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ABSTRACT

A linear program model designed to aid in site selection and the development of pupil assignment plans is illustrated in terms of a hypothetical school system. The model is designed to provide the best possible realization of any single stated objective (for example, "Minimize the distance that pupils must travel") given any number of specified constraints (for example, "No school may have an enrollment greater than its design capacity," or "Racial percentages must be within predetermined bounds"). The model uses two programs written in Fortran, both documented in full in an extensive appendix. (Author/PGD)

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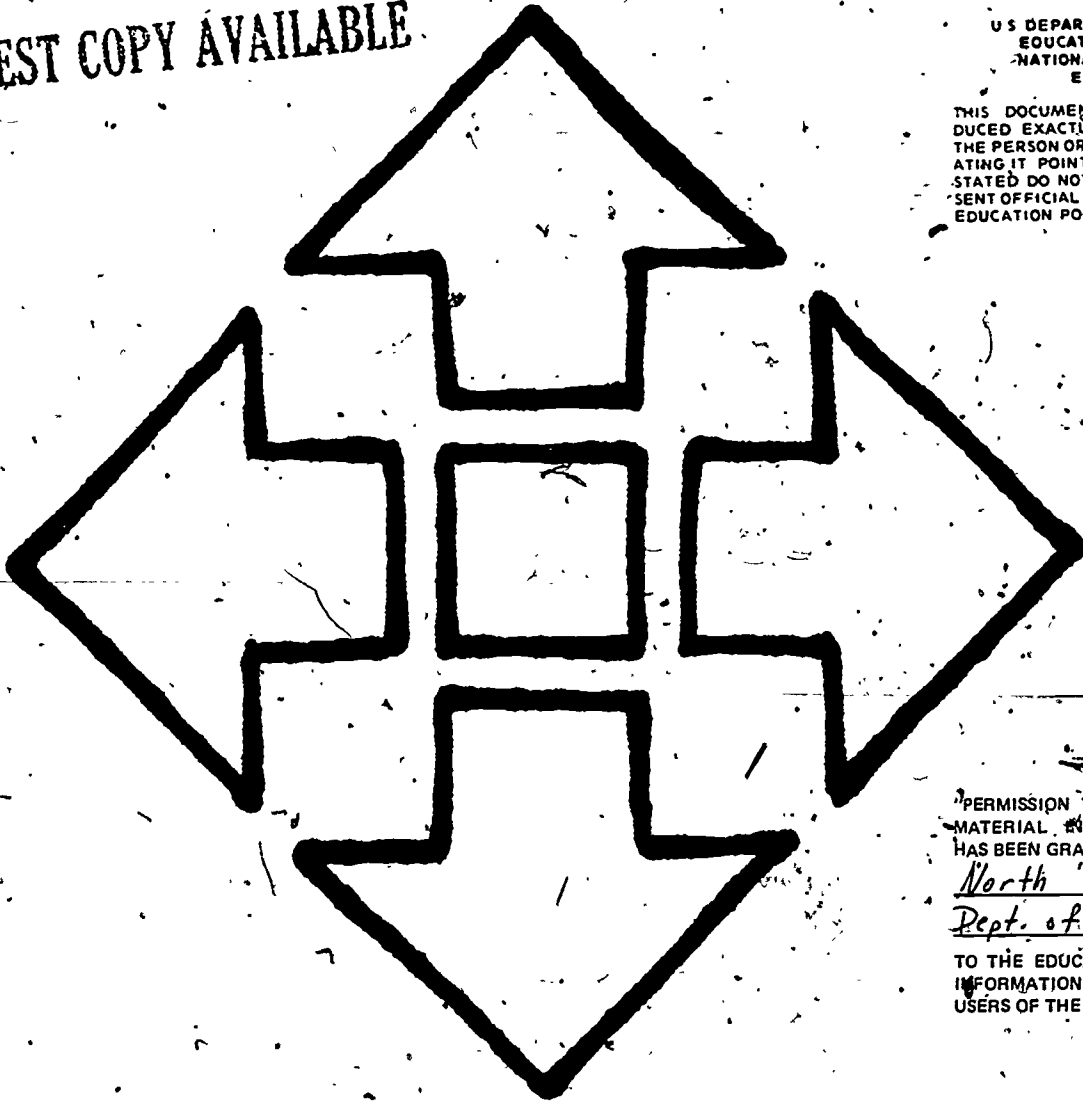
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A Programming System for School Location & Facility Utilization

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FOREWORD

As a consequence of declining enrollments, and the changing nature of our school systems, decisions regarding the location of schools, the assignment of students, and the utilization and organization of facilities have become very important.

Among the many factors contributing to this dynamic state are: new site selection, shifts in population, changes in educational programs, obtaining a desired balance of minority groups in the schools, and the merger and consolidation of two or more schools or school systems.

Local school administrators are constantly faced with the task of making decisions which maintain a delicate balance between quality education and economic efficiency. These are difficult tasks and ones which require that the best information obtainable be available when making these decisions.

Many of the quantitative tools of management available today are applicable to the problems faced by school officials. This publication describes the application of one such tool, a linear programming model developed jointly by Mr. Dave Norris, Dr. Robert Sowell, and Dr. James Young, at the Center for Urban Affairs, North Carolina State University, and Mr. Eugene Cheatham, Division of School Planning. The purpose of this model is to aid in the solution to problems of site selection and the development of pupil assignment plans for minimum transportation of pupils in a school system.

As with most management tools, linear programs utilize computers for solutions to problems posed by decision makers and this combination provides the administrator the capability to consider numerous alternatives and make decisions based on the desirability of plans resulting from the alternatives considered.

Consultants from the Division of School Planning are available to assist local units in preparing the data necessary to use the model, and in interpreting the alternatives generated by the model.

Spring, 1977

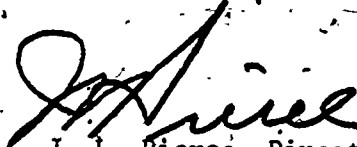

J. L. Pierce, Director
Division of School Planning

TABLE OF CONTENTS

	Page
Preface	
I. Setting Goals (Objectives and constraints)	1
II. Data Requirements	6
III. Solution Methods	7
IV. Program Results	10
V. Program Options	17
VI. Other Applications	31
VII. System Documentation	37
VIII. Example Output	57
IX. Job Control Language and Run Sequence	66
X. File Organization	71

MAPS AND CHARTS

	Page
Figure 1	2
Figure 2	8
Figure 3	9
Figure 4	11
Figure 5	12
Figure 6	14
Figure 7	16
Figure 8	20
Figure 9	22
Figure 10	24
Figure 11	25
Figure 12	27
Figure 13	28
Figure 14	30
Figure 15	33
Linear Programming System Flow	40
Program 1 (Data Entry Flow).	42
Program 2 (MPS Solution Flow).	45
Pupil Assignment Plan for the Mecklenburg County School System	60
Classification of Pupils for Each Area	61
Distances (in miles) Between Schools and Assignment Areas	62
Assignment Information for each Assignment Area	63
Assignment Information for (A) SHIA	64
Summary Statistics (A) Mecklenburg County School System	65

I. Setting Goals (Objectives and Constraints)

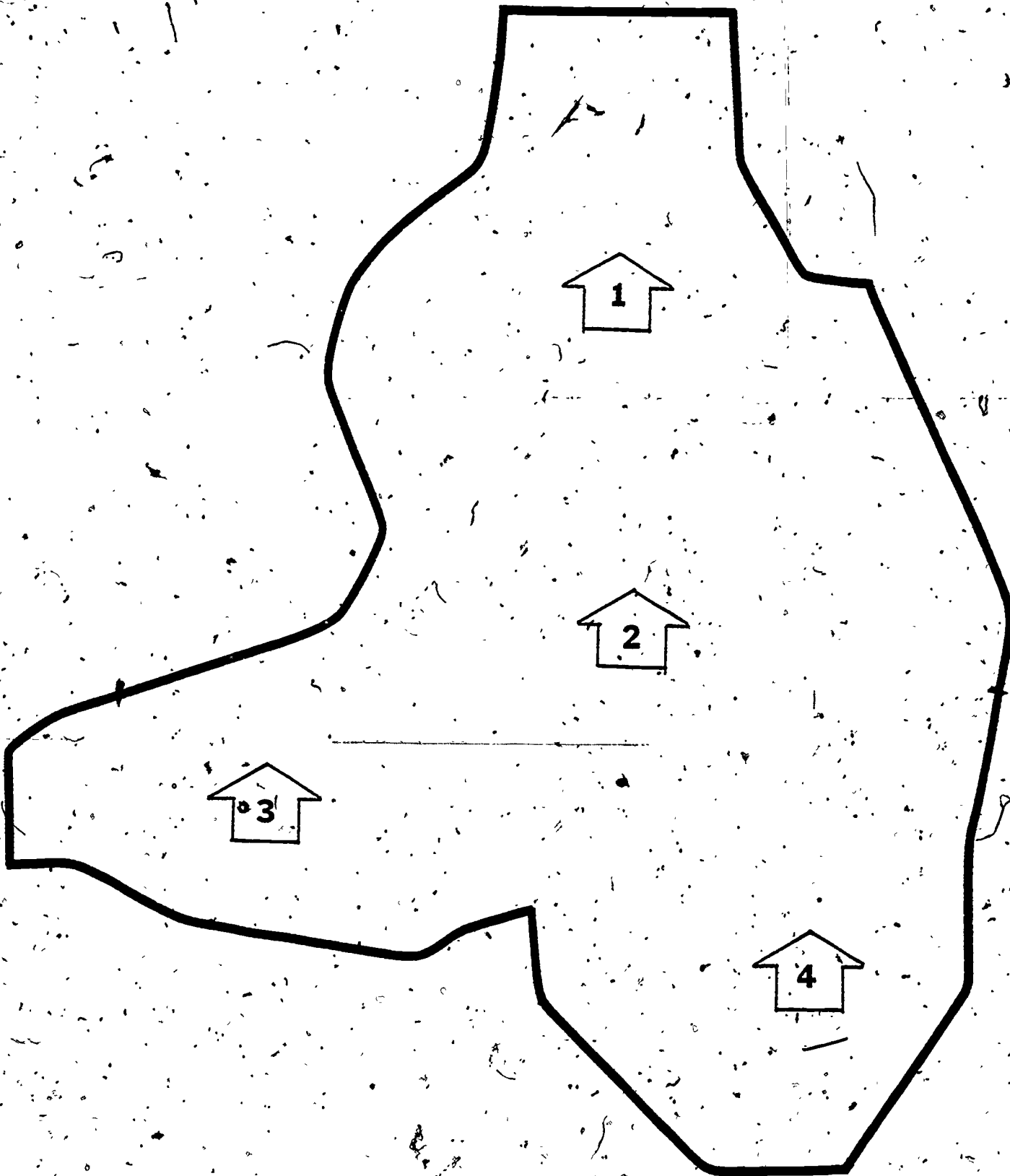
To illustrate the use of computer techniques for the development of pupil assignment plans and site selection for new schools, let's use a hypothetical example. As school administrators, let's assume that we have the ABC School System shown in Figure 1. The ABC School System has four elementary schools located as shown on the map. Our first job is to develop a pupil assignment plan for the elementary school children within the school system. What do we do? Where do we start? The number of ways in which assignments could be made are practically unlimited. Thus, we must set some goals for the plan we are to develop.

What kind of pupil assignment plan do we want? Let's look at three possible goals for the plan:

1. Every pupil can walk to a school located within a few blocks of home.
2. Each school will have an optimum number of pupils enrolled.
3. The racial, social, and economic makeup of each school will be the same as the average for the entire system.

Other goals could be listed but let's consider the above three in more detail in order to illustrate the procedure. Consider goal #1: Is it possible for all pupils to attend a school which is only a short walk from his or her home? Only if the area of the school system is very small and the population density is very high, would it be possible to have a school located near each student. Thus, the goal is impractical for the vast majority of situations. Consider goal #2: Can all the schools have an optimum enrollment? What is an optimum enrollment anyway? It would certainly be an extremely rare situation in which the location of schools and pupils was such that each school would have a desired enrollment with all pupils being able to walk only a short distance from their homes to school. Thus, the com-

Figure 1
Geographic boundaries and location of
schools in ABC School System



ination of goals 1 and 2 is impractical. Consider goal #3: Is there a homogeneous mix by race, social status, and economic level throughout the system? If not, it will be impossible to achieve the same mix in each school while still meeting goals 1 and 2. Now, although we listed only three goals for our pupil assignment plan, it is readily apparent that we cannot meet all three and perhaps we cannot meet any of the three. They are too general and they conflict with each other. We must be more specific and more realistic. Perhaps we could state them as follows:

1. Each pupil can attend the closest school.
2. Each school will have an enrollment less than its design capacity.
3. The racial makeup of each school will be within predetermined limits.

If we now consider the set of goals, we see that goal #1 could easily be met if we neglect goals 2 and 3. However, it is extremely unlikely that schools and pupils will be so strategically located that goals 2 and 3 can be met at the same time as goal 1. If we consider only goals 2 and 3, there is no conflict as long as the total capacity of the schools exceeds the number of pupils. They could both be met by proper assignment of pupils to schools. Further examination of goals 2 and 3 reveals that they are not really so much goals for a pupil assignment plan as they are limitations to be placed on the plan. These limitations are the result of the physical design of school buildings and of government and/or school system regulations concerning racial makeup of schools. The limitations must be met. In contrast, goal #1 is a desired characteristic of the pupil assignment plan but is not required. Thus, we would like for all pupils to be able to attend the closest school, but only if the school enrollments and the racial ratios are within certain limits. Since it is unlikely that goal #1 can be fully met, we would like to meet it as nearly as possible without violating the limitations imposed by 2 and 3. Thus, our original three goals might be stated as an "objective"

and two "constraints" as follows:

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: 1. No school may have an enrollment greater than its design capacity.
2. The racial percentages in each school must be within predetermined limits.

The terms "objective" and "constraints" are used by mathematicians and other scientists to describe the goals and the limitations of problems and their solutions. The "constraints" are the limitations which must be met by whatever solution is obtained. The "objective" is the goal which is to be met without violating the constraints. There can be only one objective but there may be many constraints. However, the constraints cannot conflict with each other.

Let's take a simple example which does not involve a school system.

Suppose we are planning an automobile trip from North Carolina to California and we wish to complete the trip as quickly as possible. If we could take a straight-line path for the trip and drive twenty-four hours a day at the maximum speed which the automobile can reach, we would complete the trip in a minimum of time. However, there are some limitations which must be met.

Our objective and some possible constraints may be stated as follows:

Objective : Minimize the time required for a trip from North Carolina to California.

Constraints: 1. Travel must be along paved highways.
2. Speed limits and traffic signals must be observed.
3. Stops must be made for refueling.
4. Twelve hours per day must be reserved for meals and sleep.

Once the objective and the constraints are identified, a route and schedule

may be determined which minimize the travel time without violating the constraints.

Now, back to the development of pupil assignment plans. The computer procedures discussed in this document allow the development of plans which meet certain objectives while complying with certain limitations or constraints. The objectives which may be chosen by users of these computer procedures are listed below.

Alternative Objectives:

1. Minimize the distance that pupils must travel from their homes to schools. (Includes walking distances as well as bussing distances.)
2. Minimize the distance which pupils must be bussed from their homes to schools. (Excludes distance pupils walk.)
3. Minimize the number of pupils bussed.
4. Minimize the number of pupils bussed away from closest school.

Note that only one of the above listed objectives can be chosen for the development of a particular plan. A different plan would be developed for each of the objectives.

There are also a number of constraints which may be chosen by users of the computer procedures. They are listed below.

Possible Constraints

1. No school may have an enrollment greater than its design capacity.
2. The racial percentages in each school must be within predetermined limits.
3. No pupil may be assigned to a school which is more than a predetermined distance from his or her home.
4. No pupil may be bussed to a school which is farther from his or her home than the second closest school.

Note that any combination of the possible constraints may be chosen. They may all be simultaneously specified for the development of a plan.

II. Data Requirements

Now, let's assume that as school administrators in the ABC School System, we have defined our objective and constraints for the pupil assignment plan we are to develop. What do we do next? How do we get the plan which best meets the objective? This is the function of the computer procedures described in this manual. Once an objective and appropriate constraints have been agreed upon by administrators, the problem becomes a mathematical problem which may be solved using a computer. However, the system must be described in numerical terms so that the computer can make the necessary computations. This numerical description involves data on the location and capacity of schools and the location and race of pupils. The school locations may be described in terms of X- and Y- coordinates on a map while their capacity is a specification of the maximum number of pupils to be enrolled. Each individual pupil could also be located in terms of X- and Y- coordinates on the map and his or her race specified. However, this would require a tremendous amount of unjustifiable work. The procedure used here to describe pupil locations will be to divide the total system into a number of small areas and to consider all pupils within the areas to be located at the center of population of the area. This system subdivision is illustrated for the ABC School System in Figure 2. The System has been divided into fifty-two smaller areas and population centers for the areas approximated manually. These population centers are represented in terms of X- and Y- coordinates as shown in Figure 2. The areas may be of any shape so that street or natural barriers may be followed in area definition. The areas should be more or less uniform with respect to racial housing patterns within the area. The important point to remember in area definition is that all pupils within the area will be assigned to a school as if they all lived at the population center of the area.

