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ABSTRACT

This guide deals with the total mathematics education program from K through grade 12. The primary focus is on the philosophy of mathematics education and deals with topics such as: teaching strategies, learning theory, problem solving, calculators, classroom management techniques, and staff development. Basic mathematics content strands serve as a framework for developing mathematical goals and planning a goal-based instructional program. Suggested learning activities for both formal and informal programs constitute a major portion of the guide. (MN)

MATH
in oregon schools

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Verne A. Duncan
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FOREWORD

Mathematics in Oregon schools changes from time to time in response to societal pressures such as metrication or technological advances such as computers and mini-calculators. Curriculum must respond if Oregon schools are to prepare students for their roles as individuals, producers, citizens and consumers. Sometimes changes challenge the very nature of computational processes; other times changes merely shift emphases, for example, the emergence of problem solving and problem-solving skills as major goals in mathematics education.

As planners develop curriculum at the local district level, they *must* consider the math needs of learners from kindergarten through graduation. This guide deals with the total math education program and includes many suggestions for meeting the range of individual student needs. Where electives are offered, math curriculum should be broad enough to provide a sequence of courses for non-college-bound students, college-bound students not necessarily math inclined, and college-bound students math inclined. The guide includes suggestions for both formal and nonformal programs.

Primarily the focus is on the philosophy of math education. In place of a hierarchy of math goals, basic math content strands serve as a useful framework for local district curriculum planners in developing math goals and in planning a goal-based instructional program. How students learn math, teaching strategies, classroom management techniques and many suggested learning activities fill out the framework.

Nearly all of the suggested ideas have been used successfully in Oregon classrooms. Hopefully more teachers will try some of these ideas.

Verne A. Duncan
State Superintendent of
Public Instruction

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The text illustrations come from the creative pen of Jon Maier who had just completed the ninth grade at Eugene's Kennedy Junior High School at the time he worked on them.

The Mathematics Resource Project and Director Alan Hoffer released several staff members to work on the guide. Still another project, the Oregon System in Mathematics Education, supported by the National Science Foundation, provided the assistance making it possible to bring these people together.

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GOAL-BASED PLANNING FOR MATHEMATICS

Goals are guideposts. They serve to give purpose and direction to a planning activity. Goals provide a common language for discussing the merits of various activities as those activities are carried out.

In mathematics, just as in any other instructional program offered by an educational system, a sense of purpose and direction is essential to good planning. But what are these purposes and directions? Where do they come from? Why should the math teacher be concerned? These are questions to be answered before effective planning of a math curriculum can proceed.

Each teacher must realize that planning a math curriculum cannot begin and end only in a given classroom. It needs to be done with a sense of similar planning in other classrooms and districts within the state.

The goals, goal setting, and competency-identification activities the state prescribes provide districts a common reference for the planning process. In goal-based planning, teachers must consider four levels of goals: state goals, district goals, program goals, course (and unit) goals. They must also consider student competency and state-defined competency areas.

State Goals answer the question: What does the state think a student should get out of public schooling anywhere in Oregon?

District Goals answer the question: What do the local community and its schools think a student ought to get out of local schooling, and how is that to relate to state goals?

Program Goals answer the question: What do the local curriculum planners and math teachers think a student ought to get out of math, and how is that to relate to district goals?

Course Goals—Secondary (and *Unit Goals—Elementary*) answer the question. What do the math teachers think a student ought to get out of a specific course such as Algebra I (or first grade math), and how is that to relate to program goals?

State Competency Areas, answer the question: What does the state believe to be the critical areas in which students must *demonstrate* competence for graduation?

Competency, then, answers the question: What have students demonstrated they can do with what they have learned?

The relationships among each of these components are illustrated in Figure 1 (page 3). Note that competencies may be stated broadly or specifically depending on how extensive the district desires a particular demonstration to be. The sample competency in the figure is shown as being an intended outcome of both district *and* program level goal planning. Other competencies could be shown as the intended outcomes of district or course level goal planning (broken lines).*

The system of goals and competencies just described** is designed to help the teacher and program specialist plan their own math program. It promotes a framework for planning that may be shared by all those doing similar planning. It helps in planning for individual student goals and interests, to be done within the limits of available resources. It should not be used to limit what is planned. Rather it should be used as a starting place.

*All goals may not be competencies, but all competencies may be used as goals. Goals are often written about the *acquisition* and *application* of knowledge and skills. Competencies are ALWAYS written about the *application* of knowledge and skills.

**For more information on goal-based planning, see Oregon Department of Education publications *Elementary-Secondary Guide for Oregon Schools: Part II, Suggestions* (1977) and *Planning the Education of Oregon Learners: Setting Goals* (1975)

Furthermore, frequent reexamination and evaluation of goals—and how closely they are being met—is a worthwhile investment of teacher time.* Sections of this guide give direction to the selection of learning activities and teaching strategies to meet the needs of individual students K-12 in light of established goals. Certain considerations underlie the broad framework of goals as outlined in this guide, including:

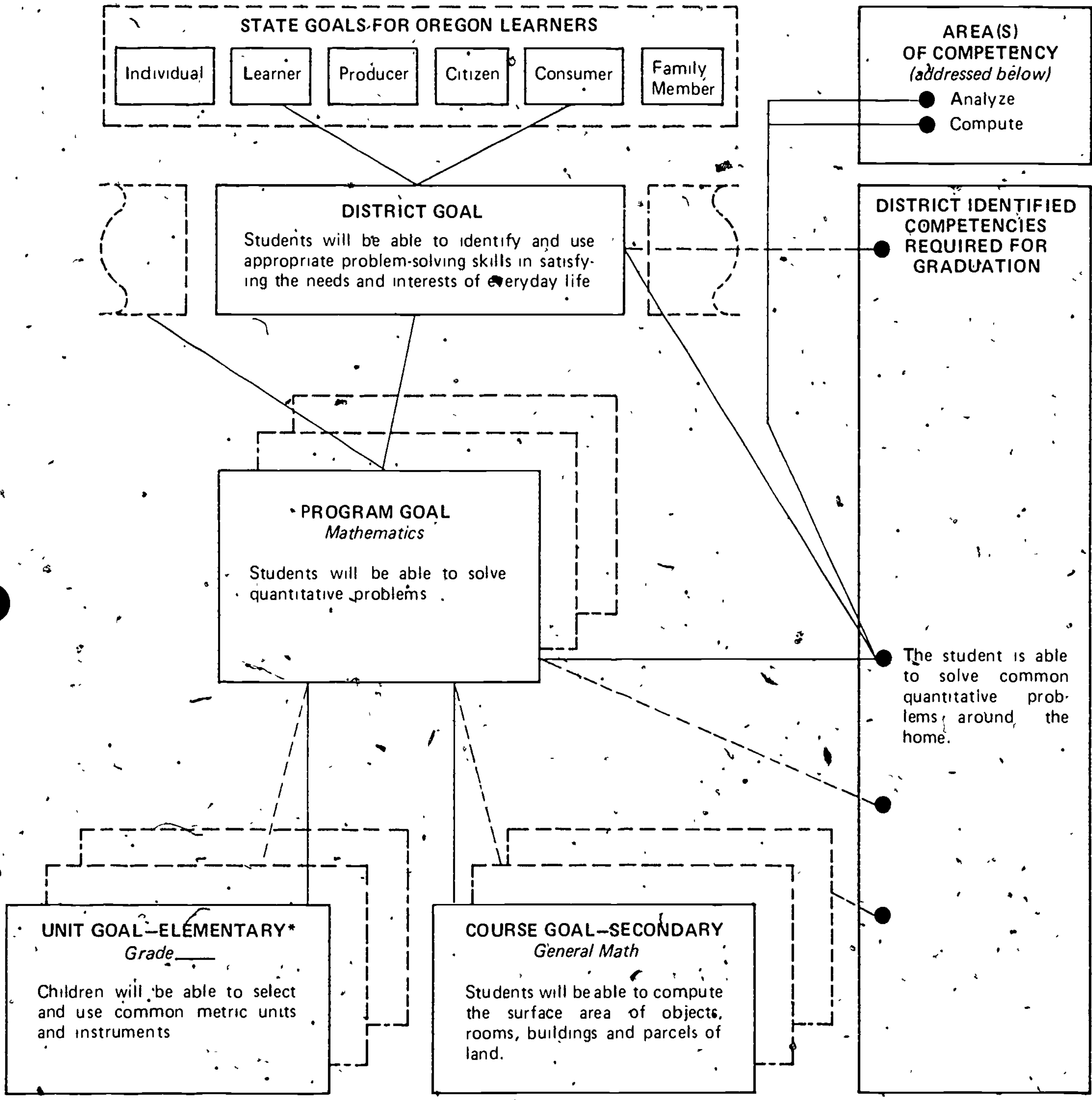
Does the mathematics program suggest activities that progress from concrete to abstract experiences and that emphasize elements from students' own environments?

Does the program encourage students to reason logically and develop problem-solving strategies?

Does the program generate positive attitudes toward mathematics and an awareness of its usefulness in solving real problems of interest to students?

*Mathematics Resource Project, *Ratio, Proportion and Scaling* (Eugene, OR: University of Oregon, 1975).

FIGURE 1
SAMPLE COMPLEMENTARY GOALS AND COMPETENCIES



*The term *unit goal* is used at the elementary level in lieu of *course goal*, since elementary classes are generally not divided along the high school course pattern.

K-12 MATHEMATICS OVERVIEW

Learning and Teaching

Learning . . . teaching . . . to think about these acts we must take into account several considerations. What influences learning? Are there levels or stages of learning? What does a teaching pattern based on what we know about learning look like? What about drill, self-concept and motivation? This section explores these questions.

The following discussion views some ideas on learning based on these premises:

- Learning is based on experience.

  Sensory learning is the foundation of all experience and thus the heart of learning.

  Learning is a growth process, developmental in nature and characterized by distinct, developmental stages.

  Learning proceeds gradually from the concrete to the abstract, hence formulation of a mathematical abstraction is a long process.

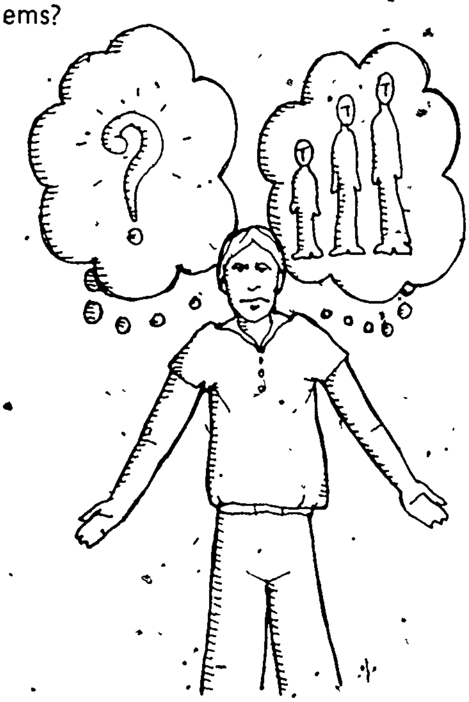
  Concept formation is the essence of learning mathematics.

  Learning is enhanced by motivation.

  Learning requires active participation by the learner.

Stages of Growth

How would your students do on the following problems?



Jim is taller than Dale. Matthew is taller than Jim. Who is tallest?

Mary runs faster than Ann. Jane runs faster than Mary. Who runs the fastest?

TABLE 1
THREE STAGES OF MATH ACTIVITY

Preoperational	Concrete Operational	Formal Operational
Students are able to manipulate physical objects representing the environment	Students are able to give simple or multiple classifications for objects conserve (recognizing constants among apparent differences) length, area and occupied volume. order objects with respect to one or more attributes. deal with number ideas and operations	Students are able to initiate thought by systematically listing possibilities reason with premises not necessarily true use proportions design experiments to control variables

Many students at age seven or eight can successfully solve the first problem but are unable to solve the second problem. Although teachers recognize the similarity between the two problems, the second problem is much more difficult for younger children since speed is a concept which they do not interpret concretely.

Jean Piaget, a developmental psychologist, and his colleagues have noted that the natural intellectual development of children progresses through stages. An understanding of these stages can give insights as to why younger children are able to deal successfully with problem 1 but not problem 2.

Piaget has identified stages of mental development described in terms of how students think during those stages.* Most K-12 students fall into three stages: preoperational, concrete operational and formal operational (see Table 1, page 5).

Of what value to teachers is knowledge of these stages?

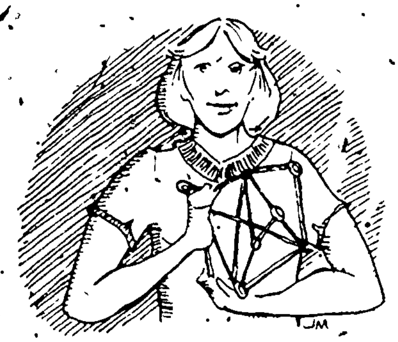
It helps teachers be more sensitive to how students may be viewing things. It may also make teachers aware that students may not be able to absorb all topics if presented by strictly verbal means. It may help teachers to see why some tasks are difficult for children and to understand how children may overcome difficulties in passing from one stage to the next. The stages themselves are not important, rather what happens in the transition. Teachers must understand, for example, why reversibility cannot be taken for granted with 4-year-olds, and why 12-year-olds have difficulty reasoning from hypotheses. What changes take place from one stage to the next, and why does it take so much time?

Each stage offers students new possibilities. Each continues something begun in an earlier stage, and begins something leading to the next. *But students may not reach a certain stage just because they are a certain age.*

Factors Affecting Development

Given that students are recognized as operating within a particular stage *for a topic,* what are the implications? Piaget includes four factors which affect the natural development of the operational stages: maturation, experience, social transmission (learning from others), and equilibration (changing one's thinking). In other words, getting *through* a stage involves more things.

Maturation means students do not develop formal thought overnight, even though they may be given careful verbal explanations. More attention to *experience* may be needed by providing numerous bases in physical realities. Students need to take a more active part in discovering facts and relationships.



A student is better able to group the properties of polyhedron by examining a physical model

Social transmission means students must learn that there are other points of view besides their own. Small group work or discussions seem to offer an excellent way to hear others' viewpoints. Social transmission also, of course, includes information received from the teacher (if students are receptive and can understand the information).

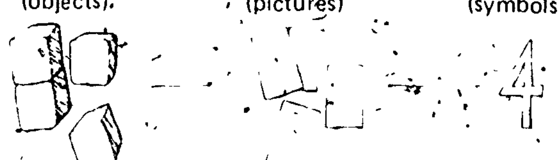
Equilibration means putting ideas together logically so they "make sense." Piaget regards *equilibration* as the fundamental factor, since it "explains" how students' mental structures change, if the other three factors allow. As long as new experiences fit into existing ways of thinking,

FIGURE 2

"IDEA OF 4"

Three Developmental Stages of Learning

Manipulative Stage	*Representational Stage*	*Abstract Stage*
(objects)	(pictures)	(symbols)



4 loose counters	pictures of 4 objects	numeral 4

*See Richard W Copeland, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York, The Macmillan Co., 1970) thought patterns need not change. Once discrepancies or tensions occur, however, and given satisfactory progress in maturation, experience or social transmission, the situation is "ripe" for changes in ways of thinking.

While this surface examination of Piaget's work is meant primarily to enter the idea of developmental stages, it is also intended to provide some background for an application to the teaching of math. For example, Robert Wirtz has taken the stages and translated them into a logical pattern of instruction.*

This model supports a practice that many teachers believe in. *Start a topic as concretely as possible, move to drawings or pictures, and gradually include work with strictly abstract symbols.*

Because of individual differences, stages of development and differences in materials, however, no unique sequence of instruction holds for all learners. When learners can meaningfully represent real-life objects by using symbols, they may already be in the third stage. But they may not possess sufficient stores of concrete experiences to fall back on when their using symbols fails to achieve goals in problem solving.

Teaching Patterns

Growth may depend upon "internalizing" events into "storage systems" corresponding with learners' environments.** Wirtz in *Mathematics for Everyone* calls this act "remembering experiences." Children begin here, then move through learning *how* to solve problems someone else gives them to posing and solving problems for themselves—"independent investigations."

Remembered experiences
↓
Problem solving
↓
Independent investigations

Both remembered experiences and independent investigations are necessary for learning. Without continuing deposits in the memory bank, students do not have sufficient tools for undertaking independent investigations. Without successful efforts at reaching satisfying goals, life may be dull and unrewarding.

*See *Mathematics for Everyone* (Washington, D.C.. Curriculum Development Associates, Inc., 1974)

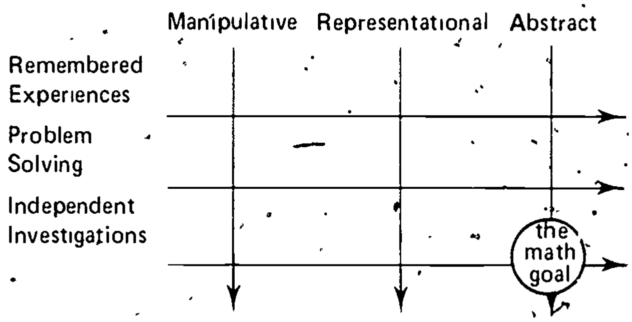
**See Jerome Bruner, *Toward a Theory of Instruction* (New York: W. W. Norton and Company, 1968).

The intermediate step—problem solving—includes learner-developed strategies whereby children deal with questions they do not yet have answers for in their memories.

But they can find answers . . . if they make conscientious efforts. (This notion of problem solving is further discussed and illustrated on pp. 9-11.)

The next task is to determine how both of these learning theories can work into a viable instructional pattern (see Figure 3).

FIGURE 3
A VIABLE INSTRUCTIONAL PATTERN



The goal for math instruction is to have students doing independent investigations at an abstract level. At this level, students must understand concepts and relate problems to their own life situations. But they need curriculum to help them move toward working at the abstract level to create and solve problems independently.

Drill

Where does drill belong in this pattern?

