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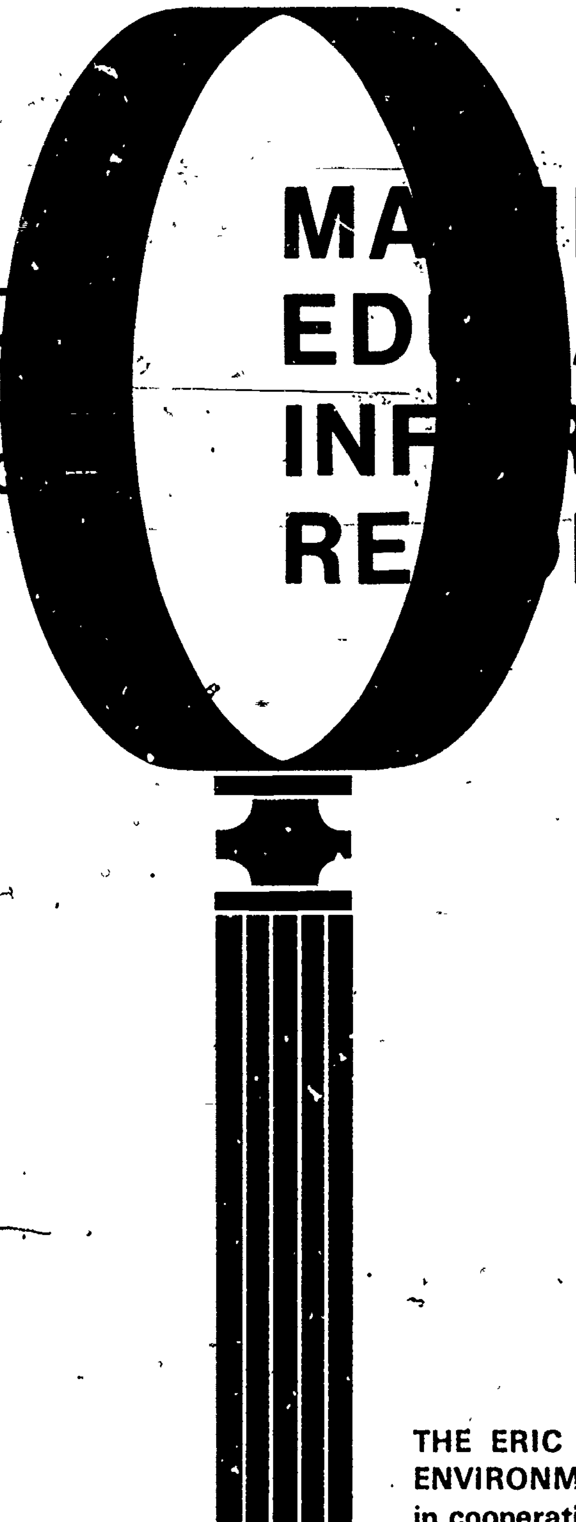
ABSTRACT

The papers in this publication were developed from speeches and reactions presented at the first National Conference on Remedial Mathematics held at Kent State University in 1974. Papers focus on identifying and describing the remedial mathematics student, classroom diagnosis, clinical diagnosis, the diagnostic process, and promising procedures and directions in remediation. In addition to reactions to each paper, a conference summary is included. (MS)



MATHEMATICS EDUCATION INFORMATION REPORT

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MATHEMATICS EDUCATION REPORTS

Papers from the
First National Conference on
Remedial Mathematics

REMEDIAL MATHEMATICS:

DIAGNOSTIC AND PRESCRIPTIVE APPROACHES

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Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the associate director.

New awareness of children's difficulties with arithmetic computation and new emphasis on basic skills in mathematics have led to renewed interest in the areas of diagnostic and prescriptive procedures for remedial work in mathematics. The papers in this publication were developed from speeches and reactions presented at the first National Conference on Remedial Mathematics held at Kent State University in May, 1974. Annual conferences on Diagnostic and Prescriptive Mathematics have been held subsequently, and interest in this area continues to grow.

The ERIC Center for Science, Mathematics, and Environmental Education hopes that the publication of these papers will encourage the continued increasing interest in remedial mathematics.

Jon L. Higgins
Associate Director for
Mathematics Education

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IDENTIFYING AND DESCRIBING THE REMEDIAL MATHEMATICS STUDENT

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Schemes which attempt to provide individualized diagnostic and prescriptive instruction have become increasingly popular. A larger number of centers and clinics have become available in an effort to provide specialized help for children having difficulty with mathematics. At colleges and universities an increasing number of courses and programs are being designed to give teachers and specialists training and experience in remediating the needs of children with disabilities in mathematics. Therefore, it seems especially timely that we not only reflect upon the current state of that part of mathematics education which is concerned with the remedial mathematics student, but that we also consider definitions of the terms we use, and theoretical constructs which may help guide our research and service activities in the future.

Which children are we concerned with at this conference? Who is the remedial mathematics student after all? How would we know him if we saw him? Why do we call him "remedial"? Are we merely talking about children who are having exceptional difficulty with mathematics? We hear about children with "special needs" and about "reluctant learners". Are these children remedial mathematics students? The profusion of terms such as "reluctant learners" illustrates, at least to some extent, the popular use of categories which have not been well-defined. Research and professional literature have been plagued with such ambiguity, and among our challenges at this conference is the attempt to find a bit of light at the end of this dark tunnel.

Though my assigned question, "How can we identify and describe the remedial mathematics student?", makes careful use of Walbesser's action verbs in a way popularized by the American Association for the Advancement of Science,[1] the question poses a dilemma for me. For, after all, which do we do first? Do we arbitrarily set forth a definition of a category which we choose to name "remedial mathematics students", then identify children which fit the category we have carefully defined? Or do we, through gross screening procedures, identify children which we are pleased to call remedial mathematics students, then carefully observe and describe those children? We have here elements of the traditional chicken-and-egg controversy, for how can we describe children we have not identified as belonging to that class; and at the same time, how can we identify children when we do not have any description of them, i.e., we do not know what to look for? Definitions are of necessity arbitrary.

As we think about the question, "How do we identify and describe the remedial mathematics student?", let us first direct our attention to the children we are concerned about, and consider the nature of our involvement with them. In this connection we will seek to define the remedial mathematics student and describe major categories, particularly in relation to the concept "remedial." We will also consider an organizational structure which reflects these categories. Finally, we will take a more detailed look at some structures which should eventually enable us to refine procedures for identifying remedial mathematics students.

As we attempt to define the remedial mathematics student, we can dismiss rather quickly any consideration of the definition sometimes applied in practice whereby the remedial mathematics student is the student who scores 1.5 or perhaps 2.0 or more years below grade level on a standardized mathematics achievement test. We know that the reasons for achieving at a lower level are so varied that this definition has limited usefulness for research or for planning instruction.

It is more common to think of the remedial mathematics student as the student who is underachieving in mathematics. In the past, a rather widely accepted measure of underachievement has been a discrepancy score comparing actual standardized achievement test results with anticipated achievement as determined by a test of aptitude or mental ability.

Though this concept of underachievement is useful as one consideration in making a gross selection of children needing extra help, it is a much maligned concept. Its inadequacy comes, in part, from the inadequacy of attempts to measure aptitude or mental ability. Those who maintain that no child overachieves find it difficult to conceive of a child underachieving. Robert L. Thorndike, in his paper entitled The Concepts of Over- and Underachievement, rather effectively destroys the traditional concepts of over - and underachievement, or at least reduces them to a problem of failing to predict achievement.[2]

Even so, the dilemma remains for the practitioner who must somehow make an initial selection of children needing extra help in mathematics. It is in this context of a school setting that Wilson defends the use of discrepancy scores for screening purposes. Though errors in measurement admittedly result in the inclusion of false negatives and false positives, the use of discrepancy scores does reduce the population so the practitioner can get started with the task of helping needy children. When a school uses aptitude and achievement tests which are both normed on the same population, the task of screening can be facilitated by use of the computer and/or tables provided by the test company.

Developmental and Remedial Needs

Another approach to screening follows from the definition of the remedial mathematics student posed by Hutchings. To begin, Hutchings proposes that we make a theoretical distinction between two kinds of needs. We shall call these developmental needs and remedial needs. In making this distinction, Hutchings defines developmental as having to do with the normal progression of the child's maturity. It may be rapid or it may be slow in relation to his peers. If it is slow, he is thought of as a slow learner or something like this. He may have very real needs, but we are not to classify those needs as remedial needs.

Developmental needs have to do with relating the child to either society or to some abstract model of maturation; they have to do with making him consistent with his fellows or with theoretically determined expectations. The child with developmental needs may learn slowly with reference to his peers but normally for himself. We see then that such developmental needs have much to do with a child's global maturation and with his progress when he begins something new.

Remedial needs, as Hutchings defines them, have to do with a deficiency which occurs in a child in some particular area in relation to other areas of his person. It is a deficiency that occurs in some part of a child's person that makes that part different than other aspects of his person. The child with remedial needs is not developing in mathematics at the rate we would expect for him in terms of his development in other areas. For example, a fourth grader who was finding all of his work very difficult would have developmental needs. A child who had progressed to the middle of fourth grade who was doing average work in all of his subjects and getting no place in mathematics would have remedial needs. This emphasis on all developing aspects of the child is reminiscent of Willard Olson's concept of "organismic age."[3]

Admittedly, the remedial child, as herein defined, and the underachiever, have a fairly large intersection; but it is not quite the same thing. There is an important potential disjunction. A critical point is that the remedial definition be based upon actual observed work, i.e., upon a child's classroom performance; whereas the underachieving definition requires instruments intended to measure psychological constructs of either global aptitude or quantitative and verbal aptitude. The measurement of aptitude is still rather primitive both in theory and practice. However, if we have a child who is doing well in reading and in the social sciences and in composition and in other subjects, but he is not doing well in mathematics, he has a remedial need in math even if his quantitative score on an IQ test is low. The point is that aspect of his achievement is different from other aspects of his achievement. In summary, Hutchings defines the remedial mathematics student as the student who has a selective deficiency in mathematics as this is displayed in his schoolwork.

Now, how do we decide when a child has or doesn't have a remedial need? Having defined him, how do we identify him? In accordance with the definition proposed, we will need to determine how a child is performing in each of several areas. Hutchings proposes that we incorporate two variables into our decision about the performance of a child. The first one is what the teacher says the child does, and this is the primary criterion of need. The other variable is achievement test results. These should not be ignored, but are a referent of occasional usefulness, especially in the location of inconspicuous affective dissonance. The judgment of a classroom teacher in this situation is essentially a clinical judgment based upon observations of the child over a period of time. To some extent, the grades given by a teacher will reflect his judgment about the nature of a child's actual performance. Scores on achievement tests are of special interest as their profile varies from the classwork profile. We see then that with Hutchings' definition of the remedial mathematics student, we are still likely to include false negatives and false positives, but now our errors of assessment are apt to be primarily errors of judgment rather than errors of measurement.

In view of the limited time and resources we have available for working with students having exceptional difficulty with mathematics, it is necessary to set priorities. If we are to maximize the social impact of our remedial service, then we must be careful to choose those whom we are most likely to benefit. The rule must be the greatest good for the greatest number. That is, we have to find the kind of child that we can service quickly and well so we can service as many students as possible. This is a morally complex issue. It's like a physician who must decide who gets a kidney machine, and it's a decision that no one has the right to make; but in a sense if nobody makes it the situation is worse than if somebody tries to make it, and that is awkward. Because that is the structure of the moral situation, we do presume to say that a child who is doing well in other things and is selectively weak in mathematics is likely to be a child that we can help. He is likely to show the greatest improvement in the least time. The reason for that is, that if he is doing comparatively well in other subjects, he probably already has parental support, some good study habits, and a reasonably positive attitude toward school. We can focus specifically on his mathematical knowledge and skills and not involve ourselves in what is essentially a massive developmental change involving his attitudes toward everything. Instead, we select the type of child that will allow us to maximize our social impact; and this is probably the child who (1) is no more than "below average" on most measures in relation to his peers, and (2) has a selective deficiency in mathematics.

As we attempt to identify remedial mathematics students as defined by Hutchings, we will observe that there is sometimes an intersection of developmental and remedial needs. In such situations we must allow the developmental category to have priority. What we are talking about now primarily is a hypothetical selection scheme for investing the limited time of specially trained diagnosticians and tutors. If a child has critical developmental problems in reading and language then we have to consider that his primary needs are developmental, even though in fact he does have a remedial mathematical situation in relation to the rest of the development. In relation to his peers he is a developmental case. Within that developmental case a remedial situation exists, but we can not say he is a remedial case. He is primarily a developmental case.

A Mathematics Clinic

It would be appropriate to make some speculations about the remedial mathematics student, as defined by Hutchings, and how he fits into the organization and procedures of a mathematics clinic. Such an agency can have very different kinds of functions, not all of which are directed toward the immediate social needs of children. Hutchings proposes three functions, each with its own administrative unit.

As we consider the first function, we do well to observe that no activity of a mathematics clinic is all service or all research; but some activities are more of one than the other. The function of the clinic which is primarily service-oriented has to be adjusted to maximum social impact by definition. This selects for the child who is generally remedial as Hutchings had defined the remedial mathematics student, i.e., the child who will show the greatest improvement in the least time. This is the child with a specific deficiency in mathematics and some general competence in other areas. It is a socially directed selection.

Another possible function of a mathematics clinic is the more usual clinical undertaking in which you deal with the child, but your interest is much more balanced between the service and research functions. In this kind of operation you might have some remedial students if their remediation were of special interest. You would also have some developmental children, and you would become much more involved in the pathological structure of a child's deficiency. In addition you would have the more heavily impaired child, or possibly a genius. In short, you would have more drastically different children than a center emphasizing the service function.

A third possible function of a mathematics clinic focuses more exclusively on seeking knowledge. In the organizational scheme of things, this might be called the Special Cognitive Section. Coming to this clinic will be children who are <u>very</u> bright or <u>very</u> different. For example, an idiot savant might be the subject of investigation. Categories such as "the remedial mathematics student" are not useful here because by definition if the child is different enough to be here he doesn't fit into recognized categories. This is apt to be a small arm of the clinic, involved heavily with individuals on a one-to-one basis, probably with instruments created by the clinic. The closest existing model for this sort of clinical function is portrayed in the recently translated Russian papers published by the School Mathematics Study Group.[4]

Figure 1 portrays the three clinical functions which have been described. These could be three sections of an administrative unit called a clinic. The Diagnostic Section, pictured in the middle, would be concerned primarily with students having developmental needs; and interests in that section would be about one-half research and one-half service. We would expect a majority of the children to be in this middle section where it is clinical in the usual sense. They would have primarily developmental problems so we would expect more global impairments and more pathologies. The testing program is likely to be exhaustive, a part of a very thorough diagnosis. It is likely that we would have interdisciplianary inputs as well, drawing upon the expertise of specialists in reading and special education, and conferring with psychologists, psychiatrists, neurologists, and the like. It is a format in which we do not have to think about maximizing social impact. We just concentrate on the child.

On the left is depicted the section with remedial students. They are of very limited research interest though they do have specific mathematics disabilities. Here we find a somewhat higher ratio of student to clinician. On the right we see the section focusing on cognitive research exclusively; the focus here is not social, nor is it so much the child; rather, the focus is on increasing knowledge.

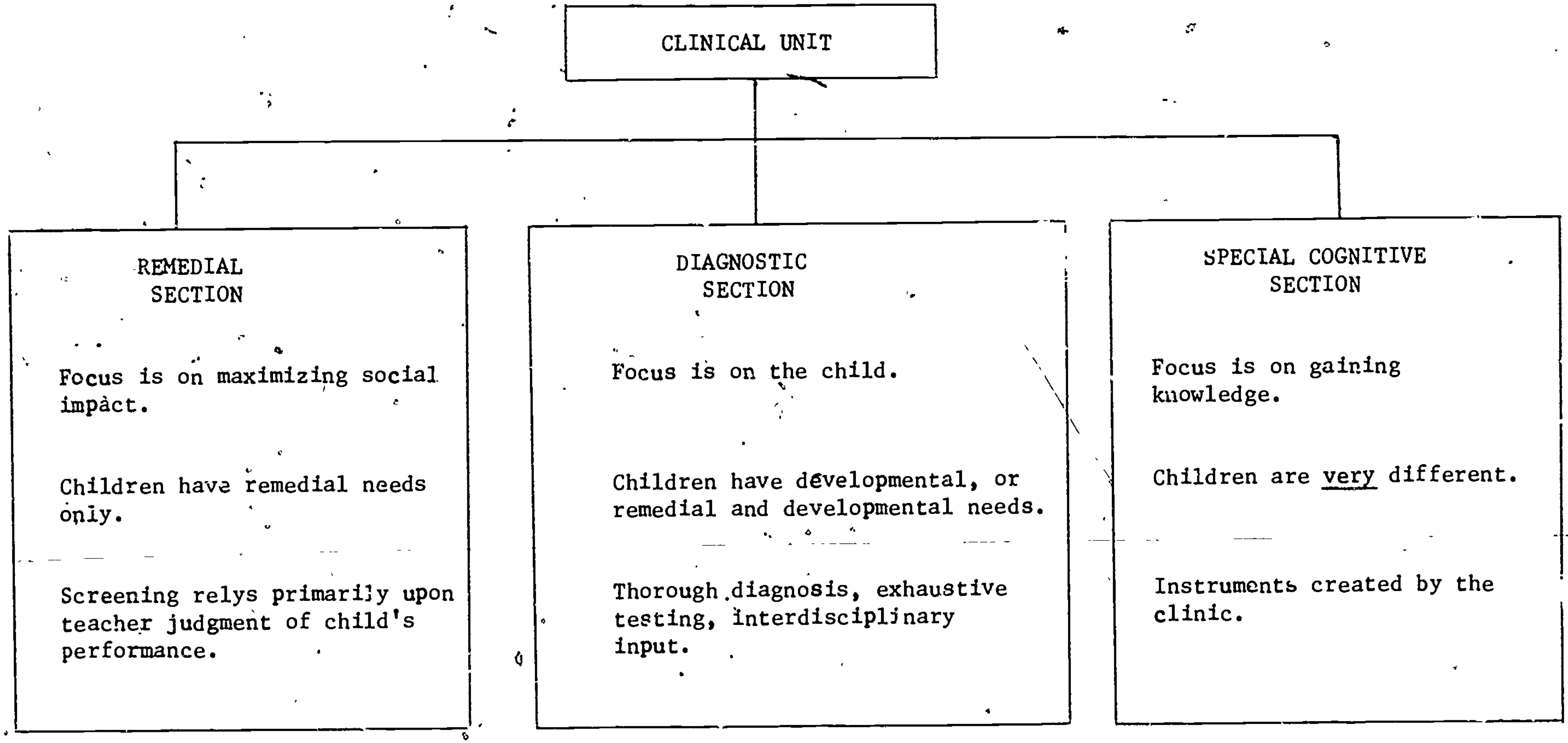

Figure 1

Organizational Chart for the Clinical Unit of a Mathematics Learning Center

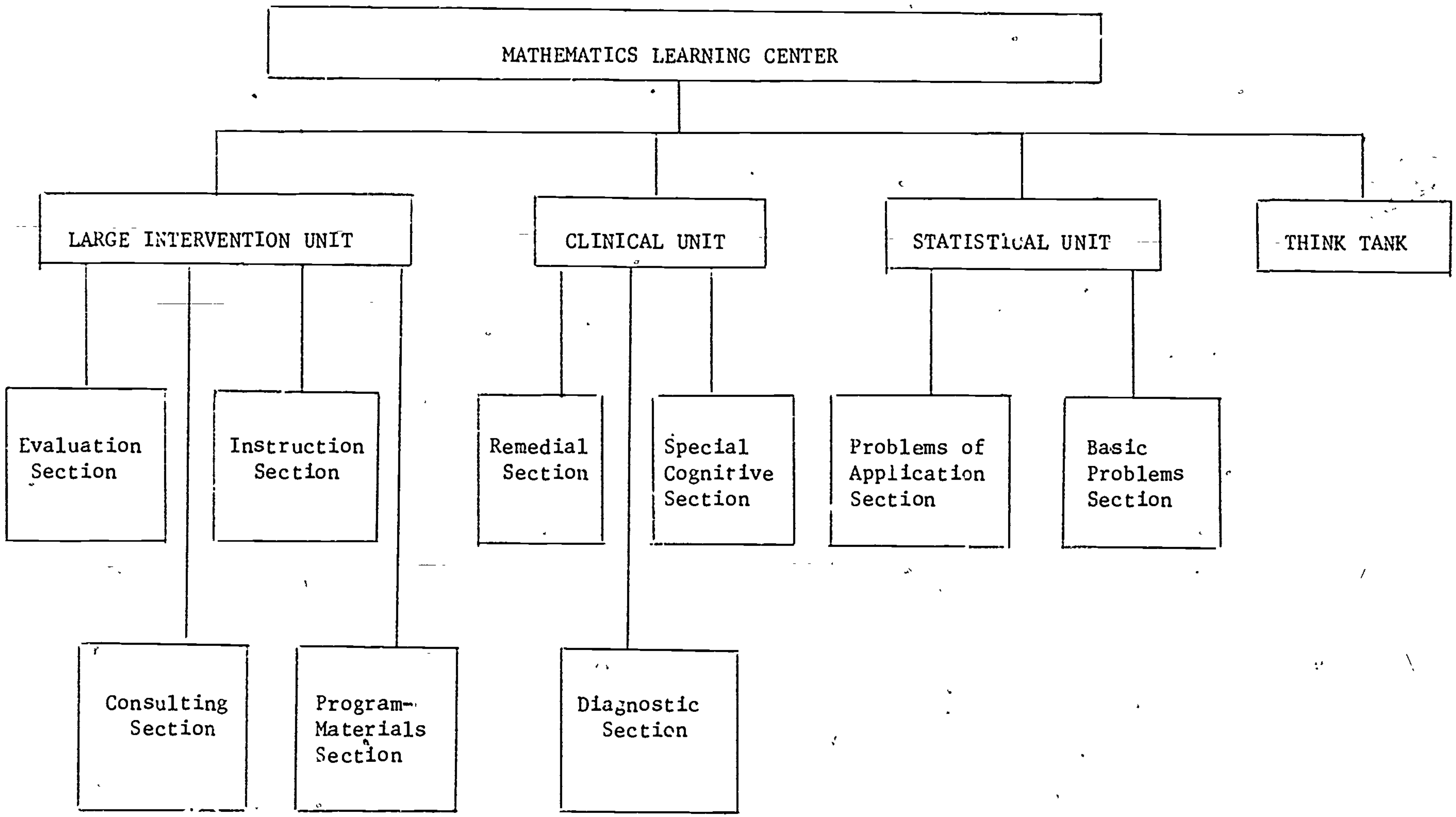

Figure 2

Organizational Chart for a Mathematics Learning Center

The organizational chart in Figure 2 shows how such a clinic could fit into a larger operation, one we might call a Mathematics Learning Center. It would be appropriate to think of the Center as an information processing agency which coordinates several arms. Note that the Special Cognitive Section of the Clinical Unit deals with a very small number of children. For example, the Russians found that even during initial instruction some students appear to omit or compress steps in mathematical sequences, and their solutions are enormously accelerated by this.[5] The Clinic might find one, two, or three children who make this temporal compression quickly. During interviews with these children researchers would find out if the children were aware of this compression, if they know when it starts, if they can extend it to other operations, or if it tends to occur with only particular mathematical processes. In this section there is a very heavy involvement with individuals specially selected for the study at hand. In contrast, the Basic Problems Section of the Statistical Unit might work with 30-40 children in the conduct of highly controlled experiments like classical psychological experiments. These studies would involve samples of the population, the administration of treatments, and careful exercise of controls.

Taking a larger view of the Mathematics Learning Center we see pictured in Figure 2, we note four major units. The Large Intervention Unit is very field oriented, providing non-statistical professional evaluations about areas of a school's mathematics program needing attention, consulting services outlining steps that need to be taken to improve the learning of mathematics, in-service instruction in a workshop format where teachers actually see demonstrations, and actual program materials which will be helpful in the local instructional program. The major involvement with remedial mathematics students is in the Clinical Unit, which we have already described. In the Statistical Unit one section focuses on problems of application; for example, statistical studies of the introduction of experimental materials in field situations. The other section conducts research in controlled, laboratory situations, studies which are in the mode of basic psychological research. A Think Tank Unit completes the Center.

We have offered a definition of the remedial mathematics student as the student with a selective deficiency in mathematics, and we have contrasted this category with students having developmental needs. We also considered the identification of such children and suggested the performance of children as judged by classroom teachers as the primary criterion. The ensuing discussion explored a possible structure for a mathematics clinic and the place of the remedial mathematics student within that clinical structure.

Let us now take a more detailed look at some structures involved in identifying and describing remedial mathematics students. As we do so, it will be helpful to make a fresh approach to the question, "How do we identify and describe remedial mathematics students?"

Structures Involved in Identifying and Describing Remedial Mathematics Students

It is not unreasonable to offer the premise that all children need diagnostic-prescriptive teaching. Further, experience prompts us to believe that almost all children need some kind of extra help at one point or another, with some children needing more help than others. But the amount of time that classroom teachers or specialists have available is limited, and they need some basis for deciding which children to spend extra time with. In this context then, it is useful to simply ask the classroom teacher, "Which children need special help in mathematics?"

To say that such a selection is useful for gross screening purposes does not mean that it is sufficient or that it is without serious limitations. Teacher judgment is accurate only within the parameters the teacher is judging. If a teacher's judgment is based on a special concern for computational skill, he or she may overlook a child's ability to analyze verbal problems. Conversely, if a teacher's judgment is based on a child's ability to analyze verbal problems, the child's computational facility may be overlooked. Given an opportunity to teach a geometry unit to fourth graders among whom the teacher had clearly identified some as needing special help with mathematics, one of the authors found that he would have selected a different set of children because his brief experience with them was limited to geometry; and geometry was not valued highly by the classroom teacher. We all value different parts of a mathematics program differently and out values will affect our judgments about which children need special help in mathematics.

In making a gross selection of children to whom we will give extra help, we may want to consider achievement test results. Comprehensive achievement tests can at least facilitate the communication which is lacking when teacher judgment is based upon unstated values. Such tests can be examined and their weaknesses found. Further, they can be supplemented. Because they tend to assess product, i.e., the result of a child's thinking, they need to be supplemented with assessment of the child's thinking processes. As has already been indicated, Wilson defends discrepancy scores between measures of aptitude and achievement as another indicator which may be considered when making very gross screenings.

It is very likely that the number of children which will be identified is rather large. Further, as we try to help such children we soon realize that we are not as well equipped to help some of them as we are to help others of them. How do we go about finding out which of the children who are not doing well in mathematics we are best equipped to help? How do we find out which children actually need help form other professionals?

For a given child, several sets of factors are involved in his difficulty with mathematics, and we are better equipped to remediate some of these factor sets than others. It therefore becomes necessary to utilize and/or develop instruments and procedures which will help us decide which factor sets are involved most heavily in a child's difficulty with mathematics. Such categories will facilitate our description of a student having difficulty with mathematics; they will also make it easier for us to decide which students we are best equipped to help. Wilson suggests four such factor sets: subject matter factors, instructional factors, learner-organismic factors, and environmental factors.

Subject matter factors relate to the mathematics a child learns. They concern those products of his learning which we identify with mathematics. Instructional factors relate to considerations such as amount of guidance and choice of models or exemplars. Learner-organismic factors are both physical and psychological, including factors as varied as considerations of learning style and specific neurological impairments. Finally, environmental factors relate not only to the settings in which formal instruction transpires but to the child's total experience with his environment. The teaching style of a teacher, e.g., the teacher who prefers to use didactic procedures rather that guided discovery procedures and the achievement orientation of parents are among factors in the child's environment which are involved in his learning of mathematics.

If such factor sets are to help us know to what extent we may be able to help a child and to what extent the help of other professionals is required, it will be necessary to articulate such factor sets. Once articulated, each factor set implies a set of diagnostic tasks or questions which should be useful in screening children having difficulty with mathematics. It will, of course, be necessary to translate such diagnostic tasks or questions into diagnostic and prescriptive procedures. Hopefully, we will be able to find or construct guiding models to help with these translations.

One of the instructional factors we might articulate is the type of exemplar used. These vary from everyday things we touch and move to highly abstract symbols, but they all can be used to exemplify the mathematics under consideration. The diagnostic task or question implied by a consideration of the type of exemplar used is, "Which exemplars are most productive with the child and which inhibit?" By asking this question we recognize that a given exemplar may or may not aid cognitive learning for a given child. We recognize that a child's affect is also involved with a given exemplar. Is there a model which will help us develop both diagnostic and prescriptive procedures with reference to the instructional use of exemplars? An expansion of an exemplar matrix developed by Edward Uprichard of the University of South Florida has proven useful to us at the University of Maryland (see Figure 3).[6] The two dimensions of the matrix are concreteness of the exemplar and sensory involvement. It may be determined that a given child responds well to instruction using exemplars suggested by certain cells of the matrix, but he does not respond well to instruction using exemplars suggested by other cells of the matrix. It should be noted that the description of a child which results from such diagnostic activity provides very useful guidance for instruction.

For another example let us consider a subject matter factor, specifically, the category of learning product. The diagnostic question implied by this factor is, "Which categories of learning products are causing the most difficulty for the child?" Learning products can be categorized in at least two ways: mathematically and psychologically. This is, a given learning product can be viewed within a set of mathematical categories such as the concept of set, or algorithms for operations on non-negative rationals; and the same learning products can also be viewed within a set of psychological categories such as concepts or principles. In our need to categorize learning products as we create diagnostic and prescriptive procedures, what models are available to guide us? Wilson is developing a content taxonomy, a summary of which appeared in his article with Vincent Glennon in the NCTM 35th Yearbook, The Slow Learner in Mathematics.[7] This is not a complete taxonomy, but we have found it to be very useful. One articulation of psychological categories is provided by Gagne in his The Conditions of Learning, in which eight categories or types of learning are described.[8]

	Concrete	Representative 3d	Representative 2d	Symbolic
Visual	1. VC	2. VR	3. VR	4. VS
Auditory	5. AC	6. AR	7. AR	8. AS
Tactile	9.	10.	11.	12.
Kinesthetic	13.	14.	15.	16.
Olfactory	17.	18.	19.	20.
Taste	21.	22.	23.	24.

