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ABSTRACT

The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, Inc., one of the national educational laboratories, and is funded by the National Institute of Education. Its major purpose is the development of curriculum materials in mathematics for grades K-6. Beginning in September, 1973, CSMP began an extended pilot trial of its Elementary Program. During the second year of the trial, a series of interviews was conducted with 18 second grade students, two each from nine classes. There are four sections to the report. The largest is Section III; four or five pages are devoted to the responses of each student. Also included are sections presenting an overview of the process of selecting and interviewing students, the interview formats, and a summary. (RH)

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tudent Interviews

Extended Pilot Trial of the
Comprehensive School Mathematics Program

Evaluation Report 2-B-3

STUDENT INTERVIEWS

Developed by CEMREL, Inc., a private nonprofit corporation supported in part as an educational laboratory by funds from the National Institute of Education, Department of Health, Education, and Welfare. The opinions expressed in this publication do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement should be inferred.

Description of Evaluation Report Series

The Comprehensive School Mathematics Program (CSMP) is a program of CEMREL, Inc., one of the national educational laboratories, and is funded by the National Institute of Education. Its major purpose is the development of curriculum materials for grades K-6.

Beginning in September, 1973, CSMP began an extended pilot trial of its Elementary Program. The pilot trial is longitudinal in nature; students who began using CSMP materials in kindergarten or first grade in 1973-74, were able to use them in first and second grades respectively in 1974-75, and will be able to use them in second and third grades in 1975-76. Hence the adjective "extended".

The evaluation of the program in this extended pilot trial is intended to be reasonably comprehensive and to supply information desired by a wide variety of audiences. For that reason the reports in this series are reasonably non-technical and do not attempt to widely explore some of the related research issues. The list of reports from the first two years of the extended pilot trial is given on the next page. The most comprehensive of these are the following:

1-A-1: Overview, Design and Instrumentation
1-A-3: Final Summary Report, Year 1
and 2-A-1: Final Summary Report, Year 2

The first of these will be particularly useful to the reader in providing a description of the program, the philosophy and goals of the evaluation and the relationship of individual reports to the evaluation effort as a whole.

Longitudinal Pilot Study of the
Comprehensive School Mathematics Program

Evaluation Report Series

Evaluation Report 1-A-1	Overview, Design and Instrumentation
Evaluation Report 1-A-2	External Review of CSMP Materials
Evaluation Report 1-A-3	Final Summary Report Year 1
Evaluation Report 1-B-1	Mid-Year Test Data: CSMP First Grade Content
Evaluation Report 1-B-2	End-of-Year Test Data: CSMP First Grade Content
Evaluation Report 1-B-3	End-of-Year Test Data: Standard First Grade Content
Evaluation Report 1-B-4	End-of-Year Test Data: CSMP Kindergarten Content
Evaluation Report 1-B-5	Test Data on Some General Cognitive Skills Related to CSMP Content
Evaluation Report 1-B-6	Summary Test Data: Detroit Schools
Evaluation Report 1-C-1	Teacher Training Report
Evaluation Report 1-C-2	Observations of CSMP First Grade Classes
Evaluation Report 1-C-3	Mid-Year Data from Teacher Questionnaires
Evaluation Report 1-C-4	End-of-Year Data from Teacher Questionnaires
Evaluation Report 1-C-5	Interviews with CSMP Kindergarten Teachers
Evaluation Report 1-C-6	Analysis of Teacher Logs
Evaluation Report 2-A-1	Final Summary Report Year 2
Evaluation Report 2-B-1	Second Grade Test Data
Evaluation Report 2-B-2	Readministration of First Grade Test Items
Evaluation Report 2-B-3	Student Interviews
Evaluation Report 2-C-1	Teacher Questionnaire Data
Evaluation Report 2-C-2	Teacher Interviews, Second Grade
Evaluation Report 2-C-3	Teacher Interviews, First Grade

Key to Indexing

1-C-2 Observations of CSMP First Grade Classes

"2" refers simply to the number within a given year and type of data

"C" refers to the type of data being reported
- A: Overview, summary and theoretical reports
- B: Student outcomes
- C: Non-test data

"1" refers to the year of the Pilot Study according to the following:

	Kindergarten	First Grade	Second Grade	Third Grade	. . .
Year 1 (1973-74)	▨	▨			
Year 2 (1974-75)		▨	▨		
Year 3 (1975-76)			▨	▨	

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Introduction

In the fall of 1973, the Comprehensive School Mathematics Program (CSMP) began a longitudinal pilot study of its Elementary Program. Over 100 teachers began using the program, either in kindergarten or first grade. During the next year of the pilot study (1974-75), most of these classes continued into second grade and first grade respectively and many new classes began using CSMP materials.

During this second pilot study year, a series of interviews was conducted with 18 second grade students; two students from each of nine of the 13 second grade CSMP classes in Metropolitan St. Louis. Each student was interviewed three or four times during the second half of the school year. The interviews were designed to investigate students' understanding of and attitude towards the content of the CSMP program. The mathematical content of the interviews was structured prior to each interview and included fundamental ideas, rather typical problems and more extensive and complex situations, though the mix of these three levels of questions varied enoumously from student to student.

There are four sections to this report. By far the largest is Section III, Student Responses, in which about 4 or 5 pages are devoted to the responses of each student. Although occasional attempts are made to summarize or interpret these responses considerable compression has obviously taken place in order for two hours of interviewing to be reduced to four or five pages. Thus it will be necessary for the reader, while reading the responses, to refer back to Section II, Interview Formats, for description of the questions being referred to.

The other two sections are very brief. Section I presents an overview of the process of selecting students and of the content of the interviews. Section IV presents a brief summary, but for this report the reader will get very little from the usual process of reading only the introduction and summary; the reader will have to come to his own conclusions about what it all means and about what issues are important.

I. Description of the Interview Series

Objectives

The basic reason for conducting this series of interviews was a rather vague dissatisfaction with complete reliance on end-of-year test data in assessing the capabilities of second grade CSMP students. This end-of-year testing was quite extensive, including problem solving and applications to novel situations, and over 400 students in both CSMP and Non-CSMP classes participated.* Nevertheless it was felt desirable to know much more about many fewer students and the general aim of the interviews was to investigate how children thought about and understood CSMP math in an informal setting.

Selection of Students

A sample of 18 students was chosen, two from each of nine second grade classes in the St. Louis Metropolitan area. Students were selected in order to obtain a total group representing a wide range of socio-economic level, interest in mathematics, general level of ability, and math achievement. This meant that within any given class the chosen pair of students was not necessarily representative of that class. The previous year's test data, relevant demographic information, and an interview with each teacher regarding the characteristics of students in her class were the sources of data for selection.

Table 1 indicates percentile ranks of the chosen interviewees both within their class and across all local second grade CSMP students on a composite of test scores. This composite was based on scores from the Cognitive Ability Scales, the End-of-Year CSMP test, the End-of-Year Test of Standard Content (tests given in first grade); and the Kuhlman-Anderson Mental Ability Test (given at the beginning of second grade). The information in these tables indicates that the ability and math achievement of these students ranged from very high to very low with representative and proportional sampling from the intermediate levels.

*Evaluation Report 2-B-1: Second Grade Test Data

Table 1

Composite Percentile Ranks of Selected Students:
Within Particular Class and Across All Local Students

Student Number for This Report	Percentile Rank Based on All Local Pilot Students	Percentile Rank Within Class
1	12.5	25
2	16.5	27
3	21.5	12.5
4	27	18
5	34.5	52
6	35	34
7	41	32
8	43	34
9	48	55
10	49	40
11	53.5	56
12	67	58
13	71	58
14	74	43
15	86	93
16	88.5	91
17	93	97
18	94	83
Mean	53.1	50.5

Character of the Interview

Each of the two students chosen from a class was interviewed by an evaluation staff member and this student-interviewer team was maintained for all of the interviews. The interviews were most often conducted either in a small special purpose place where the student and interviewer could work privately or in an empty classroom or cafeteria where the two interviews could take place in separate sections with little interference or distraction. The interviews lasted an average of 30 minutes, the actual length depending on the topics being covered and by the level of student interest.

