DOCUMENT RESUME

ED 162 552 HE 010 548

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TITLE Final Report: ISCI. Consulting Group on Instructional Design.
INSTITUTION Minnesota Univ., Minneapolis.
SPONS AGENCY Fund for the Improvement of Postsecondary Education (DHEW), Washington, D.C.
PUB DATE 5 Sep 76
GRANT OEG-0-73-6432
NOTE 57p.

EDRS PRICE MF-$0.83 HC-$3.50 Plus Postage.
DESCRIPTORS *Autoinstructional Aids; *Calculus; *College Mathematics; *Computer Assisted Instruction; Computer Oriented Programs; *Educational Technology; Higher Education; Homework; *Individualized Instruction; Information Dissemination; *Mathematics Instruction; Program Descriptions; Program Evaluation
IDENTIFIERS *Individualized Supplementary Calculus Instruction; University of Minnesota

ABSTRACT

A description and evaluation are provided of the ISCI (Individualized Supplementary Calculus Instruction) project. ISCI is a computer-based resource that can be used with any text or computer system. If a student fails to solve homework problems, he can provide information about his attempted solution through a teletype or other computer terminal. After finding the error in his attempted solution, ISCI points out that there is an error, then gives as many hints and/or explanations as the student requests, until he is able to continue the solution independently. Three facets of the ISCI project are the use of the system in Minnesota and neighboring states, program access at the University of Minneapolis Campus, and worldwide mailings about the system. Appended are a published article on the ISCI program entitled, "Tutor Resources in Mathematics," and details of ISCI evaluation in an article entitled, "ISCI: A Computer Answers Dumb Questions." (SW)

ED162552

FINAL REPORT: ISCI

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September 5, 1976

Introduction

Attached to this report is the paper, Tutor Resources in Mathematics, referred to hereafter as TRIM. This report is meant to supplement TRIM which contains a rather complete description of the ISCI program and evaluations of it. Here I will discuss the project itself: its successes, failures, and lessons to be learned for the future. The paper, TRIM, has been accepted for publication in The American Math Monthly, the journal of the Mathematical Association of America. (I judged the best medium of dissemination to be the Monthly: it is sent to every member of the Association.) Since the Monthly is very selective about its articles in mathematics education, publishing only one article per issue, it is an honor to have had the paper accepted for publication. It should appear sometime in 1977; the exact date has not yet been set.

Accomplishments

The ISCI Program works for Students. Evidence for this is to be found in TRIM, so we will not repeat it here.

The ISCI Program works for Faculty. I mean this in a technical sense: it runs. It has been through three years of testing, and for the past year and a half, it has been completely "stable." Stability is a property rarely found in computer-assisted instructional programs. They typically need frequent revisions, updating, and major overhauls when the computer system is changed. An example

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of the simplicity of the program is that a programmer at the Open University in Great Britain was able to get it running on their computer -- one totally different from any on which it had run before (Hewlett Packard vs Control Data and IBM) within four weeks after receiving the program.

<u>It is available to others</u>. Availability has two components. First, as mentioned above, the system is simple and stable enough that it can be adapted to other computer systems quite readily. Second, information about the program is being widely disseminated. This dissemination has three facets (MERITSS, Minneapolis, Mailings), and I will discuss them separately.

i) <u>The MERITSS System</u>. This is a Computer system (network) with terminals throughout the state of Minnesota and in neighboring parts of the Dakotas and Wisconsin, including (until 1976 -- see below for changes made in 1976) all public post-secondary educational campuses in Minnesota. Information about the ISCI program has been sent to math departments at every such campus, including user manuals showing how to use the program from any MERITSS terminal in the state. Results have been of two kinds: first, students from ten different campuses have used the program on more than five occasions (another 15 or 20 have used it five times or less). This was determined by a monitoring program which tells us the campus of each person using ISCI. Since I have not heard from <u>any</u> faculty at these institutions, I might guess that user manuals have been distributed informally to students who have used them independently. Secondly, faculty at some institutions have used the ISCI program as a starting point for designing their own computer-assisted instructional systems. This has occurred, for example, at the University of Illinois at Chicago Circle.

In late 1975 a new computer system, MECC, was supposed to take over from MERITSS and other systems. MECC was to become the sole computer system for all institutions of public learning. It has been a failure so far -- mainly because the equipment is still new and full of bugs -- but in a few years it should be working satisfactorily. Meanwhile, the University and state colleges still use MERITSS, and other public institutions are stuck with MECC. Although a working version of ISCI has been written to run on MECC, I have not installed it because I don't want to get involved with their problems and, more importantly, I don't want the continual (at least daily) breakdowns of MECC to turn students off to computer-assisted instruction.

ii) The University's Minneapolis Campus. This is the largest campus in the state, the "home" of MERITSS and the place where ISCI was developed, so one should expect more widespread usage here. There are two ways students have access to ISCI. First, faculty and teaching assistants in the Mathematics department can use the program by assigning or recommending it to their students, or in some way integrating it with their classroom teaching. To encourage this, information about ISCI has been sent to all teaching assistants (for they are the classroom teachers in most of the elementary courses ISCI relates to) and to certain faculty, mainly those teaching elementary courses. I know of no use of ISCI in this respect -- see page 5 below for reasons. Second, there is an office on campus which acts as a center for (free) undergraduate tutoring, and ISCI manuals are prominently displayed there. Several students pick up manuals there; two hundred were distributed during Spring 1976. Thus, this informal distribution is the only vehicle for student use of ISCI on the Minneapolis campus.

iii) Worldwide Mailings. News of ISCI has traveled in several ways: FIPSE, news releases, talks I have given at various meetings, my personal contacts, an article in the Committee on Institutional Cooperation (CIC) Bulletin. The largest such dissemination will come in 1977 with the publication of TRIM. In the Spring of 1976 I mailed approximately 100 packages to people who had written to me a... l. Contents of the package, 120 pages of written material, can be ... the cover letter (on the next page). These packages are sent free of cha... (A total of 300 have been printed with grant funds and future mailing expenses will be assumed by the Consulting Group on Instructional Design.) From this written material it is a simple matter to copy the ISCI program and get it to run. If someone desires copies of the 120 data files available, i.e., the files which enable the program to help students with 120 problems from Calculus and Precalculus, they are available on a magnetic tape at a cost of $25, which covers the cost of the tape and copying. As of July 1976, four requests for tapes have been received and filled (at no cost, since we had some tapes left over from the grant). These were from the Universities of Oregon and Colorado, the Open University in England, and John Abbott College in Quebec.

As the school year 1976-77 begins, and with the publication of TRIM, I expect to receive many more requests for information.

Failures

<u>Three False Assumptions</u>. My original concept of ISCI, and one which has been repeated often, went something like this: at the point when students stumble in trying to solve a homework problem, the help needed could be provided by a computer in an efficient and non-threatening way. Efficiency could be increased by giving help only with the first few basic problems in each topic where the students in most need of help have the most trouble. Associated with this concept are three fallacious assumptions:

a) Instructors, duly impressed by ISCI, would assign problems from it as the first few problems in each topic.

b) Instructors would encourage students to try ISCI.

c) Even if (a) and/or (b) did not happen, students could pick up

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Dear Colleague:

This package is being sent to you because of your expressed interest in the ISCI program. The package contains:

1. Tutor Resources in Mathematics, a short introduction to the ISCI program;

2. ISCI: A Computer Answers Dumb Questions, which contains details of evaluations of ISCI and pros and cons concerning its usage;

3. ISCI Software Installation Directions, including printed copies of programs and a sample user's manual.

This package will allow you to decide whether you wish to implement ISCI on your computer system. If you decide in the affirmative, see the installation directions for information about getting copies of the program and files from me.

I hope you will write to me if you have any questions, and inform me of any use you make of ISCI. In particular, these instructions doubtless contain errors and will be revised, so if you begin working with them a while after you get them, be sure to write to me for the latest version.

I am pleased to acknowledge the assistance of Joseph Gervais, who wrote the computer programs for ISCI; Judy Smith, who wrote a plurality of the problem files; and my colleague Professor H. Keynes, who has shared with me the task of developing ISCI.

Sincerely,

Leonard Shapiro

Leonard Shapiro
Assistant Professor of Mathematics

LS/me

Enclosures

manuals and use ISCI on their own. They would be motivated to do so by news of the usefulness of ISCI which would travel by word-of-mouth.

We anticipated assumption (b), and in our testing we identified certain classes of students who received no encouragement to use ISCI. These classes did as well as the others, so assumption (b) was not crucial to the success of the concept. This is not the case with (a) and (c). I claim that they are, in general, false, and that without them the concept does not work. Before giving reasons for this claim, I should point out that this problem has become apparent only in the past year when the absence of my personal influence from the math department at Minnesota made (a) false, and I was able then to observe that (c) was also false in general. Thus, I have been unable to check the truth of this claim in a formal way, and my assertions are based on personal observation, mainly at the University of Minnesota.

Why is (a) false? A study of math teachers' attitudes towards teaching is needed to fully answer this, and I have only guesses. At Minnesota, as one faculty member put it, "Math Education is the kiss of death as far as salary and promotion go." Some of the senior graduate students view ISCI as the reason I was not promoted and are afraid to have anything to do with it. Similar attitudes are probably prevalent among faculty at all Ph.D.-granting institutions. However, there are other potential reasons for faculty to ignore ISCI, reasons which apply to all institutions. Many feel that it is the fault of students if they do not learn, and the teacher's job is merely to present the material and give exams. Others do an excellent job of teaching and are very proud of their efforts; they cannot conceive of a computer being able to teath their students better than they can. This point deserves further comment. Even though ISCI was designed to interface with almost any way of teaching Calculus, each instructor feels that he has a unique and superior way of presenting material, and there is a certain reluctance to trust any other source -- even textbooks are accepted only grudgingly, and lectures are typically devoted to "explaining the text." These biases against new approaches to teaching are deep-seated and unaltered by facts.

There is one more problem, related more to staff than to faculty, of the "I can do it better myself" type. This problem was quite prevalent during the development of ISCI and will have to be contended with by others developing similar materials. It is the reaction of almost every computer-oriented person to ISCI: "But you are using only a very small part of the resources of our computer. We have sophisticated logic, graphing, detailed record-keeping, etc., available. Also, you should write the program in (insert here the name of a language which is used by only one computer system, e.g., COMPASS for Control Data machines) because that is faster and enables you to use these more sophisticated aspects of our computer." What is not admitted is that such complex approaches entail much more work in the designing stage, vast changes whenever the computer system is altered, the necessity to rewrite the program if it is to be used on a different computer system, perhaps more costly terminals, etc. This attitude is surprisingly universal (in my experience) among faculty who have had much experience with computers, and such faculty usually dismiss ISCI as a good "starting point." Unfortunately, a faculty member who contemplates using ISCI, and who has little or no computer experience, will typically consult with someone who does have computer expertise, in deciding whether to use ISCI. On the other hand, the conclusion we come to below, that ISCI cannot flourish without nourishment from an instructor, and the fact that instructors are not eager to nourish projects which they do not feel they have built themselves, lessens the advantage of a system like ISCI which is so easily adaptible to other computers.