Figure 3. Exemplar Dimensions: Sense Mode X Objectivity

Another subject matter factor is level of abstraction or generalizability. Here we are concerned with the different ways a child might think about a given learning product. At what level of maturity does he think about a problem in mathematics? With reference to level of abstraction or generalizability we might ask if a child can translate across a range of exemplars. Is he successful only when using manipulatable materials, or only when writing symbols on paper? Are there models available which will help us develop diagnostic and prescriptive procedures taking account of a child's level of abstraction with reference to specific learning products? The assessment of learning products inevitably relies upon observation of behaviors which may indicate acquisition of the mathematics under consideration. Such behavioral indicators can be categorized to suggest various levels of abstraction or generalizability. Bloom's cognitive taxonomy is a useful model here.[9] In the Glennon and Wilson chapter cited above, this use of Bloom's taxonomy is illustrated.[10]

Yet another subject matter factor is level of difficulty of the learning product. Wilson suggests there may be at least three different dimensions of level of difficulty. First, level of difficulty can be thought of in terms of the number of prerequisite leaning products. More extensive and complex hierarchies of such prerequisite learning products obtain for some learning products than for other learning products at a given level of abstraction. We can infer greater difficulty for those learning products having the greater number of prerequisite learning products. Learning hierarchies, similar to those described by Gagné, can be useful in this analysis.[11] However, the assignment of number to prerequisite learning products has not yet been accomplished in any replicable fashion.

Level of difficulty can also be thought of in terms of the number of applications of each learning product. For example, how many times does a concept (such as fiveness) appear in one form or another in a given problem? This dimension of level of difficulty is, admittedly, related to Suppes NSTEPS variable, but focuses on the number of times the <u>same</u> concept appears in a task.[12]

