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ABSTRACT

This learning module is intended to illustrate the functioning and uses of bridge circuits. The discussion and laboratory procedures suggested in the module presume familiarity with basic concepts of electronics such as voltage, current, resistance, capacitance, inductance, phase, and knowledge of such skills as breadboarding circuits from schematic diagrams, graphing data, and intermediate algebra. Topics included in the discussion are DC bridge concepts, the Wheatstone bridge, AC bridge concepts, and the Wien bridge. In the laboratory the students build and investigate some uses for a Wheatstone bridge and a Wien bridge. A glossary, list of references, and list of answers to the mathematical problems are included. (BB)

Modular Course In Electronic Instrumentation

Signal Processing Series

Bridge Circuits

Technical Education Research Centers

****PLEASE NOTE****

This preliminary draft copy of Bridge Circuits is provided to prospective interested parties free of charge. Hopefully, feedback will be generated which will result in further refinements of the overall approach used in the development of the Modular Course in Electronics Instrumentation (MCEI). The total project involves approximately twenty authors, advisors and editors in the production of over 30 modules in electronics instrumentation. Most of these modules are planned for completion by Fall, 1978. Modular topics now planned for MCEI are listed below. Interested parties are invited to send their comments, questions and criticisms to Technical Education Research Centers, 575 Technology Square, Cambridge, MA 02139.

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- Introduction to Contemporary Electronics
- Student Handbook for MCEI
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- Alternating Current Instruments
- Microcomputers
- Direct Current Transducers
- Alternating Current Transducers
- Transducer Catalog
- Amplifiers
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- Size Analysis
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- Radiometer
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- Geological Sounding
- Goldfish Conditioning
- ECG System
- Oxygen Concentration
- Polarimeter

Bridge Circuits

ONE TOPIC
IN THE
MODULAR COURSE IN ELECTRONICS INSTRUMENTATION

by

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OBJECTIVES

A number of well known and respected electronic instrument manufacturers have earned their reputations by making precise measuring instruments called bridges. These instruments are used to accurately characterize almost every kind of electrical component from resistors, capacitors and inductors to transmission lines and cables. In this application these instruments often represent the heart of any calibration and standards laboratory.

But another important application for these basic kinds of circuits exists in scientific instrumentation. Bridges can serve the roll of signal processor, transforming the physical input to a transducer to an electrical output which is proportional to that physical input. How these two functions are performed is the subject of this module. Two common bridge circuits will be used to compare some of the distinctive characteristics of DC and AC bridges. We will learn how to build and use these bridge circuits in the laboratory.

PREREQUISITES

The discussion and laboratory procedures suggested in this module presume familiarity with certain concepts, skills and instruments:

Concepts - voltage, current, resistance, capacitance, inductance and phase.

Skills - breadboarding circuits from schematic diagrams, graphing data and intermediate algebra.

Instruments - power supplies, ohmmeters, decade resistance substitution box, signal generator and oscilloscope.

DC BRIDGE CONCEPTS

Objectives: State two principle applications for bridge circuits.

Explain the idean of bridge balance in terms of the voltage difference between the output of two voltage dividers.

PURPOSES AND BEHAVIOR OF

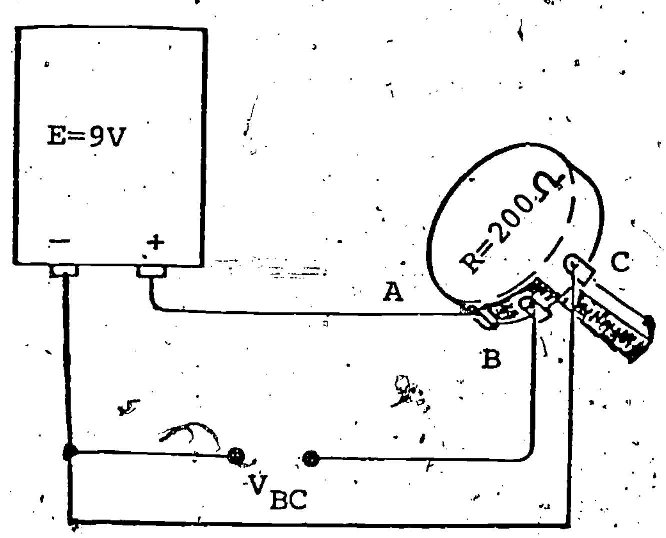
Bridge circuits generally serve two purposes. One is to make accurate measurements of the electrical quantities of resistance, capacitance and inductance. The other is to convert changes of these quantitites into changes in voltage, a more suitable quantity for processing by electronics.

The second application is perhaps the most useful for electronic instrumentation. The first stage of many electronic instruments is a transducer. A transducer is simply a device which converts a change in some physical quantity, such as temperature, light intensity, force or pressure into a change in some electrical quantity. When a transducer is used as an element in a bridge circuit the electrical output is proportional to the physical change at the transducer.

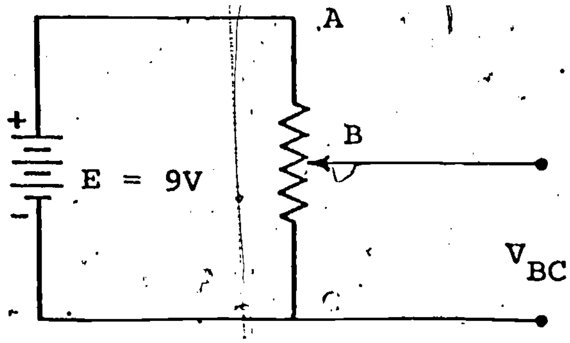
Bridges are also used as basic circuits in more complex instruments like clinical thermometers, light meters, metal detectors, oscillators, and power supplies.

THE HALF BRIDGE

We can most easily understand the bridge circuit by considering first a simple voltage divider circuit. Figure 1a shows a pictorial diagram of a voltage divider. Figure 1b shows the schematic diagram for this circuit.



1a



1b

Figure 1. The voltage divider is the basic part of a bridge. A potentiometer is a variable voltage divider as pictured in 1a. The schematic diagram of this circuit is shown in 1b.

By turning the knob on the potentiometer, the resistance between the center terminal and either end terminal can be made to vary from zero to 200 ohms. The center terminal is connected to a wiper that slides along the total length of the resistive material inside the potentiometer. The point of contact divides the total resitance into two parts which are called the <u>arms</u> of the voltage divider. The position of the wiper does not effect the total resistance across the battery which remains constant at 200 ohms. By adjusting the potentiometer, we can provde voltages from zero to nine volts.

Example 1.

Q: The potentiometer in Figure 1 has a resistance of 200 ohms. Suppose the resistance from A to B is 60 ohms. What is the current in the circuit and the output voltage, V_{BC}? What is V_{AB}?

A: From Ohm's Law the curren is I=E/R = 9/200 = .045 amps or 45 milliamps.
The resistance from B to C must be 200 minus 60, or 140 ohms, so that the voltage from B to C is

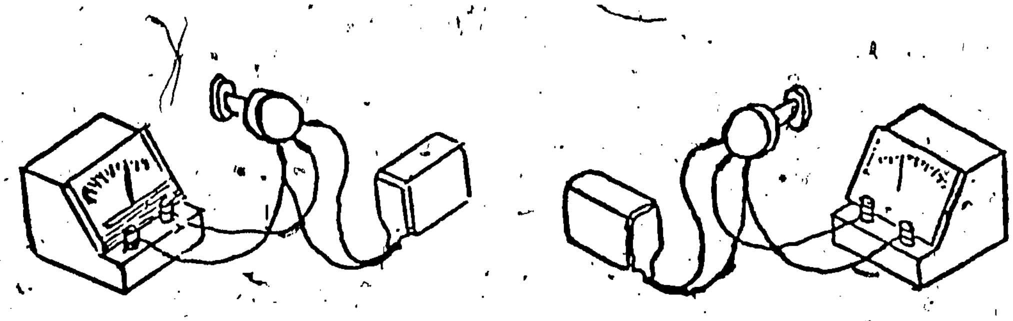
$$V_{BC} = (.045)(140) \text{ volts} = 6.3 \text{ volts}.$$

We also calculate the voltage V_{AB} using Ohm's Law as

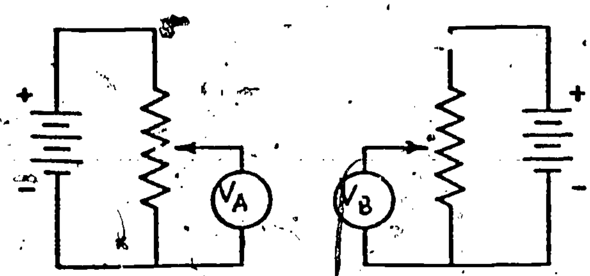
$$V_{AB} = (.045)(60) \text{ volts} = 2.7 \text{ volts}.$$

It is probably no surprise to you that the voltage V_{AB} = 2.7 volts, plus the voltage V_{BC} = 6.3 volts, adds up to give the applied voltage of 9.0 volts. We have divided the 9.0 volts into two voltages which add to nine volts.