Four interviews were planned for each student. Interviews 1 and 2 were designed to survey students' knowledge and skills in CSMP mathematics. A detailed interview plan was developed from lessons and workbooks through mid-January and this plan was followed fairly closely for all students. Thus Interviews 1 and 2 were rather test-like in nature and were to provide something like a status report for each student. The interviewers were free, however, to omit questions that they were judged unnecessary and to add questions for clarification and further probing of a student's understanding.

Interview 3 was a rather individualized follow-up and extension of information obtained from the first interview. Three general lines were followed: i) a series of questions dealing with elementary concepts was prepared in order to probe the reasons for difficulties encountered in the previous interview ii) items from the first interview were repeated. These were items on which a student had previous difficulty but might be expected to have improved, particularly if instruction on that topic had occurred during the intervening time iii) a series of problem situations dealing with applications of CSMP content was prepared for use with those students who had demonstrated a sound understanding of the content from the previous interviews. For any particular student, the interviewer, based on the first two interviews with that student, determined which areas from these three lines of inquiry would be followed. Thus it might easily be the case that two students would be asked completely non-overlapping sets of questions. This interview took place in March or April.

Interview 4 was of a summative and attitudinal nature. Key topics for each student were pinpointed. This was followed by a series of questions related to the student's attitudes towards CSMP in general and towards materials and techniques specific to the program. This interview took place in May.

While the interview was proceeding the interviewer had available an Interview Record Form, one of which was designed for each interview for shorthand coding of the student's responses. The interviews were tape recorded and after each one the interviewer prepared an Interview Summary based on the Interview Record Form, the work left behind by the student, and the tape recording.

Summary of Topics Covered

A number of topics were selected to be covered in varying degrees of thoroughness in the series of four interviews. Some were considered only once, but for most, the progress of the students was traced by following up the questions of one interview that might have uncovered weaknesses or strengths with related questions in subsequent interviews. The general topics of the interviews were:

a) the Minicomputer: the mechanics of using it; types of problems for which the student chose to use it.
b) computational skills: the ability to solve addition, subtraction, and multiplication problems; approaches used in solving problems.
c) fractions (rational numbers): finding fractional parts of objects and numbers; the mechanisms used to do so.
d) arrow diagrams: constructing and interpreting diagrams; use of information contained in and inferred from complex diagrams to assist in the solution of problems.

e) Venn diagrams: interpretation of diagrams containing three intersecting sets.
f) integers: the relative size of positive and negative integers; addition and subtraction involving a mixture of positive and negative integers.
g) equations: when two missing numbers are represented by the same or different shapes; calculations which take into account parentheses in an equation.
h) combinatorics: possible combinations of items of varying cost that can be bought for a specified amount of money.

II. Interview Formats

In this section are given the formats used in the four interviews. While they may seem rather lengthy, they are in fact somewhat abbreviated from those actually used in the interview. There were many "gestures", "pauses" and "points" in the originals and many backup questions and hints to guide the interviewer. Problems and pictures were shown on previously prepared cards and students always had a Minicomputer and paper and pencil handy. Much use was made of color, in the differentiation of arrows and strings for example, some of which is reproduced in this chapter.

On the left hand pages of this chapter the actual questions are given. Various contingencies might be followed at times, depending on a student's previous responses; some students were asked quite different questions than others. The right hand pages give explanations and further descriptions of some of the questions on the facing page. Note that Interview 3 is quite long, consisting of three parts, the third of which is further subdivided into problem sets a), b) and c).

Interview 1

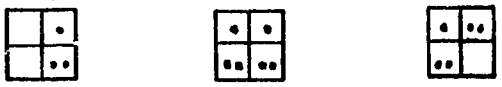
1. What number does the Minicomputer show? 

2. Show 864 on the Minicomputer. Show 701. Show 214 using only the white squares.

3. Now use your MC to add 37+16.* Then 124+398.* Add 63+28 without using the MC.*

4. Now use your Minicomputer to subtract 61-18.* Subtract 86-39 without using the MC.*

5. Now look at the Minicomputer: (Show it as arranged below.)

 (14 checkers)

Now I'm going to move some checkers and you tell me whether I've made the number on the Minicomputer larger or smaller or whether it's still the same. (Always return checkers to original position before starting the next question. When a correct answer is given ask "How much larger?" or "How much smaller?")

i) Remove two checkers on 20's square and put one of them on 40's square
ii) Move checker on 80's square to 40's square
iii) Move checker on 2's square to 20's square
iv) Remove checker on 4's square and put it and a new checker on 2's square
v) Move checker on 100's square to 200's square
vi) Move checker on 10's square to 1's square

6. Now you find out what number the Minicomputer shows. (If student doesn't know what to do, say "Make some plays." Help and correct him if necessary to arrive at 800.)

7. Show me some backward plays. (Have student make four backward plays.)
Now what number does the Minicomputer show? Can you tell me without figuring it out?

8. What does this say? $\frac{1}{2}x32$

What do you think the answer is? (Pause.) About what? How did you get that?
Could you figure out $\frac{1}{2}$ of 32 in a different way?

(If student has not already) Use your Minicomputer. Use a string picture.
(Assist, if necessary, to get a completed string picture and write "$\frac{1}{2}x32=16$" under it.)

Write another number story for this (point to string picture).
(If necessary) "Use a plus equation." or "Write a two-times equation."

(If student has been unsuccessful to this point) "What is $\frac{1}{2}$ of 6?"
"What does $\frac{1}{2}$ of something mean?"

*Have student first make an estimate. "What do you think the answer will be? About what?"

Interview 1 - Commentary

This interview dealt almost entirely with the Minicomputer. The first four questions were fairly straightforward:

Item 1: Determining what number is shown on a Minicomputer. Note that the third example is shown in what might be called "abacus notation" (ie. three 100's, two 10's and four 1's - 324)
Item 2: Showing numbers on a Minicomputer.
Item 3: Addition, with carrying, on the Minicomputer.
Item 4: Subtraction, with borrowing, on the Minicomputer. Note that students were first asked to estimate or guess what they thought the answer would be, before proceeding with the problem. Note also that students were also asked to do an addition and subtraction problem without using the Minicomputer.

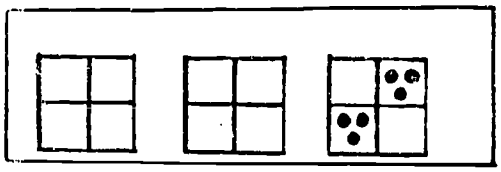
Items 5-7 probed more subtle understandings of the Minicomputer.

Item 5: Students were shown a configuration on the Minicomputer that was complicated enough that the student could not easily "read off" what number was shown. Then this display was changed in one of three ways:
i) one and only one of the checkers was moved from its original square to a higher-valued square or board,
ii) one and only one of the checkers was moved from its original square to a lower-valued square or board, or
iii) a "play" was made - a legitimate two-for-one trade which did not change the total amount on the Minicomputer (which remained unknown to the student).
The student was asked after each change whether the number on the Minicomputer was larger, smaller or the same as it had been before the change. These questions were called "Conservation on the Minicomputer" and required a rather sophisticated (for second graders) understanding of the Minicomputer as a number-storing device where, for example, $2a+2b+c+2d+e > 2a+2b+C+2d+e$ $(C>c)$.
Item 6: Students were asked to find out what was on the Minicomputer. While this could be determined by adding up the values of the individual checkers, it was too difficult to do mentally. Thus a) Did the student know enough to begin reducing the configuration to a readable one by making two-for-one forward plays? And if so, b) could he make these plays in the appropriate fashion?
Item 7: This was intended to determine a) whether or not the student knew how to make backward plays and b) whether he "conserved" when doing so. (Was he aware that making backwards plays did not change the number shown on the Minicomputer; only its representation?)

