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ABSTRACT

This document contains 12 position statements of the National Council of Teachers of Mathematics. The statements affirm positions on basic skills, mathematics and bilingual/bicultural education, class size, guidance and counseling for secondary-school counselors and mathematics teachers, involvement of and pressures on classroom teachers, individual differences, the metric system, use of calculators, competency-based teacher education, computers, and career education. (MS)

NATIONAL COUNCIL OF
Teachers of Mathematics
1906 Association Drive Reston, Virginia 22091 ■ (703) 620-9840

POSITION STATEMENTS

November 1977: E

BASIC SKILLS

The National Council of Teachers of Mathematics is encouraged by the current public concern for universal competence in the basic computational skills. The Council supports strong school programs that promote computational competence within a good mathematics program, and we urge all teachers of mathematics to respond to this concern in positive ways.

We are deeply distressed, however, by the danger that the "back to basics" movement might eliminate teaching for mathematical understanding. It will do citizens no good to have the ability to compute if they do not know what computations to perform when they meet a problem. The use of the hand-held calculator emphasizes this need for understanding: one must know when to push what button.

Consider in this regard a disturbing result of one recent national examination. Students were asked to determine 70 percent of the 4200 votes cast in an election. Almost half of the thirteen-year-olds and one out of five of the seventeen-year-olds applied the wrong arithmetic process. Some divided, some added, and some subtracted! Computational skills in isolation are not enough; the student must know <u>when</u> as well as <u>how</u> to multiply. We must address skills, but we must address them within a total mathematics program.

In a total mathematics program, students need more than arithmetic skill and understanding. They need to develop geometric intuition as an aid to problem solving. They must be able to interpret data. Without these and many other mathematical understandings, citizens are not mathematically functional.

Yes, let us stress basics, but let us stress them in the context of total mathematics instruction. (September 1976)

MATHEMATICS AND BILINGUAL/BICULTURAL EDUCATION

Every student should receive a meaningful and timely mathematics education and none should be excluded because of language or cultural differences. Schools should actively seek to identify the educational factors which diminish a student's opportunity to learn mathematics and to remove such barriers without disruption of the integrity of the student's cultural world. Special instruction in mathematics, using mate[illegible]l in the primary language of the student, should be provided until the student can function adequately in a mathematics class conducted mainly in English or in the predominant language of the area. (January 1976)

CLASS SIZE

Class size is important only insofar as it is appropriate to the mathematics learning activity taking place. Certain size groupings are best suited for certain teaching and learning activities; and, conversely, certain teaching and learning activities are best handled in certain size groupings. The activities may range from tutorial to lecture and the groups from 1-to-1 to 1-to-100s depending upon the particular learning desired and the learning style of the learner. (April 1970)

GUIDANCE/COUNSELING FOR BOTH COUNSELORS AND MATHEMATICS TEACHERS AT THE SECONDARY SCHOOL LEVEL

Today, more than ever before, the study and appreciation of mathematics are vital to the intellectual development and to the scientific, industrial, technological, and social progress of society. It is essential that teachers, counselors, supervisors, educational administrators, parents, and the general public work together to provide the best mathematics education possible for all students, regardless of sex, ethnic group, national origin, or ability. All students should be encouraged to keep options open by studying mathematics so as to make maximum use of their talents. Specifically, it is suggested that students include a maximum of mathematics appropriate to their abilities and interest in their high school programs.

The educational, vocational, personal-social choices and decisions made by students should lead to satisfying and worthwhile lives. The important members of the guidance team in each school, both the school counselor and the mathematics teacher, are responsible for helping students gain insight and understanding of themselves and their environment in this decision making. Therefore, they must work cooperatively in:

1. Planning mathematics programs for individual students.
2. Placing students in mathematics courses appropriate to their needs and abilities.
3. Anticipating developments in mathematics and fields that utilize mathematics.
4. Conferring with the school administration with regard to mathematics course offerings.
5. Planning a mathematics program designed for a specific field.

6. Securing, evaluating, and making available to students a variety of career publications.
7. Planning career-oriented activities.
8. Keeping students informed about:
   a. secondary school and college mathematics programs
   b. vocational and technical school mathematics requirements
   c. college entrance requirements in mathematics
   d. mathematics requirements for majoring in specific areas
   e. procedures for obtaining college credit for mathematics courses taken in high school
   f. career opportunities in mathematics
   g. mathematics needed for specific fields and professions

(January 1976)

INVOLVEMENT OF CLASSROOM TEACHERS

The NCTM encourages greater participation by classroom teachers at all levels in policy-making and governance roles in professional organizations. We encourage administrators to recognize the relevance of such activities to improvement of education generally and to teaching effectiveness particularly by providing teachers with adequate released time without loss to salary or leave to enable them to assume elective and appointive positions in professional educational organizations. (September 1972)

PRESSURES ON THE CLASSROOM TEACHER

We note with alarm the increasing pressures on the school classroom teacher. At a time when unresolved social problems overlap into the classroom, when teachers are expected to master both new content and new ways to communicate that content to students, and when schools and curriculum are both under continuous attack, teachers are being asked to carry still further burdens that can only affect their ability to teach.