Suppose that you had two different voltage dividers with the output voltage for each connected to a voltmeter (assume that the voltmeter draws a negligible current). Figure 2a shows a pictorial of these two circuits and Figure 2b shows their schematic diagram side-by-side.



2a



2b

Figure 2. Two voltage dividers illustrate the concept of balance. Where $V_A = V_B$ the voltage drop across the arms are equal or balanced.

CONCEPT OF BALANCE

We have labeled as V_A the voltage output of the circuit on the left, and on V_B the voltage on the right.

If you wanted these circuits to have the same voltage output, you would adjust one or both potentiometers until the voltages "read" the same on the two meters. Suppose that you use a 10 or 15 volt scale on a meter with 1 volt steps and smallest scale divisions of 0.2 volt. You adjust the two voltages until they are "equal" at 6.7 volts.

You may realize that we could compare these two voltages with fewer components. Why not use just one battery, as shown in the schematic in Figure 3? The current and voltages in each potentiometer would still be the same as they were in Figure 2.

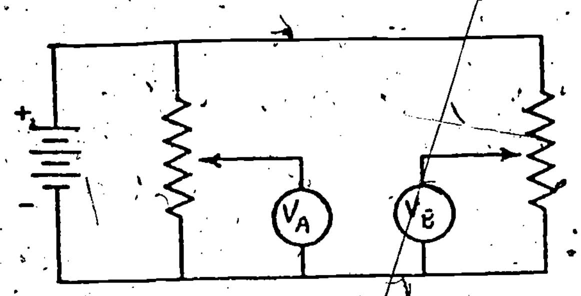


Figure 3. By using one voltage source for both voltage dividers, we obtain almost the equivalent of the Wheatstone Bridge.

In comparing V_A and V_B with this circuit, there is still a basic problem. If we think we read 6.7 volts on each meter, the actual voltage could easily differ from this by as much as 0.1 volts. Because the smallest scale divisions are 0.2 volts apart, a difference of 0.1 volts would not be easily seen on a 10 or 15 volt scale.

If we are interested in measuring only the "difference" of the two output voltages, why not place one meter directly between these points, as shown in Figure 4. Then we could use a lower scale on the voltmeter. For example, we might use a 0-1 or 0-1.5 V scale with 0.1V steps and smallest scale divisions of 0.02 volts. We could then detect a voltage difference as little as almost 0.01 volts.

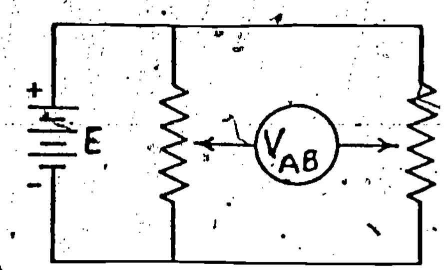


Figure 4. The condition of balance can be much more accurately determined if the voltmeter measures the difference in voltage between the two dividers.

If we place an extremely sensitive voltmeter in this circuit and adjust until the voltage V_{AB} is zero, the circuit is said to be <u>balanced</u>. This circuit is called a balanced <u>Wheatstone Bridge</u>.

THE WHEATSTONE BRIDGE

Objectives: Derive the balance conditions for a Wheatstone Bridge.

Cite three factors which influence the accuracy of a Wheatstone bridge. Explain the influence of each factor.

Explain how to extend or alter the basic range of a Wheatstone Bridge.

Define and explain the meaning of the following terms: accuracy, balance, null, precision, sensitivity and linearity.

Work problems involving all of these ideas.

The resistance bridge is usually shown with the resistances on each side of the voltage dividers as separate resistors. Figure 5 shows such a bridge circuit with each resistor labeled. We have shown the connections of the resistors at the top and at the bottom brought together. When V_{AB} is zero we say that the bridge is balanced.

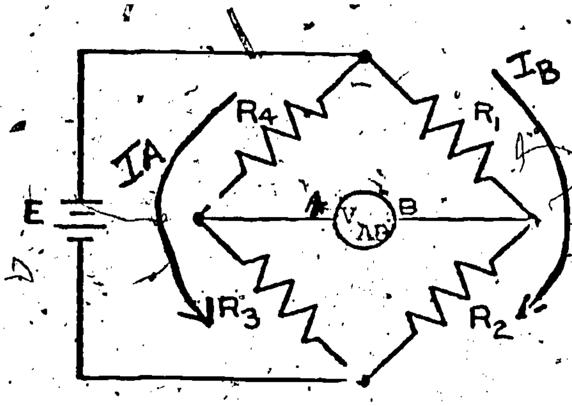


Figure 5. The Wheatstone Bridge is simply the voltage divider pair of Figure 4, but drawn in a diamond shape. Also shown are typical component designations.

As you have seen, the <u>balance condition</u> requires that the voltage across R_3, equals that across R_2. Also, the voltage across R_4 must equal that across R_1. These relationships can be expressed using Ohm's Law. Let the current through the path containing R_3 and R_4 be I_A and the current through the path containing R_1 and R_2 be I_B. Then since $V_3 = V_2$ we must have

$$I_A R_3 = I_B R_2 \qquad (1)$$

and since $V_4 = V_1$, we must have

$$I_A R_4 = I_B R_1. \qquad (2)$$

When the second of these two equations is divided by the first, the currents "cancel" and we have the balance condition for the Wheatstone bridge:

$$\frac{I_A R_3}{I_A R_4} = \frac{I_B R_2}{I_B R_1}$$

WHEATSTONE BALANCE CONDITION

$$R_4/R_3 = R_1/R_2. \qquad (3)$$

PRECISION AND ACCURACY OF A WHEATSTONE BRIDGE

We have said that the Wheatstone Bridge is used to measure resistance. But why bother with a bridge? Why not just use an ohmmeter? Because it is possible to measure resistance with greater <u>precision</u> and <u>accuracy</u> with a Wheatstone Bridge. The greater precision permits one to distinguish between nearly identical readings. This means that with a suitable bridge circuit, resistances can be measured to more significant figures. For example, an ohmmeter might give a resistance reading of 10 ohms for a resistor. With a Wheatstone Bridge, the resistance could be measured as 10.45 ohms.

Accuracy means that, within a specified tolerance, the measured resistance is the true resistance. Typical ohmmeters have a tolerance of 8% or more on their accuracy. A good Wheatstone Bridge would have a tolerance of .1% or better. Figure 6 shows pictorial and schematic diagrams of a Wheatstone Bridge circuit that are suitable for measuring unknown resistances. Three basic requirements must be met if we expect to measure resistances with high precision and accuracy.