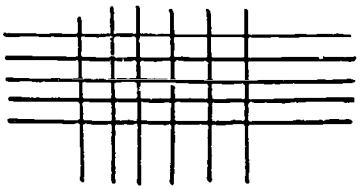
Item 8: The question probed various concepts and representations of $\frac{1}{2}x$:
i) by showing a Minicomputer representation consisting of pairs of checkers - one from each pair then yields one-half of original number;
ii) by showing a string picture - two strings with equal numbers of dots in each with the combined number of dots equal to the original number; and
iii) by showing equations - $\frac{1}{2}x32=16$, $16+16=32$, $2x16=32$.

Interview 2

1. What is 2x4? 3x3? What is 4x9?
   Could you figure it out a different way?
   (If not already used) Use your Minicomputer.
   (If not already used) Use crossing points.
   (If necessary) What multiplication idea does this show?

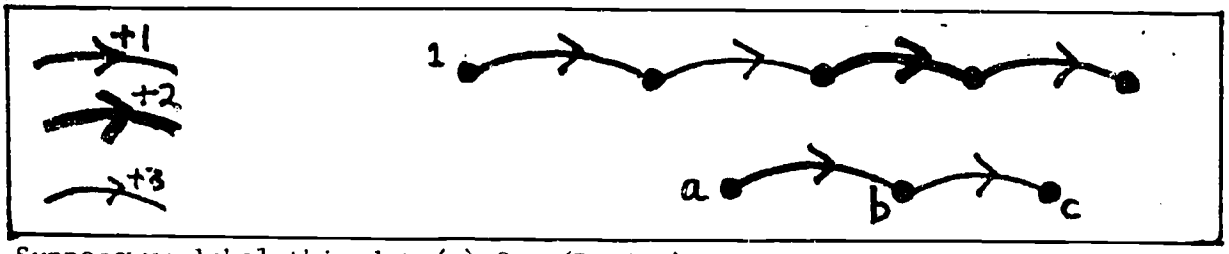


and this?



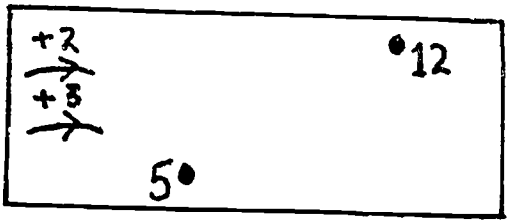
What addition story does this show? 4x9=36

2. Label as many dots as you can: (Note - labels a,b,c have been added for the reader's convenience.)

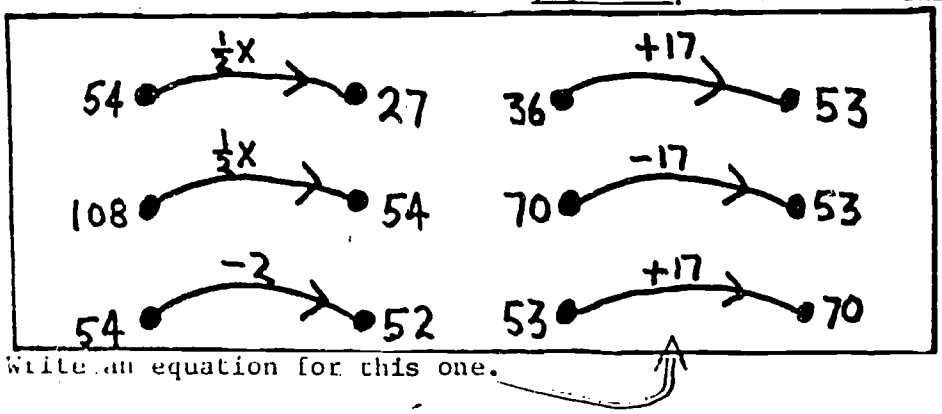


Suppose we label this dot (a) 9. (Do it.) Draw any more arrows that you can.
Label this dot (b). Draw any more arrows that you can.
Label this dot (c).

3. Build a road from 5 to 12 with +3 and +2 arrows:

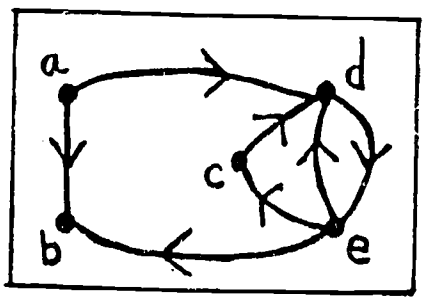


4. Which arrow tells what this is? 53-17= . How about this? 2x54=



Write an equation for this one.

5. The arrows are for "I give you a candy". The dots are for children. (Note: labels a,b,c,d,e have been added.)
   Which child will get the most candies? The fewest?
   Who will give out the most?



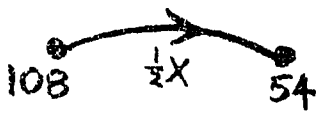
Interview 2 - Commentary (Items 1-6)

1. What is the student's understanding of multiplication and can he calculate easy multiplication facts? Can he represent the multiplication process on a Minicomputer? with crossing pictures? with repeated addition - string pictures or addition equation?

2. There are essentially two parts to this question:
   a) Labelling the four dots after 1, which requires that the student note the color of the arrow and hence its meaning, and then make the appropriate calculation, and
   b) After labelling "a" as 9 , the student must determine whether or not 9 is related to any of the other dots by +1, +2 or +3. This is clearly more difficult since the student must "check" each pair and see whether that "check" matches any of the three given relationships - potentially 3x5=15 comparisons.

3. This question dealt with the concept of a road; the calculations were intentionally simple.

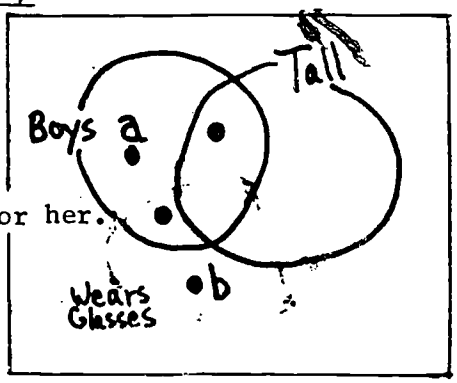
4. To do this question one had to recognize that the (undrawn) return arrow, for one of the six diagrams, would yield the required calculation. Presumably for some students this process is internalized - the student knows without even picturing a return arrow, that 108 is 2x54 from



5. This is a non-numerical use of arrows in that neither the arrows nor the dots deals with numbers. To answer the questions the student needs to know which dot is "pointed" to the most, the least, and which "points" the most.

Interview 2 (continued)

6. The red string is for "boys", the green is for "tall" and the yellow string is for "wears glasses". (Note - labels a,b have been added)

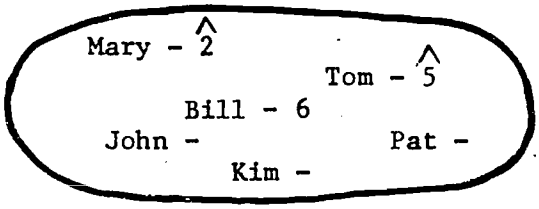


i) Is this (dot "a") a boy? tall? wears glasses?
ii) Mary is a tall girl who wears glasses. Put a dot for her.
iii) What can you tell me about this one (dot "b")
iv) How many boys are shown?