One final comment about assumption (a): in an attempt to "personalize" ISCI, we have designed it so that problem files can be written easily by any instructor. Thus, if an instructor goes far enough to get ISCI running, and he wants to use his own ideas about how to teach students, he can do so by writing his own problem files.

Of course, to some extent assumption (a) was known to be false (although I did not expect it to be so universally false.) So the burden rests on assumption (c) -- would students use it voluntarily? If so, their enthusiasm would be the best evidence to convince instructors to assign ISCI problems. To see why (c) seems to have failed (something I did not anticipate), we must look closer at the student's view of ISCI. The student we are considering knows ISCI through a user's manual he has picked up. He begins work on a topic which ISCI handles, then turns to the homework problems assigned by his instructor. Because of the failure of assumption (a), these problems do not include any which ISCI can help him with. He gets stuck on one or several non-ISCI problems. Now he can either seek help on those non-ISCI problems, or have faith in ISCI and: (1) read the manual; (2) try the (new to the student) ISCI problems on that topic; (3) find a computer terminal so he can use ISCI. Even though steps 1 and 3 are not that difficult (witness the results of our evaluations of ISCI), the addition of 2 has the psychological effect of asking a student who has failed to fail again, and with no assurance (except from the writing in a manual) that he will get help after failing with a second group of problems. Even if they feel they could get help with the second group of problems, they are too inexperienced in the subject to see that mastering the ISCI problems will give them skills needed to answer their own homework problems. All this is too much for many students, and as a result, assumption (c) fails in general.

Conclusion

Our inevitable conclusion is that ISCI cannot flourish unless nourished

This severely restricts ISCI's usefulness, but by no means does it make ISCI a failure. It is certainly a model for instructors to use in building their own systems, and as time passes (the packages were sent out only in Spring 1976), I predict we will continue to see a growth in its use. In 1977 I plan to mail a questionnaire to everyone who has received ISCI materials asking what use they have made of them. When those are received we will be able to draw a more valid conclusion about the success of the ISCI project.

Tutor Resources in Mathematics

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January 1976

Tutor Resources in Mathematics

Leonard Shapiro

Consider the math student who has been taught some material in a course, by one or more of the usual teaching instruments (text, lecture, T.V., computers, etc.). This student then tries to solve the homework problems. Often, he or she cannot. Even after reviewing the text, notes, etc., and listening to what is said in class, help is still needed. Most schools supply some kind of resources for the student in this situation, including complete printed solutions to problems, an instructor's office hours, or student tutors for that course or several math courses. For lack of a better term, let us call these "tutor resources."

This paper describes a new kind of tutor resource called ISCI (Individualized Supplementary Calculus Instruction). Studies we have conducted show that ISCI is at least as helpful as traditional tutor resources, and that for weaker students it is more helpful.

Tutor resources are not the most important component of a course, but to some students they are invaluable. On the other hand, many students will never use them. In fact, they are often too little used by the students who most need help. In this note we will argue that different resources, including ISCI, appeal to different students and serve different functions. Thus, making available to students a wide variety of resources, including ISCI, should be preferable to reliance on a few.

Although this paper is mainly intended to encourage implementation of a resource like ISCI, I hope that it can also shed some light on the question of why the traditional tutor resources do or do not help students. ISCI is a very different kind of resource, and in comparing its qualities to those of the traditional resources we may see why one or the other is more helpful to particular students. This may enable us to improve existing resources and also to guide each student to the particular one that will suit that student best.

The ISCI Program

ISCI is a computer-based resource that has been in use since Fall, 1973, at several campuses in Minnesota, Wisconsin and Illinois, on Control Data, IBM and Univac computer systems. The ISCI program has been designed to be used with any text or computer system, and once installed it needs no maintenance. Development of ISCI has been supported by a grant from the Fund for the Improvement of Postsecondary Education, an HEW agency.

In brief, ISCI works in the following way: it assumes that the student has tried to solve certain homework problems, and failed. If the student chooses to use ISCI, it will ask him (through a teletype or other computer terminal) about his attempted solution. After finding the error in his attempted solution, ISCI points out that there is an error, then gives as many hints and/or explanations as the student requests, until he is able to continue the solution independently.

The dialogue below is a sample of what ISCI looks like to a student. Underlined material was typed by the student. The problem was to find the local maxima and minima of $f(x) = x^4 - 2x^2$.

#####

WHAT PROBLEM ARE YOU HAVING TROUBLE WITH?
? <u>MM4</u>
WHERE WAS YOUR TROUBLE?
A. I COULDN'T FIND F'(X).
B. I FOUND THE DERIVATIVE, BUT I COULDN'T FIND THE CRITICAL POINTS.
C. I FOUND THE CRITICAL POINTS, BUT I COULDN'T DETERMINE WHICH WERE LOCAL MAXIMA OR MINIMA.
D. I COULDN'T DETERMINE WHICH OF THE LOCAL MAXIMA AND MINIMA WERE ABSOLUTE MAXIMA AND MINIMA.

```
   E.  MY ANSWERS DON'T AGREE WITH THE BOOK'S ANSWERS.
   F.  I DIDN'T GET AN ANSWER.
? E
LET'S SEE WHICH ONE OF YOU HAS THE RIGHT ANSWERS.

#####
THE CRITICAL POINTS I FOUND WERE
   A.  0, 1
   B.  -1, 0, 1
   C.  -1. 1
   D.  0, SQRT 2, -SQRT 2
   E.  MY ANSWER DOES NOT APPEAR
? D
THERE'S AN ERROR IN YOUR SOLUTION HERE.  ENTER  YES  FOR A HINT
ABOUT YOUR ERROR,  NO  TO TRY THE QUESTION AGAIN.
? YES
HINT:  THE CRITICAL POINTS ARE THE POINTS X SUCH THAT F(X) = 0, NOT F'(X) = 0.
ENTER  NO  TO TRY AGAIN,  YES  TO GET AN EXPLANATION OF HOW TO ANSWER THE QUESTION.
? YES
LET'S GO BACK A STEP.

#####
WHAT IS F'(X)?
   A.  X^3-2X
   B.  4X^3-2X
   C.  4X^3-4X
   D.  MY ANSWER DOES NOT APPEAR
? A
THERE'S AN ERROR IN YOUR SOLUTION HERE.  ENTER  YES  FOR A HINT,  NO  TO TRY AGAIN.
? YES
REMEMBER THAT THE DERIVATIVE OF X^N IS NX^(N-1).  DID YOU FORGET THE N?
ENTER  NO  TO TRY AGAIN,  YES  TO GET AN EXPLANATION OF HOW TO ANSWER THE QUESTION.
? STOP
```

Although only single-letter responses (A, B, etc.) are used in this simple example, words or expressions can be used for possible multiple-choice responses. Detailed explanations may be given, one step at a time, if appropriate. This student's mistake was simple and typical.

ISCI is by no means a very sophisticated error-diagnosis mechanism. It is, however, a simple and efficient method for answering the typical "dumb" questions which so often are an embarrassment to students and a time-consuming chore for teachers.

The ISCI package provides help only with homework problems for which "problem files" have been written. These are very simple to write. Most of those presently in use have been written by undergraduates. At present over 100 problem files have been written, all of them for problems in College Algebra and Calculus.

A complete technical description of ISCI is available from the author, along with a copy of the relevant computer program on magnetic tape. There will be no charge for the written materials, but there will be a nominal charge for the tape.

Prospective users are invited to use ISCI verbatim, or to alter or expand either the program or the individual problem files. Several persons with no computer programming experience have used ISCI by asking a local computer programmer to set it up, and then writing their own problem files.

Evaluations of ISCI's Effectiveness

A study of ISCI was conducted in 1974, using 400 first-year calculus students from six lecture sections, each under the supervision of a professor. Each of the lecture sections was divided into four or five recitation sections led by teaching assistants; 20 assistants in all were involved. All students had access to ISCI and to four other tutor resources: Professor's and T.A.'s during prescribed office hours, and graduate and undergraduate tutors who were available throughout the week. No changes were made in the established curriculum or teaching methods.

Students were asked to rate the helpfulness of the 11 resources available to them, including the five tutor resources. Since lecture, text and sections were usually ranked 1, 2 and 3, we considered a ranking of 4 or better for a tutor resource to indicate that it was "very helpful." Table 1 shows the results. It is clear that each tutor resource is reported as beneficial by some students; in fact, each is ranked very helpful by at least 1/3 of the students who tried it. For students with lowest-quartile grades, resources such as ISCI and undergraduate tutors are much more heavily favored, showing that these resources are most helpful to these weakest students. In fact, 52% of lowest-quartile students who used ISCI ranked it 3rd or better, while the analogous percentages for other resources were 33% (for undergrad tutors) and below.

Grade changes from the previous quarter to the current one were computed for the 350 students who took the same final exam each quarter. For each resource, grade changes for users of the resource were compared to those for nonusers. Figure 2 displays the differences in grade change for all students and for those with low math aptitude. No significant differences in grade changes of users vs. nonusers were obtained, for ISCI or any other tutor resource, even at a liberal confidence level of $\alpha = .10$.

Students were asked how their attitude toward math had changed during the quarter: more favorable, no change, or less favorable. (See Figure 3). Again, for each resource, responses of users of that resource were compared with responses of nonusers. In this case, users of ISCI responded "more favorable" significantly more often ($\alpha = .025$) than did nonusers. This significant difference still holds when we consider only students with lowest-quartile grades in their previous quarter of math.

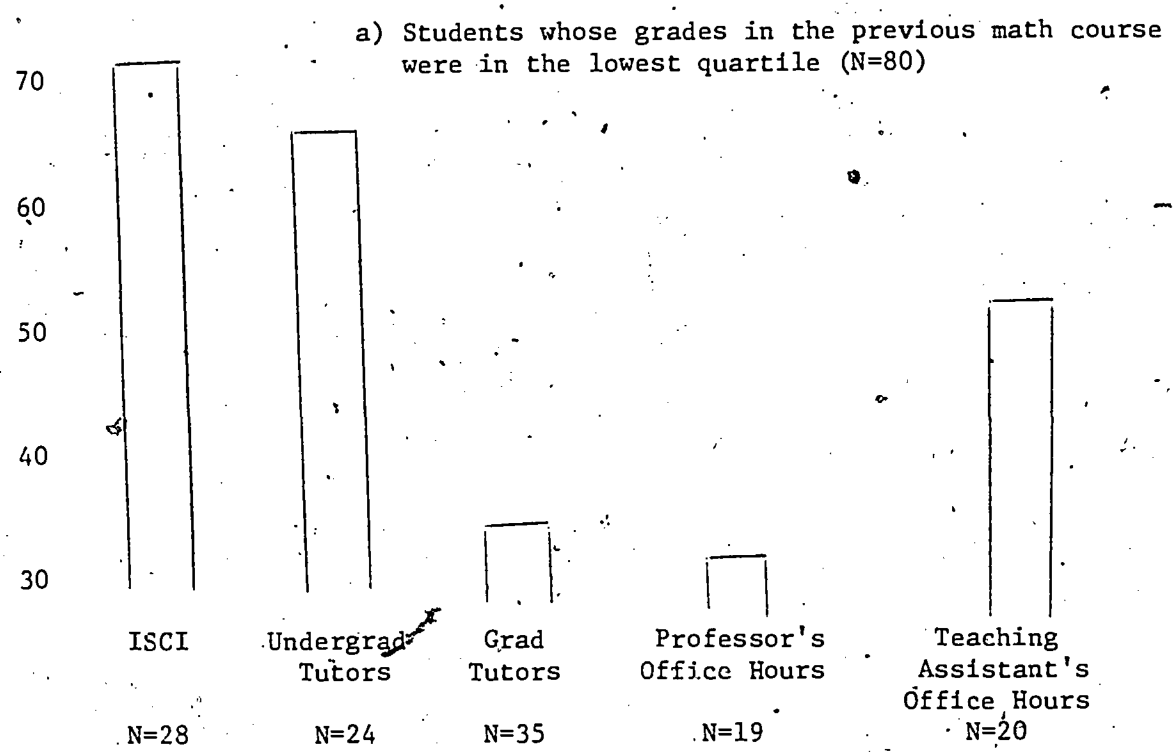
A more complete analysis of the evaluations we have conducted is available from the author.