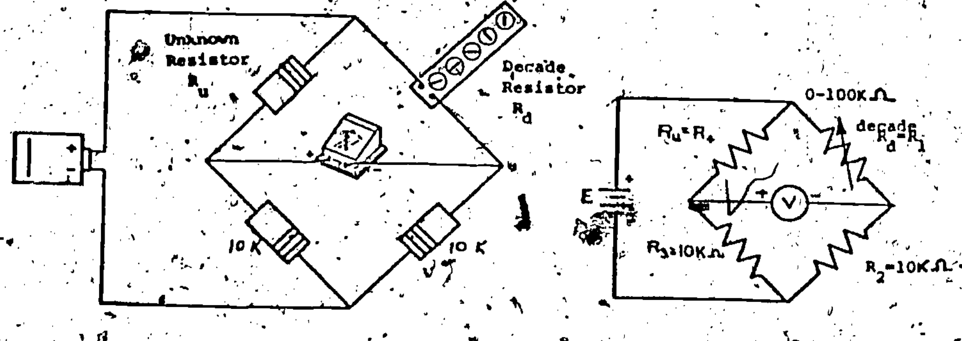


Figure 6. The bridge circuit includes a decade resistor, a voltmeter, two 10K resistors and a 9V battery.

It is necessary to know precisely when the bridge is balanced so that the relationship between the resistance in Equation 3 is valid. For this purpose a sensitive voltage detector is needed.

Secondly, if two of the resistors are precisely matched, as we have presumed for the 10k resistors in Figure 6, the unknown resistor may be determined by direct comparison with a known standard. The final requirement then is a precise standard which may be adjusted over the desired range of resistances. A precision decade resistance substitution box will serve this function.

Decade resistors of 0-100,000 ohms are available in precision steps of one ohm and accuracies of better then 0.1%. Suppose that R_4 is an unknown resistance, then

$$R_4 = (R_1R_3)/R_2. \quad (4)$$

The accuracy errors in R_1, R_2, and R_3 contribute to the error in R_4 in a rather simple way. You just add their percentage errors, and the result is the percentage error in R_4. If R_1, R_2, and R_3 are 0.1% resistors, the accuracy error in R_4 would be 0.3%. But if these were ordinary carbon resistors with 10% tolerance, then the error in R_4 could be as high as 30%

Example 2.

Q: A Wheatstone Bridge similar to the one shown in Figure 6 is used to measure an unknown resistance. Suppose R_3 and R_2 are matched to .1% and the decade resistance is calibrated to .1% tolerance. Balance occurs when the decade resistance reads 24,653±3 ohms. What is the accuracy of this measurement? How many significant figures should be quoted for the value of the unknown resistance? What is the fundamental precision of the system?

A: The combined uncertainty introduced by the tolerances of R_1, R_2 and R_3 is .1% + .1% = ±.2%. It would be misleading to quote results having a precision greater than this uncertainty,

$$\pm .2\% \times 24,653 = \pm .002 \times 24,653 = 50 \text{ ohms}.$$

The value for the unknown resistance may be specified as

$$R_x = 24,650 \pm 50 \text{ ohms}.$$

The fundamental precision of the system is

$$\pm \frac{3}{24,653} \approx \pm .01\%.$$

AN UNEQUAL ARM BRIDGE

In the bridge circuit we have just discussed we set the resistors R_2 and R_3 equal, so that the unknown resistance can be read directly from a decade at balance. What if you wanted to measure a much higher resistance than the decade could give?

Let us look again at the bridge circuit and the balance conditions. Figure 7 shows that circuit with R_x used in place of R_4.

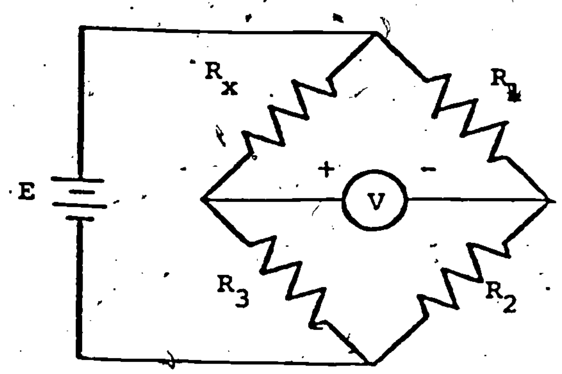


Figure 7. A schematic of the unequal arm bridge circuit.

The balance equation becomes

$$R_x/R_3 = R_1/R_2 \tag{5}$$

Multiplying both sides by R_3, we have

$$R_x = R_1(R_3/R_2). \tag{6}$$

Suppose that we select R_3 to have ten times the resistance of R_2, then we have

$$R_x = 10R_1, \tag{7}$$

and the unknown resistance would be ten times whatever the decade resistor, R_1, has at balance.

As a better example, suppose that both R_1 and R_3 were decade resistors. Then you could adjust these decades to get the right values to balance a resistance of any value. Since there are many different values of R_1 and R_3 which will provide the same ratio, and therefore give a balance, you may wonder if any particular set is better than others. The answer is yes! In the next section of the module, you will learn why.

Example 3.

Q: A Wheatstone bridge like the one shown in Figure 6 is used to measure an unknown resistance. The best null is observed when the decade resistance is on 55 or 56 ohms. Recommend a modification to the circuit that will permit determination of the unknown resistance to .3%.

A: The specified overall tolerance will be achieved if R_1, R_2 and R_3 have individual tolerances of .1%. In order to resolve sufficient significant figures on the decade resistance, an unequal arm configuration is needed. For convenience we will choose $R_2 = 100R_3$ so that $R_{unknown} = \frac{R_1}{100}$.

The best null is now observed when the decade is set at 5534.± 4. The unknown resistance should be specified as R_x=55.3±.2ohms, after rounding off insignificant figures.

Transducers are often placed in one arm of a bridge circuit as a means of balancing out an unwanted steady background signal. In these applications the bridge is initially balanced so that no signal is measured by the voltage detector in the absence of physical input to the transducer. An output voltage will be observed only when the physical input to the transducer causes the bridge to become unbalanced. In these applications it is generally important that the unbalanced voltage from the bridge be as large as possible and proportional to the physical change that is being studied. The relative size of the output voltage will be influenced by the sensitivity of the bridge. The actual output voltage of a bridge circuit is illustrated in Figure 8 for several possible combinations of resistance in the arms.

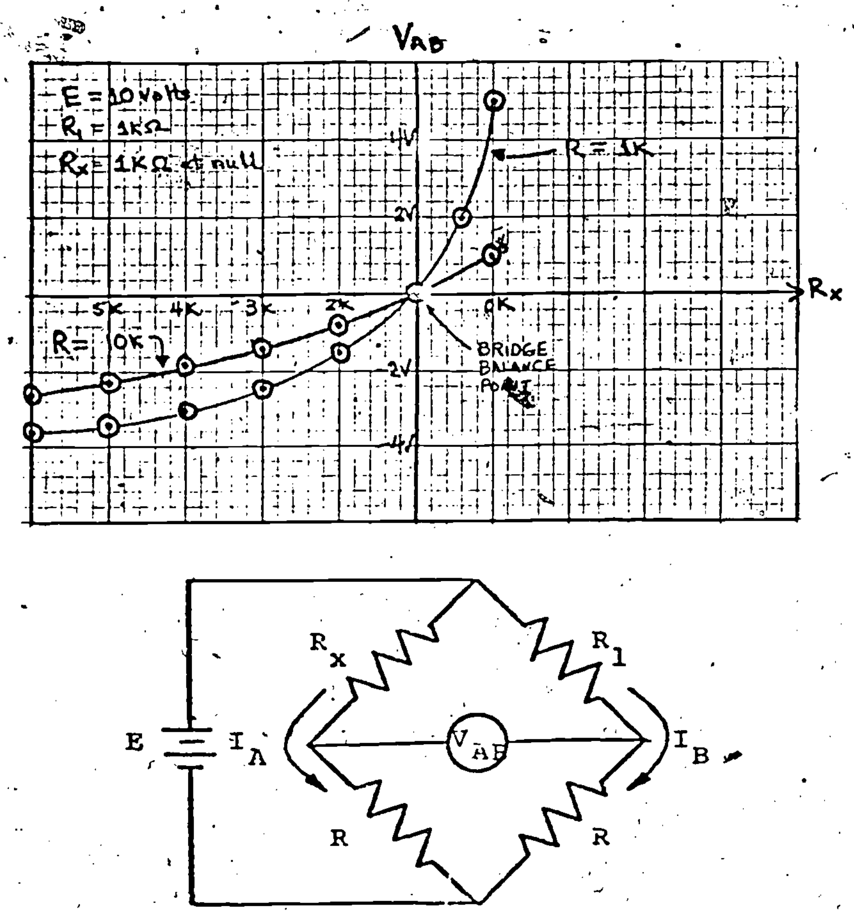


Figure 8. The unbalance voltage from a Wheatstone Bridge verses transducer resistance, R_x.