In particular we decry the imposition on the classroom teacher of hastily adopted educational innovations which essentially redefine the teacher's role into that of a manager, clerk or curriculum developer. We take the position that the classroom teacher should teach and that he or she should be supported in, rather than diverted from, that important central role.

Without addressing the merits of all novel programs we believe that many innovations have not been evaluated from the perspective of the classroom teacher--the very person who is expected to change roles and to carry all the extra burdens of the new activity.

The problems cited are not based on ill will towards the classroom teacher, rather they are imposed by people who sincerely seek to improve instruction, but who have not realized all the implications of their actions. At the same time, we do not offer excuses for school systems that are increasing student and class loads for teachers and reducing teaching staffs, and thereby sacrificing future welfare to current political expediency.

We believe that mathematics is an important and pervasive basic tool for all citizens. We believe that teaching students good mathematics is a central role

of the schools and that the classroom teacher of mathematics is a key to this. We invite support for this beleagured individual from all sides: students, parents, Board members, administrators and supervisors. The classroom teacher must be given a chance to succeed at teaching. (January 1976)

INDIVIDUAL DIFFERENCES

While recognizing differences among students in mathematical needs and abilities, the NCTM takes the position that excellence in mathematics education is equally important for all. More specifically, excellence in mathematics education for students who are college bound is no more important than excellence in mathematics for others. Also, teachers cannot start with the assumption that initial differences among students which may be due to earlier nonschool experiences will determine the long-range needs and abilities of the students. Teachers must recognize that within any type of grouping whatever, students are likely to differ greatly. Therefore it is essential, if any type of grouping is desirable at all, that there be a great deal of flexibility making possible the moving of students from one group to another.

Whereas this position recognizes different rates by which individuals learn, it further declares that the labeling of learners as "slow" serves no purpose other than describing the rate of learning. Indeed, the goals of the NCTM are violated when persistent and continuous labeling leads to grouping practices and learning situations that prohibit attainment of mathematical literacy and deprive students of the option either to pursue or to discontinue further formal study. The same position is taken with respect to other labels, such as "non-college-bound," "disadvantaged," "migrant," or "inner city."

The commitment of mathematics educators is, therefore, to create an environment in the school such that each student achieves mathematical literacy and has the option to continue or to terminate the formal study of mathematics. (April 1971)

METRIC SYSTEM OF MEASUREMENT

Adoption of Metric System

The NCTM supports adoption of the metric system and encourages that this be a system to be taught by teachers of all grades, along with other systems of measurement. The Congress of the United States is urged to pass immediately a metric bill which provides for an implementation program of not more than ten years. (September 1969, September 1972, and April 1974)

Metric Spelling

The "er" spelling should be used for words such as "meter" and "liter" as they appear in general textbooks, films, charts, and displays. However, for papers and materials of scientific and/or international significance, the spelling of "metre" and "litre" is recommended. (September 1974)

USE OF MINICALCULATORS

With the decrease in cost of the minicalculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the minicalculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics. (September 1974)

COMPETENCY-BASED TEACHER EDUCATION

Competency-based teacher education (CBTE) and performance-based teacher education (PBTE) are labels which have generated considerable activity and heated discussions among teacher educators in the past five years. It may surprise some teachers to learn that the CBTE movement, albeit altered in some respects, was popular over thirty years ago. Contemporary teacher educators tend to prefer the label CBTE over PBTE in that the "performance" concept seems to be restricted to abilities which are observed in the classroom whereas "competency" is generally interpreted to include classroom teaching skills as well as intellectual skills which might be assessed on the college campus. Throughout this resolution "competency" is used in this broad sense.

It is a fact that state departments of education, teacher organizations, and some segments of the professional community have added other facets to their definition of CBTE. The rhetoric that surrounds the initials CBTE (competency-based teacher education) or PBTE (performance-based teacher education) sometimes addresses itself to the issue of governance, sometimes to the nature of effective teaching, and sometimes to issues involving the goals of the instruction. Some of the literature equates CBTE with field-based teacher education, some with mastery of minimal objectives. These and other connotations of the label CBTE have drawn attention from the goal of improving teacher education and have led to divisive, polar positions. The issue is good teacher training that makes a strong contribution to good classroom instruction and thereby to student achievement and satisfaction, as opposed to poor teacher training that contributes little to these general goals.

