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ABSTRACT

Eighteen research reports related to mathematics education are abstracted and analyzed in this publication. Three of the reports deal with aspects of learning theory, seven with topics in mathematics instruction (problem solving, weight, quadratic inequalities, probability and statistics, area and volume conservation, cardinality), five with instructional systems, two with teacher education, and one with teaching style. Research related to mathematics education which was reported in "Research in Education" and "Current Index to Journals in Education" between April and June 1977 is listed. (MN)

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THE EFFECTS OF SMALL-GROUP DISCOVERY LEARNING ON STUDENT ACHIEVEMENT AND ATTITUDES IN CALCULUS. Brechting, Sister Mary Catherine; Hirsch, Christian R. MATYC Journal, v11 n2, pp77-82, Spring 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Douglas T. Owens, The University of British Columbia.

1. Purpose

The purpose of this study was to "...investigate the relative efficacy of a small group-discovery method and the conventional lecture-discussion method in promoting concept attainment, skill acquisition, and favorable attitudes toward mathematics in an introductory calculus course" (p. 77).

2. Rationale

The study was based upon three assumptions: (1) students should be active participants in the process of learning mathematics; (2) the student should discover for himself or herself as great a part of the mathematics as feasible; (3) learning is a social activity in which the learner is a partner in a shared activity. The study was undertaken to provide further empirical data regarding the viability of a small-group discovery approach to calculus instruction which was formulated and tested by Davidson (1971).

3. Research Design and Procedure

Subjects for the study were 46 students enrolled in two afternoon sections of an introductory calculus course during winter semester at Western Michigan University. One section was chosen to use the experimental materials (N = 21) and the other was designated the control group (N = 25). For instruction, the experimental section was divided into five groups of four or five with the flexibility of reorganizing the groups between units of instruction. With guidance from the instructor, the students formulated definitions, constructed examples and counter-examples, discovered techniques for solving classes of problems, and developed the usual skills of a traditional calculus course. The same content was covered by the experimental and control groups. The control group was apparently taught by a different instructor whose methods were representative of a conventional lecture-discussion approach.

Subjects were given a pretest of 35 multiple-choice items to measure pre-calculus competence in algebra and analytic geometry. The reliability estimate (KR-20) was 0.86. The post-test of achievement consisted of 36 multiple-choice items, in which items measuring understanding of concepts alternated with items testing manipulative skills. The KR-20 reliability estimate for the total test was 0.80, but reliabilities for the subtests were not reported.

The Aiken-Dreger Revised Mathematics Attitude Scale (Aiken, 1963) was administered twice as pre- and post-measures of attitude toward mathematics. In addition, the subjects in the experimental group were asked to complete an open-ended questionnaire regarding their learning environment.

One-way analyses of covariance with pre-calculus test scores as the covariate were performed on the data from the skills subtest, the concepts subtest, and total achievement. Similarly, analysis of covariance was used to analyze the attitude data with the pre-treatment attitude scores as the covariate. The investigators were careful to check for homogeneity of regression in each analysis.

4. Findings

Significant differences were found in favor of the treatment groups for total achievement ($p < .02$) and skill acquisition ($p < .01$). No significant difference was found for concept attainment, but the observed differences favored the experimental group. A post hoc analysis showed a significant interaction ($p < .02$) between pre-calculus competence and treatment on only those (apparently seven) items dealing with differentiation concepts.

There was no significant difference between treatment groups with regard to attitudes toward mathematics, but the observed difference favored the experimental group. Results of the open-ended questionnaire were generally favorable. Twenty-six positive comments were summarized, with an indication that there were many more, and only two negative comments were given by two students.

5. Interpretations

a. Generally, use of a small-group discovery learning method can be an effective means of improving student achievement, especially skill acquisition, in an introductory calculus course.

b. It was suggested that the difficulty of the items on the concept test may have resulted in random errors of measurement and reduced the possibility of finding a significant difference. The significant interaction on the post hoc analysis was interpreted to mean that students who scored above 14 on the pre-calculus test (the point at which the lines intersected) had a better understanding of differentiation if they were in the small-group discovery settings and students who scored 14 or less did better in the conventional lecture situation.

c. Most students had favorable attitudes toward the small-group discovery method. Improvement of attitudes toward mathematics were only slight. Attitudes toward mathematics in the control group declined.

d. It appears that small-group discovery learning is a viable alternative to conventional instruction in introductory calculus.

e. It seems desirable to investigate this method in precalculus courses.

Critical Commentary

The investigators are to be commended for doing a useful piece of research. Ascertaining alternative methods of calculus instruction is a worthwhile area for investigation.

The report implied, but never explicitly stated, that different instructors were used for small-group discovery and traditional lecture-discussion treatments. Thus, apparently, there was no control over the confounding of a possible "instructor effect" and the treatment.

A reliability estimate is reported for the total posttest. Wouldn't it be desirable also to know the reliabilities of the two subtests of concepts and skills since interpretations of these results are to be made? What was the reliability of the subtest on which the interaction effect was found?

In the investigators' discussion section, it was mentioned that an item analysis had been performed on the concepts and skills subtests. There were other instances of results being reported for the first time in the discussion section. Would it be preferable to include these as results of the study before they are used as a basis for interpretation?

The investigators had a tendency to report "observed" differences in group means and interpret "slight" differences when differences were not statistically significant. It seems preferable to more readily accept a null hypothesis of no difference in these instances.

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Davidson, N. The Small Group Discovery Method as Applied in Calculus Instruction. American Mathematical Monthly, v78, pp789-791, 1971.

PROBABILITY AND STATISTICS IN HIGH SCHOOL? Callihan, Hubert D.; Bell, Frederick H. School Science and Mathematics, v77 n5, pp418-426, May-June 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Robert E. Reys, University of Missouri.

1. Purpose

The purpose of this study was to gather information about probability and statistics offerings in Pennsylvania secondary schools. The study sought to answer specific questions such as:

(a) Do teachers think probability and statistics courses belong in the secondary mathematics curriculum?

(b) Do secondary mathematics teachers think they are adequately prepared to offer probability and statistics courses?

(c) What variables affect the probability and statistics offerings in secondary schools?

2. Rationale

Probability and statistics are very much a part of our daily lives. It is essential that students develop rudimentary concepts of probability and statistics. Such recommendations have been made by various groups: Commission on Mathematics, 1959; Cambridge Conference, 1963; and the Committee on Undergraduate Programs in Mathematics, 1971. It is important that the current impact of these recommendations on mathematics programs be assessed. Such information will provide valuable baseline data for determining where we are and provide evidence to help promote future curriculum change.

3. Research Design and Procedure

A data form and an opinnionaire designed by the researchers to determine various information related to probability and statistics were sent to 200 Pennsylvania secondary schools randomly selected from those listed in the College Blue Book.

4. Findings

One hundred fifty-eight (79%) of the data forms and opinionnaires were returned. Here are some highlights of the findings:

15% of the schools offered a complete semester course in probability and statistics.

74% felt that a probability course should be included in the secondary curriculum.

50% felt that a course in statistical inference should be included in the secondary curriculum.

90% of the schools reported they had at least one faculty member qualified to teach a course in probability and statistics.

33% of the schools not currently offering a course in probability and statistics reported they were planning to offer such a course.

17% felt that teacher preparation programs were adequately preparing mathematics teachers with regard to probability and statistics; 49% felt they were not adequately preparing teachers in this regard.

The most frequently cited reasons for not offering a course in probability and/or statistics were lack of time in the schedule (33%) and not enough mathematics teachers available (15%).

In addition to reporting these descriptive statistics, the authors reported results from several chi square tests. Here are some of the findings:

The amount of probability and statistics offered is not significantly related ($p < .40$) related to school size.

Availability of qualified instructors in probability and statistics is significantly ($p < .05$) dependent upon the school size.

Future plans of schools having no courses in probability and/or statistics are not significantly dependent ($p < .54$) upon teachers' opinions about statistics belonging in the secondary curriculum.

5. Interpretations

Most secondary mathematics teachers in Pennsylvania feel that probability and statistics belong in the secondary curriculum. More than half the schools are planning courses in probability and/or statistics in the near future. In light of this movement, it is sad that few teachers feel that teacher education programs are preparing prospective teachers to handle courses in probability and/or statistics.

Critical Commentary

This type of status study is needed across the country. It provides a glimpse of the current status of probability and statistics in the ever-changing secondary mathematics program.

In reading this research report, the following questions and concerns came to mind:

(1) Was there an inherent bias in the sample? A simple random sampling was used. The College Blue Book provided the population list of schools. This listing of schools raises a question about the representativeness of the sample. It seems likely that the sample will be weighted toward smaller schools which, although greater in number, actually enroll a small proportion of students. On the other hand, the number of large schools is much smaller, yet they collectively enroll a higher percentage of the total students in school. No mention of this bias or safeguards against it were mentioned.

(2) Operational definitions of key terms such as "qualified instructors," "portion of course devoted to probability and statistics," and "substantial part of course devoted to probability and statistics," are needed. Without a clear understanding of just what these terms mean--by researchers, participants, and readers--any related conclusions are fraught with danger. For that very reason this reviewer avoided any discussion of findings related to schools' curricula as "modern," "middle of the road," or "traditional." These are loaded with many different connotations and too many uncertainties to warrant discussion.

(3) Some additional relationships might have been explored using the same data. For instance, was there a significant relationship between schools with at least one qualified instructor and those schools which offered a complete course in probability and/or statistics? Is there a significant relationship between those who say a course in probability belongs in the secondary curriculum and whether or not their school is offering such a course?

Should similar studies be made, it would be interesting to explore several additional areas. In curriculum, for example, what is the nature of probability and/or statistics courses? Are there universally common topics? How much time is devoted to them? In teacher preparation, it would be valuable to know just how these teachers decided whether or not they had a faculty member qualified to teach a course in probability and/or statistics. What type of background did they consider necessary? How many courses (and what type) would be necessary to prepare qualified instructors? How do these suggestions compare with current mathematics education programs for preparing prospective secondary teachers?

COMPARATIVE EFFECTS OF THREE SEQUENCES OF MOVES FOR TEACHING SELECTED MATHEMATICAL CONCEPTS TO COLLEGE STUDENTS. Catanzano, R.; Godwin, W. Journal for Research in Mathematics Education, v7, pp276-289, 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by George W. Bright, Northern Illinois University.

1. Purpose

The primary purpose of the study was to test the effectiveness of three sequences of teaching moves in teaching three unrelated mathematics concepts to prospective elementary-school teachers. A secondary purpose was to gather information on the length of time required to teach each sequence, the preference of students for the sequences, student recognition of differences in the sequences, and the difficulty of using the sequence.

2. Rationale

The study, derived from Henderson's model for the teaching of mathematics, extended two earlier experiments by altering the instructional mode and by measuring multiple levels of abstraction and the sense of Dienes.

3. Research Design and Procedure

The concepts of (a) partitioning of a set, (b) function, and (c) mod 5 addition were taught by three sequences of moves:

CE: characterization-exemplification

ECE: exemplification-characterization-exemplification

ECE_I Exemplification-characterization-exemplification (interrogative).

The CE sequence contained only declarative statements. The ECE sequence contained some declarative and some interrogative statements, although the characterization moves were interrogative. The ECE_I sequence contained only interrogative statements with one metalanguage declarative.

Thirty prospective elementary-school teachers were blocked in groups of three individuals by achievement, and the subjects in each block were randomly assigned to three groups. Group A was taught concepts 1 (CE), 3 (CE), and 2 (ECE); group B, concepts 2 (ECE), 1 (ECE) and 3 (ECE); and group C, concepts 3 (ECE_I), 2 (CE), and 1 (ECE_I). One college instructor and two prospective secondary teachers taught the concepts. Each instructor taught a single concept but used all three sequences of moves. The sequences of moves but not the answers to problems or to interrogative statements were printed in instructional booklets. Instructors read all moves aloud, while students read them silently. Instructors checked student responses for correctness.