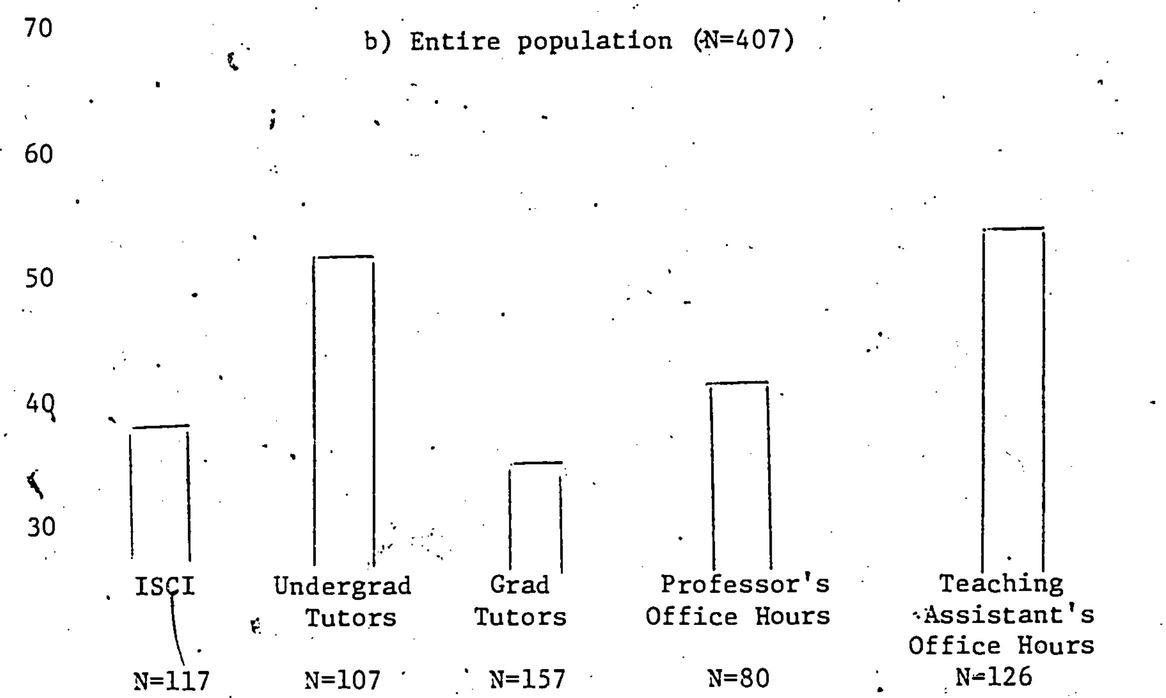
Qualities of ISCI and Other Tutor Resources

As we have mentioned above, a comparison of the qualities of ISCI and other tutor resources raises interesting and useful questions about which is more helpful to a particular student. There are at least three ways in which

Figure 1

Percentage of users of each resource who rated it as very helpful

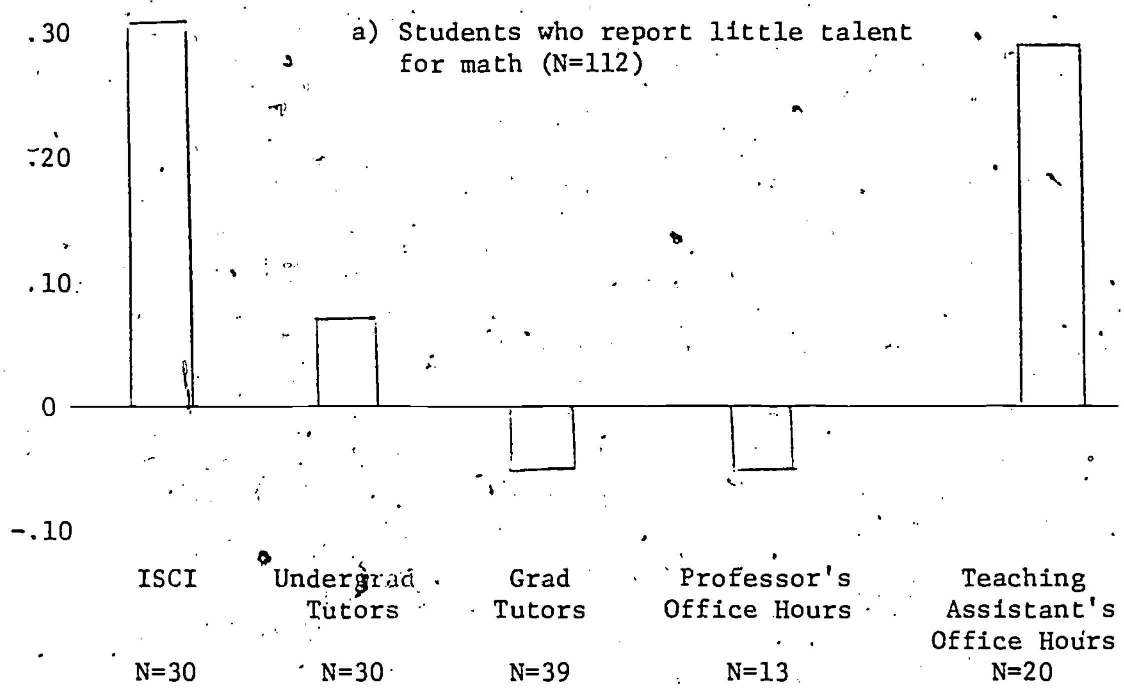


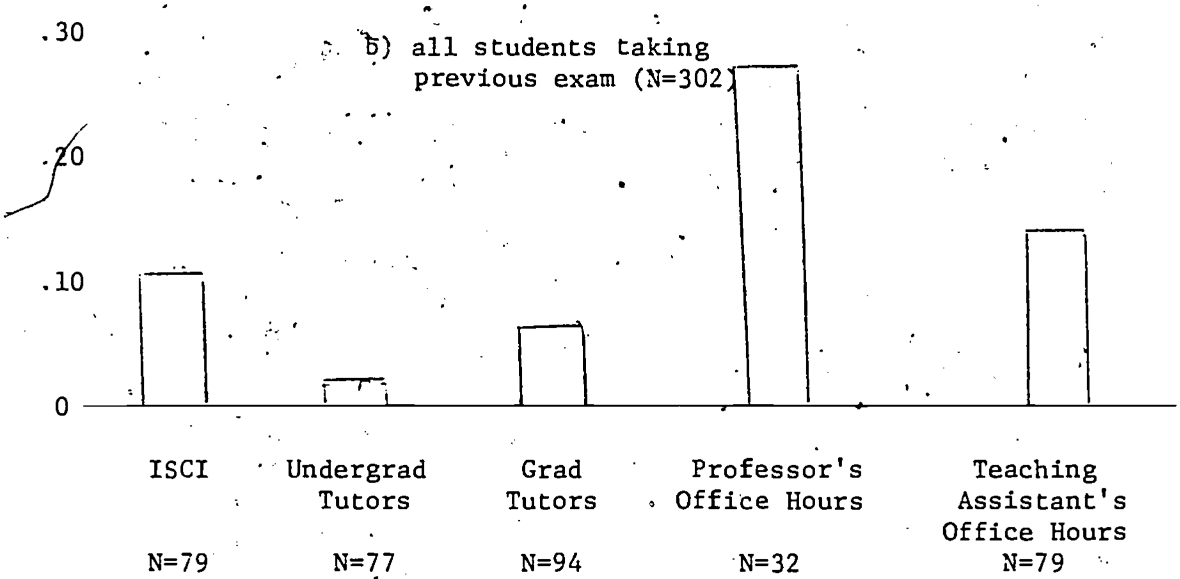


Note: N refers to the number in each group; the groups overlap in students using more than one resource.

Figure 2

Grade Changes for Resource Users Compared to Nonusers



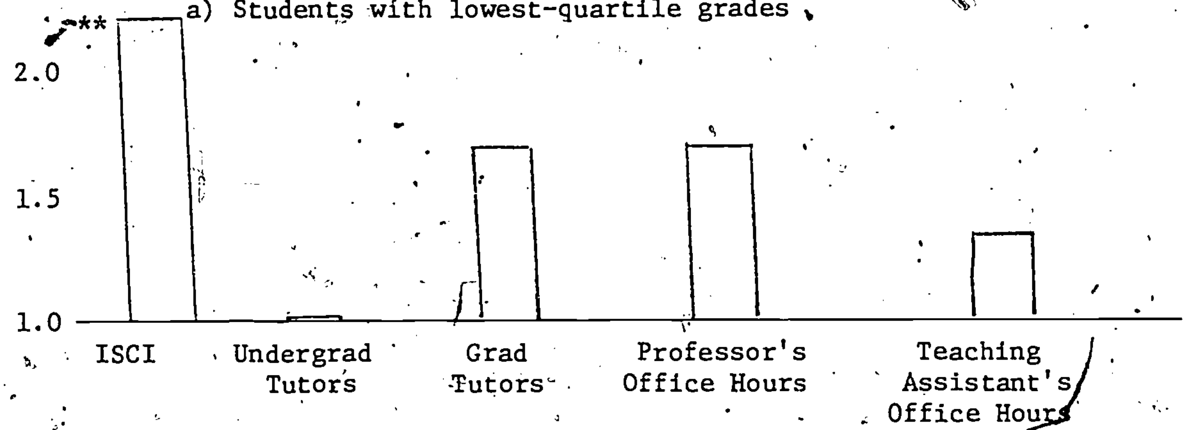


Note: Grade improvement is measured by the difference in (normalized) final exam scores from one quarter to the next. Bars pointing downward occur in situations where nonusers of a resource improved more than did users.