Once areas within the System are defined, the number of pupils of both majority and minority races are determined. These data are shown on Figure 3 for the ABC School System. The computer procedures then use the school and area data to develop the assignment plan.

The data collection steps may be outlined as follows:

1. Divide system into small areas;
2. Describe location of each area,
3. Describe location of each school,
4. Determine capacity of each school,
5. Determine number of pupils (by grade level) of majority race in each area, and
6. Determine number of pupils (by grade level) of minority race in each area.

III. Solution Methods

Assuming that we have defined our objective and constraints and have collected the necessary data, a solution can be developed mathematically utilizing a computer. The procedure for obtaining a solution consists of two primary steps. They are:

1. The estimation of distances between each school and area combination, and
2. The determination of a pupil assignment plan which satisfies the constraints and best meets the objective.

The distances between schools and areas are estimated mathematically by calculating straight line distances between their respective coordinates. These distances may be increased by an appropriate factor to adjust for the fact that actual street or highway distances will be greater than the straight-line distances. An option to be discussed later will allow the inputting of actual distances if desired.

Figure 2
 Sub-areas and population center of sub-areas
 /X&Y coordinate/ in ABC School System

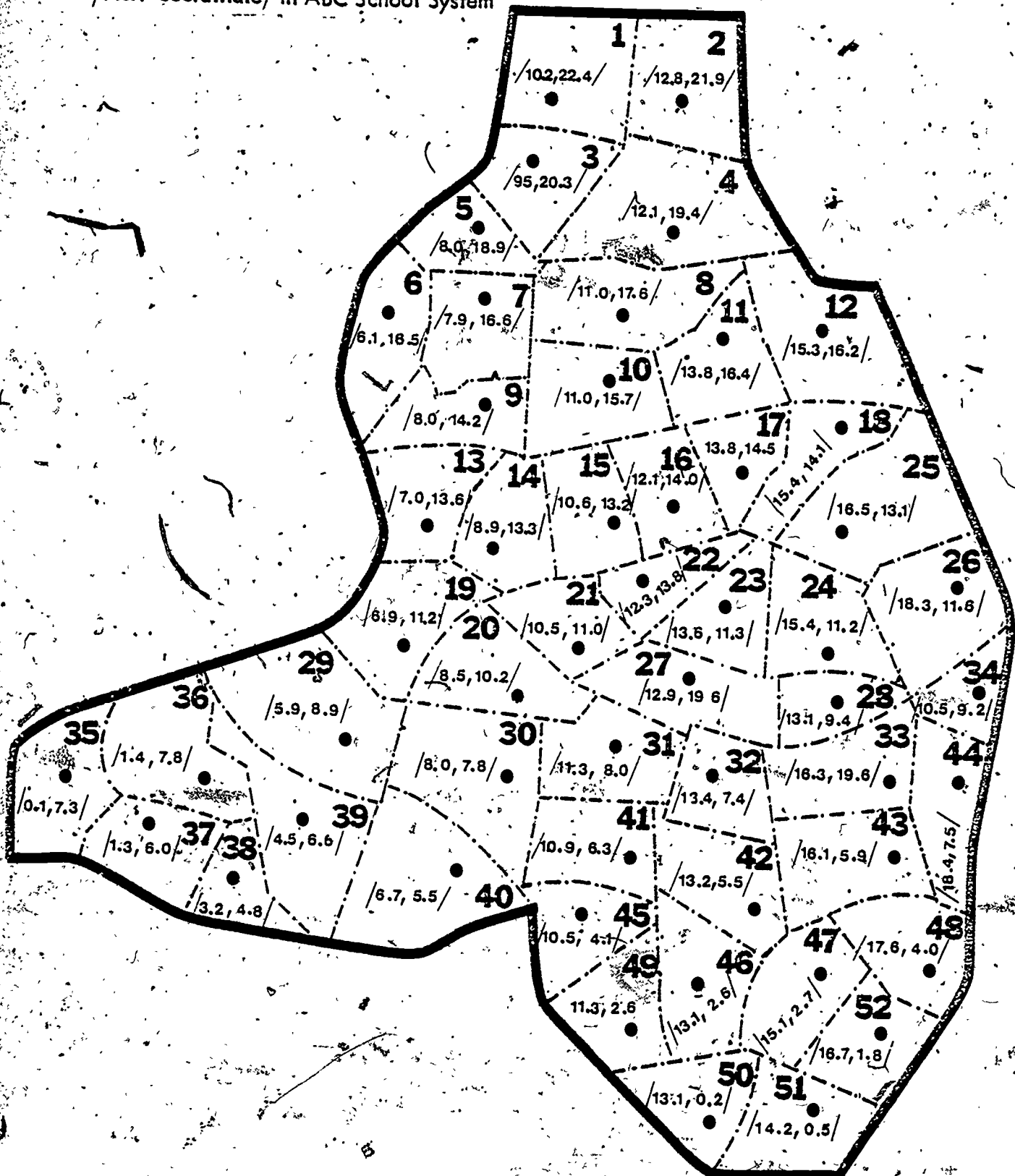
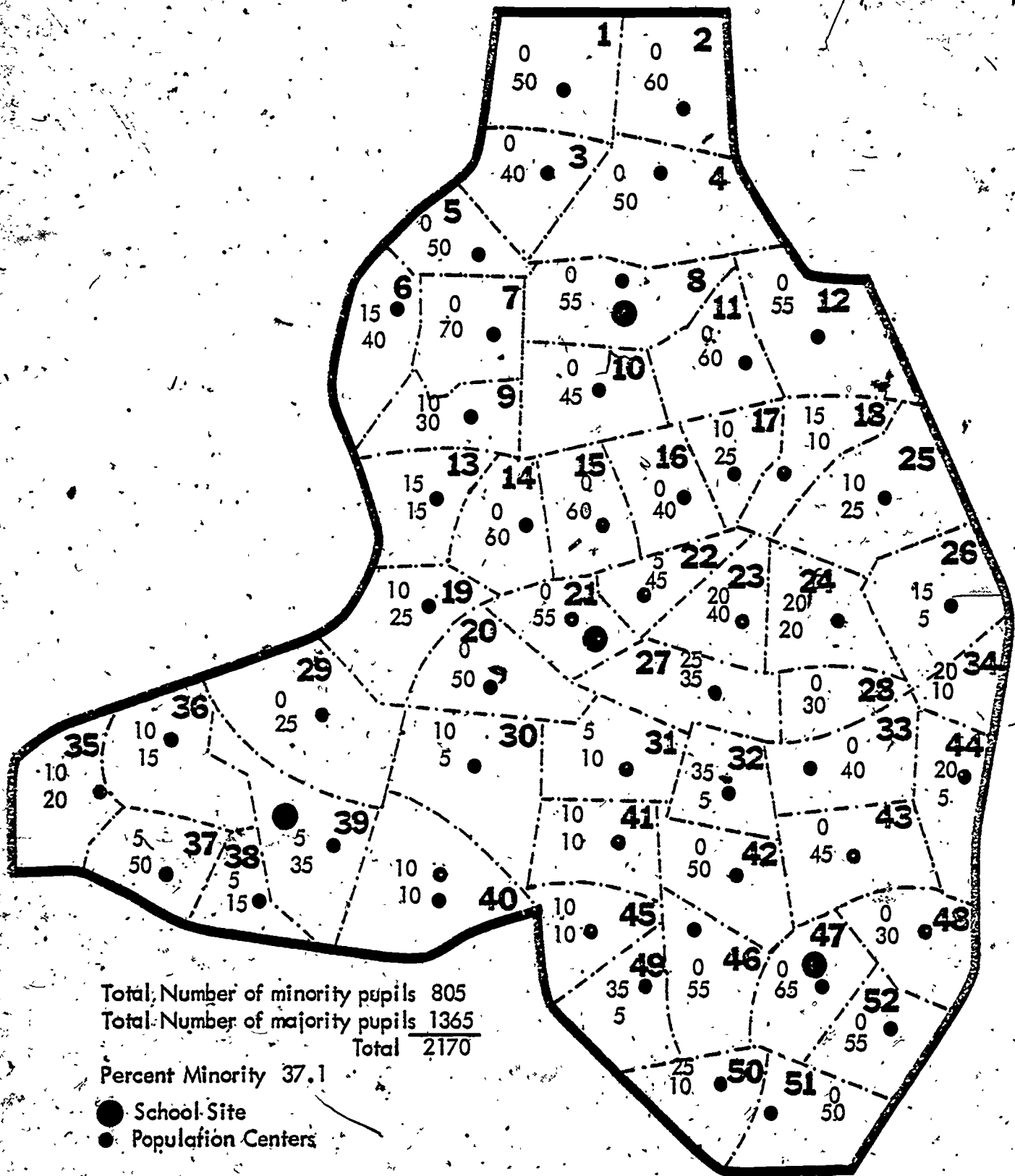


Figure 3
 Number of minority and majority pupils in each
 sub-area of the ABC School System



The pupil assignment plan which most nearly meets the objective is determined by a mathematical technique known as linear programming. This technique allows a systematic determination of the possible assignment plan which best meets the objective. The mathematical equations used for describing the objective and the constraints are given in Part IV of this manual.

IV. Program Results

Now let's look at four assignment plans developed for the ABC School System.

Plan I

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: None

The pupil assignment plan for the above objective with no constraints is shown in Figure 4. Note that the assignment areas are such that pupils in all areas are assigned to the closest school. This results, however, in wide variations in pupil enrollments and racial makeup in the four schools. The enrollments vary from a low of 215 in School 3 to a high of 760 in School 1. The percentage of minority pupils varies from 8.5% in School 1 to 90.9% in School 4. These results illustrate the necessity for using some constraints in the development of most assignment plans. Numerical values for each of the four possible objectives are given on Figure 4. Note that the total pupil-miles traveled is 3626.

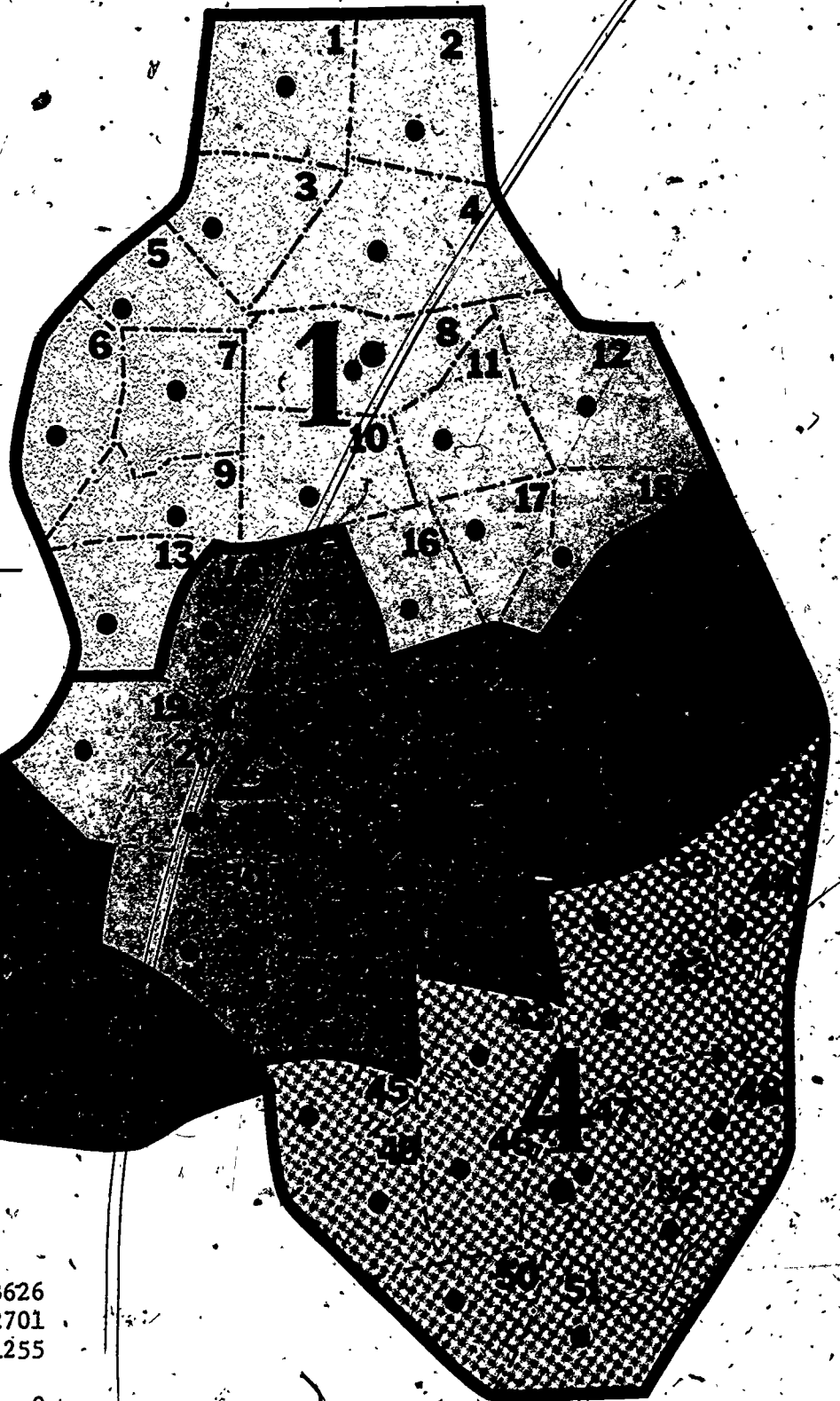
Plan II

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: No school may have an enrollment greater than its design capacity.

Figure 4
 Plan I - Assignment plan and other results--
 ABC School System

No Capacity Constraints
 No Minority Constraints
 Minimize Pupil-Miles Travelled



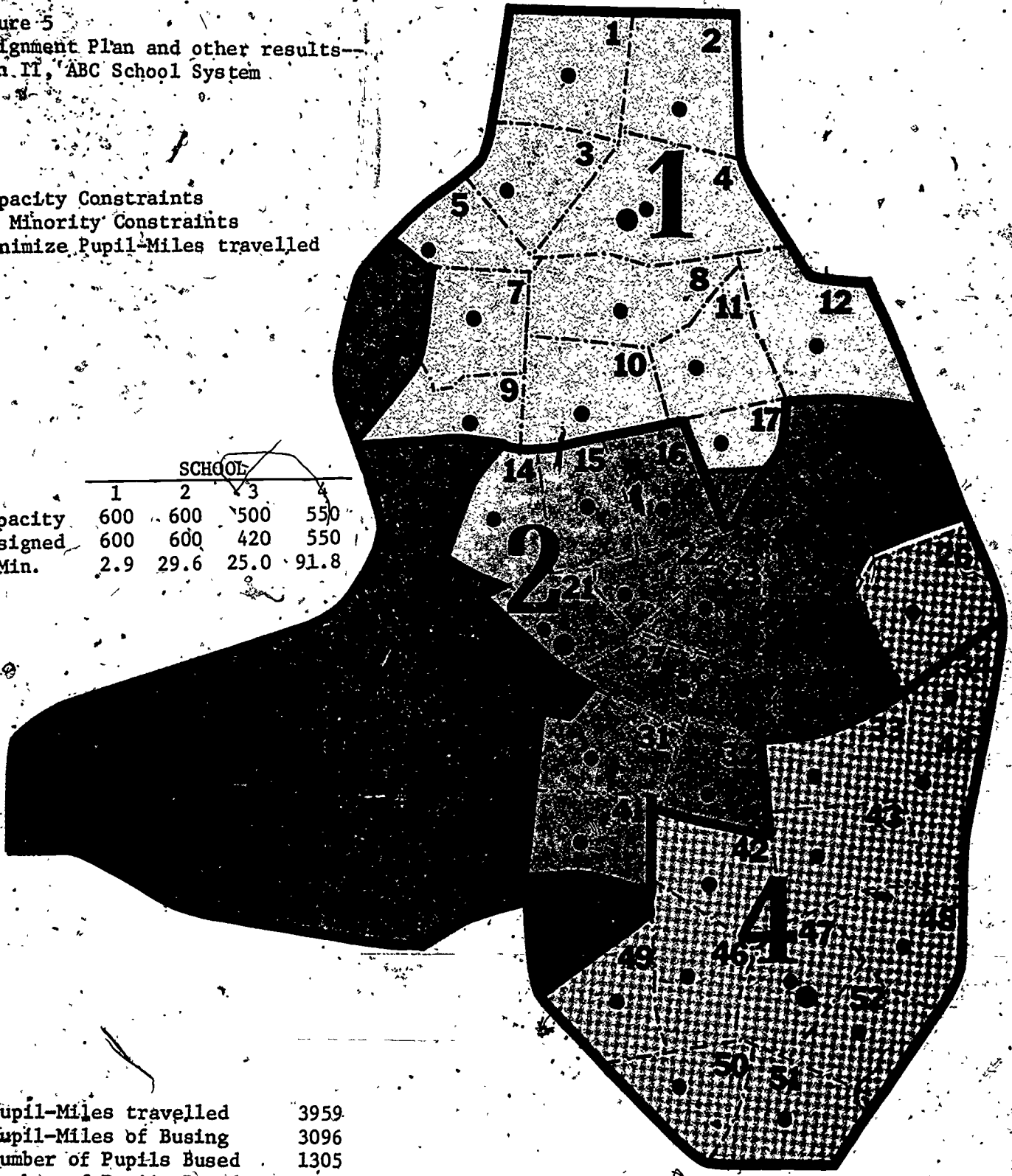
	SCHOOL			
	1	2	3	4
Capacity	-	-	-	-
Assigned	760	645	215	550
% Min.	8.5	30.2	20.9	90.9

Pupil-Miles travelled 3626
 Pupil-Miles of Busing 2701
 Number of Pupils Bused 1255
 Number of Pupils Bused
 Away from Nearest School 0

Figure 5
 Assignment Plan and other results--
 Plan II, ABC School System

Capacity Constraints
 No Minority Constraints
 Minimize Pupil-Miles travelled

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	2.9	29.6	25.0	91.8



Pupil-Miles travelled 3959
 Pupil-Miles of Busing 3096
 Number of Pupils Bused 1305
 Number of Pupils Bused
 Away from Nearest School 300

A plan for the aforementioned objective and constraint is shown in Figure 5. Capacity constraints of 600, 600, 500, and 550 have been placed on Schools 1, 2, 3, and 4 respectively. Thus, fewer pupils can be assigned to Schools 1 and 2 and more pupils must be assigned to School 3. Note that assignment zones for each school are made up of contiguous areas in most cases. Assignments are made such that Schools 1, 2, and 4 are at capacity while School 3 absorbs all the excess system capacity with only 420 pupils enrolled. The range of percentages of minority pupils in the four schools is even larger than in Plan I, varying from 2.9% in School 1 to 91.8% in School 4. The pupil miles traveled have increased from 3626 to 3959, the pupil-miles of bussing have increased from 2701 to 3096, the number of pupils bussed has increased from 1255 to 1305, and the number of pupils bussed away from the closest school has increased from 0 to 300.