7. $\hat{33}$, 13, 0, $\hat{6}$, 27

Which is the largest?
Which is the smallest?
What do you call this (point to $\hat{6}$)?

8. Here are some scores from a game.



Who had the highest score?
How many more would Mary need to have the same score as Bill?
How much higher was Mary's score than Tom's?

9. What does this equal?

$4 + \hat{7}$

10. What numbers will make this number story true?

$\triangle + \square = 8$

Anything else? (And so on. Have students write their answers.)
Are these (point to symmetric responses, like 6,2 and 2,6)

11. Which will make this number story true? Why?

$\square + \square = 10$

(After response of 5,5) Is there anything else that will make it true?

Interview 2 - Commentary (continued)

Question 6 was a straightforward application of a 3-string Venn diagram. The analytical application requires a description of an entity represented by a dot (which strings is it in and which properties it has). This can be done one step at a time. The more difficult problem is to determine where in the diagram a known entity should be represented, and this requires a synthesis of the given information which cannot be easily done one step at a time.

Questions 7-9 dealt with integers. In question 7, a response of $\hat{3}3$ for the largest and $\hat{0}$ for the smallest would indicate non-recognition of the "^", while a response of $\hat{6}$ for the smallest would indicate an order reversal in the negatives. The other two questions concerned operating on the integers; explicit addition in question 9 and implied subtraction in question 8.

Questions 10 and 11 concerned open sentences and asked the student to produce numbers which would satisfy given equations - "make this number story true".

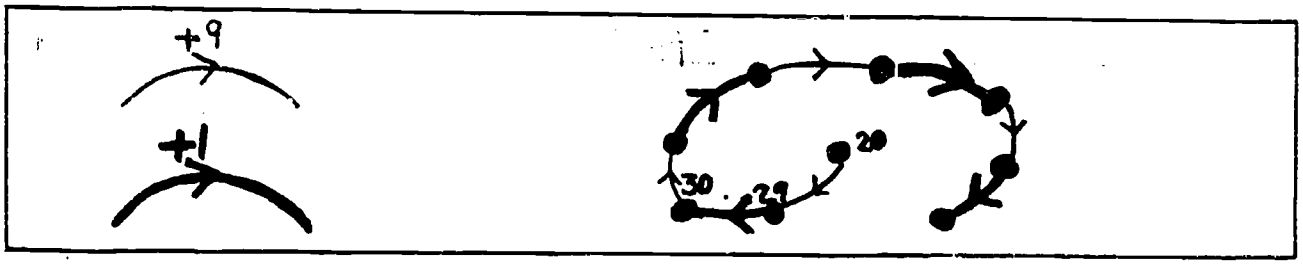
Interview 3, Part 1

1) Read the numbers on this card? Which number is the largest? Which is next largest? (Etc.)

74,	619,	38,	704,	83,	29,	562,	47

2) What number is one more than 57? What comes next? (Etc. to 61)
What number is two more than 75? And what is two more than (etc. to 81)?
What number is one less than 62? And then (etc. to 59)?
What number is two less than 33? (Etc. to 29)

3) Here is an arrow diagram.



What is this arrow (from 20 to 29) for? And this one (next arrow)?
What would this dot (30+9) be for? (Etc. for 40, 49, 50)
Suppose I drew a blue arrow from here to here (from the third last dot to the last dot).
What could the blue arrow be for?

4) What is 7+2? 4+3? (If successful): 9+6? 8+9? (If necessary): Show me how to do it.

5) How would you do this calculation? 36+52 (If successful): How about this?

$$\begin{array}{r} 22 \\ +19 \\ \hline \end{array}$$

6) What is 8-3? 9-7? (If successful): 15-8? 61-3? (If necessary): Show me how to do it.

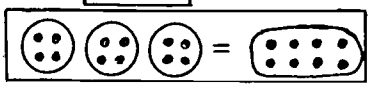
7a) Jim gets four pieces of candy and Bob gets two pieces. How many do they get altogether?

b) There are four boys playing together and they each have two pieces of gum. How many do they have altogether? (If necessary): How could we figure out how many they had altogether?

c) Suppose there were four boys playing and they each had six pieces of gum. How many do they have altogether? (If necessary): How could we figure out how many they had altogether?

d) What does this say? 5x3 What is the answer to the calculation?

e) Can you make a story to go with this? 3x4=12
(If necessary): How about this?



Interview 3 - Commentary

As noted previously, this interview consisted of three parts:
Part 1: Review of elementary concepts.
Part 2: Repetition of previous questions,
Part 3: Problem situations.

Furthermore, Part 3 was further subdivided into three sections:
Section a) Combinatorics
Section b) Arrow Diagrams
Section c) Integers

The extent to which a student covered the questions in any Part was determined by his success with the various questions in Interviews 1 and 2.

Interview 3, Part 1 - Commentary

The elementary concepts reviewed in Part 1 could, for the most part, be classified as numeration (1,2), arrow diagrams (3,10), basic computation (4-6, 8), and processes of multiplication and division (7,0). It was expected that only those few students who, in the first two interviews had what might be called fundamental problems, would be taken through these questions.

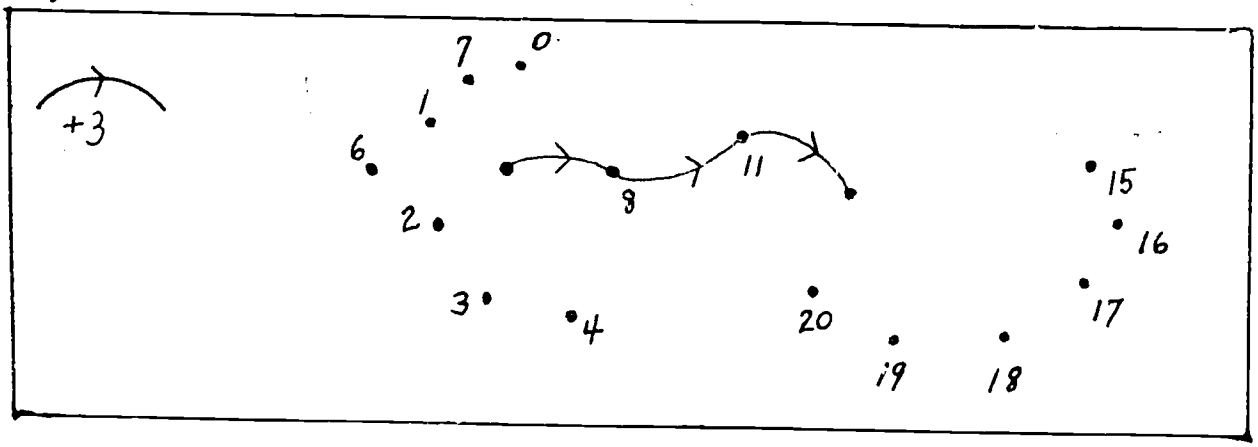
Particularly with questions 7 and 8, but for all other items as well, students were encouraged to explain their answers, to draw pictures, and to try things again if they didn't get them the first time. They were given assistance when necessary. In this way the interview was treated as a diagnostic session which might pinpoint the reasons for the difficulties encountered by these students.

Interview 3, Part 1 (continued)

8) Suppose I have eight pennies and want to divide them equally among four girls so that they each get an equal amount. How much would each get?
(If necessary, put eight pennies on table, let student use them.)
(If successful, try 20÷4 story. If unsuccessful, try 6÷3 story.)

