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ABSTRACT

This document is designed for grade eight to enrich and supplement the usual courses of instruction. Mathematics and Living Things (MALT) utilizes exercises in biological science to derive data through which mathematical concepts and principles may be introduced and expanded. Chapters included are: (1) Leaves and Natural Variation: Measurement of Length, Metric System, Ratio, and Graphing; (2) Natural Variation - "US": Addition of Measurement and Greatest Possible Error; (3) Leaf Surface Area and Water Loss: Area, Significant Numbers, Scientific Notation; (4) Muscle Fatigue: Percent; Mean, Median and Mode; Informal Extrapolation, Histogram; (5) Yeast Metabolism: Linear Graphing, Curve Fitting, Extrapolation and Interpolation, Volume of a Cylinder; (6) Growth of Mold: Rectangular Coordinates, Estimation of Area; (7) Size of Cells and Metabolism: Surface Area and Volume; and (8) Giant Trees: Formula Construction for Volume of Cylinder and Cone, Indirect Measurement. Equipment and materials needed are specified in the Commentary. (RH)

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MATHEMATICS AND LIVING THINGS

Student Text

(Revised Edition)

SMSG

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TO THE STUDENTS

Do you like mathematics? Does mathematics like you? If your answer is not "yes" to both of these questions then you are one of the people the authors had in mind when they wrote this book. Not that we were particularly looking for people who don't like math but we knew that there are some students who may not enjoy mathematics as much as they might.

A different approach to mathematics has been used in this book -- so much different that at times in the course you may wonder if this is really mathematics. There will be times when you will be involved in some very interesting activities until it may seem that you are being trained to be a scientist. And maybe you are, in some ways. After all, scientists couldn't operate without information and someone has to gather that information. This is one of the interesting things of science. A scientist often wants such a particular kind of information that he must gather it himself.

Can you imagine a scientist able to make much use of information without using some mathematics? He is continually making measurements of length, depth, time, mass or volume. All of these measurements involve mathematics.

In this course you will learn mathematics. And we hope you will really learn it and find the learning rather pleasant and not boring.

When do you think a person should learn some mathematics? Should it be when he reaches a certain age, a certain grade, a certain week in the school year? One opinion on this question is that he should learn a mathematical concept when he needs to use it. Some experts on learning say that if a person is taught something when he needs to know it and can make immediate use of it he is most apt to learn and remember.

This book was written with you in mind. It includes some interesting science activities. You will have the opportunity of gathering scientific information, recording and analyzing measurements and you will need some mathematics. The mathematics that you will need in connection with a given activity will be presented at the time you need it. The authors hope this will prove to be an interesting and effective way to learn mathematics.

THE AUTHORS

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Chapter 1

LEAVES AND NATURAL VARIATION: MEASUREMENT OF LENGTH, METRIC SYSTEM, RATIO, AND GRAPHING

1.1 Introduction to Chapter 1

Have you ever lost something that you just "had" to find? If so, just how did you go about looking for it? After the first few frantic moments, you probably began to organize some kind of plan to begin your search. You might have asked yourself questions like "Where could I have had it last?", "Where was I before I missed it?", "Where would be the best place to start looking?" Even though you may not have been able to answer these questions, you were trying to develop a logical plan to begin your search. Your plan may or may not have enabled you to find the lost article. A scientist uses this kind of reasoning to organize a plan or method to find the answers to many questions. His plan may or may not lead him to an answer, in which case he would choose another plan. The important thing is that a carefully thought out method of approach will usually give the best results. The process used in developing the method, although it does not have to be limited to a science laboratory, is used there.

In the following exercises with living things you are going to collect and organize data which will be used to lead you to important concepts of mathematics and to show how these concepts can aid in explaining the world about us.

1.2 Measure and Units

As you might guess, the scientist will find it very helpful to use numbers if he is going to use mathematics to guide his thought. Answering the question "How much?" often leads to deeper understanding than answering the question "Of what sort?"

Where, then, does the scientist get the numbers he needs? The answer, of course, is that he makes measurements. You already know something about measurement from having done a lot of it. When you weigh yourself, split a candy bar with a friend, work at a track meet, and in many other everyday instances, you use measurement.

1.3 Metric System: Length

Universally, scientists use a system of measurement called the metric system. The meter is the basic unit of length in the metric system. The meter is divided into smaller units. The centimeter is $\frac{1}{100}$ of a meter. You will use a commercially made ruler with one edge labelled in inches and the other edge labelled in centimeters. The picture in Fig. 1-3a will help you see what a unit of one centimeter looks like as compared to the common inch unit.

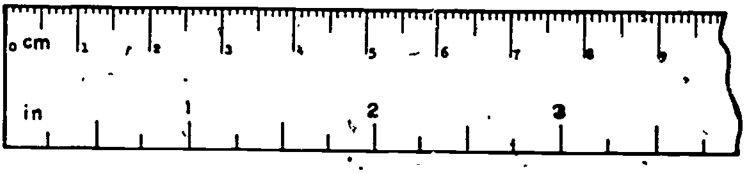


Fig. 1-3a

$$1 \text{ centimeter} = \frac{1}{100} \text{ of a meter}$$

Notice that $\frac{1}{10} \times \frac{1}{100}$ of a meter is $\frac{1}{1000}$ of a meter. The prefix "milli" means one-thousandth. Fig. 1-3b will help you see what a unit of 1 millimeter looks like. It is obtained by dividing the centimeter into 10 equal parts. Each of these parts is called a millimeter.

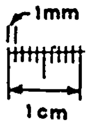


Fig. 1-3b

$$1 \text{ millimeter} = \frac{1}{10} \text{ of a centimeter} = \frac{1}{1000} \text{ of a meter}$$

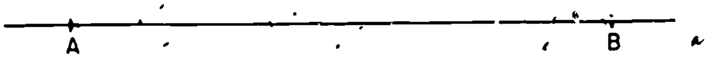
A length 1000 times the size of the millimeter is the meter.

Biologists (scientists who work with living things) are interested in many different aspects of a particular problem. In their research on a problem, biologists have discovered a basic characteristic of all living things. You have probably already heard that no two living things are alike. The following exercise will provide you with a few minutes practice with the ruler.

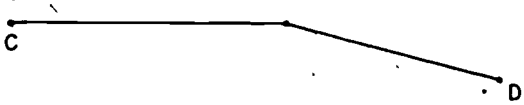
Exercise 1-3

1. Measure the length of this segment to the nearest cm.

2. Measure to the nearest cm the distance from A to B.



3. Measure to the nearest cm the shortest distance between C and D.



4. Measure to the nearest cm the distance between points x and y.

5. Measure to the nearest millimeter each of the figures in Problems 1 - 4.

6. How do the cm and mm measurements of Problem 1 compare in exactness? Which is closer to the exact measurement?

7. How do the cm and the mm measurements of Problem 3 compare in exactness? Which answer is closest to the "actual" measure? Is your answer exact? Is any measure exact?

8. Measure the length of 1 inch in millimeters. Approximately how many millimeters are there in one inch?

1.4 Activity--Measurement of Leaves

Biologists also find it necessary in many of their investigations to measure things. Is there a relationship between length and width of a leaf? How much does a tree grow in a given period of time? Families like to keep track of their children's growth through the years. "Knee high by the fourth of July" is a common expression for a good crop of corn in the Midwest if conditions of temperature and moisture have been satisfactory. Does corn in Iowa really grow as "high as an elephant's eye"? Would either of these two expressions be considered either mathematical or scientific? Why?

There are a number of things in nature that we might measure: root lengths on sprouting beans, the length of rat tails, the lengths of jumping legs on grasshoppers or frogs. Certainly you can think of many more. Leaves, however, if chosen carefully for shape, are one of the easiest and most convenient to use. Let's see if we can answer the first question in the above paragraph. "Is there a relationship between length and width of a leaf?"

[From a single tree or bush in your own yard (unless you receive special permission to get them elsewhere) select 15 to 20 leaves.]

Do not choose leaves of the same approximate size, but rather, choose a variety of sizes from the largest to the smallest and from the same plant. Be careful not to damage the plant.

The shape of the leaves is important, because some have such uneven outlines that they are difficult to measure. Elm, birch, or willow are good choices, while maple, oak, and some ivys are poor. (See Fig. 1-4a) Your teacher may suggest other local trees.

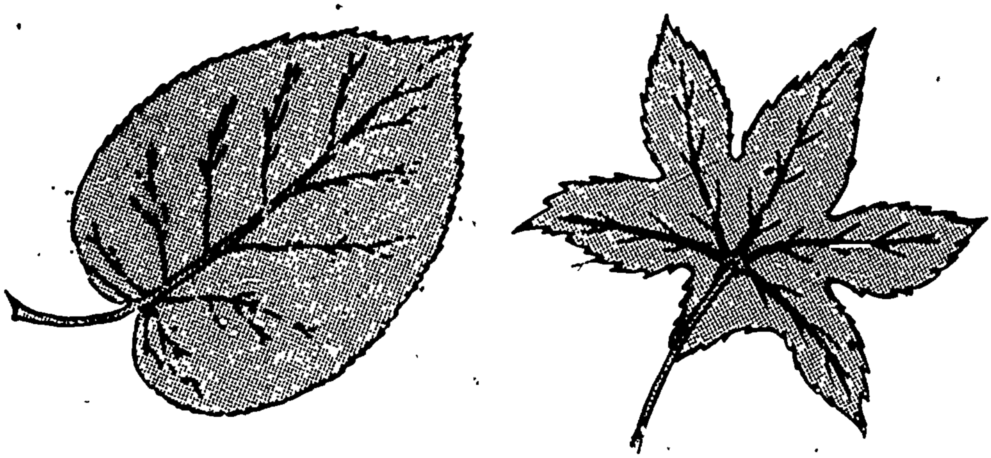


Fig. 1-4a

The leaves should be picked and brought to class on the day they are to be measured so they will be fresh. You will need a ruler calibrated with the metric scale of measure. On your ruler, each mark represents a millimeter (mm). Each number represents a centimeter (cm).

Laying the leaf on the desk, place your ruler along the longest part of the blade of the leaf. Do not include the petiole (pet' i · ol). (See Fig. 1-4b.)

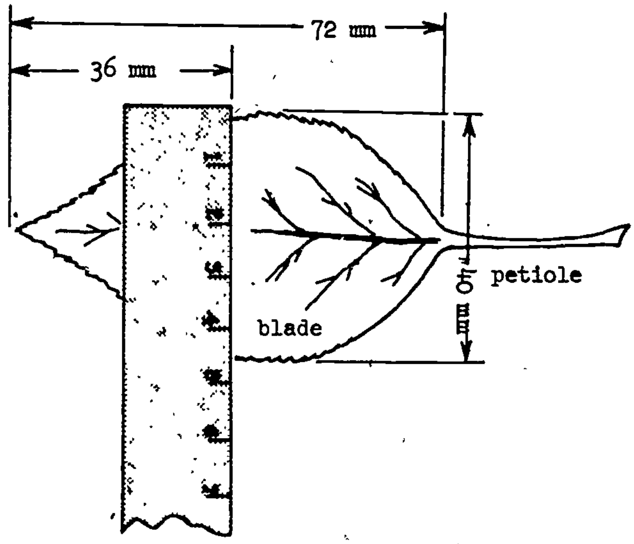


Fig. 1-4b

Record the length of the leaf blade to the nearest mm in the third column of Table 1-4. (In the next chapter you will learn about "precision of measurement" and "greatest possible error".)

Now measure the width at the mid-point of the length measure. One method of finding the mid-point is as follows:

> Fold your leaf in half -- point of blade matching base of blade -- and crease. Measure the width of the leaf along this crease. (See Fig. 1-4b.) Record this width in the <u>second column</u> of the table below.

Repeat this process for each of your leaves. Measure rapidly but carefully.

<u>NOTE</u>

Measure as carefully as you can and record the results neatly. There is no room in a scientist's procedure for "sloppy" work. Also, it is important that your data be recorded in a place where they can be kept for later use. Scientists sometimes have need to refer to data recorded years earlier. Your teacher will help you decide how this should be done, since you, too, will be needing your data for <u>later</u> mathematical problems.

<u>Notice that length is to be recorded in the third column and width in the second.</u>

Table 1-4

Column 1	Column 2	Column 3	Column 4
Leaf Number	Width of Blade in mm	Length of Blade in mm	
Example	40	72	
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

1.5 What You Really Did When You Measured Your Leaves

In the first place, measurement is a process for assigning numbers of units to objects or events. Different kinds of measurement require different processes. For instance, to find the length of a football field the measuring process might involve the use of a yardstick to mark off 100 yards and we could then say the length was 100 yards. The number assigned would be 100 in this case. Such a number is called a measure of the length of the field and in this case we could call it the yard measure.

Another possible method (but slower) would be to use a foot "stick" (ruler) and measure off 300 feet to find the length. We would then find that the length was 300 feet. In this case the measure (foot measure) is 300.

Either way we do it, the length stays the same, although the measures differ. Another way to say the same thing is: 100 yards and 300 feet are each names for the same length. In a similar way 1 hour and 60 minutes are names for the same time interval; Mach 1 and the speed of sound are names for the same speed; one centimeter equals ten millimeters.

Notice that a measurement cannot be given by a number alone. To say that the speed of a crawling ant is 200 just doesn't say anything at all, although 200 millimeters per minute might make some kind of sense. To say that the length of the football field was "100" leaves us in the dark unless extra information is added.

What information besides the number do we need to describe a measurement? We must know which unit was used. In the case of the football field, we were comparing the length of the field to the length of a yardstick. We call a yard the unit of length (distance). Thus, we describe the measurement 100 yards completely by giving the measure, 100 and the unit, yard.

To sum up our discussion we can say that scientists are concerned with measurements. Measurement is the process of comparing some object or event with some unit which we have chosen. The comparison process, which needs to be carefully described, always gives a number known as a measure. This number, or measure, will depend on the choice of unit.

Exercise 1-5
Measures and Units

1. Pick out the measures and the units in each of the following measurements:
   (A) 21 feet
   (B) 7 yards
   (C) 10 hands
   (D) 15 centimeters
2. If a fish tank were filled by emptying a gallon bucket into it 8 times, what would be the volume of the tank? What is the measure? What is the unit?
3. The fish tank of Problem 2 is filled by using a quart container rather than a gallon container.
   (A) What is the measure?
   (B) What is the unit?
   (C) Does the volume remain the same?
   (D) Is the measurement the same?

4. Change each of the following measurements to an equal measurement having a different measure and unit.

   (A) 2 hours

   (B) 2 minutes

   (C) 3 square feet

   (D) 1 square yard

5. Change the following leaf measurements to centimeters.

   (A) 25 mm

   (B) 70 mm

   (C) 4 mm

   (D) 7 centimeters, 3 millimeters

6. Change the following measures to mm.

   (A) 4.5 cm

   (B) 3.2 cm

   (C) .9 cm

   (D) 3 centimeters, 7 millimeters

1.6 Measurement of Length: General

You may think it odd to start learning about measurement in general by concentrating on length. But the truth is that length is one of the most fundamental of all physical properties and the idea of length constantly arises not only in science, but as we shall see later, also in mathematics.

To fix our thoughts, let us talk about measuring the length of our desk. We identify the length of the desk with the length of the line segment, or the distance, between two tiny marks which we have made on the two sides of the desk. The problem now is to express in mathematical language the distance betwee- these two marks.

"Easy!" you will say: "It's 3 feet 4 inches," (or whatever it may be). But you can then be challenged by this sort of question: Do you mean that the length is exactly 3 feet 4 inches? Or do you mean it is somewhere near 3 feet 4 inches?

Now it is most unlikely that you would assert that it is exactly 3 feet 4 inches, for the simple reason that you could not be exactly sure. (Remember that "exact" means exact! Not even a millionth of an inch is allowed for uncertainty either way.)

So what you mean is that it is nearly 3 feet 4 inches. But how near? Within an inch either way, or a foot, or a quarter of an inch? And it is only when questions like this are seriously considered that it begins to be necessary to be much more careful about stating what a measurement is.

Can you see now why you were instructed to measure your leaves to the nearest millimeter?

1.7 Measurement of Length: Ideas of Accuracy

You were asked to measure your leaves in millimeters. The problem of measuring a length cannot be separated from the purpose of the measurement and the use to which it is put. Let us give some examples.

The distance of the sun from the earth is often given as 93,000,000 miles. This is generally accepted as an average distance. A more precise measure is much more likely to be some figure like 93,271,412 miles! In many cases we do not consider the 271,412 miles because those six extra figures may not be needed. But it would be absurd to say that we will not bother about 271,412 miles if we were going to drive from New York to San Francisco. For this distance we must measure more accurately and we might give it as 3050 miles, even though the distance between one particular point and the Empire State Building, N.Y., might be more exactly something like 3057 miles 426 yards. So when considering driving this distance we do not bother about the 7 miles 426 yards.

And so we might argue further by showing that "the 7 miles 426 yards" would matter a very great deal if we want to measure the length of the big field nearby. Measure your height. It is not usual to quote the number of yards; you would want to know feet and inches.

All this can be summarized as follows: Whenever a length measurement is made, it is made for some definite purpose, and this purpose will usually suggest the most suitable unit of measurement.

The unit is related to the greatest length one is prepared to accept as being "close enough". Examine the following table and you will see what we mean by unit:

Measurement	A possible unit
Distance from Sun to Earth	a million miles
Distance from S.F to N.Y.	10 miles
Length of a field	1 yard
Length of a table	1 inch
Your height	$\frac{1}{4}$ inch

This table indicates that we may be satisfied if we know the number of yards in the length of a field. For most purposes we do not want to know the number of inches from the sun to the earth, or the number of yards from San Francisco to New York. Our choice of a unit for any measurement is related to the accuracy we require. When we say that the unit for the length of the field is 1 yard, we mean that we are prepared to accept 73 yards as a description of its length, even though a more accurate length may be 73 yards and 1 foot. Even though the difference in quoting 73 yards as the length is 1 foot, we are prepared to ignore it. Thus, another way of arriving at a unit for a measurement is to consider what the maximum acceptable error is. (Error as used in measurement is any amount up to one half the smallest unit used in the measurement.)

Why did you use millimeters to measure leaves? Could we have measured to the nearest yard? Would this type of measure furnish us with any usable information?

Exercise 1-7

Ideas of Accuracy

1. Suggest commonly used units for the following measurements.
   (A) the altitude of an airplane
   (B) the length of a car
   (C) the depth of the ocean
   (D) the length of an arm
   (E) the height of a tree
   (F) the height of a truck
2. What unit of measure would be commonly used when measuring the width of a window for drape rods?
3. What unit of measure would be commonly used when measuring the width of a window to fit glass?

4. What statement concerning the choice of units of measurements is demonstrated by your answers to the questions above?

1.8 Natural Variation

Now that you have a neat table of data, take a good look at it. Are all the leaves the same length? Are they all the same width? How many short leaves and long leaves did you have compared to the "in between sizes"? If you chose your leaves as instructed, you should have a few short, a few long, but most somewhere in between. This is one example of what is called natural variation. All living things show this characteristic. Take a look at your classmates. Are they all the same height? Ask yourself the same questions about them as are asked above about leaves. What about weight? In most classes one would find that there would be a few small, a few large, but most somewhere in between. Natural variation is one of the most baisc concepts in biology. Size, as illustrated by leaves, or your classmates, is only one small portion of an extremely complex pattern. If you raise pigeons you know there is a large variety of color pattern. Dogs have been domesticated and carefully bred until there are now dozens of varieties recognized. Chicken breeders have worked to produce especially desirable characteristics such as high egg yield or lots of white meat. These are examples of ways man has capitalized on a phenomenon of nature; that is, individuals differ from one to another within species.

There are many kinds of variation in living things. We have touched upon size, color, and egg production in chickens. By the way, it is also interesting to note that just because a thing is "big" does not necessarily mean it is "best". Some of the tiniest living things -- such as bacteria -- are immensely successful in nature. There are more different kinds of insects, as well as more insects (in total numbers, with the exception of bacteria) than all other living things put together. They, too, show tremendous variety, and some remarkable adaptations (structures or behavior patterns which give them the ability to survive in their own special "nook" in the world). There are many fascinating books in libraries about insects and their ways which you might enjoy reading.

In nature the more favorable variations help individuals to survive. These individuals, with their own peculiar variations, thus live longer, produce mo young, and so contribute these special adaptations to the future of the species.

Volumes have been written about natural variation and many years of study have been, and are still being, devoted to it. Natural variation is considered one of the most important concepts in the over-all understanding of the living world and man's place in it.

Will man become bald? Will all people eventually not have any tonsils, appendix, or gall bladder? Will the average size of man continue to increase?

Living things also show natural variation internally. Some are very active, move quickly, use lots of energy; others are quiet, sluggish, and seem to require much less energy for their activities.

One of the interesting things to note in a study of living things is that, in spite of variation, there is often at the same time a remarkable similarity of relationships.

For instance, in growth studies it is useful to show the relationship between the sizes of various parts of the body at birth and in adulthood. The head of a new-born baby is much larger by comparison to the rest of the body than the head of an adult.

Another relationship is that of food intake and resulting energy. In general, the more food an organism uses, the more energy is available for activities.

These relationships are expressed in ratios. To determine whether there is any relationship between length and width of the same kind of leaf, you will need to know something about ratios.

1.9 Ratio

Combinations of two numbers may be compared in several ways; e.g., 5 and 2 compared with 15 and 6.

If we add	then $5 + 2 = 7$	and $15 + 6 = 21$
If we subtract	then $5 - 2 = 3$	and $15 - 6 = 9$
If we multiply	then $5 \times 2 = 10$	and $15 \times 6 = 90$
If we divide	then $5 \div 2 = \frac{5}{2} = \underline{2\frac{1}{2}}$	and $15 \div 6 = \frac{15}{6} = \underline{2\frac{1}{2}}$

Exercise 1-9

COMPARING PAIRS OF NUMBERS						
	The Numbers		Add	Subtract (2nd from 1st)	Multiply	Divide (1st by 2nd) Record answer in simplified form
1.	5	2				
2.	90	36				
3.	65	26				
4.	102	34				
5.	27	9				
6.	39	13				
7.	51	17				
8.	72	16				
9.	162	36				
10.	9	2				

In comparing the numbers 9 and 2, we sometimes say that the ratio of these numbers is 9 to 2 or $\frac{9}{2}$. Nine is four and one-half times the number 2.

<u>Definition</u>: The <u>ratio</u> of a number c to a number d, (when d is not equal to 0) is $\frac{c}{d}$.

In the class there are 36 pupils of whom 16 are girls. The ratio of the number of girls in the class to the number of class members is $\frac{16}{36}$ or $\frac{4}{9}$.

Thus, $\frac{4}{9}$ of the class members are girls: $36 \times (\frac{4}{9}) = 16$. The ratio of the number of pupils in the class to the number of girls in the class is $\frac{36}{16}$ or $\frac{9}{4}$. There are two and one-fourth times as many pupils as girls in the class. Notice that $\frac{9}{4} = 2 + \frac{1}{4}$.

The ratio of two numbers is a comparison <u>in a definite order</u>. When we compare the numbers 9 and 4, the ratio of $\frac{9}{4}$ is not the same as $\frac{4}{9}$. Notice the definition; the ratio of a number c to a number d is $\frac{c}{d}$. How would you express the ratio of the number d to the number c ?

1.10 Ratio: Length to Width

In our experiment we want to record a ratio; the measure of the length to the measure of the width as determined for each leaf. Which measure, length or width, will be the numerator of the fraction?

The numbers you recorded for the measure of length and width are in millimeters. You know that any measure is approximate and we usually name the measure according to the smallest unit we have on our scale. If a leaf measured approximately 100 mm in length and 67 mm in width, the ratio of the length to width would be $\frac{100}{67}$. Another leaf might have a measured length of 131 mm and a measured width of 88 mm. The second ratio would be $\frac{131}{88}$. You can tell at a glance whether or not $\frac{4}{9}$ is greater than $\frac{5}{9}$ because of the common denominator. Can you tell which of our ratios is the greater, $\frac{100}{67}$ or $\frac{131}{88}$?

If we change the form of a rational number (a ratio) to another form (a decimal) we can compare the two numbers because they will have a common unit. What is the common unit (or units)? $100 \div 67 = 1.492537313+$ and $131 \div 88 = 1.488636363+$.

Now which ratio do you know is the larger? In the measurement of leaves is this difference significant?

Since our ratios are numbers found by measurement to the nearest mm, (which is, remember, approximate), answers in decimal form carried out to 10 places would not give realistic pictures of the accuracy of our measure. <u>We will divide the numerator by the denominator, carry the division out two places beyond the decimal point and round off to the nearest tenth.</u>

Thus, $\frac{100}{67} = 1.49$ and rounds off to 1.5 while $\frac{131}{88} = 1.48$ and would also round off to 1.5.

> For each leaf you have measured, determine the ratio of the measure of the length to the measure of the width. Record the decimal form of each ratio in the fourth column of your table. Label this column Ratio (Decimal form).

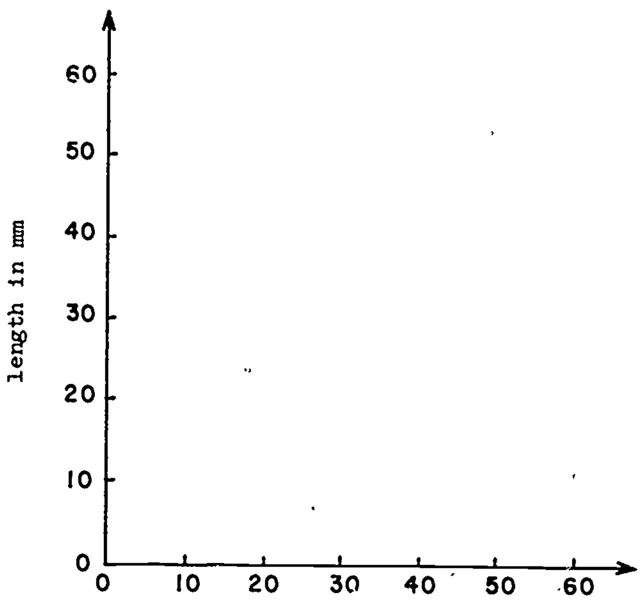
1.11 Average Ratio

Do you find several of your leaves have the same length-width ratio? Just as you average all of your grades to "guesstimate" your final grade on the report card, average the ratios by finding the sum of all of the ratios, and divide by the number of ratios (20 if you had 20 leaves). Record the average ratio immediately below the bottom of column "4". What is your average ratio? Are some ratios quite different from the average?

1.12 Graph

To better understand and use our data gathered on the measures of lengths and widths of leaves we will construct a graph by plotting the data upon it. ("Data" is the plural of the Latin word datum which means "fact".)

Our graph will use one horizontal ray and one vertical ray with a common endpoint. The horizontal ray will represent the measure of the width in mm and the vertical ray will represent the measure of the length in mm. (See Fig. 1-12a.)



Width of leaves in mm

Fig. 1-12a

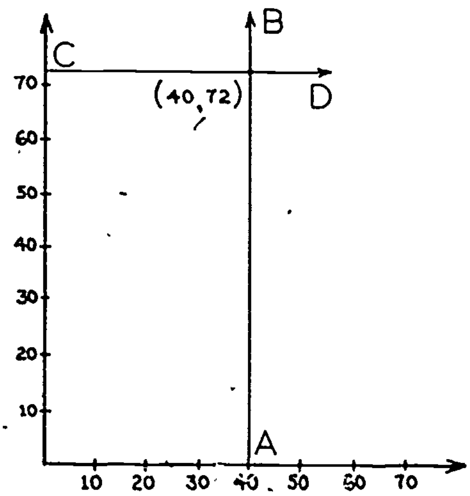
[Construct a graph similar to Fig. 1-12a.]

It would be preferable to use commercial graph paper, but you may construct your own if you are neat and careful in your preparation of the squares.

To enter the data upon the graph, think of the information on each leaf as an ordered pair.

You will enter the data as ordered pairs in the same order as you recorded it on your table. Thus, measures of width, as the first number and measures of length as the second number is your ordered pair. In the example on the first line of the table the ordered pair would be (40,72). By a mutual understanding among mathematicians, the first number in an ordered pair gives the horizontal distance from the origin (point of intersection of the reference lines -- horizontal ray and vertical ray) and the second number shows the vertical distance from the origin.

If your first leaf had a width measure of 40 mm and a length measure of 72 mm, the ordered pair is (40,72). All leaves with a width measure of 40 would be represented as line AB (see Fig. 1-12b), but our representative leaf has also a length measure of 72 mm. All leaves with a length measure of 72 mm would be represented as line CD. The only point which represents both 40 width and 72 length is the point of intersection of AB and CD. This point is labelled (40,72).



width in mm

Fig. 1-12b

> Plot the ordered pairs for each of your leaves measured. Without drawing the perpendiculars from the reference lines, mark each point and label it (width,length) as in Fig. 1-12b.

It is often difficult to determine from a table the significance of the data. Data placed on a graph, such as the one that you have just completed, enables a biologist to readily see any existing relationships.

Did you find relationship between length and width of your leaves? Notice that we are emphasizing your leaves. Here is another vital point in your understanding of the processes of science. Data pertaining to a specific thing does not necessarily mean that all things will follow a similar pattern. Your data can only be interpreted in terms of the leaves you chose for measurement. If you have time, you might try some other kinds of leaves.

The ratio of length to width can be expressed as ratio = $\frac{\text{length}}{\text{width}}$ or $r = \frac{\ell}{w}$. If we know any two of these three terms (ℓ, w or r), we can find the other term. For example, if the ratio of length to width is 1.4 ($\frac{\ell}{w} = 1.4$), then $\ell = 1.4 \times w$. Therefore, a leaf with a width of 25 mm measure would have a length of $1.4 \times 25 = 35$ mm.

Exercise 1-12

Problems Using Ratio

1. A tree in front of a fraternity house on the campus of Stanford University gave an average $\frac{\ell}{w}$ ratio of leaves as 1.4 for the leaves measured. If a leaf had a width of 50 mm what would be its length? ________

2. From the same tree as in Problem 1
   (A) If a leaf were 65 mm in width, what would be the length? ________
   (B) A width of 30 mm would give a length of ________

3. (A) If the length of a leaf from the same tree was 47 mm, what would be the expected width? ________
   (B) From a length of 65 mm, one could expect a width of ___?___. ________

4. (A) Describe a leaf which had a $\frac{\ell}{w}$ ratio of .65 . ________
   (B) Draw what you think the shape of such a leaf would be.

(Watch out for the next two problems.)

5. If a leaf had a $\frac{w}{\ell}$ ratio of .65 and a width of 35 mm, what would be the expected length? ________

6. With a $\frac{w}{\ell}$ ratio of .72 and a length of 67 mm, what would be the expected width? ________

7. (A) Given a $\frac{l}{w}$ ratio of $\frac{1.4}{1.0}$. Find the expected length of a leaf which has a width measure of 1 mm. ______

8. (A) Given the same ratio of $\frac{1.4}{1.00}$ ($\frac{l}{w}$) what would be the mathematical expectation of the length measure if the width measure were 35 meters? ______

(B) Would such a leaf be found on Earth? ______

J.F.F. (Just for Fun)

If a brick weighs 9 pounds and a half a brick, what is the weight of a brick and a half? (Reason it out.)

1.13 Activity 2. Paper Chromatography

Have you ever taken apart a watch, a clock, or a motor just to see what it was made of? If so, was it curiosity that "made" you do it? Curiosity is an admirable trait if used constructively and it is a trait common to many good scientists. Biologists and chemists, for example, are curious about the nature of matter. They want to know what special kind of living tissue a plant or an animal is made of. Long ago it was discovered that living things are made up of complex but organized mixtures of many things. The problem was to find a way to "take apart" these complex mixtures so that the simpler parts could be studied.

One tool that was found to do this job is a technique called chromatography. (chrom - a - tog - ra - phy) In paper chromatography a complex mixture is "taken apart" and its parts spread out on paper to form a paper "color graph". This technique has proven to be a valuable aid in helping scientists learn what substances there are in such things as leaves and fruit flies; in vegetables and blood.

You are going to use this technique to "take apart" a complex substance and then use the data mathematically to learn an important step in this process of chromatography.

Your teacher will provide the necessary supplies.

Procedure

After obtaining the supplies, cut a strip of chromatography paper about 1 centimeter (cm) or $\frac{3}{8}$ inch wide and 25 centimeters or 10 inches long. See Fig. 1-13a. Draw and number parallel lines preferably 5 millimeters (mm) apart. The lines must be drawn with a sharp pencil. Do not use ink in any form or it may affect the experiment. If you do not have a metric ruler, draw the lines $\frac{1}{4}$ inch apart. The lines are to be drawn on the narrow section of the paper from the end up to the wide portion. See Fig. 1-13a.

These spaces will be used to measure units of distance traveled in a unit of time. Place a paper clip on the wide end to use as a weight to hold this end in a shallow container of water. (Fig. 1-13b.)

Pour water into the shallow container until it is about $\frac{1}{4}$ inch deep and place it on your desk near the edge. (See Fig. 1-13b.) Caution: Slightly extend container over the edge of the desk so that the chromatography paper does not touch anything past the edge of the container.

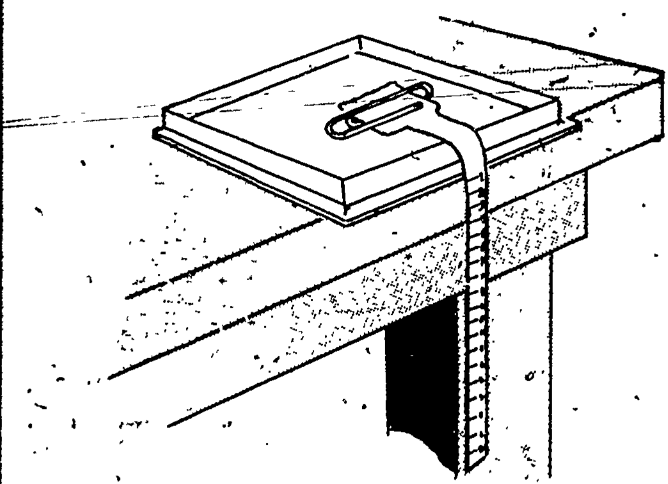


Fig. 1-13b

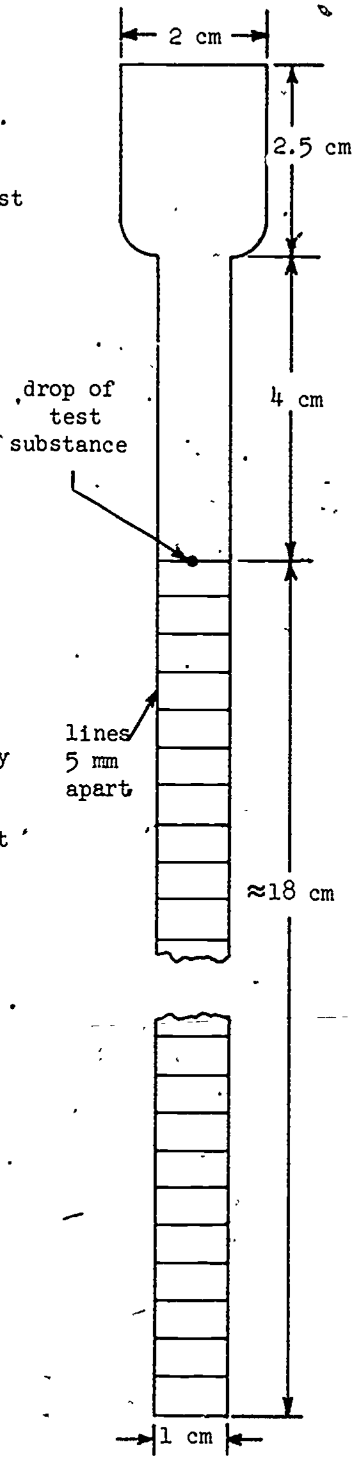


Fig. 1-13a

1. Using a toothpick or some other pointed object, place a very small drop of the substance to be tested on the first line of the paper; the line nearest the wide end.

Let the test substance dry for a few minutes. Apply another small drop of the test substance directly on top of the first drop (should appear as one drop).

2. While this second drop is drying, prepare a table on which you can record your data in an orderly fashion. You will want to record both time and distance, so label two columns for data, one column with the heading, "time (t) in minutes" and the other, "distance (d) in mm". Label this table, Table I.

Chromatography Data

Table I

time (t) in minutes	dist. (d) in mm
0	0
1	
2	
3	
4	
5	

Table 1-13a

3. You will need to have available a clock or watch that will give regular minute intervals so that time recordings will be uniform. Now insert the wide end of the paper strip in the water. Be sure that the spot of substance is <u>outside</u> the container and <u>over</u> the edge of the container.

Start recording the time <u>when the water reaches the test substance</u> (not when you first place the paper into the container).

On your data sheet, record $t = 0$ and $d = 0$. At the end of one minute, record $t = 1$ and d = distance <u>color</u> (the <u>distance</u> which the color front has traveled), not the water, has moved down the paper strip.

Our units of distance measure, as marked on the chromatography paper, are 5 mm. Every minute, record the measure of the length from the first mark to the leading edge of the most rapidly moving color. Use the 5 mm marks on your paper to determine distance.

Continue taking and recording readings for as many minutes as your teacher instructs. At the end of this time remove the strip from the container and lay it on a piece of paper on your desk. Immediately mark with pencil on the strip the maximum distance each color travelled and the maximum distance the water has travelled (if different from the most advanced color). While the mixture is moving, it is often hard to identify different colors. As the process continues, the colors gradually separate as some go farther than others. Figure 1-13c shows an example of how your chromatogram might look and a suggested method for making your distance measures.

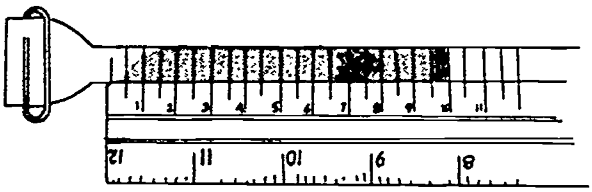


Fig. 1-13c

You may find that one color and the water have travelled the same distance, so a single mark will represent the maximum distance for both. Measure the distance from the spot on which you applied the substance (first line) to each of these marks and record this information on your data sheet. Label this data as Table II. A sample of such data may appear as shown here. See Table 1-13b.

Chromatography Data

Table II

	Distance Travelled	Ratio Front (Rf)
distance travelled by water	12.5 cm	
distance travelled by brown	11.7 cm	
distance travelled by blue	9.90 cm	
distance travelled by red	5.50 cm	

Table 1-13b

1.14 Rf Value - Ratio

A scientist is particularly interested in the kind of data that you entered in Table 1-13b, because from this information he calculates ratio values called the Ratio front (Rf).

Remember that the ratio of a number c to a number d, $d \neq 0$, is $\frac{c}{d}$. We want to find the ratio of distance travelled by color to distance travelled by water.

$$Rf = \frac{\text{Distance travelled by color}}{\text{Distance travelled by water}}.$$

From Table 1-13b the Rf of blue would be $\frac{9}{12.5} = .72$. The Rf of red would be $\frac{5}{12.5} = .4$.

Calculate the Rf values for each color on <u>your</u> chromatogram.

Exercise 1-14

Rf Values

1. Interpret the following Rf values with respect to distance travelled by water and color; e.g., Rf = .4 means color has travelled .4 as far as the water.

   (A) Rf = 0.1
   (B) Rf = 0.5
   (C) Rf = 0.8
   (D) Rf = 0.25
   (E) Rf = 0.33
   (F) Rf = 1.0

2. Listed below are some theoretical Rf values:

Substance	Rf	Substance	Rf
Compound A	0.92	Compound E	1.0
Compound B	0.40	Compound F	0.87
Compound C	0.67	Compound G	0.32
Compound D	0.25	Compound H	0.16

   (a) A chromatogram produced the following data: Identify as A, B, C, etc. the compounds in the mixture (see above).

Distance travelled by water (solvent)	=	25 cm
Distance travelled by Color I	=	25 cm
Distance travelled by Color II	=	10 cm
Distance travelled by Color III	=	4 cm

(b) Using the Rf values listed above, identify the compounds responsible for the following data.

Distance travelled by water	= 18 cm
Distance travelled by Color I	= 12 cm
Distance travelled by Color II	= 16.6 cm
Distance travelled by Color III	= 5.8 cm

In our discussion so far we have used only water as a solvent to separate the compounds from a mixture. Many other solvents are used and each will give a different set of Rf values for a given mixture. Pigments from vegetable matter or proteins from animal matter are examples of substances that need solvents other than water for separation. The various conditions and materials used would then be included in a table of Rf values. Some other factors that influence Rf values are kind of paper, temperature, and time the paper is left in the solvent.

The various compounds contained in the test substance each have a different degree of attraction to the paper. These tend to separate as they are moved along through the paper by the water, showing up as various bands of color.

Here are examples of Rf values for some common substances. Again the conditions under which these values were found is not given. Such information is generally so lengthy that the writers felt it was not practical to present it here.

Sugars	Rf	Fatty Acid	Rf	Organic Acid	Rf
Glucose	0.39	Formic	0.42	Citric	0.26
Galactose	0.44	Acetic	0.50	Lactic	0.72
Fructose	0.51	Propeonic	0.62		

1.15 Graphing Chromatography Data

Let us now look at another way o. analyzing the data you have recorded from the experiment. You recorded the distance travelled by water and color each minute.

Is this table of data easy to interpret? Can you make a general statement about the rate at which the solvent and color travel?

[Make a graph of the data you recorded in Table 1-13a using the horizontal ray for units of time and the vertical ray for units of distance travelled.]

Recall the method you used to construct a graph in Section 1-12. If necessary, refer back to that section.

Exercise 1-15

Interpreting Graph Data

1. What is the measure of the distance that the mixture travelled during the first minute? (Express your answer in mm.)
2. Find the measure of the distance travelled by the mixture during the second minute.
3. Did the mixture travel the same distance each minute?
4. How far did the mixture travel during the fifth minute?
5. What is the ratio of the distance travelled during the second minute to the distance travelled during the first minute?
6. What is the ratio of the distance travelled during the fifth minute to the distance travelled during the first minute?
7. Approximately how many mm did the mixture travel during the last minute?
8. Express as a ratio the distance travelled during the last minute to the distance travelled the first minute.
9. At what time interval does the "slowing down" process first become really noticeable?
10. Given a paper of infinite length and conditions the same as in your classroom, do you think the process would ever stop completely? Why?

Chapter 2

NATURAL VARIATION - "US" -- ADDITION OF MEASUREMENT AND GREATEST POSSIBLE ERROR

2.1 Introduction

Look around the room at your classmates. Do they all have one head, one nose, two ears, and a mouth? Sure, each one looks a little bit different from another but all have the same general characteristics.

In Chapter 1 we found a definite relationship between the length and width of the same species of leaves. In this chapter we will investigate some of the correlations in human growth and development.

Picture the tallest boy in the room with the feet of the shortest girl. Can you visualize what the shortest girl would look like if she had arms as long as the tallest boy? (Probably could tie her shoes without stooping over.)

Let's measure the height, reach, and foot length of everyone in the room and find out about the natural variation of humans.

2.2 Preliminary Preparation

You will need for this activity:

1. ≈60' of wrapping paper (30"-36" width);
2. A tape measure 6'-10' length, calibrated in $\frac{1}{16}$" ;
3. A foot ruler for each student;
4. Scotch tape to affix the wrapping paper to the wall.

This activity will be done by groups. Your teacher will determine your group. [Rows would make a good group (5 or 6 people)]. Each group should have one piece of wrapping paper - 7' long and another 2' long.

Place the wrapping paper upon a smooth wall so that the various members of the group may stand against the wall and have their height and reach recorded. (Fig. 2-4a and 2-4b)

A small piece of paper - 2 ft. by 30 in. - stuck to the floor will furnish the material to mark the "length of foot". (Fig. 2-4c)

You will be measuring the height, reach and length of foot of your partners to the nearest $\frac{1}{16}$" mark on the tape or ruler.

2.3 Greatest Possible Error

We often say when measuring, "to the nearest...". When measuring a sheet of paper you might say "to the nearest $\frac{1}{16}$th". If measuring the room width, your teacher might say "to the nearest inch". What if you were asked to measure the distance by highway to the next town (city limits sign)? What unit of measure would you choose?

In Figure 2-3a:

a. To the nearest inch $\overline{AB}$ measures 3.

b. To the nearest $\frac{1}{2}$-inch $\overline{AB}$ measures ________?

c. To the nearest $\frac{1}{4}$-inch $\overline{AB}$ measures ________?

d. To the nearest $\frac{1}{8}$-inch $\overline{AB}$ measures ________?

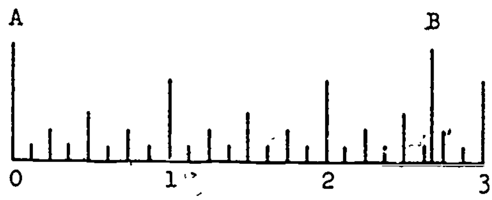


Fig. 2-3a

"B" is not "on" any of the markings on our scale. In each case we picked the unit which B was nearest to, or the half unit it was included in.

When measuring if we included in our notation the size of the smallest unit and referred to the fact that we are measuring to the nearest unit, it would aid in the understanding and use of the measurement.

Definition: The greatest possible error in a measurement is $\frac{1}{2}$ of the smallest unit of measure used in the measurement.

To restate our last measurements in Figure 2-3a, using the definition of greatest possible error:

(a) $3'' \pm \frac{1}{2}''$ (c) $2\frac{3}{4}'' \pm \frac{1}{8}''$

(b) $2\frac{1}{2}'' \pm \frac{1}{4}''$ (d) $2\frac{5}{8}'' \pm$?

The $\pm$ (read "plus or minus") symbol says two things. It means in the case of (b) Fig. 2-3a, that $\overline{AB}$ measures less than $2\frac{1}{2}'' + \frac{1}{4}''$ $(2\frac{3}{4}'')$ and greater than $2\frac{1}{2}'' - \frac{1}{4}''$ $(2\frac{1}{4}'')$. In other words the measure of $\overline{AB}$ is between

$2\frac{1}{4}"$ and $2\frac{3}{4}"$. Look at Fig. 2-3a. Is B between $2\frac{1}{4}"$ and $2\frac{3}{4}"$?

(c) in Fig. 2-3a was measured to the nearest $\frac{1}{4}"$. Thus, we recorded $2\frac{3}{4}" \pm \frac{1}{8}"$. Our measurement shows B to be between $2\frac{7}{8}"$ and $2\frac{5}{8}"$. We could express this in math symbols as $2\frac{7}{8}" >$ measurement of $\overline{AB} > 2\frac{5}{8}"$.

The more precise of two measurements is the one with the smaller possible error. To repeat, greatest possible error in a measurement is $\frac{1}{2}$ of the smallest unit of measure used.

When we use the "$\pm$" notation in a measurement, we are showing clearly both the precision and the greatest possible error, as the smallest unit used in the measurement would be twice the greatest possible error. Why?

Measurement	Smallest Unit	Greatest Possible Error
$34" \pm \frac{1}{4}"$	$\frac{1}{2}"$	$\frac{1}{4}"$
$16" \pm \frac{1}{8}"$	$\frac{1}{4}"$	$\frac{1}{8}"$
$10" \pm \frac{1}{2}"$	1".	$\frac{1}{2}"$
5 cm $\pm \frac{1}{2}$ cm	1 cm	$\frac{1}{2}$ cm
27 m $\pm \frac{1}{2}$ dm	1 dm	$\frac{1}{2}$ dm
39 cm $\pm \frac{1}{2}$ mm	1 mm	$\frac{1}{2}$ mm
31 km $\pm \frac{1}{2}$ m	1 m	$\frac{1}{2}$ m

When you measure anything you must make a choice on a unit and then cerefully determine the number of units. When you measured the leaves in Chapter 1 you measured with a metric ruler and chose millimeters as your unit. What would be the greatest possible error if you measure to the nearest millimeter?