Drill has a definite place in math instruction when:

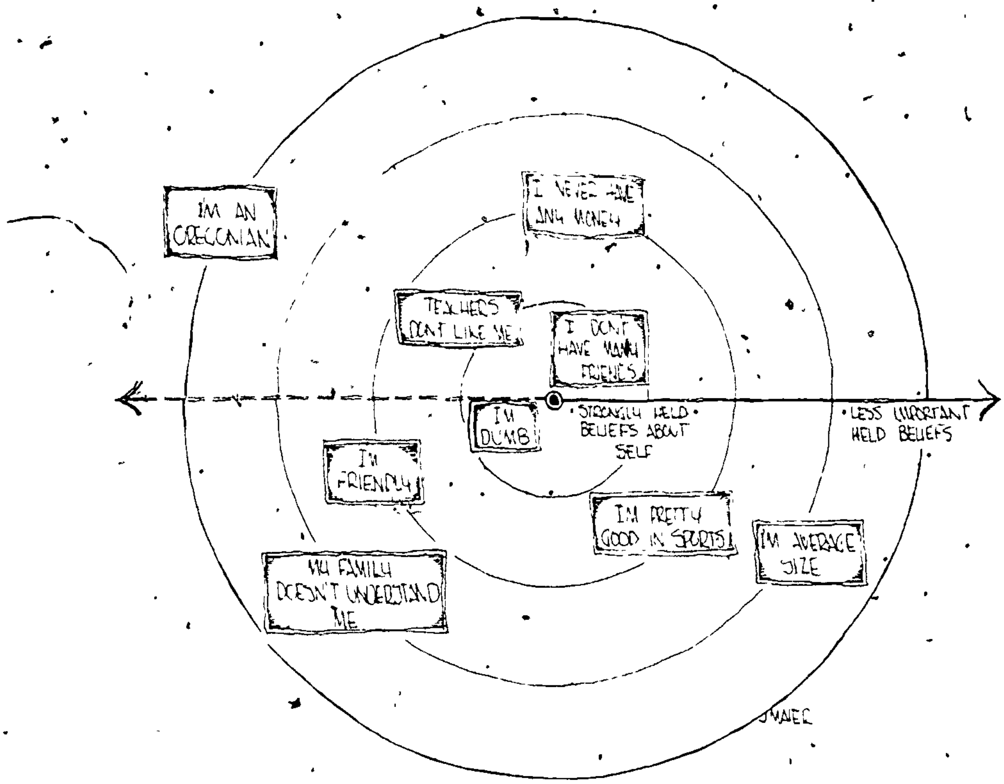
- Need for automatic response is justified
- Pupils see the value in it
- Pupils first understand the principles and skills involved in the drill exercises*

Drill or rote learning without evaluation of developmental stages or conceptional understanding may inhibit the progress in learning math. Children may even develop quite early a permanent negative attitude that further reduces their chance for success.

*See *Eugene Mathematics Program, Grades 1-12*—a publication of the Instruction Department prepared under the direction of Oscar Schaaf, Chairman, Mathematics Advisory Council, Eugene School District, July 1967.

FIGURE 4

ORGANIZATION OF SELF



Self-Concept

"First of all," he said, "if you can learn a simple trick, Scout, you'll get along a lot better with all kinds of folks. You never really understand a person until you consider things from his point of view."

"Sir?"

"Until you climb into his skin and walk around in it."*

People cannot really "climb into others' skins" and view things as they do, because the ways people view themselves and their worlds (self-concepts) are extremely complicated and unique. Beliefs people hold about themselves are well organized, not easily changed and, in the case of students, primary forces in academic achievement.

For example, if students hold the beliefs "I'm dumb" or "Teacher doesn't like me," generalized negative attitudes about their abilities to succeed in school work may override everything else. This self-perception is not easily altered, not even by an occasional success. On the other hand, students with strong positive self-concepts can cope with minor failures.

Although no clear evidence of cause and effect is known, there seem to be persistent and significant relationships between self-concepts and academic achievements at each grade level, and changes in one seem to be associated with changes in the other. While the development of self-concepts begins at an early age and is affected by experiences outside the school, school experiences play an important part in their formation. Too often schools signify failure, rejection, and daily reminders of personal limitations. Self-regard often decreases as students move through school. Success and failure in school influence ways students view themselves.

Even teachers with the best intentions can subtly produce negative self-regard in daily student-teacher interactions. Many teachers may expect more from some students than from others; those students of high expectation are asked more questions, receive more feedback, and are given more praise than students from whom less is

*From Harper Lee, *To Kill a Mockingbird* (New York: J.B. Lippincott, Co., 1960). For more about much of the information in this section, see William W Purkey, *Self Concept and School Achievement* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1970).

expected. The lower achiever receives less attention, briefer contacts, less encouragement.

The net effect may be that students begin to respond in ways which confirm teachers' expectations. Many students may not reach their potential because their teachers, not expecting much from them, are satisfied with less than average performances from them. An accumulative effect of their treatment over the school years could foster very negative self-images. Would higher (but realistic) expectations on the part of teachers have an accumulative effect in the positive direction? What can teachers do to promote and enhance positive self-concepts?

Purkey* emphasizes six factors in establishing a stimulating classroom atmosphere where positive self-images can develop and be maintained.

Providing Realistic Challenges. Students should be expected to complete challenging tasks *within reach* and exhibit academic achievement.

Freedom of Choice and Freedom from Threat. Students need to make some decisions for themselves—not planning their own courses of study, but being given occasional opportunities to choose between two or more alternatives. At the same time they need freedom to make mistakes (in the sense that mistakes are not threatening). Fear of mistakes and failure often cause anxieties that hinder performance and often do cause failure.

Respect. Students need to believe they are important and valuable, that they *can* succeed in school. High self-concepts and belief that teachers regard them favorably go hand-in-hand.

Warmth and Supportiveness. Characteristics teachers exhibit in classrooms (confidence, tolerance, understanding, friendliness, etc.) cannot be underestimated. Teachers who are calm, accepting, supportive and facilitative promote good student feelings about themselves. Teachers who are dominating, threatening or sarcastic may not.

Firm Classroom Control. Leadership qualities of teachers exhibited by classroom control are a way of promoting self-concepts by showing students that teachers care. Consistent, courteous but firm handling of "incidents"; obvious and careful preparation for class; keeping up with grading and promises; and clear explanations of certain guidelines or policies all say to the student, "You are important to me."

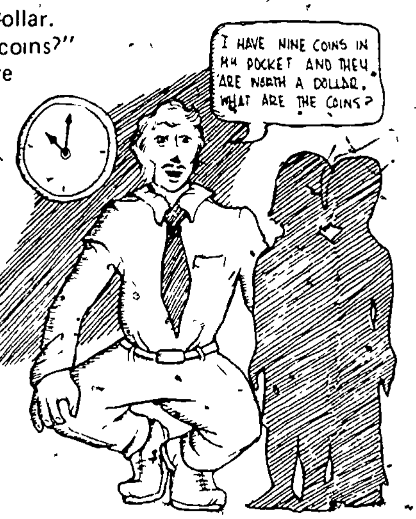
Success. Honest success experiences may be the most important factor for students' continued learning and positive feelings about themselves and school. Unfortunately, students with the lowest self-concepts often have very few success experiences. Students learn they are able from success, not from failure.

A math instruction program needs to include evaluation of attitudes and instructional approaches carefully planned to provide stimulation and challenge with an expanding positive self-image for all students.

Problem Solving

This guide addresses *PROBLEM SOLVING* as both a means and an end. Earlier in this section, problem solving was related to developmental stages of learning. In sections following this one—Elementary Grades (K-6); Middle Grades (6-9); Nonformal Mathematics (9-12); and Formal Mathematics (8-12)—problem solving is considered, first, as an important theme of teaching strategies and learning activities in response to the developmental learning stages and, secondly, as a content strand.* Where problem solving is taken up as a content strand the focus will be on the development of some problem-solving skills. Beginning here, several examples and illustrations are given to demonstrate

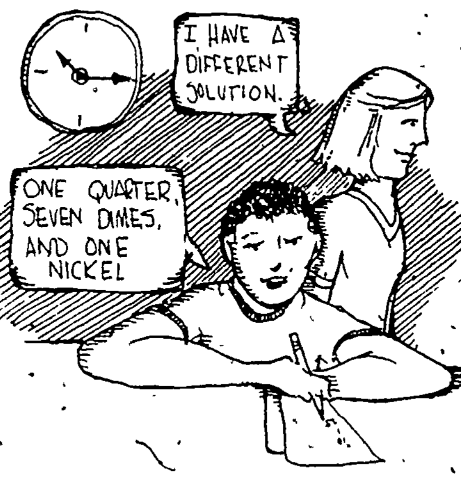
The teacher has given the students in the class the following. "I have nine coins in my pocket, and they are worth a dollar. What are the coins?" (Can you figure out what the coins were? Try!)



*See *Self Concept and School Achievement*, p 8 herein

*See p. 11 herein

some characteristics of problem-solving teaching strategies. Clearly, such strategies require problems featuring certain characteristics. These, too, will be considered here. Both aspects of problem solving will be discussed and illustrated in more specific detail in the K-6, 6-9, 9-12 and 8-12 sections.



One of the students shown has found a solution, and his partner found a different one using a half-dollar as one of the coins. (Did you find that solution?) To their surprise, neither of their responses actually named the coins in the teacher's pocket. (Now do you know what the coins were?)

What happened here illustrates several problem-solving characteristics. By noting the clock on the wall in the two scenes we observe 15 minutes have elapsed. Hence we find that the children were sufficiently motivated to persevere for 15 minutes at a task they understood. The situation did not lend itself to rote response or blind application of a previously learned technique. The children were certain that they had identified a set of coins that might have been in the teacher's pocket. (No answer book or teacher's approval was necessary.) They may have been surprised, however, to find more than one possible solution.

Something exciting has happened and additional characteristics of problem-solving teaching strategies are observed. The child in the first picture, stimulated by the search for a solution to the previous problem, thought up his own problem and the teacher was pleased to hear it. The problem he thought of was, "In how many ways can I make change for one dollar?" The student isn't afraid to work on the problem and is confident that in some way it can be solved.

We observe that both the student and the teacher ponder about the problem for longer periods of time—perhaps until 5 o'clock or even until bedtime. The problem may be a little harder and the

student may not be able to solve it right away; but to him it is a real-world problem, because the student wants to know the answer and is interested in the question.

If the children were not systematic in the attack and became overwhelmed, the teacher might have suggested a chart to organize their thinking. The third picture, once again, emphasizes the characteristic that *a good problem is difficult to leave unsolved*. Students *want* to solve it.

1¢	5¢	10¢	25¢	50¢

But suppose the child cannot solve the problem. Perhaps few, if any, students will find the number of ways to make change for a dollar. It was still a good problem, as it could have resulted in the use of a way of organizing the information which is useful in other situations. It may also result in another technique—that of simplifying the problem. Realizing the given problem was too big to be handled, they might have asked, "Could we find how many ways we could make change for a dime? A quarter?"

The examples in this section illustrate the special meaning of the word "problem" as used in this guide. People are said to have a problem when they want to do something and can't. This means that whether a situation is a problem or not depends upon the individual's acceptance of it and also upon the individual's background. To illustrate, 278 x 57 = ☐ is not a problem for most teachers, but it is a problem for most third graders—*if they care enough to try*. For most fifth graders 201 - 47 = ☐ should be an exercise—not a problem. Similarly, "If you have one cookie, and I give you two more cookies, how many will you have?" is probably an exercise—not a problem—for all but a very few first graders.

There are further examples of problem-solving situations including skills and techniques given in the sections that follow. A bibliography is included on page 81 of this guide for those wishing to find out more about problem solving.

Teaching Strategies and Classroom Management Techniques

Each day a classroom teacher is faced with the challenge of meeting the needs of a large number of students. The sections dealing specifically with K-6, 6-9, 9-12 and 8-12 will suggest ways teachers can assess and provide for these individual needs and will help answer such questions as

- What can we do to help students feel good about themselves and about math?
- How can this be done in an exciting, challenging and lively environment?
- How do we create an atmosphere that promotes problem solving, discovery and learning?
- What kinds of classroom management techniques make this environment possible?
- What resources provide the variety of ideas and materials needed?
- What is the role of the textbook?
- How can math become a real part of students' everyday experiences?
- How can we assess the effectiveness of our teaching in the classroom?

Math Content Strands

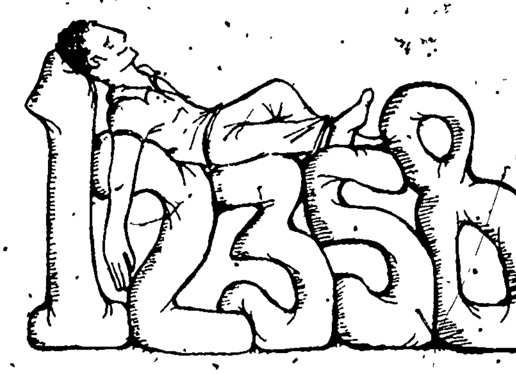
Basic content of a K-12 mathematics program will be organized in nine strands. Several of these are not truly "math content" but are included in order to call attention to their importance. The strands were identified to serve as a common base in planning and developing several statewide efforts relating to math education. The test instrument for the statewide assessment in math and the criteria for selecting instructional materials for the statewide adoption were both based on these content strands.

Good instruction in math will help students become aware of the interrelationships among the strands. Often learning activities students engage in will contribute to understandings and skills related to more than one strand and represent one attempt to help bring out the interrelations. *Listing strands as separate thoughts is done as a matter of convenience and should not be interpreted as a scope and sequence, but as elements to be included in developing a complete program.*

This section simply identifies and describes the strands. Many suggested learning activities to help students acquire understandings and skills relative to these strands will appear in the K-6, 6-9, 9-12 and 8-12 sections.

Numeration

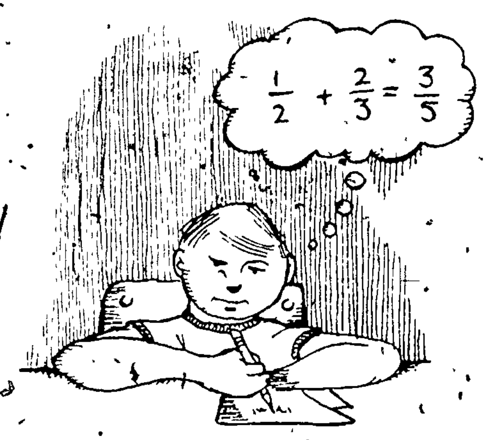
Wouldn't it be great if all students and teachers really enjoyed math? Through number concepts, number sense and numeration, students should experience a "friendliness" for numbers, their names, sizes and uses, and the patterns that can be explored with them. In the K-6, 6-9, 9-12 and 8-12 sections, content and activities concerning counting, numeration, place value, number patterns and number theory will be discussed.



Communicating Through Symbols and Models

Symbols are important tools that help people understand and communicate ideas. A symbol is just that: a representation of an idea. Different symbols serve different needs. The words you are now reading are symbols; concrete objects such as Dienes blocks and Cuisenaire rods symbolically represent ideas; numerals (symbols for numbers) are used in measurement (7.5 m), in personal finance ($12,000) and in music (3/4 time). Symbols are used in computing, working with calculators and computers, representing geometric and algebraic relationships, interpreting road signs, and picturing in two and three dimensions.

Symbols are just tools people are expected to understand and use. In arithmetic, for example, mental and verbal activities should precede using numerals and signs. Care should be taken not to overemphasize symbols, such as an excessive use of set notation.



Does this student understand the symbols?

Problem Solving

One of the strongest trends in math education is more emphasis on problem solving and developing problem-solving skills. Problem-solving skills include gathering, organizing and presenting data; recognizing or defining problems; choosing problem-solving strategies; estimating, approximating and predicting; and computing.

Selecting "problems" should be based on several criteria. Some problems should arise from the "student world" and should reflect that world in a manner students will accept as being realistic (relevant). Some problems should be activity oriented, i.e., they should make it necessary for a student to *do* something other than be limited to paper-and-pencil activity. (Note this suggestion calls for activity *in addition to* paper-and-pencil work, *not* in place of paper-and-pencil activity as part of solving problems.) Some problems should not be solved simply by recall. Some should be solved by more than one strategy, have multiple answers, and illustrate that sometimes a solution for one problem creates new problems. Selecting problems should include considerations from environmental education, consumer education, career education, science, home economics, industrial arts, etc. These provide excellent possibilities for developing many problem-solving skills and for including math applications such as geometry, measurement, estimation, graphs, statistics and probability.

Computation

All people need to compute (calculate) as one step in solving problems. From checking change received at the grocery store, to balancing a checkbook, to planning a vacation trip, to building a house, to working at various careers, computation serves an important function in solving problems.

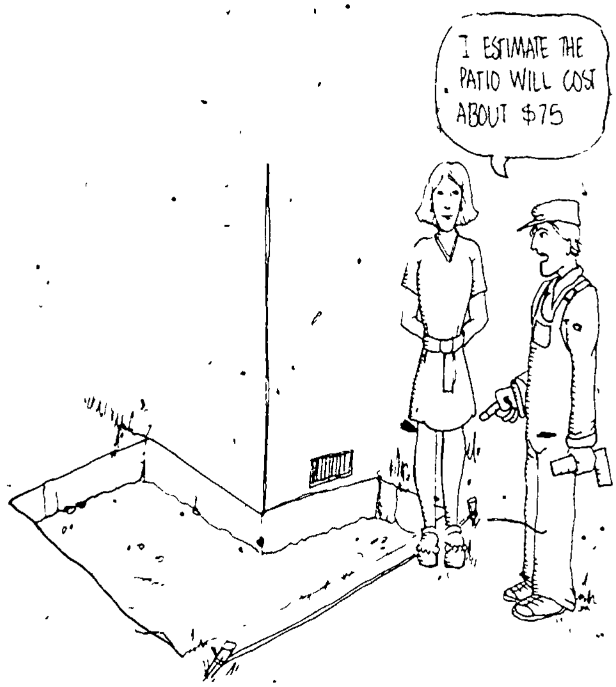
Calculators and computers help to perform computations but they cannot do all the work. It is necessary to know when to use these tools and, more importantly, how to use them—which operations to use, which buttons to press (see pp. 14-17 herein).

Computation has at least two important aspects. These compare to athletes' (1) knowing how to play a sport and (2) conditioning to perform well. In order to compute well, people must (1) *understand* numbers, algorithms and how to interpret problems as well as (2) be in *condition to perform* these activities. That is, they must make decisions about which operations to use and be able to perform computations automatically.

A list of topics related to computation could include meanings for the four basic operations, mental arithmetic including memorization of basic addition and multiplication facts, mental computation both precise and approximate (and knowing when approximate results are satisfactory), standard as well as nonstandard algorithms, ratio and proportion, and drill as fun and practice.

Applications

Using a simple application (How many weeks will it take me to save enough money to pay for a new skateboard?) or a more complex application (How can we convince the city that a traffic light is needed at the corner of Beech Avenue and 1st Street?), students will usually be more highly motivated to learn math if they can see its usefulness in real-life situations that are interesting to them.



Long ago, merchants, scientists and builders used mathematics to describe and solve situations in their immediate world. Today, applications of math are seen in other disciplines (art, science, architecture, . . .), in real-life situations (shopping, industry, personal finance, . . .) and in other branches of mathematics (statistics, geometry, . . .).