Two important features are immediately clear. Neither of the two curves are straight lines. This means that the bridge is not linear. Secondly, the two curves pass through the null point with different steepness (slope). This determines the sensitivity of the bridge.

DEFINITION OF SENSITIVITY

What shall we mean by the words, "bridge sensitivity?" The bridge is most sensitive when the smallest change in resistance of R_x produces the largest change in the voltage output of the bridge.

For this reason, we define the sensitivity of a bridge as the amount of voltage output change ΔV produced by a change ΔR in the resistance R_x. If we let S be the sensitivity, we can write the definition as

$$S = \frac{\Delta V}{\Delta R} \qquad (8)$$

You may realize, this defin[illegible]is just the slope of the graph of the output voltage [illegible] the resistance changes ΔR.

The steeper the curve, the more sensitive the bridge will be to changes in the resistance of the transducer. A steep curve is important for another reason. It improves our ability to determine the precise balance point of the bridge. For this reason we always try to maximize bridge. For this reason we always try to maximize bridge sensitivity. We can see from Figure 8 that the senstivity of the bridge near null depends on our choice for the other resistors used in the circuit. What choice of resistors will produce the maximum sensitivity?

If we actually measured the sensitivity of the bridge circuit near null for all possible values of R we discover that the senstivity behaves as shown in Figure 9. Sensitivity near null is greatest when the resistance R, of the two lower arms of the bridge, equals the resistance of R_0.

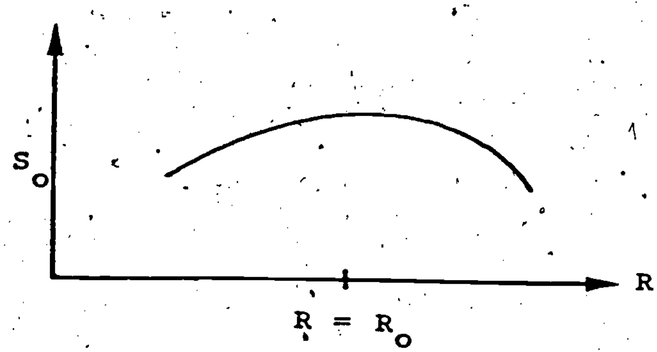


Figure 9. Sensitivity near null verses R.

Since we have assumed the lower arms of the bridge have equal resistance and the null condition requires $R_x = R_0$, the greatest sensitivity of this bridge will occur when all resistances are equal.

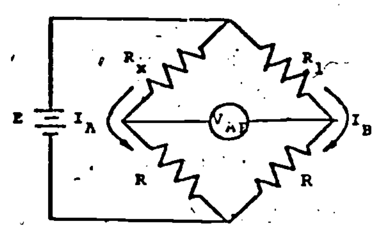
*Let us try to arrive at an equation for the sensitivity of a bridge. Look again at the bridge circuit we have been using in Figure 8.

Using Ohm's Law, we can calculate the current through each path of the bridge. Then, again by Ohm's Law, the voltage across each resistor is just the current times the resistance. The current I_A through the path containing R_x and R is given by

$$I_A = \frac{E}{R_x + R} \qquad (9)$$

The current I_B through the path containing R_1 and R is given by

$$I_B = \frac{E}{R_1 + R} \qquad (10)$$



Then the voltage across the resistor R on the left is

$$V_L = I_A R = \frac{E}{R_x + R} \qquad (11)$$

The voltage across the resistor R on the right is

$$V_R = I_B R = \frac{ER}{R_1 + R} \qquad (12)$$

The difference of these two voltages is the output of the bridge:

$$V = \frac{ER}{R_x + R} - \frac{ER}{R_1 + R} \qquad (13)$$

But the output voltage is zero unless R_x differs from R_1. Let's rewrite R_x as $R_I + \Delta R$ where ΔR is the change in R_x measured from the null condition. The output voltage which depends on ΔR is now given by

$$\Delta V = \frac{ER}{R_1 + \Delta R + R} - \frac{ER}{R_1 + R} = \frac{-ER\,\Delta R}{(R_1+R+\Delta R)(R_1+R)} \qquad (14)$$

Notice that ΔR appears two places in this equation, once in the numerator and once in a factor of the denominator. For situations of interest to us, how much difference will it make if we ignore ΔR in the denominator? Recall that we are talking about the sensitivity near null and at null $\Delta R = 0$. In our bridge problem we will be safe in ignoring ΔR in the denominator as long as it is much less than R_1+R. Therefore Equation 14 can be simplified to

$$\Delta V \simeq \frac{-ER}{(R_1 + R)^2}\,\Delta R, \text{ if } \Delta R \ll R_1+R. \qquad (15)$$

which is only valid near null. Otherwise we must use Equation 14 to calculate the output voltage. Combining Equations 8 and 15 we arrive at an equation for the sensitivity of the bridge near null,

$$S = \frac{\Delta V}{\Delta R} \simeq \frac{-ER}{(R_1 + R)^2}, \qquad (16)$$

if $\Delta R \ll R_1 + R$.

SENSITIVITY FORMULA

Example 4.

Q: Use Equation 16 to calculate the sensitivity of two bridge circuits near null using (the R values) given below.

a) R = 10K, R_x = 1K, E = 10V

b) R = 1k, R_x = 1K, E = 10V

A: For R = 10K

$$S = \frac{-10(10{,}000)}{(1000 + 10{,}000)^2} \text{ volts/ohm}$$

$$= \frac{-10^5}{1.21 \times 10^8} = -.83 \text{ mV/ohm}$$

For R = 1K

$$S = \frac{-10(1000)}{(1000+1000)^2} \text{ volts/ohm}$$

$$= \frac{-10^4}{4(10^6)} = -2.5 \text{ mV/ohm}$$

Are these findings compatible with the curves in Figure 8?

We have already observed that linearity is important if the output voltage is going to be an accurate representation of the physical input to the transducer, R_x. What condition must be satisfied in order to insure linearity? Examination of Equation 15 will show that it predicts a perfect straight line for the output voltage of the bridge. The necessary condition, that R be much less than R_1+R, is the condition for linear response of the bridge circuit. This means we can always improve linearity by increasing the value of R. (R_1 must stay the same as R_x for the bridge to satisfy the balance condition.) You should also note that the sensitivity is inversely proportional to R according to Equation 16. This means that any gains in linearity associated with an increase in R will be responsible for a corresponding decrease in sensitivity. There are two basic ways to recoup any such loss in sensitivity. The external voltage source, E, can be increased and the output signal can be amplified. The main limitation associated with increasing E is the power dissipation limits of the bridge elements. Self heating of the transducer due to high current in the bridge arms can be an important source of error in some measurements.

The amplifier would probably need to be a differential direct coupled amplifier as shown in Figure 10.

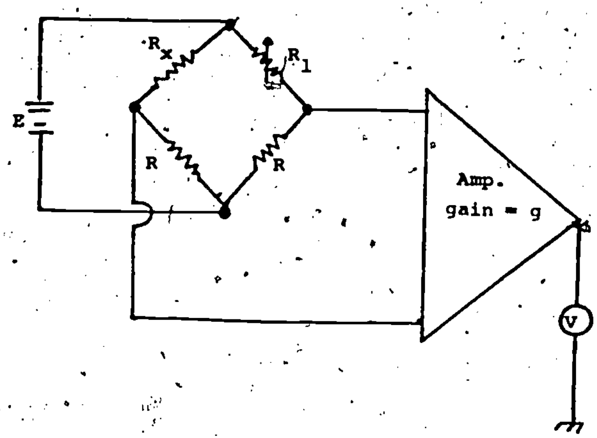


Figure 10. Using an amplifier to enhance the overall sensitivity of a bridge.