Exercise 2-3

	Measurement	Smallest Unit	Measurement with G.P.E. Expressed
1.	35 yds., 2 ft.	1 ft.	35 yds., 2 ft. $\pm \frac{1}{2}$ ft.
2.	14 ft.	1 ft.	
~~3.~~	~~14 ft.,~~ 3 in.	1 in.	
4.	3 meters	1 dm.	
5.	5 km., 270 m.	1 m.	
6.	7 km., 395 m., 47 cm.	1 cm.	
7.	38 miles, 560 yds.	10 yds.	
8.	93,562,000 miles	1000 miles	

2.4 Measuring Height, Reach and Length of Foot

Each group should have their equipment (paper on wall and floor) ready. Each member of the group should make a table similar to Table 2-5b for recording of data.

Taking turns, each student stands against the wall and has his height, reach and foot length recorded. The procedure is as follows:

1. Count off in the group $1 \rightarrow 5$ or 6.
2. No. 1, using a ruler parallel to the floor and a pencil, marks the height of each member of the group (except his own) and labels the mark with their name. (See Fig. 2-4a.)

Marking the Height

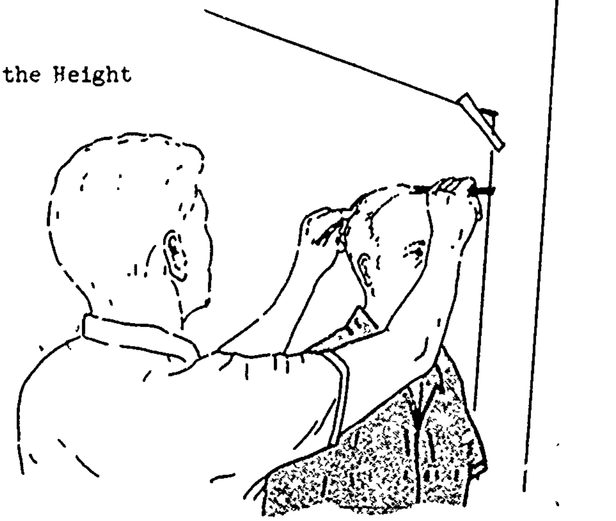


Fig. 2-4a

3. No. 2, using a pencil, marks the "reach" of each member of the group (except his own) and labels the marks with their names. (See Fig. 2-4b.)

Marking the Reach

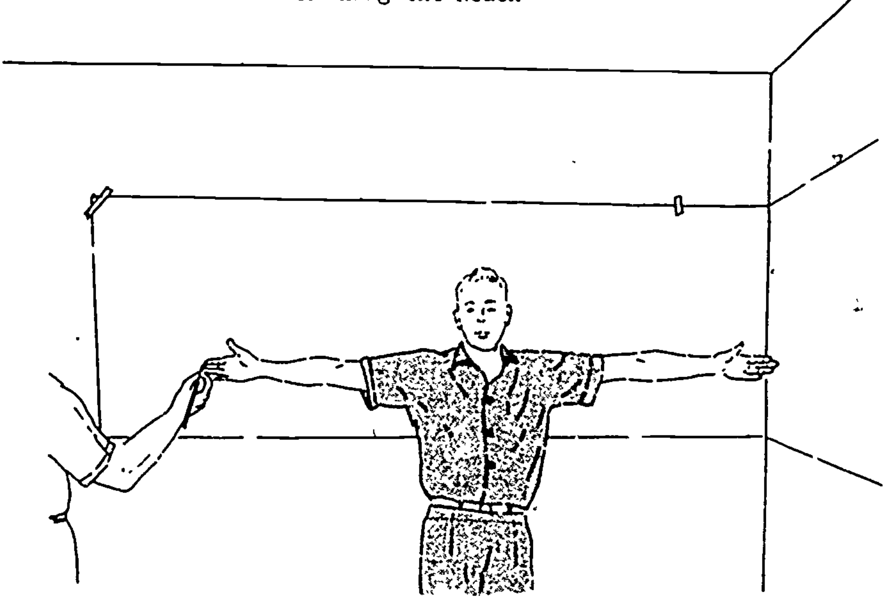


Fig. 2-4b

4. No. 3, using a pencil, marks the "length of the right foot" for each member of the group (except his own) and labels the marks with their names. (See Fig. 2-4c.)

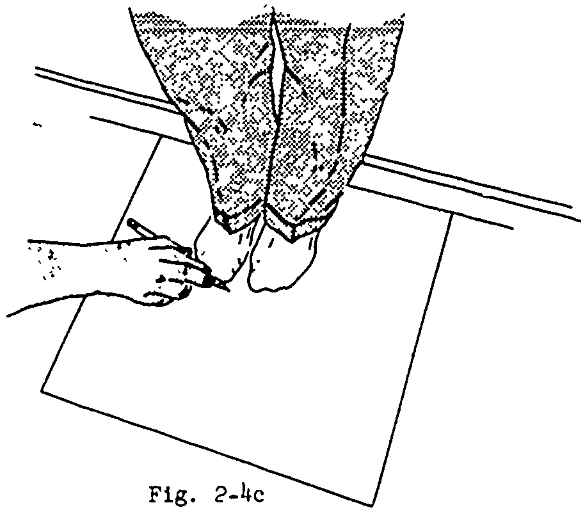


Fig. 2-4c
Marking Right Foot

5. No. 4 substitutes for Nos. 1, 2, and 3 when it is their turn to be measured.

6. After all of the marks for the height, reach and length of foot have been made, using a tape measure, Nos. 5 and 6 measure the height of each person and then the reach of each person.

7. No. 3 measures the "length of foot" for each person.

8. All measures are entered on the table and recorded with G.P.E. (greatest possible error).

2.5 Recording Data

Decimal Equivalences of Sixteenths
(to the nearest hundredth)

$\frac{1}{16}$ = .06	$\frac{5}{16}$ = .31	$\frac{9}{16}$ = .56	$\frac{13}{16}$ = .81
$\frac{2}{16}$ = .13	$\frac{6}{16}$ = .38	$\frac{10}{16}$ = .63	$\frac{14}{16}$ = .88
$\frac{3}{16}$ = .19	$\frac{7}{16}$ = .44	$\frac{11}{16}$ = .69	$\frac{15}{16}$ = .94
$\frac{4}{16}$ = .25	$\frac{8}{16}$ = .50	$\frac{12}{16}$ = .75	$\frac{16}{16}$ =1.00

Table 2-5a

Table of Data on Height, Reach, and Length of Foot

Name of Student	REACH		HEIGHT		FOOT		RATIO to nearest hundredth	
	Fraction	Decimal Equiv.	Fraction	Decimal Equiv.	Fraction	Decimal Equiv.	Reach to Height	Foot to Height
0. Example: Jones	$63\frac{3}{16} \pm \frac{1}{32}$"	63.19	$64\frac{1}{16} \pm \frac{1}{32}$	64.06	$8\frac{6}{16} \pm \frac{1}{32}$	8.38	.99	.13
1.								
2.								
3.								
4.								
5.								
6.								
7.								

Table 2-5b

Complete your table similar to Table 2-5b. After the name of each of your group members, record their reach, height and length of foot measurements. Using Table 2-5a, fill in the spaces for the decimal equivalent of each measurement. Do nothing with the last column "Ratio" until you read Section 2.6.

2.6 Computing Ratio

In the first chapter we learned that one way to compare a number to another was by a ratio. This was expressed in a formula as: ratio = $\frac{c}{d}$ (if d is not 0). (Here d is being compared to c and tells how many times larger c is than d.) To find the ratio of reach to height in Table 2.4, the formula would be: ratio = $\frac{\text{reach}}{\text{height}}$.

Example 1: If a boy has a reach of 65 inches and a height of 64 inches his ratio of reach to height would be $\frac{65}{64}$ or 1.015625. It is not necessary to carry this division to the millionths place. We must stop somewhere and it would seem that hundredths would be significant (we will cover this in Chapter 3). When we round 1.015625 to the nearest hundredth we have 1.02 . This means that his reach is 1.02 times his height.

Example 2: If a girl has a height of 66 inches and a reach of 64.5 inches, then the ratio of her reach to height would be $\frac{64.5}{66}$ or .98 . Her reach is .98 of her height.

To find the ratio of foot to height, the ratio = $\frac{\text{measurement of the foot}}{\text{measurement of height}}$.

> Compute the ratios using the decimal measurement columns and record them in Columns 7 and 8 of Table 2-5b.

2.7 Graphing Activity

Whenever it seem useful, scientists graph the data obtained from their experiments or observations. We will graph as ordered pairs the reach of each individual as one factor on the horizontal line and the height of the same person on the vertical line. (See Fig. 2-7a.)

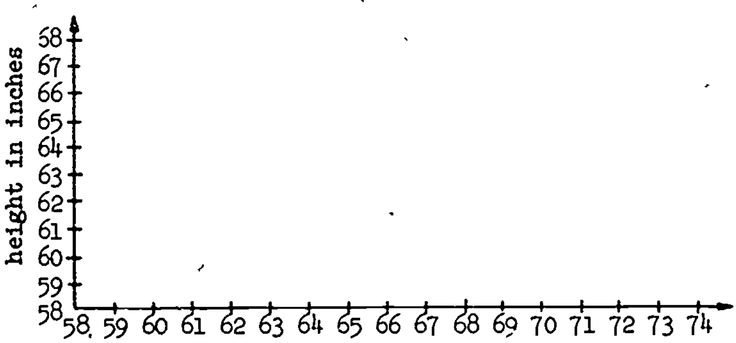


Fig. 2-7a

We should use the same scale on the horizontal as the vertical axis because we have found from Table 2-5b that the ratio between the two readings is about 1:1 (one to one). If we used a smaller scale on the horizontal axis (called the x-axis by mathematicians) than on the vertical axis (called the y-axis) we would have a line similar to Figure 2-7b.

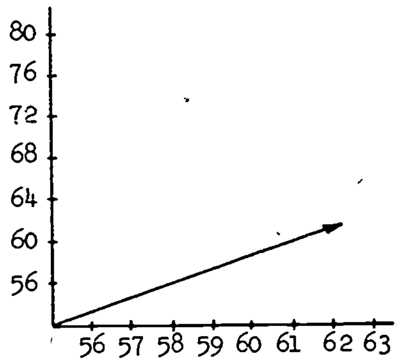


Fig. 2-7b

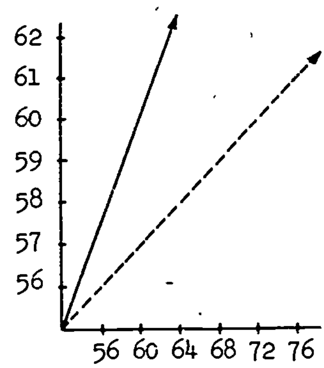


Fig. 2-7c

A glance at Figure 2-7b would indicate a higher ratio than 1:1, but a close check would indicate this is not true. Figure 2-7c would indicate a lower ratio. We prefer a line similar to the dotted line on Fig. 2-7c for a 1:1 ratio.

When planning for a graph, you must consider the following questions:

1. What information do you want to represent?
2. Can the information be represented upon a plane? (How many dimensions has a plane?)
3. How many units are needed on each reference line?
4. If the points are plotted as "ordered pairs", is the order such that the angle of the graph will represent the slope of the line? (You may take this up later in your math course.)

> Choose a scale which has a range that will include the height of the shortest student and the tallest student, making sure that it will also include the complete range of reaches. Now we are ready to plot our ordered pairs on the graph (reach,height). Reach is the first member of the ordered pair and will be on the horizontal axis; height will be on the vertical axis.

Exercise 2-7

1. Did the boys, or the girls, in your row have a closer ratio of reach to height?

2. If you used the smallest and the largest ratio of reach to height in your group, what would be the longest and shortest reach you could expect for a boy 10 ft. tall?

3. A basketball player is 7 ft., 3 in. tall. His shoulders are 13 inches from the top of his head and his arms are 9 inches from the center of his back. How high would you expect him to be able to reach?

4. About how large of a tree trunk (circumference) could the fourth person in Table 2-5b reach around and touch his fingertips?

5. If all the students in your row would extend their arms around a large tree trunk, what would be the largest circumference that they could measure if all fingers were just touching?

6. A student's foot measures 10.25 inches from heel to toe. If he walked heel-to-toe for one mile, how many steps would he take? Can this be used as a useful measuring unit?

J.F.F. If a clock strikes 4 times in 3 seconds, how many times will it strike in 9 seconds?

2.8 Addition of Measures

You have learned and used the fact that a number has many names. The number "one" could be written as 1, $\frac{2}{2}$, $\frac{9}{9}$, 1.000, $\frac{a}{a}(a \neq 0)$, and many other ways. The same is true for other numbers. "Two" could be written as $\frac{34}{17}$, II, or $\frac{.0008}{.0004}$. "Nineteen" could be written as 19, XIX, or $\frac{1083}{57}$.

Various units of measure are often found in a comparison. A map may show for its scale 1" = 50 miles, or a floor plan may have for its scale $\frac{1}{8}$" = 1 foot. Some graphs might even have for a scale 1" = 50 million people. It is common to see the sign used this way on maps, scale drawings, and various graphs and tables. This use of "equals" implies that the two measures are the same, but we recognize by common sense that they are not the same. One inch is not the same as 50 miles. "Equals" in this usage merely means that one actual measure represents the other measure in that particular circumstance.

When we say 1 foot = 12 inches we know that this case of the equal sign means both measurements represent the same length. Thus, we might read: 1 yd. = 3 ft. = 36 inches. We know it does not mean 1 = 3 = 36. It does mean that all these measurements represent the same length. They are just different names for the same length.

As in the use of the "=" sign, the use of other symbols of operation have a special meaning when dealing with measurements. From our knowledge of the distributive property we will build a correlation with measures.

To add $(13 \times 6) + (13 \times 4)$ we distribute the multiplication over the addition and write

$$13 \times (6 + 4) = 13 \times (10) = 130.$$

To add $(57 \times 17) + (43 \times 17)$ we again distribute the multiplication over the addition and write

$$(57 + 43) \times 17 = (100) \times 17 = 1700.$$

Notice in both examples the common unit (contained as a factor in both products) is not added but is used as a factor in the final product.

In the general form, $ab + ac = a(b + c)$.

In the addition of measure we can use the same general ideas. 2 inches + 3 inches means (2 + 3) inches or 5 inches, not

```
  2 inches
+ 3 inches
  5 inch inches
```

Other examples:

1. $2x + 3x = (2 + 3)x = 5x$
2. $2 \times 100 + 3 \times 100 = (2 + 3) \times 100 = 500$
3. 2 ft. + 3 ft. = (2 + 3) ft. = 5 ft.
4. 2 pennies + 3 pennies = (2 + 3) pennies = 5 pennies
5. 2 candy bars + 3 candy bars = (2 + 3) candy bars
   = 5 candy bars
   (or 1 stomach ache)

The operation (addition) is performed upon the numbers only, not upon the units of measure.

Since subtraction is the inverse of addition, we can expect the same sort of results. 5 inches - 3 inches = (5 - 3) inches = 2 inches. Again the operation (subtraction) is performed only upon the numbers.

2.81 Greatest Possible Error: Addition of Measurement

When adding measurements, the greatest possible error of a sum is the sum of the greatest possible errors.

For example:

$$\begin{array}{r} 25 \text{ mm} \pm .5 \text{ mm} \\ + 12 \text{ mm} \pm .5 \text{ mm} \\ \hline 37 \text{ mm} \pm 1.0 \text{ mm} \end{array}$$

This means that the sum of the measures is someplace between 36 mm and 38 mm.

Computation involving measurements is very important in today's world. Many rules have been laid down giving the accuracy or precision of the results obtained from computation with approximate measures. Too many rules, however, might create confusion and would never replace basic knowledge of approximate data. If the meaning of the greatest possible error is understood, the precision and accuracy of the result of computation with approximate data can usually be found by applying common sense and judgment. Common sense would tell us that with a large number of measurements the errors will, to a certain extent, cancel each other.

The general principle is that the sum or difference of measures cannot be more precise than the least precise measure involved. Therefore, to add or subtract numbers arising from approximations, perform the operation and ᵊn round the result to the unit of the least precise number.

Exercise 2-81

1. Perform the following computations:

(a) $\begin{array}{r} 37 \text{ mm} \pm .5 \text{ mm} \\ + 13 \text{ mm} \pm .5 \text{ mm} \\ \hline \end{array}$

(b) $\begin{array}{r} 41 \text{ cm} \pm .5 \text{ cm} \\ + 39 \text{ cm} \pm .5 \text{ cm} \\ \hline \end{array}$

(c) $\begin{array}{r} 64 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ + 3? \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ \hline \end{array}$

(d) $\begin{array}{r} 3 \text{ yds. } 2 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ + 9 \text{ yds. } 1 \text{ ft.} \pm \frac{1}{2} \text{ in.} \\ \hline \end{array}$

(e) $\begin{array}{r} 19 \text{ mm} \pm .5 \text{ mm} \\ - 17 \text{ mm} \pm .5 \text{ mm} \\ \hline \end{array}$

(f) $\begin{array}{r} 39 \text{ cm} \pm .5 \text{ cm} \\ - 38 \text{ cm} \pm .5 \text{ cm} \\ \hline \end{array}$

2. Find the greatest possible error for the sums of the measurements in each of the following. (Assume that the unit of measurement is the least unit used in any one measure; e.g., $5\frac{1}{2}$" measurement - least unit is $\frac{1}{2}$" - G.P.E. is $\frac{1}{4}$".)

(a) $5\frac{1}{2}$ in., $6\frac{1}{2}$ in., $3\frac{0}{2}$ in.; C.P.E. of sum is $\pm\frac{3}{4}$ inch.

(b) $3\frac{1}{4}$ in., $6\frac{1}{2}$ in., 3 in.

(c) 4.2 in., 5.03 in.

(d) 42.5 in., 36.0 in., 49.8 in.

(e) 0.004 in., 2.1 in., 6.135 in.

(f) $2\frac{3}{4}$ in., $1\frac{5}{16}$ in., $3\frac{3}{8}$ in.

3. Add the following measures:

(a) 42.36, 578.1, 73.4, 37.285, 0.62

(b) 85.42, 7.301, 16.015, 36.4

(c) 9.36, 0.345, 1713.06, 35.27

4. Subtract the following measures:

(a) 7.3 - 6.28

(b) 735 - 0.73

(c) 5430 - 647

2.82 Greatest Possible Error - "Sum of - computer"

61 mm ± .5 mm	35 mm ± .5 mm
- 47 mm ± .5 mm	13 mm ± .5 mm
14 mm ± 1.0 mm	17 mm ± .5 mm
	23 mm ± .5 mm
	+ 11 mm ± .5 mm
	99 mm ± 2.5 mm

Notice that the greatest possible error of a sum or difference of several measures is the sum of the greatest possible errors of the measures involved.

This error is, remember, the "grea - - - - test" possible error. For almost all calculations the error would be less and in fact the errors often cancel each other out but it is possible to get such an error as the G.P.E. Therefore, by the definition of the term, this is the "greatest".

As a matter of fact, the extreme values would occur very infrequently and the other occurences of greatest possible error values would increase as you approach the value in the middle (zero). To illustrate this important concept, make the following slide rule.

1. About halfway down on a piece of tag board, manila folder or similar material, draw a straight line 8" long.
2. Mark the left ends -1, the right ends +1, the center "0" and the rest as in Figure 2-82a.

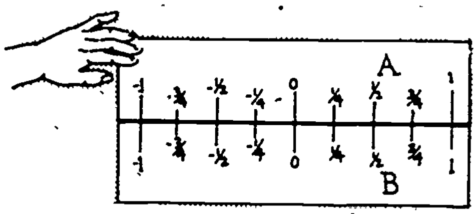


Fig. 2-82a

3. Cut carefully along the line so that you can make a slide ruler.
4. Make a table as in Table 2-82a.

Sum of 2 G.P.E.	2	$1\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
No. of ways event can happen									
% of Total									

Total No. of Tallies

Table 2-82a

In the rational number system, between any two numbers an infinite number of numbers can be written. In this table we will work only with the numbers shown on our slide ruler and make a conjecture about the rest of the rational numbers.

5. Line up the +1 on scale A with the +1 on scale B. Note and record with a tally mark on a table such as 2-82a those <u>non-negative sums</u> which are indicated by the numbers which are vertically aligned; e.g., $1 + 1 = 2$, $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$, etc. (Disregard all sums less than 0 for now. They will act nearly the same as those from 0 to +2.)

6. Move rule A to the left until the +1 lines up with the $+\frac{3}{4}$ on B. Again note and record on your table with a tally mark, all of the non-negative sums which occur. (You should now have one tally mark below each sum.) (See Fig. 2-82b.)

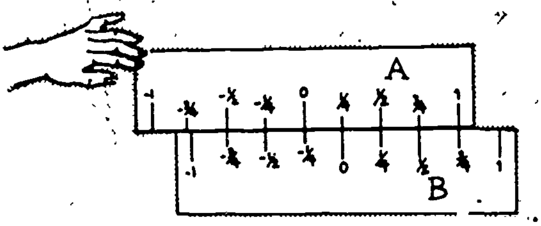


Fig. 2-82b

7. Move rule A to the left until the +1 lines up with the $+\frac{1}{2}$ on B. Repeat the tallying of sums.

8. Continue to move A to the left, one mark at a time, tallying sums which occur after every setting.

9. When the +1 on A lines up with the -1 on B, you have completed the table. Make a tally below the zero on your table. (See Fig. 2-82c.)

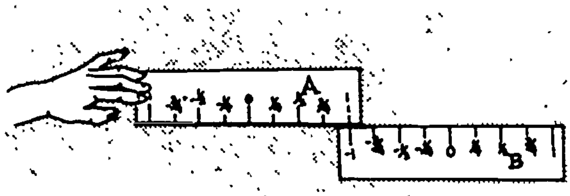


Fig. 2-82c

Count the total number of tallies. What per cent of the total number turned out to have a sum of zero? What per cent of the total turned out to have a sum of 2 ?

[Record the per cent of the number of ways each sum is arrived at under the tallies.]

Observe the information on our table and consider the possible percentages if we had used more values on our slide rule. Do you see that often the sum of possible errors balance each other and the extremes seldom occur?

You may want to continue the tally for a greater number of values between +1 and -1 and also for the sums between 0 and -2.

A SECOND ACTIVITY

2.9 (Shoes, shoes, shoes...)

This activity is designed to use your knowledge of "addition of measurement". Read the following outline completely before starting the activity.

1. Construct a table similar to Table 2-10 for recording the data.

2. Fix to the floor a piece of wrapping paper $\approx 6'$ long.

3. Near one end of the paper carefully mark the outline of the back and front of the shoe of the first person. (See Fig. 2-9.)

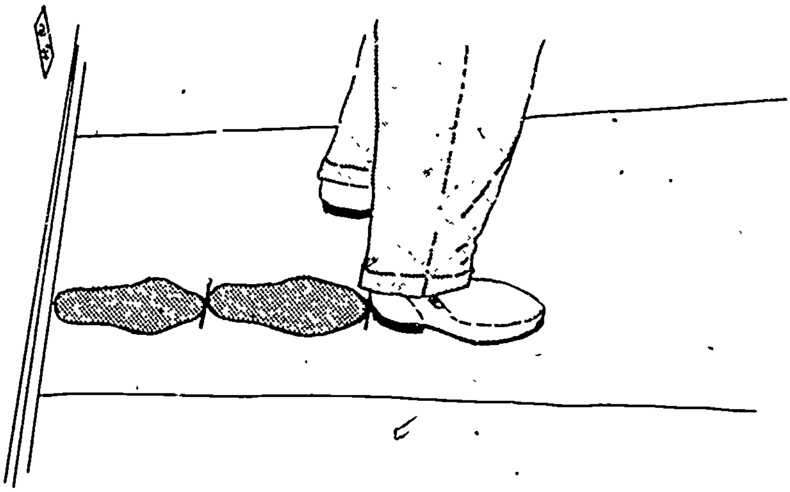


Fig. 2-9

4. The second person carefully places his shoe, with his heel just on the line, directly in front of the last mark.

5. The length of his shoe is recorded by marking the paper directly in front of his shoe.

6. Then continue for the entire group (could be the same group as the last activity).

7. Each person carefully measures the length of his own shoe marks to the nearest $\frac{1}{16}$" and records it. (Don't forget the G.P.E.)

8. Under the supervision of the entire group, using the tape measure, carefully measure the length from the first to the last mark (the total length of shoes) with just <u>one</u> measure. Record this.

2.10 Recording Data

1		2	3	4	5
Name		Length of Foot	Greatest Possible Error	Smallest Possible Measure	Greatest Possible Measure
Example: George		$11\frac{13}{16}$	$\pm\frac{1}{32}$	$11\frac{25}{32}$	$11\frac{27}{32}$
1.					
2.					
3.					
4.					
5.					
6.					
7.					
Sum of measures 1 - 7					
Result of Inst. 8 above					

Table 2-10

Record the name and foot length of each person in your row in a table similar to Table 2-10. Subtract the greatest possible error from measured foot length and enter in column four (smallest possible measure). In the last column enter the sum of the foot measure and the greatest possible error for each student. Total columns two, three, four and five. These represent the smallest measure you can have, measuring to the nearest $\frac{1}{16}$ -inch, in column four and the greatest measure you can have. Now record your single measurement made with the single measure. Is this measurement ($\pm\frac{1}{32}$ -inch) between the totals you have for the last two columns? If it is not, you have not measured to the nearest $\frac{1}{16}$ -inch.

2.11 Summary

You have measured leaves of various sizes and ages. You have measured the height and reach of yourself and your classmates. Have you ever wondered about how things grow? How does a tiny seed grow into a giant redwood tree 367.8 feet tall? (See Chapter 8.) How does a "white dot", which you can just barely see when you open an egg, grow into a full-fledged chicken?

It would take many more volumes than this small book to answer these questions, even to the extent of our present knowledge, but we can examine a few important factors.

First of all, the tiny seed, the white dots in the eggs, are alive! They are made up of very tiny living bits of matter called cells. Cells are extremely complicated (in spite of their small size), very active, constantly changing units, consisting of a very thin membrane enclosing a substance called protoplasm.

Protoplasm is extremely complex. Briefly, it is made up of particles which must be constantly replenished, chemicals which are constantly reacting with each other, the much talked about DNA molecules duplicating themselves, and many other highly dynamic substances.

All living things are made up of these "lively" little units called cells. One of their most remarkable properties is their ability to divide into more cells, sometimes nearly exactly duplicating themselves, other times changing gradually into specialized forms, such as nerve cells, bone cells, skin cells, leaf cells, or root cells. This is called growth.

Much emphasis has been placed on the dynamism of cells. This means there is constant activity going on. All this activity requires energy.

In Chapters 1 and 2 you were introduced to the idea of "natural variation" through the consideration of the size of leaves and humans. Most of the following exercises will show you some of the ways living things obtain materials to provide this energy, as well as the materials for growth and repair.

2.12 Optional Activity - Growth from Seeds

When you measured leaves earlier you discovered that living things show evidence of natural variation. You found short leaves and long leaves, but you also found that most of them were "about average". You also found that you and your classmates illustrate natural variation in animal (that's you) growth and development. In this activity you will find that different parts of a plant will grow at different rates, but that when you add them together you will be able to see a part of a total growth pattern. To do this you will add lengths.

Preliminary Preparation

The following procedure must be done at least 5 days before the first activity day, in order that the plants will be ready to use. Read all of Section 2.12 before starting the activity.

Materials Needed

- Bean seeds which have been soaking for 24 hours. (Black-eyed peas are beans, easily available, and are recommended.) Other recommendations are pumpkin and watermelon seeds.
- Milk cartons. Size does not matter -- half-pint cartons are adequate.
- Plant food - may be plant tabs, liquid food, etc.
- Labelling tape.
- Med. heavy acetate (outdated X-ray film).
- Paper towel.
- Soda straws.

Procedure

(work in teams of two)

1. Cut a milk carton similar to Figure 2-12a. This will give support for our sprouting chamber and will also allow us to view the growth.

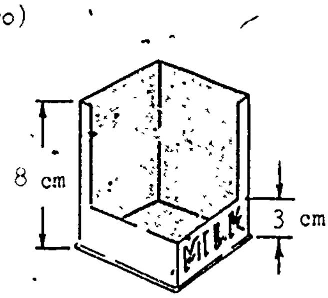


Fig: 2-12a

2. Cut the heavy acetate in strips about $2\frac{3}{4}$ inches wide (the long way so they will be about 11 inches long).
3. Cut about 4 thickness' of paper towel $2\frac{1}{2}$ inches wide to act as a wick to carry moisture to the plants.

4. Lay the paper towel strips (four thicknesses) on one piece of plastic. Place two soda straws on the paper towel strips to form 3 chambers of equal widths. "Paint" a thin strip of rubber cement on the straws to hold them against the towelling. Don't slop cement on the towels - it will impede the absorption.

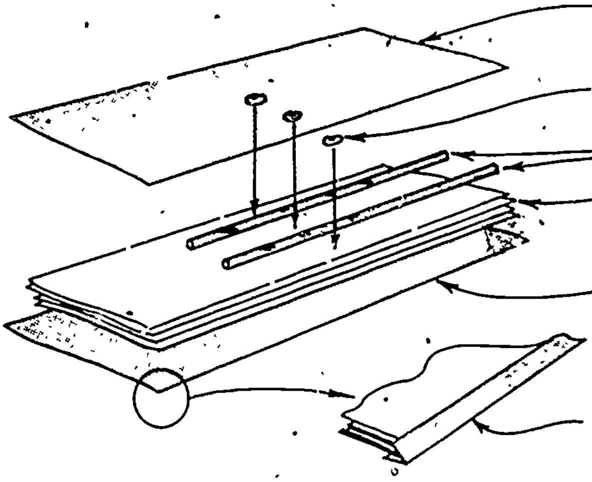


Fig. 2-12b

5. Place one soaked seed in the middle of each chamber (the seed must be at least 10 cm above water level).

6. Carefully place the other sheet of plastic on top and use mending tape to seal the sides but not the top and bottom.

7. Pour water into the milk carton you have prepared (about $1\frac{1}{2}$ cm deep). Stand the plastic "wafer" in the water.

8. Check the water level regularly and do not let it get below the paper towel wick.

Your plants should be ready to be measured in about 5 days. All of the seeds probably will not sprout unless you happen to be a very good gardener, but don't worry about those that do not. Use as many as do grow.

It is also all right to use the late-comers as they sprout, although this is not done in careful scientific studies of growth patterns. Why? If you have started measuring, and on the second day a new one sprouts, record its measurements in your table for Day 2 to avoid confusion.

2.13 Measurement of Seed Growth

Materials Needed

Rulers, preferably with metric scale
Data tables
Graph paper (optional)

Procedure

First Day: Examine the seeds you "planted" about 5 days ago. They should now have sprouts of measureable length. Which direction do the first sprouts go, up or down? If the seed was planted upside down it would still grow in the same direction. How do you suppose the seed "knows" which way to grow?

We are concerned with two things in this exercise, so careful records must be kept in order to keep your data straight. Daily growth of root and stem and total growth of the plant are the needed data. This will be done on a number of plants and the average calculated. Living things are so variable in their development that only by using this averaging method can a valid estimate be reached. Take another look at your plants. Some of them may not even have the stem and leaves yet, while others are growing more rapidly. You can see now why as large a number as possible must be used. Figure 2-13 illustrates growth after 6-10 days.

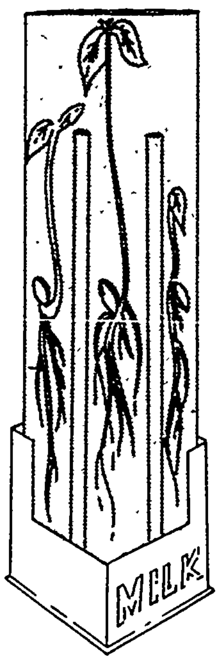


Fig. 2-13

Step 1. Using your metric ruler place the zero point at the edge of the seed (the one you think healthiest) and measure the length of the root to the nearest mm. Record this measurement to the nearest mm in the table for Day One (Table 2-13).

Step 2. When the stem also appears, measure and record its length daily.

Step 3. Be ready to give your measurement to the class so they can record the data when your teacher calls for the information.

Second and succeeding days

Step 4. Repeat steps 1, 2 and 3 every day until you have recorded five to ten readings on a table like Table 2-13 (one for each day).

Step 5. For calculations for columns 4 and 5 and totals and averages, see Section 2.9 .

Exercise 2-13

1. Which day showed the greatest total growth?

2. On which day did your ratio of stem to root change from zero?

3. Did the $\frac{\text{stem}}{\text{root}}$ ratio ever equal one? Exceed c.?

4. Do you think this ratio exceeds 2 for any plant?

5. Is the total growth at a constant rate? (That is, the same amount each day.)

6. If you had different types of seeds -- did each type show the same growth pattern?

7. On which day did you first notice the green coloring?

8. What does the presence of the green color signify?

9. If we used weight rather than length for our measures, what do you think the $\frac{\text{stem}}{\text{root}}$ ratio would be for a carrot, potato, and pine tree?

10. Where do the stem and roots get their nourishment to grow before the plant food is added?

Table 2-13

Measurement of Seed Growth - Day _____

Plant of Team	Total length of original root to nearest mm	Total length of stem to nearest mm	Total length of plant to nearest mm	Daily Growth (e.g. T.L.Day 1 minus T.L.Day 2	Ratio $\frac{\text{T.L. Stem}}{\text{T.L. Root}}$
Example	12 mm ± .5 mm	0 mm	12mm ± .5mm	12 mm ± .5 mm	0
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					
11.					
12.					
13.					
14.					
15.					
16.					
17.					
18.					
19.					
20.					
Total					
Average					

Chapter 3

LEAF SURFACE AREA AND WATER LOSS: AREA, SIGNIFICANT NUMBERS, SCIENTIFIC NOTATION

3.1 Introduction

In your work so for you have measured leaves and discovered that they are variable in size, but still show a unity of design (similar ratios in most cases). All living things, in order to grow, must have food and oxygen, and all green plants obtain their food by manufacturing it themselves. This vital process is called photosynthesis (photo-sin'-the-sis: building with light), and can be carried on only by green plants. Photosynthesis has been called the most important chemical reaction in the world. Can you figure out why?

It might be important to clarify a misconception regarding "plant food". The minerals and other fertilizers which plants obtain from the soil are not classified as "food" in the sense that they are needed to provide energy, even though it is a term commonly used. For the most part, these minerals are used by the plant cells to assist chemical reactions, to become significant parts of important substances (such as magnesium in chlorophyll, nitrogen in proteins), and in other ways to maintain the proper chemical balance in protoplasm. The food provided by photosynthesis, on the other hand, is the major source of energy for the cells of all living things.

Photosynthesis is a process in which two raw materials, carbon-dioxide (di-ox-ide) and water, are chemically combined with the help of light and chlorophyll (chlo'-ro-phill: green pigment in plants) to produce sugar. By-products of this process are oxygen and water. This is another of the many activities taking place in cells - but only in green plant cells.

Since water is one of the major raw materials needed for photosynthesis, it must reach the leaf cells somehow. Recall the leaves you measured. Do you remember that they had "veins"? These veins make up a system of tiny tubes which connect leaves - through the petiole, stem and then to the roots. It is through the roots that plants usually absorb water. Some rootless plants absorb their moisture from the air. Some orchids are an example, as are lichens, the moss-like plants you see growing on fence posts and dry limbs of trees and rocks.

Many people do not realize that a portion of the water absorbed is lost by the plant into the atmosphere. This water that is lost escapes in the usually invisible form of water vapor. (If you get up early enough in the morning and see "dew-drops", those drops on the plants may not really be "dew". Dew is pure water condensed from the air. The drops on the plants may be water lost by the plant, in which case they will contain minerals and often leave a white deposit when they evaporate.) This loss of water vapor from plants is known as transpiration (tran'-spi-ra'-shun).

Most of the transpiration occurs through the stomates (sto'-mates) - openings in the leaves, so small they can only be seen with the microscope. The number of stomates present in leaves may range from a few thousand to over a hundred thousand per square centimeter.

Transpiration rates show great variation from one kind of plant to another, and for the same kind of plant under different growing conditions, such as temperature, water in the soil, and the amount of light.

The biological problem we are going to attempt to find an answer to will be concerned with the amount of transpiration from the leaves of a particular kind of tree during a given amount of time. We will do this by collecting some water from a known number of leaves, measuring the surface area of a leaf from the same tree, estimating the number of leaves on the tree, then determining the amount of water loss by the plant. Will this be an accurate measure of the water loss of the entire plant? Why?

3.2 Activity --- Leaf Surface and Water Loss

Materials Needed

Large plastic bags

A broad-leafed plant, preferably a tree
-- to be selected by the teacher

Graph paper

Ruler (preferably metric scale)

String or rubber bands cut in one place to give an elastic string for tying the bag to the branch

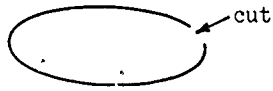


Fig. 3-2a

Labeling tags - these can be easily made out of a piece of heavy paper and a rubber band or piece of string (see Fig. 3-2b)

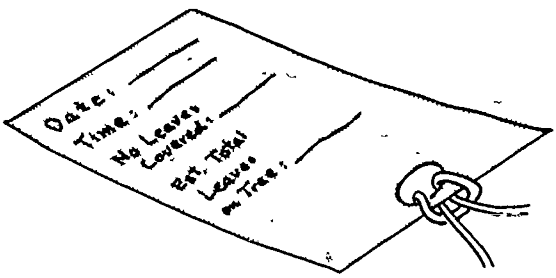


Fig. 3-2b

Procedure

You will be collecting part of the data while outdoors so first prepare a table similar to Table 3-2.

Table 3-2

Leaf Data

Tree Identity	Number of leaves covered	Est. total leaves on tree	Volume of water transpired	Area of average leaf

Study Figure 3-2c before leaving the classroom. The following instructions refer to this figure.

First Day

1. Label your tags as instructed by your teacher.

2. Working in teams as designated by your teacher, locate an appropriate branch on a broad-leafed tree. Count the leaves that will be used. Record the number of leaves in your data table.

3. Slip the plastic bag over the branch in such a way that the leaves you have counted will be sealed inside the bag and not crowded together.

4. Carefully gather the edges of the bag together around the stem and tie tightly with the string or cut rubber band. Whether using string or rubber band, it is best to pass the tie around the branch twice. The plastic bag is to remain on the tree for 24 hours.

5. Tie your label at the place where you tied on the bag. (See Figure 3-2c.)

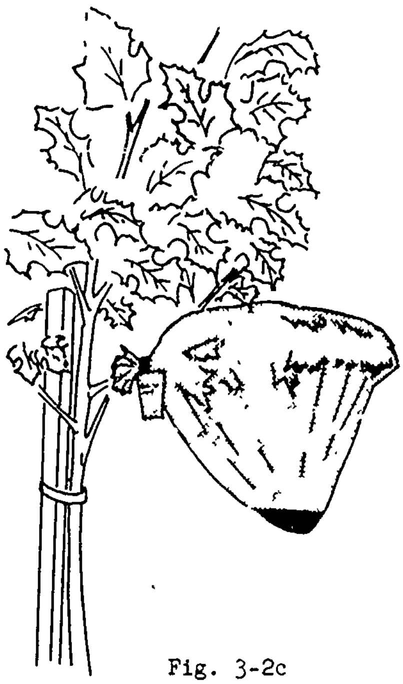


Fig. 3-2c

6. Look over the tree, remember how many leaves you enclosed in the bag, and estimate approximately how many leaves there are on the tree. Record your estimate.

7. Each person selects an average size leaf to take back to the classroom.

8. When back in the classroom, and while your leaf is still fresh, place the leaf on the graph paper laying it out as flat as possible, and sketch with pencil the outline the leaf. (See Figure 3-2d.). Discard the leaf.

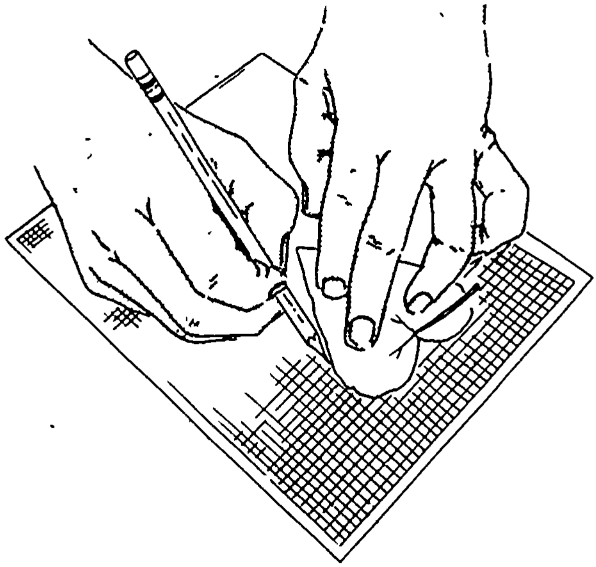


Fig. 3-2d

You should now have several important bits of data:

1. The number of leaves enclosed in your plastic bag.
2. Your estimated number of leaves on the tree.
3. A piece of graph paper with a closed curve marked on it, the boundary of which represents an outline of an average leaf (from the same tree).

Second Day

1. Remove the plastic bags and labels from your tree, being extremely careful not to spill any of the water. Bring the bag back to the classroom for measuring the amount of water collected. (If more convenient, do steps 2 and 3 where the activity took place.)
2. Pour the water, being careful not to spill any, into a jar or bottle. Gently shake or tap the bag to get as many droplets from the bag as possible. (Do not worry about the very small droplets left adhering

to the sides of the bag. At most, they will amount to only one or two milliliters.)

3. Seal the bottle with a tight lid. Attach to the bottle the label from the tree, making sure the number of leaves enclosed in the bag is indicated on your label and store it away for future use.

When you have collected and measured the accumulated moisture from the plastic bag you will have enough data to estimate how much moisture has transpired from your tree.

Now you are ready to proceed with the mathematical calculations to determine the area of your average leaf which was outlined on your graph paper.

3.3 Simple Closed Curves

It was mentioned earlier that your leaf outline was a closed curve. A curve is a set of points which can be represented by a pencil moving on a paper and made without lifting the pencil off the paper. A simple closed curve is one in which the drawing starts and stops at the same point, and no other point is touched twice. (See Figure 3-3a.)

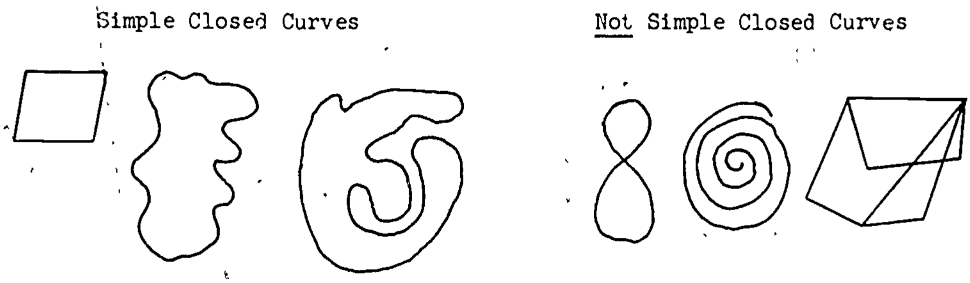


Fig. 3-3a

The most practical method that we have at our disposal for finding the area of a closed curve is to draw inside the closed curve the largest rectangles which will fit, compute their areas and then find or estimate the areas of the enclosed region which are not in our rectangles so that the areas may be added.

Exercise 3-3

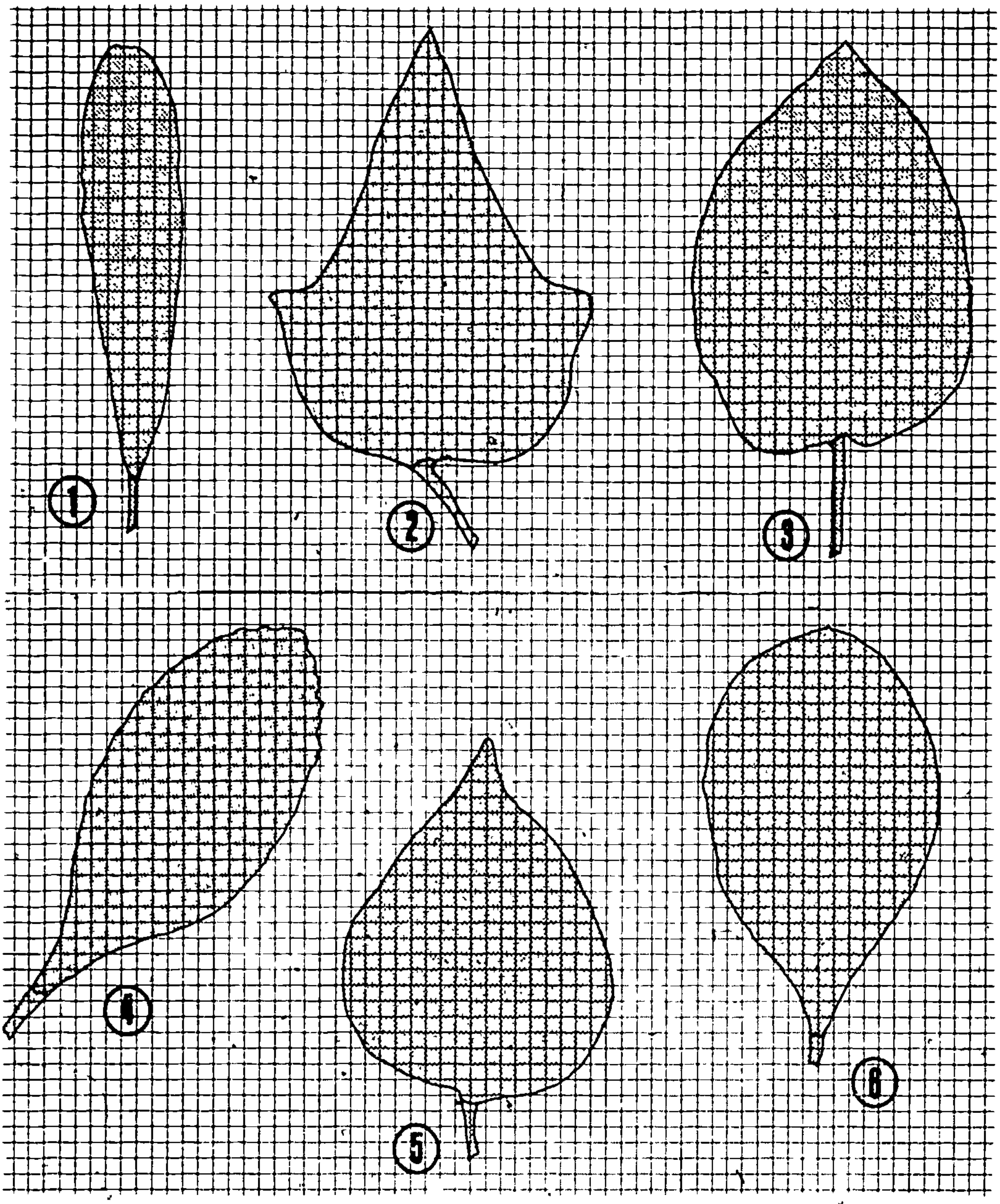
Trace on "10 × 10 to the inch" graph paper the leaf outlines in Figure 3-3b.

First construct the largest possible square or rectangle that will enclose only whole graph paper squares. Count the number of squares in a horizontal row and vertical row within this figure. The product of these two numbers will be the total number of squares within the figure. Record this product in the center of the figure, as shown in Figure 3-3c.

Construct as many smaller squares or rectangles around the perimeter of the first, count and record the number of squares in each. This should leave uncounted only those partial or whole squares that fall between the leaf outline and the sides of the geometric figures you constructed. It will take some estimating on your part to determine the number of squares around the sides, but with a little practice you will find that it can be done quite accurately.