A third dimension of level of difficulty is in terms of discriminability of learning products. Learning products can be viewed along a continuum of discriminability, with the least confusable being the least difficult, and the most confusable being most difficult. For example, higher decade addition may be more difficult after children have been introduced to multiplication of a two-digit number by a single digit number because the problems look very similar. The algorithms may actually be confused (see Figure 4).

```
      6   5           6   5
      x   7           +   7
  -----------     -----------
  4   5   5       1   4   2
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Figure 4

It is possible that the more difficult basic facts of arithmetic may be as difficult as they are, at least in part, because they are more easily confused with other basic facts than are some of the easier basic facts. This dimension seems to involve phenomena the learning theorists refer to as proactive or retroactive inhibition. Some sample factors, implied tasks or questions, and models available for guiding the creation of diagnostic and prescriptive procedures, as discussed above, are summarized and extended somewhat in Table I.

It should be noted that some of these models are useful in classroom situations even now. We have observed teachers clustering children for instruction by the categories in Wilson's content taxonomy. We have also seen teachers index instructional resources by the taxonomy. As a result, a teacher working in a given area such as multiplication of integers can quickly find relevant journal articles, exemplar suggestions, etc. We have even seen teachers design hierarchies similar to Gagné's learning hierarchies, and use these for creating diagnostic tests and sequencing instruction.

Conclusion

We started with students identified merely as needing special help in mathematics. We were then confronted with the fact that not only is the number of such students large, but we as mathematics educators are better equipped to work with some of these students than with others. Therefore, we focused our thinking on categories which can help us describe a given student more adequately, sets of factors relevant to his learning of mathematics.

This attempt to better articulate symptomatic and etiological syndromes of learning difficulties in mathematics is an attempt to clarify the field that is implied by the question, "What factors (and in what combinations) facilitate or inhibit learning for a particular child?" The setting implied is a clinic focusing upon research and professional training. A large number of children are not needed; what is needed is a few children and thinking people working with them. As those who work with children develop a clearer picture of how different factors relate to a child's learning of mathematics, they can also develop diagnostic procedures. For a given factor, these procedures will help identify which categories and levels obtain with reference to specific children; and by using such procedures it will be possible to modify screening procedures to make them more effective.

A final word of caution seems appropriate. Any attempt to identify and describe the remedial mathematics student inevitably reflects a view of curriculum. Glennon has attempted to picture the tension that exists when we ask the question, "What mathematics?"[13] He suggests a triangular model portraying the tension that exists between emphasizing (1) mathematics which is a series of related ideas, (2) mathematics which is necessary for the business and common life situations of the adult population, and (3) mathematics that eventuates from the expressed needs of the child. In NCTM's yearbook, The Slow Learner in Mathematics, Glennon applies his model more specifically as he asks, "What mathematical knowledge is of most worth for the slow learner?"[14] We may find Glennon's model to be helpful as we ask a similar question, "What mathematics is of most worth to the remedial mathematics student?" As we agree upon a definition of the remedial mathematics student, will our definition, in essence, specify the student having difficulty understanding mathematics as a system of related ideas? Will our definition of the remedial mathematics student select for the student having difficulty with that mathematics reputed to be most essential for the business and common life situations of the adult populations? Or will we somehow attempt define the remedial mathematics student in such a way that we, in essence, specify the student who does not want to learn mathematics? Our judgments, whether based upon observations of classroom performance or upon more formal test procedures, will reflect our views of curriculum. We tend to see what we value highly. We tend to test what we think is important. It may be that before we can more adequately come to grips with the problem of identifying and describing the remedial mathematics student, we will have to face difficult questions such as: How important is it that each student understand the mathematical relationships involved at every point in his study of mathematics? How important is it that every student be able to do the mathematics needed for the business and common life situations of the adult population? What mathematics is needed in a day of mini-calculators? And how important is it that every student like mathematics?

TABLE I

Sample Factors, Implied Tasks, and Models Available for Creating Diagnostic and Prescriptive Procedures

SAMPLE FACTOR SETS	IMPLIED TASKS (Questions)	MODELS AVAILABLE
1. Subject matter factors		
a. Category of learning product	Which categories of learning are causing the most difficulty for the child?	Wilson's content taxonomy and Gagné's types of learning
b. Level of abstraction or generalizability of the learning product	At what level of abstraction does the child think with reference to a specific learning product?	Bloom's cognitive taxonomy
c. Level of difficulty of the learning product		
d. . . .		
2. Instructional factors		
a. Type of exemplar used	Which exemplars are most productive with the child? Which inhibit?	Exemplar matrix (adapted from Uprichard)
b. Expository vs. discovery teaching	Which instructional strategy is most productive with the child?	Worthen's research definitions
c. Amount of guidance required in discovery teaching	What amount of guidance is most productive with the child during discovery teaching?	
d. . . .		
3. Learner-organismic factors		
a. Cognitive		
(1) Capacity		
(a) General intelligence		

Table I (continued)

SAMPLE FACTOR SETS	IMPLIED TASKS (Questions)	MODELS AVAILABLE
3. Learner-organismic factors (cont.)		
a. Cognitive		
(1) Capacity		
(b) Specific intelligence		
(2) Cognitive style		
(a) Reflexive vs. impulsive		
(b) Wholistic vs. incremental		
(c) Auditory minded vs. visual minded		
(d)		
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(3)		
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.		
b. Affective		
(1) Specific Attitudes toward math		
(2) Personality		
(a) Need structure (motivational structure)		Eric Erickson's eight stages of man and Maslow's hierarchy of needs
(b) Incentive mode		
(c) Self concept		
(d)		
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.		
(3) Social relations		
(a) Peer		
(b) Adults		
(4)		
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Table I (continued)

SAMPLE FACTOR SETS	IMPLIED TASKS (Questions)	MODELS AVAILABLE
3. Learner-organismic factors (cont.)		
c. Physical		
(1) Partially sighted		
(2) Double vision		
(3)		
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4. Environmental		
a. Socio-economic		
b. One parent family		
c. Environmental press		
d.		
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REACTION PAPER
IDENTIFYING AND DESCRIBING THE
REMEDIAL MATHEMATICS STUDENT

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Ashlock, Wilson and Hutchings point out the need to identify the remeidal student along with descriptions of some ways of performing that task. In their initial statements they state that "a larger number of centers and clinics have become available in an effort to provide specialized help for children having difficulty with mathematics". pg. 1. Furthermore, it is stated that more and more college courses and programs related to remediation are available. In a 1973 survey of the top one hundred teacher-producing colleges or universities in the United States, only nine indicated the existence of the proposed availability of diagnostic and remediation clinics.

These results indicate a survey is needed to determine the existence of programs which train personnel to diagnose and remediate mathematical difficulties found in students. Before conducting this survey, criteria that define desires aspects should be established. The survey should provide answers to the following questions:

1. How many diagnostic clinics or programs or centers are currently operating?
2. How many diagnostic clinics or programs or centers are proposed?
3. When will the proposed clinics or programs or centers be operational?
4. How many colleges or universities currently have complete or partial training programs for diagnostic personnel?
5. How many colleges or universities propose establishing a complete or partial program in which diagnostic personnel will be trained?

If indeed there is an increase in the number of available clinics, then one of the underlying questions which needs to be answered is, "Where are the staff and personnel to be trained?" Another way of asking this same question would be, "Are there adequate facilities established or proposed to train staff and personnel?" A discrepancy between the shown number of training facilities and the implied number of people available with suitable backgrounds appears to exist.

Trained personnel, if they are to be effective, need to know not only how to handle diagnostic situations but also how to identify the remedial mathematical student. Thus, the questions stated by Ashlock, Wilson, and Hutchings become even more significant.

> Who _is_ the remedial mathematics student after all? How would we know him if we saw him? Why do we call him remedial? Are we merely talking about children who are having exceptional difficulty with mathematics? We hear about children with "special needs" and about "reluctant learners." Are these children remedial mathematics students?

These questions point to a need to identify the remedial mathematics student. The problem is whether to define the category of so called remedial mathematics students and find those students who fit it or to find children that agreeably have mathematical difficulties and then establish a description of this group. Many current educational concepts indicate that schools should be altered to fit the needs of the student as opposed to making the students find a place where they fit in some existing structure. It therefore seems more realistic to observe children who are having difficulties and classify them. With this scheme of doing things, the students who are frustrated by any mathematics work would most definitely be included in the observations and thus in this classification group. Of course the possibility exists that not all types of students encountering difficulties with mathematics will be observed, resulting in an incomplete description of the category.

Whichever of the two classification schemes is used, the existing description of remedial students will still need to be examined, altered or discarded. For example, Wilson's discrepancy score is defined as a comparison between achievement test scores and predicted achievement scores derived from aptitude tests or tests of mental ability. However, there is a growing trend to shift to competency based instruction. If this continues, then the underachiever in mathematics will need to be redefined. For example, are the classifications "underachiever" and "capable of demonstrating competencies at a much slower rate" synonymous? If they are, when does this sameness occur? Do these classifications merge when a student does not progress as rapidly as he should? Does the classification of slower rate and underachiever occur when the student takes twice as long to reach the goals as had been anticipated? On the other hand, if the switch to competency based instruction is made, is it necessary to be concerned with the underachiever at all? Will the classification disappear by virtue of the implementation of competency based instruction wherein the concern ceases to be whether or not something is done within a period of time but rather whether or not something is done? The time factor would not be eliminated since each student would be encouraged to work at an acceptable pace commensurate with his ability. The emphasis on reaching an objective in a certain time period because others could, would be diminished.

Hutchings' proposed distinction between developmental and remedial needs seems most reasonable. A child classified as having developmental needs could progress slowly with respect to other students and yet the rate of development would be normal and consistent within that child's growth profile. Developmental needs refer to a child who generally does not progress as rapidly as his peers. This lack of progress is reflected in all subjects.

Conversely Hutchings states that a student with remedial needs would essentially develop at one rate for all areas except one or two in which the progress would be slower. In other words, the rate of this child's development in one subject is slower than the rate of growth in all other areas. These definitions of developmental and remedial needs as posed by Hutchings seem useful and reasonable. Since it is felt that these definitions could simplify classification and selection of students needing special assistance in mathematics, their adoption is urged and supported.

Having defined the remedial mathematics studen, in a seemingly useful way, Hutchings offers two important considerations about a child's performance: the first of these is based upon what the teacher says a child does. Should we pursue additional training either at the pre- or in-service level to assist the teacher in accepting this role? Or, realizing that many teachers already perform such duties, would it be better to provide an outline of characteristics or procedures to follow as an aid for teachers as they identify the remedial mathematics student? Given current demands on teachers' time, energy and attitudes, is the task of identifying the remedial mathematics student, in any sense other than the most general, a burden which would hamper their other classroom performances? Is the teacher being asked to perform the role of a mathematics specialist or clinician?

Hutchings' second consideration is the evaluation of achievement test results. This also precludes that the teacher make clinical judgments based on prolonged observations of the student. Here again, is the classroom teacher qualified to perform such a task? Can the teachers effectively make such judgments based upon achievement test result interpretation? In fathering information as a part of the decision making process, how much will students in the class be neglected either consciously or unconsciously?

Hutchings concludes identifying the remedial mathematics student with " . . . now our errors of assessment are apt to be primarily errors of judgment rather than errors of measurement". The immediate question is, "Which of the errors is more likely to occur"? Can teachers be made aware of the selection process? Can teachers be effectively and efficiently trained to do a better job of judging student needs through observation than would be achieved through test scores?

Since it is reasonable to expect that each classroom teacher would not have the time or often the training to perform these tasks, it is evident that a mathematics specialist much like the reading specialist is needed. This mathematics specialist would be in residence at the school and could aid the classroom teacher with ideas and techniques that could be used to enhance a student's learning situation. Remedial mathematics students could be referred to the specialist so that more attention could be directed toward that student's special needs. The trained mathematics specialist would be familiar with specific learning patterns of students having difficulty with mathematics. Since it is unreasonable to expect that each classroom teacher would be capable of performing such tasks, additional personnel and training would be necessary, implying the creation of a mathematics specialist whose specific purpose would be to strengthen the mathematical weaknesses of selected students.

As the situation now stands, the mathematics clinician is yet to come but the desire to assist students having mathematical difficulties now exists, necessitating reassignment of priorities. Need the resources be so limited? Is there no way that the limiting factors of time, lack of general training, lack of specialists, lack of identification definitions, etc, can be eased? Certainly the classroom teacher tries to find children who can be serviced quickly and efficiently so that as many students as possible can be treated. However, we are obligated to provide a means of assistance for those who are not so fortunate as to receive help when needed. Like the physician who must (even though no one should have such a right) decide who gets a kidney machine, we must decide who gets the mathematical help so vitally needed by so many. Unlike the kidney machine example, the student who does not receive the mathematical help survives and contacts others, perhaps spreading his potentially contagious sickness, at least in the sense of a negative mathematical attitude.

The assumption is that the child who is selectively weak in mathematics but doing well in other subjects is likely to show greatest improvement in the least amount of time. This is plausible for some but not for others. The attitude factor appears to have been overlooked in this classification. Certainly many students who do well in everything except mathematics can be helped but there are just as many who will not improve mathematically because of developed negative attitudes.

Hutchings also implies a need for a mathematics specialist when he mentions " . . . a hypothetical selection scheme for investing the limited time of specially trained diagnosticians and tutors". Here again is the clinical setting with a limited number of students and extensive materials and equipment. However, because of the limited number of clinics, teachers and students cannot all be reached. Certainly the clinic could be used for resource or research but, either way, a vast majority of needy students go untreated. There is a need to increase the number of clinics now to avoid not treating many while searching for efficient ways to treat the many.

In discussing clinics, reference is repeatedly made to limited numbers of students who are specially selected for studies involving " . . . samples of the population, the administration of treatments, and careful exercise of controls". pg. 13. For research purposes, this type clinic might be fine, but from a practical standpoint it has limitations. Extensive funding would be needed. Selection systems would be extremely complex and pressures for admission would be extensive. Only a few students would be treated.

Realizing that it is currently not practical to have a clinic in each school, is there the possibility of a compromise situation? Could the funding demands, needs for vast numbers of qualified personnel and admission pressures be reduced while increasing the amount of research and the number of students treated? A mobile diagnosis clinic appears to perhaps provide a plausible temporary solution. This mobile unit would be outfitted with tests, equipment, materials, personnel, etc. and would be delivered to a specified site to be operated as a means of assisting schools in the identification of remedial mathematics students. As opposed to teachers seeing demonstrations, this mobile unit would be used to involve and train local people to begin functioning as diagnosticians. It is well established that participation yields far better results than observation. Conceivably the portable unit process would create attitudes in personnel which would stimulate a desire to want to identify the remedial mathematics students.

Leaving the clinical situation, appealing as it may be, and returning to the real classroom, the identification problem still exists. As Ashlock, Wilson and Hutchings state, "It is likely that the number of children which will be identified is rather large . . , how do we go about finding out which of the children who are not doing well in mathematics we are best equipped to help? How do we find out which children actually need help from other professionals?" pg. 16. Basically this can be answered by listing three levels of diagnosis: informal, classroom and clinical.

As is implied by the terms used, the three diagnostic levels are hierarchial in nature. A child adding two numbers and getting the wrong answer consistently would be an example of the informal level. It could also be that a child is sleeping in class and some decision is made about that. Or perhaps a student is noticed as having a tendency to squint when attempting to read board material. These situations indicate that the informal level of diagnosis would be the classification of problems in which the teacher or specialist would be somewhat certain of the source or area of difficulty.

The classroom level of diagnosis is usually more formal. Perhaps the informal diagnosis was incorrect or incomplete. The child who added incorrectly does so with most pairs of numbers. The child who slept did so for several days in a row. Not only did the child squint but he also tilted his head to one side. In the classroom diagnosis situation, the teacher usually would not have the time or the wherewithall to adequately perform the task presented. For example, suppose the sleeping child is required to work every night. Does the teacher have the time to council the student? Or consider the child who is in an algebra class when the teacher discovers that he cannot add. Does the teacher have the time to provide the student with the necessary background information that will permit that student to experience success?

Finally, there is the clincial diagnosis level. This would be reserved for more difficult situations. Perhaps the child who could not add in algebra would be referred to the clinic and additional information would be gathered and an appropriate program of remediation developed. (See Figure 1:)

Cognitive Domain	Non-attainment of reversibility as defined by Piaget	Teacher using incorrect models
Affective Domain	Poor home life	Non-stimulative learning situation
Psychomotor Domain	Physical impairment	Wrong sized chair
	Non-educational (factors that cannot be attributed to education.	Educational (factors that might be caused by education).

Figure 1

This development would describe a child's progress not only in school and non-school settings but also at varying depths and areas of emphasis. Each cell in Figure 1 contains an example of the described conditions.

It is stated that several models are not only available for guiding and creating diagnostic and prescriptive procedures, but also some of these models are presently being used in classroom situations.

Teachers clustering students and indexing instructional resources are noted as having been observed. This statement gives rise to the question of whether or not the observations were in a typical classroom where the teachers do not normally have resource personnel such as a college person, clinician, or mathematics diagnosis specialist. It would seem that a "typical" teacher in a "normal" setting would be too overburdened to provide such extensive help. That is, most teachers would probably need help, a situation not too commonly resolved since much of the help must come from personnel not commonly available to classroom teachers. Neither the college people nor the mathematics specialists currently available are numerically large enough to begin to satisfy the growing diagnosis needs.

The concluding questions listed by Ashlock, Wilson, and Hutchings of the importance of understanding mathematics, the ability to do the mathematics of everyday life, and attitudes, along with what mathematics is needed in today's mini-calculator world cause a recycling to the question of competencies. Would a list of competencies help solve at least a part of the dilemma caused by questions such as the ones listed?

What points of agreement, questions or alternates have been generated? Hutchings' differentiation between remedial and developmental students seems reasonable and useful. The general screening proposals appear plausible but an additional process centering around informal, classroom and clinical diagnosis should also be considered. As these screening processes are defined, it seems imperative that techniques to aid in the screening should be built into pre- and in-service teacher training programs. Perhaps this compilation of techniques would be established in competencies.

The classroom teacher, already weighted down with many tasks appears headed for another burden, that of more extensive diagnosis. Is this potential burden reasonable? Does the evidence indicate that mathematics specialists, equipped and trained in a manner similar to the reading specialists, are needed? Should program guidelines be established to help in the fundamental training of the mathematics specialist? Will these specialists, who should be in residence in each school, need a resource person to whom they can turn for advice or to whom a more challenging case could be referred?

The expense and time requirements to develop the personnel needed to adequately meet the mathematics remediation demands appear staggering. Perhaps a mobile diagnosis laboratory, fully equipped with test materials and staffed by qualified personnel could be used initially. This mobile unit would serve several schools on a regular visitation basis.

Whatever choice is made, we must respond to current demands that diagnostic facilities be provided to point out students' mathematical deficiencies. Futhermore, we must develop the capability of treating each student showing a need; not just those who can get to isolated clinics; not just those who can be dealt with quickly; not just those who are doing well in all subjects except mathematics; and not just those who exhibit a positive learning attitude. Mathematics is an essential part of our society and we owe it to ourselves to provide each citizen with a program which will not permit him to be mathematically handicapped.

CLASSROOM DIAGNOSIS

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I. Dilemmas Resulting from Evolving Expectations

Evolution is an everyday concept to twentieth century man. Application of the concept to numerous facets of the life of present day man is commonplace. Hardly an eyebrow will rise when the term is used to refer to changes in the mathematics curriculum.

One cannot adequately respond to the question, "Evolving towards what?" However, one can discuss the evolutionary process and the resultant ramifications implied for change in related concepts, strategies and roles. The central thesis of this paper is that the evolving expectations or objectives for the mathematics curriculum are not changing concurrently with teaching roles and evaluation strategies. In short, development of new pedagogical conceptualizations is lagging far behind goal expectations. Three major dilemmas face us: (1) Can we create pedagogical models which will deliver results concommitant with these expectations? (2) Can we create evaluation models to appraise the delivery? and (3) Can we train teachers through whom application of these models may be successfully executed?

What are the Goals

To whom shall we turn for the answer to this question? The man on the street? The mathematics educator? The psychologist? Each of these recognizes the need for basic skills for present day demands but, also, each recognizes that the evolution of twentieth century society will create new, yet unknown demands.

In colonial days and continuing into the late nineteenth and early twentieth century, mathematics had a stong application emphasis because ours was a developing nation and the needs were reflected in the curriculum of the schools. (DeVault and Kriewall, 1969) An early work by Brueckner (1930), which was a major contribution to arithmetic diagnostic literature, placed strong emphasis on the identification and remediation of numerous errors in basic computations.

As mass education became the rule, mathematics educators sought to extend the goals beyond efficient skill development. In 1948, Spizer stated:

> Thus, for years, "Teach with understanding" has been one of the maxims of arithmetical instruction. In spite of this emphasis, the understanding of those who study arithmetic has been unsatisfactory. In an effort to improve understanding, another maxim, "Teach with meaning," has recently been adopted.

In a later work, Brueckner and Bon (1955) referred to the mathematical and social phases of arithmetic. They discussed understanding, meaning, skill and application. These emphases are reflected in nearly every contemporary exposition on teaching arithmetic by mathematics educators (Grossnickle and Reckzeh, 1973; Holmes, 1968; Riedesel, 1973; Underhill, 1972) and also special educationists (Spencer, 1969). These directions were supported by the SMSC Conference on Mathematic Education for Below Average Achievers (1964) and the USOE Conference for low achievers held in 1964; the latter recommended, "In order to make these children employable, they must develop some abilities which cannot be duplicated by machines."

Development of meaning and understanding within the learner's capacities to perform has been the accepted expectation of the psychological community. It has been most elaborated in the writings of and about Piaget and Bruner. The abilities of the learner to transfer and generalize have been emphasized. Bruner (1966) states, "There are too many particulars to teach and to master."

John Q. Public seeks mastery of basic skills. He wants Johnny to compute correctly and efficiently. He wants Johnny to know <u>when</u> to compute. Broudy (1961) discusses this expectation in an analytic manner and presents a most convincing argument that control of knowledge achieved through learning is by logical and practical necessity a function of one's ability to think in new situtations. Application of skills is facilitated through efficiency but "if we keep efficiency of response constant, differences in mastery correspond to differences in the levels of theoretical insight required to get the correct response . . . mastery can be characterized as insightful action made habitual." (Browdy, 1961)

Thus, it is an assumption that two broad goals of mathematics are (1) to develop mastery of basic skills, and (2) to make correct application of those skills.

The success of learners is related to the level of insight and meaning possessed.

What is Teaching?

No lengthy treatise is in order here. The reader is simply reminded that teaching is a facilitation and directing process towards specified goals. A teacher may or may not be present. Goal orientation is the point to be noted as it will be alluded to later.

What is Evaluation?

A series of quotes will make this point:

> It is important that tests be constructed so as to provide as thorough coverage of the elements which have been taught as is feasible. (Morton, 1953)

> Diagnosis is not an end in itself. Rather, it is a means to more effective differentiated instruction. Only when we have

diagnosed the difficulties and determined the needs of children, quantitatively, can we provide the kind of instruction designed to remedy those difficulties and meet those needs . . . remedial teaching is basically good teaching, differentiated to meet specific instructional needs. (Weaver, 1954)

The chief contributions of testing and evaluation to arithmetic instruction are:

1. The selection and clarification of objectives which serve as guides for testing and instruction.
2. The determination of the rate of growth and the progress made by each learner in achieving accepted objectives.
3. Provision of a basis on which teachers can set up educational experiences adapted to the needs, interests, and ability of the learners.
4. Motivation and guidance of learning, especially by helping children to evaluate their own responses and behavior.
5. The location, diagnosis and treatment of learning difficulties.
6. The basis for coordinating improvement programs in related fields such as arithmetic, reading, science and social studies. (Brueckner, 1959)

It (the Yearbook Committee) shares a deep conviction that the purpose of evaluation is to provide feedback and guidance to the whole educational process at every level. It believes that the going system of evaluation, which has largely drifted into the service of marking and grading and crediting, must be replaced by a system dedicated to the fundamental needs of the learner and teacher as well as those of the curriculum designer and policy maker. (ASCD, 1967)

Evaluation is a feedback process. Disgnosis is a feedback cycle aimed at determining specific learner needs and identifying specific learner difficulties.

Recapitulation

Goals, teaching and evaluation comprise the specific components of a diagnostic cycle Beginning with a set of specified goals, teachers design and execute learning experiences aimed at achieving them. Diagnosis yields data on the degree of success.

There exist two problems.

Several spokesmen (Rappaport, 1959; Burns, 1965; Morton, 1953; Glennon, 1968; Spitzer, 1948; Brownell, 1956) recognize the disparity between meaningful arithmetic instruction as a goal and the weakness of evaluation strategies employed to measure progress toward that goal. They point out with dismay that instruments for measuring understanding are not available. Teachers are placed in the difficult position of being encouraged to teach for understanding but then having their teaching success measured only with efficiency and skill instruments. A vicious cycle exists between what is espoused and what is rewarded. Teachers are lead to doubt the intent of leaders and to question the goal since it isn't even measurable! Present diagnostic instruments are weak in that they cannot pinpoint causes of learning difficulties. Suggestions have been made by numerous writers relative to the need, and relative to the potential in the theories of Gagné and Bruner, but few examples of enlightened diagnostic thought have surfaced. There is a high priority need for new conceptualizations which will enable classroom teachers to appraise needs and prescribe remedial procedures.

In the words of Jerome Bruner (1966), "Knowing is a process, not a product." We need evaluation strategies which diagnose learner understanding of mathematical processes.

II. Elaboration Upon the Relationships Between Goals, Teaching and Evaluation

A more satisfactory set of conditions will not evolve until we refine our thinking in such a way as to deal with goals, instruction and evaluation by systematically building upon, synthesizing and integrating new knowledges of learners, content and pedagogy.

Refinement of Goals

Beyond the general goals of transferability, generalizability, skill development and application, we are left with the basic notion of meaning. Does meaning relate to what is retained? Perhaps there is a real distinction which can be made between memorizing and remembering. The former is related to recall, the lowest level of Bloom's taxonomy. Remembering, on the other hand, may be contextual. Ideas or concepts are retained because of insights gained through contextual relationships.

To gain support, examine Bruner's (1966) statements about the emphases of education:

> It would seem, from our consideration of man's evolution that principal emphasis in education should be placed upon skills--skills in handling, in seeing and imagining, and in symbolic operations . . .
>
> A curriculum should involve the mastery of skills that in turn lead to the mastery of still more powerful ones, the establishment of self-reward sequences . . . The reward of deeper understanding is a more robust lure to effort than we have yet realized . . .
>
> If there is any way of adjusting to change, it must include, as we have noted, the development of a metalanguage and "metaskills" for dealing with continuity in change . . .
>
> If we are to do justice to our evolution, we shall need, as never before, a way of transmitting the crucial ideas and skills, the acquired characteristics that express and amplify man's powers.

Definitions

The embodiment of an idea through manipulations of objects, use of pictures or diagrams, or use of symbols, roughly correspond to Bruner's enactive, iconic and symbolic ways of knowing and are called models. A model is said to be known when it can be traced to some original state(s) in one or more concrete embodiments (corresponding with enactive).

Conjecture and Assumption

New learning is meaningful when it is based on a series, synthesis or integration of known models.

Characteristics of Good Teaching

Good teaching of mathematics is defined here as instruction which facilitates learner comprehension of a structure of a discipline in a Brunerian sense and which develops learner understanding of process:

> "Optimal structure" refers to a set of propositions from which a larger body of knowledge can be generated, and it is characteristic that the formulation of such structure depends upon the state of advance of a particular field of knowledge.
>
> . . . since the merit of a structure depends upon its power for simplifying information, for generating new propositions, and for increasing the manipulability of a body of knowledge, structure must always be related to the status and gifts of the learner. (Bruner, 1966)

According to Bruner, this structure is characterized in three ways: the (1) mode of representation, (2) economy, and (3) power. That is to say (1) "a" structure possessed by a given learner is characterized by his level of "knowing", i.e., at what levels are its constituent parts known, (2) how much information must be processed to acquire new knowns, and (3) what are his capabilities for generating new knowns.