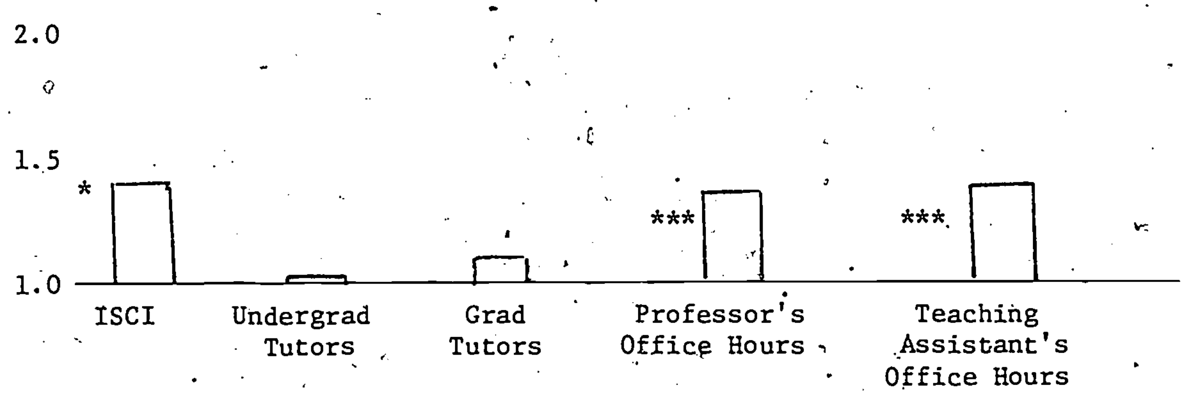
Figure 3

Ratio of percentages of users and of nonusers of each resource, who claimed their attitude toward math had become "more favorable" during the course.

a) Students with lowest-quartile grades



b) Entire population



Note: The leftmost bar in the upper graph, with height 2.2, means that if we consider only students with lowest quartile incoming grades, the proportion of students using ISCI who responded "more favorable" attitude was 2.2 times the proportion of nonusers of ISCI who responded "less favorable."

Using original data from which these tables were calculated, the hypothesis: "users and nonusers of a resource were equally likely to claim a more favorable attitude" could be rejected in the four cases marked by asterisks above. A single * refers to an α of .01, ** means α = .025, *** corresponds to α = .05. For N's see Figure 1.

ISCI differs significantly from other tutor resources of printed solutions to problems, instructor's office hours, and student tutors.

1. Personal Remediation: A computer-assisted program (like a non-hurried and sensitive human tutor) can encourage the student to find his own mistake by following whatever path the student's solution takes and giving judicious hints. Alternatively, the student can just be told what he did wrong, or be shown a correct solution. The latter alternatives are certainly more efficient, but do they teach any more than the ability to imitate? Can allowing the student to find his own error, and the extra time this requires, effectively teach an important aspect of problem-solving?

2. Accessibility of Resources: The accessibility of a resource depends on the individual student as well as on the available facilities. For example, a student may feel so threatened by a human tutor or an instructor (especially if that instructor will be giving the student a grade) that those tutor resources are not acceptable alternatives to that student. Computers do not have this specific disadvantage, but fear of computers in general may keep students away from computer terminals. Facilities that are overcrowded or open very few hours per day can be discouraging, although overcrowding can be used to advantage for some students if they can be encouraged to study together.

But is ease of access an unqualified good? This too depends on the individual student. It has happened to me several times that a student, at the end of a lecture, tells me he has had difficulty solving the problems in a particular topic. I point out where he can get help, including the ISCI program. When I see him again, he will say: "The program (or tutor) was lousy. I tried it but it was so slow (or crowded) that it was faster for me to do it myself." So for this student the necessity to seek out, or wait for, a resource was beneficial: it encouraged him to try harder to do the problem himself. This is not an argument for eliminating all tutor resources -- if no aid had been available to the student under any circumstances he may have been too discouraged to continue. Furthermore the student I am describing is by no means typical, and others cannot be so easily persuaded to do without help.

3. Range of Material: Human tutors, ideally, can provide help with any difficulties a student has in a particular course. But this is not the case with printed solutions or ISCI. The latter resources usually cannot easily handle aspects of mathematics such as proofs of theorems, or graphing. An instructor may feel that all students should be helped with all problems. But with limited resources, what kind of help should be given? In the final analysis it is up to the professor to set such priorities, and to choose those resources which best fit the needs of the student in her particular class, within the constraints of limited resources.

Conclusion

Different tutor resources have different properties and appeal to different students. It is of course an immense (and probably impossible) task to gather enough data to be able to predict student preferences, although this may be of interest to educators. The best we as teachers can do is to make available to students a sufficient variety of resources and not require of every resource that it be popular with a large majority of students.

THE ISCI PACKAGE: A COMPUTER ANSWERS DUMB QUESTIONS

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February, 1975

The ISCI Package: A Computer Answers Dumb Questions

Leonard Shapiro

Contents: Page:

Contents (continued): Page:

Tables

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February 1, 1975

The initials ISCI stand for Individualized Supplementary Calculus Instruction. The package was originally designed as a supplementary resource in Calculus courses, although it could be used to help students in any problem-solving course. It assumes that the student has tried (and failed) to solve certain homework problems. Upon accessing ISCI through a computer terminal, the student will be asked via teletype about the attempted solution. After finding the error in the attempted solution ISCI points out that there is an error, then gives as many hints and/or explanations as the student requests, until the student is able to continue the solution independently.

Appendix A contains a sample of what ISCI looks like to a student.

We feel that the concept underlying ISCI, of a simple but comprehensive package to be used voluntarily by students for diagnosing errors in homework problems, is new, at least in the field of mathematics, and we hope it will find wide application.

The ISCI package has been in use since Fall 1973 in Calculus courses at the University of Minnesota, and more recently at institutions of higher learning throughout Minnesota, and at the University of Illinois at Chicago Circle. In this report we would like to describe our experience with ISCI, including the results of evaluations we have conducted. We will also describe the package itself, so that others may copy it in whole or in part, or may use it as a model for building similar packages. We have paid particular attention to making the package as simple as possible, and as such we feel it can run cheaply on almost any modern interactive computer system. It requires no knowledge of computers on the part of instructors or students, and once set up needs no maintenance. Moreover, we have tried to maximize its compatibility with other systems so that few changes will need to be made in order to adapt it to a new system (versions of it currently operate on CDC and IBM products).

It should be pointed out that the ISCI package provides help only with certain homework problems: those for which "problem files" have been written. The structure of these files is given in section IV below; a sample file is in Appendix B. As can be seen, they are very simple to write. At present 100 files have been written, all of them for problems in Calculus, and several persons are now engaged in writing new files.

We would like to thank the Fund for the Improvement of Postsecondary Education, and especially David Justice, for their support and interest, without which the package would not have been developed. Many faculty, students and teaching assistants have also contributed to the project, especially my colleague Prof. H. Keynes, Profs. D. Strawn, J. Johnson, R. Burris, J. Lewis and N. Rickert, Mr. J. Gervais, and Ms's. J. O'Halloran, P. Baumann, J. Smith and L. Molde, and the staff of MERITSS.

I. History

The ISCI package has been developed with the support of the Fund for the Improvement of Postsecondary Education, a Federal agency within HEW.

The initial version of the ISCI package was written during the Summer of 1973. It was first used, on an experimental basis, by Calculus classes in Fall 1973.

Although the package still needed revision, it was made available during winter 1974 to 800 students in Calculus along with the other usual resources (office hours, tutors, written materials). This class followed a standard lecture-recitation format, taught by 6 Professors and 20 TAs. No changes were made from the usual course content or instructional methods. At the end of the quarter, students were asked to complete a questionnaire, and their grades in this and their previous math class were recorded for analysis. During spring 1974 a controlled experiment was conducted, wherein only a randomly selected half of the 150 students in a Calculus class was allowed to use the ISCI package.

Through the University of Minnesota's statewide time-sharing network MERITSS, using a CDC 6400 computer, the ISCI package was made available in March 1974 to colleges throughout Minnesota. Since the package does not require the user to identify him/herself, it has been difficult to ascertain who is using the program, but it has been widely used and we have received communications from several individual schools making use of the package. We will now inform these faculty members about the structure of ISCI in order to encourage them to contribute to

II. Does ISCI Help Students?

We will try to answer this question in three ways:

1. Do students feel that it helps them?
2. Does it lead to an improved attitude toward Mathematics?
3. Does it improve grades, as measured by the difference between common final exam scores in two consecutive quarters of Calculus?

Before discussing the evidence on these questions, we would like to emphasize one point. The ISCI package was developed with simplicity of operation and wide applicability as primary goals. These properties it shares in common with the other tutorial resources (office hours and student tutors) in use at the time our studies were made. This is to be contrasted with more sophisticated tutorial systems, e.g. student tutors trained for, and devoted to, helping with a particular course, or computer-based systems dedicated to particular texts and implementable only on certain systems. As one might expect, increased applicability and simplicity exact a price in benefits to students. But it is our contention that each of the resources we study, including ISCI, is of significant benefit to a certain group of students. We also feel that a selection of such simple resources may be preferable to dependence on a single more sophisticated one.

1. Do students feel that ISCI helps them? In the Winter 1974 survey students were asked to rank the 11 resources available to them, in order of helpfulness. Results are shown in Table 1. These results show that the standard resources (classroom and text) clearly are the most helpful to students, with mean rankings of 2 or 3. Other resources, including the ISCI program, cluster about a rank of 5. As mentioned, however, it is more useful to determine whether each resource

Table 1

Cumulative percentage of students' ranking of 11 resources, where 1 = most helpful resource, 11 = least helpful resource. Total population = 407.

Resource* Rank	1	2	3	4	5	6	7	Mean	Mean** of user > once
Professor's lectures/389	49	78	88	93	97	98	99	2.03	
TA sections/373	16	53	74	85	90	92	93	3.12	
Text/375	23	44	68	82	89	92	94	3.21	
Written review material/272	1	2	5	15	34	52	65	6.88	
ISCI Manual/263	4	13	25	43	56	63	71	5.81	
Extra Problems list/291	7	16	32	50	67	77	80	5.07	
Math Dept. tutors (grad. students)/157	8	14	21	34	49	65	79	5.47	5.35/94
Institute of Technology tutors (undergrads.)/107	19	23	26	52	68	78	89	4.57	4.59/77
Professor's office hours/80	10	20	25	42	55	70	85	5.07	4.79/32
TA's office hours/126	10	23	42	57	69	83	89	4.42	4.07/79
ISCI package/117	7	12	22	37	55	73	82	5.29	4.92/79

* The number following the resource is the number of usable responses. The last 5 resources, below the double line, required students to go to a specific place at certain times to get help. Thus only those who indicated they had used each resource were counted.

** Mean for those who used the (optional) resources more than once/number of such users.

is viewed as helpful by some students. Since lectures and text most often occupied ranks 1, 2 and 3, it is reasonable to consider "helpful" as corresponding to a ranking of 4 or above. For all tutorial resources, at least 1/3 of those who used the resource ranked it as fourth or higher (i.e. ranked it 1, 2, 3 or 4) - the actual amounts ranging from 34% to 56%.

