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ABSTRACT

This unit is designed for elementary school students in intermediate grades. It is designed to make mathematics enjoyable, promote systematic thinking, practice and reinforce arithmetic skills, and provide opportunity for independent research. The student manual contains twelve lessons with background information and activities. In addition, the appendices contain a number of activities that students can do. No prior background with probability is required. (RH)

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SCHOOL MATHEMATICS STUDY GROUP

PROBABILITY FOR INTERMEDIATE GRADES

Student Text

(Revised Edition)

SE 023 021

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PROBABILITY FOR INTERMEDIATE GRADES

Contents

1. THINKING ABOUT CHANCE

You probably have heard or even made statements such as these:

1. More likely than not we will go to the park on Saturday.
2. Chances are good that we will get to do it.
3. John and Billy have equal chances to win.
4. I am almost certain that I can come to your house after school.

These sentences are alike in one way. They have words and ideas which are used in mathematics. These words and ideas are used in a part of mathematics called probability. In probability, we are interested in things which happen by chance. By using mathematics we can often estimate quite accurately what will probably happen.

We will experiment with such things as coins, spinners, colored blocks, and dice to learn what to expect. Later we will learn what to expect by working with numbers instead of using experiments.

What Do You Know About Chance?

Do you know the answers to these questions?

Who will win the World Series this year?

Will all the members of our class be in school next Monday?

How many people in our class will have perfect spelling papers this week?

Will I see a Ford truck on my way home from school this afternoon?

We cannot be certain of the answers to questions such as these because they are chance events. However, there are some things about chance events which we do know.

Some things are more likely to happen than others. For example:

Which is more likely, that one of the pupils will be absent or that the teacher will be absent?

Which is more likely, that you will have cereal for breakfast or that you will have cereal for lunch?

Which is more likely, that a boy will build a model airplane or that a girl will build a model airplane?

Some things are more likely to happen than not. Think of answers to these questions:

In Phoenix, Arizona, in July, is it more likely than not that the sun will be shining at noon?

Is it more likely than not that you can find the sum of 324 and 465?

Is it more likely than not that your neighbor has a TV set?

Some things are certain and some things are impossible. Which are these?

A man can live without any liquid for three months.
I will use my brain some time this week.
My dog can write his first and last name in Russian.
All new cars this year will use water for fuel.
Tomorrow, today will be yesterday.

Our ideas about chance might be classified Certain, Uncertain, or Impossible. In front of the following sentences, write C, U, or I for Certain, Uncertain, or Impossible.

_____ 1. Sun will set in the east.

_____ 2. A river flows downhill.

_____ 3. We will see the sun today.

_____ 4. Sun will rise

_____ 5. A river flows uphill.

_____ 6. A river is deeper today than yesterday.

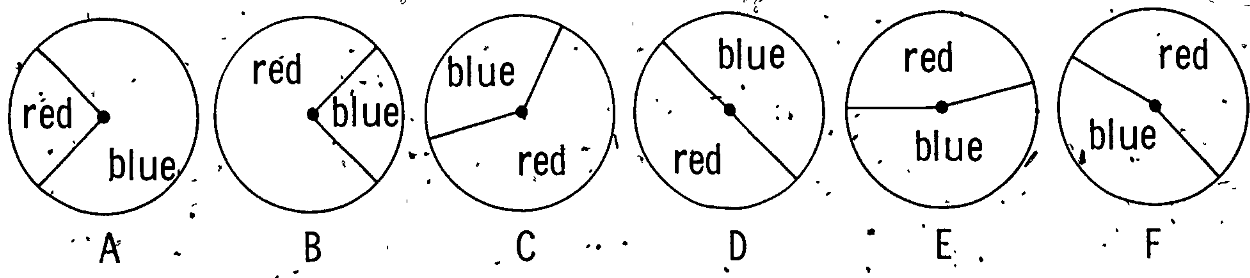
_____ 7. I will sleep 8 hours on Monday.

_____ 8. I will sleep sometime this week.

_____ 9. I will not sleep at all this week.

Exercises - Lesson 1.

Use these pictures for Exercises 1 through 3.



1. Circle the letter of the spinner whose pointer is more likely to stop on red than blue.

   (a) A or B ?
   (b) C or D ?
   (c) E or F ?
   (d) B or C ?
   (e) D or E ?
   (f) C or F ?

2. Study spinner D and answer these questions.

   (a) Could you get 100 reds in 100 spins on this spinner? ________

   (b) Are you likely to get 100 reds in 100 spins on this spinner? ________

   (c) About how many reds do you expect from 100 spins? ________

3. Suppose a pirate captain said to you, "I will give you just one chance on a spinner. If the pointer stops on blue, into the sea you go. If it stops on red, you may go free."

   (a) If the captain let you choose one of these six spinners, which would you choose for your chance? ____________

   (b) If the captain allowed you to make the spinner, how would you color the dial? ____________

   (c) If the captain were very angry, how do you think he would color the dial? ____________

Things To Do At Home - Lesson 1.

1. Look for stories in the newspaper that use some of these words.

probable	probability
chance	equal chance
likely	unlikely

Bring them to share with your class.

2. Place a newspaper on a table. Shake two pennies in your cupped hands and drop them on the paper. Make a tally mark on a piece of paper to show the number of heads or tails. (See the example below.)

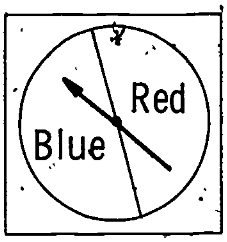
2 heads	1 head and 1 tail	2 tails
𝍸 𝍸 ||	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	𝍸 𝍸 𝍸

Do this 60 times and record the results in a table like the one above.

1. How many times did you get heads on both coins? ________
2. How many times did you get tails on both coins? ________
3. How many times did you get one head and one tail? ________
4. Are your answers to the first three questions the same number? ________
5. Are any two of your results equal? ________
6. Do you expect to get fewer "2 heads" than "1 head and 1 tail"? ________

Lesson 2.

Activity 1: Spinning a spinner with dial $\frac{1}{2}$ blue and $\frac{1}{2}$ red.



Directions:

The pointer of the spinner is to be spun and a record made of whether it stops on red or blue. (If it stops exactly on a line between red and blue, make no record, but spin again.)

One member of the committee will serve as recorder. Here is a sample record of 20. spins to show how the count should be kept:

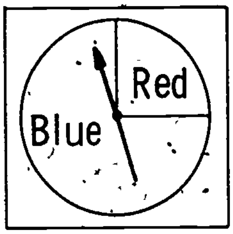
Number of Red		Number of Blue		Total Number of Spins
𝍸 III	8	𝍸 𝍸 II	12	20

The other members of the committee will take turns spinning the pointer until a total of 50 spins have been recorded as in the form above. In this class and in math and science classes in higher grades, you will need to keep a record of results from experiments. Practice now by organizing your results and recording them neatly.

Make a table on a full sheet of paper similar to the sample on this page. Head it "Activity 1". Record the results of 50 spins. The person who records the spins and the persons doing the spinning should sign the sheet. Then give it to your teacher.

Lesson 2.

Activity 2: Spinning a spinner with dial $\frac{1}{4}$ red and $\frac{3}{4}$ blue.



Directions:

The pointer of the spinner is to be spun and a record made of whether it stops on red or blue. (If it stops exactly on a line between red and blue, make no record, but spin again.)

One member of the committee will serve as a recorder. Here is a sample record of 20 spins to show how the count should be kept:

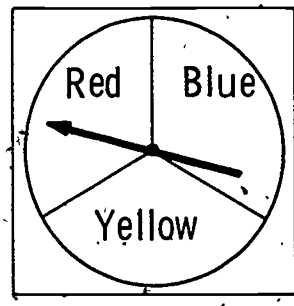
Number of Red		Number of Blue		Total Number of Spins
~~||||~~ |||	8	~~||||~~ ~~||||~~ ||	12	20

The other members of the committee will take turns spinning the pointer, until a total of 50 spins have been recorded on another sheet as in the form above. Be sure to put Activity 2 for its heading.

When all 50 spins have been recorded, each committee member should sign the report. Give it to your teacher..

Lesson 2.

Activity 3: Spinning a spinner with dial $\frac{1}{3}$ red, $\frac{1}{3}$ blue, and $\frac{1}{3}$ yellow.



Directions:

The pointer of the spinner is to be spun and a record made of whether it stops on red, blue, or yellow. (If it stops exactly on a line separating two colors, make no record, but spin again.)

One member of the committee will serve as recorder. Here is a sample record of 20 spins to show how the count should be kept:

Number of Red		Number of Blue		Number of Yellow		Total Number of Spins
𝍸 I	6	𝍸 IIII	9	𝍸	5	20

The other members of the committee will take turns spinning the pointer, until a total of 50 spins have been recorded on another sheet as in the form above, headed "Activity 3".

When all 50 spins have been recorded, each committee member should sign the report. Give it to your teacher.

Lesson 2.

Activity 4: Tossing a coin.

heads tails

Directions:

The coin is to be tossed and a record made of whether it falls with heads or with tails showing.

One member of the committee will serve as recorder. Here is a sample record of 20 tosses to show how the count should be kept:

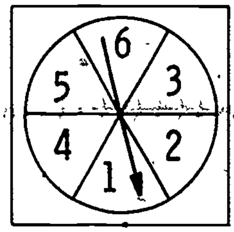
Number of Heads		Number of Tails		Total Number of Tosses
卌 ||||	9	卌 卌 |	11	20

The other members of the committee will take turns tossing the coin, until a total of 50 tosses have been recorded on another sheet as in the form above. Head the sheet "Activity 4".

When all 50 tosses have been recorded, each committee member should sign the report. Give it to your teacher.

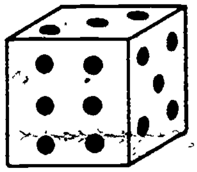
Lesson 2.

Activity 5: Spinning this spinner



or

Tossing a die and counting the number of dots on the top face.



Directions:

The pointer of the spinner is to be spun or the die is tossed and a record made of the results.

One member of the committee will serve as the recorder. Here is a sample of 20 turns to show how the count should be kept:

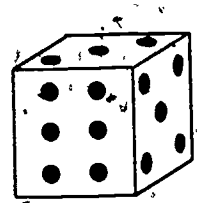
No. of 1's	No. of 2's	No. of 3's	No. of 4's	No. of 5's	No. of 6's	Total Number
////	~~////~~	/	//	///	~~////~~	
4	5	1	2	3	5	20

The other members of the committee will take turns, until a total of 50 spins or tosses have been recorded on another sheet as in the form above, headed "Activity 5".

When 50 results have been recorded, each member of the committee should sign the report. Give it to your teacher.

Lesson 2.

Activity 6: Tossing a die and noting whether the number of dots on the top face is even or odd.



Directions:

The die is to be tossed and a record made of whether the number of dots on the top face is an even number or an odd number.

One member of the committee will serve as a recorder. Here is a sample record of 20 tosses to show how the count should be kept:

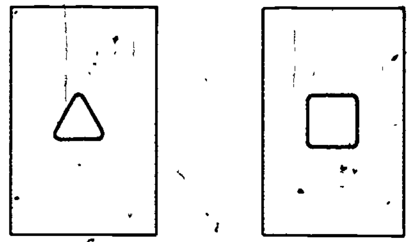
No. of times even		No. of times odd		Total Number of Tosses
𝍸 ||	7	𝍸 𝍸 |||	13	20

The other members of the committee will take turns tossing the die, until a total of 50 tosses have been recorded on another sheet as in the form above, headed "Activity 6".

When all 50 tosses have been recorded, each member of the committee should sign the report. Give it to your teacher.

Lesson 2.

Activity 7: Choosing one of the two cards pictured here.



Directions:

The two cards are to be placed face down on the desk. (Be sure that the person who is to choose does not know which card is which.) One card is chosen and a record is made of the picture on it.

One member of the committee will serve as recorder. Here is a sample of 20 choices to show how the count should be kept.

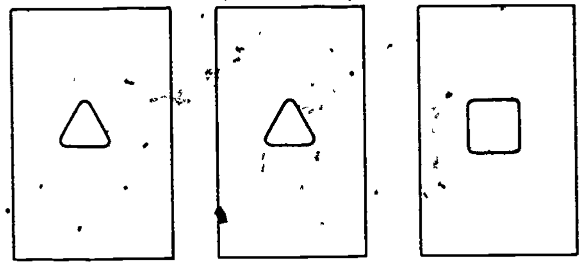
Number of △'s		Number of □'s		Total Number of Choices
卌 ||||	9	卌 卌 |	11	20

The other members of the committee will take turns choosing cards until a total of 50 choices have been recorded on another sheet as in the form above, headed "Activity 7".

When all 50 choices have been recorded, each member of the committee should sign the report. Give it to your teacher.

Lesson 2.

Activity 8: Choosing one of the three cards pictured here.



Directions:

The three cards are to be placed face down on the desk. (Be sure that the person who is to choose does not know which card is which.) One card is chosen and a record is made of the picture on that card.

One member of the committee will serve as recorder. Here is a sample record of 20 choices to show how the count should be kept:

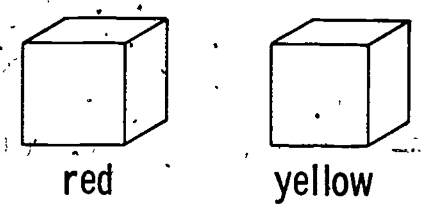
Number of △'s		Number of □'s		Total Number of Choices
卌 卌 |||	13	卌 ||	7	20

The other members of the committee will take turns choosing cards, until a total of 50 choices have been recorded on another sheet as in the form above, headed "Activity 8".

When all 50 choices have been recorded, each member of the committee should sign the report. Give it to your teacher.

Lesson 2.

Activity 9: Choosing a cube from a box containing one yellow and one red.



Directions:

From a box containing one red cube and one yellow cube, neither visible, a single cube is chosen and its color recorded.

One member of the committee will serve as recorder. Here is a sample record of 20 choices to show how the count should be kept:

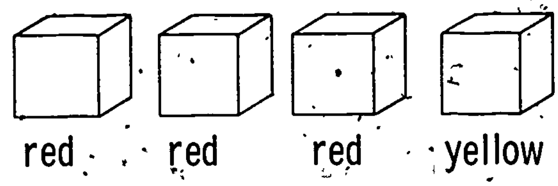
Number of Red		Number of Yellow		Total Number of Choices
卌 卌 |	11	卌 ||||	9	20

The other members of the committee will take turns choosing cubes, until a total of 50 choices have been recorded on another sheet as in the form above, headed "Activity 9".

When all 50 choices have been recorded, each member of the committee should sign the report. Give it to your teacher.

Lesson 2.

Activity 10: Choosing a cube from a box containing one yellow and three reds.



Directions:

From a box containing three red cubes and one yellow cube, none visible, a single cube is chosen and its color recorded.

One member of the committee will serve as recorder. Here is a sample record of 20 choices to show how the count should be kept:

Number of Red		Number of Yellow		Total Number of Choices
𝍸 𝍸 𝍸 I	16	IIII	4	20

The other members of the committee will take turns choosing cubes, until a total of 50 choices have been recorded on another sheet as in the form above, headed "Activity 10".

When all 50 choices have been recorded, each member of the committee should sign this report. Give it to your teacher.

Exercises - Lesson 3.

Here are ten statements about chance events. If you think a statement is true, put a T in the blank after the statement. If you think the statement is not true, put an F in the blank.

1. If a tossed coin does not stand on its edge, it is certain to be either heads or tails. ________

2. If we toss a coin once, we are as likely to get a head as a tail. ________

3. If we toss a coin 100 times, it may be heads 0 times or 100 times or anything in between. ________

4. If we toss a coin 1000 times, it is very unlikely that we will get 900 tails. ________

5. Whether we get heads or tails when we toss a coin is a matter of chance. ________

6. You might toss a coin 1000 times without getting a single head. ________

7. If a box contains two blue marbles and one red one, and you pick one marble without looking, the chances are 2 out of 3 that it will be blue. ________

8. In Exercise 7, you have one chance in three of picking a red marble. ________

9. In Exercise 7, your chances of picking a green marble are zero. ____________

10. Joe is eight years old. It is more likely that he is four feet tall than ten feet tall. ____________

Read the following statement carefully and then answer Questions 11 to 15.

A spinner has a dial which is one-fourth white and three-fourths red.

11. If you spin the pointer 10 times, are you likely to get the same number of reds as whites? ____________

12. Are you likely to get more whites than reds? ____________

13. If the chances of getting red are 3 out of 4, what are the chances of getting white? ____________

14. Can you be certain of getting at least one red in 10 spins? ____________

15. Is it very likely that you will get no reds in 10 spins? ____________

Read the following statement carefully and then answer Questions 16 through 20.

James has three green marbles and two blue marbles in his pocket.

16. How many marbles must he remove to be sure of getting a blue marble? ______

17. How many marbles must he remove to be sure of getting both the blue ones? ______

18. How many marbles must be removed to be sure of getting both colors? ______

19. How many marbles must be removed to be sure of getting a green one? ______

20. If James removes one marble, there are three chances out of ___?___ that it will be a green one. ______

Think about some things that are certain to happen. Think about some things that might happen, and about other things that just can't happen. Then answer questions 21, 22, and 23.

21. List three things that you know are certain to happen.

a. ____________________

b. ____________________

c. ____________________

22. List three things that may or may not happen.

a. ____________________

b. ____________________

c. ____________________

23. List three things that cannot happen.

a. ____________________

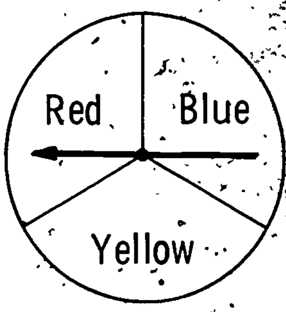
b. ____________________

c. ____________________

Things To Do At Home - Lesson 3.

1. Make a spinner with a dial $\frac{1}{3}$ red, $\frac{1}{3}$ blue, and $\frac{1}{3}$ yellow. Use your imagination. Spinners can be made from paper plates, plastic tops from a paper cup, cardboard, ice cream sticks, and so on. Place a bead under the pointer, or anything to keep it up off of the dial. Stick a pin through the pointer, the exact center of the dial, and into something like an eraser.

   Spin the pointer 100 times. Keep track of the outcomes in a table such as this:



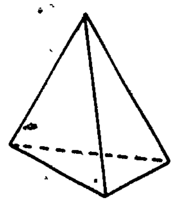
	Blue	Red	Yellow
Tally			
Total			

a. How many times did the pointer stop on blue? ________
   On red? ________ On yellow? ________

b. Is each of these colors equally likely? ________

c. If you spin the pointer 900 times, you expect it to stop on red about 300 400 500 times. (Circle your answer.)

2. Use the directions in the Appendix to construct a tetrahedron. Color one face red, one blue, another yellow, and the last green.



Toss the tetrahedron 100 times and note the face that is down. Keep track of the outcomes in a table such as this:

	Red	Blue	Green	Yellow
Tally				
Total				

a. How many times did the tetrahedron fall on Red ? ________

Blue? ________ Green? ________ Yellow? ________

b. Add the number of times it fell on red and on blue. ________

Add the number of times it fell on green and on yellow. ________

Is each of these sums about $\frac{1}{2}$ of the total number of tosses, or about $\frac{1}{4}$ of the total number of tosses? About ________.

c. If you throw the tetrahedron 1000 times, about how many times do you expect it to fall on red? ________ On either blue or red? ________

3. Make up a game that two people can play so that each person will have an equal chance of winning. Explain it to the class. Describe any material needed, such as spinners or dice. Be sure your rules are clearly stated.

4. Toss a die or a cube which has its faces numbered from 1 through 6. The Appendix has directions for constructing a cube (hexahedron), or you may use a die from some game.

Toss it 180 times. Use a table to keep a record of the results.

	Number of 1's	Number of 2's	Number of 3's
Tally			
Total			

	Number of 4's	Number of 5's	Number of 6's
Tally			
Total			

a. Did each face come up at least once? ________

b. The faces are either odd (1, 3, 5) or even (2, 4, 6). On 180 tosses, about how many outcomes would you expect to be even numbers? ________ In your experiment, how many were even numbers? ________

c. On a die or cube, each of the six faces has an equal chance to be up. In 180 tosses, how often would you expect 2 to be up? ________ 5 to be up? ________ In you experiment of 180 tosses, how many times was 2 up? ________ 5? ________

d. If you tossed a die 6000 times, about how often would you expect each face to be up? ________

Lesson 4.

Activity 11: Tossing a coin -- a team activity.

Directions:

The coin is to be tossed ten times and a record made of whether it falls heads or tails.

One member will toss the coin and another member will record the result. Here is a sample record of 10 tosses to show how to record the results.

Sample Record

Number of Heads		Number of Tails		Total Number of Tosses
////	4	~~////~~ /	6	10

On another sheet make a table similar to the one above. Keep track of your results for 10 tosses. Each member of the team should sign the sheet. Your teacher will ask for it soon, because your results will be used in a class activity.

Lesson 4.

Class Data Sheet for Tossing a Coin -- To accompany Activity 11.

