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ABSTRACT

This document reviews the use of mathematical models of learning for research in the learning of science concepts. Three mathematical models for the representation of concept learning are presented. (SL)

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MATHEMATICAL MODELS IN SCIENCE EDUCATION RESEARCH

by

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MATHEMATICAL MODELS

IN SCIENCE EDUCATION RESEARCH

A Rationale for Mathematical Models in Science Education

For a number of years science educators including Watson (1), Tyler (2), Pella (3), Hurd (4), Glass (5) and Novak (6) have advocated the development of theoretical bases for research in science education. However, literature reviews (Novak, King & Tamir [7]; Johnson, Curran & Cox [8]; Voelker [9]) support the contention that research on science concept learning suffers from a lack of underlying models. And more recently, Bowen (10) has discussed the need for a paradigm in science education research. While major research efforts in science education have focused on the cognitive domain, progress has been relatively slow in the development of theoretical models which give power to individual studies and to groups of studies. An improved conceptualization of the domain of science education and the cognitive subset of that domain is needed. Voelker (9) has stated that research in science education would be enhanced if studies tested the application of a specific theory to a specific science concept learning situation.

The authors believe that mathematical models of concept learning have the potential to provide meaningful theoretical bases for research in science education and that science educators generally should be informed regarding their research implications. Mathematical models are examples of Nagel's (11) "theory in the second sense"; the fundamental assumptions being considered are descriptive laws or generalizations presented in a mathematical manner. Mathematical models are primarily formulations of fundamental assumptions under idealized conditions. Such models are still relatively new in behavioral and social science research and are even less evident in contemporary educational research. In psychological research, however, mathematical modeling has been used to compare certain hypotheses such as all-or-none learning and incremental learning. Snow (12) has written that "mathematical models are extremely powerful tools, not only for systematizing research on individual theoretical formulations but also for controlling comparisons between competing formulations [p. 96]."

Research on mathematical models attempts to maintain the specificity and simplicity of the models themselves, while aspiring to a degree of generality that is necessary to any useful model of learning behavior. Atkinson (13) stated that over time mathematical modeling must develop

> the kind of engineering knowledge that will enable investigators to select situational variations and rules of correspondence that are simplified and yet relevant both to the model and to the behavior it attempts to predict. It is especially true of research on mathematical models that such relatively minor situational and experimental variations as the location and ascribed significance of stimuli, responses, and reinforcing events must be considered in detail [p. 162].

A major limitation of mathematical models in science education research may well be that the general principles developed by studying a phenomenon in its most simplified form might prove to be considerably more complex in application. Even when the model is correct, and all the variables are known, prediction of the course of actual events may be very different due to the enormous number of interacting variables. Mathematical models do enable researchers to examine behavior in fine detail; analysis shifts from testing null hypotheses toward testing formal predictions by "goodness of fit" techniques.

Empirical Studies Concerning Concept Learning: An Historical Perspective

Early in this century, serious efforts were made to move away from a priori dogma in educational practice to analysis of empirical data. During this time psychologists, following the lead of people like Edward L. Thorndike, began to apply a broad range of results from psychological research to problems of classroom learning. However, the movement toward scientific operationalism has not been constant and unchallenged: negative reactions to achievement testing by the use of standardized instruments and to "objectivity" were particularly apparent after World War II. Currently there is a movement for greater flexibility in educational evaluation which is less limited to superficial notions of "hard data" (14).

However, over the past two decades there has continued to be a growing emphasis in educational research upon objective data, statistical analysis and application. Many empirical studies, sometimes of excellent design and execution, have been conducted to evaluate the learning of students and the effectiveness of particular methods of instruction. In the late nineteen-fifties mathematical models of learning were developed from empirical studies. Researchers developed these models in attempts to quantitatively describe specific kinds of learning. Mathematical models were generally developed cautiously without sweeping claims that they were adequate for all kinds of learning.