Plan III

Objective : Minimize the distance that pupils must travel from their homes to schools.

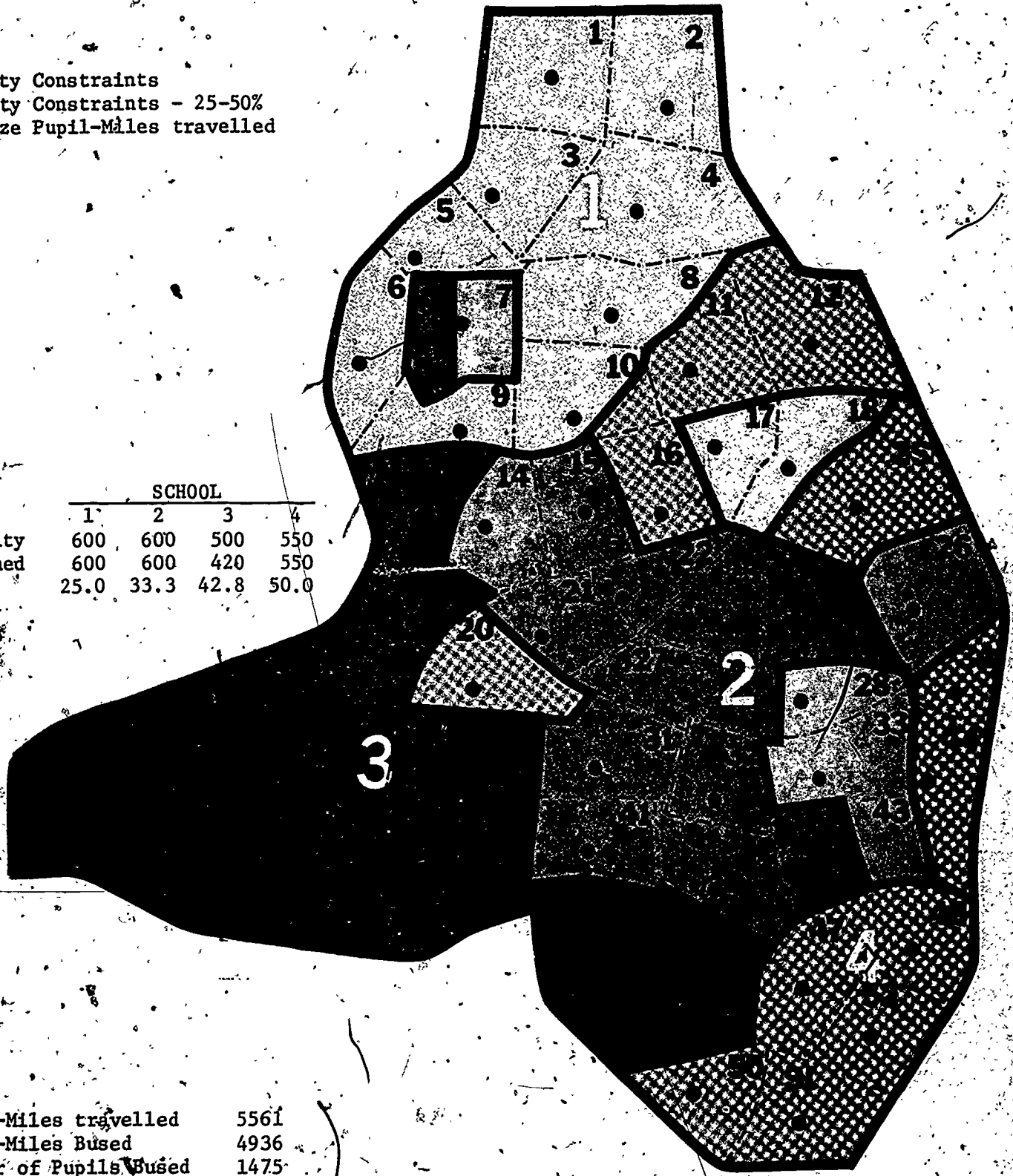
Constraints: 1. No school may have an enrollment greater than its design capacity.
2. The percentage of minority pupils in each school must be between 25% and 50%.

Figure 6 illustrates a plan for the ABC School System when a racial constraint of from 25% to 50% is placed on each of the four schools in addition to the capacity constraints. Note that now a number of satellite areas are necessary in order to give the desired racial balance. For example, pupils in area 7 are split between Schools 2 and 3 rather than being assigned to the nearby School 1. The attendance zone for School 1 includes a satellite consisting of areas 17 and 18 and a second satellite consisting of areas 26, 28, 33, and part of 43. The attendance zone for School 4 includes a satellite

Figure 6
Assignment plan and other results—
Plan III, ABC School System

Capacity Constraints
 Minority Constraints - 25-50%
 Minimize Pupil-Miles travelled

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	25.0	33.3	42.8	50.0



Pupil-Miles travelled 5561
 Pupil-Miles Bused 4936
 Number of Pupils Bused 1475
 Number of Pupils Bused
 Away from Nearest School 700

consisting of area 20, a second satellite consisting of area 25, and a third satellite consisting of areas 11, 12, 16, and part of 10.

The plan shown in Figure 6 results in a minority percentage in School 1 at the lower limit of 25 and a percentage in School 4 at the upper limit of 50. The pupil-miles traveled have increased from 3959 to 5561, the pupil-miles of bussing have increased from 3096 to 4936, the number of pupils bussed has increased from 1305 to 1475, and the number of pupils bussed away from the closest school has increased from 300 to 700.

Plan IV

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: 1. No school may have an enrollment greater than its design capacity.

2. The percentage of minority pupils in each school must be between 34% and 40%.

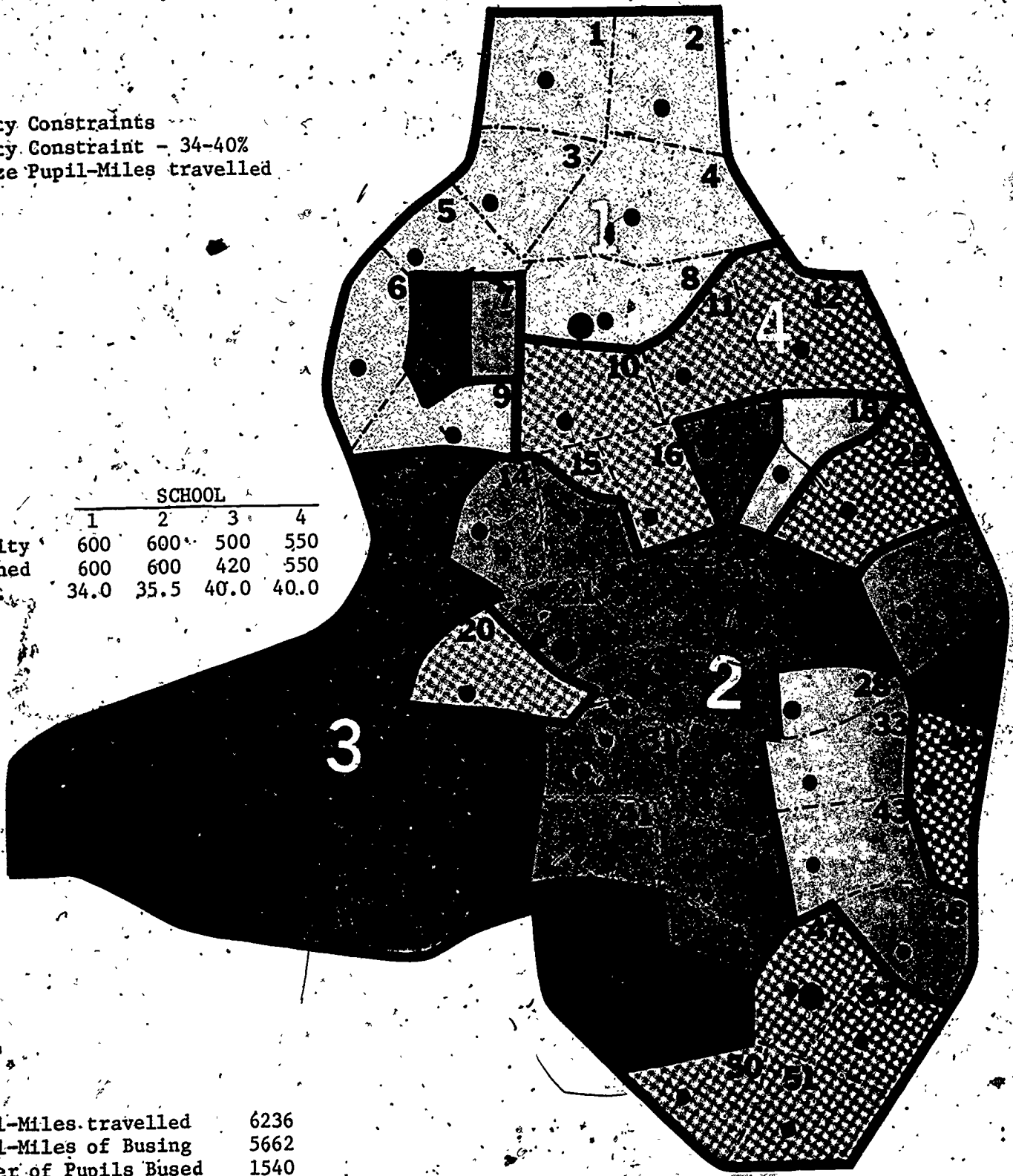
Figure 7 illustrates a plan in which the allowable range in racial percentages has been reduced to 34% to 40% minority pupils. A comparison of Figures 6 and 7 reveals that the plans are very similar but that the satellite areas have primarily increased in size. School 1 continues to have the lowest allowable minority percentage of 34% while both Schools 3 and 4 have the maximum allowable minority percentage of 40%. The pupil-miles traveled have increased from 5561 to 6236, the pupil-miles of bussing have increased from 4936 to 5662, the number of pupils bussed has increased from 1475 to 1540, and the number of pupils bussed away from the closest school has increased from 700 to 855.

Summary of Plans I - IV

The assignment plans illustrated in Figures 4-7 are the plans which minimize mathematically the pupil-miles of transportation in the total

Figure 7
 Assignment plan and other results--
 Plan IV, ABC School System

Capacity Constraints
 Minority Constraint - 34-40%
 Minimize Pupil-Miles travelled



	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	34.0	35.5	40.0	40.0

Pupil-Miles travelled 6236
 Pupil-Miles of Busing 5662
 Number of Pupils Bused 1540
 Number of Pupils Bused
 Away from Nearest School 855

system subject to the listed constraints. There may be features of the plans, however, which are unacceptable for reasons which have not been considered in the computer solution. An example in point might be the assignment of pupils in area 7 in Plan IV (Figure 7). Area 7 is divided between Schools 2 and 3 rather than being assigned to nearby School 1. Table I reveals that area 7 has 70 majority race pupils and no minority race pupils. These are not assigned to School 1 due to the difficulty in maintaining the minimum allowable percentage of minority pupils there. However, splitting area 7 between Schools 2 and 3 does not seem desirable since two satellite areas are formed, neither of which has enough pupils to fill a bus. In the next section we will look at some ways to make alterations in the plan to meet criteria which cannot be readily expressed mathematically.

Other areas in Plan IV (Figure 7) which may need further inspection include areas 17, 18, 20, 25, 34, and 44. If the number of satellite areas can be reduced or other aesthetic features of the assignment plan improved with only a small effect on the stated objective, these probably should be accommodated.

V. Program Options

In this section we will consider some optional features of the computer program which may be used in the development of pupil assignment plans. Some of these features will be used to improve the plans developed in the previous section for the ABC School System.

1. Input Actual Distances

In Section D we stated that the distances between schools and areas were estimated mathematically by calculating the straight line distances between their respective coordinates. This straight line distance was then increased by an appropriate factor in order to approximate actual street

or highway distances. This procedure for estimating distances will be satisfactory in most cases but there are situations in which the calculated distances will be unrealistic.

Let's again use the ABC School System to illustrate a situation in which it might be desirable to input actual distances rather than depend on the calculated values. Consider the distances between School 3 and areas 45, 46, 49, 50, and 51. The straight line distances would be along lines which pass through an area outside the School System. Let's assume that area contains some natural (river, lake, mountain, etc.) or artificial (railroad, park, etc.) barrier which makes a route along approximately a straight line undesirable or impossible. Then we might provide the computer with actual measured distances by a feasible route. The actual distances may be provided for as many or as few area to school combinations as is desired by planners. In the plan to follow for the ABC School System it will be assumed that in order to travel from School 3 to areas 45, 46, 49, 50, or 51 it is necessary to go by way of area 41. Measured distances for these routes were provided to the computer.

Plan V

Objective : Minimize the distance that pupils must travel from their homes to schools.

- Constraints:
1. No school may have an enrollment greater than its design capacity.
 2. The percentage of minority pupils in each school must be between 34% and 40%.
 3. Actual distances are used from School 3 to areas 45, 46, 49, 50, and 51.

Figure 8 illustrates a plan for the ABC School System in which the actual distances for the listed areas were used. The objective and con-

straints were otherwise the same as in Plan IV (Figure 7). Note that area 45 is no longer assigned to School 3 but becomes a satellite area of School 4. Comparison of Figures 7 and 8 shows that Plan V is similar to Plan IV but there are minor shifts in the assignment areas for all four schools. School 1 continues to have the lowest allowable minority percentage of 34% but now School 3 also has the minimum percentage. School 4 continues to have the maximum minority percentage of 40%. The pupil-miles traveled have increased from 6236 to 6300, the pupil-miles of bussing have increased from 5662 to 5727, the number of pupils bussed was unchanged, and the number of pupils bussed away from the nearest school has actually decreased from 855 to 836.

2. Elimination of Selected Assignment Possibilities

Plan V (Figure 8) has a number of rather small satellite areas which are undesirable. We will now use an option for eliminating selected assignments in an attempt to improve the aesthetic qualities of Plan V. We will look at a series of six restrictions to make Plan V more acceptable. First, let's see what will happen if we prohibit the assignment of pupils in area 7 to either School 2 or 3 and prohibit the assignment of pupils in areas 17 and 18 to School 1.

Plan Va

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: 1. No School may have an enrollment greater than its design capacity.