Team	No. of Heads in 10 Tosses	No. of Tails in 10 Tosses
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
Total		

1. Which team had the largest number of heads in 10 tosses? _______
   the largest number of tails in 10 tosses? _______
2. About how often would you expect heads in 10 tosses? _______
   in 100 tosses? _______
3. Each team tossed the coin 10 times. In all, how many times was the coin tossed? _______ What is one-half of this number? _______
4. Out of all these tosses, how many times did heads occur? _______
   Is this about one-half the number of tosses? _______
5. Would we need to record the number of tails on our Class Data Sheet? _______

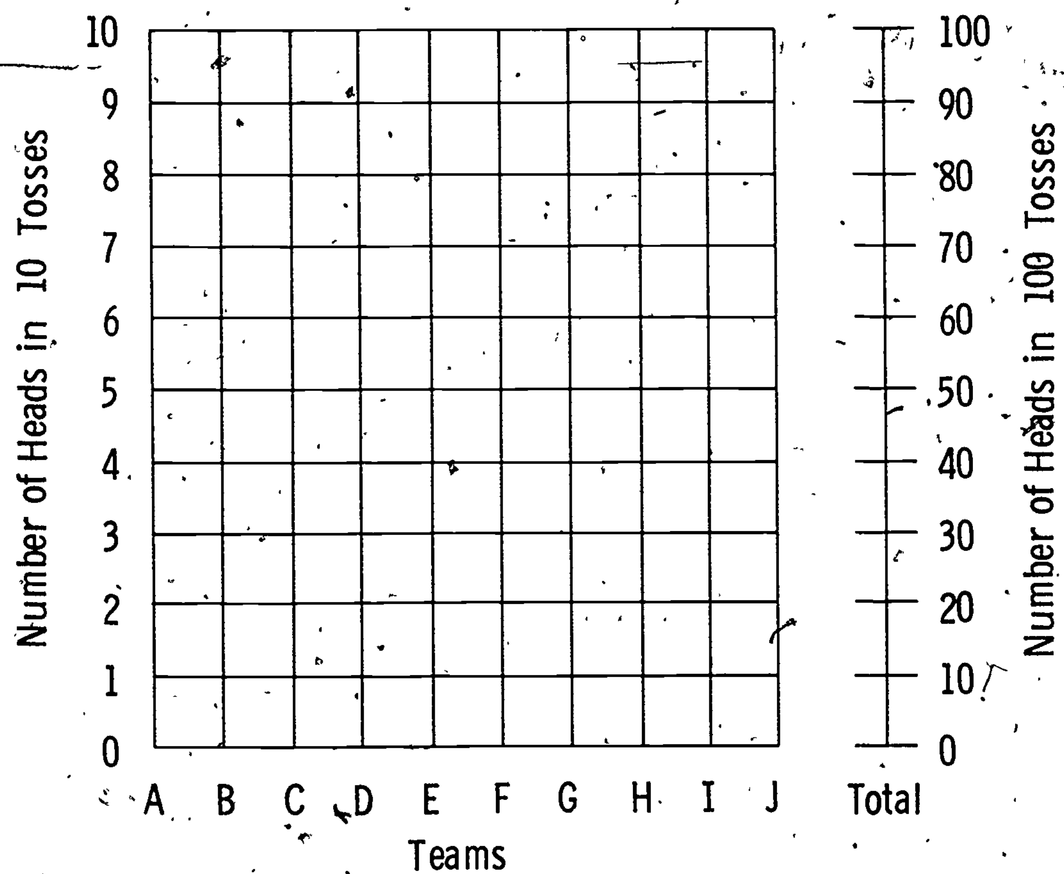
Lesson 4.

Graph Sheet for Tossing a Coin -- To accompany Activity 11.

Directions:

Graph each team's data with an "O". (Use the scale on the left.)

Graph the class total data with an "X". (Use the scale on the right.)



Compare the graph of each team's results of 10 tosses with the graph of the total data of 100 tosses.

1. In general, which is more nearly what you would expect, the various teams' results or the total results? ________

2. Is the number of heads in 100 tosses more nearly what you would expect than the results your team had in 10 tosses? ________

3. Is a larger number of tosses more likely to result in what you expect than just a few tosses? ________

Lesson 4.

Activity 12: Tossing a die -- a team activity.

Directions:

The die is to be tossed sixty times and a record made of the number of dots on the top face.

One member will toss the die and another will record the result. Here is a sample record of 20 tosses.

Sample Record

	No. of ⚀	No. of ⚁	No. of ⚂	No. of ⚃	No. of ⚄	No. of ⚅
Tally	//	////	///	//	///	~~////~~ /
Total	2	4	3	2	3	6

Use a sheet of paper to make a table similar to the one above. Toss the die 60 times and record the results in your table. Each member of the team should sign the sheet. Your teacher will soon ask for it because your results will be used in a class activity.

Lesson 4.

Class Data Sheet for Tossing a Die -- To accompany Activity 12.

Team	No. of ⚀	No. of ⚁	No. of ⚂	No. of ⚃	No. of ⚄	No. of ⚅	Total No. of Tosses
A							
B							
C							
D							
E							
F							
G							
H							
I							
J							
Total							600

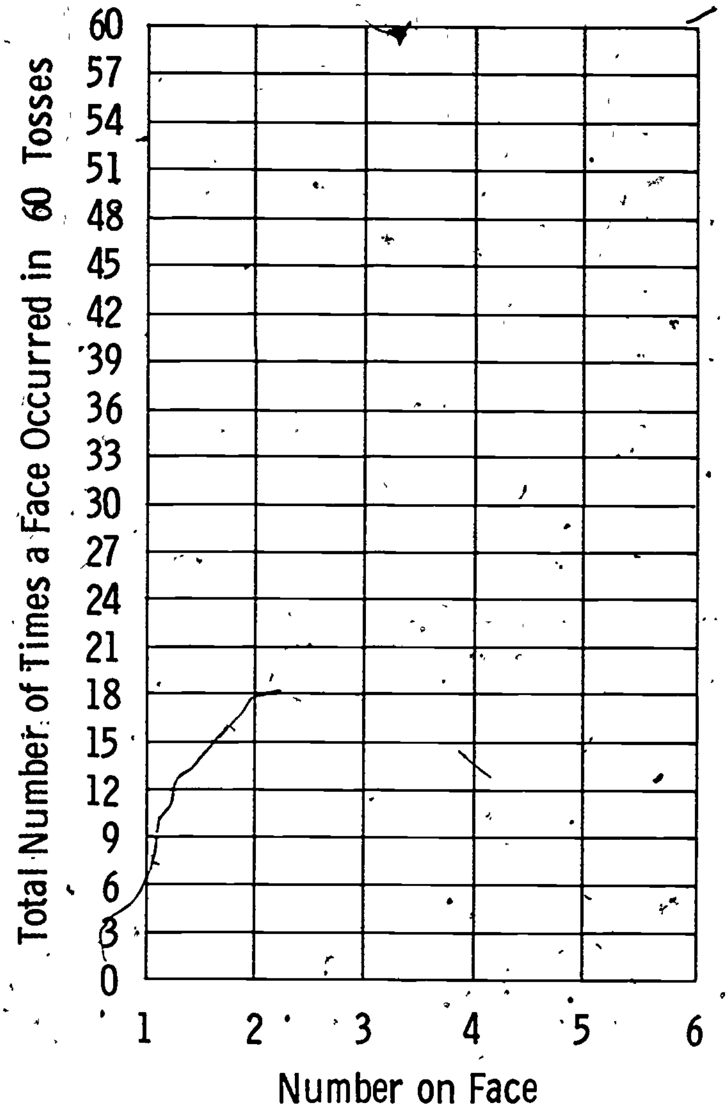
1. From the totals, which face of the die was up the most? ________
2. Are any two or more totals the same? ________ Which? ________
3. Did any team fail to get all six numbers? ________
4. Would you expect that on 60 tosses, each number would come up at least once? ________
5. If we tossed a die 1000 times, could we be sure that every number would come up at least once? ________
6. In 600 tosses of a die, about how many times would you expect each face to be up? ________

Lesson 4.

Graph Sheet for Tossing a Die -- To accompany Activity 12.

Directions:

Graph your team's data with an "O".



1. Did you have any results which you think are unusual?

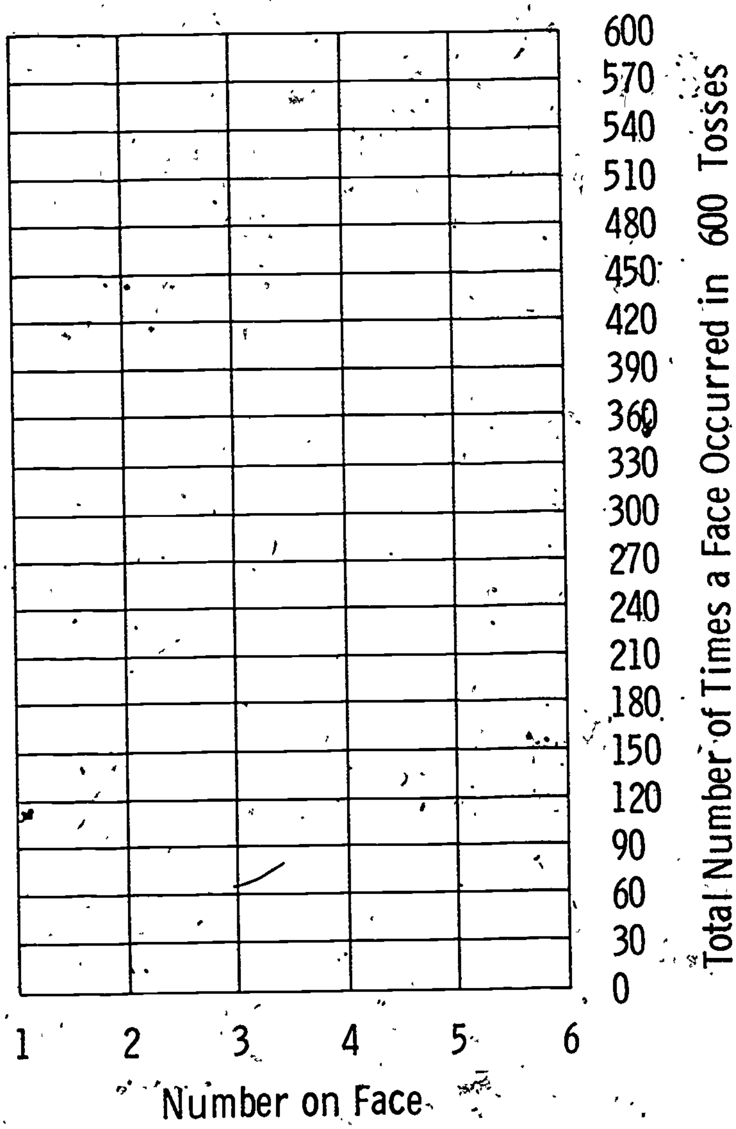
2. What makes you think these results are unusual?

Lesson 4.

Graph Sheet of Total Class Results for Tossing a Die -- To accompany Activity 12.

Directions:

Graph the total class results with an "X".



This graph form is the same form that you used to graph your team's results. However, the numbers on the right side are ten times those on your team's graph form. This is a graph of 600 tosses instead of 60 tosses. Compare your team's graph with this graph.

1. Does your team's graph seem to be farther from the expected results, or are the total class results farther from the expected results? ________
2. Can you explain why this is so? ________

GRAPH OF COMMITTEE ACTIVITIES

Activity	1 Spinner 1/2 Red 1/2 Blue	4 Coin	6 Die	7 2 Cards 1 with △ 1 with □	9 2 Cubes 1 Red 1 Yellow	2 Spinner 1/4 Red 3/4 Blue	10 4 Cubes 3 Red 1 Yellow	3 Spinner 1/3 Red 1/3 Blue 1/3 Yellow	8 3 Cards 2 with △ 1 with □	5 Spinner or Die
Choice	Blues	Heads	Even	Triangles	Reds	Blues	Reds	Reds	Squares	Ones
50										
48										
46										
44										
42										
40										
38										
36										
34										
32										
30										
28										
26										
24										
22										
20										
18										
16										
14										
12										
10										
8										
6										
4										
2										
0										

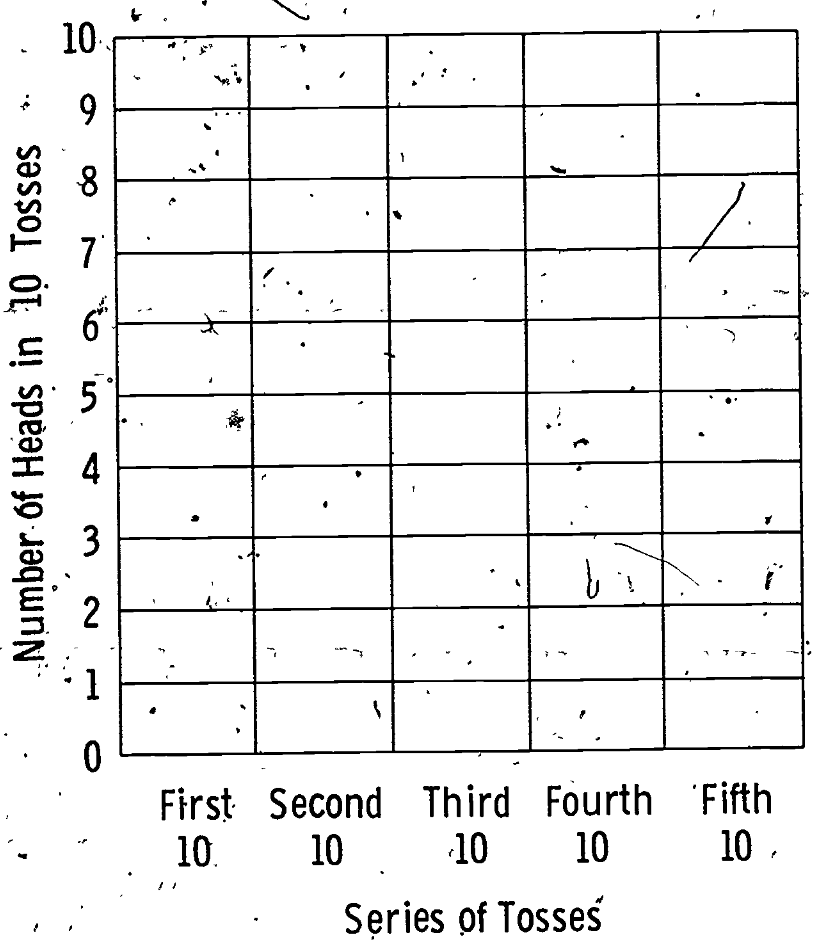
Exercises - Lesson 5.

1. Jim tossed a coin 50 times in groups of 10 tosses. He kept a record of the results by tallies. H stands for heads. T stands for tails.

Tosses

	First 10		Second 10		Third 10		Fourth 10		Fifth 10	
	H	T	H	T	H	T	H	T	H	T
Tallies	~~IIII~~ II	III	IIII	~~IIII~~ I	~~IIII~~ I	IIII	III	~~IIII~~ II	~~IIII~~ I	IIII
Number of heads	7		4		6		3		6	

a. Draw a bar graph to show the number of heads Jim got on each group of 10 tosses.



b. Out of 50 trials, how many heads did he toss? ________

c. On which series did he have the most heads? ________

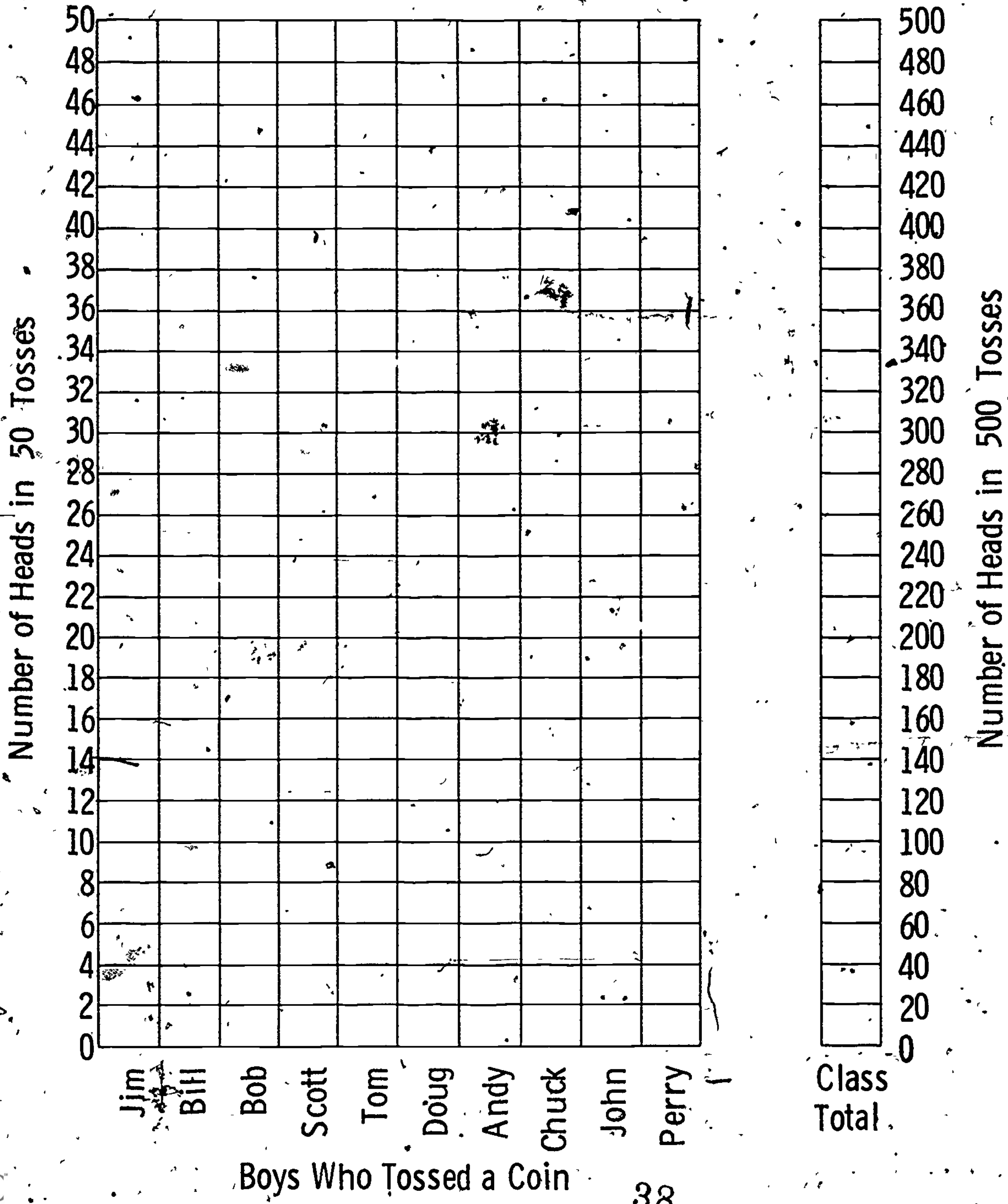
d. On which series did he have the most tails? ________

e. Did the number of heads equal the number of tails on any one series? ________

2. Nine other boys in Jim's class did the experiment, too. Their results for 50 tosses were:

Bill	35 heads	15 tails		Andy	20 heads	30 tails
Bob	19 heads	31 tails		Chuck	27 heads	23 tails
Scott	18 heads	32 tails		John	17 heads	33 tails
Tom	30 heads	20 tails		Perry	23 heads	27 tails
Doug	21 heads	29 tails				

Make a bar graph to show the number of heads these ten boys tossed. Use the scale on the left. Make another graph of the total number of heads in 500 tosses. Use the scale on the right.



2. (Continued)

a. Who are the boys who tossed more heads than tails? ____________

____

b. Who tossed the smallest number of heads? ____________

Who tossed the largest number of tails? ____________

Is this surprising? ________ Why? ____________

____

c. Which boy, do you think, might have been most surprised by the results he had in 50 tosses of the coin? ____________

Why? ____

d. Each of the ten boys tossed a coin 50 times. This makes a total of ________ tosses. How many of these tosses were heads? ________

Without counting, how many of these tosses were tails? ________

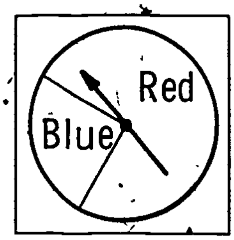
e. How many boys tossed more than 25 heads? ____________

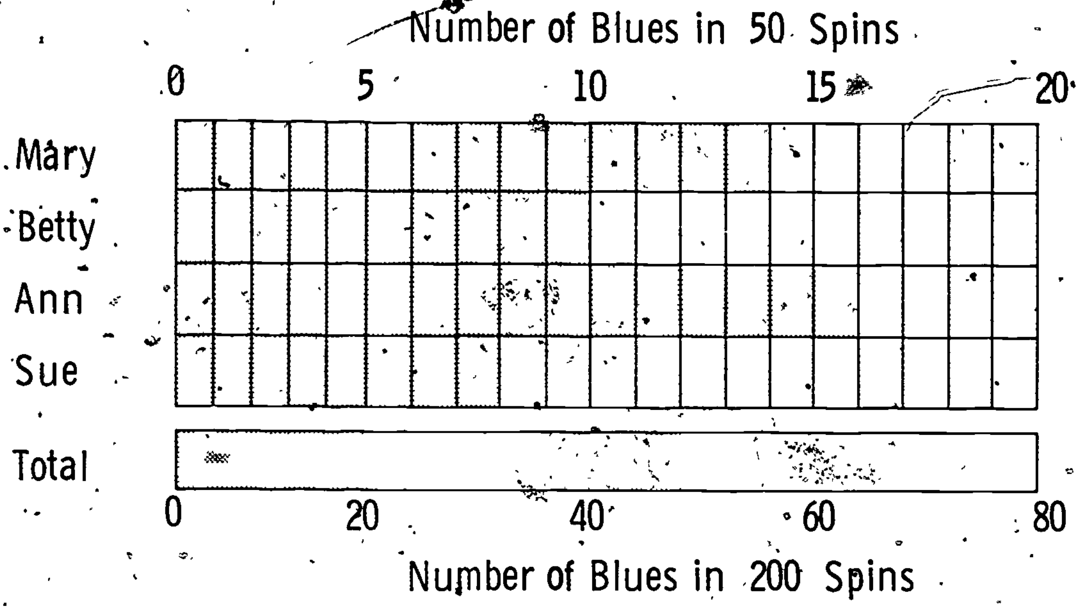
f. Draw a horizontal line across the graph so that about as many of the boys have results above the line as below. At what number does this line intersect the left edge of the graph? ____________

g. At what number does this line intersect the graph of the class total? ________ Is this about the same as the number of heads the boys tossed in 500 tosses? ________

h. Would you rather tell how many heads you expect in 10 tosses of a coin or in 500 tosses of a coin? ____________ tosses.

3. Below is a bar graph of the results some girls found in using the spinner at the right. You can read it in the same way you do other bar graphs. Look at it carefully, and you will see how to do this. Use it to answer a through h.





a. Who had the smallest number of blues in 50 spins? ________

b. Who had the largest number of blues in 50 spins? ________

c. How many reds did Betty get in 50 spins? ________

d. Which of these fractions tells about how much of the dial is blue?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ ____________

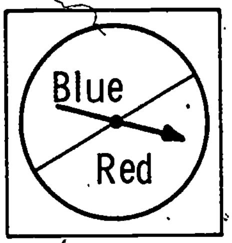
e. Did any girl get 25 or more blues? ________

f. How many times in all was the spinner spun by the girls? ________

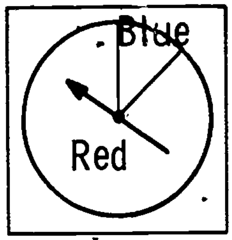
g. How many of these spins ended on blue? ________ Is this about the number of blues you would expect on 200 spins? ________

h. Would you rather guess the number of blues on 20 spins or on 200 spins? ________ spins.