This building block is discussed in another module. The addition of an amplifier having gain, g, increases overall sensitivity to

$$S \simeq \frac{-gER}{(R_1+R_0)^2} \qquad (17)$$

QUESTIONS

1. A single thermistor is used in a DC bridge to produce an output voltage proportional to the change in temperature. The bridge is driven by a 3 volt DC source. The initial resistance of the thermistor is 1000 ohms and its temperature coefficient is $\Delta R/\Delta T = -1$ ohm/°C.

   a) What is the largest change in voltage possible at the output of the bridge if $\Delta T = 10°C$?

   b) Draw a schematic of the bridge circuit that will produce this output.

2. In question 1, a bridge circuit was designed that maximized sensitivity. It is known that this design will not have good linearity.

   a) What error is produced by non-linearity in the bridge for a temperature change of 100°C? To answer this question compare the predictions of Equations 14 and 15 and express the difference as a percentage.

   b) How could the bridge be modified to limit the non-linearity error in this measurement to less than 1%?

   c) How much gain would need to be added to the detector to recoup the lost sensitivity caused by the modification in b?

3. A Wheatstone Bridge like the one shown in Figure 6 has a voltage detector just sensitive enough to measure 50 microvolts of signal at the output. Express the precision of the bridge in terms of the minimum ΔR that will produce this output.

4. The Wheatstone Bridge discussed in Question 3 is to be used to calibrate some unknown resistances. It is decided that the accuracy of the bridge should be checked against some resistance standards that are available in the lab. The standard resistances are $R_1 = 10.17$ ohms, $R_2 = 101.34$ ohms, $R_3 = 999.97$ ohms, and $R_4 = 10,010$ ohms.

   a) Which of these resistors would you choose for this check? Explain your choice.

   b) Suppose the bridge is found to have an accuracy of .1%, what is the nearest significant bridge reading expected in measuring each of these standard resistors?

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5. The answers to Question 4 should have revealed a few of the limitations of the equal arm Wheatstone Bridge. Design an unequal arm Wheatstone Bridge capable of measuring unknown resistances of the range of 1 to 10 ohms with 1% accuracy. Assume you have available a .1% decade which can be adjusted in 1 ohm steps from 0 to 100,000 ohms. Assume also that a voltage detector is available which will provide all the sensitivity needed for these measurements. Draw a schematic diagram similar to Figure 7 showing the values and tolerances of each resistor. Explain the operation of your proposed circuit for measurements in this resistance range.

6. Check the unequal arm bridge design in Question 5 to see if there is a problem with excessive power dissipation in the low resistance arms of the bridge. Specify the maximum voltage that can be used to drive the bridge if the power dissipation in any element is to be less than the .1 watts.

**7. In the unequal arm configuration of the Wheatstone Bridge the derivation of bridge sensitivity will lead to two different results depending on whether the ΔR is associated with a change in the decade resistor or a change in the unknown resistor (as in the case of a transducer). Show that the expression for sensitivity, when the bridge is used with a transducer in the location of R_x, is given by

$$S = \frac{\Delta V}{\Delta R} \simeq \frac{-ER_2^2}{R_3(R_1+R_2)^2} \quad \text{if } \Delta R \ll \frac{R_3}{R_2}(R_1+R_2)$$

Proceed in your derivation along the lines used in Equations 9 through 16 with a modification in the logic at the point where the substitution is made for R_x near null. Recall that in the unequal arm configuration the balance condition is $R_x = \frac{R_1 R_3}{R_2}$ so that near null R_x should be replaced by $\frac{R_1 R_3}{R_2} + \Delta R$.

You can also show that this sensitivity is just $\frac{R_2}{R_3}$ times the sensitivity obtained when ΔR is associated with the decade resistor instead of a transducer at location R_x.

AC BRIDGE CONCEPTS

Objectives: Explain why AC bridges are often used instead of DC bridges.

Identify three fundamental differences between AC and DC bridges.

Distinguish the terms resistance, reactance and impedance.

Explain the fundamental differences between resistive and reactive circuit elements.

Explain precautions that should be recognized when adding voltages and current amplitudes in AC circuits.

Explain how Ohm's Law can be used in modified form in AC circuits.

Work problems involving all of these ideas.

WHAT IS AN AC BRIDGE?

An AC bridge is a variation of the Wheatstone Bridge, where the bridge is driven with an AC signal rather than DC and the arms of the bridge contain generalized resistance-like elements called impedances. These changes are illustrated in Figure 11.

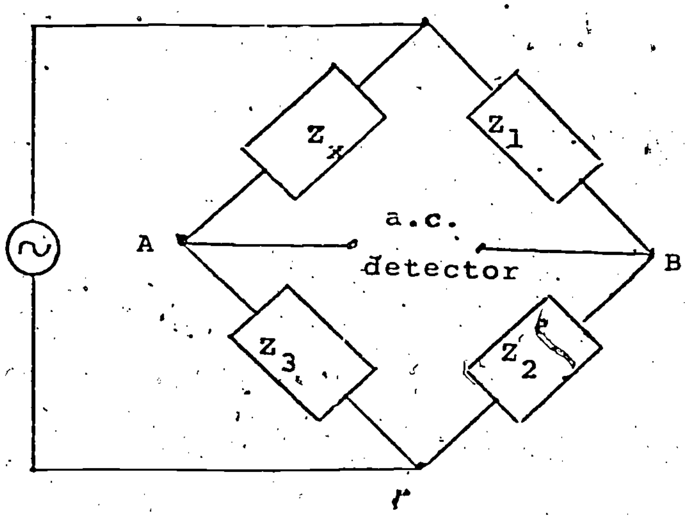

Figure 11. General schematic of an AC bridge.

The possible variations of this circuit are enormous since each impedance can be composed of a series/parallel network of resistors, capacitors and inductors. The mathematics that are used to discuss AC bridges involves the use of complex numbers which we wish to avoid using here. We will therefore restrict our attention to one common type of AC bridge which we can discuss in simpler terms.

ADVANTAGES OF AC BRIDGES

We have just mentioned that the sensitivity of DC bridges can be enhanced by adding an amplifier to the output of the bridge. When a Wheatstone Bridge is driven by AC, the balance conditions are still the same, but now the output is AC and can be amplified using an AC amplifier. This will eliminate errors due to offset and drift in the amplifier and effects an overall improvement in signal to noise. For this reason alone it is common to use AC for driving a Wheatstone Bridge. Some of the other advantages are more subtle and require a closer look at the properties of transducers and the balance conditions for AC bridges. In general, transducers produce both resistive and reactive electrical responses to a physical input. We have seen how the Wheatstone Bridge can be used to measure the resistive variations in transducer output. What is the origin of the <u>reactive component</u> in a circuit element or transducer? How can these characteristics be investigated? How is reactance different than resistance? How do these two ideas combine to give meaning to the word impedance? What do these ideas have to do with balance conditions in AC bridges? These are the questions we must now address.

Capacitors and inductors respond to the passage of an electrical current in a way that is fundamentally different than a resistor. At DC neither device plays a roll in circuit analysis except at the transition between DC levels. The capacitor look like an open circuit and the inductor looks like a short circuit. Since these two circuit elements are the origins of reactance in all networks we are already able to give this quantity a value at DC. If the reactance is due to a capacitor it is infinite and if it is due to an inductor it is zero. If we performed all measurements at DC we would never be able to determine the capacitance or inductance of a circuit element or transducer. The reactance would always have the same value regardless of the size of these components. Even this simple DC analysis shows that inductive reactance is different than capacitive reactance. The value of these two quantities may be calculate at any frequency using Equations 18a and 18b.

$$X_L = \text{Inductive reactance} = 2\pi fL \qquad (18a)$$

$$X_C = \text{Capacitive reactance} = \frac{1}{2\pi fC} \qquad (18b)$$

where f is the frequency in Hertz, L the inductance in Henries and C the capacitance in Farads. The unit for reactance is Ohms the same unit used for resistance.