We are convinced that there are good and bad CBTE programs just as there are good and bad non-CBTE programs.

Recommendation 1. The National Council of Teachers of Mathematics (NCTM) recommends that the CBTE, however defined locally, not be used exclusively by certification bodies until more research and evaluation of its outcomes are available.

In its continuing effort to improve mathematics teaching, the NCTM attempted to contribute to the effort of better delineation of competencies needed by the beginning teacher of mathematics in its document, "Guidelines for the Preparation of Teachers of Mathematics." We note that these "Guidelines", while including a necessary subset of these competencies, do not include those which can be observed only in the classroom.

Recommendation 2. The NCTM recommends that:

a. The competencies identified in the "Guidelines" be utilized as base-line competencies for purposes of teacher education.

b. Efforts to identify and assess additional competencies, in particular those observable only in the classroom, be encouraged.

Any assessment of teacher performance must recognize that the teacher functions as an integrated whole, not as a composite of small, narrowly defined behaviors. Care should be taken to avoid the checking off of individual competencies as the sole measure of a teacher's performance. This type of "check-off" assessment fails to demand that the teacher consistently demonstrate a particular ability. Alternatively, assessment systems devised to take into account how the teacher functions as an integrated whole should be a basic procedure in teacher evaluation.

Recommendation 3. The NCTM recommends that evaluation in teacher education programs be characterized by systematic assessment of all competencies over a period of time to identify consistent and effective performance.

Many of the competencies necessary for the successful teaching of mathematics are intrinsically related to the nature of mathematics. Their identification and assessment requires the skills of those working in the discipline.

Recommendation 4. The NCTM recommends that the identification and assessment of performance related to mathematics teaching should be the responsibility chiefly of the professionals in the field of mathematics education: college professors of mathematics and mathematics education, school mathematics teachers, and mathematics supervisors.

Some regions have mandated an approach to certification without specifying the need to include representatives from the groups designated in Recommendation 4.

Recommendation 5. The NCTM recommends that:

a. Representatives from the mathematics education community be involved in the development of competencies and assessment procedures related to mathematics teaching.

b. If NCTM Affiliates have prepared guidelines, those guidelines should be used as a framework against which proposals can be judged. Where guidelines are not available, the NCTM guidelines should be used in a similar way.

In making all five of the above recommendations, the NCTM reasserts the need to encourage a variety of creative approaches to the complex problem of teacher education. (September 1976)

COMPUTERS IN THE CLASSROOM

Although computers have become an essential tool of our society, their diverse and sustained effects on all of us are frequently overlooked. The astounding computational power of the computer has altered priorities in the mathematics curriculum with respect to both content and instructional practices. Improvements in computer technology continue to make computers, minicomputers, and programmable calculators increasing accessible to greater numbers of students at reasonable cost.

An essential outcome of contemporary education is computer literacy. Every student should have firsthand experiences with both the capabilities and the limitations of computers through contemporary applications. Although the study of computers is intrinsically valuable, educators should also develop an awareness of the advantages of computers both in interdisciplinary problem solving and as an instructional aid. Educational decision makers, including classroom teachers, should seek to make computers readily available as an integral part of the educational program. (September 1976)

CAREER EDUCATION

In recognition of the prime importance of work in our society and the role that mathematics plays in the lives of all individuals, The National Council of Teachers of Mathematics supports the position that Career Education should be a major goal of all who teach and learn mathematics.

For the purposes of this statement, Career Education is the composite of all learning experiences, classroom and non-classroom, that promote:

1. The learner's understanding of the values of the work ethic and how these values contribute to his or her personal development;
2. The learner's awareness of the nature of various careers and how mathematics is used in those careers; and
3. The learner's attainment of mathematical concepts and skills, with the ability to apply that knowledge to the solution of career related problems.

Career Education may be achieved by increasing the emphasis placed by schools on career awareness, exploration, decision making, and planning. Teachers, parents, and guidance counselors should encourage each student to pursue the study of mathematics to the highest level of his or her ability, making it clear that the value of knowledge and skills so gained cannot always be judged on the basis of immediate need or use.