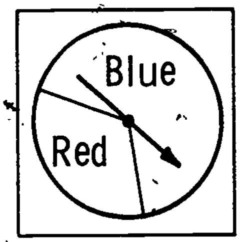
4. Here are some more spinners and graphs.



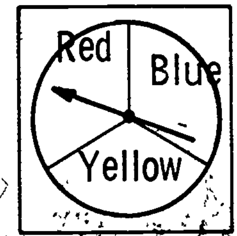
1



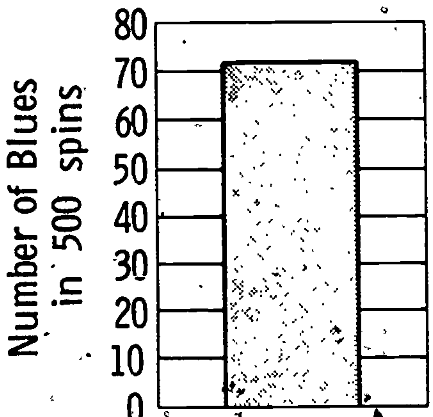
2



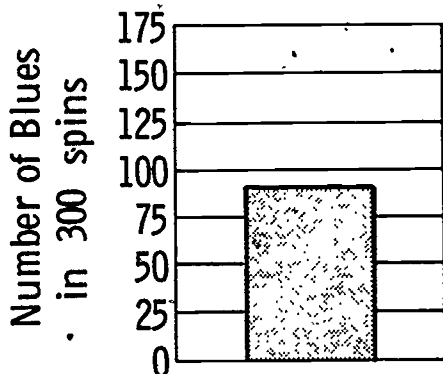
3



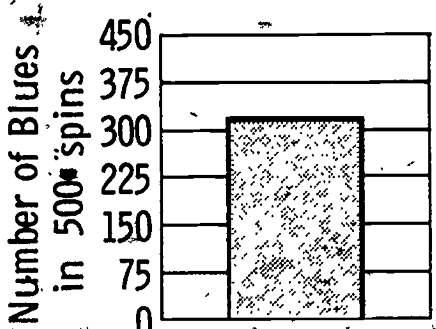
4



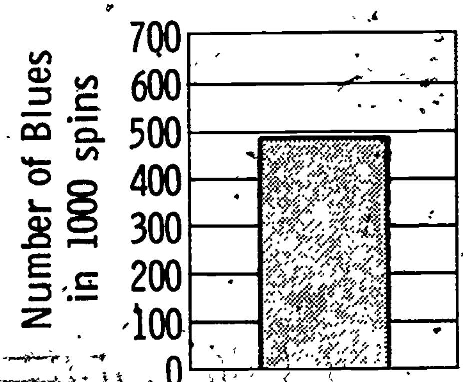
Graph A



Graph B



Graph C



Graph D

a. Graph A was probably made by using data from spinner ________.

b. Graph B was probably made by using data from spinner ________.

4. (Continued)

c. Graph C was probably made by using data from spinner ________.

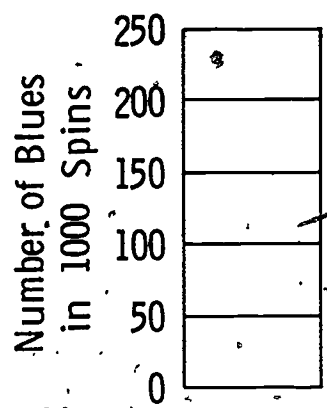
d. Graph D was probably made by using data from spinner ________.

e. Which spinner would you choose if you wanted to be most likely of getting blue? ________

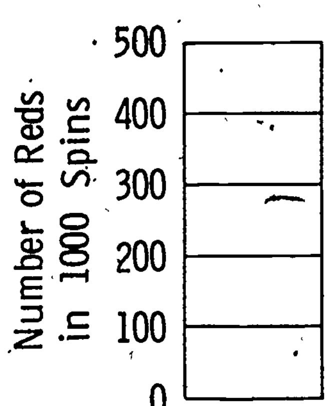
f. On which spinner is red more likely than blue? ________

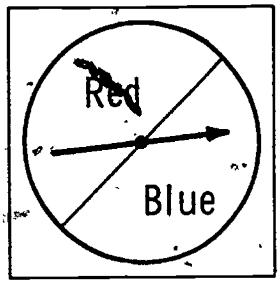
g. One of the spinners is spun 10,000 times. Blue was the result 3,300 times. Which spinner would you expect was used? ________

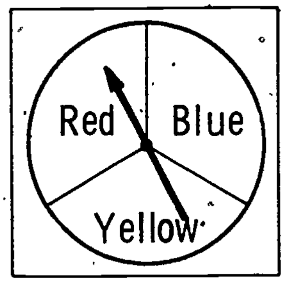
h. Spinner 2 is spun 1,000 times. Draw a bar graph to show the number of blues you would expect.

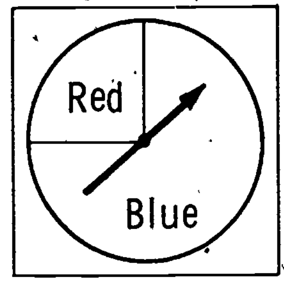


i. Spinner 3 is spun 1,000 times. Draw a bar graph to show the number of reds you would expect.









Look at these spinners.

You know that you can use fractions to compare the chances of different results.

Complete this table.

$\frac{1}{2}$ of dial red	<u>means</u>	1 chance in 2	<u>means</u>	Chance of red = $\frac{1}{2}$
$\frac{1}{2}$ of dial blue	<u>means</u>	___ chance in 2	<u>means</u>	Chance of blue = ___
$\frac{1}{3}$ of dial red	<u>means</u>	1 chance in ___	<u>means</u>	Chance of red = ___
$\frac{1}{3}$ of dial blue	<u>means</u>	___ chance in 3	<u>means</u>	Chance of blue = ___
$\frac{1}{3}$ of dial yellow	<u>means</u>	___ chance in ___	<u>means</u>	Chance of yellow = $\frac{1}{3}$
$\frac{1}{4}$ of dial red	<u>means</u>	___ chance in ___	<u>means</u>	Chance of red = ___
$\frac{3}{4}$ of dial blue	<u>means</u>	___ chances in 4	<u>means</u>	Chance of blue = ___
All of dial red	<u>means</u>	red is certain	<u>means</u>	Chance of red = ___
___ of dial red	<u>means</u>	red is impossible	<u>means</u>	Chance of red = 0

Exercises - Lesson 6.

1. James spins the pointer of a spinner 100 times and gets 35 reds. Which of the following statements is most likely to be true?
   (a) The dial of the spinner is all red.
   (b) The dial of the spinner is one-half blue.
   (c) The dial of the spinner is one-eighth red.
   (d) The dial of the spinner is one-third red.

2. Mary spins the pointer of a spinner 100 times and gets 25 red, 25 blue, and 50 yellow. Which of the following statements cannot be true?
   (a) The dial of the spinner is one-fourth yellow.
   (b) The dial of the spinner is one-third green.
   (c) The dial of the spinner is one-fourth blue.
   (d) The dial of the spinner is all red.

3. A spinner has a dial that is one-third red, one-half white, and one-sixth blue. Which of the following cannot result from exactly 100 spins?
   (a) 30 reds, 50 whites and 20 blues.
   (b) 40 reds, 40 whites and 20 blues.
   (c) 50 reds, 5 whites and 10 blues.
   (d) 60 reds, 40 whites and 0 blues.

4. You wish to get exactly 5 reds and 5 blues in 15 spins. Which of the following dials could not give this result?
   (a) One-half red and one-half blue.
   (b) One-third red, one-third blue and one-third yellow.
   (c) One-fourth red, one-fourth blue and one-half yellow.
   (d) One-fifth red, two-fifths blue and two-fifths yellow.

5. In which of the following statements is the chance of red equal to $\frac{1}{4}$?
   (a) One chance in two of red.
   (b) Two chances in four of red.
   (c) One chance in five of red.
   (d) Two chances in eight of red.

6. Which of the following spinners is likely to give about the same number of reds and yellows?
   (a) One-half red, one-fourth yellow, one-fourth blue.
   (b) One-third red, two-thirds yellow.
   (c) One-third red, one-third yellow, one-third blue.
   (d) Four-fifths yellow, one-fifth red.

7. If the dial of a spinner is all red, we say the chance of red is equal to:
   (a) any other chance.
   (b) one chance in two.
   (c) one-half.
   (d) one.

8. If the dial of a spinner is all blue, we say the chance of red is equal to:
   (a) one.
   (b) zero.
   (c) one chance in one.
   (d) one-half.

9. The dial of a spinner is one-third red, one-third yellow, and one-third blue. Which of the following statements are true?
   (a) Red, yellow, and blue are equally likely to occur.
   (b) The chance of getting red is equal to $\frac{1}{3}$.
   (c) One spin must result in either red or yellow or blue.
   (d) The chance of getting green is equal to zero.

10. If the chance of red on a spinner is equal to zero, which of the following statements could be true?
   (a) The dial is all red.
   (b) The dial is all blue.
   (c) The dial has at least two colors.
   (d) The dial has at least three colors.

11. Complete this table.

All of dial red	means	red is certain	means	Chance of red = 1	means	P(R) = 1
None of dial red	means	red is impossible	means	Chance of red = 0	means	P(R) = ___
$\frac{1}{2}$ of dial red $\frac{1}{2}$ of dial blue	means	1 chance in 2 of red ___ chance in 2 of blue	means	Chance of red = ___ Chance of blue = ___	means	P(R) = ___ P(B) = ___
$\frac{1}{4}$ of dial red $\frac{3}{4}$ of dial blue	means	1 chance in ___ of red ___ chances in ___ of blue	means	Chance of red = $\frac{1}{4}$ Chance of blue = ___	means	P(R) = $\frac{1}{4}$ P(B) = ___
$\frac{1}{3}$ of dial red $\frac{1}{3}$ of dial blue $\frac{1}{3}$ of dial yellow	means	___ chance in 3 of red 1 chance in ___ of blue ___ chance in ___ of yellow	means	Chance of red = ___ Chance of blue = ___ Chance of yellow = ___	means	P(R) = ___ ___ ___
___ red cubes and ___ blue cube	means	2 chances in ___ of red ___ chance in 3 of blue	means	Chance of red = ___ Chance of blue = $\frac{1}{3}$	means	___ P(B) = ___

12. A spinner has a dial which is evenly divided into red, white, and blue spaces. Write a number sentence that describes the chance of getting blue. ______________

13. Write a number sentence that answers the question, "What is the probability of yellow on the spinner in Problem 12 ?" ______________

14. In Problem 12, $P(R) =$ __________.

15. Write this number sentence (about Problem 12) in words: $P(W) = \frac{1}{3}$.

____

16. A bag contains several marbles. Some are red, some white, and the rest blue. If you pick one marble without looking, the probability of red is $\frac{1}{3}$ and the probability of white is $\frac{1}{3}$. What is the probability of blue?

____

17. A bag contains one red marble, two white marbles, and three blue marbles. If you pick one marble without looking, what is the probability that the marble will be red? ______________

18. In Problem 17, what is the probability that the marble will be white? ______________

19. In Problem 17, what is the probability that the marble will be blue? ______________

20. In Problem 17, how many white marbles must be added to the bag to make the probability of white equal to $\frac{1}{2}$? ______________

21. Write the following number sentence in symbols: "The probability of yellow is equal to three-fourths." ______________

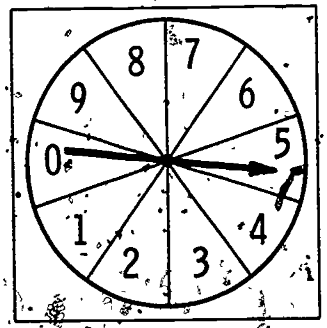
22. A wooden cube has a dot on two of its faces. If it is tossed on the floor, what is the probability that a face with a dot on it will be on the bottom when it stops rolling? ____________

23. In Problem 22, what is the probability that a face without a dot will be on the bottom? ____________

24. The dial of a spinner is divided into three colors: red, white, and blue. If $P(R) = \frac{1}{2}$ and $P(W) = \frac{1}{4}$, what is the probability of blue? ____________

25. In Problem 24, is the probability of red greater than, less than, or equal to the probability of blue? ____________

The dial of this spinner is divided into 10 equal regions.

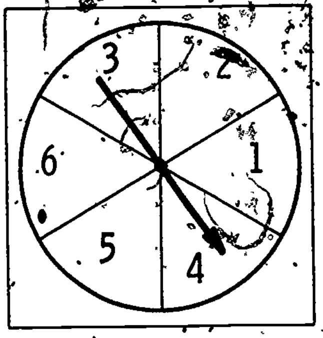


26. P(3) = ____________.

27. P(10) = ____________.

28. Is P(4) = P(8) ? ____________

The dial of this spinner is divided into 6 equal regions.



29. P(2) = ____________.

30. P(5) = ____________.

Brain Teasers

1. John has ten pairs of socks in a drawer. Nine pairs are red and one is blue. If he picks the socks one at a time without looking, how many socks must he pick to be sure he has two socks of the same color? _______

2. A bag contains several marbles. Some are red, some white, and the rest blue. If the probability of picking red is $\frac{1}{6}$ and the probability of picking white is $\frac{1}{3}$, what is the probability of picking blue? _______

3. In Brain Teaser 2, what is the smallest number of marbles that could be in the bag? _______

4. In Brain Teaser 2, could the bag contain 48 marbles? _______

5. In Brain Teaser 2, if the bag contains 4 red marbles and 8 white marbles, how many blue marbles does it contain? _______

Exercises - Lesson 7.

John and Paul each have one white and one green marble. John picks one of his marbles without looking and then Paul picks one of his. The four possible outcomes are listed in the table below. Complete the table on the right to show the outcomes in a shorter way.

	John's Marble	Paul's Marble
1.	White	White
2.	White	Green
3.	Green	White
4.	Green	Green

	John's Marble	Paul's Marble
1.	W	W
2.	W	___
3.	G	___
4.	___	___

1. What is the probability that John picks a white marble? __________
2. What is the probability that Paul picks a white marble? __________
3. What is the probability that both boys pick white marbles? __________
4. What is the probability that both boys pick green marbles? __________
5. What is the probability that the boys pick a marble of the same color? __________

6. a. P(WW) = ________; P(Not WW) = ________.

   b. P(WG) = ________; P(Not WG) = ________.

   c. P(GW) = ________; P(Not GW) = ________.

   d. P(GG) = ________; P(Not GG) = ________.

   e. P(WG or GW) = ________; P(Not WG and Not GW) = ________.

7. a. P(GG) + P(WW) $= \frac{1}{4}$ + ____ = ________.

   b. P(GW) + P(WG) = ____ + ____ = ________.

   c. P(WW) + P(Not WW) = ____ + ____ = ________.

8. a. If John picks a white marble, what is the probability that Paul will <u>not</u> pick a white marble? ________

   b. Does John's outcome have any effect on Paul's outcome? ________

9. a. If John picks a green marble, what is the probability that Paul also picks a green marble? ________

   b. Does Paul's pick depend on what John picks? ________

10. What is the probability that Paul will not pick a marble of the same color as John's? ________

A bag contains three marbles, one red, one white and one blue. Imagine that you choose one marble without looking.

11. P(R) = ________.

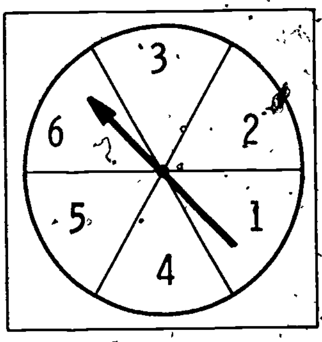
12. P(R or W) = ________.

13. P(Not B) = ________.

14. 1 - P(R) = ________.

15. P(R) + P(W) + P(B) = ________.

The dial of this spinner is divided into six equal regions.



16. P(3) = ________.

17. P(Not 3) = ________.

18. P(5 or 6) = ________.

19. P(1 or 3 or 4 or 5 or 6) = ________.

20. P(2) = ________.

21. Are the events in Problems 19 and 20 complementary events? ________

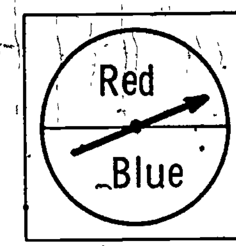
22. 1 - P(3) = ________.

23. 1 - P(Not 3) = ________.

24. P(3) + P(Not 3) = ________.

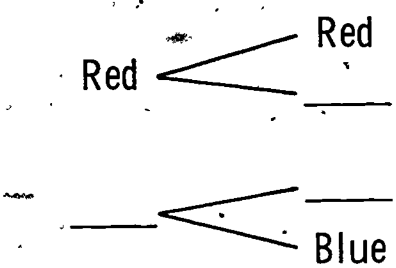
Exercises - Lesson 8.

1. Complete the tree diagram and the table to show the possible outcomes of two spins with this spinner.



Tree

First Spin   Second Spin



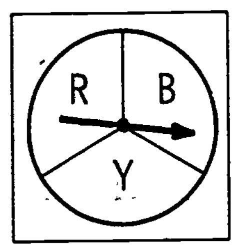
Second Spin

		Red	Blue
First Spin	Red	R	

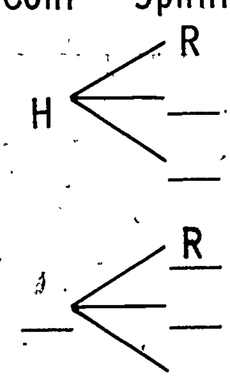
a. P(RR) = ________.   c. P(BR) = ________.

b. P(Not RR) = ________.   d. P(BB or RR) = ________.

2. Complete this tree diagram and the table. Show all the possible outcomes of the toss of a coin and one spin on this spinner. The dial is divided into three equal regions.



Coin   Spinner



Spinner

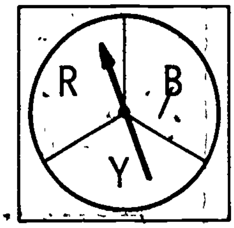
Coin		R		Y
	T			

a. P(HY) = ______.   c. P(TR) = ______.

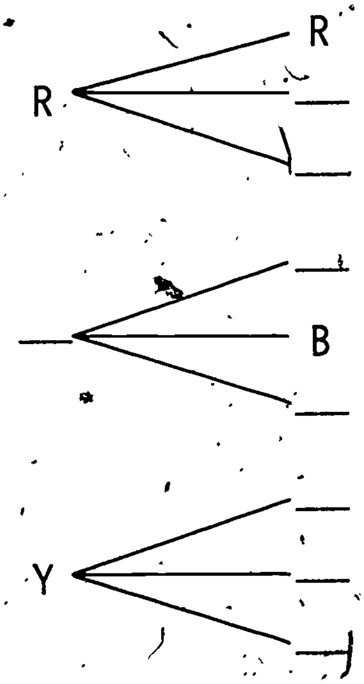
b. P(TB) = ______.   d. P(HB) = ______.

e. P(Not HB) = ______.

3. Complete this tree diagram and the table to show all the possible outcomes of two spins. The dial is divided into three equal regions.



First Spin — Second Spin



Second Spin

First Spin		B	Y
R			
B			

The total number of outcomes is ________.

a. P(RB) = ________.

b. P(YR) = ________.

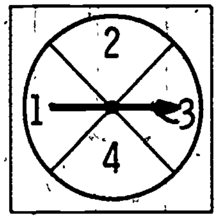
c. P(Not YR) = ________.

d. P(BY or RR) = ________.

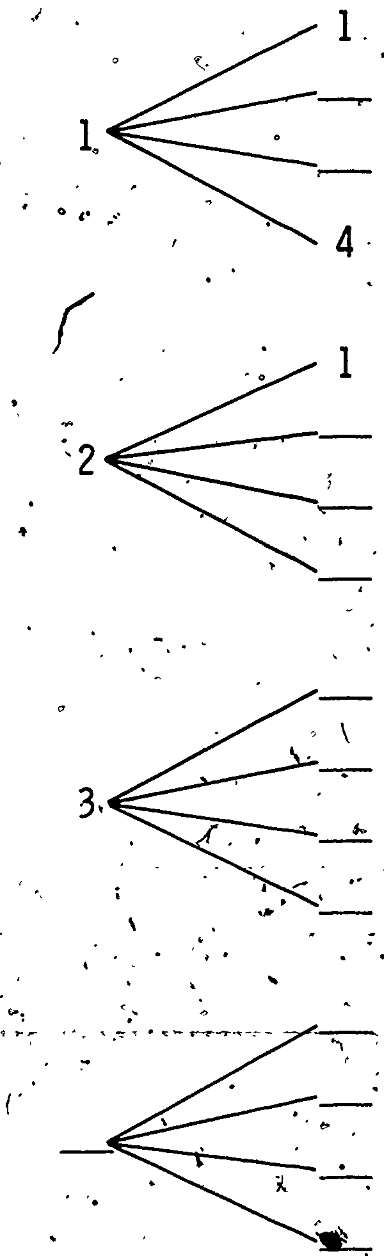
e. P(RY or BY or YY) = ________.

f. How many possible outcomes are there if this spinner is spun three times? ________

4. Complete this tree diagram and the table to show all the possible outcomes of two spins with this spinner. The dial is divided into four equal regions.



First Spin Second Spin



Second Spin

		1			4
First Spin	1	1,1		1,3	
	4				

The total number of outcomes is ________.

a. P(3,4) = ________.

b. P(2,3 or 3,2) = ________.

c. P(Not 4,2) = ________.

d. P(Not 1,2 and Not 3,2) = ________.

e. P(Two odd numbers) = ________.

f. P(Not two odd numbers) = ________.

g. P(Three odd numbers) = ________.

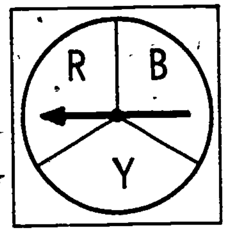
h. P(Two odd numbers or two even numbers) = ________.