9) This is called a rectangle. Let me see you shade 1/3 of it.

10) Label these dots (point to the two unlabelled dots).
Can you draw more arrows?



Interview 3, Part 2

1. Repetition of selected questions from Interviews 1 and 2.

2. Calculations

   a) 23+45, 73+49

   b) 78-43, 56-39

   c) $\frac{1}{2}$x12, $\frac{1}{3}$x18, $\frac{1}{2}$x62, $\frac{1}{2}$x74

   d) 15-(4+2), 6+(2x8)

Interview 3, Part 2 - Commentary

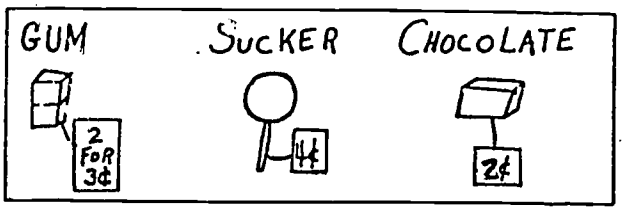
In this part the interviewer was to follow up on topics from Interviews 1 and 2 which were considered questionable for the student or to which inconsistent responses were given. That is, the student was somewhere between having virtually no knowledge or skill (in which case one would go back to Part 1) and having a rather sure knowledge of the concept or mastery of the skill (in which case one would go on to the problem situations of Part 3).

The calculations listed in question 2 were like those in which instruction and practice were given to the students during the interval between the second and third interviews. For the first three kinds of calculations, the interviewer was to note the preferred method of solution: mentally, Minicomputer, or paper and pencil (which might involve dots and strings for 2c).

Interview 3, Part 3

Section a)

I have a story here about a very lucky boy (or girl). He/she was walking down the street one day and found 10¢ lying on the ground. He/she decided to go to the candy store and spend all of the 10¢ on candy. Here is a picture of the candy the store sold.



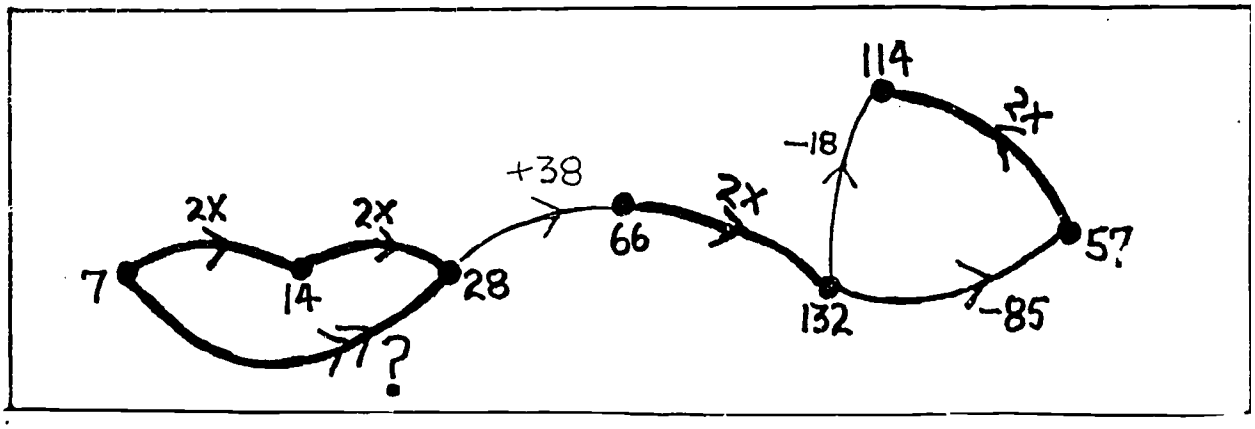
a) If you were that lucky boy (girl) what would you buy? Remember you have to spend all 10¢. (Pause.) You can use this paper (blank) to help you if you want. (Allow lots of time for student to consider the problem. Then after a solution, however arrived at and with whatever assistance): Is there any other way you could spend the 10¢ on the candy shown in the picture? (Repeat until student or combinations are exhausted.)

b) (If no response): Suppose he/she bought 2 suckers. What else could he/she buy?

c) (If still no response draw and explain arrow diagram): Can you draw another road from 0 to 10?

d) If student gets 4 or 5 combinations quickly, ask him to use arrow diagrams to spend exactly 15¢ and have
   (1) exactly 3 pieces of chocolate (2 ways)
   (2) exactly 2 suckers (1 way)
   (3) no suckers (3 ways possible)

Section b)



Here is a very complicated arrow diagram. Let's see if we can use it for some calculations. You look at this carefully and see if you can use it to figure out:

a) 2×66? (If incorrect or no response, assist student)
b) 28+38?
c) 114+18? (If necessary, draw a return arrow) What could this be for?
d) $\frac{1}{2}$ x114? (If necessary, draw a return arrow) What could this be for?
e) 57+85?
f) Draw a $\frac{1}{2}$× arrow.
g) What could this ( ) arrow be for? (If "+21") What else?
h) (If g) correct): Could you use the diagram to find $\frac{1}{4}$x28?

Interview 3, Part 3 - Commentary

Part 3 was intended for students who had shown a good grasp of content covered in the first two interviews, though computational facility was not necessary. There were three separate sections and in each one the student was shown a "situation" about which many interesting questions could be asked. The questions which are listed were not necessarily followed as shown - the questions tended to get much more difficult towards the end - and the interviewers used their discretion in determining when to stop (if too difficult) or when to pursue an entirely different line of questioning.

Section a)

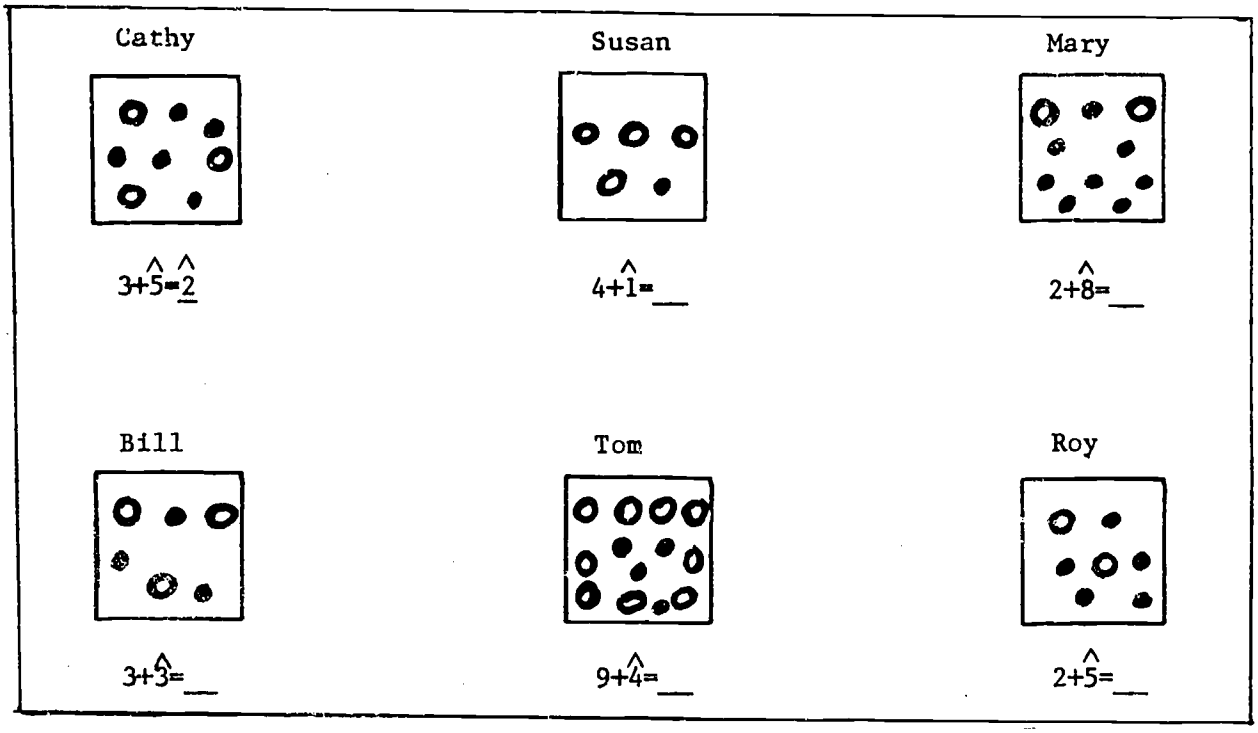
This was a situation in which, essentially, combinations of numbers were to be found which added to a certain amount. More interesting than how many questions the student was able to get correct (which depended on which particular questions the interviewer might ask), was the way in which the student went about investigating the problem. Did he build a road with arrows, for example, or have a way of recording each different combination in question a)?