One of the major criticisms of the research was that learning theorists have ignored the prescriptive aspects of instruction. On the other hand, Atkinson (15) has stated that "the danger lies in that if the surge in this direction goes too far, we will end up with a massive set of prescriptive rules and no theory to integrate them." Critics have also argued that the analysis of learning in idealized laboratory environments should be redirected and should be studied in real-life situations. There are research studies, however, that appear to bridge this gap. Suppes (16, 17, 18), Suppes and Rosenthal-Hill (19), Atkinson (13), Atkinson and Paulson (20), and Treagust (21), for example, have utilized mathematical models of learning that have not been restricted to simple tasks in the learning laboratory. These models have been applied directly to the learning of subject matter ranging from concepts in elementary mathematics to a second language at the college level.

Mathematical Models for Optimizing Instructional Strategies

In recent years, Atkinson (20, 22, 23) and Suppes (24, 25, 26) have developed some theoretical contributions to systems of computer-assisted instruction (CAI). Two primary factors have facilitated these developments. First, the dramatic growth of computer technology has provided a new instructional medium having the potential to facilitate individualized instruction. Second, computers have assisted in the formulation of mathematical models of learning and instruction.

A major focus of the research effort of Atkinson and Suppes has been the development and testing of instructional strategies, expressed as mathematical models, for simple learning processes such as initial reading and elementary mathematics. Optimization models, comprising an important class of mathematical models, prescribe the sequence of instructional events which will produce optimal learning for individual students within certain boundary conditions. Optimization models are difficult to investigate in a rigorous way for complex learning but are suitable for fairly simple learning tasks. Optimization models are not concerned with how well data from subjects on a conceptual task compare with data from the mathematical model of the underlying conceptual process. Rather, the models are aimed at finding a strategy which leads to the best optimizing procedure for learning. Optimization of relatively simple learning processes has been studied by comparing three models: the incremental model, the all-or-none model, and the random-trial increments model.

In the <u>incremental model</u>, the state of the learner with respect to each concept is determined by the number of times the concept to be learned has been studied. At the start of the experiment, a concept has some initial probability of error; each time the concept is presented its error probability is reduced by a factor a, which is less than one. Stated as a mathematical equation, the probability of an error on the n + 1st presentation of a concept to be learned is related to its probability on the nth presentation in the following way:

$$q_{n+1} = a(q_n)$$

Thus the error probability (q) for a given concept depends on the number of times it has been reduced by the factor a; that is, the number of times it has been presented. Learning is the gradual reduction in the probability of error by repeated presentations of concepts to be learned. This is represented diagrammatically in Figure 1.

Insert Figure 1 about here.

In the <u>all-or-none model</u>, mastery of a concept is not gradual. At any point in time, a student is either in the learned state or the unlearned state with respect to the concept to be learned. When a concept is presented, an incorrect response is given when the subject is in the unlearned state unless the subject makes a correct response by guessing. When an unlearned concept is presented, the subject may move into the learned state with probability c. This probability does not change until the concept moves into the learned state. Stated as a mathematical equation:

$$q_{n+1} = \begin{cases} q_n \ (=1), & \text{with probability } 1-c \\ 0, & \text{with probability } c. \end{cases}$$

Here the error probability in the learned state is 0, the error probability in the unlearned state is 1. Once a concept is learned, it remains in the learned state throughout the course of instruction. Some concepts are learned the first time they are presented, others may be presented several