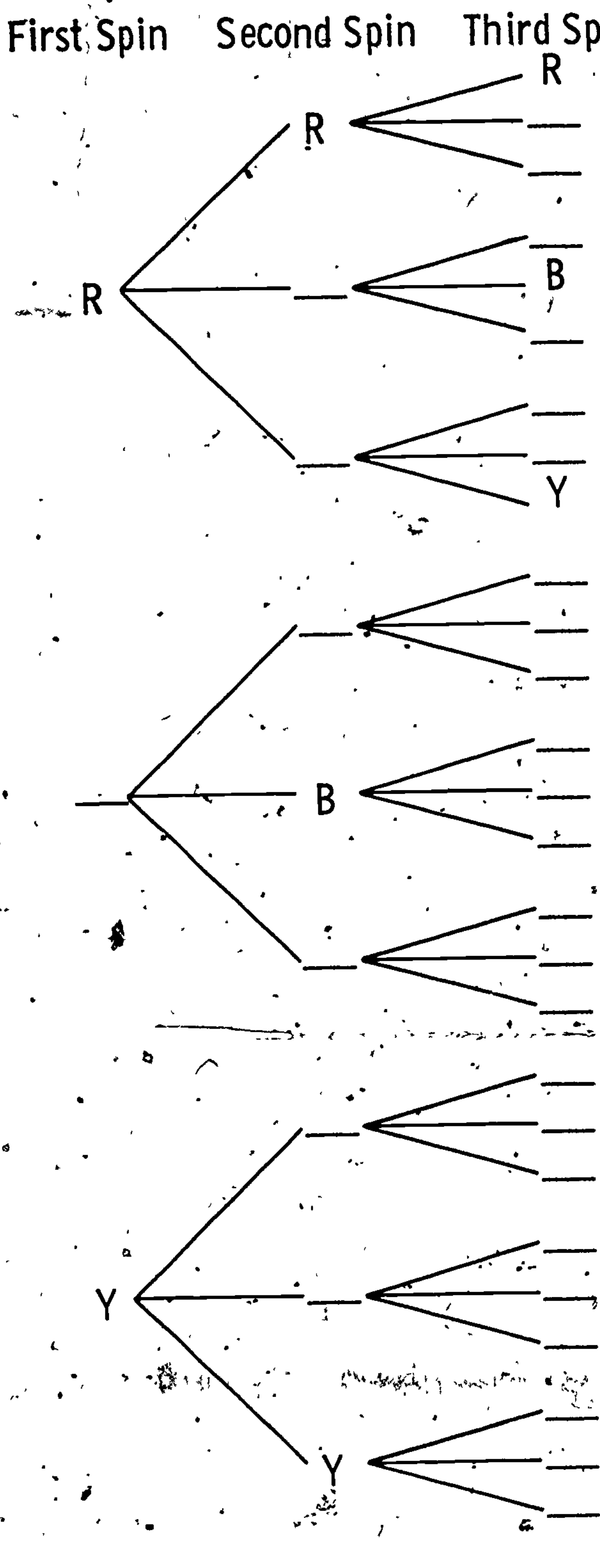
i. P(At least one 1) = ________.

j. P(Not more than one odd number) = ________.

k. This spinner is spun four times. How many possible outcomes are there? ________

5. Complete this tree diagram and the tables to show all the possible outcomes of three spins. The dial is divided into three equal regions.





Second Spin

First Spin	R		Y
R			
Y			

Third Spin

First and Second Spins	R	B	
RR			
RB			
BR			
YR			

a. P(RBB) = ________.

b. P(YRB) = ________.

c. P(Not BRB) = ________.

d. P(YYR) = ________.

e. P(YRR or RRR) = ________.

f. The probability that the colors will be the same on three spins is ________.

6. Complete the table to show all the possible sums of the dots on two dice.

Number on Second Die (columns); Number on First Die (rows)

	1	2	3	4	5	6
1	2	3				7
2	3					
3		5				
4				8		
5					10	
6						12

a. One way to get a sum of 7 is to get a 1 on the first die and a 6 on the second die. We can write this as (1,6). There are five more ways to get a sum of 7. List them below.

(,), (,), (,), (,), (,).

b. How many entries are there in the table? ________

c. How many possible entries are there when you toss two dice?

____

d. Of the entries in the table, how many are 6's ? ________

e. What is the probability of getting a sum of 6 when two dice are tossed? ________

f. How many of the entries are odd numbers? ________

g. What is the probability of getting a sum that is an odd number?

____

h. How many of the sums are either 5's or 9's ? _______

i. What is the probability that the sum will be either 5 or 9 ? _______

j. (1) P(sum = 3) = _______.

(2) P(sum = 8) = _______.

(3) P(sum = 12) = _______.

(4) P(sum = 2) = _______.

(5) P(sum = 11) = _______.

(6) P(sum = 2 or sum = 12) = _______.

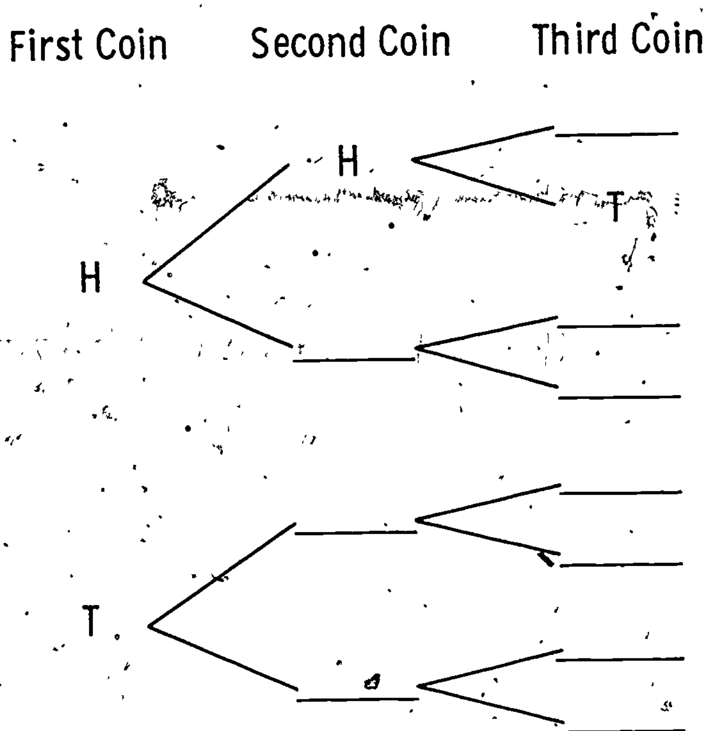
(7) P(sum = 6 or sum = 8) = _______.

(8) P(sum = 5 or sum = 9) = _______.

(9) P(sum ≠ 7) = _______.

(10) P(sum > 9) = _______.

7. Fill in the tree diagram and the tables to show all the outcomes when three coins are tossed. Use it to answer questions a through j.



Second Coin

First Coin	H	T
H		
T		

Third Coin

First and Second Coins	H	T
HH		
HT		
TH		
TT		

a. What is the total number of outcomes when three coins are tossed? ________

b. How many of these outcomes include three heads? ________

c. What is the probability of getting three heads when three coins are tossed? ________

d. How many of these outcomes include two heads and one tail? ________

e. What is the probability of getting two heads and one tail when three coins are tossed? ________

f. What is the probability of getting three tails when three coins are tossed? ________

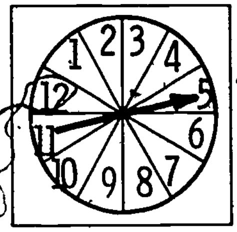
g. What is the probability of getting at least one tail when three coins are tossed? ________

h. What is the probability that you will not get three heads or three tails when you toss three coins? ________

i. What is the probability that you will get heads on the first coin? ________

j. (1) P(3 heads) = ______.
(2) P(2 heads, 1 tail) = ______.
(3) P(Not 3 heads) = ______.
(4) P(3 heads or 3 tails) = ______.
(5) P(At least 2 tails) = ______.
(6) P(No heads) = ______.
(7) P(3 tails) = ______.
(8) P(No tails) = ______.
(9) P(At least 1 head) = ______.
(10) P(At least 1 head or 1 tail) = ______.

8. Complete the table to show all the possible sums of numbers on two spins. The dial is divided into twelve equal regions.



Second Spin

First Spin	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3										13
2			5									
3												
4												
5												
6						12						
7												
8												
9												
10												
11												
12												

a. How many entries are represented in this table? ________

b. Of these entries, how many are 9's ? ________

c. What is the probability of getting a sum of 9 on two spins? ________

d. P(sum = 5) = ________. e. P(sum = 12) = ________.

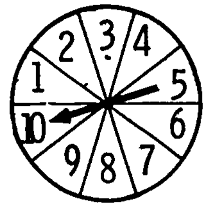
f. P(sum is odd) = ________. g. P(sum > 18) = ________.

h. P(sum < 4) = ________. i. P(sum ≠ 2) = ________.

j. Which sum has the greatest probability? ________

k. Which two sums have the least probability? ________ and ________

9. Complete the table to show the possible <u>products</u> of numbers on two spins. The dial is divided into ten equal regions.



Second Spin

	1	2	3	4	5	6	7	8	9	10
1	1	2								
2		4		8						20
3										
4										
5			15							
6										
7										
8										
9										
10										

a. How many entries are represented in this table? ________

b. How many times is there a product equal to 16 ? ________

c. What is the probability that a product equals 16 ? ________

d. P(24) = ________.

e. P(27) = ________.

f. P(even number) = ________.

g. P(16 < product < 30) = ________;

h. P(multiple of 5) = ________.

i. List the products which have a probability of $\frac{1}{100}$:

______, ______, ______, ______, ______, ______

Lesson 9 - "Both, and ; either, or"

A bag contains 3 marbles, one red, one green, and one white. Let's draw a marble, note its color, and return it to the bag. Shake the bag and draw again.

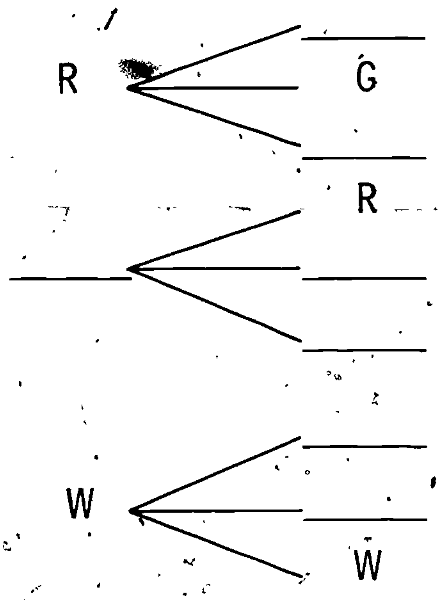
P(red on first draw) = $\frac{1}{3}$.

P(green on second draw) = $\frac{1}{3}$.

We want to find the probability of getting both a red marble on the first draw and a green marble on the second draw. Complete the table and tree diagram of the possible outcomes.

		Second Draw		
		Red	Green	White
First Draw	Red	R R		
	Green			
	White			

First Draw   Second Draw



Both the table and the tree show 9 possible outcomes. Only one of them is Red on the first draw and Green on the second draw (RG).

P(RG) = P(Both red on first and green on second) = ________.

P(WR) = ________.

P(WW) = ________.

Let's think of this same bag with 3 marbles, one red, one green, and one white.

But now, what is the probability of <u>either</u> red on the first draw <u>or</u> green on the second draw <u>or</u> <u>both</u> red on the first and green on the second?

Look at the table or the tree diagram we just made. There are 3 possible outcomes in which the first draw is red:

________, RG, and ________.

There are also 3 outcomes in which the second draw is green:

RG, ________, and ________.

Notice that you listed one of the outcomes twice. Which one? ________

Therefore, there are only ________ outcomes with <u>either</u> red on the first draw <u>or</u> green on the second, or both red on the first and green on the second.

They are ______, ______, ______, ______, and ______.

We can count to see that

P(either red on first or green on second or both) = ________.

1. P(either R on first or W on second or both) = ________.

2. P(either G on first or G on second or both) = ________.

3. P(R on second or W on first or G on first) = ________.

4. P(R on first or G on first or W on first) = ________.

Think of the same bag and three marbles, red, green, and white.

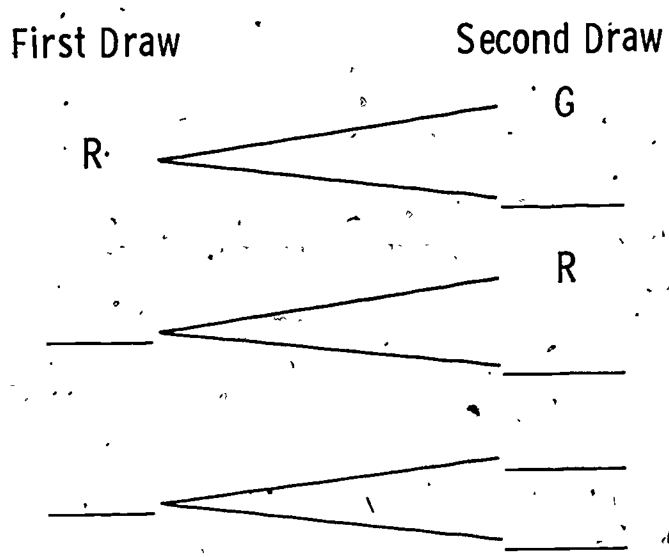
This time we do not replace the first marble before we draw the second marble. What is the probability of red on the first draw and green on the second draw ? ________

Complete this table to show the outcomes. Remember that if we draw a red on the first draw and do not replace it, it cannot be drawn on the second draw. So R,R is impossible. Is G,G possible? ________

Second Draw

		Red	Green	White
First Draw	Red	impossible	R G	
	Green			
	White	W R		

A tree diagram would also show the possible outcomes. As you complete it, be careful to include only the possible outcomes.



1. How many possible outcomes are there? ________

2. What is the probability of red on the first draw? ________

3. What is the probability of green on the second draw? ________

4. What is the probability of both red on the first draw and green on the second draw? (Look for <u>RG</u> in the table.) ________

5. What is the probability of white on the first draw? ________

6. What is the probability of both white on the first draw and white on the second draw? ________

7. List the outcomes which show a red on the first draw, <u>or</u> a green on the second draw, <u>or</u> <u>both</u>. ________, ________, ________

8. P(either red on first or green on second or both) = ________.

9. P(either green on first or white on second or both) = ________.

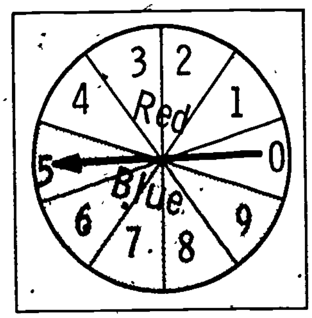
10. P(WW) = ________.

11. The probability that white will be drawn on either the first draw or second draw or both is ________. Does the phrase "or both" affect this statement? ________ Why or why not? ________

____

12. The probability that white will not be drawn at all on the two draws is ________.

Exercises - Lesson 9.



Use this spinner. The dial is divided into ten equal regions.

1. What is the probability of red? ________
2. What is the probability of blue? ________
3. What is the probability of a 7 ? ________
4. P(5) = ________.
5. P(even number) = ________.
6. P(odd number) = ________.

Let's try our phrases "both, and" and "either, or" to find:

7. the probability of both red and even;
8. the probability of either red or even or both.

Complete this table to help you find these probabilities.

		Numbers									
		0	1	2	3	4	5	6	7	8	9
Colors	Red	R,0				R,4					
	Blue							B,6			

(Check your spinner; is a red 7 possible? ________)

9. How many possible outcomes are there? ________
10. List the outcomes which are both red and even:

R,0, ________, ________.

11. Now answer Question 7. P(both red and even) = ________.

12. List the outcomes which are red:

_______, _______, _______, _______, _______.

13. How many outcomes are red? _______

14. List the outcomes which are even:

_______, _______, _______, _______, _______.

15. How many outcomes are even? _______

16. How many outcomes in Problems 12 and 14 are listed twice? _______

17. Now answer Question 8. P(either red or even or both) = _______.

18. P(both red and 2) = _______.

19. P(both blue and 4) = _______.

20. P(either blue or even or both) = _______.

21. P(either red or odd or both) = _______.

22. P(both red and $<$ 4) = _______.

23. P(red or $>$ 4) = _______.

24. P(blue or $>$ 4) = _______.

25. P(blue or red) = _______.

26. P(both blue and red) = _______.

27. P(both red and $>$ 4) = _______.

Brain Teaser - Quacky Quotients

This is a game for two players. You need two spinners numbered 1 to 9. (You can use the spinners labeled 0-9 and spin again whenever either spinner stops on a zero, or you can cut out a new dial, divided into 9 equal parts, and place it under the pointer of any spinner.)

Each player spins once. The one with the larger number is called player A and the other is player B.

Each player spins his spinner. The number on player A's spinner is divided by the number on player B's spinner.

If the first digit of the quotient is a 1 or 2 or 3 (or .1 or .2 or .3), player A wins a point. If the first digit of the quotient is 4 or 5 or 6 or 7 or 8 or 9 (or .4 or .5 or .6 or .7 or .8 or .9), B wins a point. Thus A wins on 3 of the 9 possibilities while B wins on 6 of the 9 possibilities.

Is it a fair game? Play it and then decide. Here are examples:

A spins an 8, B spins 3. $\frac{8}{3} = 2. + \ldots$ So A wins.

A spins a 7, B spins a 9. $\frac{7}{9} = .7 + \ldots$ B wins.

A spins a 1, B spins a 4. $\frac{1}{4} = .25$. A wins.

A spins a 3, B spins 5. $\frac{3}{5} = .6$. B wins.

Spin 20 times and see who wins. Play several games before you decide if this is a fair game. Then make a table to show all the possible quotients in this game. From this table you can see which player has an advantage. What is the probability that player A will win a point on one set of spins? ______ that player B will win on one set of spins? ______ Why could we not use 0 on the dial in this game? ______________________

The Pascal Triangle - Lesson 10.

List some of the activities we have tried which have had two equally likely outcomes.

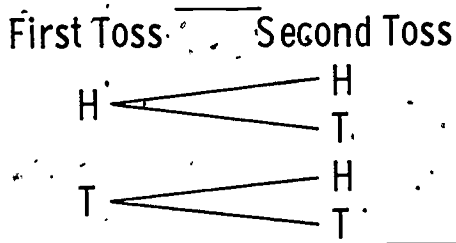
a. Spinner $\frac{1}{2}$ red, $\frac{1}{2}$ blue ;   d.

b.   e.

c.   f.

We will make tree diagrams for one of these activities, tossing a coin. Show the possible outcomes for two tosses.

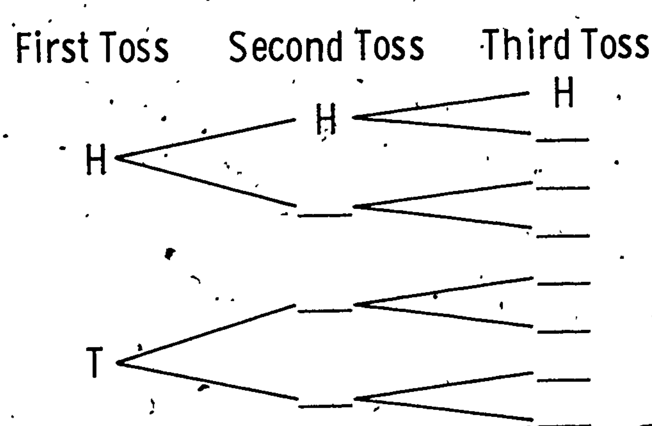
Tree



Table

	2H, 0T	1H, 1T	0H, 2T.
	H H	H T	T T
		T H	
Number of outcomes	1	2	1

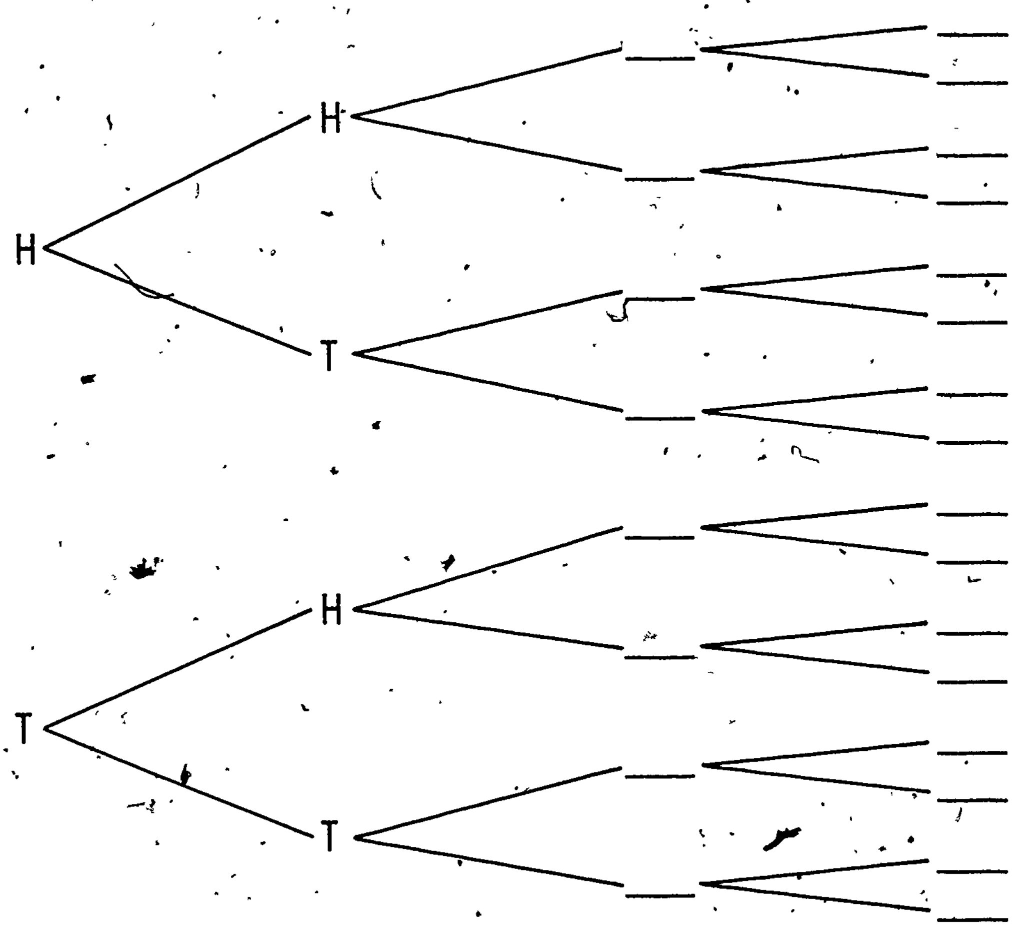
Complete the tree and table for 3 tosses.