Add all of the numbers to get the total number of graph paper squares enclosed by the leaf outline.

Fig. 3-3b



Find in square inches the area of the leaf outlines in drawings 1 through 6. (Do not include the area of the petiole.)

Now that you have had some practice finding areas inside leaf outlines, find in square inches the area of your leaf outline. (See Fig. 3-3c.)

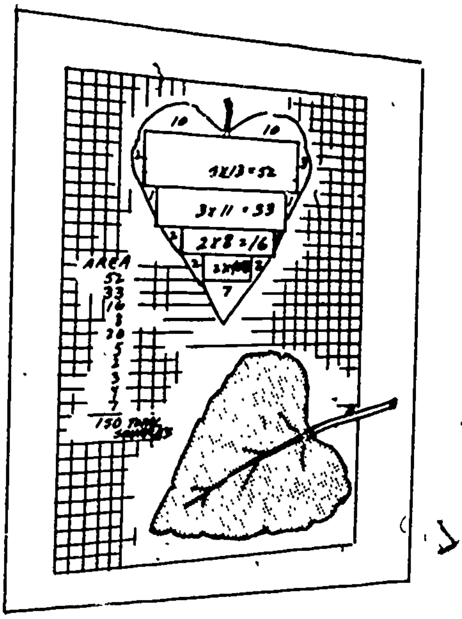


Fig. 3-3c

3.4 Conversion - Metric English

Since we use both English and metric units of measurement in the United States, it is often necessary to convert from one system to the other. In the laboratory this is particularly true when the solution to a problem is expressed in units of one (usually the metric) and the instruments with which one works are calibrated in the other. You will feel a real sense of accomplishment when you find that you can readily change measurements in one system to the comparable measurements in the other system. In a following exercise you will gain some practice in conversion, especially if the graph paper you use is marked in the English system.

The following conversion values are given to help in the solution of the problems found in the activity. A more complete treatment of metric-English conversion is given at the end of this chapter.

$$2.54 \text{ cm.} = 1 \text{ in.}$$

$$100 \text{ cm.} \approx 39.37 \text{ in.}$$

$$\left(\frac{100 \text{ cm.}}{2.54 \text{ cm.}} \approx 39.37\right)$$

As you already know, the smaller units of length in the metric system are obtained by dividing the meter by powers of 10. Thus, we define

$$1 \text{ decimeter} = \frac{1}{10} \text{ meter}$$

$$1 \text{ centimeter} = \frac{1}{10} \text{ decimeter} = \frac{1}{100} \text{ meter}$$

$$1 \text{ millimeter} = \frac{1}{10} \text{ centimeter} = \frac{1}{100} \text{ decimeter} = \frac{1}{1000} \text{ meter}$$

For longer units of length we simply multiply by powers of 10. Thus, by definition,

$$1 \text{ dekameter} = 10 \text{ meters}$$

$$1 \text{ hectometer} = 10 \text{ dekameters} = 100 \text{ meters}$$

$$1 \text{ kilometer} = 10 \text{ hectometers} = 100 \text{ dekameters} = 1000 \text{ meters}$$

To emphasize the simplicity of the relations involved in the metric system, we write these units in terms of meters, using scientific notation. (Will be covered in Section 3.7 .) Then the relationships look like this:

$$1 \text{ millimeter} = 10^{-3} \text{ meter}$$

$$1 \text{ centimeter} = 10^{-2} \text{ meter}$$

$$1 \text{ decimeter} = 10^{-1} \text{ meter}$$

$$1 \text{ meter} = 10^{0} \text{ meter} \quad (10^{0} = 1)$$

$$1 \text{ dekameter} = 10^{1} \text{ meters}$$

$$1 \text{ hectometer} = 10^{2} \text{ meters}$$

$$1 \text{ kilometer} = 10^{3} \text{ meters}$$

The definition and abbreviations for the metric units of length are summarized in Table 3-4a. In this table notice how useful scientific notation is in showing relationships to the basic metric unit of length, the meter.

Table 3-4a

Linear Metric Units

Name of Unit	Abbreviation	Equivalent in meters	Meter Equivalent in Scientific Notation
1 millimeter	1 mm	$\frac{1}{1000}$ m	10^{-3} m
1 centimeter	1 cm	$\frac{1}{100}$ m	10^{-2} m
1 decimeter	1 dm	$\frac{1}{10}$ m	10^{-1} m
1 meter	1 m	1 m	10^{0} m
1 dekameter	1 dkm	10 m	10^{1} m
1 hectometer	1 hm	100 m	10^{2} m
1 kilometer	1 km	1000 m	10^{3} m

Notice that all of the metric units other than meter use the word "meter" together with a prefix. These prefixes are also used to name other units of measure (weight, volume) in the metric system.

In the U.S.A., hectometer, dekameter, and decimeter are seldom used. The meter, centimeter, millimeter, and kilometer are in very common usage and we shall devote most attention to them.

Two other prefixes which we should mention are mega, meaning million, and micro, meaning one-millionth. Thus,

Prefix	Meaning
mega	$1,000,000 = 10^{6}$
micro	$\frac{1}{1,000,000} = 10^{-6}$

You often hear now of 3 megatons (3 million tons), 1 megacycle (1 million cycles). Even the slang "megabuck" uses this classical Greek prefix!

In these days of nuclear and atomic studies very small quantities are often studied and lengths as small as one-millionth of a meter are common. The micron is defined as:

$$1 \text{ micron} = \text{one-millionth of a meter} = 10^{-6} \text{ m} = \frac{1}{1,000,000} \text{ m}$$

Thus, 3 microns = $3 \times 10^{-6} = \frac{3}{1,000,000}$ meters = 3×10^{-3}mm = $\frac{3}{1000}$ millimeters, since 1 micron = 10^{-6} meters = 10^{-3} millimeters. The usual symbol for micron is the Greek letter " μ " (read Mu). Hence

$$14\ \mu = 14 \text{ microns} = 14 \times 10^{-6} \text{ m} = \frac{14}{1,000,000} \text{ m} .$$

We will now return to the original objective of developing some conversions that will be of direct use in the solution of the problems in this exercise, and look at the relationships involved in area. The inch has been defined as being exactly equal to 2.54 cm and since centimeters can be changed to other metric units of length merely by multiplying or dividing by 10, then

$$1 \text{ inch} = 25.4 \text{ mm} = 2.54 \text{ cm} = 0.254 \text{ dm} = 0.0254 \text{ m} .$$

Notice the placement of the decimal as we go from short to longer units of length.

3.41 Metric Units of Area

We have learned how to find the area of the interior of a simple closed curve for a variety of simple curves. We chose the area of a square region as the best unit to use in measuring the area of the interior of such a closed curve.

The metric unit for measuring areas is also a square region. We use as a basic unit the area of a square region with each edge of length one meter. The area of the interior of this square region is called one square meter (abbreviation m^2).

If you have a centimeter rule available you should draw a square, each side 1 cm, in order to get some idea of the size of one square centimeter. From 1 m = 100 cm $\approx$ 39.37 inches we see that 1 cm $\approx$ .3937 inch, because

if 100 cm $\approx$ 39.37 inches, then 1 cm $\approx \frac{1}{100}$ of 39.37

or .3937 inch. Thus, 1 cm $\approx$ .4 inch .

Compared to the basic unit of one square meter, the square centimeter is small indeed. Remember that 1 cm = $\frac{1}{100}$ m , hence

$$1 \text{ cm}^2 = \frac{1}{100} \times \frac{1}{100} \text{ m}^2 = \frac{1}{10,000} \text{ m}^2 .$$

Here is another instance in which you may prefer to use exponential notation and write instead

$$1\ cm^2 = 10^{-4}\ m^2 .$$

Since the meter is the basic unit of length and the square meter the basic unit of area, it is important to give the various units of area in terms of square meters. The most commonly used units are listed in Table 3-41b.

Table 3-41b

Unit of Length	Unit of Area	Equivalent Area in Square Meters
millimeter	square millimeter	$\frac{1}{(1000)^2}\ m^2$
centimeter	square centimeter	$\frac{1}{(100)^2}\ m^2$
kilometer	square kilometer	$(1000)^2\ m^2$

Here again we see that it is convenient to use exponential notation and write,

$$1\ mm^2 = 10^{-6}\ m^2 = \frac{1}{10^6}\ m^2$$

$$1\ cm^2 = 10^{-4}\ m^2 = \frac{1}{10^4}\ m^2$$

$$1\ km^2 = 10^6\ m^2$$

In Figure 3-41a the diagram on the left is a picture of a <u>one centimeter square closed region</u>; the one on the right is a <u>one millimeter square closed region</u>.

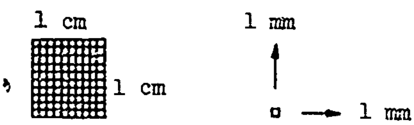


Fig. 3-41a

<u>Exercise 3-41</u>

1. How many square millimeters are there in a square centimeter?

2. How many square centimeters are there in a square meter?

3. How many square millimeters are there in a square meter?

4. Draw a 3 cm square. Draw also a rectangle whose area is 3 square centimeters. Which is larger?

5. A rug is 2 meters by 3 meters. Find its perimeter and area.

6. The floor of a boy's room is in the shape of a rectangle. The length and width are measured as 4 meters and 3 meters. There is a closet 1 meter long and 1 meter wide built into one corner. What is the floor area of the room (outside the closet)?

7. Convert the areas found in Exercise 3-3 to square decimeters.

   [Now convert to dm^2 the area of <u>your</u> leaf outline and record in Table 3-2.]

3.5 <u>Volume - Metric Units</u>

The metric unit for measuring <u>volume</u> is a cubical solid. The length of each edge of this cube is 1 meter. The volume of this cube is thus 1 cubic meter (abbreviation 1 m^3).

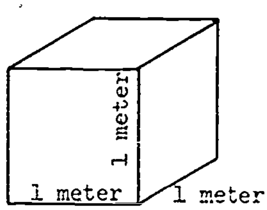


Fig. 3-5a

The cubic meter is a rather large unit of volume. A much smaller unit is the cubic centimeter (cm^3 or cc). As we have seen, the centimeter is about .4 in ; hence, the cm^3 is a cube about the size of a small sugar cube. To suggest the size of the cubic meter, note that

$$1 \text{ cc} = \frac{1}{100} \times \frac{1}{100} \times \frac{1}{100} \text{ m}^3 = \frac{1}{1,000,000} \text{ m}^3 .$$

In scientific notation,

$$1 \text{ cc} = 10^{-6} \text{ m}^3 \quad \text{and} \quad 1 \text{ m}^3 = 10^{6} \text{ cc} .$$

By similar calculations we can find the commonly used multiples and subdivisions of the cubic meter. These are displayed in Table 3-5a.

Table 3-5a

Unit of Length	Unit of Volume	Equivalent Volume in Cubic Meters
millimeter	mm^3	$\frac{1}{(1000)^3}$ m^3
centimeter	cc or cm^3	$\frac{1}{(100)^3}$ m^3
kilometer	km^3	$(1000)^3$ m^3

Here, even more than before, we see the convenience of the exponent notation in writing

1 cubic millimeter $= 10^{-3} \times 10^{-3} \times 10^{-3}$ cubic meters $= 10^{-9}$ cubic meters

1 cubic centimeter $= 10^{-2} \times 10^{-2} \times 10^{-2}$ cubic meters $= 10^{-6}$ cubic meters

1 kilometer $= 10^{3} \times 10^{3} \times 10^{3}$ cubic meters $= 10^{9}$ cubic meters

Look at the calculations above. You see that

$$10^3 \times 10^3 \times 10^3 = (10^3)^3 = 10^{3 \times 3} = 10^9$$

$$10^{-2} \times 10^{-2} \times 10^{-2} = (10^{-2})^3 = 10^{(-2) \times 3} = 10^{-6}$$

$$10^{-3} \times 10^{-3} \times 10^{-3} = (10^{-3})^3 = 10^{(-3) \times 3} = 10^{-9}$$

<u>General Property</u>: If a and b are any positive or negative integers, then

$$(10^a)^b = 10^{ab} .$$

Exercise 3-5

1. Complete each of the following:

   Example. There are $(1000)^3$ or <u>1,000,000,000</u> m^3 in 1 km^3.

   (a) There are 10^3 or ________ mm^3 in a cc.

   (b) There are $(\frac{1}{100})^3$ or ________ m^3 in a cc.

   (c) There are $(\frac{1}{1000})^3$ or ________ m^3 in a mm^3.

   (d) There are $(10^6)^3$ or ________ mm^3 in a km^3.

2. A rectangular solid has dimensions of 6 cm, 7 cm, and 80 mm. Calculate the volume of the interior of this solid. Recall that the volume of the interior of a rectangular solid is equal to the product of the measure of the length, width, and height, when the measurements are expressed <u>in the same unit</u>.

3. What is the volume of the interior of a rectangular solid whose height is 14 mm and whose base has an area of 36.5 sq cm ?

3.51 Metric Units of Mass and Capacity

The metric unit for the measure of <u>mass</u> is defined as the mass of water contained by a vessel with a volume of one cubic centimeter.

> The mass of one cubic centimeter of pure water at 4° Celcius (Centigrade)(1) is called a gram.

This definition of mass is a very convenient definition, for when we know the volume of a container we immediately know the mass of water it can contain. For example, if the volume of the interior of a container is 500 cc, then the mass of water it can contain is 500 grams. The important thing to note in this definition is that the numerical measures are the same.

---

(1) The temperature scale on which the freezing and boiling points of water are 0 degrees C and 100 degrees C respectively. At an International General Conference on Weights and Measures it was recently decided to recognize the name of the man who invented the scale - hence, Celcius.

When we speak of the volume of a box or other container, we frequently use the term capacity. By the capacity of a container we simply mean the total volume which the container will hold.

In talking about the volume of liquid a container will hold, we frequently use special units, such as pint, quart, and gallon, in the English system. Thus, we may say the capacity of a tank is a certain number of gallons, and its volume is so many cubic feet.

In the metric system the usual unit of capacity is the liter (abbreviated ℓ). One liter is defined as the capacity of a cubical box with edge of length 10 cm (1 decimeter). Thus, one liter means a volume of 1000 cc . We say its capacity is one liter. It can contain a mass of 1000 grams (or one kilogram) of water.

One liter is approximately one quart, or more precisely,

$$1 \text{ liter} = 1000 \text{ cc} \approx 1.056712 \text{ qt} .$$

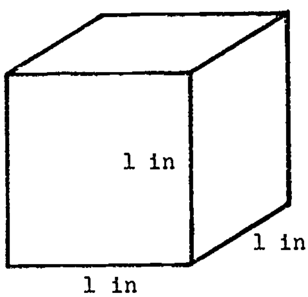
The other most common metric measures of capacity are the

milliliter (ml) = .001 liter = 1 cc
kiloliter (kl) = 1000 liters .

A mass of 1000 kilograms is called one metric ton. How many grams would equal one metric ton? The metric ton is the mass of 1 kiloliter of water.

There are several abbreviations commonly used for the gram. The abbreviations, g or gm , are both generally accepted.

In the English system of measures units also may be expressed in various ways. For example, a cubic inch could be abbreviated cu. in. or since it is the product of an inch times an inch times an inch it can be expressed as in^3 .



The same is true of other units - ft^3, yd^3, etc.

Exercise 3-51

1. The volume of a jar is 352.8 cc . What is the mass of the water it can contain, expressed in:
   (a) grams?
   (b) kilograms?

2. (a) What is the capacity in milliliters of a rectangular tank of volume 673.5 cc ?
   (b) What is its capacity in liters?

3. A cubical tank measures 6 feet 9 inches each way and is filled with water.
   (a) Find its volume in in^3 .
   (b) Find its volume in ft^3 . Recall that $1728\ in^3 = 1\ ft^3$.
   (c) Find the weight of the water. $1\ ft^3$ of water weighs about 62.4 lb.

4. The dimensions of the tank in Problem 3 are about 2 meters each way.
   (a) Find its volume in cubic meters.
   (b) Find its contents in liters. Recall that there are 1000 liters in a cubic meter.
   (c) What is the mass of the water? Recall that 1 liter of water has a mass of 1 kilogram .

5. How did the time needed to solve Problem 4 compare with the time needed to solve Problem 3? What is the main advantage of computing in the metric system?

6. A tank has a volume of 2500 cc .
   (a) What is the capacity of the tank in milliliters?
   (b) How many kilograms of water will the tank hold?
   (c) How many metric tons of water will the tank hold?

7. A cubical box has edges of length 30 cm .
   (a) What is the volume of the box in cc ?
   (b) What is the capacity in liters?
   (c) How many kilograms of water will the box hold? (Assume that it is water tight, of course!)

8. The volume of the sun is estimated to be about 337,000 million million cubic miles, or 3.37×10^{17} cubic miles.

(a) Using the fact that 1 mile $\approx$ 1.6 kilometers, express the volume of the sun in cubic kilometers. (Simply indicate multiplications in your answer if you wish.)

(b) Express the sun's volume in cc, leaving your answer in the form of an indicated multiplication.

9. The British Imperial gallon, used in Canada and Great Britain, is equivalent to $\approx$ 1.20094 U.S. gallons, or

1 British Imperial gal. $\approx$ 1.2 U.S. gal.

(a) When you buy 5 "gallons" of gasoline in Canada, how many U.S. gallons do you receive?

(b) How many Imperial gallons are required to fill a barrel which holds 72 U.S. gallons?

[Now return to the water that was collected from the leaves on a tree. Measure this as a volume in milliliters and record in Table 3-2.]

Brief Summary of Relations among Units

The following table summarizes much of the work in the metric system. From it you can derive all the multiples and subdivisions of units of area, length, volume, mass, and capacity.

Table 3-5c

Linear

10 millimeters	(mm)	=	1 centimeter (cm)
100 centimeters	(cm)	=	1 meter (m)
1000 meters	(m)	=	1 kilometer (km)

Capacity

1000 milliliters (ml) = 1 liter (ℓ) = 1000 cc

Mass

1000 milligrams	(mgm)	=	1 gram (gm)
1000 grams	(gm)	=	1 kilogram (kgm)
1000 kilograms	(kgm)	=	1 metric ton

Some important conversion relations between corresponding English and metric units are listed for reference in Table 3-5d.

Table 3-5d

Linear

1 inch	=	2.54 cm (definition of the inch)
1 meter	≈	39.37 in
1 cm	≈	.39 in
1 km	≈	0.62 miles
1 mile	≈	1.61 km

Capacity

1 liter	≈	1.0567 qt
1 liter	≈	0.2642 gal
1 gal	≈	3.785 liters

Mass

1 kilogram	≈	2.2 lbs
1 pound	≈	454 grams

3.6 Scientific Notation

Very often people working with scientific information find that the numbers they must work with are immense. In astronomy the measures of the distances from star to star are in millions of millions of miles. In fact, astronomers have converted the vast measures of distance into a unit called a light year. One light year is the approximate distance light will travel in 1 year. The speed at which light travels is approximately 186,000 miles per second. To find out how far light would travel in 1 year, you would need to find the product: $186,000 \times 60 \times 60 \times 24 \times 365 = 5,865,595,000,000$ miles. Why were those particular numbers used in this multiplication? Can you read the answer?

Astronomers often need to compute with light years. Can you imagine the work involved if you were asked to square 5,865,595,000,000? Since no one, at least for a few more years, is going to measure this distance with a yard stick, we can for purposes of computation round off to 5,900,000,000,000. All of those zeros should give us a clue to an easier way of writing such a large number. Let's use our knowledge of exponential notation.

If $100 = 10 \times 10 = 10^2$

and $1000 = 10 \times 10 \times 10 = 10^3$

and $10{,}000 = 10 \times 10 \times 10 \times 10 = 10^4$

then $100{,}000{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 10^{11}$

Our number of 5,900,000,000,000 could be expressed as a number times a power of ten. If we write it as 5.9×10^{12}, it is expressed in scientific notation.

Definition: A number is said to be expressed in scientific notation if it is written as the product of a decimal numeral between 1 and 10 and the proper power of 10. If the number itself is a power of 10, the first factor is 1 and need not be written.

Exercise 3-6

1. (a) Is 15×10^5 scientific notation? Why, or why not?
   (b) Is 3.4×10^7 scientific notation? Why, or why not?
   (c) Is 0.12×10^5 scientific notation? Why, or why not?

2. Write the following in scientific notation:
   (a) 5687 (b) 14 (c) $3\frac{1}{2}$ million (d) 135
   (e) 14,650 (f) 35,000,000,000
   (g) $7\frac{1}{2}$ billion (h) the product of 50 and 70

3. Write the following in decimal notation:
   (a) 3.7×10^6 (b) 4.7×10^5 (c) 5.721×10^6
   (d) 2.25×10^7 (e) 2.8×10^8 (f) $.653 \times 10^{10}$
   (g) 14.75×10^4 (h) $36.2 \quad 10^3$ (i) 386×10^5
   (j) $.046 \times 10^7$ (k) 6.821×10^3 (l) $.0038 \times 10^3$

4. Since the earth does not travel in a circular path, the distance from the earth to the sun varies with the time of the year. The average distance has been calculated to be about 93,000,000 miles.
   (a) Write the above number in scientific notation. The smallest distance from earth to the sun would be about $1\frac{1}{2}$ per cent less than the average; the largest distance would be about $1\frac{1}{2}$ per cent more than the average.

(b) Find $1\frac{1}{2}$ per cent of 93,000,000.

(c) Find approximately the shortest distance from earth to the sun.

(d) Find approximately the greatest distance from earth to the sun.

3.7 Significant Digits

When you are calculating with approximate numbers, the accuracy of the conputation is dependent on the number of significant digits.

As an example we will use a leaf whose traced outline enclosed approximately 150 squares on a sheet of graph paper. This is not an exact number, as in counting the number of people in your classroom, because you had to think of parts of squares adding up to whole squares to arrive at your total. This type of counting is an approximation, the same as any measurement is an approximation. If our graph paper was marked in $\frac{1}{4}$-inch squares, then our 150 squares represent $150 \div 16$ (number of $\frac{1}{4}$-inch squares in a square inch) = 9.375 square inches.

This leads us to an interesting dilemma. We guessed we had about 150 squares. Not $150\frac{1}{2}$ or $150\frac{1}{4}$ or 150.17 or 150.179387654, but 150 seemed as closely as we wanted to guess in all honesty. When the figure of 150 quarter-inch squares is changed to square inches we arrive at a number 9 plus 375 thousandths which sounds rather exact. Can we increase the accuracy of our measurement by doing some mathematical computation? If so, we go one more step and change this answer in square inches to square decimeters. We find, by using the conversion ratio of 1 square inch = .064516 square dm, that 9.375 square inches = $9.375 \times .064516$ = .6048375 square decimeters.

Did you imagine when you were given approximately 150 squares that it would come out a number like .6048375 ? Read that number in words. You must be about ready to say "Let's round it off a bit." But how far; to .604838, or .6048, or maybe .605 ? Of course, .60 is more than half way to "1" so maybe we should just say "1". The truth is, 1 square decimeter is over $15\frac{1}{2}$ square inches and our 150 squares was only a little over 9 square inches . If we round off too far, our numbers become too inaccurate to use.

Our purpose in this unit will be to decide upon some useful rules which will aid us in deciding how many digits should be used in a problem.

If we were to measure desks with an ordinary yard stick marked off in inches, $\frac{1}{2}$ -inches, $\frac{1}{4}$ -inches, and $\frac{1}{8}$ -inches, we could find the length of the desk to the nearest $\frac{1}{8}$ -inch. If we also had an expensive engineer's tape marked off in 64ths of an inch (some are marked in 100ths of an inch) we could get nearer in our measurement to the actual length of the desk. We would say the measurement with the smallest unit was the <u>more</u> <u>precise</u>. Thus $32\frac{3}{64}$ inches is a more precise measurement, or has greater precision than $32\frac{1}{8}$ inches.

Is 5264 inches as precise a measurement as $1\frac{1}{2}$ inches? Why?

Is 375 feet more precise than 2 yards? Why?

Does $64\frac{1}{16}$ inches have greater precision than $128\frac{1}{4}$ inches? Why?

The <u>precision</u> of a measurement depends upon the <u>magnitude</u> (size) of the smallest unit used in the measure, not upon the <u>number</u> of <u>units</u> in the measure.

In science, decimal notation is often used. We will make an agreement about what is meant when we write a number in decimal notation. When we write that a length has a measure of 17.62 inches we understand that the measurement has been made with an error no greater than .005 inches (one half the smallest unit used in measuring). Thus the measure 17.62 is correct to the second decimal place to the right of the decimal point. In the $\pm$ notation this would be equivalent to writing $(17.62 \pm .005)$ inches for the measurement. By this agreement, each of the four digits in 17.62 serves a real purpose, or is "<u>significant</u>".

In measures like 1462, 3.1 and .29637 all the digits are understood to be significant. But in a numeral like 0.008 the three zeros simply serve to fix the decimal point. In this case we say only the 8 is a significant digit.

In the number 2.008, all four digits (2, 0, 0, 8) are significant. In a number like 0.0207 the first two zeros are <u>not</u> significant but the third is. Thus, 0.0207 has three significant digits, 2, 0, 7.

When we write 2960 ft. or 93,000,000 miles, it is <u>not</u> <u>clear</u> which, if any, of the zeros are significant. Therefore we agree that they are <u>not</u> significant in order to avoid assuming a degree of accuracy which may not be present. Thus 2960 ft. has three significant digits (2, 9, 6) in its measure. The measurement is precise to the nearest 10 ft. and the greatest possible error is 5 ft.

For our purposes whenever we wish any of the zeros at the end of a numeral like 28,000 or 2960 to be significant, we will indicate the final zero which is significant by placing a short bar under it. Thus, $296\underline{0}$ ft. indicates a measurement correct to the nearest foot. The measure has four significant digits (2, 9, 6, 0). The measurement $93,\underline{0}00,000$ miles is correct to the nearest 100,000 miles. The numeral has three significant digits (9, 3, 0).

Definition: A digit in a decimal numeral is said to be a "significant digit" if it serves a purpose other than simply to locate (or emphasize) the decimal point.

Some Examples

Measurement	Number of Significant Digits
.057	2
3.492	4
3.004	4
167	3
160005	6
$49\underline{0}00$	3
$4900\underline{0}$	5
47.40	4
47.4	3
47.000	5

When a number is written in scientific notation we agree that all of the digits in the first factor are significant.

Thus,

$$73,000 \text{ ft.} = 7.3 \times 10^4 \text{ ft.}$$
$$73,\underline{0}00 \text{ ft.} = 7.30 \times 10^4 \text{ ft.}$$
$$73,00\underline{0} \text{ ft.} = 7.3000 \times 10^4 \text{ ft.}$$

Also, the measurements

2.99776×10^{10} cm/sec for the velocity of light has 6 significant digits
2.57×10^{-9} cm for the radius of the hydrogen atom has 3 significant digits
2.8×10^{11} dollars for the National Debt in 1957 has 2 significant digits
4.800×10^{8} has 4 significant digits.

In the last case, the two final zeros are significant. If they were not, the number should have been written as 4.8×10^8. It is this possibility of indicating significant digits in scientific notation which is another of the advantages of the notation.

Exercise 3-7

	Measurement	Number of Significant Digits
(1)	8	______
(2)	.01	______
(3)	457	______
(4)	3.32	______
(5)	3005	______
(6)	36.70	______
(7)	6700	______
(8)	23.000	______
(9)	620.03	______
(10)	.0028	______

J.F.F. If a bottle and its cork cost two and a half cents, and the bottle alone costs two cents more than the cork, what is the cost of the cork?

3.71 Multiplying and Dividing Measurements

In Section 3.3 "Estimating Area" we reviewed the formula for the area of a rectangle. Suppose in measuring the length and width of a rectangle with a ruler marked only in inches we found the measurements to be 30 inches and 20 inches. These would be recorded as length, $30'' \pm .5''$; width, $20'' \pm .5''$. Thus the length might be as small as 29.5 inches and the width as small as 19.5 inches. The length might be as large as 30.5 inches and the width as large as 20.5 inches.

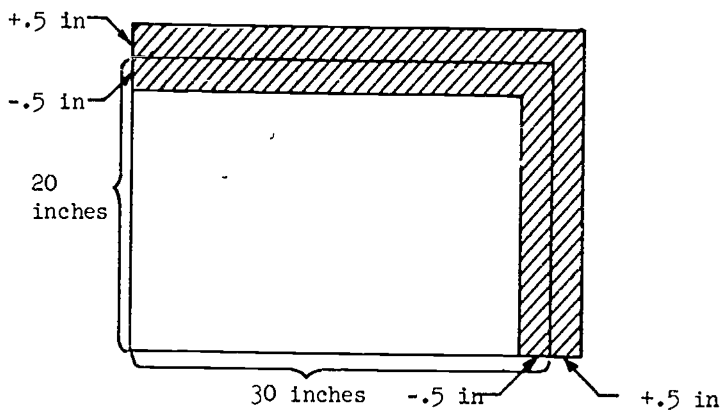


Fig. 3-71a

Look at Figure 3-71a to see what this means. The outside lines show how the rectangle would look if the dimensions were as large as possible. The inner lines show how it would look if the length and width were as small as possible. The shaded part shows the difference between the largest possible area and the smallest possible area with the given measurements.

Let us see what the differences are. The given measurements are 20 in × 30 in. The smallest possible dimensions are (20 - .5) in × (30 - .5)in and the largest possible dimensions are (20 + .5) in × (30 + .5) in. With a given measured area of 20×30 square inches or 600 square inches, the least possible area would be:

$$(30 - .5)(20 - .5) \ \text{in}^2 .$$

By the distributive property we have

$$(600 - 10 - 15 + .25) \ \text{in}^2 = 575.25 \ \text{in}^2 .$$

The greatest possible area would be:

$$(30 + .5)(20 + .5) \ \text{in}^2 = (600 + 10 + 15 + .25) \ \text{in}^2$$
$$= 625.25 \ \text{in}^2 .$$

The difference between the greatest and the least is 50 square inches.

Therefore, if we wish to be very careful about our statements, we must make clear what is meant when we say the area of the rectangle is 600 in^2. As we have seen, this answer is not correct to 1 square inch, but, on the other hand, it is correct to something less than 100 in^2. If we wish to indicate the situation the best way we know it, we may write the area as $(600 \pm 25) \ \text{in}^2$. (We have chosen here to round the greatest possible area 625.25 in^2 to 625 in^2. You might prefer to write 575.25 $\text{in}^2 \leq$ the area $\leq$

$625.25\ in^2$.) If we choose to write the area as $600\ in^2$, we must interpret the numeral as having one significant digit. This says the area is given to within $100\ in^2$ and hence lies between $550\ in^2$ and $650\ in^2$. This is correct, but not quite as good a result as our answer ($600 \pm 25\ in^2$).

It is difficult to give a rule for the multiplication of approximate measurements which will be completely satisfactory for every circumstance. However, when data are expressed in decimal form, a rough guide can be suggested for finding a satisfactory product.

The number of significant digits in the product of two numbers is not more than the number of significant digits in the less accurate factor.

Note that this states that the number of significant digits is not more than the number of significant digits in the less accurate factor -- it does not assure you that there will be that many!

As an illustration of this principle, consider the following problem: What is the area of a rectangle with sides measured as 10.4 cm and 4.7 cm?

To find the area we might multiply 10.4 by 4.7 to obtain 48.88 . Now there are three significant digits in 10.4 and only two significant digits in 4.7 . Hence, the product cannot have more than two significant digits and we round the area to $49\ cm^2$. Hence, the area of the rectangle is said to be $49\ cm^2$. If we wish to find a better estimate of the possible error, we must use the " $\pm$ " notation we used earlier.

Using the distributive property,

$$(10.4 + .05)(4.7 + .05) = 48.88 + .52 + .235 + .0025 = 49.6375 \text{ maximum}$$

and

$$(10.4 - .05)(4.7 - .05) = 48.88 - .52 - .235 + .0025 = 48.1275 \text{ minimum.}$$

We see that $48.1\ cm^2 <$ the area of the rectangle $< 49.6\ cm^2$. If we use only two significant figures, we see that the area lies between 48 and $50\ cm^2$. Hence, $(49 \pm 1)\ cm^2$ is an acceptable answer.

You might ask, why not round the numeral 10.4 to 10 and work only with two significant digits in each factor? Then we would get

$$10\ cm \times 4.7\ cm = 47\ cm^2$$

for the area, and we see that this is not correct to two significant figures.

For such reasons as this, we ordinarily agree to the following general procedure when multiplying two factors which do not have the same number of significant digits.

If one of the two factors contains more significant digits than the other, round off the factor which has more significant digits so that it contains only one more significant digit than the other factor.

Suppose we wish to find the circumference of a circle with the diameter equal to 5.1 mm. The circumference $C = \pi d$. What value of π shall we use? Since the diameter 5.1 mm is given to two significant digits we use three significant digits for π or $\pi \approx 3.14$. Then

$$C = \pi d \approx 3.14 \times 5.1 \approx 16.014,$$

which we round to 16, since only two digits are significant in the product. Hence, the circumference is approximately 16 mm .

If we were dealing with a large circle with diameter measured as 1012 inches, then we would use $\pi \approx 3.1416$ and round the result of the multiplication $C \approx (3.1416)(1012) \approx 3179.2992$ to four significant digits (3179).

Division is defined by means of multiplication. Therefore, it is reasonable to follow the procedure used for multiplication in doing divisions involving approximate data.

When a multiplication or division involves an exact number, such as 2 in the formula for the circumference of a circle ($C = 2\pi r$), the approximate number determines the number of significant digits in the answer.

We ignore the exact number in determining the significant digits in the answer. An exact number is a number that is not found by measuring, but can be found by counting.

Exercise 3-7la

1. Multiply the following approximate numbers:
   (a) 4.1×36.9
   (b) 3.6×4673
   (c) $3.76 \times (2.9 \times 10^4)$

2. Divide the following approximate numbers:
   (a) $3.632 \div 0.83$
   (b) $0.000344 \div 0.000301$
   (c) $(3.14 \times 10^6) \div 8.006$

3. Suppose a rectangle is $2\frac{1}{2}$ inches long and $1\frac{1}{2}$ inches wide. Make a drawing of the rectangle. Show on the drawing that the length is $(2\frac{1}{2} \pm \frac{1}{4})$ inches and the width $(1\frac{1}{2} \pm \frac{1}{4})$ inches. Find the largest area possible and the smallest area possible, and find the difference, or uncertain part. Now find the area using the measuring dimensions, and find the result to the nearest $\frac{1}{2}$ square inch.

4. Find the area of a rectangular field which is 835.5 rods long and 305 rods wide.

5. The circumference of a circle is stated $C = \pi d$, in which d is the diameter of the circle. If π is given as 3.1 1593, find the circumferences of circles whose diameters have the following measurements:
   (a) 3.5 in.
   (b) 46.36 ft.
   (c) 6 miles

6. A machine stamps out parts each weighing 0.625 lb. How much weight is there in 75 of these parts?

7. Assuming that water weighs about 62.4 lb per cu ft, what is the volume of 15,610 lbs of water?

J.F.F. Suppose the distance around the equator is 25,000 miles and that the surface is quite smooth and circular in section. If a steel band made to fit tightly around it is then cut, and a piece eighteen feet long welded into it, how loose will the ring be? In other words, what will be the size of the gaps all around between the inside of the ring and the earth's surface? Could you slip a piece of paper under it? Crawl under it?

There are many rough rules for computing with approximate data, but they have to be used with a great deal of common sense. They won't work in all cases. The modern high speed computing machine, which adds or multiplies thousands of numbers per second, needs to have special rules applied to the data which are fed into it.

In the example we used at the first of Section 3-7, we needed to change the measure of approximately 150 $\frac{1}{4}$ -inch squares to square decimeters. The numbers we needed to use in our computations were:

(150) $\frac{1}{4}$ -inch squares

(16) $\frac{1}{4}$ -inch squares in each square inch

.064516 dm^2 per in^2 .

How many significant digits in 150, 16, and .064516?

Your answer should be 2, 2, and 5, respectively. Based upon the preceding discussion, how many significant digits should be used in any one factor in this problem? What is the greatest number of significant digits which may be in the product?

Following our rule for factors having different numbers of significant digits we will round off .064516 to .0645 (one more significant digit than the other factors). Now our computation for changing 150 $\frac{1}{4}$ -inch squares to square decimeters would be:

150 (2 significant digits) ÷ 16 (2 significant digits)
≈ 9.4 (2 significant digits)

9.4 (2 significant digits) × .0645 (3 significant digits)
≈ .61 (2 significant digits)

Answer: .61 dm^2

Exercise 3-71b

1. Take the leaf outline areas from Exercise 3-3 which you converted to dm^2 units and round them off to the correct number of significant digits.

2. If the leaf outline enclosed 180 squares on a graph paper with quarter-inch squares, calculate its area in square decimeters.

3. (a) If the approximate area of a leaf were 210 quarter-inch squares, what would be the area in square decimeters?
   (b) Express the answer to (a) in square centimeters.
   (c) Express the answer to (a) in square millimeters.

4. If a leaf were outlined on graph paper with $\frac{1}{2}$ -inch squares and the approximate area was computed at 8 squares, what would be the approximate area in square decimeters?

5. (a) Using your knowledge of significant digits re-write the area of your average leaf outline (from Table 3-2).
   (b) Express the answer to (a) in cm^2.
   (c) Express the answer to (a) in mm^2.
   (d) Express the answer to (a) in m^2 (be careful).
   (e) Express the answer to (a) in km^2.

So far we have concerned ourselves only with the surface area of a single leaf. To continue our investigation of transpiration and the math that applies, we will calculate the amount of water lost by one leaf by using the leaves in our previous problems. Recall the statement that was made at the beginning of this chapter on the amount of water lost by leaves. Let us assume that in temperate regions an average water loss would be <u>.4 grams per square decimeter per hour</u>. If we use, for example, a leaf with a surface area of $.6\ dm^2$, then we can calculate the water loss as:

$$.4\ g\ (\text{per } dm^2 \text{ per hour}) \times .6\ dm^2 \times 24 \text{ hours}.$$

Then $.4 \times .6 \times 24 \approx 5.8$ grams of water lost by this leaf in 24 hours.

> You can calculate the theoretical volume of water lost by the tree you selected earlier in this activity by using the following outlines. Refer to Table 3-2 for the data needed.

Exercise 3-71c

Complete the following computations:

(1)	Number of leaves on the tree	______		(1)
(2)	Average area of a leaf	______	dm^2	(2)
(3)	Area of leaves on the tree	______	dm^2	(1) × (2)
(4)	Amount of water transpired per day per dm^2 of leaf area	9.6 gm/	dm^2	(4)
(5)	Amount of water transpired per day by tree (theoretical)	______	gm	(3) × (4)

To find the amount of water lost by the tree based on the amount of water you collected in the plastic bag, use the following outline:

(6)	Number of leaves on the tree	____		same as (1)
(7)	Number of leaves enclosed in the plastic bag	____		(7)
(8)	Ratio of number of leaves on the tree to number enclosed in the plastic bag	____		(1) ÷ (7)
(9)	Volume of water trapped in bag	____	ml	(9)
(10)	Daily transpiration of tree (based on your measurement)	____	gm	(10)
		(1 ml = 1 gm)		

Compare your answer on line 10 to your answer on line 5. If these values are different, can you identify the factor or factors listed at the beginning of this chapter that might account for the differences?

We can continue our investigation by considering water loss by a complete plant or acres of plants. Calculations of this nature will involve rather large numbers. Our measurements will not be exact but at least a reasonable estimate. We will continue to use the concepts of significant numbers and scientific notation.

Exercise 3-71d

A young (2-year) mulberry tree was found to have approximately 400 leaves with an average surface area of 1.3 square decimeters. One reference gives the average number of stomates per cm^2 (lower side only) on mulberry leaves as 48,000. Let us assume that this tree has an average water loss of .5 grams per dm^2 per hour. Solve the following. (Express answers in scientific notation.)

1. The number of stomates per dm^2.
2. The number of stomates per average leaf.
3. The approximate number of stomates on the tree.
4. The water loss per day from an average leaf.
5. The water loss per day by the tree.
6. The water loss per year (365 days) by the tree (in grams).
7. The number of liters of water lost by the tree in one year.
8. The number of gallons of water lost by the tree in one year.

Chapter 4

MUSCLE FATIGUE: PERCENT; MEAN, MEDIAN AND MODE; INFORMAL EXTRAPOLATION, HISTOGRAM

4.1 Introduction

Almost everyone is aware that any kind of work (even thinking) causes fatigue (getting tired). You also know that when you get tired, you can rest awhile, recover, then go on.

There are a number of questions we could ask ourselves, finding the answers by simple tests. How much of a specific kind of exercise is one able to do before he must rest? Does "work" (in this case strenuous exercise) cause the heart to beat faster? If so, how much faster? Can one keep from breathing more deeply after such exercise? Will a simple little hand exercise cause fatigue? (Most "hale", "hearty", and "brawny" boys scoff at the suggestion that they could get tired from such a minor exercise.) If it does cause fatigue, can one "recover" in a short time? Everyone can! After all, just fingers!

We will try a simple exercise to find out just what does happen. The question of increasing rate of heart beat we will explore in a later activity.

4.2 Counting vs. measurement

Since you will be counting hand movements, let's pause briefly to consider counting as compared with measurement.

Recall that in the previous exercise you were asking of nature, "How long?" and "How much?" You asked, "How long and how wide is each leaf?" You also asked, "How much water is transpired by a given tree?" In all these instances you were never able to say, "It is exactly this long," or "It is exactly that much." In each case you were forced by the nature of measurement to answer with an estimate, a close estimate, with some precision and yet always with a "greatest possible error."

Counting, on the other hand, asks the question, "How many?" Here is a different story. You would not be likely, if asked to count your classmates, to come up with the answer: "There are $34 \pm .5$ people in the class." Here you can be exact. There are exactly 34 people.

You are going to be counting hand motions, so this time exact numbers will be used.

4.3 Activity 1; Muscle Fatigue

Carefully read the following rules for the complete activity before starting the exercises.

Work in teams of three. The only equipment you will need is a watch or clock with a second hand, paper and pencil. (Your school clock may be sufficient.)

First person will be the "guinea pig."

1. Use both hands in the experiment, the right hand first.

2. Sitting in a comfortable position, with an arm straight in front of you resting on the desk top, fingers together, open and close the right hand as fast as you can, being quite sure each time that your fingers TOUCH THE DESK when the hand is open and the fingertips touch the palm when closed. (See Figure 4-3a.)

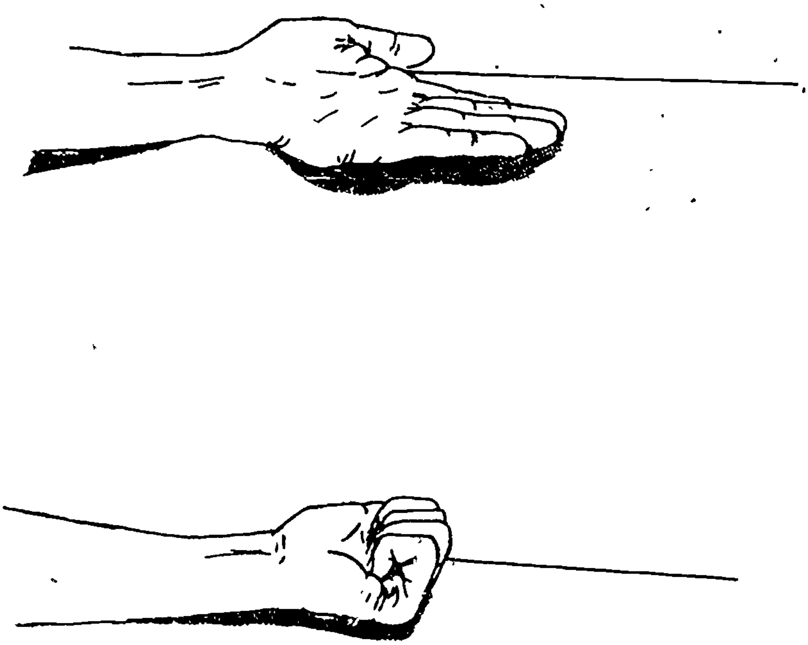


Fig. 4-3a

Second person will be the timer and recorder.

1. The timer will watch the second hand, start the "guinea pig," and call time at the end of 90 seconds. He will also record the number counted by the "counter" at the end of each 30-second period. The "guinea pig" will now be allowed to rest for 30 seconds, then start again on another 90-second period. Again the count will be recorded at the end of each 30-second period.

The person doing the exercise will "work" for a full 90 seconds, rest for 30 seconds, and then repeat the "work" for another 90-second period. It is most important that the person doing the exercise follow these instructions. It is also the timer's responsibility to see that the "guinea pig" is following the instructions given in the activity; that is, the fingers must straighten completely, touch the table top, and then close until the tips touch the palm.

Third person is the "counter."

1. The "counter" will watch carefully and count each time the "guinea pig's" fingers touch the table top. It is important that the counter count quietly, but aloud, so the "timer-recorder" can hear and record the count at the end of each 30-second period. The recorder will now have to subtract the first recording from the second and the second from the third for each 90-second period. This will give the consecutive 30 second counts which should be recorded in Table 4-3a.

Table 4-3a

Muscle Fatigue - Opening and Closing Fingers

Name of "guinea pig" ________ Rt.- or left-handed ________						
		Number of Times Fingers Opened Each Time Period				
Time Period	Time in Seconds	Total Count Rt.-hand	Right Hand		Total Count Left Hand	Left Hand
1st	30					
2nd	30					
3rd	30					
Rest	30	Rest	Rest	Rest	Rest	Rest
4th	30					
5th	30					
6th	30					

Make a table for each member of your team and rotate jobs until everyone has had a chance to be "guinea pig."

Surprised? The writers were! Did this "simple little" hand exercise cause fatigue? How about recovery? Did you "recover" fully in the 30-second rest period? Which muscles did you find became fatigued? Can you locate them approximately? Did you use the same muscles for opening the fingers and closing them?

4.4 How Do Muscles Work?

Let's explore briefly and in general terms what muscle action is, and and what happens when muscles become fatigued. Muscles are made up of many tiny living cells specialized for motion by being unusually long and thin compared to other cells of the body such as skin cells, fat cells, or bone cells. All living cells can contract (shorten and thus thicken) to some extent. Muscle cells because of their specialized arrangement in long fiber-like threads laid side by side, have a much greater ability to shorten than other types of cells. When many muscle fibers contract at the same time, something has to "give." Muscles that are attached to bones are always attached in such a way that there is at least one joint between the attachment of one end of a muscle and the attachment of the other end. Therefore, the joint will change as opposing muscles contract and relax. What do you think might happen if both ends of a muscle were attached to the same bone? Do muscles push, pull, or both? Let's try a simple test. Hold your arm straight in front of you, palm up, and bend your elbow. Which muscle contracted? Now, straighten your elbow. Could you tell which muscle contracted this time? Perhaps you discovered that muscles cannot "push," so which muscle must have contracted to "pull" your arm straight? All of your skeletal muscles (muscles attached to bones) are arranged in similar opposing pairs. Visceral muscles (those that are not attached to bones, such as heart and stomach muscles) simply contract and relax in place. Some of them are in layers that oppose each other. The muscles of the small intestine, for instance, are in two layers; one circular, and one longitudinal (lengthwise) and the result of their contractions causes food to be pushed along the intestine.

Muscles need energy in order to contract. This is supplied by food containing chemicals which are loaded with potential energy in the form of chemical energy. When the food reaches the cells, it is oxidized (oxygen reacts with it) to change the chemical energy into the energy of motion (kinetic energy). Notice that oxygen is needed to react with food. How do the food and oxygen reach the cells?

Another thing important to cells is that they get rid of toxic (poisonous) waste products. The more active muscles are, the greater is the amount of wastes produced which must be removed. How are the waste products carried away from the cells?

