DOCUMENT RESUME

ED 162 434 EA 011 041

AUTHOR	Beck, E. M.; Summers, Gene F.
TITLE	Additive, Multiplicative, and Mixed Models for Studying Community Change.
PUB DATE	78
NOTE	17p.; Paper presented at the Seminar on Nonmetropolitan Industrial Growth and Community Change (San Francisco, California, 1978)
EDRS PRICE	MF-$0.83 HC-$1.67 Plus Postage.
DESCRIPTORS	Change Agents; *Community Change; *Community Study; *Mathematical Models; Social Change
IDENTIFIERS	Additive Models; Mixed Models; Multiplicative Models

ABSTRACT

The conceptual and theoretical implications of employing different functional models of social change at the community level are outlined in this paper. While no new theoretical or methodological ground is broken, a class of models is recommended that appear infrequently in sociological literature, yet are well suited for representing social change. In particular, the ramifications of additive, multiplicative, and mixed models are explored. Regardless of whether the variable being examined is rate of social change or amount of social change, the additive model has little to recommend its use. If the researcher is interested in the amount of change, then a mixed model seems the best form to be employed. If the interest is in the rate of change, the multiplicative form may be more desirable. Social change can be viewed as a contingent process in which the state of development interacts with change agents to produce a new stage of development. Furthermore, change agents themselves do not operate in isolation, but in a complementary fashion to produce social change. (Author/JM)

ADDITIVE, MULTIPLICATIVE, AND MIXED MODELS FOR STUDYING COMMUNITY CHANGE*

E.M. Beck
University of Georgia
Athens, Georgia

and

Gene F. Summers
University of Wisconsin
Madison, Wisconsin

*For presentation at the One Day Seminar on Nonmetropolitan Industrial Growth and Community Change held in conjunction with the annual meetings of the Rural Sociological Society, San Francisco, California, 1978

Abstract

In this brief paper we outline the conceptual and theoretical implications of employing different functional models of social change at the community level. In particular we explore the ramifications of additive, multiplicative, and mixed models in such research. Suggestions are made as to the appropriate functional forms to be utilized in studies of change.

Additive, Multiplicative, and Mixed Models
For Studying Community Change

Introduction

In this brief paper we will examine the conceptual implications of several models for studying social change where the unit of analysis is the community. While no new theoretical or methodological ground is broken, we will recommend a class of models which infrequently appear in the sociological literature yet are well-suited for representing social change. Specifically we will argue that it is conceptually reasonable to view social change as a contingent process where the initial state of development interacts with change agents to produce a new stage of development. Furthermore, we will contend that change agents themselves do not operate in isolation, but in a complementary fashion producing social change.

Some Preliminaries

Let us assume that interest is focused on explaining change in a dependent variable Y over some period of time. Without loss of generality we will assume that Y is measured on a set of n communities at two points in time: its current value Y^t and its base period value Y^{t-1}.[1] Change in Y is defined as the simple difference between these two values,

$$\Delta Y \qquad - Y_i^{t-1} \tag{1}$$

...eve that there ... e agents, or ...ependen... (X_k) and that c... amount of change in the ... nt variable. Agai... e ... the amount of change in the jth of k independent variables in the ith community,

$$\Delta X_{ij} = X_{ij}^{t} - X_{ij}^{t-1} \qquad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,k \end{matrix} \tag{2}$$

While the basic argument is that ΔY is some function of these ΔX_k's, given our imperfect knowledge it is unlikely that we can specify the exact determinants of ΔY. It this instance it may be more realistic to assume that one or more important independent variables have been inadvertently excluded from the model. The effects of these excluded variables can be summarized into one term, u_i. The argument is, then, that change in the dependent variable is some function of changes in a set of independent variables --- social change agents --- and a disturbance term representing the collective effects of excluded variables (deleting the community subscripts):

$$\Delta Y = f(\Delta X_1, \Delta X_2, \ldots , \Delta X_k; u) . \tag{3}$$

From this specification two questions arise. First, what sort of functional form should be used to link ΔY to the predetermining variables? Second, is the expression on the r.h.s. of (3) complete; in other words, are there additional terms which should be included along with the predetermining variables already in the equation?

