) on this factor for only four of its eight items, with generally much larger loadings on Factor V, a speed factor. Scattered analogies and opposites items (#1, 10, 29, 30, and 32) also have loadings greater than .4 on this factor, but only eight of 38 analogies and opposites items load highest on Factor II. Only one quantitative item has its highest loading on Factor II.

Factor III, accounting for 21.4% of the common variance, reflects *vocabulary: words and concepts in isolation*. Of the 18 loadings above .5, 16 are contributed by opposites and analogies items. The five highest loadings on Factor III are all contributed by opposites items. The two sets of sentence-completion items, which are associated with the comprehension factor (Factor II), appear in a single timed section (I) of the test with the analogies and opposites items; this situation might be expected to lead to moderate loadings from sentence-completion items on Factor III. Completion items with loadings greater than .4 on this factor are numbers 22, 27, 28, 53, 55. However, only items 27 and 55 load highest on this factor. There is no evidence that single-blank completion items relate differently to this verbal reasoning factor than do double-blank completion items. Items in the opposites format generally exhibit their highest loadings on Factor III, with 18 of 20 such items loading higher on this factor than on any other; 12 of 18 analogies items also load highest on this factor. Of the reading passage items, only two display loadings greater than .3 on Factor III. The two reading passage items having their highest loadings on Factor III are short questions, each having one-word answer choices involving rather difficult vocabulary. Loadings of these items therefore also tend to support the interpretation of Factor III as an ability to deal with words in isolation.

Factor IV contributes 7.5% of the common variance, and displays loadings greater than .4 on items 3, 5, 6, 8, 9, 11, 17, 20, and 21 of Section III. In addition, items 1, 10, 29, and 32 exhibit loadings in the .30's. Of these items, only numbers 6 and 10 do not involve algebraic notation for the variables "x" and "y". Each of the five items having its highest loading on this factor involves algebraic notation. Putting behind us the temptation to christen Factor IV "the x factor," we dub it *elementary algebra*. The only other items in Section III involving algebraic notation are items 30, 36, and 38, with loadings of .22, .28, and .29 on Factor IV, and the more advanced items from number 47 to the end of the test, which those pupils high in general quantitative ability were most likely to attempt. These latter items load almost exclusively on Factor I, the general quantitative factor.

Factor V, accounting for 6.2% of the common variance, is defined by the fact that its highest loadings are from items 48-55, the sentence-completion items at the end of the 25-minute first section of the test. Loadings of these items on this factor range from .40 to .68, higher (except for item 55) than their loadings on any other factor, including verbal Factors II and III. In contrast to the role of speed in the quantitative test (an instrument in which the items at the end appear to be the best measures of the quantitative factor), these sentence-completion items show strongest covariation along a dimension orthogonal to other verbal items, suggesting that speed is a less central component of verbal ability, and that Section I, with 55 items in 25 minutes, may be more speeded than is consistent with optimal measurement of verbal reasoning. While *speed of response to discrete verbal items* may be an imp tant trait that adds to the validity of the instrument, the fact that 15% of the items in Section I appear to be strongly influenced by this factor may be of importance in the interpretation of the measure. The only other loadings greater than .2 on the factor appear in items also near the end of Section I, supporting the interpretation of Factor V as a measure of speed.

Factor VI, accounting for 4.3% of the common variance, identifies a dimension of variance underlying quantitative items 12-15 and 22-25 -- two sets of data interpretation items, the first based on a rather unusual graph and the second on a parcel-post rate table. Each of the five items having

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its highest loading on this factor is a data interpretation item. This factor seems to reflect *ability to extract information* from a rather complex ground, since data interpretation items involving simpler tables or items requiring more extensive manipulation of information in addition to extraction from tables and graphs (items 16, 26, 33, 42-46) exhibit lower loadings on Factor VI and correspondingly higher loadings on Factor I.

Factor VII, accounting for 3.1% of the common variance, has no large loadings from any item, but a large number of positive and negative loadings ranging in absolute value from .1 to .3 -- loadings that are positive for verbally-presented mathematics items, such as 6, 7, 10, 27, 28, and 31, and negative for more abstract problems in which algebraic notation appears, such as 8, 36, 48, 49, and 51-53. Only item 7, which is a relatively easy, practical word problem dealing with sales and commissions, has its highest loading on this factor. Three opposites and analogies items from verbal Section I display small positive loadings on this factor.

The mathematics content of the items with positive loadings on this factor tends to be practical: ratios, percentages, and averages. This fact, coupled with the textbook "word problem" nature of the items, led to designating the factor as *applications: word problems*, taken in a similar textbook sense.

Finally, Factor VIII accounts for 2.5% of the common variance. Items from the last passage of Section II are the only ones to show moderate (greater than .2) loadings on this factor, suggesting that it represents a factor of *reading speed: comprehension*. Since all but one of these final items (item 38, which loads highest on Factor VIII) display higher loadings on Factor II (reading comprehension) than on Factor VIII, it was concluded that reading

Speed does not play an important role in Section II. Examination of biserials suggests that reading speed seems neither to decrease final-item relations with the intended construct, as seemed to be the case for Section I, nor to increase those relations, as did quantitative speed in Section III. Although the content of these items is scientific, and the other scientific passage also exhibits low loadings on the comprehension factor, Factor VIII does not exhibit strong relations to the earlier scientific items. Thus, a content-related interpretation is less tenable than one based on speed. Furthermore, the interpretation of Factors V and VIII as speed factors instead of difficulty factors seems most plausible in light of (1) the appearance of large loadings at the points where the test becomes speeded, and (2) the fact that difficult items appearing earlier in the test do not load on these factors. In addition, the increase in loadings seems to correspond more directly to the sudden increase in the test's speededness rather than to the more gradual increase in the difficulty of items. A schematic summary of factor loadings of form K items is given in Table 4. Detailed tables of factor loadings appear in Appendix C.

Form K2 - Internal Structure

A second principal-factor solution, again using Tucker's communality estimates, was computed for form K2, and a total communality estimate of over 64 (43% of 150, the total test variance) was obtained. Inspection of roots and trial varimax rotation of 6, 8, 9, and 10 factors resulted in the retention of the 10-factor solution as most meaningful. These 10 factors, with a total estimated communality near 61, account for 41% of the total variance or 95% of the common variance of the test. After orthogonal rotation according to the varimax criterion, the first three factors account for about

Table 4

Summary of Factor Loadings of Item Groups (Form K)

	General Quantitative I	Words in Context II	Words in Isolation III	Algebra IV	Section I Speed V	Data Interpretation VI	Applications vs. Algebra VII	Section II Speed VIII
Analogies			+					
Opposites			+++					
Completion		+++						
Analogies		+	+++					
Opposites		–	+++					
Completion		+			+++			
Narrative		+++						
Humanities		+++						
Argumentative		+++						
Biological Science		++						
Social Studies		+++						
Physical Science		++						+
Algebraic Notation	++			++				
Data Interpretation	++					+		
Arithmetic (word problems)	+						(+)*	
Geometry and Miscellaneous	+++						(-)*	

Code: +++ = more than half of the loadings from these items greater than .4 and more than half have their highest loadings on this factor.

++ = either half of loadings greater than .4 or half load highest on this factor.

+ = more than half of loadings greater than .3 or all greater than .2.

* = Positive loadings from some word problem items, negative from abstract function items.

72% of the common variance, the first five for 84%, and the first seven for 92%. Again the factors are defined by item type, but there is little to suggest any differentiation among verbal items with respect to content classification. The content classification for quantitative items, on the other hand, is more closely related to the factor structure.

The first three factors -- one quantitative and two verbal -- are similar to those of form K, reflecting the general skills that are tapped by the test and accounting for a relatively large (72%) portion of the common variance of the items. (See Table C-9.) The remaining factors, only one of which accounts for more than 5% of the common variance, reflect the speededness of each of the three test sections, two aspects of data interpretation, an ability to comprehend scientific material, and an ability to change response sets. Each of these ten factors is discussed in more detail below.

Factor I, which accounts for nearly 30% of the common variance, is characterized by loadings greater than .4 for 40 of the 55 quantitative items and no loadings greater than .4 for any of the 95 verbal items. Only 3 of the ve bal items load highest on this factor, as contrasted with 47 of the quantitative items. All of the remaining 8 quantitative items, which do not load highest on this factor, have been classified as data interpretation items. Factor I, therefore, was termed *general quantitative ability*, although there is some indica that algebraic items are instrumental in its definition. This indication is suggested by the fact that, of Factor I's seven highest loadings (greater than .7) four come from items classified as algebraic in content. Factor I also seems to reflect a speededness component linked to quantitative ability, since seven of the final 11 items have loadings greater than .6 (whereas less than one-fourth of the earlier items have loadings this high).

Nearly 23% of the common variance is contributed by the second factor. Factor II is clearly a verbal factor, since none of the quantitative items displays its highest on II (and none loads greater than .4), but 42 of the verbal items show their heaviest loading on this factor. Twenty-five of the 40 reading comprehension items exhibit their highest loadings on this factor, along with seven of the first nine sentence-completion items. Of the 15 reading comprehension items that do not relate most strongly to Factor II, 13 are items associated with two scientific passages, instrumental in defining two subsequent factors. The six highest loadings on Factor II come from reading comprehension items (#5, 18, 23, 24, 28, 29) associated with passages containing humanities, narrative, or argumentative content. Factor II is, therefore, thought to reflect an ability to deal with connected discourse. This factor seems to relate closely to what is commonly referred to as reading comprehension and has thus been termed *reading comprehension: connected discourse.*

Factor III, accounting for 19.4% of the common variance, emerges as a second verbal factor, with none of the quantitative items having loadings greater than .2. Furthermore, only two reading comprehension items (#26, 33) load highest on this factor and none have loadings greater than .4. Similarly, only four of the 17 sentence-completion items have loadings greater than .4 and only four of these items (#22, 27, 53, 55) have their highest loadings on Factor III. This pattern suggests that Factor III is most clearly defined by the relatively large number of high loadings on opposites items and, to a lesser extent, by high loadings on analogy items. This factor, therefore, is thought to reflect an ability to deal with words in isolation or vocabulary, narrowly defined. This contention is supported by the nature of the opposites items, which rely heavily on vocabulary knowledge, and on the nature of analogy items, which require both a knowledge of vocabulary and an ability to determine relationships. The fact that the analogy items have a lower proportion of high loadings than do opposites items supports the interpretation of Facto III as a *vocabulary: words and concepts in isolation* factor. The interpretation is further substantiated by examining the two reading comprehension items (#26, 33) and the sentence-completion items (#22, 27, 53, 55), which have their highest loadings on Factor III. In general, these items seem to include relatively difficult vocabulary in either the item stem or the answer choices. On the other hand, 10 of the 17 analogy items and 13 of the 20 opposites items have loadings greater than .4, and 10 analogies and 14 opposites items have their highest loadings on this factor.