We should also note that some resources may be more useful to certain groups of students, e.g. those with low math ability. Table 2 shows the ranking of resources by students who took the previous quarter of Calculus and had final grades in the lowest quartile. Again, at least 1/3 of users of each tutorial resource ranked it fourth or higher - the actual amounts ranging from 36% to 71%. Note also that the relative standing of some resources, e.g. ISCI, has increased significantly. More detail on the question of who used which resource can be found in section III.

In summary, it is clear that the unique features of the ISCI package are viewed by some students as helpful, just as each traditional tutorial type of resource has appeal to certain students. However, only the standard lecture-text resources seem to have wide appeal.

In the spring 1974 controlled experiment, students were asked to rank the ISCI package as poor, fair, average, good, excellent. Among ISCI users, 34% responded excellent, and 50% responded good. Other tutorial resources were not included in that study.

2. <u>Does use of ISCI lead to an improved attitude toward Mathematics?</u> In the winter 1974 survey, students were asked whether their attitude toward Mathematics had become more favorable, more negative, or had not changed during the course. For each tutorial resource we computed the percentage of users who responded "more favorable" and the percentage of nonusers who did so.

Table 2

Following the format of Table 1, but restricted to students with lowest-quartile grades from previous quarter Math course. Total population = 80.

Resource: Rank:	1	2	3	4	5	6	7	Mean	Mean of users > once/N
Professor's lectures/77	47	82	87	95	99	100	100	1.91	
TA sections	19	56	79	81	90	91	93	3.04	
Text/72	26	39	71	85	88	89	92	3.33	
Written review material/49	0	0	8	20	43	55	74	6.53	
ISCI Manual/38	3	13	21	32	55	63	74	5.71	
Extra Problems list/45	0	13	24	47	67	80	82	5.22	
Math Dept. tutors (grad. students/35	0	4	20	36	64	84	84	5.08	4.89/23
Institute of Technology tutors (undergrads.)/24	33	33	33	67	76	86	91	3.81	3.31/18
Professor's office hours/19	0	17	17	33	50	75	83	5.25	5.43/8
TAs office hours/20	13	25	25	50	56	75	81	5.06	4.67/14
ISCI Package/28	19	29	52	71	76	86	86	3.95	3.86/17

The ratio of these two percentages can be found in Table 3. In the spring 1974 survey, 68% of ISCI users said their attitude toward math had become more favorable during the course, and 43% of nonusers made the same claim. One must be cautious, however, since a priori one would expect that students who became more enthusiastic during the course would be motivated to use ISCI and other resources.

If the original data from which Table 3 was derived are analyzed, the hypothesis that users and nonusers of a resource are equally likely to have an improved attitude toward Mathematics can be rejected in 4 cases, as noted in Table 3.

3. Does use of ISCI improve grades? There is no statistically significant evidence on this question, in any of the surveys for ISCI, or for that matter for any other tutorial resource. In the winter 1974 evaluation, final exam scores of students who had taken the previous quarter of Calculus were compared with their final exam scores in winter quarter, after normalization into z-scores. Changes in these normalized grades were computed for users and nonusers of each resource. The entries in Table 4 are the differences between average z-score change for users (more than once) and average z-score change for nonusers, of each resource. None of this data was statistically significant, even at the .10 level. Similar, non-significant, data can be found in the spring 1974 evaluation.

Many reasons can be suggested for the non-significance of these results. In math (more so than in most subjects) a student's performance depends heavily on what has been learned in previous math courses. Thus a longer study, or one which measured grade changes more carefully, or use of a case-study method, might show which if any resources are effective in improving grades. These tutorial resources, requiring special effort on the part of students to get to them, may not have been used enough to affect the entire exam grade of a significant number of users. The ISCI program, for example, provided help only with the simpler problems in certain topics. Also, the resources are meant to be used by students

Table 3

	Entire Population	Lowest Quartile Incoming Grades	Low Talent*
Graduate tutors	1.1/157	1.6/35	0.7/56
Undergraduate tutors	1.0/107	1.0/24	0.8/40
Professor's office hours	1.3/80 (1)	1.6/19	1.0/27
TAs office hours	1.3/126 (1)	1.3/20	1.4/34
ISCI	1.4/117 (3)	2.2/28 (2)	1.3/38

* Students were asked, "how much talent do you feel you have for learning math?" Low Talent denotes the 112 students who answered, "some but not much" or less.

(1) Significant at .05 level
(2) Significant at .025 level
(3) Significant at .01 level

Ratio of percentages of improved attitude change for users and nonusers/number of users

The entry 1.6/19 in the 2nd column, third row, means that if we consider only students with lowest quartile incoming grades, the proportion of the 19 students who used Professor's office hours and answered that their attitude toward math had become more favorable during the quarter was 60% higher than the proportion of nonusers of Professor's office hours who answered "more favorable."

Table 4

	Entire Population	Lowest Quartile Incoming Grades	Low Talent
ISCI	.10/79	.13/17	.30/30
Graduate tutors	.07/94	-.01/23	-.07/39
Undergraduate tutors	.02/77	-.07/18	.08/30
Professor's office hours	.29/32	-.32/8	-.07/13
TAs office hours	.15/79	.41/14	.29/20

Difference in z-score changes between users more than once and nonusers/number of users more than once

The "z-score change" measures the normalized improvement in grade between consecutive quarters of Calculus. The entry .07/94 in column 1 means that the average z-score change of the 94 students who used graduate tutors more than once was .07 higher than for nonusers of graduate tutors.

III. Advantages and Disadvantages of ISCI

As one can see from the previous discussion, the ISCI package is seen as valuable by some, but by no means all, students. Here we will try to point out some of the unique features, both positive and negative, of the ISCI package.

Non-Evaluative Assistance

A typical difficulty for students in mathematics is their reluctance to ask a question for fear that it will turn out to be a "dumb" question. ISCI responds to even the most trivial student error with the same simple message. It makes no value judgments, and does not enter into the grading process in any way. As presently used, the student's name is not even requested, although this feature is available. If we consider only the 290 students in the winter 1974 survey who felt they had a high talent for Mathematics, 28% of those who used ISCI ranked ISCI fourth or higher among the 11 resources. For the 112 "low talent" students, 56% of ISCI users ranked it fourth or higher. Similar differences exist when we compare upper to lower quartile students (cf. Table 2). The latter groups of students, whom one might expect to be more sensitive about asking possibly "dumb" questions, clearly rate ISCI highly.

Efficient Assistance

Since ISCI can be set up to offer help only with the more basic homework problems in a course, it is not very useful to the better student, who can do such problems without special aid. Thus, whatever extra effort is spent in setting up the package will benefit those students who need the most help.

The ISCI package is designed to be used only in areas and problems where the student has difficulty. Moreover, the student is expected to do most of the work alone, before accessing the package. Interaction with the program is limited to short answers about the work the student has already done.

When a student has made an error in a solution, ISCI follows the solution and comments only on the error made. It does not do the problem for the student (it is designed to be very slow and inefficient if a student tries to use it for this purpose), and it gives only as much advice as is requested.

Extended Availability of Assistance

A student can get help from ISCI at any time an operating computer is available. Depending on the installation, this may result in an increase in the number of hours help is available, especially during evening and weekend hours.

Cost-Effectiveness

This can be an advantage or disadvantage, depending on the local computer system and what it charges for student use. Problem files, which need to be stored, are typically 7000 alphanumeric characters long. The program (on a CDC 6400) use 3000 ten-character words. In our evaluations, when an average of eight problems were available on each of 6 topics, the number of problems accessed per student averaged between 6 and 8. Students' reported time spent per problem averaged 20 to 30 minutes. This corresponded to approximately one-half second of central processor time (CDC 6400) per problem. Of course the half-second figure could be greatly improved, if necessary, by writing an assembly language version of ISCI.

Hardware Problems

While the ISCI program itself needs no maintenance, it depends on the availability of a computer system and teletype terminal. This can be a problem. At Minnesota, for example, the computer's operating system was updated at the beginning of Winter 1974 (when our main evaluation was conducted), and for the first half of the quarter the computer was not working ("crashed") for extended periods of time. Now that the system is stable, the University is planning to change to a new computer (Univac 1110) next year. Without these problems, ISCI would have looked much better in the Winter 1974 survey, but to some extent these problems are endemic to computer-based instructional programs.

The ISCI package does not need Cathode-Ray-Tube terminals, or upper and lower case letters -- it can run on any teletype terminal. Such cheap terminals are typically available for student use at schools, but they type at 10 characters per second. This is rather slow, and students remark that the more expensive 30 character per second machines are much easier to "talk with." However, this may serve to keep terminal use concentrated on those (often weaker) students who are willing to tolerate this inconvenience.

Limited Structure

In its interaction with the student, the ISCI program follows a very simple pattern, as one can see from appendix A, or, in more detail, in section IV. Questions are essentially multiple choice (although an unrecognized response is stored for possible future addition to the package). The ISCI-student interaction proceeds through a series of distinct questions. Within each question, hints and/or explanations can be given, or transfers can be made to another question, depending on a student's response and requests for information. However, the questions are independent: what the program does within a given question cannot depend on answers to a previous question. Furthermore, ISCI has no graphing ability, since it operates on simple teletype terminals.

In the first version of ISCI we included many of these capabilities: pattern recognition, branching conditioned on previous responses, etc. But as we gained more experience with student use of the package we found that students benefited most from the simplest features of the program; their mistakes were amazingly simple and consistent. As we reviewed logs of student usage, in almost every question at most 2 or 3 of the many possible responses we had available were used more than once. Furthermore, these deficiencies of ISCI are counter-balanced by ease and low cost of installation, maintenance and (for the student) operation.

In summary, ISCI is not a sophisticated error-diagnosis mechanism; it is a simple and efficient package for solving the typical, "dumb" questions which so often are an embarrassment to students and a time-consuming chore for teachers.

As mentioned, ISCI can only handle specific homework problems. One may decide to write files for problems from the particular text one is using, but with changes in texts not uncommon, this is dangerous. Of course if use of ISCI and similar packages becomes more widespread, one may be able to copy problem files from other schools using a given text.

We decided to write "typical" problems from certain topics in Calculus. We included in the user's manual a review of the definitions and techniques used in each topic (see appendix C for sample pages from the user's manual), so ISCI could be used with any text. This means, however, that the student must go through a lengthy procedure before getting help with a topic: get a manual, read the section on that topic, try a problem, then, if (s)he cannot solve the problem, find an empty terminal, etc. Another difficulty encountered here is the differing order of presentation in different texts. For example, should problems on the chain rule include trig functions?

Who Uses ISCI; Why and Why Not?

We have already discussed some of the influences on students to use or not use ISCI: it handles only simple problems, the problems it handles may not be in the text, etc. Some of these reasons influence the weaker students to use the program more than other students (see Table 5 for data to this effect); other reasons influence the entire class.

Students in the winter 1974 survey were asked: "If you did not use ISCI, or used it only once, check off any of these reasons which apply to you." Results are given in Table 6. Obviously the most common reason given was, "I was too busy with other things to try it."

Given the data from users, it is fair to assume that if some of these people had taken the time to try ISCI they would have found it worth their time. What could influence these people?

Two obvious answers are: more familiarity with the system, and more encouragement from instructors. The data we have about the effect of these actions is not all clear.