Section b)

The arrow diagram used in this section was extremely complicated. This situation was similar to what are called "Detective Stories" in the CSMP materials. The first few questions involve rather straightforward (once the information is located!) reading of the diagram. Questions then become increasingly difficult requiring the construction, mentally or physically, of return arrows and composite arrows. Considerable pointing and tracing occurred in this section and some long intervals of time were needed for "searching" activities.

Section c)

Some boys and girls were playing a game with a basketball. Each person shot the basketball as many times as they could in one minute. Here is a picture of their score cards. (Note: originally in color; ○=red, ●=blue)



Each time the ball went in the basket they put a red dot on their score sheet and each time the ball didn't go in the basket they put a blue dot on their score sheet. The number sentences below their score sheets show how they figured out what their score was. Cathy made three baskets and missed five times and her score was negative two.

a) What were the rest of the scores?
b) Which person had the best score? Who was next? (etc.)
c) If Cathy took some more shots, and made a basket with each shot, how many more shots would she need to have the same score as Susan.
d) What is the difference between Tom's score and Mary's score?
e) Which player took the most shots? Which player took the fewest shots?
f) If they had to pick a player to shoot just one shot, who do you think would have the best chance of making a basket with the shot?
g) Suppose in this game the boys had been playing against the girls. Which team won? (If necessary): How could we figure out what the score for the boys' team was?
h) A new player, Bob, took 10 shots at the basket and his score was negative 2. How many baskets did he make out of his 10 shots? (If necessary): Draw in the red and blue dots on this score card to show how he did.
i) Altogether, Bob did this 6 times, and each time his score was negative 2. If he kept adding dots to his score card each time, what would his score be altogether after the 6 times?

Interview 3, Part 3 - Commentary (continued)

Section c)

This situation was one involving the use of positive and negative integers. Questions a) through e) were concerned with reading the data and making calculations with it. Questions f) through i) were applications and extensions of these ideas and brought out very sophisticated concepts:

f) This asks the question "Which child has the highest probability of making a shot?" Based on available information, the answer is not the child with the highest score but rather the child with the highest ratio of hits to misses.

g) This question can be answered by several methods: adding together the three scores for each team, adding together the hits and the misses across team members and then calculating the sum or by a cross-team member cancelling process.

h) Algebraicly this is the simultaneous solution to

$$\left.\begin{array}{l} x+y=10 \\ x-y=-2 \end{array}\right\}$$ where x = number of hits, y = number of misses

i) This is the aggregation of six scores of $\hat{2}$, ie. $6\hat{x}2$ or $\hat{2}+\hat{2}+\hat{2}+\hat{2}+\hat{2}+\hat{2}$.

Interview 4

A. Review individual file and repeat of follow-up according to previous responses.

B. Attitudes towards CSMP (possible questions).

1. Do you do well in school? What do you do best in? How good are you in math? Do you think you are learning a lot in math? Is it easy or hard for you? Why?

2. Is this math different than what kids in some other classes do? How is it different? Do you think it is better than their math or not? Is it as much fun or more fun? Do you get any chances in school, besides math class, to do math?

3. What kinds of things do you like to do in school? (If necessary): What about math? What sorts of things do you like about math class? Do you like doing workbooks? Would you rather work on your own with a workbook or with the rest of the class on a regular lesson? Why?

4. Do you ever take things (Minicomputer, workbook, games, etc.,) home with you? Do you show them to your parents (brother, sister, friends)? What do they think of them? Is it hard to explain how your math works?

5. Do you like using the Minicomputer? Why (or why not)? What kind of calculations can you use it for? Can you do those (just named or, for example, 73+48 or 81-63) better with the Minicomputer or with just a pencil and paper?

   (Find-out a) reliance on it
   or b) do only harder problems with it
   or c) not use it at all.)

C. Sample materials.

I have a whole bunch of math things here. Look at this page and tell me what you think of the problems. Do you like doing this kind? Are they easy or hard? How about these here? (Leaf through file of Workbooks 4 and 5.)

Interview 4 - Commentary

This interview was intended to serve two purposes:

a) To follow up topics and content which the interviewer felt should be probed further (to clear up uncertainties for example) or should be repeated (to check for student learning during the intervening time). This aspect was therefore completely individualized and might consume much or little time.

b) To try to find out how the student felt about various aspects of CSMP. The questions given in B. were intended to be illustrative and to be samples from which the interviewer could choose. But in the main, this part of the interview was to be open-ended and to attend to what seemed to be important to the student. The last set of questions served to focus this attitudinal aspect of the interview on specific mathematics activities. This greater specificity would more easily allow the student to say "I like (don't like) this activity because..."

III. Interview Responses

The students have been numbered from 1 to 18 according to their composite test scores, with student 1 being the lowest of the eighteen and student number 18 the highest. The interview responses are presented on a student-by-student basis; interviews 1-4 for student 1, then interviews 1-4 for student 2, etc. These responses were written up within a day of each interview based on shorthand notes, audio tapes and the paper and pencil work done by the students.

Student 1

A. Teacher's Views

Student is the second youngest in a family of about 11 children. He gets along rather well with his classmates.

He likes school and stays busy during free time with puzzles or dot-to-dot pictures. He has a rather good attendance record but he rides the bus, and if he misses it, he has no way to get to school.

He is actually repeating most of the first grade subject matter but the teacher decided to have him continue with the second graders in math.

B. Mean percentile rank of tests through November = 12.5

ent 1, Interview Summary #1

Behavior and Appearance

Student appeared to be nervous at the beginning; he did not respond to any questions asked of him while walking to interview room. He was very attentive throughout the interview. At times during the interview, facial expressions seemed to indicate panic when he did not have any idea of an answer.

Summary of Responses

He could not read numbers on the MC with any consistency. He tried to add the numbers one board at a time but did not remember what he'd done at the end. He can not mentally put numbers together; i.e., 300+20+4 is as far as he can get. When reading the numbers on MC he has a habit of reading the 4's (40's or 400's) square as 8 (80 or 800). The closest he came to reading a number correctly was 324 shown on white squares as 300+20+1+1+1+1. He did not have any problem in putting the three numbers on the MC.

He could not add numbers on the MC. He can make 2+2=4, 4+4=8, and 1+1=2 moves but didn't have any idea of an 8+2=10 move. With the first problem he put both numbers on MC correctly but did not make correct moves and could not read his answer. In the second problem he did not put the numbers on MC correctly and could not make correct moves nor read his answer.

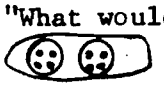
He did the first addition problem 63+28 by writing the problem down horizontally and got an answer of 99. I gave him the problem 47 for which his answer was 71. He added the ten's column first and then +35
added 7+5=11 but didn't carry.