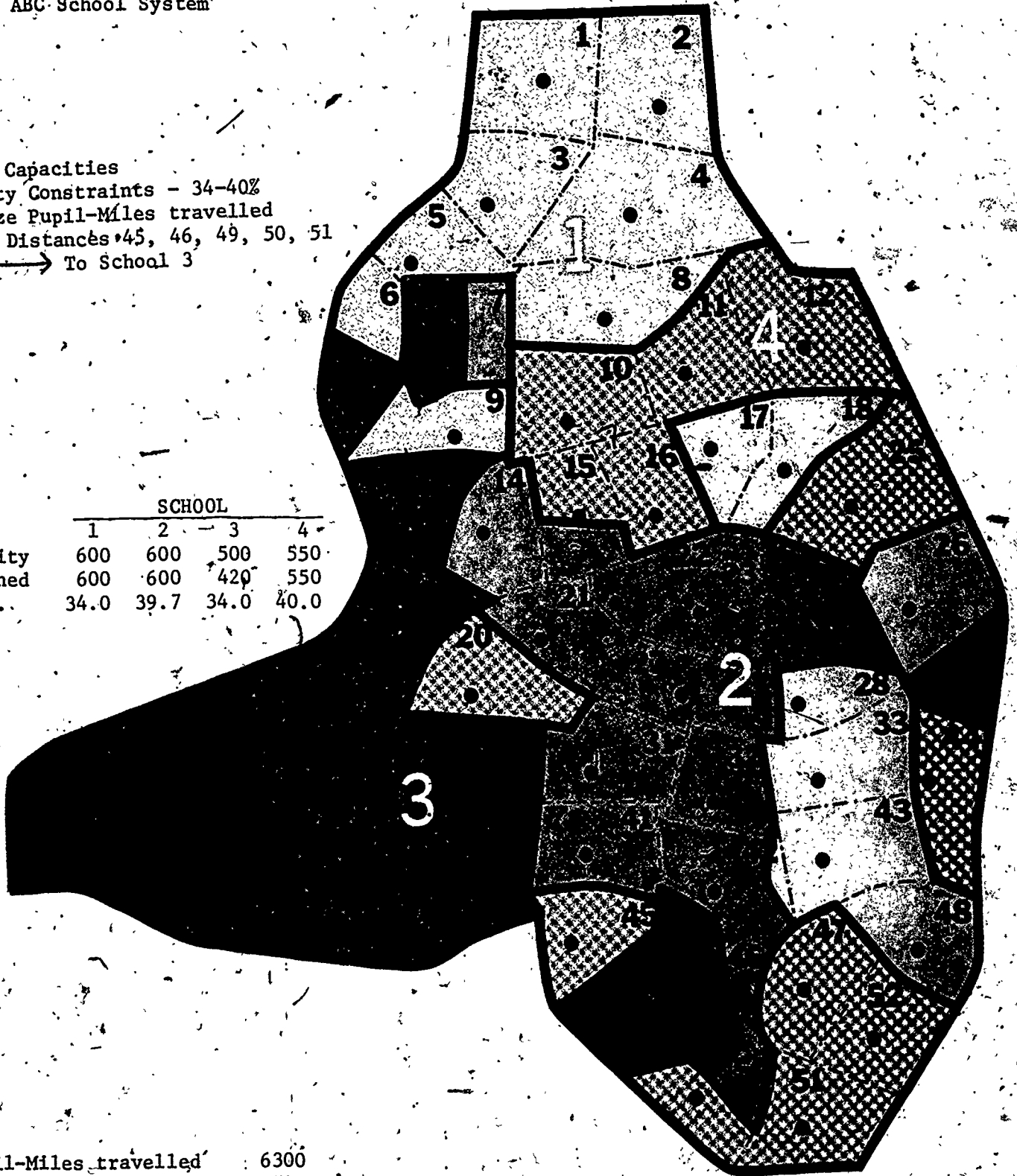
2. The percentage of minority pupils, in each school must be between 34% and 40%.

3. Actual distances are used from School 3 to areas 45, 46, 49, 50, and 51.

Figure 8
 Assignment plan and other results--
 Plan V, ABC School System

School Capacities
 Minority Constraints - 34-40%
 Minimize Pupil-Miles travelled
 Actual Distances 45, 46, 49, 50, 51
 Areas. → To School 3

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	34.0	39.7	34.0	40.0



Pupil-Miles travelled 6300
 Pupil-Miles Bused 5727
 Number of Pupils Bused 1540
 Number of Pupils Bused Away
 From Nearest School 836

4. Assignment of area 7 to School 2 or 3 is prohibited.

5. Assignment of areas 17 and 18 to School 1 is prohibited.

Figure 9 illustrates the results for Plan Va. Note that most of area 7 is now assigned to nearby School 1 while a small portion of the area joins the already existing satellite area of School 4. Areas 17 and 18 are now assigned to School 2. Comparison of the statistics for Plans V and Va shows that the pupil-miles traveled have increased from 6300 to 6356, the pupil-miles of bussing have increased from 5727 to 5782, the number of pupils bussed is unchanged, and the number of pupils bussed away from the nearest school has decreased from 836 to 822.

The next step in our attempt to improve the assignment plan will be to prohibit the assignment of area 45 to School 4. This is prohibited since there are only 20 pupils in area 45 and the area does not join any other area assigned to School 4.

Plan Vb

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: (1) - (5) Same as Plan Va.
(6) Assignment of area 45 to School 4 is prohibited.

Figure 10 illustrates the results for Plan Vb. Area 45 is now assigned to School 3 and the number of satellite areas has been reduced by one. The statistics indicate an increase of only 3 pupil-miles of transportation and pupil-miles of bussing while 3 additional pupils are bussed away from the nearest school.

Our next step will be to prohibit the assignment of area 44 to School 4.

Figure 9

Assignment plan and other results--
Plan Va, ABC School System

School Capacities.

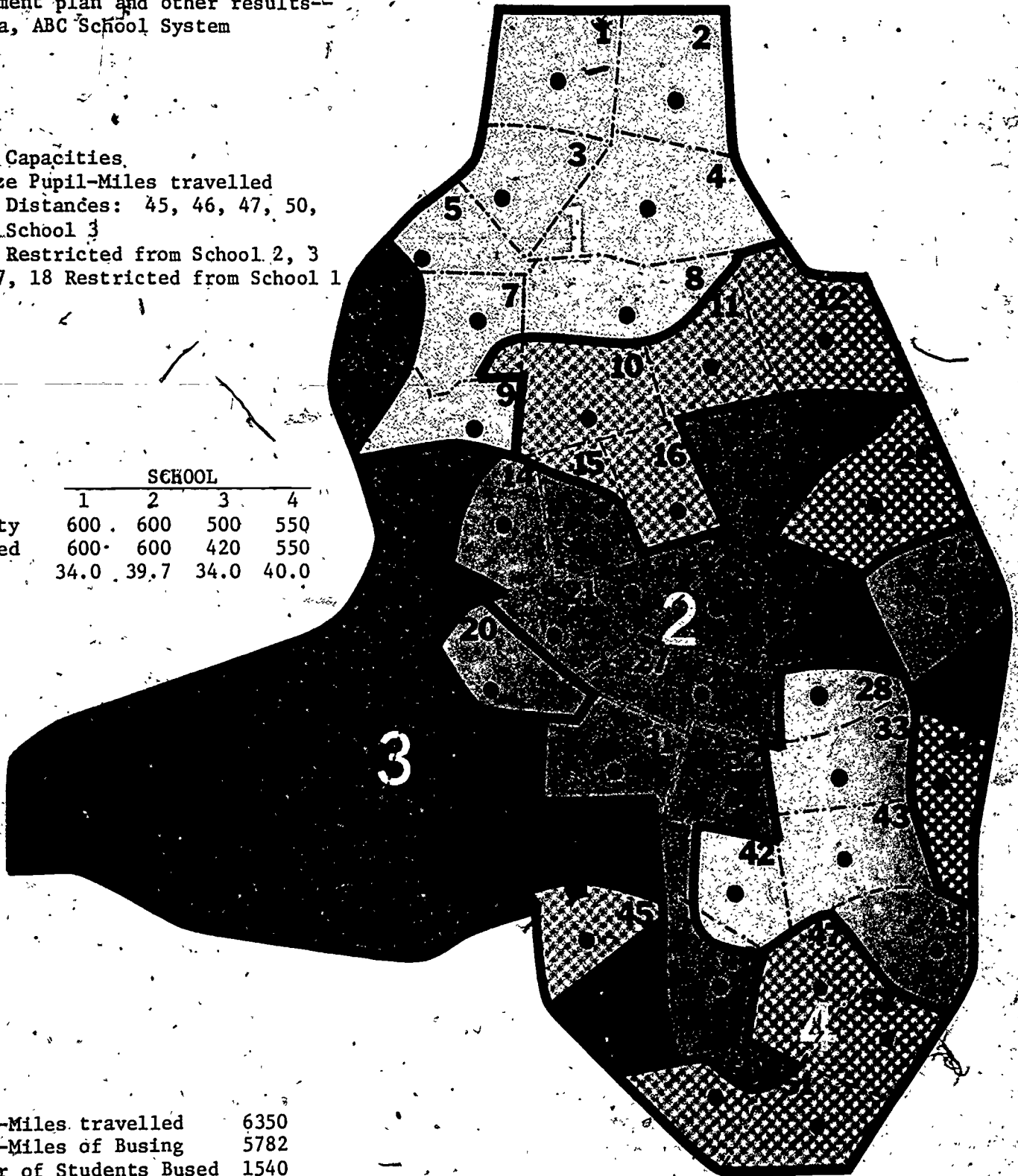
Minimize Pupil-Miles travelled

Actual Distances: 45, 46, 47, 50,
51 to School 3

Area 7 Restricted from School 2, 3

Area 17, 18 Restricted from School 1

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	34.0	39.7	34.0	40.0



Pupil-Miles travelled 6350
 Pupil-Miles of Busing 5782
 Number of Students Bused 1540
 Number of Students Bused
 Away from Nearest School 822

Plan Vc

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: (1) - (6) Same as Plan Vb.
(7) Assignment of area 44 to School 4 is prohibited.

Figure 11 illustrates the results for Plan Vc. Area 44 is now assigned to School 2 and the number of satellites again reduced. In this case the pupil-miles traveled have increased from 6359 to 6406, the pupil-miles of bussing have increased from 5785 to 5819, the number of pupils bussed has decreased from 1540 to 1525, and the number of pupils bussed away from the nearest school has decreased from 825 to 790.

Our next step will be to prohibit the assignment of pupils in area 20 to School 4.

Plan Vd

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: (1) - (7) Same as Plan Vc.
(8) Assignment of area 20 to School 4 is prohibited.

Figure 12 illustrates the results for Plan Vd. Area 20 is now split between Schools 2 and 3. The pupil-miles traveled have increased from 6406 to 6412, the pupil-miles of bussing have decreased from 5819 to 5809, the number of pupils bussed has decreased from 1525 to 1512, and the number of pupils bussed away from the nearest school has decreased from 782 to 762.

The fifth step is to prohibit the assignment of pupils in area 25 to School 4.

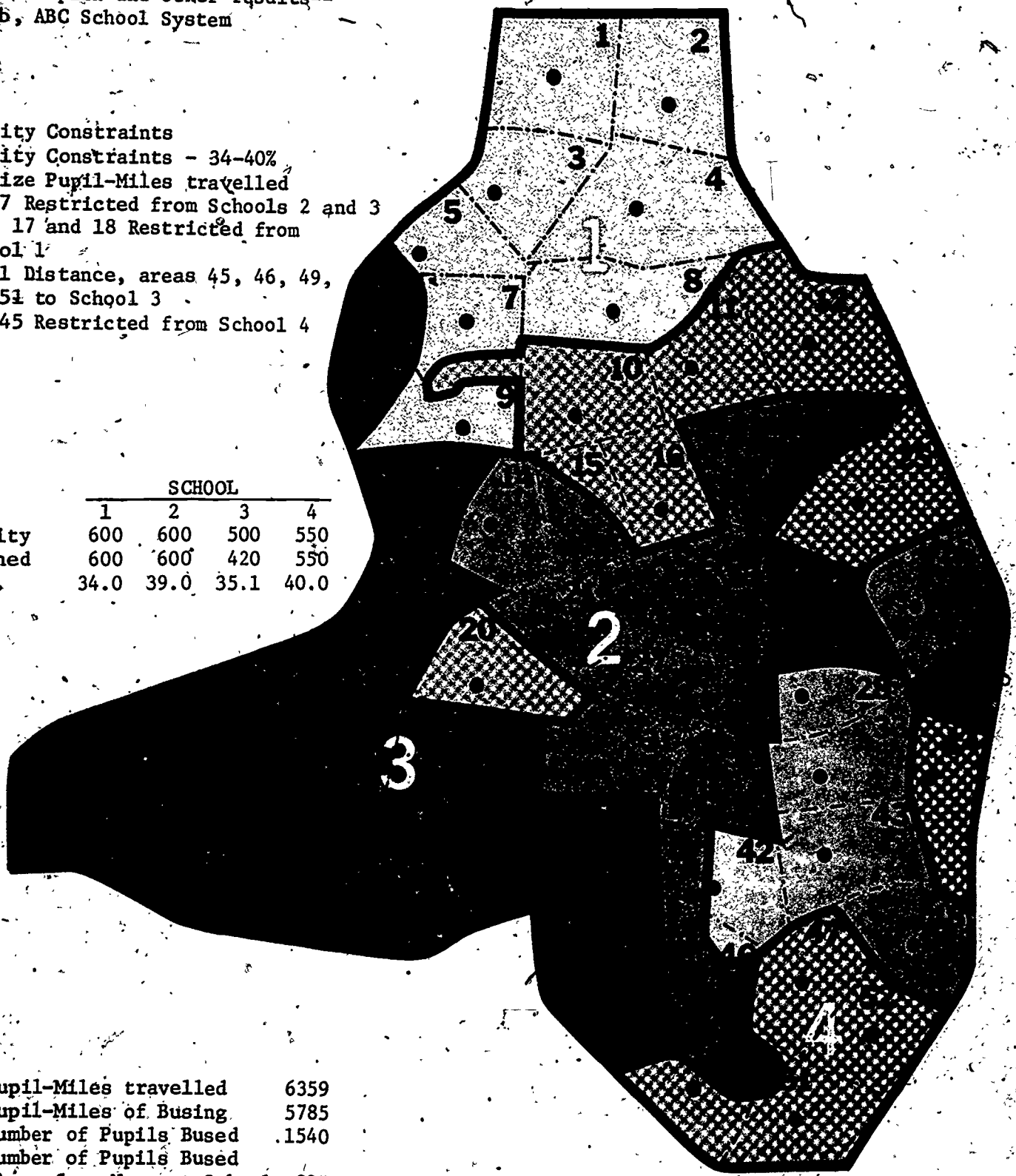
Plan Ve

Objective - : Minimize the distance that pupils must travel from their homes to schools.

Figure 10
 Assignment plan and other results--
 Plan Vb, ABC School System

- Capacity Constraints
- Minority Constraints - 34-40%
- Minimize Pupil-Miles travelled
- Area 7 Restricted from Schools 2 and 3
- Areas 17 and 18 Restricted from School 1
- Actual Distance, areas 45, 46, 49, 50, 51 to School 3
- Area 45 Restricted from School 4

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Min.	34.0	39.0	35.1	40.0



Pupil-Miles travelled	6359
Pupil-Miles of Busing	5785
Number of Pupils Bused	1540
Number of Pupils Bused Away from Nearest School	825

Figure 11
 Assignment plan and other results--
 Plan Vc, ABC School System

Capacity Constraints

Minority Constraints - 34-40%

Minimize Pupil-Miles travelled

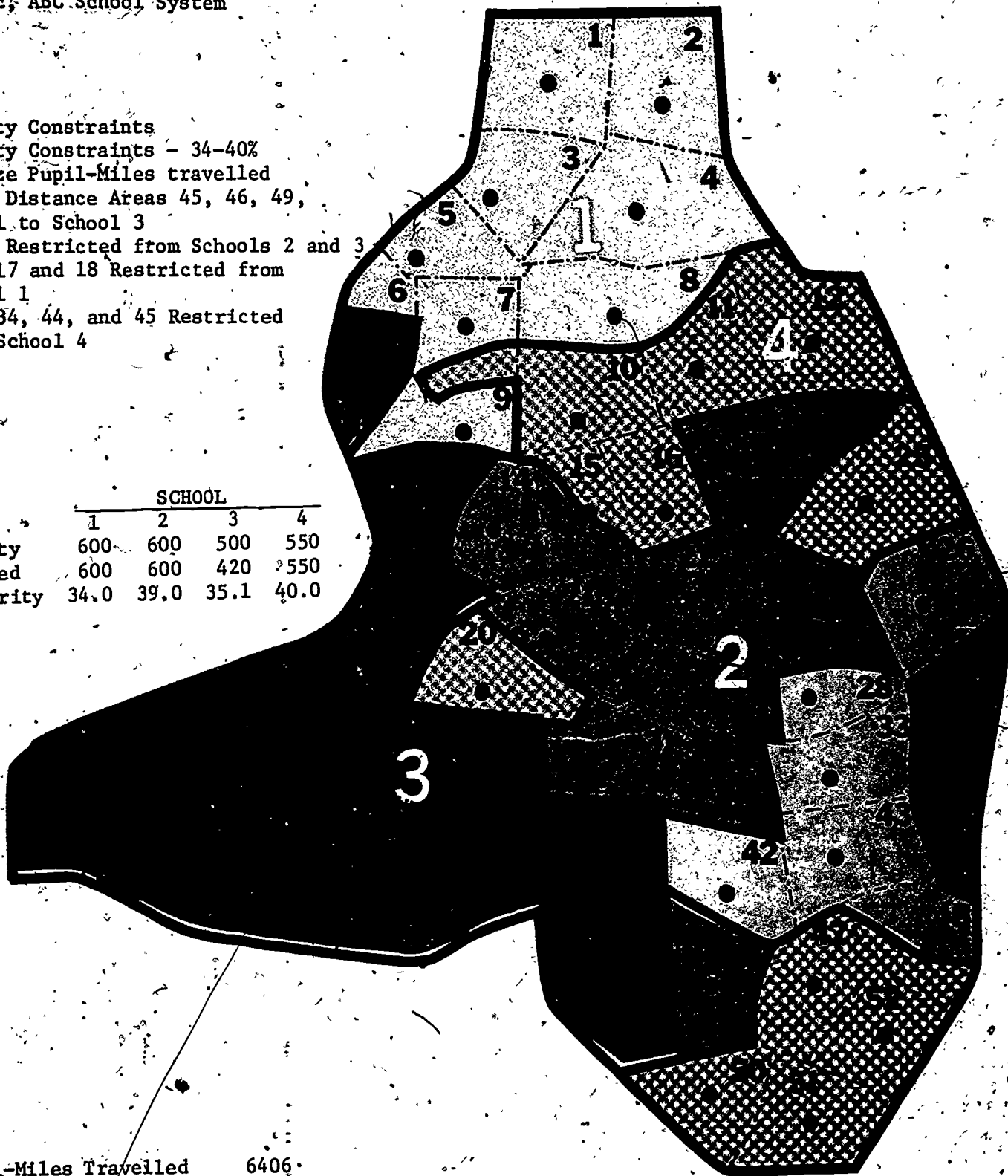
Actual Distance Areas 45, 46, 49,
 50, 51 to School 3

Area 7 Restricted from Schools 2 and 3

Areas 17 and 18 Restricted from
 School 1

Areas 34, 44, and 45 Restricted
 from School 4

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Minority	34.0	39.0	35.1	40.0



Pupil-Miles Travelled 6406
 Pupil-Miles of Busing 5819
 Number of Pupils Bused 1525
 Number of Pupils Bused
 Away from Nearest School 790

Constraints: (1) - (8) Same as Plan Vd.