The use of AC is essential for measuring the value or detecting the presence of either of these two electrical properties. It is sometimes important to realize that there is no such thing as a pure resistance. Every resistor will begin to manifest the charcteristics of a capacitor and/or an inductor at sufficiently high frequencies. AC bridges are used to investigate these imperfections in real resistors. It is also true that there is no such thing as a pure inductance or capacitance. At sufficiently high frequencies, resistive effects will begin to dominate their electrical characteristics. AC bridges are used to these effects as well. Transducers often exhibit a combined resistive and reactive response to a physical input. It is only possible to separate these two pieces of information using an AC bridge.

The application of AC signals to a pure resistive circuit element results in a voltage waveform that is precisely in phase with the associated current passing through the device. The current and voltage reach their maximum values precisely at the same time and are simultaneously zero at another time. In pure reactive elements the AC currents and voltages are precisely 90 degrees out of phase at all times. When the voltage reaches its maximum value, the current is passing through zero and vice-versa. This is the fundamental difference between resistive and reactive circuit elements. A consequence of this fact is that electrical power (current x voltage) is zero in any pure reactance. It can neither receive nor deliver any net power in an AC circuit. These phase shifts complicate the balance conditions for AC bridges. If balance is to be achieved, the AC voltages at points A and B in Figure 11 must not only have the same amplitude, they must also be in phase at all times.

The peculiar phase relationship between current and voltage also complicates the rules that must be used when these two quantities are added together to obtain the electrical quantity called impedance. The impedance of a circuit element, such as a speaker, is often specified by a quantity like 8 ohms. If you measure a speaker using a DC Wheatstone Bridge or an ohmmeter, the number obtained would probably be more like 4 ohms or less. Impedance can not be measured at DC because DC ignores the reactive contribution of the speaker's inductance and measures only the resistance. The reactive contribution dependes on the frequency used in the measurement and so will the impedance. Since all three of these quantities are measured in ohms it is easy to be confused. In simple elements, it is often possible to use impedances or reactances to compute the relationship between AC currents and voltages in much the same manner as Ohm's Law is used with resistance in DC circuits. See Figure 12.

DISTINCTION BETWEEN RESISTANCE AND REACTANCE

TWO BALANCE CONDITIONS IN AC BRIDGES

FORMULA	APPLICATION	PHASE SHIFT	FREQUENCY DEPENDENCE
$X_C = \frac{V}{I}$	Capacitive elements	I leads V by 90 degrees	$= \frac{1}{2\pi fC} \propto \frac{1}{f}$
$X_L = \frac{V}{I}$	Inductive elements	I lags V by 90 degrees	$= 2\pi fL \propto f$
$R = \frac{V}{I}$	Resistive elements (Ohm's Law)	I and V in phase	None
$Z = \frac{V}{I}$	Complex Networks	all shifts from +90 to -90 degrees possible	Complex

Figure 12. How to use Ohm's Law for comparing the magnitudes of currents and voltages in resistive, reactive and complex circuit elements.

Apparent discrepancies are often observed when the familiar rules for adding voltages or currents in DC circuits are used improperly in AC circuits. Two examples are shown in Figure 13.

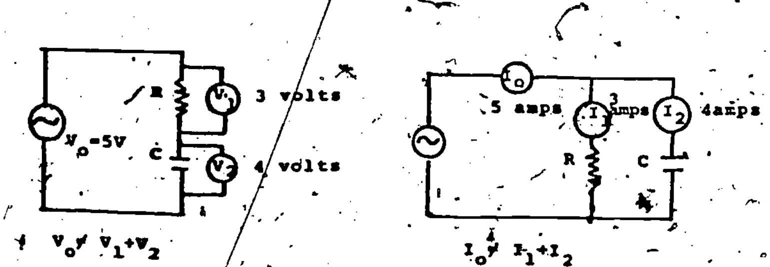

Figure 13. Two examples of apparent violation of basic circuit principles in AC circuits.

Such discrepancies always occur when the amplitudes of AC quantities are added without recognizing that each of the quantities involved attained their respective maximum values at different times. At any instant in time the voltages and currents will add correctly in the familiar way (see Figure 14) but one can not simply add the largest values of AC quantities and ignore the phase relationship between the waveforms.

As we have already said, formal analysis of these concepts requires the use of vectors and/or complex mathematics. Several appropriate resources are mentioned in the reference material for the pursuit of these ideas. On the other hand, one can easily verify these concepts by direct measurement in the lab.

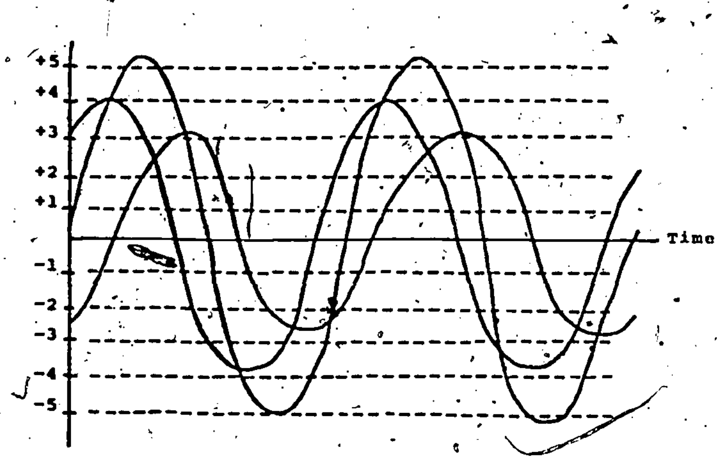

Figure 14. Three AC waveforms with peak amplitudes of 3, 4, and 5 volts. The phase shifts are such that at any instant the magnitudes of the 3 and 4 volt waveforms can be added together to obtain the magnitude of the 5 volt waveform.

THE WIEN BRIDGE

Objectives: Explain the relationship between the two general requirements for balance in AC bridges and the balance conditions of the Wien Bridge. (The relationship can be seen through an explanation of the circuits response to differenct frequency AC signals.)

An actual AC bridge that illustrates all of the concepts we have just discussed is shown in Figure 15.

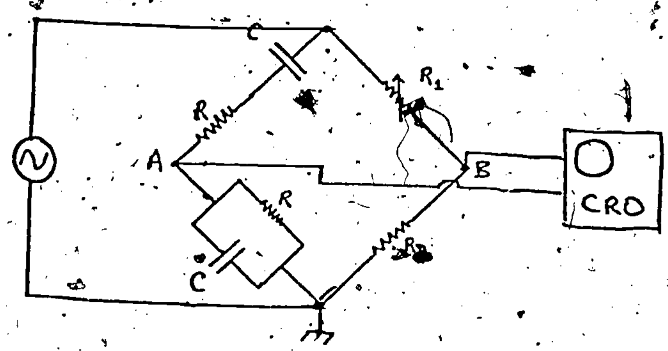

Figure 15. The Wien Bridge with cathode ray oscilloscope detector and adjustable AC signal source attached.

AC input to the bridge circuit is provided by a variable frequency signal generator and the unbalanced output, if any, is detected with an oscilloscope. There is a frequency at which the two balance conditions for an AC bridge will be satisfied. That frequency is given by Equation 19,

$$f = \frac{1}{2\pi RC}, \qquad (19)$$

if the two Rs and the two Cs are identical. If the components in the left hand side of the bridge are not matched, balance is still possible but the frequency will differ from that predicted by Equation 19. If the components in the left hand side are properly matched a null should be observed when R_1 is set equal to twice R_2.

In this AC bridge the two parameters that must be adjusted to obtain a null are relatively independent or <u>orthogonal</u> to each other. This is a highly desirable situation in AC bridges. If substantial interaction exists between the two parameters chosen for adjustment, it is practically impossible to devise a procedure for finding the two null conditions of the bridge. There is no substitute for a liberal dose of formal mathematical analysis of the circuit to unravel these mysteries of AC bridge design.