The foregoing conceptualization of learner-unique structure incorporates the content heirarchy schema proposed by Gagné, but it is more than that. From the simple but elegant notion of mathematical prerequisites, we have evolved pedagogical principles which embrace and go beyond. We have evolved our thinking from the structure of mathematics to a structure of mathematics. This is the evolutionary wedding and acknowledgment of the dual interplay of content and learner-perception-of-content. While, in a strict sense of Gagné, the structure will be the eventual outcome of common knowns abstractly, the multiplicity of combinations of knowns of the many structural parts makes structure in this sense a very personal and unique possession. As Bruner (1960) so aptly states it, "In seeking to transmit our understanding of such structure to another person--be he a student or someone else--there is the problem of finding the language and ideas that the other person would be able to use if he were attempting to explain the same thing. If we are lucky, it may turn out that the language we would use would be within the grasp of the person we are teaching." Thus, good teaching is facilitation of cognitive movement of growth which occurs within and extends the perceived structure of the learner.

Mathematics instruction characterized as "meaningful" is based on exploration and analysis of process. How computations are performed is equally as important as the result of the computation. Reiteration of process will develop an underlying comprehension of mathematics as a process, as an approach to structuring reality and of categorizing, classifying, and studying environmental relationships. Meaning is synonymous with a grasp of process interfacing of the "real environment" and its symbolic mathematical models or representations. This is accomplished through carefully established sequences based on modeling and contextual readiness. Contextual readiness is the extent of familiarity of new instructional settings through which topics are presented to learners. More will be said of this later.

This brings us to the major job of the teacher, designing sequence.

Sequence

The chaining of activities with many nuances of pedagogical significance constitutes learning sequences. The creation of successful chains encompasses much more than the logical sequencing proposed by Gagné and traditional concerns for development-and-practice balances. Spitzer (1948) recognized this distinction when he differentiated between drill and practice, and McClellan (1961) and Rosenberg (1962) were addressing this issue in distinguishing between logical and psychological presentations of content. In the present treatment, we will concern ourselves with the roles of intuition and readiness in relation to other factors of successful sequence design.

Intuition

The specific brand of intuition referred to here is the one described by Bruner (1960) which appeared as "pre-mathematics" in the Cambridge Conferences on School Mathematics (1963) and Teacher Training (1967). He stated, "Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft. Perhaps the first thing that can be said about intuition when applied to mathematics is that it involves the embodiment or concretization of an idea, not yet stated, in the form of some sort of operation or example." The 1963 Conference Report recommended that a spiraling curriculum attend to the introduction of new mathematics concepts first at the pre-mathematical level. These early associations would be related to the learner's general experience; presentations would not be wrong, only incomplete structurally. As Lovell (1971) states " . . . thinking is greatly dependent on the total perception of the situation, and the child is largely unaware of the processes by which he arrived at his ideas. There is, as it were, a basic awareness, not yet formalized. Usually intuitive thinking depends upon considerable familiarity with the ideas involved, and almost always such thought is unable to detach itself completely from physcial reality."

Good teachers have always used intuitive or pre-mathematical approaches to concept development. However, the approach has not been given adequate recognition as an important and integral part of systematically designed sequence. It has had the status of a "highly recommended" practice.

Readiness

Just as the goals of mathematics and concept of structure are evolving to encompass new knowledge of the learner and that which is to be learned, so, too, is the concept of readiness evolving. Readiness has been translated by and large as the possession or lack of prerequisite subskills or subconcepts in the sense of Gagné, or as attainment of a certain level of maturation. The former can be verified by examining evaluation instruments designed to measure mathematical readiness and diagnosis. Such instruments focus on the possession of certain concepts and skills. The nature of the questions have reflected the "easiest" or most obvious ways to measure the concepts rather than a systematic treatment of conceptualization levels founded on a theoretical base. This is no longer adequate. The latter, maturational, point of view is recognizable in many sources with a Piagetian orientation. These sources stress what children cannot do rather than offer the positive aspects of what children can do. This problem has a single referent: children's conceptual levels are equated with adult conceptual levels. (Rousseau's battle all over again!)

The child often knows a concept in a way which is different from the way in which his adult counterparts know it. His set of knowns are at lower levels on the enactive, iconic, symbolic chaining of experiences. He often possesses meaning but his meaning has not evolved to the highest or symbolic level. Therefore, he is stamped "not ready." Not ready for what? Not ready for the adult, symbolic treatment of successive levels of the conceptual hierarchy.

With an evolved conceptualization of structure based on learner-uniqueness develops a concurrent redefining of readiness. Bruner (1966) recognized this need for change. He claimed that one teaches for readiness. As expressed in the Twenty-fourth Yearbook of the National Council of Teachers of Mathematics: "Teachers in all grades should view their task in the light of the idea that the understanding of mathematics is a continuum . . . . (1) Teachers should find what ideas have been presented earlier and deliberately use them as much as possible for the teaching of new ideas. (2) Teachers should look to the future and teach some concepts and understandings even if complete mastery cannot be expected." What, then are the components of readiness to which we should attend? There are at least five; we have made varying degrees of progress in our recognition of their respective roles and importance:

A. Content Readiness
B. Pedagogical Readiness

C. Maturational Readiness
D. Affective Readiness
E. Contextual Readiness

Let's consider each of these separately in the next section we will try to integrate some of them into a systematic treatment of sequence.

A. Content readiness refers to that about which we know most and treat with the greatest level of expertise. It is the analysis of the discipline by sequential acquisition in a logical sense according to the hierarchical strategy proposed by Gagné. Most of the work of Brueckner and his disciples was a refinement of this element. For example, place value must precede two digit addition, and regrouping or renaming of numbers must precede addition with regrouping (carrying). However, there are certain additional concepts related to the nature of the discipline which reflect unifying ideas. These ideas can be capitalized upon in sequence. These include such items as (a) properties, (b) symbols. In the intuitive treatment of number system isomorphisms commutativity, associativity, distributivity, renaming, regrouping, identity elements are recurring themes which, if handled masterfully, will facilitate learner readiness. New situations can be related to previously encountered and known situations. Use of expanded notation, operation signs, exponents, multiple symbols (as with rational numbers), and exponentiation could play a much stronger role if planned into sequence in a systematic manner.

B. Pedagogical readiness is an area of slight exploration but of great promise. The recurring themes of concrete, semi-concrete and abstract models carry learners forward. They acquire higher levels of knowing on subsequent encounters. Instructional materials such as the abacus can be used to build bridges from whole numbers to decimal fractions; pedagogical distinctions in problem solving such as measurement and partition division or take-away, comparative and additive subtraction can facilitate learning; use of proportional (Cuisenaire Rods) builds readiness for non-proportional aids (poker chips); new symbols may be created; properties can be emphasized. These are important elements included intuitively by good teachers: it is time to capitalize upon them systematically.

C. Maturational readiness in the sense of Piaget places constraints on the child's level of knowing but does not, as far as we presently understand, preclude all knowing of a given concept. This was Bruner's central thesis when he gave his now famous, but oft misunderstood, pronouncement. Learner progress is limited but not halted by his capabilities in conservation, reversibility, attending to sequencing, and attending to two or more variables simultaneously.

D. Affective readiness refers to the learners attitudinal predisposition towards that which is to be learned. While the importance of this type of readiness has long been recognized, only slight progress in appraising and accommodating it has been made.

E. Contextual readiness refers to the setting in which new concepts are introduced. Extra-school experiences or experiences of the child's not-at-school world should be related whenever possible to school learning. Whenever a concept of the discipline can be placed into a context (preferably extra-school) familiar to the learner, he will be more ready for it than when it is not. This is one element of the intuitive role of concept development.

Sequencing Theory

A sequence of learning experiences should accommodate the learner's perception of structure, account for varying readiness needs and provide pre-mathematical (intuitive) associations with new concepts.

Given a level of content readiness ascertained through diagnostic evaluation, associations with new concepts should be sequenced in such a way as to account for three variables: treatments, model experiences, and symbols.

There are two categories of treatments: intuitive (pre-mathematical) and mathematical. When introducing integers, a thermometer or a football field or an elevator in a building might be used. The pre-mathematical treatment would use neither new words mathematics vocabulary) nor new mathematical symbols associated with the concept; the mathematical treatment would.

There are three types of model experiences: concrete, semi-concrete and abstract. Concrete learning experiences are "hands-on," tactile embodiments; semi-concrete experiences are primarily visual embodiments (no "hands-on"); and abstract learning experiences are characterized by symbolic treatments (no tactile or visual reinforcement). Notice that all teacher-demonstration, T.V., filmstrip and other observer activities are semi-concrete.

There are four categories of symbols of interest: (1) non-mathematical verbal, (2) mathematical verbal, (3) non-mathematical written, (4) mathematical written.

Non-mathematical verbal refers to the use of only familiar vocabulary. A "new" mathematical idea is introduced without the use of its proper name or label; the mathematical verbal, on the other hand, introduces the learner to the appropriate term or name or label for the mathematics idea under consideration. The relation between written non-mathematical and mathematical vocabulary and symbols is differentiated similarly. Examples related to integers:

(1) "Two floors up"
(2) "Negative four"
(3) U2
(4) ⁻4

An hypothesized "ideal" sequence is as follows:

I. Treatments
   A. Intuitive (I)
   B. Mathematical (M)

II. Model Experiences
   A. Concrete (C)
   B. Semi-Concrete (S)
   C. Abstract (A)

III. Symbols
   A. Non-Math Verbal (NV)--no new words or symbols
   B. Non-Math Written (NW)--no new symbols or words
   C. Math Verbal (MV)--new words and symbols
   D. Math Written (MW)--new symbols and words

Chronological Sequence

	I		II		III
1.	I	-	C	-	NV
2.	I	-	S	-	NV, NW
3.	I	-	A	-	NV, NW
4.	M	-	C	-	MV
5.	M	-	C	-	MV, MW
6.	M	-	S	-	MV, MW
7.	M	-	A	-	MV, MW

Examples related to integers:

1. Thermometers as "degrees rise" or "degrees fall," each child with model, no new symbols or words
2. Pictures of buildings--"floors up" or "floors down," children use pictures and write "U2" or "D3"
3. Talk of examples but without presence of concrete or semi-concrete embodiments, "U2" or "D3" or "F3" or "B6"
4. Use same model as the one above or some other similar model, use "positive three degrees" or "negative eight degrees"
5. Same as 4, introduce "+3" and "-8"
6. Same as 2 above except use "+2" instead of "U2"
7. Same as 3 except use "+2" instead of "U2"

As mentioned earlier, content readiness is built into this model; so is pedagogical readiness: the cyclical treatment of model modalities or embodiments is evident as are systematic treatments of symbols and language. Teachers can increase readiness by using familiar contexts, terminology and instructional materials. Maturational readiness establishes growth limits during a given encounter, and intuition is evident as an integral, planned component of the sequence. Affective readiness in not formally accommodated; it is still a teacher responsibility, however, "Instruction consists of leading the learner through a sequence of statements and restatements of a problem or body of knowledge that increase the learner's ability to grasp, transform, and transfer what he is learning. . . . by giving the child multiple embodiments of the same general idea expressed in a common notation we lead him to 'empty' the concept of specific sensory properties until he is able to grasp its abstract properties." (Bruner, 1966)

Artfully orchestrated sequence is a harmonic interplay of content and pedagogy. Readiness, in all its refinements, represents the major goal and consideration of quality sequencing. Readiness is based on the content and pedagogical experiences of children in previous encounters, attunes itself to the contextual and affective climate of today and anticipates learner needs in future encounters. Artful sequencing is a bridge-building pathway through the continuum of abstract structure by careful repetition of concrete, semi-concrete and abstract counterpoint repeating thematically until the learner knows the melody. Continued practice on themes and variations of sequence builds bridges or associations between the harmonic themes which, collectively, constitute the symphony of mathematical understanding of structure and result in appreciation and knowledge of its intricate complexities and interrelationships.

Conjectures and Assumptions

A. Intuition is extended through repeated bridging from knowns to unknowns or from the "believed" to the "uncomprehended," i.e., one learns to "believe" through continued (repeated) associations between the known and the unknown. Continued bridging attempts between the believed and the related-but-not-believed produces new belief or insights about not-previously-perceived relationships.

B. Economy results when knowledge is known, and the economy is highly and positively correlated to the types of models which are used in constituting what is known.

C. The learner possesses power to the extent that he can chain, integrate and synthesize knowns in new patterns.

D. Learners will grow to higher levels of known (concrete to semi-concrete to abstract) on successive chaining or bridging encounters (repetition) due to maturation and intuitive familiarity.

Characteristics of Good Evaluation

From the evolving goals and concomitant changes mandated in instruction comes the compelling need to evolve a more refined conceptualization of evaluation and, more specifically, diagnosis. As reviewed earlier, recognition of the need is not new. Calls for improved evaluative and diagnostic procedures have been called for since the 1940's.

During the period when Brueckner's scholarly work was produced in 1930 and on through the 1940's, the goals of mathematics were still basically computational in nature. Use of concrete and semi-concrete models were seen as helpful instructional tools; they were a means to an end. This perception of aids in teaching mathematics is borne out by study of his early work, in his much later work (1959) and in his work with Bond (1955). From these works one can quickly ascertain that models were not ends in themselves. This point is poignantly made by an analysis of his diagnostic model. Survey tests were very broad abstract diagnostic tools used to determine class level of functioning for planning the instructional program; analytical tests were group or individually administered tests of narrowly refined ranges of abstract work with systematic evaluation of small incremental steps in computation; clinical procedures were suggested for individual cases and consisted of several diagnostic elements all aimed at the identification of computational skills at the abstract level. Only after diagnosis was completed was the teacher encouraged to use models to develop understanding and overcome difficulties. In Brueckner's (1959) own words, "Analytical diagnostic tests locate specific areas of weakness. However, they do not in themselves reveal the nature of the underlying difficulty, how well the child understands the steps in the solution, the effectiveness of his thought processes and his methods of work, or the kinds of computational errors as such that cause incorrect work." He then goes on to describe several specific procedures to determine difficulties clinically but the procedures make no reference to concrete and semi-concrete models. This conceptualization of the diagnostic process represents a very limited use of readiness, prerequisites, understanding and objectives or goals.

Recognition of modeling as an evolution of stages of understanding began to take shape as a movement in the 1930's when Morton (1938) presented a case for modeling as an important part of the "meaningful arithmetic" program. Recognition of the need for evaluation of these outcomes by Spitzer (1948) and Morton (1953) signaled an important shift in the perceptions of models: they were no longer simply means; they were now means and ends. This was an important historical shift. It represented recognition of process as an important goal of mathematics instruction. Computational skills were still important, but they now shared the limelight with an equally important part of the mathematics instructional program, sequence. Sequence was now recognized as a duality of content and pedagogy, a very significant step.

Recent contributions have been made by Reisman (1972) and Underhill (1972). Reisman incorporates concrete and semi-concrete models in clinical diagnostic procedures, and Underhill has proposed application of this approach in class instruction and diagnosis. One of Underhill's major contributions has been emphasis on the interfacing of content and pedagogy in the instructional sequence by emphasizing use of three or four behavioral objectives for every concept. This helps give adequate recognition to important concrete, semi-concrete, abstract and application outcomes of the instructional program.

Problems for Classroom Teachers

To successfully implement a diagnostic approach to instruction, the task must fall within the management capabilities, mathematical expertise, time limitations and physical resources of average teachers. This is, indeed, a challenge. The evaluation model proposed by Underhill (1972) can be used with the management capabilities of average teachers; it also falls within the range of reasonable limitations of time and physical resources. The major problem is one of creating appropriate sequences. While teachers can create appropriate models, they need a detailed sequence into which it can be placed. There are presently two main alternatives: (1) a sequence developed by Brueckner or his followers, or

(2) a textbook. The former is available from Brueckner's writings and from the writings of numerous people who agreed with him. Textbooks have associated with them the problem of identifying prerequisites; this information is not always clear. A more desirable alternative would be the creation of a new type of sequence which would facilitate teaching for individual needs and evaluation (diagnosis). Such a sequence is proposed in the next section and is related briefly to Underhill's (1972) model for class instruction and diagnosis.

Assumption

Diagnosis and sequence are cyclical. Diagnosis should evaluate learner progress through a given sequence, and the results of diagnosis should be useful in planning new sequence(s).

Recapitulation

Conceptualizations of mathematics goals, teaching and diagnosis have been too limited. The evolution of goals has had impact on teacher behavior, and a great need exists in the feedback process to determine the validity of goals, the merit of given sequences and the extent to which learners succeed. New or modified conceptualizations are needed.

III. A Model For Classroom Diagnosis

Classroom diagnosis should be a "best fit" of three sets of conditions: parameters related to sequence, parameters related to the environment and parameters related to teachers. The "best" model of diagnosis is one which meets the following conditions or standards:

A. Parameters related to teachers. A "good" model should

1. Function within the usual range of management skills possessed by classroom teachers.
2. Function within the usual level of mathematical competence possesed by classroom teachers.
3. Function within the usual time allocated to evaluation by classroom teachers.
4. Facilitate decision-making relative to the needs of individual learners.

A model should be easy to understand and easy to implement. This means that instruments and procedures should be easy to design, construct, administer, score and interpret; prescription should follow readily. Diagnostic results should facilitate grouping of learners in such a way that typical teachers can manage the total process.

B. Parameters related to the environment. A "good" model should

1. Function within usual space allocations and building and classroom design.
2. Require a minimal outlay of additional resources.

A model should be practical from a resources viewpoint. It should be within the reach of typical schools as well as atypical schools.

C. Parameters related to sequence. A "good" model should

1. Include a systematic accounting of content prerequisites.
2. Facilitate evaluation of meaning or understanding.
3. Include a systematic accounting of pedagogical spiraling.
4. Facilitate sequence decision-making.
5. Operate within the context of a minimal number of overriding principles.
6. Facilitate general placement within a program (survey).
7. Facilitate identification of specific problem areas (analytical).

A model should relate sequence and structure. It should allow a teacher or group of teachers to isolate general and specific needs of learners and to plan appropriate learning experiences for them. Factors of readiness should be accounted for to the extent possible.

Evolution of a Model

This model is a synthesis of ideas from three major thinkers: Leo Brueckner, Jerome Bruner, and Robert Gagné.

Brueckner's contributions are primarily in (1) the concept of a survery test, (2) the concept of an analytic test, and (3) an elaboration upon and extension of Morton's operations hierarchies. The survey and analytic tests were described earlier. Suffice it here to say that the survey test was a general test and the analytic test was a specific test.

Whereas Brueckner described a mathematics survey test as one which was used to ascertain general achievement in a broad area such as whole number operations or work with decimal fractions, the evolved definition of a survey test is a test which is computational in nature but which determines the level of performance with a K-6 addition continuum. For example, addition is an operation which is typically taught on the set of whole numbers, then non-negative rational numbers using common fractions and then decimal fractions; integers may be added to this continuum in the next decade. This development where the learner is performing within this continuum at the computational level.

Brueckner described an analytic test as one designed to determine a specific level of difficulty. The evolved use of analytic testing would have the same objective. Whereas in Brueckner's approach analytic testing was a detailed look at abstract computational skills, the proposed use of analytic tests would be to use semi-concrete models from which learner understanding and meaning can be inferred.

Through the work of Morton, Brueckner and Gagné, an analysis of content can be hypothesized. From suggested continua, prerequisites, and thus one aspect of readiness, can be ascertained. Through a restructuring and expansion of existing content chains, a new continuum for each operation can be hypothesized. This results in a chain for a K-6 addition continuum, one for subtraction, and so on. A sample for addition is appended.

Bruner's concept of <u>knowing</u> has been integrated into each continuum in such a way as to create a pedagogical spiral about each operational continuum.

ADDITION CONTINUUM

Concrete/semi-concrete/abstract spiral

The evaluation (diagnostic) model attempts, in two phases, to determine at what level on the <u>pedagogical</u> spiral learner needs exist. Phase I uses survey testing to determine the approximate level of functioning on the continuum. Phase II examines learner <u>understanding</u> on a narrow range of the pedagogical spiral after the approximate location of the problem area has been determined.

Suppose as in the sample addition continuum appended there are 47 increments, K-6. Depending on the age/grade level of the children, a probability estimate is made on the basis of norms and previous experience of the portion of the 47 increments in which nearly all children of that level will fall. It is suggested, for example, that children in the first half of grade three will usually perform in the increment range 10 through 19. A survey test is constructed, three questions for each incremental level. (A sample survey test is appended.) Using a criterion level of two out of three as suggested by Brueckner, each child is determined to be functioning at a given increment.

In Phase II, and analytic test is constructed. Suppose the child encountered difficulty on increment 14 on the survey test. Then the analytic test would cover levels 13, 14 and 15. The analytic test would contain 18 questions in sets of three in this order: level 13 semi-concrete, level 13 abstract, level 14 semi-concrete, level 14 abstract, level 15 semi-concrete, level 15 abstract. (A sample analytic test is appended.) Scoring would be as with the survey test; the criterion of 2 out of 3 would be used to determine the level at which the child is performing. It is assumed that learners would be asked to use only familiar models in diagnosis.

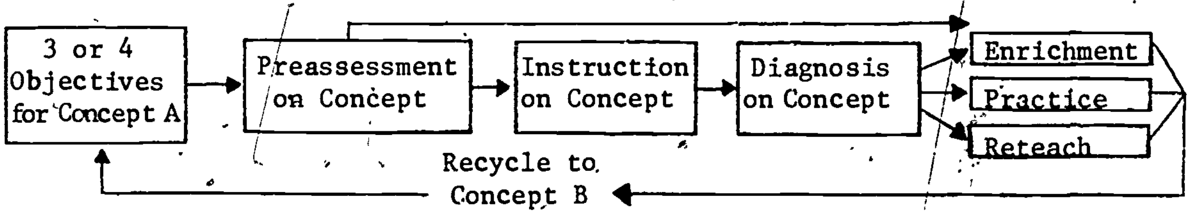
Instruction would begin at the pedagogical (model) level <u>preceeding</u> the one at which the child failed to meet the criterion level, as suggested by Bruckner and Bond (1955) and Underhill (1972). For example, if the child met criterion level on every set of three until he got to level 14 abstract, instruction would begin on level 14 semi-concrete to help him bridge the gap or make the transition from his understanding of semi-concrete (iconic) to abstract (symbolic). If the child met criterion level on every set of three until he got to level 15 semi-concrete, an instructional sequence should be designed which would begin him at level 15 concrete (enactive).

Implementation of the Model

The model can be implemented in several ways; three will be presented. Two utilize the model for individualizing mathematics instruction proposed by Underhill (1972). One is for use in a self-contained classroom, and the other two are for use in cooperative teaching arrangements.

Self-Contained Classroom

Early in the school year, a survey test is administered for each operational continuum. After the lowest level of learner performance within the continuum is know, instruction begins on concept A. After objectives have been stated, learners are preassessed either formally or informally to determine which learners, if any, already possess the concept; they are channeled into enrichment activities of a lateral type. Instruction then occurs using grouping or individual approaches as appropriate and within the management capabilities of the teacher.



When the teacher feels that several learners have mastered the objectives, diagnosis takes place to determine what further work is needed. When most learners have mastered the objectives, the teacher recycles through the model for the next concept in the operational continuum. Learners who failed to reach criterion level can do independent work or the teacher may hold weekly remedial sessions with selected learners. In this manner, the teacher uses a concrete/semi-concrete/abstract approach in the instructional phase and then decides who needs an additional set of experiences at each level on the basis of diagnostic feedback. The teacher rotates from one operational continuum to another throughout the year.

Cooperative Teaching-Groups

Assume there are three teachers working together. A survey test is administered. The levels of the children are determined as before. After the preassessment, one teacher works with those children who already possess mastery of the concept; the other two work with the remaining learners in the instructional phase. After diagnosis, one teacher works with each of the three groups. They may choose a variation depending on the size of each group.

Cooperative Teaching - Individualized

A major problem in implementing individualized programs is the great job of meeting diverse needs. Three teachers working together can administer a survey test on an operational continuum. Each teacher works with learners performing at specified levels. For example, if in the addition continuum learners are performing at levels 12 through 23, one teacher might work with levels 12-14, one 15-18 and one 19-23. In this way, a given teacher works with a range much narrower than is typically encountered in a heterogeously grouped class. Through use of flexible grouping. progress of each learner is facilitated through each continuum. Records maintained for all learners will allow them to move in each continuum at a pace commensurate with abilities and efforts. Through monitoring, students can be regrouped as they change from one operational continuum to another and as they move within a continuum at varying rates. Individual monitoring will also permit and facilitate flexible grouping due to individual plateaus and periods of acceleration or deceleration.

Summary and Questions

Developments of the past thirty years foster awareness of new relationships, hypothesizing of new alternatives, refinement of goals, and redefinition of old conceptualizations. This paper has sought to collate, integrate and synthesize, to the extent possible in a brief paper, some of these developments and to hypothesize some new relationships. It has not been the intent of this paper to answer questions, but, rather, to formulate new questions and to stimulate new thinking. If experience verifies some of the postulations presented, that is good; if experience leads to discovery of other more valid approaches or to more precisely evolved conceptualizations, that is also good. As our knowledge of learners, learning and teaching expands, and as our goals alter to meet the demands of a new social order, our conceptualizations should evolve into more precisely understood relationships and our instruction a more harmoniously orchestrated symphony of sequences.

Many questions remain unanswered; many conceptualizations remain vague.

The most important questions, I feel, are those related to sequence. Is there an "ideal" sequence? Is it possible to create one grand design for sequence which is related to a small number of variables which interface in an orderly manner? Can certain sequence variables be accommodated more easily by learners of various ages, experiences, mental maturity? What is the interplay of intuition and symbols in acquiring mathematics concepts? How do various learning styles interact in sequences? Does a pedagogical spiral exist which can be as explicitly stated as those presently designed for content? Can we define and systematically account for <u>all</u> components of readiness? What are the roles of overlearning, retention and regression in the diagnostic process?

Finally, there are critical concerns about teachers. Are our expectations realistic? Can we create models for teachers to implement with ease? What kind of pre and in-service training is needed? Hopefully, the model presented and the discussion included will help answer some of these questions and serve as a catalyst in formulating others.

APPENDIX

A. Hypothesized K-6 Addition* and Subtraction# Continuum

1. Pre-number order relationships
2. Matching
3. Pre-number inequalities
4. Pre-number seriation
5. Classification--negation
6. Concepts of numbers less than 10
7. Inequalities of numbers less than 10
8. Seriation of numbers less than 10
9. Sums less than 10
10. Place value - ones and tens (proportional)**
11. Sums 10 thru 18, both addends less than 10
12. 2 digit plus 1 digit WITHOUT regrouping (proportional)
13. Two two-digit numerals without regroupings (proportional)
14. 2 digit and 1 digit numerals with regrouping (proportional)
15. 2 digit and 2-digit numerals with regrouping (prop. and non-prop.)
16. 3 two-digit numerals, sum of ones greater than 20 (prop. and non-prop.)
17. Place value - hundreds, tens and ones (prop. and non-prop.)
18. Three-digit numerals without regrouping (non-prop.)
19. Three-digit numerals w th regrouping (non-prop.)

In the following group, use denominators 2, 3, 4, , 6, , 10, 12, 15

20. Unit fractions
21. Unit fraction inequalities
22. Non-unit fractions
23. Equivalent fractions (building sets)
24. Non-unit and unit fraction inequalities
25. Seriation (unit and non-unit)
26. Sums less than one, same denominator
27. Mixed numerals
28. Sums of mixed numerals, no regrouping, same denominator
29. Regrouping of fractional parts
30. Sums between one and two, same denominator
31. Sum of mixed numeral and non-unit fraction, regrouping, same denominator
32. Sum of mixed numerals with regrouping
33. Three non-unit fractions, sum between 2 and 3, same denominator
34. 3 mixed numerals, sum of fractional parts between 2 and 3, same denominator
35. Equivalent fractions (using sets of multiples and identity element)
36. Sum less than one, different denominators

*Assume two addends unless noted otherwise.

#To determine the subtraction sequence, change each statement to an inverse statement. Omit all categories which involve three numbers.

**Proportional indicates visual and/or physical size relationship between models used for ones and tens or parts and wholes.

37. Mixed numerals, no regrouping, different denominators
38. Mixed numerals, regrouping, different denominators
39. 3 non-unit fractions, sum between 2 and 3, different denominators
40. 3 mixed numerals, sum of fractional parts between 2 and 3, different denominators

In the following group, use prime numbers 2, 3, 5, 7

41. Prime factorization of numbers with 3 prime factors
42. Use prime factorization to find LCM of 2 three-factor numbers
43. Sums less than one using prime factorization to find LCD
44. Renaming (reducing) fractions with prime factorization and identity
45. Sums of decimal fractions with even endings, tenths and hundredths
46. Sums of decimal fractions with ragged endings, tenths and hundredths
47. Sums of decimal fractions, other cases

Grade Level	Continuum Increments*
1.0-1.4	6-11
1.5-1.9	6-13
2.0-2.4	8-15
2.5-2.9	9-17
3.0-3.4	10-19
3.5-3.9	10-30
4.0-4.4	16-36
4.5-4.9	16-40
5.0-5.4	23-43
5.5-5.9	23-44
6.0-6.4	23-46
6.5-6.9	23-47

*If the number of continuum increments is less than or equal to 10, construct three items for each increment. If the number of continuum increments is more than 10, construct three items for every other increment.

Sample Items for the Addition Continuum

6. Circle the numeral which names five: 6 3 5

7. Circle the numeral which names a number less than six: 7 4

8. Circle the smallest number named. Place an "X" on the largest: 3 9 4

9. 3 + 4 = ____

10. 36 = 16 ones and ____ tens

11. 6 + 7 = ____

12. 42 +3

13. 61 +37

14. 67 +5

15. 67 +88

16. 26 47 +38

17. 367 = 2 hundreds 7 ones, ____ tens

18. 134 +265

19. 486 +157

20. Write the numeral which represents one-fourth:

21. Circle the numeral which represents the smallest number: $\frac{1}{2}$, $\frac{1}{3}$

22. Write the numeral which names three-fourths:

23. Fill the blanks: $\frac{2}{3} = \frac{\_\_}{6} = \frac{12}{\_\_}$

24. Is $\frac{2}{3}$ more than $\frac{3}{4}$ or is $\frac{2}{3}$ less than $\frac{3}{4}$?

25. Circle the smallest number named. Place an "X" on the largest:

$\frac{2}{3}$, $\frac{2}{4}$, $\frac{5}{6}$

26. $\frac{2}{6} + \frac{3}{6}$ = ____

27. Write the numeral for three and four-fifths:

28. $2\frac{1}{5}$ $+4\frac{2}{5}$

29. Write $\frac{4}{3}$ as a mixed numeral.

(Express the following as mixed numerals when possible)

30. $\frac{4}{5} + \frac{3}{5} =$ ____

31. $6\frac{2}{3}$
$+\ \frac{2}{3}$

32. $7\frac{4}{5}$
$+2\frac{3}{5}$

33. $\frac{5}{8} + \frac{6}{8} + \frac{7}{8} =$ ____

34. $3\frac{3}{5}$
$2\frac{4}{5}$
$+4\frac{4}{5}$

35. Determine the LCD by creating sets of multiples. Then express the given fractions with the LCD by using the identity element for multiplication (name for 1): $\frac{2}{3}, \frac{3}{5}$

36. $\frac{1}{4} + \frac{2}{3} =$ ____

37. $2\frac{1}{2}$
$+1\frac{2}{5}$

38. $3\frac{3}{4}$
$+2\frac{7}{8}$

39. $\frac{2}{3}$
$\frac{3}{4}$
$+\frac{5}{6}$

40. $2\frac{1}{2}$
$5\frac{2}{3}$
$+6\frac{3}{4}$

41. Determine the prime factorization of 75

42. Use prime factorization to find the LCM of 12, 18, 30

43. Use prime factorization to find this sum: $\frac{1}{8} + \frac{1}{12}$

44. Rename (reduce) this fraction using prime factorization and identity.

$\frac{8}{12}$

45. .13
+.28

46. .65
+1.8

47. 2.678
+1.2

Sequence Level	Addition Survey Test for Grade Level 2.0 - 2.4

(8) In this group of problems, draw a circle around the name of the smallest number. Draw an "X" over the largest. Example: 6 9 (4) If six is a smaller number than nine and four, draw a circle around six. If six is a larger number than nine and four, place an "X" on top of six.

(1) 4 5 2 (2) 9 5 7 (3) 1 8 7

(9) Write the sums in the blanks.

(1) 5 + 2 = ____ (2) 1 + 7 = ____ (3) 3 + 6 = ____

(10) How many tens and ones are in the number at the left? Example: 43 = 3 ones and 4 tens.

(1) 52 = ____ tens and 12 ones.

(2) 71 = ____ ones and 6 tens.

(3) 25 = 1 ten and ____ ones.

(1 .) Write the sums in the blanks.

(1) 7 + 5 = ____ (2) 8 + 6 = ____ (3) 9 + 5 = ____

Determine the sums of each of the following.

(12)	(1) 36 +3	(2) 72 +7	(3) 61 +4