He couldn't subtract with or without the MC. When using the MC he put the numbers 61 and 18 on the MC using the same colored checkers. He made backward moves (incorrectly) and did not take any checkers off the board. He could not read the number on the MC. As in the addition problem, he did the subtraction problem on paper by writing 86-39=11. I gave him 78 for which he got an answer of 33.
-35

His estimates were nowhere near the correct answer. When I made moves on the MC he correctly identified the moves which made the number smaller and larger but did not want to say by how much. For the two legitimate moves he said the number was larger each time.

He tried to figure out the number in his head and finally tried to make moves when I asked if he could make plays. He tried to make all plays by getting groups of two checkers, when he got to 8+8 he said "8+8=10" and made a carry move. Finally got the MC with one checker on several squares and said answer was 800 (MC did not show 800)

I put 800 on MC using one checker and asked him to make backward moves which he did. He picked up a checker from desk in one hand and checker on 800 square in the other hand and said "800+800=400", put the checker from desk on 400 square and checker from 800 square on the desk. He did exactly the same thing for 400+400=200, 200+200=100 100+100=80. "What number is on MC" - his reply was 80, which was correct for his moves.

Could not read $\frac{1}{2}$x32 correctly. When asked "What would the answer be" he guessed at 99. "What would $\frac{1}{2}$ of 6 be" - his reply "12". I gave him the following string diagram  and asked for a number sentence and he gave 4+4=8. For another

5+3=8. Any others, using multiplication? No.

"What is $\frac{1}{2}$ of 4?" - reply - "8."
"How did you find that?" - reply - no response.
"Could you use your Minicomputer to find that?" - reply - "No."

Student 1, Interview Summary #2

Behavior and Appearance

Student appeared to be a little bit more at ease, although I still feel he is anxious during interview.

Summary of Responses

He does not have a firm grip on the concept of multiplication, 2x4="6", 3x3="6", 4x9="10". When asked how he figured 4x9=10 - "guessed". When given 4x3=__, he tried to figure out by counting on fingers 4 groups of 3, but counted a couple extra fingers. He couldn't come up with any other way of finding the answer, although he said "take away", but could not show me what he meant. When asked to do it (4x3) on MC, he put a checker on the 1's, 2's and 40's square and proceeded to make some plays, taking a checker from the table and the checker from the 1's board while saying "1+1=2", he put the checker from the table on the 2's square and put the checker that was on the 1's square on the table. He repeated this pattern for the rest of the moves, "4+4=8", "8+8=10", "10+10=20", "20+20=80", 80+80=40". When moving checkers from the 20 to the 40's square, he calls the 40's square 80 and the 80's square 40. He did make a correct play 2+2=4.

I gave him a crossing point diagram of 3x2 and asked for a number story, and he said "3+2=6". I asked "Do you mean plus or times" and he said "times". I gave him a 5x5 crossing point diagram and asked for a number story and got "3+3+3=9". Asked for another number story using multiplication and got "6+3=9".

Student 1

He started labeling the dots 1, 2, 3, and I asked "What does the green arrow stand for?" and he said +3 and went back and correctly labeled the dots. I labeled the 9 dot and asked for other arrows. I gave him an example and drew a +1 arrow from 9 to 10. He could not draw any more. He did label the 8 dot correctly but started to draw a +3 arrow from the last unnamed dot to a dot he drew to the left of the diagram. He could not correctly draw any arrows connecting the two strings of dots. He drew a +3 arrow from 8 to 7. He could not label the remaining dot and I drew a return arrow from 8 to 5 and asked what it would be. He had no idea. I wrote -3 and asked "What would the dot be?" He said 6. I said "No, 5." and labelled the dot. The look on his face seemed to indicate he did not believe me when I said 5+3=8. He did not see anything wrong with the diagram.

He did build a road from 5 to 12, using +2,+2,+1,+1 arrows. He did not put a dot for 12 at the beginning but started to draw the arrows and when he reached 10 asked where he was going.

He did not seem to understand the arrow diagrams. To solve 53-17 he pointed to 10 and 53 and said the answer was 73. He chose the correct arrow for 2x54 but said the answer was 54. I did not ask for a number story for 53+17=70. I asked him what 2x3 was and he said "10".

He correctly identified which child got the most candies, but said he received 2 pieces. He said (e) got the fewest and said he got 1 piece (1 piece is correct). (a) gave out the most pieces of candy, "7" (he counted all the arrows). When asked about the arrows going to and coming from (c) he correctly said, "(c) said you take my candy to (d)," but he said the same thing about (e) saying that to (e).

He correctly identified the first dot shown as a boy, not tall, not wearing glasses. He placed Mary in just the wearing glasses category. He correctly described two other dots pointed out to him. When asked to place a dot for I. he asked what the strings stood for, (he had forgotten and could not read). He correctly placed a dot for himself. He said there was only one boy shown in the strings.

Student 1, Interview Summary 3

Behavior and Appearance

Student was very quiet. He does not volunteer any information. He still has that panic look on his face when a difficult question (for him) is asked.

Summary of Responses

He could not read a series of numbers correctly. The four he tried were wrong but did not make any consistent errors. (He read 619 and 704 as 74.) He started correctly counting by 1's - 57,58,59,56,49. When asked what's 2 more than 75, he replied 96 and didn't know what 1 less than 62 was.

He correctly identified the +9 and +1 arrows but could not label the dots. He labeled the dots as follows 98,99,100,300, without any apparent regard to the color of the arrows.

All the simple addition and subtraction was done by counting on fingers. The only addition problem he got wrong involved carrying, for which he added the tens column first, then the ones and ended with 312 but could not read the answer. He didn't know 61-3 and I didn't ask 15-8.

Student 1

I don't think he understood story problems as he answered (after thinking about them for a while) 2 (for 4+2) and 2 (for 4x2). He did show correctly how to find 4x6 by drawing 4 loops and putting 6 dots in each (one dot per loop at a time).

He did not know what the x sign was. I said "times" and he said "5 times 3". His story for 3x4=12 was "Jimmy has 3 popsicles, Bob has 4 suckers".

He said he didn't know what $\frac{1}{3}$ was so I asked him to shade $\frac{1}{2}$ of the rectangle which he did fairly accurately but didn't know of any way to check his work.

He said "Give 1 penny to each girl" for 8÷4 story. When I put 6 pennies on the desk he correctly moved them into 3 groups of 2.

When asked to draw all the +3 arrows he quickly drew an arrow from 1 to the unlabeled 5 dot. When asked what the end dot was, he labeled it correctly as 14 and then drew an arrow from 14 to 15 and 15 to 16. I stopped him and asked him if that was correct, what is 15+3. He said 18 and drew an arrow from 15 to 18. He looked around and said there weren't any more arrows he could draw.

I asked him to read a series of numbers on the MC. He could read most of the numbers with one checker on each board but could not read any number with more than one checker on a board (ie. 42, 22, yes.- 62, no)

Uncertainties: He can not read numbers and thus I am not sure if he has idea of order. Not sure about concept of multiplication.

Follow-Up: He does not have basic concept of number. He can not add or subtract with any 2 digit numbers involving carrying or borrowing. All problems must involve small numbers. Arrow diagrams seem to help him although numbers must be small. He cannot read numbers on MC.

The interview took place in an empty study room.