Many times the action is so strenuous that the supply of food and oxygen and the removal of wastes cannot keep pace with the demand. The result is fatigue or tiredness. When we rest, this gives the body a chance to "catch up" by supplying more food and removing the accumulated wastes; then the muscles are ready to go again.

4.41 Interpreting Data

You have already learned that biological data, in order to be really meaningful, and to achieve the degree of precision of some of the other sciences, must be interpreted mathematically. Therefore, our next step is to consider some of the analyses which can be made from our table of data. Topics such as percent of increase and decrease, mean, informal extrapolation and frequencies will be developed and used.

A table of sample data is shown below.

Table 4-41a

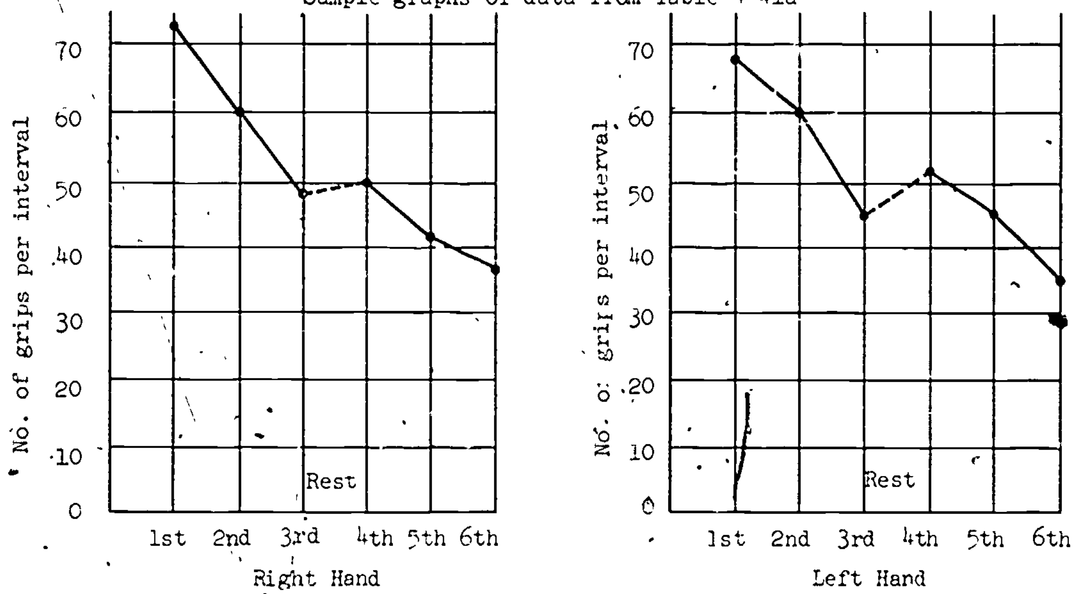
Table of Sample Data

Name of "guinea pig" ____________ Rt. or left-handed ________

Time Period	Time in Seconds	Total Count Rt. Hand	Number of Times Fingers Opened Each Time Period: Right Hand		Total Count Left Hand	Left Hand	
1st	30	72	72		68	68	
2nd	30	132	60		128	60	
3rd	30	179	47		173	45	
Rest	30	Rest	Rest	Rest	Rest	Rest	Rest
4th	30	50	50		52	52	
5th	30	92	42		97	45	
6th	30	129	37		132	35	

On a system of coordinates graph your data using the y-axis as number of grips per 30 seconds of time and the x-axis as the number of time intervals.

Sample graphs of data from Table 4-4Ia



Connect with a solid broken line points for first, second, and third; leave a dotted line between third and forth.

4.5 Percent

Many of you are familiar with the word "percent", and you may know something about its meaning. If your teacher says, "90 percent of the answers on this paper are correct," would you know what he means? The word "percent" comes from the Latin phrase <u>per centum</u>, which means "by the hundred." If the paper with 90% of the answers correct has 100 answers, then 90 answers out of the 100 are correct. The ratio $\frac{90}{100}$ could be used instead of the phrase "90 percent" to describe the part of the answers which are correct. The word "percent" is used when a ratio is expressed with a denominator of 100.

$$90 \text{ percent} = \frac{90}{100} = 90 \times \frac{1}{100} .$$

For convenience the symbol, %, is used for the word "percent." This symbol is a short way of saying "$\times \frac{1}{100}$."

$$\frac{90}{100} = 90 \times \frac{1}{100} = 90\%$$

$$\frac{16}{100} = 16 \times \frac{1}{100} = 16\%$$

$$\frac{37}{100} = 37 \times \frac{1}{100} = 37\%$$

$$\frac{77}{100} = 77 \times \frac{1}{100} = 77\%$$

$$? = 13 \times \frac{1}{100} = 13\%$$

Suppose that the paper has 90 correct answers out of the 100; 6 incorrect answers out of the 100; 4 answers omitted out of the 100. Since we know the total number of problems we can express this information in terms of percent.

$\frac{90}{100} = 90 \times \frac{1}{100} = 90\%$ (90% of the problems were correct.)

$\frac{6}{100} = 6 \times \frac{1}{100} = 6\%$ (6% of the problems were incorrect.)

$\frac{4}{100} = 4 \times \frac{1}{100} = 4\%$ (4% of all the problems were omitted.)

$\frac{90}{100} + \frac{6}{100} + \frac{4}{100} = \frac{100}{100}$ (all answers)

$90\% + 6\% + 4\% = 100\%$ (all answers)

Another name for the number <u>one</u> is 100%.

The number <u>two</u> can be written

$$\frac{2}{1} = \frac{200}{100} = 200\% .$$

An eighth grade class of 25 pupils is made up of 11 girls and 14 boys. The ratio of the number of girls to the number of pupils in the class can be expressed in many ways. For instance:

$$\frac{11}{25} = \frac{22}{50} = \frac{33}{75} = \frac{44}{100} = \frac{55}{125} = \frac{66}{150} .$$

If we wish to indicate the percent of the class that is girls, which fraction gives the information most easily? Why? The ratio of the number of boys to the total number in the class may be written:

$$\frac{14}{25} = \frac{c}{50} = \frac{d}{75} = \frac{56}{100} = \frac{e}{125} = \frac{f}{150} .$$

What numbers are represented by the letters c, d, e, f in the last proportion? Notice the two ratios $\frac{11}{25}$ (girls) and $\frac{14}{25}$ (boys). What is the sum of the two ratios? Find the sum of the two ratios $\frac{44}{100}$ and $\frac{56}{100}$. Express the two ratios and their sums as percents, using the symbol, %. The entire class is considered to be 100%.

Any number $\frac{a}{b}$ can be expressed as a percent by finding the number c such that $\frac{a}{b} = \frac{c}{100} = c \times \frac{1}{100} = c\%$.

Example: If $\frac{a}{b} = \frac{3}{4}$, then $\frac{3}{4} = \frac{c}{100}$

$$\frac{3}{4}(100) = c$$

$$75 = c \quad \text{and} \quad \frac{a}{b} = 75\% .$$

Exercise 4-5a

1. Write each of the following numbers as a percent.

(a) $\frac{1}{2}$	(d) $\frac{1}{5}$	(g) $\frac{3}{2}$	(j) $\frac{7}{5}$
(b) $\frac{1}{4}$	(e) $\frac{2}{5}$	(h) $\frac{2}{2}$	(k) $\frac{5}{4}$
(c) $\frac{3}{4}$	(f) $\frac{3}{5}$	(i) $\frac{4}{4}$	(l) $\frac{5}{2}$

2. Consider the following group of students.

Student	Hair Color
William	blond
Joanne	brown
Mary	blond
John	red
Joe	brown
Betty	brown
Roy	blond
Don	brown
Margaret	red
David	brown

(a) What percent are boys?
(b) What percent are blonds?
(c) What percent are redheads?
(d) What percent are not redheads?
(e) What percent are brown-haired girls?
(f) What percent are redheaded boys?

J.F.F. A man bought a radio for \$40, sold it for \$60, purchased it back for \$70, and finally sold it for \$90. How much money did he gain or lose on the complete transaction?

4.51 Applications of percent, increase and decrease

Percent is used to express ratios of numerical quantities in everyday experience. It is important for you to understand the notation of percent, and also, to be accurate in computing with numbers written as percents. Percent is used to indicate an increase or a decrease in some quantity. Suppose a Junior High School had an enrollment of 600 students in 1950. If the enrollment increased to 800 by 1960, what was the percent of increase? If we let "x" stand for the percent of increase, then

$800 - 600 = 200$ and actual increase, building a proportion--

$$\frac{x}{100} \text{ (\% of increase)} = \frac{200}{600} \begin{matrix}\text{(amount of increased enrollment)}\\ \text{(original enrollment)}\end{matrix}$$

$$x = 100 \times \frac{200}{600}$$

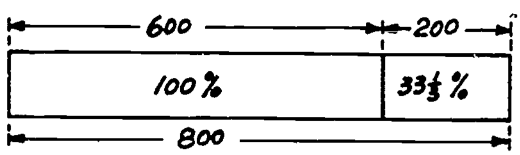
$$x = \frac{20000}{600}$$

$$x = 33.33 \text{ or } 33\tfrac{1}{3}.$$

There was an increase of $33\frac{1}{3}\%$ over the original enrollment.

Notice that the percent of increase is found by comparing the actual increase with the earlier enrollment figure of 600, not with the later enrollment figure of 800. The enrollment increased from 600.

A graphic representation of the problem would be:



The 800 was made up of the 600(100%) plus the increase of 200 ($33\frac{1}{3}\%$). So the enrollment of 800 in 1960 was $133\frac{1}{3}\%$ of the enrollment of 600 in 1950.

Suppose now that the enrollment in another Junior High School was 250 in 1950. If the enrollment was 200 in 1960, what was the percent of decrease? If x represents the percent of decrease, then $250 - 200 = 50$, the actual amount of the decrease. The proportion needed is:

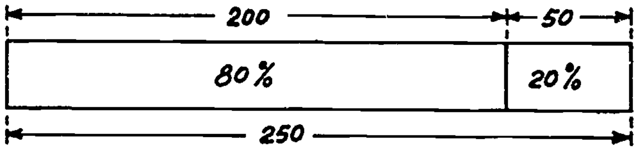
$\frac{x}{100}$ (% of decrease) $= \frac{50}{250}$ (actual decrease) (original enrollment)

$$x = 100 \times \frac{50}{250}$$

$$x = \frac{5000}{250}$$

$x = 20$. Therefore, a 20% decrease from the original enrollment of 250.

A graphic representation of the problem:



Could you have mentally arrived at the answers in our two examples by comparing ratios? Look at the ratio $\frac{200}{600}$. This can be simplified to $\frac{1}{3}$ or $33\frac{1}{3}\%$. What number must replace x so that the ratio $\frac{x}{100}$ will also be $\frac{1}{3}$ or $33\frac{1}{3}\%$? We hope your guess was $33\frac{1}{3}$. Now look at the second example. What number must replace x in $\frac{x}{100}$ so that you would have the same ratio as $\frac{50}{250}$ ($\frac{1}{5}$ or 20%)? Did you guess 20 ?

Exercise 4-51

1. A junior high school mathematics teacher had 176 pupils in his classes. The semester grades of the pupils were 20 - A's; 37 - B's; 65 - C's; 40 - D's; and 14 - F's. Find to the nearest tenth percent, the percent of grades that were:

   (a) A (b) B (c) C (d) D (e) F

   What is the sum of the answers in parts (a), (b), (c), (d), and (e)? Does the sum help to check the answers?

2. Bob's weight increased during the school year from 72 lbs. to 81 lbs. What was the percent of increase?

3. During the same year, Bob's mother reduced her weight from 140 lbs. to 126 lbs. What was the percent of decrease?

4. On the first of September John's mother weighed 130 lbs. During the next four months she decreased her weight by 15%. However, during the first four months of the next year her weight increased 15%. What did she weigh on the first of May? Are you sure of the correctness of your answer?

4.6 Mean-Median-Mode

In many cases data collected from scientific experiments is recorded in table form. Some information can be determined easily by looking at all the data in table form. Sometimes, however, the large number of items in a table makes it confusing. In this case, it may be better to describe the data by using only a few numbers. Finding an average of such a set of numbers is often very helpful in studying the data given to you.

When you calculate the average from a set of numerical grades by adding the numerical grades and dividing the sum by the number of grades, you find a number which is used as a representative of the numbers in the set. This useful average, with which you are already familiar, is called arithmetic mean or the mean.

Let us look at the table of the data printed below. The table gives a list of the heights of 11 pupils in order from tallest to shortest.

Table 4-6

Heights of 11 eighth grade pupils

Pupil	Height in inches
1	68
2	65
3	62
4	60
5	60
6	59
7	58
8	56
9	55
10	54
11	52

In describing this set of data, can we find one number which we can use to represent the numbers in these measurements? One such number would be the arithmetic mean. For this table, the average height (arithmetic mean) is

$$\frac{\text{sum of the heights}}{\text{number of pupils}} = \frac{649}{11} = 59 .$$

This commonly used measure can be found without arranging the data in any special way.

Median

Another way to obtain one number representing the numbers in a set of data is to find a number such that half of the numbers in the set are greater and half are less than the number found.

The median of a set of numbers is the middle one of the set when the numbers in the set are arranged in order, either from smallest to largest or from largest to smallest. In the set of heights in Table 4-6 the middle number is 59 when the list is arranged in order. 59 is then the median of the set. Half of the numbers are greater than 59 and half are less.

If the number of elements in the set is even, there is no middle number. Thus, we must define the median for this situation. If there is an even number of elements in the set, the median is commonly taken as the mean of the two middle numbers. For example, in the set of numbers, 8, 10, 11, 12, 14, 16, 17, 19 the two middle numbers are 12 and 14. The median is 13, the mean of 12 and 14, although 13 is not in the set. Sometimes several items are the same as the median. The set of scores 12, 13, 15, 15, 15, 15, 16, 18, 19, 20 has 10 numbers in it. The two middle ones are 15 and hence the median is 15. But the third and fourth scores are also 15 so that 15 is not a score such that 5 scores are smaller and 5 are larger than it.

In the set of salaries below, the median is $7650.

$6150
$6300
$6900
$7200
$7500
$7800
$8100
$8100
$8100
$9450

The arithmetic mean is $7560. The median and the arithmetic mean are nearly equal. But, if the largest salary had been $15,450 instead of $9450, the arithmetic mean would have been $8160 and the median would still have been $7650. This illustrates that the usefulness of the median in describing a set of numbers often lies in the fact that one number (or a few numbers in the set) does not affect the median as it does affect the arithmetic mean.

Mode

Which height occurs more than any other in Table 4-6? How many pupils have this height? This height is called the mode.

In sets such as the natural numbers 1, 2, 3, 4, 5, ..., no number occurs in the set more than once. But in a set of data some number, or numbers, may occur more than once. If a number occurs in the set of data more often than any other member, it is called the mode. (We might say it is the most fashionable or most popular.) There may be several modes.

In the set of salaries $6150, $6300, $6900, $7200, $7500, $7800, $8100, $8100, $8100, $9450, the mode is $8100. But in the set of scores 19, 20, 21, 21, 21, 24, 26, 26, 26, 29, 30 there are two modes, 21 and 26. (These are equally fashionable.) If there had been another score of 21 in this set of scores, what would the mode have been? In Table 4-6 if the 6th pupil were 60 inches tall, how would this affect the mode?

Exercise 4-6

1. Find the mode of the following list of chapter test scores: 79, 94, 85, 81, 74, 85, 91, 87, 69, 85, 83.

2. From the scores in problem 1, find the:
   (a) mean, (b) median.

3. The following annual salaries were received by a group of ten employees:

   $4,000, $6,000, $12,500, $5,000, $7,000, $5,500,
   $4,500, $5,000, $ 6,500, $5,000.

   (a) Find the mean of the data.

   (b) How many salaries are greater than the mean?

   (c) How many salaries are less than the mean?

   (d) Does the mean seem to be a fair way to describe the typical salary for these employees?

   (e) Find the median of the set of data.

   (f) Does the median seem to be a fair average to use for this data?

4. Following are temperatures in degrees Fahrenheit at 6 p.m. for a two-week period in a certain city: 47, 68, 58, 80, 42, 43, 68, 74, 43, 46, 48, 76, 48, 50. Find the
   (a) mean, (b) median.

4.7 Informal Extrapolation

Extrapolation (ex - trap' - ō - la' - tion) is the estimation of the value of a function beyond an interval from values of the function within the interval. A simple example: Suppose you could measure the height to the window sill of a building as 5' and the height to the second floor as 15' and the measure of the height to the window sill of the third floor as 25', but then your tape would not reach any further. If asked, what would you estimate for the measure of the distance to the window sill of the fourth floor? Eighth floor? 150th floor? You see, you don't actually know the measure of this length but if the conditions seem the same, you can extrapolate and estimate a value.

Given the series 2, 4, 6, and 8 ..., extrapolate for the sixth term.

If a trackman could run:

100 yards in 10 seconds,
200 yards in 22 seconds,
300 yards in 36 seconds,
400 yards in 52 seconds,
500 yards in 70 seconds,

how long would you estimate it would take him to run 600 yards?

This type of extrapolation is not very involved. We are just making an educated guess of the next term based on previous information. We must be careful because, if the data is limited, it is easy to arrive at a wrong conclusion. If we are given two terms 2 and 4 and asked to find the third term, we might think of it as 2, 4, 6 if we added 2 each time. Some might even say 2, 4, 16, if they squared each previous number to arrive at the next number. A wild one could be 2, 4, $4\frac{2}{3}$ if you added 10 and divided by 3. You see, there is no limit to the possibilities of the next term if your data is not large enough.

Sometimes even with plenty of data it is difficult to see a pattern and extrapolate another term. Here is a fancy one to demonstrate our point.

Given:

200 = 2.30103
300 = 2.47712
400 = 2.60206
500 = 2.69897
600 = ?

The type of extrapolation we will do is very relaxed. With a given amount of obtainable data we will extrapolate for values not readily obtainable.

Exercises 4-7

1. Find the mean for the first three readings in your table of data, "Muscle Fatigue" (Table 4-3a) and enter it in the column to the right of the readings.

2. Find the mean for the last three readings, (4th, 5th, and 6th time intervals) and enter to the right. Do the same for the data of the other hand.

To solve the following problems use only the data (Table 4-3a) from the column labeled "Right Hand."

3. What is the difference between the two means?

4. What is the "% of loss" between the average of the first three readings and the average of the last three?

5. What is the difference between the first and second readings?

6. What is the difference between the second and third readings?

7. Find the average of the differences found in problems 5 and 6.

8. Based upon your readings for the first, second, and third time intervals and the average loss per interval, extrapolate and estimate a reading for a fourth interval if you had not rested.

9. What was your actual recorded value for the fourth interval?

10. To extrapolate further, compute and mark with "x" on your graph estimated points for 5 more time intervals.

11. Did you extrapolate in a straight line? Should you? Why?

12. Which time interval would show your P.O.F.? ("Poop out" factor.)

13. What does your graph illustrate as an answer to our original question? Does exercise cause fatigue and does one recover quickly?

14. Did you "recover" fully in the 30-second rest period?

15. What differences were apparent in the right and left hands, if any? How do you account for the differences?

16. How might a physical therapist use data of this kind to be able to better help his patient?

17. Could you "feel" where the muscles were located that contracted to close the fingers? Open them?

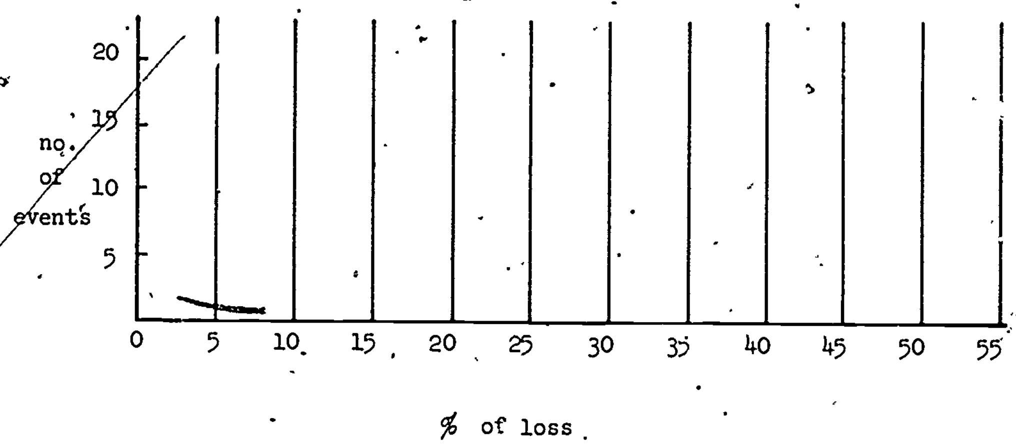
18. If class time permits, or at home, re-do the experiment except do not stop until you have completed six or more consecutive 30-second intervals. Graph your results, in another color, on your present graph.

4.8 A Histogram

Often a histogram will show data in such a manner as to give further insight into the problem. A histogram (histo-gram) is a graphical representation of a frequency distribution. It is made up of a series of rectangles, the length of which is the frequency and the width the range of a frequency. Wow!!!

The definition sounds formidable but the graph itself is most useful. We want to show the frequency (fre' - kwen - see), which is the ratio of the number of actual occurences of an event to the number of possible occurences. Make a graph as in Figure 4-8a.

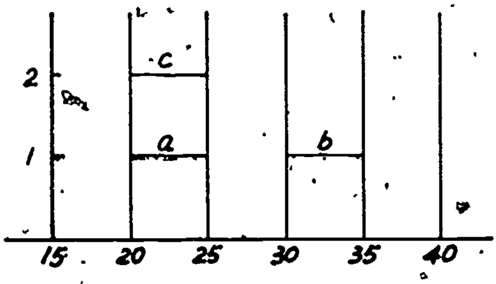
Fig. 4-8a



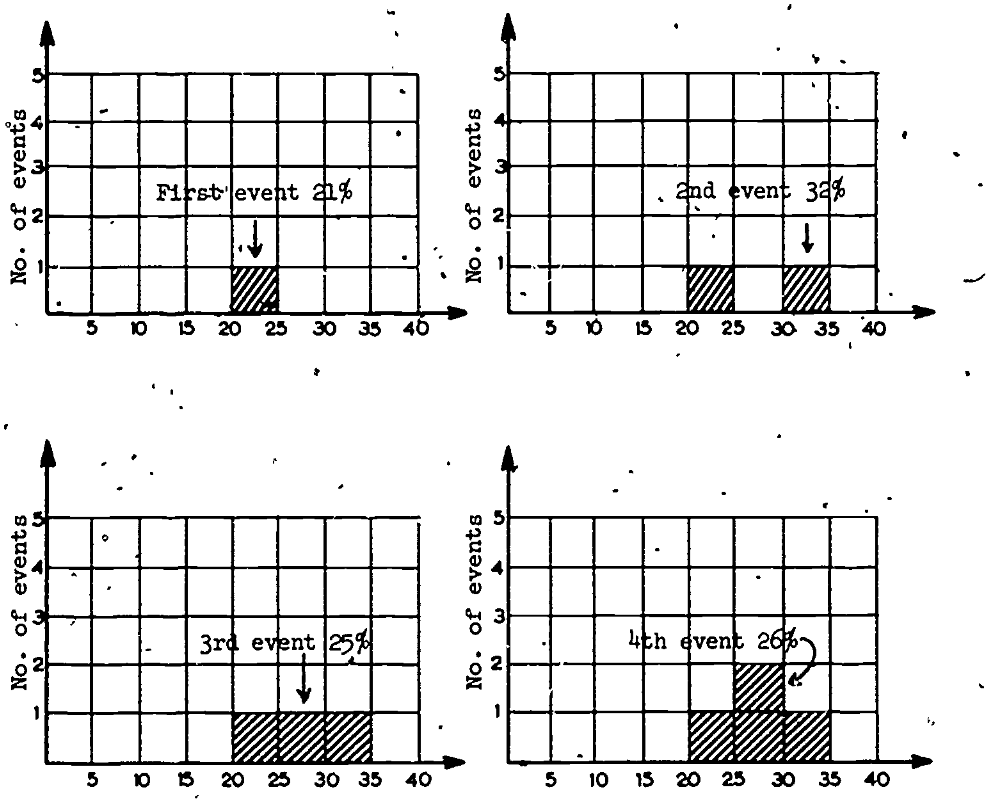
Each student will write his "% of loss" value on a piece of paper and hand it in to the teacher. (This is your answer for problem 4 in Exercise 4-7.) The teacher will then have the percentages read out loud. As each percentage is read, draw a horizontal segment on your graph (a line which goes between the ranges of %).

As a general rule for our histogram, if a percentage is the same as one of the lines, mark the space to the left of the line. In Figure 4-8b below, line "a" would be drawn for a value of 25% , line "b" for a value of 35%, and line "c" for a value of 21%.

Figure 4-8b



If, for example, the first four percentages read were 21%, 32%, 25% and 26%, the information would be recorded as shown in the following graphs:



A histogram, such as you have just developed, clearly represents which values occurred most frequently, which occurred infrequently, and the relationship to each other.

To complete our histogram we need to determine the percentage of occurrences for each group of events. To find the percentage of each group of events:

1. Reread the total number of events (number of % read by teacher).
2. Record the number of events in each category.
3. Find the percentage of occurrences of events in each category, to the total number of events.

Example: If there are 40 students in a class and 5 students had a % marked between 15% and 20%, then this category shows an occurrence of $\frac{5}{40}$ or 12.5% . If 6 had a % marked between 20% and 25%, then its occurrence would be $\frac{6}{40}$ = 15%.

There is a way to avoid dividing by the same number for the identical problems. It uses a reciprocal. Can you figure it out?

4.9 Optional Activity: Exercise and Pulse Rate

Remember one of the questions at the beginning of this chapter: "Does strenuous exercise cause the heart to beat faster?" Of course, you know the answer to this. However, if we ask instead, "How much faster does the heart beat in order to help the body recover from strenuous exercise?", could you think of an "experiment" to answer the question? It's not so easy, is it? You are already beset by other questions: Faster than what? How much exercise? What kind of exercise? Let's try again. "How many beats per minute above normal will the heart increase as a result of stepping up on a chair and back down 10 times?

In preparation for the experiment you should practice counting a pulse. Look at the diagram in figure 4-9a and notice the position of the fingers on the wrist. With a little practice you can locate the spot just above the wrist joint and on the thumb side of the tendons of the wrist where a pulse can be felt. Try locating this spot either on your own wrist or on the wrist of a classmate. Practice until you can readily locate the spot.

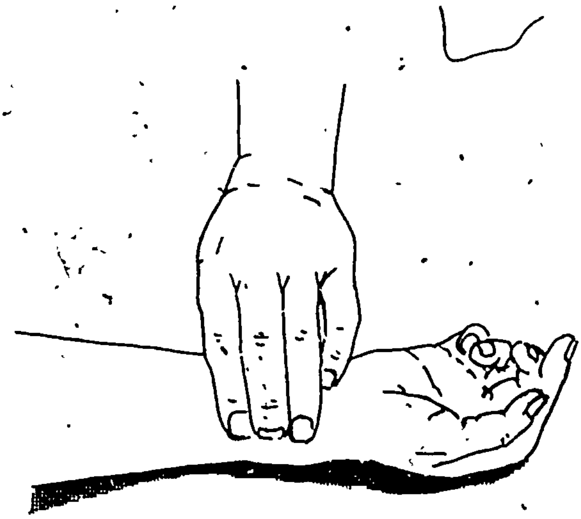


Figure 4-9a

To do the experiment you will need a partner. (This is a strenuous exercise. Be sure neither you nor your partner have a medical reason which would keep you from participating.)

First one does the exercises while the other counts pulse and records; then trade positions so that each member has done the exercise. Prepare a table (see example - Table 4-9) for your data.

Table 4-9a

Data of Pulse Rate

Team Member	Column I Normal Pulse Rate	Exercise for 30 sec	Column II Pulse Rate after exercise	Rest for 30 sec	Column III Pulse Rate after 1 min.	Rest for 30 sec	Column IV Pulse Rate after 2 min.
A							
B							
	30 sec	30 sec	30 sec	30 sec	30 sec	30 sec	30 sec

$3\frac{1}{2}$ min total lapse time

Be sure to read carefully the following instructions before proceeding with the exercise.

For the purpose of this explanation we will identify the members of the team as A and B.

A will be the first to exercise. So B will:

1. Take A's pulse for 30 seconds, multiply by 2, and enter in column 1 of your table.
2. Count while A exercises according to instructions which follow.
3. Immediately after A's exercise, take his pulse again for 30 seconds, double and record in column 2.
4. Allow A to rest for another 30 seconds.
5. Count A's pulse for 30 seconds, double, and record in column 3.
6. A will continue to rest for 30 more seconds.
7. Count again and record A's pulse.

Look at the diagrams in Figure 4-9b to see the stance for both the up and down positions. Do not change from one foot to the other on the chair while doing the exercise and be sure that while on the floor and on the chair the knee of the leg you are standing on is straight.

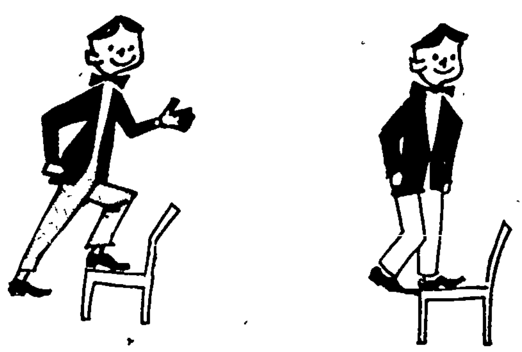


Figure 4-9b

To do the exercise, the subject should place one foot on the chair as in the first drawing, Figure 4-9b. Then, when the timer calls "Go," the subject should step up on the chair (see Figure 4-9b) and back to the floor 10 times. Movement should be at a fairly rapid but uniform pace.

Now trade jobs with your partner and repeat the exercise and recording of data.

Exercises 4-9

1. What was your recorded normal pulse rate?

2. What was your pulse rate count immediately after exercise?

3. What was the percent of increase in the pulse rate?

4. What was the ratio of second reading to first reading?

5. Express the ratio of problem 4 in a form of ? to 1 (second reading ÷ first reading).

6. How many counts did your pulse rate diminish (decrease) between the second and third reading?

7. Why did the number of pulse counts go up between the first and second readings and down between the second and third reading?

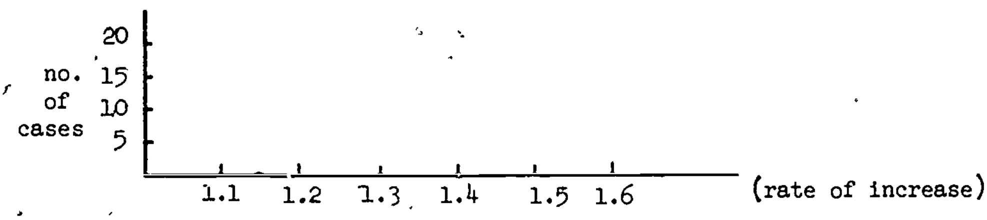
8. What was the % of decrease between the second and third readings? (Second reading - third reading) ÷ second reading.

9. Was your final reading (fourth) greater than, less than, or equal to your normal pulse rate? How much difference is there in number of counts?

10. Graph the four readings using the horizontal scale as "number of readings" and the vertical scale as "pulse rate."

11. Solely on the record of the second, third, and fourth readings of your pulse rate, what would the graph seem to predict for a fifth and sixth reading? Do you think this prediction is valid?

12. Class project. Make a frequency histogram of the number of people per ratio of pulse rate increase (second reading ÷ first reading).



13. What is the "mean" ratio of increase in your class?

14. Could you say then that the normal rate of increase is known? Why?

Chapter 5

YEAST METABOLISM:
LINEAR GRAPHING, CURVE FITTING, EXTRAPOLATION AND INTERPOLATION, VOLUME OF A CYLINDER

5.1 Introduction

So far in your activities you have either discovered for yourselves or learned through discussion several important biological concepts. Remember your very first experiment? You were asked to measure leaves, and in so doing discovered natural variation. Remember that you were asked to measure the same kind of leaves. Some of you may have chosen a thin, shiny type, others a greyish, fuzzy type, still others a thick, waxy type. In each case, the leaves were especially adapted to specific kinds of environmental circumstances, amount of water, temperature, etc. In Chapter 2 you learned something about what is involved in growth of all living things. You learned that building materials are needed, that cells not only increase in number, but also specialize, and - most important of all - that cells are constantly active and need energy for their activities.

Next you learned that leaves have tiny openings called stomates, which not only control water loss, but also allow gases (oxygen and carbon dioxide) to enter and leave the leaf. Then you also found that when you used muscles strenuously, the cells could not get oxygen or get rid of waste materials fast enough to keep pace, fatigue set in, and you had to rest to allow your body to "catch up".

So far you have analyzed data using tables, graphs, and mathematical computations to determine such things as variability in size and growth in living things (if your reach equals your height does that mean you are a "square"?), average number of stomates in an average leaf, how many times the average person can move fingers before fatigue sets in and how long it takes to recover after exercise. If you did the optional activity in Chapter 4, you had to breathe fast to get more air, and your heart beat faster to deliver the oxygen to your cells! Have you actually measured the most basic thing of all in all of these activities -- the amount of gases used or released? You did have to breathe more heavily after strenuous exercise (take in air and release carbon dioxide); stomates in leaves open and close to regulate the amount of gases entering and leaving the leaves - but, in each case how much?

Just what is this process of breathing? What does it involve? Is there any way it can be measured? A much better name for what is commonly called breathing is respiration.

5.2 What is Respiration?

Living things must "breathe". Even plants "breathe"! Many people have the mistaken idea that plants "breathe" carbon dioxide, while animals breathe oxygen. If we look a little more closely at what is meant by "breathing", perhaps we can get a clearer picture. When you take a breath, you are pulling into your lungs a fresh supply of oxygen (as well as some other gases). This mechanical process is what is technically called breathing. As discussed before, this oxygen is then picked up by the blood circulating through your lungs and carried to each individual cell where it reacts with the food you have eaten to produce the energy needed for cells to work. As a natural by-product of this reaction, carbon dioxide is produced as a waste material, is transported -- again by the blood -- back to your lungs, where it is expelled (breathed out) from the body. All living cells, whether plant or animal, (with very few exceptions) obtain their energy in this way, and the process is known as respiration -- one phase of metabolism. Obviously small organisms do not "breathe" in and out of lungs, nor do plants, but most living things, nevertheless, must have some way to obtain oxygen and get rid of carbon dioxide; that is, some way to allow an exchange of gases between the cells and the environment.

It might be wise here to clear up the confusion about plants "breathing" carbon dioxide. During the day, when there is light, green plants use carbon dioxide in photosynthesis and release oxygen into the atmosphere (an extremely significant fact biologically), but at the same time they are also using oxygen for respiration and releasing carbon dioxide. It is just that when there is light, photosynthesis overbalances respiration. At night, however, green plants carry on respiration only -- just as animals do all the time.

5.3 Activity -- Measuring Yeast Metabolism

There are many ways one can measure this gas exchange, but most are quite complicated, and require complex apparatus. However, by using a simple plant cell such as yeast, and some very simple apparatus, we can measure the carbon dioxide given off by the yeast cells in a given amount of time, thereby determining its rate of metabolism. Yeast is a microscopic plant cell which does not contain chlorophyll (see Chapter 1), so does not produce its own food. Therefore, we must supply it with a food source -- sugar in this case. Also, yeast metabolism is not as simple as this discussion might indicate. It can "work" either aerobically (a'er ō' bic ly) -- that is, with oxygen, or anaerobically (an a'er ōbic ly) -- without oxygen. In either case, however, carbon dioxide is released in measurable quantities by the yeast and furnishes us with adequate data for measurement of metabolism.

Materials Needed

Disposable plastic syringe (without needle)

18 inches of tubing the right size to fit tightly over the nozzle of the syringe, either alone or with some type of fitting

12-inch plastic ruler, with the metric scale, and preferably with a groove, lengthwise, down the center

Yeast - either dry or cake (check the date on it to be sure it's "fresh")

Sugar

Food coloring

Scotch tape

Block covered with white paper (see illustration)

Clock or watch for measuring one minute intervals

Procedure

1. Mark the tubing with a pen 5 cm from one end. Lay the tubing in the groove of the ruler so that the mark coincides with the zero end of the metric scale. Fasten with Scotch tape.

   See the right half of Figure 5-3a.

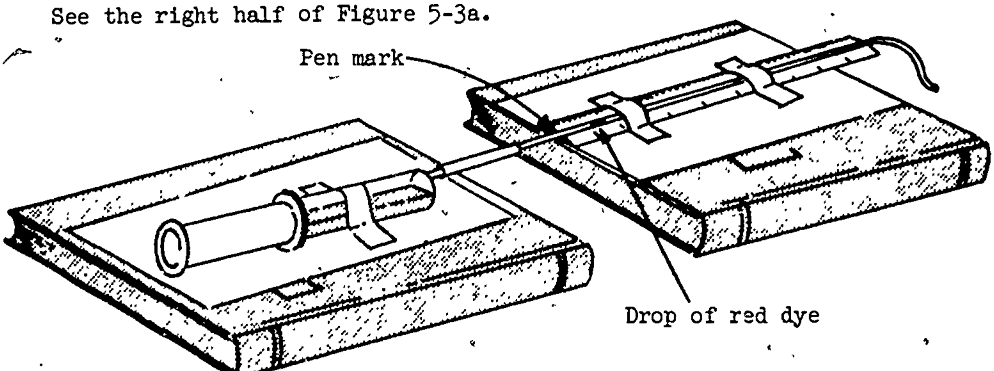


Fig. 5-3a

2. Mix some dye by adding 2 drops of red food coloring to 3 or 4 drops of water in a small bottle cap, or other container.

This next step is tricky, so you may need to practice a little.

Dip the end of the tube nearest the zero point of the ruler into the dye, and using the soda-straw technique, carefully draw up about 1 cm of the red dye. (You may use the syringe as a "vacuum pump" and pull the dye along.)

With a little practice you should be able to place it about where you want it, although the exact location is not vital. It should be located near the one or two cm mark on your ruler.

Lay the ruler with its tube containing the red drop on the block, or book, with the zero point of the ruler at the edge of the block.

3. Filling the syringe with yeast culture.

This is also tricky, so be sure to read all of the instructions and study the diagrams carefully before proceeding.

(1) Make sure the plunger works smoothly in your syringe.
(2) Push the plunger completely into the syringe.
(3) Dip the nozzle of the syringe into the yeast culture (prepared by the teacher) and slowly draw the liquid into the syringe until the syringe is a little less than half full. See Figure 5-3b. If you have a 12-ml (milli-liter, —the same as cubic centimeter, cc) syringe, draw up 5 ml of culture. If yours is a 2 ml syringe, draw up .75 ml .

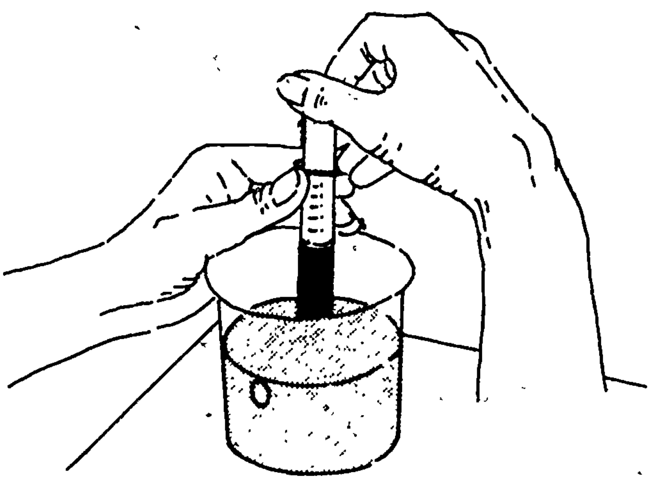


Fig. 5-3b

4. Now tip the syringe, so that the nozzle is up, and slowly draw the plunger back to the last mark, thus pulling air into the remainder of the syringe. See Figure 5-3c. You now have a known volume of yeast culture in your syringe, as well as a known volume of air.

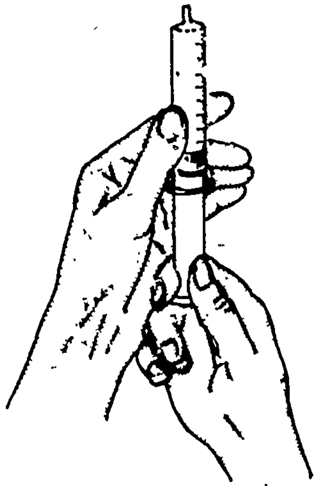


Fig. 5-3c

5. Carefully (without spilling the yeast) lay the syringe on the second block, so that the nozzle is in the vicinity of the end of the tubing, and tape into position. Be sure the nozzle is placed in such a way that yeast culture does not leak from the syringe.

   The blocks must be movable so that the two pieces of apparatus can be positioned easily. See Figure 5-3a.

You are now ready to begin the experiment. Work in teams of three. First person acts as "timer", second person as "reader", and third person as "recorder". Everyone should be ready before the next step.

BEFORE DOING STEP 6 READ THE BALANCE OF THE INSTRUCTIONS.	← READ

6. Carefully attach the end of the tubing to the nozzle of the syringe. It should be a tight fit, and moistening it a little should help to slip it on.

7. As soon as possible, but not so hastily as to cause confusion, the timer should call "begin". At this instant the "reader" should tell the recorder at what point on the metric scale the advance end of the red drop is located. The recorder records this as location of drop at zero time.

   The timer should then continue to call time at one minute intervals for ten minutes (or as long as necessary to get sufficient readings), the reader giving the reading in millimeters at the end of each minute.

8. The recorder will record each one minute reading for ten minutes. See Table 5-3b.

This is most easily recorded in a table. Record the temperature of the room, as this is significant biologically, although it will not be used in this experiment. (You might be interested sometime in trying the experiment under varying temperature conditions, i.e., laying the syringe in hot water, or ice water, and taking readings at several temperatures.)

At the end of ten minutes (when the drop should have moved along much of the ruler), disconnect the tubing from the nozzle and, tipping the ruler gently, allow the drop to run back down to near its original location.

Reconnect the tubing, and repeat the readings for another ten minutes. By this time you should have perfected your technique and be able to achieve greater precision.

[Again record your results in the table, then calculate the difference between each reading.]

For example, suppose your readings were as follows:

At zero time the drop was at 22 ± .5 mm, at 1 minute it was 38 ± .5 mm, at 2 minutes it read 53 ± .5 mm .

Table 5-3a shows the above sample data.

Table 5-3a

Sample of Data and Calculations of 3 Possible Readings

Time in Minutes	Reading in mm including greatest possible error	Difference between consecutive readings
0	22 ± .5	16 ± 1.0
1	38 ± .5	15 ± 1.0
2	53 ± .5	
↓ 10	↓	↓

Table 5-3b

Measurement of Rate of Carbon Dioxide (CO_2) Production of Yeast

Volume of Yeast Culture _____ Room Temp. _____

Time in Minutes	First Run: Reading in mm including greatest possible error	First Run: Difference between successive readings	Second Run: Reading in mm including greatest possible error	Second Run: Difference between successive readings
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Find the measure of the total distance traveled for the first run by subtracting the recorded reading at zero minutes from the recorded reading at ten minutes. Enter this information at the bottom of the table and label it "Total Distance traveled". Repeat for the second run.

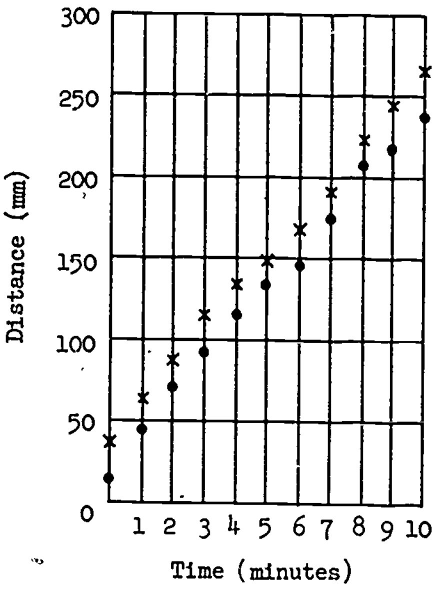
Does the moving drop indicate gas production by the yeast cells?

Do you think you could calculate from your data how much gas was produced per minute?

Does your data show whether gas was produced at approximately the same rate each minute with these conditions (of temperature, amount of yeast, etc.)?

5.4 Graphing the Data

To graph the data obtained and recorded in Table 5-3b, we will use for our reference lines "time and distance". "Time" will be represented on the horizontal reference line. "Distance", on the vertical reference line. If your graph paper is rectangular in shape, use the smaller end as the horizontal reference line. See Figure 5-4a.



Sample of Data - Gas Production

Fig. 5-4a

Plot the data recorded in Table 5-3b as ordered pairs (Time, Distance). Use a small dot for the first run information and a small neat "X" for the second run information.

These points represent the observed data. That is, at a specific time we noted a specific distance which the drop had traveled. However, we know that the drop has actually traveled through the entire recorded distance. If we had made more readings between readings we would have more points of information to graph. To continue the thought, more readings, between more readings, between more readings, ... more readings would each give us points to graph.

5.5 Curve Fitting

The definitions of lines, rays, or segments all included the expression "...a set of points ..." . If we continued to plot points between points we would eventually have points so close together they would touch and appear as a line segment. Would it be truly a line segment? Could we not always plot one more point in between any two points? Of course, by looking at our demonstration as distinct points we can approach the problem somewhat in the manner argued by Zeno in the "Achilles and the Tortoise" paradox.

Zeno, in approximately 450 B.C., presented an argument. He suggested that even though Achilles could run 10 times as fast as a tortoise, he could never catch the tortoise if the tortoise were given a head start. If the tortoise were given a 1000-yard head start over Achilles, they would appear as in Figure 5-5a.

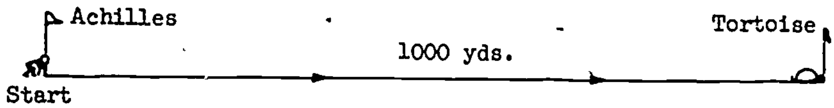


Fig. 5-5a

When Achilles had run 1000 yards, the tortoise would still be 100 yards ahead. See Figure 5-5b.

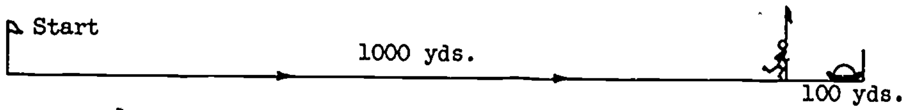


Fig. 5-5b

After Achilles had run the 100 yards, the tortoise would still be 10 yards ahead; when Achilles had run the 10 yards the tortoise would be 1 yard ahead, etc.

Now do you really believe Achilles could not catch up? What is wrong with Zeno's argument? Or do you agree with it?

Meanwhile, back to our problem. Since the drop did move through all of the distance from start to 10 minutes, we are justified in fitting a line in such a way as to include all of the recorded data and points in between.

In so doing, we are implying that we have a continuous relationship and we can interpolate the position of our drop at any instant of time between zero minutes and ten minutes. This is in contrast to the discrete (dis-crete') set of points found in the previous chapter on muscle fatigue.

Using solid line segments for the second run and dotted line segments for the first run, connect all points from the same run together.

Perhaps you may find that there is a discrepancy in the neatness of pattern of the two lines. This is why you were asked to make two runs.

(Scientists frequently make many, many more.) Living things are peculiar phenomena -- they are so complicated that many things can alter metabolism. Perhaps you are familiar with Basal Metabolism tests often used for medical diagnosis. To prepare for this, the patient is requested not to eat for several hours before the test and to get a good night's rest. In other words, every attempt is made to stabilize the individual's metabolism before the test is made. Similarly, your yeast culture may need time to stabilize before your readings will be consistent. Your first run, then, in addition to being good practice of technique, is also to give the culture time to adjust to its new environment.