Factor IV accounts for a significantly lower proportion of variance (7.7%) than any of the previous three factors. It was, however, fairly easily interpreted by virtue of its high loadings from the last third of Section I (which includes both opposites and sentence-completions) and the absence of loadings from any other items -- either verbal or quantitative. Specifically, 15 of the last 18 items on Section I have loadings greater than .2. These are also the only loadings greater than .2 on this factor. Seven of these items (and six of the last eight) exhibit higher loadings on this factor than on any other. Factor IV is, therefore, interpreted as *response speed* associated with Section I of the test, thus resembling Factor V of form K.

Factor V, which contributes 4.4% of the common variance, is by virtue of its loadings interpreted as a second speed factor (*discrete reading speed*) associated with Section II. The only items having loadings greater than .2 and/or loading highest on Factor V are the final reading comprehension items in Section III.

The last seven have loadings greater than .2 and the last six have their highest loadings on Factor V. This factor is comparable to Factor VIII of form K.

Factor VI contributes 4.4% to the common variance of the test. Of the 13 quantitative items that show loadings of .2 or higher, nine are data interpretation items. All six items with loadings greater than .4 are of this item type and the three highest loadings on Factor VI come from items 25, 27, and 42 -- all data interpretation items. Data interpretation items 25, 27, and 41 have higher loadings on this factor than on any other. Besides the data interpretation items, only four other quantitative items have loadings greater than .2 on this factor (although none of these loadings represents the highest loading for any of the four items). Three of the four items are the last three in Section III, suggesting that a slight component of speed might be present in the factor. Item analysis statistics, which show that the percentage of examinees attempting each item decreases very rapidly at this point in the test, support this interpretation: approximately two-thirds of the examinees attempted the antepenultimate item and fewer than half of them attempted the last two items. However, since Factor VII also seems to be chatacterized by loadings from data interpretation items, it was necessary to determine the distinguishing features of each of these "data interpretation" factors. These distinct characteristics are discussed below.

Factor VII, the second data interpretation factor, accounts for only a slightly smaller percentage of common variance (3.9%) than Factor VI, the first data interpretation factor. Factor VII is defined by six items, all data interpretation items having loadings of .4 or more. Only four items (#23, 24, 26, and 39) have their highest loadings on Factor VII, but all are data interpretation items, and Factor VII's two highest loadings (which are both greater than .6) are from data interpretation items.

The items instrumental in defining Factor VI (i.e., items 25, 27, 41, and 42) require the examinee to extract information from graphs or tables and to perform one or more simple arithmetic calculations in order to answer the items. Items 23, 24, 26, and 39, on the other hand, require only extraction of information from the graphic or tabular material. Factor VI, therefore, has been called *data interpretation: extraction and manipulation*, whereas Factor VII has been termed simply *data interpretation: extr tion*. These factors are comparable to the undifferentiated Factor VI of form K.

Only seven items have loadings greater than .2 on Factor VIII, which accounts for 3.1% of the test's common variance. All the loadings come from seven items (7-13) associated with the highly technical first scientific passage, which load higher on Factor VIII than on any other. This phenomenon suggests that the factor reflects an ability to comprehend scientific or technical prose. Somewhat contradictory to this interpretation is the fact that items associated with the second scientific passage do not load on this factor. The apparent anomaly may be explained by the fact that the latter items appear at the end of Section II and load on the speededness factor (Factor V) discussed above. Apparently, then, doing well on these items is more dependent on speed, under existing time limits, than on ability to comprehend scientific material. Factor VIII, however, may be explained by appeal to content. It is, of course, possible that this factor is specific. Factor VIII has thus been termed *reading comprehension: scientific/technical*.

Factor IX, accounting for 2.5% of the variance, is a bipolar factor defined exclusively by ten quantitative items having loadings greater in absolute value than .2. Although none of these items load highest on this factor,

Table 5

Summary of Factor Loadings of Item Groups (Form K2)

	General Quantitative I	Words in Context II	Words in Isolation III	Section I Speed IV	Section II Speed V	Data Interpretation: Extraction and Manipulation VI	Data Interpretation: Extraction VII	Reading Comprehension: Scientific/Technical VIII	Quantitative Speed IX	Opposites [illegible] (Easy Opposites) X
Analogies		++	++							
Opposites			+++							(+)**
Completion		++								
Analogies			+++							
Opposites		+	+++							(+)**
Completion				+++						
Narrative		+++								
Scientific		+						+++		
Argumentative (Social Studies)		+++								
Humanities		+++								
Humanities		+++								
Scientific		+			+++					
Algebra	+++								(+)***	
Data Interpretation	++					+	+			
Arithmetic	+++									
Geometry and Miscellaneous	+++									

Code: +++ = more than half of the loadings from these items greater than .4 and more than half have their highest loadings on this factor.

++ = either half of loadings greater than .4 or half load highest on this factor.

+ = more than half of loadings greater than .3 or all greater than .2.

* = Defined here as items employing symbolic notation (e.g., x, y)

** = Initial opposites items only

*** = Final quantitative items only

the interpretation of it as a *quantitative speed* factor is straightforward. Two early quantitative items (#6,7) have loadings less than .2, while eight of the last ten items have positive loadings greater than .2. Item analyses statistics support this interpretation, since loadings greater than .2 do not appear for any items attempted by more than 90% of the examinees, but they are apparent for all of the six items attempted by less than three-quarters of the sample. No comparable factor was retained in form K, although Factor VIII of the 10-factor trial solution had a similar interpretation.

The final factor, Factor X, which accounts for only slightly more than 2% of the common variance of the test, was somewhat difficult to interpret. Only five items, all of which are of the opposites format, have loadings greater than .2 on this factor. These items include the first two in the first opposites section and the first three in the second opposites section. Somewhat problematic in interpreting this factor, however, is the confounding of item difficulty and position. That is, these five items are also the easiest of the 20 opposites items. This factor has thus been dubbed *easy opposites*, although the authors' own response tendencies were to search for synonyms instead of antonyms when attempting initial items of this nature. This tendency probably results in part from the response set that is established by the analogy items that precede each of the opposites sections. The factor might thus represent a warm-up phenomenon for this item type. Nevertheless, interpretation in terms of difficulty seems more parsimonious. Table 5 summarizes the loadings of groups of items on the factors for form K2. Detailed tables appear in Appendix C. A summary of the factors in forms K and K2 is presented in Table 12.

Factor Extension Analysis

To determine the structure of each of eight experimental tests or, more specifically, to determine whether any experimental test reflected dimensions not shared with the operational forms, a factor extension analysis was performed. The factor structure in each operational form was extended to each of the four experimental tests that had been administered in a spiral design with each operational form.

After the matrix of factor loadings was obtained, item responses to each of the four associated optional tests were located in the factor space of the operational test. This factor extension technique (Dwyer, 1937; Harman, 1938) results in an augmented matrix, in which the factor extension matrix consists of the loadings of each experimental test item on each operational test factor.

The product of the extension matrix and its transpose is an estimate of the intercorrelation matrix of the items of that experimental test, based on only those dimensions of item covariance that are shared with the operational test. The residual correlations and reproduced item communalities may be examined to determine the degree to which the experimental test reflects other dimensions not shared with the operational test. The variance accounted for by each factor of the operational test gives a measure of the relative importance of that factor in accounting for the common variance shared by the two instruments. In general, this shared common variance will be less than the total common variance of the experimental test. To the extent that factors of the operational test can be interpreted meaningfully, the pattern of factor loadings in the extension matrix may be employed to examine the construct validity of the experimental instrument. It should be noted that for each operational test, the same factor matrix, based on 8,000 cases, has been employed with each of the four 2,000-candidate random subsamples. Thus the validity of the analysis depends on the assumption that the structure of operational forms is identical for each experimental test group, a reasonable assumption considering the sample size and the spiralling assignment technique.

We first discuss the experimental verbal tests, given with form K of the operational test. For each experimental test, we examine the degree to which its common variance is "explained" by that of form K, and the relationship of each test's items to the structure of that form. The experimental mathematics tests, given with form K2 of the operational test, are then related to the slightly more complex structure of form K2. Comparison of results between operational tests depends even more on randomness of sample selection and size than do comparisons among experimental tests within each operational form. As noted on page 6, the analysis also depends on the presumed representativeness of the two operational test forms.

Results of Extension Analysis: Verbal Instruments

Candidates receiving operational test form K were divided randomly into four subgroups. Each was administered a different 25-minute experimental verbal instrument after completing the three operational sections. The experimental tests differed in item type and content, as indicated in Table 2 and summarized in Table 6.

Table 6

Summary of Experimental Verbal Tests

Experimental Tests	No. of Items	Description
X_{10}	25	humanities/social studies content (three short passages, two long passages)
X_{11}	30	identical to X_{10} except for five additional questions
X_{12}	25	science content (three short passages, two long passages)
X_{13}	30	identical to X_{12} except for two additional short passages and five additional questions

For the present study, samples of 2,000 candidates were used for each subgroup. Table 7 displays means, standard deviations, and correlations for these samples.

Table 7

Descriptive Statistics for Experimental Verbal Tests

					Correlation	
Test	No. of Items	Mean	Sigma	α	V	Q
V	95	51.24	16.04	.94		
Q	55	30.60	10.19	.92	.51	
X_{10}	25	16.04	5.12	.85	.83	.44
V	95	51.23	16.03	.94		
Q	55	31.07	10.18	.92	.52	
X_{11}	30	17.18	6.01	.87	.83	.47
V	95	51.26	16.18	.94		
Q	55	30.73	10.36	.92	.52	
X_{12}	25	16.69	4.90	.83	.77	.57
V	95	50.25	16.23	.94		
Q	55	30.25	10.52	.92	.52	
X_{13}	30	18.10	5.63	.84	.77	.55

The means and standard deviations indicate that the four subsamples are quite comparable. The X_{13} group is approximately one point lower than the other groups on the verbal test, a difference barely significant at the .05 level because of the large sample size. This group is also lower on the quantitative test than is the X_{11} group ($t_{3998} = 2.5$). However, the difference of .82 of a point does not suggest any systematic difference in the samples. Correlations of the verbal and quantitative tests are almost identical in the four subgroups. A systematic difference appears between the correlations of the humanities passages tests X_{10} and X_{11} and the science passages tests X_{12} and X_{13} with the operational tests. The experimental tests based on humanities passages correlate .83 with the operational verbal score, but those based on science passages correlate at a significantly lower .77. However, the humanities passages show correlations of about .45 with the quantitative score, while science-based passages display coefficients a full .10 higher. Although these latter correlations are comparable to the V-Q correlations obtained from form K, the lower reliability of the shorter experimental tests suggests that even greater discrepancies might have resulted from a longer operational form. Thus, any plan to allow candidates to select passages should take into account the different characteristics of science-based passages -- a difference apparent in the factor structure of both operational forms, the correlations discussed above, and the following extension analysis.