It is instructive to briefly consider the action of each side of the bridge in satisfying the two balance conditions. The right hand side of the bridge is a purely resistive voltage divider network. The AC signal appearing at point B will be in phase with the applied AC signal no matter what frequency is chosen. The amplitude is adjustable over a wide range depending only on the relative values selected for R_1 and R_2. If a null is ever going to be achieved with this bridge, the signal at point A will have to be in phase with the applied AC signal and hence with the signal at point B. Once this match in phase is achieved, the amplitude of the two signals can be matched by adjusting R_1.

The resistive and reactive components in the left hand side of the bridge are arranged so that the phase of the signal at point A is a function of the applied frequency. At very low frequency, the amplitude of the signal at point A approaches zero and the phase shift approaches 90 degrees ahead of the driving signal. At very high frequencies, the amplitude of the signal approaches zero but now the phase shift approaches 90 degrees behind the driving signal. Somewhere between these two extremes, the amplitude reaches a relative maximum and the phase shift goes to zero. As soon as we have discovered the frequency where the phase shift at point A goes to zero the bridge can be balanced. Simply adjust R_1 so that the amplitude at point B matches the amplitude at point A and null is achieved.

QUESTIONS

8. The current through each element in any series circuit must have the same value at any instant. Suppose an AC current of 1 amp passes through a series connected resistor of 4 ohms and capacitive reactance of 3 ohms.

    a) Using Figure 12, predict the magnitude of AC voltage that would be measured across both elements.
    b) What phase shift must exist between these two voltage waveforms?
    c) Carefully sketch both voltage waveforms with the proper phase relationship for one complete cycle.
    d) Add these two waveforms together to find the waveform of the total voltage across the series combination.
    e) Again, refer to Figure 12 and compute the impedance for this series network.

9. The voltage across each element in any parallel circuit must have the same value at any instant. Suppose an AC voltage of 12 volts is applied to a parallel combination of 4 ohms resistance and 3 ohms inductive reactance.

    a) Using Figure 12, predict the magnitude of AC current that would be measured through both elements.
    b) What phase shift must exist between these two current waveforms?
    c) Carefully sketch both current waveforms with the proper phase relationship for one complete cycle.
    d) Add these two waveforms to find the waveform of the total current through the parallel combination.
    e) Compute the impedance for this parallel network using Figure 12.

10. The AC bridge shown in Figure 14 is called a Wien Bridge. One of the balance conditions is shown in Equation 19. Show that this balance condition is equivalent to the statement that at null the capacitive reactance of C equals the resistance of R.

**11. Analyze the left hand side of the bridge in Figure 14 using the methods developed in Questions 8 and 9 to show that at null the impedance of the upper arm is twice that of the lower arm. This conforms with the second balance condition for the Wien Bridge, i.e. $R_1=2R_2$.

LABORATORY

Objectives: Build a Wheatstone Bridge and use it to

a) measure several unknown resistances,
b) measure and maximize the sensitivity,
c) measure output voltage and observe non-linearity and
c) balance out the initial offset resistance of a thermistor.

Build a Wien bridge and use it to

a) verify two balance conditions for AC bridges,
b) study and record the frequency dependence of the phase shift produced in the arms of an AC bridge, and
c) balance out the initial offset reactance of an AC transducer.

MATERIALS AND EQUIPMENT

1	DC power supply with several output voltages
1	Signal Generator
1	Oscilloscope
1	Decade Resistance Box, 0 to 100k
1	Thermistor
3	10k resistors, two matched
2	Matched 1k resistors, 3.9k resistors, 39k resistors and 100k resistors
2	Capacitance Substitution Boxes or several matched capacitors .001 μfd $<$ c $<$ .ol μfd
	Assorted coils of copper wire
1	1:1 Audio Transformer

EXPERIMENTAL PROCEDURE

The laboratory will focus attention on the two basic types of bridge circuits we have studied: Wheatstone and Wien bridges. We will use the Wheatstone Bridge to measure several unknown resistances, to measure and maximize the sensitivity of the basic bridge, to measure and observe the non-linearity of the output voltage and to use the bridge to null out the offset voltage produced by the initial resistance of a thermistor. The Wien Bridge will be used to investigate the two balance conditions necessary for null in an AC bridge, to study the frequency dependent phase shift produced by impedance networks in the arms of the bridge and to build a metal detector with a reactive transducer

1. For our first experiment we will use the Wheatstone Bridge to measure a few "unknown" resistances. Build the circuit shown in Figure 6 using the DC power supply to drive the bridge and the oscilloscope for detecting the output voltage. The oscilloscope should be direct coupled to the bridge. The zero position for the trace should be set to the vertical midpoint of the scale and the senitivity should be set initially at about 1 volt/division. Use two matched 10k resistors in the bridge.

2. The next step in using a Wheatstone bridge is finding the balance or null condition. From the discussion we know that the unknown resistance will equal the setting of the decade resistance at null if the 10k resistors are matched. One systematic procedure for finding null begins by setting all the decades to their highest value, nine. When the power supply is turned off the oscilloscope trace should still be vertically centered on the screen. When the power supply is turned on, the trace will deflect away from the zero postion. Adjust the oscilloscope sensitivity so that the deflected beam remains on the screen.

   Proceed to decrease the decade resistance in the largest possible steps and observe the output voltage from the bridge. As the output moves toward null, continue decreasing the setting of the decade. If the output moves away from null, the unknown resistance is greater than the decade resistor and measurement with this circuit is impossible. When the output passes through null, return that decade switch to its previous setting. Then drop down to the next lower significant decade and repeat this procedure.

   As we get closer to null it will be necessary to continue increasing the sensitivity of the oscilloscope. This will improve our ability to determine the precise null point and hence the precise value of the unknown resistance. A useful trick for distinguishing small departures from null is to switch the power supply on and off while looking for a deflection of the trace. At null no deflection should be observable.

At some point it will no longer be possible to improve the null as observed on the oscilloscope. The setting of the decade resistor, rounded off to the last adjusted decade, is the best value this circuit can give for the unknown resistance. Can you suggest another systematic method for finding the null or balance condition?

3. Determine the precision of the bridge. Express this in terms of the smallest steps on the decade resistance that will produce an observable change in the output. Report sources of error in the measurement process that will degrade the precision of the measurement. Can the procedure be improved to overcome the error? Considering the precision of this bridge, what is the lowest percentage tolerance warranted for the decade resistance and fixed resistors? Explain your reasoning.

4. Determine the sensitivity of the bridge. Use the definition of sensitivity in Equation 8. Measure the change in voltage on the output and divide by a corresponding change in the resistance of the decade resistor. Calculate the predicted sensitivity according to Equation 16. Record and compare your calculation with your measurement. Replace the 10k resistors in turn with the 1k, 3.9k, 39k and 100k matched pairs and repeat the sensitivity measurement and calculation. Summarize your observations in a table.

5. From the information on bridge sensitivity obtained in Step4, create a graph similar to Figure 9. What conclusions can be drawn from these data about maximizing bridge sensitivity? Verify that changing the power supply voltage has the expected effect on bridge sensitivity.

6. Verify that the balance conditions for this bridge are the same with an AC drivign signal as they were for DC drive. Replace the power supply with the variable frequency signal generator and devise a nulling procedure for AC signals. Do the results agree? Is it easier to find null?

7. It is possible to use a thermistor as the unknown resistor in our bridge circuit. Since the resistance of the thermistor changes as the temperature changes, this circuit can be used as an electronic thermometer. Optimize the sensitivity of the circuit, calibrate the output and record your design.

8. Build a Wien Bridge using the schematic shown in Figure 15 where R_1 is the decade resistance box or a 100k pot, R_2 is 39k, R is 10k and C is somewhere between .01 and .001 microfarads. The circuit shows the bottom corner of the bridge grounded. This presumes that neither of the oscilloscope connections are referenced to ground, i.e. the oscilloscope has differential inputs. If two points on the bridge circuit are grounded through connections to the oscilloscope and signal generator, the circuit will behave in an unexpected manner. Unformtunately this situation often arises without recognition by the user. Multiple grounding of this circuit will effectively short out one of the essential bridge arms through an external "ground loop" and no null will be possible. There are several ways to avoid this problem. One easy solution is to introduce the AC signal through an audio transformer and leave the bottom corner of the bridge ungrounded. You can then safely ground any one point in the bridge circuit with one of the oscilloscope leads and measure the voltage differences at any other point in the circuit. Differential input is no longer needed.