(13)	(1) 26 +33	(2) 53 +30	(3) 82 +17
(14)	(1) 37 +6	(2) 85 +9	(3) 57 +4
(15)	(1) 26 +37	(2) 45 +18	(3) 57 +36

Addition Analytical Test for Sequence Levels 13, 14, 15

(13S) Draw pictures to determine these sums using a large circle, 0, to represent each ten and a small circle, o, to represent each one as you have been instructed. Show regrouping or carrying when it occurs.

(1) 36 +21	(2) 16 +42	(3) 45 +34

(13A) Determine these sums <u>without</u> pictures or drawings.

(1) 27 +31	(2) 62 +17	(3) 84 +12

(14S) Draw pictures as on the questions at the top of this page.

(1) 29 +3	(2) 65 +8	(3) 88 +6

(14A) Compute the sums <u>without</u> use of pictures.

(1) 37 +7	(2) 41 +9	(3) 53 +8

(15S) Use pictures, circles, to determine these sums as before. Be sure to show regrouping when it occurs.

(1) 16 +25	(2) 37 +28	(3) 24 +28

(15A) Determine these sums without pictures.

(1) 65 +18	(2) 36 +47	(3) 58 +25

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REACTION PAPER
CLASSROOM DIAGNOSIS

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Professor Underhill began his presentation of a model for classroom diagnosis by developing a concise theoretical base. He concluded with a clear description of a prototype model which could be derived from the theoretical foundation. Now, it is our responsibility to discuss the crucial question: Can this model provide the information which is necessary for the prescription of remedial programs? As we subject his suggestions to a thorough analysis let us look for the stong features of his proposal, ways of modifying his model which will produce other desirable outcomes, and for opportunities to apply his model to still other situations.

There are several significant features of his proposal which need to be mentioned at the outset. One is the positiveness of the approach itself. As one would expect, the model provides for the identification of areas of deficiency in a pupil's attainment of specific concepts and skills. But, as one might not expect, the model also focuses on the determination of areas of strength. This positiveness to approach is important for two reasons. First, it identifies a foundation on which the remedial work can be based. Secondly, such an approach promotes a better pupil-teacher relationship. As the teacher plans the remedial work for the correction of specific deficiencies the teacher is aware that the child has already acquired certain concepts and skills which are related to the increment being investigated. Thus, the teacher begins the remedial program with positive expectations of the pupil Too often this is not the case, because from the outset the teacher is convinced that the pupil cannot or will not learn. And, as you know, the pupil performs accordingly. This latter teacher attitude cannot be tolerated if we expect the remediation to be effective. Therefore, as we scrutinize the proposed model, we must keep in mind that we are trying to determine areas of strength as well as trying to identify areas in which the pupil is experiencing some degree of difficulty.

The second notable feature of the model is its brevity. Each continuum is not unduly long, and the tests for each increment of the continuum have only a few items. This means that the total testing program for each increment can be administered without consuming an undue amount of time. As we strive to assess the effectiveness of the proposed model, it will be natural for us to expand the list of things we think we need for diagnostic purposes to a point where there is only one logical conclusion about conducting diagnostic work in a classroom. Namely, it's impossible to do it. On one hand, we must recognize the complexity of the diagnostic process, but on the other, we cannot allow this complextity to stifle our efforts to develop a model for utilizing diagnostic procedures within the context of a typical classroom setting. Since we cannot allow ourselves to fall into the trap of requiring that a diagnostic model provide more information than we actually need, we should restate the assessment question as follows: Does the model provide sufficient information for a functional diagnosis of a child's learning difficulty? We should keep the classroom diagnosis with the parameters Professor Underhill has suggested, and leave a more thorough analysis to a specialist.

A third significant characteristic of the proposed model is its parallelism to a recommended learning sequence; that is, an instructional sequence which begins with the concrete, progresses to the semi-concrete and finally concludes one spiral of a continuum at the abstract level. The obvious advantage of this parallelism between the sequence of instruction and the diagnostic model is the ease with which one can move in and out of each program. Also, this parallelism between the components of the instructional and diagnostic programs makes it relatively easy to construct appropriate assessment items for each increment of the continuum. These are important considerations and should not be lightly discarded. We must, however, raise certain questions about the usefulness of the data we can obtain from such a model. Namely, does the dual parallelism between instruction and diagnosis provide the information we need for an adequate diagnosis? One particular point relates to concepts and skills that do not lie altogether in any one continuum; that is, concepts and skills that establish relationships among two or more areas, such as addition and subtraction. In one sense, this may not be an unsurmountable problem in that it is possible to construct integrative continuums. But these new continuums pose other problems regarding their integration with the standard continuums which have been identified by Professor Underhill. Most viable instructional sequences move back and forth among continuums. Therefore, one must ask, "will the complexity of the instructional program create problems for the diagnostic process?" In particular, would the order of movement among continuums have an effect on the type of diagnostic instrument that should be constructed? Theoretically the dualism between instructional and diagnostic programs seems ideal. But, in practice, does the complexity of the former negate the effectiveness of the latter? We should address ourselves to that question.

Another significant feature of Professor Underhill's model is that the data collected relative to a pupil's performance on each increment selected for investigation is obtained two levels. On one hand the student is administered an abstract or computational test, and on the other hand he is expected to perform similiar tasks at the semi-concrete level. This dual diagnosis is a definite improvement over the usual assessment strategy. But we must ask, "Has he chosen the best two levels?" I am making the assumption, as I feel that Professor Underhill has, that the administration of test components at three levels--concrete, semi-concrete, and abstract--would result in considerable duplication of information about a pupil's performance on a given increment. Therefore, I am not suggesting that we consider a three tier diagnostic instrument. You may, however, wish to question the validity of this assumption. The selection of the abstract level seems to be necessary. So let's turn our attention to the question of semi-concrete vs. concrete. The semi-concrete has an obvious advantage in that it is easy to administer, whereas, the administration of concrete exercises would require the services of a monitor or proctor to record the pupil's responses. Now, does the ease of administering the semi-concrete take precedence over the need for vital information which could possibly only be obtained from the administration of concrete tasks? There seem to be several reservations about the use of a semi-concrete component. First, are the skills which are required of a pupil to complete a semi-concrete task explicitly taught in the average elementary school classroom? In one of Professor Underhill's examples he asked the pupil to draw pictures representing the addition of two 2-digit numbers with regrouping. If the skills required to complete this task are not taught or given considerable emphasis would the administration of such a test item really provide valid information for diagnostic purposes? One indication that children are not generally proficient at the semi-concrete has been the results obtained from the Florida Assessment Project. On these tests, children consistently have a higher level of performance on abstract items than on semi-concrete items. Another possible reservation about the semi-concrete level is a question as to whether the semi-concrete activity really represents an essential skill which a child should possess. For example, would it be more important in assessing a child's proficiency with addition of 2-digit numbers with regrouping to know whether a child could draw pictures of this process or whether he could actually demonstrate this with some manipulative object such as bundles of straws or maybe even money? From a standpoint of making a determination as to what a child has learned and what he needs to learn next, I fe[illegible] it would be more important for us to know his proficiency with regards to the manipulation of objects rather than his skill in drawing pictures. A third point along this line is that the semi-concrete is still at a somewhat abstract level in that it involves the use of more symbolism and notation than is required at the concrete level. Thus, knowledge of symbolism and notation becomes a variable which might understandably prevent a child from performing adequately at this level. This is also one of the same variables that one encounters at the abstract level. A final reservation regarding the semi-concrete is that the evaluation is made only on the end product, that is, the answer which the child produces. A semi-concrete test item can not provide information relative to the procedures or the methods that the child used in arriving at his answer--either a correct answer or an incorrect answer. This is a severe limitation in that the assessment process does not provide information about the cause of the deficiency. This is really a crucial question for us to consider. Can the diagnostic model provide information related to causes of deficiencies, in addition to the identification of the deficiencies themselves?

At the end of Professor Underhill's presentation, he lists several k[illegible] questions for us to consider. All of these questions seemed to be re[illegible]d to instr[illegible]tional sequence. The implication of these questions seems to be that if we direct our efforts toward the refining of the instruction se[illegible]ence, with consideration begin given to both content and pedagog[illegible]al factors, then the design of an effective diagnostic model will be self-evident. Phrasing this suggestion in a slightly different manner, he appears to be saying that the design of a diagnostic model which assesses the output of the instructional program should be essentially the same as the design of the instructional program itself. Since this dualism is really the basic core of his recommendations, we should examine the validity of the dualistic approach. The basic reason for questioning the appropriateness of the parallelism between instruction and evaluation is that, as we all know, it is often the case that children's behavior, which is the output, very often has little resemblance to the nature of the input, that is the instruction itself. Thus, if we accept as our goal the determination of causes for deficiencies in children's performance of certain tasks, shouldn't we look at the output (children's behavior) for a basis on which to construct and to design appropriate diagnostic instruments? Let's pursue this premise by examining several possible reasons for some children's inabilities to perform certain tasks.

As we search for probable causes for a child's inability to perform certain tasks, primary consideration must be given to the influence or effect of affective factors. As Professor Underhill developed the theoretical base for his model, he briefly touched on the significance of affective factors to the total teaching-learning process. But, in the design of his diagnostic model no explicit consideration is given to the determination of such effects. This I feel, is unfortunate, because affective factors have a considerable bearing on a pupil's performance on a diagnostic test. They should be reflected in the resulting diagnosis. To illustrate the need for considering factors such as interest, motivation, and fear in the diagnostic process, I would like to cite a specific case.

John, a bright fifth grader, was referred to me by his parents as having a problem in division. With this sketchy information I began the diagnostic process. The procedures I used clearly resembled the model Professor Underhill has proposed. First, I gave a brief paper and pencil test. After about three minutes John had answered none of the questions of the test. He simply looked at the paper and made little doodle marks with his pencil. Realizing it was futile to continue along this line, I administered a concrete component. In presenting these tasks I gave John some blocks, and asked him to show me how to use the blocks to solve a simple division problem. Again there was no positive response from John. He did, however, build a house with the blocks. If my diagnosis of John's performance had stopped at this point, I would have been forced to conclude that John had no understanding of division. My prescribed remediation program would have been to begin teaching division to John all over again, perhaps using a different approach or technique. But in the actual case I included a third component in the diagnostic procedure, a game in which he had to solve simple division problems. John's behavior on this component amazed me. In each case John answered the division problem with such speed and accuracy that it was obvious that he had committed the division facts to memory. Now, what was my assessment of John's proficiency with basic division concepts and skills? Was it that he had no knowledge or skill in this area? Or rather was it a matter of a lack of interest, motivation, or perhaps a simple refusal on John's part to perform typical textbook exercises?

I have a feeling that John's case is not atypical. As we strive to develop an effective diagnostic model, one which will identify causes of poor performance, as well as the nature of the errors themselves, we should make provision for assessing affective factors. That is, we must find ways of determining whether or not affective factors such as interest, motivation, or fear of paper and pencil tests are really the cause for the child's poor performance on assigned tasks. Along this same line, most of you have observed that children sometimes perform quite differently on paper and pencil tests than they do on applications, games, or maybe even such activities as flash card drills. For example, you might have noted that certain children will quickly give correct sums for addition problems on flash card drills, but then when given the same problems on a written exercise they go back to using their fingers or perhaps making tally marks on the paper. Thus, if all of our diagnostic components are of the paper and pencil type and are modeled after typical classroom situations which replicate features of the instructional program, then we will likely get an invalid evaluation of a child's mastery of the concepts and skills under investigation. I would like to recommend, for your consideration in the discussion which follows, that we examine the feasibility of including game or application exercises, as well as the typical textbook type of activities, as an integral component of diagnostic instruments. This is seemingly an easy task until you begin to consider the various parameters Professor Underhill has mentioned, that is, the various parameters related to teachers and environment.

Now, let's look at another possible technique for determing probable causes of incorrect answers given by pupils on written exercises, one that is more directly related to student responses than to the instructional program itself. Several years ago, I had the opportunity to observe a second grade class in which the teacher was introducing addition of a 2-digit number and the 1-digit number with regrouping. The teacher's instruction, I felt, was quite good. Since most of the students correctly worked all of the problems in a follow-up exercise, I believe that this was indeed the case. But, there were six pupils in the class who missed most if not all of the problems in the exercise. In the space below I have identified each of the six children with his answer to one of the problems in the drill activity. In each case, the illustrative answer is consistent with the answers given for the other problems in the exercise set.

Betty	Tom	Jerry	Sue	Brenda	Carev
46	46	46	46	46	46
+7	+7	+7	+7	+7	+7
52	51	71	43	413	467

You will note that there are six different incorrect answers, this suggests the possibility of six different causes. And further, the variance in the answers suggests the need for six different remediation programs, each specifically aimed at a particular cause. Betty, for example, in finding the sum of 46 and 7 arrived at an answer of 52. What was her problem? The answer was only one off. Possibly she added 6 and 7 to get 12. This might be viewed as a chance error, if it were not for the fact that all of Betty's answers were exactly one off. Therefore one must ask, "What was the cause for this consistent error?" A possible explanation, one that was actually verified during an interview with Bett , is that her scheme for computing facts was defective. For example, in computing the sum of 6 and 7, Betty counted as follows: I'll start at 7 and count 6 more. In doing so she said, "7, 8, 9, 10, 11, 12." The cause of Betty's difficulty was attributable to the employment of a slightly defective counting procedure and not to any lack of understanding of the regrouping process. I am sure that if Betty had been administered a semi-concrete or even a concrete task, she would have worked the problem correctly.

Now, let's consider Tom, the one who gave 51 as the answer to the problem. Detecting the cause behind this error was a difficult task. But, actually, Tom was exhibiting remarkable insight into the nature of adding a 2-digit and a 1-digit number. Tom had worked the problem like this. He looked at the problem and said, "The answer is going to be in the 50's. Then he wrote the 5 in the tens place. Next he correctly added 6 and 7. He wrote down the 1 of 13 in the unit's place. He did not write the 3, because he realized that his answer could not have three digits. This is an indication of another unique cause that probably would not have been identified by semi-concrete or concrete procedures.

Jerry, our third student, had an answer of 71. How could he have arrived at the answer? One seemingly wild scheme would have been for Jerry to have added 4 and 6 and 7 to get an answer of 17 and then write it backwards. Unlikely, but it is definitely a possibility. A more likely answer would be that Jerry, in working the problem, added 6 and 7, got 13, wrote the 1, carried the 3, and added 3 and 4 to get 7 in the tens place. Which of these would be more likely cause for his incorrect answer? Is it that he has no understanding of the addition process itself, or is the cause a matter of a simple reversal in writing a 2-digit numeral within the context of an addition problem? Knowing the real causes would be absolutely essential to providing an effective remediation program for Jerry.

Sue, our fourth student, gave an answer of 43. Now there are two likely explanations for this error. One explanation is that she added the 6 and 7 to get 13, wrote the 3, and then forgot to carry the 1. On the other hand, Sue could be one of those students who works from left to right. In this case she could have brought down the 4, added the 6 and 7 to get 13, and, much like Tom, realized a 3-digit answer was inappropriate, so she wrote only the 3, omitting the 1. In this case we have two likely causes for an answer.

The fifth pupil, Brenda, gave an answer of 413. You will note her answer is shifted slightly to the right. That is, the 4 is in the tens column, the 1 is in the units column, the 3 is sticking out to the right. This probably indicates that the cause of Brenda's difficulty is, in part, due to the fact that she works from left to right. Therefore, in any remediation program, if it is to be effective, one must teach Brenda to work from right to left.

Larry, the sixth student, gave an answer of 467. What was the nature of his difficulty? Again, two possible explanations occur. One falls in the affective domain in that he simply did not want to answer any of the questions. Perhaps he did not feel like working on this particular day, so he simply just wrote down the digits in the problem to have something to hand in. The other possible explanation is that this was an indication that Larry had no real understanding of the addition process itself. In Larry's case it might be possible to determine if the first situation was actually the cause of the difficulty by administering a game. In the second case, the use of a concrete or semi-concrete task would give some insight as to whether Larry really had an understanding of addition itself. By discussing with you an actual classroom situation, I have intended to do two things.

First, I have tried to provide a common situation which we can use in our discussions to assess the effectiveness of the proposed model. To be more specific, we can attempt to answer the following question: If the proposed diagnostic model had been used in the situation described above, would it have provided the classroom teacher with sufficient information about each pupil's mastery of this increment of the addition continuum? I would like to suggest that the answer to this question should be "no". My rationale for this conclusion is that the total model components and methods of analyzing the data would provide no specific clues as to the causes for the errors.

The second reason for presenting this particular case is to lay the foundation for a strategy which might be utilized, within a typical classroom, to predict the probable cause for a certain error type. Basically, the design of the proposed diagnostic technique is tied to the following assumption: it is possible to determine the nature of the cause of a student's deficiency in performing certain skills by analyzing the answers. If we can accept the validity of this assumption, then a new model for diagnosing learning difficulties might be developed along the following lines. Before I outline the model, I would like to restate one important feature of the strategy: each diagnosis is based primarily on the nature of the incorrect answers. By contrast, in most diagnostic models the diagnosis is based solely on the number of problems missed.

1. Administer a brief test. All of the tests items are related to one increment of a continuum. The test may be written or a game. If a game is used, a wrong answer response should not deter the continuation of the game.
2. Incorrect answers are identified.
3. Refer to a predetermined table or chart which relates specific errors and/or combinations of errors to a probable cause.
4. Having arrived at a possible cause, select appropriate remediation activities from a catalog of tested prescriptions.

In making this suggestion I realize that it sounds somewhat mechanical, perhaps too impersonal or cut and dried. But, then on the other hand, it somewhat resembles the procedures or techniques used in the diagnosis of a medical illness. That is, a particular combination of symptoms (incorrect answers) suggests a certain illness (a cause of a learning difficulty). And there are usually several recommended medications (remediation activities) for each illness. The effectiveness of this diagnosis method can be attested to by most parents who have used a baby book or medical encyclopedia. When one considers the teacher parameters Professor Underhill has identified, a model such as the one I have described may be the only feasible solution.

In offering this model for your analysis, I must acknowledge the existence of a vital area of concern--a problem which must be solved prior to the implementation of the model. Since part of our charge at this conference is to discuss needs for further research in the area of diagnosing learning difficulties, I'll present this problem as a research question:

> Is it possible to collect a data base such that, given a particular combination of responses to a limited number of selected questions, one would be able to predict, with 80% accuracy, the probable cause of the learning difficulty?

A thorough investigation of this question is of utmost importance to the further study of impediments to the learning of mathematics, regardless of its relationship to the diagnostic model discussed above.

In closing, I would like to summarize my remarks by stating three criteria for any classroom diagnostic model. First, the model should provide positive information about a child's mastery of a given topic, as well as detect deficiencies. Secondly, the diagnostic procedures must not be unduly complicated or consume too much valuable instructional time. And, finally, the model must provide for the determination of probable causes for the learning difficulty.

CLINICAL DIAGNOSIS OF CHILDREN WITH MATHEMATICS DIFFICULTIES

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The preceding papers have looked at the individual child who is having difficulty with mathematics. We have tried to define a role for the classroom teacher in diagnosing and remediating children's mathematics difficulties. No matter how sophisticated the classroom teacher becomes in mathematics teaching, there still will be children with severe mathematics difficulties with which the classroom teacher cannot cope within the classroom environment.

Elementary schools need some place where children with severe mathematics difficulties can be referred for individual help. Thus, there is a need for a clinic setting that is equipped to systematically and efficiently diagnose children's mathematics difficulties and then to prescribe procedures to follow in effectively remediating each child's mathematics difficulties. Within the clinical setting, the child must receive the necessary help so that he can return to and function successfully in the normal classroom atmosphere. The purpose of this paper, therefore, is to define a role for the mathematics clinic; identify procedures and techniques and then isolate areas of needed research. Before attempting to discuss these three areas, it is probably necessary to define what is meant by "clinical diagnosis." Webster defines clinical as connected with a clinic and defines a clinic as a place where patients are studied or treated by physicians specializing in various ailments and practicing as a group; an organization or institution that offers some kind of advice or treatment. Relating these definitions to mathematics we would say that clinical diagnosis of mathematics would be a clinic setting where a person's mathematics difficulties are carefully and systematically investigated to determine the nature of the difficulty, to isolate the specific mathematics difficulties, and to prescribe a treatment for the difficulty.

In order to diagnose mathematics difficulties we should probably examine how children learn mathematics. Lovell (1971) states that our knowledge of the growth of human thinking is as yet insufficient to provide a basis for scientific pedagogy, and an intuitive understanding of children on the part of the teacher must complement what we know of them in a scientific sense. Learning could be responding to each item to be learned and memorizing each idea independently. However, it would seem logical to put ideas into classes and then respond to a class as a whole. In comparing Gagné and Piaget, we find similar and useful ways for conceptualizing learning and for determining the level at which children can learn. They both see knowledge as accumulating in an orderly sequential and hierarchial manner. Both suggest that there is an invariant order in which concepts may successfully be acquired. Rosenthal and Jacobsen (1968) have documented the fact that teacher expectations and parental expectation are communicated to children and that it affects their achievement. How teachers conceptualize mathematics will have considerable influence upon how much children will learn, and which children will learn.

Within the literature, there is a great deal of variation in ways in which the term "concept" is used; consequently there is a variety of description of the essential conditions for learning concepts by children. Perhaps if we can agree upon what a concept is, and how a concept is learned, then it would be possible to design experimental studies to find out how to diagnose children's mathematics difficulties. What is concept learning? Is concept learning organizing ideas into taxonomic classes and utilizing mnemonic devices to facilitate learning?

Berlyne (1965) believes that an individual forms a concept when an overt behavior comes to depend upon certain properties of a stimulus pattern while disregarding other properties. It means forming equivalence classes of stimulus situations, which share some characteristics but are distinct in other aspects.

Gagné (1965) states that concept learning makes it possible for the individual to respond to things or events as a class. But it is important to conduct the discrimination learning within stimulus situations that represent the actual range of the concept being learned. The effect of concept learning then is to free the individual from control by specific stimuli.

Gagné continues by defining principles as chains of concepts that make up what is generally called knowledge. Different conditions are applicable to the learning of concepts and the learning of principles. Concepts are learned prior to principles and, in a sense, are simpler to learn. If concepts and principles are two different capabilities, then it is also quite possible that the conditions for learning are also different and that the techniques for diagnosing should also be different.

Kendler (1964) defined concept learning as the acquisition of a common response to dissimilar stimuli. He also states that clues and associations function as mnemonic devices. Carroll (1964) defined a concept as an abstraction from a series of experiences which defines a class of objects or events.

Gagné (1971) after studying definitions of concepts, summarized the following general properties:

1. A concept is an inferred mental process.

2. The learning of a concept requires discrimination of stimulus objects (distinguishing "positive" and "negative" instances).

3. The performance which shows that a concept has been learned consists in the learner being able to place an object in a class.

Gagné (1965) conceptualizes all learning as a function of prior learning, or prerequisite learnings. Learning of a particular concept only occurs if the concepts (or learnings) that are prerequisite to the concept have been acquired. Every bit of learning is thought of as generating a hierarchy in which prerequisite learnings can be identified, these prerequisites must be "learned" before the learning on the next level can occur. Gagné (1965) hypothesizes that hierarchies can be derived logically in certain content areas and describes mathematics as one of the areas.

Using Gagné's prerequisite learnings approach, hierarchies can be created by beginning with the final learning task and then asking oneself the question: "What kinds of learnings or understandings would an individual need to possess if he were able to perform this learning task successfully?" Through systematic analysis, hierarchies of learnings can be generated in which lower members of the hierarchy serve as prerequisite learning to higher members.

For a number of years the members of the Mathematics Education Team of Kent State University have used Gagné's prerequisite learning concept as a technique to help taxonomize mathematics ideas into what we call a check list of mathematics.[1] We were unable to define a mathematical concept functionally and Gagné's (1965) discussion of concept did not seem applicable to the situation. A "chain" as described by Gagné might be more easily applied but it also did not seem adequate. Consequently, we used the idea of concept clusters as the major headings. We defined a concept cluster as a taxonomized list of all mathematics ideas necessary to comprehend and function with any major mathematical idea. Within each concept cluster we use the idea of prerequisite learning to develop a sequence. Each concept cluster then, is a hierarchial sequence of mathematics development commencing from the concrete level through the semi-concrete, semi-abstract to the abstract level. It is a development from the real world into the abstractness of pure mathematics.

There is an inter-relatedness among the many concept clusters so that a person does not study and learn all of the ideas itemized under one concept cluster and then proceed to the next concept cluster. Instead there is a hierarchial scheme within each concept cluster that must be interlaced with other concept clusters. For example, the concept cluster place value must be continually expanded as the concept cluster addition and its inverse is being developed. Note how the check list is based upon the systems of numbers and the characteristics that are necessary for a mathematical system. The logical structure of mathematics is very systematically integrated into the hierarchy of mathematics ideas. The mathematics check list implies that the children must attain an understanding and an ability to function with whole numbers before he can proceed to the set of integers and then to the set of rational numbers. Since the set of whole numbers is a basic subset of the set of integers and the set of rational numbers, the understanding and operation upon the set of whole numbers is essential before moving to the study of integers or rational numbers. Note how the development of each subsequent set of numbers is parallel in sequence.

[1] This check list of mathematics is available for $1.00 by writing: Professor James W. Heddens, Department of Elementary Education, Kent State University, Kent, Ohio 44242.

Carefully examine the two operations and the two inverse operations for common ideas that children need to know. There seem to emerge five basic ideas that need to be taught and these ideas are exactly the same for the operations of addition, and multiplication, and the inverse operations of subtraction and division. The five basic ideas seem to be:

1. The student needs to understand what the operation or the inverse operation means. This will be referred to as a "definition" of the operation.

2. The student needs to memorize the basic facts for each operation.

3. The student needs to understand place value in order to apply it to each operation.

4. The student needs to understand the structures of mathematics (properties) and how they are applied to the operations and inverse operations.

5. The student needs to understand regrouping.

Placing all this information into a table will help us put the ideas into perspective (see Table 1).

In diagnosing a child's mathematics difficulties in respect to the basic operations for the whole numbers, it becomes necessary to evaluate the child's level of functioning under each of the above listed ideas.

Fernald (1943) was probably one of the first to suggest a procedure for diagnosing children's mathematics difficulties. Her suggested procedure outline was:

I. A carefully given individual intelligence test is particularly important in connection with mathematical disability because retarded mental development may be the cause of the difficulty.

II. A general achievement test covering various subjects is given. A study of the test results not only gives a profile showing the relative development of the individual in different school subjects but also indicates the weak points in specific subjects.

III. The tests to determine the nature of the individual's disability come under the following heads:

   1. tests in simple combinations
   2. tests for skill in complex situations involving simple combinations
   3. tests in problem solving

Table 1

WHOLE NUMBERS

	Definition	Basic Facts	Place Value	Structure	Regrouping
ADDITION	A + A = S	100 Facts	Same for all ↓	Closure; Identity element of addition; Commutative property of addition; Associative property of addition	Same for all ↓
SUBTRACTION	Inverse S − A = A	100 Facts	↓	(shaded)	↓
MULTIPLICATION	F x F = P	100 Facts	↓	Closure; Identity element of multiplication; Commutative property of multiplication; Associative property of multiplication; Distributive property of multiplication over addition	↓
DIVISION	Inverse P ÷ F = F	90 Facts	↓	(shaded)	↓

In diagnosing children's mathematics difficulties, it is necessary to eliminate as many extraneous factors as possible. The reading factor may be controlled by using oral diagnostic procedures only. Verbal behavior on the part of the child does not necessarily indicate that he has a well developed concept, therefore overt behavior might be a more valid means of evaluating and diagnosing. Mathematics ideas begin vaguely or hazily and grow and develop with experience and maturation. The questions become what is the child's developmental level and how can that level be determined.

Consider any one of the entries in the mathematics check list. During the session planning for the diagnoses, the clinician devises a set of observable behaviors that not only reflects the objective but also the level of functioning according to Bloom's Taxonomy of Educational Objectives. The overt behavior of the client would indicate his level of maturity for the selected mathematics idea. Each task must be precisely stated so that by observing the client, the clinician can evaluate the level of the functioning.

There are mathematics tests available that have been classified as diagnostic tests. Usually, a diagnostic mathematics test is a series of mathematics examples that the student is to solve. The tester usually compares the student's responses with a set of standard responses and from the data collected, the tester assigns a grade level score or a chronological age score. Does a diagnostic test really isolate a child's mathematics difficulties?

The clinician is trying to identify clues to reasons underlying the difficulty which can be corrected. He is trying to estimate the child's mathematics potential, to estimate his mathematics level, to estimate his strengths and weaknesses and trying to isolate the possible causes of the difficulty.