Students need to have the appropriate skills before applications can be attempted, but a well-chosen application or a student-initiated question can provide the interest and motivation for developing and practicing a particular skill. Applications can provide an experience with a calculator or a measuring device and can be used as an in-the-field experience to give students a feeling for what is currently happening in the world around them.

The lists of applications that follow in the K-6, 6-9, 9-12 and 8-12 sections could include estimating materials or results, working with money and wise purchasing of goods, statistics about sports, ideas about ecology, ...

Geometry

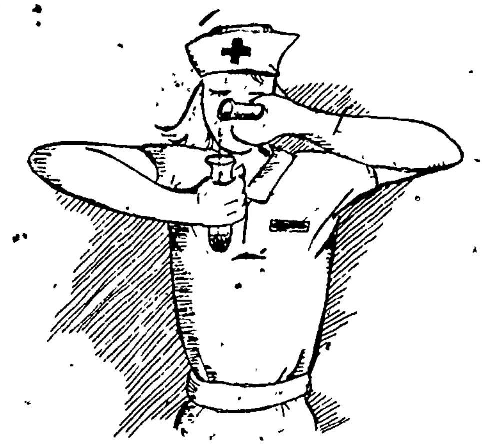
Environment surrounds people with models of geometric figures. Fostering an awareness of geometry in young children through examples from nature, fine arts and man-made structures lays a foundation for formal study later. Patterns may be found in blossoms and leaves and symmetry in dance figures and tapestry designs. Bridges, trains, transmission lines and city buildings repeat geometric figures again and again.

Coordinate geometry should be introduced in the middle grades making a wider variety of graphs possible. Measuring geometric figures makes the solution of many practical problems possible and builds necessary skills for future carpenters, engineers and architects.

Caution: Even though the geometry is informal, the language should not be casual to the point of being incorrect. All activities should be building carefully for future studies.

Measurement

Nearly everyone experiences measurement every day traveling in automobiles (distance, speed), buying items in a store (money, size). Math educators are expected to provide students with experiences in metric measurement skills, measurement estimation skills and wise consumer-spending techniques, as well as an understanding of measurement instruments, measurement units and their relationships. Examples of these topics as well as other measurement concepts such as temperature and time will appear in the content strands component of the K-6, 6-9, 9-12 and 8-12 sections.*



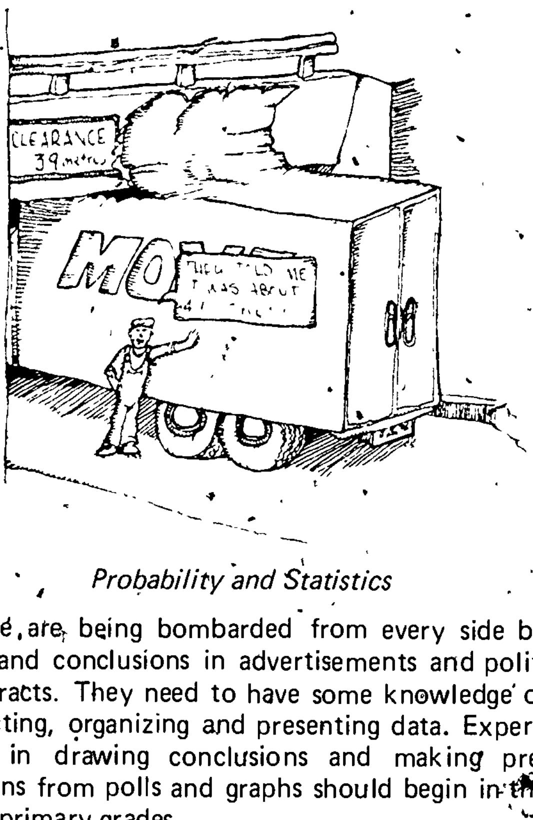
*For more complete discussion of metric measurement, see *MEASUREMENT. . . with Metric* (Salem, OR. Oregon Department of Education, 1977).

Estimation and Approximation

Most mathematics done outside the classroom involves estimation and approximation. (Some authorities claim 75 percent of nonoccupational uses of math are mental.) "The attendance at the school dance last night was about 500." "My new clothes cost about $65." "It'll take about 15 minutes to walk from the school to the swimming pool."

Estimation (making educated guesses) and approximation (improving the accuracy of estimates) are skills that can be developed. Students will need to understand the units of measure involved, to be willing to accept the idea that approximate answers can convey a great amount of information (see the examples in the preceding paragraph) and to practice continually and refine the skills involved—computing with single digits, multiplying and dividing with powers of ten and rounding numbers. Using calculators and determining reasonable answers make processes of estimation and approximation even more important.

Suggested activities involving the processes of estimating and approximating are given in the K-6, 6-9, 9-12 and 8-12 sections.



Probability and Statistics

People are being bombarded from every side by data and conclusions in advertisements and political tracts. They need to have some knowledge of collecting, organizing and presenting data. Experience in drawing conclusions and making predictions from polls and graphs should begin in the early primary grades.

People are naturally interested in the chances an event will or will not occur. Coin tossing, card drawing and dice rolling are examples of "hands-on" activities appropriate for students as a prelude to the more formal study of probability in the upper grades.

Usual elementary math topics getting a workout in probability and statistics include counting, comparing, bar graphs, operations with fractions and decimals, averaging, line graphs, . . .

Calculators and Computer Literacy

Most people agree the major goal of math education is to prepare students to cope with the math problems they may encounter on the job, in future education, in day-to-day activities. But should calculators be used in this preparation? David Moursund below answers the question for elementary schools.* Could a similar case be made for using them in secondary instruction?

Janet, a school secretary, has been given a $10 bill and sent off to a donut store with instructions to bring back 6 dozen donuts for a faculty party. As Janet enters the donut store, she sees a sign stating that donuts are $1.93 per dozen, with a 20% discount on orders of 5 dozen or more.

Note that this is a problem situation, rather than a problem, since there is no clear statement of what problem is to be solved. Let's suppose that Janet has no other money with her, and the store does not accept checks or credit cards. Can she complete her assigned task?

There is a subtle but important difference between this "real-world" problem and the typical "school-world" problem that might be assigned from the same problem situation. A school-world problem would be to determine how much change Janet will receive from the $10 bill. It requires intricate calculation, and would be a challenging problem at the upper elementary school level. A calculator might well be a useful aid for solving that problem.

*"Calculators in the Elementary School" in *Math Learning Center Report* (Salem, OR: Math Learning Center, February 1977). Adapted and used with permission

But Janet does not concern herself with school math. She merely asks a clerk the cost of 6 dozen donuts. The clerk responds immediately, either from memory or by consulting a table taped to the cash register. When the purchase is completed the cash register likely computes the amount of the change. In any event the clerk carefully counts the change, starting at the cost of the donuts and ending at $10.

Let's complicate the situation by supposing the store is very crowded, so that Janet cannot readily ask a clerk the price of 6 dozen donuts. Instead she decides to make a mental estimate of the cost, to see if her $10 will suffice. She thinks to herself, "If donuts were $2 per dozen, 6 dozen would cost $12. But I will get a 20% discount. And 20% of $12 is certainly more than $2, since 20% of $10 is $2. Hence 6 dozen donuts will cost less than $10." If Janet can perform this type of mental estimation/computation, we must conclude that she is functionally literate in math—indeed, that she is quite competent.

(Problem Solving)

The donut example illustrates the major aspects of problem solving:

1. Understand the problem. This requires a broad range of skills. Janet had to read and understand the donut pricing policy, which included the difficult concept of "percent discount." She had to understand money. She had to formulate a clear problem (do I have enough money?) from the problem situation.

2. Formulate a solution procedure. Typically there are many different ways to solve a problem. Janet considered the options and immediately picked the easiest one—she decided to ask a clerk for the cost of 6 dozen donuts. Or, she rejected this possibility because the store was crowded, and decided to make a mental estimation of the cost of 6 dozen donuts.

3. Carry out a solution procedure. It is important to see that formulating a solution procedure and carrying out a solution procedure are two distinct steps. Often, of course, they are intermingled. As one attempts to carry out a solution procedure, one may be led to change the procedure. As one attempts to formulate a procedure, one must keep in mind what types of steps one knows how to carry out.

4. Understand and use the results of step 3. In more complicated problems, one often loses sight of the original problem and gets bogged down in carrying out a complicated solution procedure. In Janet's case, step 4 was trivial. She noted that the cost of the donuts was less than $10, so she could indeed proceed to order 6 dozen donuts.

(Computation)

Step 3 of the general problem-solving process involves carrying out a solution procedure. In mathematical problem situations this often involves arithmetic computation. It follows logically, then, that, if one cannot compute, one cannot solve such math problems. For this reason calculation is certainly an essential and fundamental part of the mathematics curriculum.

But wait! Clearly, unless one is adept at steps 1, 2, and 4 one cannot solve problems. Computational skill by itself has little value.

Also, recall the donut problem again. Janet's first solution procedure required no computation on her part (except perhaps to note that the clerk's response was less than $10). Moreover, the clerk solved the computation problem by memory or by reading a table. Later the clerk made change by use of a machine or a standard change-counting procedure.

Now where do calculators fit into all of this? A calculator is an aid to computation. There are many standardly used computational aids and methods:

1. Exact mental arithmetic. (Six dozen donuts at $2 per dozen is 6 x $2 = $12.)

2. Mental estimation and approximate mental arithmetic. (Janet did this to conclude a 20% discount on $12 would result in an amount under $10.)

3. Pencil-and-paper arithmetic, using procedures one has memorized for addition, subtraction, multiplication, division, etc.

4. Table reading. One might also include "ask someone for the answer" as part of this category.

5. Calculators, computers, and other mechanical aids. The donut store's cash register is likely a calculator or computerized device.

Perhaps you have anticipated the next question. Outside of school-world problems, what percentage of real-world computation falls into each category? More to the point, how useful and important are pencil-and-paper arithmetic computation skills?

In this situation it is safer to beard the lion in his den, or to plead the fifth amendment. Hopefully, readers will provide their own answers and opinions.

Suppose you were designing the elementary school mathematics curriculum, and category 5 (calculators, computers and other mechanical aids) did not exist. That is, suppose there were no calculators, computers or other mechanical aids to computation. Then your curriculum would place considerable emphasis upon pencil-and-paper algorithms. There would be no alternative to this, since many computational situations cannot be handled via categories 1, 2, and 4.

This may be why there is so much emphasis upon pencil-and-paper computation in our current curriculum, because we ignore the existence of calculators and computers and strongly emphasize the role of computation in the overall process of problem solving.

(A Change Agent)

Calculators should have a major impact upon the elementary school mathematics curriculum merely because they exist and because they are inexpensive and readily available. Even if one is not going to allow calculators to be used in the elementary school, one must reexamine the curriculum and its goals in a world where calculators and computers are readily available. This reexamination will likely result in an increased emphasis upon the overall process of problem solving, and in particular upon step 2 (formulate a solution procedure). It will likely also cause a shift in emphasis in the methods and modes of computation. Because pencil-and-paper computation skill is decreasing in importance in the world, one can spend more instructional time upon exact and approximate mental computation.

Actually, it seems silly to reject outright the use of calculators in the elementary school curriculum. Perhaps they can be useful aids in the teaching of non-calculator topics that we consider to be important. And perhaps we should be teaching students something about calculators.

(Teaching with Calculators)

Computer-assisted instruction has received much publicity in recent years, although its impact to date upon education has been small. What about calculator-assisted instruction?

One of the most obvious applications of calculators is for drill and practice on number facts—that is, as an automated flashcard device. For under $20 one can buy a machine that looks like an electronic calculator, but instead is an automated flashcard machine. It can automatically generate problems of varying degrees of difficulty and provide feedback on the correctness of students' responses.

A very useful calculator application, using an ordinary calculator, is as an answer checker, or feedback mechanism. To illustrate the latter, suppose one set as a goal to have students learn to make rapid mental estimates of the product of two 3-digit numbers, with an error of less than 20% of the exact product. Students could be given a page of pairs of 3-digit numbers. For each pair of numbers the task is to write down an estimate of the product and then to compute the percentage of error. A calculator is used for the latter task. Somewhat more sophisticated would be to ask students to estimate the product and write it down; then use a calculator to compute the exact product and write it down, then to estimate the percentage error and write it down; then with a calculator to compute the exact percentage error and write it down. After doing a sequence of such problems, students should analyze their success and the nature or pattern of their errors in the product and the percentage-of-error estimations.

Finally, consider the teaching of "trial and error" or "examine specific cases and make a conjecture." This is one of the most important ideas in mathematical problem solving, but for the most part it is ignored in elementary school mathematics education. A calculator can change this situation. For example, sixth grade students could readily learn to solve the following problem by using a calculator:

Neglecting air resistance, a formula for the distance an object falls (due to gravity, and near the earth's surface) is approximately $16t^2$ feet, and where t is the time in seconds. About how long will it take for an object to fall one mile?

Students want to find a number t so that $16t^2$ is approximately 5280. Initial mental estimates of $t = 10$ (too low) and $t = 50$ (too high) bracket the answer. Systematic trial and error, making suitable use of a calculator, can quickly bracket the answer to between 18 and 19 seconds.

(Teaching About Calculators)

If calculators are available, then one should consider teaching students about them. The goals below could serve as a starting point for a school wanting to incorporate calculator education into its curriculum. The intent is that these goals be pursued systematically (at least weekly) throughout the school curriculum.

1. Mechanical aspects: Students have had experience with a variety of calculators, with major differences in location of the on/off switch, types of power supply, keyboard layout, etc. For example, students know about disposable batteries, rechargeable batteries, and direct use of "wall" current to power a calculator. Students can determine how a new or unfamiliar machine functions. Students know about calculators' resistance to physical abuse (or lack of resistance, as the case may be).

2. Capabilities: Students understand both the capabilities of calculators in general, and the capabilities of several specific calculators. Students have sufficient calculator experience to have reasonable skill in making use of these capabilities. By the time students complete elementary school, they can use calculators to

carry out the computations in any problem involving addition, subtraction, multiplication or division of decimal numbers. Students understand and can make effective use of a calculator's "constant" feature and whatever "memory" features are typically found on inexpensive calculators. Students understand and can make use of √‾, %, and other function keys appropriate to an elementary school education. Students understand that many calculators have additional function keys whose meaning and use are taught in secondary school mathematics courses.

3. Limitations. Students understand the general capabilities of the simplest 4-function calculator, multi-function calculators, programmable calculators and computers. Thus students understand the limitations of the simpler and less expensive machines. Students understand some of the limitations of calculator arithmetic (for example, 1/3 times 3 is not 1). Students have insight into the unsuitability of a simple calculator as an aid to handling problems involving large amounts of data, large amounts of computation, or large amounts of repetition. Such problems are more suited to programmable calculators or computers.

4. Calculator errors. Students understand various sources of error in using a calculator. Those include errors in the procedure being followed, errors in reading and keying in data and operations, and errors in reading the output display. Students have experience in detecting, correcting and avoiding such errors.

5. Problem solving: students understand the overall process of problem solving, the role arithmetic computation plays in problem solving, and thus the role calculators can play in problem solving. Students understand that calculators are a minor part of mathematics education and of the field of mathematics.

6. When to use. Students have experienced the use of calculators in a wide variety of computational and problem-solving situations. These span the range from the very simplest "number fact" (more appropriately memorized) to very large and complex computations (more appropriately done by computer). Based upon training and experience, students can classify computational situations into categories: (a) memorized number fact, (b) mental arithmetic, (c) pencil-and-paper arithmetic, (d) simple 4-function calculator, (e) multi-function or programmable calculator, (f) computer.

Students who may then enter secondary schools already knowing about calculators are in step with a changing world around them. They may apply their knowledge and skills in either nonformal or formal math courses, there too learning more about calculators and perhaps computers. True, math goals must be reassessed and accommodating curriculum developed. Some modest equipment may have to be bought. And teachers may need in-service. But these inexpensive, useful, easy-to-use and readily available math tools make it easier than ever before to present *balanced* math curriculum addressing all nine content strands.

Reading and Mathematics

Most educators agree reading in any subject matter cannot and should not be separated from comprehension and concept development in that subject, and considerable evidence shows student achievement is improved if reading is integrated with content instruction.* Helping students read math textbooks or other printed sources of information with greater comprehension is, therefore, an important goal of any program in math instruction.

Teachers of mathematics—be they at kindergarten or community college level—can do many things to improve pupil reading skills.

- *Determine "mathematical readiness" for reading.* Direct, concrete experiences provide foundations for successful reading experiences with math content. Piaget and others have noted that young children grasp abstract concepts slowly, in a pattern that builds on direct experiences with a variety of materials and events. Active experiences with concrete materials form the basis for concept development.

  As learners interpret and analyze their concrete experiences again and again, while relating them to the oral language they hear and use at the same time, they gradually develop summarizing ideas or concepts symbolized by spoken words. Symbols growing out of learners' firsthand experiences while accompanied by oral language acquire lasting meaning.

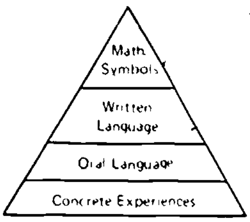
  Written words—such as add, product, area, etc.—are further extensions of oral language and are even more abstract. Each must first be turned back to oral language through decoding requiring word attack skills and sight vocabulary. Math

*For example, see Lewis Aiken, Jr., "Language Factors in Learning Mathematics" in *Review of Educational Research* (summer 1972) and James Laffey, ed., *Reading in the Content Areas* (Newark, DE: International Reading Association, 1972).

symbols—such as +, x, > , etc.—are also abstract representations which acquire meaning through "hands-on" experiences and oral discussion.

Students having been given learning experiences relevant to their own points of view will gain longer retention, better application of new ideas, increased problem-solving abilities, and skills in dealing with mathematics at the written or abstract level. This foundation for "reading math" can be illustrated by the pyramid in Figure 5. A suggested teaching pattern accompanies the pyramid. Read up from the base of the pyramid.

FIGURE 5
"READING MATH"



Introduce math symbols (numerals, √‾, %, +, ÷, etc.). Match to written words.

Write the words on chalkboard. Discuss any appropriate decoding skills (phonetics or structural analysis).

Provide many opportunities for pupils to use the appropriate vocabulary orally.

Involve pupils in concrete experiences with objects and events and use appropriate words or phrases during that involvement.

Plan to diagnose students reading skills in a math setting. Individual students vary greatly in their abilities to decode and comprehend math materials. Before teachers can plan ways of helping students, teachers must have a knowledge of students' reading abilities. Analysis of available reading test scores in the students' permanent records will help identify special needs and problems. Finding time to talk with each student is also helpful, individually asking each to read aloud a familiar math assignment and to answer a few brief questions about what has been read. If pupils have special problems, seek the help of a reading specialist or language arts teacher. Such persons may be able to help design teaching strategies and prepare appropriate learning materials. They may also be encouraged to include math terminology in the vocabulary lists they prepare for students to study.