9. Find the null conditions for the Wien Bridge. Again a procedure is needed to systematically determine the balance or null point of the bridge. Since two conditions must be satisfied for balance we should expect to simultaneously minimize bridge output for two adjustable parameters. The two parameters that are within our power to adjust are the frequency of the AC signal source and the decade resistor. If either of these is very far from balance, the null in the output voltage may be very slight. It may be necessary to minimize the output through adjustment of the frequency parameter, then adjust the decade for best null and repeat this cycle through several iterations before finding the best null.

10. Verify that the balance conditions for the Wien bridge agree with theory. Does the measured frequency agree with the prediciton of Equation 14? Is R_1 twice the size of R_2? Are observed discrepancies resonable? Explain.

*11. Measure the output amplitude and phase shift as a function of the frequency of the AC signal source. These measurements require considerable expertise in using the oscilloscope. Phase shift measurements are always made with respect to a reference. The AC signal source will serve that function here. Trigger the oscilloscope on the bridge input waveform and the phase shift at the output of the bridge can be obperved on the oscilloscope display. Figure 16 shows a block diagram for doing this.

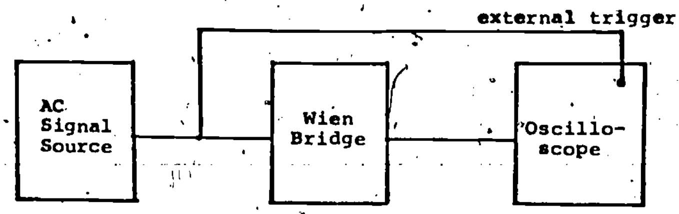

Figure 16. A block diagram of the experimental setup for measuring output phase shift from the Wien Bridge.

Begin the measurements with the bridge balanced, then vary only the frequency of the signal source. Amplitudes and phase shifts change most rapidly near null. They both should be zero when the bridge is balanced. Gather enough data so you can sketch a graph of both of these quantities for all frequencies.

**12. Add an inductive transducer to the basic Wien Bridge and discover how the null conditions are modified. A good place to add this circuit element is in series with R_2 in that arm of the bridge. The transducer may just be a coil wire which will sense the proximity of metallic or magnetic objects. If the transducer is going to be sensitive as a sensing device, the inductive reactance will need to be about the same size as R_2. Use Equation 18a to estimate the approximate inductance desired. It may be difficult to construct an open coil of wire having this much inductance. You may find it useful to construct a variety of coils and experiment with them in this circuit to see how each modifies the balance conditions. Compare theamount of unbalance signal that is, produced when a "standard" metal object is placed in the center of each coil. Alternatively, it may be easier to study the effect of changing the size of R_2 on the relative sensitivity of the bridge. Other circuit locations for the inductor appear to be possible but study of the phase shifts and amplitudes that are produced as a function of frequency at points A and B will show that null is not always possible. A little experimentation will soon convince you that, in the long run, the analytical approach will epxedite AC bridge design. See the Reference material for some suggested reading on this subject.

GLOSSARY

AC BRIDGE — an electrical c.rcuit having four arms, each arm consisting of an impedanc., which are connected together in a loop having four junctions. This circuit is driven at two opposing junctions by an AC signal source and the output of the circuit is detected by an "C detectors connected between the other two junctions. It .s generally necessary to satisfy two balance conditions in order to null the output.

ACCURACY — a specification of any measurement process which expresses the degree to which the process is able to determine the actual value of the measurement. This results in an uncertainty which is usually express.d as a percentage of the measured quantity. It is often referred to as the tolerance of the measurement process. For example, resistors are color coded according to the values measured in processes whose tolerance is 5%, 10% or 20%.

ARMS — in bridge circuits, four impedances are connected in a loop at four junctions. These four junctions define the points at which external connec.ions are made to the basic bridge circuit. The four circuit portions that lie between each of these four corners are called the arms of the bridge.

BALANCE CONDITION(S) — in bridge circuits, the combination of circumstances that must be met if the output is to be zero for non-zero input. These circumstances usually involve necessary relationships between the circuit parameters such as the ratio of resistances or the frequency of the driving signal.

BALANCED — in bridge circuits, the state in which the output is zero or null when the input is non-zero.

COMPLEX MATHEMATICS — the body of theory which defines the rules for manipulating quantities that must be specified by two independent numbers. It is especially useful in the analysis of the electrical quantity called impedance, which is composed of the two quantities, resistance and reactance.

COMPLEX NUMBERS — numbers which are composed of two independent parts. The theory of this number system is called complex mathematics. It is important in electronics because impedance can be represented by a complex number, $Z=R+jX$, where R is the resistance, X is the reactance, and j is the symbol used to distinguish one part of the number pair.

IMPEDANCES — circuit elements which contain resistive and/or reactive parts.

LINEARITY — a term used to describe the relationship between two interrelated circuit parameters. In the range in which they are proportional, the circuit exhibits linear response. Otherwise the response is non-linear.

ORTHOGONAL — in AC bridge circuits, the term used to define those bridges whose design permits adjustment of either balance parameter without alteration of the other balance condition.

PRECISION — a term used to express the resolving power of a measurement process. It may be conveniently stated in terms of the smallest change in the measured quantity that can be discerned through the measurement process. Another way of stating the idea is by specifying the minimum distinguishable difference between two measured quantities. The precision of a Wheatstone Bridge could be specified as ±.1 ohms, meaning that changes this large are needed to measurably alter the bridge output.

PRECISION DECADE RESISTANCE SUBSTITUTION BOX — a switchable resistance box in which tighter than normal standards have been followed in order to insure that the total resistance selected is accurate to a predetermined tolerance such as 1% or .1%.

REACTIVE COMPONENTS — circuit elements in which the applied AC currents and voltages are 90 degrees out of phase. These effects are observed in inductors and capacitors.

SENSITIVITY — a ratio between the changes in two inter-related circuit parameters. In bridge circuits, the change in the output voltage divided by the corresponding change in the impedance of one of the arms.

VECTORS — physical quantities which have both magnitude and direction. They are sometimes used to represent AC voltage where the magnitude is used to represent the peak voltage and the direction is used to represent the relative phase.

VOLTAGE DETECTOR — a measuring device which is sensitive to small voltages.

WHEATSTONE BRIDGE — a circuit consisting of four resistances connected together in a loop at four junctions. Two opposing junctions or corners are used for connection to an external voltage source. The remaining two junctions provide output to a voltage detector. The output is zero for certain resistance ratios which makes the circuit useful for measurement and calibration of unknown resistors.

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2. Stout, M. B., Basic Electrical Measurements, Prentice-Hall, Inc., Second Edition, 1960.

   Classical resource on bridge circuits.

3. Wedlock, Bruce D. and James K. Roberge, Electronic Components and Measurements, Prentice-Hall, Inc., 1969.

   Models for and measurement of real capacitors, inductors and resistors.

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ANSWERS

1. 7.5 mv, $R_2=R_3=1k$.
2. 5%, 9k, g = 5.
3. About 5 ohms.
4. a) R_1 can be verified to within 50%
   R_2 can be verified to within 5%
   R_3 can be verified to within .5%
   R_4 can be verified to within .05%, use this for testing the accuracy of the bridge.

   b) 10; 100; 1000; 10,010.
5. $R_2 = 100R_3$, both R_2 and R_3 should be .5% tolerance or better.
6. $E_{max} = \sqrt{P_{max}R_3}$
8. a) 4 volts, 3 volts.
   b) 90^o
   c) d) } See Figure 14.
   e) 5 ohms
9. a) 3 amps, 4 amps.
   b) 90^o
   c) d) } See Figure 14.
   e) 2.4 ohms