John Wilson, when he was at Syracuse University, made as much or more of a contribution to diagnosing children's mathematics difficulties than anyone else in the field. Wilson (1967) suggested a model, which was limited to the cognitive domain, as a guide to diagnosis mathematics learning by seeking answers to the following questions:

1. What specific mathematics learning products might be present/absent, correct/incorrect, mature/immature?

2. What overt behaviors will indicate the presence, correctness and maturity of each of these specific learning products?

3. What kind of psychological learning product does each of the specific mathematics learning products represent?

Glennon and Wilson (1972) have also developed a sequence to use in diagnosing children's mathematics difficulties. They also have related Bloom's Taxonomy and Gagné's work to their mathematics sequence as a model for clinical diagnosis.

There is probably no one diagnostic procedure that can be labeled clinical diagnosis, for each clinic will have developed its own data collecting technique. However, there appear to be several levels of clinical diagnoses, initial screening and detailed analysis. A standardized diagnostic test might be used as an initial screening device so that the clinician will have some guidance to help zero in on specific areas of difficulty.

In our procedures at Kent State the clinician transfers the general data collected from the standardized test to the columns on the right-hand side of the mathematics check list. The items evaluated on the standardized test will be scattered along the sequence of the mathematics check list. This will provide a clue to the clinician as to the specific areas that need to be evaluated to isolate the mathematics difficulties. Prior to the in-depth detailed diagnostic examination, the clinician should mark the items to be used and then design the specific behaviors the client will be asked to demonstrate.

After the initial screening and while the clinician is administering the detailed analysis, he should direct his attention to the client's behavior and the task of recording that overt behavior. He should not worry about interpreting or analyzing the data during the data collecting examination.

The record of the overt behavior is then studied to identify within each concept cluster the exact point at which the child experienced difficulty. The behavior must be interpreted either as an understanding difficulty or a skill difficulty. The analyses of each concept cluster are then studied in respect to each other to identify common difficulties. For instance, the idea of place value may be the difficulty that has hindered the child in understanding regrouping in each of the basic operations. From the study of each individual item, the analysis moves to concept clusters and then to the total program.

As soon as a client is referred to a diagnostic clinic, the clinician begins to amass data upon which a diagnosis can be made. The letter of referral should provide the initial data for the clinic record.

Frequently a team of specialists is used in the work-up of the clinic record. The assignment of specialists is requested by the assigned clinician. Psychological difficulties should be referred to a psychologist. Ocular difficulties should be referred to a physician. Auditory difficulties should be referred to an audiologist. Reading difficulties should be referred to a reading specialist. Mathematics and learning difficulties should be left to the educationist.

Prior to the initial meeting the clinician needs to very carefully identify the kinds of information that he deems necessary for a complete diagnostic report. The clinician takes the leadership in organizing the study, making referrals, collecting data and interpreting the data to the parents and to the school.

The initial meeting should begin with a minutely planned interview that encourages introspection to provide both affective and cognitive data. A skillful interview will provide specific information about the child's characteristics, habits and environment. The interviewer should not worry about interpretation of the data but should accurately record facts as related by the client. On the directive—non-directive continuum, the interviewer should remain sensitive and respond to the feelings of the client. The clinician must be relaxed and not be disturbed by silences. The clinician must keep in mind that it is natural for the student to be reticent at the beginning of the interview. The interviewer should have definite objectives in mind so that the interview does not degenerate to a conversation with random questioning which may have a desultory effect. Interviewing is a two-way communication that requires mutual understanding. The interviewer should avoid technical words and psychological jargon that is not understood by the client. The keys to a fruitful interview are a good quality of questions and the ability to listen. The clinician should write up the interview immediately upon completion of the interview so that important data are not lost. The clinician needs to record both verbal responses and overt behaviors.

Standardized tests are evaluation instruments that can provide valuable data. Non-verbal intelligence tests have been used as predictors of an individual's ability to achieve in mathematics. However, there has been some skepticism about the value of intelligence tests. We must be wary of over-interpretation of test scores in view of the unreliability of tests and the standard error of measure.

In diagnosing, the clinician tries to ascertain the reasons for, as well as the nature of the mathematics difficulty. Even the so-called diagnostic tests do not yield much understanding of the causes of mathematics deficiencies. Diagnostic tests give a certain amount of detail on the kinds of mathematics difficulties, but they do not explain why the child is having the difficulty. There is very little useful diagnostic insight that is actually extracted from a standardized diagnostic test. A general score may be obtained from an area, such as addition, but the test does not specify the type of difficulty. The diagnostic test may have an example of addition with regrouping and if the client misses that one example what does this mean? To check inferences based on an analysis of errors, the teacher needs another method: introspection. The clinician needs to question the client about how he arrived at his answer. What the clinician does with the diagnostic information obtained from a diagnostic test is of prime importance. The most important use of a diagnostic mathematics test is an aid in preparation for the informal testing session.

Using a check list of mathematics concepts as developed at Kent State University or the content taxonomy of Glennon and Wilson (1972), the clinician can zero-in on specific mathematics difficulties. Gagné (1971) states that the acquisition of the principle is tested by asking the student to demonstrate its application to a particular case which he has not encountered during the learning. A verbal behavioral on the part of a child does not assure that he has a well-developed concept of the given mathematics. We must keep in mind that concept development is not an "all or none" situation.

Thus the clinician must develop a series of situations that requires the client to demonstrate an overt behavior that can be classified, indicating the child's level of understanding or algorithm development. Thus at the informal test level the clinician is assessing the maturity level. Is the child functioning on the knowledge, comprehension, application, analysis, synthesis or evaluation level? Is the child functioning on the concrete, semi-concrete, semi-abstract or abstract level? To develop a full clinic record, in-depth diagnosing is necessary in each concept-cluster.

At this point in time, the clinician must bring together the data obtained from each referral, the initial interview, the standardized tests, and the informal testing procedures.

The information accumulated in a clinical diagnosis is useless unless it can be organized and synthesized so that it can be readily understood. The precise form may vary, but a format must be selected that will make optimum use of the information. One format may be:

1. Cover page
2. One page containing a concise summary of essential data (i.e., name, age, address, school, parents, name and address)
3. Formal tests administered to the client (tests, names, description of tests, description of student behaviors)
4. Informal tests administered to the client (purpose of informal tests, description of diagnostic activity, description of student overt behavior)
5. Pupil responses (attitude, emotional tone, pupils' comments, self-perception, attention span, mannerisms, etc.)
6. Analysis and interpretation of data (A report of the findings is brought together into a unified whole. An explanation should be derived and provided regarding the mean obtained from the test data. Interpretations should be supported by the data.)
7. Specific recommendations. (See Table ?)

The clinicians' procedures are suggested by Glennon and Wilson (1972) as well as the suggested model. No research has been developed to verify a hierarhical sequency of mathematics concepts. Two distinct notions have influenced the sequencing of mathematics concepts: the notion of prerequisite learnings and the notion of the logic inherent in mathematics content. How does one evaluate a given hierarchical mathematics sequence? Data needs to be gathered that assists in determining the adequacy of a given mathematics sequence. Empirical data to substantiate a given mathematics sequence could be obtained by studying students who are just being introduced to a concept as contrasted with students who have mastered a concept.

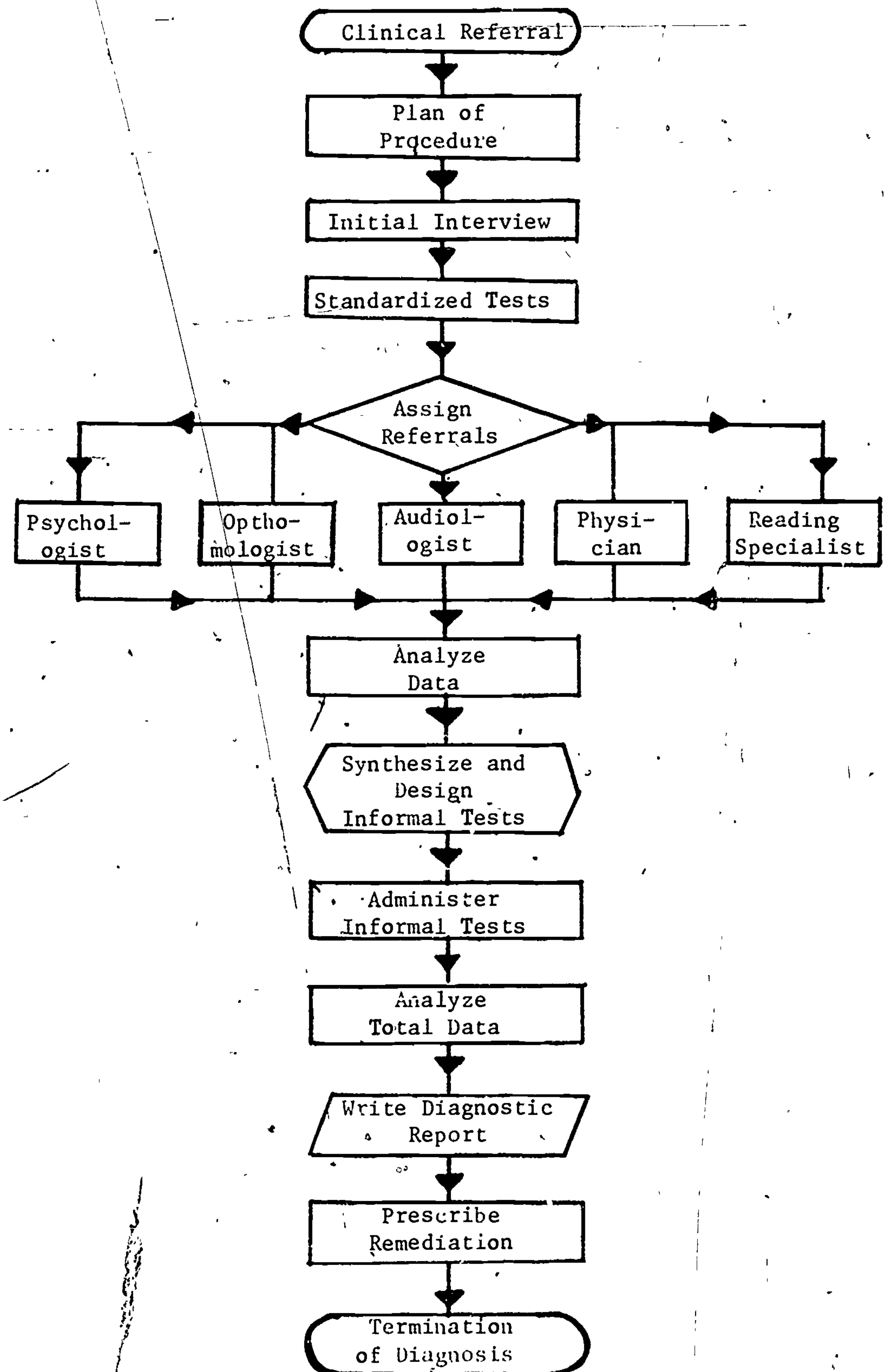
Clinical Referral
Plan of Procedure
Initial Interview
Standardized Tests
Assign Referrals
Psychologist
Opthomologist
Audiologist
Physician
Reading Specialist
Analyze Data
Synthesize and Design Informal Tests
Administer Informal Tests
Analyze Total Data
Write Diagnostic Report
Prescribe Remediation
Termination of Diagnosis

Table 2

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REACTION PAPER: CLINICAL DIAGNOSIS OF
CHILDREN WITH MATHEMATICS DIFFICULTY

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Professor Heddens' paper was designed to (a) define role for the mathematics clinic, (b) identify procedures and techniques and (c) isolate areas of need research. At a general level he has accomplished this in a laudable fashion. However, at a "specific" level there are a number of questions that need to be explored before a workable pattern of "clinical diagnosis" can be accomplished.

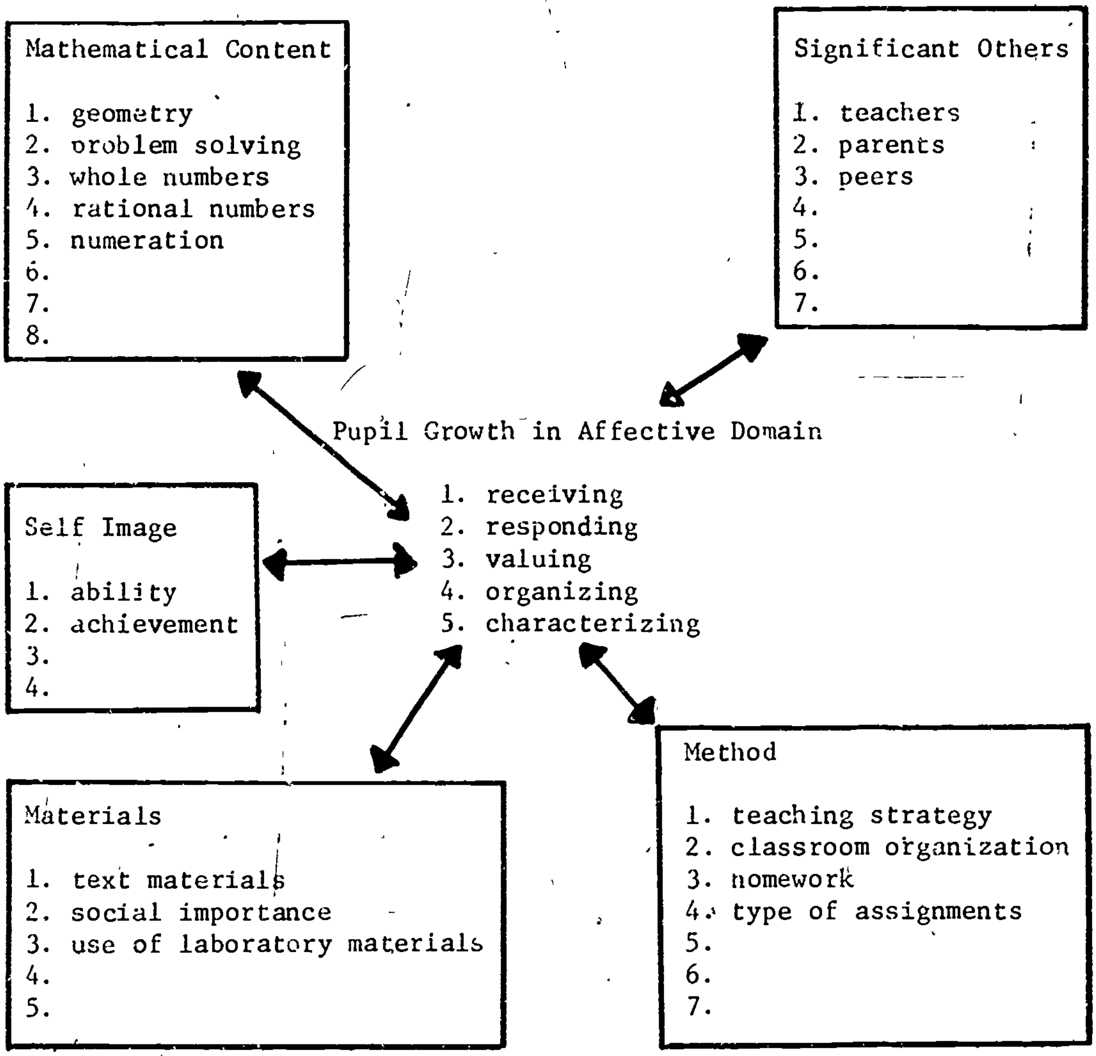
Since I feel myself in basic agreement with the suggestions made in the paper, I would like to take this opportunity to specifically react to or raise questions concerning a number of assumptions or statements.

(1) A given school system, teacher, parent, child, etc. needs to know an appropriate answer to "what is mathematics?" There are many mathematics programs today which have very little to do with mathematics. They present learning in a sterile setting in which mathematics is viewed as finishing a number of pages or tack cards so that a greater number of pages or tack cards may be completed. I very strongly believe, and I find that same underlying theme in the Heddens' paper, that we must consider all aspects of mathematics in clinical diagnosis and treatment. Thus, the creative aspects of mathematics, skill in searching out mathematical patterns, non-routine problem solving, and the study of functional relationships must be considered as carefully as the skill aspects.

(2) I would question the state of knowledge concerning the statement "both (Gagné and Piaget) suggest that there is an invarient order in which concepts may successfully be acquired." There are a number of non-cognitive factors which may contribute to 'order' of a learning sequence. For us to make careful use of a number of Gagné-type hierarchical sequences moving from concrete through to abstract there are a number of ideas from the affective domain that need to be considered. Research concerning diagnostic instruments in the affective domain needs to be continued and these results used to develop appropriate motivational procedures. Such a model is diagrammed in Figure 1.

(3) How should material be presented in a remedial setting when a child has previously studied the material in a given manner? That is, should the remedial setting be different from the original setting? There is some observational evidence which indicates that if a child has failed several times with a particular algorithm, he or she will continue to fail as long as that algorithm is used. However, we need to know specifically the probability of using particular remedial techniques with particular children.

A MODEL FOR MEASURING MATHEMATICAL
OUTCOMES IN THE AFFECTIVE DOMAIN



From: Riedesel, C. Alan, Guiding Discovery in Elementary School Mathematics, Second Edition. Englewood Cliffs, New Jersey: Prentice-Hall, 1974.

Figure 1

(4) What factors effect the order of mathematical sequences? We all have had experience with children who could not answer a low level question but knew with quite good understanding the answer to a higher level question. For example, we found many children in center city Atlanta and Jackson who did not know the addition facts, but knew the multiplication facts. I would hypothesize that the reason for this is that there isn't much of a time savings for knowing the addition facts, but it takes a long time to find the answer to 8 x 9 = ☐.

(5) What is the sequence in terms of properties and "facts?" Should a child ever "drill" on any facts until he knows all of the ideas which can aid him in answering an addition, subtraction, multiplication or division question?

(6) The hand-held calculator is creating a minor revolution in the adult world. As these machines continue to drop in price and increase in capacity, it is conceivable that there are a number of things that can be taught using these devices in a different order. For example, it might be in the next few years that "regular" sequence of topics might focus upon problem solving with all four operations of arithmetic before teaching the computational procedures using them.

(7) How can instruments be developed to predict the most productive learning style, materials, motivational approach, etc. for an individual child? For example, which child learns best from a remedial treatment motivated by the social utility of the topic; which child learns best from a games approach to a topic?

(8) Could parents be used in parts of the clinical approach to solving the difficulties that children have? For years, we have ignored the role that a parent could play in improving a child's mathematical achievement. A clever staff at a mathematics clinic could well diagnose a particular difficulty, develop a game or other suitable means of teaching the idea and send helpful suggestions home to the parents. In fact, there is a whole realm of research and development that could be conducted concerned with the parent as a tutor. Certainly, this would not work with all children and all parents but, there might be a significant number of parents and children who could work together on the mathematical difficulties of the child.

(9) What type of research should be conducted to find answers to clinical procedures, diagnostic techniques and the like? Certainly the typical experimental pattern of research has little to contribute in this field. We are not as interested in whether treatment A works better than treatment B for a group of students. The question is more what will work for Billy who is like this.... I am reasonably convinced that the reason that most studies reveal very little information is that most things are normally distributed. In that manner, treatment A is best for John, treatment B is best for Alice, treatment C is best for Ken, while treatment D is best for Jane.

(10) Do we know hierarchies well enough for a clinician to decide or is it necessary to give the student some choice? Would it be wise to have concrete, semi-concrete, semi-abstract, and abstract material available to each item checked? Many of us have had experience with a child who misses a low level concept and then shows a great deal of understanding of a high level concept.

(11) How can a "group spirit" be maintained in a clinical setting? Research is reasonably clear that grouping patterns do not cause much of an effect on achievement. However, working with others often increases student interest in a topic.

(12) How can we select the "right type" of persons to be trained as clinicians? There is a very strong need to have someone who is extremely enthusiastic, pupil supportive and flexible. There is also a great need for organization. I hope that we never sacrifice good teaching for organization. With the child who is not learning mathematics, we need someone who will "turn him on, not off."

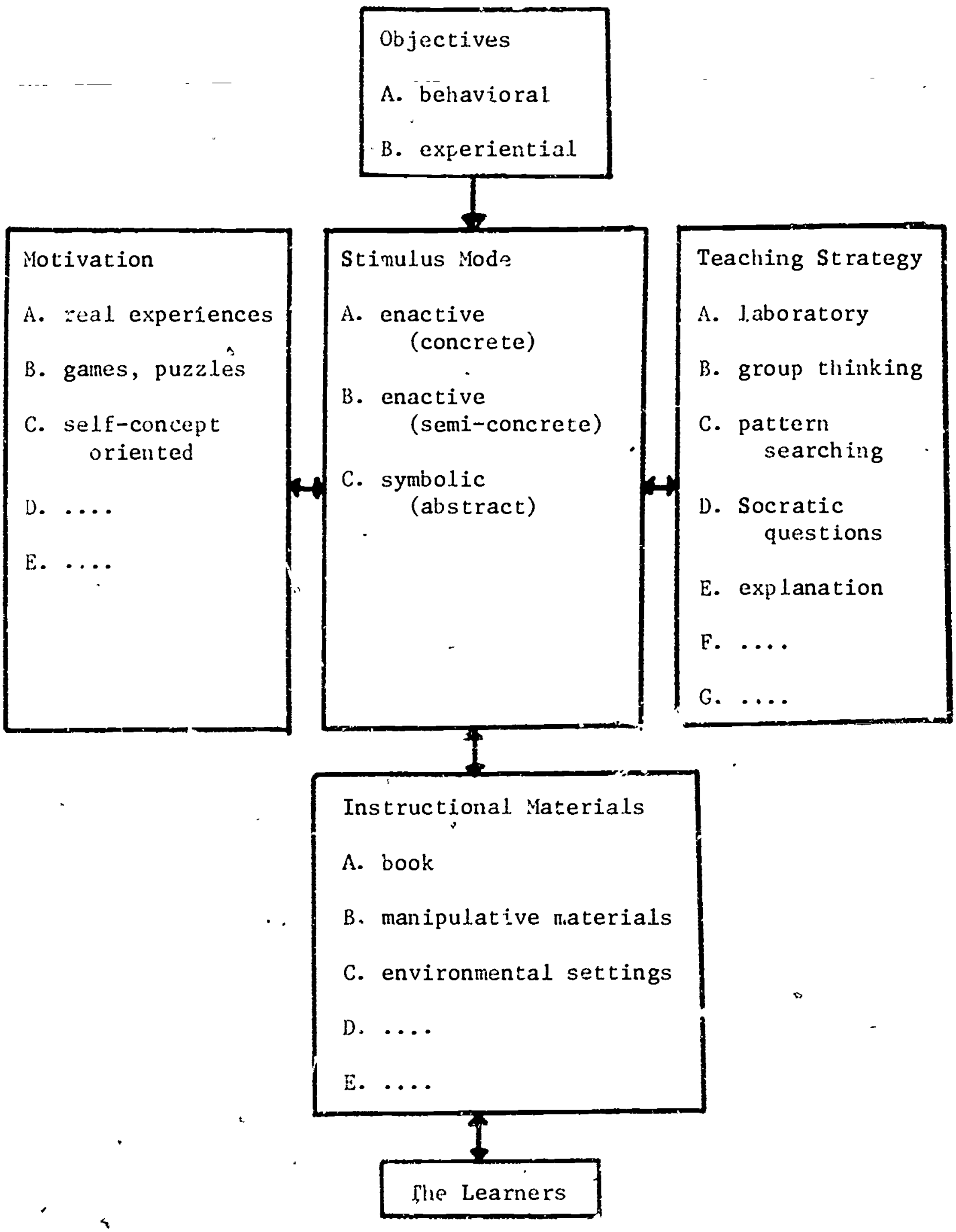
(13) What about other factors such as stimulus mode, teaching strategy, etc.? How do these fit into the clinic? How should a clinician operate using the Content Sequence diagrammed in Figure 2?

(14) What portion of the child's treatment should be classroom vs. clinic?

(15) How can a prevention mode be tied into the clinic? For example, the unit sequence in Figure 3 below should go a long way in preventing failure.

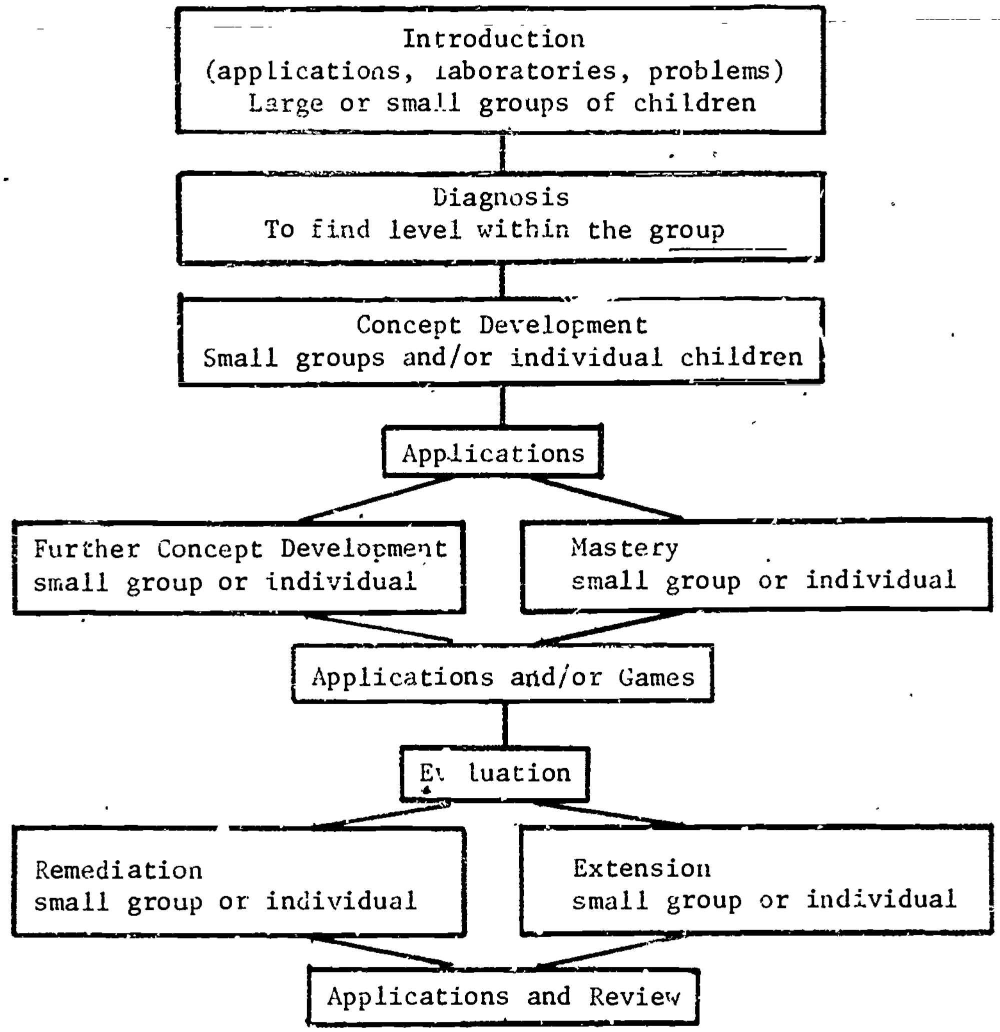
There are many questions that the development of clinics will raise. I hope that we benefit from the experiences of the reading people and don't spend all of our time testing and very little of our time teaching.

CONTENT SEQUENCE



From: Riedesel, C. Alan, Guiding Discovery in Elementary School Mathematics, Second Edition. Englewood Cliffs, New Jersey: Prentice-Hall, 1974.

Figure 2



From: Riedesel, C. Alan, Guiding Discovery in Elementary School Mathematics, Second Edition. Englewood Cliffs, New Jersey: Prentice-Hall, 1974.

Figure 3

THE DIAGNOSTIC PROCESS IN MATHEMATICS INSTRUCTION

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Too often the diagnostic process in education has been viewed as a unique procedure used by all teachers in the same way regardless of organizational setting, the background of learners, or the kind of difficulty the learner is having. In this paper, I attempt to do three things: First, to clarify the assumptions and steps in the diagnostic process in education. This is done by referencing the analogous process of diagnosis and pr ription in medicine with respect to known illnesses. Second xamine a framework of how teachers are expected to use the di ic process in school settings in order to classify both the known educational illnesses and the strategies designed to combat their effects. And third, to discuss some of the important illnesses which have been identified during the past few years.

The Diagnost : Process

Webster's Dictionary (1970) defines diagnosis as: "The act or process of deciding the nature of the diseased condition by examination of the symptoms" (p. 388). In this medical definition, there are four assumptions: First, there is at least one recognizable symptom (abnormal condition) which one can examine. Second, there is someone trained and capable of recognizing and examining symptoms. Third, that symptoms are caused by illnesses, and through systematic investigation the nature of the disease can be determined by that trained person. And fourth, that on the basis of the information gathered, alternate prescriptions are available both to overcome the symptoms and to cure the illness itself.

To translate this process to mathematics education is not easy. However, if the primary outcome of mathematics instruction is proficiency[1] (Bloom, 1963), then the first step—the existence of symptoms—rests primarily on defining lack of mathematical proficiency. That is, from students' reactions to a variety of problems requiring or suggesting the use of mathematics, adequacy or inadequacy can be determined. This implies there are proficiency expectations (and boundary conditions) for individual students. Such expectations must be based on instructional goals.[2] The boundary conditions are in part prescribed by the organizational constraints outlined in the next part of this paper.

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[1] The term "proficiency" is used here to encompass several notions such as: achievement, ability to use, attitude . . .

[2] The importance of adequate proficiency in terms of goals—societal and individual—far transcends this paper. (See illness #2, page 86.) For purposes of this paper, I have assumed that some notions about adequate proficiency can be agreed upon.

For the second assumption, the classroom teacher is obviously the person who must perform the tasks of the diagnostician. Similarly, the third and fourth assumptions of the medical diagnostic process translate readily. There is sufficient knowledge of some common educational diseases and their symptoms that they can be identified by the teacher and there are alternate procedures available to the teacher.

Before leaving the medical analogy, it will be helpful to recall the four basic strategies used in the medical treatment of illness: First, disease eradication. Second, preventive medicine—the periodic examination of patients prior to the existence of clearly identifiable symptoms. Third, patient-initiated examinations. This hypochondriac strategy for many individuals (running noses, etc.) is important because it allows the doctor to assure the individual of his normalcy. And fourth, the treatment of major illness through drastic remediation procedures both for the treatment of the symptoms and the treatment of the underlying disease.

Translating these basic medical strategies to education yields the following needed developments: First, better instructional programs, better learning environments, more competent teachers, etc. need to be constructed so that the illnesses causing inadequate proficiency may be eradicated. Second, preventive techniques must be developed to identify diseases in their initial stages before they become serious. Third, our students need opportunities to complain and to be assured of their normalcy. And fourth, dramatic and radical procedures for the treatment of certain symptoms and diseases as must be developed.

In summary, to study the diagnostic process in mathematics instruction, the medical analogy suggests that illnesses be identified and examined, teachers be trained to identify and treat the illnesses, and that four general strategies be used to combat diseases.

A Framework for Identifying Educational Illnesses

As a social organization, the public school has an organizational task structure and technology (OT/T) (Perrow, 1970). Schools with self-contained classrooms, children sitting in chairs arranged into six rows with six chairs in a row, a single teacher (in front), a single textbook, and a set of worksheets to be used, differ considerably from schools which are organized into units of 120 children and 8 adults, the adults assigned varying responsibilities, learning taking place in several locations, tables as well as or instead of desks, various texts and materials, and tasks assigned in a variety of ways. The OT/T contrast between these two stereotypes is clear. However, what is not clear is that the diagnostic process also differs in these kinds of settings. Indeed, both the symptoms observed and the perceived illnesses differ.

Organization sociologists have identified two dimensions which underly most organizations. The first is related to the assumption one makes about the uniformity of raw material entering the organization.

Thus for education, students may be assumed either to be basically uniform in terms of capability or to be highly variable. The second is related to the assumption one makes about how well the process for transforming this raw material into a finished process is understood. For education, this translates into whether "how to instruct" is well understood or not. In Figure 1, four kinds of schools are identified based on these assumptions.

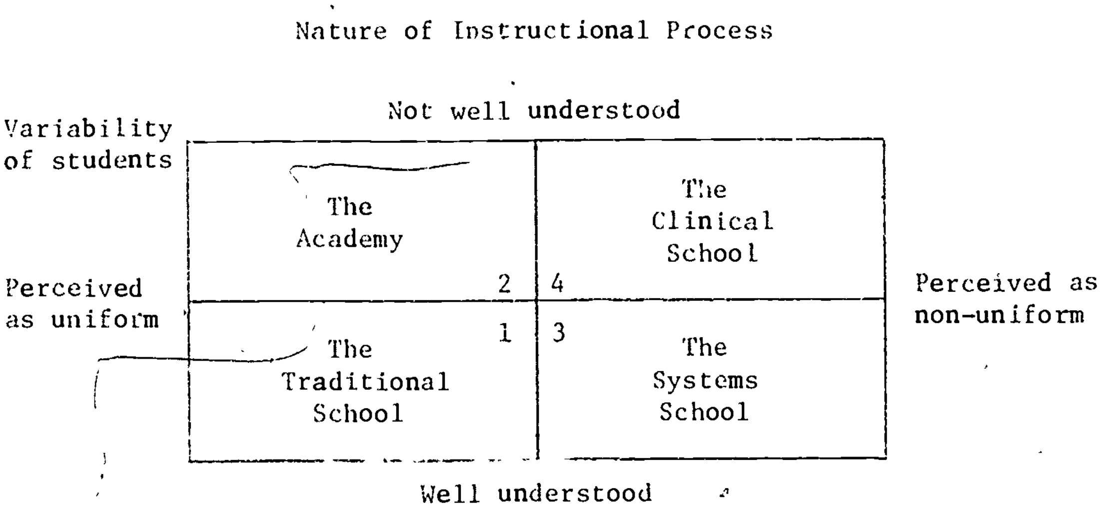

Figure 1.--Kinds of schools based on variability of students and nature of the instructional process.

Although these schools could be contrasted on several characteristics, I have chosen four for this paper: staff training, locus of decision making, technology, and the role of diagnosis.

For the traditional school, Category 1, individuals entering the school are assumed to be basically uniform and the process of instruction is well understood. The staff has been minimally trained with the same competencies. Most decisions are made at a considerable distance from the production line. Objectives and materials are chosen at the district or even at the state level. Even how instruction takes place (time and sequence) are spelled out. The staff's role is to carry out the production in a routine manner. The technology (materials, etc.) from class to class is stable and minimal. The diagnostic process in this setting has a very low priority. Most symptoms are ignored. If serious symptoms occur, it is assumed that there is nothing wrong in the system. The main illness is inaccurate placement (he should have been in general math, not algebra). Remediation involves eliminating the raw material (flunking).

The second school is the academy. Here individuals are assumed uniform but the process of instruction is not well understood. The staff is more professional with varying talents and capabilities (usually with specialized competence in a variety of subject areas). Decisions about what is to be taught are still removed from the instructional setting (common textbooks are often used), but how instruction takes place is up to each teacher. The technology provided each teacher is minimal and uniform. However, each teacher is encouraged to adapt. The diagnostic process has low priority. If something g wrong, it is assumed that the teacher is able to vary instruction to overcome those difficulties. In essence, the teacher is seen as a superior craftsman able to turn out a high quality product.

The third type of school might be referred to as the highly technical or "systems school." Here it is assumed that the students vary considerably, but the process of instruction is well understood. The staff consists of highly trained technicians with a variety of important roles. Each carries out specified instructional routines. Decisions about what a student is to learn are at the level of instruction based on proximal data. Decisions about how instruction is to take place are highly engineered. The technology of such a school is sophisticated, often based on behavioral objectives, and a variety of prescribed routines for reaching those objectives. Diagnosis becomes an important part of the overall engineering of the instructional program. (For example, see Figure 2.) Teachers are then systems managers waiting for the warning lights to go on with a variety of prescribed routines available to them to overcome any deficiencies.

In the clinical school, Category 4, the raw material is assumed to be non-uniform and the process of instruction is assumed to be not well understood. The staff consists of highly trained professionals with a variety of backgrounds and competencies Decisions about both what is to be taught and how it is to be taught are made by the staff. The teacher is a clinician who may use either the sophisticated technology of the systems school as tools for diagnosis or may rely on his experience. The prescriptions are not mechanical.

One reason for talking about these four kinds of schools is that some writers refer to the importance of the diagnostic process in the teaching of mathematics without reference to schools. By looking at these four kinds of schools which do exist in our society, it becomes apparent the diagnosis and remediation are viewed quite differently in different schools. Diagnosis may be central to one's conception of schooling or it may be peripheral. A second reason is to provide a framework for discussing several common educational illnesses; namely, that known illnesses can be identified with respect to either the process of instruction, or the variability of the students we teach.

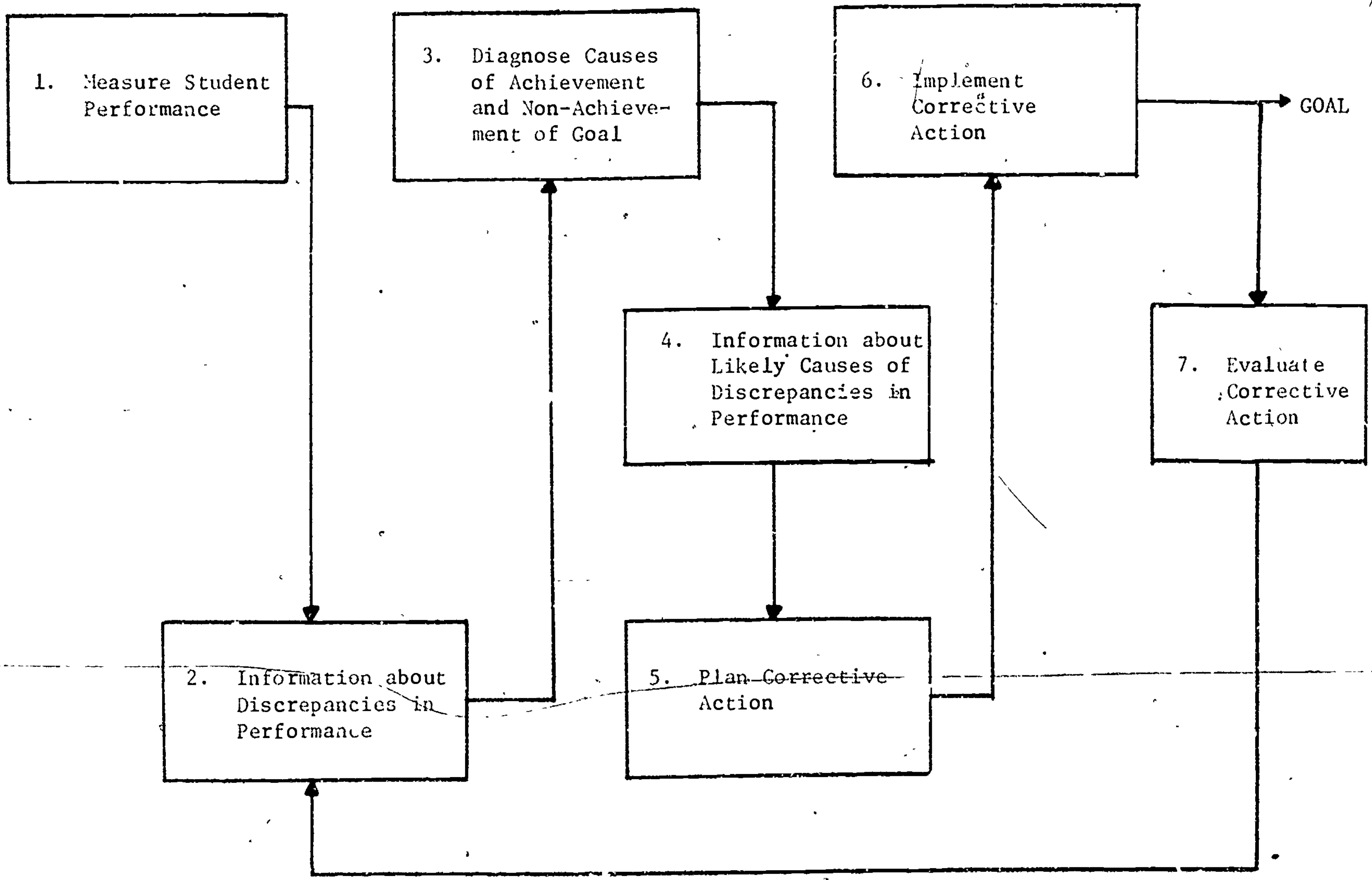

Figure 2. The model of the accountability system

Some Diseased Conditions Common in Mathematics Instruction

In this section of the paper, ten important illnesses which have been identified are discussed. Many of them have been clarified for me through the research of the Wisconsin Research and Development Center (Harvey & Romberg, 1973). Of the ten, seven are associated with the nature of the instructional process and only three with student variability.

Instructional Process—Disease #1; Inhumanity: The first and most important disease today is the inhumanity of many school systems. That some schools foster a nightmarish learning environment which is joyless and repressive has been well documented.[3] Too often children are not viewed as human beings with individual personalities, interests and desires. In the motivation project in the R & D Center, it has been shown that spending ten minutes a month talking with each individual child about his goals has produced dramatic results in learning (Klausmeier, Jeter, Quilling & Frayer, 1973). Spending some time with each child is essential and will produce results in learning in any school setting. It also provides an opportunity for students to complain and discuss their perceived illnesses.

Instructional Process—Disease #2; Inadequate Conceptualization of Mathematics: Although it is unlikely that there will ever be total agreement as to what constitutes mathematics, it is not simply a collection of concepts and skills nor is it an encompassed detailed list of instructional objectives. One rationale for the modern mathematics revolution was to build upon the conceptual structure of mathematics so that the "collection of tricks" to be mastered in the traditional programs would have meaning. Unfortunately, in too many instances the unifying notions introduced have now been reduced to more tricks to be learned which are more abstract and less relevant to reality than the old tricks. And, in the process students now become less proficient at some of those old tricks.

Personally, I prescribe to the notion that mathematics is something one does in order to solve problems. This is best expressed by R. C. Buck in his list of goals:

1. To provide understanding of the interaction between mathematics and reality.

2. To convey the fact that mathematics, like everything else, is built upon intuitive understandings and agreed conventions, and that these are not externally fixed.

3. To demonstrate that mathematics is a human activity and that its history is marked by inventions, discoveries, guesses, both good and bad, and that the frontier of its growth is covered by interesting unanswered questions.

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[3] For example, see Holt, 1964; Silberman, 1970; Sobel, 1969; and particularly, on mathematics learning, Berieter, 1971.

4. To contrast "argument by authority" and "argument by evidence and proof;" to explain the difference between "not proved" and "disproved," and between a constructive proof and a nonconstructive proof.

5. To demonstrate that the question "Why?" is important to ask, and that in mathematics, an answer is not always supplied by merely giving a detailed proof.

6. To show that complex things are sometimes simple, and simple things are sometimes complex; and that, in mathematics as well as in other fields, it pays to subject a familiar thing to detailed study, and to study something which seems hopelessly intricate (Buck, 1965, pp. 949-56).

Any conceptualization of mathematics that does not lead toward these kinds of goals is inadequate.

Instructional Process—Disease #3; Opportunity to Learn: Many school children have never had an opportunity to learn many of the concepts and skills that we assume are being taught. Often teachers, in attempting to meet the standards set up by external agents, cover the contents of a textbook by skipping large sections—thus, leaving out concepts, explanations and opportunity for practice. The tendency to skip over the important ideas (like explaining concepts) in order to get to the computational skills is too common. This is done in spite of the fact that, if the concepts which underlie the skill had been well developed first, the skill itself would have taken relatively small time to teach. It is also apparent that for all practical purposes many concepts or skills, while covered, were not taught. In the Concept Attainment Abilities Project (Harris & Harris, 1973), nouns of mathematics which teachers thought students at fifth grade understood were not understood by most students. Words such as numerator, denominator, dividend or quotient conveyed little meaning to most students (Romberg & Steitz, 1970). In another study related to the concepts of geometry, students were often unable to identify positive examples of triangles, parallelograms, or squares when these figures were not presented in common format (Frayer, 1970). It should also be noted that most achievement test results can readily be explained by opportunity to learn.[4] Furthermore, even if the concept or skill has been well covered in the instructional materials, it does not mean that the student has had an opportunity to learn it. What is being argued is that "the message is in the receiver and not the sender." If an individual does not receive the message in the way in which the information was intended, he is likely to misunderstand. He has not had an opportunity to learn it as intended.

Instructional Process—Disease #4; Level of Complexity: It is surprising to many people that students are able to grasp and work with very advanced notions such as those of topology at an early age and yet at the same time are unable to carry out the long division algorithm. Only recently has it become clear that the level of complexity of a task is not well reflected in this historic sequencing of instruction. Some tasks, such as most computational algorithms, are much more complex than learning most concepts.

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[4] For example, see Fletcher, 1971.

Instructional Process—Disease #5; Lack of Small Group Learning: Learning should not take place independent of others, particularly at the elementary grade levels. If one is to believe the developmental psychologists, it becomes apparent that to learn during the concrete operational stage, one must talk about the things with peers. Recently, I have been carrying out a series of research studies related to the role of overt verbal behavior within small group activity settings in the learning of mathematics. And while this line of research is only in its infancy, it is already apparent that talking is critical in the acquisition of concepts. The elementary classroom needs to be a loud and noisy place.[5]

Instructional Process—Disease #6; Lack of Skill Maintenance: Any football coach could describe to us the importance of drills to maintain skills. Periodic, short drills sharpen learned skills. Too often in mathematics skills which have been learned become dull through lack of use. Because of that, past-learned skills often interfere with the learning of new skills at a later date.

Instructional Process—Disease #7; Stereotyping: Often mathematics textbooks through illustrations and word problems portray mathematics as something one uses only if he is an affluent, well educated, white male. Racial minorities and women are often portrayed in stereotyped cultural roles.

These seven diseases are related to illnesses associated with our knowledge of how to organize instruction. The next four illnesses relate to individuals.

Student Variability—Disease #1; Inadequate Performance on Prerequisite Behaviors: The identification of prerequisite behaviors for specific learning objectives in mathematics is important. Children may not be ready to learn a new concept or skill if they have not mastered concepts or skills which are prerequisite.

Student Variability—Disease #2; Inadequate Conceptualization of the Cognitive Development of the Child: Children at the elementary grades perceive the world differently than do adults. Kenneth Lovell's (1971) remarks about one study carried out under my direction make this clear. Shepler (1970) was able to teach students many probability concepts quite well. However, on one task students had not reached criteria. Shepler concluded that it was due to inadequate instruction. Lovell pointed out that it was much more likely that they had not done well because they were as yet not at the abstract reasoning stage.

Student Variability—Disease #3; Assumed Abilities of an Individual as They Relate to His Acquisition of Mathematical Concepts and Skills: The ways that individuals process information are quite complex. Any assumptions about comparable aptitudes, abilities, learning styles, personalities, moral reasoning, etc. are probably wrong. Given a problem situation, the variety of ways that individuals perceive the information, and carry out several processes, is not well understood. Some personality dimensions (learning styles) clearly influence learning. There are also affective mediators which influence the way in which individuals react to problems. And finally there are individual differences in cognitive processes.

Summary

The several notions discussed in this paper are related to the diagnostic process. Illnesses exist in the teaching of mathematics. It is impossible to argue that in our schools today that students are achieving at adequate levels of proficiency. Symptoms of several illnesses are apparent. The illnesses and symptoms presented here are serious problems in education. It should be our goal to eradicate these illnesses if it is possible by creating better learning environments, by being more human with our students, and by developing a mathematically better, more psychologically, sociologically and pedagogically sound mathematics program. We can also develop better preventive systems. We can create schooling in which positive reinforcement for adequate progress is clear and in which the student has the opportunity to complain. And if serious illness does occur, we need teachers who are able to accurately diagnose and adequately prescribe treatments.

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REACTION PAPER: THE DIAGNOSTIC PROCESS IN MATHEMATICS INSTRUCTION

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I am flattered to have been asked to react to a paper by Professor Romberg. I need not offer many complimentary remarks about his scholarly and perceptive approach to the analyzation of critical areas of need in the field of mathematics education because the paper just delivered is testimony to his insightfulness, creativity, and general high quality of his professional endeavors.