	3H, 0T	2H, 1T	1H, 2T	0H, 3T
	H H H	H H T		
Number of outcomes	1	3	3	1

Now make one more tree (for four tosses).

First Toss | Second Toss | Third Toss | Fourth Toss



List the outcomes in the table.

	4H, 0T	3H, 1T	2H, 2T	1H, 3T	0H, 4T
Number of outcomes	1	4	6	4	1

Let's organize our data from the tables into a triangular display.

___ First Toss

___ Second Toss (Look at the first table.)

___ Third Toss (From the second table.)

___ Fourth Toss

___ Fifth

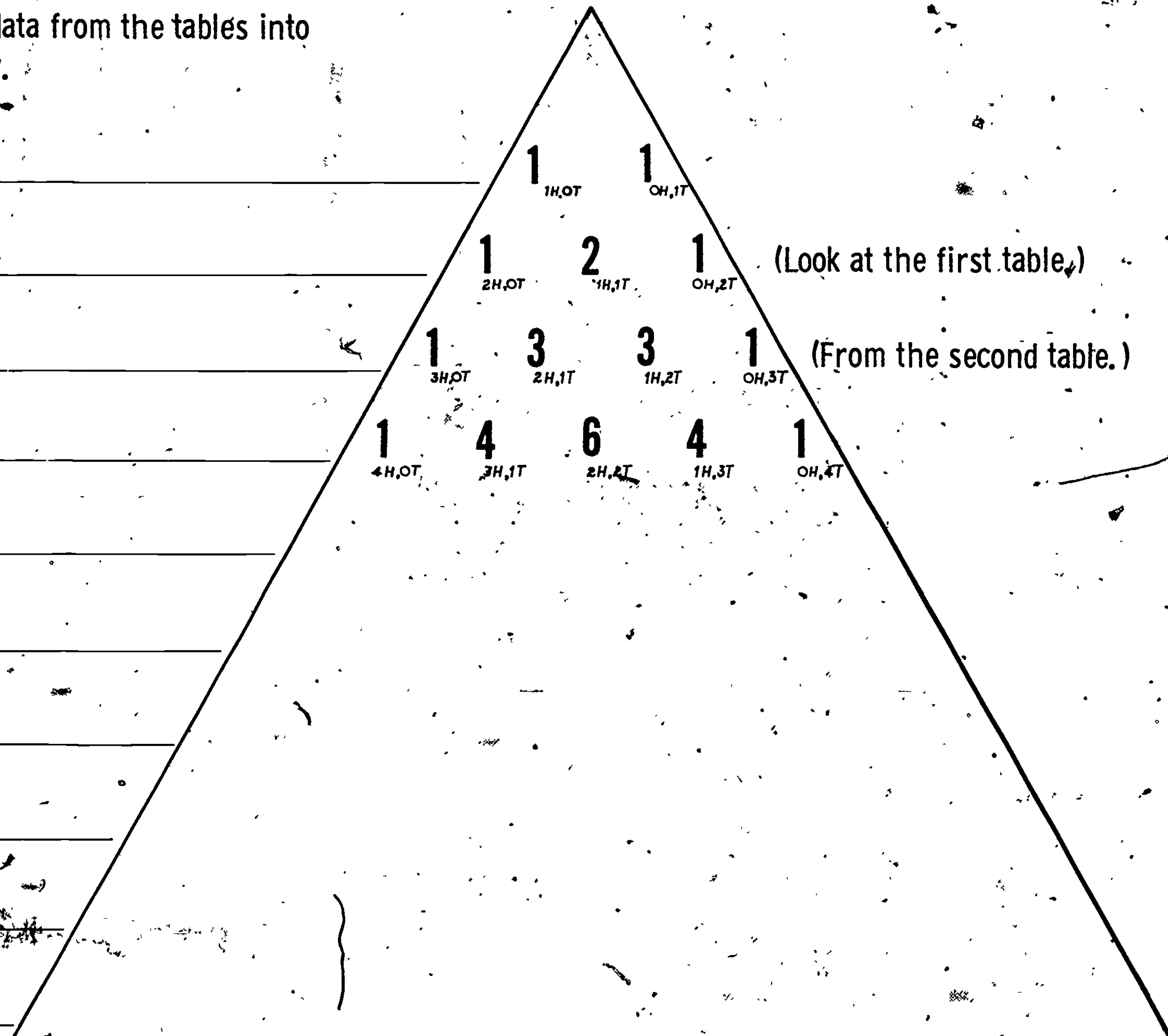
___ Sixth

___ Seventh

___ Eighth

___ Ninth

___ Tenth



Look at our display. Do you see a pattern? ______ Do you see several different patterns? ______
Can you complete the fifth row (5 tosses of a coin) without making a tree or a table? ______
Go back to the fifth row and try it.

Your fifth row should be

$$1_{5H,0T} \quad 5_{4H,1T} \quad 10_{3H,2T} \quad 10_{2H,3T} \quad 5_{1H,4T} \quad 1_{0H,5T}$$

Did you write this row so that it continues the triangle? ________

This pattern was noticed by a 17th-Century mathematician, Blaise Pascal. So it is named the Pascal Triangle.

Go back to the triangle and complete rows 6 through 10. Don't rush through it. Place a ruler so that it touches the ends of the 1's. Draw a light line down toward the "Tenth Toss". Do the same on both sides; it will help you keep your display in order. Always work neatly. Your results will be used to answer some questions.

Now go back to the first row of the triangle. Write in, just to the left of the words "First Toss", the number 2. The 2 stands for the total number of outcomes for one toss of a coin.

Just to the left of "Second Toss", write a 4 to show it represents 4 outcomes for 2 tosses of a coin. What should you write to the left of "Third Toss"? ________ Complete by writing the total number of outcomes for each row. (The 7th row is 128 and the 10th row is 1024.)

Do you also see a pattern to this column of numbers? ________

What is the pattern? ____

____

____

Use the Pascal Triangle to answer these questions.

1. How many outcomes are there for 2 tosses? ________
2. How many outcomes of exactly 2 heads are there in 2 tosses? ________
3. In 2 tosses, P(2H) = ________.
4. In 3 tosses, P(3H) = ________.
5. In 7 tosses, P(7H) = ________.
6. In 7 tosses, P(7T) = ________.
7. In 4 tosses, how many possible outcomes are there? ________
8. In 4 tosses, how many outcomes can you expect to be 2H,2T ? ________
9. In 4 tosses, P(2H,2T) = ________.
10. In 4 tosses, P(3H,1T) = ________.
11. In 6 tosses, P(4H,2T) = ________.
12. In 6 tosses, P(2H,4T) = ________.
13. In 10 tosses, P(8H,2T) = ________.
14. In 10 tosses, P(2H,8T) = ________.
15. In 10 tosses, P(5H,5T) = ________.
16. If you toss 10 coins at one time, is the probability of 4 heads and 6 tails more likely or less likely than 7 heads and 3 tails?

    ________ likely

Brain Teaser. If you toss 15 coins at one time, the greatest probability would be for ________ heads, ________ tails, and for ________ heads, ________ tails.

Let's see how we can use the Pascal Triangle to help us answer other problems. Look at the third row.

If the question were, "How many different combinations of exactly 2 Heads are there?", you would say ________.

This triangle can be used to answer many questions.

Use the third row again. There are 3 people. How many different combinations of 2 people can you choose from 3 people? (Look at 2H,1T.)

____

Exercises - Lesson 10.

Use your Pascal Triangle to answer these questions.

1. From 4 people, how many committees of 2 can be chosen? (Look at 4th row, 2H,2T). ________

2. From 4 people, how many committees of 3 can be chosen? ________

3. From a group of 7 people, there are:

   a. How many different committees of 3 people? ________

   b. How many different committees of 4 ? ________

   c. How many different committees of 1 ? ________

   d. How many different committees of 6 ? ________

   e. How many different committees of 7 ? ________

Exercises - Lesson 10 (continued).

4. a. In how many different ways can a committee of 6 be chosen from a group of 8 pupils? ________

   b. In how many different ways can a committee of 7 be chosen from a group of 8 pupils? ________

   c. In how many different ways can a committee of 8 be chosen from a group of 8 pupils? ________

5. The teacher asks for 3 volunteers to go with her to the library. However, 9 pupils volunteer.

   a. In how many different ways can the teacher select the 3 to go with her? ________

   b. If the teacher could select 4 from the 9 pupils, how many different groups could there be? ________

6. There are 5 flavors of ice cream: chocolate, vanilla, strawberry, black walnut, and peach.

   a. In how many different ways can you choose 2 of these flavors? ________

   b. In how many different ways can you choose 3 of these flavors? ________

   c. In how many different ways can you choose 4 of these flavors? ________

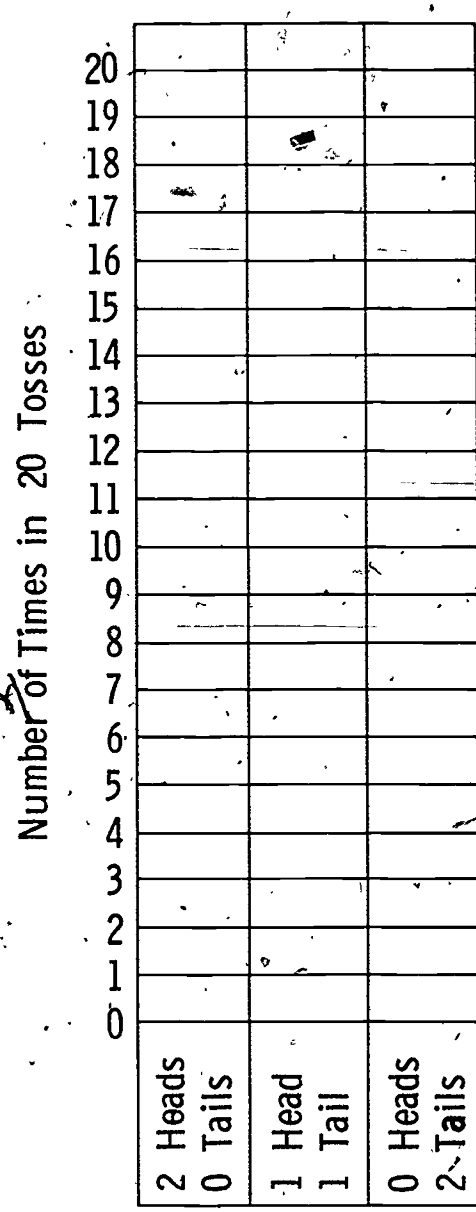
Things To Do At Home - Lesson 10.

1. The Pascal Triangle can be used to show the different possible outcomes in making selections. It is a pattern and we expect results similar to it. To compare the actual results with the Pascal Triangle, toss two coins and record the outcomes on the chart and on the graph.

CHART

Toss Number	2 Heads 0 Tails	1 Head 1 Tail	0 Heads 2 Tails
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
Totals			

BAR GRAPH



a. Did 2 heads occur about as often as 2 tails? ________

b. Did 1 head, 1 tail occur about twice as often as 2 heads? ________

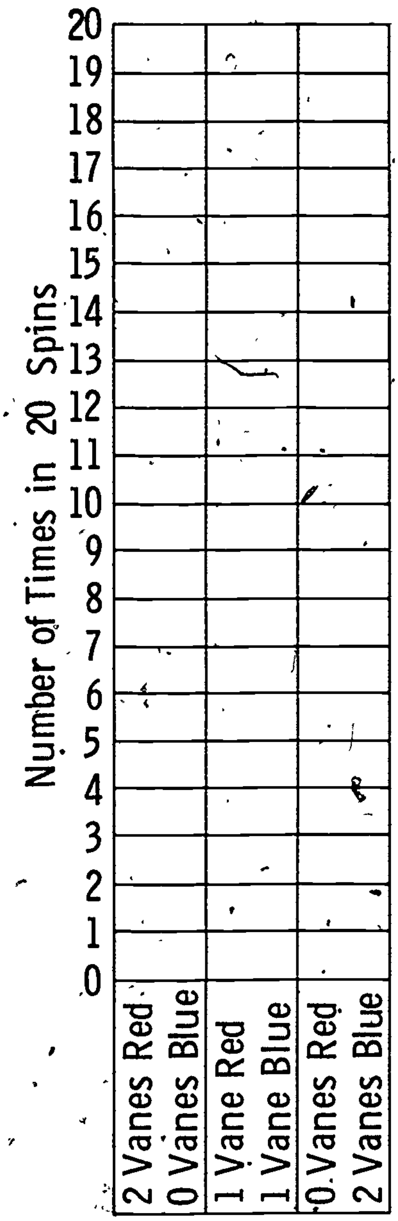
c. Were your results about what you thought they might be? ________

2. Make a windmill with two vanes. See the Appendix to learn how to make it. Color each vane red on one side and blue on the opposite side. Blow on the vanes and then place the windmill on a flat surface to see which vanes are up. Mark the chart to show how the vanes stop: both vanes red, one vane red and one vane blue, or both vanes blue. Show the totals on the graph.

CHART

Spin Number	2 Vanes Red 0 Vanes Blue	1 Vane Red 1 Vane Blue	0 Vanes Red 2 Vanes Blue
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
Totals			

BAR GRAPH



a. Would you expect the results of this experiment to be about the same as with the coin tossing experiment with two coins? _________

b. Were your results about what you expected them to be? _________

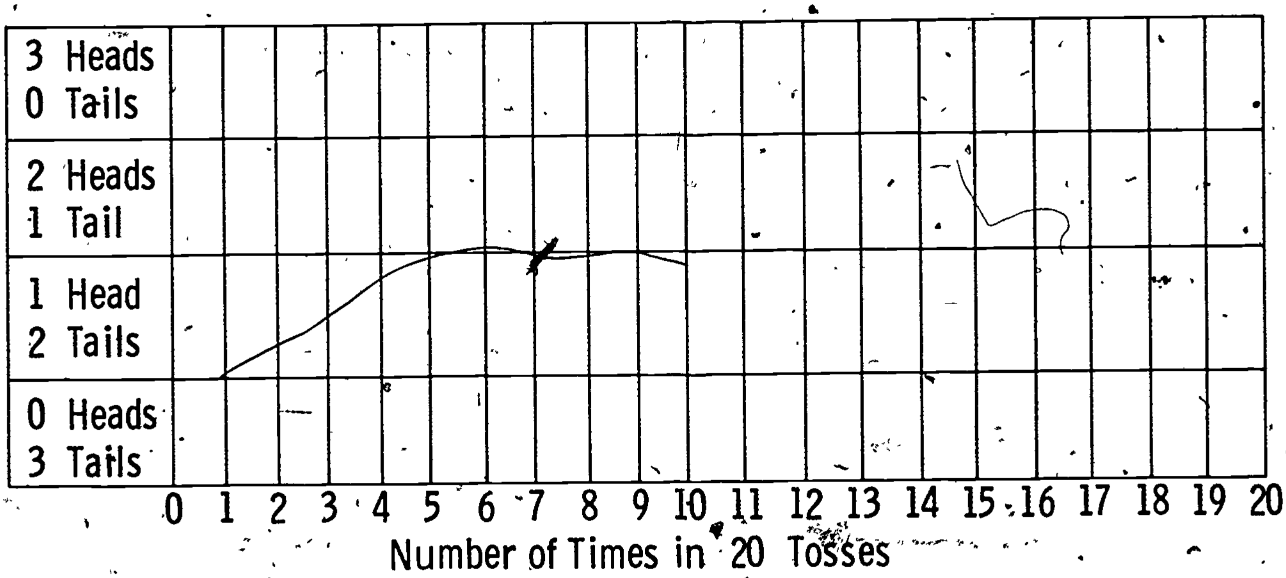
c. Circle your answer. About how many times would you expect an outcome of 1 vane red and 1 vane blue if you did this experiment 1000 times? 400 500 600

3. Toss three coins. Mark the chart and complete the graph from the totals. Compare your results with row 3 of the Pascal Triangle.

CHART

Toss Number	3 Heads 0 Tails	2 Heads 1 Tail	1 Head 2 Tails	0 Heads 3 Tails
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
Totals				

BAR GRAPH



a. From the Pascal Triangle, do you expect 2 heads, 1 tail, will occur about as often as 2 tails, 1 head? ________

b. Were your results about what you expected? ________

c. If you tossed 3 coins 600 times, about how many times would you expect all 3 coins to land with the heads up? ________

4. Add one more vane to the windmill you made so it will have three vanes. Color this third vane red on one side and blue on the opposite side. Blow on the vanes and then place the windmill on a flat surface to see which vanes are up. Mark the chart to show how the vanes stopped and complete the graph from the totals. Compare your results with the Pascal Triangle.

CHART

Spin Number	3 Vanes Red 0 Vanes Blue	2 Vanes Red 1 Vane Blue	1 Vane Red 2 Vanes Blue	0 Vanes Red 3 Vanes Blue
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
Totals				

BAR GRAPH

3 Vanes Red 0 Vanes Blue																				
2 Vanes Red 1 Vane Blue																				
1 Vane Red 2 Vanes Blue																				
0 Vanes Red 3 Vanes Blue																				

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Number of Times in 20 Spins

a. Were your results about what you thought they would be? ________

b. If you did this experiment 900 times, about how many times would you expect all 3 vanes to stop with red up?

100   200   300   400   500

You might like to try this experiment with a windmill of 4 or more vanes.

Conditional Probability - Lesson 11.

A coin is tossed 3 times. What is the probability that at least two are heads? Listing the outcomes in a table, we have:

3H	2H	1H	0H
H H H	H H T	H T T	T T T
	H T H	T H T	
	T H H	T T H	

There are a total of 8 outcomes. The outcomes with at least 2 heads are: ________, ________, ________, ________. There are ________ outcomes which have at least two heads. So

P(at least 2 heads) = ________.

Now suppose someone whispered to you, "The first toss was a head." What is the probability of 2 heads now? Look at the table. The outcomes with a head on the first toss are: ________, ________, ________, ________. How many outcomes have a head on the first toss? ________

Of these, how many have at least two heads? ________

P(at least 2 heads if you know the first toss is a head) = ________

This could be abbreviated to

P(at least 2H | H on first toss) = ________.

This is read, "The probability of at least two heads given a head on the first toss." In this example, the probability of 2 heads increased from ________ to ________ when we knew that the first toss was a head.

What is the probability of at least 2 heads if we know that the first toss shows a tail? A list of the outcomes with a tail on the first toss is:

THH, THT, TTH, TTT

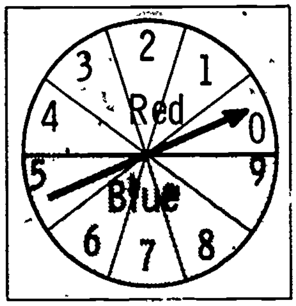
Of these, how many had at least 2 heads? ________

P(at least 2H | T on first toss) = ________.

Continue to use the table of outcomes for 3 coins to answer the following questions.

1. P(at least 1H | T on first toss) = ________.
2. P(3T) = ________.
3. P(3T | T on first toss) = ________.
4. P(at least 1H) = ________.
5. P(at least 2H | 2H on first two tosses) = ________.
6. P(at least 1H | T on second toss) = ________.
7. P(exactly 2 tails) = ________.
8. P(exactly 2T | T on second toss) = ________.
9. P(exactly 2H | H on first toss) = ________.
10. P(at least 1H | exactly 1 tail) = ________.

Exercises - Lesson 11.



Use this spinner. The dial is divided into 10 equal regions.

1. On one spin

a. P(Blue) = ______.
b. P(Red) = ______.
c. P(2) = ______.
d. P(Not 2) = ______.
e. P(7) = ______.
f. P(Not 7) = ______.
g. P(even) = ______.
h. P(odd) = ______.

2. The 10 possible outcomes are:

R0, R1, R2, R3, R4, B5, B6, B7, B8, B9

a. P(both red and even) = ______. To answer (a.), find those outcomes which are red (R0, R1, R2, R3, R4), and of those which are even (R0, R2, R4). Thus 3 of the 10 possible outcomes are both red and even.

b. P(both blue and even) = ______.

c. P(red | even) = ______. Remember that this means, "The probability of red given that it is even."

d. P(red | odd) = ______.

3. Use the same spinner. List the numbers which are multiples of 4.

______, ______, ______ (be sure that you list three)

a. P(multiple of 4) = ______.

b. P(both red and a multiple of 4) = ______.

c. P(either red or a multiple of 4) = ______.

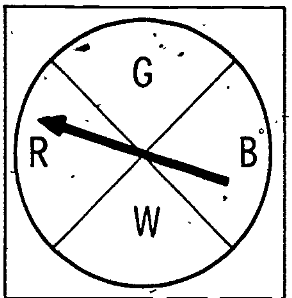
d. P(both even and a multiple of 4) = ______.

e. P(red|multiple of 4) = ______.   f. P(multiple of 4) = ______.

g. P(multiple of 4|red) = ______.   h. P(multiple of 4|blue) = ______.

i. Which has the greatest probability, (1), (2), or (3) ? ______
(1) P(even number)
(2) P(even number|red)
(3) P(even number|blue)

4. Complete this table of outcomes of two spins on this spinner. It has four equal regions.



Second Spin

		R	W	B	G
First Spin	R				
	W				
	B				
	G				

On two spins, the:

a. Total number of outcomes = ______.

b. P(RR) = ______.

c. P(at least one R) = ______.

d. P(both R on first and G on second) = ______.

e. P(R on first|G on second) = ______.

f. P(R on second|G on first) = ______.

g. P(at least one R | R on first) = ______.

h. P(2R | R on first) = ______.

i. P(at least one W | R on second) = ______.

j. Which has the smallest probability, (1), (2), or (3) ? ______

(1) P(WW | W on first)

(2) P(WW | W on second)

(3) P(WW)

Lesson 12 - Ghosts, Goblins and "Coins that Remember".