Direct student attention to the special reading and vocabulary problems of math content. Point out to students that reading math materials in which details must be observed requires a slow, deliberate pace different from the rapid style used in reading other materials where general meaning may be the objective.

Call attention to words having technical meanings different from their meanings in general contexts. Several examples include conservation (recognizing constants among apparent differences), product (result of multiplication), reciprocals (two numbers whose product is one), and root (a solution for an equation).

Have students read! Students may learn something if they have opportunities to learn it. Far too many students rely on teacher direction for new learning. Assigning written explanatory materials in addition to written exercises will help students become self-directed.

Reading experts* suggest several methods to encourage good reading habits with content material.

- Have students read a short passage silently and then answer oral questions.
- Prepare written questions as reading study guides for students.
- Allow a few minutes of silent reading of materials as the routine *before* exercises are assigned.
- Give students some guidance before a reading assignment. Discuss what to look for; special vocabulary problems; special information related to the assignment.
- Form small groups of mixed reading abilities occasionally. Have more capable students read orally while others follow along, then discuss passages or answer questions as a class. Use task cards or other materials requiring students to follow written directions in order to complete an interesting or motivating investigation.

Discuss special features of textbooks or other books used regularly in class. Students will be able to read and use a table of contents, index, glossary and special reference tables in their texts if they are to be "independent readers." Studying the organization of explanatory material is worthwhile—the role of underlined, italicized, colored, or shaded print and the use of other special features.

*See, for example, Jane Catterson, "Techniques for Improving Comprehension in Mathematics" in *Reading in the Middle School*, edited by Gerald Duffy (Newark, DE. International Reading Association, 1974).

Give special attention to reading charts, diagrams, tables and graphs. Point out that reading such devices is not necessarily done from left to right or top to bottom. Experiences in making charts, graphs, tables, etc., is especially helpful preparation for reading such devices. Many opportunities may be found in the math curriculum for collecting data and organizing it in meaningful ways.

Many students encounter difficulty with story problems in their math books. Perhaps teachers could help students attack word problems.

Have them read the problem *several* times—once quickly for the main idea, once slowly to interrelate the data, once to go back and pick out specific details. Having them paraphrase the problem is also useful.

Be certain students have a good conceptual base. A clear understanding of the required number of operations themselves is an essential foundation for working with story problems.

Have students themselves *write* story problems. Individuals or small groups of students can prepare and exchange word problems. Such experiences may help students relate story problems to their own lives and prepare themselves to interpret the story problems they encounter elsewhere.

Avoid assigning word problems of the same type or that use the same algorithm over a period of several days. Such a practice encourages students *not* to read for understanding.

Force students to read for understanding by exposing them to some problems having insufficient data and to others giving too much information. Often teachers give "tidy" problems, making it easy for students *not* to read and think.

Give students many opportunities to discover, discuss and "store in their memory banks" a collection of usable problem-solving strategies. Here is one list* of such strategies:

Make a diagram, number line, chart, table, picture, model, or graph to organize and structure the given information.

Guess and check. Organize the trial and error investigations into a table. Look for patterns.

Translate the phrases of the problem into mathematical symbols and sentences. Write an equation.

Break the problem into "cases." Solve part of the problem at a time.

Check to recall if you have worked with a similar problem. What methods did you use?

Can you solve a simpler but related or analogous problem?

Keep the goal in mind. Could you work backward toward a solution?

Pupils who have acquired such a memory bank of problem-solving strategies will more likely approach word problems with the confidence necessary to attack any special reading difficulties the problem may possess.

Staff Development

Teaching and learning go hand in hand. Among the exciting developments in the past few years have been math workshops teachers plan and conduct for each other. In these workshops an emphasis has been placed on developing techniques and materials for classroom use based on theories about how children learn mathematics. Teachers are learning strategies which help develop good computational and problem-solving skills as well as a positive attitude toward math. Every teacher needs opportunity for this kind of continued growth. Consider the following checklist:

When was the last time

as a classroom teacher, you	as an administrator, you
—created a worksheet for your students?	— attended a math lesson?
—really felt good about a math lesson?	—taught students a math lesson?
—used an interesting approach to skill building?	—helped teachers plan in-service meetings or workshops in math?
—shared one of your ideas with another teacher?	—encouraged your teachers to visit another math class?
—found a good idea when visiting another teacher's classroom?	—utilized the skills of your teachers having special math interests?
—asked the librarian to order a math book for your students or for your own professional growth?	—helped a teacher order math materials other than textbooks?
—had students solve a problem using things other than paper & pencil or textbooks?	—recommended a resource center for one of your teachers to visit?
—took or helped plan a workshop or in-service?	—helped teachers take advantage of released time to develop these interests?
—asked for released time to do any of the above?	

Teachers of math can do much to help pupils become better readers of content presented. If

*Mathematics Resource Project, *Geometry and Visualization* (Eugene, OR: University of Oregon, 1976)

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instruction in math reading helps students become more self-directed, more confident in problem-solving situations, and more positive about their abilities to use math, then it will be time well spent.

Models of staff development may encourage teachers to take advantage of some of these resources and services.

A four week summer program may be sponsored by a district, a consortium of districts or an IED. Teachers may receive salary or credit as options. Each day may consist of six hours divided as follows:

One hour in getting familiar with materials or equipment or techniques or math topics

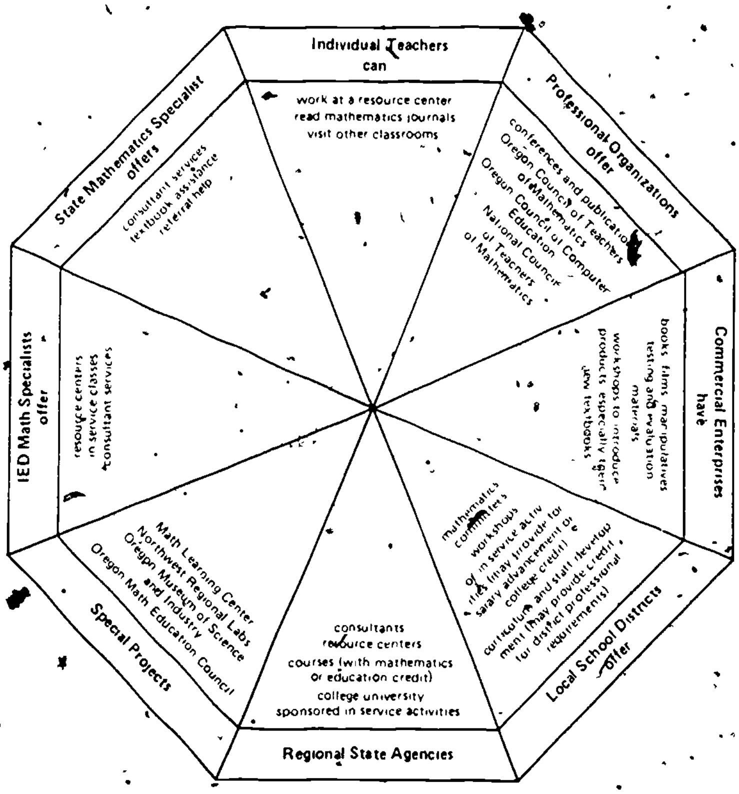
One hour in planning and organizing, such as district communication in math

Four hours in creating and preparing activities and materials for use with students.

A series of sessions totaling about ten clock hours on measurement and the metric system. Teachers may receive in-service credit. College credit may be optional. Weekly sessions may be held after school. Two Saturdays or a Saturday and two or three after-school sessions are other possibilities. An important factor influencing the potential success of such in-service workshops is participant involvement in planning, *including planning the schedule*. Teachers learn about the metric system and develop activities and materials appropriate for their own classes.

FIGURE 6
GROWTH OPPORTUNITIES

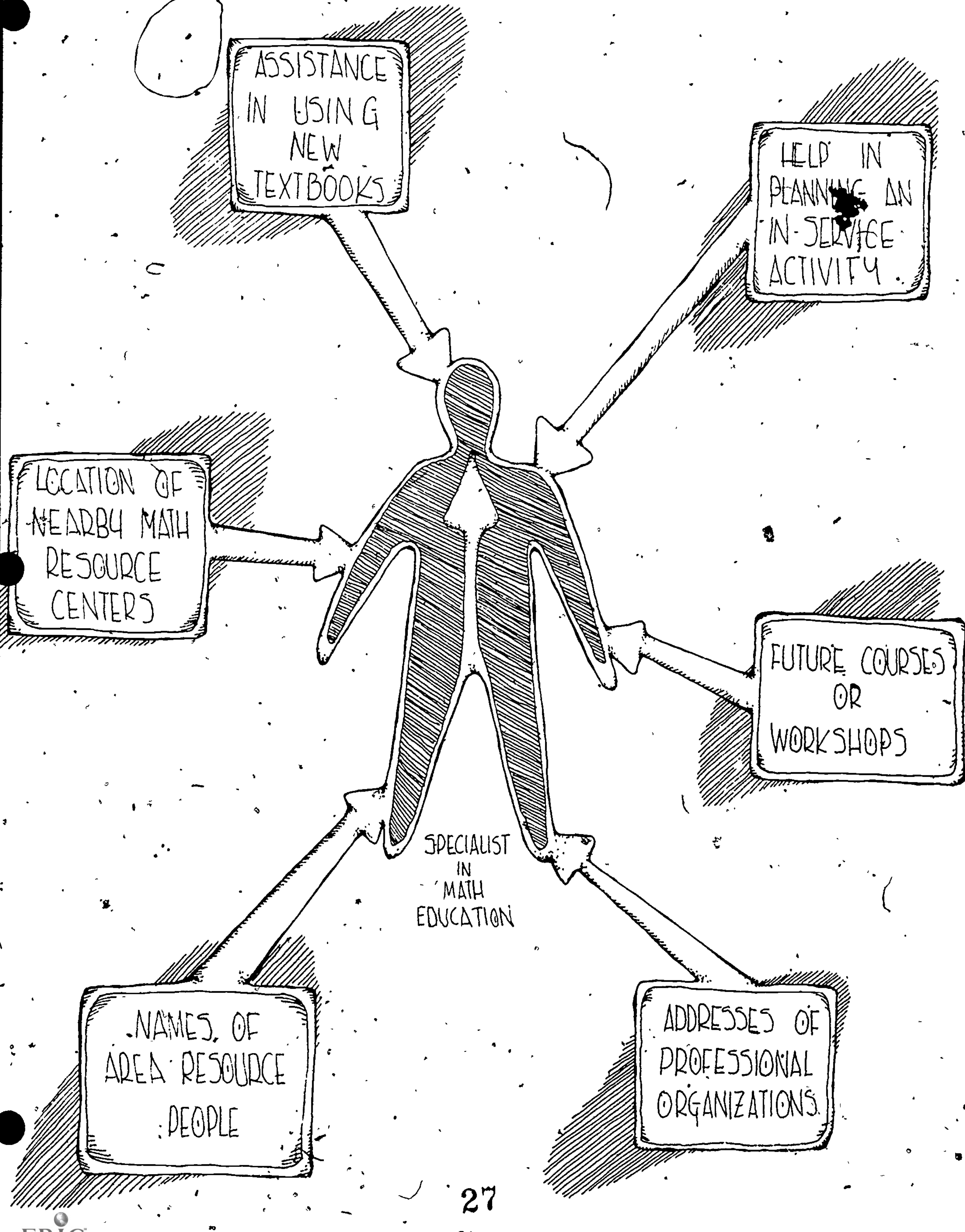
A variety of resources and opportunities also are available for continued growth. Consider the following activities shown below by type of resource.



These activities may focus on a particular math topic from the content strands and may develop materials and strategies for teaching that topic. Detailed descriptions of some successful programs may be of assistance in planning activities. These descriptions are available from the state specialist in math education. Teachers not aware of math resource people or centers in their areas may contact the specialist.

SERVICES AVAILABLE FROM STATE MATH SPECIALIST

Oregon Department of Education
Salem, Oregon 97310
(503) 378-3602



ELEMENTARY GRADES (K-6)

Learning and Teaching

The time from youngsters' opening day at kindergarten to junior high visitation probably marks an intensity and range of growth second only to the first year of life. In order to apply learning and teaching theories to this period, it seems reasonable to consider, first, maturation stages affecting the learning of math. Richard Copeland's book* helps elementary teachers define and guide youngsters through these stages. He has grouped his suggestions around four areas basic to beginning math: space, number, logical classification, measurement. Within each area are sets of perceptual skills to be understood and developed. For example, conservation of number and reversibility of thought are concepts basic to understanding the addition and subtraction of numbers. Copeland says it is often assumed that if children can say how many objects are in a given collection, they understand what the number means. However, if the objects are spread out and the number "changes," the invariance or conservation of numbers is not developed enough to understand numbers. Furthermore, children need to be able to "see" both parts of the whole of a given set simultaneously (reversibility of thought) before they can understand addition and subtraction.

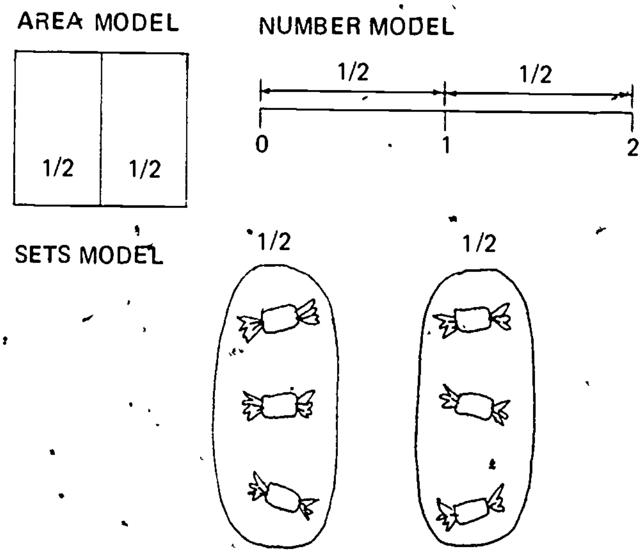
Thus, before teachers teach basic arithmetic, a readiness component needs to be included that insures diagnosis and prerequisite skill development. This readiness component is not limited to primary grade levels. Many fifth and sixth grade teachers have found in the middle of a geometry unit, for example, many students do not understand or perceive space from different perspectives. This kind of problem may result from a faulty concept of conservation of space. It is quite likely students did not have an experience-oriented background with three dimensional objects so that a representation (usually a diagram) of those objects has meaning.

Assuming readiness activities are successful and students are prepared for the content to be presented, what else can a learning-and teaching model do? The continuing emphasis on self-concept throughout all school curriculum clearly has an important function in the teaching of math. Teachers who give attention to motivating a class before attempting to teach are usually rewarded. Motivation may assume many forms, but its purpose is always the same: to make the student an active, positive participant in the process at hand. Consequences of ignoring or misreading student signals that indicate lack of interest or understanding can be an accumulative and long-term "attitude" that works against both self-concept and math skill.

A learning-teaching theory must also insure a logical development of an algorithm, as opposed to a memorized one. Providing a physical experience with "number-ness" may be the primary focus for very young children. As children move through successful experiences, they will need less and less time at the manipulative level; however, teachers should also be ready to return to a less abstract form should it be necessary. As soon as transitions can be made successfully from concrete levels to more abstract levels, for any given concept, youngsters usually will prefer to make the shift.

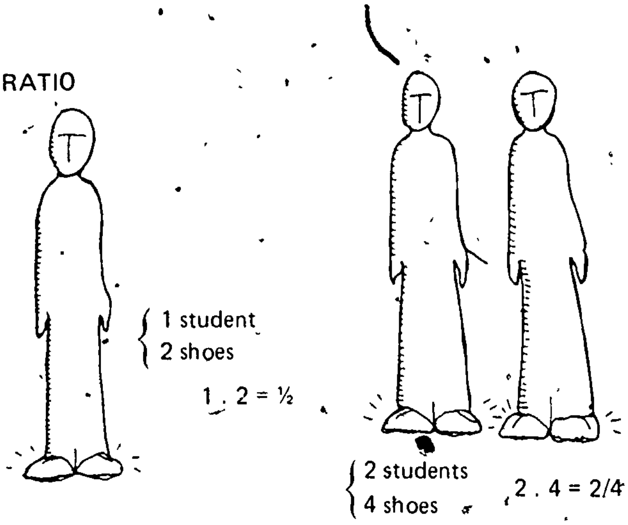
Physical and representational models should be included in all content strands. Consider common fractions. Framework with this part of the curriculum is not presented until upper elementary grades (although informal fraction experiences are pursued throughout the youngsters' school life), so readiness stages should present no problems. However, long before any attempt is made to teach the operations, the concept of fractions should be explored and developed through a variety of physical and representational models.

FIGURE 7
PHYSICAL & REPRESENTATIONAL MODELS



*See page 6 herein.

RATIO



Once comprehension of fractions is established, some or all of these models offer a reference point for the teaching of operations. Many materials now available offer concrete experience so that students are doing complex operations long before they are formalized. With this approach, students' written work serve as records at an abstract level of what has occurred at a manipulative or representational level. Again, once students are able, they usually are anxious to make the shift to the abstract level.

Perhaps the best way to summarize these ideas is to describe sixth graders about to leave elementary school. These students

have demonstrated competency in all math skills required,

like themselves,

like math.

Teachers with whom these students have worked

made sure students were "ready" before introducing a concept,

made students a part of the process and fostered enjoyment of each step,

designed a logical approach to any skill development—beginning with manipulative, concrete experiences and gradually shifting to formal algorithms (any special methods for solving problems or exercises),

provided opportunities for students to discover the "beauty" of mathematics and its application to their own environment.

Several samples of activities which have proven useful in good classroom teaching of elementary math follow, along with comments on learning and teaching implications, problem-solving elements and teaching strategies. The classroom activities are described on each left-hand page. Following straight across horizontally to each right-hand page, readers will find appropriate comments about learning and teaching implications, problem-solving elements and teaching strategies as these considerations relate to the suggested activity.

The activities are divided into two sections: Sample K-2 on pages 26-29 and Sample 3-6 on pages 32-39.