Let us address the latter issue first since the remainder of the paper will be devoted to the former. We argue that the r.h.s. of (3) is not complete because it states that the amount of change in the dependent variable is unaffected by the initial level of the dependent variable Y^{t-1}. We do not find this assumption very plausible. Rather we believe as a broad principle that the amount of change will depend on the state of the community in the base period t-1. For example, it could be anticipated that the degree of change in aggregate educational attainment in a community is inversely related to the level of education in the base period. It would be expected that communities with high levels of educational attainment would, <u>ceteris paribus</u>, experience less change in education than communities with lower aggregate education at the initial period. In a sense we are positing a "ceiling" effect such that there is an inverse relationship between a change in Y and its initial value. At a more general level, we believe that in most situations it would be difficult to argue that the amount of change is independent of the base period value of the dependent variable.

Given this position (3) must be re-specified to include the t-1 value of the dependent variable,

$$\Delta Y = f(Y^{t-1}; \Delta X_1, \Delta X_2, \ldots, \Delta X_k; u) \quad (4)$$

Now that the terms in the function have been identified, an appropriate form to be employed in analysis must be selected.

Functional Forms

<u>Linear, Additive Models</u>. The simpliest change model would [illegible]press ΔY as a linear, additive function of the initial value of Y, [illegible] the independent variables and the disturbance term:[2]

$$[illegible] + \beta_1 \Delta X_1 + \beta_2 \Delta X_2 + \ldots + \beta_k \Delta X_k [illegible] u \quad (5)$$

The part[illegible] of ΔY with respect to the predeter[illegible] variables

$$\frac{\partial [illegible] Y}{\partial [illegible] X_k} [illegible] \qquad \frac{\partial \Delta Y}{\partial Y^{t-1}} = \Gamma \quad (6)$$

show that the eff[illegible] of the kth social change agent, as deno[illegible] by the partial derivative of ΔY with respect to ΔX_k, is a constant (β_k) for all of the k independent variables. Similarly, the effect of the base period value of the dependent variable is a constant. Further, the effects of the predetermining variables are <u>additive</u>: the effect of an explanatory variable does not depend, nor is contingent, upon <u>any</u> of the remaining variables in the model.

These characteristics of the linear, additive model are also obvious when (5) is re-written so that only the current value of Y appears on the l.h.s.:

$$Y^t = \alpha + (\Gamma+1)Y^{t-1} + \beta_1\Delta X_1 + \beta_2\Delta X_2 + \ldots + \beta$$ [illegible]

[illegible]expression shows that after an adjustment is ma[illegible]se period [illegible]e of Y, the current value of the dependent variab[illegible]itive [illegible]ination of the seperate effects of each of the s[illegible] agents.

In sum, although the linear, additive model has [illegible]ertai[illegible] [illegible]ance, it is restrictive and its implicit underl[illegible] theore[illegible] [illegible]ndations [illegible] unrealistic and unacceptable. The model presen[illegible]d in ([illegible] states tha[illegible] [illegible]he social change agents do <u>not</u> interact with one another, that the effec[illegible] of these agents are <u>not</u> conditioned by the initial state of the commu[illegible]ity, and lastly, that the social change effects are constant thro[illegible]ut their range. Since we do not believe that any of these three propositio[illegible] are reasonable for most studies of social change, the linear, additive [illegible] is rejected as being a viable representation.

<u>Mixed Models.</u> Many of the limitations of the linear, additive model can be rectified by introducing multiplicative interaction terms into the function. The interactive model of social change is composed of a mixture of additive and multiplicative terms. For simplicity's sake, and without loss of

generality, let us assume that we are dealing with only two independent change agents, ΔX_1 and ΔX_2, and that we believe that there are significant interaction effects among ΔX_1, ΔX_2, and Y^{t-1} in producing change in Y, whence

$$\Delta Y = \alpha + \Gamma Y^{t-1} + \beta_1 \Delta X_1 + \beta_2 \Delta X_2 + \theta_1 (\Delta X_1)(Y^{t-1}) + \theta_2 (\Delta X_2)(Y^{t-1}) + \theta_3 (\Delta X_1)(\Delta X_2) + \theta_4 (\Delta X_1)(\Delta X_2)(Y^{t-1}) + \varepsilon$$