The duration of the experiment was seven days, three for the treatments followed by a four-day delay before the retention test. For each concept an immediate acquisition test was given composed of three subtests. Subtest 1 contained problems taken directly from the treatments. Subtest 2 contained "problems with replacement sets of numbers different from those presented during instruction" (p. 282). These problems represented the primitive generalization level in the sense of Dienes. Subtest 3 contained problems requiring vertical transfer, called the process of abstraction by Dienes. A 15-item (5 per concept) short-term retention test was given four days after the end of the treatments. A nine-item questionnaire was given immediately prior to the short-term retention test. A randomized block design ANOVA was used to analyze the data.

4. Findings

Two significant F ratios were found for the scores related to the partitioning concept; for subtest 1, CE > ECE, and for the retention test, ECE > CE. For the mod 5 addition, $1/2(CE + ECE) > ECE_I$ on subtest 2.

Nine of ten subjects in group B, which was taught solely by ECE, reported that they had received the same kind of instruction for each concept. Twelve of twenty subjects in groups A and C reported that they had received different types of instruction.

5. Interpretations

The data, with a few very specific exceptions, support the conclusion that the three sequences of moves were equally effective. Students seemed to be able to recognize the same or different modes of instruction. Trends in the data suggested that the CE sequence was most effective for immediate acquisition, while the ECE sequence supported short-term retention best.

Critical Commentary

(1) The study seems to be carefully constructed, well-executed, and important as part of a long-range study of the effects of sequences of teaching moves. The report of the study, however, is incomplete and contains some information which appears contradictory. Several questions also need to be raised concerning the experimental design and interpretations.

(2) The relationship of this study to the previous research is clearly established. The experimenters are to be congratulated both for building on earlier research and for assuming the reader's familiarity with the terminology of and information contained in the earlier works. However, because of its age and the source of publication, the Dienes reference on generalization and abstraction should have been summarized more completely.

(3) The criteria for selecting the concepts to be taught may not have been satisfied completely. Certainly mod 5 arithmetic could be considered a function from $\{0,1,2,3,4\} \times \{0,1,2,3,4\}$ to $\{0,1,2,3,4\}$.

The reader is never told whether allusions were made to function concepts during the instruction on mod 5 arithmetic.

(4) There was considerable discrepancy in the numbers of moves used for each sequence. For example, in teaching partitioning the CE sequence consisted of 5 moves; ECE, 14 moves; and ECE_I, 10 moves. Explanation of this discrepancy was never offered. Too, the criteria for selection of specific moves were not presented. If 5 moves are sufficient for CE, then 14 moves for ECE may constitute overteaching.

(5) Students answered the interrogative statements (presumably by writing answers in the booklets) and instructors checked responses [illegible] correctness. How were incorrect responses handled? Were answers [illegible] stated or were more questions asked? What percentage of students' [illegible] responses were wrong?

(6) The explanation of the experimental design is the most confusing part of the report. According to the text, "the three different groups A, B, and C, had been taught three different ways (the treatments)" (p. 281). This reviewer interprets that to mean that each group received all three treatments. According to Table 1, however, group B received only ECE and groups A and C received two treatments each. The information in the table seems to be more consistent with later discussion, so the textual description was ignored. The assignment of ECE to a single group seems to be a serious confounding factor that was never recognized.

(7) The claim (p. 282) that ECE was superior on subtest 2 is not borne out by the data of Table 2. The total subtest 2 score for the CE sequence was higher than the total for the ECE sequence. Also, no explanation was given for the different distributions of test items across subtests in the immediate acquisition tests for the three concepts.

(8) It seems odd that of 14 sets of test scores (subtests, composite tests, and retention tests), only 2 showed any effects of the blocks used to construct the experimental groups. (F-statistics were not reported for the blocks, so they were computed by the reviewer.) Since the blocks were identified on the basis of previous achievement, then the lack of effect of blocks suggests that the mathematics concepts used may not be representative of mathematics taught to these subjects. Such a conclusion, if accurate, would call into question all implications for classroom instruction.

(9) The authors seem to confuse observations and conclusions. For example, it was observed that the presentation time of the CE sequence "ranged from 6 to 8 minutes" (p. 285). One of the conclusions, however, was that "the CE sequence was found to take from 6 to 8 minutes to execute" (p. 287). Similar repetitiveness appeared in the reporting of statistical <u>results</u> as conclusions. Consequently, the synthesis of the results is very weak.

(10) The apparently greater effectiveness for the ECE sequences on the retention test may be explained by the fact that ECE was given in the same group three times. Consistency of sequence may promote retention independent of the particular sequence used.

(11) The preference of subjects in groups A and C for sequences CE and ECE_I, respectively, may be explained by the fact that these sequences were the first sequences that these subjects were exposed to.

(12) The discussion of the need for including specific kinds and orders of moves in the ECE sequence is not based on any clearly identified data from the experiment. The basis for this discussion needs to be more clearly identified.

(13) Most of the criticisms outlined above result from omissions in the report of the experiment. Researchers must realize that their work is not done until the report of the experiment is written. As much care, attention to detail, and technique are required for writing the report as for conducting the experiment. The lack of attention to detail in the writing of this article seriously detracts from the overall impact of the research.

SMALL GROUP INSTRUCTION: A STUDY IN REMEDIAL MATHEMATICS. Chang, Ping-Tung. The MATYC Journal, v11 n2, pp72-76, Spring 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward Uprichard, University of South Florida.

1. Purpose

The purpose of this study was to investigate the differential effects of two instructional processes (small-group method and lecture-demonstration method) in order to develop an improved instructional approach for teaching remedial mathematics at the community college level.

To fulfill the primary purpose of this study, it was necessary to address the following subproblems:

(a) To determine whether the instructional processes (small group method and lecture-demonstration method) had a positive impact on achievement.

(b) To determine whether the instructional processes (small-group method and lecture-demonstration method) had a positive impact on the attitudes toward mathematics.

2. Rationale

Considerable interest is being shown among various college mathematics faculties in developing effective methods for teaching remedial mathematics. Methods presently employed vary from such strategies as the lecture-demonstration method to self-paced instruction. Currently, some research is emerging which supports a category of methods customarily referred to as "small group" instruction.

Olsen, Phillips, Olmstead, and Thoyre have studied the effectiveness of various forms of the small-group approach, and the results support the contention that the small-group method is an appropriate strategy for teaching mathematics.

3. Research Design and Procedure

In order to accomplish the purpose of the study, the author used two classes of students enrolled in Math 99 at Gordon Junior College. The methods of instruction were randomly assigned so that one class (experimental group) was taught by the "small group" method, and the other (control group) was taught by the "lecture-demonstration" method. Each class contained 14 students.

Math 99 is a remedial course in basic arithmetic and elementary algebra for students who are not sufficiently prepared to begin a college-level mathematics course. Students are assigned to Math 99 using the following criteria:

(1) A combined SAT score of 650 or less, and

(2) A score below 43 on Test D of the Comparative Guidance and Placement Test (CGP):

The pattern for conducting the class using the small-group method (student-centered) was:

(1) Following the administration of Pretest I, the class was divided into small groups of three or four students each, with the students receiving higher scores on the pretest being assigned evenly among the groups.

(2) The instructor lectured the first five to ten minutes of each class period in order to review and introduce new subject matter, and the next thirty to forty minutes were allotted for small group activities.

(3) The last five to ten minutes of the period were used for class discussion.

(4) The primary function of the instructor was to serve as a consultant and facilitator to stimulate discussion within and among the small groups.

The pattern for conducting the class using the lecture-demonstration method (teacher centered) was as follows:

(1) The instructor lectured at least thirty to thirty-five minutes of each class period in order to review and introduce new materials and skills.

(2) The last fifteen or twenty minutes of the period were used for class discussion.

(3) The primary function of the instructor was to present and explain new subject matter and skills to the class, and to help individuals in class during the class discussion.

The following instruments were used:

(1) Pretest and Posttest I (arithmetic achievement at beginning and end of course respectively).

(2) Pretest and Posttest II (elementary algebra achievement at the beginning of 3rd week and end of course respectively).

(3) Dutton's Attitude Toward Mathematics test (attitude at beginning and end of course respectively).

Pretest I and Posttest I are two equivalent forms in arithmetic, and Pretest II and Posttest II are two equivalent forms in elementary algebra. These forms were constructed by the author and their equivalency was established using the Pearson Product Moment Coefficient with a pilot group of students during fall quarter of 1974. A one-way analysis of variance was used to test all hypotheses at the .05 level of significance.

4. Findings

The major findings of the study are summarized below.

(1) There were no significant differences in the Pretest scores between treatment groups in any of the three areas.

(2) The Posttest scores in Arithmetic and Elementary Algebra were significantly affected by the small-group method as compared to the lecture-demonstration method.

(3) No significant difference was found between the two treatment groups on the Attitude Posttest.

(4) There was no significant gain between Pretest and Posttest on elementary algebra within the control group.

(5) There were significant differences between Pretests and Posttests on arithmetic and elementary algebra within the experimental group, and arithmetic achievement within the control group.

(6) No significant difference was found in mathematics attitude within the treatment group.

5. Interpretations

The findings of the study support the premise that the small-group method of instruction is an appropriate strategy for teaching remedial mathematics at the college level, and that students taught by the small-group method demonstrate significantly greater achievement in arithmetic and elementary algebra than the students taught by the lecture-demonstration method employed in this study.

Critical Commentary

The author states that his study was limited (N = 28), but that the results should give encouragement for future investigations on a larger scale of the effectiveness of this instructional technique (small group) for remedial mathematics in colleges. However, the author fails to articulate the specific contributions this study makes to the literature above and beyond the work of Olsen and Phillips, who studied the same problem with respect to remedial mathematics instruction at the college level. For example:

(1) What hypotheses could be advanced as a result of this particular study that were not generated from previous research?

(2) What specific directions should future research on this topic take as a result of the present study?

(3) Was the present study a replication of the work of Olsen and/or Phillips?

Chang's study, although exploratory in nature, has some experimental design problems. The author states that he used a one-way analysis of variance to test all hypotheses at the .05 level of significance. I guess one could use this particular analysis to test for differences between treatment groups on the pretests (arithmetic, algebra, and attitude) and then again on the posttests. However, a simple one-way analysis of variance is not appropriate in testing for pretest-posttest differences <u>within</u> treatment groups on the dependent measures. Pretest scores and posttest scores for the same treatment group are not independent. The author's design would have been more efficient had he used an analysis of covariance with pretest scores acting as a covariate for posttest scores. He could have used this type of analysis three times (arithmetic, algebra, and attitude) or opted for a more sophisticated design (multivariate).

A QUESTION OF TEACHER COMPETENCY. Crable, Elain; Dinnan, James A. Southern Journal of Educational Research, v10 n1, pp43-49, Winter 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Donald J. Dessart, The University of Tennessee, Knoxville.

1. Purpose

The authors stated that the purpose of the study was "to discover if teachers were prepared with some necessary fundamentals for the teaching of reading and mathematics, and if they can demonstrate the ability to learn these fundamentals."

2. Rationale

It is claimed in this study that "methods instructors of future teachers are spending time on the frills of teaching by demonstrating innovative approaches and assuming the basic knowledge of the subject area to be innate." The authors felt that the "basic concepts" related to reading and mathematics are being neglected in teacher training.

3. Research Design and Procedures

During the first day of instruction in a reading class composed of 125 advanced undergraduate or graduate students of whom 90 percent consisted of teachers or students preparing for teaching, the following instructions were given without additional comment:

(1) Print the alphabet.