In the spring 1974 controlled experiment, every experimental section had an in-class demonstration of the system, where students ran the program themselves. In those sections 51% of the students returned on their own to use the program. In winter 1974, when (most) students were offered only a manual with written instructions on where and how to use the program, only 31% used it. During winter 1974 the percentage of users, in sections where manuals were handed out, varied from 14% to 52%, with the lower usage associated with instructors who knew nothing about the program. In spring 1974, the three experimental sections each handled encouragement differently: in experimental group one (denoted E_1), students were told of ISCI only at the initial demonstration, it E_2 it was mentioned once as each of the 6 topics it covered was introduced, in E_3 it was mentioned whenever

Table 5

Percentage of various groups using each tutorial resource.

	Low Talent	High Talent	Lowest Quartile	Low Middle Quartile	High Middle Quartile	Highest Quartile
Graduate tutors	50.0	34.1	43.8	38.5	39.0	29.9
Undergraduate tutors	35.7	22.4	30.0	37.2	20.8	22.1
Professor's office hours	24.1	17.9	23.7	23.1	20.8	11.7
T.A.'s office hours	30.4	31.4	25.0	35.9	33.8	27.3
ISCI package	33.9	26.9	35.0	33.3	22.1	28.6

N.B. Quartile refers to final exam scores in the previous quarter of Calculus; Low Talent is explained in Table 3.

Table 6

Percentages of students giving various reasons for not using ISCI.

	Low Talent	High Talent	Lowest Quartile	Low Middle Quartile	High Middle Quartile	Highest Quartile
Too confusing	.8	.6	1.2	0	2.6	0
It didn't work	1.7	3.1	3.8	1.2	3.9	2.6
I was too busy	41.1	50.6	50.0	50.0	46.8	44.1
No help needed	11.6	32.1	11.3	21.8	32.5	40.0
Poor reputation	5.4	4.4	3.7	3.8	6.5	3.9
No empty terminals	5.4	2.4	1.2	5.1	6.5	1.3
Dislike computers	12.5	10.6	6.3	10.2	16.9	13.0
Thought it too complex	6.3	5.5	7.5	6.4	2.6	2.6
Number in each group	290	112	80	78	77	77

possible. The usage in these three sections was: E_1, 59%; E_2, 46%: E_3, 48%, respectively (approximately 26 students in each of the 3 sections). Those users who used ISCI more than once in each section were: E_1, 56%; E_2, 64%; E_3, 42%.

During fall 1974 many Calculus students were informed (usually by an announcement in Lecture) about the location of tutor help, ISCI manuals, etc., but none of the professors and very few TAs knew much about ISCI, and usage of ISCI was very low.

This evidence clearly does not support the hypotheses that more familiarity or more encouragement yields more usage. But it does suggest that the use of ISCI is self-motivating once a student has had either first-hand experience with the package, or assurance from some instructor that it will be worthwhile.

Of course there remains the problem of encouraging only those (weaker) students who will really benefit from ISCI, and this applies equally to other resources. One possible solution, originated at Minnesota's Metropolitan Community College, is to give manuals and assign ISCI problems to students who score below a certain level on quizzes. Another is to provide lists of problems available on ISCI to all students, as homework, and allow students to pick up a manual if they wish.

Diagnostic Ability

When the ISCI program has identified a student's error and the student requests an explanation, the explanation typically begins by naming the technique which is to be explained, and giving a textual reference where this technique is explained in detail. Thus the student (or counselor interested in why the student is having trouble), upon looking at the printout of his/her session with ISCI, can often see common references to areas of difficulty for the student. As the ISCI problem set expands, suggestions can be made to the student to try specific other problems available on ISCI in order to master the troublesome technique. The instructor has a similar opportunity, to look through the log of student usage and determine which problems, or which techniques, are causing students the most trouble.

IV. Description of the ISCI Software

The software of the ISCI package has 3 components: a main program, problem files (one for each homework problem the package can handle), and a log. The log records, in abbreviated form, all student-ISCI interactions.

A detailed description of this software is being prepared. Here we will discuss only its most important component, namely the problem files.

The problem file (see appendix B for a sample) begins with a statement of the problem, followed by a sequence of questions. Each question includes directions about how to handle possible responses. The abbreviated flowchart below illustrates the pattern which the program follows. It may be worthwhile to follow this flowchart using appendices A and B.

The problem file is structured as follows: The first line contains, in columns 1-4, the problem number. This is followed by text lines containing the statement of the problem. The body of the file is a series of questions, each beginning with a QQ prefix, then AN's alternating with control prefixes (CR, IR, AG, DE, EA, or IY), then EX's and perhaps an EN. The prefix EP signifies End of Problem and can come after any question, or even right after the statement of the problem (in case the problem is out of order, in which case the statement of the problem would be a message to that effect).

As mentioned, each question (QQ) is followed by a series of AN prefixes, whose text represents possible answers. Following each AN prefix there is a control prefix. After this series of AN-Control Prefix pairs comes one or more sections of explanation, each section marked by an EX or EN.

In summary, the prefixes other than control prefixes are used as follows: (Text = columns 5-64 following the prefix.)

QQnn Text is question number nn

AN Text is a possible answer

EX	Text is one section of the explanation of how to answer the question at hand
EN	Used in place of EX, when no explanation is given (see below for details)

The action taken in response to most control prefixes should be clear from the flowchart, so we will mention only their typical, though by no means exclusive, uses.

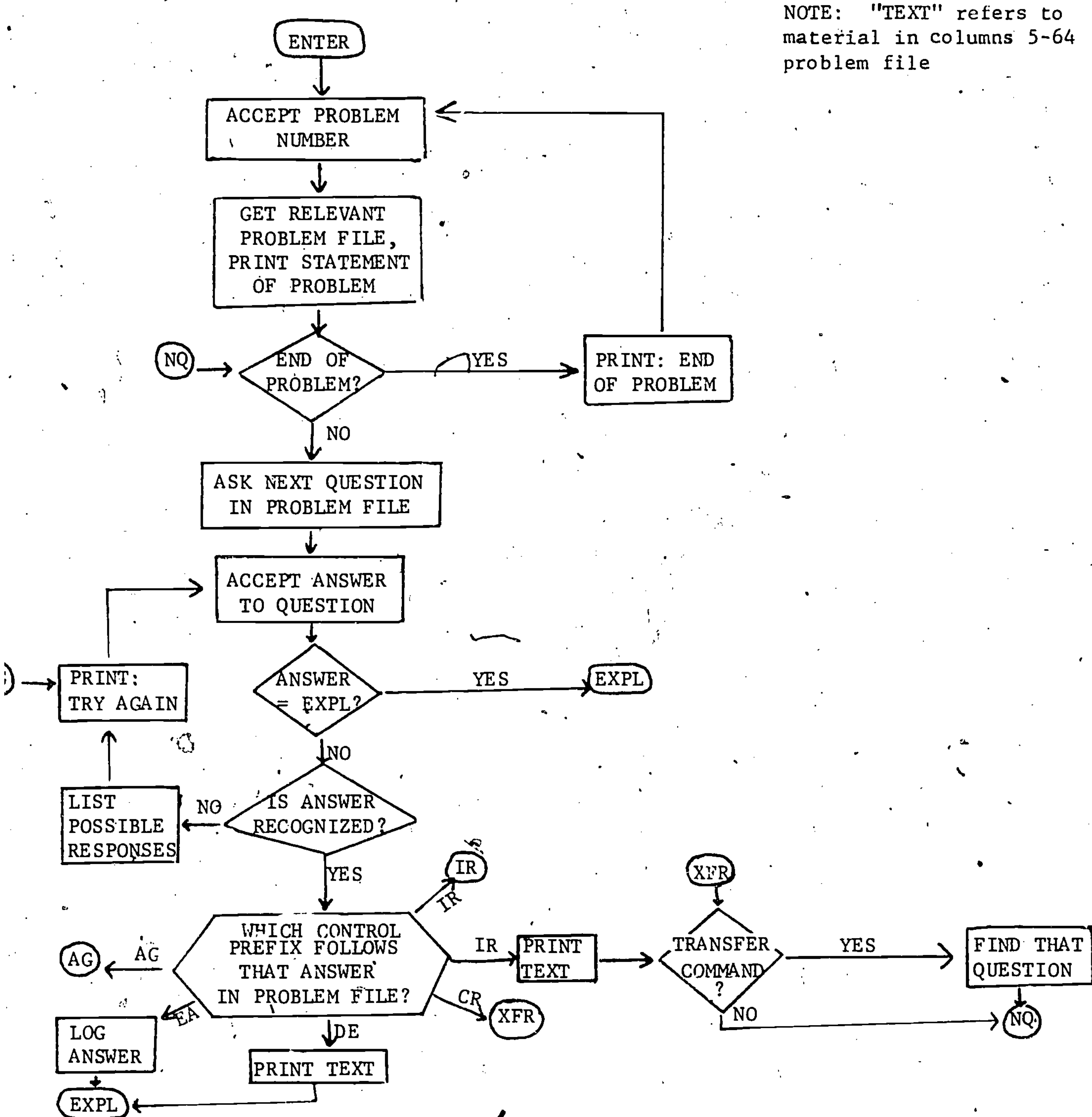
CR	(Correct Response) Text is usually encouragement. This is also used for transfers.
IR	(Incorrect Response) Text is a hint (not an explanation) about why answer was incorrect.
AG	(Try AGain) Text is hint or explanation of why this answer is incorrect.
	(The choice of whether to use IR or AG is a matter of style.)
DE	(Direct to Explanation) When the question is of the form "do you understand...?" and the response is NO.
EA	(Extra Answer) When the student has answered, "My answer does not appear," or some such. Note: after all problem files have been thoroughly tested, one may wish to drop this prefix and depend upon students to use COMM for comments.
IY	(Incorrect Yes-no) Not on flowchart. Similar to IR, but used when there are only two possible responses, as yes-no. Sets a flag so that when student indicates an understanding of the current question, the program goes on to the next question rather than trying the current question again.

If the student responds EXPL to a question, or after processing of an IR, DE or IY prefix, the program begins typing an explanation of how to answer the problem correctly. This explanation is given one part at a time, each part marked by an EX prefix. After the explanation is finished, the program usually asks whether the student is satisfied with this explanation, or wishes to make a comment. This latter feature can be suppressed, however, if EN is substituted for the last EX; then the program proceeds directly to the next question, without asking if the student is satisfied.