Here β_1, β_2, and Γ are the "ma[illegible] [illegible] action parameters. In this mo[illegible] we have [illegible] two-way interactions as well as the three-way inter[illegible] ΔX_1, [illegible] and Y^{t-1} [illegible] interactions between ΔX_1 and Y^{t-1} and betw[illegible] and [illegible] as well as the three-way interaction, are especially re[illegible] In many [illegible]ances it is the state of the community in the base period wh[illegible] provide[illegible] stimulus for social change. For example, if Y is the pover[illegible] rate, [illegible]rticularly large value at t-1 may encourage the introduction of social change agents designed to reduce the severity of the problem. In this example there would be interaction among these agents and the level of poverty in the base period, t-1. In fact, any time change in the social change agents is stimulated by the initial level of the dependent variable, there exists interaction effects such as these.

Some of the conceptual implications of this mixed model are revealed by inspecting the partial derivatives of (8)

$$\frac{\partial \Delta Y}{\partial Y^{t-1}} = \Gamma + \theta_1(\Delta X_1) + \theta_2(\Delta X_2) + \theta_4(\Delta X_1)(\Delta X_2) \tag{9}$$

$$\frac{\partial \Delta Y}{\partial \Delta X_1} = \beta_1 + \theta_1(Y^{t-1}) + \theta_3(\Delta X_2) + \theta_4(\Delta X_2)(Y^{t-1}) \tag{10}$$

$$\frac{\partial \Delta Y}{\partial \Delta X_2} = \beta_2 + \theta_2(Y^{t-1}) + \theta_3(\Delta X_1) + \theta_4(\Delta X_1)(Y^{t-1})$$

(9) shows ... effect of the bas... period value of the dependent variable ... aries accordi... the values of ΔX_1, ΔX_2, and their interac... ...larly, (10) ...es that the effect of the first independent variable ... $\Delta Y/\partial\Delta X_1$) is ...near combination of a constant and the effects of ...$_2$, ...$^{-1}$, and the ...ction between ΔX_2 and Y^{t-1}. Likewise, (11) shows tha... the effect of the ... social change agent is a linear function of ... $^{t-}$... and their interaction.

In mixed models it is clearly ...ssible, as shown by these partial derivatives ... discuss the effects ... these social change agent without simultaneo... considering the values ... the other social change agents as well as the state of the community in the base period, t-1. While mixed models are attractive, there is another set of interactive models which also merit consideration.

Multiplicative Models. Fully multiplicative models, also known as Cobb-Douglas models, posit that the dependent variable is a multiplicative, complementary function of the predetermining variables in the model rather than an additive combination of "main" effects and interactions. Again restricting attention to the situation where there are only two independent variables, the multiplicative model of change would be

$$\Delta Y = \alpha(Y^{t-1})^{\Gamma}(\Delta X_1)^{\beta_1}(\Delta X_2)^{\beta_2}\,u \quad . \qquad (12)$$

The coefficients of this model have a straightforeward interpretation as elasticities. Γ is the percentage change in ΔY associated with a 1% change in the base period value of Y. Similarly, β_k is the percentage change in ΔY that would r...lt from a 1% change in ΔX_k. Also it should be noted that the sum of the p...ameters of the social change agents indicates whether ΔY changes at an increasing ($|\beta_1+\beta_2| > 1$) or decreasing ($|\beta_1+\beta_2| < 1$) rate for changes in the two change agents.

As can be seen from the partial derivatives of (12), the effect of any one variable in the model is dependent upon the remaining variables, including the scaling factor (α):

$$\frac{\partial\Delta Y}{\partial Y^{t-1}} = (\alpha)(\Gamma)(Y^{t-1})^{\Gamma-1}(\Delta X_1)^{\beta_1}(\Delta X_2)^{\beta_2} \tag{13}$$

$$\frac{\partial\Delta Y}{\partial\Delta X_1} = (\alpha)(Y^{t-1})^{\Gamma}(\beta_1)(\Delta X_1)^{\beta_1-1}(\Delta X_2)^{\beta_2} \tag{14}$$

$$\frac{\partial\Delta Y}{\partial\Delta X} = (\alpha)(Y^{t-1})^{\Gamma}(\Delta X_1)^{\beta_1}(\beta_2)(\Delta X_2)^{\beta_2-1} \tag{15}$$

Comparing (13)-(15) with those of the mixed model, (9)-(11), shows that in both models the effects of each predetermining variable on ΔY are conditioned by the remaining variables in the model. The major difference is that in the multiplicative model the effect of any one variable is dependent on the <u>interaction</u> among <u>all</u> other predetermining variables, whereas in the mixed model the effect is dependent upon an additive function of main effects and interaction effects.