(1a) However you printed it, do it another way.

(2) Write the first ten numbers.

(2a) However you wrote them write them another way.

(3) Identify the difference between a vowel and a consonant.

(4) Write the first word you think of when I say these words: in, over, up, on, father, one, old, high, open, straight.

4. Findings

It was found that:

(1) No one printed the alphabet as it appears in "school" or "book" type (clarendon typeface). Letters such as "a", "g", and "q" were misprinted in over 80 percent of the cases.

(2) Ninety-eight percent missed the zero in its prime position: 0 1 2 3 4 5 6 7 8 9.

(3) Eighty-seven percent were not able to give an acceptable answer for the difference between a vowel and a consonant.

(4) Ninety-four percent supplied paradigmatic replies to the ten stimulus words.

5. Interpretations

The following interpretations regarding the mathematical portion of the study were offered by the authors:

(1) "This study strong [sic] implies that failure in reading mathematics stem [sic] from a weakness in basic instruction administered by unsure teachers."

(2) There is confusion among teachers and students as to whether or not zero is the first of the first ten numbers.

(3) "It seems mathematics' [sic] authors forget that the whole concept of mathematics is built upon the prime numbers, 0-9."

(4) "All fractions and decimals fall between 0 and 1."

(5) "To go further all Algebra and Geometry is solving for the zero."

(6) "When teachers are asked to identify the basic foundations of written language and mathematics, it becomes apparent that they fail to recognize simple components."

Critical Commentary

The purpose of this study as stated by the authors (see "Purpose" above) is certainly interesting, but it was never seriously addressed in the report. Furthermore, on the basis of student replies to the single instruction to write the first ten numbers (which is impossible to do without further definition), the authors reached a number of unwarranted conclusions (see "Interpretations" above). These conclusions not only had little, if any, relationships to the data of the study but in themselvers are nonsensical.

With a minimal review of this report, one can easily conclude that it should never have reached the publication stage. It is most regrettable that a more critical review had not been exercised by those responsible for its publication.

THE USE OF ADVANCE ORGANIZERS FOR FACILITATING LEARNING AND TRANSFER FROM QUADRATIC INEQUALITIES. Eastman, P. M. School Science and Mathematics, v77 n5, pp377-384, May-June 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Joe Dan Austin, Emory University.

1. Purpose

"The purpose of this study was to determine the effects of advance organizers on the transfer of learning from material on quadratic inequalities" (p. 377).

2. Rationale

Ausubel has argued that advance organizers can facilitate the learning and retention of unfamiliar but meaningful verbal material. The justification for this position is that the advance organizer--presented prior to the learning task and at a higher level of abstraction--can provide concepts that relate the unfamiliar new material to material already familiar to the individual. Eastman argues that this reasoning suggests that advance organizers may also facilitate the transfer of learning. He argues for this transfer because "the cognitive structure of the individual receiving the advance organizer would then be better organized than a person not receiving the advance organizer" (p. 377). Eastman found no studies that considered this hypothesis.

3. Research Design and Procedure

The author used a design appropriate for a two-way analysis of variance. One factor concerned the presence or absence of the advance organizer. The two levels were called advance organizer and introductory overview. The second factor had two levels and related to the method of presenting the material. The two levels were called analytical and graphical. Eighty-seven tenth-grade geometry students were randomly assigned to the four groups. Students in each group worked through linearly programmed booklets on solving quadratic inequalities. The booklets were 24 to 30 pages long. The two introductory overview groups had a one-page table of contents as the first page. The booklets were studied for two class periods. During the third class period a 24-item multiple-choice transfer test was administered.

4. Findings

A two-way analysis of variance was computed on the 80 complete test scores. ["The data from seven Ss were incomplete and were therefore eliminated" (p. 379).] Using an α-level of 0.05, no significant differences were found.

5. Interpretations

The author argues that even though no significant differences were found, this "does not imply that advance organizers were ineffective" (p. 380). This is justified by noting that the pooled averages favored the advance organizer groups over the introductory overview groups. This view is reiterated in the summary as "neither do the results suggest that advance organizers, in general, are ineffective in producing transfer, rather that the results are inconclusive" (p. 381). Here the author also concludes by stating that the hypothesis that advance organizers enhance "the transfer of learning gains no support from this study" (p. 381).

Two reasons for the lack of significant differences are postulated. These were (1) the short length of the advance organizers compared to the length of the treatments, and (2) the advance organizers may not have been constructed appropriately to relate the material to cognitive structures of the individuals.

Critical Commentary

The author presents a nice introduction to advance organizers. He also provides a good discussion of the differences between advance organizers and introductory overviews. The argument that advance organizers should help facilitate the transfer of learning has a quite logical appeal. However, the author should have reviewed--even if briefly--the literature on transfer of learning to better support his argument.

Several basic questions must be asked before the results can be evaluated. Specifically, the following questions arise:

(1) Why was the topic of quadratic inequalities used? What relevant background did the students have that related to this topic? How do we know the topic was "unfamiliar"?

(2) What type of students were involved? What was the male-female distribution, the ethnic make-up, and the general ability level?

(3) What is the rationale for the analytical vs. graphical factor? This factor first appears in the null hypotheses and is not discussed in the literature review or the introduction.

(4) What is the reliability of the 24-item transfer test?

(5) In what way were the data incomplete for the seven deleted subjects? All seven appear to have been from the introductory overview groups.

(6) Did the teacher or researcher interact with the subjects in any way?

(7) Did the analytical groups have the same programmed booklets aside from the first page? The same question applies to the graphical groups.

The previous questions make it somewhat difficult to evaluate the results. If these questions can be satisfactorily resolved, then it is difficult to argue with the author's final conclusion that the hypothesis that advance organizers enhance the transfer of learning gains no support from this study. While one study is unlikely to completely resolve a problem, the author's claim that the results do not imply that advance organizers were ineffective is at best naive. It seems strange to cite the pooled averages as evidence for this when the two averages were only 10.11 and 11.35 with a standard deviation of about 4.

A minor inconsistency appears in the discussion section. Here the author claims the results fail to confirm the hypotheses. Since the hypotheses were stated in null form, they were confirmed, i.e., not rejected.

The two reasons postulated for not rejecting the null hypotheses seem plausible. They cannot, of course, be supported by the analysis. (An obvious third possiblity that the author seems reluctant to consider is that advance organizers have no effect on the transfer of learning.) In fact, the second reason--the advance organizer may not have been appropriately constructed--points out a major problem with research on advance organizers; namely, how does one define an advance organizer and construct one for a particular topic? In this study the advance organizers are reproduced in the appendix. Unfortunately, so little information is given on the subjects or their backgrounds that it is hard to judge whether these advance organizers do relate the new material to the general background of the subjects.

In summary, the author has raised an interesting question on the relation of advance organizers to the transfer of learning. Unfortunately, so many questions are left unanswered that one is unsure how much this study contributes to a meaningful answer to this question.

THE EFFECTS OF TWO METHODS OF PRESENTING A PEDAGOGIC MODEL TO PRESERVICE TEACHERS. Ensey, Lowell; Cooney, Thomas J. Journal for Research in Mathematics Education, v8 n2, pp107-114, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by David F. Robitaille, University of British Columbia.

1. Purpose

The study sought answers to two questions:

(1) Would education students with no student-teaching experience benefit more from a study of Henderson's moves for teaching concepts than would a comparison group with student-teaching experience?

(2) Would the mode of presentation of such moves to the student teachers (videotape or written transcript) have a differential effect upon the performance of the two groups?

2. Rationale

Use of Henderson's model for teaching mathematical concepts is based upon the assumption that "teachers should possess knowledge of the teaching act in general and knowledge of teaching moves in particular." The present study was designed to address the questions of when and how such knowledge should be presented to student teachers.

3. Research Design and Procedure

Two groups of undergraduate student teachers participated in the study. The first group, G_1, had had no previous student-teaching experience, whereas those in group G_2 had completed their student teaching. The students were assigned to two treatment groups: students in T_1 viewed videotaped lessons illustrating instances of concept moves, while students in group T_2 read typed transcripts of such lessons. Before and after the treatments were administered, each student taught a 15- to 20-minute microlesson to three seventh grade students. These microlessons were audiotaped.

The audiotapes were analyzed for three types of information: The number of moves utilized which dealt specifically with the mathematical concept being taught, the total number of moves employed, and the number of different moves employed. Outside observers also analyzed 22 of the tapes, and an index of observer agreement was determined: 0.81 for the total number of moves employed, and 0.86 for the content-specific moves. Following the treatment period, all students were administered a 10-item test of their ability to classify moves.

Thirty-six students were included in the data analysis (G_1=20, G_2=16). "A 2x2x2 design with repeated measures on the third factor was used to detect differences in the interactions among the factors" of groups, treatments, and microlessons.

4. Findings

The mean score on the 10-item posttest which was administered to all students was 9.2. Separate scores for each group are not reported.

Analysis of variance results showed no significant interactions between groups and treatments for total number of moves employed, for content-specific moves, or for the number of different moves employed. In each case, a significant difference was found between the results obtained on the pre- and post-treatment microteaching sessions. However, "the design of the study does not allow one to determine conclusively that [this difference] can be accounted for solely by treatment effects."

5. Interpretations

The authors, in attempting to interpret the lack of significant differences, suggest either that the treatments were not sufficiently different or that the hypothesized interactions do not exist. They point out that the questions of when and how to introduce students to concept moves remain unanswered. They suggest that further research is needed to determine the effect of treatment differences in increasing the number and types of concept moves employed.

Critical Commentary

On the whole, the study was well conceived and well designed. The statistical techniques employed seem to be appropriate and the authors have been careful to interpret their results conservatively. The study was based upon the first author's master's thesis, and most of the paper's weaknesses would seem to be due to factors frequently associated with research of that kind. These would include, for example, small sample size and brevity of treatment period.

The treatments "consisted of five 1-hour sessions, one session per day." The first three days were identical for both T_1 and T_2. On the last two days, students in T_1 spent 15 minutes per day viewing a videotape while those in T_2 read transcripts of the same material. In other words, the two treatment groups were performing essentially the same activity 90% of the time. Even if the treatments had been more dissimilar, one week seems a very brief treatment period in which to hope to obtain significant behavioral changes of the kind the authors were interested in.

A larger sample size would have increased the power of the statistical tests employed. This might well have been impossible if all of the available student teachers were used in the study; however, we are not given this information. Similarly, we are not told what method was used to assign students to treatment groups.

The formula which was used to obtain a measure of inter-rater agreement seems to give artificially high results. The formula

$$\frac{2 \times (\text{number of moves agreed on})}{(\text{observer total}) + (\text{investigator total})}$$

gives an agreement index of 0.67 in the case where the observer sees 20 moves, the investigator sees 10, and they agree on 10. A more conservative agreement index, 0.50, is obtained if we use the formula attributed to McGrew (1972, p. 24):

$$\frac{\text{No. of agreements(A\&B)}}{\text{No. of agreements(A\&B)} + \text{No. seen by B only} + \text{No. seen by A only}}$$

Using their formula, the authors may have obtained artificially high reliability estimates which may, in turn, have affected the ANOVA results.

This study should be replicated, taking into account the considerations raised here, as well as those raised by the authors themselves in the paper. The research question initially posed by the authors are sufficiently important to warrant further study. Of course, the most important questions to be raised in this area concern the impact of such training upon teachers after they have graduated and are teaching their own classes.

Reference

McGrew, W. C. An Ethnological Study of Children's Behavior. New York: Academic Press, 1972.

MATHEMATICS PRESENTATION AS A FUNCTION OF COGNITIVE/PERSONALITY VARIABLES. Gordon, Marshall. Journal for Research in Mathematics Education, v8 n3, pp205-210, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James Fey and Joanne Rossi Becker, University of Maryland.