Sample K-2 Classroom Instruction

Sample K-2 Classroom Instruction

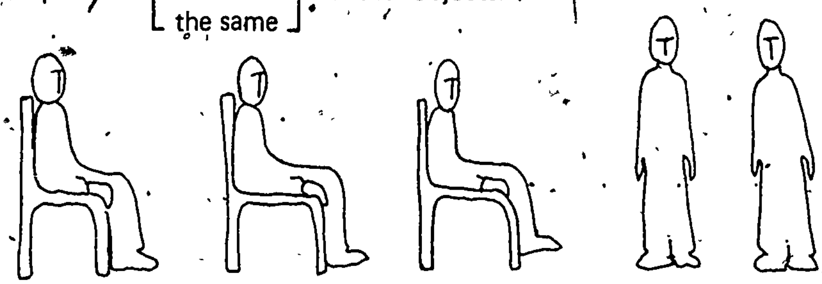
CLASSROOM ACTIVITIES

Early in the year the kindergarten students help the teacher collect 12 bottles of different shapes and sizes.

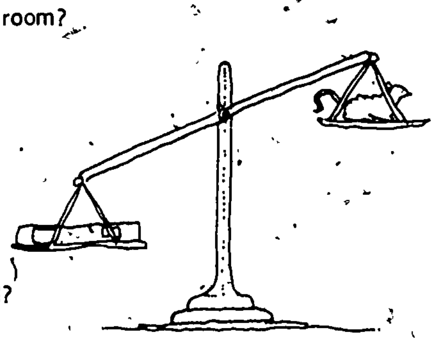
For a month, pairs of children play with the jars using scoops, measuring cups and a large dishpan of rice. During this period of free exploration, the teacher stresses ground rules: (1) No throwing. (2) Share materials. (3) Clean up and put materials away properly. While the children are scooping and pouring, the teacher can learn about the children by keeping a checklist of observations.

+√−	Chris	Amy	Darcy	Sam
cooperates with partner.	√	+	+	−
enjoys work	+	+	√	
counts meaningfully		√		
spontaneously verbalizes about task	√	√	−	√
follows ground rules	+	√		−

Later in the year, the teacher provides activities involving matching and comparison [more / less / the same] of real objects.



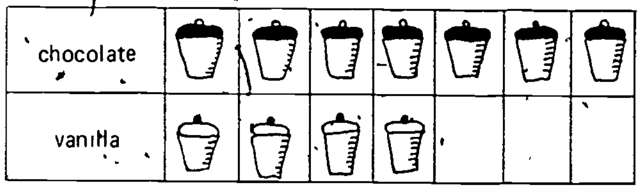
Are there more students or chairs in the room?



Which weighs less, the book or the gerbil?

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
Involving the children in this collecting task is a sound motivational activity.		
Free exploration can be translated to "enjoying math."	Both cleaning and storing materials can provide a sound problem-solving environment, especially if the youngsters have choices to make.	Establishing ground rules is essential to successful group and individual activities.
These activities provide opportunity to diagnose and develop one-to-one correspondence and conservation (constancy) of number.		
Children's first experiences should involve real objects. Children in this class could actually match chairs and students, chocolate and vanilla puddings.		Entire class instruction is an efficient way of getting the activity started as well as offering students the shared experiences of others.
Only after many concrete experiences would the teacher introduce comparison from pictures. "Hands-on" activities should precede work from a textbook no matter how attractive and colorful the pictures appear.		Small group lab activities allow the teacher the opportunity to assist youngsters having special problems.

CLASSROOM ACTIVITIES

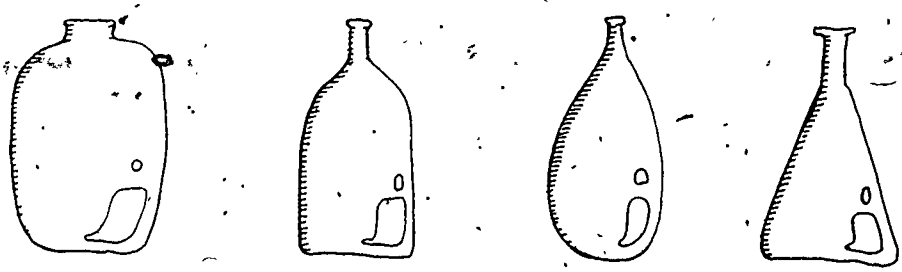


Did more children choose to make chocolate or vanilla pudding?

Even a child who cannot count the seven chocolate puddings can see more chocolates and fewer vanillas.

The teacher directs the initial comparison activities and involves the entire class. Later, the teacher introduces several lab stations involving comparison. One of the stations uses the 12 jars collected earlier in the year. The children select two jars and fill one with rice. This rice is poured into the empty jar. The children then determine if there was too much, the same amount, or too little rice.

The jars could also be used at another time during the year to help students with counting and ordering.



Record the number of scoops needed to fill each jar. Place them in order from largest to smallest.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
	Graphing is a problem-solving tool which helps learners see relationships and recognize patterns. Opportunities to make and use graphs can develop naturally from sorting and classifying experiences and from learners' interest in comparing groups.	
As before, the primary purpose for these activities is to allow youngsters ample opportunity to move through developmental growth stages successfully—e.g., ordering, conservation (constancy) of space.	A structured problem allows for predicting and verifying skills to be fostered	

Sample 3-6 Classroom Instruction

Sample 3-6 Classroom Instruction

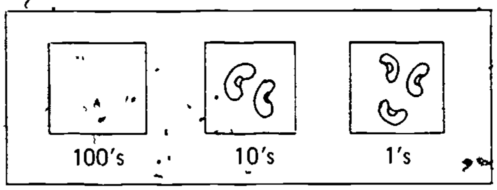
CLASSROOM ACTIVITIES

This section examines one week of mathematics instruction in an elementary classroom. The statements and notes on the left-hand page are from the teacher's lesson plan book. These activities review certain place value and addition concepts through "hands-on" experiences and prepare pupils for forthcoming work in subtraction with regrouping.

Monday

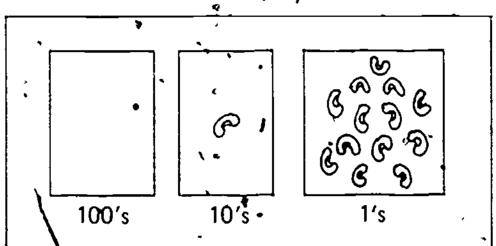
Introduce lima bean computer to the entire class. Have computers and beans available for each child to work with. Include activities such as the following:

Have pupils show 23 with as few beans as possible.



Solution

Have pupils show 23 a different way.



One Solution

Have pupils determine how many different ways it is possible to show 23. (There are 3 ways.)

Repeat the activities in part 1 using such numbers as 27, 30, 36, 40.

Ask pupils to suggest a way to use beans and the computer to show the addition of: 23 and 6
35 and 8
26 and 13

Discuss the regrouping required in some of the problems.

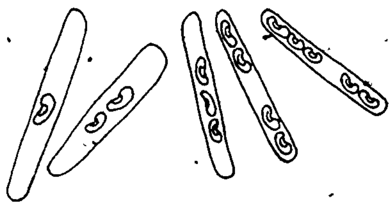
Observe as pupils work a few more similar problems and determine which pupils are experiencing difficulty.

Tuesday

Based on observations made Monday, the students are placed in two groups.

Group A—Pupils experiencing no difficulty

Group B—Pupils experiencing some difficulty

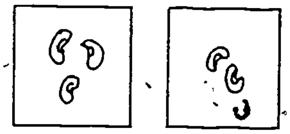
LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
The teacher selected this introductory activity because she believed the activity was one they could perform. Initial success encourages and motivates students. The teacher was aware of necessary prerequisite skills by providing earlier experiences with the more concrete models of beans and beansticks (beans glued to popsicle sticks). 	Problem solving is involved since students do not know how many solutions (if any) until they experiment. Also students can use different methods of attack to solve the problem.	Earlier in the school year, beans and base ten beansticks were used to develop place value and addition concepts. The teacher is hopeful that most of the pupils will now be able to deal with these and other concepts using a slightly more abstract physical model provided by the lima bean computer. The teacher will be certain pupils can demonstrate an understanding of the concepts presented using a physical model before asking them to work at the abstract level with problems in the basic text.
	Pupils are asked to find a way of using the lima bean computer and beans to show the addition of 23 and 6, rather than being *told* a method to use.	Carefully worded questions by the teacher can help lead pupils to make those discoveries for themselves.
Since student achievement varies, the teacher evaluates students' grasps of the concept so that subsequent activities meet individual needs of students.		

The math period is split into two parts:

Part I

Group A is given a worksheet with problems similar to those given on Monday. Students are asked to work problems with the help of the lima bean computer if needed, and to record their results. Recording results may vary.

23 + 9 =



10's 1's
Susie's record

23 + 9 = 32
Bill's record

Group B meets with the teacher and is verbally asked questions similar to those asked on Monday, but pupils are encouraged to use beansticks and beans to help solve the problems. When the teacher has determined that the pupils can successfully use the beansticks and beans, they are given the worksheet assigned Group A to complete.

Part II

Group B completes the worksheet independently.

Group A meets with the teacher. Together they determine and practice a method of using the lima bean computer to solve subtraction problems such as:

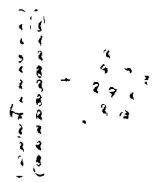
35 − 4
41 − 6
36 − 19

Wednesday

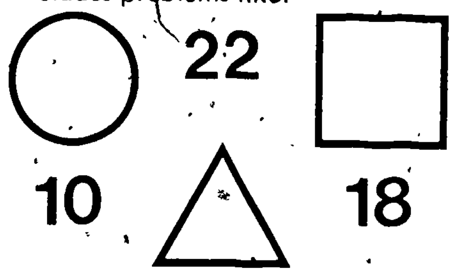
Students are again placed in two groups. Any students from Group A who yesterday experienced difficulty with the written assignment may work with Group B today.

Group A is given a page of subtraction from the textbook to complete, with optional use of the lima bean computer.

Group B meets with the teacher and works through subtraction with borrowing using beansticks and beans.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
The use of beansticks and beans, a remembered experience, helps the teacher meet the needs of those pupils not ready for a more abstract physical model, thus assuring all pupils success with the assigned activity.		Pupils who continue to find the manipulatives useful in solving (or checking) their problems will be encouraged to use them as needed.
		
Beansticks is a remembered experience since students can count each bean to determine the sum.	Again, pupils are lead to discover a method for themselves, rather than being told "how."	Recent statewide math assessment* of 4th graders revealed pupil difficulty in subtraction with "borrowing." Here are concrete experiences to provide a foundation for understanding the concept.
Students in this class feel good about themselves and about math. The teacher is careful to introduce concepts at a concrete level. The activity is designed so that students become less and less dependent on the physical activity until the action is internalized as a mental operation.		The teacher works to provide for individual differences through flexible regrouping. The environment is active and exciting. The text is used when pupils are ready to function at the level of abstractness it requires. *_Education for Oregon Learners: Where We Stand/Results of the 1976 Assessment of Mathematics_ (Salem: Oregon Department of Education, 1976).

Before the lab activities begin, a "drill and practice" activity sheet* is distributed which includes problems like:



Insert numbers in the figures so they add to the indicated sums.

Thursday and Friday: Lab Days

Six different laboratory activities have been organized. Each will take from 20 to 30 minutes to complete. Students will rotate to a new station on Friday, and will continue to rotate to different activities until they have completed all six activities.

Activity 1 Jill and Judy are seeing how long it takes them to put all the numbers in order on a 100's board. (See photograph.)

Stan and Stephanie are also involved in the same 100's board activity, but are working in a different part of the room.

According to this same organizational plan, each of the following lab activities is assigned to two pairs of students.

*Robert Wirtz, *Drill and Practice at the Problem-Solving Level* (Washington, D.C. Curriculum Development Associates, 1974).

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
Children, like adults, are usually more motivated when their tasks are varied. The students are actively involved in the activities and not mere spectators.		While the children are involved with these problems, the teacher then has the opportunity to distribute the lab activities to each group and give any necessary advice, directions, encouragement, etc. Also, since the pupils will complete their labs at different times, they can return to the "drill and practice" problems when finished.
	Many lab activities can be used to show applications of math. For example, students involved in "store" activities and "catalogue problems" will gain much in the way of consumer education. Perhaps data from a "counting cars" experiment can even be used in an ecology report presented to the rest of the school.	Students often perform better at some tasks than on others. For example, some do very well in solving problems in a math lab environment but do not perform as well when asked to do subtraction with borrowing.
In these activities students can choose their own methods of attack.	Sources of problems are almost unlimited. In addition to catalogues, others include the local newspaper, local sports teams, local businesses, almanacs, *Guinness Book of World Records*, *Oregon Blue Book*, etc. Other curricular areas are fine sources of problems—science, social studies, career education, etc.	Lab activities, when organized in the manner suggested here, allow students to solve problems at their own rate.

Activity 2 How many different 4-sided figures can you make on your geoboard? (See photograph.)

Activity 3 "To Market, to Market" (See photograph and task card.) A classroom store is easy to set up if students are requested to bring empty cereal boxes, egg cartons, etc. Below is a reduction of a task card.

To Market, to Market

GET Grocery items

DO

1. Buy 4 items. Spend less than $1.25.
   Make a chart like this on your paper.

item	price
total	

   Record what you bought. Record prices and total.
2. Buy 4 things. Spend more than $1.25.
   Record the same way as before.
3. Buy 5 items. Spend as close to $1.60 as you can.
   Record the same way as before.

   Take turns. Make up some store problems.
   Have your partner solve them.

Activity 4 Experiment to determine which sum is most popular when two dice are thrown. Graph the results.

Activity 5 Conduct a survey involving cars that go past the school in 15 minutes. How many are occupied by one person? Two persons? etc. Graph the results.

Activity 6 Use hand-held calculators and Sears catalogues. Be a BIG spender! Spend between $990 and $1,010. Record each item and its cost.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
"Slower" students do not feel threatened by the accomplishments of their classmates. For example, Erik and Sonja were delighted when they found 24 different quadrilaterals on the geoboard. Two weeks later, when Stan and Stephanie did the same activity, they were just as pleased to have found 11.	Activity 2 is open-ended. Students do not know the outcome and must choose their own methods of attack.	
	Task card problems vary from simple one-step/one-answer problems to multi-step problems with many possible answers. Exposure to the teacher-created problems can lead pupils to investigate problems of their own creation.	Small group activities on "lab days" provide excellent opportunities for students to work and contribute to the group effort at their individual levels. The task card shown includes more than one level of problem within the same activity. The last suggestion may even lead individual students into some independent investigations.
	In activity 4 students collect data and then organize the information to solve the problem.	
		Many students would have difficulty with the catalogue problem unless a hand calculator was available. The problem is essentially one in estimating and then refining the estimates in order to fall within the $990-$1,010 range. Thus, a hand-held calculator is quite appropriate for this problem-solving activity.

MIDDLE GRADES (6-9)

Learning and Teaching

Most teachers find middle grade students highly heterogeneous and emotionally unsettled. These students rapidly change during short periods of time. These differences complicate the teacher's task because motivational patterns, social needs and teacher-student relations are not uniform.

This period of development between childhood and adolescence has been termed "preadolescence" or "early adolescence." Until recently, very little time and effort have been devoted to the study of this period. It is now being recognized, however, that early adolescence is more than a phase that youth pass through on the way to adolescence. *Early adolescents are persons with specific qualities and characteristics who, at this particular time of their lives, have certain rules to play, skills to develop, tasks to perform and things to learn.*

The task of the middle-grade teacher would be complicated enough if dealing only with early adolescents. However, most middle-school classes are a "mixed bag." If 40 sixth-graders and 40 ninth-graders were selected at random, the following combinations might likely result:

Sixth Grade Class	
10 childish girls	16 childish boys
8 early adolescent girls	4 early adolescent boys
2 adolescent girls	
Ninth Grade Class	
4 preadolescent girls	2 childish boys
16 adolescent girls	8 early adolescent boys
	10 adolescent boys

As students enter the middle grades, they have been traditionally viewed as more intellectually mature and their courses became "high school" courses with more emphasis on subject matter. While it is true that important changes are occurring in the early adolescents' mental powers, both quantitatively and qualitatively; equally significant other changes taking place must not be neglected. Some of these significant developmental characteristics are:

- Profound and rapid body changes coupled with the experience of physical sexual maturity. They wonder about themselves and strongly identify with classmates of same sex. They are learning their roles in heterosexual relationships.
- a striving for independence and a concern about relationships with other people. They move from adult protection to interdependence with peers.
- a growth in mental ability enabling them to think more abstractly: facility with combinational logic, ability to utilize a second symbol system (symbols for symbols), and acceptance of contrary-to-fact premises as bases for argument.
- possession of great mental, physical and emotional capacity to experiment—to try things out.
- striving for personal values. Rather than passive observers, early adolescents become participants.

The implications of these characteristics for math teaching are not well-defined but do suggest some actions that teachers may want to consider.

- Because students are at different stages of development, they may not all be able to work at the same level of abstraction. Some may have to start with concrete experiences while others are ready to operate more abstractly.
- Small group activities enhance peer communication—providing another rationale for the laboratory approach.
- Activities with all experimentation and movement around the room are, in general, good supplements to the math program.
- Certain social issues of concern to students could be incorporated with problem-solving activities.
- At certain times students could be offered choices (choosing a laboratory activity, a special project of interest to them, or even the opportunity to work in groups or individually on some homework assignments).
- To let students experience success, teachers could give some problems all students can do, and follow them with gradually more difficult ones that many will then at least try to do. Some problem-solving activities have the advantage that everyone can get results but the more able students can extend their investigations in greater depth.

The classroom episode at the end of this section indicates how some of these learning and teaching implications are incorporated in a week-long unit on probability.

Sample 6-9 Classroom Instruction

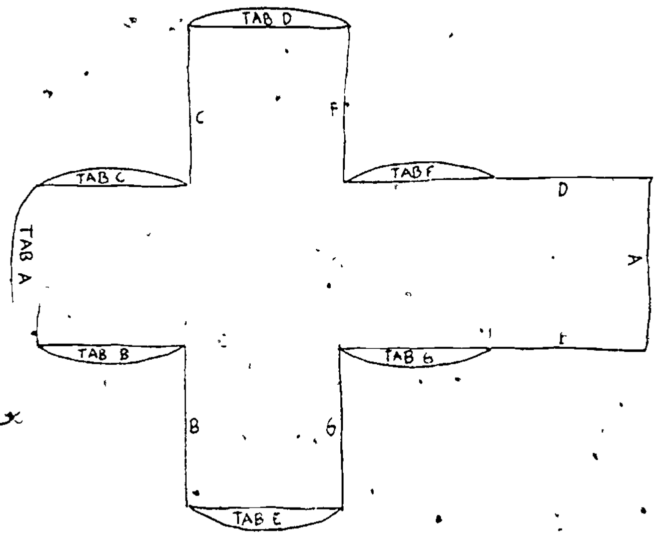
Sample 6-9 Classroom Instruction

CLASSROOM ACTIVITIES

This section examines five days of mathematics instruction in a junior high school classroom. Again the statements on the left-hand page list the activities, and the comments on the right-hand page call attention to related learning theory, problem-solving elements and strategies for conducting such activities.