While the multiplicative model has certain attractive features, such as the interpretation of its parameters as elasticities, it does bring into focus certain issues which demand consideration. First, since the model states that the process generating change in Y is simultaneously contingent upon all of the predetermining variables in the equation, the lack of change in any one of the variables will produce no change in the dependent variable. Thus as (12) shows if there is no change in either of the social change agents, or if the base period value of the dependent variable is zero, change in Y will also be zero. This may or may not be a resonable specification depending on the sociological context of the research. Second, the model does present some difficulties in estimation which can be shown by taking (12) and solving for the current value of Y,

$$Y^t = Y^{t-1} + \left[(\alpha)(Y^{t-1})^{\Gamma}(\Delta X_1)^{\beta_1}(\Delta X_2)^{\beta_2}u\right] \qquad (16)$$

This function is intrinsically nonlinear and can not be easily estimated without employing an iterative nonlinear estimation solution.

An Alternative Specification

It could be argued that communities are in a more-or-less constant process of change. If this position is assumed, then we may be interested in discovering the reasons why communities change at different rates. That is, what are the determining factors which influence the rate of community change. Now the appropriate dependent variable is no longer ΔY, but (Y^t/Y^{t-1}), the rate of change in Y (plus one).[3] Clearly we could express the rate of change as a linear, additive function of the predetermining variables in a model similar to (5), but such a model would have the same conceptual limitations as discussed in regard to (5).

As a better first approximation we could apply the mixed model to this new dependent variable:

$$Y^t/Y^{t-1} = \alpha + \Gamma Y^{t-1} + \beta_1\Delta X_1 + \beta_2\Delta X_2 + \Theta_1(\Delta X_1)(Y^{t-1}) + \Theta_2(\Delta X_2)(Y^{t-1}) + \Theta_3(\Delta X_1)(\Delta X_2) + \Theta_4(\Delta X_1)(\Delta X_2)(Y^{t-1}) + u \qquad (17)$$

In this form we are arguing that the rate of change in Y is a function of the social change agents, the base period value of Y, and the interactions among these predetermining variables. The partial derivatives of this model would be similar to those presented in (9)-(11) and will not be presented here. It is instructive to take (17) and solve for the current value of Y:

$$Y^t = \alpha(Y^{t-1}) + \Gamma(Y^{t-1})^2 + \beta_1(\Delta X_1)(Y^{t-1}) + \beta_2(\Delta X_2)(Y^{t-1}) + \Theta_1(\Delta X_1)(Y^{t-1})^2 + \Theta_2(\Delta X_2)(Y^{t-1})^2 + \Theta_3(\Delta X_1)(\Delta X_2)(Y^{t-1}) + \Theta_4(\Delta X_1)(\Delta X_2)(Y^{t-1})^2 + v \quad (18)$$

where $v = u(Y^{t-1})$. (18) shows that the mixed model of change implies that the current value of the dependent variable is a function of the interaction of Y^{t-1} with each of the predetermining variables. This further implies that Y^t is not only affected by the linear effects of Y^{t-1}, but also the quadratic effects. This may or may not be a reasonable specification of the process by which current values of the dependent variable are generated, but in any regard this specification appears to be so complex as to inhibit a simple substantive interpretation of the model's parameters.

If we apply the fully multiplicative functional form to this dependent variable,

$$Y^t/Y^{t-1} = \alpha(Y^{t-1})^\Gamma(\Delta X_1)^{\beta_1}(\Delta X_2)^{\beta_2}\ u \quad (19)$$

and then solve for the current value of Y,

$$Y^t = \alpha(Y^{t-1})^{\Gamma+1}(\Delta X_1)^{\beta_1}(\Delta X_2)^{\beta_2}\ u \quad (20)$$

we find that Γ is the percentage increase in the rate of change for a 1% change in the base period value, yet $(\Gamma+1)$ is the percentage change in the current value of Y given a 1% change in Y^{t-1}.[4] Thus we can readily express the effect of the initial period either in terms of the rate of change (Y^t/Y^{t-1}) or the current value, Y^t. Furthermore, a comparison of these two equations shows that β_k is the percentage change in <u>either</u> (Y^t/Y^{t-1}) <u>or</u> Y^t.