1. Purpose

The purpose of this study was to determine whether a prospective teacher's conceptual level or field dependence is related to his or her choice of rule/example sequences for teaching mathematics concepts. The specific research hypotheses were:

(1) High conceptual-level teachers tend to provide more mathematics examples prior to discussion of a rule than low conceptual-level teachers, who tend to discuss rules early in the lesson.

(2) Relatively field-dependent teachers would prefer to have the students present the mathematics rule, whereas relatively field-independent teachers would prefer to present the mathematics rule themselves.

2. Rationale

Starting from a 1966 finding that effective teachers seem to be those who can produce more alternative ways of teaching mathematics concepts, Gordon argues that Hunt's measure of conceptual level should identify such productive thinkers. He claims further that high conceptual level should predict teacher preference for example-to-rule style of teaching rather than a more structured, less flexible, rule-followed-by-examples sequence.

By a quite different line of reasoning, Gordon argues that research on field dependence suggests that field-dependent teachers are more likely to choose a socially interactive style of teaching in which students are more likely than teachers to formulate and state rules in concept learning situations.

3. Research Design and Procedure

Subjects for the study were 70 elementary education majors in a preservice methods course. Each subject's conceptual level was assessed by Hunt's Paragraph Completion Test and field dependence was assessed by Witkin's Group Embedded Figures Test. Then subjects were asked to write five sentences that would express how they would teach each of two mathematical rules (one in arithmetic, one in geometry) and to indicate at what point in their presentation they would want the rule to appear. From these descriptions of their conjectured teaching preferences, each subject was categorized as preferring rule-to-examples or examples-to-rule teaching sequences. A chi-square analysis tested the null hypothesis that conceptual level and rule/example sequence preference are unrelated. From the same descriptions of teaching plans, each subject was categorized as preferring student or teacher presentation of concept rules. A chi-square analysis tested the null hypothesis that field dependence and rule presenter preference are unrelated. For each hypothesis, separate analyses were performed on the arithmetic concept data.

4. Findings

From both the arithmetic and geometry rule-teaching plans, data suggested rejecting the null hypothesis of no dependence between conceptual level and rule/example sequence preference ($p < .01$). It appears that field-dependent teachers prefer having students formulate rules, while field-independent teachers prefer stating the rules themselves.

5. Interpretations

Gordon interprets the results as suggesting that the cognitive-personality variables of conceptual level and field dependence are related to presentation style preference in mathematics teaching and that these variables should be considered, in some unspecified way, in planning instruction. He suggests that the next line of research should investigate ways of raising conceptual level and developing field dependence.

Critical Commentary

The Gordon study is yet another attack on the challenging, but consistently unyielding, problem of identifying personal traits related to teaching effectiveness. However, basic theoretical and technical flaws in the study make it difficult to see how the hypotheses or the findings contribute to progress in understanding reaching.

Conceptual level and field dependence are intriguing personality/cognitive constructs for which measurement instruments are available; but the rationale for studying these teacher traits in order to understand teaching effectiveness is unconvincing. Investigation of the relation between teacher conceptual level and classroom effectiveness is justified by an extremely tenuous connection to earlier research on productive thinking ability. The completely separate investigation of field dependence and teaching effectiveness is supported by an even weaker argument involving teacher preference for classroom social interaction.

Even more curious is the decision by Gordon not to investigate teacher classroom effectiveness, but rather teacher preference for various strategies of using rules and examples in concept instruction. There is an implicit assumption that <u>egrul</u> sequencing is uniformly more effective than <u>ruleg</u>, and a similar assumption that student presentation of rules is more effective than teacher presentation. None of the very extensive research on rule/example sequencing is cited; it would not support the assumptions.

The use of sketchy written teaching plans made by preservice teachers, rather than observation of their classroom performance, is perhaps the final major weakness that leads one to ignore the findings and interpretation offered later in the report. At best the data extracted from analysis of those few sentences suggest that high-conceptual preservice teachers express a natural preference for _egrul_ sequencing in their teaching plans and the field-dependent teachers express a preference for student generation of concept rules (in arithmatic). But, even here the various scoring procedures are questionable. What are the validity and reliability of the Conceptual-Level and Embedded Figures instruments? What is the rationale for partitioning scores into high, medium, or low on the Embedded Figures measure? Was scoring of expressed teaching preferences reliable? Both _egrul_ and rule presenter measures seem to involve high rater inference.

It might be, as the author conjectures, that high conceptual level and field dependence are traits that predict effectiveness in mathematics teaching and can be cultivated by appropriate training. But the present study gives no reason to believe either.

THE RELATION BETWEEN MATHEMATICS ACHIEVEMENT AND SPATIAL ABILITIES AMONG ELEMENTARY SCHOOL CHILDREN. Guay, Roland B.; McDaniel, Ernest, D.
Journal for Research in Mathematics Education, v8 n2, pp211-215, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Michael C. Mitchelmore, University of the West Indies, Jamaica.

1. Purpose

To investigate the relationship between elementary school mathematics achievement (simple mathematics concepts and calculation skills) and high- and low-level spatial abilities.

2. Rationale

Recent research reviews indicate that, starting in early adolescence and continuing into adulthood, males outperform females on tests of mathematical achievement and spatial ability. A possible relation is suggested by Smith (1964), who concluded that spatial ability is positively related to high-level mathematical conceptualization.

The relation between mathematics achievement and spatial ability has not been studied at the elementary-school level. Smith (1964) concluded that spatial ability was not related to low-level mathematical conceptualization stressing simple calculation skills. It was therefore expected that no substantial relation would be found between mathematics achievement and spatial ability among elementary-school children.

To clarify the issue, two levels of spatial ability were defined: low-level spatial abilities, requiring only the visualization of two-dimensional configurations; and high-level spatial abilities, requiring the visualization and mental manipulation of three-dimensional configurations.

3. Research Design and Procedure

Low-level spatial abilities were measured by two group tests. In Serial Integration (SI), children selected from four figures the one which was formed by four lines previously projected one at a time. In Embedded Figures (EF), children selected from four figures the one containing a simple two-dimensional pattern previously projected for 5 seconds. KR-20 reliabilities were 0.75 for SI and 0.74 for EF.

High-level spatial abilities were measured by two individual tests. In Coordination of Viewpoints (CV), children viewed a simple geometric object and then selected from three drawings the view of the object from a specified position. In Surface Development (SD), children viewed a geometric object as in CV and selected from three drawings the development of its surface. KR-20 reliabilities were 0.56 for CV and 0.66 for SD.

SI, CV, and SD were administered to 14-16 children from each of grades 2-7, a total of 90 children. EF was administered only to the children in grades 5-7 at one school. Children were divided at the median of each grade level into high or low mathematics achievers on the basis of their total mathematics score on the Iowa Tests of Basic Skills.

Scores on SI, CV, SD, and EF were analysed in separate analyses of variance with the factors of Mathematics Achievement, Sex, and Grade.

4. Findings

No significant interactions were found. Mean scores of high mathematics achievers were significantly higher than those of low mathematics achievers on all four tests ($p<0.05$ or better). Mean scores of males were significantly higher than those of females on CV and SD, but not significantly different on SI and EF. There were significant main effects for grade on all four tests.

In grades 2-4, 1 out of 9 correlations between mathematics achievement and spatial test scores was significant. In grades 5-7, 10 out of 12 correlations were significant ($p<0.05$ or better).

5. Interpretations

The findings suggest that, among elementary school children:

(1) high mathematics achievers have greater spatial ability than low mathematics achievers;

(2) the relation between mathematics achievement and spatial abilities is not a function of grade level or sex;

(3) the relation between mathematical and spatial thinking exists for low- and high-level spatial abilities;

(4) males have greater high-level spatial ability than females, but males and females have similar low-level spatial abilities.

(5) the sex differences in (4) are not a function of grade level.

Critical Commentary

This study is a useful contribution, but it ignores the basic question of the rationale for a relation between mathematics achievement and spatial ability at any level. Smith (1964) rather overstates his case; the view of Krutetskii (1976) that some high-achievers in mathematics use visualization very efficiently (but others do equally well with non-visual methods) seems more acceptable. The correlations between mathematics and spatial test scores observed in this study could be due to a common relation to abstract reasoning (or even verbal ability); no evidence for the construct validity of SI, EF, CV, or SD is presented.

There are some further questions about the details of procedure and interpretation:

(1) How was the sample selected? Were grades 2-4 children selected from the same school as grades 5-7? Why was EF not administered to grades 2-4? Why were there not equal numbers at the various grade levels? Why were there not equal numbers of males and females?

(2) What was the relation between sex and mathematics achievement? What were the cell sizes in the factorial design? If the design was unbalanced, the interpretation of the ANOVA results would be considerably complicated, if not invalidated.

(3) Why was information on mathematics achievement wasted by dichotomizing scores? Would not multiple regression have been a more powerful method of analysis? (It would also have allowed one to take account of any relation between sex and mathematics achievement.)

(4) Why is there an apparent discontinuity in spatial test scores and mathematical-spatial test correlations between grades 2-4 and grades 5-7? Were the data obtained in different schools? Or were the spatial tests very unreliable in grades 2-4? (The CV and SD scores are close to chance levels in grades 2 and 3.)

(5) Could the difference between the results for high- and low-level spatial abilities be related to differences in method of test administration?

The report is rather brief, even by JRME standards. A few extra sentences here and there could have answered the above doubts without taking the paper outside reasonable limits on length.

References

Krutetskii, V.A. The Psychology of Mathematical Abilities in Schoolchildren. Chicago: University of Chicago Press, 1976.

Smith, I.M. Spatial Ability. San Diego: Knapp, 1964.

AREA AND VOLUME CONSERVATION AMONG THE ELDERLY: ASSESSMENT AND TRAINING. Hornblum, Judith Newman; Overton, Willis F. Developmental Psychology, v12 n1, pp68-74, January 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James Hirstein, University of Illinois.

1. Purpose

Studies have consistently shown that the elderly perform poorly on Piagetian tasks of logical thought. The question under investigation is whether this decline is due to a loss of cognitive competence or to a loss of specific performance abilities.

2. Rationale

The common explanation for poor performance by the elderly, that cognitive structures are lost, presents problems for Piaget's theory of integrated stages. An alternative explanation is that structural competence remains present but is unactivated. While most studies of conservation in the elderly are assessment studies, one must go beyond verifying conservation deficiencies to explain a decline in performance. The degree of difficulty in training elderly nonconservers would indicate the status of their cognitive structure: easy training would imply that an existing structure is activated while difficult training would imply an absence of that structure.

3. Research Design and Procedure

Subjects for the study were noninstitutionalized females from Philadelphia senior citizens centers; their participation in the study was voluntary. Initial screening was done for educational level (at least sixth grade) and intelligence (correlated Quick Test score of 85). Each subject was interviewed in her own home. The first part of the study was an assessment of 60 subjects on six tasks involving area and volume conservation. In the second part of the study, assessment results were used to select subjects for a training program. The first assessment was used as a pretest which was followed by an individual training procedure and a posttest.

The first assessment included three tasks on area conservation and three tasks on volume conservation. The order of task presentation was randomized among subjects. Each task consisted of four conservation trials. For each trial, the subject verified the equivalence of two stimulus figures. Then the shape of one figure was changed and the subject was asked whether the quantity in question was the same or different for the two figures after the transformation. Within each task, the quantity remained the same in three trials and was different in one trial. Based on responses and justifications, each subject was judged to be a nonconserver (N), partial conserver (P), or conserver (C) on each task. A judgment across all six tasks was made for each subject according to the following criteria:

(a) overall conservers were those judged C on at least 5 tasks with P on a sixth,

(b) overall nonconservers were those judged N on at least 5 tasks with P on a sixth,

(c) all others were called overall partial conservers.