The eight factors of operational form K were extended into the space of tetrachoric interitem correlations for each experimental test. Table 8 gives the proportions of variance accounted for by this procedure. Item loadings of each experimental test on the operational factors appear in Appendix D.

Table 8

Variance of Experimental Verbal Tests Explained by Operational Test Factors: Form K

		X_{10}	X_{11}	X_{12}	X_{13}
No. of items = Total Variance		25	30	25	30
Common Variance*		10.91	13.34	11.57	13.83
Common Variance as a Percentage of Total Variance		43.64	44.47	46.30	46.09
Percentage Common Variance Explained by:					
Factor I	General Quantitative	2.17	5.22	11.58	9.03
II	Reading Comprehension: Connected Discourse	51.95	46.38	46.64	35.17
III	Vocabulary: Words in Isolation	14.80	14.75	5.76	7.71
IV	Elementary Algebra	2.39	.38	2.74	2.77
V	Discrete Verbal Response Speed	2.84	2.85	1.82	2.94
VI	Data Interpretation : Information Extraction	1.39	1.30	.95	2.25
VII	Applications: Word Problems	2.10	.96	.85	.91
VIII	Reading Speed: Comprehension	2.85	3.24	5.71	4.19
Subtotal Verbal Factors II, III, V, VIII		(72.44)	(67.22)	(59.93)	(50.01)
Subtotal Quantitative Factors I, IV, VI, VII		(8.04)	(7.86)	(16.12)	(14.96)
Total (Percentage Common Variance)		80.49	75.08	76.05	64.97
Percentage Total Variance		35.13	33.39	35.21	29.94

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[*] Common variance estimated as the sum of Tucker's highest adjusted off-diagonal estimates of communality.

Arranging the tests as in Figure 1 illustrates the manner in which content and speed dimensions jointly affect variance shared with the operational test. This illustration is analogous to a two-way analysis of variance.

Figure 1

Interaction of Test Length and Content in Accounting for Variance: Form K

(a)

Percentage of Variance Explained by Verbal Factors

	Humanities	Science
25 Items	72.44	59.93
30 Items	67.22	50.01

(b)

Percentage of Variance Explained by Quantitative Factors

	Humanities	Science
25 Items	8.04	16.12
30 Items	7.86	14.96

Averaging over the two humanities tests in column 1 of Figure 1a, we find that verbal factors extended from the operational test account for 68.8% of common variance and an average 55.0% for the science passages in column 2. Averaging over rows, we see that the two shorter tests have an average of 66.2% of common variance accounted for by verbal factors, versus 58.6% for the two longer tests. The pattern apparent in Figure 1 suggests an interactive effect of test length (speededness) and test content. That is, lengthening reading comprehension tests seems to change the structure more drastically for tests with scientific content than for those with humanities content.

When the variance explained by factors extended from the quantitative operational test is exsmined, speed appears to have a smaller effect on the variance explained, although science passages in column 2 of Figure 1b again seem to be more affected by test length, but content has an opposite effect on common variance from that of verbal factors. Quantitative factors explain an average 8.0% of the variance of the humanities items in column 1, but 15.5% of the variance of the two science tests. It appears that science passages are different from humanities passages and that the difference increases in the face of speededness, if argument by analogy to interaction in the analysis of variance is appropriate.

Although form K does not exhibit a factor identifiable as "science passages," Factor VIII, a speed factor, has largest loadings on the items of a science passage at the end of Section III. Form K2 does exhibit a science factor separate from reading speed. However, since form K2 was not given in conjunction with experimental verbal tests, it was not possible to determine how much factors of K2 would explain the variance of tests X_{12} and X_{13}. If form K2 should relate differently to these tests, it would be necessary to question how parallel the two operational forms actually are.

Residual Correlations among Items within Experimental Verbal Tests (X_{10} - X_{13})

Test X_{10} is experimental in that it contains three reading passages, each approximately one-third as long as those in the operational test. Each passage in X_{10} has been classified as humanities or social studies in content. In terms of the magnitude of residual correlations* among items in X_{10}, the eight factors extracted from operational form K appear to do an excellent job of explaining the variation in test X_{10}, since only 17 of the possible 300 residual relationships are greater than .10.

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*The difference between the observed correlations among experimental test items and the corresponding correlations reproduced from the loadings of experimental items on the factors in the operational form.

The only unexplained relationship among items is a speed component peculiar to this test. This component is suggested by the predominance of large (greater than .10) residuals among the last eight items (12 residual relationships greater than .10). Items associated with the new, shorter passages in test X_{10} are explained quite adequately by the factor structure of the operational test, since only one of the [illegible] residual relationships is greater than .10.

The experimental reading comprehension test, X_{11}, is the same as X_{10}, except for an additional question added to each passage, thus increasing the test length to 30 items. Fifty-five of the 435 residual correlations are greater than .10 and 10 are greater than .20 for this test. A component of speed associated with X_{11} is apparent since 30 of 36 residual correlations among the last nine items are greater than .10. The variance contributed by the 12 items based on the short passages seems less well explained by the factors from the operational test than is that of the nine corresponding items in test X_{10}, since six of the 66 residual correlations among these 12 items are greater than .10. Relationships among the items for the longer passages also seem less thoroughly explained. It appears then that speeding the test by adding items may introduce two components related to speed -- one having to do with reaching or attempting the final items and one with time spent considering each item. That is, an additional component, which we call *distributive speed*, emerges when examinees have less time per item.

Test X_{12}, a 25-item test requiring reading of scientific-technical material, is less fully explained by the eight factors than is X_{10}, a

25-item reading comprehension test containing humanities/social studies content. Thirty-nine of 300 residuals are greater than .10, and six are greater than .20. Only four of the 36 residuals among the items following short passages are greater than .10; so it appears that decreasing the length of passages does not add any component of variance that is not explained by the eight factors extracted from the operational test.

A high proportion (16 of 28) of large residual correlations among the last eight items, which are associated with the final passage, suggests a distinct component of speed peculiar to this test, since item 18 is the point at which the number of examinees not attempting items increases sharply, although this unexplained variance could also be due to the nature of the passage.

An even higher proportion (17 of 28) residuals greater than .10 (four are greater than .20) appear in the relationships among the eight items associated with the next to last passage -- items that are attempted by nearly all the examinees. This pattern suggests that this long scientific passage contributes test variance not explained adequately by the eight factors found in the operational form. This particular passage seems to be the most technical of any of the passages in the sense that it contains a relatively large number of highly technical terms (e.g., names of chemicals, types of bacteria).

The first short passage also contains more technical terms than any of the other short passages, and it, too, is not as fully explained as the other less technical passages. In summary, the only relationships among large residuals seem to be their connection with specific passages. That is, no specific item type seems less well explained than any other.

Test X_{13}, which is the same as test X_{12} except for two additional short passages and five additional questions, is characterized by a relatively large number of large residuals. Of the 435 residual correlations, 66 are greater in absolute value than .10 and, in fact, eight of these are greater than .30. There are few (5 of 91) large residuals among the items associated with the five short passages. Three of these residuals result from unexplained (in terms of loadings on the operational tests' factors) relationships among the three questions about the first passage -- the short passage that contains the most technical terminology.

Relationships among items for the two final passages in X_{13} -- both long -- are considerably less adequately explained by the extended factors. All the residual correlations among items within each passage are greater than .10 and the preponderance greater than .20. Test X_{13} appears quite speeded when the last passage appears, so (as with test X_{12}) it is difficult to say whether the unexplained component reflects speed or ability to deal with the type of material contained in the passage. Since the residuals associated with items relating to this passage are considerably greater in number and size in test X_{13} than in the otherwise identical X_{12}, it appears that the unexplained variance results primarily from a unique component of speed. On the other hand, most examinees do have time to consider the next-to-last passage and it seems to contain an unexplained component related to its highly scientific/technical terms. Probably this unexplained variance also results from a speed component, although nearly every examinee attempted each item associated with the passage. That speed plays a role becomes apparent by comparing the residuals among this passage's items for tests X_{12}

and X_{13}. Each residual from test X_{13} is larger than the corresponding residual from X_{12}. This component is termed *distributive speed*, i.e., the time spent considering each item.

Results of Extension Analysis: Quantitative Instruments

Candidates receiving operational test form K2 were divided randomly into four subgroups. Each subgroup was administered a different 25-minute experimental mathematics instrument after completion of the three operational sections of the examination. These experimental tests are based on different item types, as specified in Table 2 and summarized in Table 9.

Table 9

Summary of Experimental Quantitative Tests

Experimental Tests	No. of Items	Description
X_{14}	30	"regular" items, no data interpretation items
X_{15}	40	quantitative comparison items
X_{16}	35	quantitative comparison items followed by 12 regular items
X_{17}	20	data interpretation items

For the present study, samples of 2,000 candidates were used for each subgroup. Table 10 displays means, standard deviations, and correlations for these samples.

Table 10

Descriptive Statistics for Experimental Quantitative Tests

Test	No. of Items	Mean	Sigma	α	Correlation V	Correlation Q
V	95	52.24	16.47	.94		
Q	55	32.43	10.74	.93	.57	
X_{14}	30	14.91	6.57	.90	.51	.90
V	95	52.34	16.15	.94		
Q	55	32.68	10.27	.92	.56	
X_{15}	40	20.52	8.19	.91	.47	.87
V	95	51.75	16.29	.94		
Q	55	32.42	10.40	.92	.58	
X_{16}	35	17.52	7.37	.90	.51	.88
V	95	51.93	16.42	.94		
Q	55	32.38	10.51	.92	.54	
X_{17}	20	10.63	4.27	.82	.52	.84

The means, standard deviations, and correlations of V and Q scores indicate the four subgroups are highly comparable, with verbal means ranging from 51.75 to 52.34 and quantitative means from 32.38 to 32.68 -- about two points higher than for the form K quantitative means. V-Q correlations ranging from .54 to .58 are .03 to .07 higher than the V-Q correlations obtained for form K, a difference significant for all but the group taking X_{17}.