Hence we can readily interpret the parameters of (19) either in terms of the rate of change in the dependent variable or in terms of the current value of Y. In either of these instances the parameters retain their interpretation as the percentage change in the dependent variable resulting from a 1% change in the predetermining variables.

Concluding Comments

We have outlined the features of three functional forms which could be used to study community change. While by no means exhausting the potential forms which could be employed, these three have simple mathematical properties which lend themselves to analysis. Regardless of the functional form chosen, we argued that the base period value of the dependent variable should be included as one of the predetermining variables in the model.

We noted that researchers may be interested in either of two types of variables: the amount of change in a dependent variable, ΔY, or the rate of change in the dependent variable, Y^t/Y^{t-1}. Regardless of which variable is of interest though, the linear, additive model has little to recommend its use. Although it is easily estimated and its parameters have a straightforward interpretation, this model is far too restrictive and implies unreasonable conceptual limitations on the analysis. If the researcher is interested in the amount of change (ΔY), then a "mixed" model seems to be a reasonable functional form to be employed. In this formulation the amount of change is expressed as a function of the "main" effects of the predetermining variables and their interaction terms. On the other hand, if interest is on the rate of change, the multiplicative form may be more desirable. In this model the rate of change is determined by the joint interaction among all the predetermining variables. One added advantage of this specification is that its parameters have a simple interpretation as the percentage change in the dependent variable resulting from a 1% change in the predetermining variables.

While it would be impossible for us to specify a suitable functional form that is applicable to all substantive research problems, the mixed and multiplicative models seem like reasonable choices for a wide variety of research problems.

Notes

[1]Without loss of generality the arguments presented here can be extended to models with jointly-dependent variables as well as models involving more than two points in time. If there are multiple points in time, the estimation of the models discussed here are complicated. See Nerlove's (1971) paper for a discussion of the estimation of a time series of cross sections.

[2]If we wish to estimate (5) we run immediately into the problem of Y^{t-1} appearing on both sides of the equation. To estimate this model we first rewrite the function solving for the current value of Y:

$$Y^t = \alpha + (\Gamma+1)Y^{t-1} + \beta_1\Delta X_1 + \beta_2\Delta X_2 + \ldots + \beta_k\Delta X_k + u .$$

If Y^{t-1} is considered a fixed exogenous factor, then this equation can be estimated directly and the parameters of (5) can be easily retrived. If Y^{t-1} is endogenous, however, it will be quite likely that there will be a correlation between its value and the disturbance term thus rendering ordinary least squares estimates inconsistent. For a discussion of this issue we any standard econometrics text such as Kmenta (1971) or Theil (1971)

[3]The rate of change is defined as: $\Delta Y/Y^{t-1}$. Since

$$\Delta Y/Y^{t-1} = (Y^t - Y^{t-1})/Y^{t-1} = (Y^t/Y^{t-1}) - 1 ,$$

we find that (Y^t/Y^{t-1}) is the rate of change in Y plus one:

$$(Y/Y^{t-1}) = (\Delta Y/Y^{t-1}) + 1 .$$

For an example of a study using this type of dependent variable see Greenwood's (1975) investigation of urban migration.

[4]To estimate (19) we take the natural logarthm of (20),

$$\ln Y^t = \ln \alpha + (\Gamma+1)\ln(Y^{t-1}) + \beta_1\ln(\Delta X_1) + \beta_2\ln(\Delta X_2) + e$$

where e = ln u. If Y^{t-1} is a fixed exogenous variable this equation can be estimated using ordinary least squares. See note #2 for a further discussion of this point and Greenwood's (1975) study of migration for an illustration of the usage of this functional form.

Citations

Greenwood, Michael J.
1975 "A Simultaneous-Equations Model of Urban Growth and Migration." Journal of the American Statistical Association 70 (December): 797-810.

Kmenta, Jan
1971 Elements of Econometrics. New York: Macmillan Publishing Co.

Nerlove, Marc
1971 "Further Evidence on the Estimation of Dynamic Economic Relations From a Time Series of Cross Sections." Econometrica 39 (March): 359-382.

Theil, Henri
1971 Principles of Econometrics. New York: John Wiley & Sons,