Observe the graphs of your fellow students. Do they all look approximately alike? Are they generally in a straight line or do they curve?

5.6 Interpolation and Extrapolation

As you are already familiar with ratios and ordered pairs of points on a plane, we can use this knowledge to devise a simple method of determining one of the values of an ordered pair if the other is known. This problem arises in two related ways in scientific experiments: first of all you may wish to estimate a value from an experiment when you know earlier and later values (or higher and lower values)(called interpolation), and secondly, you may wish to estimate what a particular value will be which is outside the range, or beyond the range of your information (called extrapolation).

In Chapter 1 you plotted points on a graph. The coordinates of these points were ordered pairs in which the first term was the measure of the width and the second term the measure of the length. Figure 5-6a is an illustration of such a graph.

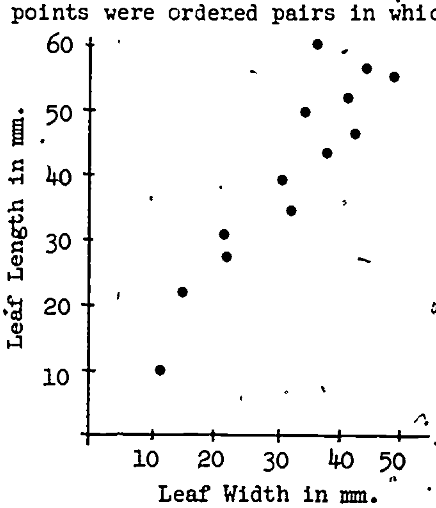


Fig. 5-6a

While we were working in Chapter 1, we estimated a value beyond our known values. This is extrapolation. We might also have decided to estimate a value within the range of the data. To do this would be interpolation.

There are two basic ways to make such estimates. One is from observing the graph, and the other is from the use of the table of data.

The first way is simply to take the plotted points and with the aid of a transparent plastic ruler, draw a line which appears to best represent the data. Such a line may not go through very many points but conversely few points will be very far from your line. As an example, consider the points and lines in Figure 5-6b.

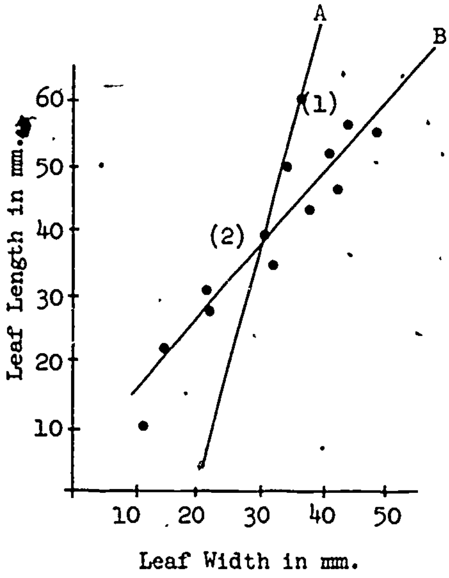


Fig. 5-6b

Line A is closer to points 1 and 2 but does not "fit" the data as well as line B.

Line A, in order to be closer to points 1 and 2, is farther than B from a larger number of points.

Later in your study of mathematics you will find that there are more advanced mathematical techniques which enable a scientist to fit the "best" line to any group of ordered points derived from the experiment.

Line B in Figure 5-6b seems to be the "best fit". Using line B to interpolate for a given width of 30 mm, we do the following:

1. From the 30 mm mark on the horizontal reference line, draw a light perpendicular line up until it intersects line B.

2. From the point of intersection of line B and the perpendicular from the horizontal axis "drop" a perpendicular to the vertical reference line.

3. The perpendicular from B appears to intersect the vertical axis at approximately 37 mm.

4. Our interpolated value for the length of a leaf with a width of 30 mm is then 37 mm.

Notice that in effect you are reversing the method that you used to plot the original points.

Exercise 5-6a

From Figure 5-6b and "best" line B, answer the following questions.

1. A width of 40 mm would give an expected length of _____ mm.
2. Find the expected length for a leaf width of 15 mm.
3. If the width is 45 mm, the expected length would be _____ mm.
4. If the length is 45 mm, the expected width would be _____ mm.
5. Where does line B intersect the axis?
6. What would be the mathematical interpolation of your answer to Problem 5?
7. Would the information you have about biology help you to justify your answer to Problem 6? Why?

The second major method in linear interpolation is from a table. The following examples will illustrate this method.

5.61 Interpolation and Extrapolation from Tables of Values

Interpolation without the use of a graph is based upon a proportion. An example will best illustrate the principle involved. Given Table A:

Table A

First term	Second term
23	49
26	?
27	65

These are ordered pairs; (23,49) and (27,65). We want to find a second term for (26, ?).

We will build our proportion by comparing the difference between the terms. The difference between the first terms is 4 or (27 - 23), and we want to go "3" of that difference or (26 - 23). The difference between the second terms is 16 or (65 - 49), and we want to go "X" of that difference. Thus, our proportion would be:

first term	second term
$\frac{3}{4}$ (difference desired)/(total difference)	= $\frac{X}{16}$ (difference desired)/(total difference)

Solving the proportion for "X" we have

$$\frac{3}{4} = \frac{X}{16}$$

$$\frac{3}{4} \times 16 = X$$

$$12 = X$$

If X = 12, then adding 12 to the second term, 49, we have 61 . The desired ordered pair is then (26,61).

Here is a second illustration.

Table B

first term	second term
150	176
152	182
154	187
156	193
157	?
158	198

(Brackets in the table: 1 from 156 to 157, 2 from 156 to 158; X from 193 to ?, 5 from 193 to 198.)

Find the second term for 157 .

Our proportion is $\frac{1}{2} = \frac{X}{5}$ (198 - 193)

$$\frac{1}{2} \times 5 = X$$

$$2\frac{1}{2} = X .$$

The second term for the ordered pair of 157 would be $195\frac{1}{2}$ $(193 + 2\frac{1}{2})$. We would round off to 196. Remember the rule for significant digits? All of the first and second terms listed have three significant digits; therefore our interpolated value must not have more than three significant digits. Thus the ordered pair is (157,196) .

A third illustration:

Table C

first term	second term
110	41
120	79
130	113
140	146
150	176

Find the second term when 133 is the first term in (133, ?).

Proportion of first terms: second terms:

$$\frac{3}{10} = \frac{(133 - 130)}{(140 - 130)} = \frac{X}{33} = \frac{X}{(146 - 113)}$$

$$\frac{3}{10} \times 33 = X$$

$$\frac{99}{10} = X$$

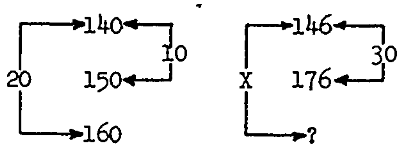
$$9.9 = X$$

$$10 \approx X$$

The second term for 133 would be 113 + X or 113 + 10 = 123 . The ordered pair would be (133,123) .

To extrapolate (find a second term beyond our given table), we developed a similar proportion. Using the data from Table C, find a second term for a first term of 160 in (160, ?) .

first term second term



A proportion is built:

$$\frac{10}{20} = \frac{30}{X}$$

$$10X = 600, \quad X = 60$$

Adding the value 60 to the second term 146, we have the second term for the ordered pair (160,206).

Exercise 5-61a

Use the data from the following table to answer the questions in this exercise.

first term	second term
10	100
20	201
30	303
40	406
50	510
60	615
70	721
80	828
90	936
100	1046

1. What would be the second term for the ordered pair (30,__)?
2. What would be the first term for the ordered pair (__,721)?
3. Find the second term for a first term of 45.
4. Find a second term for the ordered pair of (63,__).
5. Find the second term for a first term of 87.
6. Find the first term for a second term of 252.
7. Find the first term for a second term of 980.
8. The first term is 110. Find the second term.
9. The first term is 103. Find the second term.
10. The second term is 836. Find the first term.

> Based upon the observation of many other similar experiences in your classroom, draw a segment over your graph (of data from table of CO_2 by yeast) which you believe would do a "better" job of generalizing your data. Using a ruler for the segment, move it around until you believe it "best" fits the data.

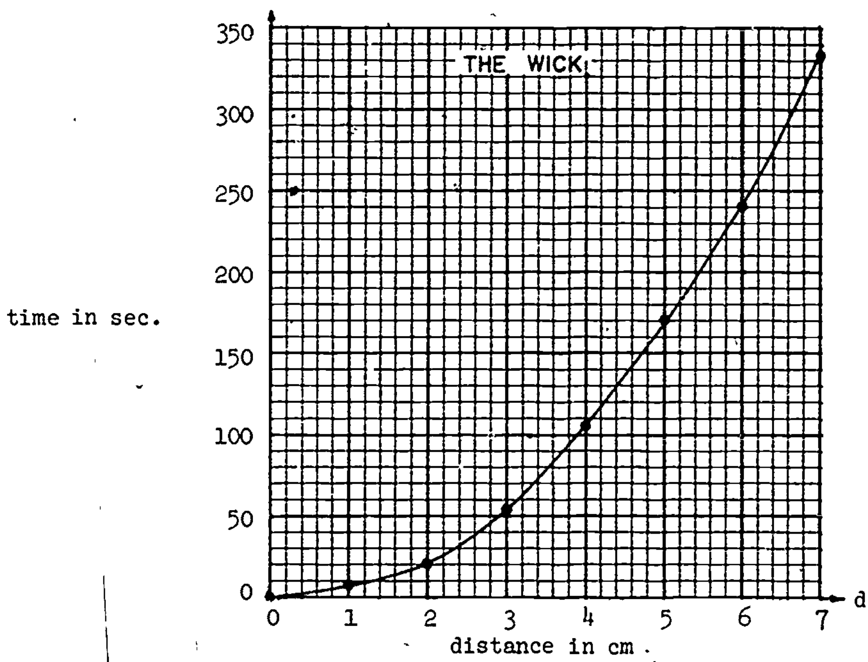
Of course it will not touch all of your observed points. Do you think it is possible that from visual observation alone, some of your readings of the movement of the drop might not be exact? Could the time have been off a second or two?

Scientists and mathematicians frequently generalize in such an experiment as you have just finished. They, the scientists and mathematicians, have various devices and formulas for determining the best curve or segment for their data. We have been using a very informal way of "curve fitting". This is called an <u>empirical curve</u> -- a curve that is drawn to approximately fit a set of data. It is usually assumed to represent, approximately, additional data of the same kind.

Do not be confused by our use of the word <u>curve</u>. A rectangle is considered to be a closed curve in the language of mathematics. A line fitted to some points on a graph, whether a straight, broken, or bending line is considered a curve.

Exercise 5-61b

1. If the condition of your experiment did not change, how far would you expect the mark in the tube to advance by the end of one hour? (Pretend the tube is of sufficient length.)
2. Under the same conditions as in Problem 1, what distance could you anticipate by the end of one week?
3. Given a tube of infinite length, do you think the action shown in your experiment would continue for one hour? One week? One month? One year? Why?
4. Below is a graph of data gathered in a wick experiment. A strip of chromatography paper was calibrated in centimeters. It was suspended over a jar with water in it. The bottom of the paper just touched the water. Time readings were taken as the water moved up each centimeter of length.



(a) Each small square on the d-axis represents what part of a centimeter?

(b) How far did the water move in the first minute? (Give answer in tenths of a centimeter.)

(c) How far did the water move in the first 20 seconds?

(d) How long did it take the water to move up to the 5-centimeter mark?

(e) How long did it take the water to move up to $2\frac{1}{2}$ centimeters?

(f) The smooth curve in this graph indicates a continuous function. What does this mean to you regarding values in between the points?

You can probably recognize by now the fact that your data on the yeast activity indicated a continuous function. This means you should draw a "best" straight line through each of your two sets of graphed points.

5.7 Volume

In the introduction to this chapter we asked whether one could measure "how much" gas is produced in metabolism. We asked: "Can such a thing be measured?" Have you answered the question? One factor which you need to know before proceeding is this: in most organisms (including yeast), the volume of oxygen used is approximately equal to the volume of carbon dioxide released. In yeast, however, this is true only when an unlimited supply of oxygen is available. Now, if you can compute the volume of carbon dioxide produced by the yeast and the amount of oxygen in the syringe at the beginning of your experiment, you should be able to determine theoretically when respiration ended and fermentation began (this happens when all free oxygen is used). Since your tube is a long, slender cylinder, you need to know how to compute the volume of a cylinder.

5.71 Volume of a Rectangular Solid

Let us start by reviewing some of the fundamentals used to determine the volume of a rectangular solid. When we consider the volume of a box that is 1 inch high, 2 inches wide, and 3 inches long, we are considering the number of cubical units in the box. Our box would be exactly like one made by placing 6 one-inch cubes side by side, 2 cubes wide and 3 cubes long. We could mentally picture this box to appear as in Figure 5-71.

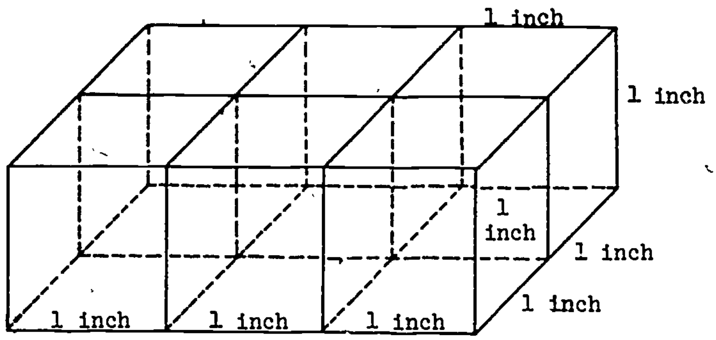


Fig. 5-71

In the figure we can easily see the 6 cubical units that make up this rectangular model. Can we find this "volume" mathematically so we will be spared the bother of drawing a model? Of course! You all remember that volumes are determined by finding the area of the base and then multiplying by height. In the case of our example, the base area is width times length or $2 \text{ in} \times 3 \text{ in}$, and this is equal to 6 square inches (6 in^2). The area of the base (6 in^2) $\times$ the height (1 in) = volume (6 in^3). Notice that the volume is always expressed in cubic units (units^3) whereas length, width, and height are measured in units of length such as centimeters, meters, inches, feet, yards, etc.

Exercise 5-71a

1. Find the volume of a rectangular object that is 4 cm wide, 8 cm long, and 2 cm high.

2. Given a bottle with flat sides (planes), a square base and the inside edges of the base 1 cm, what would be the volume of liquid when filled to a depth of:
   (a) 1 cm ?
   (b) 2 cm ?
   (c) 4 cm ?

3. Given another bottle with flat sides but with a square base of 2 cm on each internal edge, what would be the volume of liquid when filled to a depth of:
   (a) 1 cm ?
   (b) 3 cm ?
   (c) $\frac{1}{2}$ cm ?
   (d) $2\frac{1}{2}$ cm ?

4. Given a metal box (parallelopiped), rectangular base, with internal measurements of 4 cm for length, 2 cm for width, and 25 cm high, what would be the volume of liquid when filled to a depth of:

(a) 1 cm ? (c) 5 cm ?

(b) 2 cm ? (d) 25 cm ?

5.72 Volume of a Cylinder

How can we find the volume of a cylindrical solid? In one sense there is a fairly easy method. If the solid is like a tin can and will hold water (or sand) we can fill it up and then pour it into a standard container. However, we would like to know what the answer is without having to do this every time. For some cylinders, this method would be very impractical, perhaps impossible.

Recall how we found the volume of a box. We first considered a box one unit high. The number of cubic units in this box would be the same as the number of square units in the base. Thus, the measure of the volume was clearly the measure of the area of the base times one. If the box had an altitude of two units, then the measure of the volume would clearly be twice as much as the measure of the area of the base. That is, it would be 2 times the measure of the area of the base.

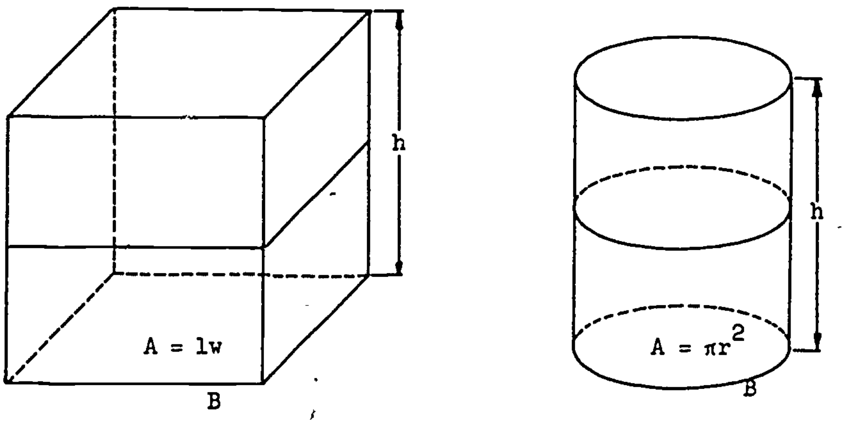


Fig. 5-72

In general, if the area of the base were B square units and the altitude of the box were h units, then the volume would be $B \times h$ cubic units.

Exactly the same situation occurs with a cylindrical solid. The measure of the volume of the cylinder is simply the measure of the area of the base times the measure of the altitude. The area of the base of a cylinder is πr^2 square units. So the volume is $\pi r^2 \times h$ cubic units.

We now have one basic principle which applies to boxes, to right prisms, and to right cylinders (cylinders whose "sides" are perpendicular to the base). The measure of the volume is the measure of the area of the base times the measure of the altitude. In mathematical terms, this is frequently written as follows:

$$V = Bh$$

where B represents the measure of the area of the base and h represents the measure of the height.

You should learn and remember how to compute the volume of a cylindrical solid. To compute the volume of any solid of this type, we simply multiply the measure of the area of the base by the measure of the altitude. The altitude is the (perpendicular) distance between the parallel planes which contain the bases. If you think of the geometrical figure and what it is you want to find, then most problems of this type are very easy.

As an example, assume you want to find the volume of a cylinder whose radius has a measure of 7 and whose height (or altitude) has a measure of 10.

$$V = Bh; \quad \text{for a cylinder,} \quad B = \pi r^2$$

Therefore,

$$V = \pi r^2 h$$

$$V \approx \frac{22}{7} \times 7 \times 7 \times 10$$

$$V \approx 1540 \ .$$

The volume of the cylinder is about 1540 cubic units. ($\frac{22}{7}$ is only a handy approximation of the value of π.)

A note on computation. Sometimes when making computations involving π, it is easier to use a decimal approximation for π only at the last step of the arithmetic. Remember, multiplication has the commutative property. That is, the order of multiplication does not affect the product. In this way we use long decimals as little as possible in our computations. Consider $\pi \times 5^2 \times 8$.

If we commute to $5^2 \times 8 \times \pi$

then $25 \times 8 \times \pi$

$$200 \times \pi \approx 628 \ .$$

If we don't commute then $(3.14 \times 25) \times 8$

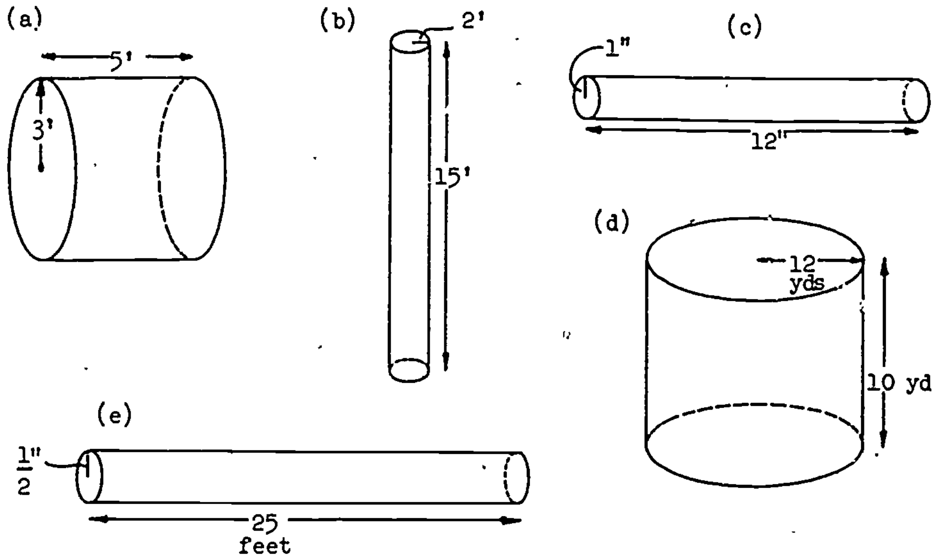
$$78.50 \times 8 \approx 628 \ .$$

Exercise 5-72

1. Information is given for five right cylinders. The letters r and h are the measures of the radius of the circular base and the height of the cylinder respectively. Using 3.1 as an approximation for π, find the volumes of each cylinder.

	Cylinder	Radius (r)	Height (h)	Volume
(a)	A	4 in	8 in	?
(b)	B	8 ft	4 ft	?
(c)	C	10 cm	30 cm	?
(d)	D	7 yds	25 yds	?
(e)	E	12 in	12 in	?

2. Find the volumes of the right cylinders shown here. The dimensions given are the radius and the height of each cylinder. The figures are not drawn to scale. (Use $\pi \approx 3.1$.)



3. A silo (with a flat top) is 30 feet high and the inside radius is 6 feet. How many cubic feet of grain will it hold? (What is the volume?) Use $\pi \approx 3.14$.

4. A cylindrical water tank is 8 feet high. The diameter (not radius) of its base is 1 foot. Find the volume (in cubic feet) of water which it can hold. Leave your answer in terms of π . If you use an approximation for π , what is your answer to the nearest (whole) cubic foot?

5. There are about $7\frac{1}{2}$ gallons in a cubic foot of water. About how many gallons will the tank of Problem 4 hold?

6. Find the amount of water (volume in cubic inches) which a 100-foot length of pipe will hold if the inside radius of a cross-section is 1 inch. Use $\pi \approx 3.14$. (A cross-section is the intersection of the solid and of a plane parallel to the planes of the bases and between them.)

J.F.F. A man walked one mile south, then one mile east, and then one mile north and found he had returned to his starting point. Where did he start (if he was south of the equator)?

5.8 Volume of Gas Produced by Yeast Activity

One experiment (done by a biology instructor) used the following equipment and measurements:

Total capacity of syringe -- 12 cubic centimeters (cc)
Diameter (inside measure) of tubing = 3 mm
When started, the syringe contained 5 cc of liquid and 7 cc of gas (air).

If the tube is a cylinder with a diameter of 3 mm, what is the area of the base of such a cylinder? How many mm^3 are needed to make 1 cm^3? If you poured 1 cm^3 of liquid into a tube of diameter 3 mm, to what length would the tube be filled?

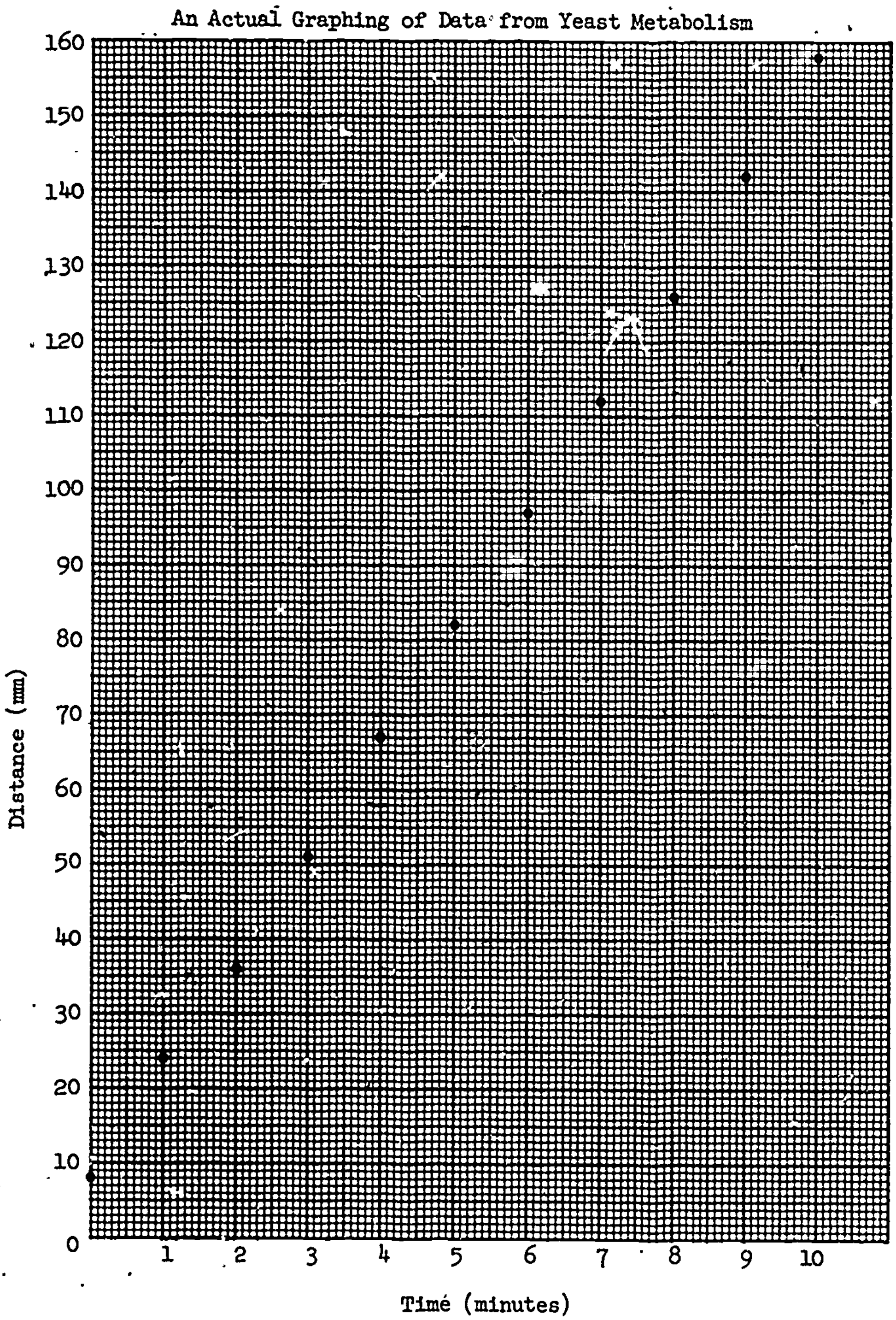

An Actual Graphing of Data from Yeast Metabolism
Distance (mm)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
1
2
3
4
5
6
7
8
9
10
Time (minutes)

Fig. 5-8

Exercise 5-8a

Using the data shown above, answer the following questions. (Use 3.14 for π.)

1. The length of tubing from the syringe to the bubble (drop of liquid) was 10 cm. What is the volume of gas (air and carbon dioxide) in the tubing (answer in cubic mm)?

2. The volume of gas in the syringe was 7 cc. Express this as cubic mm.

3. What is the total volume of gas at the time the experiment started?

4. How much gas (in cubic mm) was produced during the first minute?

5. How much gas was produced by the end of 2 min?

6. At the end of the experiment (first record in table of data) how much gas (in cubic mm) had been produced?

7. What is the total volume of gas in the system at the end of the experiment?

8. The volume of liquid (expressed in cubic mm) in our syringe was _____?

9. What is the ratio of volume of liquid to gas produced in 10 minutes?

10. If the liquid continued to produce gas at the same rate, in how many minutes would the volume of gas produced equal the volume of liquid?

11. Theoretically, how much gas would be produced in our system in one hour?

12. Gangbusters: At the same rate, how much gas would be produced in our system in 1 year (365 days)?

    Express your answer in scientific notation and in terms of cubic mm. Two significant digits.

    Do you think such an experiment could continue for a year? Why?

13. Express your answer to Problem 12 without scientific notation in cubic meters.

14. Now, go back to Sections 5.2 and 5.7, first paragraph, and review the information given there. Assuming that 20 percent of the air in the syringe was oxygen, at what theoretical point in the generation of CO_2 by the yeast did respiration cease and fermentation begin?

Exercise 5-8b

[Using the apparatus in your own activity,]

1. Carefully measure the inside diameter of the syringe.
2. Compute the volume of gas in the system when it was zero time.
3. From your table of data - compute the volume of gas produced in:
   (a) 1 minute
   (b) 5 minutes
   (c) 10 minutes
4. Interpolate from the answer to Problem 3 for the volume of gas produced in 7 minutes.
5. From your table of values, compute the volume of gas produced in 7 minutes.
6. How do the answers to Problems 4 and 5 compare?
   Which do you believe is the more accurate? Why?
7. Extrapolate for the expected volume of gas produced in 30 minutes.
8. In your experiment, if an unlimited supply of oxygen had been available, what would have been the theoretica approximate volume of oxygen used?

Chapter 6

GROWTH OF MOLD: RECTANGULAR COORDINATES, ESTIMATION OF AREA

6.1 Introduction

Another major concept of living things is growth. The amazing hing is that all living things have quite similar growth patterns. All living things get larger; most of them become more complex, that is, they develop specialized parts such as arms, legs, hearts, brains. Most plants develop roots, stems, leaves, and flowers. Even populations grow. Here we are not necessarily speaking of human populations, but of all kinds of populations. One example is a colony of bacteria (the mass of bacteria that arises from one single bacterium that happens to find a suitable place to grow -- or is deliberately placed there by an experimenter, a neat trick which requires special techniques). Rabbits in Australia are another example. A tremendous number (population) of rabbits existed there until recently when a natural control, a fungus disease, was introduced which killed off most of them. All of the rabbits were descended from a few that were introduced many years ago.

The number of people in the world is steadily increasing -- you have all heard or read about the "population explosion". We are interested here in seeing the pattern of growth.

Do populations grow at a constant (linear) rate? If they do not grow at a constant rate, what might be some of the factors influencing changes? What is meant by the term "population explosion"?

These are questions you shoul be able to answer after you have investigated the growth of a population of mold.

However, before you can proceed intelligently with this activity, there are some additional mathematical procedures you need to know. One of these is how to work with rectangular coordinates. You will also be using some of your previous experience: estimating area by visual observation, extrapolation and interpolation.

6.2 Coordinates

If the teacher asked for the student in the third row and the fourth seat to stand up, would you know who should rise? Would the same person stand if the teacher called for the student in row four, seat three? Did you start numbering rows from the left or right side, your right or the teacher's right? Did you number seats from the front or the back of the room? Why front? Could you not number from the back? Why not start with the middle seat in the room and number both ways? Have we done anything for you besides cause confusion?

A system of locating points in a plane is very useful and is often needed, but definite points of reference (a place to start from in this case) must be included in the system.

Use as your "starting point" the front seat in the end row to the teacher's left, as he faces the class (got it?).

Exercises 6-2

1. The first name of the student in Row 3, Seat 4, is ________.
2. The first name of the student in Row 4, Seat 3, is ________.
3. How many students are in Row 2?
4. How many students would be in a "4th seat" back?
5. How many students would have the same number for both row and seat?

This system of identification of points works very well only if the numbers used are both positive. However, from our work with the number line and in observing various facets of business and mathematics, we know that often negative numbers appear and need to be plotted.

Businessmen talk about "gains and losses". We know ourselves that having no money (zero) is better than owing five dollars (-5). A man who owns a \$20,000 house (+20,000) may have a mortgage on it for \$17,000 (-17,000). The net value then is the difference: \$3,000 (+3,000). The number line shows these relationships clearly.

6.21 Coordinates on a Line

A rational number can always be associated with a point on the number line. The number associated in this way with a point on the line is called a coordinate of the point.

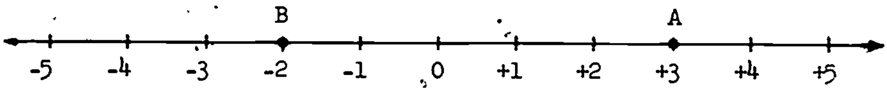


Figure 6-21a

In Figure 6-21a we note that point A is denoted by the number (+3). Point B is denoted by (-2). We write A(+3) to mean that A is the point with coordinate +3. Likewise, B(-2) means that B is the point with coordinate (-2) on the line.

The coordinate we have assigned to a point in this way tells us two things. It tells us the <u>distance</u> from the origin to the point. It also tells us the <u>direction</u> from the origin to the point.

Exercises 6-21

1. Draw a segment of a number line 6 inches in length. Mark off segments of one inch in length and place the origin at its midpoint. On the line locate the following points:

   A(-1), B($\frac{5}{2}$), C(1), T(0), L(- $\frac{3}{2}$), P(-2).

2. (A) In Problem 1, how far is it in inches between the point labeled T and the point labeled L?
   (B) Between P and B?
   (C) Between L and B?
   (D) From the origin to A?

3. Using a number line with 1 inch as the unit of length, mark the following points:

   R($\frac{1}{3}$), S($\frac{5}{6}$), D(- $\frac{3}{2}$), F(0), E($\frac{3}{2}$).

4. If the line segment in Problem 3 were a highway and if it were drawn to a scale of 1 inch representing 1 mile, how far in miles would it be between these points on the highway:
   (A) F and R? (B) D and E?

5. Draw a number line in a vertical instead of horizontal position. Mark your number scale with positive numbers above the origin and negative numbers below the origin. Label points to correspond with the rational numbers 0, 1, 2, 3, -1, -2, -3, -4.

From the above exercise you can see that the decision of "where to start and which way to go" is up to us. We select a zero point, then a "unit distance" to the number one. From then on the remaining points with coordinates are determined. Arbitrarily, it is understood, that unless otherwise stated, to the right or up from zero is positive and to the left or down from zero is negative.

6.22 Coordinates in the Plane

You have learned that a single coordinate locates a point on the number line. A point like S in Figure 6-22a is not on the number line and cannot be located by a single coordinate. However, we see that S is directly above the point A(+3).

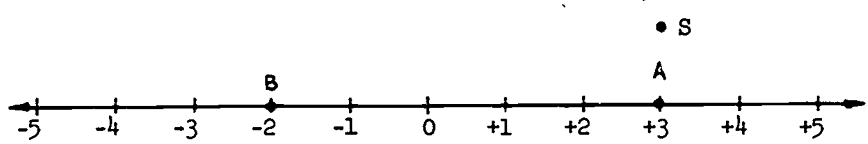


Figure 6-22a

To locate point "S", draw a vertical number line perpendicular to the horizontal number line and intersecting it at the zero point. This point of intersection is named the origin. Number the vertical line as shown in Figure 6-22b.

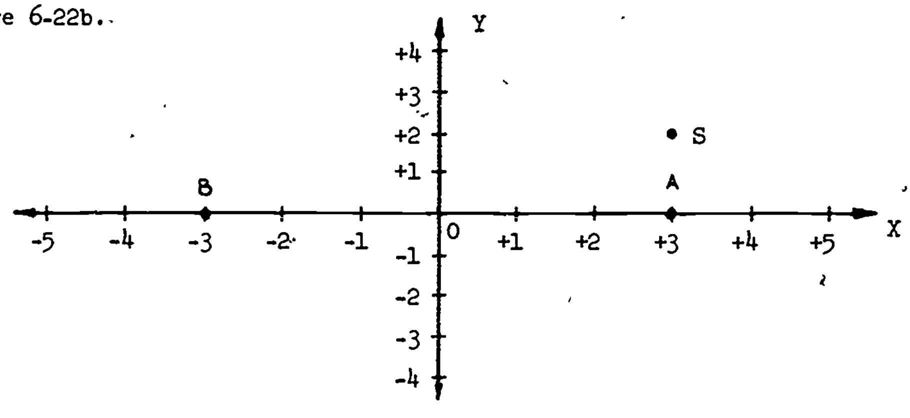


Figure 6-22b

The horizontal number line is called the X-axis and the vertical number line is called the Y-axis. When we refer to both number lines we call them the axes (notice the spelling).

To determine the coordinates of the point S, construct and mark line segments as in Figure 6-22b and draw a line segment from point S perpendicular to the X-axis. It intersects the X-axis at (+3). Now draw a perpendicular line segment from point S to the Y-axis. It intersects the Y-axis at (+2). Point S is said to have an x-coordinate of (+3) and a y-coordinate of (+2), which we write as (+3,+2). We use parentheses and always write the x-coordinate before the y-coordinate (an arbitrary rule all mathematicians follow) - the reason why such pairs of numbers are called "ordered pairs".

In Figure 6-22c observe how the coordinates of points A, B, C and D were located.

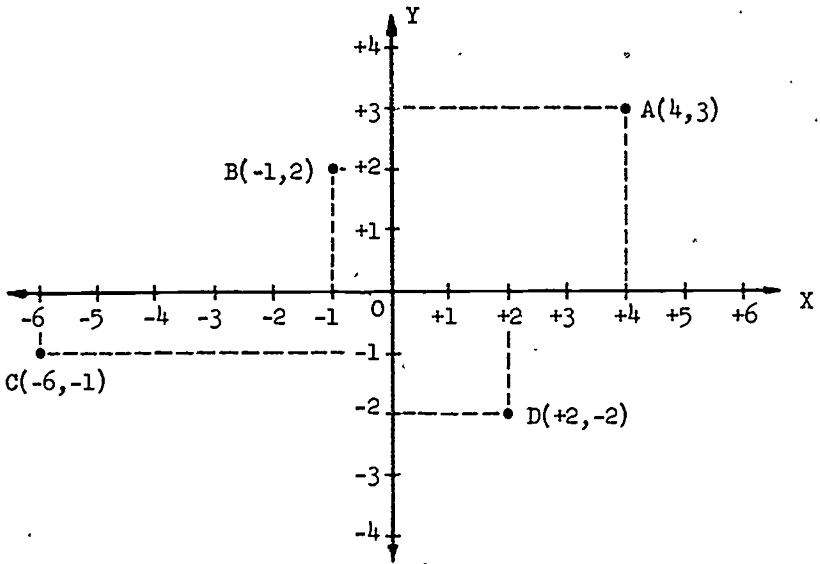


Figure 6-22c

Thus P(x,y) represents the point P in terms of its coordinates. This may be done for any point P in the plane. This system of coordinates is called a rectangular system because the axes are at right angles to each other and distances of points from the axes are measured along perpendiculars from the points to the axes. Each ordered pair of rational numbers is assigned to a point in the coordinate plane. Locating and marking the point with respect to the X-axis and the Y-axis is called plotting the point.

The idea of a coordinate system is not new to you. When you locate a point on the earth's surface, you do so by identifying the longitude and latitude of the point. Note that the order in which you write these numbers is important. For example, suppose you looked up the longitude and latitude of your home town and accidentally switched the numbers around. It is possible that your description would place the location of your home town in the middle of the ocean.

The system of rectangular coordinates is often referred to as Cartesian coordinates. The name Cartesian comes from the man Rene Descartes (Ren-ay' DAY-CART') who is credited with first devising the system. Descartes was born near Tours, France, on March 31, 1596, and died at Stockholm, Sweden, on February 11, 1650. His main contribution to mathematics was his development of analytical geometry. (Analytical geometry is a branch of mathematics which stresses the interrelationships of algebra and geometry.)

Descartes moved to Holland in approximately 1628 and spent almost 20 years there studying philosophy and mathematics. He said science could be compared to a tree; metaphysics is the root, physics the limb, and the three main branches are medicine, mechanics, and morals, these being the three applications of their knowledge -- the human body, the external world, and the conduct of life.

Exercises 6-22

1. Given the following set of ordered pairs of rational numbers, locate the points in the plane associated with these pairs and label each point with its coordinates.
{(4,1), (1,0), (0,1), (2,4), (4,4), (-1,-1), (-3,3), (4,-3), (-5,3), (0,-5), (-6,0)}

2. On graph paper draw a pair of axes and label them. Plot the points in the following sets. Label each point with its coordinates. Use a different pair of axes for each set.

   Set A = {(6,-3), (-7,-1), (-9,-7), (5,-1), (-8,10), (0,0), (-1,-1), (4,3)}

   Set B = {(1,1), (6,-5), (-3,-3), (4,-10), (-9,-6), (-8,0), (0,-5), (-2,-5)}

3. (A) Plot the points in the following set:

   Set C = {(0,0), (-1,0), (+1,0), (-2,0), (+2,0), (-3,0), (+3,0)}.

   (B) Do all of the points named in Set C seem to lie on the same line?
   (C) What do you notice about the y-coordinate for each of the points?
   (D) Are there any points on this line for which the y-coordinate is different from zero?

4. (A) Plot the points in the following set:

Set D = {(0,0), (0,-1), (0,1), (0,-2), (0,2), (0,-3), (0,3)}.

(B) Do all of the points named in Set D seem to lie on the same line?

(C) What do you notice about the x-coordinate for each of the points?

(D) Are there any points on this line for which the x-coordinate is different from zero?

6.23 Rectangular Coordinates

A system of rectangular coordinates separates the plane into 4 regions. These regions are usually named by a Roman Numeral as designated in Figure 6-23.

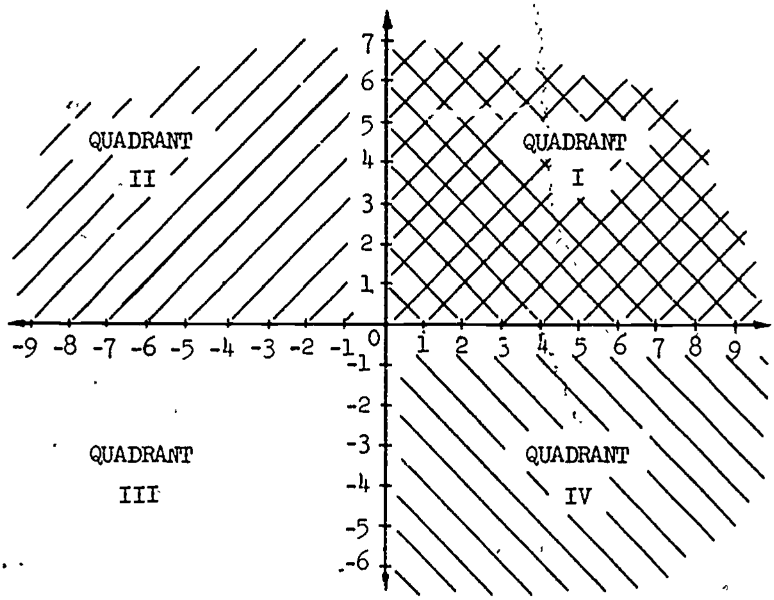


Figure 6-23

The coordinate axes are not a part of any quadrant; thus, (0,2) is on the Y-axis and is not in either Quadrant I or Quadrant II. According to our explanation, would the origin be in any quadrant? All quadrants?

The numbers in ordered pairs may be positive, negative, or zero, as you have noticed in the exercises. Both numbers of the pair may be positive. Both numbers may be negative. One may be positive and the other negative. One may be zero, or both may be zero.

Any point, whose coordinate is an ordered pair in which one or both numbers are zero, is on one of the axes. Such a point would not be in any quadrant.

Exercises 6-23a

1. Given the following ordered pairs of numbers, write the number of the quadrant, if any, in which you find the point represented by each of these ordered pairs. (A number without a sign is understood to be a positive number.)

Ordered Pair	Quadrant
(A) (3,5)	________
(B) (1,-4)	________
(C) (-4,4)	________
(D) (-3,-1)	________
(E) (8,6)	________
(F) (7,-1)	________
(G) (-3,-5)	________
(H) (0,7)	________

2. (A) Both numbers of the ordered pair of coordinates are positive. The point is in Quadrant ________.
   (B) Both numbers of the ordered pair of coordinates are negative. The point is in Quadrant ________.
   (C) The x-coordinate of an ordered pair is negative and the y-coordinate is positive. The point is in Quadrant ________.
   (D) The x-coordinate of an ordered pair is positive and the y-coordinate is negative. The point is in Quadrant ________.

3. (A) If the x-coordinate of an ordered pair is zero and the y-coordinate is not zero, where does the point lie?
   (B) If the x-coordinate of an ordered pair is not zero and the y-coordinate is zero, where does the point lie?
   (C) If both coordinates of an ordered pair are zero, where is the point located?

4. Points on either the X-axis or the Y-axis do not lie in any of the four quadrants. Why not?

Exercises 6-23b

1. (A) Plot the points of set L = {A(2,1), B(2,3)}.
   (B) Use a straightedge to join A to B. Extend line segment AB.
   (C) Line AB seems to be parallel to which axis?

2. (A) Plot the points of set M = {A(2,3), B(5,3)}.
   (B) Use a straightedge to join A to B. Extend line segment AB.
   (C) Line AB seems to be parallel to which axis?

3. (A) Plot the points of set N = {A(0,0), B(2,3)}.
   (B) Join A to B. Extend line segment AB.
   (C) Is line AB parallel to either axis?

4. (A) Plot the points of set P = {A(4,4), B(2,0)}.
   (B) Join A to B. Extend line segment AB.
   (C) Plot the points of set Q = {C(6,3), D(0,1)}.
   (D) Join C to D. Extend line segment CD.
   (E) What is the intersection set of lines AB and CD?

5. (A) Plot the points of set R = {A(0,0), B(6,0), C(3,4)} on the coordinate plane.
   (B) Use a straightedge to join A to B, B to C, C to A.
   (C) Is the triangle (1) scalene, (2) isosceles, or (3) equilateral?

6. (A) Plot the points of set S = {A(2,1), B(-2,1), C(-2,-3), D(2,-3)}.
   (B) Use a straightedge to join A to B, B to C, C to D and D to A.
   (C) Is the figure a square?
   (D) Draw the diagonals of the figure.
   (E) The coordinates of the point of intersection of the diagonals seem to be ___________.

7. (A) Plot the points of set T = {A(2,1), B(3,3), C(-2,3), D(-3,1)}.
   (B) Use a straightedge to join A to B, B to C, C to D, D to A.
   (C) What is the name of the quadrilateral formed?
   (D) Draw the diagonals of quadrilateral ABCD.
   (E) The coordinates of the point of intersection of the diagonals seem to be ___________.

6.3 Activity -- Growing Mold

The experience and knowledge you gained in the section on Rectangular Coordinates will be used in this activity. You will read a coordinate system to identify the position of the mold.

Material Needed

Aluminum pie or cake tin approximately 9 inches in diameter (one per team. Keep "team" small - 1, 2, or at most 3 on a team.)

10 × 10 to the inch graph paper to cover bottom of tin (one per team). This may require fastening two sheets together so that the bottom of the tin will be completely covered.

10 × 10 to the inch graph paper for daily graphing of the data and final graph (eight sheets per student).

Gelatin (colorless if possible), one envelope per 10-12 teams.

Bouillon cube (preferably beef), one cube per 10-12 teams.

Saran Wrap, enough to cover each tin.

Rubber bands (one per team), large enough to fit snugly around tin, or Scotch tape if tins have tapered sides.

Scissors and rulers.

Read all of the following procedure carefully before you start any of the activity.

Procedure

1. Prepare the tin:

   1.1 Cut the graph paper to fit into the bottom of the tin as smoothly as possible at the edges (place the tin on the graph paper and trace around the edge).

   1.2 With the origin near the center of the circle, construct X and Y axes on your paper (refer to Section 6.2), using a lead pencil or waterproof ink.

   1.3 Fit the paper into the bottom of the tin. Rubber cement or glue spread on both the tin and the back of the graph paper helps to hold it in place. See Figure 6-3a.