Correlations of the experimental tests with verbal and quantitative total scores are also quite similar, ranging from .47 to .52 for the verbal score, with quantitative comparison items exhibiting the lowest correlation and data interpretation items the highest correlation to the verbal score, and from .84 to .90 for the operational quantitative total with regular items sh the highest value and data interpretation items the lowest. Tests X_{15} and X_{16}, which contain quantitative comparison items, are intermediate in correlation with the operational quantitative score (r = .87 and .88, respectively). These differences are too small to be important, but their directions are consistent with item format differences and further corroborate the adequacy of the sampling. The 10 factors of the operational test were extended into the space of tetrachoric interitem correlations for each experimental test. Table 11 gives the proportions of variance accounted for by this procedure. Detailed extension loadings appear in Appendix D.

Table 11

Variance of Experimental Quantitative Tests Explained by Operational Test Factors

	X_{14}	X_{15}	X_{16}	X_{17}
No. of Items = Total Variance	30	40	35	20
Common Variance*	17.46	19.21	17.37	9.32
Common Variance as a Percentage of Total Variance	58.19%	48.03%	49.62%	46.61%
Percentage Common Variance Explained by				
Factor I (General Quantitative)	67.06	59.08	57.03	46.28
Factor II (Reading Comprehension)	3.22	4.26	5.97	4.93
Factor VI (Data Interpretation 1)	6.68	3.90	5.27	10.90
Factor VII (Data Interpretation 2)	1.72	1.97	3.21	8.86
Factor IX (Quantitative Speed)	4.50	5.59	6.97	4.16
Subtotal (Percentage Common Variance)	(83.18)	(74.80)	(78.45)	(75.13)
Verbal Factors III, VIII, X	4.90	3.58	3.76	4.51
Verbal Speed Factors IV, V	1.91	3.07	4.24	3.08
Total (Percentage Common Variance)	89.99	81.45	86.45	82.72
Percentage Total Variance	52.37	39.12	42.90	38.56

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*Common variance estimated as the sum of Tucker's adjusted highest off-diagonal estimates of communality.

It is interesting that test X_{14} has a higher proportion -- over 58% -- of common variance than do other tests based on other item formats. Test X_{16}, which contains 12 items similar to those of test X_{14}, came in a distant second, with common variance nearly 50% of total variance. Test X_{17}, consisting of data interpretation items, exhibits the lowest proportion of communality, and tests involving quantitative comparison items occupy an intermediate position. For comparison, the operational quantitative test, containing a mixture of regular and data interpretation items, has a total estimated communality of 25.80, or 46.91% of total variance.

Clearly, test X_{14}, consisting of items most like those of the operational quantitative test, has the most in common with that test--the general quantitative factor accounting for 67.06% of the estimated common variance of X_{14}, or 39.02% of its total variance.

Test X_{17}, consisting solely of data interpretation items, shows the lowest relationship to the general quantitative factor, but has nearly 20% of its common variance explained by the two data interpretation factors, VI and VII. Tests X_{15} and X_{16}, containing quantitative comparison items, show a pattern of relationships to the factors that is generally more like that of regular items than the pattern of data interpretation items.

Factor IX, quantitative speed, represents that component of speededness predictable from performance on the operational test. It is reasonable that test X_{16}, although slightly shorter than X_{15}, shows a stronger relationship to speededness on the operational test than does X_{15}, since the final 12 items on X_{16} are of the same type as the items in X_{14} and those in the operational test, while X_{15} consists entirely of quantitative comparison items. Thus we might expect examination of residuals to provide evidence of speededness unique to the quantitative comparison item type.

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Residual Correlations among Items within Experimental Quantitative Tests (X_{14} - X_{17})

Residual correlations among items within each experimental test were examined to determine which, if any, relationships among items are not explained by the factor structure of the respective operational tests. Test X_{14}, which contains regular quantitative items most like those in the operational test, seems to be thoroughly explained by the ten factors found in form K2. Of 435 residual correlations, only 12 are greater in absolute value than .10 and none is greater than .20. These residuals display no major relationships among the combinations of items with residual correlations greater than .10. There are a few interpretable relationships among pairs of items exhibiting residual relationships. These relationships are highly specific, however, relating to the content of the items. For example, two items dealing with areas of triangles in a coordinate geometry framework show a residual relationship, as do two geometry items dealing with angles.

In test X_{15}, which contains 40 quantitative comparison items, 52 of the 780 residual correlations are greater in absolute value than .10 (three are greater than .20). Residuals among combinations of items suggest that two major components of test X_{15} are not totally explained by the factors in form K2. One of these components, by virtue of high residual relationships among the final seven items (14 of the 21 residual relationships are greater than .10), appears to be a speed factor that is distinct from the general quantitative speed factor extracted from the operational test.

The second major unexplained relationship is interpreted to be an ability to deal with data sufficiency items. Although there are no items classified as "data sufficiency" items in test X_{15}, the fourth suggested answer (choice D) for each quantitative comparison item in the test is "the relationship cannot be determined from the information given." In a sense, then, each item has a data sufficiency aspect. A data sufficiency component is suggested by the residuals among the seven items for which choice D is the keyed response, since a disproportionately high number (6 of 21) of the residual relationships among these items are greater than .10. Because operational test K2 contains no data sufficiency items, although form K has seven, it is not possible to further test this interpretation. On form K, data sufficiency items do not form a separate factor, but appear with data interpretation items. A number of additional, highly specific, relationships among pairs of items are apparent also.

Test X_{16}, a mixture of quantitative comparison and regular quantitative items, exhibits residuals greater than .10 in 43 of 595 cases. Of the 15 pairs of residuals among the six quantitative comparison items correctly keyed as choice D, five are greater than .10. This pattern is consistent with that found in test X_{15} and gives further support to a data sufficiency interpretation. The pattern of residuals among the final items (24 of 66 greater than .10) of this experimental test again suggests a component of speed distinct from the quantitative speed factor found in the operational test.

The residuals among items in test X_{17}, data interpretation, suggest that a relationship among the final five items is not adequately explained by the 10 factors that were extended into this test. In all, 25 of 190 residuals are greater in absolute value than .10. However, 8 of the 10 residuals among the last five items are greater than .10 and 6 are greater than .20. Interpretation of this unexplained component, however, is somewhat problematic since each of the last five items is based on a rather unusual graph. The component is, therefore, either a component of speed specific to this test, or a component underlying this atypical graphic material, or both. Since item statistics suggest that speed becomes a factor even before these last five items, there is a strong temptation to attribute the unexplained variance to the unusual graphic material. Again, as with the previously discussed experimental tests, a number of specific relationships among pairs of items remain unexplained, although many of these pairwise relationships are unlikely to represent generalizable variance.

Summary

Structure of Operational Forms

A factor analysis was obtained for each of the two GRE Aptitude Test forms administered in October 1975. These forms appear to be representative of a larger number of GRE Aptitude Test forms by virtue of the methods by which they were constructed and assembled. Input to the analyses consisted of item intercorrelations among all of the 150 verbal and quantitative items in the operational part of each test. The analysis of form K yielded an eight-factor solution which accounted for 94% of the common variance and 38% of the total variance. A 10-factor solution, accounting for 95% of the common variance and 41% of the total variance, was retained for form K2. Table 12 summarizes the factor structure of each form. The similarity of structure in the two forms is apparent from the first three rotated factors, which together account for approximately three-quarters of the common variance in each form. These three factors -- one quantitative and two verbal -- represent the global skills tapped by the GRE Aptitude Test. The quantitative factor is general in nature by virtue of its high loadings on most of the quantitative items. The two verbal factors define abilities to deal with connected discourse (reading comprehension passages and sentence completion items) and with words in isolation (opposites and analogies), respectively. None of the remaining factors explain more than 10% (and most less than 5%) of the common variance.

Additional similarities between the two forms include factors involving the speededness of each of the separately timed verbal sections of the tests. The speededness associated witn Section I (discrete verbal items) accounts for 6.2 and 7.7% of the common variance of forms K and K2, respectively, whereas the factor reflecting the speededness of Section II (reading comprehension passages) explains smaller portions of common variance (2.5 and

Table 12

Summary of Factors Found in Two Operational Forms

Factor	Factor Name Form K	 Form K2
I	General quantitative	General quantitative
II	Reading comprehension: connected discourse	Reading comprehension: connected discourse
III	Words-in-isolation: vocabulary	Words-in-isolation: vocabulary
IV	Elementary algebra	Discrete verbal response speed (Section I)
V	Discrete verbal response speed (Section I)	Reading speed: comprehension (Section II)
VI	Data interpretation	Data interpretation: extraction and manipulation
VII	Applications: word problems	Data interpretation: extraction
VIII	Reading speed: comprehension (Section II)	Reading comprehension: scientific/technical
IX	———	Quantitative speed (Section III)
X	———	Easy opposites

4.4%, respectively). The less prominent factors appearing in each form are as follows: for form K, a factor accounting for slightly more than 4% of the common variance involves the ability to extract information from graphic or tabular material; for form K2, two data interpretation factors underlying an ability to extract information versus an ability to both extract and manipulate information.

The remaining factors from each form highlight differences between the two forms in terms of their structure. A factor termed *elementary algebra*, which explains 7.5% of the common variance, appears in form K but not in form K2. This factor is defined not so much by the necessity to use algebraic manipulations to answer the questions, as by the presence of algebraic notation. Comparable items in form K2, instead of defining a separate factor, show very strong loadings on the general quantitative factor. Form K contains an additional dimension reflecting an ability to solve verbally presented quantitative items ("word problems" or applications).

The 10-factor solution computed for form K2 contains four dimensions that are not apparent in form K. These factors, each of which accounts for about 4% or less of form K2's common variance, have been interpreted, in order of their emergence, as (1) an ability to both extract and manipulate information from graphs and tables, (2) an ability to comprehend scientific/technical material, (3) an ability to work quickly in the quantitative domain, and (4) a skill or response tendency related to opposites items. Factor (1) above, unlike the other data interpretation factor found in both forms, reflects an additional component of variation related to manipulating data extracted from graphs or tables or performing calculations based on those data. Factor (2), a component underlying performance on items associated with scientific/technical reading passages, appears as a separate factor in the analysis of form K2 but not in form K, although, in both analyses, items associated with scientific/technical passages have smaller loadings on the *reading comprehension: connected discourse* factor than do the items from nontechnical passages.