Monday

Introduction to Dice and Probability
Provide squared paper, scissors and tape. Have each student cut out a pattern which folds up to make a cube. Tape cube, and label the faces 1, 2, 3, 4, 5 and 6.



Have each student toss the die 50 times and record the top number. Ask if results seem reasonable and discuss.

Ask and discuss: Should a 5 come up more often than a 4? Than a 3?

Ask students: How would you change your die so a 5 comes up more often?

Have students renumber their die so that a 5 comes up:

- about half the time
- about a third of the time
- all the time
- none of the time.

Summarize the activities; introduce the term probability.

Assign a textbook page or give a worksheet with dice probability activities and give pupils some time to work on the examples.

End with this thought question: Suppose you throw two dice and add the two top numbers. What sum will you predict to be the most

LEARNING & TEACHING IMPLICATIONS	PROBLEM SOLVING ELEMENTS	TEACHING STRATEGIES
These activities were chosen to introduce middle-grade pupils to the topic of probability, because they are active, "hands-on" experiences of high interest. Pupils of varying capabilities can all have *success* with these activities.	*Use of a concrete model,* such as the die, can be part of a strategy used to solve problems.	The teacher has chosen to introduce probability through experimentation before assigning written probability exercises from the textbook.
	One of the basic skills for problem solving is learning how to *record and organize* data. After the experiment has been performed, students could be encouraged to share their method for recording die outcomes with the entire class.	Experimentation, discussion, conjecturing and further experimentation to verify student guesses provide for changes in activity within a class period while encouraging students to make predictions.
	Making a guess is another strategy often used to solve problems. The guess may help determine starting or ending points for the experiment.	

popular; a 3, an 8, or a 13? The following problem might also be given at this time as the challenge of the week:

> Construct a die 3.5 x 4 x 4.5 cm. Number the faces, throw the die several times, observe what happens and record the results. Summarize and state your conclusions.

Tuesday

Pupils work in pairs. Each pair rolls two dice 50 times, adds the two numbers on top each time, and records each sum. When 50 rolls are recorded, the data are used to make a bar graph.

Have each pair of pupils record the results of the 2-dice activity in table form on the chalkboard.

Groups ↓	Sums 0	1	2	3	4	5	6	7	8	9	10	11	12	13
Joe & Sue			2	1	3	4	10	11	6	6	3	3	1	
Frank & Bob			1	3	5	5	9	12	6	4	2	2	1	

Have pupils add the columns and make a graph of the class results. Use the class graph to answer the following questions:

> How does the class graph compare to each pair's graph of their own results?
>
> Did anybody get a sum of 0, 1, or 13? Why?

The totals for each sum represent what percent of the total results? Give this as a homework assignment.

Wednesday

Discuss the results of the 2-dice activity done yesterday. Why does 7 seem to be the most popular sum?

Have pupils work in pairs to determine and record the different ways each sum can show up on the tops of two dice (suggest pupils imagine one die is red and the other green). Encourage pupils to

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
		This is an extension of the activity pursued during class. Students could be asked to think about this question (posed by the teacher) *or* ask and answer their own questions. The more students are challenged to do the latter, the greater the chances of fostering independent investigation.
Notice that this investigation provides students an opportunity to interact with classmates. They could be given an opportunity to choose partners.		As before, the teacher has chosen an active investigation. This activity has been adapted from the one on Monday. The last activity on Monday was designed to prepare students for this investigation.
	Here again, students are *recording, organizing,* and *graphing data* they have obtained through experimentation. In order to *see patterns* and draw conclusions, these activities are essential to problem solving.	The teacher is "free" during this time and can ask pupils to explain their predicted outcomes for the activity. Also during this time, the teacher can assist students in organizing data.
	This comparison can show students that even though their individual results were different, each one succeeded at the task given by the teacher. Here we see computation in context, i.e., used as a problem-solving skill—a means to an end.	Such problems often include opportunities for computation.
Once again pupils are working in a social situation in which they have an opportunity to discuss the investigation.	*Looking at this pattern* will give students a chance to use another problem-solving strategy.	
	The concrete model of red and green dice may help them answer that question more quickly. Ask	The teacher has a chance to move about the room and interact with pupils as they pursue the investigation.

organize their data in some way and be ready to report their organized results to the class.

Have the pupils report their findings to the class. Summarize in chart form on the blackboard. Use the data to answer the following:

How many outcomes have a sum of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12?

Which outcomes occur 6 of 36 times? 5 of 36 times? 1 of 36 times?

Prepare a ditto like the one below.

	Sums with 2 Dice											
	2	3	4	5	6	7	8	9	10	11	12	Total Attempts
Frequency of each sum during class experiment												
Frequency of each sum is what percent of total attempts?												100%
Number of possible outcomes for each sum												36
Number of possible outcomes for each sum is what percent of total outcomes possible?												100%

Use totals from the activity during class yesterday to fill in row 1. Use homework data to complete row 2. Use data from the activity today to fill in row 3. Assign row 4 as homework.

Thursday

Allow some time at first of period for pupils to work on the challenge of the week.

Discuss the completed chart from Wednesday.

How do the percentages in row 2 compare with the percentages in row 4?

Are there other observations that pupils can make about the completed chart? (Do they notice any symmetry in the rows of results?)

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
	the students, "Have you seen the sum before? How are they the same or how are they different?" Answering these questions would have been more difficult without a chart with the organized information.	
Individuals may contribute to the group effort at their own level allowing all to feel a sense of success. This contributes to self-confidence and positive attitudes.	A number of problem-solving activities are involved. gathering, organizing and presenting data defining problems choosing strategies estimation and prediction computation	Such activities provide opportunities for both small group work and large group instruction.
The beginning activity on Thursday is another example of a pupil-oriented rather than a teacher-directed learning situation.	Patterns are much more apparent when the data have been organized into chart form.	The teacher is able to meet individually with pupils at their desks to check on the completion of homework, to assist them with the challenge of the week or to encourage extended investigations if pupils have completed the challenge. (For example, they might consider an investigation using three dice.)

Use remaining time to work on a textbook assignment or worksheet.

Friday
Lab Day

Six different laboratory activities have been organized. Each will take about 30 minutes to complete. Students will rotate to a new station next week and will continue to rotate different activities until they have completed all six of them.

Activity 1 Megan and Karen are investigating the stretch of a rubber band by attaching different weights to one end. (See photograph.) Martin and Steve are also involved in the same "rubber band stretching" experiment, but are working in a different part of the room.

According to this same organizational plan, each of the following lab activities is assigned to two pairs of students.

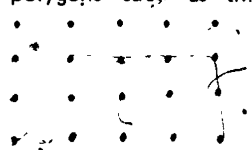
Activity 2 How many different polygons can you make on your geoboard which have an area of 4?

Activity 3 What's the smallest number of moves required to move the six discs to a different post? (See photograph.)

Follow these rules:

Only one disc may be moved at a time.

A larger one may never be placed on a smaller one.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
Most of these activities are organized so that each pair of students has the opportunity to use a manipulative or concrete model when first starting the problem. Laboratory experiments seem to be most successful when they allow students to work through the stages of development from concrete to abstract.	Setting up the laboratory experiment will allow students both to choose among and use problem-solving strategies practiced that week. All of the activities will utilize strategies such as *using concrete models, looking for patterns* and *recording the information in an organized form.*	
Lab activities, when organized in the manner suggested here, allow students to solve problems at their own rate. "Slower" students do not feel threatened by the accomplishments of their classmates. For example, Judy and Jamie were delighted when they found 30 different polygons with an area of 4. Two weeks later, when Keith and Oscar were involved with the same activity, they were just as pleased to have found 13.		Many lab activities can be used to show the application of mathematics. For instance, the stretching experiment is an example of how mathematics is helpful to show a scientific result.
Teachers can *see* misconceptions about polygons such as this as  students are making the shapes. The concrete model many times makes the diagnosis of difficulties easier for a teacher.		When the teacher *sees* a shape that is not a polygon on the geoboard, it is an appropriate time to ask of the two students, "Can you show me a shape of an area of 4 that's *not* a polygon?" This may lead to a discussion of the correct definition of a polygon.
Sometimes students are more willing to try problems in a format which leaves no record of errors made during the attempt.	There are different ways to solve the "moving of the discs" problem. If students are frustrated with the task as it is stated, perhaps the teacher could suggest they begin with *easier cases* and *look for a pattern.* After a pattern has been discovered, a hand calculator could be used to determine how many moves it would take to move 30 discs. (This extension should be given *only* to those students who are able to handle this kind of challenge.)	

Activity 4 "Earth Airways" (See task card.)

GET – Globe of world
String
Ruler

Earth Airways

1 Find the scale Write it down

2 Use the string to help you find the approximate airline distance from your nearest airport to

a San Francisco
b New York
c London, England
d Sydney, Australia
e Seoul, Korea

3 Guess how far it is around the world at the equator Use the string to check your estimate

4 You are now the owner of Earth Airways You own only one airplane, a Boeing 747 Jumbo Jet You also have only one air crew to fly the airplane

Your job is to plan an around the world flight that meets these specifications
a The crew may fly a maximum of 8 hours a day
b Average cruising speed is 600 miles per hour
c You must cross the equator 2 times
d You must complete the trip in 7 days.

Keep a detailed record of the miles you fly and airports you use

5 Airlines charge their passengers about 10 cents per mile At this same rate, how much would Earth Airways charge for the around the world trip? Record

EXTENSION
The Boeing 747 is capable of carrying 490 passengers If the around the world trip is completely sold out, how much money will Earth Airways receive?

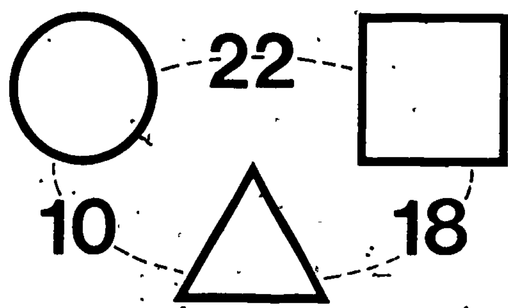
Activity 5 Make two dice patterns out of tagboard and "load" each of them by taping a paper clip inside one of the faces. Try to number the faces so that the most popular sum will be 11.

Activity 6 Study the example.

$$\begin{array}{r} 684 \\ -486 \\ \hline 198 \end{array} \qquad \begin{array}{r} 198 \\ +891 \\ \hline 1,089 \end{array}$$

Use a calculator and try using this same procedure with other 3-digit numbers. What about 4-digit numbers? 5-digit numbers? Write about your results.

Before the lab activities are begun, a "drill and practice" activity sheet* is distributed which includes problems like:



Insert a number in each figure so that pairs will add to the indicated sum.

*Robert Wirtz, *Drill and Practice at the Problem-Solving Level* (Washington, DC: Curriculum Development Associates, 1974).

LEARNING & TEACHING IMPLICATIONS	PROBLEM SOLVING ELEMENTS	TEACHING STRATEGIES
		The "distances on a globe" probably shows a relationship between mathematics and geography.
	Here's a chance for students to use a technique not commonly encouraged: *trial and error.*	Hand-held calculators can also be appropriately used to assist with investigations like the "digit-reversal" problem in which paper-and-pencil computation would likely hinder the discovery of unusual math patterns.
Students, like adults, are usually motivated when their tasks are varied. Also, they often perform better on some tasks than on others. For example, some students do very wall in solving problems in a math lab environment but do not perform as well when asked to do exercises involving long division of decimals.	Again the strategy of *trial and error* will be utilized.	While students are involved with these problems, the teacher then has the opportunity to distribute the lab activities to each group and give necessary advice, directions, encouragement, etc. Also, since students will complete their labs at different times, they can return to the "drill and practice" problems when finished.

NONFORMAL MATHEMATICS (9-12)

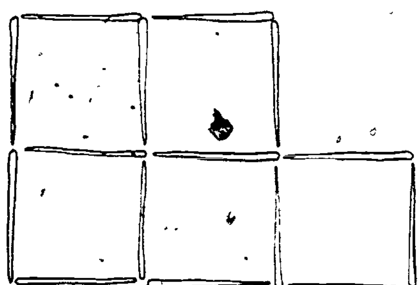
Problem Solving

The less structured content of these courses allows teachers excellent opportunities to incorporate problem-solving activities in their classes. These activities can improve students' computational skills, reinforce geometrical concepts, and illustrate mathematical applications. A selection of good problems provides welcome variations in classroom activities. Problems can be chosen to motivate and provide some success for students at all levels.

In solving problems it is often helpful to use some basic strategies. What follows is an example of how one teacher emphasized these strategies in teaching problem solving at this level.

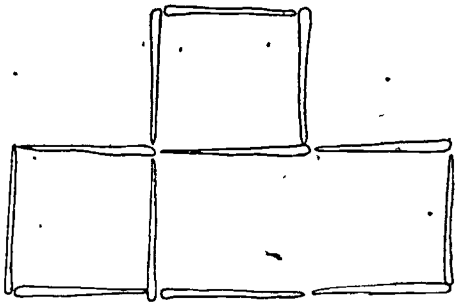
The teacher first identified the following strategies to be emphasized: using trial and error, using concrete models and drawing pictures, ~~using~~ charts, using variables and other notation, working backwards, simplifying the problem, looking for a pattern or remembering a similar problem. She then selected problems that could be solved by using one or more of these strategies and arranged them in order of difficulty.

The strategy for the month of September was *using a concrete model or drawing a picture*. After a short discussion of this technique, students were given some toothpicks and some problems including this one:



Remove three toothpicks so that exactly three squares remain.

More problems were given each week with increasing levels of difficulty. In addition to the high level of enthusiasm, the 1½ to 3½ hours per month (depending on the nature of the course and the students) resulted in increased student confidence and familiarity with one strategy. She also has the chance to reteach the definition of square when several asked, "Do they have to be perfect squares?"



Students found patterns in the removal or rearrangement of toothpicks as more challenging problems were given, and they created their own problems.

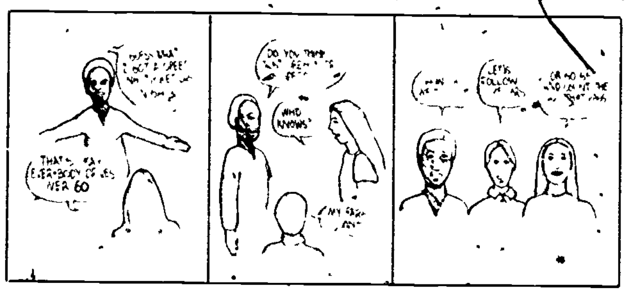
After a few months, students will have learned several strategies and can choose those appropriate for a particular problem.

Which of the strategies would your students use if you gave this problem in class?

> A woman crosses a river, paying $24 each time. When she gets to the other side, the money she has is doubled. After crossing four times, she runs out of money. How much money did she start with?

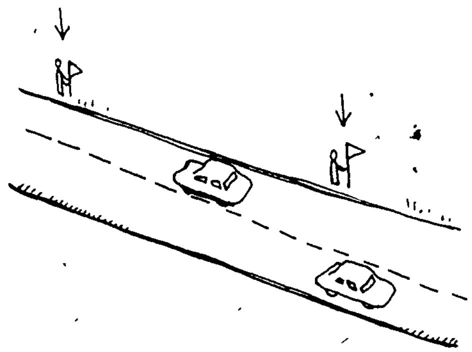
To attack this problem, some people may work backwards in some pattern while others choose the trial and error method. The solution will perhaps also come after drawing a sketch, selecting variables or notation or using a chart. Whichever strategies are chosen, some computation must be used to arrive at the solution.

Mathematics teachers can also make use of conversation among students as well as items from newspapers or concerns of citizens in their area as a source of problems.



In this class, the teacher has turned a casual discussion into a problem the class may solve. He first let them brainstorm as a large group to see the many ways to pursue a solution, such as breaking up into small groups or letting individuals work on their own.

In this classroom, one group of students decided to find the time it took to travel a fixed distance and then calculated the speed of the cars. They set out two flags along the highway at a premeasured distance apart and used a stopwatch to calculate the time each of 100 cars took to travel that distance. Back in the classroom, the resulting speeds were calculated and graphed on a chart. In sharing the statistics, they also asked the other class members to figure percentages of the sample that were speeders. These percentages were compared with percentages that other groups had obtained.



Sample Nonformal (9-12) Classroom Instruction

Sample Nonformal (9-12) Classroom Instruction

CLASSROOM ACTIVITIES

A one-week episode is presented to illustrate some learning and teaching concepts and at the same time serves as an example of teaching strategies and classroom management techniques.

Day One

One team of students is given the task of measuring and recording data from a bouncing ball, e.g., determining the ordered pairs (height of drop, height of bounce).

One team of students is given the task of reading two thermometers, one Fahrenheit and one Celsius, in several different situations, e.g., classroom, outdoors, in refrigerator or freezer, under running water.

Remainder of students are given the pencil-and-paper task of determining several lengths and widths of a rectangle of constant area; e.g., 48 cm^2; 108 cm^2.

Similarly display and discuss data from the thermometer team and dimensions found for given rectangular area.

Assignment: Construct graphs for each of the three projects. Selected individuals work on acetate sheet(s) for later overhead projection.

Enrichment: Collect data and graph on single axis information from bouncing ball experiment using variety of balls and bouncing surfaces.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
Introductory stages of new concepts begin with an active involvement with physical materials. This is consistent with the theories of Piaget, Bruner and others. Many secondary school students are able to move to representational or abstract levels more quickly than younger students It is very important, however, to remember that learners of all ages learn new concepts best from experiences with concrete materials, then move to the representational and abstract levels when they are ready.	A nice balance of several problem-solving skills is being developed in such activities as gathering, organizing and presenting data recognizing and defining problems choosing strategies estimation and prediction computation	Different types of activities allow for taking advantage of individual differences in performance skills. Students can get success experiences in those types of activities that best suit their abilities. These activities include skills of data organization and scale reading that are out-of-school oriented and have direct analogs in problem solving. With controlled movement on the part of several students, the entire class is more easily managed. Different groupings of students within the class period tend to give environmental variety so that boredom and attendant mischief is avoided. Student involvement in the presentation and posting of information gives further opportunity for recognition of students with limited ability.
	Perhaps some students will suggest other investigations which occur to them as a result of their experimentations.	The art-oriented student can demonstrate graphic display talents by producing the overhead transparencies. Some screening of these would be necessary to make sure they are basically correct.