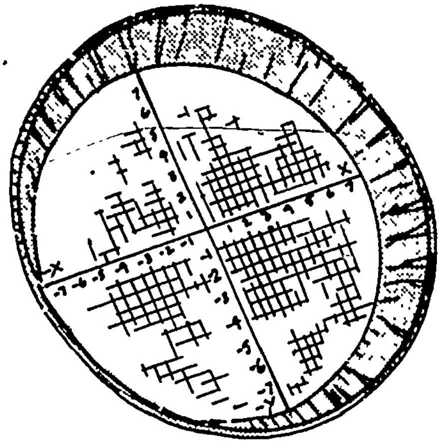


Figure 6-3a

2. Prepare the gelatin mix: (sufficient for 10-12 teams)
For smoothest mixture proceed in the following order.
(A) Mix envelope of gelatin with 2 tablespoons of water from cold tap.
(B) Mix bouillon cube with 2 tablespoons of hot water.
(C) Combine the two mixes A and B.
IMPORTANT: THIS MIX MUST STILL BE WARM ENOUGH TO POUR WHEN ALL TINS ARE READY. As it cools, it "sets". It is recommended that one student prepare this mixture for the entire class while other students are preparing the tins. This mixture is the growth medium (the substance from which the mold will receive nourishment).

3. Pour into each tin, a team at a time, enough medium to cover the graph paper with a thin layer. (Do not worry if the paper buckles slightly.) Pour any excess mix back into the container. Put the tins aside to cool and "set" (2-5 minutes). While it is "setting", it will be contaminated with mold from the air. (Mold spores floating in the air will fall on the mix and, finding a favorable medium, will begin to grow.)

4. Cut a piece of Saran Wrap or similar transparent wrap large enough to cover the tin and fold down on all sides.

5. After a few minutes (2-5), cover the tin with the Saran Wrap and fasten with a rubber band or Scotch tape.

6. Store in a dark place, a drawer or a cupboard, where the temperature is fairly uniform.

7. Observe daily and record your observations every day.

Recording

On the first day that mold is visible and each day thereafter, use the following method: (the mold will probably first appear on about the 2nd, 3rd, or 4th day). The mold will appear as tiny dots which may be any of several colors, depending on the species of mold which happens to fall on the medium.

1. Each student should have a piece of graph paper on which he has prepared a coordinate system similar to that in the tins. There should be one for each student for each day.

2. One student on each team should study the tin for evidence of mold, read the ordered pair which best identifies the location of each colony, and give to his teammates an estimate of the number of squares (each equals one-one hundredth of an inch square) covered. For instance: at the beginning there will probably be several "pin-points" of mold. (See Figure 6-3b.) They should be considered as one square. The best anyone can do in a case such as this is estimate, but this will be accurate enough for your purposes. EACH COLONY SHOULD BE RECORDED ON THE GRAPH AT THE PROPER COORDINATE IN NUMBERS OF SQUARES COVERED. When all the colonies have been recorded, note the total number of squares covered.

3. Each day

   3.1 Record the total number of squares covered.
   Record your data, daily, in line B, on a table such as 6-3.

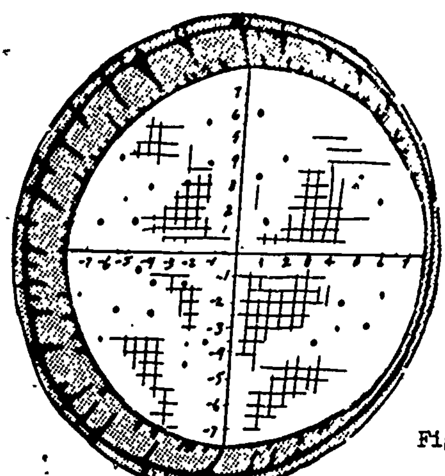


Figure 6-3b

Table 6-3

Record of Mold Growth

A	B		C		D	
DAYS	TOTAL AREA TO DATE (SQUARE UNITS)		INC. AREA OVER PREVIOUS DAY (SQUARE UNITS)		PERCENT OF INCREASE - INC. AREA / TOTAL AREA	
Start (Friday)						
2nd. Saturday 3rd. Sunday	(Extra-polate)	(Inter-polate)	(Extra-polate)	(Inter-polate)	(Extra-polate)	(Inter-polate)
4th. Monday 5th. Tuesday 6th. Wednesday 7th. Thursday 8th. Friday						
9th. Saturday 10th. Sunday						
11th. Monday 12th. Tuesday You may go on...						

To complete your table each day, record the values for Column C by finding the difference between that day's area and the previous day's area (e.g., Wednesday's reading - Tuesday's reading). To fill in Column D find the percent of increase of area to total area (increase in area ÷ total area to date × 100). After several days your mold growth might look like Figure 6-3c.

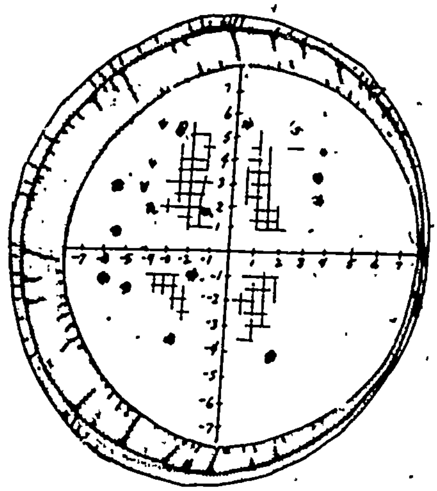


Figure 6-3o

3.2 Record the difference between that day and the previous day's growth.

3.3 Plot the total growth data from Column B on a graph (one graph to be used continuously throughout the experiment) using day number for the X-axis and area of squares covered for the Y-axis. Note: The pattern of growth should show best if you use one one-half inch unit of your X-axis for each day, and let each one-half unit of the Y-axis represent 25 squares covered.

3.4 After the second daily addition to the graph, predict the point of the next day's growth.

There will be two parts of your table in which actual data cannot be obtained, the two weekends. (Don't take the mold growth apparatus home.) Instead, do two mathematical operations, extrapolation and interpolation. If necessary, review Chapter 5, Section 5.6. Approximate a guess ahead of time for the values of Saturday and Sunday. Base your approximation upon the previous data obtained, Friday for the first weekend and Monday through Friday for the second weekend. Interpolate on Mondays for the preceding Saturday and Sunday. Our interpolation will probably not coincide with our extrapolation. There are several mathematical interpolation formulas which you may look up in an advanced mathematics text or dictionary, but we will not use any at this time. Extrapolate and interpolate by the processes learned in Chapter 5.

4. When you have completed the seventh day's recording and graphing, study your graph carefully. This will help you to answer the following questions.

   1. Were your predictions (extrapolations) in all cases correct?
   2. If not, at what points did they fail?
   3. What tentative explanations can you give for any deviations you found from a linear (straight line) graph?

The curve which you have recorded on your graph is a typical "growth curve". What does it look like to you? Your teacher can tell you the technical name used to describe this curve.

In addition to your graph of total growth, plot a second graph using for reference lines, "Days of Record" for the horizontal line and "Percent of Inc. Area" for the vertical line. See Figure 6-3a. The points plotted as ordered pairs would be first term: "days of record" and second term: "% of inc. area" (from Column D).

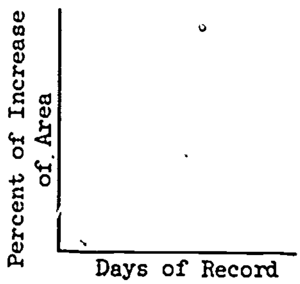


Figure 6-3a

On your graphs record your extrapolated and interpolated points for each Saturday and Sunday by using special marks such as "X" or "O".

It would be a coincidence if the extrapolation points and interpolation points coincided. The interpolated points should somewhat "correct" the extrapolated guess.

Exercises 6-3

The following sample data was obtained from actual measurements of various types of growth. Included are measurements of diameter, percentage of adult weight, number of bacteria in millions per ml of broth, dry weight in grams of corn in a field.

Graph each of these sets of data and compare the resulting graphs with your graph of mold growth.

1. Growth of Gourd Fruit

Diameter in mm (vertical reference line)	Time in days (horizontal reference line)
4	1
5	2
6	3
8	4
9.5	5
12	6
15	7
20	8
26	9
31	10
36	11
41	12
45	13
46	14
47	15
47.5	16
47.5	17
49	18

2. Growth of a Chick

Percentage of adult weight (vertical)	Time (in days) (horizontal)
4	21 (Hatching)
4.5	30
5	40
7	50
20	75
38	100
55	125
65	150
78	175
82	200

3. Average Growth of Corn Plants in a Field

Weight (dry weight in grams) (vertical)	Time (in weeks after planting) (horizontal)
10	1
20	2
60	3
100	4
140	5
210	6
280	7
350	8
440	9
560	10
630	11
700	12
750	13
800	14
840	15
900	16

4. Population Growth of Bacteria

Number of Bacteria (in millions) per ml of broth (vertical)	Time in hours (horizontal)
4×10^6	0
4×10^6	6
5×10^6	12
6×10^6	18
8×10^6	24
20×10^6	30
70×10^6	36
93×10^6	42
99×10^6	48
102×10^6	54
103×10^6	60
104×10^6	66
104×10^6	72

After 72 hours a new culture of bacteria was started by inoculating (introducing) a new batch of broth from the above culture.

4×10^6	0
5×10^6	6
6×10^6	12
7×10^6	18
18×10^6	24
70×10^6	30
80×10^6	36
90×10^6	42
100×10^6	48
102×10^6	54
103×10^6	60
104×10^6	66
104×10^6	72

The above two sets of data should be graphed side by side. This will show how a population of bacteria, after being slowed by limiting factors, then started anew in a medium, will renew its population growth. See Figure 6-3b.

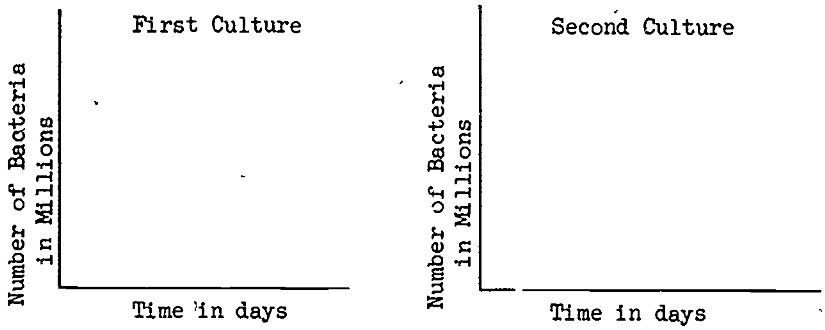


Figure 6-3b

5. Human Growth · Body, liver, and brain
Per cent of full size

Age	Brain (Symbol •)	Liver (Symbol ×)	Total Wt. of Boys (Symbol ♂)	Total Wt. of Girls (Symbol ♀)
Birth	27	17	8	8
1	65	25	17	17
2	75	31	21	21
3	82	35	24	24
4	88	37	27	27
5	94	42	30	36
10	98	60	52	60
15	100	83	87	95
18	100	95	100	100

Plot all of this data on the same set of reference lines using separate symbols (• , × , ♂, and ♀ , for example).

Let the horizontal distance represent age and the vertical, per cent of full size. One unit should represent one year horizontally and 10 per cent vertically.

6. Now that you have graphed and noted the pattern of a "typical growth curve", can you guess what a curve of the population growth in the United States might look like to cause scientists and sociologists so much concern over what they call the "population explosion"? Draw what you think such a graph might look like, using for your X-axis the years from 1660 to 1960 in 20-year intervals, and for the Y-axis, the population in intervals of 10 million people. Your teacher has data based on fact which may be graphed to compare with your "guess".

6.4 Second Activity - Growth of Crystals

Growth is defined as an increase in size and/or complexity. We have so far in this book been concerned with living things, their growth and development. This activity deals with growth of another sort--very rapid and fascinating to watch.

You will need:

- Household liquid bluing
- Household ammonia
- Table salt (sodium chloride)
- Water

You also will need:

- Milk cartons
- Pie plates
- Paper toweling
- Razor blade
- Plastic rulers--metric
- Acetate (X-ray film)
- Charcoal briquets or commercial brick
- Graph paper-- 10 × 10 to the inch

Many different activities can be done with the solution we will mix. This section will give the general directions and then your teacher will determine just what activity you will do and which group you will be a part of.

Solution

The mixture is a saturated solution of salt (sodium chloride). The proportions are:

6 parts of salt
6 parts of water
6 parts of bluing
1 part of ammonia

By "part" we mean any equal portion by volume. You may use teaspoons or some other quantity measure.

Mix them all together (may be added in any order); stir well. Some of the salt may not dissolve. It does not matter. This solution is the basic ingredient for all of the activities which follow.

Area

Construct an apparatus as illustrated in Fig. 6-4a.

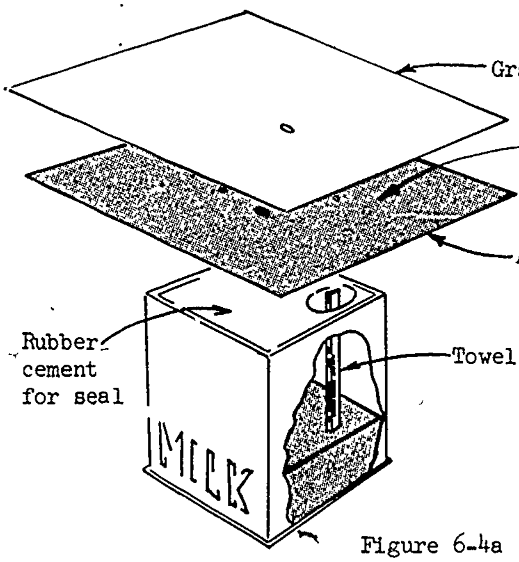


Figure 6-4a

Make a wick of paper toweling 8 layers $\approx$ 1 mm thick, 5 mm wide, 9 mm long. Staple it.

Using rubber cement, glue a piece of rectangular coordinate graph paper (10 × 10-inch) to a piece of acetate (X-ray film). Use lots of cement. Be sure the paper is well glued to the acetate, especially where the wick will come through.

Remove the lid from a milk carton (half-pint), not the whole top, just the round lid. This will give a surface to which you can glue the plastic (X-ray film).

Cut a small triangle in the center of the plastic and graph paper; make this triangular hole slightly smaller than the paper wick and insert the wick so it is through the hole about $\frac{1}{16}$ inch on the graph paper side.

Pour a mixture of the solution into the milk carton. This should be about $2\frac{1}{2}$ cm to 3 cm deep.

Cover the top of the mild carton with rubber cement or glue and insert the wick in the hole of the mild carton and lower the wick with plastic attached. Press the plastic firmly so the glue will make good contact.

Record the area covered against the time in a table.

Time of Day	Total Minutes	Approximate Radius	Area (Circle)

At first you may be able to count the squares. As growth occurs you may decide to change your approximation to fit the area of a circle $(A = \pi r^2)$.

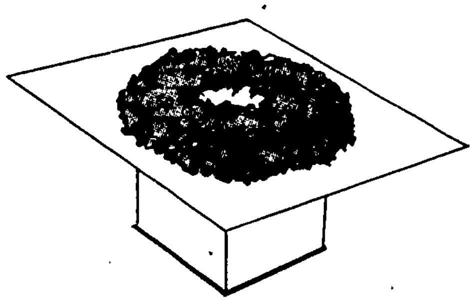


Figure 6-4b

Some class member should keep a close watch. The first specks of growth may appear in 15 to 20 minutes, depending upon the relative humidity and temperature of the classroom. A lamp placed above ($\approx$ 10 cm) will aid the evaporation if the humidity is high. Provision should be made to record the time and area growth periodically through the school day.

DON'T TOUCH THE GROWTH. It is rather fragile and will crumble if disturbed.

6.5 Wick (Optional Activity)

A device similar to Fig. 6-5b may be used to draw up the solution and allow it to evaporate (the same mixture described in Sect. 6.4).

Cut a milk carton as Fig. 6-5a.

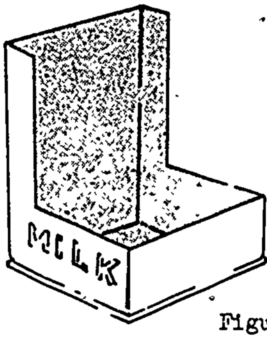


Figure 6-5a

Fasten a ruler to the tall side (top); tape a paper clip to the top. See Fig. 6-5b. Labeling tape may be used to hold the ruler to the carton and also the paper clip to the ruler. The carton may tend to tip over from the weight of the ruler until the solution is added.

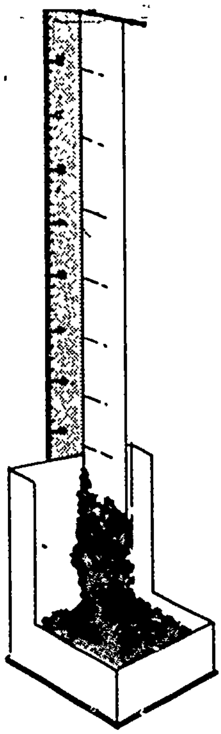


Figure 6-5b

Cut a piece of paper towel so that you have the equivalent of 4 thicknesses 2-3 cm wide. This may be folded or separate strips. Staple along the edge with 5 or 6 staples to hold together (do not tape).

With a sharp pencil (not ink) mark 1 cm spacings on the toweling.

Use a paper clip for a weight at the bottom end (1 cm end) of the towel and tape the other end to the paper clip at the top of the ruler.

When you are all rigged up, carefully pour in the solution to a depth of approximately $2\frac{1}{2}$ cm.

If the carton is placed in an aluminum pie plate, you may feel safer leaving it overnight. If the atmosphere of your classroom is dry, you will see results in about 20 minutes and in 24 hours truly startling results.

6.6 Mass Production

A. Uncontrolled

Materials needed

1. The solution
2. Aluminum pie pan (A glass or plastic pan will be all right. Some metals react to the solution.)
3. Host material (A piece of a porous substance, any size will work, but for best results use something abouc the size of $\frac{1}{4}$ a common brick. Any of these will work: a brick, cinders, charcoal briquets, coal, cellulose sponge, or a crumbled paper towel.)

Figure 6-6a

Procedure

Place host material in the pie pan. Pour the solution over the host material letting the excess run off into the pan.

Observations

The growth of salt crystals may start in just a few minutes. This is dependent upon the lowness of humidity, amount of heat, and air currents. Generally, it will take a day or two before your "garden" will have enough growth to show. The growth patterns will vary greatly. Remember when you add solution that it will dissolve any crystals that it touches. Food coloring may be dropped on the growth to add color to your production.

B. Controlled

Materials needed

1. The solution
2. A small empty milk carton like the one you get with your school lunch
3. Host material--a piece of any porous substance that will fit inside your milk carton but must not come less than one inch from the top. Any of the following will work: a piece of brick, cinders, charcoal briquets, coal, cellulose sponge, or crumbled paper towel.

Procedure

Remove the lid from the milk carton about one-half inch from the top. Place host material in milk carton, making sure the top of the material is about 1 inch below the top of the carton. Pour the solution over the host, letting the excess spill into the bottom of the container. Be careful not to slop on the outside of the milk carton (it will also act as a host).

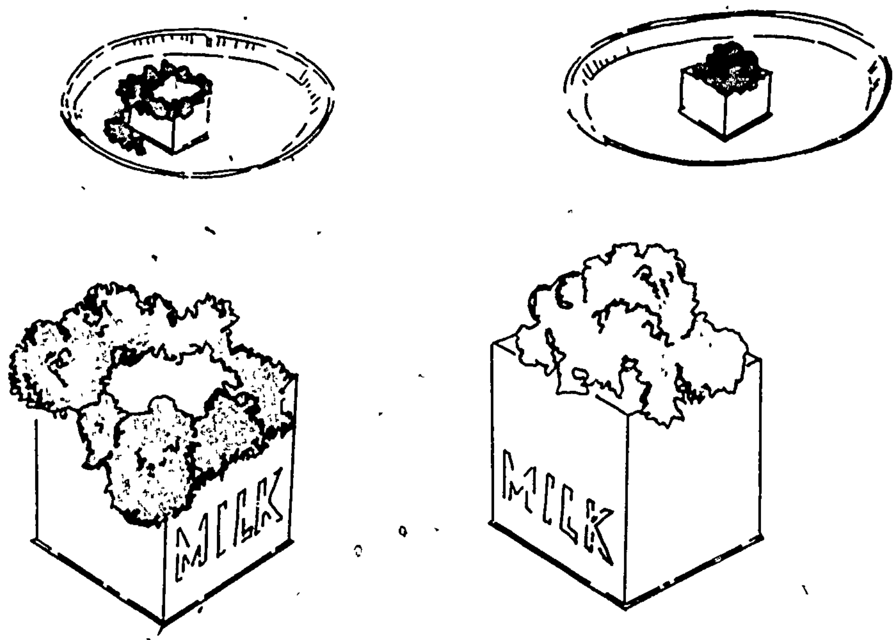


Figure 6-?b

Observations

The container will fill with the crystals, and as the growth continues, it will mushroom over the sides. When new solution is added to your "plant", it will dissolve any crystals it touches. Too much solution may cause the whole "garden" to dissolve.

6.7 "Drip and Splatter" (Optional Activity)

Materials needed

1. The solution
2. Empty milk carton ($\frac{1}{3}$ or $\frac{1}{2}$ pint)
3. Pieces of large cotton string about 2 feet long
4. Paper towels

Procedure

Remove the top from the milk carton. Use your pencil to punch a hold in each side of the carton about $\frac{1}{2}$ inch from the top. Cut two pieces of string each about 10 inches long. Tie one end of one of the strings to one of the holes in the top of the carton, the other end to the hole on the opposite side. Tie the other string to the other two holes. Now turn the carton upside down and punch a hole in the bottom just large enough to get the remainder of your string through. Tie a knot in the string on the inside of the carton and pull the string outward so the knot "plugs" the hole from the inside. Cut off the string so it hangs about $\frac{3}{4}$ inch below the carton. Your container should look similar to Figure 6-7a.

Hang the container from four to six feet above the floor and place paper towels on the floor so the solution will slowly drip on the towels. Pour some of the solution into the container (so that is is about $\frac{1}{2}$ inch deep). Make sure the drops fall slowly enough so the blotter does not become saturated. Some splash may occur so have newspapers spread around and under the drops. This type of growth is a rapid approximation of tne action in caves where stalagmites are formed. Be patient. The drops may be slow in starting and may appear just as a wet mess on the toweling, but by the next morning you will see a lively growth.

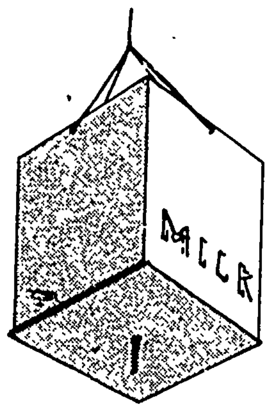


Figure 6-7a

Observations

The crystal growth will appear on the blotters in many varied designs. You may want to change the height that the drops fall to change your pattern.

6.8 Activity - Stalactites

Materials needed

1. The solution
2. Empty milk carton ($\frac{1}{3}$ or $\frac{1}{2}$ pint)
3. Piece of string about 2 feet long
4. Old newspapers
5. Razor blade

Procedure

Remove the top from the milk carton. Use your pencil to punch a hole in each side of the milk carton about $\frac{1}{2}$ inch from the top. Cut two pieces of string about 10 inches long. Tie one end of the string to one of the holes in the top of the carton, the other end to the hole on the opposite side. Tie the other string to the other two holes. Now turn the carton upside down and carefully make a slit in the bottom about $\frac{1}{2}$ to $\frac{3}{4}$ inch long (see Fig. 6-8). This should be done with a razor blade making sure that the slit is very thin--if too wide, the solution will all run out. Place about 3 cm of solution in the milk carton and hang it over a newspaper. Be very careful when adding solution to the carton because a slight bump will cause the crystal formation to fall. This particular activity can be continued over a long period of time. Some of the crystals will fall, but the stalactite will continue to grow.

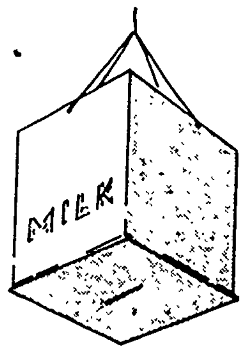


Figure 6-8

6.9 Drop Patterns (Optional Activity)

Materials needed

1. The solution
2. Eye dropper or soda straw
3. Paper towels

Procedure

Place paper towel on floor and drop one drop of the solution on the towel from a height of one foot. Mark on the towel the height from which the drop fell. Try another drop from two feet, making sure that the towel was moved over so that the second drop did not fall near the first one. Repeat the drops from other heights and mark the height on the towel from which each one fell. Remove the towel to dry.

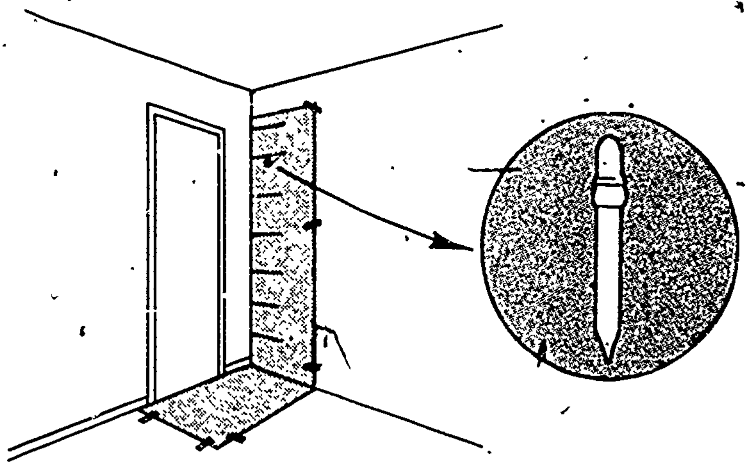


Figure 6-9

Observations

When you come to class the next day, check the different patterns of crystals on the blotter. Did the height from which the drop fell change the pattern?

After seeing the patterns, you may want to repeat the activity using dark or black construction paper. This will make the crystals easier to see.

Chapter 7

SURFACE AREA AND VOLUME
SIZE OF CELLS AND METABOLISM

7.1 Introduction

In several of your activities you have become acquainted with the fact that in order for living cells to function efficiently (indeed -- to remain "living" at all) certain materials must enter the cell and others must be removed. Molecules (mol - e - culz), we will define for this discussion as particles of matter which are made up of different kinds of atoms chemically combined -- atoms such as carbon, hydrogen, oxygen and nitrogen -- among others which are necessary "building blocks" for living things. Living cells, in order to grow, to repair tissues, to produce energy, to divide -- in fact, to do any of the processes essential to life -- need molecules of protein, carbohydrate, and fat. Most of you are aware that your cells get their protein from meat, eggs, cheese, etc., and carbohydrates from sugars and starches. From these life processes there are by-products (wastes) which must be removed. In quantity waste products act as poisons to the cells. All of these processes require metabolic activity. Remember the discussion of metabolism in Chapter 5?

These molecules, whether going into or out of a cell, must pass through a very thin layer called the cell membrane. Every cell has such a membrane the extent of which, of course, constitutes the surface area of a cell.

Since metabolic processes must take place throughout the cell -- even in the innermost portion -- enough materials must be able to enter and leave through the membrane to adequately maintain the entire volume of protoplasm inside the membrane.

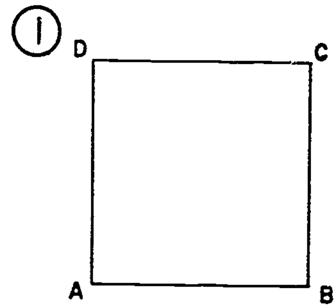
A given amount of surface area (cell membrane) can allow only a given number of molecules to pass through during a given amount of time.

In this chapter you will discover the relationship between surface area and volume, and will see why cells are limited to a maximum size. An elephant is considerably larger than a mouse (and you are somewhere in between). Could this mean that the elephant has larger cells than the mouse, or does he have more cells, or both?

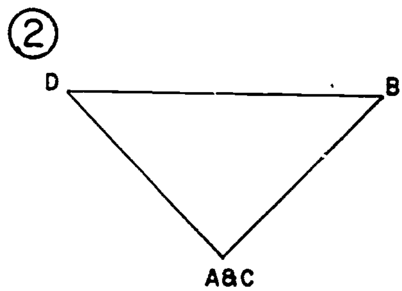
7.2 Activity - Construction of Solids

(a) A regular Polyhedron. The cube.
Constructed by a paper folding activity.

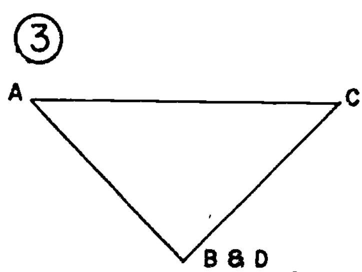
1. Cut a piece of paper to a square. Work carefully - be sure it's a square. (Dimensions will be specified by teacher.) Label the corners as in Diagram (1).



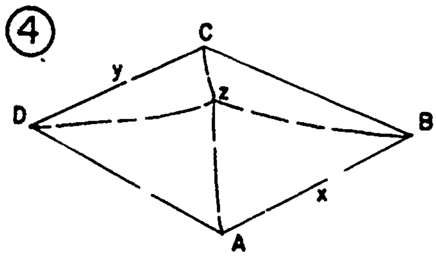
2. Fold diagonally so that A and C are together. Be sure that all edges line up. See Diagram (2).



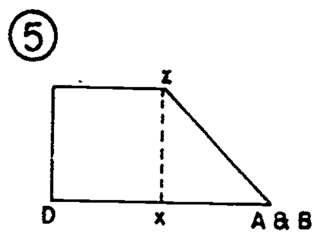
3. Open and refold diagonally so that B and D are together. See Diagram (3).



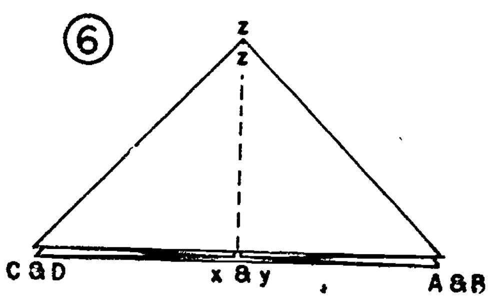
4. Open with intersection of diagonal lines in "up" position. See Diagram (4).



5. Bring A and B together, folding edge x inward. Crease along zx as in (5).

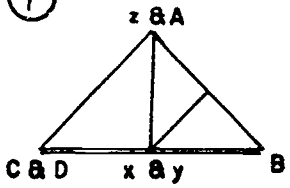


6. Bring D and C together, folding edge y inward. Crease along zy as in (6).

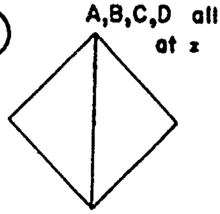


7. Fold A and D to z . Turn paper over and fold B and C to z . See Diagrams (7) and (8).

(7)

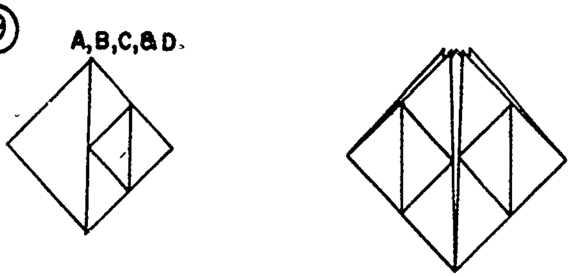


(8)



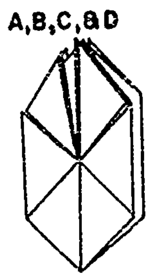
8. With points A, B, C and D pointing away from you, fold the top right and left corners inward as in two steps of Diagram (9).

(9)

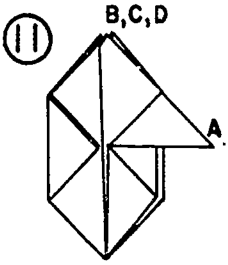


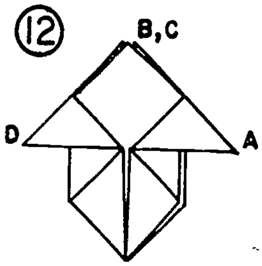
9. Turn paper over and repeat step 8 so that folds appear as in Diagram (10).

(10)



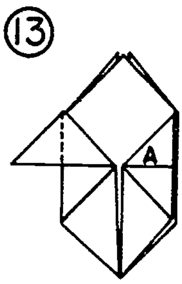
10. Fold points A and D as in Diagrams (11) and (12).

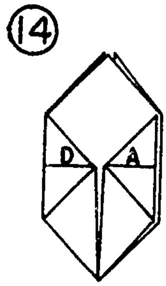




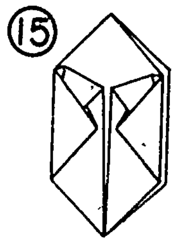
11. Turn paper over and repeat for points B and C.

12. Fold points A and D; B and C inward as in Diagrams (13) and (14). These folds will now appear as triangular shaped "flaps".





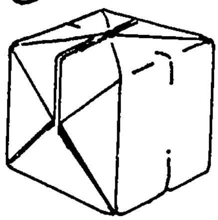
13. Tuck each "flap" into the fold directly under the "flap". See Diagram (15).



14. One "end" has a small hole. Blow air into the hole while gently expanding the paper into a hollow solid.

15. Crease the "edges" between corners to form a cube as in Diagram (16).

(16)



[Measure one edge of your finished cube. Is there a mathematical relationship between this edge and an edge of the original square of paper that you started with? Can you express this relationship as a ratio?]

If we let the letter s stand for an edge of the cube, then: The volume of a cube = s^3. What would be the formula for the surface area of a cube given the measure of an edge = s ?

Exercise 7-2

Comparing Measures of a Cube

Complete the following table by filling in calculated values for the question marks. (Do not write in this book.)

Table 7-2

Surface Area and Volume of a Cube

Edge of Cube (s)	Surface Area in sq. in.	Volume in cu. in.	Ratio - S.A./Volume
12"	864	1728	1 : 2
9"	486	729	1 : ?
6"	?	216	? : 1
3"	54	?	1 : ?
?	24	8	? : ?
?	?	1	? : ?

7.21 Construction of Regular Polyhedrons

Many solid geometrical figures are easily constructed and give interesting comparisons of their volumes and surface areas. As you learned in Section 7.1, biologists have found that the relationship between volume and surface area is often a most important factor in the size of cells as well as in the amount of total growth of various living plants and animals. Later in this section we will study more about the relationship of surface area vs. volume in the growth of living organisms. The following diagrams can be used to construct the five known regular polyhedrons. A regular polyhedron is a polyhedron which has faces that are congruent regular polygons and whose polyhedral angles (those angles formed by the faces of the polyhedron which have a common vertex) are congruent.

In Figure 7-21a the faces are four in number and are equilateral triangles. The polyhedral angles are also four in number. Each angle is formed by 3 faces.

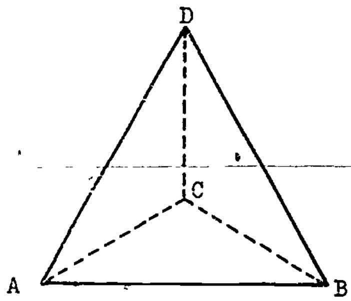


Fig. 7-21a

The diagrams in Figures 7-21b and c will be an aid in the drawing and construction of these polyhedrons. Your teacher will assign to each of you a specific measure for an edge. It is suggested that these models be made of manila tag paper (a manila folder) or colored construction paper. You will need a ruler, protractor, scissors, and glue for construction of the models.

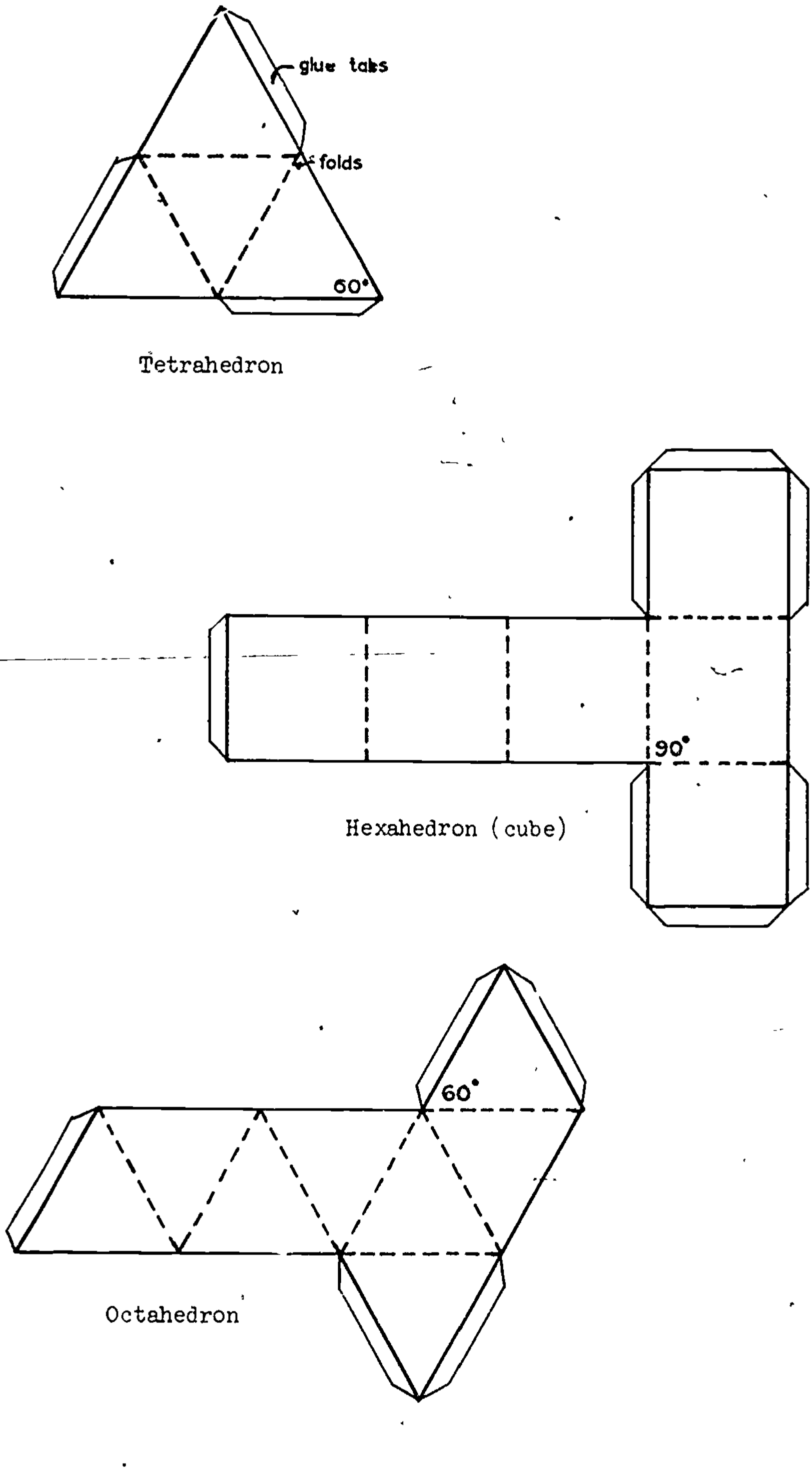
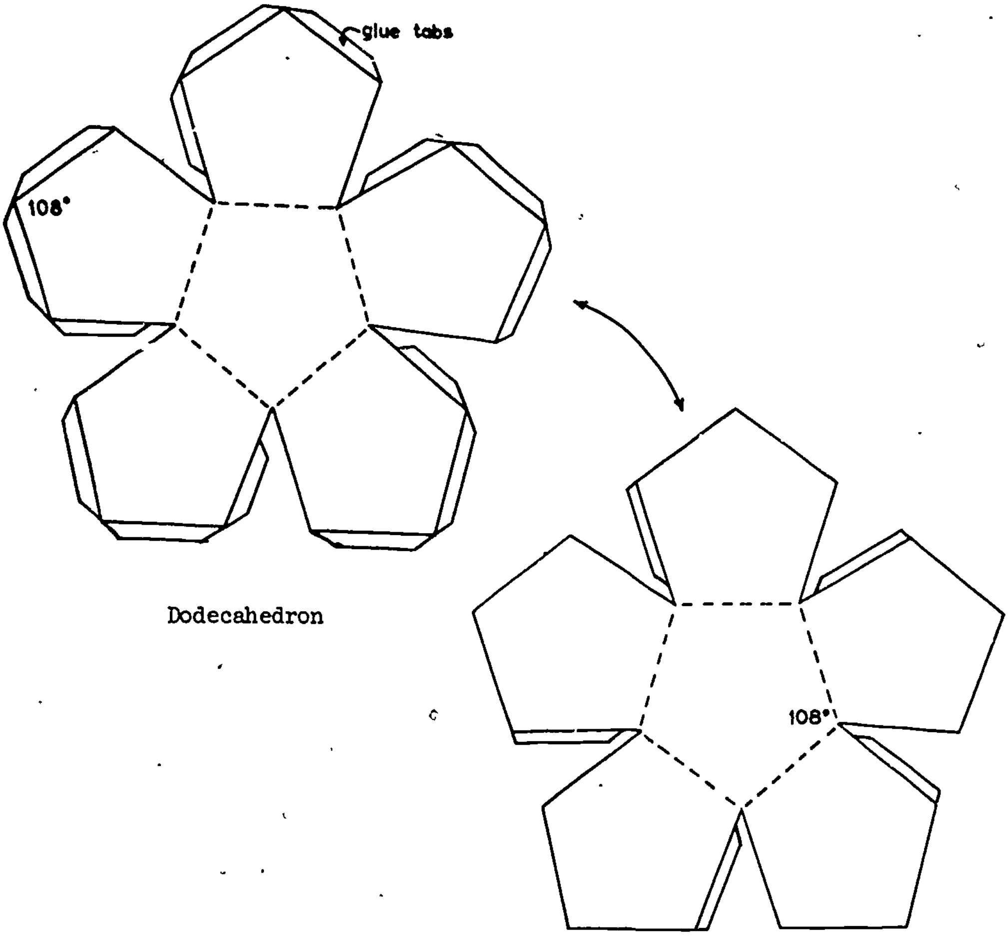
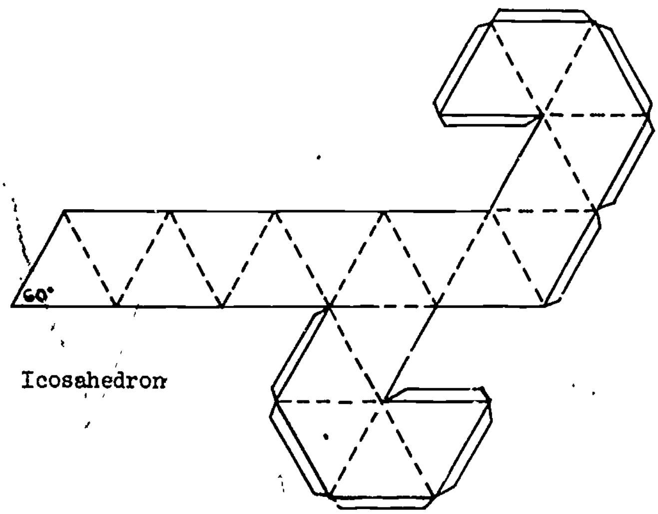

glue tabs
folds
60°
Tetrahedron
90°
Hexahedron (cube)
60°
Octahedron

Fig. 7-21b



Dodecahedron



Icosahedron

Fig. 7-21c

7.3 Surface Area Formulas for Regular Solids

Copy the following table and save it for future data.

Table 7-3a

Regular Polyhedrons

Polyhedron	Number of Faces	Number of Edges	Number of Vertices	Shape of Face	Surface Area if Edge measures 2"
Tetrahedron	4	6	4	triangle	
Hexahedron	6	12	8	square	
Octahedron	8	12	6	triangle	
Dodecahedron	12	30	20	pentagon	
Icosahedron	20	30	12	triangle	

The surface area of the hexahedron (cube) is easy to compute, since the surface is made up of 6 squares. If an edge were 2 inches, then each surface would be 4 in^2 and the total surface area would be 24 in^2. You have computed areas of squares many times before, but how do you find the area of a triangle when only the measure of an edge is given? Remember, the formula for area of a triangle is $\frac{1}{2}bh$. One half of the base is readily computed, as the base is an edge, but we also need a measure for height which is not an edge (unless a right triangle).

The faces of all regular polyhedrons are themselves regular and thus all of the triangular faces are equilateral triangles. (Equal angles and equal edges.) Using this information we will find "h" and hence the area of each face.

Three of the five regular solids have triangular faces, one has squares for its faces and one has pentagons. As we have already computed the surface area and volume of the cube, it is only the other four solids we need to consider now.

One of the main purposes of this chapter is to demonstrate to you just how a formula can be constructed. The formula for the area of a triangle was originally developed for you in somewhat the following manner.

Given: Rectangle ABCD and dimension of length = b; dimension of width = h. The area of the rectangle would be $A = b \times h$.

Draw $\overline{BD}$.

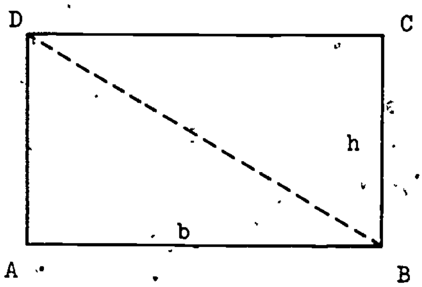


Fig. 7-3a

The diagonal ($\overline{BD}$) separates the rectangle into two congruent triangles. Therefore, triangle ABD would have $\frac{1}{2}$ of the area of ABCD .

The formula for an area of a right triangle $= \frac{1}{2} b \times h$.

A similar development was done using a parallelogram. Area of a parallelogram $= b \times h$. Draw a diagonal which divides the parallelogram into 2 congruent triangles, then the area of each triangle is equal to $\frac{1}{2}$ base × height . You are already familiar with such a formula for the area of a triangle, but this formula requires you to know the measures of the base and the height. We will develop a formula for the area of an equilateral triangle when only the measure of one of its equal sides is known. The formula will be one which requires only one dimension, the base.

Looking at the solids that you have constructed, you can see that the edges are all bases of the triangular faces (tetrahedron, octahedron, and icosahedron). The measure of the edge was given for construction of the solids. Now, if we can develop a formula which requires only the measure of an edge, we can find the surface area of the solids.

Study the following development carefully. Don't skim. Understand every step before you go on and you will gain an appreciation of accomplishing an interesting task.

To construct an equilateral triangle:

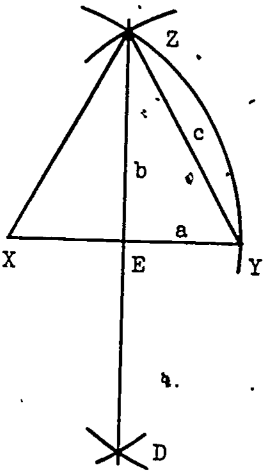


Fig. 7-3b

1. Draw segment xy with a measurement of 2 inches.
2. Place point of compass on x and pencil end on y.
3. Swing an arc from y at least 90°.
4. Place point of compass on y and pencil end on x.
5. Swing another arc which intersects the first arc.
   Label intersection of arc point "z".
6. Draw $\overline{xz}$ and $\overline{yz}$.

Our equilateral triangle, XYZ, has an edge of 2 inches. We need to find a height in order to compute the area. Remember a height is a measure of the perpendicular distance from any vertex to the opposite side. We could use any vertex, as this triangle is equilateral, but for demonstration purposes we will "drop" a perpendicular segment from "z" to the opposite side $\overline{xy}$.

To review the construction procedures,

1. Swing equal arcs from x and y which intersect preferably on the side of $\overline{xy}$ opposite to z.
2. Call the intersection "D".
3. Draw $\overline{ZD}$.
4. Where $\overline{ZD}$ intersects $\overline{xy}$, label point of intersection E. $\overline{ZE}$ would be a height to use in computing the area of triangle XYZ.

Now that we have a face of our polyhedron divided up into 2 right triangles (called right because they have a right angle formed at the foot of the perpendicular), we need to digress for a few minutes and explain just what Pythagoras did about 2500 years ago.