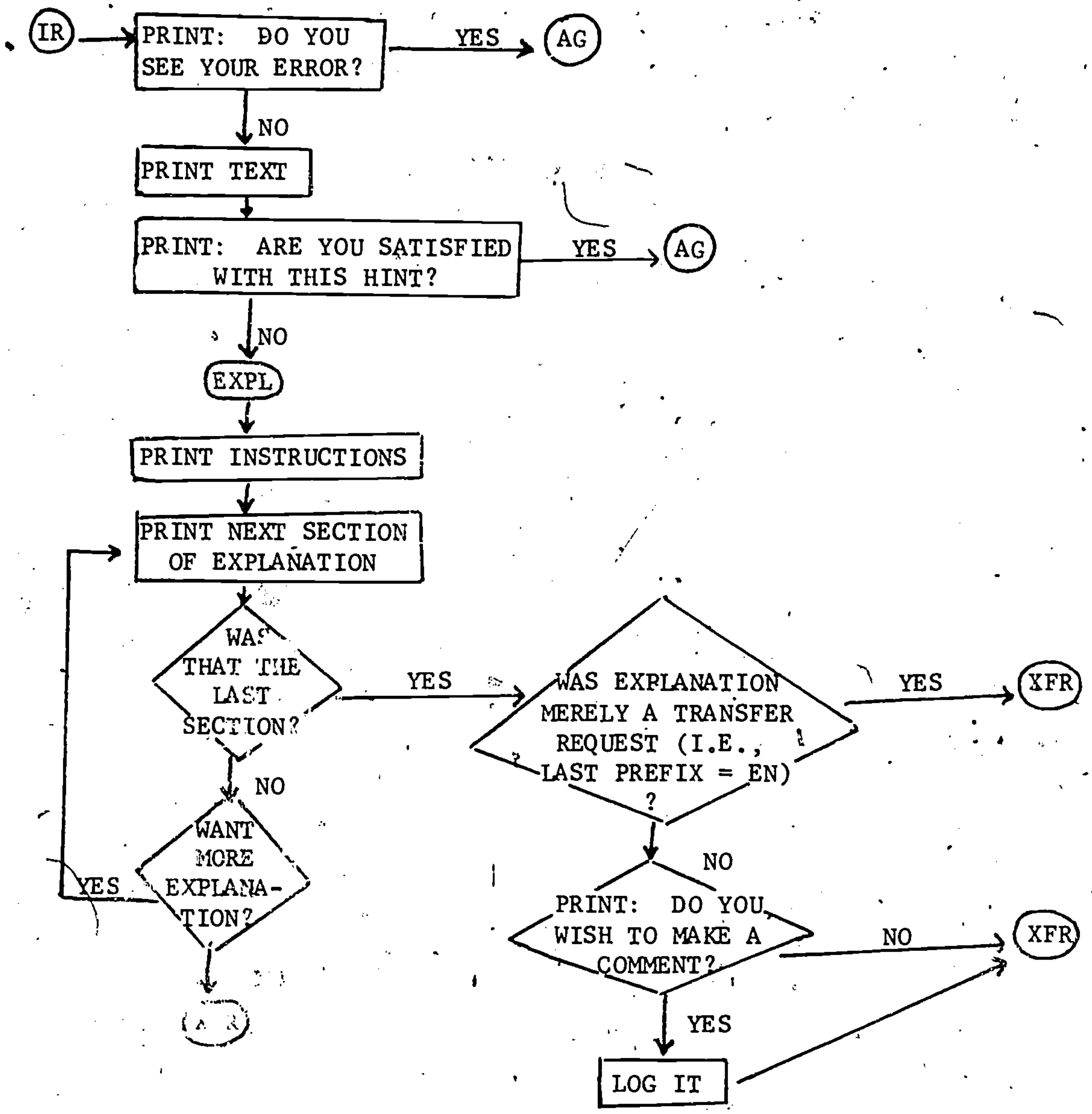
Transfers, indicated by a question number (or AG) in columns 3 and 4, can follow CR, IY, EX or EN. If the program has just processed one of these prefixes and is ready to move on to the next question (i.e., transfer to location XFR in the flowchart), it will take one of the following actions: 1) go to the next consecutive QQ (or EP) if columns 3 and 4 are blank, or 2) ask the student to try the current question if columns 3-4 are AG, or 3) move to QQmm if columns 3-4 are mm. For example, if a prefix is IY36 and the student, after the text following that prefix has been printed, says (s)he now understands the question, then Question 36 will be asked next.

Flowchart of ISCI Program

NOTE: "TEXT" refers to material in columns 5-64 problem file



(continued next page)



At any time the student can enter COMM (to make a comment), PROB (to go on to another problem) or STOP (to end the session).

Appendix A

Sample ISCI-Student Interaction

Appendices A and B are contrived to demonstrate the properties of ISCI and are not meant to be typical. In the following, student input is underlined.

```
FIND THE DERIVATIVE OF F(X) = (1 - SQRT X) (1 + SQRT X)

#####
IN SOLVING THIS PROBLEM, DID YOU USE THE QUOTIENT RULE, THE
PRODUCT RULE, OR DID YOU MULTIPLY F(X) OUT?  (ENTER QUOT,
PROD, OR MULT.  IF NONE OF THESE APPLY, ENTER OTHER)
? WHAT?

POSSIBLE RESPONSES TO THIS QUESTION ARE:
QUOT
PROD
MULT
OTHER
PLEASE REMEMBER THAT YOU CAN ENTER EXPL (TO GET AN EXPLANATION),
COMM (TO MAKE A COMMENT),  PROB (TO GO TO ANOTHER PROBLEM),
OR STOP.

PLEASE TRY THE LAST QUESTION(MARKED BY #####) AGAIN.
? QUOT
OK

#####
USING THE QUOTIENT RULE, WHAT DID YOU CHOOSE FOR U(X) AND
V(X) SO THAT F(X) = U(X)/V(X)?
   A. U(X) = 1 - SQRT X     V(X) = 1 + SQRT X
                                              2
   B. U(X) = (1 - SQRT X)(1 + SQRT X)    V(X) = 1 + SQRT X
   C. I CHANGED MY MIND; I WANT TO USE THE PRODUCT RULE.
   D. MY ANSWER DOES NOT APPEAR.
? A
WITH YOUR CHOICES FOR U(X) AND V(X), U(X)/V(X) =
(1 - SQRT X)/(1 + SQRT X), WHICH IS NOT F(X).
PLEASE TRY THE LAST QUESTION(MARKED BY #####) AGAIN.
? C
THAT'S A GOOD IDEA.

#####
USING THE PRODUCT RULE, WHAT DID YOU CHOOSE FOR U(X) AND V(X)
SO THAT F(X) = U(X)V(X)?
```

A. U(X) = 1 - SQRT X   V(X) = 1 + SQRT X
B. U(X) = 1 - X   V(X) = 1
C. U(X) = 1 - X   V(X) = 1 + SQRT X
D. MY ANSWER DOES NOT APPEAR.

? C
THERE'S AN ERROR IN YOUR SOLUTION HERE. ENTER YES TO TRY AGAIN,
NO FOR A HINT ABOUT YOUR ERROR.
? NO
WITH YOUR CHOICE OF U(X) AND V(X), U(X)V(X) =
(1 - X)(1 + SQRT X), WHICH IS NOT F(X).

ENTER YES TO TRY AGAIN, NO TO GET AN EXPLANATION OF HOW TO ANSWER
THE QUESTION.
? TRY AGAIN
PLEASE REPLY: YES OR NO
? YES

PLEASE TRY THE LAST QUESTION(MARKED BY #####) AGAIN.
? A
VERY GOOD

#####
WHAT DID YOU FIND FOR U'(X) AND V'(X)?

A. $U'(X) = 1 - (1/2)X^{-1/2}$ AND $V'(X) = 1 + (1/2)X^{-1/2}$
B. $U'(X) = -X^{-1/2}$ AND $V'(X) = X^{-1/2}$
C. $U'(X) = -(1/2)X^{-1/2}$ AND $V'(X) = (1/2)X$
D. U'(X) = -SQRT X AND V'(X) = SQRT X
E. MY ANSWER DOES NOT APPEAR.

? EXPL
I WILL NOW TRY TO EXPLAIN HOW TO ANSWER THIS CORRECTLY.
EACH TIME I TYPE A QUESTION MARK, PLEASE ENTER ONE OF THE FOLLOWING:
MORE IF YOU WISH MORE EXPLANATION,
AGAIN IF YOU WISH TO GO BACK AND TRY THE QUESTION AGAIN.

YOU FOUND U(X) = 1 - SQRT X AND V(X) = 1 + SQRT X.
? MORE
WRITE SQRT X AS $X^{1/2}$, SO $U(X) = 1 - X^{1/2}$ AND
$V(X) = 1 + X^{1/2}$.
? MORE
REMEMBER THAT THE DERIVATIVE OF X^N IS NX^{N-1}, AND THE DERIVATIVE OF A CONSTANT IS ZERO.
? MORE
THEN $U'(X) = -(1/2)X^{-1/2}$ AND $V'(X) = (1/2)X^{-1/2}$, SO
THE CORRECT ANSWER WAS "C".

THIS ENDS MY EXPLANATION. I HOPE YOU ARE READY TO CONTINUE WITH
THE NEXT QUESTION. IF SO, ENTER YES. IF YOU ARE STILL CONFUSED
AND WISH TO WRITE A COMMENT TO THE INSTRUCTOR, ENTER COMM.
? YES

#####

APPLYING THE PRODUCT RULE TO F(X) = U(X)V(X), WHAT IS F'(X)?
PLEASE SIMPLIFY YOUR ANSWER AS MUCH AS POSSIBLE.

A. 0
B. 1
C. 1 - X - (1/(4X))
D. -1
E. MY ANSWER DOES NOT APPEAR.

? D

GREAT, YOU'RE DONE. NOTICE AT THE START YOU COULD HAVE MULTIPLIED OUT F(X), TO GET F(X)=1-X. THAT WOULD HAVE MADE THINGS EASIER, BUT THIS WAS GOOD PRACTICE.

Appendix B: Sample Problem File

This file is in "arrow" format, which means that each exponent, instead of being on a separate line, is preceded by a special mark (↑). This format, which uses less storage space, also requires the program to do a bit more work, and may not be practical on machines which have no internal encode/decode mechanism.