(9) Assignment of area 25 to School 4 is prohibited:

Figure 13 gives the results for Plan Ve. Area 25 is assigned to area 2 along with its adjoining areas 18 and 24. The number of satellite areas have now been reduced to three (one each for Schools 1, 2, and 4). Comparison of the statistics for Plans Vd and Ve indicate that the pupil-miles traveled have increased from 6412 to 6421, the pupil-miles of bussing have increased from 5809 to 5811, the number of pupils bussed has decreased from 1512 to 1508, and the number of pupils bussed away from the closest school has decreased from 782 to 762.

Our last step for improvement of Plan V will be to prohibit the assignment of areas 34 and 44 to School 2.

Plan Vf

Objective : Minimize the distance that pupils must travel from their homes to schools.

Constraints: (1) - (9) Same as Plan Ve.
(10) Assignment of areas 34 and 44 to School 2 is prohibited.

Figure 14 illustrates the results for Plan Vf. The number of satellite areas has now been reduced to two (one each for Schools 1 and 4). The remaining satellite areas consist of several contiguous source areas having a sufficient number of pupils for development of efficient bus schedules. The pupil-miles traveled have increased from 6421 to 6465, the pupil-miles of bussing have increased from 5811 to 5840, the number of pupils bussed has decreased from 1508 to 1493, and the number of pupils bussed away from the nearest school has increased from 762 to 766.

Plan Vf appears to be a quite acceptable assignment plan for the ABC School System. It has been developed by first determining the plan (Plan V)

Figure 12
 Assignment plan and other results ->
 Plan Vd, ABC School System

Capacity Constraints
 Minority Constraints - 34-40%
 Minimize Pupil-Miles travelled
 Actual Distance Areas 45, 46, 49,
 50, 51 to School 3
 Area 7 Restricted from Schools 2 and 3
 Areas 17 and 18 Restricted from
 School 1
 Areas 20, 34, 44, and 45 Restricted
 from School 4.



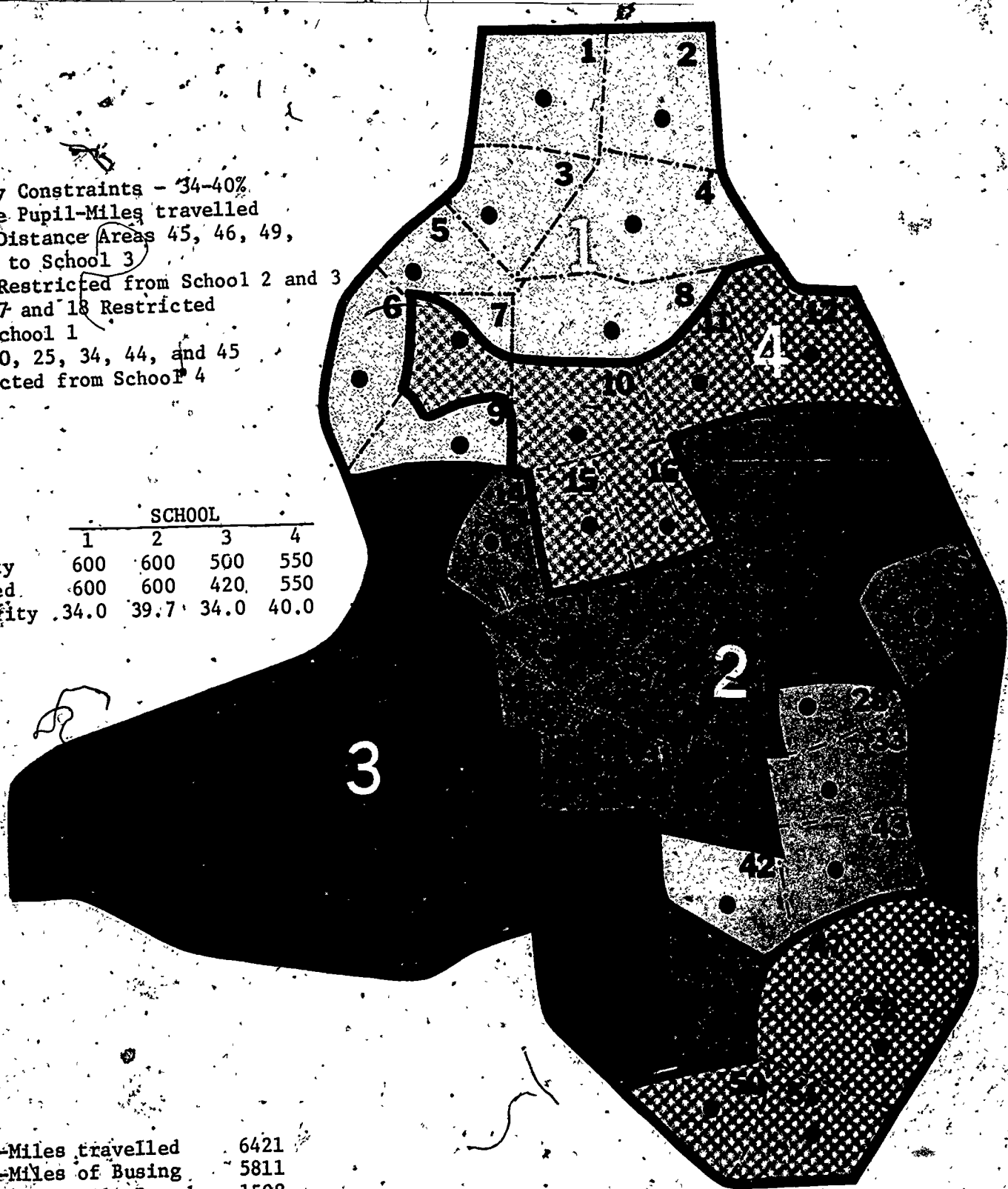
	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Minority	34.0	39.7	34.0	40.0

Pupil-Miles travelled 6412
 Pupil-Miles of Busing - 5809
 Number of Pupils Bused 1512
 Number of Pupils Bused
 Past Nearest School. 782

Figure 13
 Assignment plans and other results--
 Plan Vc, ABC School System

Capacity Constraints - 34-40%
 Minimize Pupil-Miles travelled
 Actual Distance Areas 45, 46, 49,
 50, 51 to School 3
 Area 7 Restricted from School 2 and 3
 Areas 17 and 18 Restricted
 from School 1
 Areas 20, 25, 34, 44, and 45
 Restricted from School 4

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Minority	34.0	39.7	34.0	40.0



Pupil-Miles travelled 6421
 Pupil-Miles of Busing 5811
 Number of Pupils Bused 1508
 Number of Pupils Bused
 Past Nearest School 762

which minimizes pupil-miles of transportation subject to the desired capacity and racial constraints. The base plan was then improved aesthetically by sequentially prohibiting undesirable features. Let's now compare the statistics for Plans V and Vf as shown in Table 1. The additional constraints which have been placed on Plan Vf result in a 2.6% increase in the pupil-miles of transportation (the characteristic being optimized), a 2.0% increase in the pupil-miles of bussing, a 3.1% decrease in the number of pupils bussed, and an 8.4% decrease in the number of pupils bussed away from the nearest school. Thus, for a very nominal increase in pupil-miles traveled, the aesthetic features of the plan have been considerably improved. An added advantage of Plan Ve is the reduction in the number of pupils bussed which might eliminate the need for one bus.

Table 1

	Plan V (Figure 8)	Plan Vf (Figure 14)	% Change
Pupil-miles traveled	6300	6465	+ 2.6
Pupil-miles of bussing	5727	5840	+ 2.0
Number of pupils bussed	1540	1493	- 3.1
Number of pupils bussed away from nearest school	836	766	- 8.4

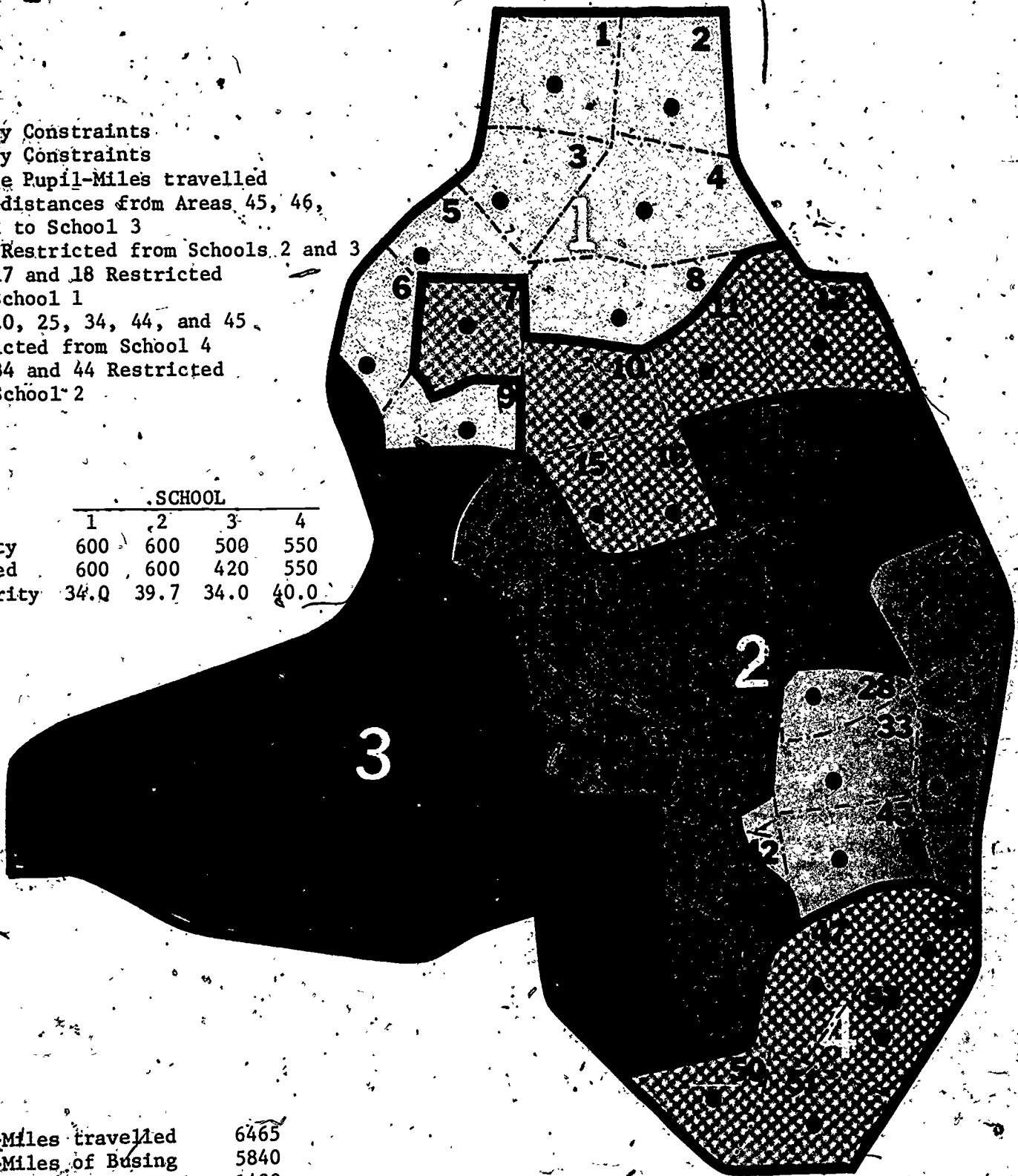
3. Bussing Farther than the Second Closest School Prohibited

Another optional feature of the computer program is the ability to prohibit the bussing of a pupil to a school farther from his or her home than the second closest school. For our hypothetical ABC School System, it is not possible to prohibit bussing farther than the second nearest school and still maintain a narrow range of racial percentages. We can get an idea of what the option does, however, by looking at Plan II in Figure 5. In this plan, area 6 is assigned to School 3 though it is closer to both School 1 and School 2. Also, area 45 is assigned to School 3 though

Figure 14
 Assignment plan and other results--
 Plan V-f, ABC School System

- Capacity Constraints
- Minority Constraints
- Minimize Pupil-Miles travelled
- Actual distances from Areas 45, 46, 50, 51 to School 3
- Area 7 Restricted from Schools 2 and 3
- Areas 17 and 18 Restricted from School 1
- Areas 20, 25, 34, 44, and 45 Restricted from School 4
- Areas 34 and 44 Restricted from School 2

	.SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	420	550
% Minority	34.0	39.7	34.0	40.0



Pupil-Miles travelled 6465
 Pupil-Miles of Busing 5840
 Number of Pupils Bused 1493
 Number of Pupils Bused
 Away from Nearest School. 766

it is closer to Schools 2 and 4. These assignments would be prohibited if the option for not allowing bussing past the second closest school were utilized.

VI. Other Applications

Thus far, we have discussed the use of the computer procedures for the development of pupil assignment plans based on current pupil populations and existing school facilities. There are many other situations in which the procedures may provide valuable information.

Many of the applications proposed in this section will emphasize costs. Transportation costs are directly related to pupil-miles of bussing required by the pupil assignment plan. The North Carolina Board of Education determines each year the cost of bussing in terms of dollars per pupil mile. Pupil-miles of bussing associated with each assignment plan are the number of miles pupils travel by bus one-way per day. Total cost of transportation for the year is determined by multiplying the pupil-miles of bussing by two, then by the number of school days per year, and finally by the cost per pupil-miles of bussing.

1. School Location Analysis

Factors which should be considered in the selection of school sites include the transportation requirements and the pupil assignment plans which would be required by alternative sites. These need to be considered along with such things as the availability of land, utilities, etc. The computer procedures described here can be used to quickly evaluate the effects of alternative school sites on pupil assignment plans and transportation costs to be expected by the School System.

As an example, let's assume that School 2 of the ABC School System has deteriorated physically to the point that it must be replaced either at its present site or at an alternate site. Figure 15 illustrates an assignment plan (Plan VI) for the ABC School System if School 2 is replaced by School

2A at a different location. The objective and constraints are the same as were used in the development of Plan IV (Figure 17). Comparison of the statistics for Plan VI and Plan IV show that the alternate school site could result in a reduction in the pupil-miles traveled from 6236 to 6149, a reduction in pupil-miles of bussing from 5662 to 5600, an increase in the number of pupils bussed from 1540 to 1574, and a reduction in the number of pupils bussed away from the nearest school from 855 to 837.

Although the pupil-miles of transportation and numbers of pupils bussed would not be greatly affected by the location of School 2, observation of the assignment plans indicates a much more acceptable plan aesthetically if school site 2A is used. As shown in Figure 15, Plan VI has three small isolated satellite areas which are undesirable as compared with seven in Plan IV. Placing restrictions on the assignment of these three areas using procedures discussed in the previous section could easily result in an acceptable assignment plan if the school were located at site 2A.