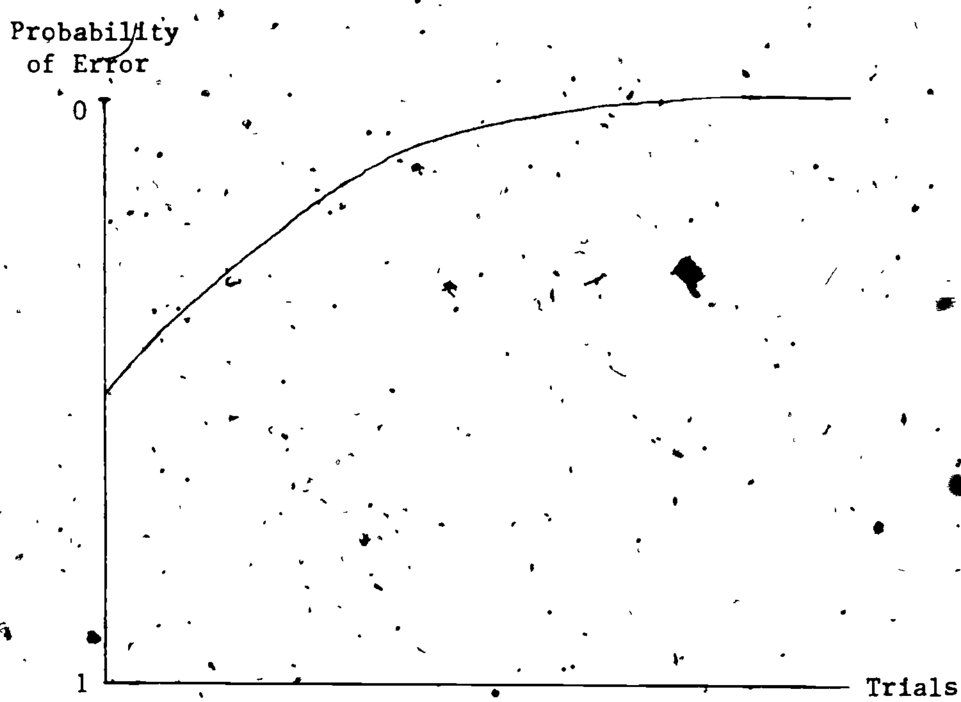

Figure 1. Idealized individual learning curve for the incremental model.

times before they are finally learned. Therefore the list as a whole is learned gradually, but for any particular presentation of a concept to be learned, the transition from the unlearned to the learned state occurs in a single trial. This is represented diagrammatically in Figure 2.

Insert Figure 2 about here.

The <u>random-trial increments model</u> is a compromise between the incremental and all-or-none models (27). For this model, the mathematical equation is stated as:

$$q_{n+1} = \begin{cases} q_n, & \text{with probability } 1-c \\ a(q_n), & \text{with probability } c \end{cases}$$

where c is the probability that some event that produces learning occurs on any trial n and a is the reduction factor relating to the number of presentations of the concept.

If $c = 1$, the random-trial increments model reduces to the incremental model; if $a = 0$, it reduces to the all-or-none model. However, for $c < 1$ and $a > 0$, the random-trial increments model generates predictions that are quite distinct from both the incremental and the all-or-none models.

Optimal strategies were developed for the incremental model and for the all-or-none model with the assumption that each concept to be learned had the same learning parameters and initial error probabilities. With the incremental model, the reduction in error probability on each trial was used to deduce the optimal strategy for presentation of items to be learned. It involves presenting all items once, randomly reordering them, and repeating the procedure until either the time allocated for instruction has