A factor of quantitative speed, accounting for 2.5% of the common variance, emerges clearly as a separate factor in form K2, although final quantitative items (i.e., those not reached by all examinees) continue to display high loadings on the general quantitative factor, suggesting a relationship between speed and ability in the quantitative domain. The final factor unique to form K2 is a difficult-to-interpret component characterized by loadings from the initial (and also the easiest) items in each of the opposites sections. This factor has been referred to, therefore, as *opposites warm-up* or *easy opposites*, for lack of any more appropriate term.

Structure of Experimental Verbal Tests

In general, each of the experimental verbal tests is explained fairly well in terms of the factors extracted from operational form K. There are however, several interpretable dimensions of the experimental tests that are not adequately explained.

Test X_{10} is very well explained, since more than 80% of its estimated common variance has been accounted for by the eight factors in the operational test (52% is explained by the reading comprehension factor) and since there are extremely few significant residual relationships among the items in this test when the eight factors have been removed. The relationships among the items associated with the shorter, experimental passages (whose content was humanities/social studies) are as well explained as the longer passages used in current operational forms. This is not surprising, when we recall that the connected discourse factor displays loadings from operational "passages," as short as one sentence, as well as from comprehension passages of standard length. The only unexplained component is a speed factor apparent from a disproportionately large number of significant residuals among the final items in [illegible] test. Interestingly enough, this component is distinct from the reading comprehension speed factor in the operational form.

The second experimental verbal test, X_{11}, is similar in content to X_{10} in every respect except number of items. That is, additional item is associated with each passage in X_{11}, increasing the total number of items from 25 (in X_{10}) to 30. Adding these additional items seems to introduce a component that is not explained by the factors in the operational form. About 5% less common variance than in test X_{10} is explained for test X_{11}, and a proportionately greater number of significant residuals are found among the final items in test X_{11}. This suggests that a component of speed specific to this test has been introduced. An additional component of speed is suggested by virtue of the relatively greater number of residuals among the shorter passages in this test than in X_{10}, the shorter test. Since these items were reached by virtually every examinee and since they are, except for three additional items, the same as those in X_{10}, it appears that this additional component of speed reflects the time that the examinee is afforded to consider each item. This component has been termed *distributive speed.*

Test X_{12}, the 25-item test containing both long and short scientific/technical reading passages, is less adequately explained by the operational factors (76% of its common variance is explained) than test X_{10}, its non-scientific counterpart. Like tests X_{10} and X_{11}, test X_{12} also has a component of speed that is orthogonal to the reading comprehension speed component in the operational form. Somewhat lower loadings on the reading comprehension factor and a higher proportion of large residuals suggest that scientific/technical passages contribute variance that is not well explained by the operational test's factors. Furthermore, it appears that the more technical the passage, the greater the variance.

Test X_{13}, the longer of the two tests containing scientific/technical passages, is less thoroughly explained by the operational test factors than any of the other tests spiralled with form K; only 65% of its common variance is accounted for. Examination of residuals suggests that this test contains three dimensions not common to the operational form. Two of these have been interpreted as speed factors distinct from those in the operational test and are consistent with the interpretation of factors in test X_{11}. As with test X_{12}, the remaining unexplained dimension is related to the scientific/technical aspect of the passages. Overall, the variance of the tests containing scientific/technical reading passage items is not as well explained by the factors in the operational test as the variance of the humanities/social studies passage items. This is attributed mainly to their (the scientific items') lower loadings on both the general verbal factors, although they load somewhat higher on quantitative factors than the humanities/social studies items.

Adding items to the experimental tests -- both the humanities/social studies and the scientific/technical tests -- has the effect of increasing the portions of common variance of the experimental tests not shared with the operational form. Increasing the length of the scientific/technical tests seems to decrease this shared variance more then does lengthening the humanities/social studies tests.

Structure of Experimental Quantitative Tests

The experimental quantitative tests are, in general, very well explained by the operational test factors with which they are associated. In each case the 10 operational factors explain more than 80% of the estimated common variance of the experimental test. In fact, the case of test X_{14}, which contains items paralleling the non-data-interpretation items now used in the operational test, fully 90% of its common variance was explained. Examination of residuals for X_{14} reveals that only relationships highly specific to pairs of items remain unexplained.

Test X_{15} is the least well explained of the four experimental quantitative tests, although 81% of its common variance is accounted for. Examination of residuals suggests two dimensions in this test that are not shared with the operational form K2. Neither dimension, however, is directly related to the item type (quantitative comparison) found in the test. One dimension is a component of speed apparently distinct from the quantitative speed factor in the operational test. The second unexplained dimension underlies those items for which the correct response is "the relationship cannot be determined from the information given" -- the final option in each of the items. This dimension, therefore, seems to be related, at least tangentially, to the data sufficiency factors that have been found in tests containing data sufficiency items.

Test X_{16}, containing a mix of regular and quantitative comparison items, exhibits the same patterns of unexplained variance as tests X_{14} and X_{15}. The variance of test X_{17}, which contains only data interpretation items, is primarily explained by the general quantitative factor, but also to a lesser extent by the two data interpretation factors, which account for nearly 20% of the tests' common variance. The only unexplained dimension appears to underly items based on a rather unusual (atypical of usual data interpretation stimuli) polar graph having a nonlinear scale. Whereas the component may possibly also be interpreted as a speed factor because of this graph's position at the end of the test, item statistics suggest the former alternative as more plausible.

Recommendations

Recommendation One

The shorter reading comprehension passages have been found to add no variance that is not shared with the common variance of the operational test. In light of the finding that the reading comprehension portion of the current test contains a component of speed independent of ability in this domain, it seems reasonable to include a number of shorter passages at or near the end of the reading comprehension section. Including such passages would result in a smoother distribution of the number attempting each item, since fewer examinees would be "caught" reading a passage upon expiration of the time limit. This smoother distribution should reflect slightly more adequately the assumed normality of the underlying trait.

Recommendation Two

Because inclusion of scientific/technical reading comprehension passages and items adds a component of variance independent of ability to comprehend nonscientific (humanities/social studies) passages, it is recommended that the implications of any plan allowing examinees to choose among several types of passages be carefully considered. Studies to assess the differential validity of the two types of passages for examinees intending to pursue scientific and nonscientific fields of study seem to be in order.

Recommendation Three

Although the common variance of the experimental test containing quantitative comparison items is less adequately explained than for the other experimental tests, the unexplained variance of these items does not directly reflect the item type. Indeed, these items exhibit higher loadings on the general quantitative factor than data interpretation items already in use. It is recommended, therefore, that the decision to include or omit items of this type be based on considerations other than the factor structure of the test.

Recommendation Four

Although distinct components of speed were found to be associated with each of the three sections of the operational forms, the component found in Section I (discrete verbal items) is problematic because of its relatively large contribution to the test's total common variance. Since the GRE Aptitude Test is purported to be primarily a power test, it is suggested that Section I be reexamined in light of its relative speededness -- especially since speed and ability emerge here as uncorrelated traits.

Recommendation Five

It is recommended that the relevance of the results of this study for test development and assembly be noted. The structure of the GRE Aptitude Test, at least as it has been determined factorially here, suggests that there are possibly useful and important dimensions that have not been considered in the test's present content specifications. Furthermore, some of the classifications now used do not contribute to determining the test's factor structure. This does not imply that the item and content classifications now used are not of value, but that there are others that should also be considered in balancing test forms. Specifically, the following suggest themselves as important:

(1) the presence of algebraic notation in quantitative items

(2) the distinction between practical application word problems and quantitative items more theoretical in nature

(3) the difference between data interpretation items requiring only extraction of information and those requiring both extraction and manipulation of information

(4) the value of the item option type keyed as correct (e.g., the data sufficiency option type — "cannot be determined from the information given" — in quantitative comparison items suggested itself as an additional dimension of variance in the present research).

In the interest of improving parallelism, forms K and K2 could be balanced with respect to these dimensions as well as on the item type requiring the examinee to indicate which combination of inferences may be drawn from given information.

Recommendation Six

Finally, it is recommended that this study be viewed in light of the exploratory spirit in which it was undertaken. That is, the study was intended to result in a preliminary factor analytic description of the test and a statement about the relationship of several experimental tests to the factors that emerged in that description. More elaborate alternative approaches could have been implemented instead of, or in addition to, those used in the present study. These could have included alternative methods of (1) computing interitem relationships (e.g., phi or gamma coefficienta), (2) extracting factors and estimating communalities (e.g., maximum likelihood or MINRES), and (3) rotating to simple structure (e.g., Yates' geomin or oblique methods). In addition, approaches using subsets of homogeneous items instead of individual items might have also been informative, while avoiding some of the methodological difficulties associa with analyses based on individual items. It is recommended, therefore, that these alternative types of analyses be conducted in the future when feasible. We like to believe, however, that, although alternative methods might have added to our current understanding of the structure of the GRE Aptitude Test, the operational implications outlined in recommendations one to five above would not have been substantially different.

Appendix A

Examples of Selected GRE Aptitude Test Items

Appendix A

Examples of Selected GRE
Aptitude Test Items

VERBAL

ANALOGIES

Questions of this type test the ability to understand relationships among words and ideas.

Directions: In each of the following questions, a related pair of words or phrases is followed by five lettered pairs of words or phrases. Select the lettered pair which best expresses a relationship similar to that expressed in the original pair.

1. COLOR : SPECTRUM :: (A) tone : scale
(B) sound : waves (C) verse : poem
(D) dimension : space (E) cell : organism

ANTONYMS

Questions of this type test the extent of the candidate's vocabulary.

Directions: Each question below consists of a word printed in capital letters, followed by five words or phrases lettered A through E. Choose the lettered word or phrase that is most nearly opposite in meaning to the word in capital letters. Since some of the questions require you to distinguish fine shades of meaning, be sure to consider all the choices before deciding which one is best.

3. PROMULGATE: (A) distort (B) demote (C) suppress
(D) retard (E) discourage

SENTENCE COMPLETION

This type of question provides a measure of one aspect of reading comprehension the ability to recognize logical and stylistic consistency among the elements in a sentence

Directions. Each of the sentences below has one or more blank spaces, each blank indicating that a word has been omitted. Beneath the sentence are five lettered words or sets of words. You are to choose the one word or set of words which, when inserted in the sentence, best fits in with the meaning of the sentence as a whole.