Twenty-two subjects judged N on one of the area conservation tasks (Surfaces) were used in the training study. The subjects were assigned to two treatment groups. The treatment consisted of a 20-trial program involving tasks similar to the original Surfaces task missed by all 22 subjects. One treatment group received feedback regarding the responses given during the training program. The other group received the same program without feedback. Immediately following the training program, the original six conservation tasks were readministered as a posttest. The Surfaces posttest was considered near transfer with respect to the treatment; the other five tasks were considered far transfer items.

4. Findings

In the first assessment, subjects were determined to be in the three conservation categories as shown in the following table:

Number of Subjects	Range For The Six Individual Tasks	Surfaces Task	Overall
Nonconservers	14 - 31	31	8
Partial Conservers	1 - 12	3	32
Conservers	26 - 45	26	20

Each subject was assigned a score from 0 to 5 on each task for analysis of the training results. Mann-Whitney U tests were used to compare the posttest results of the two treatment groups. In general, the feedback group scored significantly better than the nonfeedback group: $p < .001$ for the near transfer task, $p < .05$ or better on four of five far transfer tasks, and $p < .01$ on the overall posttest score. Ease of training was measured by counting the errors made in training trials. The eleven feedback subjects made a total of eight errors after the first incorrect response, while the eleven nonfeedback subjects made a total of 60 errors after the first incorrect response.

5. Interpretations

The assessment results indicate that the elderly perform poorly on tasks of area and volume conservation, which agrees with other findings regarding the logical thought of the aging. However, based on ease of training, the authors conclude that" the elderly, in general, maintain the relevant strategies in competence and that simple verbal feedback activates those strategies into performance" (p. 73). The question of whether repeated activation of these strategies can increase the performance level for long periods of time is suggested for further study.

Critical Commentary

The question under consideration is highly significant in studying the mental development of elders. The problem is well stated and the performance-competence approach is commendable. The criticisms of the study derive from procedural concerns.

Criteria for naming categories of overall performance are necessarily arbitrary, and the rigid standard for the label "overall conserver" may give misleading percentages. Of 360 tasks performed in the original assessment, 222 (61.7%) of the responses were judged to be conserving responses. Yet only 33.3% of the subjects were classified as "overall conservers." Apparently with elders, as with youngsters, conservation in one domain does not imply conservation in other domains. This suggests that the six tasks do not measure the same ability. A statement of the relationships among individual task results would be much more meaningful than an overall decision regarding general conservation ability. Furthermore, with 61.7% conserving responses it is difficult to contend that these subjects are poor conservers. In the absence of performance data on these tasks by younger adults, it is not clear that the results obtained represent a significant decline in conservation ability among the elderly.

It is generally recognized that the justification of a response is critical in distinguishing between conserving and nonconserving responses. Although the experimenters solicited justifications, they do not report how a justification is reflected in any recorded score. Hence, it is not possible to determine whether the improvement attributed to the training procedure results from subjects giving more adequate justifications or simply from subjects changing their responses. The qualitative assessment, which should be the important concern here, suffers from the attempt to quantify conservation on a linear scale.

PROCESSES INVOLVED IN MATHEMATICAL PROBLEM SOLVING. Kantowski, Mary Grace. Journal for Research in Mathematics Education, v8 n3, pp163-180, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by W. George Cathcart, University of Alberta.

1. Purposes

(a) Clinically to uncover information about, and gain a greater insight into, processes involved in solving complex, non-routine geometry problems.

(b) To providing direction for future research.

2. Rationale

The solving of true mathematical problems (not just simple algorithmic applications) has long been a goal of mathematics education. Two aspects of problem solving, process (behaviors that direct the search for the solution) and product (actual solution) are both essential components of the problem-solving experience.

Most problem-solving research has been experimental in design and has focused on the product. Educators have recognized a need for a clinical analysis of problem-solving processes and how these processes develop. Kantowski's study is an attempt to meet this need.

3. Research Design and Procedure

Eight subjects from a private school in an Atlanta surburb were selected from among the high-ability ninth-grade algebra students. The study was conducted over an 8-month period.

The study was completed in four phases:

PHASE 1: Pretest--subjects were asked to think aloud as they solved eight problems. Verbal protocols were recorded and coded for sequence of processes used and scored according to processes and correctness of result.

PHASE 2: Instructional readiness (3 lessons per week for 4 weeks). This phase was designed to acquaint subjects with the heuristic method of instruction and introduce them to the use of heuristics in problem-solving. A second test was administered at the end of the 4 weeks.

PHASE 3: Heuristic instruction in geometry (4 months). A series of individual tests were administered periodically during this phase.

PHASE 4: Posttest--consisted of geometry and verbal problems and "prerequisite knowledge" tests which included facts and concepts necessary for the solution of the posttest problems.

During each phase verbal protocols were recorded and later analyzed according to a modification of a coding scheme developed in earlier research. Points on the following variables were added to yield a process-product score: suggesting a solution plan, persistence, looking back, absence of structural errors, absence of executive errors, absence of superfluous deductions, and correctness of result.

Percentages of problems in which the above processes were used were calculated for problems with scores above and below the median for each subject.

4. Findings

The use of goal-oriented heuristics was consistently more evident in solutions with process-product scores above the median than below the median. The use of goal-oriented heuristics generally increased as problem-solving ability developed.

Regular patterns of analysis and synthesis were noted in the solutions of problems with scores above the median. Usually these regular patterns were immediately preceded by a goal-oriented heuristic. Successful problem-solvers exhibited significantly more regular patterns of analysis and synthesis than less successful problem-solvers.

Failure to introduce a heuristic often led to many superfluous analyses. On the other hand, the suggestion of a heuristic not related to the goal often resulted in superfluous syntheses. The introduction of a goal-oriented heuristic tended to decrease the number of superfluous syntheses. The better problem-solvers showed less evidence of superfluous syntheses than poorer problem-solvers.

Persistence improved as problem-solving ability developed. Persistence seemed to be affected by prerequisite knowledge and personality factors. Reflective subjects were more persistent than impulsive subjects.

The use of "looking back" strategies did not increase as problem-solving ability developed, nor was it related to success in problem-solving.

5. Interpretations

One purpose of this study was to find regularities which could form the basis of hypotheses for future research. More clinical studies are suggested with subjects representing a wider range of ability, age, and problem-solving sophistication, and using different mathematical content. Experimental studies are recommended in which the role of heuristics in problem-solving is systematically investigated.

More research is needed to devise a method of scoring problem-solving processes which would be valid, reliable, and could be used efficiently and objectively with the large number of subjects required for experimental studies.

While the use of heuristics was related to success in problem-solving, it is not known what effect the heuristic instruction during two phases of this study has on this relationship. The effects of heuristic instruction versus expository instruction should be studied with the use of heuristics as the dependent variable.

An investigation of the relationships between process variables and other dependent variables should be carried out.

Experimental studies are needed to support the observation that regular patterns of analysis and synthesis followed the introduction of a goal-oriented heuristic. The relationship between the use of heuristics and level of prerequisite knowledge needs further study. Similarly the relationship between the number of superfluous syntheses and the use of heuristics needs to be examined more closely. "Does the successful solution of novel problems result from the use of heuristics?" is another question worthy of examination. The effect of making students aware of problem-solving processes such as the use of regular patterns of analysis and synthesis should be examined. An aptitude-treatment-interaction study is recommended involving conceptual tempo and problem-solving styles. More research is needed to explore the relationship between the use of looking-back strategies and success in problem-solving.

Critical Commentary

The processes involved in solving problems are of concern to most teachers and other mathematics educators. Kantowski is to be commended for studying with care this important topic.

Clinical studies of the type reported here usually suffer from obvious limitations due to size, characteristics, and selection of the sample. However, the author is fully cognizant of these and other limitations in her study and is careful not to overgeneralize.

We are not told anything about the nature of the instruction other than that it was heuristic and that it lasted 4 months. It is unfortunate that there were no controls on the instructional phase. The author leaves one with the impression that it was the heuristic nature of the instruction which resulted in improvement in problem-solving success. It could just as easily have been due to the change in routine, Kantowski's personality, or something else in the instructional program. This is the only sense in which any overgeneralization occurs. The author is aware of the limitations of her instructional phase because she explicitly recommends further research on the effect of heuristic teaching, but at other times, implicitly, she seems to place more confidence in it than perhaps is warranted.

One of the major purposes of the study was to generate hypotheses for future experimental studies. Many very good suggestions for further research are given which, hopefully, will be followed-up. However, only a minority of these suggestions are actual hypotheses.

In summary this was a carefully conducted clinical study of a very important topic. The suggestions made could form the basis for many worthwhile future studies which could extend our knowledge of problem-solving processes significantly.

THE PERSONALIZED SYSTEM OF INSTRUCTION IN INTRODUCTORY CALCULUS.
Klopfenstein, Kenneth F. American Mathematical Monthly, v84, pp120-124, February 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Bert K. Waits, The Ohio State University.

1. Purpose

The Problem: To investigate the effectiveness of the Personalized System of Instruction (PSI) in beginning college calculus.

Questions: Can PSI be used effectively to teach beginning calculus? If so, how should it be implemented to maximize effectiveness?

2. Rationale

The contextual framework for the investigation is the Keller PSI method of instruction (Keller, 1968). PSI is distinguished by (a) self-paced instruction, (b) mastery required before advancement, (c) course content is communicated primarily through written materials, and (d) repeated tests are available with immediate feedback (scoring). Kulik, Kulik, and Carmichael (1974) reviewed the evaluative research on PSI courses and concluded that the effectiveness of the PSI method "should no longer be a matter of serious debate."

3. Research Design and Procedure

The subjects were 53 students (primarily college freshman) enrolled in a PSI beginning calculus section (Calculus I) and an unspecified number of students enrolled in a "comparison" Calculus I section taught in a traditional lecture-discussion style. The experiment was conducted Winter Quarter, 1974, at Colorado State University.

The variables measured were achievement and attitude. An analysis of "success rates" (number of students completing the course) was conducted. Achievement was measured by a 20-item posttest administered at the beginning of the subsequent quarter (Spring) in the follow-up course (Calculus II). Students' attitudes relating to method of instruction, course content, and fulfillment of expectations were measured during the seventh week of the Winter Quarter by the Neidt Attitude Scale (Neidt and Sjogren, 1968). It was given to 28 students in the PSI section and 27 students in the comparison section.

The PSI students were required to complete 20 units (including 4 review units) based on material from a standard college calculus text (Thomas, 1972) in the 10-week quarter. The students were required to complete all units to receive a final course grade. A, B, and C course grades were assigned based on the last unit test, which was a comprehensive review unit and sampled the entire course. Students who did not complete the course (complete all units) were given an incomplete grade and were allowed to complete the course in the Spring Quarter (either in the PSI mode or the lecture-discussion mode).

It was stated that the PSI students likely had higher mathematical aptitudes and were probably better motivated to study mathematics than the students in the comparison section based on standard college aptitude tests (ACT and SAT tests) and declared majors. No adjustment was made in the analysis for these apparent differences.

4. Findings

(a) The posttest results indicated that the A and B students from the PSI section (all students completing the PSI course in the Winter Quarter received A or B grades) performed slightly better than the A and B students from the comparison section as well as the other traditional sections. The differences were not significant.

(b) The student attitudes in the comparison (traditional) section were slightly more favorable than those of the students in the PSI section. Again the differences were not significant.

(c) The "success rate" strongly favored the traditional lecture-discussion sections. Only 17 (32%) students from the original PSI group completed the course in one quarter. A total of 31 students (58%) from the original PSI group completed the course in one or two quarters with passing grades. It was stated that 20 students (38%) withdrew from the PSI section. It was also stated that 83% of the students in all of the traditional sections completed the course in one quarter with passing grades. The "success rate" data for the comparison section were not specified.