1. Do you believe that there are ghosts?
2. Do you believe that there are goblins?
3. Do you believe that a coin can remember?

You probably answered "No" to all these questions. Yet often we hear people talk as if they believe that coins can think and remember. They really do not understand the ideas in the law of large numbers. Most people call it the law of averages, and they often draw wrong conclusions from it.

You have heard people say:

A. "I have tossed an honest coin four times. Each time it came up heads. The law of averages says that the next toss will be tails."

Do you believe that the next toss is more likely to be tails than heads?

B. "My teacher uses a spinner to assign positions for the baseball game of "work up". I haven't been assigned as a pitcher yet this year. Therefore, by the law of averages, I'm sure to be assigned as pitcher today."

Do you think that this pupil is more likely than not to be chosen as a pitcher?

C. "I have been tossing an honest die. In 23 tosses, the face with one dot on it has never been up. By the law of averages, it is very likely that it will come up on the next toss."

Do you think the face with one dot is more likely than any other face?

Let's look at each of these examples of a misunderstanding of the "law of averages". Look back at statement A.

A. A coin does not have a memory. It cannot "remember" that it has been heads on the last four tosses. There is an equal chance for heads or for tails on the next toss.

We can use mathematics to prove that it is "unusual" to have a coin show four heads in four tosses. We can draw a tree diagram, make a table, or look at the fourth row in the Pascal Triangle. How many different outcomes are there when 4 coins are tossed or when one coin is tossed 4 times? How many of these outcomes consist of four heads? So, $P(4 \text{ heads}) = \frac{1}{16}$.

However, this also means that we expect 4 heads in a row, once every 16 times that we toss 4 coins. The coin while flying through the air on the fifth toss cannot say to itself, "Well, that's 4 heads in a row; I better twist a bit more and be sure to land tails or I'll mess up the 'law of averages'." The probability of heads on the next toss is of course $\frac{1}{2}$, the same as any other individual toss. Some people who misunderstand the law of averages think the probability of tails is much greater than $\frac{1}{2}$ after a coin has been heads several times in a row. Do you know people like this? They have forgotten that what happens on one toss has NOTHING to do with what will happen on the next toss.

Refer to statements B and C.

B. If there are 9 positions on the baseball field, then the probability of getting any one position is 1 out of 9. The fact that this pupil has not been a pitcher yet does not cause the spinner to favor one position for him over the others. He still has only 1 chance in 9 of being a pitcher today.

C. This person is overlooking one simple fact about a die-- it cannot think! It cannot say, "Let's see now. I know the probability of any face is $\frac{1}{6}$. My face with one dot on it has not been up in 23 tosses, so on the next toss I'll land so that the face with the one dot is on the top."

This person is thinking, "One face hasn't been up for a long time, so that face is more likely to come up than any of the others." This is a mistake about the law of averages that people often make. He doesn't really believe that dice can think, yet he is acting as if they could. Each face on a die has just as much chance to be up as any other face. If the face with one dot has not been up in 100 tosses, it still has no more chance than any other face to be up on the next toss. In fact, it has just one chance out of six.

Can you think of other correct or incorrect statements that you have heard about the law of averages? List some of them.

Why do so many people misunderstand the law of averages? It is too bad, but we all believe things and arrive at conclusions which just aren't true.

Which of these statements are false?

1. Lightning never strikes twice in the same place.
2. If you handle a frog, you'll get warts.
3. The end of the Panama Canal on the Pacific Ocean side is farther west than the end on the Atlantic Ocean side.
4. Horses are smarter than pigs.
5. George Washington threw a dollar across the Potomac River.
6. Columbus discovered America.

Many people believe some of these statements. Did you believe any of them? If you did, it isn't at all surprising. However, the six statements are all false. Most of us believe some things which really aren't true. Why is this so? There are many reasons. Among them are:

1. We are told or we have read something which is not true, but we remember it.
2. We did not understand what we were told or what we have read.
3. We reasoned incorrectly.
4. Our experience caused us to believe something that wasn't true.

5. We jumped to a conclusion without knowing enough facts.
6. We failed to check our belief against the facts.

This list could go on and on. There may be other reasons that you can think of. People arrive at false ideas about the law of averages for many of these same reasons. We can be fooled unless we are very careful. We might believe that an outcome, such as heads on a toss of a coin, is bound to happen if it hasn't happened for many tosses. It is easy to understand how our brain fools us in this case. It tells us that for a large number of tosses of a coin, heads will occur about half of the time -- and this is true. This is an example of the law of large numbers. Then we observe that heads hasn't occurred for several tosses and we make the mistake of thinking that heads must now start occurring more often to "catch up" with the number of tails. This is not true. Remember, a coin can't think. On each toss, there is just as much chance for heads to turn up as for tails.

By using mathematics, we can learn many interesting things. For example, from 15 children in your room, there are 6,435 different ways you can have 7 children on a committee. If you choose a 7-member committee from 30 students, you have a choice of 2,035,800 different committees. Another example is if a coin has been tossed and heads have occurred 7 out of 10 times, chances are less than $\frac{1}{2}$ that tails will "catch up" in 100 tosses. The mathematician can tell what will probably happen in cases such as this.

The next time that you hear some statement about the "law of averages", listen carefully. Try to find what the person believes and see if he is using it correctly.

Exercises - Lesson 12.

Mark these TRUE or FALSE.

_______ 1. You have been spinning a spinner that has a dial which is $\frac{1}{2}$ black and $\frac{1}{2}$ red. The last four spins have landed on black. It is more likely that the spinner will show red on the next spin than black.

_______ 2. The last five new pupils who came to our school were boys. The chances are better than equal that the next new pupil will be a girl.

_______ 3. The hospital reported that the last seven babies born there were girls. It is more likely that the next baby born there will be a boy than that it will be a girl.

_______ 4. The weatherman says that on the average it rains 4 days during the month of July. Today is the 27th of July and it has not rained all month. Therefore, it will rain tomorrow.

_______ 5. An auto dealer has 250 new cars and he knows that one out of every five new cars he sells is colored black. This week he has sold a blue, a white, a green, and a grey car. It is more likely than not that the next car he sells will be a black one.

Things To Do At Home - Lesson 12.

This experiment may help you to see why some people draw wrong conclusions from the law of averages.

Problem:

How many times, on the average, do you think that you would have to toss a coin before it comes up heads? Use the chart on the next page.

Procedure:

Toss a coin. Count the number of tosses until you get a head. For example: If you get a head on the first toss, write 1 in the column just to the right of "1st head". Start over. If you do not get a head until the fourth toss, write a 4 just to the right of "2nd head". Continue until you have completed column A. Repeat for columns B through E. Each column provides spaces to record the tosses for 10 heads.

After you have tossed 50 heads, add the number of tosses to get each group of ten heads. Divide each of these sums by 10 to find the average number of tosses needed to get one head.

Then, add the sums from the five columns and divide by 50. This gives the average number of tosses to get one head. Is this average closer to 2 than the average for each of the five columns? How many times did it take more than 5 tosses to get a head? How many times did it take 2 tosses to get a head? How many times did it take only 1 toss to get a head?

Number of Tosses to Get a Head

	A.	B	C	D	E
1st head					
2nd head					
3rd head					
4th head					
5th head					
6th head					
7th head					
8th head					
9th head					
10th head					
SUM					
SUM ÷ 10					

Perhaps now you can see why some people misunderstand the law of averages. With many tosses of a coin, we do see that about half of the tosses are heads. That is, it takes 2 tosses, <u>on the average</u>, to get heads. But, when you tossed a coin, you found that sometimes you tossed a head on only 1 toss. Other times, you had to toss the coin several times to get a head. This should help you understand that these people fail to see that the "average" is made from numbers that differ quite widely and that there is NOT a "law" which says that you <u>must</u> get a head after tossing 5 tails, for example.

APPENDIX - Probability Devices

This appendix suggests devices which you might like to make. You can think of games which can be played with them. Decide how to organize and record the data. These devices can help you in your study of probability.

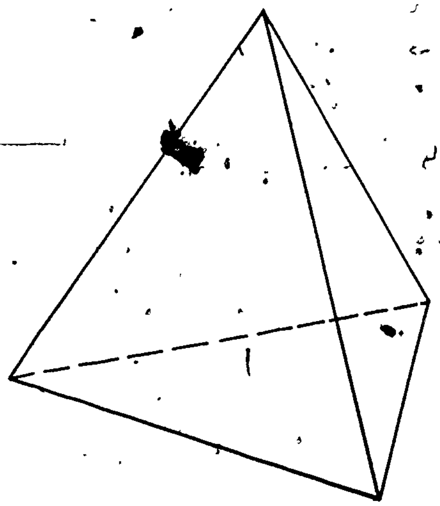
1. Regular Solids

We have used coins, dice, cubes, cards, and other materials to help us learn about probability, but there are many devices which are just as useful. The patterns on the next six pages are for the construction of regular solids which can be used in probability experiments. The patterns can be traced on tagboard and the solids will then be sturdy enough to toss or roll.

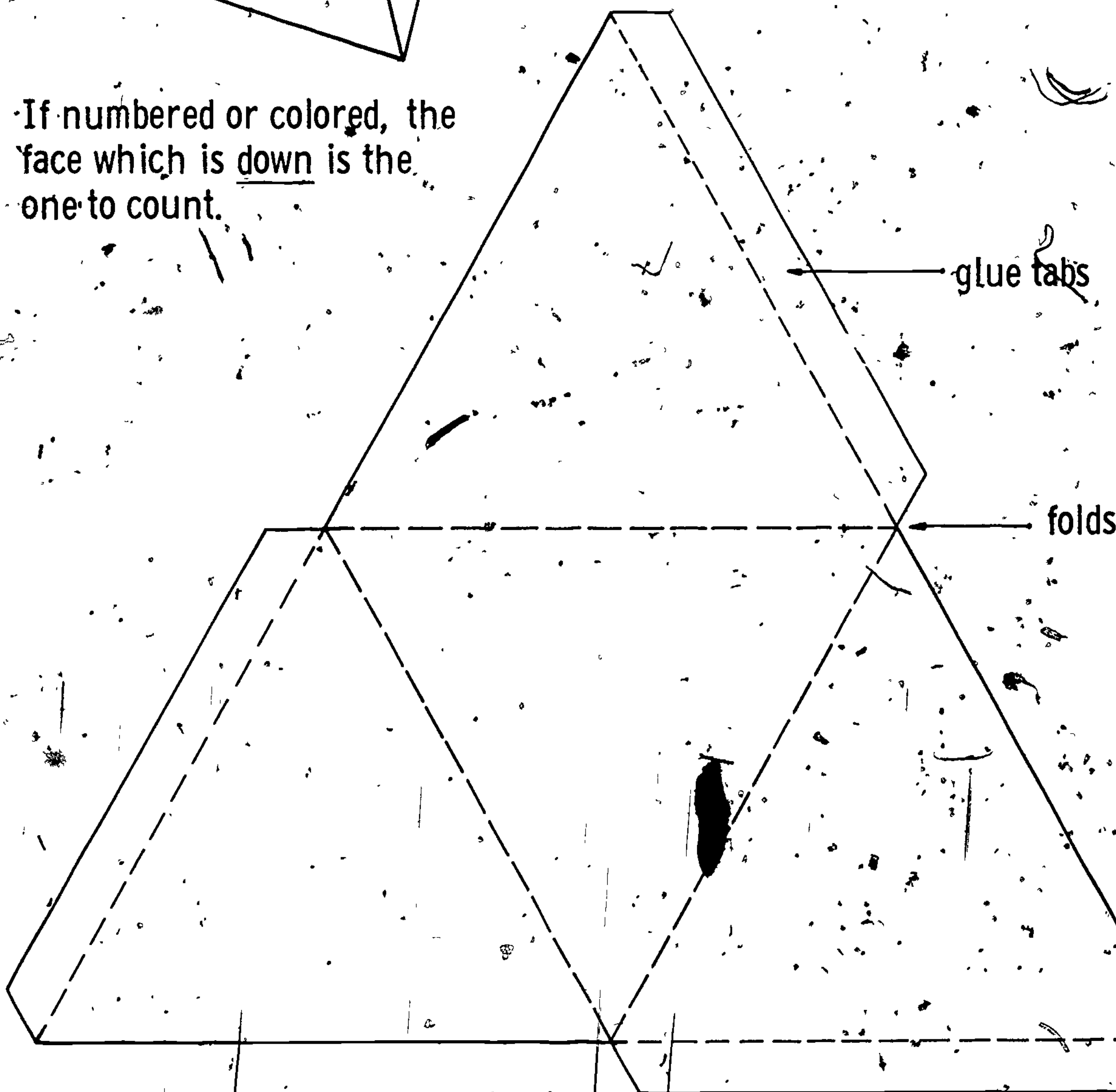
The tetrahedron, octahedron, and hexahedron (cube) are not difficult to construct. Just fold on the dotted lines and glue the tabs.

The dodecahedron is more difficult to construct. Make the first half of it by cutting to the dotted line at each arrow. Then fold on the dotted lines and glue the tabs. Complete by folding the second half and gluing it to the first half, tab by tab. Do not make both halves and then try to put them together.

The icosahedron appears difficult to construct, but it isn't. Cut to the dotted line at each arrow. Then fold and glue the tabs in order, one by one, and it will work out nicely.

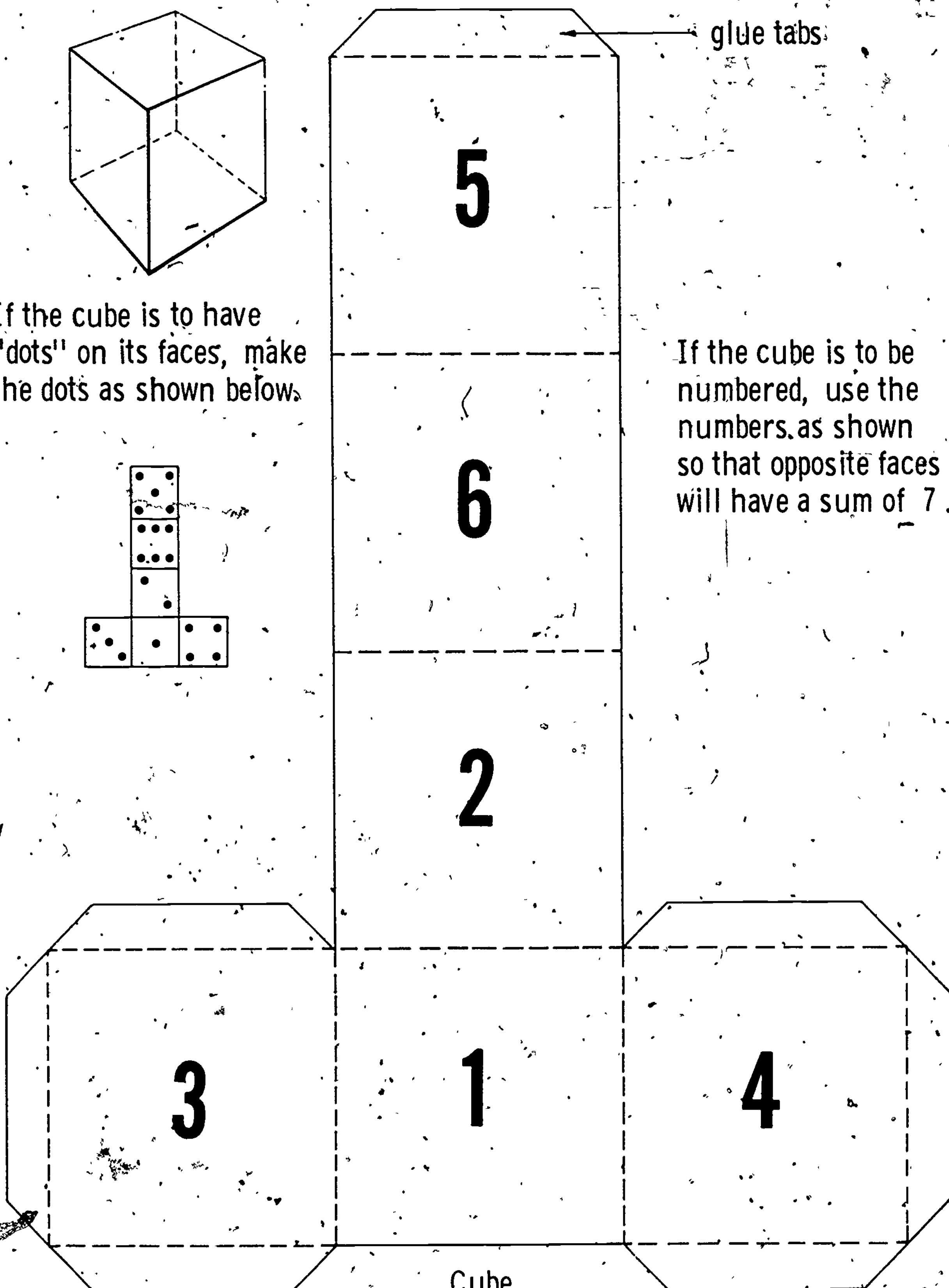


If numbered or colored, the face which is down is the one to count.



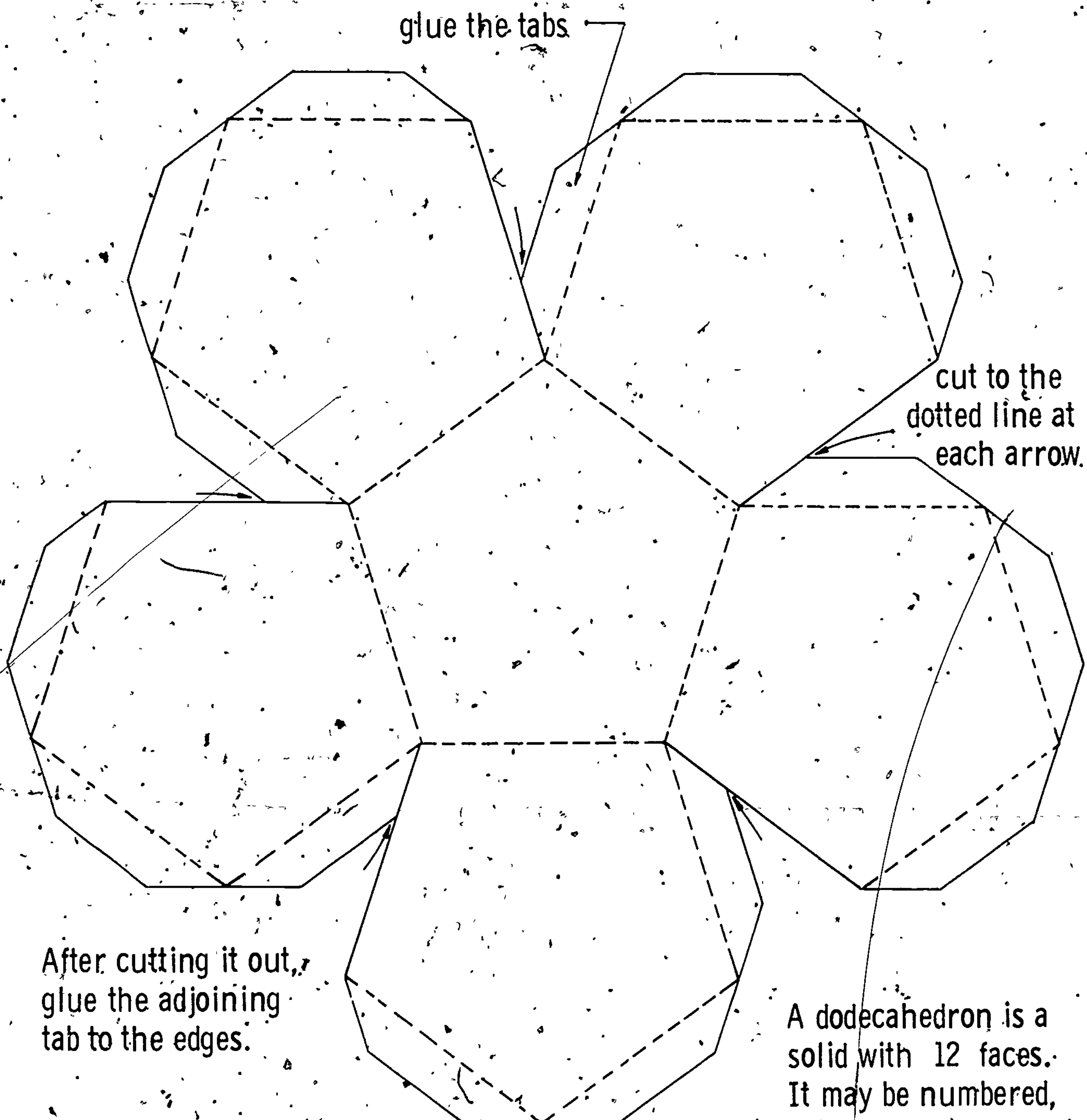
Tetrahedron

If the cube is to have "dots" on its faces, make the dots as shown below.



If the cube is to be numbered, use the numbers as shown so that opposite faces will have a sum of 7.