7.4 Pythagorean Property

The ancient Egyptians are said to have used a particular right triangle to make corners "square". This triangle had sides 3 units long, 4 units long, and 5 units long (see Fig. 7-4a). When such a triangle is made of tautly stretched rope, the angle between the two shorter sides is a right angle.

While the Egyptians are thought to have made use of this fact, it was left to the Greeks to prove the relationship involved for all right triangles.

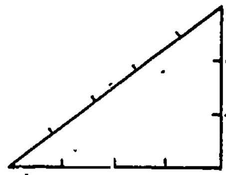


Fig. 7-4a

The Greek philosopher and mathematician, Pythagoras, who lived about 500 B.C., became interested in the problem. Pythagoras is credited with the proof of the basic property which we will study in this section; this property is still known by his name: the Pythagorean Property.

It is thought that Pythagoras looked at a mosaic of tile similar to the one pictured in Figure 7-4b. He noticed that there are many triangles of different sizes that can be found in the mosaic. But he noticed more than this. If each side of any triangle is used as one side of a square, the sum of the areas of the two smaller squares is the same as the area of the larger square. In Figure 7-4c two triangles of different sizes are inked in and the squares drawn on the sides of the triangles shaded.

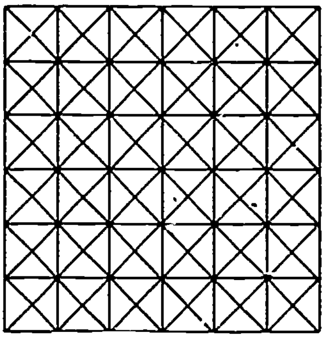


Fig. 7-4b

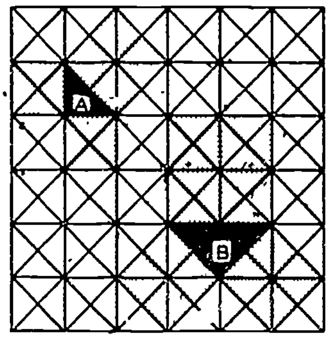


Fig. 7-4c

[Count the number of the smallest triangles in each square.]

Triangle A, in the upper left-hand corner has 4 small triangles underlined in the square drawn on each leg. It also has 8 small triangles included in the square drawn on the hypotenuse (hy-pot' e-nuse), the side of a right-angled triangle opposite the right angle. Count the triangles in the squares drawn on triangle B. For each triangle that is inked in, how does the number of small triangles in the two smaller squares compare with the number in the larger square? If you draw a mosaic like this, you will find that this is true not only for the two triangles given here but for even larger triangles. If you have graph paper with lines "4 by 4 to the inch", by drawing diagonal lines through the squares you can make a triangular mosaic similar to Pythagoras'.

Pythagoras probably noticed the same relation in the 3-4-5 triangle that the Egyptians had used for so long to make a right angle. The small squares are each 1 square unit in size. In the three squares there are 9, 16 and 25 small squares. Notice that 9 + 16 = 25. Pythagoras was able to prove that in any right triangle, the area of the square on the hypotenuse (longest side) is equal to the sum of the areas of the squares on the other two sides. This is the Pythagorean Theorem, or, as we call it, the Pythagorean Property.

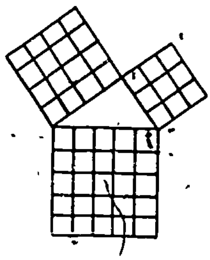


Fig. 7-4d

So far this has been shown only for two very special right triangles. It is true for all right triangles, and you can prove it for yourself after you have studied geometry in high school.

Exercise 7-4a

1. Show for each set that the square of the first number is equal to the sum of the squares of the other two numbers, for example in the set of numbers 10, 8 and 6;

$$10^2 = 100 = 8^2 + 6^2 = 64 + 36 = 100, \quad \text{thus}$$

$$10^2 = 8^2 + 6^2 .$$

(a) 5, 4, 3
(b) 13, 12, 5
(c) 25, 7, 24
(d) 20, 16, 12

2. Make a drawing of the triangles with the sides of length given in part (a) of Problem 1. Use your protractor to show that this triangle is a right triangle. (Use cm as the units.)

3. Draw right triangles, the lengths of whose shorter sides (in centimeters) are:

   (a) 1 and 2 (b) 4 and 5 (c) 2 and 3 .

   Measure, to the nearest millimeter, the lengths of the hypotenuses of these triangles.

4. Use the Pythagorean Property to find the area of the square on the hypotenuse for each triangle in Problem 3.

The Pythagorean Property is a very useful mathematical tool and has been used now for hundreds of years. Written mathematically, the property says: "If given a right triangle with legs of length "a" and "b" and hypotenuse of length "c", then the relationship between the sides of the triangle are $c^2 = a^2 + b^2$." Remember, the Egyptians used this property in just one form; 5, 4 and 3 were the sides of the triangles with which they worked. If $c = 5$, $b = 4$ and $a = 3$, then $c^2 = a^2 + b^2$ and $5^2 = 3^2 + 4^2$ or $25 = 9 + 16$. The Egyptians found that if they always used triangles with sides of this length, then they always had a right triangle. They needed the right angle in the right triangle in order to "square" their property lines. It is reported the Nile overflowed almost every year and washed away most identifying land marks in their fields, therefore, they needed to re-survey frequently.

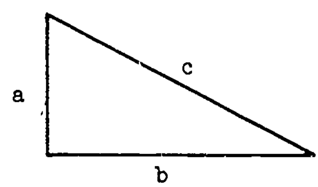


Fig. 7-4e

Pythagoras' <u>proof</u> of the property of right triangles opened up many more facets of its use.

1. <u>If</u> a triangle has the length of its sides in the relationship of $c^2 = a^2 + b^2$, <u>then</u> you know it is a right triangle.
2. <u>If</u> a triangle is a right triangle, <u>then</u> you know the length of its sides are in the relationship to each other of $c^2 = a^2 + b^2$.
3. <u>If</u> you know the length of <u>two</u> sides of any right triangle, <u>then</u> you can find the measure of the third side from the relationship of $c^2 = a^2 + b^2$.

We can use a familiar triangle to show this. If the two short sides are 3 units and 4 units, what is the square of the hypotenuse?

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25 .$$

Since c^2 is equal to 25, we will know c if we can find a number whose product is 25 when it is multiplied by itself. Of course, $5 \times 5 = 25$, so $c = 5$; 5 is the positive square root of 25. If a number is the product of two equal factors, then each factor is a square root of the number. The symbol for the positive square root is: $\sqrt{\quad}$. The numeral is placed under the sign. For example, $\sqrt{25} = 5$.

What is $\sqrt{9}$? $\sqrt{16}$? $\sqrt{36}$? $\sqrt{30}$? The first three are easy to understand since $3 \times 3 = 9$; $4 \times 4 = 16$; and $6 \times 6 = 36$, but there is no integer that can be multiplied by itself to give the product 30. In fact, there is no rational number whose square is 30 .

Can you find a number multiplied by itself that will give a product close to 30 ? Yes, $5 \times 5 = 25$, which is close to 30 . What is 6×6 or 6^2 ? This tells us that 30 is greater than 5^2 but less than 6^2. We can find the square root of a number to the desired number of significant digits by division. Recall the relationship between the divisor, dividend and quotient. For example, if $\frac{30}{5} = 6$, then

$30 = 6 \times 5$. If $\frac{72}{9} = 8$, then $72 = 8 \times 9$.

If $\frac{64}{8} = 8$, then $64 = 8 \times 8$.

If $\frac{a}{b} = c$, then $a = b \times c$. Now if $b = c$ then each is the square root of a . Right?

We know that the square root of 30 is greater than 5 (because $5^2 = 25$) and less than 6 (because $6^2 = 36$). If we divide 30 by 5 we get a quotient of 6 (if the divisor = the quotient we would have the square root).

$$\frac{30}{5} = 6$$

Average the first divisor 5 with the first quotient 6

$5 + 6 = 11$; $11 \div 2 = 5.5$

and use this average as a second divisor of 30 .

$$\frac{30}{5.5} = 5.45$$

$30 \div 5.5 = \underline{5.45}$. Now find a new average of the second divisor 5.5 and the second quotient 5.45 .

$$5.5 + 5.45 = 10.95 \; ; \quad \frac{10.95}{2} = \underline{5.47}$$

Use this second average as a third divisor

$$\frac{30}{5.47} = 5.484$$

$30 \div 5.47 = 5.484$. Average 5.47 and 5.484 to get $\underline{5.477}$ as a closer approximation of the square root of 30 .

You can continue on in the same manner to get the square root of any number to as many significant digits as you desire.

The square roots of most numbers, if expressed as a decimal, are approximate. If we were to list the whole numbers from 1 to 50, only 1, 4, 9, 16, 25, 36, and 49 have square roots which are rational numbers. The balance of the numbers would have values which we would need to round off. Thus, whenever we use a decimal value for a square root, we must designate whether it is exact or approximate. The square root of 4 is 2 because $(2)^2 = 4$. The square root of 3 is ≈ 1.7320, because $(1.7320)^2 = 2.999824$ and $(1.7321)^2 \doteq 3.00017041$.

The table at the end of this chapter gives the squares and the square roots of the counting numbers from 1 to 100 .

Exercise 7-4b

When approximate values are used in these problems, use the symbol $\approx$, in the work and answer.

1. Use the table of squares and square roots given at the end of this chapter to find the approximate value of:

   (a) $\sqrt{5}$ (d) $\sqrt{92}$

   (b) $\sqrt{41}$ (e) $\sqrt{7}$

   (c) $\sqrt{13}$ (f) $\sqrt{3}$

2. Using the division method described in Section 7.4, find the approximate square root of the following numbers to 4 significant digits.

   (a) $\sqrt{27}$ (check your answer with the table of square roots)

   (b) $\sqrt{77}$ (again check your answer after you have done the computation)

   (c) $\sqrt{139}$

   (d) $\sqrt{68.5}$

   (e) $\sqrt{9.99}$

3. Using the table of square roots, find the approximate values of:

(a) $\sqrt{\frac{16}{4}}$ (c) $\sqrt{\frac{100}{25}}$ (e) $\sqrt{\frac{5}{8}}$

(b) $\sqrt{\frac{25}{9}}$ (d) $\sqrt{\frac{3}{36}}$

4. Use the Pythagorean Property to find the length of the hypotenuse for each of these triangles.

(a) Length of a is 1", length of b is 2".

(b) Length of a is 4', length of b is 5'.

(c) Length of a is 2", length of b is 3".

(d) Length of a is 5 yd, and the length of b is 6 yd.

(e) Length of a is 1 unit, and the length of b is 3 units.

5. Sometimes the hypotenuse and one of the shorter sides is known. How can you find the length of the other side? As an example, use this problem. The hypotenuse of a right triangle is 13 ft and one side is 5 ft. Find the length of the third side.

$c^2 = a^2 + b^2$

$13^2 = 5^2 + b^2$ substitute values for "c" and "a"

$13^2 - (+5^2) = b^2$ subtract (5^2) from both sides

$169 - 25 = b^2$

$\sqrt{144} = \sqrt{b^2}$

$12 = b$, therefore the third side is 12 feet long.

Find the third side of these right triangles. The measurements are in feet.

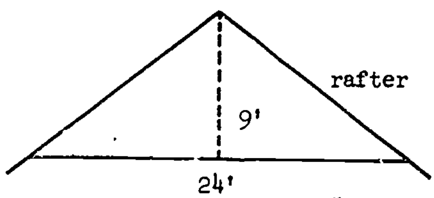
(a) $c = 15$, $b = 9$

(b) $c = 26$, $a = 24$

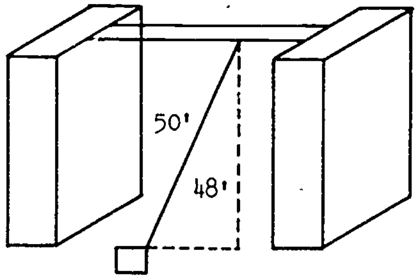
(c) $c = 39$, $b = 15$

6. A telephone pole is steadied by three guy wires. Each wire is to be fastened to the pole at a point 15 ft above the ground and anchored to the earth 8 ft from the base of the pole. How many feet of wire are needed to stretch 3 wires from the ground to the point on the pole at which they are fastened?

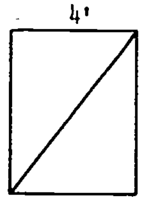
7. A roof on a house is built as shown. How long should each rafter be if it extends 18" over the wall of the house? (See figure at right.)



8. A hotel builds an addition across the street from the original building. A passageway is built between the two parts at the third-floor level. The beams that support this passage are 48 feet above the street. A crane operator is lifting these beams into place with a crane arm that is 50 ft long. How far down the street from a point directly under the beam should the crane cab be? (See figure at right.)



9. A garden gate is 4 ft wide and 5 ft high. How long should the brace that extends from C to D be? (See figure at right.)



10. How long is the throw from home plate to second base in a softball game? The bases are 60 ft apart, and a softball diamond is square in shape. Give your answer to the nearest whole foot. (Ignore the curve or arc of the ball.)

11. Draw a square whose sides are one unit long. What is the length of the diagonal? Check by measurement. Now draw a right triangle with the sides 1 unit long. What is the length of the hypotenuse?

7.5 Surface Area of an Equilateral Triangle

From the information we learned about the Pythagorean Property, we know that in any triangle with a right angle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides (Figure 7-4d). If a and b represent the short sides (legs) of a right triangle, and c the hypotenuse (side opposite the right angle), then $c^2 = a^2 + b^2$.

In our example we have a right triangle (in fact, two of them, ZEY and ZEX). Letter the sides of triangle XYZ so that $\overline{ZY}$ is "c", $\overline{ZE}$ is "a", and $\overline{EY}$ is "b". Our triangle is now as Figure 7-5a. Our project, remember, is to compute the surface area of solids. The 3 solids with equilateral triangles for faces are considered first. After finding the area of one face we can quickly find the surface area of the entire solid. If the edge of the regular solid were given as 2", then the edge of each triangular face would be 2". Therefore, in a right triangle EYZ

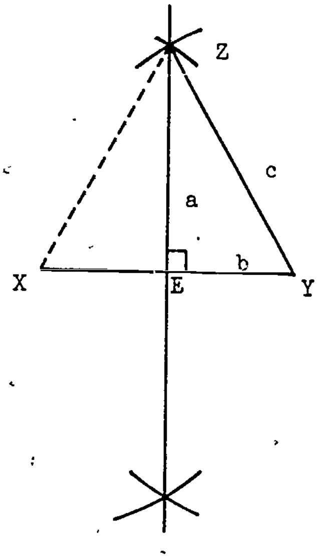


Fig. 7-5a

If $c = 2$
then $c^2 = 4$
and $b = 1$
and $b^2 = 1$

Why? (Be sure you know why.) See Figure 7-5a.

By the Pythagorean Property

$c^2 = a^2 + b^2$. Using our values for c^2 and b^2 we have

$4 = a^2 + 1$

$3 = a^2$ (adding -1 to both sides of the equation)

$\sqrt{3} = a$ (taking the square root of both sides) .

Now we have all of the information we need to find the area of each face of our polyhedron if the faces are equilateral triangles with sides of 2".

The area of a triangle is $\frac{1}{2}bh$. In our triangle the base is 2 and the height is $\sqrt{3}$. The area is $\frac{1}{2} \times 2 \times \sqrt{3}$, or simply $A = \sqrt{3}$. The surface area of a tetrahedron with edge of 2 would then be 4 times the area of each face, or $4 \times \sqrt{3}$.

The surface area of an octahedron with an edge of 2 would be 8 times the area of each face or $8\sqrt{3}$.

At the end of this chapter there is a table of squares and square roots of numbers. The $\sqrt{3} \approx 1.732$, thus the surface of the tetrahedron would be 4×1.732 or ≈ 6.928 sq. in.

The measure of a side may not always be 2", of course. Before we go on to other polyhedrons we will derive the formula for the area of any equilateral triangle given the measure of the side.

Given: An equilateral triangle XYZ with a side measure of "a". Then $\overline{ZY}$, $\overline{XY}$, and $\overline{XZ} = s$. In triangle EYZ, $\overline{ZY} = s$, and $\overline{EY} = \frac{1}{2} s$.

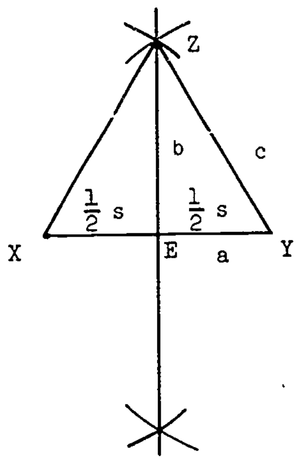


Fig. 7-5b

STOP. Check the drawing. See what we are talking about.

By the Pythagorean Property:

$c^2 = a^2 + b^2$ when

c = length of hypotenuse and

a = length of one leg

b = length of the other leg.

In Figure 7-5b, $c = s$, $a = \frac{1}{2}s$ or $\frac{s}{2}$

$b = $ "h", height of the triangle .

Substituting into the Pythagorean formula

$s^2 = (\frac{1}{2} s)^2 + h^2$	substituting values in formula
$s^2 = (\frac{s}{2})^2 + h^2$	
$s^2 = \frac{s^2}{4} + h^2$	$(\frac{s}{2})^2 = \frac{s}{2} \times \frac{s}{2} = \frac{s^2}{4}$
$s^2 - \frac{s^2}{4} = h^2$	subtracting $\frac{s^2}{4}$ from both sides
$s^2 (1 - \frac{1}{4}) = h^2$	distributive property
$s^2 (\frac{3}{4}) = h^2$	another name for a number
$\frac{s\sqrt{3}}{2} = h$	square root of both sides
or $\frac{1}{2}s\sqrt{3} = h$	another name

What does $h = \frac{1}{2}s \times \sqrt{3}$ mean? It means, that if you know the side (s) measure of an equilateral triangle, the height will be $\frac{1}{2}s \times \sqrt{3}$.

We want to find a formula for the area of an equilateral triangle. Using this formula for the height of an equilateral triangle, given the measure of a side, we can substitute into the area formula and get a new formula for the area of an equilateral triangle if we know the measure of a side.

Formula for area of a triangle = $\frac{1}{2} \times$ base $\times$ height; if we identify the base as "s" then the area = $\frac{1}{2} \times s \times$ height. We have just found that the height of an equilateral triangle = $\frac{1}{2} s \times \sqrt{3}$.

If we substitute this value for height into the original formula for area we have $\frac{1}{2} \times s \times (\frac{1}{2} s \times \sqrt{3}) = A$.

Area of an equilateral triangle = $\frac{1}{2} \times \frac{1}{2} \times s \times s \times \sqrt{3}$ commutative property

Area of an equilateral triangle = $\frac{1}{4} \times s^2 \sqrt{3}$ or

Area of an equilateral triangle = $\frac{s^2 \sqrt{3}}{4}$ another name

A square root table shows that $\sqrt{3} \approx 1.732$ then

$$\frac{\sqrt{3}}{4} \approx \frac{1.732}{4} \approx .4330 .$$

therefore,

"Area of equilateral triangle $\approx .4330\ s^2$ when s = the measure of an edge."

Example:

Given: Equilateral triangle with a measurement of 2 inches on an edge.

Formula for area of equilateral triangle: Area $\approx .4330 \times s^2$.

Given: $s = 2$ inches .

Area $\approx .4330 \times 2^2 \approx .4330 \times 4 \approx 1.732\ in^2$.

An equilateral triangle with edge of 4" would have an area of

$$\approx .4330 \times 4^2 \approx .4330 \times 16 \approx 6.9280\ in^2 .$$

Exercise 7-5a

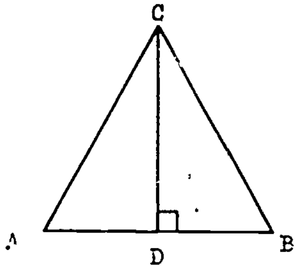
1. What is the area of an equilateral triangle if the measurement of its edge is 5"?

2. If an equilateral triangle has an edge with a measurement of 4.1", what would be the area?

3. Given an equilateral triangle with an edge measurement of $\frac{1}{4}$", what would be the area?

4. If the area of an equilateral triangle is 10.825 in^2, what would be the approximate measurement of its edge?

5. Brain buster. Given equilateral triangle ABC, with altitude CD and an area of 21.2770 in^2. Find the measurement of $\overline{AB}$ and $\overline{CD}$.



6. A regular tetrahedron with an edge measurement of 4 cm has a surface area of ________?

7. Find the surface area of a regular tetrahedron whose edge measurement is 11".

8. If the surface area of a regular tetrahedron is 339.472 in^2, what is the measurement of its edge?

7.51 Surface Area - Tetrahedron, Octahedron, and Icosahedron

Now that we have a formula for the area of an equilateral triangle, we can compute the surface area of the tetrahedron, octahedron and icosahedron. (We already know the formula for the area of a square and thus, the surface area of a hexahedron.)

A few examples:

An octahedron has edges which measure 1". Surface area ≈ 8 (number of faces) $\times .4330 \times 1^2$.

Surface area $\approx 8 \times .4330 \approx 3.464 \text{ in}^2$.

An icosahedron has an edge measurement of 4".

Surface area ≈ 20 (number of faces) $\times .4330 \times 4^2$.

Surface area $\approx 20 \times .4330 \times 16$

$\approx 20 \times 6.928$

$\approx 138.56 \text{ in}^2$.

Watch this problem, it's different.

The surface area of a tetrahedron is 84.868 in^2.

Find the measure of an edge.

Substitute in the formula for the surface area of a tetrahedron.

Surface area ≈ 4 (number of faces) $\times .4330 \times s^2$

$84.868 \approx 4 \times .4330 \times s^2$	substitution in formula
$84.868 \approx 1.732 \times s^2$	multiplying $(4 \times .4330)$
$49 \approx s^2$	both sides divided by 1.732
$7 \approx s$	square root of both sides

The measurement of the edge is 7 inches.

We will do more problems after we have developed the formula for the last of the known polyhedrons, the dodecahedron.

Exercise 7-51

Complete the following table:

	Faces	Figure	Edge Measurement	Area of Face	Total Surface Area
Tetrahedron	4	Triangle	3 inch		
Hexahedron	6	Square	3 "		
Octahedron	8	Triangle	3 "		
Dodecahedron	12	Pentagon	3 "		
Icosahedron	20	Triangle	3 "		

J.F.F. Draw carefully on a piece of tagboard or cardboard a square divided as shown in Figure 7-51a. What is the area?

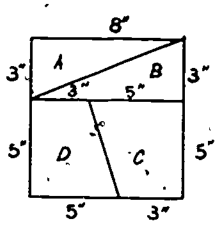


Fig. 7-51a

Cut it into parts A, B, C and D and put together as in Figure 7-51b.

What is the area?

Are you sure?

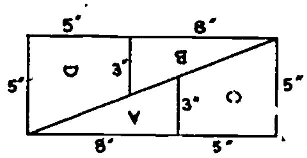


Fig. 7-51b

7.6 Surface Area of a Regular Dodecahedron

A regular dodecahedron has pentagons for its faces. To find the surface area of a regular dodecahedron (dodeca meaning twelve) we would need to multiply the area of one face by 12 (the number of faces).

Our immediate problem is to find the area of a regular pentagon if we know the measure of its edge. Given a regular pentagon ABCDE and the center "X". The center is that point equidistant from the sides of the pentagon and also equidistant from the vertices. Draw $\overline{XA}$, $\overline{XB}$, $\overline{XC}$, $\overline{XD}$ and $\overline{XE}$. Drop a perpendicular segment from X to $\overline{AB}$ and call the intersection with $\overline{AB}$ "F".

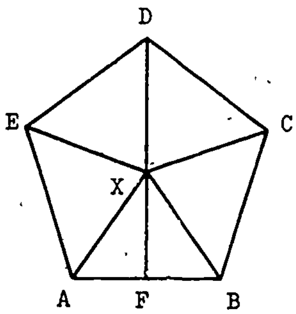


Fig. 7-6a

By our definition of center, the point X is equidistant from all vertices A, B, C, D, E. Thus we would have five congruent triangles (congruent meaning figures with identical size and shape). If we don't consider $\overline{XF}$, each angle with X as a vertex would have a measure of $\frac{1}{5}$ of 360° or 72°. Why? $\overrightarrow{XF}$ also bisects $\angle$ AXB. Why? Thus $\angle$ FXB would be one-half of 72° and would have a measurement of 36°.

Why would triangle FBX be one of ten congruent triangles in the given pentagon?

We now have our area problem narrowed down. If we can find the area of triangle FBX we will have one-tenth of the total area of the pentagon.

In triangle FBX we know: $\angle$ FXB = 36° and $\overline{FB} = \frac{s}{2}$.

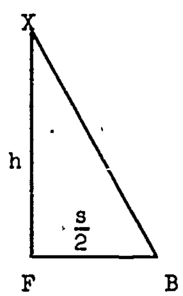


Fig. 7-6b

If we can find the length of $\overline{FX}$ in relationship to "s" we could compute the area of any such triangle (and thus a pentagon).

In a later portion of mathematics you may study a topic called "trigonometry". (Trig' o nom' e tree -- meaning "measurement of angles".) By the use of trigonometric ratios we can determine a formula for the area of a right triangle if we know <u>one angle and a side</u>.

The "tangent" of an angle in a triangle is described as the ratio of:

> <u>The measure of the length of the side opposite the angle divided by the measure of the length of the side adjacent to the angle.</u>

In Figure 7-6b, the tangent of $\angle FXB = \frac{s}{2} \div h$. On a table of trigonometric values, we would find the tangent of an angle of 36° is $\approx .7265$. Therefore, substituting into the tangent formula,

$$\text{tangent of the } \angle = \frac{\text{side opposite}}{\text{side adjacent}}$$

$.7265 \approx \frac{s}{2} \div h$	substitution
$.7265 \approx \frac{s}{2h}$	division by a fraction
$.7265 \times 2h \approx s$	multiplying both sides by 2h
$1.4530 \times h \approx s$	multiplying $2 \times .7265$
$\frac{1.4530}{1.4530} \times h \approx \frac{s}{1.4530}$	dividing both sides by 1.4530
$h \approx \frac{s}{1.4530}$	$\frac{1.4530}{1.4530}$ = another name for one

The area of triangle FBX = $\frac{1}{2} \times \text{base} \times \text{height}$. The base is $\frac{s}{2}$ and the height has been found to be $\frac{s}{1.4530}$ (for an angle of 36°).

Substituting into the formula for the area of a triangle, we have

Area of a triangle	$= \frac{1}{2} \times b \times h$	Area of a triangle
Area of triangle FBX	$\approx \frac{1}{2} \times \frac{s}{2} \times \frac{s}{1.4530}$	substitution of terms
	$\approx \frac{1 \times s \times s}{2 \times 2 \times 1.4530}$	multiplication of fractions
	$\approx \frac{s^2}{5.8120}$	another name

Now remember triangle FBX is one-tenth of the pentagon, so $10 \times \frac{s^2}{5.8120}$ would equal <u>the area of the pentagon</u>.

A neater way to use this formula would be to change the part, $\frac{10}{5.8120}$, to a rational number with a denominator of 1. If $\frac{10}{5.8120} \approx 1.720$, then in our formula for the area of a regular pentagon, Area $\approx 1.720 \times s^2$ = the length of a side.

Area of a Regular Pentagon $\approx 1.720 \times s^2$
when s = measure of side.

Exercise 7-6

1. Given a regular pentagon with a side measurement of 5". Find the area.
2. If the side of a regular pentagon has a measurement of 7", find the area.
3. With a side length of $3\frac{1}{2}$ in., find the area of a regular pentagon.
4. If the perimeter of a regular pentagon is 40 cm, what would be the area?
5. The perimeter of a regular pentagon is 25 in. Find the area.
6. If the area of a regular pentagon is 208.12 in^2, find the length of one side.
7. Find the perimeter of a regular pentagon if the area is 497.08 cm^2.
8. A dodecahedron is a regular solid with 12 pentagons as its faces. Find the surface area of a regular dodecahedron which has an edge with a measurement of 6 cm.

7.7 Volumes of Regular Polyhedrons

In this chapter we have developed the formula for finding the surface areas of the 5 regular solids if we know at least the measure of an edge.

We would like to do a similar development for the volumes of these solids but the mathematics needed is beyond the scope of this study. However, we do need to know the volumes of these solids, so Table 7-7 furnishes the needed formulas.

Table 7-7

Surface Area and Volume of Regular Polyhedrons

s = measure of an edge

Name	Number of Faces	Surface Nature of Face	Surface Area (approx.)	Volume (approx.)
Tetrahedron	4	equil. tri.	$1.732 \times s^2$	$0.118 \times s^3$
Hexahedron	6	squares	$6.000 \times s^2$	$1.000 \times s^3$
Octahedron	8	equil. tri.	$3.464 \times s^2$	$0.471 \times s^3$
Dodecahedron	12	pentagons	$20.646 \times s^2$	$7.663 \times s^3$
Icosahedron	20	equil. tri.	$8.660 \times s^2$	$2.182 \times s^3$

Exercise 7-7

Use Table 7-7 "Surface Area and Volume of Regular Polyhedrons" for needed information.

Example:

If an octahedron had an edge measurement of 2 in, its surface area would be computed by substituting into the formula for surface area of an octahedron. Surface area $\approx 3.464 \times s^2$, $s = 2$ in, therefore,

$$\text{Surface area} \approx 3.464 \times 2^2 \approx 3.464 \times 4 \approx 13.86 \text{ in}^2$$

$$\text{Volume} \approx .471 \times 2^3 \approx 0.471 \times 8 \approx 3.77 \text{ in}^3 .$$

1. A tetrahedron has an edge measurement of 2 in.
   (a) Find its surface area.
   (b) Find its volume.

2. Find the surface area of a dodecahedron whose edge measurement is 50 mm.

3. What is the measure of the volume of an icosahedron which has an edge measurement of 3 cm?

4. Find the surface area and volume of an octahedron which has an edge measurement of 5 cm.

7.8 Comparison of Surface Area and Volume

The cube gives us a good physical model to consider when comparing surface area and volume. The other four regular polyhedrons have complicating factors in their formulas that restrict their use in this chapter.

However, with the relationships you have noted in the cubes you constructed you should now be able to think in terms of "practical" biological applications of surface area and volume.

At the first of this chapter you started equivalent tables of values based on the cube. We will repeat this table and add some information to it. (See Table 7-8a.) Fill in the missing items in Table 7-8a based on the given edge measurements.

Table 7-8a

Volume and Surface Area of a Cube [S.A. = $6s^2$; Vol. = s^3]

Measure of the Edge (inch)	Surface Area	Volume	Ratio $\frac{S.A.}{Vol.}$	Decimal Equiv. of ratio
12	864	1728	1 : 2	.50
9	486	729	2 : 3	.66
6	216	216	1 : 1	1.00
3	54	27	2 : 1	2.00
1	6	1	6 : 1	6.00
.5				
.1				
.01				
.001				

Do you notice a definite pattern in the ratio of surface area to volume? Could you make a general statement regarding this ratio as the volume of a solid decreases?

Using the measurements of the edges with which you constructed the five regular polyhedrons as given in the table below, complete the following table.

Table 7-8b

Name	Number of Faces	Length of Edge (in mm)	Surface Area Formula	Surface Area	Volume Formula	Volume
Tetrahedron	4	61	$1.732 \times s^2$		$.118 \times s^3$	
Hexahedron	6	30	$6.000 \times s^2$		$1.000 \times s^3$	
Octahedron	8	38	$3.464 \times s^2$		$.471 \times s^3$	
Dodecahedron	12	15	$20.646 \times s^2$		$7.663 \times s^3$	
Icosahedron	20	23	$8.660 \times s^2$		$2.182 \times s^3$	

Exercise 7-8

[Graph the data from Table 7-8b. Make two graphs.]

Graph A: Titled "Surface Area of Regular Polyhedrons". The horizontal axis will represent "Number of Faces". The vertical axis -- "Surface Area" in mm^2.

Graph B: Titled "Volume of Regular Polyhedrons". On the horizontal axis -- "Number of Faces", on the vertical axis -- "Volume in mm^3".

Answer the following questions.

1. Do the points plotted in Graph A approximate a straight line?
2. Which geometric solid has the greatest surface area?
3. Which solid has the least surface area?
4. Do the points plotted in Graph B approximate a straight line?
5. What could you "generalize" from Graph B?
6. What can you generalize from Graph A?
7. If a solid had three faces would its surface area be more or less than the surface area of a tetrahedron of the same volume?
8. Could a solid have 3 faces only? 2 faces only?
9. As the number of faces increases, what could you say about the surface area?
10. Could a solid (not necessarily a regular solid) have more than 20 faces?
11. Could a solid have an infinite number of faces?

12. What solid would appear to approach having an infinite number of faces?

13. You have a contract for $10,000 to package 1000 cm^3 of ambergris. The packaging material costs $20 per cm^2. What shape package would you like to use? (You get to keep the money you don't spend on packaging.)

14. Tough - Using the information in Problem 13, what is the most amount of money you could expect to make?

7.9 Applications to Biology

At the beginning of this chapter we talked about the fact that metabolic processes must take place throughout the cell and that certain materials must get inside the cells for these processes. You also learned that these materials make their way into the cell through the cell membrane.

Most living cells measure between 100μ and 150μ (microns -- refer to chapter 3 if you don't remember what a micron is) at their longest dimension. (There are notable exceptions: some nerve cells have extensions reaching from the base of your spinal column to the end of your big toe!)

The usual shape of a plant cell is a parallelopiped (shoe box), while common shapes in animal cells are dodecahedrons and spheres.

Assume a plant cell is a cube with a side measure of .002 inch (a close approximation to 50 microns). Estimate the ratio of surface area to volume, then use the formulas and compute the ratio.

As explained earlier, determining the volume of a dodecahedron is quite complex, but we can give you the formula for the volume of a sphere and ask you to calculate two different sizes just to see how rapidly volume increases in proportion to the surface area.

The formula for surface area of a sphere is:

$$S.A. = 4\pi r^2 .$$

The formula for volume of a sphere is:

$$V = \frac{4}{3}\pi r^3 .$$

Now see if you can calculate the surface area-volume relationships of two animal cells, one with a diameter of .008 inch and the other with a diameter of .016 inch.

Since building your cubes and determining relationships between surface area and volume and knowing the relationship between surface area and volume of a sphere, can you predict what might happen if cells continued to grow indefinitely? How would you now answer the question pertaining to cells in an elephant and a mouse? Which would you predict (since you are not likely to have the opportunity to actually compare cell size) that an elephant would have larger cells or more cells or both?

Interestingly enough, in addition to cell size, the total size of the organism is related to rates of metabolism. We have not yet considered metabolism in terms of rate (how fast it works). This is dependent upon a number of things and results in certain patterns of behavior. If you tried the yeast experiment with different temperatures, what effect did you discover as you increased the temperature?

Another factor involved in metabolic rate is the amount of material (necessary molecules of food and oxygen) available to the cell. Certain organisms (shrews, for instance) have a very high rate of metabolism, so must have great quantities of food per day in relation to their body size. "Eating like a bird" really means (for some birds) eating the equivalent of one's own body weight in one day! If you weigh 100 lbs, could you eat 100 pounds of food in one day, day after day, without soon outweighing the world's fattest "fat man"? Obviously, your rate of metabolism is lower than that of a bird or a shrew. For the same reason, the tinier animals have a much faster heartbeat. Why? An elephant, on the other hand, has a slower heartbeat and needs less food per day for its size than either you or a bird.

Now, let's see if, step by step, we can find out a little about surface area, volume, and metabolism in some typical animals.

First, most studies are made in terms of weight and we can, in general, equate weight with volume.

Secondly, most studies of metabolism are made in terms of heat loss per day. One of the purposes of metabolism, remember, is to produce energy for life processes. Much of this energy is in the form of heat, and much of this heat is lost.

Physiologists are still pursuing some knotty problems relating to heat loss per unit of weight (and surface area) but if we confine our discussion to homeothermic (hōm ē, ō therm' ik) animals, those which must maintain a constant temperature, we find some very interesting comparisons.

Metabolic activity per 24 hours of some common mammals* is shown below.

	(Weight in Kilograms)	(Calories of Heat Lost per Kilogram)
Rabbit	2.0	58
Man	70.0	33
Horse	600.0	22
Elephant	4,000.0	13
Whale	150,000.0	about 1.7

* Mammals are animals which are identified by the following characteristics:

1. Their young are born alive.
2. They nurse their young through mammary (milk) glands.
3. They all have hair at some time in their lives. (Even a newborn whale has some hair!)

Looking at the above table, can you predict whether a mouse would lose more heat per unit of body weight or less than man?

Thinking in terms of surface area and volume of each of the animals in the table, and remembering that surface area accounts for loss as well as gain of materials (or heat), explain why a rabbit must eat more for its size per day than an elephant.

Nectar (the sugar solution manufactured by flowers) is perhaps the most concentrated food available in nature. It is the staple diet of hummingbirds and bees. Why?

You have built models of various geometric figures and calculated surface area and volume relationships of some of them.

Let's consider a cell with a radius of 1 unit and a cell with a radius twice as large -- or a radius of 2 units.

Surface area of a sphere $= 4\pi r^2$.

Using radius of 1, S.A. $= 4\pi \times (1)^2 = 4\pi \times 1$.

Using radius of 2, S.A. $= 4\pi \times (2)^2 = 4\pi \times 4$.

Volume of a sphere $= \frac{4}{3}\pi r^3$.

Using radius of 1, Vol. $= \frac{4}{3}\pi (1)^3 = \frac{4\pi}{3} \times 1$.

Using radius of 2, Vol. $= \frac{4}{3}\pi (2)^3 = \frac{4\pi}{3} \times 8$.

Notice that when the radius becomes twice as large, the surface area becomes four times greater, and the volume eight times greater.

It is important here to re-emphasize that -- although you used only a few shapes -- living cells are actually found in many diverse shapes. Examples are:

1. The before-mentioned nerve cells, have long fibers, called axons, reaching from the main body of the cell to whatever portion of the body that nerve cell must "serve".

2. An amoeba (a microscopic animal) has no specific shape, but changes shape as it moves through the water.

3. A paramecium (another microscopic animal) is shaped like a slipper; in fact, it is often called the "slipper animalcule".

4. Some leaf cells are like columns; in fact, a group of them is called the "palisade" layer.

5. Bacteria cells may be rod-shaped, tiny spheres, or spirals.

One could go on and on describing the varied shapes of cells, as they have been observed by scientists throughout the years (after the microscope was discovered).

The important thing to remember is that no matter what the shape, the cells are limited in size by the metabolic needs of the protoplasm and the ability of molecules to get in and out through the cell membrane to serve those needs.

SQUARES AND SQUARE ROOTS OF NUMBERS

Number	Squares	Square Roots	Number	Squares	Square Roots
1	1	1.000	36	1,296	6.000
2	4	1.414	37	1,369	6.083
3	9	1.732	38	1,444	6.164
4	16	2.000	39	1,521	6.245
5	25	2.236	40	1,600	6.325
6	36	2.449	41	1,681	6.403
7	49	2.646	42	1,764	6.481
8	64	2.828	43	1,849	6.557
9	81	3.000	44	1,936	6.633
10	100	3.162	45	2,025	6.708
11	121	3.317	46	2,116	6.782
12	144	3.464	47	2,209	6.856
13	169	3.606	48	2,304	6.928
14	196	3.742	49	2,401	7.000
15	225	3.873	50	2,500	7.071
16	256	4.000	51	2,601	7.141
17	289	4.123	52	2,704	7.211
18	324	4.243	53	2,809	7.280
19	361	4.359	54	2,916	7.348
20	400	4.472	55	3,025	7.416
21	441	4.583	56	3,136	7.483
22	484	4.690	57	3,249	7.550
23	529	4.796	58	3,364	7.616
24	576	4.899	59	3,481	7.681
25	625	5.000	60	3,600	7.746
26	676	5.099	61	3,721	7.810
27	729	5.196	62	3,844	7.874
28	784	5.292	63	3,969	7.937
29	841	5.385	64	4,096	8.000
30	900	5.477	65	4,225	8.062
31	961	5.568	66	4,356	8.125
32	1,024	5.657	67	4,489	8.185
33	1,089	5.745	68	4,624	8.246
34	1,156	5.831	69	4,761	8.307
35	1,225	5.916	70	4,900	8.367

Number	Squares	Square Roots	Number	Squares	Square Roots
71	5,041	8.426	86	7,396	9.274
72	5,184	8.485	87	7,569	9.327
73	5,329	8.544	88	7,744	9.381
74	5,476	8.602	89	7,921	9.434
75	5,625	8.660	90	8,100	9.487
76	5,776	8.718	91	8,281	9.539
77	5,929	8.775	92	8,464	9.592
78	6,084	8.832	93	8,649	9.644
79	6,241	8.888	94	8,836	9.695
80	6,400	8.944	95	9,025	9.747
81	6,561	9.000	96	9,216	9.798
82	6,724	9.055	97	9,409	9.849
83	6,889	9.110	98	9,604	9.899
84	7,056	9.165	99	9,801	9.950
85	7,225	9.220	100	10,000	10.000

Chapter 8

GIANT TREES: FORMULA CONSTRUCTION FOR VOLUME OF CYLINDER AND CONE, INDIRECT MEASUREMENT

8.1 Introduction

The "General Grant" giant sequoia redwood tree has enough wood to build 45 five-room houses. The "General Sherman", largest known tree in the world, could furnish enough lumber to build 50 houses. Obviously, these trees have not been cut down and houses built from their lumber. How do you think these facts have been obtained?

Both of the trees mentioned are in national parks and will not be turned into lumber, but every year literally millions of trees are felled, sent to the saw mills, and processed into usable lumber for construction.

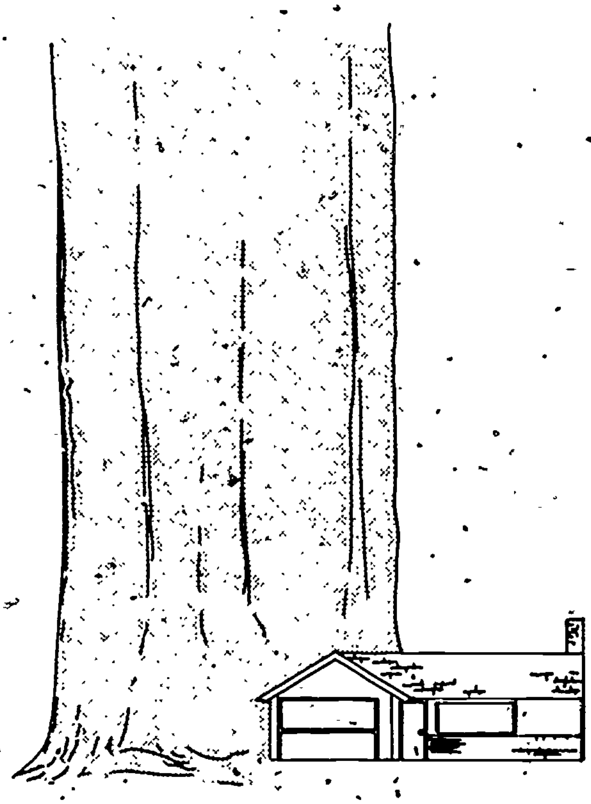


Figure 8-1

Oftentimes, the U. S. Government, state governments, and private individuals offer their timber land for sale. People interested in bidding on the timber rights (the right to take the trees) must have an accurate method of estimating the value (the worth of the lumber which can be made from the trees) of the obtainable wood.

Each tree is itself a valuable product and an entire tract of trees can be expensive to a purchaser. Timber buyers want to be as sure as possible that the prices they bid reflect the values they can receive. Thus a careful estimate is made of the volume of wood obtainable.

To start with an entire forest would be a job of monumental proportions for us. We will start with just one tree. If we could figure the volume of wood in a tree we would be well on our way toward solving the whole problem.

8.2. Indirect Measurement

If you and your little brother were to stand beside each other (probably hard for your parents to believe) in the sunlight, the lengths of your shadows would not be the same. His shadow would be shorter than yours in exactly the same ratio as his height is less than your height. (If he is taller than you, then the opposite would be true. We'll pretend you are the taller.)

In fact, we can estimate the height of your little brother very closely if we know your height and the measurement of both shadows. Let's assume you stand together on the sidewalk. Your shadow is 4 feet long and your brother's is 3 feet. His height then is $\frac{3}{4}$ of your height. If you are 6 feet tall, then he is $\frac{3}{4}(6)$ or $4\frac{1}{2}$ feet tall.

At the same time of the day, the relationship between <u>your height</u> and the height of <u>your house</u> or a tree or any other object would be in the <u>same ratio</u> as the length of <u>your shadow</u> and the <u>shadow</u> of the object.

This relationship is often included in many manuals and guides dealing with hiking, camping, exploring, etc. The mathematical concept is part of a topic called similar triangles.

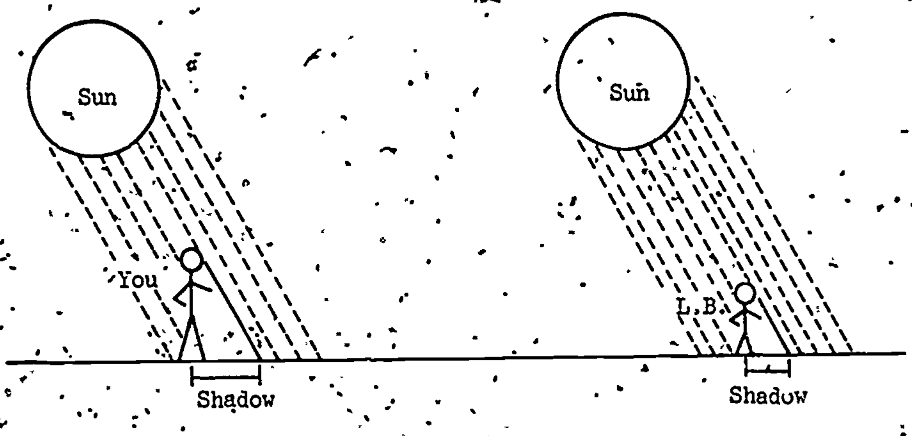


Figure 8-2a

You and your shadow, and L. B. and his shadow, are shown as legs of right triangles. See Figure 8-2b.

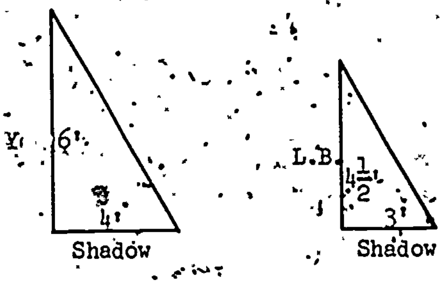


Figure 8-2b

Superimposing the two triangles so that each leg lies on the same line, and each hypotenuse lies on the same line we have a figure as in Figure 8-2c.

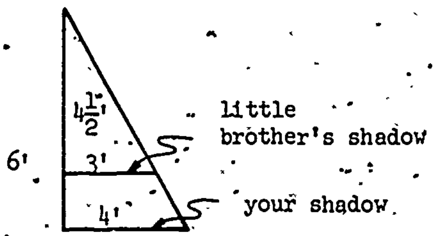


Figure 8-2c

The two triangles are similar; that is, in a one-to-one correspondence between the vertices, the corresponding angles are congruent and the ratios of the measures of corresponding sides are equal.

Answer the following questions. Measure the required distances in Figure 8-2d to check your answers.

Exercise 8-2a

1. What is the ratio of the base to the height?
2. If $\overline{xf}$ is 60 mm, what is the measure of $\overline{fi}$?
3. $\overline{xc}$ is 30 mm. What is $\overline{c\ell}$?
4. $\overline{ej}$ measures $37\frac{1}{2}$ mm. What is the measure of $\overline{xe}$?
5. Find the measure of $\overline{an}$ if $\overline{xa}$ = 10 mm.