2. Facility Utilization

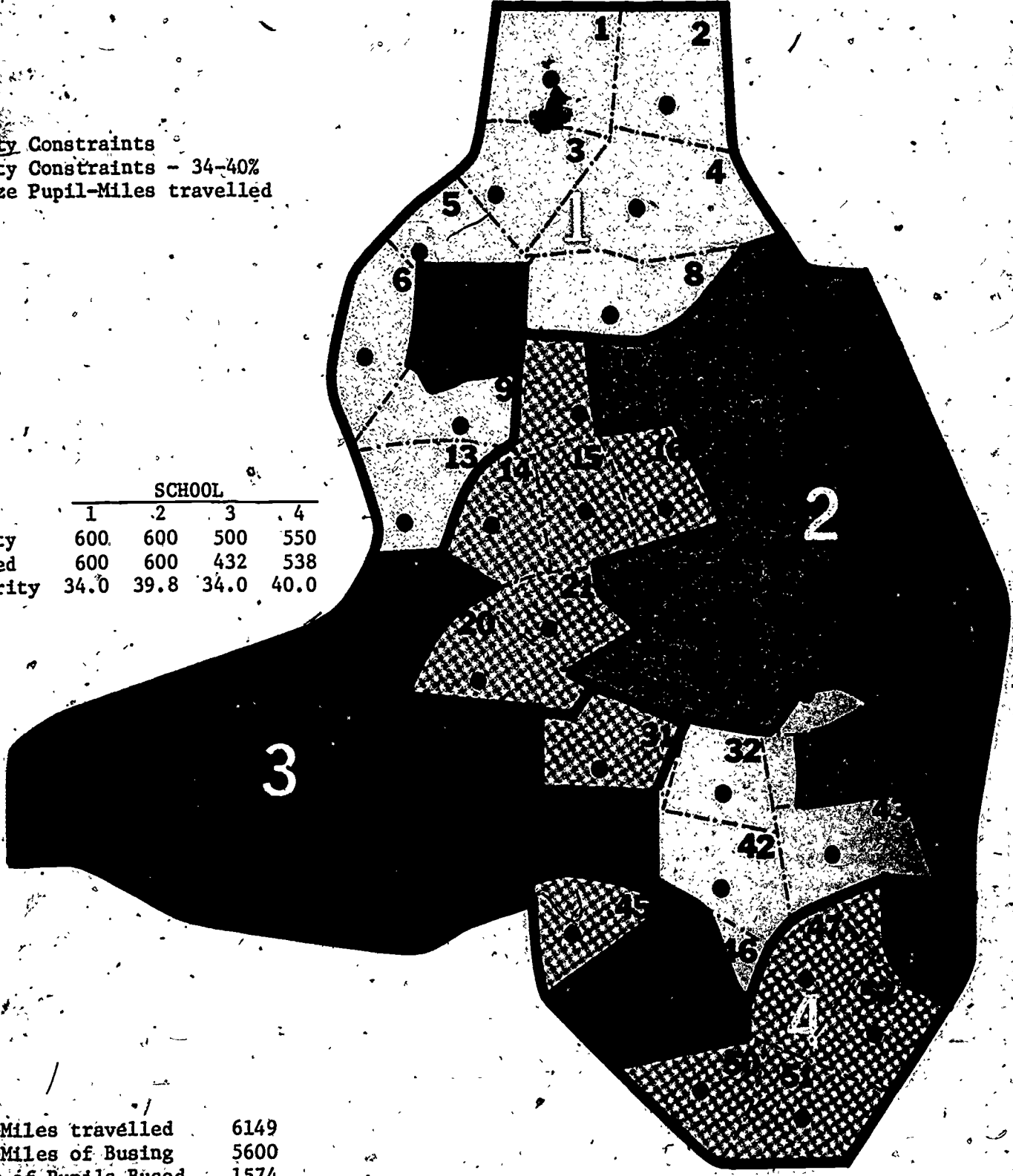
In the ABC School System we have considered thus far, the total capacity of the four schools is greater than the number of pupils in the system. In most of the plans we have developed, three of the schools have been filled to capacity and the remaining pupils have been assigned to the fourth school. The school with the extra capacity has been School 3 in each case. Thus, the computer results give us an estimate of the utilization factor for each school in the system. This can be used by planners to analyze a number of capacity problems which school administrators face.

Let's assume that the enrollment in the ABC School System increases to a total of 2600 pupils. Then we must increase the total capacity of the

Figure 15
 Assignment plan and other results--
 Plan VI, ABC School System

Capacity Constraints
 Minority Constraints - 34-40%
 Minimize Pupil-Miles travelled

	SCHOOL			
	1	2	3	4
Capacity	600	600	500	550
Assigned	600	600	432	538
% Minority	34.0	39.8	34.0	40.0



Pupil-Miles travelled 6149
 Pupil-Miles of Busing 5600
 Number of Pupils Bused 1574
 Number of Pupils Bused
 Away from Nearest School 837

schools in the system. This can be done by building an additional school or by adding capacity to existing schools. If a new school is to be built, the computer procedures can be used to estimate the effects of various alternative locations on pupil assignment plans and transportation costs. By removing the capacity constraint on a new school and determining how many pupils would be assigned there, planners can evaluate the school size which would result in the least transportation costs for the system. This information can be used along with construction costs, land costs, etc. in making the final decisions on the location and size of the school to be built.

If the capacity of existing schools is to be increased either by permanent construction or by adding mobile classrooms, determination of the best location for the additional space can be made by removing capacity constraints from the existing schools and seeing where pupils are assigned to minimize pupil-miles of travel.

In school systems with declining enrollments there may be situations in which the utilization of certain schools is very low in the optimum assignment plan. This information may be used to select schools which should be closed to reduce operating costs and thus maintain complete programs at remaining schools.

3. Organizational Problems

School systems are organized in many different grade level arrangements. These may vary within a given system. For example, one school may contain grades 8-12 while another school may contain grades 9-12. The pros and cons of program aspects of different grade level organizations will not be addressed in this report. However, the computer procedure presented can be quite useful when evaluating the economics of different grade level structure. The procedure could be particularly useful when the grade level organization throughout the system is the same. It could be used to develop assignment

plans and estimate costs of transportation for any number of alternative organizations that the planners would like to consider.

One popular organizational structure involves the concept of feeder schools. Under this concept pupils who start to school in the same elementary school would eventually go to the same high school. The computer procedure can be used to develop plans for the feeder concept by first developing pupil assignment plans for the high school. The attendance area identified for each high school would represent the area which would be considered in developing assignment plans for the junior high schools which feed that high school. Thus the problem of developing assignment plans for an entire school system would consist of one plan for the high schools, one per high school for the junior high schools, and one per junior high school for the elementary schools. The process of arriving at the most desirable plans might require several iterations through the computer program. It might be necessary, for example, to choose a high school plan which is not the most desirable in order to obtain desirable junior high and elementary assignment plans.

4. Facility Forecasting

The matter of optimum utilization of school facilities has been discussed. When the location and population of pupils and location and capacity of schools are known, this problem is not simple. When one considers the changes in pupil population, both rate changes and location changes, and addition and deletion of facilities, this problem becomes even more complex. In much the same way that the program can be utilized to study facility site selection and facility utilization, it can be used to study the effects of future changes in pupil population and facilities on pupil assignment plans and transportation of pupils.

More importantly, this program can be used to develop input during the

planning stages for site selection of new schools and for decisions which local government must make regarding such issues as zoning for new residential development or low-income housing projects. In the latter case the program can be used to determine effects of pupil assignment on existing schools from the point of view of capacity and racial makeup of the schools.

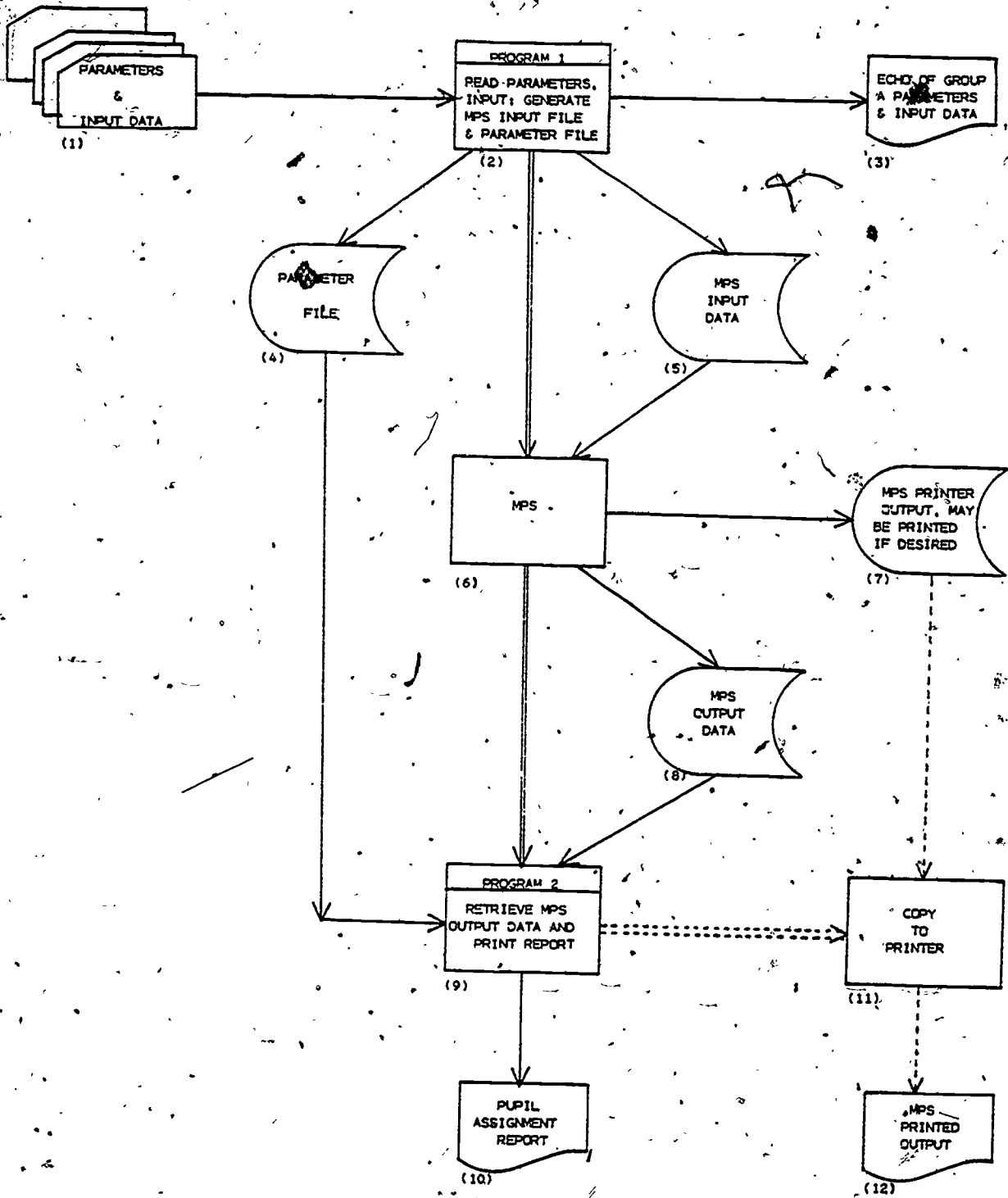
LINEAR PROGRAMMING SYSTEM
FOR SCHOOL LOCATION AND FACILITY UTILIZATION
SYSTEM DOCUMENTATION

The linear programming system for school location and facility utilization uses the MPS/360 version 2 compiler which is currently in use at the Triangle University Computing Center. The MPS system uses a revised simplex method to calculate probable solutions. The two fortran programs that are a part of the system are written for the Fortran IV G Level Compiler and are used to format the input data for MPS and then to transform the MPS output into a readable report format.

SYSTEM NARRATIVE

The first program (2) reads the parameter cards (1) and the input data. The parameters set the constraints for the solution and specifies the amount of input data that is to be read in. The first program also generates a parameter file (4), the MPS input data file (5), an echo of the following parameters: number of schools, number of pupils, number of assignment areas, maximum busing distance, minimum busing distance, maximum minority ratio, minimum minority ratio, number of days in the school year; cost per pupil mile, school system name, city, North Carolina Board number of cost per pupil mile, number of socio-economic factors, ratio for each factor, and an echo of the area input data. (3) the second step executes MPS (6) which reads in the input data (5) and generates the MPS output data (8). The print produced by MPS is saved in a temporary file (7) so it may be printed if desired but printing is not necessary in most cases. The third step executes the second program (9) which accesses the MPS output data with special subroutines available at TUCC. [Postin, Array, Colnam, Vector] the report generated (10) indicates the assignments made in a readable format. The last step is optional and may be executed if the MPS printed output is desired. A utility program (11) that simply copies datasets is employed to read the temporary dataset (7) and produce a printed copy.

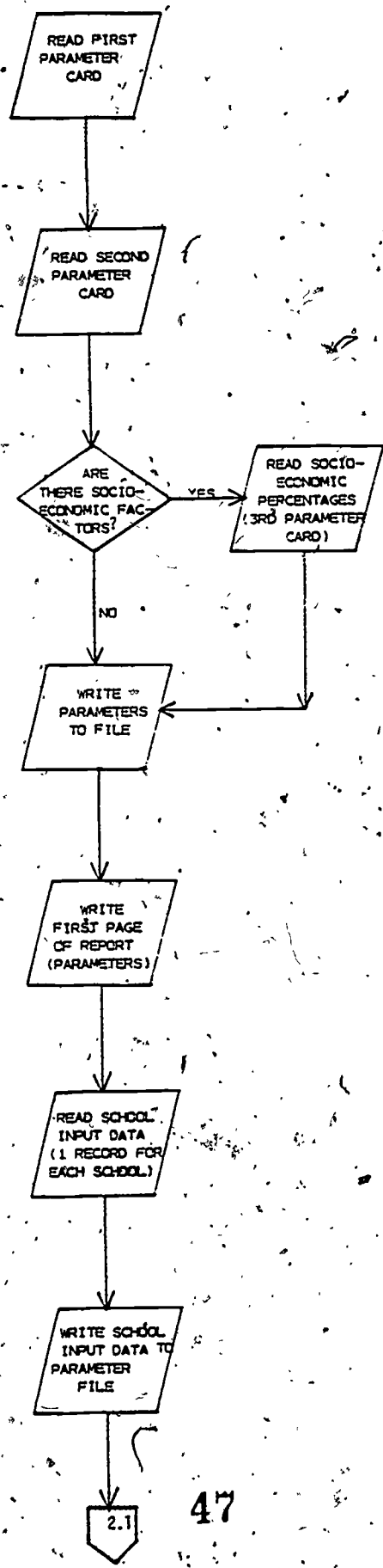
LINEAR PROGRAMMING SYSTEM FLOW

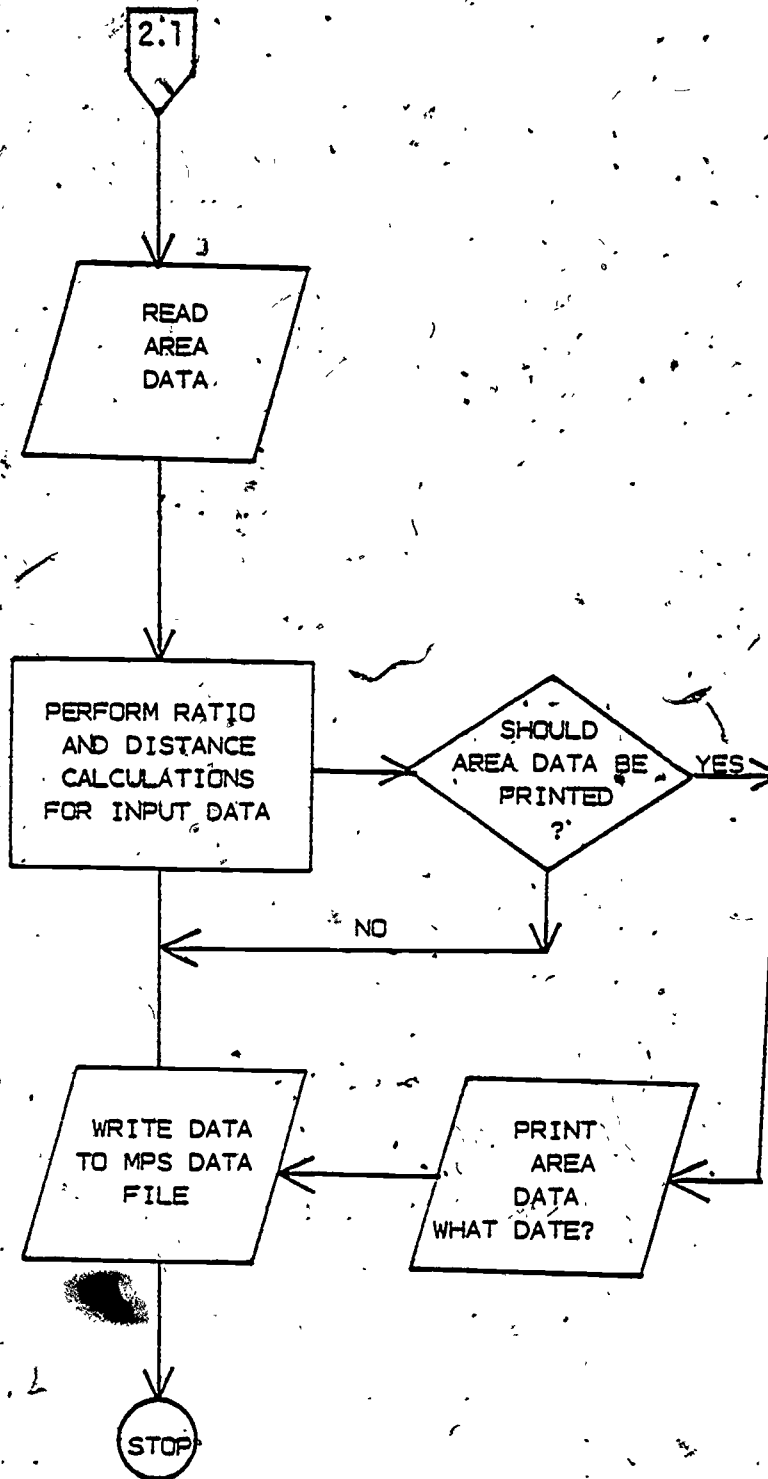


PROGRAM 1 (DATA ENTRY)

The program reads the first two parameter cards and from one of the parameters determines if there is an additional parameter card. If so, the third parameter card is read. The parameters are written to a file to be used as input to the second program. A report page of the parameters is printed out and the school input data is read in. There must be one card for each school specified in the parameter card. This information is also written to the parameter file.

The area data is read in and processed. This information is contained on two cards for each area and there must be two cards for each area from the parameter card. The input for each area contains its location with X, Y coordinates and the majority and minority pupils for each grade, kindergarten through 12th grade. The program calculates the minority ratio and the distance to each school from each area. This information is written to the MPS input dataset. If called for the minority ratio for each grade in an area is printed out.