5. Interpretations

The PSI calculus section was much less successful than the literature suggested. Withdrawal and failure rates were abnormally high, and an unacceptably large number of students (48%, excluding students who withdrew from the PSI section) required two quarters to complete the one-quarter course.

Three factors were identified which may have contributed to the lack of success in this experiment of the PSI method in teaching beginning calculus.

(a) PSI calculus students were required to learn all material at a high level of mastery while only some of the traditional calculus students (the A and B students, approximately 25% of the total population) were required to "master" some (at least 85%) of the material.

(c) The PSI calculus students tended to put off studying the course material until it was too late.

It was suggested that, assuming PSI is a sound instructional system, the main problem for further research is to determine how the implementation of PSI in beginning calculus should be modified to achieve the success of PSI reported in the literature for other disciplines.

Critical Commentary

The inherent requirement of unit mastery necessary in the PSI system makes meaningful comparisons with traditionally taught sections difficult. This study is no exception. There are a number of unanswered questions.

1. How were the PSI students selected? Randomly? By student choice?

2. How "insignificant" were the differences in achievement posttest scores? No analysis was given.

3. How "insignificant" were the differences in students' attitudes? No analysis was given.

4. How much "better" were the PSI students initially? No quantitative data were given. Were the differences significant?

5. Was the 20-item achievement posttest valid?

Other questions are worthy of discussion and further research.

1. Are the PSI achievement standards applied to college calculus realistic?

2. What would have been the results if lectures were given in the PSI section (assuming the PSI format would remain otherwise unchanged)?

3. Are there perhaps objectives of traditional calculus instruction that are not measured by the usual achievement tests (and thus could not be measured in any similar study comparing the PSI system with traditional instruction)?

4. Is it perhaps the case that the PSI calculus system of instruction is <u>inherently</u> not effective for many students? Perhaps a PSI system works well for courses that are primarily "memorization" courses--such as some beginning science or language courses. Can the PSI system be modified (thus it would no longer be, by definition, a PSI system) in order to be more effective in a "problem-solving" type of course, such as calculus?

The Klopfenstein study clearly indicates that the question is <u>not</u>, "How can PSI be implemented in beginning college calculus?"; rather, the question remains, "Can the PSI method of instruction be effectively used to teach beginning college calculus?" There is a definite need for further thoughtful research on this question.

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ELEMENTAL GROUPINGS HELP CHILDREN PERCEIVE CARDINALITY: A TWO-PHASE RESEARCH STUDY. Maertens, Norbert W.; Jones, Rowan C.; Waite, Ardis. Journal for Research in Mathematics Education, v8 n3, pp181-194, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Douglas T. Owens, University of British Columbia.

Phase One

1. Purpose

The purpose of Phase One of the study was to provide evidence for answering the following questions:

(a) Can children more easily identify the cardinality of sets of objects which are elementally grouped than sets which are not elementally grouped?

(b) Can children more easily perceive the cardinality of elementally grouped sets arranged in uniform patterns than in non-uniform patterns?

(c) Are there differences in first-, second-, and third-grade children's abilities to perceive the cardinalities one to nine of sets?

2. Rationale

An underlying assumption of the investigators is that young children should be provided with examples of sets that enable them to perceive the cardinality of the sets rapidly, meaningfully, and without counting. Perceiving the whole set is an important step in linking early number concepts to the use of symbols. Previous research indicated that the maximum number of objects in a horizontal row that can be perceived at one time without counting or regrouping is four or five. While not explicitly stated, the investigators accept the definition of sets of cardinality one through four as elemental groupings.

3. Research Design and Procedure

The subjects in Phase One were the 239 children in grades 1, 2, and 3 of an elementary school in a semi-suburban, middle class neighborhood. Subjects in each grade were randomly assigned to three treatment groups. Treatment conditions for a set of seven objects are illustrated by the diagram below.

Treatment A (N=80) Elementally grouped Uniform pattern	Treatment B (N=79) Elementally grouped Nonuniform pattern	Treatment C (N=80) Nonelementally grouped
* * *	* * *	* * * * * * *
* * * *	*	
	* * *	

Nine 35 mm slides of red-orange paper dolls on a blue background were viewed by pupils for 1/2 second with a 2-second interval between slides to allow each child to respond orally to the previous slide. After practice slides (1, 2, 3) each child viewed the slides for one treatment condition in the sequence 1, 4, 2, 7, 3, 9, 5, 8, 6. Data were analyzed as orthogonal comparisons with unequal group sizes.

4. Findings

The investigators found that when using sets with cardinality one through nine:

(a) Identifying the cardinality of elementally grouped sets was significantly easier than identifying the cardinality of nonelementally grouped sets.

(b) Sets of elementally grouped objects in uniformly patterned arrangements were significantly more easily identifiable than those in nonuniform patterns.

(c) Second- and third-grade children scored significantly higher than first-grade children in perceiving sets having larger cardinality.

Phase Two

1. Purpose

Phase Two of the study was designed to answer the following questions:

(a) Can young children be trained to perceive increasingly larger numbers of objects in nonelemental groupings?

(b) Can young children be trained to view elementally arranged sets of ojbects?

(c) What effect does pretesting prior to training have on children's ability to perceive elementally and nonelementally grouped arrays?

2. Rationale

The question of efficacy of training children to perceive the cardinality of elementally grouped and nonelementally grouped sets grew out of Phase One.

3. Research Design and Procedure

The sample for Phase Two was 58 children in two second-grade classrooms. The children were from a different community than in Phase One, but of similar socioeconomic background.

Two treatments, one for elementally grouped sets (E) and one for nonelementally grouped sets (NE), were designed for a group situation. In the first of three 20-minute sessions of each treatment, the subjects viewed 28 slides representing cardinalitites one through nine. None of the slides were those used for testing. The second session consisted of the same slides in a different order and large flash cards containing various numbers, colors, and objects. The third training session consisted of slides, flash cards, and worksheets. The worksheets were given as a timed game in which children were asked to work quickly and write the number of objects under each group without counting.

The same test was given as a pretest and a posttest. It consisted of 18 items of elementally grouped objects of Phase One (Treatment A) and nonelementally grouped objects (Treatment C), for cardinalities one through nine. The investigators used a modified Solomon Four-Group Design as shown with group sizes in the figure below. Analysis of variance was used to analyze the posttest data.

Group	Number	Pretest	Training
A	15	Yes	NE
B	14	Yes	E
C	15	No	NE
D	14	NO	E

4. Findings

The results are given in each case for cardinalities 5-9 because the items 1-4 were similar for elemental and nonelemental groupings. Results of the analyses were:

(a) Performance on nonelemental groupings was not significantly different for subjects who received training in nonelementally grouped arrays (Groups A and C) than for subjects who received training in elementally grouped sets (Groups B and D).

(b) Differences in performance on elemental groupings were significant in favor of those who received training on elemental groupings (Groups B and D) over the treatment group on nonelemental groupings (Groups A and C).

(c) Subjects who took a pretest prior to training (Groups A and B) performed significantly better than subjects who had not taken the pretest (Groups C and D) on the posttest of elementally grouped items only. There was no identifiable difference for the nonelementally grouped posttest.

The investigators reported under "other findings" that for all subjects there was a marked decrease in ability to perceive nonelemental groupings greater than four. Further, training had a greater effect on perception of odd numbers of elementally grouped objects than on even numbers. In most cases, regroupings occuring after three were more accurately perceived than regroupings after four.

5. Interpretations (Phases One and Two)

Training in nonelemental groupings appeared to have a negligible effect on performance on nonelemental groupings. Training did have a positive effect on children's ability to perceive elemental groupings. This seems to indicate that "...use of materials that are not elementally grouped for children only leads to rote patterning instead of meaningful mental images for numbers" (p.192). Judicious use of elemental groupings in curriculum activities is one way that children can and should increase their understanding of number. From the training it was apparent that some media and some arrangements were more readily perceived than others.

The investigators give a considerable list of suggestions for further research. For example, how easily are elemental groupings with cardinalities greater than nine perceived? Do children need to know multiples of elemental numbers in order to identify the cardinality of larger groupings? A number of questions remain unanswered about the effect of arrangement, spacing, objects represented, color of objects, and background on performance.

Critical Commentary

This was a valuable study in that it contributed to knowledge about one aspect of young children's number concept. Perceiving the cardinality of a set is a very important facet of number concept, but it is not clear that other means of determining cardinality, such as counting, are necessarily rote activities.

The investigators report apparent informal observations as "other findings" without reporting the supportive data. Observations such as this are a valuable means of generating hypotheses for further study. Wouldn't it be preferable to report the data, such as group means, on which the observations were based?

Apparently there were errors in reporting in Tables 1 and 2 which give the results of Phase One. The tables gave the same entry four times for "Source" of variation, while four different results were clearly being reported.

RELEVANT AND SOME IRRELEVANT FACTORS IN THE CHILD'S CONCEPT OF WEIGHT.
Murray, Frank B.; Johnson, Paul E. Journal of Educational Psychology, v67 n5, pp705-711, 1975.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Leslie P. Steffe, University of Georgia.

1. Purpose

Murray and Johnson's purpose was to propose and test a descriptive model for the child's concept of weight. The model describes the relevant (or "logical") conditions under which an object's weight changes, and some of the important irrelevant conditions under which it does not change.

2. Rationale

The relevant conditions under which an object's weight changes are defined by the equation $w = g.\ m_1 \cdot m_2 . 1/d^2$. In the equation, w represents the weight of some object of mass m_1; g represents a gravitational constant; and d represents the distance between the center of the object and the center of the appropriate planet of mass m_2. The sources of irrelevant conditions under which an object's weight does not change can be quite varied. Their importance resides in the fact that students may consider conditions which are irrelevant to a change in an object's weight as relevant. Why they do so is of importance to psychological research.

The relevant and irrelevant conditions chosen for study were described by a collection of transformations on a clay ball. The logically relevant transformations defined by the equation for the weight of an object were (a) a change in the mass of the object and (b) a change in the distance between the planet and the object (vertically upward or vertically downward). Logically irrelevant transformations (not defined in the equation) were (a) a change in temperature, (b) a change in the number of connected pieces (continuity), (c) a change in proximity to a larger or smaller object (context), (d) a change in shape, and (e) a change in horizontal position (nearer to or farther from the child).

3. Research Design and Procedure

One hundred twenty white second-grade children were used as subjects. They were balanced across socioeconomic groups (high, middle, and low) and sex. All children had used the Minnemast curriculum unit on weight. Each subject was tested individually with 18 weight problems concerning a clay ball. In each problem, the subject had to judge whether the clay ball was more, less, or the same in weight as it was before the ball was changed in some way. The order of alternatives was balanced across problems. The subjects were randomly assigned to two groups which differed in that Group 1 would received forward transformations (State A to State B) whereas Group 2 received the reverse transformations (State B to State A). In the following table, Group 1 would receive the "forward" transformation of having clay added to a ball, while Group 2 would have the reverse transformation of having clay removed from a ball. In case of the attribute temperature, Group 1 children would start with a warm ball and would be asked to imagine the ball hard and cold outside. Group 2 children would first be asked to imagine the ball hard and cold outside then would be asked to imagine it coming inside. For all attributes and problems, then, the initial and final states were reversed for Group 1 and Group 2.

TRANSFORMATIONAL TYPES ON A CLAY BALL

Attribute	Mass	Temperature		Continuity	Context
Problem	1 and 2	3 and 4	5 and 6	7 and 8	9 and 10
Transformation	Add clay	Imagine hard and cold	Imagine hot and soft	Divide into three pieces	Place next to bigger ball then smaller ball

Attribute	Shape	Vertical Upward	Vertical Downward	Horizontal Movement
Problem	11 and 12	13 and 14	15 and 16	17 and 18
Transformation	Roll into a "sausage"	Imagine on a mountain or on an airplane	Imagine in a tunnel below ground or thrown into a deep hole	Imagine in room down hall or move on a table top

4. Findings

(1) The results of the relevant transformations of adding or subtracting clay to a clay ball essentially were relevant aspects of "weight" for the children. With few exceptions, the children knew that the "weight" also would change.