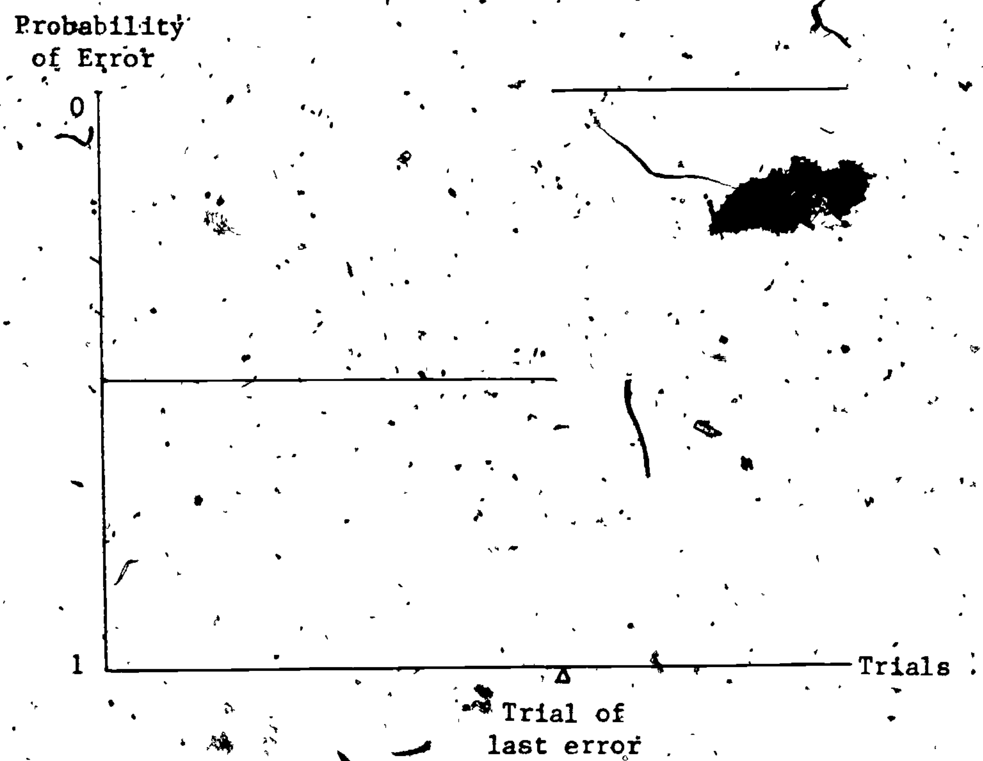

Figure 2. Idealized individual learning curve for the all-or-none model.

been exhausted or the task has been learned. With the all-or-none model, once a concept has been learned there is no further reason to present it. Since all unlearned items are equally likely to be learned if presented, the optimal presentation strategy selects the item least likely to be in the learned state for presentation. If the last response was incorrect, the item was certainly in the unlearned state at that time. If the last response was correct, then it is more likely that the concept was in the learned state. In general, the more correct responses that have been made since the last error on the concept, the more likely it is that the concept was in the learned state. The situation is more complex in the random-trial increments model.

Chant and Atkinson (23) described a number of researchers who were interested in the application of these three optimization techniques to models of learning and instruction. Atkinson (22) described a CAI program designed for spelling lessons in the primary grades. This application of CAI involved a regular program of practice and review designed to complement teaching by the classroom teacher. In another experiment, Atkinson and Paulson (20) described optimization strategies for an instructional program to teach 300 Swahili vocabulary items to college-level students. The objective of both CAI programs was to teach students the correct responses to each item in a given list.

Atkinson's (22) experiment compared the incremental model and the all-or-none model. Data from this experiment indicated that the all-or-none strategy was more efficient than the incremental model at a level predicted by the theory, and was far better than strategies that presented the items to be learned in a predetermined manner. Atkinson and Paulson (20) compared the all-or-none and the random-trial increments models for presentation procedures. Results indicated that the random-trial increments model was more sensitive than the all-or-none strategy in identifying and presenting those items that would benefit most from additional study. In other words, the random-trial increments model provided a better optimization procedure for learning in that study. According to Atkinson (22):

> . . . the development of effective optimization strategies and viable theories of learning will be an interactive enterprise, with advances in each area influencing the concepts and data base of the other. For too long psychologists studying learning have shown little interest in instructional problems, whereas educators have made only primitive and superficial applications of theory [p. 594].

Using these research methods to examine the learning of selected science concepts may well facilitate the development of one kind of theoretical basis for research in science education.