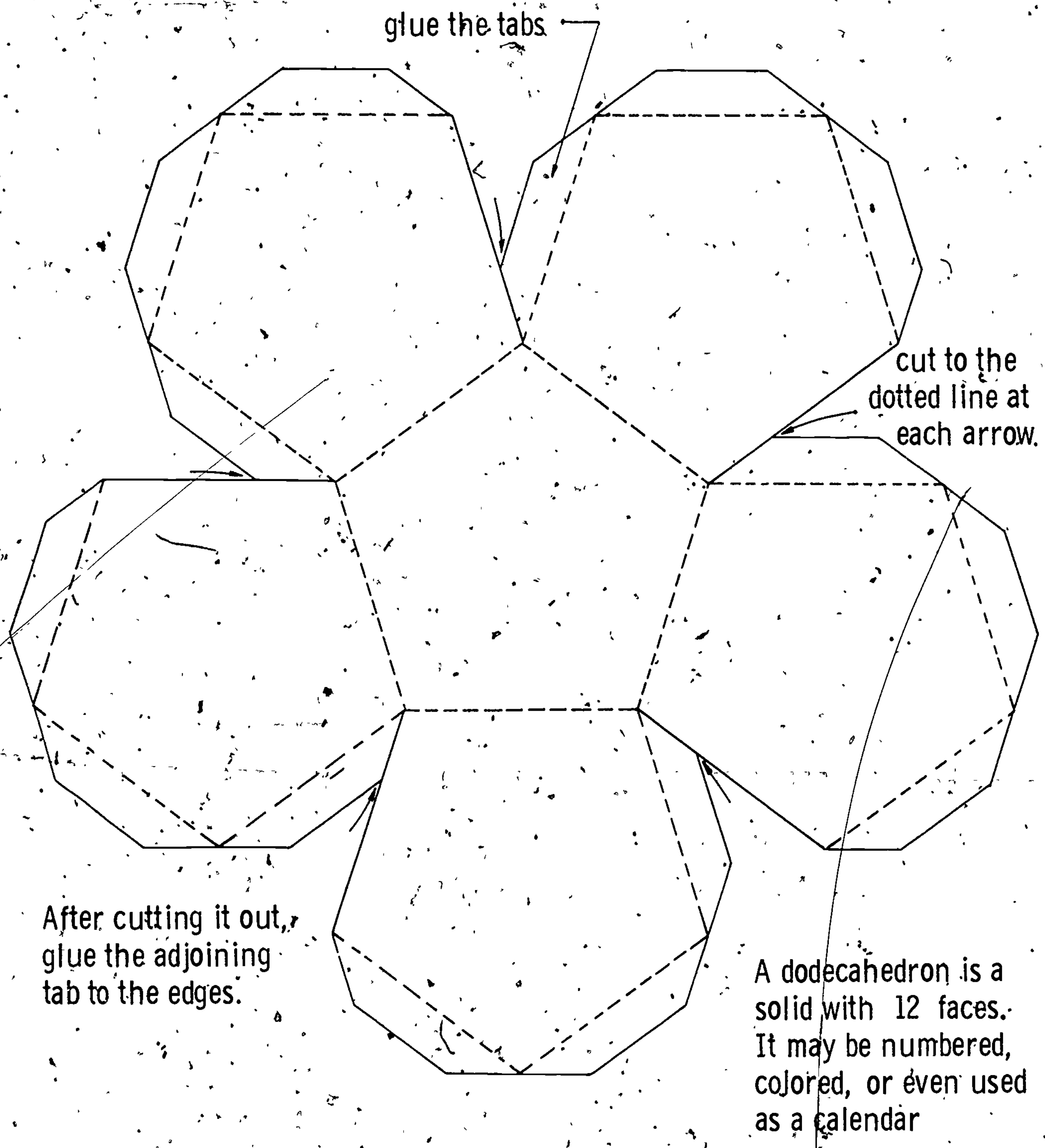
Cube
or
Hexahedron



After cutting it out, glue the adjoining tab to the edges.

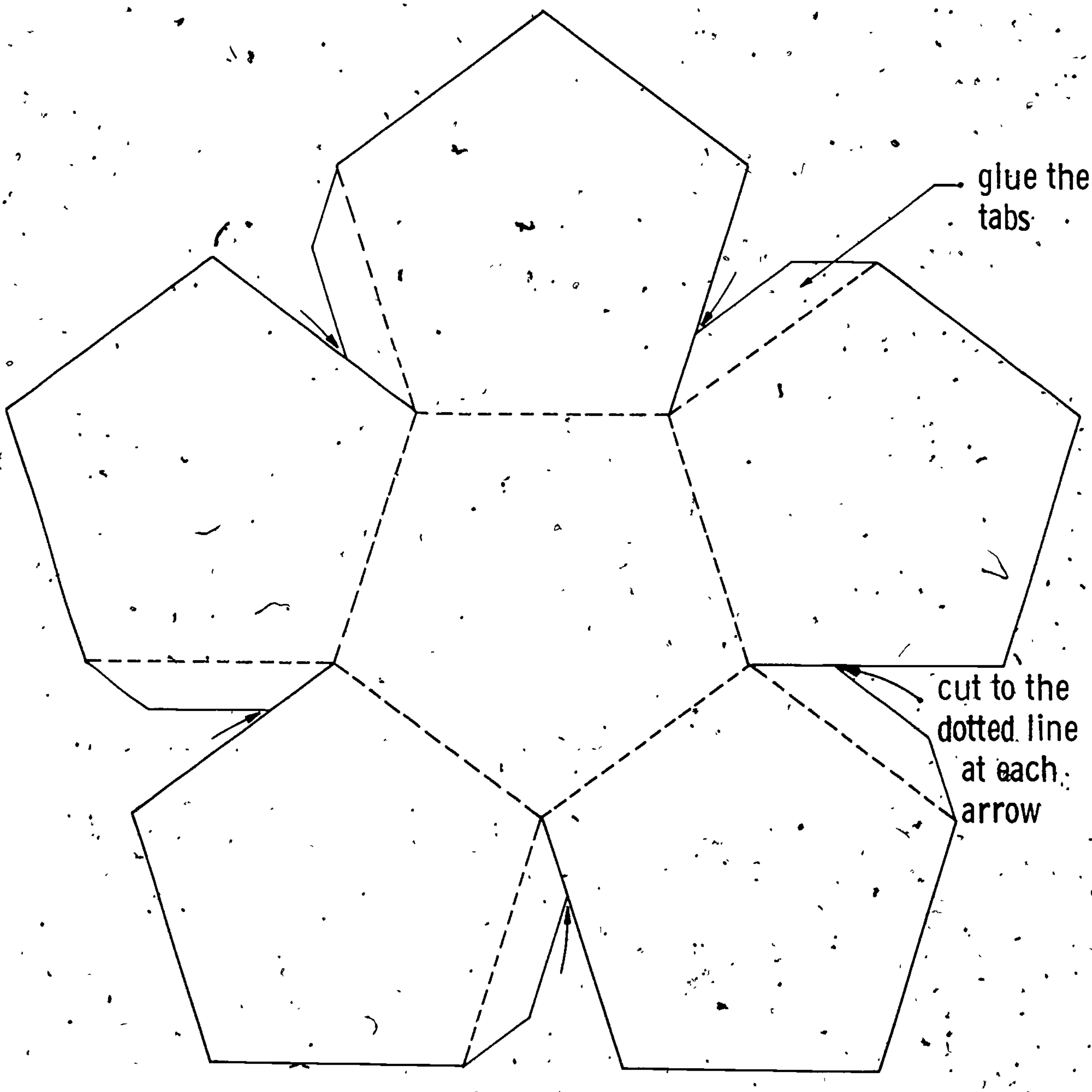
A dodecahedron is a solid with 12 faces. It may be numbered, colored, or even used as a calendar

First half of a Dodecahedron

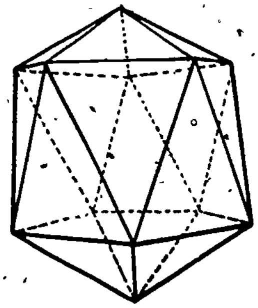


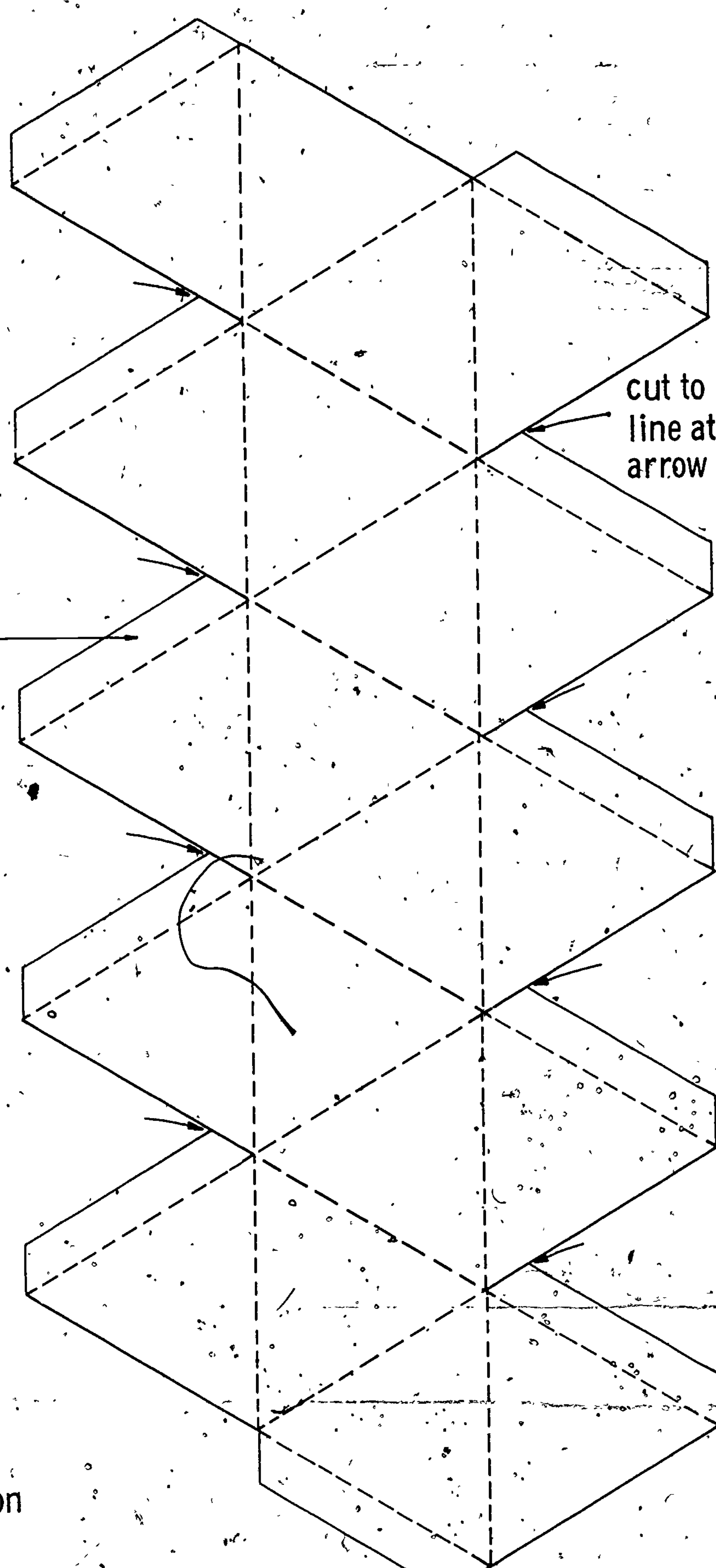
First half of a Dodecahedron

After cutting this out, do not glue adjoining tabs to the edge. Instead, glue one tab at a time to the first half of the dodecahedron.



Second half of a Dodecahedron



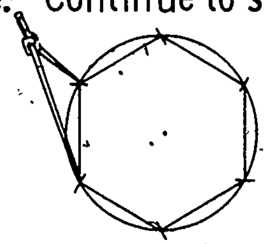


Icosahedron

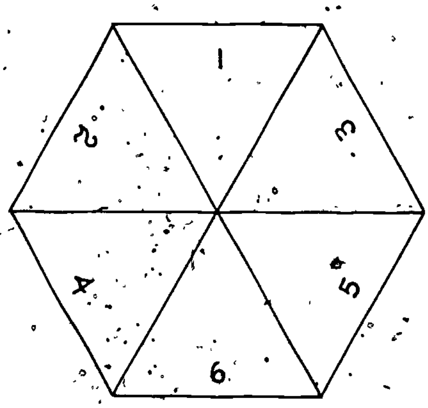
2. Tops

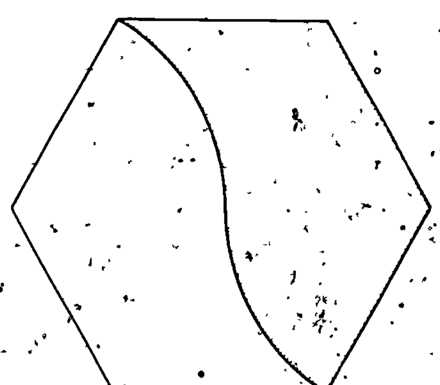
For a small top you need a piece of cardboard, a wood dowel or sucker stick, a straight edge, and a compass. Use a two-inch piece of dowel or sucker stick. Sharpen one end to a point with a pencil sharpener.

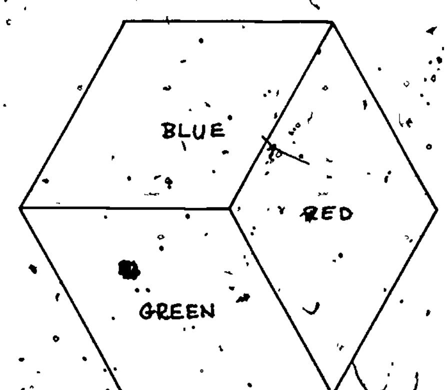
Use a compass and straight edge to make the dial from cardboard. Mark a point for the center and draw a circle with radius $1\frac{3}{4}$ inches. Mark any point on the circle and, with that point as center, strike an arc with the same radius to intersect the circle. Continue to strike arcs around the circle.

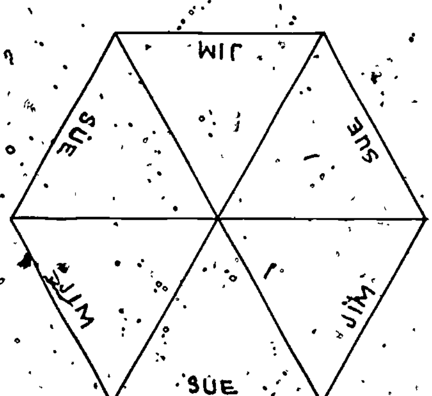


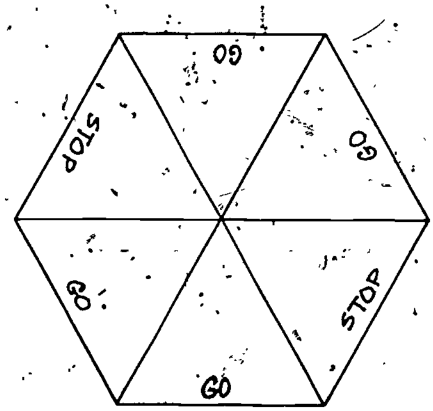
Connect points marked and cut off the outer part of the region to make a hexagonal region. Divide it as desired. Here are some suggestions:







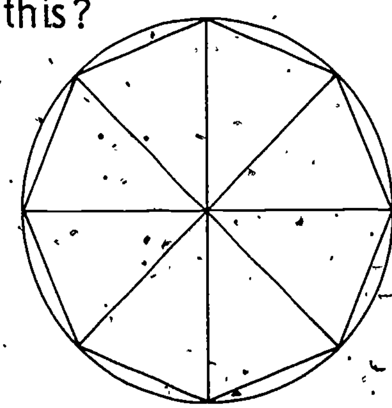


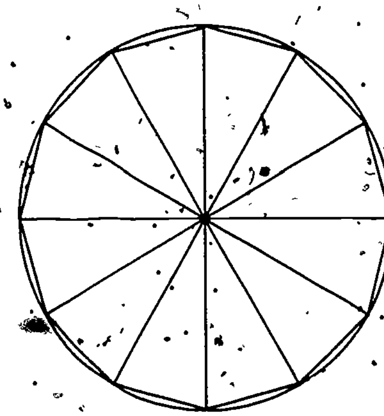


Poke the sharpened end of the stick through the center of the hexagon. (A drop of glue will help to keep it from wearing too large a hole, but experiment first to find the best balance for the top.) Spin the stick between thumb and forefinger. Spin the top on a flat surface. The edge that stops against the table is the one that is counted.

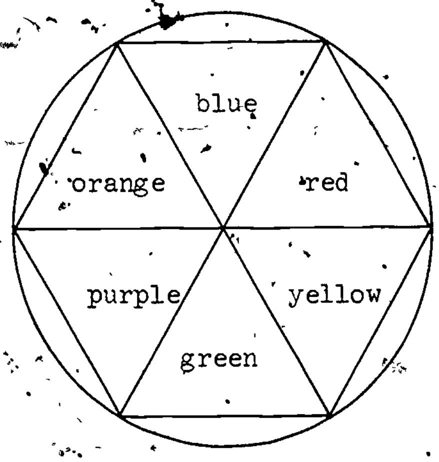
A large top may be made in the same way. An ordinary pencil makes an adequate stick. It is a good idea, however, to slit the cardboard along the division lines at the center before inserting the pencil. Use glue to fasten the cardboard to the pencil.

The top can also be made with eight edges or with twelve. Do you know how to do this?



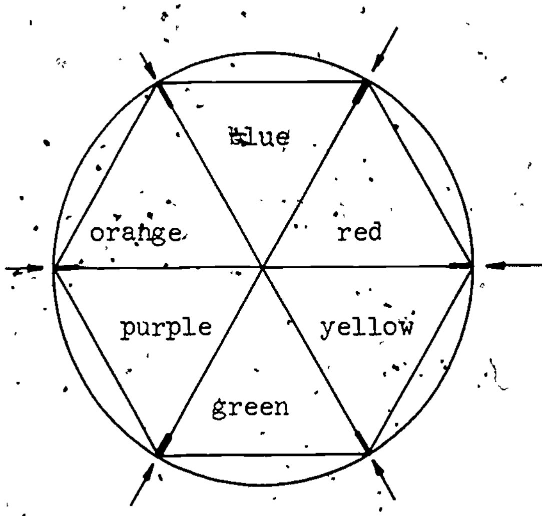


You can make a top with two or three dials. Use a sucker stick or thin dowel $3\frac{1}{2}$ inches long and sharpened at one end. Make a circular cardboard dial 6 inches in diameter. Inscribe a hexagon, but do not cut it out. Color as shown:



Insert the stick through the center and glue the dial to the stick.

When the glue is dry, place over the dial a "bearing" made of a piece of milk carton 1 inch square with a hole in the middle. Make another dial 6 inches in diameter, but if possible use cardboard that is slightly lighter in weight. Inscribe a hexagon, color, and cut as shown:

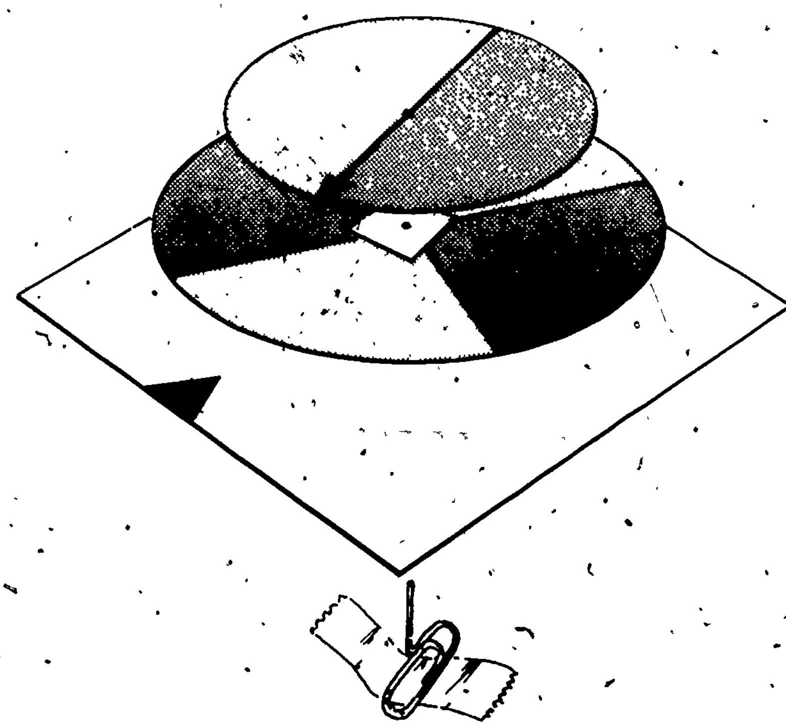


Make cuts $\frac{5}{8}$ inches long on the lines, as shown by arrows. Bend the cardboard up on the right of each cut to make a triangular "wind-catcher".

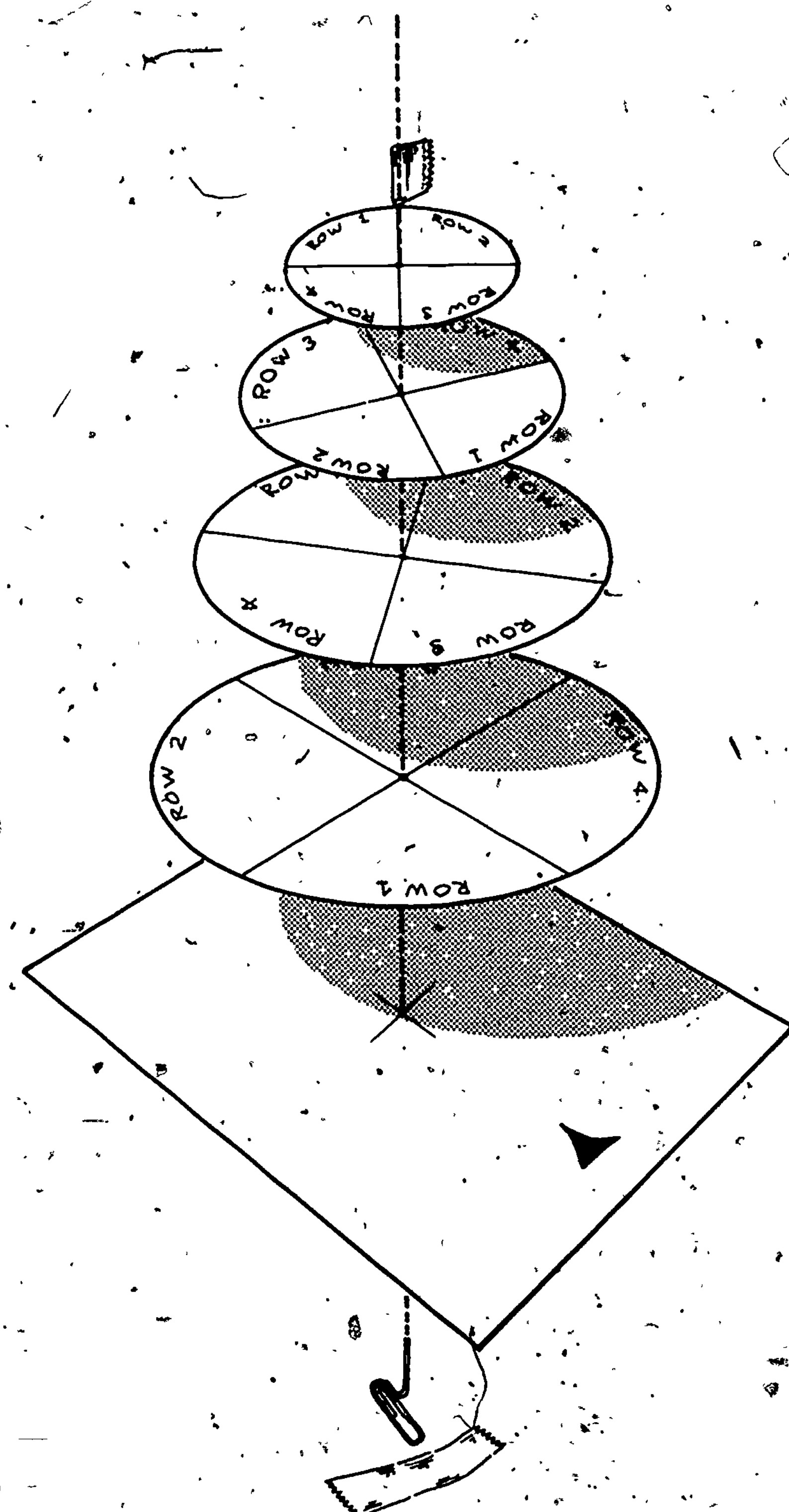
Place this dial on top of the bearing. Before spinning the top, line up the two dials so that the colors match. The bent-up cardboard will let you do this. Spin. Record the number of spins necessary before the colors again match. How many spins would you expect it to take?