(2) The results of the relevant transformation of increasing or decreasing the distance between the clay ball and the planet Earth essentially were irrelevant aspects of "weight" for the children. For the eight problems (13, 14, 15, and 16 for Groups 1 and 2) approximately

50 of the 60 children doing each thought the weight would remain the same.

(3) The only irrelevant transformation that was, in fact, essentially irrelevant for the children was horizontal movement.

(4) The difference in the proportion of subjects responding "same" and "not same" between the problems with horizontal movement of the clay ball and any other type of transformation was significant statistically.

5. Interpretations

Murray and Johnson in their discussion of the results, stated

(1) ".... if the child believes that a certain transformation increases weight, he should judge that the undoing of that transformation should decrease weight.... The data on the differences in more and less responses between the groups clearly indicate that the subjects exhibited this consistency or reversibility for (a) addition-subtraction, (b) continuity, (c) temperature increase and decrease, (d) context transformation, and slightly less clearly for the shape transformation."

(2) ".... where the physicist's definition of weight is that it is the function of two masses and the distance between them, ... it appears that for the child of 8 years weight is a function, in varying degrees, of the object's mass, temperature, continuity, context, shape, and not its vertical or horizontal location."

(3) "These data provide no prescriptions to the curriculum writer for correcting these misconceptions; rather, they merely point out that these errors exist and that perhaps they should be treated."

(4) "It is clear that although Minnemast conservation training did not generalize across other transformations, it was successful in some degree for training conservation of weight under the shape transformation."

Critical Commentary

Murray and Johnson should be congratulated for their excellent research. The issues are seemingly important for the measurement strand of mathematics curricula and certainly for science curricula that treat weight at the early ages. While some data were reported by the authors concerning acquisition of the concept of weight by 8-

year-old children, there is an important unanswered question concerning the nature of their concept. Murray and Johnson speak of treating errors that exist in the 8-year-olds' conception of weight. But are those "errors" evidence of a primordial conception of weight and, as such, precious indicators of factors a child must encounter in the construction of the concept?

The evidence presented by the authors seems to indicate an affirmative answer to this question, as the children had been in a unit of weight in the Minnemast curriculum. The curriculum did produce non-generalized conservation of weight under the shape transformation. Direct tutelage on the weight concept for the purpose of eradication of the "errors" may do untold damage to the construction of the concept. The "errors" may point to a set of factors quite critical for a child to encounter in a variety of situations and contexts.

If one accepts the assumption that mathematical (and, in some cases, physical) concepts go through "stages" for the learner much as do concepts in development, a natural question to ask is whether there are identifiable states, other than reported in the study, of the conception of weight for children. A natural strategy in such a search would be to look across ages for the answer. If such states can be identified, then profitable training studies could be done attempting to elucidate the mechanism of transition from one state to another. Such training studies are potentially beneficial in the search for effective instructional strategies regarding weight.

SELF-PACED MATHEMATICS INSTRUCTION: A STATISTICAL COMPARISON WITH TRADITIONAL TEACHING. Peluso, Ada; Baranchik, A.J. American Mathematical Monthly, v84 n2, pp124-129, February 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Harold Mick, Virginia Polytechnic Institute and State University.

1. Purpose

To compare the effects of two modes of self-paced instruction with traditional classroom instruction as measured by final examinations and success in subsequent mathematics courses.

2. Rationale

The open-admissions policy at CUNY motivated the adoption of the Keller Plan, and, in general, self-paced instruction at Hunter College. The department of mathematics expected large numbers of students with deficient backgrounds in mathematics to enroll in the Basic Structures of Mathematics course. Previous research involving the Keller Plan had shown that students enjoyed mathematics more when they were permitted to progress at their own rates and to interact with tutors.

3. Research Design and Procedure

Basic Structures of Mathematics, a one-semester course intended for non-science students, had a syllabus consisting of the elements of logic, relations and operations, elementary number theory, set theory, the rudiments of real numbers, some analytic geometry, and an introduction to functions and their graphs. Each of the 617 college students who registered for the course in the fall of 1972 was assigned to one of three modes of instruction:

Classroom (C): Regular classroom instruction, consisting of three lecture hours per week, with one instructor for approximately 35 students.

Learning Center (LC): Self-pacing instruction utilizing tapeslide shows, with one instructor and one tutor available for each set of approximately 35 students.

Learning Center-Keller Plan (LCK): Similar to LC but with a Keller Plan modification in which each group of 10 to 12 students was identified with a student tutor.

Comparisons of three modes of instruction were conducted relative to: the scores of the final examination administered at the end of the semester; the number of students who passed or did not pass a subsequent mathematics course; and the value of the pretest as a predictor of final examination performance. Of the original 617 students to enroll, 185 had been pretested in arithmetic computation. The design and procedures associated with the Basic Structures course were essentially replicated for a beginning calculus course scheduled the same semester.

4. Findings

Basic Structures of Mathematics results. A one-way analysis of variance showed a significant difference favoring the self-pacing modes on final examination scores ($F_{2,487} = 33$, $p < .05$) among the three instructional modes of the 490 students who completed the Basic Structures course, but showed no significant difference ($F_{1,285} = 1.056$) between the two self-pacing modes. When the sample was restricted to the 185 students who had been pretested in arithmetic computation, a two-way ANOVA showed significant differences in final examination scores ($F_{2,157} = 15.9$, $p < .05$) among the three instruction modes favoring the self-pacing modes. An ANOVA comparing the self-pacing modes showed a slight superiority of LC over LCK ($F_{2,82} = 3.76$, $p < .05$). Of the approximately equal proportions of students from the three modes of instruction (C-29%, LC-31%, LCK-33%) who chose to take a subsequent mathematics course, those in the self-pacing modes were more likely to pass their next mathematics course ($X^2_2 = 6.3$, $p < .05$). Finally, a regression analysis performed on predicting final examination scores as a function of pretest scores showed that performance in C and LCK was strongly related to prior mathematical skills but this was not the case for LC. In fact, several students entering LC with low pretest scores performed very well on the final examination.

Calculus results. Data from the 395 students enrolled in a first-semester calculus course were not subjected to an ANOVA analysis, but it was observed that 93% of the students enrolled in C took the final examination while only 65% of those students in LC and 59% in LCK took the final examination. The percentages for those passing the course were 88%, 50%, and 51% for C, LC, and LCK respectively, and the average final scores were 62.33, 60.88, and 61.96 for C, LC, and LCK respectively. When the sample was restricted to the 142 students who were pretested with a calculus readiness test, a two-way ANOVA indicated significant differences in final examination scores ($F_{2,120} = 11.6$, $p < .05$) among the three instructional modes favoring the self-pacing modes. Follow-up data were inconclusive in terms of the likelihood of taking the subsequent calculus course and of the performance in that course. A regression analysis again indicated that performance in LC was relatively independent of the precalculus pretest.

5. Interpretations

In the Basic Structures course the two self-pacing modes of instruction were found to be superior to traditional teaching, both in terms of final examination performance and success in subsequent mathematics courses. In particular, the Learning Center mode of instruction appeared to raise weak (as measured by a pretest) students' performance up to the level of that of strong students. These modes of instruction were also compared in teaching beginning calculus, with more mixed results. The slight advantage of the self-pacing modes as measured by final examination performance must be weighed against their substantially lower successful-completion rate.

Critical Commentary

The results of this study should have implications for establishing priorities and determining future curriculum decisions at Hunter College, but the study has little, if any, significance for the mathematics education community at large. Expenditures of time, money, materials, and talent directed at individualizing mathematics learning are deserving of praise, and according to this study, these resources have returned dividends toward meeting the needs of students with deficient or average mathematics background at Hunter College. However, the study lacks sufficient experimental controls and overall quality to be generalizable to a larger population.

The relation of this study to previous research associated with self-pacing instruction was sketchy at best. For instance, if it was the intent of the authors to extend and build on current knowledge and theory, why didn't they review or report the literature? The only research study cited found that students enjoy Keller courses much more than conventional courses because of self-pacing and interaction with tutors, but no analysis was made regarding measures of student attitudes. There also appears to be a contradiction in the interpretation and use of the term "self-pacing." The entering freshmen who had been pretested and assigned to self-pacing instruction had to finish the basic structures course and beginning calculus within the same time interval as the traditional group in order to be included in the statistical analyses comparing relative effectiveness of the three modes of instruction. While this design does control the time variable, it permits self-pacing only within the interval of one semester. Moreover, all students allegedly completed each study unit which involved behavioral objectives, self-tests, content in the form of slide-tape lessons, posttest for checking understanding, and sample examinations with answers. Considering the time it took to complete all the self-pacing lessons, it may have been the case that the traditional classes had more time and opportunities for "self-pacing" than the self-pacing classes. If so, what assurances does the reader have that the traditional classes covered all the material on the final examination, and in a manner consonant with the self-pacing sample examinations? Did the final examination consist of questions exemplary of the sample examinations with answers given each student in the self-pacing modes of instruction? Were the questions mostly related to rule-type learning?

From a statistical point of view, why weren't all students originally pretested? And if the original selection of students to the three modes of instruction was made at random (random assignment was not mentioned), then why limit the analysis just to pretested students as occurred with the calculus results? Generally the authors appeared biased, as indicated by their failure to report statistical analyses of the calculus study for all students originally enrolled, for not reporting any data from the subsequent calculus course, and for not emphasizing that the pretest differences favored the self-pacing modes in both analyses of final examination scores.

A VIABLE INDIVIDUALIZED LEARNING SYSTEM. Rubillo, James M. <u>MATYC Journal</u>, v11 n2, pp89-95, Spring 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Frank Matthews, University of Houston.

1. Purpose

The purpose of the study was to compare the effectiveness of a traditional mode and an individualized mode of presenting college algebra.

2. Rationale

Along with many other departments across the country, the Mathematics Department of Bucks County Community College is involved in investigating alternatives to the traditional mode of classroom instruction.

3. Research Design and Procedure

The subjects were 150 students who registered for college algebra. These subjects self-selected themselves either into three experimental (E) sections which were labeled "Individualized Learning" (78 students) or into three control (C) sections (72 students). The experimental sections were organized around a series of 12 booklets partitioning the course content. Each booklet contained a list of objectives, preview test, pre and post self-tests, textual presentation of content with exercises, a list of alternate materials, and review exercises. Completion of each booklet was followed by an examination, with 70% required for continuation. Instructors conducted small-group discussions and individual tutorials along with completing administrative duties during class. Students completing more than two-thirds of the content were allowed up to an additional 8 weeks.

A 25-question multiple-choice pretest covering concepts considered to be prerequisite was administered and a *t*-test was used to test for initial differences between the groups. In the twelfth week of the semester, a locally modified attitude scale was administered and mean scores compared. At the end of the semester a 60-question multiple-choice final was given and the results compared. Finally, the success rates in the two approaches were compared.

4. Findings

On the pretest the experimental group had a mean score one unit higher than the score for the control group. This was not, however, significant at the 0.05 level.

For the 102 students who took the attitude test there was a difference, in favor of the experimental group, significant at the 0.02 level. Scores for the 98 students taking the final examination resulted in a higher mean for the experimental group which was significant at the 0.05 level.

A chi-square analysis of a contigency table for the number of students successfully completing the course indicated that significantly (χ^2=4.35, significant at 0.05 for df=1) more students completed the experimental program.