CLASSROOM ACTIVITIES

Day Two

Review previous activities and conclusions using overhead transparencies produced by students in assignment.

Pendulum Demonstration

Rig ringstand-trapeze apparatus for pendulum experiment, provide for varying length, mass and displacement of pendulum. (See picture.) Establish fixed time interval, e.g., one minute; record number of swings (frequency) of pendulum as length, mass and displacement are varied. Use students to time, count, modify pendulum and record data. Students will also want to make suggestions as to modifying pendulum. Teacher coordinates all activities as the class works as a unit to construct tables of data.

Discuss tables in terms of variation as time permits. Key question, "How does *frequency* change (vary) as *length* increases or decreases?"

Assignment: Construct graphs for length and frequency, mass and frequency, and displacement and frequency. Selected individuals work on acetate sheets for overhead projection.

Day Three

Review examples of variation using "Key Question" format. Ask students for experience-related examples of variation (may require testing of some suggestions).

Ask students to determine length of given rectangle used on first day for width not previously used. First illustrate graphical techniques for this determination then formula usage as being more precise.

Examine pendulum data and graph for possible formula. One such formula is $T = 2\pi\sqrt{\frac{l}{g}}$; T = time in seconds of a complete vibration

l = length of the pendulum

g = acceleration of gravity

Ask for prediction of frequency for given lengths of pendulum. (Motivate by observing setting for grandfather clock, e.g., 60 swings per minute.)

Check out hand calculators (classroom set assumed available).

Class activity: To determine several values for lengths given frequencies, also frequencies given lengths. Include some extreme cases.

Assignment: Complete several ordered pairs given one element.

Enrichment: Collect data from gas station for litres and cost table, or marble (golf ball) on inclined plane data for height and roll time table.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
		Pendulum demonstration is teacher-centered but student-conducted. This illustrates another good strategy for involving students in positive classroom activities of the teacher's choosing.
	Again, several problem-solving skills are utilized. Problems from many sources help provide interest through variety. Such selections can help to relate math to other curricular areas and the local environment, and help students to see the math they work on in the classroom as relevant.	
Note that several developmental stages of learning are involved in this activity		Give opportunity for students to bring experiences to classroom.
	Use of graph is a problem-solving skill used elsewhere in everyday experiences.	
		Develop student creativity.
	62	Math becomes a part of students' everyday experiences.

Day Four

Supply formulas for automobile stopping distance as function of speed, temperatures conversion °F–°C, and others as previously studied by class. With or without calculators, have class determine several ordered pairs for each formula and determine type of variation involved.

Midway in period have calculator fun time exploring number patterns on calculator; e.g., Fibonacci sequences,* word games, etc. Paired teams of three students each: one mental arithmetic versus one with calculator for five exercises, then conditions reversed for five exercises. Total least errors wins.

Return to complete formula work or to previously identified enrichment activities if formula exercises completed with 100 percent correct designation of variation type.

Day Five

Summary Day

Provide many circular objects for which circumferences and diameters can be fairly determined with available measuring devices.

Group class into teams of two's or three's. Each group determines diameter-circumference table for four to five objects.

Have master grid on chalkboard on which each group records its data.

Ask key question(s): "How does _____ vary as _____ gets larger (smaller)? Determine formula, i.e., $C=\pi d$.

Assignment: All students draw graphs for their own group's data. With calculator, determine group's best guess for π. With calculator determine class(es) best guess for π.

Evaluation: At the end of the five-day episode, the teacher determines those parts that worked well and those parts needing modifications, making written suggestions for improvement.

The teacher liked the results and shared the unit and accompanying equipment with a fellow math teacher.

*One source of calculator activity task cards is *Mathlab Junior High* by McFadden, Anderson and Schaaf (Eugene, OR. Action Math Associates, 1975).

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
		Formula problems for this series of exercises can be from the text and be familiar (as from previously studied topics; e.g., i=prt, d=rt, $V=s^3$, $S=6s^2$, $A=s^2$, P=2w + 2l, A_t=1/2 ph).
	Several levels of computation may be involved—estimation, mental, paper and pencil, calculator. Help students to choose the level that will be appropriate in various situations.	Even the relaxation time from routine can be mathematically rewarding and skill enhancing. The contest between paired teams drill both mental facts and calculator proficiency. An alternative, based on given classroom types, would be to allow paper and pencil to those teams doing the mental arithmetic. Also number and operational difficulty may vary according to class ability.
Students need some opportunities to analyze data for patterns and generalization, and to create their own formulas. Successes will be exciting for both students and teachers and highly motivational.	Once more we see various problem-solving skills being developed.	Note an interesting change of pace occurring during the week and even within a single class period.

FORMAL MATHEMATICS (8-12)

Problem Solving

Problem-solving activities in the secondary math program should involve problems which offer challenges to students and provide them with opportunities to use and develop problem-solving strategies. Well chosen problems will require using skills in arithmetic, algebra, and so on, and will allow students to discover new concepts in mathematics. The teacher's judicious selection of problems can often heighten students' interests in materials and provide much needed variation from the more routine drill exercises of the math text.

One difficulty in selecting problems is to find or develop ones at the correct level for students. The problem should not require a background so different from students' previous experiences that they become completely discouraged, yet it should not be so simple as to be merely one more exercise with little challenge at all. This difficulty is often overcome by taking a problem and adapting it to student's levels. Consider the classical problem from calculus:

> An open box is made from a rectangular sheet of metal by cutting out equal squares from each corner and then folding up the resulting flaps. If the sheet is 12 cm by 15 cm, find the size of the squares so that the volume will be a maximum.

This problem can be adapted to student use at all levels in algebra, geometry or analysis.

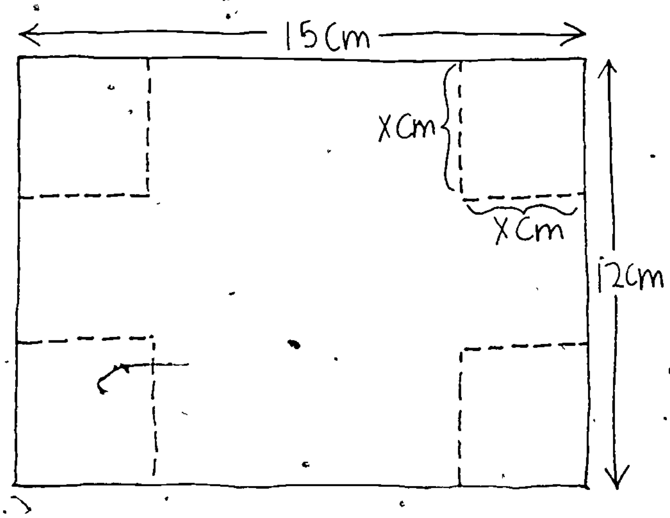
If students were not yet at the stage of setting up formulas, the problem could be handled as a lab activity by making actual models from square centimetre paper (restrict the size of squares cut out to have sides of length 1 cm, 2 cm, etc.).

To find the largest box, comparisons could be made by filling the models with granulated styrofoam or centimetre cubes, or by computation of the volumes.

Another useful strategy would be to list all the results of the construction of boxes on a chart, such as is partly shown below

Length of side of cutout square	Height	Width	Length	Volume
1 cm	1 cm	10 cm	13 cm	130 cm^3
2 cm	2 cm	8 cm	11 cm	176 cm^3
3 cm	3 cm	6 cm	9 cm	162 cm^3

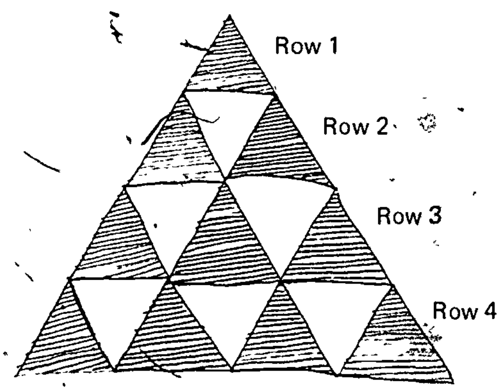
For students who are studying the development of formulas and equations, the problem could be directed toward this end. In this case, the use of a sketch would be a helpful strategy in attacking the problem.



Letting x be the side length of the square to be cut out, the height of the box becomes x, the width (12 - 2x) and the length (15 - 2x). The formula for volume is $V = x(12 - 2x)(15 - 2x)$ or $V = 4x^3 - 54x^2 + 180x$. If a chart had been made as above, it would be a natural extension of the chart to find this information as another line where the square size was "xcm." For more advanced students, the equation for volume could be graphed to find the maximum, and for students in analysis, the answer could be obtained by use of the derivative.

For use in a geometry class, consideration of the surface area of the box might be considered, along with observations of a comparison between surface area and volume of the various boxes.

Another example follows the type of problem often appearing in elementary school materials. Find the number of triangles (counting only the smallest ones) in this figure.



For algebra students, problems involving numbers of triangles could be generated in several ways. This is one example of such a development. The students are given a piece of isometric graph paper (or a triangular grid), and asked to shade in the top triangle (row 1), then the two triangles which are "pointing upwards" in row 2, and the three triangles "pointing upwards" in row 3. The question is asked, What percent of row 1 is shaded?; then, What percent of the first two rows is shaded? and, finally, What percent of the first three rows is shaded? The exercise can be continued through several more rows extending the picture downward.

Then questions such as, If 100 rows were considered, what percent would be shaded? . . . 1000 rows? . . . 1 million rows? Will the percent of shaded area always become smaller as additional rows are considered? Will the percent of shaded area become almost zero? Will it become less than 50 percent?

To arrive at an answer to all these questions, the strategy of making a chart would be one way to begin.

No of Rows	No. of Triangles Shaded	Total No of Triangles	Percent of Area Shaded
1	1	1	100
2	3	4	75
3	6	9	66 2/3

The students would be encouraged to look for patterns in the sequence of numbers of shaded triangles, the sequence of numbers of shaded sides and the sequence of percentages.

From these patterns, formulas could be developed when the number of rows is an arbitrary *n* and the information added to the charts.

n	$\frac{n(n+1)}{2}$	n^2	$\frac{n(n+1)}{2} - (n^2)$
			$\frac{1}{2} + \frac{1}{2n}$

Percentages could be calculated for several numbers, and a graph of those percentages made. Additional calculations for large numbers such as 100, 1000, etc., would show further what is happening to the percentages, and the concept of limits could be touched by exploring what would happen as n "approached the infinite." The students could attempt to set up other problems about the triangles in the pattern and try to develop formulas related to them by simplifying, looking for patterns, making charts and graphs and generalizing.

For more advanced students in junior- or senior-level mathematic courses, problems are often posed by the text being used. Unfortunately, many of these problems are often little more than exercises of the material of the section just covered or problems completely analogous to ones solved as examples in the text, so the responsibility for finding and using meaningful problems remains with the instructor.

The section which follows examines one week of math instruction at the high school level. The statements and notes on the left-hand pages are from the teacher's lesson plan book. On the right-hand pages are comments which relate to learning theory, problem solving and teaching strategies.

Sample Formal (8-12) Classroom Instruction

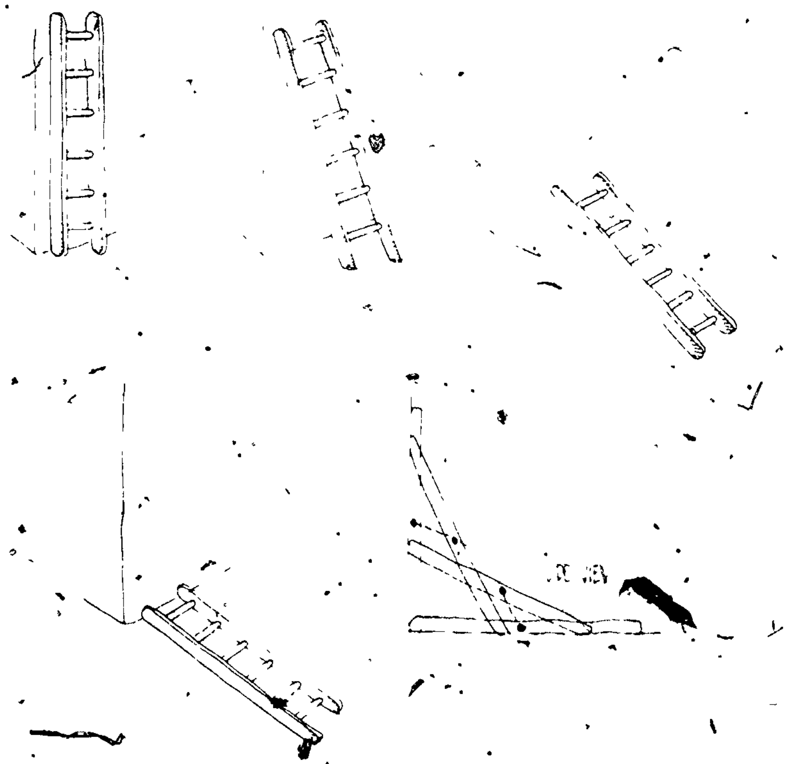
Sample Formal (8-12) Classroom Instruction

CLASSROOM ACTIVITIES

Five days of mathematical activities and instruction are presented. This suggests some learning activities and at the same time illustrates planning based on the learning theory discussed in earlier sections, development of problem-solving skills and several teaching strategies.

Monday

Every Monday is challenge problem day. The old challenge problem is discussed and a new one is given. Last week the challenge was to describe what seems to be the locus of the midpoint of a ladder as it moves down a wall and to prove any hypotheses formulated.



The new "challenge of the week" was presented: What is the largest rectangle that can be cut out of a triangle?

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
The challenge problems provide variety, enrichment and motivation. Students can share their discoveries with the class.	One student independently investigated varying the angle between the wall and the ground. Another student investigated the path of other points on the ladder.	The students were given enough time in class to begin on the investigation and to begin formulating some of their own questions. (Does it matter where, the rectangle is placed? Does it matter what kind of rectangle we started with?) This attempt at understanding the problem is stressed in George Polya's book, *How to Solve It* (New York: Doubleday and Company, 1957).

For the remainder of the period and for homework, students are given some review exercises involving the various properties of quadrilaterals and their diagonals. In addition, some review exercises are provided involving the following property:

> The segment joining the midpoints of two sides of a triangle is parallel to the base and equal to one-half of it.

Tuesday

At the first of the period, students are given review exercises in coordinate geometry. Midpoint and slope formulas are given special attention.

Next, the teacher informs the class that the activities which follow are intended to review and extend the ideas they've learned in the chapter on quadrilaterals.

The teacher draws a square on the chalkboard and connects the midpoints of the sides in succession. The resulting figure is obviously a square. Questions posed:

> What figure will result if we do the same thing with a rectangle? (The students are asked to predict and then find out by making an accurate drawing.)
>
> What if we started with a rhombus?
>
> What if we started with a kite?
>
> What if we started with . . . ?

Students are now encouraged to investigate on their own and to keep track of their discoveries. Only after considerable time has been provided for the investigations should a discussion of the discoveries take place. In the discussion, attention needs to be given to diagonal relationships in quadilaterals and how the shape of the "inscribed" figures depend upon these relationships.

The homework assignment is to prove that the figure resulting from connecting the midpoints of a rectangle is a rhombus.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
The teacher is attempting to provide the necessary prerequisites for the remainder of the week.		The pencil-and-paper activities provide a change of pace.
The warm-up activity reviews concepts and skills that will be needed in the next lesson. Some attempt is made to give students an awareness of the direction the instruction is taking.		
		At first the activity is teacher-directed. The teacher's enthusiasm and questions motivate the class investigation.
Students encounter the problem concretely before attempting to generalize or analyze it abstractly.	Students predict and make drawings to check these predictions.	The activity is individualized by allowing students sufficient time to investigate the open questions. During this time the teacher can provide additional challenges to individual students (e.g., Can you find other figures that produce a square? a rectangle?).
The teacher gives attention to the underlying structure and interrelationships in geometry.	Students involved in such activities are practicing several problem-solving skills.	This approach gives students an opportunity to discover mathematical relationships.
The teacher identifies one student and asks her to make a flexible model of a quadrilateral with elastic bands at the midpoints. The teacher is aware that students need opportunities for success.		
Models and drawings are important in developing concepts. Learners need experiences with physical objects before moving on to abstract levels of thought.		

Wednesday

At the beginning of the period, Sylvia demonstrates the flexible quadrilateral model she has constructed.

The class is then asked to prove the properties they discovered on Tuesday. It is suggested they work together in pairs.

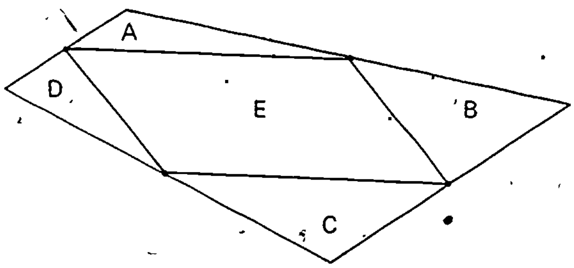
After allowing sufficient time, students are asked to share their proofs with the rest of the class. After this sharing session, the teacher should summarize the results and give further emphasis to certain generalizations. For example, students have probably proven that the "inscribed" figure will always be a parallelogram. However, they may have not identified the conditions for which the inscribed figure is a rhombus, rectangle or square.

As a final "wrap-up," the teacher assists the class in proving the general theorem by means of coordinate methods. For homework, students are given exercises from the text involving inscribed figures in quadrilaterals and midpoints in triangles.

Thursday

At the beginning of the period, students are allowed at least ten minutes to work on the challenge problem given on Monday.

The following activity is presented and the students are given the rest of the period for their investigations.



Draw a large quadrilateral with all sides of different lengths. Find the midpoint of each side and connect them.

Cut out the large quadrilateral.

Snip off the regions A, B, C, and D.

Fit the regions A, B, C, and D completely inside region E (no overlapping).

Will this always work?*

Repeat the experiment several times by starting with different quadrilaterals.

*Bonus exercise: Prove that it will always work.

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
The goal of the teacher is to help students become independent problem solvers. The teacher provides ample time for students to experiment and share their discoveries with others and helps them to achieve self-reliance by giving them assistance only when necessary.		While students work, the teacher circulates among them and inspects the previous night's homework and today's proofs. This informal evaluation assists the teacher in planning the follow-up discussion.
The class has been involved in both an inductive approach and a deductive approach to the theorem. Students are also exposed to two methods of deductive proof of the general theorem.		The teacher structures the class activities so that there is a "change in pace." Students work by themselves or in small groups before participating in a whole class discussion.
		Students work quietly and individually on the challenge.
	Hint cards are available for students who feel too frustrated to continue without some help. Hints include: "Draw a figure," or "Try a special triangle like an equilateral or isosceles.	The teacher uses this time to pass out the equipment needed for the following activity.
	The problem contains familiar terms all students can understand.	