6. Early — of hearing loss is — by the fact that man's other senses are able to compensate for moderate amounts of loss, so that he frequently does not know that his hearing is imperfect.
(A) discovery..indicated (B) development..prevented
(C) detection..complicated (D) treatment..facilitated
(E) incidence..corrected

7. Swamp drainage is used to prevent, or at any rate to —, the breeding of malaria-bearing mosquitoes.
(A) end (B) remedy (C) postpone
(D) inhibit (E) exclude

READING COMPREHENSION

More than half of the testing time for the verbal sectio
GRE Aptitude Test is devoted to reading comprehens
reading passages are taken from a variety of fields. a
ing comprehension is tested at several levels. Som
questions merely test understanding of the plain sense
has been stated Others ask for interpretation, analysi
plication of the principles or opinions expressed by the

Directions: Each passage is followed by questions ba
its content. After reading the passage, choose the best
to each question. Answer all questions following a pas
the basis of what is *stated* or *implied* in that passage.

In the years following the Civil War, economic exp
for the first time was provided with adequate resource
competent technique, and busy prospectors were daily
ering new sources of wealth. The coal and oil of Penns
and Ohio, the copper and iron ore of Upper Michigan, t
and silver, and the lumber and fisheries of the Pacific
provided limitless raw materials for the rising indus
The Bessemer process quickly turned an age of iron
age of steel and created the great mills of Pittsburg
which issued the rails for expanding railways. The rea
binder, the sulky plow, and the threshing machine cr
large-scale agriculture on the fertile prairies. Wild gra
provided grazing for immense herds of cattle and she
development of the corn belt enormously increased the
of hogs; and with railways at hand the Middle Border
into Omaha and Kansas City and Chicago an endless
of produce.

As the line of the frontier pushed westward, new tow
built, thousands of claims to homesteads were filed, an
ulator and promoter hovered over the prairies like bu
seeking their carrion. With rising land values money wa
made out of unearned increment, and the creation of
was a profitable industry. The times were stirring, and
a shiftless fellow who did not make his pile. If he had be
late to file on desirable acres, he had only to find a c
homesteader who had failed in some legal technicali
"jump his claim." Good bottom land could be had e
late-comers if they were sharp at the game.

This bustling America of 1870 accounted itself a dem
world. A free people had put away all aristocratic pri
and, conscious of power, had gone forth to possess t
frontier But America's essential social philosophy, w
found adequate to its needs, was summed up in three w
preemption, exploitation, progress. Its immediate and pr
business was to dispossess the government of its rich ho
Lands in the possession of the government were so mu
waste, untaxed and profitless; in private hands they
be developed. They would provide work, pay taxes, s
schools, enrich the community. Preemption meant explo
and exploitation meant progress.

It was a simple philosophy and it suited the simple i
ualism of the times. The Gilded Age knew nothing of enl
ment; it recognized only the acquisitive instinct. That
at least the frontier had taught the great American demo
and in applying to the resources of a continent the les
had been so well taught, the Gilded Age wrote a pro
characteristic chapter of American history.

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8. According to the passage, increased corn production was mainly responsible for an increase in the
   (A) number of sheep (B) output of farm implements
   (C) supply of hogs (D) amount of pasture land
   (E) number of cattle

9. According to the passage, the progress of the 1870s depended mainly on
   (A) the existence of rich government land holdings
   (B) the spread of education
   (C) the development of a philosophy of individualism
   (D) private development of natural resources
   (E) the elimination of aristocratic privileges

10. The author implies that the social philosophy of the United States in the 1870s was basically
    (A) humanistic (B) materialistic (C) aristocratic
    (D) democratic (E) hypocritical

11. As used by the author, the term "Gilded Age" refers to an age of
    (A) social progress (B) intellectual enlightenment
    (C) frontier living (D) great fortunes
    (E) aristocratic privilege

12. With which of the following aphorisms would the exploiters of the 1870s probably have been in strongest agreement?
    (A) A penny saved is a penny earned.
    (B) Nothing ventured, nothing gained.
    (C) Grasp all, lose all.
    (D) He who dances must pay the fiddler.
    (E) The love of money is the root of all evil.

QUANTITATIVE

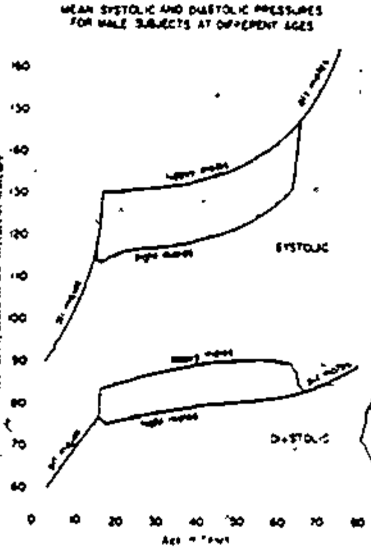
Directions: In this section solve each problem, using any available space on the page for scratch work. Then indicate the best answer in the appropriate space on the answer sheet.

Note: Figures which accompany problems are intended to provide information useful in solving the problems. They are drawn as accurately as possible except when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

All numbers used are real numbers.

13. If p, q, and r are consecutive whole numbers, which of the following must be true?
    (A) $p + q + r$ is even. (B) $p + q + r$ is odd.
    (C) pqr is even. (D) pqr is odd.
    (E) If $p + q - r$ is odd, then pqr is odd.

Questions 14-19 refer to the following graph.



Blood Pressure is customarily expressed as $\frac{\text{systolic pressure}}{\text{diastolic pressure}}$

Note Pulse Pressure is systolic pressure minus diastolic pressure

14. A rule of thumb says that the systolic pressure should be "age plus one hundred." At which of the following ages does this rule of thumb differ most from the data for heavy males?
    (A) 20 (B) 30 (C) 40 (D) 50 (E) 60

15. What is the mean pulse pressure of a light male aged 40?
(A) 14 (B) 30 (C) 39 (D) 44 (E) 53

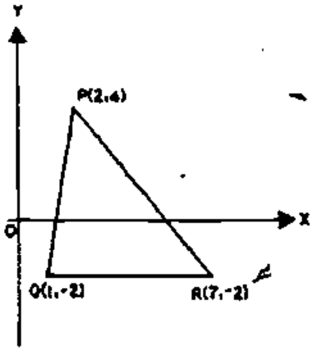
16. What is the youngest age for which mean blood pressure can be read from the graph?
(A) 1 (B) 4 (C) 7 (D) 9 (E) 17

17. At which of the following ages is the mean pulse pressure greatest?
(A) 4 (B) 17 (C) 40 (D) 65 (E) 75

18. Of the following, which is the best estimate of the mean pulse pressure of "average weight" males aged 30?
(A) 30 (B) 40 (C) 53 (D) 84 (E) 124

19. What is the percent increase, to the nearest percent, in the mean systolic pressure for heavy males from the age of 50 to the age of 60?
(A) 0% (B) 1% (C) 4% (D) 8% (E) 10%

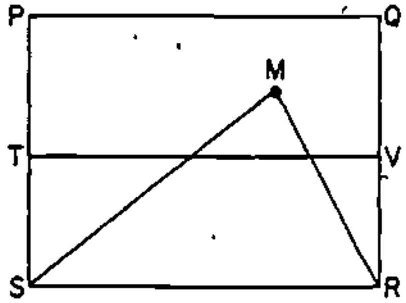
20. If the cost for manufacturing r articles was M dollars in 1964 and N dollars more in 1965, what was the increase in cost, in dollars per article?
(A) $\frac{N-M}{r}$ (B) $\frac{N}{r}$ (C) $\frac{N}{M}$ (D) $\frac{M-N}{r}$ (E) rN



21. What is the area of triangle PQR in the figure above?
(A) 8 (B) 12 (C) 18 (D) 24 (E) 36

22. If $\frac{2}{3}$ of the members of a committee voted on a c
measure and passed it by a vote of 26 to 24, what p
(to the nearest percent) of the entire committee vo
favor of the measure?
(A) 35% (B) 48% (C) 52% (D) 67% (E) 78%

23. Given that a and b are real numbers and $b \neq$
$D(a, b) = \frac{a}{b}$ and $M(a, b) = a \times b$.
Then $M(D(3, 12), M(6, 2)) =$
(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) 3 (D) 12 (E) 48



24. The area of rectangle PQRS above is 48, QV = VR
PT = TS. If point M is somewhere inside the rect
PQVT, and if x is the area of triangle MRS, which c
following includes all the values and only the values
(A) $6 < x < 24$ (B) $12 < x < 24$ (C) $12 < x <$
(D) $24 < x < 48$ (E) $48 < x < 96$

ERIC Full Text Provided by ERIC

Appendix B

Description of Tucker's Procedure for Estimating Communalities

Appendix B*

Description of Tucker's Procedure for Estimating Communalities

Tucker's adjusted highest off-diagonal correlation provides a communality estimate obtained by multiplying the highest off-diagonal correlation, r_{jk_m}, between [illegible] target variable, j, and other variables by a weighting constant. The constant is a ratio with numerator equal to the sum of the absolute values $|r_{jk}|$ for $k \neq j, k_m$, and denominator equal to the sum of the absolute values $|r_{k_m k}|$ for $k \neq j, k_m$. Thus, of the pair of variables j, k_m, the variable exhibiting higher correlations with all other variables in the system receives the higher communality estimate. In the case of a unifactor matrix, or a higher dimensional matrix in which the two variables are collinear and differ only in length, this estimate is exact, since we can rotate so that both variables have nonzero loadings on factor one and zero loadings on all others, leading to the following expression, where p is the number of factors and a_{kp} is the factor loading of the kth variable on the pth factor:

$$|r_{jk_m}| = |a_{j1}|\,|a_{k_m 1}|$$

$$|r_{jk}| = |a_{j1}|\,|a_{k1}| \quad ; \quad \sum_k |r_{jk}| = |a_{j1}| \sum_k |a_{k1}|$$

$$|r_{k_m k}| = |a_{k_m 1}|\,|a_{k1}| \quad ; \quad \sum_k |r_{k_m k}| = |a_{k_m 1}| \sum_k |a_{k1}|$$

$$\text{Then } \hat{h}_j^2 = |r_{jk_m}| \frac{\sum_k |r_{jk}|}{\sum_k |r_{k_m k}|} = |a_{j1}|\,|a_{k_m 1}| \frac{|a_{j1}| \sum_k |a_{k_m 1}|}{|a_{k_m 1}| \sum_k |a_{k_m 1}|} = a_{j1}^2 = h_j^2$$

Clearly, this procedure is less appropriate when high correlations obtain among noncollinear items and is not robust in the face of doublet variance. The sum of these estimates should provide a reasonable estimate of total communality if doublet variance is not an important source of item collinearity.

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* This derivation is from Carlson (1976). Since documentation on this procedure is not widely available, it is presented here.