The National Council of Teachers of Mathematics, recognizing that incorporation of concepts and approaches to Career Education into the school curriculum requires that teachers develop special knowledge and skills, recommends that Career Education training of mathematics teachers be given special and immediate attention at both preservice and inservice levels. (September 1977)

2. Homework and drill are very important pedagogical tools used to help the student gain understanding as well as proficiency in the skills of arithmetic and algebra, but students should not be burdened with excessive or meaningless drill. We therefore recommend that teachers and authors of textbooks step up their search for interesting problems that provide the opportunity to apply these skills. We realize that this is a difficult task, but we believe that providing problems that reinforce manipulative skills should have high priority, especially those that show that mathematics helps solve problems in the real world.
3. We are aware that teachers must struggle to maintain standards of performance in courses at all levels from kindergarten through college and that serious grade inflation has been observed. An apparently growing trend to reward effort or attendance rather than achievement has been making it increasingly difficult for mathematics teachers to maintain standards. We recommend that mathematics departments review evaluation procedures to ensure that grades reflect student achievement. Further, we urge administrators to support teachers in this endeavor.
4. In light of recommendation 3, we also recognize that the advancement of students without appropriate achievement has a detrimental effect on the individual student and on the entire class. We therefore recommend that school districts make special provisions to assist students when deficiencies are *first* noted.
5. We recommend that cumulative evaluations be given throughout each course, as well as at its completion, to *all* students. We believe that the absence of cumulative evaluation promotes short-term learning. We strongly oppose the practice of exempting students from evaluations.
6. We recommend that computers and minicalculators be used in imaginative ways to reinforce learning and to motivate the student as proficiency in mathematics is gained. Calculators should be used to supplement rather than to supplant the study of necessary computational skills.
7. We recommend that colleges and universities administer placement examinations in mathematics prior to final registration to aid students in selecting appropriate college courses.
8. We encourage the continuation or initiation of joint meetings of college and secondary school mathematics instructors and counselors in order to improve communication concerning mathematics prerequisites for careers, the preparation of students for collegiate mathematics courses, joint curriculum coordination, remedial programs in schools and colleges, the exchange of successful instructional strategies, the planning of in-service programs, and other related topics.
9. Schools should frequently review their mathematics curriculum to see that it meets the needs of its students in preparing them for college mathematics. School districts that have not conducted a curriculum analysis recently should do so now, primarily to identify topics in the curriculum that could be either omitted or de-emphasized, if necessary, in order to provide sufficient time for the topics included in this statement. We suggest, for example, that the following could be de-emphasized or omitted from the curriculum:
   (*a*) Logarithmic calculations that can better be handled by calculators or computers
   (*b*) The extensive solving of triangles in trigonometry
   (*c*) Proofs of superfluous or trivial theorems in geometry
10. We recommend that algebraic concepts and skills be incorporated wherever possible into geometry and other courses beyond algebra to help students retain these concepts and skills.

This position statement was prepared jointly by the National Council of Teachers of Mathematics, 1906 Association Dr., Reston, VA 22091, and the Mathematical Association of America, 1225 Connecticut Ave., NW, Washington, DC 20036.

NCTM-MAA
Position Statement
on

RECOMMENDATIONS for the PREPARATION of high school students for COLLEGE MATHEMATICS COURSES

The following statement, adopted by the Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics, is a brief outline of the basic ingredients of adequate preparation for collegiate-level mathematics.* The statement does not break new ground; it reflects standards that have been generally accepted for over a decade. It is intended to support the continuing efforts of conscientious teachers everywhere to provide students with sound and stimulating mathematical training. It is specifically designed to provide a benchmark for our efforts and those of others to assess and react to recent reports of a general decline in the performance of students in mathematics.

A joint committee of the Mathematical Association of America and the National Council of Teachers of Mathematics consulted with secondary school and college teachers in various parts of the country to study recent trends in the preparation of students. The comments from these consultations on which there was strongest consensus are the basis for this statement and its ten recommendations.

The Mathematical Association of America and the National Council of Teachers of Mathematics wish to emphasize that the statement and recommendations, as they refer to secondary school programs, are addressed only to those programs for students planning to go to college and that they are not intended to be more comprehensive. During the past twenty years many important changes have taken place in both the content and teaching of mathematics at the secondary school level. Many excellent new programs have been adopted and taught effectively by teachers in elementary and secondary schools. Nevertheless, any consideration of the relative merits of new versus traditional school curricula has been deliberately avoided. A study of this issue would have

* *Collegiate mathematics* refers to courses in calculus (or calculus and analytic geometry), probability and statistics, finite mathematics, and higher-level mathematics courses.

exceeded both the charge to the committee and its limited resources. This statement and these recommendations incorporate many of the best features of both of these curricula and are addressed to all mathematics programs regardless of pedagogical heritage.