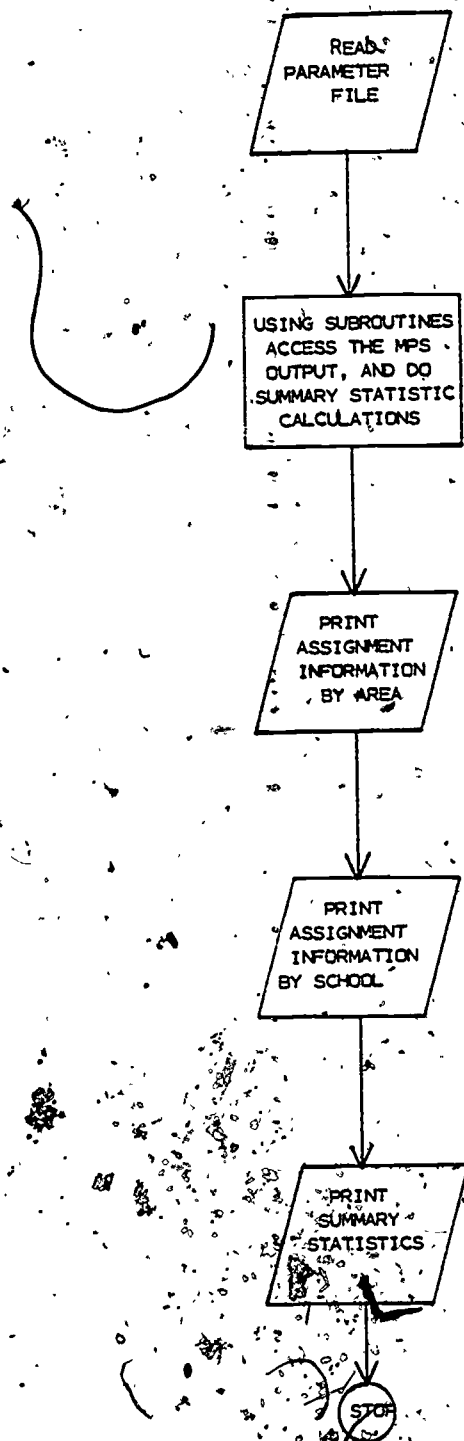


PROGRAM 2 (MPS SOLUTION)

The second program accesses the MPS output dataset using special MPS subroutines. The data is printed out and summary statistics are accumulated for a one page summary.

NOTE: For information on the subroutines see IBM publication:

MPS/360 Read Communications Format (Readcomm) (H20-0372)



USERS GUIDE

The linear programming system for school location and facility utilization requires some input parameters that describe to some extent the input data that follows. It should be noted that very little error checking is done, so to ensure maximum reliability of the output, the input data should be checked thoroughly. Following is a brief description of the input parameters and input data.

INPUT PARAMETERS

NUMBER OF SCHOOLS (Figure 1A, item 1)

This parameter indicates the number of schools that are being considered. Maximum number of schools is twenty-six.

NUMBER OF ASSIGNMENT AREAS (Figure 1A, item 2)

This parameter indicates the number of areas that contain pupils that need to attend the schools from the first parameter. Maximum number of areas is one hundred sixty.

PLAN NUMBER (Figure 1A, item 3)

This is merely an input that will appear on the report generated to indicate the plan number.

LOWER LIMIT ON MINORITY COMPOSITION (Figure 1A, item 4)

This parameter sets the minimum minority ratio allowed for a school make-up.

UPPER LIMIT ON MINORITY COMPOSITION (Figure 1A, item 5)

This parameter sets the maximum minority ratio allowed for a school make-up.

MINIMUM DISTANCE FOR BUSING (Figure 1A, item 6)

This parameter sets the minimum distance for which a pupil may be bused to a school.

MAXIMUM DISTANCE FOR BUSING (Figure 1A, item 7)

This parameter sets the maximum distance a pupil will be bused to a school (one way).

FIRST GRADE TO CONSIDER (Figure 1A, Item 8)

Since this model will treat kindergarten through the 12th grade, and most schools do not hold all twelve grades, it was necessary to determine which grades were to be considered. This parameter sets the first grade to be included in the assignment plan.

LAST GRADE TO CONSIDER (Figure 1A, item 9)

This parameter sets the last grade to be included in the assignment plan.

DISTANCE ADJUSTMENT FACTOR (Figure 1A, item 10)

This parameter is a number which is added to the distance calculation since the calculations are based on straight line distances and the majority of distances will not be straight line.

DISTANCE FLAG (Figure 1A, item 11)

This flag allows input of particular distances such as when the distance calculation will not be correct.

MINIMUM DISTANCE FLAG (Figure 1A, item 12)

This flag allows the bypass of the second closest school option.

PRINT FLAG (Figure 1A, item 13)

This flag allows printing of the input data to be bypassed.

NAME OF SCHOOL SYSTEM (Figure 1B, item 1)

This is the name that will appear in the report title.

CITY LOCATION

This is the name of the city for the school system that will appear in the report title.

NUMBER OF PUPILS IN THE SCHOOL SYSTEM (Figure 1B, item 3)

Printout only

NUMBER OF DAYS IN SCHOOL YEAR (Figure 1B, item 4)

Used for final cost calculations

COST PER PUPIL MILE (Figure 1B, item 5)

Estimated cost of busing one pupil one mile

NORTH CAROLINA BOARD OF EDUCATION (Figure 1B, item 6)

Source of cost per pupil mile

NUMBER OF SOCIO-ECONOMIC FACTORS (Figure 1B, item 7)

Determines if an additional input card should be read. Also determines the number of factors.

NOTE: Figure 1C - Has not been implemented yet. The card must be present if the above item so indicates, however, nothing is done with the data.

FIGURE 1C LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION AND FACILITY UTILIZATION

FILE LAYOUT

Application _____ INPUT PARAMETER _____ Date _____ Job No. _____ Page No. _____

1	2	3	4	5	6	7	8	9	10	11	12	
99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999
1 2 3 4 5 6 7	8 9 10 11 12	13 14 15 16 17 18	19 20 21 22 23 24 25	26 27 28 29 30 31	32 33 34 35 36 37	38 39 40 41 42 43	44 45 46 47 48 49	50 51 52 53 54 55	56 57 58 59 60 61	62 63 64 65 66 67	68 69 70 71 72	73 74 75 76 77 78 79 80

1. Percent of socio-economic category 1.
2. Percent of socio-economic category 2.
3. Percent of socio-economic category 3.
4. Percent of socio-economic category 4.
5. Percent of socio-economic category 5.
6. Percent of socio-economic category 6.
7. Percent of socio-economic category 7.
8. Percent of socio-economic category 8.
9. Percent of socio-economic category 9.
10. Percent of socio-economic category 10.
11. Percent of socio-economic category 11.
12. Percent of socio-economic category 12.

INPUT DATA

Two kinds of input data are necessary after defining the input parameters. The first describes the schools, the second describes the areas that are involved in the assignment plan. The school input data consists of a school identification so the school may be identified in the printouts generated, the X and Y location from a grid map and the school's capacity. (see figure 2A) The area data consists of an area identification which is used the same way the school ID is used, the X and Y coordinates from a grid map, and the number of majority and minority pupils for kindergarten through the twelfth grade. There is also a place to indicate the socio-economic category that describes the area, however, at this time this information is not in use. Figure 2B and 2C show the formats for entering the area data. For each area, there must be two cards, one from figure 2B and one from figure 2C. The two cards must also appear consecutively, figure 2B first.

FIGURE 2B LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION AND FACILITY UTILIZATION

FILE LAYOUT

Application INPUT DATA. Date _____ Job No. _____ Page No. _____

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16				
9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999	9999				
1 2 3 4	5 6 7 8	9 10 11 12	13 14 15 16	17 18 19 20	21 22 23 24	25 26 27 28	29 30 31 32	33 34 35 36	37 38 39 40	41 42 43 44	45 46 47 48	49 50 51 52	53 54 55 56	57 58 59 60	61 62 63 64	65 66 67 68	69 70 71 72	73 74 75 76	77 78 79 80

1. Area ID.
2. Socio-economic factors (not used).
3. X coordinate.
4. Y coordinate.
5. Number of majority pupils in kindergarten.
6. Number of minority pupils in kindergarten.
7. Number of majority pupils in first grade.
8. Number of minority pupils in first grade.
9. Number of majority pupils in second grade.
10. Number of minority pupils in second grade.
11. Number of majority pupils in third grade.
12. Number of minority pupils in third grade.
13. Number of majority pupils in fourth grade.
14. Number of minority pupils in fourth grade.
15. Number of majority pupils in fifth grade.
16. Number of minority pupils in fifth grade.

In order for the program to run correctly, the input parameters and data must be in a predetermined sequence. Following is the input sequence of cards.

NUMBER OF CARDS

FORMAT

1

Figure 1A

1

Figure 1B

1*

Figure 1C

1 For each school in item 1,
Figure 1A

Figure 2A

1

Figure 2B

1 } One pair for each area in
1 } item 2, Figure 1A

Figure 2C

1**Card for each distance to be
overridden

Figure 1A

* Only if item 7 from figure 1B is not equal to zero.

** Only if distances are to be overridden. The number of area is placed in item 1, the number of the school is placed in item 2 and the distance is placed in item 3.

LINEAR PROGRAMMING SYSTEM
FOR SCHOOL LOCATION AND FACILITY UTILIZATION
EXAMPLE OUTPUT

The output from a computer run consists of six formats. The first format is an echo of the initial conditions (format 1). The number of schools that are being considered in the problem (1A), the combined capacity of the schools (1B) and the number of areas from which a pupil may be assigned (1C) are a few of the constraints which are echoed. The second format is a printout of the area data (format 2). The area identification (2A), the number of pupils which will attend each grade (2B) and the ratio of minority pupils in each grade (2C) are reported. The third format is a printout of the distances that were calculated between each area and school (format 3). The area ID (3A), X and Y coordinates (3B), distance to each school (3C), school ID (3D) and school X and Y coordinates (3E) are on the report. The three formats described thus far are produced by the first program. The remaining three formats are produced by the second program using the MPS output data. The fourth format lists the assignment information by area (format 4). The area ID (4A), number of pupils in the area (4B), percentage of pupils that are part of a minority (4C), school ID to which the pupils have been assigned (4D) and the number of pupils from the area that were assigned to the school (4C). The fifth format lists the assignment information by school (format 5). The school ID (5A), areas from which pupils are assigned to the school (5B), number of pupils in the area (5C) and the number of pupils from the area that were assigned to the school (5D) are given. Also included is a breakdown by grade of the pupils assigned (5E) and the percentage of each grade's enrollment in relation to the whole school (5F). The final format is a statistical summary of the solution arrived at by MPS (format 6). The school system name (6A) is presented in the title. The total distance in miles pupils will have to travel one way to attend the school to which they were assigned for one day (6B), which includes

both busing and non busing distances, the total distance pupils will be bused one way on one day (6C), the number of pupils bused (6D), the number of pupils that are bused away from the school closest to them (6E), the cost of busing for one day (6F) and the school year cost of busing are summarized in this final report.

PUPIL ASSIGNMENT PLAN
FOR THE
MECKLENBERG COUNTY SCHOOL SYSTEM
MATTHEWS, NORTH CAROLINA

* INITIAL CONDITIONS

- * (A) NUMBER OF SCHOOLS 3 NUMBER OF PUPILS 150000, (B) NUMBER OF ASSIGNMENT AREAS, (C) 69
* NO PUPIL WILL BE BUSED MORE THAN 20.00 MILES ONE WAY
* NO PUPIL WHO LIVES WITHIN 1.50 MILES OF A SCHOOL WILL BE BUSED TO THAT SCHOOL
* THE RACIAL MAKEUP OF EACH SCHOOL MUST BE BETWEEN 29.20 AND 69.20 PERCENT MINORITY
* THE SCHOOL YEAR CONSISTS OF 166 DAYS

* BUSING COST DATA

- * BUSING COSTS ARE BASED ON AN AVERAGE COST PER PUPIL MILE OF \$00.35
* (OBTAINED FROM THE NORTH CAROLINA BOARD OF EDUCATION, #0476)

| THIS PROGRAM WAS DEVELOPED FOR THE SCHOOL PLANNING DIVISION
| OF THE NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION
| (EUGENE CHEATHAM, COORDINATOR), BY THE CENTER FOR URBAN
| AFFAIRS, NORTH CAROLINA STATE UNIVERSITY UNDER CONTRACT
| DIRECTED BY DAVID A. NORRIS. ANALYSTS RESPONSIBLE FOR
PROGRAM DEVELOPMENT, WERE JAMES H. YOUNG AND ROBERT S. SOWELL

CLASSIFICATION OF PUPILS FOR EACH AREA

FORMAT 2

AREA: A001

GRADE	NO. OF PUPILS	PERCENT MINORITY
K	(B) 11.	(C) 45.455
1	11.	45.455
2	5.	0.0
3	7.	57.143
4	1.	100.000
5	1.	100.000
6	4.	100.000
7	9.	77.778
8	10.	50.000
9	13.	84.615

AREA: A002

GRADE	NO. OF PUPILS	PERCENT MINORITY
K	(B) 11.	(C) 72.727
1	11.	72.727
2	20.	75.000
3	1.	0.0
4	20.	45.000
5	24.	75.000
6	15.	60.000
7	3.	0.0
8	15.	66.667
9	27.	51.852

AREA: A003

GRADE	NO. OF PUPILS	PERCENT MINORITY
K	(B) 6.	(C) 83.333
1	6.	83.333
2	8.	37.500
3	3.	33.333
5	5.	40.000
6	3.	66.667
7	3.	100.000
8	8.	75.000
9	12.	58.333

FORMAT 3

SCHOOLS (WITH X AND Y COORDINATE)

AREA	XCOORD		YCOORD		
	XCOORD	YCOORD	SH1A	SH2A	SH3A
			63.00	46.00	02.00
			57.00	16.00	36.00
A001	56.10	61.10	2.3	7.8	10.5
A002	53.00	65.00	3.7	8.7	11.9
A003	45.90	53.50	5.1	9.1	11.6
A004	43.50	62.00	5.8	7.6	13.5
A005	40.70	58.20	6.5	6.6	13.6
A006	41.20	48.80	6.8	4.0	12.4
A007	26.50	56.00	10.6	8.1	17.1
A008	33.20	47.20	9.1	4.9	14.5
A009	17.90	48.70	13.3	8.9	10.0
A010	18.70	31.50	14.2	8.0	18.4
A011	27.90	35.80	11.7	5.3	15.7
A012	23.50	17.10	16.3	0.5	17.8
A013	36.90	32.50	10.4	2.8	13.1
A014	45.00	33.50	8.6	0.8	10.8
A015	50.00	35.00	7.4	1.2	9.3
A016	34.00	17.00	14.3	6.5	15.0
A017	43.00	17.00	13.0	5.6	12.6
A018	41.80	25.80	10.9	3.2	12.0
A019	55.10	29.30	8.4	3.3	8.0
A020	59.80	31.00	7.6	4.3	6.6
A021	58.50	22.90	10.0	5.3	7.8
A022	55.80	13.90	12.7	7.0	9.9
A023	79.50	18.10	12.3	11.0	5.2
A024	85.20	11.70	14.6	13.4	7.1
A025	89.90	15.30	14.4	14.1	6.4
A026	87.00	25.40	11.5	12.3	3.4
A027	80.00	34.30	9.4	11.6	1.3
A028	85.00	38.00	8.4	11.3	1.0
A029	109.50	31.50	15.4	18.5	8.1
A030	94.20	27.60	12.4	14.2	4.3
A031	91.90	36.50	10.3	13.3	2.9
A032	78.90	48.10	5.3	10.2	3.6
A033	92.90	57.30	8.7	14.9	6.9
A034	89.50	49.50	8.0	13.2	4.5
A035	99.60	48.50	10.9	16.0	6.3
A036	57.90	57.70	1.5	7.2	9.4
A037	64.40	56.10	0.5	7.9	7.7
A038	54.50	53.60	2.7	5.7	9.5
A039	56.50	51.40	2.5	5.4	8.6
A040	60.50	53.50	1.2	6.6	8.0
A041	62.00	51.50	1.6	6.5	7.3
A042	64.80	51.90	1.6	7.1	6.8
A043	69.80	51.20	2.6	8.2	5.7
A044	55.90	48.20	3.3	4.6	8.4
A045	59.20	47.80	2.9	5.1	7.4
A046	61.40	48.50	2.5	5.9	7.0
A047	62.60	48.80	2.4	6.1	6.7
A048	64.00	49.60	2.2	6.5	6.5
A049	56.90	44.00	4.2	3.2	7.6
A050	59.50	45.20	3.6	4.7	7.0
A051	60.90	46.20	3.2	5.2	6.8
A052	63.00	45.50	3.1	5.6	6.2
A053	64.60	46.00	3.2	6.1	5.8
A054	64.90	47.50	2.8	6.4	6.0
A055	65.90	46.90	3.0	6.8	5.6
A056	67.00	46.00	3.4	6.7	5.2
A057	61.80	44.10	3.8	5.1	6.3

FORMAT 4

SCHOOLS TO WHICH PUPILS IN EACH AREA ARE ASSIGNED

AREA	NO PUPILS IN AREA	PERCENT MINORITY		
(A) A001	(B) 36	(C) 25.00	(D) SH1A	(E) 36.
A002	60	45.00	SH1A	60.
A003	26	30.77	SH2A	26.
A004	30	36.67	SH2A	30.
A005	31	32.26	SH2A	31.
A006	39	35.90	SH2A	39.
A007	78	41.03	SH2A	78.
A008	65	50.77	SH2A	65.
A009	58	51.72	SH2A	58.
A010	59	42.37	SH2A	59.
A011	51	62.75	SH2A	51.
A012	41	36.59	SH2A	41.
A013	34	41.18	SH2A	34.
A014	49	69.39	SH2A	49.
A015	71	26.76	SH2A	71.
A016	48	22.92	SH2A	48.
A017	26	30.77	SH2A	26.
A018	106	28.30	SH2A	106.
A019	34	11.76	SH2A	34.
A020	58	20.69	SH2A	58.
A021	89	23.60	SH2A	89.
A022	37	32.43	SH2A	37.
A023	61	42.62	SH3A	61.
A024	99	50.54	SH3A	99.
A025	14	57.14	SH3A	14.
A026	45	82.22	SH3A	45.
A027	15	100.00	SH3A	15.
A028	84	100.00	SH3A	84.
A029	21	57.14	SH3A	21.
A030	44	13.64	SH3A	44.
A031	145	67.59	SH3A	145.
A032	101	83.17	SH3A	101.
A033	40	55.00	SH3A	40.
A034	19	47.37	SH3A	19.
A035	10	70.00	SH3A	10.
A036	66	0.0	SH1A	66.
A037	62	32.26	SH1A	62.
A038	95	0.0	SH1A	95.
A039	55	0.0	SH1A	55.
A040	105	0.0	SH1A	105.
A041	56	0.0	SH1A	56.
A042	74	0.0	SH1A	74.
A043	85	20.24	SH3A	85.
A044	85	0.0	SH2A	85.
A045	101	0.99	SH1A	51.
A046	73	90.41	SH1A	73.
A047	50	2.00	SH1A	50.
A048	66	0.0	SH1A	66.
A049	95	2.11	SH2A	95.
A050	77	0.0	SH2A	77.
A051	61	11.48	SH2A	61.
A052	95	100.00	SH1A	81.
A053	75	94.67	SH1A	75.
A054	44	97.73	SH1A	44.
A055	119	96.64	SH1A	119.
A056	97	97.94	SH2A	97.
A057	88	45.45	SH2A	88.
A058	101	100.00	SH1A	99.
A059	73	100.00	SH3A	73.
A060	74	100.00	SH3A	74.
A061	54	100.00	SH3A	54.