Homework: If you start with a square, you can fold the region over to completely fill region E. It is also possible to start with other quadrilaterals and accomplish the same thing. What quadrilaterals *can* you start with?

Reminder: Quadrilaterals quiz on Friday.

Friday

The students are given ample time to share the discoveries they've made regarding the "folding" experiment given yesterday.

A 20-30 minute quiz is given on quadrilaterals. A hint on the challenge problem is given at the bottom of the quiz sheet.

> Start with a triangle. Fold in the corners so that a rectangle is formed. Is this the largest rectangle possible?

LEARNING & TEACHING IMPLICATIONS	PROBLEM-SOLVING ELEMENTS	TEACHING STRATEGIES
Students use transformations—both translations and rotations to fit the regions inside region E. Thus activities all week integrate three approaches to geometry—synthetic, coordinate and transformations. The proof can also be done easily with transformations.		The teacher provides for individual differences. Students who quickly discover how to cover region E can try the bonus exercise. Students are not rushed into the proof. Again, the teacher functions as a facilitator of learning not as a dispenser of information.
The hint can motivate students to continue work on the challenge problem. During discussion Monday, the teacher can relate the solution of the challenge problem to the solution of the folding experiment given Thursday.		The hint encourages those students who finish early to work quietly by themselves.

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ASSESSMENT

In the Goal-Based Planning for Mathematics section of this publication, five sets of desired outcomes were identified: (1) state goals; (2) district goals; (3) program goals; (4) course goals—secondary/unit goals—elementary; and (5) district-identified competencies. Personal goals of individual students were also mentioned. These were followed by specific suggestions (learning activities, teaching strategies, resources, alternative instruction, etc.) designed to assist the planner in implementing a goal-based curriculum.

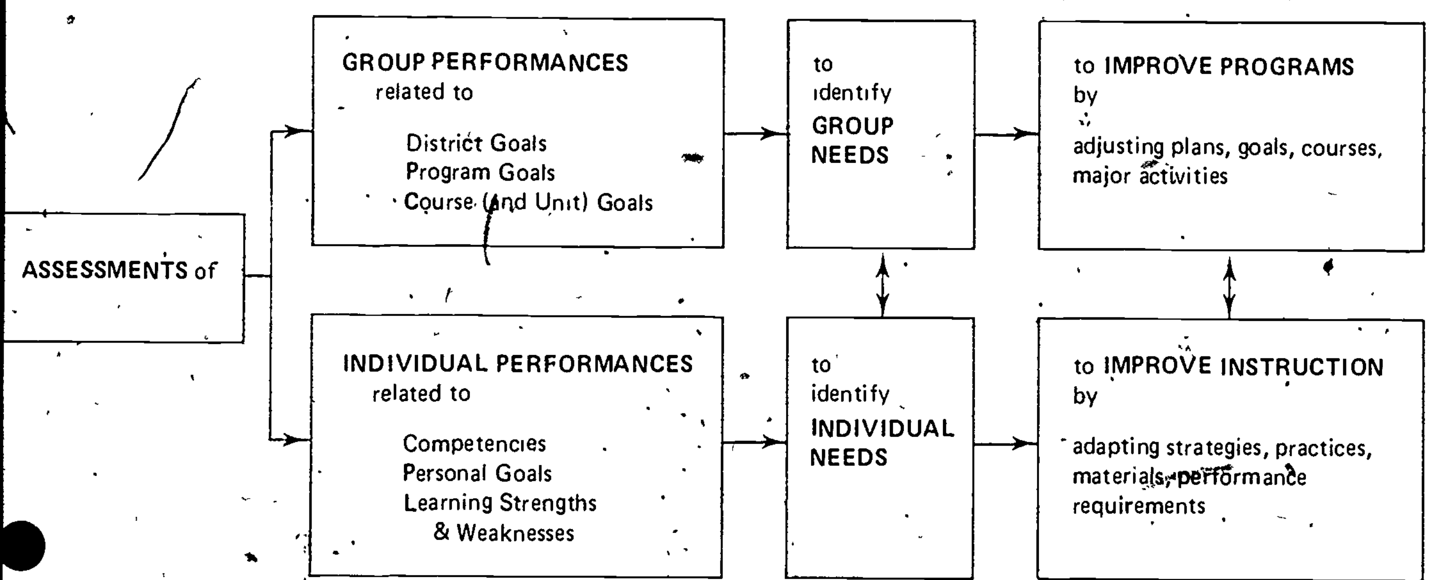
Once instructional plans are implemented, the teacher must pose the question: Are students attaining desired outcomes, and is the math program helping them to reach those outcomes? The quality of the answers to these questions depends on for what purpose (group or individual) and how well assessment activities are designed and carried out.

If it is desirable to know the kind of overall job the math program is doing, then the performance of groups of students is significant. Assessment focuses on whether an acceptable majority of students is attaining established goals. The needs of groups of students can then be identified and program planning improved accordingly. If, however, it is desirable to know how well individual students are attaining desired (or required) outcomes, then the performance of each individual student is significant. Assessment focuses on the needs, interests, and learning strengths and weaknesses of individual students as they strive to develop and demonstrate desired outcomes. The needs of individual students can then be identified and learning activities, teaching strategies, resources, etc., adjusted accordingly.

These relationships are shown in Figure 8. Assessment of each of the elements shown in the figure will provide answers to particular kinds of questions.

FIGURE 8

ASSESSMENT OF GROUP AND INDIVIDUAL PERFORMANCE



Assessment of district goal attainment answers the question: To what extent are students attaining the outcomes of schooling the community and its schools desire?

Assessment of program goal attainment answers the question: To what extent are students attaining the outcomes math teachers and curriculum planners desire?

Assessment of course goal or unit goal attainment answers the question: To what extent are students attaining the outcomes math teachers desire for, say, Algebra I or first grade math?

Assessment of competency attainment answers the question: To what extent is a student demonstrating desired *applications* of what has been learned?

Assessment of personal goal attainment answers the question: To what extent is a student attaining those outcomes he or she has designated as personally important?

Assessment of learning strengths and weaknesses answers the question: What characteristics reflected by a student's performance can be seen as enhancing or inhibiting attainment of desired outcomes?

In seeking answers to these questions, student performances that can be accepted as indicators of attainment of desired outcomes must be clear. These performance indicators serve to guide the assessment activity in producing the most needed information.

To be in compliance with state requirements, each district must assure that assessment activities are carried out in relation to three points. Assessment of student attainment of competencies for graduation and identification of the learning strengths and weaknesses are two of these. In addition, a district may select its own math program for a special kind of assessment the state requires. If this happens, it will be necessary to analyze the goals of the program to determine the extent to which students must develop or apply reading, writing, and computing skills in attaining those goals. Assessment will then focus on describing how well the necessary skills are being developed or applied.*

Suggested assessment strategies may take many forms. Below is one list of evaluation techniques.**

Basis: student performance on
- a. teacher-made
  - –pretest
  - –diagnostic test
  - –quiz
- b. standardized test
- c. homework
- d. project
- e. student evaluation report (pupil reaction to own progress

Basis: teacher observation
- f. casual observation (in and out of class)
- g. small group
- h. interview

Basis: both student performance and teacher observation
- i. seatwork
- j. boardwork
- k. classroom dialogue

*For more information on assessment procedures, see *Planning the Education of Oregon Learners. Completing the Management Cycle* (Salem Oregon Department of Education, 1975). See also pages 21-26 in *Elementary-Secondary Guide for Oregon Schools: Part II, Suggestions.*

**Mathematics Resource Project, *Ratio, Proportion and Scaling* (Eugene, OR. University of Oregon, 1975).

Certain evaluation methods are, of course, better suited to measuring given goals (or objectives) than others. Tests and written assignments yield certain information, but teacher observation during small group work may be needed to gather such information as whether students use problem-solving strategies in attacking new problems. Actual teacher observations and interactions would also be needed to evaluate objectives such as "pupils use concrete objects to support answers."

Finally, since a major goal of mathematics education is to promote a positive attitude toward mathematics, any assessments of program effectiveness or pupil progress would be incomplete without some consideration of feelings and emotions. How does one determine if pupils are acquiring a "friendliness" with number and shape? Acute teacher observations yield valuable information. Teachers should walk around the classroom and listen to comments. They should observe students' behavior and work habits. They may ask students to identify the topics they best understand and enjoy. Teachers should note pupils' willingness to accept or initiate special projects (reports, models, demonstrations, bulletin boards, etc.). They may have pupils write their own comments and evaluations for their written assignments. Finally, and perhaps most importantly, they may note pupil initiative, willingness, and perseverance in trying new math problems.

BIBLIOGRAPHY

Aichele, Douglas B. and Robert E. Reys. *Readings in Secondary School Mathematics.* Boston: Prindle Weber and Schmidt, 1971.

This book contains edited articles from several sources on such topics as curriculum, math learning theory, pedagogy, math labs, evaluation, research, issues, and trends. Includes an excellent bibliography.

Biggs, Edith E., and James R. MacLean. *Freedom to Learn: An Active Learning Approach to Mathematics.* Ontario, Canada: Addison-Wesley (Canada) Ltd., 1969.

This book emphasizes an activity-centered approach to elementary mathematics instruction and gives many suggestions for organizing lab lessons in the elementary classroom.

Copeland, Richard W. *How Children Learn Mathematics: Teaching Implications of Piaget's Research.* New York: The Macmillan Company, 1970.

This book is intended to help teachers discern a child's stage of development as a basis for determining the type of mathematics for which he or she is ready. It summarizes the ideas of Piaget in a form most teachers will find readable.

Duckworth, Eleanor. "Piaget Takes a Teacher's Look" in *Learning* (October 1973).

Interview by Eleanor Duckworth, Atlantic Institute of Education, Halifax, Nova Scotia, on special assignment for *Learning.*

Kidd, Kenneth P., Shirley S. Myers, and David M. Cilley. *The Laboratory Approach to Mathematics.* Chicago: Science Research Associates, Inc., 1970.

This book gives many suggestions for lab activities and classroom organization to facilitate the laboratory approach to mathematics instruction.

Laycock, Mary and Gene Watson. The *Fabric of Mathematics: A Resource Book for Teachers* (Revised Edition). Hayward, CA: Activity Resources Company, Inc., 1975.

This resource book contains many suggested classroom activities for teachers interested in using concrete materials to introduce and reinforce elementary mathematics concepts.

McFadden, Scott, Keith Anderson, and Oscar Schaaf. *Math Lab—Junior High.* Eugene, OR: Action Math Associates, Inc., 1975.

This book consists of pages which can be reproduced to make activity cards presenting nine topic strands, including perimeter and area, volume, calculators, and applications. The tasks presented are problem solving in nature and many require hands-on investigations.

McFadden, Scott, Judy Johnson, Mary Ann Todd, and Oscar Schaaf. *Math Lab—metric.* Eugene, OR: Action Math Associates, Inc., 1976.

This series of three books (separate editions for grades 1-2, 3-4, 5-6) contains numerous activity cards for use in elementary math classrooms. Fifteen math topics are covered at each grade level, and the tasks presented are activity based.

Mathematics Resource Project. *Geometry and Visualization.* Eugene, OR: Department of Mathematics, University of Oregon, 1976. (Available from Creative Publications, P.O. Box 10328, Palo Alto, CA 94303.)

This lengthy collection of classroom materials for middle school students contains units on Lines, Planes and Angles; Polygons and Polyhedra; Curves and Curved Surfaces; Similar Figures; and Area and Volume. Teaching emphases of the materials include visual perception, graphic representation, calculators, applications, estimation and approximation, and laboratory approaches.

Mathematics Resource Project. *Mathematics in Science and Society.* Eugene, OR: Department of Mathematics, University of Oregon, 1976. (Available from Creative Publications, P.O. Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students is organized into six units: Mathematics and Astronomy, Mathematics and Biology, Mathematics and Environment, Mathematics and Music, Mathematics and Physics, and Mathematics and Sports. Problem solving, teaching for transfer, and laboratory approaches are stressed.

Mathematics Resource Project. *Number Sense and Arithmetic Skills.* Eugene, OR: Department of Mathematics, University of Oregon, 1975. (Available from Creative Publications, P.O. Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students contains resource ideas on whole number operations, numeration systems, fractions, and decimals. Teaching emphases of the materials include calculators, applications, problem solving, mental arithmetic, algorithms, estimation and approximation, and laboratory approaches.

Mathematics Resource Project. *Ratio, Proportion and Scaling.* Eugene, OR: Department of Mathematics, University of Oregon, 1975. (Available from Creative Publications, P.O. Box 10328, Palo Alto, CA 94303.)

This collection of classroom materials for middle school students contains resource ideas on the topics, ratio, proportion, scaling, and percent. Teaching emphases of the materials include calculators, applications, problem solving, mental arithmetic, estimation and approximation, and laboratory approaches.

Moursund, David. "Calculators in the Elementary School" in *Math Learning Center Report* (February 1977).

The *Math Learning Center Report* is published monthly during the school year and relates current mathematics education happenings in the Pacific Northwest. The MLC reports are available from the Math Learning Center, 325 13th Street NE, Room 302, Salem 97301.

National Council of Teachers of Mathematics, Inc. *The Arithmetic Teacher.* Reston, Virginia: National Council of Teachers of Mathematics, Inc.

This magazine is one of two official journals of the National Council of Teachers of Mathematics. It is published nine times yearly (effective September 1977) and is distributed to NCTM members. It contains a wealth of ideas that can be put to use in the elementary and middle school mathematics classroom.

National Council of Teachers of Mathematics, Inc. *The Mathematics Teacher*. Reston, VA: National Council of Teachers of Mathematics, Inc.

This magazine is one of two official journals of the National Council of Teachers of Mathematics. It is published eight times yearly and distributed to NCTM members. It contains information on recent happenings in secondary mathematics education and includes a special activities section each issue.

Oregon Council of Teachers of Mathematics. *The Oregon Mathematics Teacher*. Salem, OR: Oregon Council of Teachers of Mathematics.

This magazine is the official journal of the Oregon Council of Teachers of Mathematics. It contains many classroom tested ideas for use in both elementary and secondary schools. The journal is available through membership in OCTM; for information, contact Specialist, Math Education, Oregon Department of Education.

Oregon Department of Education. *Education for Oregon Learners. Where We Stand.* Salem, OR: Oregon Department of Education, 1976.

This publication summarizes the results of the 1976 Oregon statewide assessment of the fourth grader's math skills.

Oregon Department of Education. *MEASUREMENT . . . with Metric.* Salem, OR: Department of Education, 1977.

Oregon Department of Education. *Planning the Education of Oregon Learners: Completing the Management Cycle.* Salem, OR: Department of Education, 1975.

This publication discusses the function of statewide assessment and its relationship to identifying needs. It traces how identified priorities affect state program operation and how results are evaluated and reported.

Oregon Department of Education. *Planning the Education of Oregon Learners: Setting Goals.* Salem, OR: Department of Education, 1975.

This publication presents the statewide goals for education adopted by the State Board of Education in January 1974, and discusses the rationale for setting these "life role" goals, the method of involving representatives of Oregon communities in the setting of these goals, and the relationship of the statewide goals to the goals of local school districts.

Polya, G. *How to Solve It.* Princeton, NJ: Princeton University Press, 1973.

This book contains a comprehensive discussion of the nature of problem solving and suggests many specific problem-solving strategies.

Purkey, William W., Sr. *Self Concept and School Achievement.* Englewood Cliffs, NJ: Prentice-Hall, Inc., 1970.

"Must" reading for teachers interested in what can be done to improve student self concept.

Rising, Gerald and Richard Wiesen. *Mathematics in the Secondary School Classroom* New York: T. Y. Crowell Company, 1972.

This publication contains many in-class topics for mathematics, as well as discussions of classroom techniques. It is divided into pre-algebra, algebra, geometry, and twelfth-grade levels for ease of use.

Schaaf, Oscar. *Eugene Mathematics Program Grades 1-12.* Eugene, OR: Eugene Public Schools, 1973.

Sobel, Max A. and Evan M. Maletsky. *Teaching Mathematics: A Sourcebook of Aids, Activities, & Strategies.* Englewood Cliffs, NJ: Prentice-Hall, Inc., 1975.

This book presents a collection of experiments and activities for secondary mathematics classrooms. Also included is a bibliography of resources and references.

Wirtz, Robert W. *Drill and Practice at the Problem-Solving Level: An Alternative.* Washington, DC: Curriculum Development Associates, Inc., 1974.

This resource is a collection of activity pages for elementary children which stress the development of problem-solving strategies. The accompanying teacher's guide contains an explanation of the Wirtz matrix and the philosophy underlying the program.

Wirtz, Robert W. *Mathematics for Everyone.* Washington, DC: Curriculum Development Associates, Inc., 1974.

The book is one source of more information on the Wirtz matrix.

QUESTIONNAIRE

Your views are important! After you read and examine this publication, please forward your comments to Specialist, Math Education, Oregon Department of Education. This response form is provided for your convenience. Your views will be considered as we plan future publications, revise this guide, and plan in-service workshops.

1. What type of work do you do?

   _____ Classroom teacher _________ Consultant to classroom teachers

   _____ School administrator _________ College teacher of teachers

   _____ Other ________________________________

2. Does this publication fulfill its purpose as stated in the Foreword?

   _____ Completely _________ Partly

3. Is the writing clear and direct?

   _____ Clear (communicates adequately)?

   _____ Not clear? Where? ________________________________

   _____ Too much information? Where? ________________________________

   _____ Too little information? Where? ________________________________

4. Did the graphics complement what you read?

   _____ Helpful? How? ________________________________

   _____ Not helpful? Why not? ________________________________

5. Each of the topics on the left specifically presented a set of ideas. Check only the column(s) you feel are *NOT* sufficiently sequential, clear, necessary or specific.

	Not Sequential	Not Clear	Not Necessary	Not Specific
Learning & Teaching	_____	_____	_____	_____
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Teaching Strategies	_____	_____	_____	_____
Math Content Strands	_____	_____	_____	_____
Calculators/Computers	_____	_____	_____	_____
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If you have checked a box(es), please comment on how improvements could be made.

____

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6. As a result of reading this publication, have you made (or do you plan to make) changes in any of the following? Check all that apply.

   _____ Instructional planning _____ Learning activities _____ Teaching strategies

7. How clearly does the topic "Staff Development" identify needs and responsibilities for planning and providing teacher in-service?

   for teachers:

   _________ Clear?

   _________ Not clear? What is needed? ________________________________

for administrators:

_____ Clear

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