3. Spinners

Spinners which come with most games have a fixed dial and an arrow which spins. You can make this type of spinner. Another spinner which is easily constructed is one in which the dial spins, as shown in the drawing. Construct the base out of heavy tag board. Use a heavy paper clip for the post and then various dials may be exchanged and used on the base.

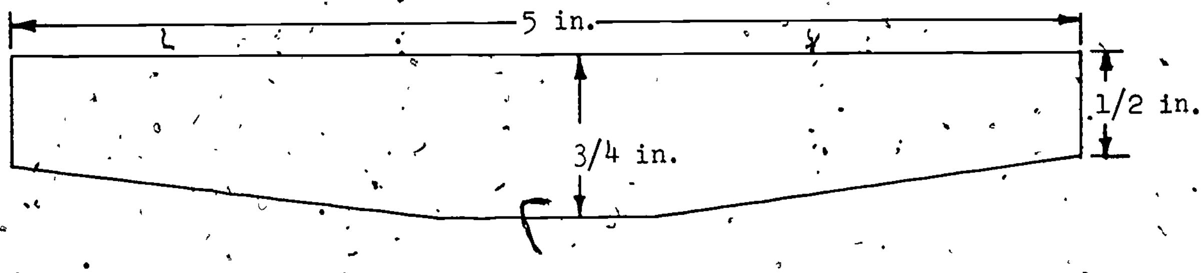


Another spinner with dials which go from large to small can illustrate how items can be put into an order. Each heavy cardboard dial is divided into a number of equal parts. The number of parts is the same as the number of items to be ordered. For example, to experiment to see how four rows in a classroom might be dismissed for lunch, each of four dials is divided into four equal parts. (A dial is made for each row.) Spin the dials and record how they line up with the arrow on the base. This device can be used to illustrate an orderly way of arriving at and listing the various arrangements.


ROW 1
ROW 2
ROW 3
ROW 4

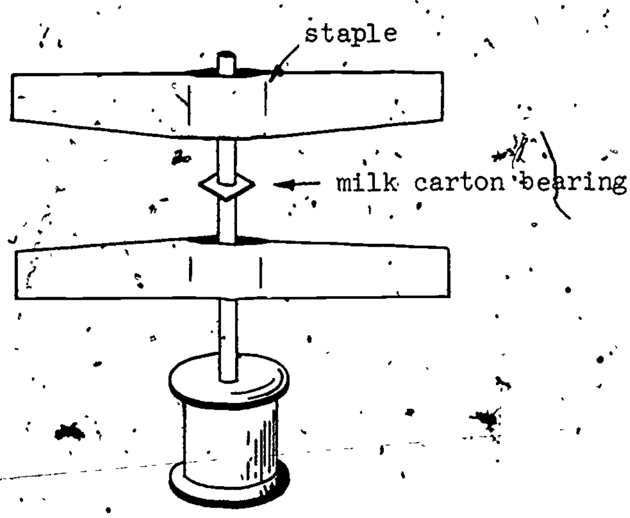
4. A Windmill

Cut four pieces of file card or tagboard to make two vanes.



A

Staple two pieces together in two places as shown in sketch B. Color one side of this vane red with a pencil or crayon. Insert a sucker stick in the middle between the two pieces. Use a punch to make a hole in a piece of milk carton one-half inch square for a "bearing". Staple the other two pieces of file card together to make a second vane. Mark one side as before, and insert the top of the stick between them. Paste a strip of gummed paper or tape over one end of a small spool. Put the end of the stick into the hole at the other end of the spool. Hold the spool and blow the file card vanes. They should turn quickly and independently.



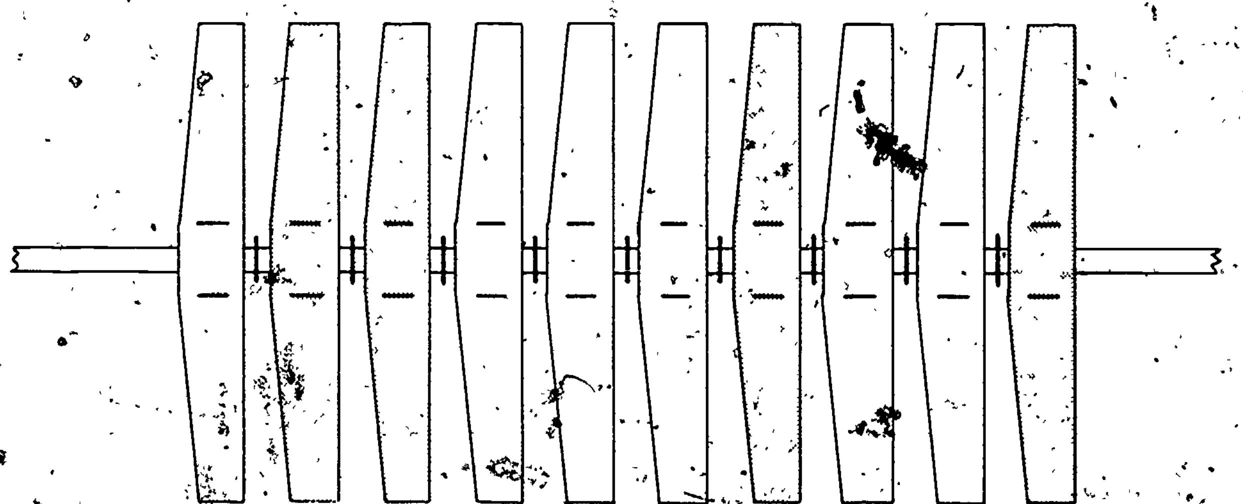
B

Practice blowing a few times. When vanes have stopped turning, lay them gently on a flat surface so that both vanes are flat on the surface. Then record whether both vanes are red, one is red and one is white, or both are white. After fifty trials, do the the results of the experiment fit with the expected results?

Whirly-bird

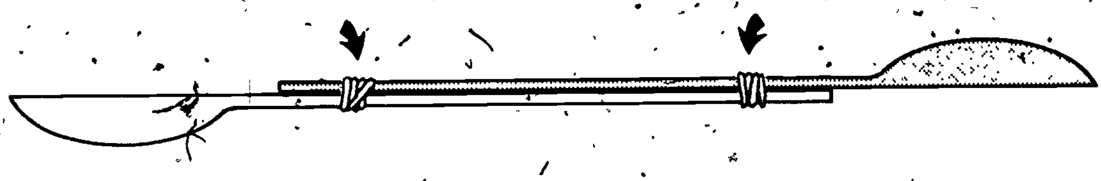
This is another type of windmill.

Use a section of quarter-inch dowel about two feet long. Wax it by rubbing it with a piece of crayon. Follow the directions as given for a windmill and make as many file-card vanes and milk-carton bearings as desired. Hold the dowel at both ends, like a harmonica, and blow. When vanes have stopped turning, lay the Whirly-bird gently on a flat surface so that all vanes are on this surface. One vane corresponds to one coin, so ten vanes can be used to duplicate an experiment of "tossing ten coins". Can you think of other ways to color the vanes so that other experiments can be done?



5. Spoon device

Use two plastic picnic spoons of different colors. Lay the handle of one on the handle of the other so that the bowls are at opposite ends and face opposite ways. Fasten with two rubber bands as shown at arrows.

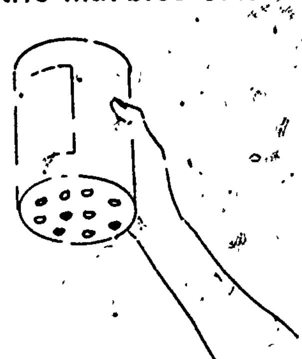


Roll the handles between your palms and drop on a table from a height of a foot or so. The spoon on top counts. Is it just as likely that one spoon will be up as the other?

6. Sampling Boxes (Urns)

Many probability experiments require a sampling to be taken in a random manner. This device uses various colored marbles.

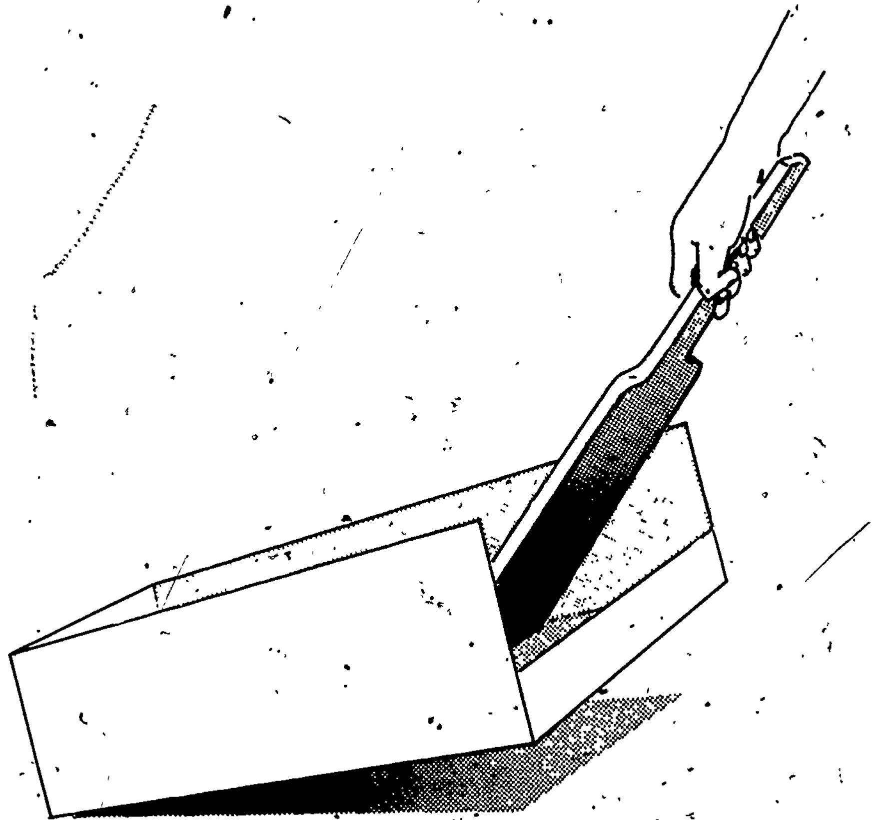
An oatmeal box serves well as the container (urn). Cut round holes, ten, for example, in the bottom of the box. These holes should have a diameter slightly smaller than the marbles so the marbles can be seen in them. Chinese Checker marbles serve well and come in packages of 6 colors, 10 of each.

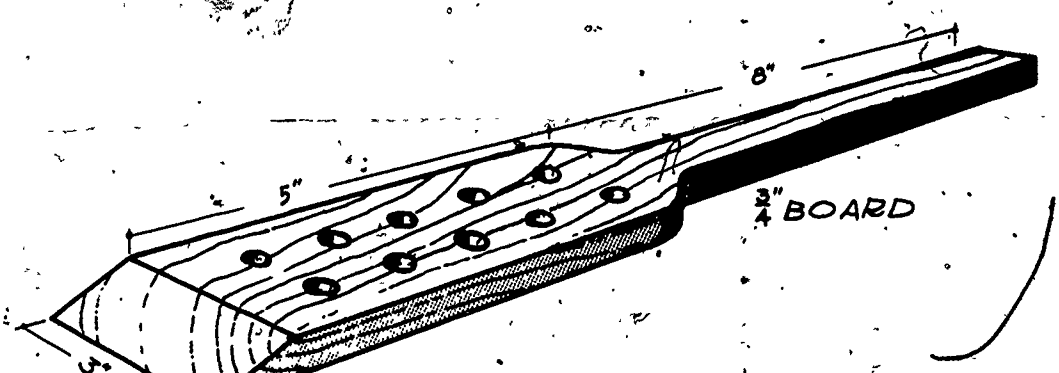


Example: Tell a friend you have 60 marbles in the container. Do not tell him how many there are of each color. Use, for example, 40 white and 20 black. Turn the urn up 10 times and record the number of black and white which show in each sampling. From the total, predict the probable number of white and black marbles in the urn.

A second sampling box can be made by using a box (rectangular or cylindrical) with a lid. It should be large enough to allow objects such as marbles or small balls to roll around. In a corner of a rectangular box or at the bottom edge of a cylindrical one, cut a hole just large enough to let one object go through easily. With the hole up, shake the box. Turn it over in your hand to let one object come out.

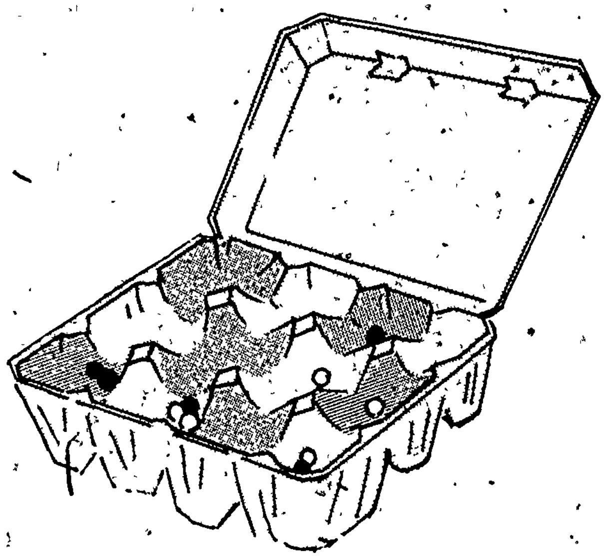
A third sampling device, a paddle, is made as shown in the drawing. The holes are slightly smaller in diameter than the marbles. Cut a box as shown (a shoe box works well). Place marbles in the box. Scoop with the paddle until all "holes" are filled. This gives a sample of the entire collection of marbles.




8"
5"
3"
$\frac{3}{4}$" BOARD

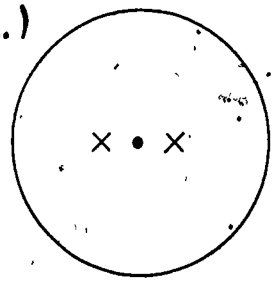
Egg-Carton Sampler

An egg carton and some marbles can be used for experiments. For example, color alternate pockets of the carton black. Place 5 black marbles and 5 white ones inside the carton. Close the lid, turn the carton upside down, and allow the marbles to roll around. Flip the carton upright and open the lid. Record the information you are interested in, for example, the number of black marbles in black pockets, etc.



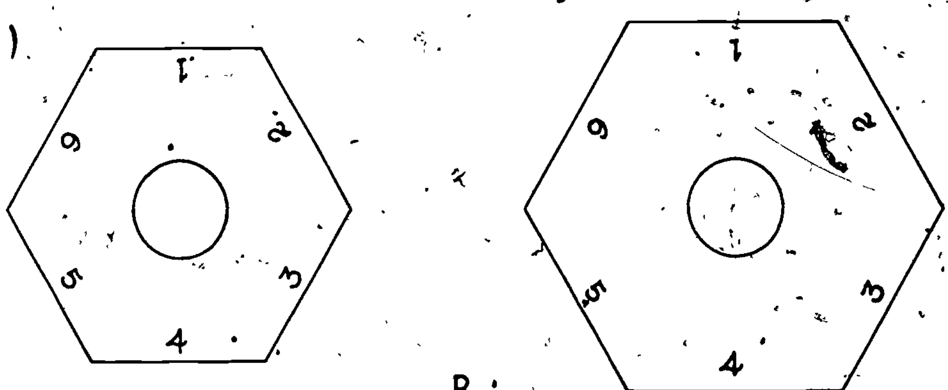
8. Hexawhirl

This gadget works like an old-fashioned button-on-a-string toy. Cut from cardboard two circles with radius $1\frac{3}{4}$ inches. In each, punch two holes just big enough to insert a piece of strong string. The holes should be punched on one of the diameters of the circle, each $\frac{1}{2}$ inch from the center. (See points marked X in Figure A.)



A

Make two or more cardboard hexagons using a $2\frac{1}{2}$ inch radius for the first, a 3 inch radius for the second, etc. Cut a hole with radius $1\frac{1}{4}$ inches in the middle of each hexagon and number the sides 1 through 6. (See Figure B.)



B

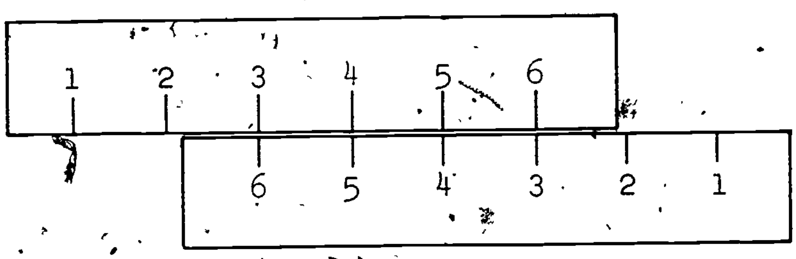
Use a strong string 50 inches long. Insert one end through a hole in one of the circles, through the smaller hexagon, then the larger hexagon (making sure the numbered sides face the same way), and then through the other circle. Leave a loop of string beyond the circle and insert the string through the other hole of the second circle, back through the larger hexagon, the smaller hexagon, and the first circle. Tie the ends of the string together to make a second loop. Adjust the cardboard pieces so that the loops on each

7. A Probability Scale

When tossing two dice or other regular solids, it helps to construct a table for counting how many ways a certain sum or product can be obtained. Two cubes, for example, give the following sums.

Sums	Possible Combinations						No. of Ways
2	(1,1)						1
3	(1,2),	(2,1)					2
4	(1,3),	(2,2),	(3,1)				3
5	(1,4),	(2,3),	(3,2),	(4,1)			4
6	(1,5),	(2,4),	(3,3),	(4,2),	(5,1)		5
7	(1,6),	(2,5),	(3,4),	(4,3),	(5,2),	(6,1)	6
8		(2,6),	(3,5),	(4,4),	(5,3),	(6,2)	5
9			(3,6),	(4,5),	(5,4),	(6,3)	4
10				(4,6),	(5,5),	(6,4)	3
11					(5,6),	(6,5)	2
12						(6,6)	1
						Total	36

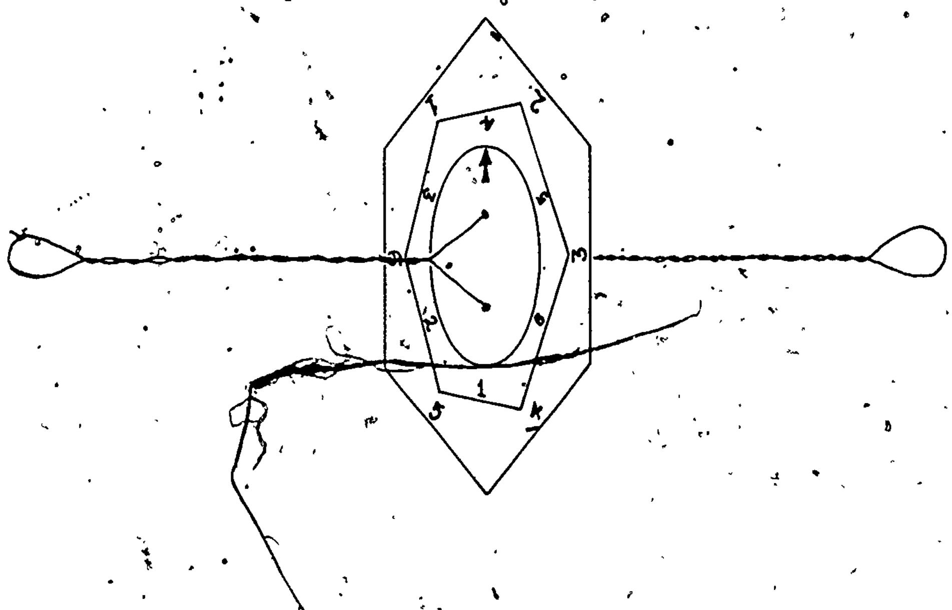
This information can be placed on two number lines on strips of cardboard as in the sketch.



These two strips can be placed in pockets of a larger piece of cardboard so that by sliding the scales along, one quickly sees the number of possible combinations. This figure shows the 4 possible combinations of a sum of 9.

Scales for other solids may also be constructed.

side of the cardboard are the same length. There should be just enough space between the circular pieces for the hexagons to turn on the string. Fasten the circles to the string with a drop of glue. Make an arrow on the circle next to the smaller hexagon. (See Figure C.)



C

To operate the hexawhirl, hold a loop in each hand and swing the cardboard pieces around and around (25 or more times) until the loops of string are twisted. Pull the loops until the twisting is undone, release to allow string to twist the other way, and pull again. With practice you can make the hexagons spin rapidly between the circles. Stop, and see which sides of the hexagons are in line with the arrow (2 on the smaller, 3 on the larger, for instance). Experiment to find out if the results are similar to those obtained by throwing two dice.