Appendix C

Structure of Operational Tests K and K2

Table C-1

Rotated Factor Loadings*
(Form K, Section I)

Item Type	Item Number	Factor I	Factor II	Factor III	Factor IV	Factor V	Factor VI	Factor VII	Factor VIII	Estimated Communality	Computed Communality
Analogies	1		45							27	28
	2		34	38						35	32
	3		26	25						22	19
	4		26	53						36	39
	5		29							18	17
	6		26	47						32	30
	7			30						14	16
	8			29						14	13
	9			39						11	17
Opposites	10		55	36						55	49
	11	20	29	56						53	47
	12		34	34						27	25
	13		28							19	14
	14		31	67						56	56
	15		34	63						57	55
	16	28		46						33	32
	17		24	62						47	46
	18			44						18	21
	19			53						28	31
Sentence Completions	20		63							46	52
	21		52	34						43	40
	22		55	43						57	53
	23	21	44	21						33	31
	24		40	29						34	29
	25	21	35		24					31	27
	26		26	24						17	14
	27		34	62						54	56
	28		49	42						51	45
Analogies	29		41	25				21		35	30
	30		42	62						57	57
	31		25							13	12
	32		44	41						47	40
	33		30	52						44	39
	34		32	55						45	44
	35		27	45				20		33	35
	36			33					23	15	22
	37			38						10	17
Opposites	38		32	36		24			31	43	41
	39		29	48				28		49	46
	40		30	57						52	49
	41		29	63		21				57	54
	42		34	54		25				55	52
	43			55						34	36
	44		28	49		21				42	40
	45		29	71		21				67	65
	46			68						44	52
	47			59						29	39
Sentence Completions	48	21	23	21		68				55	63
	49	28	33	26		58				64	62
	50		41	38		58				73	70
	51		22			60				38	49
	52			25		58				43	47
	53		33	51		52				62	66
	54	23	33	26		58				58	58
	55		21	46		40				43	45

*All loadings less than .20 (and all decimal points) have been omitted from this and all subsequent tables.

Table C-2

Rotated Factor Loadings (Form K, Section II)

Item Type	Item Number	Factor I	II	III	IV	V	VI	VII	VIII	Estimated Communality	Computed Communality
Narrative Passage	1		41							23	22
	2		46							27	25
	3		50							30	32
	4		31							12	11
	5		23							12	8
	6		43	24						27	26
	7		57				24			34	41
Humanities Passage	8		45							21	23
	9		46							29	25
	10		50							34	31
	11		47							33	29
	12		60							39	42
	13		44							22	22
Argumentative Passage	14		45	23						34	30
	15		38							18	17
	16		35	29						26	27
	17		39							27	22
	18		63	22						45	48
	19		43	24						33	29
	20		44	37						36	37
Biological Science Passage	21		41							28	23
	22	25	38							28	26
	23		27							13	11
	24		29							20	13
	25		30							19	12
	26		25							15	13
Social Studies Passage	27	21	46	26						41	36
	28		46							38	29
	29		64							57	51
	30		50							40	36
	31		50	22						42	34
	32		29							16	13
	33		36							30	18
Physical Science Passage	34		44						26	41	34
	35		36	31					24	32	31
	36	25	48						40	53	50
	37		50	21					29	51	44
	38		34						35	30	31
	39	21	35						30	32	30
	40	22	25						23	26	20

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Table C-3

Rotated Factor Loadings
(Form K, Section III)

		Factor									
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII	Estimated Communality	Computed Communality
Algebra	1	30			38					31	28
Algebra	2	29			24					22	22
Arithmetic	3	26	33		59					46	55
Algebra	4	42								25	25
Arithmetic	5		27		45					41	35
Geometry	6	53	25		47			30		70	67
Arithmetic	7	27	23				21	38		23	26
Algebra	8	51	22		47					56	54
Algebra	9	46	23		49					57	51
Arithmetic	10	36	23		32			25		38	38
Geometry	11	49	22		45					58	54
Data Interpretation (DI)	12	20	26				54			63	44
DI	13		27				46			45	35
DI	14	26					31			24	21
DI	15	38			23		31			33	34
DI	16	54	27		25					48	48
Arithmetic	17	44	23		41					45	42
Geometry	18	44								30	28
Algebra	19	63			30					51	55
Arithmetic	20	36	25		47					40	43
Algebra	21	49	31		48					69	60
DI	22	25	25				49			50	39
DI	23	30	25				58			69	50
DI	24	43	30				30			54	43
DI	25	29	38		26		33			65	44
DI	26	26					22			11	14
Arithmetic	27	55					21	23		41	44
Arithmetic	28	42						21		31	26
Geometry	29	52			35					44	43
Algebra	30	40			22					25	25
Algebra	31	51	27					21		44	45
Geometry	32	63			38					65	59
Miscellaneous	33	43	23		21					35	33
Algebra	34	43								26	22
Geometry	35	39			22					31	28
Algebra	36	51			28					47	39
Geometry	37	57								47	38
Algebra	38	70			29					70	60
Miscellaneous	39	48								35	31
Arithmetic	40	54								36	32
Geometry	41	55								34	36
DI	42	40								28	23
DI	43	46	23				22			45	36
DI	44	66								53	54
DI	45	56								48	41
DI	46	50								33	28
Geometry	47	63								40	44
Algebra	48	79								62	66
Geometry	49	67								53	49
Geometry	50	66								46	48
Miscellaneous	51	77						-28		89	75
Miscellaneous	52	80						-21		89	74
Miscellaneous	53	80								89	75
Geometry	54	77								61	63
Miscellaneous	55	83								87	75
Percent of common variance		28.3	26.7	21.4	7.5	6.2	4.3	3.1	2.5		

Table C-4

Rotated Factor Loadings
(Form K2, Section I)

Item Type	Item Number	Factor I	II	III	IV	V	VI	VII	VIII	IX	X	Communality Estimated	Computed
Analogies	1		38									22	20
	2		36									16	17
	3	22	33	23								21	21
	4		36	48								34	37
	5		22									10	9
	6	22	44	43								40	43
	7		21	61								50	42
	8			56								48	35
	9			65								20	23
Opposites	10		52					27			31	46	40
	11	35	34	25							29	62	34
	12		40	38								31	33
	13			52								35	35
	14		22	40								27	27
	15	27		40								34	29
	16			57								35	35
	17			66								40	46
	18		29	53								36	38
	19			65								37	44
Sentence Completions	20		55									31	38
	21	26	45									38	34
	22		39	40								37	34
	23		49	25								33	36
	24		42	21								26	26
	25		33	24								16	18
	26		31									21	15
	27		30	35								26	25
	28		31	22								16	15
Analogies	29		46									37	32
	30		39	33								37	30
	31	22										17	12
	32			43								19	26
	33		35	65								57	56
	34	26	24	36								33	28
	35			51								29	26
	36		32	62								56	53
	37			69								40	47
Opposites	38	21	37	22	39						33	61	49
	39		46	37	27						26	63	48
	40		37	28	36						38	64	49
	41		35	50	26							60	51
	42		38	41	21							36	38
	43		21	64								59	54
	44			35	21		22					35	28
	45		23	68	21							57	58
	46			59								39	41
	47			52								32	34
Sentence Completions	48		23		89							84	92
	49	21	25		85							94	86
	50	20	30	22	73							93	71
	51	21	21		64							67	53
	52	27		42	57							82	62
	53			51	45							59	50
	54		22	30	43							52	37
	55			64	36							51	59

Table C-5

Rotated Factor Loadings
(Form K2, Section II)

		Factor										Communality	
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII	IX	X	Estimated	Comput
Narrative Passage (Minority Theme)	1		53									35	33
	2		45	23								30	28
	3		49									32	25
	4		51	27								30	34
	5		56									31	35
	6		34									25	16
Scientific Passage (Biochemistry)	7	34	36						45			55	38
	8	[illegible]	32						47			36	23
	9	27	31						47			42	30
	10	30	28						47			42	29
	11		31						34			26	23
	12		28						38			30	17
	13			20					28			15	12
Argumentative Passage (Social Studies)	14		27									13	12
	15	20	49	21								55	34
	16		49	33								48	40
	17	25	42	25								45	34
	18		62	22								70	45
	19		31									14	12
	20		39									30	21
Humanities Passage (Synthesis)	21		53									39	33
	22		42									16	19
	23	20	55	23								37	40
	24		55	23								39	39
	25		43									27	24
	26		28	35								22	20
	27	26	44	28								38	35
Humanities Passage (Mythology)	28		55									38	38
	29	26	60	30								69	60
	30	22	49	28								49	43
	31		47	24								38	34
	32		33									15	16
	33		20	24								19	12
	34	24	44	26		20						47	41
Physical Science Passage (Geology)	35	23	33	20		60						57	48
	36	28	32	23		58						69	49
	37	28	37	25		64						79	58
	38		27			48						39	32
	39		29	26		53						56	42
	40	25		25		48						40	35

Table C-6

Rotated Factor Loadings
(Form K2, Section III)

Item Type	Item Number	Factor I	II	III	IV	V	VI	VII	VIII	IX	X	Communality Estimated	Computed
Arithmetic	1	45										30	26
Arithmetic	2	65	35									63	62
Geometry	3	42	23					21				33	27
Algebra	4	44	28					24				36	38
Arithmetic	5	37	23									23	23
Arithmetic	6	58								-26		48	43
Arithmetic	7	55								-24		41	39
Arithmetic	8	32	26									23	19
Arithmetic	9	39	27									37	29
Arithmetic	10	57	21									50	43
Geometry	11	56	20									40	39
Algebra	12	58										32	36
Algebra	13	57	23				26					56	49
Algebra	14	77										78	68
Arithmetic	15	68										56	53
Geometry	16	52										28	28
Miscellaneous	17	42	33									32	30
Algebra	18	47	20									33	28
Miscellaneous	19	35	27									26	22
Algebra	20	75										57	60
Arithmetic	21	38										18	19
Geometry	22	75										60	61
Data Interpretation (DI)	23	32	25					66				64	51
DI	24	34	30					60				63	47
DI	25	41					44	21				71	46
DI	26	25						42				29	22
DI	27	37					44					53	37
DI	28	40					28	32				44	38
DI	29	39					24	34				44	38
DI	30							23				25	16
Algebra	31	63										45	43
Geometry	32	52										39	32
Geometry	33	64										50	46
Arithmetic	34	35	23									24	19
Geometry	35	62										48	45
Arithmetic	36	73										61	58
Algebra	37	77										71	65
Geometry	38	59										40	41
DI	39		22					30				25	19
DI	40	43					41					57	37
DI	41	37					43					52	39
DI	42	50					44					68	52
DI	43	47					40					47	44
DI	44	38					36					38	31
Algebra	45	46	23									35	33
Geometry	46	54								27		41	41
Arithmetic	47	61								23		51	44
Geometry	48	68										58	54
Geometry	49	64										65	46
Miscellaneous	50	67								45		78	65
Miscellaneous	51	76								44		95	77
Geometry	52	69								24		57	59
Arithmetic	53	40					24			31		38	31
Algebra	54	62					25			32		60	59
Algebra	55	43					33			26		46	41
Percent of common variance		29.7	22.7	19.4	7.7	4.4	4.4	3.9	3.1	2.5	2.1		

Table C-7

Principal Factor Solution
Roots and Variance Contributions
for Forms K and K2

	Form K.		Form K2	
Factor	Root	Cumulative Percent of Total Common Variance	Root	Cumulative Percent of To Common Variance
I	33.07	55.2	35.35	55.1
II	10.27	72.4	9.58	70.0
III	4.84	80.5	4.44	77.0
IV	2.25	84.2	3.11	81.8
V	1.89	87.4	2.01	84.9
VI	1.70	90.2	1.85	87.8
VII	1.22	92.3	1.34	90.0
VIII	1.04	94.0	1.10	91.6
IX	.85	95.4	1.08	93.3
X	.83	96.8	.97	94.8
XI	.81	98.2	.82	96.1
XII	.74	99.4	.80	97.4

Table C-7 (Cont.)