I had planned to make several comments about the medical analogy in Professor Romberg's paper, but so much has been said today about the parallelism between the diagnostic procedures in medicine and education that I will refrain from further caustic remarks or vain attempts at humor at the expense of the medical profession.

However, within this analogy Professor Romberg describes four strategies for treatment of "mathematical illness," which have implications for mathematics education that need further clarification. One can infer from the strategies:

1. eradication of poor quality education
2. development of preventive techniques with early identification of symptoms
3. acceptance of student initiated requests for assistance, and
4. treatment procedures for the "cures" for "mathematical illnesses".

The classroom teacher of mathematics will be responsible for implementing at least the last three strategies. However, the classroom teacher of mathematics is already overburdened with too many students, too much administrative paper work, and too many school responsibilities outside the classroom to perform effectively. The added responsibilities of teaching through the activity approach and individualizing instruction, to say nothing of using behavioral objectives, have driven many good teachers from the classroom and to other fields of employment. Thus, we cannot expect the classroom teacher of mathematics to accept the added responsibility of diagnosis graciously.

Professor Romberg hinted at a solution to this situation in his summary statement of the medical analogy, ". . .teachers must be trained to identify and treat the illnesses. . .," and clarified in his presentation that there is a need for a professional who might be called a mathematics specialist.

Other speakers today have mentioned the need for teachers who could function as clinicians in mathematics diagnosis, but often the speakers have been somewhat apologetic about suggesting the hiring of such professionals. Let's stop being apologetic where we are thinking of the welfare of students!

If there is a surplus of teachers, it should be possible to reduce the pupil-teacher ratio. If reducing the ratio has budgetary implications, school administrators should be committed to the improvement of public education and rise to the challenge of finding funds to hire mathematics specialists.

If programs need to be developed at the university level to educate mathematics teachers in diagnostic procedures, mathematics educators should be willing to set up courses of study for this need.

Thus, a commitment must be made by all members of the education community if mathematics students are to benefit from the services of specialists as reading students have benefitted from the aid of reading specialists. The reading specialist has served two important functions in the schools, the diagnosis and remediation of pupils with severe reading handicaps and the reduction of the reading class size which promotes a better learning environment. Both of these benefits to instruction could be an integral part of the mathematics program if mathematics specialists were present in the schools. Reduced class sizes would allow classroom teachers to use more diagnostic-prescriptive techniques in their classes, individualize instruction, and use many more activity oriented lessons. The other benefit would be the individual attention given to those students who exhibit either developmental or remedial needs as described by Hutchins. This is not to imply that classroom teachers of mathematics should perform no diagnostic acts. Every member of the mathematics faculty has the right and responsibility to diagnose mathematical illnesses as they appear. However, the role of the classroom teacher will differ from the specialist. Let's consider the following diagnostic model to make this clearer.

A Diagnostic Model

Every mathematics teacher performs informal diagnostic acts daily. As the teacher observes students doing seatwork, checks homework, or grades quizzes, errors are often noticed. Patterns of errors allow the teacher to make immediate hypotheses as to the cause of the error(s). As an example of this, consider the little girl who has added a whole page of subtraction examples which involved regrouping. The teacher during the informal stage of the diagnostic model of teaching mathematics would probably simply remark to the girl that she had made a silly error. The child would be expected to redo the assignment by correctly subtracting the examples. Many students completing this same assignment would probably be given similar guidance based upon informal observations of errors. However, the first little girl turned in her paper a second time and the examples look like this:

45	23	71	96	32
− 8	− 6	−19	−47	−28
43	23	68	51	16

Most teachers could quickly spot that the little girl had subtracted the number with the least value from the greatest regardless whether the greatest number, in any particular place value, appeared in the sutrahend or minuend.

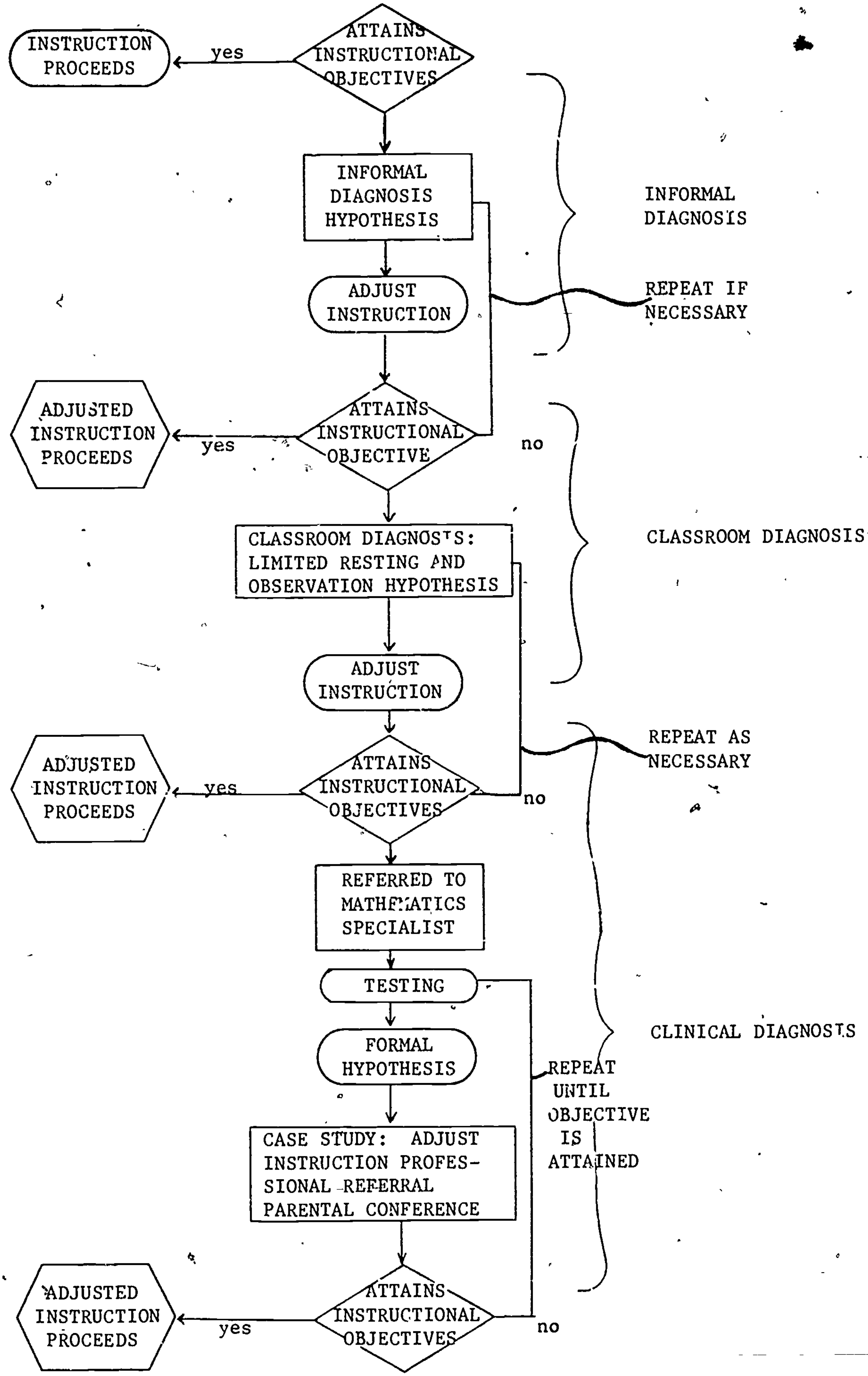

Figure 1. A Diagnostic Model

As a result of this second informal diagnosis, the teacher might adjust her teaching style, use different manipulative aids to teach the concept again, or readjust the assignment to reflect an easier level of subtraction. Teachers would, of course, adjust the instruction in the manner which would most likely overcome the apparent difficulty and prevent future frustrations.

A teacher's success in performing informal diagnosis depends a great deal upon the knowledge of mathematics and needs of the children possessed by the teacher. However, there are always situations where informal diagnosis is not sufficient to overcome difficulties. Some children require more stringent diagnostic procedures.

Suppose that the little girl who could not subtract continued to have difficulty in many areas. The teacher begins to see a pattern in the student's behavior. In the subtraction exercise the little girl completed the exercises incorrectly without asking for assistance. This pattern of completion of exercises with little regard for correctness persisted over several weeks and with several mathematical skills and concepts. Thus, the child seemingly responded to the adjustments in the instructional techniques based on informal diagnosis, but as time passed another problem became apparent. Since this pattern seemed serious the teacher began to work directly with this girl during seatwork assignments, checked her cumulative records, administered survey tests of the previous year's mathematical work, and called the parents for an informal conference.

The purposeful collection of data by the classroom teacher for the purpose of adjusting instruction is called classroom diagnosis. Classroom diagnosis differs from informal diagnosis by being more direct, methodical, and thorough. The hypotheses formed on the basis of data collected during classroom diagnosis are more likely to be correct because of the effort by the teacher to collect pertinent data for a specific child. However, instructional adjustments made on the basis of classroom hypotheses may need to be re-evaluated if the child fails to respond to the prescription.

Let's look again at the little girl who couldn't subtract. Based upon the information that the teacher had collected from many sources, the girl was a "nice little girl who tried to please," had a poor achievement record in previous grades, and had a good attitude toward school according to the parents. Thus the teacher attempted to adjust instruction by first giving her subtraction examples which required no regrouping. With much effort, by both the teacher and student, subtraction with regrouping was "understood." Each time a new concept or skill was introduced, the little girl continued to complete the assignment. However, most of the examples in the assignments were incorrectly done. The teacher recognized that the little girl should be asking for help, but she continually failed to ask ever though there seemed to be a good personal relationship between her and the teacher. Since the teacher could not cope with the apparent symptoms nor determine the cause of the girl's behavior, she referred the child to the mathematics specialist.

The mathematics specialist operates in a clinical environment. That is, there are small numbers of students, a wide variety of mathematics tests, a collection of screening materials for vision, auditory and perceptual difficulties and a list of qualified professional people from other fields to support the specialist. Thus, in clinical diagnosis the classroom diagnosis is expanded in depth and preciseness.

The little girl mentioned in the previous paragraph might initially be given a criterion referenced mathematics test for her grade level so that her achievement level could be established in terms of behavioral objectives. She might undergo testing with the available vision machines to determine potential vision problems. Informal counseling would be a continual part of the diagnostic procedure since many achievement problems are rooted in attitudinal or psychological difficulties. The little girl might be referred to an appropriate professional if the parents consent to the action. These are only examples of factors which might be assessed by the clinician.

Thus, this diagnostic model is a three-stage model. The classroom teacher of mathematics would be involved in the informal and classroom stages of the model, and the mathematics specialist would be responsible for clinical procedures. This implies that two levels of training are needed. The classroom teacher must be trained to identify the common mathematical illnesses of all types (the general practitioner, if you please). The mathematics specialist, on the other hand, will be more highly trained in mathematics and mathematics education so that students with those "illnesses" which are more difficult to diagnose and treat may be "cured".

To reiterate the battle cry once more, let me say that if you are committed to this model or a similar one then you must be willing to promote the hiring of mathematics specialists for every school.

Let me now return to Professor Romberg's paper. I found the descriptions of the different organizational task structures and technologies of public school very informative and revealing. However, my ignorance of this type of description of educational settings has caused me some concern. I do understand the dimension in which students are assumed to be basically uniform in terms of capability or to be highly variable in ability. However; the other dimension has me somewhat confused as I compare the academy with the clinical school. I am familiar with "academies" where teachers are hired with little training in instructional techniques and, therefore, I can accept the statement that the process of instruction is not well understood.

In the clinical school, however, the same statement about the lack of understanding of instructional processes is made. The given description of the professional activities of the staff is inconsistent with a lack of understanding of instructional processes. In the statement, "the teacher is a clinician who may use either sophisticated technology ...or may rely on his experience," a great deal of knowledge of instruction, mathematics and learning is indicated. Thus, I wonder if it might not be better to describe this school as one in which the instructional process is understood, but the staff is not blindly committed to one strategy regardless of the needs of the students. That is to say, these teachers know enough about the instructional process to realize how little is known about the process.

Once I was able to convince myself that this was a valid distinction between the academy and the clinic schools, I certainly agree with Professor Romberg's conclusion that diagnosis and remediation would be viewed differently in each school. However, I do want to add that if we, as mathematics educators, feel that diagnosis and remediation programs should be available to all students then we should be committed to promote the establishment of the school setting which is most conducive to diagnostic procedures because these will be the schools in which mathematics specialists are employed without hesitation.

Professor Romberg's list of ten important illnesses is a valuable addition to our knowledge of diagnosis. Identification, classification and isolation of illnesses is a necessary activity if the cures for the illnesses are to be determined. The extensive nature of this list is more evidence of the need for highly trained mathematics specialists who can diagnose and treat specific illnesses effectively. However, this list implies the need for two more lists; one from the affective domain and another from the psychomotor domain.

The mathematics specialist, of course, is not expected to treat illnesses in these domains, but illnesses in either of these domains may cause mathematical illness. Thus, the specialist must be able to recognize the symptoms of vision disorders, psychological problems, auditory impairment, etc. so that the student might be screened and referred to a specialist outside of education who can treat these non-mathematical illnesses. Thus, the necessary training of the mathematics specialist becomes more extensive, and the need for this type of teacher becomes more acute.

The mathematics specialist does have a responsibility to plan instructional programs which reflect a student's affective and psychomotor needs. Shields has indicated that a student's motivational readiness (locus of control, fear of failure, expectancy of success, etc.) and preference for a type of response (verbal, non-verbal, vocal, manipulative, etc.) as well as the cognitive levels of process must be considered in planning instructional programs for children.

This concern for the affective, psychomotor and cognitive needs of the child do reflect concern for the child as a person. However, there is a danger that diagnostic procedures could undermine this concern for the child. Professor Romberg has identified his first disease as "the inhumanity of many school systems," as documented by Holt (1964), Silberman (1970), Sokel (1969) and Berieter (1971). I can see instances where the institution of diagnostic clinics could promote this feeling of inhumanity. Consider the feelings of a student who has been identified as in need of clinical assistance to overcome a mathematical illness. The shuffling of this student from the regular classroom to the clinic and from the clinic setting to the classroom could make him feel very insecure and could be very degrading.

Let's again look at the diagnostic model proposed earlier in this paper, and to simplify the discussion consider only classroom diagnosis. What are the steps taken in classroom diagnosis?

1. Determination of different capabilities and performance levels.

2. Determination of specific behaviors indicating illness and a description of the illness.

3. Determine relevant data about the illness.

4. Form treatment hypotheses which are concise, precise, and specific.

These steps certainly are not dehumanizing to the student, but the actions taken due to the hypothesis may be. Let's look at two models showing this specific aspect of diagnosis.

MODEL I

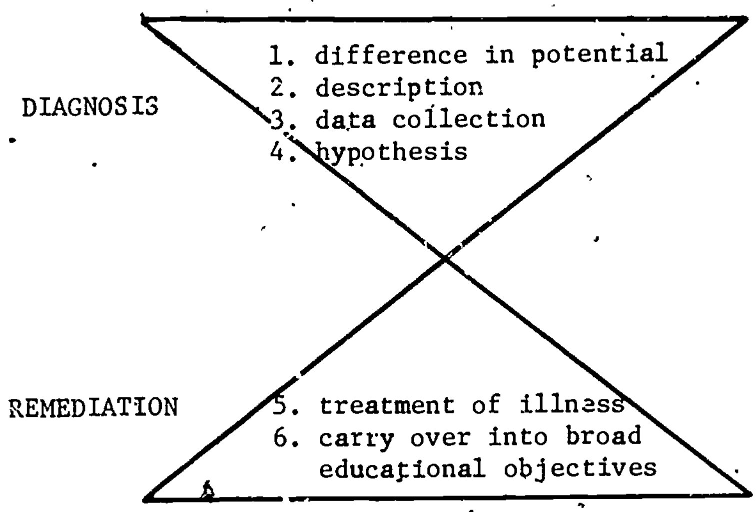

Figure 2. Potential for Humanizing

MODEL II

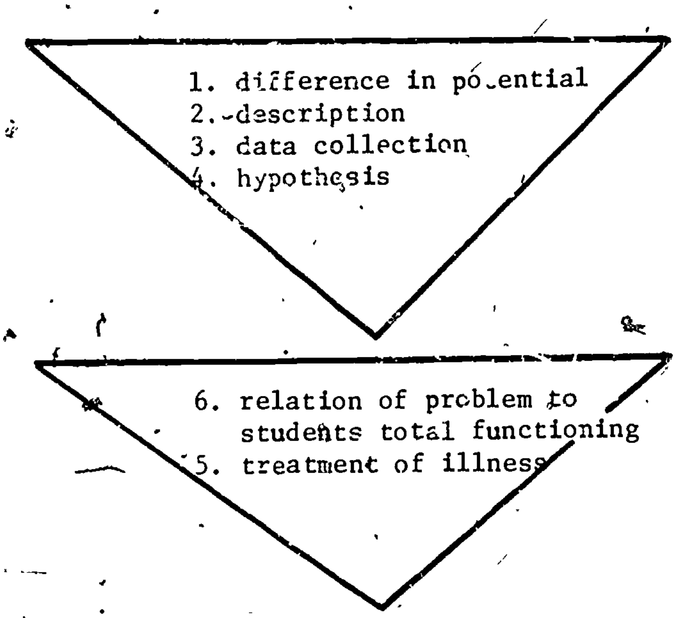

Figure 3. Humanizing

Models I and II do not differ in the diagnosis phase but are radically different in the remediation phase. Model I is the usual means of diagnosis because the teacher has little time to sit down and discuss the student's mathematical illness with him in relation to other problems pertinent to his total functioning.

This is not meant to be a criticism of classroom teachers, but rather is a limitation of classroom diagnosis.

The existence of clinicians or mathematics specialists does not guarantee that the needs of the child in relation to his total functioning will be considered. However, mathematics specialists would be more likely to have time to spend the ten minutes talking with individual students that produced the dramatic results in learning at the Wisconsin R & D Center (Klausmeier, Jeter, Quilling and Frayer, 1973). These ten-minute discussions are exemplary of how the adoption of a carefully conceived diagnostic model and the existence of functioning mathematics specialists in the program might produce a more productive yet humanistic approach to the instruction of mathematics.

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REMEDIATION OF LEARNING DIFFICULTIES IN SCHOOL MATHEMATICS: PROMISING PROCEDURES AND DIRECTIONS

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Identifying strengths and weaknesses is only part of any program for helping students with learning difficulties in mathematics. The remedial measures which result from diagnosis are often the more exacting and more extensive part of such a program. The purpose of this paper, therefore, is to identify procedures which hold promise for remediation in school mathematics and o suggest directions for future research and study. To accomplish this purpose, the task is undertaken in two parts. The first is a selective review of the literature and the second is an exposition of three general areas of need.

Any effort to discuss promising procedures and directions for remediation in school mathematics must begin with a clarification of the term remediation. Remediation may be used to describe instructional procedures for alleviating either the symptoms of a learning difficulty or the causes of a learning difficulty. In the former case, since the symptoms of a learning difficulty are absent, immature, or incorrect mathematics concepts and skills, remediation consists of instructional procedures specifically designed to help the learner acquire, fully develop or correct these concepts and skills. In the latter case, remediation consists of procedures designed to eliminate the factors or conditions which precipitated the undesirable symptoms. In either case, however, for instructional procedures to be referred to as remediation, they must follow previously unsuccessful instruction. Of these two types of remediation, both the literature and existing remedial programs almost exclusively have emphasized remediation as alleviating symptoms; therefore, other than to acknowledge its existence and call for further study, this paper will not further consider remediation as alleviating causes of learning difficulties in mathematics.

Selective Review of the Literature

The existing literature on the remediation of children's learning difficulties in mathematics ideally should provide solid information upon which remedial programs could be designed and their relative success assured. If such information were available, this paper could serve as a guiding document for individual and commercial efforts to develop remedial programs. Unfortunately, the literature is meager; it contains few solid conclusions and provides little guidance for the construction of guaranteed remedial programs.

Several reasons for this state of affairs can be speculated. One reason may be insufficient understanding of the learning process in general. As Harvey and Romberg (1973) stated,

> the more one attempts to understand the mathematical learning process in a classroom, the more one realizes how ignorant we are of how children learn and how we can manage instruction to get them to learn (p. 249).

Another reason may be the disjointed nature of most research efforts. Limited by available resources of time, money and manpower, most research is confined to the investigation of a few diverse variables in special areas of interest, and as a result, numerous aspects of broad research questions are left unattended. A third reason may be the apparent lack of theoretical models for mathematical remediation, models which would identify sets of variables and provide a common focus for research.

Although one of the goals for reviewing the literature in any area is to obtain knowledge and information, another perhaps equally important goal is to stimulate hypotheses and provide direction for future research efforts. Therefore, the literature related to remediation in school mathematics is reviewed for conclusions and implications, and directions for future research and study are suggested. This review is organized into three parts: learning readiness, instruction and individual differences.

Readiness for Learning

In the literature readiness is frequently mentioned as a prerequisite for learning mathematics; since any factor which affects the learning of mathematics will likely have implications and provide directions for remediation, research studies related to readiness for mathematics learning are reviewed. These studies are classified along three dimensions of readiness: content, cognition and affect.

Content readiness.--Studying the readiness of fifth-grade children for division by two-place divisors, Brownell (1951) found that children often experience difficulties because they lack prerequisite concepts and skills. Although cautious about generalizing this finding, Brownell stated that division

> is not the only topic involving the use of previously acquired facts and skills, and it is not the only topic likely to suffer from imperfect mastery at earlier points in learning (p. 22).

Robert Gagné (1962) in the report of a study on mathematics learning, proposed a similar idea; he hypothesized that the mastery of any intellectual skill is dependent upon the mastery of a set of prerequisite skills. Numerous studies have supported his hypothesis (Bloom, 1973; Phillips and Kane, 1973; Walbesser and Eisenberg, 1972).

Success in learning the concepts and skills[1] of mathematics thus appears, at least in part, to be dependent upon the learner's prior acquisition of prerequisite concepts and skills. In designing remedial instruction, learners' mastery of prerequisite knowledge should be monitored and, if necessary, provided.

A second possible aspect of content readiness is the learner's maturity or immaturity of understanding. In examining primary children's number ideas, Brownell (1928) identified four developmental levels of understanding, ranging from counting to "meaningful habituation," from immaturity to maturity. Informally associating maturity-immaturity of understanding with readiness, Brownell (1938) stated that although first graders possess greater number knowledge than we suppose, they generally function only in concrete settings, "representing immature procedures." Maturity-immaturity has been mentioned by other researchers as a prior consideration for learning mathematics (Glennon and Callahan, 1968; Wilson, 1967).

If (a) levels of maturity of understanding can be operationally defined and (b) more mature levels of understanding can be shown to be dependent upon the prior attainment of less mature levels, then implications would exist for what is meant by mastery of prerequisite concepts and skills and for designing remedial instruction.

Cognitive readiness.--Several theorists have suggested the existence of stages in cognitive development. Whitehead (1929) discussed the stages of romance, precision and generalization, while Bruner (1964) referred to enactive, iconic and symbolic stages. Jean Piaget (1951), perhaps the most widely known and researched of these theorists, conceived of cognitive development as a gradual adaptation to the environment; he proposed four stages of cognitive development (sensory-motor, pre-operational, concrete operations, and formal operations) and suggested that although these stages may occur at different ages, their order is invariant and they are relatively independent of instruction. Without enumerating the many studies, research on Piaget's theory has generally been supportive.

Basing her conclusions on studies of the human brain, Farnham-Diggory (1968) has taken exception to Piaget. She concluded that cognitive readiness "is not a simple, chronological function (p. 620);" furthermore, she indicated that humans develop strategies for coping with the world and that cognitive development is the transformation of these coping strategies through growth and experience. According to Kilpatrick and Wirszup (1969), the views and work of Soviet psychologists have generally agreed with Farnham-Diggory.

Whether cognitive development is viewed in terms of invariant stages (relatively independent of instruction) or as flexible arrangements of coping strategies (responsive to instruction), it appears that a child undergoes developmental changes in his cognitive structure;

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[1]The terms "concepts" and "skills" are used rather loosely here and later in this paper; no technical definitions (like those of Gagné) should be inferred.

regardless of which view is accepted, remedial instruction, as well as initial instruction, should include consideration of the learner's cognitive readiness. If the stages or coping strategies can be determined which are necessary for learning given mathematical ideas, then learners' possession of these should be insured prior to instruction, and, in the case of coping strategies should be incorporated within the instruction. In overview of the literature on cognitive readiness, it appears that little is known about the nature or development of cognitive structure and much work is needed.

Affective readiness.--Emotions have long been identified as a contributing factor in learning mathematics. Schonell (1938) reported that from his clinical observations in teaching arithmetic, normal emotional reactions are more necessary than normal intellectual ones.

Little doubt exists in the literature concerning the relationship of various emotional factors to mathematics learning. Employing various measuring instruments, numerous studies (Bassham, Murphy and Murphy, 1962; Faust, 1963; Husen, 1967; Lindgren, 1964; Moore, 1972; Tyan, 1968; Shapiro, 1961) have found a positive relationship between attitude toward mathematics and achievement. Other researchers (Anderson, 1972; Bachman, 1970; Bodwin, 1957; Coopersmith, 1966; Fink, 1962; Moore, 1972) have reported a significant positive relationship between self-concept and mathematics achievement. And still other investigators (Feldhusen, 1965; McCandless and Castaneda, 1956; McGowan, 1960; Philips, 1962) have found a significant negative relationship between anxiety and achievement.

Although attitude, anxiety and self-concept appear to be related to mathematics learning, the direction of the causality remains uncertain; Neale (1969) and Aiken (1970) have explored this issue in depth with respect to attitude and mathematics learning. In fact, whether "affective readiness" is a viable concept depends upon whether the various affective factors can be shown to cause or be prerequisite to effective mathematics learning.

Experience in clinical work (Brueckner and Bond, 1955; Fernald, 1943; Glennon and Wilson, 1972) has suggested that the issues of causality and affective readiness are somewhat academic, for it has been found that a direct attack on the mathematics learning difficulty helps the learner overcome emotional problems. What seems to be important is that pupils experience as much success in learning mathematics as possible, succ s which, in turn, will lead to more positive affect and further success in learning mathematics. This conclusion, however, needs to be verified by empirical research.

Instruction

In the literature much research has been conducted on various instructional practices in mathematics. Most of these studies, however, were concerned with initial instruction. Since their findings may have implications for remediation and suggest directions for future research, many of them are reviewed. Other than research studies, several sources were identified in the literature which present instructional principles for remediation in school mathematics. These principles, in general, were abstracted from the personal experiences and observations of individuals in remedial clinics. Although they were not research-based in the usual sense, such principles may provide directions for future research.

Meaning.--Much experimental research has suggested the need for "meaningful" mathematics instruction, i.e., instruction which stresses understanding rather than rote memorization. Summarizing the findings of research on meaningful instruction, Riedesal (1970) listed its advantages over rote-rule instruction as increased retention, greater transfer, and superior understanding of mathematical principles. In separate reviews, Dawson and Ruddell (1955), Spitzer (1970) and Weaver and Suydam (1972) made similar conclusions. Conflicting findings, however, have been found for low-ability or low-IQ groups (Burns, 1968; Krich, 1964; Miller, 1957; Shipp and Deer, 1960; Tredway and Hollister, 1963) and no studies were identified in which meaningful instruction was used within the context of remedial instruction.

Although it would appear that stress upon meaning in remedial instruction would have particular advantage, one should note that the supportive research dealt only with arithmetic computation and initial instruction situations. Research is needed in areas other than the basic skills and especially in instructional situations where initial instruction was ineffective.

Drill.--In a study by Brownell and Chazal (1935), the researchers concluded that (1) drill and practice should follow understanding and (2) drill tends to fix and make efficient that which is actually practiced. Although no studies were identified which re-examined the second conclusion, the effectiveness of preceding drill with meaningful instruction has been supported in several other investigations (Anderson, 1949; Brownell and Carper, 1943; Howard, 1950; Pincus, 1956).

If meaningful instruction is found to be effective in remedial situations, then it appears that meaningful instruction followed by drill and practice would be an appropriate procedure for remediation. It is hardly the goal of remedial instruction to have students become proficient at immature procedures like finger-counting or block-pushing. Therefore, Brownell and Chazal's second conclusion, if given further support, has implications for identifying those aspects of remedial instruction most appropriate for drill.

Materials procedure.--Examining primary children's number ideas, Brownell (1928) suggested that encouraging pupils to use drawings and objects may help those having difficulty learning number combinations. By 1970, Brownell's conjecture had received considerable support; Suydam and Weaver (1970) reported that researchers generally have concluded that understanding is best facilitated by the use of concrete materials, followed by semi-concrete materials such as pictures, and finally by an abstract presentation with words and symbols.

Although no research studies were identified in which this procedure was explicitly examined in remedial situations, Risdon (1956) reported it to be appropriate in a remedial case study. The evidence thus suggests that the concrete-pictorial-symbolic procedure may be appropriate for remedial instruction; further study is suggested.

Crutches.--In a survey of Scottish teachers, the Scottish Council for Research in Education (1939) found that in general teachers first allowed the use of crutch figures in the early stages of instruction and then later discouraged this practice. Concluding that it is necessary to let pupils make full use of crutches, Schonell and Schonell (1965) observed that in the counting process involuntary movements like moving lips or tapping seem to be necessary "to put the mental mechanism into operation." Supporting this conclusion, two well designed studies (Brownell, 1940; Brownell and Moser, 1949) found the use of crutches to be more effective in learning subtraction with borrowing than no crutches; they also found that for most children the crutches could be discarded "without too much trouble."

The use of crutches during the early stages of instruction thus appears to be an appropriate instructional procedure. Once again, caution must be exercised in generalizing this conclusion to remedial instruction. Studies need to be conducted in instructional situations where initial instruction was ineffective.

Reinforcement.--Suydam and Weaver (1970) indicated that "knowledge of results" is one of the best ways to reinforce learning. Several researchers (Hillman, 1970; Miller, 1970; Paige, 1966) have reported that immediate, knowledge-of-results reinforcement resulted in higher mathematics achievement than delayed or no reinforcement. Suggesting there is reason to doubt the importance of immediate feedback, Wittrock (1973) cited research indicating that immediate feedback sometimes reduces learning.

With respect to low-achieving students, two studies (Glavach and Stoner, 1970; Hillman, 1970) have supported the use of reinforcement. In a study of reinforcement with underachieving primary children, Masek (1970) reported increases in arithmetic performance and task-orientation when teachers emphasized reinforcements like verbal praise, physical contact, and facial expressions; performance rates were reduced when reinforcement was withdrawn and again increased when reinforcement was reinstated.

Although it appears that reinforcement can promote learning in mathematics, especially for low-achievers, the nature and scheduling of that reinforcement remain unclear. Further research needs to be conducted, particularly in remedial situations.

Sequence.--Studies concerned with sequencing in mathematics instruction generally have investigated one of two approaches—sequencing by learning hierarchies or sequencing by modes of representation. According to the theory of learning hierarchies, higher-order skills and knowledge emerge from lower-order ones, and the systematic ordering of these knowledges and skills into levels from lower-level to higher-level is referred to as a learning hierarchy. After a review of the literature, Walbessar and Eisenberg (1972) concluded that the existence of hierarchical structures of knowledge seems to be supported. Several studies have found sequencing instructional tasks according to hypothesized learning hierarchies to be an effective procedure (Elfner, 1973; Geisert, 1973; Hinds, 1973; Miller, 1970); other studies have found these sequences to be more effective than alternative ones (Hegedus, 1973; Jones, 1973; Phillips, 1973). Both Gagne (1968) and Pyatte (1969) have indicated that determining the hierarchical ordering is a major problem. King (1970), after reviewing research on sequencing instruction, may have identified the crucial point; he concluded that varying highly refined sequences of instructional stimuli does not make much difference in the effectiveness of the instruction, as long as the concept order is preserved.

Most of the afore-mentioned literature suggests that sequencing instruction according to learning hierarchies is an effective procedure. Although my own experiences at the Mathematics Learning Clinic at Arizona State University tend to confirm this as an appropriate procedure for structuring remedial instruction, more research is needed. As suggested by King's work, the simplicity of using a hierarchical listing of only concepts as the basis for remedial instruction has special appeal; however, this too needs to be the subject of further research.

Fewer studies have examined the sequencing of mathematics instruction by modes of representation. According to Bruner (1966), knowledge can be processed and represented in three ways: (a) through actions, (b) through summary images, and (c) through abstract symbols; he suggested that although it may be possible for some learners to by-pass the first two stages, an optimum instructional sequence will progress in that order. Suydam and Weaver (1970) reported that researchers generally have agreed that understanding is facilitated by the use of concrete materials, followed by semi-concrete materials such as pictures, followed by an abstract presentation with words and symbols; in a remedial case study, Risdon (1956) reported this procedure to be effective. These studies seem to support the general concrete-pictorial-symbolic progression in instruction; precise sequencing of instruction through modes of representation, however, appears to be a more complex issue.

Reimer and Lottes (1973) have initiated a series of involved studies designed to examine modes of representation in learning and instruction. One of their approaches was to create a matrix or cluster of instructional objectives for a given mathematical concept in which the "condition" (or given) and the "performance" each varied according to the three modes of representation; the resulting 3-by-3 matrix was then examined for the properties of mathematical functions (Farris, 1970; Hirschbuhl, 1971; Klein, 1970). Findings of these studies were mixed.

Sequencing instruction by modes of representation may be an effective procedure; however, considerably more research is needed. Although it seems reasonable that sequencing by learning hierarchies and modes of representation are parts of a more general theory of sequencing mathematics instruction, no theoretical or developmental effort was identified. It appears that much work is needed on the sequencing of both initial and remedial instruction.

Instructional Principles for Remediation

Critical observation of remedial mathematics instruction in various clinics has led to a number of guidelines or principles for remediation. Although many of them are not empirically-based, they may provide directions for future research. The more commonly mentioned principles are as follows:

1. Treatment must be based upon a diagnosis (Bernstein, 1959; Brueckner, 1955);

2. Protect and strengthen the child's self-image (Ashlock, 1972);

3. Remediation should consider the affective as well as the cognitive (Bernstein, 1959; Brueckner, 1955; Schonell and Schonell, 1965);

4. The learner should help in the planning of remedial instruction and be aware of its purposes (Ashlock, 1972; Bernstein, 1959; Brueckner, 1955);

5. Remedial instruction should be geared to the learner's readiness (Brueckner, 1955; Reisman, 1972; Schulz, 1972);

6. Corrective treatment must be individualized (Bernstein, 1959; Brueckner, 1955; Schonell and Schonell, 1965);

7. Remedial instruction should differ from previous instruction (Ashlock, 1972; Junge, 1972);

8. Remedial instruction should employ a variety of procedures and activities (Ashlock, 1972; Schonell and Schonell, 1965);

9. Emphasize ideas which help the learner organize his learning (Ashlock, 1972; Reisman, 1972);

10. Instruction must be structured in small steps (Ashlock, 1972; Junge, 1972; Schonell and Schonell, 1965);

11. Successive learning tasks must be properly sequenced (Schonell and Schonell, 1965; Schulz, 1972);

12. Practice should follow understanding and be distributed (Ashlock, 1972; Schonell and Schonell, 1965);