ASSIGNMENT INFORMATION

FOR

(A) 301A

PLAN-P#25

FORMAT 5

CAPACITY OF SCHOOL = 1510.

NUMBER OF PUPILS ASSIGNED = 1510.
PERCENT MINORITY = 48.441

PERCENT UTILIZATION = 100.000

AREAS ASSIGNED TO THIS SCHOOL

AREA	NO PUPILS	NO THIS SCHOOL	AREA	NO PUPILS	NO THIS SCHOOL	AREA	NO PUPILS	NO THIS SCHOOL	AREA	NO PUPILS	NO THIS SCHOOL	AREA	NO PUPILS	NO THIS SCHOOL
(B) A001	(C) 36.	(D) 36.	(B) A002	(C) 60.	(D) 60.	(B) A036	(C) 66.	(D) 66.	(B) A037	(C) 62.	(D) 62.	(B) A038	(C) 95.	(D) 95.
A039	55.	55.	A040	105.	105.	A041	56.	56.	A042	74.	74.	A045	101.	101.
A046	73.	73.	A047	50.	50.	A052	95.	95.	A053	75.	75.	A054	44.	44.
A055	119.	119.	A058	101.	98.	A064	79.	79.	A065	55.	55.	A068	83.	83.
A069	100.	100.												

ESTIMATED NUMBER OF STUDENTS FOR EACH GRADE

GRADE	NO. OF PUPILS	PERCENT OF TOTAL
6	(E) 383.	(F) 25.364
7	345.	22.849
8	393.	25.996
9	389.	25.791
TOTAL:	1510	

S U M M A R Y S T A T I S T I C S

(A) MECKLENBURG COUNTY SCHOOL SYSTEM

PLAN P#25

FORMAT 6

P U P I L - M I L E S O F T R A N S P O R T A T I O N P E R D A Y

(B) MINORITY PUPILS 10046.70
 MAJORITY PUPILS 9863.35

 TOTAL 19910.04

P U P I L - M I L E S O F B U S I N G P E R D A Y

(C) MINORITY PUPILS 9881.05
 MAJORITY PUPILS 9539.68

 TOTAL 19420.73

N U M B E R S O F P U P I L S B U S E D

(D) MINORITY PUPILS 2023.
 MAJORITY PUPILS 2017.

 TOTAL 4040.

N U M B E R O F P U P I L S B U S E D A W A Y F R O M N E A R E S T S C H O O L

(E) MINORITY PUPILS 578.
 MAJORITY PUPILS 591.

 TOTAL 1169.

B U S I N G C O S T S (B A S E D O N A N A V E R A G E C O S T O F \$ 00.35 P E R P U P I L - M I L E)

(F) COST PER DAY \$ 13,595.00

(G) COST PER YEAR \$ 2,256,689.00

JOB CONTROL LANGUAGE FOR THE
LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION
AND FACILITY UTILIZATION
RUN SEQUENCE

```
//LPSSLFU JOB ACC.OUN.TCODE,PROGRAMMER,T=5,PRTY=0
//*PW=
/**
/** STEP 1 (RSS)
/** EXECUTE FIRST PROGRAM
/** READ PARAMETER CARDS AND INPUT DATA
/** PRODUCE PARAMETER OUTPUT FILE (G.FT09001)
/** MPS INPUT FILE (G.FT04F001)
/**
/** RSS EXEC FTGCC
/** C.SYSIN DD *
```

```
{ PROGRAM DECK HERE }
```

```
/*
//G.FT04F001 DD DSN=&MPSINPT,
// UNIT=DISK,
// DISP=(,PASS)
// SPACE=(TRK,(50,10),RLSE),
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=1680)
//G.FT09F001 DD DSN=&MPSPARA,
// UNIT=DISK,
// DISP=(,PASS),
// SPACE=(TRK,(24,8),RLSE),
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=1680)
//G.SYSIN DD *
```

```
{ INPUT DATA HERE }
```

```
/*
/** STEP2 (MAG)
/** CATALOGUE SECOND PROGRAM FOR EXECUTION LATER
/**
/** MAG EXEC READCOMM, COND.L=((5,LT,RSS.C),(5,LT,RSS.L),(5,LT,RSS.G))
/**
/** READCOMM, IS A PROCEDURE WHICH CONCATENATES THE MPS LIBRARY
/** SO THE SUBROUTINES USED CAN BE FOUND
/**
/** G.SYSIN DD *
```

```
{ INPUT DECK HERE }
```

```
/*
//L.SYSIN DD *
INCLUDE SYSLIB(READCOMM)
ENTRY MAIN
NAME SOLREP(R)
```

```
/*
/**
```



```

/** STEP3 (PUPASGN)
/** EXECUTE MPS AND THEN THE CATALOGUED FORTRAN PROGRAM.
/** FILES USED-
/** CPC.SYSPRINT MPS PRINTED
/** EXEC.SYSPRINT OUTPUT
/** EXEC.INPUT MPS INPUT DATA
/** EXEC.FT03F001 PRINTER
/** EXEC.FT09001 PARAMETERS
/** EXEC.FT12F001 MPS OUTPUT
//PUPASGN EXEC MPS, COND.EXEC=( (5,LT,RSS.C), (5,LT,RSS.L), (5LT,RSS.G),
// (5,LT,MAG.C), (5,LT,MAG.L) ), REGION-ZOOK
//CPC.SYSPRINT DD DSN=&MPSPRNT,
// UNIT=DISK,
// DISP=(,PASS),
// SPACE=(TRK,(50,5),RLSE),
// DCB=(RECFM=FB,LRECL=133,BLKSIZE=1596)
//CPC.SYSIN DD *
{ MPS CONTROL CARDS } SEE MPS/360 CONTROL LANGUAGE (H20-0290-2)
/*
//EXEC.SYSPRINT DD DSN=&MPSPRNT,
// DISP=(MOD,PASS)
//EXEC.FT03F001 DD SYSOUT=A
//EXEC.INPUT DD DSN=&MPSINPT,
// DISP=(OLD,DELETE)
//EXEC.FT09F001 DD DSN=&MPSEARA,
// DISP=(OLD,DELETE)
//EXEC.FT12F001 DD DSN=&MPSOUTP,
// UNIT=DISK,
// DISP=(,PASS),
// SPACE=(TRK,(90,9),RLSE,CONTIG),
// DCB=(RECFM=FB,LRECL=36,BLKSIZE=7200)
/**
/** STEP4 (TT)
/** COPY MPS PRINTED OUTPUT
/**
//TT EXEC COPY
//INPUT DD DSN=&MPSPRNT,
// DISP=(OLD,DELETE)
//

```

MPS PROGRAM

The MPS control program for normal use of the system is as follows:

```

0001          PROGRAM('ND')
0002          TITLE('PUPIL ASSIGNMENT')
0003          INITIALZ
0066          MOVE(XDATA, 'ABC-SCHL')
0067          MOVE(XPBNAM, 'PBFILE')
0068          MOVE(XOBJ, 'F1')
0069          MOVE(XRHS, 'CONSTRTS')
0070          ASSIGN('SCHOOL', 'INPUT', 'CARD')
0071          ASSIGN('READCOMM', 'FT12F001', 'COMM')
0072          PREPOUT('READCOMM')
0073          CONVERT('FILE', 'SCHOOL', 'SUMMARY')
0074          SETUP
0075          PRIMAL
0076          SOLUTION('FILE', 'READCOMM')
0077          FREECORE
0078          SOLREP
0079          EXIT
0080          PEND
    
```

There are two statements whose parameters in quotes in the parentheses must refer to data input as described earlier. The parameter 'ABC-SCHL' in statement number 003 represents the name given to the problem in the first eight characters of figure 1B, item 1 of the input. Statement 067 must contain the name of the function which the user wishes to minimize. This may be F1, F2, F3, or F4. The parameter 'FT12F001' in statement 71 refers to the file onto which the MPS solution is written for later use by program 2. The number 12 refers to the value assigned to Ifile in the fourth statement of program 2. If the user wishes to change this file number, he must change the value of Ifile in program 2.

When several runs are made on the same problem, with only minor changes in the input data, it may be possible to reduce computer time required to solve the linear program by starting with a solution from one of the earlier runs. This can be accomplished by the addition of a single MPS statement

Insert

which should be placed between statements 074 and 075. The data deck which

contains the earlier solution should follow the MPS program.

The card deck with this MPS solution can be obtained by inserting the statement

Punch

between statements 0075 and 0076 in the MPS program.

There may also be situations for which the user will want to evaluate a solution obtained by some other means. This solution should be punched in the same format as that provided by MPS. Again the MPS program which uses this solution must contain the insert statement and the data must follow the MPS program.

LINEAR PROGRAMMING SYSTEM
FOR SCHOOL LOCATION AND FACILITY UTILIZATION
FILE ORGANIZATION

PARAMETER FILE

Contents

The file contains the parameter information that was read into the first program.

Recording Information

Dataset Organization	(DSORG):	PS
Record Format	(RECFM):	FB
Record Length	(LRECL):	80
Block Length	(BLKSIZE):	1680
Medium	(UNIT):	DIST (3330)

LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION AND FACILITY UTILIZATION

FILE LAYOUT

Application PARAMETER FILE Date Job No. Page No. 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
---	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------	---------------

- 1. School ID:
- 2. Capacity:



LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION AND FACILITY UTILIZATION

FILE LAYOUT

Application PARAMETER FILE Date _____ Job No. _____ Page No. 4

1	2	3	4	5	6				
99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999	99999999
1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16	17 18 19 20 21 22 23 24	25 26 27 28 29 30 31 32	33 34 35 36 37 38 39 40	41 42 43 44 45 46 47 48	49 50 51 52 53 54 55 56	57 58 59 60 61 62 63 64	65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80

NOTE: This format represents a matrix of the number of assignment areas by the grades being considered in the problem. Each entry in the matrix is the minority ratio for the area and grade that comprise the subscripts.

- Example:
- 1 - Minority ratio for area 1 grade 6
 - 2 - Minority ratio for area 1 grade 7
 - 3 - Minority ratio for area 1 grade 8
 - 4 - Minority ratio for area 1 grade 9
 - 5 - Minority ratio for area 2 grade 6
 - 6 - Minority ratio for area 2 grade 7



MPS Input

Contents

The file contains input data for MPS that has been reformatted by the first program.

Recording Information

Dataset Organization (DSORG):	PS
Record Format (RECFM):	FB
Record Length (LRECL):	80
Block Length (BLKSIZE):	1680
Medium (UNIT):	DISK (3330)

LINEAR PROGRAMMING SYSTEM FOR SCHOOL LOCATION AND FACILITY UTILIZATION

FILE LAYOUT

Application MPS INPUT Date _____ Job No. _____ Page No. 9

1	2	3	4	5	6
9999	9999	9999	9999	9999	9999
1 2 3 4	5 6 7 8	9 10 11 12	13 14 15 16 17 18	19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36	37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80

1. Area ID.
2. School ID.
3. Area ID.
4. '1'.
5. School ID.
6. Number of pupils for area.



MPS Output

Contents

Data created by MPS as solution to problem.

Recording Information

Dataset Organization (DSORG): PS
Record Format (RECFM): FB
Record Length (LRECL): 36
Block Length (BLKSIZE): 7200
Medium (UNIT): DISK (3330)

NOTE: MPS/360 Read Communications Format

(READCOMM)
(H20-0372)

MPS Print

Contents

Printed output normally produced by MPS

Recording Information

Dataset Organization (DSORG): PS
Record Format (RECFM): FB
Record Length (LRECL): 133
Block Length (BLKSIZE): 1596
Medium (UNIT): DISK (3330)

MATHEMATICAL MODEL

The objective is to make pupil assignments from a given number of source areas to a given number of schools such that some function of distance or number of pupils is minimized while maintaining a desired racial mix in each school.

The following notation is used in the model:

M = Number of assignment areas

n = Number of schools

N_i = Number of pupils in source areas, $i = 1, 2, \dots, M$

r_i = Fraction of pupils in source area who belong to a minority group
 $i = 1, \dots, M$

X_{ij} = Fraction of pupils in source area i assigned to school j , $i = 1, 2, \dots, M$; $j = 1, 2, \dots, n$

C_j = Capacity of school j , $j = 1, 2, \dots, n$

U = Upper limit on fraction of pupils assigned to a school who belong to the minority group

L = Lower limit on fraction of pupils assigned to a school who belong to the minority group

d_{ij} = Distance from source area i to school j

D = Minimum distance pupils are eligible for busing

The mathematical statement of the linear program which maximizes total pupil-miles traveled is as follows:

Minimize

$$Z = \sum_{ij} d_{ij} N_i X_{ij}$$

[1]

Subject to the constraints

$$\sum_{j=1}^n X_{ij} = 1 \quad \text{for } i = 1, 2, \dots, m \quad [2]$$

$$\sum_{i=1}^m N_i X_{ij} \leq C_j \quad \text{for } j = 1, \dots, n \quad [3]$$

$$\sum_{i=1}^m r_i N_i X_{ij} \leq U \sum_{i=1}^m N_i X_{ij} \quad \text{for } j = 1, 2, \dots, n \quad [4]$$

$$\sum_{i=1}^m r_i N_i X_{ij} \geq L \sum_{i=1}^m N_i X_{ij} \quad \text{for } j = 1, 2, \dots, n \quad [5]$$

$$X_{ij} \geq 0 \quad \text{for } i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

There are basically four types of constraints in the linear program as represented by equations [2], ..., [5]. The first constraint, equation 2, insures that all pupils are assigned to some school. The second constraint, equation 3, restricts the number of pupils assigned to each school to be less than or equal to the capacity of that school. The third and fourth constraint, equations 4 and 5, maintain an upper and lower limit, respectively, on the percentages of pupils in each school which represent a minority group. Other objective functions which can be evaluated include:

Minimization of pupil miles based.

Minimize

$$Z = \sum_{i,j} d_{ij} N_i X_{ij}$$

$$\text{Where } d_{ij} = \begin{cases} d_{ij}, & \text{for all } ij \text{ pairs such that } d_{ij} > D \\ 0, & \text{otherwise} \end{cases}$$

Minimization of number of pupils bused

Minimize

$$Z = \sum_{ij} N_{ij} X_{ij}$$

ij

$$\text{where } N_{ij} = \begin{cases} N_i, & \text{for all } ij \text{ pairs such that } d_{ij} > D \\ 0, & \text{otherwise} \end{cases}$$

Minimization of number of pupils bused past nearest school

Minimize

$$Z = \sum_{ij} N_{ij} X_{ij}$$

ij

$$\text{where } N_{ij} = \begin{cases} N_i, & \text{for all } ij \text{ pairs such that } d_{ij} > d_{ij}^* \text{ (where} \\ & d_{ij}^* \text{ is the distance from area } i \text{ to the nearest} \\ & \text{school) and } d_{ij} > D \\ 0, & \text{otherwise} \end{cases}$$