Additional Roots

Form K		Form K2
Factor	Root	Root
13	.64	.68
14	.61	.64
15	.56	.62
16	.51	.54
17	.49	.51
18	.47	.46
19	.45	.45
20	.42	.42
.	.	.
.	.	.
.	.	.
77	.00	.01
78	.00	.01
79	-.01	.00
80	-.01	-.01
.	.	.
.	.	.
.	.	.
137	-.30	-.30
138	-.31	-.30
139	-.32	-.31
140	-.33	-.31
141	-.33	-.32
142	-.34	-.33
143	-.34	-.34
144	-.35	-.35
145	-.37	-.36
146	-.37	-.38
147	-.38	-.39
148	-.41	-.40
149	-.43	-.44
150	-.48	-.46

Table C-8

Roots and Variance Contributions for Rotated Factors - Form K

Factor	Root	Percent of Common Variance	Cumulativ Percent
I	15.94	28.3	28.3
II	15.05	26.7	55.0
III	12.06	21.4	76.4
IV	4.20	7.5	83.9
V	3.51	6.2	90.1
VI	2.39	4.2	94.3
VII	1.72	3.1	97.4
VIII	1.42	2.5	99.9
	56.29		

Table C-9

Roots and Variance Contributions for Rotated Factors - Form K2

Factor	Root	Percent of Common Variance	Cumulativ Percent
I	18.14	29.8	29.8
II	13.75	22.6	52.4
III	11.83	19.4	71.8
IV	4.69	7.7	79.5
V	2.69	4.4	83.9
VI	2.59	4.3	88.2
VII	2.38	3.9	92.1
VIII	1.88	3.1	95.2
IX	1.53	2.5	97.7
X	1.35	2.2	99.9
	60.83		

Appendix D

Factor Extension Matrices for Experimental Verbal (X_{10} - X_{13}) and Experimental Quantitative (X_{14} - X_{17}) Tests

Table D-1

Factor Extension Matrix
for Test X_{10}

		Factor							
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII
Short Passage One	1		64		20				
	2		42						
	3		29						
Short Passage Two	4		46						
	5		49						
	6		29						
Short Passage Three	7		52						
	8		58						
	9		48	26					
Long Passage One	10		53	36					
	11		41						
	12		56						
	13		53	28					
	14		51	26					
	15		37					20	22
	16		52	26					
	17		62	23					
Long Passage Two	18		52	26					21
	19		36	28					
	20		42	31					
	21		49	40					
	22		39	33					
	23		47	33					
	24		34	36					
	25		48	39		22			23

Table D-2

Factor Extension Matrix

for Test X_{11}

		Factor							
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII
Short Passage One	1		70						
	2		43						
	3		22	20					
	4		30	40					
Short Passage Two	5								
	6		46						
	7		50						
	8		40	22					
	9		31						
Short Passage Three	10		54						
	11		51						
	12		55	22					
Long Passage One	13		51	31					
	14		40						
	15		54						
	16	20	50	22					
	17		48	24					
	18		41						21
	19		51	22					
	20		49	31					
	21		58						28
Long Passage Two	22		43	34					22
	23		37	30					
	24		44	33					
	25	23	46	39					
	26	24	47	35					
	27	23	40	32					
	28	22	45	35					
	29		30	33					
	30	27	42	38					23

Table D-3

Factor Extension Matrix

for Test X_{12}

Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII
		Factor							
Short Passage One	1		56	24	24				
	2		24						
	3		51						
Short Passage Two	4								
	5		49						
	6		60						
Short Passage Three	7								
	8		35	22					
	9	24	45		21				
Long Passage One	10	23	41						
	11		57						21
	12		46						
	13		77						23
	14	23	53						
	15		45						
	16	28	43						
	17	23	58						
Long Passage Two	18	24	52						37
	19	37	40						
	20	32	42						
	21	25	48	24					27
	22	27	48	22					27
	23	36	37						
	24	46	37	21					21
	25	32	42	25		23			

Table D-4

Factor Extension Matrix

for Test X_{13}

		Factor							
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII
Short Passage One	1	22	49	24					
	2		22						
	3		49						
Short Passage Two	4								
	5		50						
	6		49		21				
Short Passage Three	7		25						
	8		39						
Short Passage Four	9		44						
	10		26						
	11	23	38						
Short Passage Five	12								
	13		41	21					
	14		47	21	21				
Long Passage One	15		38						
	16		50				26		
	17		46						
	18		57						25
	19	22	51	22	21				
	20		45						21
	21		38	23					21
	22		48	21					27
Long Passage Two	23	27	43	23	20				30
	24	37	33						
	25	30	42	22					
	26	25	39	24					
	27	30	40	28		21			
	28	28	30	31					
	29	39	28	25					
	30	30	32	31		23			

Table D-5

Factor Extension Matrix

for Test X_{14}

		Factor								
Item Type	**Item Number**	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**	**IX**
	1	43	25							
	2	63								
	3	48								
	4	65	25							
	5	71								
	6	68								
	7	82								
	8	69								
	9	65								
	10	47								
	11	54					33			
	12	74								
Regular	13	68								
	14	56								
Quantitative	15	47	39							
	16	72								
	17	59					32			
	18	52								
	19	56					33			
	20	58		23			34			
	21	76								
	22	77								
	23	59								
	24	57					28			
	25	76								
	26	58		27						33
	27	62								
	28	46								35
	29	63		25			29			30
	30	55		29			36			42

Table D-6

Factor Extension Matrix

for Test X_{15}

		Factor									
Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII	IX	X
Quantitative Comparison	1	47	31					21			
	2	41	26					22			
	3	52	20								
	4	60					-21				
	5	38						26			
	6	47					-21				
	7	58									
	8	64									
	9	40						25			
	10	58	26								
	11	51									
	12	55	27								
	13	52									
	14	59									
	15	58					-21				
	16	54									
	17	55									
	18	41									
	19	63									
	20	72									
	21	62									
	22	67									
	[illegible]	48									
	24	50									
	25	65									
	26	55									
	27	61									
	28	56					26				
	29	57								30	
	30	45					21			34	
	31	51								21	
	32	27									
	33	53								21	
	34	52								26	
	35	37								29	
	36	34								28	
	37	49			23					26	
	38	65									
	39	51									
	40	48								28	

Table D-7

Factor Extension Matrix

for Test X_{16}

Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII	IX
Quantitative Comparison	1	41	38							
	2	47	31							
	3	64	32							
	4	54								
	5	55	31							
	6	53	25							
	7	58	28							
	8	49								
	9	56								
	10	37								
	11	65								
	12	56								
	13	49								
	14	45	20							
	15	65					22			
	16	62								
	17	52					23			
	18	55								21
	19									
	20	[illegible]								
	21	48								
	22	62								23
	23	52								
Regular Quantitative	24	31								34
	25	61			22					
	26	71			26					
	27	41								25
	28	59			23					
	29	60					24			
	30	65			22					29
	31	40					31			27
	32	40					33			24
	33	44					31	58		52
	34	56					26			27
	35	58					22			28

Table D-8

Factor Extension Matrix

for Test X_{17}

Item Type	Item Number	I	II	III	IV	V	VI	VII	VIII	IX	X
Data Interpretation	1	47					20			-21	
	2	35									
	3	59									
	4	48									
	5	31	23								
	6	25									
	7	42									
	8	44					21				
	9	46	25								
	10	39									
	11	36						21			
	12	54					25	22			
	13	50	23				32				
	14	54					37				
	15	53					27				
	16	50					28				
	17	53									
	18	41					39	70		37	
	19	49					24				
	20	56					31				

References

Altman, R.A., Carlson, A.B., and Donlon, T.F. Developing a plan for the renewal of the GRE morning test: A report on investigations into the possible shortening of the GRE-V and GRE-Q and the creation of a modular aptitude test. GRE Report 74-3. Princeton, New Jersey: Educational Testing Service, 1975.

Carlson, A.B. Criteria for farm managers. Research Bulletin 67-31. Princeton, New Jersey: Educational Testing Service, 1967. (Appendix H)

Coffman, W.E. A factor analysis of the verbal sections of the Scholastic Aptitude Test. Research Bulletin 66-30. Princeton, New Jersey: Educational Testing Service, 1966.

Dwyer, P.S. The determination of the factor loadings of a given test from the known factor loadings of other tests. Psychometrika, 1937, 2, 173-178.

Graduate Record Examinations. Sample Aptitude Test. Princeton, New Jersey: Educational Testing Service, 1975.

Harman, H.H. Extensions of factorial solutions. Psychometrika, 1938, 3(2), 75-84.

Kaiser, H.F. The varimax criterion for analytic rotation in factor analysis. Psychometrika, 1958, 23, 187-200.

Pruzek, R.M., and Coffman, W.E. A factor analysis of the mathematical sections of the Scholastic Aptitude Test. Research Bulletin 66-12. Princeton, New Jersey: Educational Testing Service, 1966.

Swinton, S.S., and Powers, D.E. Relevant and irrelevant speededness. Research Memorandum 76-3. Princeton, New Jersey: Educational Testing Service, 1976.

Yates, A. An oblique transformation method for primary factor pattern simplification which permits factorial complexity in exploratory analyses. Paper presented at the meeting of the Psychometric Society, Palo Alto, 1974.