The Application of Mathematical Models in Concept Learning

Psychological research involving concept learning consists of both concept formation and concept identification. While some authors claim this distinction is difficult to draw or that the distinction is semantic, other authors define concept formation as the inventive act by which categories are constructed and define concept identification as the search for attributes or rules that distinguish examples from non-examples in the category one seeks to discriminate (28-31). In concept identification tasks it is assumed that a subject already knows what the given concept means; his only task is to discover the defining attributes or rules of the concept in order to predict whether or not a presentation belongs in that category. Since much of science classroom learning is within the realm of concept identification (for example, identifying fauna and flora using taxonomic keys or deciding which laws apply in solving a physical problem), a more thorough investigation of the nature of mathematical models of concept identification and their applications would appear to be appropriate research in science education.

In this regard Treagust and Lunetta (32) designed a study to examine the application of a mathematical learning theory to a four-category science problem consisting of identification of broadleaf trees. It was hypothesized that this inquiry might lead to the development of a model that will facilitate understanding of concept learning and instruction in science education. The model under investigation did appear to be generalizable to science stimuli where the dimensions of the concepts had a binary nature.

A variety of models have been proposed to explain the major phenomena that have been observed in two-category concept identification research studies within the psychology laboratory. Early studies evaluated whether these learning processes were all-or-none or incremental; in the all-or-none model there is no improvement before the subject learns, whereas in the incremental model the performance of a subject improves step by step with practice. The results of a wide variety of concept learning experiments (reported by Suppes and Rosenthal-Hill [19]) generally concluded that an all-or-none model provided a first approximation to response data, but that a more complex model was needed to go beyond the first approximation.

Bourne and Dominowski (33) reported that the first concept identification tasks, where subjects selected from a pool of rules and/or attributes that were possible contenders for solving the conceptual problem, were developed by Bruner. Other researchers have developed more elaborate hypothesis formation and selection strategy theories in two-category concept identification tasks, and have also developed mathematical models in an attempt to formulate their findings. Mathematical models were initially developed for simple learning situations with animals and were applied to human concept learning tasks later with considerable caution. Caution is essential since human learning is so much more complex; hence, workable models will also be relatively complex.

Three Mathematical Models for Research in the Learning of Science Concepts

Three such systems have been developed for the representation of human concept learning by mathematical models:

1) an information processing approach utilizing computer simulation,
2) a stochastic approach utilizing Markov chains, and
3) an information theory approach.

(1) <u>The information processing model</u> is explicitly designed for computer simulation. An information processing system (IPS) consists of a memory containing symbols and mechanisms for receiving, organizing and interpreting stimuli and feedback from the environment. A computer is

a familiar example of an IPS, and in this approach to concept learning, human thinking is also an IPS. Computer programs can be written to perform tasks which in humans require thinking and learning. The model can be tested by comparing the "learning" of the computer program with that of the subject when both are performing the same learning task. If the computer output and subject's learning strategy do not compare well, other procedures can be incorporated into the information processing model to improve the goodness of fit of the model.

Another example of an information processing approach is the Wisconsin model of concept learning and development initially formulated by Klausmeier (as reported by Kerlinger and Carroll [34]). It defines four levels of concept attainment, outlines the possible uses and extensions of attained concepts, specifies the cognitive operations involved in learning concepts at each of the four levels and postulates internal and external conditions of learning related to the specific levels. The levels of concept mastery, the operations, and the conditions of learning have been identified through behavioral analyses of concept-learning tasks and through empirical research in laboratory and school settings. The Wisconsin model is concerned with a system of concepts and related experimentation involving subjects ranging in age from about three years to young adults. The model describes different levels of attainment of the same concept and specifies the operations essential to attaining concepts at successively higher levels.

(2) <u>The stochastic model</u> involves a random process that is observed repeatedly; the probability of the outcomes may be different from one trial to the next. One of the basic assumptions of the stochastic model of concept learning is that man's cognitive processes operate as an approximate ergodic

source (i.e., the effect of one cognitive process on another is limited to a certain finite maximum that can be ascertained as in the completion of an incomplete sentence) using a stochastic process and Markovian chain reasoning. The probability of identifying concepts depends directly on what has happened before. Concept identification is indirectly related to trials or time. In the simplest case the probability distribution on trial n depends only on the outcome of trial n-1: this produces a finite Markov chain. Greeno (35) claims that normal human behavior treated as a sequence of behaviors is Markovian; although this is a subtle consideration it has major importance to the research of learning behaviors.