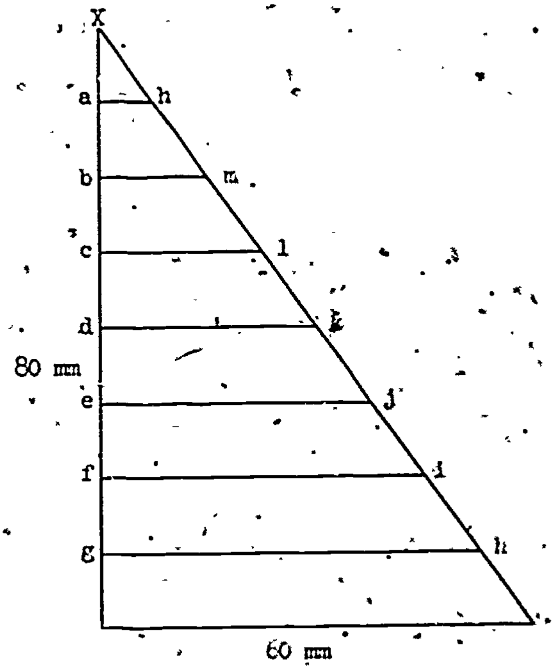


Figure 8-2d

Now we will move quickly from little brother to a very tall tree. Let's assume the same time of day and same place on the world for you and the very tall tree. Why? The ratio of your shadow to your height is $\frac{4}{6}$ or $\frac{2}{3}$, thus, if we measure the shadow of our tall tree, that shadow will be $\frac{2}{3}$ or $\frac{4}{6}$ of the height of the tree. If a tree has a measurement of 180 feet, then its shadow will be $\frac{2}{3}$ of 180 feet or 120 feet. However, we want to find <u>the height of a tree if we know the shadow</u>. If the ratio of your height to your shadow is 6 : 4 we can use the proportional value to find the height of the tree when we know its shadow length.

$$\frac{\text{Your height}}{\text{Your shadow}} = \frac{6}{4} = \frac{3}{2} = \frac{\text{Tree's height}}{\text{Tree's shadow}}$$

Let the tree's height be represented by x, then we have the proportion $\frac{3}{2} = \frac{x}{120}$ to solve for x.

Solving the proportion for tree height

$$\frac{3}{2} \times 120 = x$$
$$180 = x$$
$$180 = \text{height of tree in feet.}$$

In Figure 8-2e, a proportion formed is $\frac{x}{y} = \frac{a}{b}$. Three of the four possible values are needed. Which 3 measurements could be easily determined? a, b, x, or y?

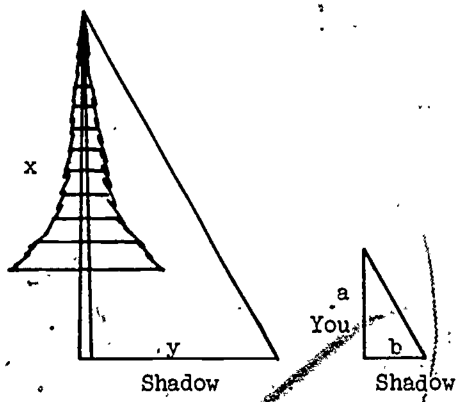


Figure 8-2e

Exercise 8-2b

1. Bob, a 6-foot tall man, has a shadow measure of three feet. A ponderosa pine tree by his log cabin has a shadow measure of 124 feet. What is the approximate height of the pine?

2. The "Rockefeller tree", a coast redwood tree, casts a shadow of 60 feet when a man 72 inches tall casts a shadow of 12 inches. What is the approximate height of the tree?

3. A young sugar pine's shadow is 9 feet. Frank, eight feet tall (a "big man" in that part of the country), standing beside the tree has a shadow 12 feet long. What is the height of the tree?

4. John, a hiker, armed with two meter sticks and a level, measured the shadow of a tall tree as 63.24 meters. He then measured the shadow of one meter stick held vertically to the earth as 72 cm. What was the height of the tree?

5. The shadows of Bill, Pat, and a white fir tree are in the ratio of 18, 16, 360 respectively. If Bill is 6 feet tall, how tall are Pat and the tree?

6. Choose a tree in the schoolyard (or flagpole if there is no tree). Working with a partner, measure your shadow and the tree's shadow (a) before school, (b) at noon lunch hour, and (c) after school. Your height and the tree's height remain the same. Only the shadow lengths

vary. Solve each separate proportion for the tree height. Find the average. Does this seem like a good way to closely approximate the height of the tree?

8.3 Volume

Now that we can estimate the height of a tree, what other information do we need to find the volume of wood in the tree?

Remember the formula for computing the volume of a rectangular parallelepiped (rectangular solid)? No? We will quickly review. If in Figure 8-3a,

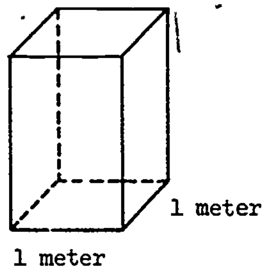


Figure 8-3a

the area of the base of the box were 1 m^2 and if it were filled to a height of 1 m, the volume would be 1 m^3. If filled to a height of 2 m, the volume would now be 2 m^3.

In Figure 8-3b the dimensions of the base are: length 5 m and width 4 m. The area of the base of the box would be ______m^2.

Filled to a height of 1 m the volume would be ________m^3. Two meters height would give ________m^3.

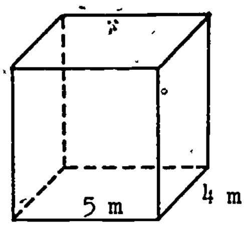


Figure 8-3b

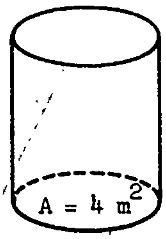


Figure 8-3c

In Figure 8-3c, if the area of the base is 4 m^2 the volume would again be computed by finding the product of: (area of base) × (height).

The volume of a right circular cylinder (sides perpendicular to a circular base) = B × h where B is the area of the base.

Years ago you learned the formula for the area of a circle $A = \pi r^2$ when r stands for the radius of the circle. We will use this well-known relationship to develop another formula for the area of a circle.

8.4 Area of a Circle Given the Diameter

Lumbermen need to compute the volume of wood in logs. With thousands of logs to measure, the loggers have turned to using a device similar to a caliper to quickly convert the diameter of the log into a usable volume measure. (See Figure 8-4a.) In a later section of this chapter we will discuss this measuring device.

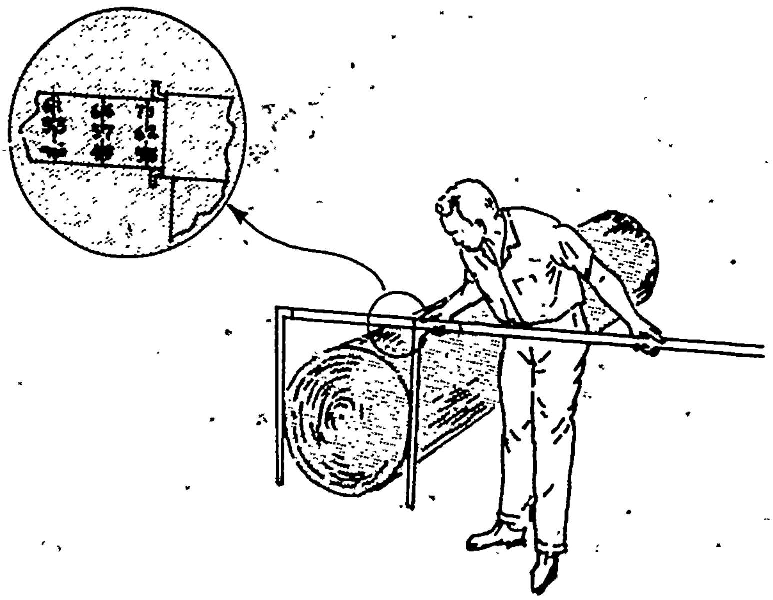


Figure 8-4a

We know several formulas which deal with a circle.

1. Area of a circle = $\pi \times \text{radius}^2$ or $A = \pi r^2$
2. Circumference of a circle = $\pi \times$ diameter or $C = \pi d$
3. Diameter of a circle = $2 \times$ radius or $D = 2r$

(In the remainder of this chapter A will be used as the symbol for area of a circle.)

If the diameter of the circle is the most easily obtainable measurement then we can develop a formula for the area of a circle given the diameter. We will work step by step from the original formula (using radius) to the formula which uses diameter.

$$\text{If diameter} = 2 \times \text{radius}$$
$$\text{then radius} = \frac{\text{diameter}}{2} \quad \text{or} \quad \left(\frac{d}{2}\right).$$

In the formula

$$A = \pi r^2$$
$$A = \pi \left(\frac{d}{2}\right)^2 \quad \text{substitute } \frac{d}{2} \text{ for } r$$
$$A = \pi \left(\frac{d}{2} \times \frac{d}{2}\right)$$
$$A = \frac{\pi d^2}{4} \quad \text{or} \quad A = \frac{\pi}{4} \times d^2.$$

The value of π is approximately 3.14159. Therefore, $\frac{\pi}{4} \approx \frac{3.14159}{4} \approx$ .78539 . Equivalent formula for the

<u>Area of a circle is</u> $\approx .785 \times d^2$

<u>Exercise 8-4</u>

1. The diameter of a circle is 2 feet; find the area.
2. If the diameter of a circle is 14 inches, what is the area?
3. Given the diameter of a circle as $\frac{1}{2}$ foot, find the area in square feet.
4. If $144 \text{ in}^2 = 1 \text{ ft}^2$, give the answer to Problem 3 in square inches.
5. If the radius of a circle is $2\frac{1}{2}$ feet, find the area using the formula $A = .785\, d^2$.

6. A washer has dimensions as shown in Figure 8-4a. Find the surface area of the shaded portion. (The answer is not .785 .)

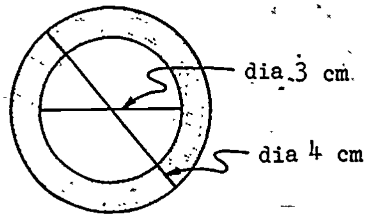


Figure 8-4a

7. In the Grant Grove of Kings Canyon National Park stands the General Grant tree with base diameter of 40.3 feet and the Texas tree, equally tall, but with base diameter of 22 feet. What is the ratio of the cross sectional areas of the General Grant tree to the Texas tree?

8. Gary stepped off the distance around a giant redwood stump. He measured his step as 30 inches and it took 38 steps to go completely around the tree. What would be the area of the top of the stump?

8.5 Area of a Circle Given the Circumference

Without a caliper device for measuring diameter, the circumference of a tree would be the most easily obtainable dimension. (See Figure 8-5a.)

A flexible tape measure is excellent to use in measuring the circumference. However, a piece of string stretched around the tree, marked, then measured with a yard or meter stick, will also give a usable measurement.

We will derive another formula for the area of a circle. This formula will be useful when the circumference is the measurement most easily obtained.

We know $A = \pi r^2$ and we found that $A = \frac{\pi}{4} d^2$. The circumference of a circle is equal to π times the diameter or $C = \pi d$ (formula 2 in 8-4).

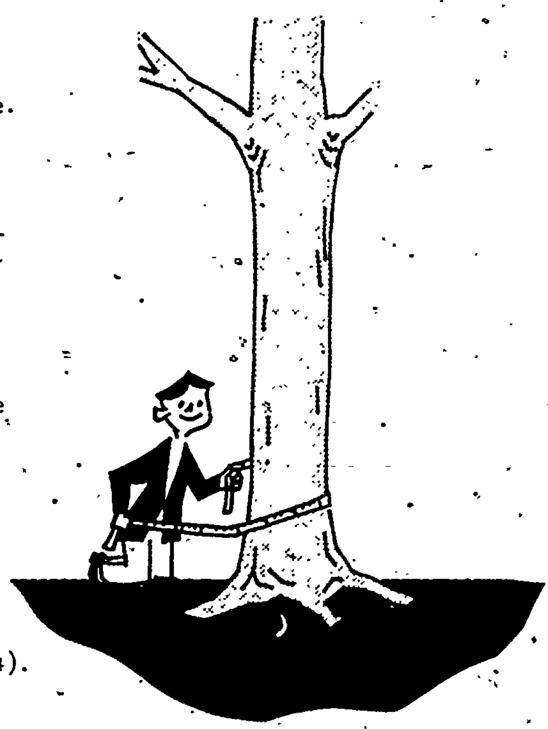


Figure 8-5a

Thus $C \div \pi$ would equal the diameter or $\frac{C}{\pi} = d$.

If $A = \frac{\pi}{4} \times d^2$	formula derived in Section 8-4
then $A = \frac{\pi}{4} \times (\frac{C}{\pi})^2$	substituting $\frac{C}{\pi}$ for d
$A = \frac{\pi}{4} \times \frac{C}{\pi} \times \frac{C}{\pi}$	meaning of "square"
$A = \frac{\pi \times C \times C}{4 \times \pi \times \pi}$	multiplication of fractions
$A = \left(\frac{\pi}{\pi}\right) \frac{\times C \times C}{\times 4 \times \pi}$	commuting 4 and π
$A = \frac{C^2}{4\pi}$	$\left(\frac{\pi}{\pi}\right)$ another name for "one"
$A = \frac{1}{4\pi} \times C^2$	another name for the same number

The part $\frac{1}{4\pi}$ will always be the same in any problem. Only the circumference will vary. Therefore, if we change $\frac{1}{4\pi}$ to the decimal value, it will be convenient to use in computations.

$$\frac{1}{4\pi} \approx \frac{1}{12.5664} \approx .07958$$

Using the value for $\frac{1}{4\pi}$ we have

$$\text{Area of a circle} \approx .07958 \times C^2$$

which can be rounded off to $\approx .08 \times C^2$

<u>Area of a circle $\approx .08 \times C^2$</u>

Our new <u>formula for the area of a circle</u> is the one to use when we know or can most conveniently find the <u>measure of the circumference</u>.

Exercise 8-5

1. The circumference of a given circle is 2 feet. Find the area.
2. If the circumference of a circle is 8 feet, what is the area?
3. The measure of the distance around the trunk (circumference) of the Colonel Herman cedar at Hume Lake is 23.2 ft. What is the area of such a circle?

4. Smorgasbord -- (Take your choice)

$A = \pi r^2$ or $A = \frac{\pi d^2}{4}$ or A $.7854d^2$ or A $.08C^2$

(Use $\pi \approx 3.14$)

When A = area of a circle, r = radius, d = diameter and c = circumference

(A) Find A if r = 3 cm

(B) Find A if d = 5 cm

(C) Find A if c = 9 cm

(D) If d = 6' find A

(E) If r = 7" find A

(F) If c = 6' find A

5. Erik, a nine-year-old, and his fifteen-year-old brother, Andy, could just touch each other's finger tips when they reached around a sugar pine. They then measured their combined reach. It was 10.5 feet. What would be the cross sectional area of the tree?

8.6 Volume of a Cylinder

Often, lumbermen need to know how many board feet of lumber can be obtained from logs, not standing trees. For their purposes, they assume a log is a cylinder. Of course, for several reasons it really isn't a cylinder. What is it?

We noted in Section 8-2 that the volume of a right cylinder is found by multiplying the area of the base by the height of the cylinder. The formula is $V = B \times h$ when $B = \pi r^2$. Thus, the volume of a cylinder = $\underline{\pi r^2}h$. The underlined portion is simply the formula for the area of a circle and we already know 3 different formulas for finding the area of a circle.

Area of a circle = πr^2 is used when the radius is known

Area of a circle $\approx .785\ d^2$ is used when the diameter is known

Area of a circle $\approx .08\ C^2$ is used when the circumference is known

The formula for the volume of a cylinder can then be revised to use any of the above formulas. If the V of a cylinder = $\pi r^2 h$ then, since πr^2 = area of a circle or a base, we can use

V of cyl = $\pi r^2 h$ when the radius is known

or V of cyl $\approx .785\ d^2 h$ when the diameter is known

or V of cyl $\approx .08\ C^2 h$ when the circumference is known.

Exercise 8-6

1. Find the measure of the volume of a log 20 ft long and 4 ft in diameter.

2. In Sequoia National Park, a home built in a fallen giant redwood tree has been named "Tharp's Log". It's not known for sure if Hale Tharp actually lived in the tree, but James Wolverton, another early day trapper and cattleman, lived in the log for several years. The hollowed out part is over 56 ft long and varies in height from 8 ft to 4 ft. Wolverton had it furnished with furniture, doors, and a fireplace. Before the tree fell, it was estimated to be 311 ft tall and have a base diameter of 24 ft. Assuming it was solid, what would be the volume of wood in a 40 ft cylindrical section from the base end of Tharp's tree?

3. Calculate the volume of wood in the bottom 24-foot section of the newly-discovered "tallest tree in the world" having a base circumference of 44 ft and a height of 367.8 feet.

4. Calculate the volume of wood in the bottom 24-foot section of a Michigan pine tree having a 39-inch base diameter and a height of 90 feet.

5. What is the ratio of the volumes of Problems 3 and 4?

8.7 Board Feet

Lumber (wood ready for use) is seldom sold by the cubic foot. It is almost always handled as "board feet". A board foot is defined as the equivalent of a board 1 inch thick, 12 inches long and 12 inches wide. Thus, a board 1 inch by 12 inches by 60 inches would be 5 board feet.

Several ways to compare board feet and cubic feet are:

$$1 \text{ bd ft} = 1 \text{ in by } 12 \text{ in} \times 12 \text{ in} = 144 \text{ in}^3$$

$$1 \text{ cu ft} = 12 \text{ in by } 12 \text{ in} \times 12 \text{ in} = 1728 \text{ in}^3$$

$$1 \text{ bd ft} = \frac{1}{12} \text{ ft}^3 \quad \text{or} \quad 12 \text{ bd ft} = 1 \text{ ft}^3$$

A log 20 ft long and with a diameter of 3 ft would have a volume measure of 141 ft^3. Multiplying this volume in ft^3 by 12 would give us the number of "bd ft" in the log. $141 \times 12 = 1692$ board feet.

Trees are cut into logs of various lengths and then at the sawmill into many varied sizes of lumber depending upon the needs of the lumber yards and

and the size and shape of the logs. We will not work with the number of board feet in various sizes of lumber. Instead, our problems will deal with the total number of board feet in a tree.

To be realistic, lumbermen figure that approximately $\frac{1}{3}$ of the total volume of a log is lost at the sawmill. The loss comes from the amount of bark, the "slabs" cut from the sides, and the sawdust made from the saw cuts.

> To continue our computation, using the same log as in the last example, the 1692 board feet would become $1692 \times \frac{2}{3} = 1128$ board feet of usable lumber since $\frac{1}{3}$ of the original volume is lost.
>
> In all computations, when figuring the board feet in a log or tree, we will consider only $\frac{2}{3}$ of the total volume.

The formula we have been using to find the volume of a cylinder, given the diameter and height is: $V \approx .785 \times d^2h$. If the diameter is in feet and the height in feet, then the volume is given in ft^3. Can you see why? We will develop a formula for computing the usable lumber in a log.

Vol. of a cylinder	$\approx .785\ d^2h$	
Bd ft in cylinder	$\approx .785\ d^2h \times 12$	
Usable bd ft	$\approx .785\ d^2h \times 12 \times \frac{2}{3}$	
	$\approx .785 \times 12 \times \frac{2}{3} \times d^2h$	commutative property of multiplication
	$\approx 6.28\ d^2h$	

Usable bd ft of lumber in a log $\approx 6.28\ d^2h$

(d and h must be expressed in feet)

The board feet in a log 20 ft long and with a diameter of 4 ft could thus be computed by substituting into our formula:

$$V \approx 6.28 \times 4^2 \times 20 \approx 6.28 \times 320 \approx 2009 \text{ bd ft.}$$

Right about now it is possible that someone in the class (surely not you) is saying, "Wow! What a lot of formulas. I'd rather stick to good old πr^2. I can remember that one." A good point. πr^2 is the one to memorize, but the others are good to use when needed. To illustrate this last remark let us solve a sample problem using the traditional formula for one solution and the formula we just developed for another solution. You decide which is the more practical to use.

Our problem is to find the number of usable board feet of lumber in a log that has a diameter of 3 feet and a length of 15 feet (assuming a cylindrical shape).

First Solution

Formula is: $\pi r^2 h \times 12 \times \frac{2}{3}$

Data given: $d = 3$ ft

$h = 15$ ft

If $d = 3$ ft then $r = 1.5$ ft

Substituting in formula for r and h

$$\text{bd ft} \approx 3.14 \times 1.5^2 \times 15 \times 12 \times \frac{2}{3}$$

$$\text{bd ft} \approx 3.14 \times (2.25 \times 15) \times (12 \times \frac{2}{3})$$

$$\text{bd ft} \approx 3.14 \times (33.75 \times 8)$$

$$\approx 3.14 \times 270$$

$$\approx 848 \text{ bd ft}$$

Second Solution

Formula is: bd ft $\approx 6.28\, d^2 h$

Data given: $d = 3$ ft

$h = 15$ ft

Substituting in formula for d and h

$$\text{bd ft} \approx 6.28 \times 3^2 \times 15$$

$$\approx 6.28 \times (9 \times 15)$$

$$\approx 6.28 \times 135$$

$$\approx 848 \text{ bd ft}$$

Are you convinced? In the formula we developed, the "fixed" values of π, 12, and $\frac{2}{3}$ have been multiplied and their product becomes the factor 6.28 in our formula.

Exercise 8-7

1. Find the usable bd ft in a log 18 ft long and with a diameter of $3\frac{1}{2}$ feet. Use both methods as described above.

2. Mr. French has six logs on his truck. Three logs have a diameter of 42 inches each, the other three have diameters of 48 inches each. They are all 18 feet long. What is the value of the load at $55.00 per thousand usable board feet? Use the short method.

3. Find the usable board feet in the following logs:

   (A) Base diameter 4 ft, length 14 ft;

   (B) Base diameter 14 ft, length 22 ft;

   (C) Base circumference 18 ft, length 16 ft.

Loggers always measure the diameter at the narrower end of the log. Can you imagine why? This formula, $V \approx 6.28d^2h$, is neat and handy, but can you imagine a logger walking up to each log with a tape measure and electric calculator to determine the bd ft in each log? The bd ft in each log must be computed in many cases as the logs are purchased from sellers by the number of board ft in them.

A table of values can be constructed and used so that only the measurement needs to be made in the field.

Make a table similar to Table 8-7 and fill in the values. Some values have been filled in for you. Round off to the nearest foot.

Table 8-7

<u>Usable Board Feet in log</u> = <u>$6.28\ d^2h$</u>

Length in feet

Diameter in feet	8	10	12	14	16	18	20
1	bd 50 ft	63	75	88	100	113	126
2	201						
3			678				
4							2010
5		1570					
6					3617		

A table such as this has been constructed and placed on an instrument called a caliper which is explained in the following discussion.

The caliper rule described in section 8-4 (see Figure 8-4a) was calibrated from data similar to Table 8-7. Notice the inset portion of Figure 8-5a. To save space on the ruler the last zero was not printed. Each number is understood <u>to be multiplied by a factor of 10</u>. Each line is for a different length of log for each <u>inch</u> of diameter. We will enlarge the inset portion. See Figure 8-7a.

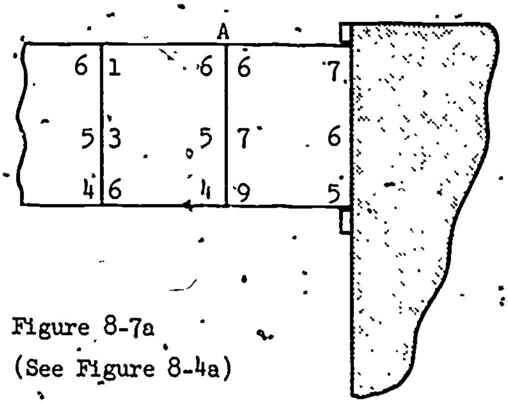


Figure 8-7a
(See Figure 8-4a)

Line A in Figure 8-7 is for a log with a 30 inch diameter and 16 ft, 14 ft, or 12 ft length. The caliper would be read as follows: If the log were 16 ft long, it would contain 660 board feet. If it were 14 ft long, it would contain 570 board feet. A 12 ft log would contain 490 board feet.

Your table has values for even numbers of feet only, but you can <u>interpolate</u> for values between the known values. For example: using data from Table 8-7,

Diameter of log	Board feet of 16 ft log
2 ft (24 in)	402
3 ft (36 in)	904

To find the approximate number of board feet in a 16 ft log of 30 inch diameter, we could interpolate for a value between 24 inches and 36 inches.

12 [24 in, 30 in (6 in), 36 in] 502 [402, ? (x), 904]

A proportion could be $\frac{6}{12} = \frac{x}{502}$. Solving for x, $\frac{6}{12} \times 502 = x$ and $x = 251$. <u>Adding</u> the value of x to 402, we find the estimated board feet for a 16 ft log with a 30 inch diameter to be 653. The logger's caliper gave 660 board feet. Which is correct? Both are close enough for most practical purposes.

<u>Exercise 8-7b</u>

Interpolate or extrapolate from Table 8-7 for the following problems.

1. A 16 ft log measures 52 inches in diameter. Interpolate from your table to determine the usable board feet contained in the log.

2. Compute the usable board feet for the log in Problem 1 by using your formula for the volume of a cylinder, the factor of $\frac{2}{3}$, to account for sawmill loss, and compare your answer to that of Problem 1.

3. How much difference, based on the answers to Problems 1 and 2, in dollar and cent value does this represent at $30.00 per thousand board feet?

4. Interpolate from your table to determine the lumber in an eighteen foot log 68 inches in diameter.

5. Extrapolate from your table to determine the usable board feet in a log 16 ft long by 7 ft in diameter.

6. Extrapolate from your table to determine the usable board feet in a 22 ft log which is 5 ft in diameter.

7. Interpolate from your table to find the usable board feet in a 15 ft log $5\frac{1}{2}$ ft in diameter.

8. Check the validity of your interpolations or extrapolations by computing directly.

Would the lumberman's caliper rule described here help in finding the board feet of a standing tree? If you place the rule on the trunk of a tree with a known height and read the board feet, what geometrical shape are you assuming for the tree?

Could a similar caliper rule be made which, when placed around the base of a tree, would give a measurement for the board feet in the tree (height known)?

8.8 <u>Volume of a Cone</u>

The "tallest living thing", a coast redwood, was recently discovered. It is in a grove of redwoods, all of which are over 300 feet tall, in northern California's Humboldt County. Scientists from the National Geographic Society discovered it.

The height of this tremendous tree is 367.8 feet. To gain an appreciation of its height, see Figure 8-8a. An article describing the tree and its discovery is in the July, 1964, issue of the <u>National Geographic</u>.

The General Grant tree is 267 feet tall. It is a member of a different species than the coast redwoods.

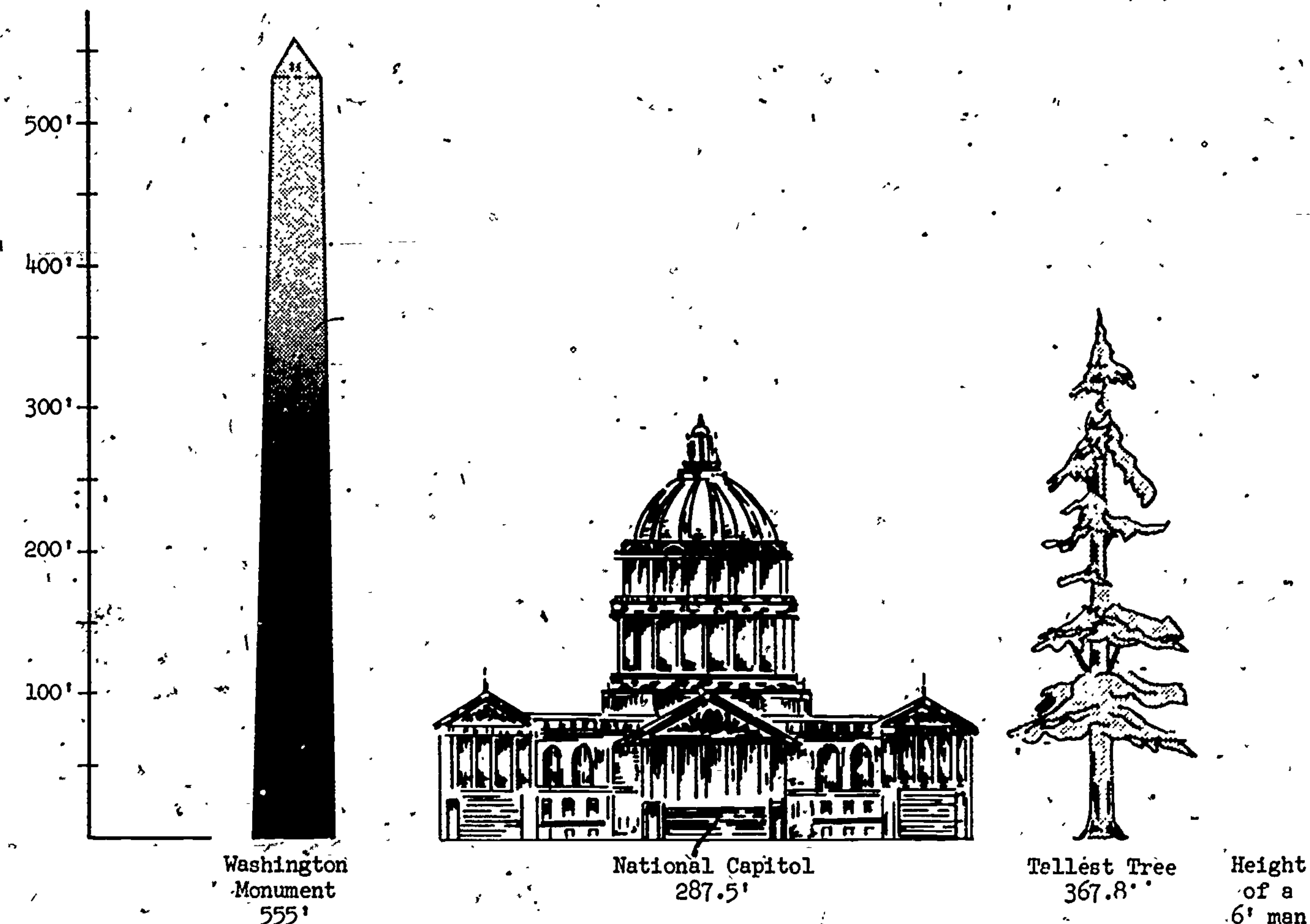

500'
400'
300'
200'
100'
Washington Monument 555'
National Capitol 287.5'
Tallest Tree 367.8'
Height of a 6' man

Figure 8-8a

Which tree do you think has the greater volume of wood, the General Grant tree or the Humboldt County tree? Wait a minute . . . here is one more bit of information. The General Grant tree has a circumference at the base of over 100 ft, while the tree in Humboldt County has a base circumference of 44 ft. Now do want to change your guess on which has the greater volume of lumber?

The volume of a cylinder is found by multiplying the area of the base by the height (A = bh). We could now figure the volume of a tree, if it were only a cylinder. But the tree trunk is approximately a cone.

Many trees which are grown for lumbering purposes are conifers. Some common varieties are: sugar pine, ponderosa pine, lodge pole pine, redwood, red fir, Douglas fir and white fir. Some other types of trees whose wood is used for lumber are not as conical in shape. Walnut, oak, maple, ash, hickory, and others often have quite irregular shapes. However, while standing as trees (before they cut out logs) a cone or portion of a cone still is the closest geometrical approximation of the shape.

A tree trunk although considered to be a cone, does not resemble an ice cream cone. Why?
(We know a tree trunk has no ice cream in it!)

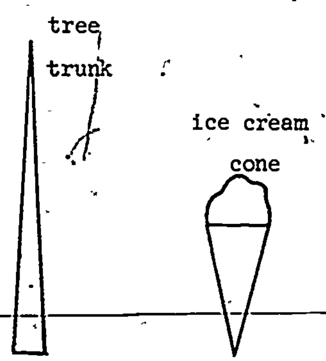


Figure 8-8b

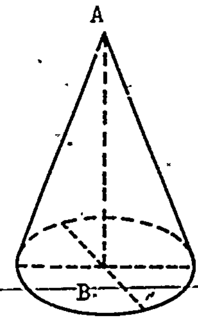


Figure 8-8c

It can be shown that the formula for the volume of a right circular cone (right meaning that in Figure 8-8c $\overline{ab}$ is perpendicular to the base forming right angles at the base) is $V = \frac{1}{3}\pi r^2 h$ or exactly $\frac{1}{3}$ of the volume of the circular cylinder having the same height and area of base. See Figure 8-8d.

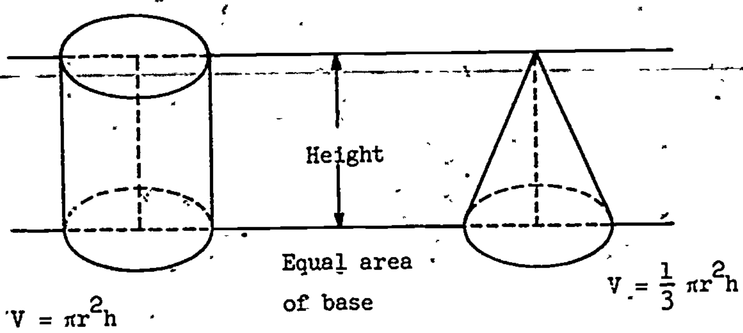


Figure 8-8d

In using the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone in computing the volume of a tree, which measurements must we make of the tree?

Can you readily estimate the height?

Can you readily measure the radius?

As radius, diameter and circumference are all related to a circle, is there a measurement, other than radius, which may be easier to find?

For our tree volume problem it would seem that the circumference of the base and the measure of the height, by an indirect method, would be the most easily obtainable measurements.

We will do all of the involved computation once and develop a new formula for volume of a tree using the circumference and height as the elementary measurements.

$$\text{Volume of a cone} = \frac{1}{3}\underline{\pi r^2 h}$$

The underlined portion represents the area of the base (πr^2). We found in Section 8-5 another formula for the area of a circle is $.0796 \times C^2$ when C = the measure of the circumference. Substituting this in our formula we have

$$\text{Volume of a cone} \approx \frac{1}{3} \times .0796 \times C^2 \times h$$

$$\approx \frac{.0796}{3} \times C^2 h \quad \text{multiplication of rational numbers}$$

Volume of a cone $\approx .0265 \times C^2h$ when given the measure of the circumference and height

The tallest tree has a height measure of $\approx$ 368 ft and a circumference at the base of 44 ft. To find the volume of the trunk, substitute in the formula for the volume of a cone.

$$\begin{aligned} V &\approx .0265 \times 44^2 \times 368 \\ &\approx .0265 \times (1936 \times 368 \text{ ft}) \\ &\approx .0265 \times 712{,}448 \\ &\approx 18{,}880 \text{ ft}^3 \text{ of wood} \end{aligned}$$

For lumbering purposes we would like to know how many bd ft are in such a tree (don't worry, they won't cut the tree down. The owners, the Arcata Redwood Company, are making plans to preserve it).

Recall that 1 cu ft = 12 bd ft and $\frac{1}{3}$ is lost at the sawmill so

V in cu ft $\approx$ 18,880

V in bd ft $\approx 18{,}880 \times 12$

Usable bd ft $\approx 18{,}880 \times 12 \times \frac{2}{3}$

Possible usable bd ft $\approx$ 151,040 bd ft

in the world's tallest tree

This, of course, is just a calculated guess, as we have not seen the tree. What is assumed in this calculation?

Exercise 8-8a

Using the formula $V = .0265\, C^2h$, 1 cu ft = 12 bd ft and $\frac{1}{3}$ loss of total volume at the mill, find the approximate usable bd ft in the following problems on trees.

1. A sugar pine is 200 ft tall and has a base circumference of 30 ft.
2. A red fir has a base circumference of 14 ft and a height of 110 ft.
3. A ponderosa pine with a height of 220 ft has a circumference at the base of 31 ft.

Is it possible some of you are using an idea from a previous section? Do you find you are computing with some of the same numbers over and over again? What numbers were the same in each of the 3 problems? Did you use

.0265? Did you use 12 as a factor? How about $\frac{2}{3}$? Can you "compact" these numbers by multiplying and making a new factor in our formula for bd ft? Think about it. . . . . Now do it.

4. A Jeffrey pine 120 ft tall has a base circumference of 10 ft. Find the usable bd ft.

5. Norm, a "timber cruiser", (title for the man who estimates the value of trees) found a tract of ponderosa pines: 5 with an average height of 200 ft and a circumference of 25 ft; 7 with an average height of 100 ft and a circumference of 20 ft; 15 with an average height of 150 ft and a circumference of 12 ft. How many usable bd ft of lumber were in the tract?

6. If a lumber mill bids $20 per thousand board feet for ponderosa pine, how much would be the bid for the tract described in Problem 5?

For the following problem use this information when necessary.

sugar pine	$30	per thousand	($30 per m)
ponderosa pine	$20	per thousand	($20 per m)
fir and cedar	$ 9	per thousand	($9 per m)

7. One thousand acres are open for bids on the timber rights. The following numbers of trees are marked for felling:

3000	sugar pines	at an average of	12,000 bd ft each
7000	ponderosa pines	at an average of	15,000 bd ft each
9000	cedar	at an average of	8,000 bd ft each
17000	fir	at an average of	4,000 bd ft each

Find the bid on the timber rights. (Would scientific notation help?)

8.9 The Largest Living Things in the World

Fortunately, the trees used as examples in this chapter: the General Sherman, the General Grant, and the newly discovered "tallest living thing in the world" are--at least for the moment--protected for the future. The first two were discovered many years ago, recognized for their magnificence, and were finally protected by law by enclosing them and their surrounding groves and mountains in national parks.

Less fortunately, many of these giant redwoods were not protected soon enough and were cut by loggers before Congress was finally convinced that the remaining ones (and they are few indeed!) should be protected. To this day, only the known groves (there may be others "undiscovered" or at least still confined to privately owned land) are located in state or national forests. Lumbermen still have access to many, and the fallen logs and some of the younger trees are used.

Interestingly enough, when these huge trees were discovered, only those who saw them really believed they existed. In 1875, seventeen years after they were first seen by white man, one of the great trees was felled and portions of it cut into sections for shipping to the Centennial Exhibition in Philadelphia. There it was reassembled for the exhibition but was considered by the general public to be a hoax. The evidences that the log had been "put together" caused the public to believe it had been constructed from many trees. Little did they know that already a hollowed out fallen log had been lived in by men who found it a spacious residence, that another would be used to shelter 32 horses, that hollow stumps were to be used as schoolrooms, or that someday a single smooth stump would be used as a dance floor!

These mammoth trees have the impressive (but fitting) name of Sequoiadendron giganteum. They are commonly called Giant Sequoias and were named for a famous Indian chief: Sequoyah, the man who first worked out an alphabet for his people, the Cherokee Indian tribe. The Giant Sequoias are found only in a few groves in the Sierra Nevada Mountains of California.

The "tallest living thing in the world", the redwood tree just recently discovered in the Coast Range Mountains in northern California, belongs to a different group of trees. It has not yet been officially named.

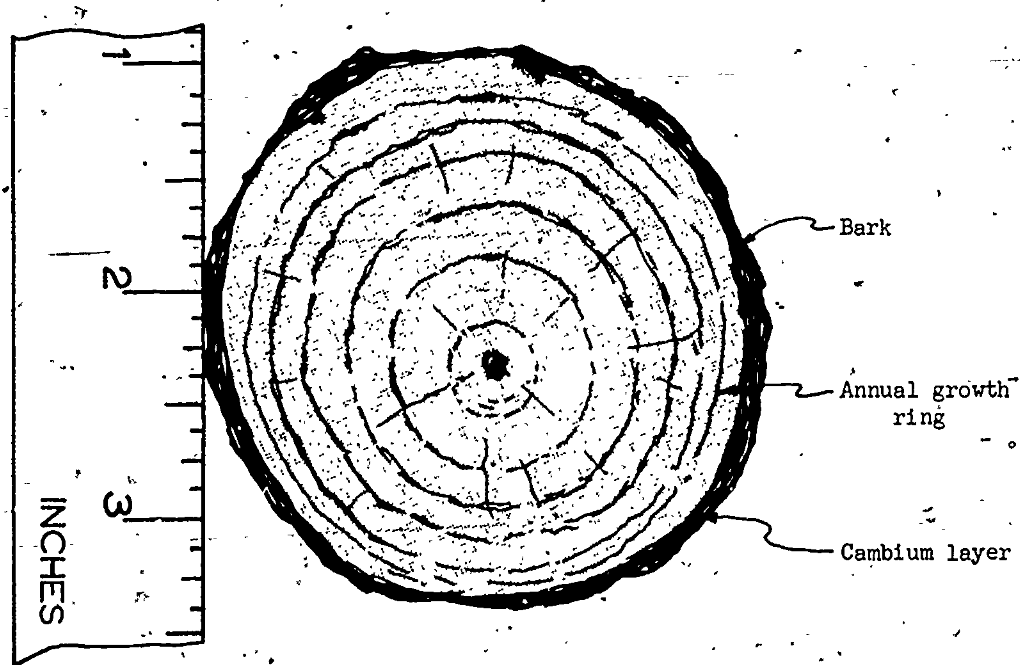
Taxonomists (scientists who decide which organisms are related to each other and how closely) used to call both of these kinds of trees Sequoia. The giants in the Sierras were called Sequoia gigantea and the coast redwoods were given the name Sequoia sempervirens. However, further studies showed that they were not enough alike to be so similarly named, so they left the coast redwood as it was: Sequoia sempervirens, and renamed the giant sequoias: Sequoiadendron giganteum.

The coast redwood grows much taller than the giants but does not reach such an impressive diameter (as you learned as you found how to estimate board feet). See Figure 8-1. The coast redwoods are far more widespread than the giant redwoods. Many coast redwood trees are being "logged out" today. Their lumber, which resists decay, has many important uses in the building industry and in the construction of outdoor furniture. You have all seen redwood picnic tables and chairs.

Not too long ago (during World War II) a species of redwood tree that was known to have existed over 60 million years ago was discovered in a remote area of China. (One scientist said it was like finding a group of dinosaurs--alive!) Scientists are still trying to puzzle out its relationship to the two redwoods found in California. There is strong evidence that it could be the ancestoı of the present coast redwood. Fossil evidence tells us that redwoods were once growing throughout the northern hemisphere, but only these three small groups, the giant sequoias, the coast redwoods, and the dawn redwoods are known to still exist.

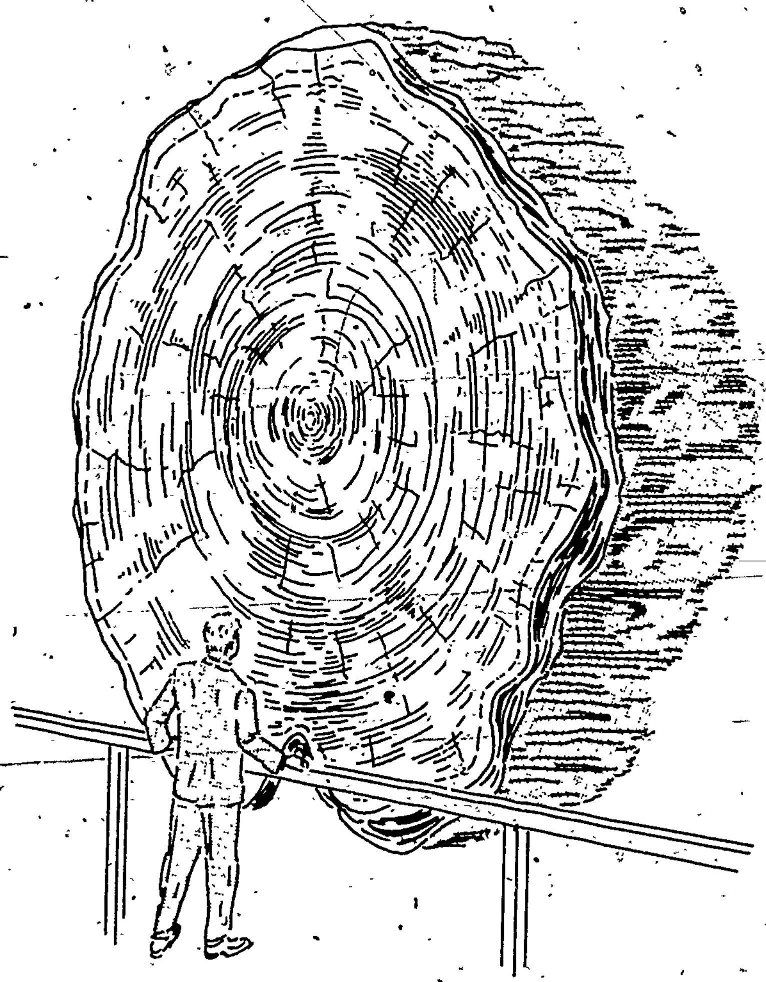
Seeds of the dawn redwood have been brought to the United States and planted in special areas such as college and university campuses, botanical gardens, and national parks. Even some private individuals have been able to obtain some of the seeds and grow them in their own yards. They are very similar to the coast redwood, except they are <u>deciduous</u> (lose their "leaves" in the fall)! The more "modern" redwoods remain green throughout the year. Such trees are called <u>evergreen</u> trees.

How long did it take for these huge trees to grow so big?



Cross section of a tree approximately 7 years old
(drawing magnified to show the growth rings)

Figure 8-9



Cross section of a redwood approximately 2000-3000 years old

Figure 8-9

(drawing greatly reduced in order to show entire diameter)

You are all familiar with "tree rings". As trees grow, they add rings of cells year by year, large cells during the wet season when lots of water is available, and small cells during the dry season. These variations in cell size are easily visible in cut logs as "rings". Many of you have seen slices of logs in national parks or museums. Trees that are still standing can have thin "cores" removed by a boring tool which does not damage the tree. These cores can then be examined for the growth rings, and the age of the tree estimated. The General Sherman Tree is estimated to be between 3,500 and 4,000 years old! Just think, it was at least 1500 years old when Jesus was born and nearly 1,000 years old when Pythagoras was working with triangles! The oldest known coast redwood is just a young thing--only 2200 years old! Incidentally, the truly oldest living thing found so far is a white bark pine, located in the White Mountain Range of southern California. It has been found to be over 4,000 years old!

How tall the redwoods were at the time Pythagoras lived would be difficult to estimate, because they do reach a limit of size forced upon them by the amount of water that can reach the topmost branches. However, the growth of coast redwoods has been measured at 1 foot per year during their younger years. The "tallest tree" previously mentioned has not yet been "cored" to determine its age. It stands in a newly discovered grove (see Section 8-8) which includes several trees nearly as tall, all of which are taller than the former "tallest tree", the Rockefeller Tree.

Here, then, we see further examples of basic biological concepts. There is great variation in height and girth. Diversity is illustrated by the fact that two different species might have originated from one early ancestor. Growth and the accompanying cell activities continue year after year for thousands of years. Food manufacture goes on in the scale-like leaves way up on those high branches and the food is transported down through the hundreds of feet of trunk to keep the roots alive. The roots, in their turn, absorb water which moves up through those hundreds of feet to the leaves. Since the cells are active and dynamic, energy is needed, so respiration takes place in each living cell. (The central part of a tree is composed of dead cells--hence the tree's ability to survive a fire which hollows it out!) These trees are peculiarly adapted to the mountain areas in which they grow by having shallow root systems which absorb surface (rain) water, rather than penetrating deeply in search of underground water.

Scientists, as usual, are still curious about many unanswered questions pertaining to these trees. How does the water get to the top of the tree? No one really knows yet, although several theories have been proposed. Have the redwood trees found the "fountain of youth"? Do they ever die of old age? Hence inquiry is still going on here as it is in all other phases of science.

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