13. Reinforcement must be positive and immediate (Ashlock, 1972; Schulz, 1972);

14. Make wide use of manipulative and representational materials (Ashlock, 1972; Brueckner, 1955; Junge, 1972; Schonell and Schonell, 1965);

15. Gradually raise the level of thinking from manipulations to visualizing to symbolizing (Brueckner, 1955; Junge, 1972);

16. Encourage the learner to use aids as long as they are of value (Ashlock, 1972; Schonell and Schonell, 1965);

17. Growth should be made apparent to the learner and self-appraisal should be encouraged. (Ashlock, 1972; Brueckner, 1955).

Individual Differences

Throughout the literature, much concern has been expressed for individual differences in learning. The numerous attempts historically to adapt classroom organizations to accommodate a wide range of learning differences have testified to this concern. Today there appears to be more interest in factors like learning styles or cognitive styles which might account for the individual differences in learning.

In the literature a number of studies, for example, have investigated students' impulsive-reflectiveness. Shulz (1972) suggested that

> Since speed of response is the dominant observable and rewarded behavior in most academic classrooms, many students may have been unfairly penalized for their slow styles (p. 5),

thus creating learning difficulties for these students. Although Ackerman (1973) found the impulsive-reflectiveness of low-achieving boys did not affect their mathematics achievement, Cathcart and Liedtke (1969) reported that reflective students achieved higher in mathematics than impulsive students. Pendleton (1972) found that reflective students tended to use focusing strategies on a concept attainment task in mathematics, while impulsive students tended to use scanning strategies. The results of my own studies (in progress) appear to suggest that impulsive students tend to make different types of errors than reflective students. If future studies continue to find significant relationships between impulsive-reflectiveness and various aspects of mathematics learning, then attempts should be made either to adapt instruction or, as Kaga, Pearson and Welch (1966) suggested, to modify individuals' cognitive styles.

It appears that if factors like cognitive style can be shown to account for individual differences in learning, implications for remedial instruction would exist.

Observations and Further Directions

Based upon this review of the literature in the areas of learning readiness, instruction and individual differences, several observations and related directions for future study can be made. First, although few solid conclusions could be reached, the literature contains a number of suggestions which, if successfully supported by further research in remedial settings, could offer limited guidance in designing remedial programs. Second, a preponderant number of studies were concerned only with the computational aspects of mathematics; since learning difficulties occur in the non-computational aspects as well, future research should explore remedial procedures in these areas. Third, the lack of any unified, systematic effort to study mathematics learning and instruction was readily apparent. Although the broadening or replication of existing research efforts like those at the Wisconsin Research and Development Center could provide a vehicle for such a unified effort, the development of theoretical models for remediation seems like a more realistic approach. Fourth, it was not apparent from the literature whether or in what ways it is within the capabilities and constraints imposed upon the classroom teacher to operate an effective remedial program in mathematics; nor was it apparent at what severity of learning difficulty the classroom teacher should request or suggest clinical help. Research in both classrooms and remedial clinics is needed on these issues.

Three Needs in Remediation

Based upon the review of the literature a number of needs in remediation may be identified. In the remainder of this paper, three of these needs are examined; specifically the need for a theoretical model, the need for identifying optimum remedial procedures for a given learner, and the need for a research strategy appropriate to developing a theoretical model. Although no definitive means for meeting these needs is presented, each of them is explored in greater detail and suggestions are provided.

A Theoretical Model

Besides explaining the complex interactions of learner, symptoms and instruction, a theoretical model of remediation in mathematics serves as a focus for a systematic, unified research effort. There is a great need, therefore, for the development of such a model.

In designing a remediation model, two considerations should be examined. The first is the general nature of the model. Diagnostic-remedial models may be classified as psycho-medical or behavioral. Psycho-medical models are distinguished by their separation of diagnosis and remediation. One example of this type of model is implicit in the procedures of some practitioners in special education. After administering tests of visual perception or digit span, for example, they might conclude that a child is visual-minded and therefore needs a sight approach in reading. The crux of this procedure is the notion that an aptitude can imply an instructional procedure. However, in light of the generally unproductive findings of aptitude-treatment-interactions (Cronbach and Snow, 1969), the validity of this notion is dubious. Probably the simplest example of a psycho-medical model is the medical profession. Rarely do the results of the diagnosis (influenza or a broken arm) have much to do with the appropriate remedial procedure; a whole new set of principles appears to be operating.

Behavioral models of diagnosis-remediation are distinguished by their preoccupation with observable behaviors and the intimate relationship between diagnosis and remediation. An example of a behavioral model is implicit in the procedures of other practitioners in special education; their approach, rather than to determine aptitudes, is to identify specific behaviors associated with the disability and then proceed to modify these behaviors.

Based upon these descriptions, a behavioral model appears more appropriate to mathematics remediation. Rather than the development of a theoretical model for remediation, what is being suggested is a comprehensive model for diagnosis-remediation in which appropriate remedial procedures are a by-product of diagnosis.

A second consideration in designing a remediation model is the specification of variables deemed crucial. Since remediation has been defined as instructional procedures for alleviating the symptoms of an individual's learning difficulty, any model designed to specify an appropriate remediation should include variables related to the learner, the symptoms and the instruction. Learner and symptom variables would be independent, while the instruction variables would be dependent. In essence what is being described is a functional model in which information about the learner and his learning difficulty are input and information about instructional procedures is output.[2]

Whether it is possible to design such a model is open to question, but until some model of remediation is developed, it appears certain that research on remediation in mathematics will continue to lack unity of purpose and direction.

Identification of Optimum Remedial Procedures

In the absence of a guiding model for remediation, it would be beneficial to have a method for identifying remedial procedures most appropriate for a given individual. One such method might be to design a battery of tests for this purpose. In each test concepts would be taught using a variety of instructional procedures, several concepts per procedure. Whichever instructional procedure resulted in the greatest achievement for an individual would be assumed to be his preferred one. For example, a test could be constructed such that several concepts were presented using a didactic strategy, other equivalent concepts using a socratic strategy, and still others using a discovery strategy. If an individual achieved best using one of the instructional strategies, that strategy would be assumed to be the most appropriate one for instruction.[3] Although this example suffers from several problems, this method may have merit if these can be solved. Regardless, there remains a need for ways to identify instructional procedures most effective for a given individual.

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[2]Wilson (1967) has proposed a model in which information about the learning difficulty is input and an approximation to instructional procedures is output.

[3]It should be recognized that the conclusion is for "initial" instruction; to infer about "remedial" instruction would require that the concepts on the test be ones which the individual failed to attain as a result of prior instruction.

A Research Strategy for Developing a Theoretical Model

The need for a theoretical model of remediation has already been pointed out, and two considerations in developing such a model were identified. Since it may not be possible to readily deduce such a model, there is need for a research strategy to aid in developing a remediation model. One such strategy is therefore presented for consideration.

If one observes instructional situations like remediation, particularly in the clinical setting, one might note that the teaching-learning process can be analyzed into a series of behaviors. Teachers exhibit certain behaviors; in response students exhibit certain other behaviors; and in response teachers exhibit still other behaviors. A number of interaction analysis schemes have been used to classify and describe these behaviors and their order; however, such schemes have not examined why certain behaviors follow certain other behaviors. These reasons appear to be at the heart of the teaching process. When a teacher responds to a student's behavior in a predictable way, the teacher is ordinarily employing one or more principles of teaching, regardless of her awareness of that principle.

Suppose, for example, a teacher consistently responded to a child's computational errors with a statement suggesting that the answer was wrong but that some part of the problem was correct. Suppose further that upon being questioned she indicated that students seem to maintain greater interest in learning mathematics if their errors are pointed out to them in a positive way. This teacher has very tersely pointed out a principle of teaching.

If a number of these principles were identified (by observation as well as questioning) and substantiated by empirical research, knowledge of the teaching-learning process could be significantly expanded. Furthermore, if higher-order principles could be identified which organize these principles, a theory of instruction or remediation might be inductively derived. Although it is not known whether they were derived in the manner here described, a number of principles for remediation were identified earlier in this paper; if they are substantiated by research, they may form a beginning cluster of remedial principles that may eventually lead to a theoretical model of remediation.

Conclusion

The compound purpose of this paper was to identify instructional procedures which hold promise for remediation in school mathematics and to suggest directions for future research and study. To meet that purpose, (a) research in the areas of readiness, instruction and individual differences with respect to mathematics learning was reviewed, and (b) three needs in remediation were examined.

That much work is needed in the area of remediating children's learning difficulties in school mathematics is obvious. The challenge for both classroom and clinic has been sounded. Questions need to be posed and answers sought; the problems encountered should be a challenge to creative scholars and practitioners at all levels.

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REACTION PAPER: REMEDIATION OF LEARNING DIFFICULTIES IN SCHOOL MATHEMATICS: PROMISING PROCEDURES AND DIRECTIONS

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As noted by Professor Engelhardt, the past studies in learning readiness and individual differences have lead to few solid conclusions. The failure to make substantial advances seems to be due to inadequate development in three basic curriculum and instruction areas. These are:

1. Educational Research—There have been only a limited number of studies that meet the quality standards necessary to meet the demands required by a diagnostic approach to instruction—identification, prediction, description, prescription and evaluation of results.

2. Instructional Theory—Models for conducting a rational and systematic study of classroom instruction have not been set forth with adequate detail and precision.

3. Instructional Practice—Those persons responsible for instruction at the practitioner's level follow trial and error procedures for making instructional decisions that do not allow them to document or benefit from their experience.

The purpose of the writer's reaction paper is to respond to Professor Engelhardt's request for a remediation model by: 1), describing the general nature of a goal-referenced diagnostic model for instruction that could be used to individualize mathematics instruction and help practitioners begin to make a more rational study of their instructional practices, and 2) specifying some variables for the educational researcher that are crucial to the teacher operating in an individualized instructional setting.

A Goal-referenced Diagnostic Model

Consider Figure 1. It presents an instructional model that is an extension of the goal-referenced model proposed by Bloom (1968), Glaser (1968) and Lindval (1961). This model would be classified by Professor Engelhardt as a behavioral model because it assumes that feedback and correctives would be given to individual students based upon their performance on carefully designed oral interviews and written diagnostic tests.

In general the model assumes that the teacher should individualize instruction in order to deal with the wide range of individual differences referred to by Professor Engelhardt's review of the research.

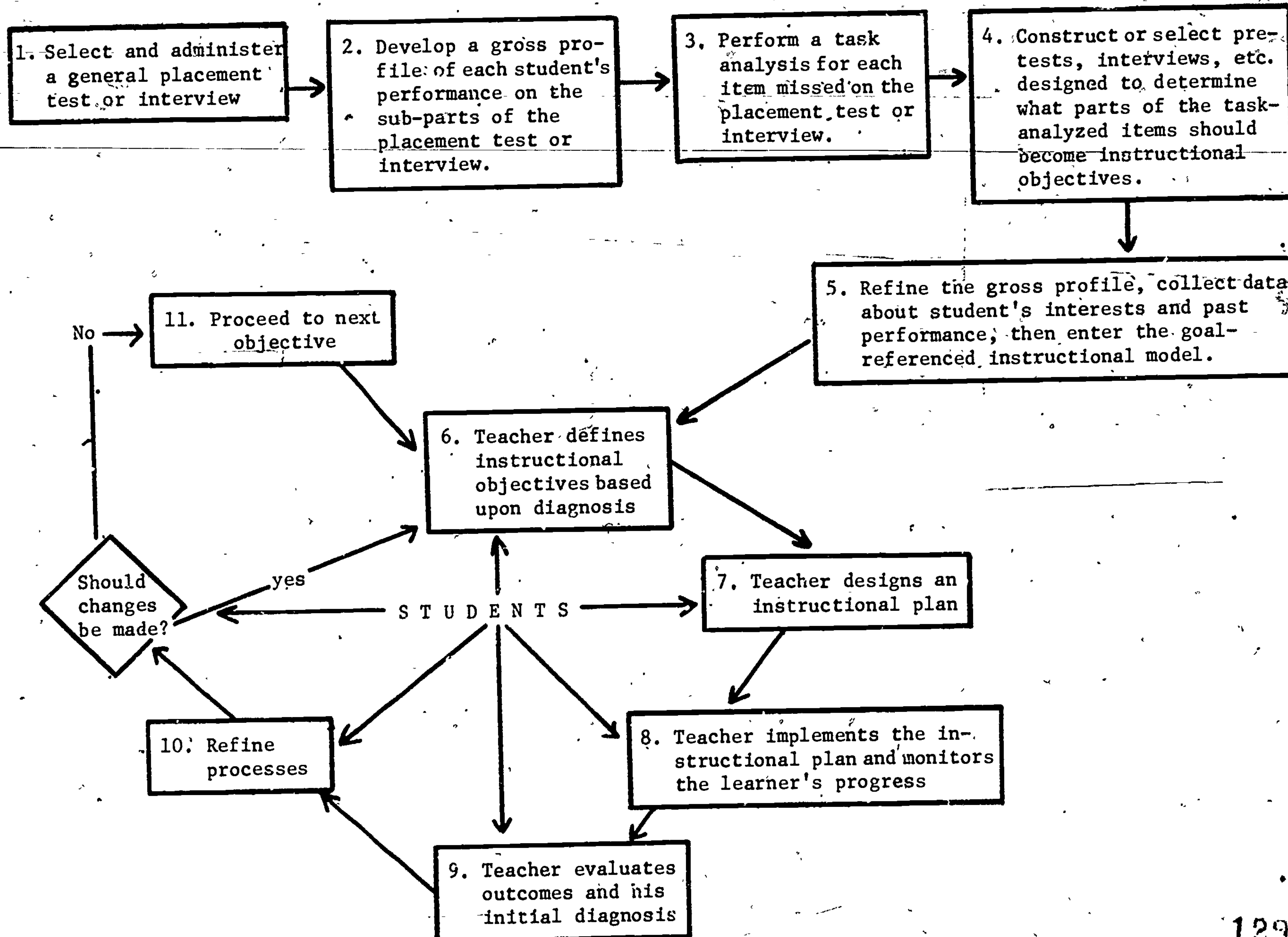

Fig. 1.--A Model For Using Goal-Referenced Diagnosis (Trueblood 1971)

As he indicated there have been numerous attempts to adopt classroom organization to accommodate the wide range of learning differences that teachers encounter. By and large, however, these adaptations have been what Bloom (1973) described as school learning situations which are group based. This means the students are pressured to learn a given set of knowledge, skills, etc. on a group basis and at the same rate.

The open education movement is the most current attempt to deal with the wide range of individual differences in learning rates. It has resulted in the emergence of four basic types of individualized instruction. They are presented in Figure 2. Each type is identified by specifying the source from which the instructional objectives and means originate.

Source of Instructional Means	Source of Instructional Objectives	
	Teacher	Learner
Teacher	Individually Prescribed Instruction (IPI)	Personalized Instruction (PI)
Learner	Self-directed Instruction (SDI)	Independent Study (IS)

Fig. 2.--A matrix showing the source from which objectives and instructional means originate for four types of individualized instruction (Trueblood 1971).

When one compares these four basic types of individualized instruction with the steps in the goal-referenced diagnostic model shown in Figure 1, an important and significant question arises--

> What role does diagnosis play for teachers wishing to use this type model within the context suggested by the four basic types of individualized instruction?

To begin to answer this question, consider the general nature of how practitioners would characterize each of the four basic types of individualized instruction.

IPI.--This type of individualized instruction does not refer only to the individually Prescribed Instruction Project of the Learning Research and Development Center at the University of Pittsburgh but rather to a point of view about meeting the individual needs of learners. It maintains that a highly trained professional teacher should identify each learner's needs and prescribe appropriate feedback and correctives to meet those needs. Therefore, IPI is characterized by the following.

1. Carefully developed diagnostic tests designed to identify the specific needs of each learner.

2. Clearly specified objectives that are formulated by the teacher using the results from the diagnostic tests.

3. Carefully constructed learning activities, materials and progress checks designed to help the learner achieve each outcome.

4. Individually determined pacing where the student is given the amount of time he needs to work with the learning activities and materials provided until he achieves mastery level specified for each outcome. Bloom (1973) refers to this as mastery learning.

5. Efficiently designed management and record system to record each learner's progress and provide him with the feedback and correctives needed for each step of the learning process.

PI.--The basic point of view supporting the use of this type of individualization is that learners should learn how to set their own goals and therefore should be given some choice regarding the instructional objectives and sequence they pursue. Given this type of freedom, it is reasoned that learners will tend to be more motivated and will learn the process involved in goal clarification as well as seeing and feeling the results of their decision making. Hence, PI is characterized by the following:

1. The learner and teacher choosing and defining learning outcomes: a contract is usually employed.

2. The teacher using a wide range of resources inside and outside the school to help his students reach their outcomes.

3. The individual learner and teacher selecting and discussing evaluation criteria to be used to determine when the learner has fulfilled his contract.

4. The student meeting his contract on a self-determined schedule.

SDI.--The basic rationale of those teachers employing this type of individualized instruction is that each individual's strengths, learning style and strategies are somewhat unique. Denying the learner an opportunity to fully develop his individuality by always prescribing his learning activities does the learner an injustice. One of the goals of education should be to learn to be a self-directed and self-actualizing person and this is partially acquired by learning how to employ one's own resources to solve problems. Hence, SDI is characterized by:

1. A carefully developed diagnostic testing program that can be used by the teacher to identify his learner's need.

2. Clearly stated objectives that facilitate the teacher's attempt to communicate instructional outcomes to the learner.

3. A fully equipped learning materials laboratory with a wide variety of activity options that can be placed at the disposal of the learner who chooses the instructional options to meet his diagnosed needs.

4. Self-determined pacing where the teacher allows the student to take the time he needs to master the knowledge, concepts and skills identified by the diagnostic testing program.

IS.--The basic philosophy underlying this type of individualization asserts that the basic purpose of education is to develop adults who can identify their own objectives which are consistent with their value system and then choose appropriate means for achieving these objectives. Therefore, IS is characterized by learners:

1. Diagnosing their needs and formulating the objectives they desire to pursue.

2. Selecting their own learning activities from a well-developed learning resource center within and outside of the school.

3. Evaluating their progress in consultation with the teacher.

Crucial Implementation Variables

In addition to describing the general nature of the goal-referenced diagnostic model as applied to the four basic types of individualized instruction, the writer will follow the suggestion of Professor Engelhardt and specify some of the variables deemed crucial to the model's successful implementation.

Evaluation Instruments.--No need is more crucial than the development of evaluation instruments to supplement the current standard diagnostic tests. The emphasis of any such formative developmental effort should include development and validation of diagnostic interviews such as those developed by Brownell & Moser (1949), systematic observation schedules that focus upon understanding cues exhibited by learners as they learn specific concepts, skills, attitudes, etc., and a self-report format needed to support the IS option. In addition to the development of evaluation instruments, teachers should be taught the skills needed to use the most recent technology to record, recall and assemble diagnostic profiles for individual learners.

In-service teacher education should begin to focus upon teaching teachers how to construct and validate criterion referenced tests which apply directly to their classroom situation. This effort should be accompanied by a teacher aide program that will supply the classroom teacher with the help needed to administer, score and record the results of the classroom teacher's diagnostic testing program. In short, diagnostic models and evaluation instruments are useless without the personnel support needed to make the teacher testing program function in an effective manner. It should be noted that a good diagnostic program will cost more not less than current group-based procedures. Therefore, money is another crucial element in the development of an effective goal-referenced diagnostic mathematics program.

Prerequisite Learning.--As Professor Engelhardt reported, several studies support the idea that instruction should be presented according to the learner's readiness and that successive learning tasks should be properly sequenced. Accordingly in the goal-referenced diagnostic model, this means that the objectives for instruction should be sequenced based upon the subparts of a placement test or interview.

Bloom (1973) agrees with the importance of this step. He reported that cognitive entry behavior seems highly related to the student's ability to reach a specified achievement level on succeeding tasks as well as to their willingness to spend class time on the instructional tasks they are given. Bloom's findings, it should be noted, are related to his mastery learning model which is very much like the type of individualized instruction labeled IPI in Figure 2. His findings were recently supported by Wruble (1974) in a study of the effects of prerequisite addition skills on learning two subtraction algorithms.

It would seem that much research effort should be directed toward the development of adequate instructional hierarchies. In this area Gagné provides some very helpful data, research procedures and analysis techniques. Some very limited work has been done with comparing instructor-generated sequences and learner-generated sequences. The initial and very limited data seems to indicate that the more mature learners do as well on student-generated sequences as on instructor-generated sequences. This finding lends some support for conducting studies on the SDI and IS types of individualized instruction, including the strategies used by students to select their own learning sequences and means of instruction. We should probably not, as we have tended to do in the past, view IPI as the only type of individualized instruction that can be used with learners. A study recently conducted by Houser (1974) lends support to allowing mature students to sequence their own learning in a computer-assisted instructional setting.

Time as a Central Variable.--Bloom (1973) has reported that in terms of using his mastery model the percent of time a student is willing to spend learning a task seems highly related to the student's knowledge acquired over preceding instructional units, his interest in the subject, and the quality of the feedback and correctives he is given to improve his performance. This has direct bearing upon the quality of instructional material to be used in IPI. It seems to suggest that under favorable learning conditions students will put in more time on purposive learning activities than under unfavorable conditions. This means some care must be taken in providing quality instructional materials or the students will decrease the time they spend working with the materials and hence increase the time it takes to reach mastery.

Other Considerations

The writer wishes to close his presentation by suggesting some other topics that should be investigated relative to the basic four types of individualized instruction. These are:

1. How is self-concept effected by exposure to the four types of individualized instruction? Does it improve?

2. How does time spent on learning vary with the amount of individual attention given to students? Does it increase or decrease?

3. In some schools peer tutoring is being used to give students individualized attention. How are the tutors' and tutees' achievement effected by this procedure?

4. Lastly, much research remains to be done relative to the preparation of teachers. If our past experience provides any guidance, it is that the teacher is one of the most important variables in the classroom. Therefore, considerable resources and effort should be spent on the identification of the competencies teachers need to implement the various types of individualized instruction.

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CONFERENCE SUMMARY

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Introduction

This conference has explored some parameters, progress and potential for upgrading diagnostic-remediation procedures within mathematics education. Diagnostic-remedial procedures are accepted as an essential part of the medical profession with its goal of attaining, and retaining, the healthy individual. These procedures have generally received less emphasis in education. One area within the field of education that has shown some concern for diagnostic-remedial procedures is reading. Many schools, or school systems, have reading specialists with special preparation and skill in diagnosing and remediating reading difficulties. School mathematics generally has no such tradition. Both medicine and reading-education have been used by some conference participants as possible models for aspects of diagnosis-remediation in mathematics education.

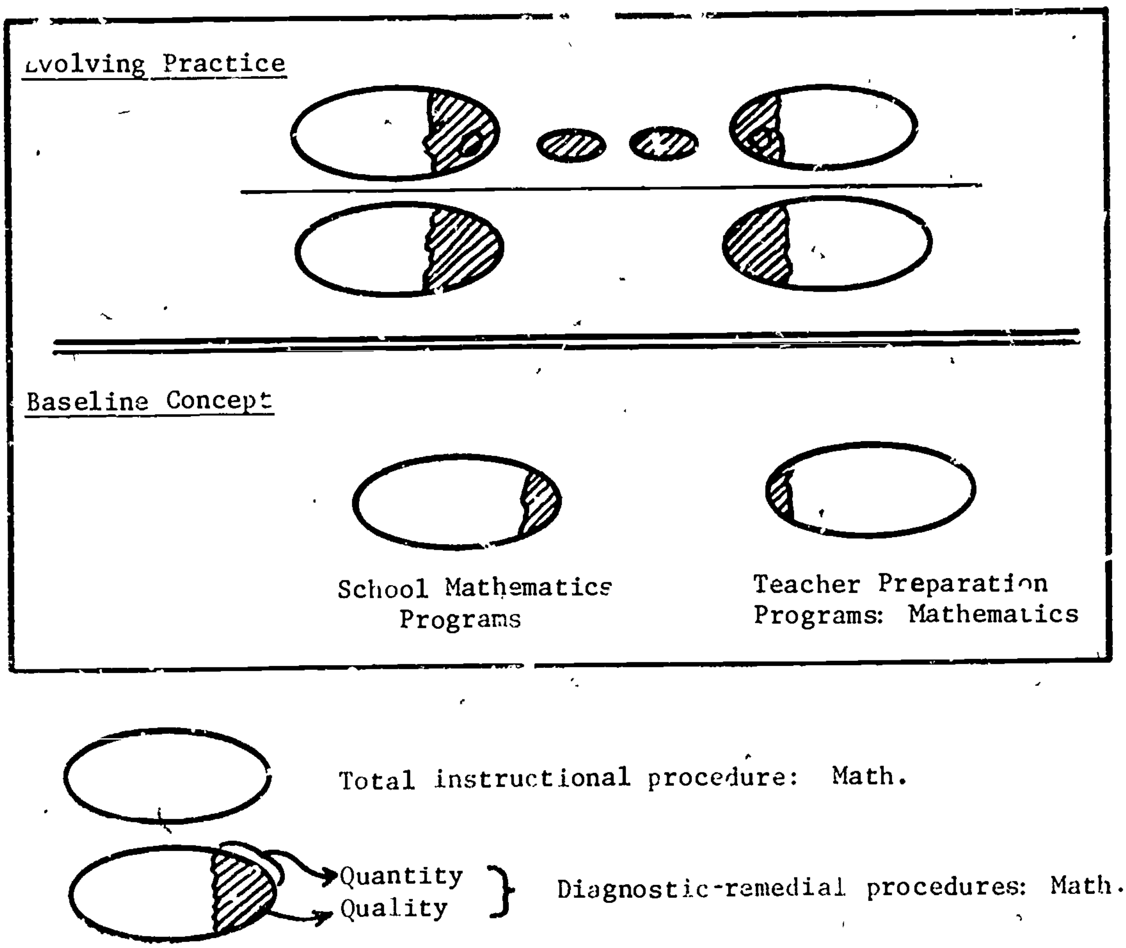
The diagnostic-remedial episode is a ubiquitous occurrence in social contact. It seems to happen quite naturally when concerned individuals interact. A golfing partner may observe a flaw in his partner's game and offer advice on correcting it. Parents may observe some deficiency in a child's learning and offer corrective instruction. Likewise, diagnostic-remedial episodes seem to occur quite naturally in the classroom. Various factors may, however, diminish the effectiveness of the episode.

One inhibiting factor may be the mental health of the parties involved. There may be some "knowers" who harbor hidden delight in possessing something not possessed by the "non-knower" and may tend to obstruct rather than facilitate the flow of knowledge. Some "non-knowers" may have such deep guilt feelings that various facades and covers are constructed to parry the diagnostic probes of the "knower." Whatever the problems, the effectiveness of the diagnostic-remedial episode may be seriously impaired by the mental health of the parties involved.

Another factor that may have an effect on the diagnostic-remedial episode is the ability of the "knower" to perform the appropriate procedures in the episode. These probably include an awareness of the body of knowledge involved in the episode; a command of the means available to probe in order to determine the existing condition of the "non-knower;" and a repertoire of techniques and strategies for correcting the situation. It was these abilities, vis-a-vis mathematics, that formed the focus of this conference. The assumption underlying the conference was that upgrading the diagnostic-remedial episode will have a beneficial effect on learning mathematics.

Participants were generally oriented toward two foci in regard to diagnostic-remedial concerns. One focus was on the application of the procedures with children; the other was on the preparation and training of teachers to carry out the procedures. There was some general feeling that the present level of proficiency in diagnostic-remedial procedures at each level does not rise far above ground-zero. In light of the previous comments about the natural tendency of the act, however, some especially healthy (mentally), intellectually bright, and talented teachers—both in the schools and in the preparation programs of colleges—may naturally reflect a fairly high level of proficiency in carrying out such procedures. With this in mind, the hatched area of the circular regions at the bottom of Illustration 1 may suggest a quantity of time and the quality of the procedure within the total universe of mathematics instruction now going on in schools and in preparation programs.

Illustration 1



Upgrading Diagnosis-Remediation

If the bottom circular regions in Illustration 1 reflect the general state of the art, those regions above them in the illustration indicate where the conference participants believe we ought to go. . .and in some cases are going. The middle pair of circular regions reflect the belief of participants that more and better diagnostic-remedial procedures ought to occur in the schools, with the corollary that there should be better preparation of teachers to do it. There was general agreement among participants that the classroom teachers hold the key to success in upgrading mathematics instruction. A conservative estimate was that 90% of the "remedial" mathematics cases could be, and should be, handled by the classroom teacher.

Two overriding questions emerged, if an increase in the quantity and quality of the diagnostic-remedial episode in mathematics is desirable. What is known about such episodes from a scientific point of view? (Perhaps it is just an art-form between two mentally healthy individuals?) If we come to know more about the variables that effect the quality of the procedures, how can this knowledge be delivered where the action is . . .the classroom? As with medicine, a country may develop the most advanced scientific and technical procedures, but it may profit it little if effective delivery systems are not simultaneously developed.

Many participants commented on the level of knowledge that exists in regard to diagnostic-remedial procedures in mathematics. It would seem fair to say that there is not a great deal of systematic, accumulated knowledge. Some smatterings of research evidence and some sensitive and insightful thoughts on the subject exist. Some isolated individuals at various points in time have attempted to pull together some of the research and thoughts, but the level of scientific knowledge regarding the diagnostic-remedial episode in mathematics is not great. This conference may have been the first concerted attempt to pull the pieces of knowledge together.

As part of the first steps in developing systematic and scientific information about diagnostic-remedial procedures, some participants prepared papers that presented theoretical models. These models, if validated, could serve as a conceptual catalyst for developing practical procedures, and also contribute to an accumulation of systematic knowledge. Theoretical models tend to have certain qualities that curtail their usefulness. They may be so unrefined that they can neither offer guides for specific applications nor structures for the accumulation of knowledge. On the other hand, they may be so over-refined that they may simply sink from usefulness through their own complexity. The models presented seemed to me to have some of these problems, but they are still useful first steps.

Delivery systems were the concern of many of the public and private school people in the working sessions. There was a real desire to find out about the latest developments in various components of diagnostic-remedial procedures. How do you identify the remedial cases? How do you differentiate between the serious learning problem and the less serious corrective case? Where and how do you intervene to begin the remedial procedures? What are some testing techniques that may be useful in evaluation? And on-and-on.

Means for delivery generally involved the pre-service preparation of teachers and the in-service preparation of teachers. There was some indication among conference participants that some opportunity to learn diagnostic and remedial techniques in mathematics is being provided in pre-service preparation programs. This is probably not too general now, but with these competencies stated in the recent NCTM's Guidelines for Preparation of Teachers, such opportunities will become more common. Delivery to in-service teachers tends to pose special problems. Though most conference papers did not get into this problem, few new suggestions seemed to arise out of the working sessions. Traditional "circuit-riding" forays by college professors seemed to be the most often discussed procedure. There was also mention of the use of electronic media in carrying the message. The following section comments on clinics and clinicians. There was a suggestion that these could be useful in carrying out in-service work with classroom teachers.

Mathematics Clinics

The top row of circular regions in Illustration 1 indicates another trend that is reflected in this conference. On the left-hand side is conveyed the fact that some schools have rooms with resource teachers who perform some clinical activities. Although the classroom teachers handle the vast majority of remedial students, there is a means to provide supplementary services for the more serious learning problems in mathematics. Most such services are being provided through Title I projects. . .many in the larger metropolitan areas. Participants often discussed the desirability of having well-trained clinicians available in every school district. Public school people tended to question whether this was realistic, given the present financial constraints on education.

Reflected on the top right-hand side of Illustration 1 is the emergence of mathematics clinics associated with professional schools of education. Some of these exist within field-centered courses in mathematics methods; others are separate mathematical clinics. It may be useful to consider the emerging functions of such clinics.

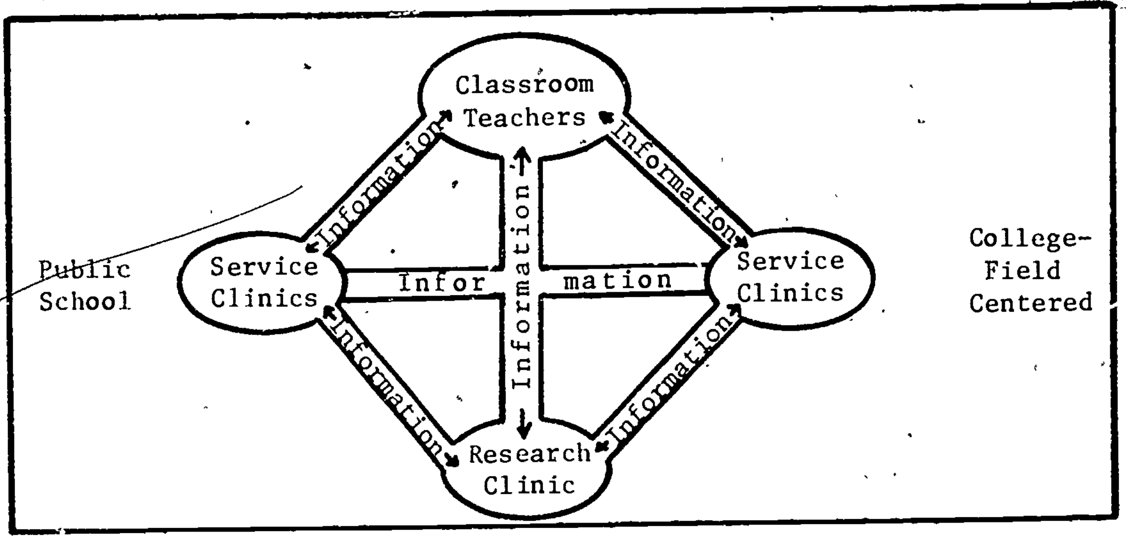
The function of these clinics appear to focus in two general areas, research and service. The service functions tend to fall into two areas, service to teacher preparation and service to children who are referred to the clinic for help. The research function is aimed at the general objective of accumulating knowledge about learning problems in mathematics and their remediation.

The Need for Communication-Cooperation-Coordination

It would seem that if mathematics learning is to be upgraded through improvement of diagnostic-remedial procedures, all parties concerned must be involved. This is suggested in Illustration 2. Each party plays an important role if progress is to be achieved.

Illustration 2

Math Diagnosis-Remediation Units: Regional Network



The classroom teacher receives the latest thinking on identifying learning problems, intervention techniques, testing procedures, instructional strategies, etc. from the service-oriented and research-oriented clinics. They may also refer students to, and work closely with, clinics in the region. The teachers' observations and insights into student learning problems at the classroom level are invaluable to the research-oriented clinic.

The service-oriented clinics may be either college field operations or public school clinics staffed by a trained clinician. These clinics are field-centered, pragmatic in philosophy, and need to be closely associated with the classroom teachers. As enclaves of excellence in the diagnostic-remedial procedures they can act as models for teachers in these procedures, thus incorporating an in-service function. They also are an excellent source of much evidence on what remedial procedures "work" and "don't work" which is invaluable to the theory and research-oriented clinic.

The research-oriented clinic should probably be University based. The University setting can provide various support services such as medical, psychological, statistical and computer services. Emphasis would be on theoretical considerations with collection of empirical data to confirm, or question, the validity of their models. Any degree of success would require coordination, cooperation and communication w··h classroom teachers in the region as well as the service-oriented c··nics.

Illustration 2 also attempts to reflect possible priorities for allocations of resources in this effort. The classroom teachers should receive top priority. There would be need for more service-oriented clinics than research-oriented clinics.

What is envisioned is a regional network of educational units committed to the common cause of upgrading diagnostic-remedial procedures in mathematics. These units (classroom teachers, service-oriented clinics, and a research-oriented clinic) would cooperatively plan research on various aspects of the diagnostic and remedial episode; would disseminate research results and share observations on student learning and useful instructional practices; and develop pipelines for free flow of information being generated on the topic. Success of any one of the parts in the network would depend on communication, coordination and empathetic cooperation.

A Final "C"—Criticalness

Participants from the public schools indicated a need for help in conveying to their publics the critical importance of upgrading mathematics learning in the schools. They felt this was essential if significant amounts of resources were to be committed to improving diagnostic-remedial procedures.

College participants similarly felt that educators in the public schools could help them sell their administrations on committing resources to the development of the kinds of programs suggested during the conference.

The pervasive concern was that the feeling of critical importance of mathematics literacy and learning, shared by all participants, often did not permeate our society as a whole. Only if this spirit of criticalness could be conveyed would there be hope for the commitment of resources required to carry out the thrusts in diagnosis and remediation that emerged at the conference.