The idea of finite chains in learning psychology has had at least three effects. 1) The use of Markov chains with few states to represent learning has encouraged the development of ideas about learning processes involving very small changes in a learner's state of knowledge. 2) Finite Markov chains have provided the basis of a vigorous methodology for investigating stages in the process of learning. By noting the pattern of change in the parameters of a Markov model, the investigator can make relatively strong inferences about the nature of the psychological processes involved in learning. 3) It is convenient to represent a complex process as a homogeneous collection of elementary processes. In this manner finite Markov chains have been applied successfully to the theory of problem solving, especially concept identification, by researchers such as Bower, Restle, Suppes and Trabasso.[9]

An example of the stochastic approach is Bower and Trabasso's model, which postulates two concurrent processes during concept identification, namely a selection of stimulus dimensions and a learning process by which responses are assigned to the values of a selected stimulus dimension; both processes are assumed to be all-or-none. Bower and Trabasso's model has been supported by data in a series of experiments, initially in situations involving simple concepts with a single relevant dimension and later in the learning of concept problems with two relevant dimensions (21, 36, 37, 38).

(3) The third system to represent concept learning, the <u>information theory model</u> proposed by Moser and his associates (39, 40), draws on the early work of Shannon, Broadbent and other information theorists. Moser has incorporated additional theorems and algorithms to describe how human memory operates for processing information in the acts of learning or cognition. The basic concepts of the model are that cognitive behavior is Markovian and that the human memory operates in a logarithmic fashion to receive, output and store information.

Moser reports several experiments where students' behavior in the science classroom was quantitatively recorded. For example, Fazio (as reported in Moser [39]) investigated the structuredness (which relates to the influence of complexity of form) of the overt concrete problem-solving behavior of college students working on three related electric circuit tasks. It was hypothesized that learning would be greater with more structured output. The results of this experiment (and others) agreed with the prediction of the information-theory model. In another study, Dunlop (as reported in Moser [40]) looked for behavior changes in fifteen-year-old youths resulting from instruction in a classification task. Dunlop concluded that abstract and concrete kinds of perceptual tasks are processed in different ways by groups of children who are at formal and concrete operational levels of development respectively.

The difficulty with computer simulation models is that a human subject is not easily describable as an information processing system. Yet, computer simulation models are more convincing than Markov models in that, although existing models are imperfect, their ultimate aim appears to be the simulation of psychological reality. A case in point is Simon's (41) attempt to provide a more concise definition of Piaget's notion of "operational structures" using an information processing approach. Simon conjectures that an objective referent for "operational structures" will consist of some of the general features of the means that children in a given society learn to use for storing information. Data from information processing programs for human intellectual processes indicate that some memory structures and associated processes are relevant to Piaget's approach to intellectual development.

The incorporation of memories into the pool of hypotheses, the increasingly detailed level at which data are identified, and the more complex language in which models are formulated bring the most recent stochastic concept learning studies close to the computer simulation approaches. Both approaches appear to aim ultimately at drawing inferences about psychological processes from fine-grain data.

Summary

This paper has reviewed some research in both educational and psychological fields to discuss the relevance and application of mathematical models of concept learning to the field of science education. Most of the research on concept learning has been conducted within the idealized psychology laboratory, though there is considerable evidence that such research has potential application in science education. While many science educators strongly support such lines of research, little research has been done in science education involving mathematical models of concept learning. This problem may be due, in part, to a lack of awareness of the relevant psychological work that this paper has reviewed. The areas of research covered include:

1) an historical perspective of empirical studies concerning concept learning;
2) mathematical models for optimizing instructional strategies;
3) concept learning and the development of mathematical models relevant to research in science education.

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