5. Interpretations

The investigator states, "The Individualized Learning System is a viable learning alternative." He notes that the experimental program discussed produced better attitudes, a higher completion rate, and higher final examination scores for the students involved in the study. Finally, he states that an instructor wishing to employ such an approach can be assured that students are achieving and learning at an acceptable level.

Critical Commentary

This article provides a reasonable description of a particular approach toward individualization and a reasoned defense of its viability. Readers should, however, use care if they wish to use it to show the superiority of individualized approaches over traditional methods. This is an error which the author carefully avoids in his conservative interpretation of the results--for which he should be commended.

The equivalence of the two groups is at least questionable. A level of 0.15 is quite stringent when the important error would be to assume equivalence where the groups are different. In fact, there is less than 1 chance in 5 that such a difference would occur if the existing scores were divided at random into two such groups. The potential for error here is at least high enough that some effort to control for an effect of pretest scores should have been made in the analysis of final examination scores. The result of that t-test was close enough (2.004 with a cutoff score of 1.985) so that any contribution from the pretest scores could greatly affect the significance level.

A different set of problems occurs with the attitudinal data. First there is a subject selection difficulty. Participants in the experiment did volunteer for a different kind of experience and may have begun with better attitudes. In addition, the design made no pretext of controlling for a Hawthorne effect which is likely in such a situation.

No discussion occurs of the sample-size variation contractions at each stage and the potential for error from such contractions. This is particularly strange when you consider that five students more successfully completed the experimental course than took the required final examination. The reason for this difference is crucial. If they were students who received "incompletes," they were not among the better students and their inclusion would reasonably be expected to have an effect on the t-test by lowering the difference of the means. In addition, if one were to compare the rate of successful completion within the semester (without the extra 8 weeks) one would get a X^2 of 1.94 which is not significant at any reasonable level. If, however, they were exceptional students for whom the final examination was not required the result would be strengthened. The reason for this shift in ns would be interesting.

Finally, the generalizability of the results depends on the amount of correspondence with the program studied. Individualized instructional systems vary in many parameters and little is known about which parameters are crucial. The system discussed utilized different text materials, more tests, retests, flexible scheduling of tests, variability in pacing, a variety of instructional modes, and up to an extra 8 weeks of time. A variety of subsets of these variables and some others have occurred in other studies of the same genre. Agreement has occurred that the alternatives are reasonable although not always cost-effective.

On the whole the article provides support for the instructors who wish to use a novel mode of instruction, and assists them in justifying the validity of such an approach. It does not, however, provide them with much guidance as to the optimal way to approach it. It is to be strongly hoped that further development in the area will lead to more such guidance.

ASPECTS OF TEACHERS DISCOURSE AND STUDENT ACHIEVEMENT IN MATHEMATICS. Smith, Lyle R. Journal for Research in Mathematics Education, v8 n3, pp195-204, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by John W. Gregory, University of Florida.

1. Purpose

The purpose of the study was to examine aspects of discourse of first-year algebra teachers in order to identify verbal behaviors related to student achievement from a lesson on direct variation.

2. Rationale

The author suggests that very little had been done in the way of verbal behavior analysis in mathematics classrooms. No rationale for the variables of analysis employed nor for selection of content was given.

3. Research Design and Procedure

Twenty teachers of first-year algebra were randomly selected from a large school system. One class of each teacher (selection not explained) served in the investigation. Each teacher agreed to teach a twenty-minute lesson which was to focus on (1) translating statements involving an expression of direct variation into mathematical equations, (2) solving for the constant of variation, and (3) solving for the remaining variable given the constant and a value for one variable.

The sampling unit of classes provided a sample of 455 students who received an administration of an investigator-written, 14-item test over the three objectives. Validity and reliability of the instrument were not disclosed. When the test was administered is also not clear.

Mean class scores and adjusted mean class scores (utilizing the California Achievement Test scores on file as a covariate; timing of the CAT administration not explained) were subjected to correlational analyses with the following verbal behavior measures serving as independent variables:

(1) frequency and percent of relevant examples presented

(2) a mean rating score (from 1 to 10) combining the number of lesson objectives the teacher attempted to meet, a rating of "degree to which the objectives were dealt

with," and the "degree to which lesson objectives were focused on and the sequence in which they were dealt with"

(3) average frequency of "OKs" per minute of teacher talk

(4) frequency of irrelevant examples

(5) average vagueness terms per minute of teacher talk (including ambiguous designation, negated intensifiers, approximations, error admission, indeterminate quantification, multiplicity, and possibility statements)

(6) total lesson time (undefined, it seems to include some student work-time)

(7) total examples and application (both relevant and irrelevant)

(8) average mazes per minute of teacher talk (including false starts, redundancy, and tangles of words)

(9) total frequency of teacher-initiated responses (definition unclear)

(10) total frequency of student-initiated responses

The independent measures were obtained either from actual timing or from trained coder ratings of typescripts. Three coder interpretations were averaged for each variable for each lesson. Reliability of the coders was determined and maintained upon the basis of percentage of agreement on the total frequency of each variable and percentage of agreement on the location of instances of the behavior.

4. Findings

Mean performance on the achievement test and measures constituting the independent variables were sufficiently frequent and variant for analysis. Some of the ranges of interest to the reviewer were "average 'OKs' per minute" - 0.00 to 4.92 (one teacher used in excess of 110 "OKs"); "relevant examples" - 4.0 to 49.0; "teacher-initiated responses" - 4.0 to 153.0; and "percent teacher talk" - 55.8 to 94.0 percent.

Correlation coefficients, using an unidentified a priori alpha risk, exhibited the following significant (reviewer-defined $p < .10$) relationships with class achievement:

- frequency and percentage of relevant examples ($\underline{r}$ = .65)
- lesson objective rating ($\underline{r}$ = .58)
- average "OKs" per minute ($\underline{r}$ = .44)

- number of irrelevant examples ($r = -.52$)

- average vagueness terms per minute ($r = -.41$)

5. Interpretations.

It is the investigator's contention that the three variables relating positively with achievement involve "...organization, structuring and clarity of lessons." The lesson objective rating and use of relevant examples were thought to provide evidence of careful planning and execution. An analysis of the use of "OK" revealed that "OK" was "used as a form of punctuation mark by some teachers, an 'OK' at the beginning of a statement indicating that a new train of thought had begun, and an 'OK' at the end of a statement indicating that the statement was complete."

The appearance of a negative correlation between frequency of vagueness terms and achievement is consistent with work done by investigators of social studies classes referred to by the investigator.

Critical Commentary

Having been revised as recently as 1976, it is unfortunate that the author did not include a consideration of the numerous investigations which have been conducted relative to verbal behavior analysis in mathematics classes. The reader, and author, should consider the work completed and being continued by individuals associated with the Georgia Center for Research on Teaching and Learning Mathematics. An inspection of this work would reveal that the study under review has just touched upon some of the more sophisticated analyses and results of others in mathematics education.

Questions brought to mind by both the procedures and results of this study are:

(1) Is same-day testing appropriate in light of the necessity for student practice? Would retention testing lead to similar results?

(2) Is linear regression an appropriate technique of analysis for frequency measures? Can we really assume that the highest frequency is the optimum?

(3) Is it the frequency of relevant examples or their placement relative to definitions and practice which lead to achievement? Would analysis of the occurrence of relevant non-examples along with relevant examples lead to more interpretable findings?

(4) Is the rate (average frequency per minute) an appropriate measure in view of varying individual speech patterns? Does averaging averages of coder scores in order to produce a rate-per-minute compound the problem of interpretation?

(5) Can habitual language ("OKs") and vagueness terms be replaced functionally by use of wait-time (silence)?

THE EFFECTS OF THREE INSTRUCTIONAL STRATEGIES ON PROBLEM SOLVING BEHAVIORS IN SECONDARY SCHOOL MATHEMATICS. Vos, Kenneth. Journal for Research in Mathematics Education, v7, pp264-276, November 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Stephen S. Willoughby, New York University.

1. Purpose

To investigate the effects of three instructional strategies (repetition, list, behavioral instruction) used to increase the effective use of five problem-solving behaviors (drawing diagrams, approximating, constructing equations, classifying data, constructing charts).

2. Rationale

Problem-solving ability is generally recognized as an important goal of mathematics education. Most research on the subject has concentrated on translating word problems into number sentences and on answers. This research was designed to study ways of affecting the methods of solution.

3. Research Design and Procedure

The independent variable was instructional strategy. The three strategies were:

(1) Repetition: Only problem tasks were given.

(2) List: The problem task was presented. A checklist of suggested procedures to follow was provided and children were told to check off all procedures they "tried or thought about trying." Some behavioral instruction was provided and they were instructed to return to the problem task.

(3) Behavioral Instruction: Behavioral instruction was provided and then the task was presented.

Five behaviors identified in a pilot study and taught in the treatment period were: drawing diagrams, approximating and checking, writing equations, classifying data, and constructing charts.

Instructional material was presented through self-directed, written material in the form of twenty problems (about 20 minutes each) used over a fifteen-week period. Six classes at three grade levels in a private school in Iowa were involved. The classes (with numbers of students) were: Algebra II (25); Geometry (29); Algebra I, section 1 (22); Algebra I, section 2 (28); Math Survey (21); and Elementary

Algebra (8). The total N was 133. Low-ability students were assigned to the last two classes. The very small number of students in Elementary Algebra resulted from unexpected schedule conflicts. In each case, students were randomly assigned to one of the three treatments.

The dependent variables were:

(a) Scores on STEP forms 2A and 3A, Mathematics Part II (pretest) and Part I (posttest).

(b) Scores on a Problem-Solving Approach Test (PSAT) constructed by Vos (posttest). The students were not required to solve the problems, but rather were to choose from a list of five approaches the best and next-best approaches to each problem.

(c) Scores on a Problem-Solving Test (PST) apparently constructed by Vos (posttest). Students were to show their work and were scored both on a right-wrong basis and partial score basis.

Analysis of variance was used to analyze the data. For the PST there also was an analysis of the five instructed problem-solving behaviors and of other problem-solving behaviors. For the PSAT, a Newman-Keuls method was used to analyze the differences between all pairs of posttest means within each class.

4. Findings

There were no significant differences between treatment groups on the STEP. The author reports differences significant at the .20 level for three of the six classes on the PST. On a purely descriptive basis, the author reports that in the low-ability classes the list treatment had the highest proportion of occurrences of the taught behaviors, while in the other algebra classes the behavior instruction treatment had the highest proportion, and in the geometry class the repetition treatment had the highest proportion of occurrences.

Some differences on the PSAT were reported at the .05, .10, and .20 levels of significance generally favoring the behavior instruction and the list procedures over the repetition.

5. Interpretations

The author calls attention to the unusual nature of the tests he constructed and the instructional materials. He mentions the short time of treatment and the possibility that some differences reported were more a result of regular classroom instruction by the teacher than a result of the treatment. He also calls attention to the fact that students in more advanced classes tend to be better problem solvers.

Critical Commentary

Presumably, if we knew that certain strategies were more useful than others, knowing how to get students to use them would be important information. Therefore, the question asked in this study appears to be a reasonable question. However, forcing students to choose strategies from a multiple-choice list seems to be a doubtful procedure for determining what strategies they use. In fact, I used a strategy of counting on one of the test items shown and of drawing a picture on the other and in neither case was my strategy listed as a possibility. I suspect some students may have had a similar difficulty. A case study procedure would seem to have been more appropriate than the experimental procedures used, especially in light of the "fishing trip" nature of the entire study.

As to the statistical procedures used, given the large number of times analysis of variance was tried, the omission of post analysis of variance tests, the apparent complex lack of any hypotheses, and the willingness of the author to report levels of significance as great as .20, there appears to be a rather high probability that the reported results could have been produced by a set of random variables.

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