See section IV for details on the meaning of prefixes.

```
PQ5
    FIND THE DERIVATIVE OF F(X) = (1 - SQRT X) (1 + SQRT X)
QQ 1IN SOLVING THIS PROBLEM, DID YOU USE THE QUOTIENT RULE, THE
    PRODUCT RULE, OR DID YOU MULTIPLY F(X) OUT? (ENTER QUOT,
    PROD, OR MULT.  IF NONE OF THESE APPLY, ENTER  OTHER )
AN  QUOT
CR20OK
AN  PROD
CR  GOOD WORK
AN  MULT
CR10GREAT
AN  OTHER
EA
EN 3I SUGGEST YOU TRY THE PRODUCT RULE, WUTH U(X) = 1 - SQRT X,
    AND V(X) = 1 + SQRT X.
QQ 2USING THE PRODUCT RULE, WHAT DID YOU CHOOSE FOR U(X) AND V(X)
    SO THAT F(X) = U(X)V(X)?
       A. U(X) = 1 - SQRT X     V(X) = 1 + SQRT X
       B. U(X) = 1 - X          V(X) = 1
       C. U(X) = 1 - X          V(X) = 1 + SQRT X
       D. MY ANSWER DOES NOT APPEAR.
AN  A
CR  VERY GOOD
AN  B
CR11GREAT, YOU MULTIPLIED F OUT; THIS SIMPLIFIED F CONSIDERABLY
        FORGET ABOUT BREAKING F(X) UP INTO U(X) AND
        V(X) AND JUST LOOK AT F(X)= 1-x
AN  C
IR  WITH YOUR CHOICE OF U(X) AND V(X), U(X)V(X) =
    (1 - X)(1 + SQRT X), WHICH IS NOT F(X).
```

```
AN  D
EA
EX  TO USE THE PRODUCT RULE, YOU WANT TO EXPRESS F(X) AS THE PRODUCT
    OF TWO OTHER FUNCTIONS WHICH ARE EASIER TO DIFFERENTIATE
EX  SINCE F IS THE PRODUCT OF (1 - SQRT X) AND (1 + SQRT X), THE
    MOST LOGICAL ANSWER IS "A", U(X) = 1 - SQRT X AND V(X) =
    1 + SQRT X.
QQ 3WHAT DID YOU FIND FOR U'(X) AND V'(X)?
       A. U'(X) = 1 - (1/2)X↑-↑1↑/↑2 AND V'(X) = 1 + (1/2)X↑-↑1↑/↑2.
       B. U'(X) = -X↑-↑1↑/↑2 AND V'(X) = X↑-↑1↑/↑2.
       C. U'(X) = -(1/2)X↑-↑1↑/↑2 AND V'(X) = (1/2)X↑-↑1↑/↑2
       D. U'(X) = SQRT X AND V'(X) = SQRT X
       E. MY ANSWER DOES NOT APPEAR.
AN  A
AG  REMEMBER THE DERIVATIVE OF A CONSTANT IS ZERO.
AN  B
AG  REMEMBER THE DERIVATIVE OF X↑N IS NX↑N↑-↑1.
AN  C
CR  VERY GOOD.
AN  D
AG  NO; WRITE SQRT X AS X↑1↑/↑2 AND THEN DIFFERENTIATE.
AN  E
EA
EX  YOU FOUND U(X) = 1 - SQRT X AND V(X) = 1 + SQRT X.
EX  WRITE SQRT X AS X↑1↑/↑2, SO U(X) = 1 - X↑1↑/↑2 AND
    V(X) = 1 + X↑1↑/↑2.
EX  REMEMBER THAT THE DERIVATIVE OF X↑N IS NX↑N↑-↑1.
EX  THEN U'(X) = -(1/2)X↑-↑1↑/↑2 AND V'(X) = (1/2)X↑-↑1↑/↑2, SO
    THE CORRECT ANSWER WAS "C".
QQ 4APPLYING THE PRODUCT RULE TO F(X) = U(X)V(X), WHAT IS F'(X)?
    PLEASE SIMPLIFY YOUR ANSWER AS MUCH AS POSSIBLE.
       A. 0
       B. 1
       C. 1 - X - (1/(4X))
       D. -1
       E. MY ANSWER DOES NOT APPEAR.
AN  A
IR  CHECK YOUR CALCULATIONS, WATCH YOUR SIGNS.
AN  B
AG  CHECK YOUR CALCULATIONS.  YOU'RE CLOSE.
AN  C
AG  THE PRODUCT RULE SAYS F'(X) = U'(X)V(X) + U(X)V'(X); YOU
    HAVE U(X)V(X) + U'(X)V'(X).
AN  D
CR  GREAT, YOU'RE DONE.  NOTICE AT THE START YOU COULD HAVE
    MULTIPLIED OUT F(X), TO GET F(X)=1-X.  THAT WOULD HAVE
    MADE THINGS EASIER, BUT THIS WAS GOOD PRACTICE.
AN  E
EA
EX  THE PRODUCT RULE SAYS F'(X) = U'(X)V(X) + U(X)V'(X).
EX  YOU CHOSE U(X) = 1 - SQRT X AND V(X) = 1 + SQRT X, AND WE FOUND
    U'(X) = 1(1/2)X↑-↑1↑/↑2 AND V'(X) = (1/2)X↑-↑1↑/↑2.
```

```
EX  SO U'(X)V(X)= [-(1/2)X↑-↑1↑/↑2] [1+SQRT X]
      =[-(1/2)X↑-↑1↑/↑2 - 1/2].
EX  LIKEWISE U(X)V'(X)= [1-SQRT] [(1/2)X↑-↑1↑/↑2]
    =[(1/2)X↑-↑1↑/↑2 - 1/2]. NOW ADD U'(X)V(X) AND
    U(X)V'(X) TOGETHER TO GET F'(X).
EX  THEN F'(X)= -1. SO "D" WAS CORRECT AND YOU'RE DONE.
    NOTICE THAT WHEN YOU MULTIPLY F(X) OUT, YOU GET
    F(X) = 1 - X + (SQRT X) - (SQRT X) = 1 - X. THAT WOULD HAVE
    BEEN MUCH EASIER TO DIFFERENTIATE, BUT THIS WAS GOOD PRACTICE.
EP
QQ10WHAT IS F(X) AFTER MULTIPLYING IT OUT?
      A. 1 + X
      B. 1 - X
      C. 1 - X↑2
      D. 1 - (SQRT X) + (SQRT X) - X
      E. 1 - 2(SQRT X) - X
      F. MY ANSWER DOES NOT APPEAR.
AN  A
AG  ALMOST. WATCH YOUR SIGNS.
AN  B
CR  GOOD
AN  C
AG  I THINK YOU FORGOT THAT (SQRT X)↑2 = X.
AN  D
CR  GOOD. NOTICE THAT THIS IS THE SAME AS "B".
AN  E
AG  YOU'RE CLOSE. WATCH YOUR SIGNS.
AN  F
EA
EX  (1 - SQRT X)(1 + SQRT X) = 1 - (SQRT X) + (SQRT X) - X, SO
    "B" WAS CORRECT.
QQ11WITH F(X) = 1 - X, WHAT DID YOU FIND FOR F'(X)?
     A. -1
     B. 0
     C. 1
     D. MY ANSWER DOES NOT APPEAR.
AN  A
CR  GREAT, YOU'RE DONE.
AN  B
AG  REMEMBER, THE DERIVATIVE OF A CONSTANT IS ZERO.
AN  C
AG  REMEMBER THE DERIVATIVE OF AX↑N IS ANX↑N↑-↑1.
AN  D
EA
EX  RECALL THAT THE DERIVATIVE OF A CONSTANT IS ZERO.
EX  ALSO RECALL THAT THE DERIVATIVE OF AX↑N IS ANX↑N↑-↑1.
EX  THEN F'(X) = 0 - 1 = -1, SO "A" WAS CORRECT.
EP
QQ20USING THE QUOTIENT RULE, WHAT DID YOU CHOOSE FOR U(X) AND
    V(X) SO THAT F(X) = U(X)/V(X)?
      A. U(X) = 1 - SQRT X     V(X) = 1 + SQRT X
      B. U(X) = (1 - SQRT X)(1 + SQRT X)↑2    V(X) = 1 + SQRT X
```

C. I CHANGED MY MIND; I WANT TO USE THE PRODUCT RULE.
D. MY ANSWER DOES NOT APPEAR.

AN A
AG WITH YOUR CHOICES FOR U(X) AND V(X), U(X)/V(X) =
(1 - SQRT X)/(1 + SQRT X), WHICH IS NOT F(X).
AN B
AG THIS WILL WORK, BUT YOUR U(X) IS MORE DIFFICULT TO
DIFFERENTIATE THAN F(X).
AN C
CR 2THAT'S A GOOD IDEA.
AN D
EA
EX 2THERE IS NOTHING WRONG WITH USING THE QUOTIENT RULE. HOWEVER,
THE FUNCTIONS U AND V THAT YOU MAKE UP ARE MORE COMPLICATED
THAN THOSE NEEDED FOR THE PRODUCT RULE. LET'S TRY APPLYING THE
PRODUCT RULE.
EP

Appendix C: Sample Pages
from ISCI User Manual

The Chain Rule

(1) Prologue: Before learning the chain rule, you should be very accustomed to differentiating functions with respect to variables other than x. For example, if $f(u)= u^{30}$ then $\frac{df}{du}$, or $f'(u)$, is $30u^{29}$. If this seems at all new or odd to you, go back and find some differentiation exercises, substitute your favorite letter (or u, if you don't have a favorite letter) for x, and differentiate the result with respect to that letter. The answer should be equal to the answer to the original problem, with that letter in place of x. After doing each problem, repeat "There is nothing special about the variable x."

(2) As explained in your text, the chain rule allows you to differentiate a given function g(x) by writing it as the composition of two simpler functions f and u such that $g(x) = f(u(x))$, then multiplying the derivatives of the two functions f and u:

Chain Rule If $g(x) = f(u(x))$, then
$g'(x) = f'(u(x))\cdot u'(x)$,
whenever the derivatives
on the right both exist.
(the dot refers to ordinary multiplication)

(3) If you don't forget that u is a function of x, the chain rule can be shortened to:

$$g'(x) = f'(u)\cdot u'(x)$$

(4) In practice you will be given the function g(x) and asked to find its derivative. Your first, and usually most difficult, task will be to find appropriate functions u(x) and f so that $g(x) = f(u(x))$, then apply the chain rule. Of course u and f must be chosen so that f' and u' are easy to compute, otherwise applying the chain rule will not help to simplify matters.

(5) The ISCI program will assume that you have applied the chain rule in these four steps:

I. Identify some expression within the function g(x) as your candidate for u(x). Be sure that all occurrences of x are included in u(x).

II. Explicitly write out f so that $g(x) = f(u(x))$. (The simplest way to do this is to take g(x) and replace u(x) by the letter u. Then you will get $g(x) = f(u)$.)

III. Check whether u and f are both easier to differentiate than g. (Remember that f(u) is to be differentiated with respect to u--see the prologue.) If they are not, go back to step I and try another u.

IV. Apply the chain rule, $g'(x) = f'(u)\cdot u'(x)$; then replace u by its expression u(x).

(6) <u>Example</u> $g(x)=(x^2+4)^5$.

Step I: Pick $u(x)=x^2+4$

Step II: Take $(x^2+4)^5$ and replace x^2+4 by u, to get $u^5=f(u)$.

Step III: $f'(u)=5u^4$ and $u'(x)=2x$ are both easy to compute, so we can proceed

Step IV: $g'(x)=f'(u)\cdot g'(u)=5u^4\cdot 2x$. Replacing u by x^2+4, we get

$g'(x)=5(x^2+4)^4\cdot 2x=10x(x^2+4)^4$.

Chain Rule Problems:

You should be able to differentiate problems 1 - 9 with one application of the chain rule.

Note: Problems 6 - 11, involving trig functions, may not have been covered yet, in some texts.

CR1 $(x^3 + 2x - 1)^{32}$

CR2 $\sqrt{x^2 + 1}$

CR3 $(1 - x^2)^{-3}$

CR4 $(t^{-2} + t^2)^{-2}$

CR5 $\left(\frac{1}{1+t}\right)^4$

CR6 $\sin (1 + x^2)$

CR7 $\sin^2 x$

CR8 $\tan (x^2 + 2)$

CR9 $\sin (1/(1 + 2x))$

CR10 $\tan^2 (2x)$

CR11 $\sin (x \cos x)$