Necessary Course Work

Mathematics is a highly structured subject in which various concepts and techniques are greatly dependent on each other. The concepts of arithmetic and algebra, however, are basic to all of mathematics. Further work in mathematics and in all areas in which mathematics is used as a tool requires correct performance, with understanding, of basic arithmetic operations, the manipulation of algebraic symbols, and an understanding of what the manipulations mean.

Any student who is unable to perform arithmetic calculations and algebraic operations with accuracy and reasonable speed, to understand which operations to use in a given problem, and to determine whether the results have meaning is severely handicapped in the study and applications of mathematics. The prevalence of inexpensive pocket calculators makes the performance of complicated calculations less tedious but the use of calculators does not lessen the need for students to understand which concepts and operations are needed to solve a problem, to make sensible estimates, and to analyze their results.

For further work in mathematics, and in many other areas from business to psychology, from biology to engineering, the ability to use algebra with skill and understanding is also essential. Having a passing grade in algebra is not enough. Both understanding and competence in the skills of algebra are necessary. Neither conceptual understanding nor technical skill alone will suffice in today's world, let alone in tomorrow's. Algebra is a useful subject which will help to solve problems in the real world. Opportunities to apply algebraic skills should be provided whenever possible, especially to problems that show the utility of mathematics.

Algebra courses in secondary school should include, in addition to the basic topics—

(*a*) polynomial functions;
(*b*) properties of logarithms,
(*c*) exponential and logarithmic functions and equations;
(*d*) arithmetic and geometric sequences and series;
(*e*) the binomial theorem;
(*f*) infinite geometric series;
(*g*) linear and quadratic inequalities.

For most students, adequate coverage of the topics in algebra requires at least two years of study.

Students who will take calculus—and this now includes many students who will take college work in business, premedicine, economics, biology, statistics, engineering, and physical science—may or may not need trigonometry, depending on the type of calculus course appropriate for their particular programs But they will need a good deal of what is often called precalculus, including especially a sound understanding of the concept of a function, which is also fundamental for work beyond the most elementary level in probability and computing.

Those students needing trigonometry should study—

(*a*) trigonometric functions and their graphs;
(*b*) degree and radian measure;
(*c*) trigonometric identities and equations;
(*d*) inverse trigonometric functions and their graphs.

For such students, the equivalent of one semester should be devoted to the study of the topics in trigonometry.

All students who go on to take collegiate mathematics will find their college work easier if they have been introduced to some axiomatic system and to deductive reasoning. Traditionally this has been accomplished in a geometry course. Geometry courses in secondary school should include, in addition to basic topics—

(*a*) fundamental properties of geometric figures in three dimensions;
(*b*) applications of formulas for areas and volumes;
(*c*) experience in visualizing three-dimensional figures.

Other courses (the word *course* refers here and elsewhere in this statement to a semester course unless otherwise noted) beyond algebra, trigonometry, and geometry should be available to students who have adequate background and time to take them. A course in coordinate (or analytic) geometry is ideal, since it combines algebra with geometry and provides a useful preparation for calculus. In addition to coordinate geometry, courses in the following topics are valuable: probability, statistics, elementary finite mathematics (or linear algebra), an introduction to computers and computing, and applications of mathematics.

If coordinate geometry is offered, it should include, in addition to the basic topics—

(*a*) conic sections;
(*b*) rational functions and their graphs;
(*c*) polar coordinates;
(*d*) parametric equations and their graphs.

Inductive as well as deductive reasoning, techniques of estimation and approximation, and an awareness of problem-solving techniques, with special emphasis on the transition from the verbal form to the language of mathematics, should be emphasized in all courses.

Calculus, where offered in secondary schools, should be at least a *full year* course and be taken only by those students who are strongly prepared in algebra, geometry, trigonometry, and coordinate geometry.

We recognize that many secondary schools have a curriculum similar to that outlined above. We emphasize again that, in order to be properly prepared for collegiate-level courses in mathematics, students need to develop skills (1) in applying standard techniques and (2) in understanding important concepts.

Recommendations

The Board of Governors of the Mathematical Association of America and the Board of Directors of the National Council of Teachers of Mathematics make the following recommendations:

1. Proficiency in mathematics cannot be acquired without individual practice. We therefore endorse the common practice of making regular assignments to be completed outside of class. We recommend that parents encourage their children to set aside sufficient time each day to complete these assignments and that parents actively support the request of the teacher that homework be turned in. Students should be encouraged to develop good study habits in mathematics courses at all levels and should develop the ability to read mathematics.