Student 1, Interview Summary #4

Student still can't read numbers on the MC if there is more than one checker on the 10's board. He correctly put 38+26 on the MC but could not make any correct plays. On paper he added 54+25=79 and 37+45=83. He correctly carried but made the mistake of 7+5=13. For 61-18 he got an answer of 57. $\frac{1}{2}$ x6="23" and 3x4= "24". On the 3x4 problem he took 4 fingers and counted them 3 times and then took 3 fingers and counted them 4 times. He can not use an arrow diagram to find 3x5 so I did not attempt to have him use return arrows to solve a problem. He knows positive numbers and zero are larger than negative numbers but does not have the order of these negatives yet.

He was not very verbal in answering any of the attitudinal questions. He doesn't (or didn't want to answer) know how well he's doing in school. He says he likes math best and he likes to do addition problems (pluses). He doesn't feel the math he does is any different than what other kids do. He does take home some work but doesn't go over them with his parents. He prefers to work on the workbooks alone. He also prefers to do the addition problems on paper without the MC.

He says he likes most of the work samples shown in the workbooks.

4RI	pg. 1	
4RII	pg. 6	$\frac{1}{2}$ of a number using strings - like - hard
4RIII	pg. 1	matching points in Venn Diagram - like - easy
	pg. 3	roads - like - hard
	pg. 13	secret messages - likes best - easy
5RI	pg. 5	using $\in$ and $\notin$ doesn't like
5RII	pg. 2	subtract with MC - doesn't like
	pg. 5	multiplication with MC - doesn't like
5RIII	pg. 1	addition table - doesn't like

At the end of the interview he asked if he could have one of the workbooks to take home and work on.

Student 2

A. Teacher's Views

Student comes from a large family with 8-10 children. He loves to come up and talk to teacher and to the class. He is especially interested in the Boy Scouts.

Student is pretty well behaved in school--he tries hard. He has a low intelligence level--doesn't really belong in second grade. He is not reading yet, although he has begun to develop a sight vocabulary.

His attendance is pretty good. He doesn't have a favorite subject in school. He tries to pay attention in math but he starts to daydream if it gets too deep for him.

Student has shown great progress recently in math and reading.

B. Mean percentile rank of tests through November = 16.5

Student 2, Interview Summary #1

Behavior and Appearance

Student was of average height and fairly stocky. He appeared quite nervous throughout the interview - unsmiling when the Interviewer said supposedly humorous things, did not often look at Interviewer, fidgeted considerably. He brightened and sat up straighter the few occasions when he could explain to the interviewer his correct answer. Did not appear particularly enchanted when the interviewer told him he was returning the next day.

Summary of Responses

The student displayed only the most rudimentary knowledge of the MC; he could read and write numbers which did not contain zeros. It was not possible to determine whether or not he knew in principle that to read a number shown in nonstandard form on the MC it was necessary to simplify, since, after trying very unsuccessfully to do it mentally and being then told to make some plays, he wasn't able to make any successful forward or backward plays. He merely shifted and added and removed checkers without pattern. He had no conception of conservation on the MC nor of course any ideas about calculating with it.

Uncertainties: In the next interview the Interviewer will ask him to make plays with very simple configurations. He will also bring a game, toy, puzzle or some sort of icebreaker to put the student at ease. It is doubtful, in my opinion that his nervousness had much effect on his performance. The student did appear to understand the concept of halving and the means of verifying (which in fact were the same - find a number which adds to itself to give the original). With considerable help he was able to complete a 13+13 set diagram and to write the corresponding addition equation, though 2x was apparently unfamiliar to him (2x3=6 [this after I said "No - 2+3=5"], 2x4=7, 2x5=8, etc.)

Follow-Up: Since this student is making little or no progress with the MC, it seems important to investigate carefully his numerical skills and concepts (Has he nevertheless picked up, albeit slowly and imperfectly, the important ideas?).

The interview was carried out in the nurse's office - a quiet and private room - and without interruption.

udent 2, Interview Summary #2

Behavior and Appearance

Student was much more at ease after discussion about Boy Scouts and siblings. Smiled and volunteered information and chatted on several occasions. More inclined to explain when asked and to stick to his guns. Inquired about whether interviewer would talk to other children.

Summary of Responses

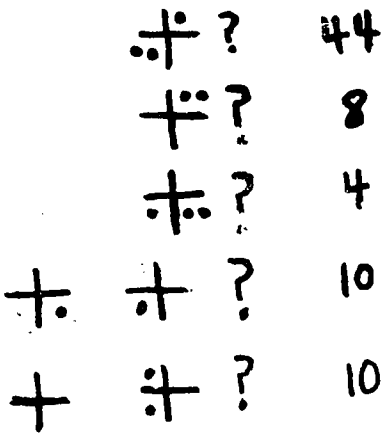
Labelling dots on arrow diagrams could be done only slowly with assistance and correcting explanations for the first two. Similarly with the candy picture-except that student never did seem to grasp what was intended. Return arrows were unfamiliar and the dots seemed to have some sort of "location value" which prevented student from drawing an arrow from 10 to 12 which would have been longer than previous arrows (although with help he was able to draw road to that point). Made frequent directional errors both in drawing and interpreting.

String pictures were also difficult for student. He did not seem to understand that an entity without a given characteristic should be represented outside the boundaries for that characteristic. He also tended to actualize a given dot for convenience without regard to strings.

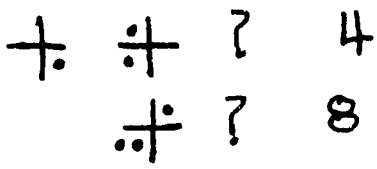
Student could name $\hat{6}$ as "negative six" and could pair off negative and positive checkers to get the outcome of the game. He could not add or tell the difference (won by how much?) between integers, nor did it appear that he was able to order them.

The correct solution was given for $\square+\square=10$ but the suggestion (6,4) was also accepted. Student listed several correct pairs for $\triangle+\bigcirc=8$ and recognized that (1,7) was different from (7,1). Twice he attempted to "change the rules" by saying, for example, 10-2=8 ie. the objective was for something to equal 8.

Some additional questions were asked on the MC as follows:



Student 2



Make "play" with ? No
Other plays with other configurations? No.

Uncertainties: Ordering integers should be attempted in other formats. Very quick answers given.

Follow-Up:
1. Interpreting very simple arrow diagrams of numerical and non-numerical nature.
2. Concept of not vis-a-vis string pictures.
3. Concept of multiplication.
4. Ability to make simple plays on MC.

The interview was characterized by an excessive amount of teaching and explaining by the interviewer without which there would have been virtually no progress made in any area. It is unlikely that this amount of assistance will result in any noticeable differences in student's performance. The interview was again carried out in a small, quiet room.

Student 2, Interview Summary #3

Behavior and Appearance

Student was again rather shy but after a minute or two, after playing with a commercial "cube" game and with a successful answer or two, he came to life: friendly, volunteering information and attempting to explain an answer when asked. It was extemely difficult for him to explain what he was doing-he seemed able to string only a few words together at a time. Without doubt he enjoyed the interview - was proud of the considerable success he attained. At the close of the interview he asked if he could keep my pen (red felt pen).

Summary of Responses

The interviewer was impressed by the progress this student has made in the ten weeks since the last interview. (This was verified by teacher who said he had also made great strides in reading). His understanding of place value in larger numbers is still incomplete. He read the sequence of numbers successfully, (except for "740" instead of "704") and ordered them successfully, except for the reversal of "38" and "83". He could count forwards by 1's (or "one less than"). His sequence of answers backwards from 62 was 63, 64, 65...61,59 (corrected), 60, 69,...89, 79, 69, 59. However, later in interview when asked to calculate mentally 61-3 he was successful. He mentally added 7+2, 4+3, 9+6 (with fingers) and 36+52 (!) successfully, making an error with 9+8. For 22+19 with paper and pencil he put 311 (ie. 2+1=3, 2+9=11). He mentally subtracted 8-3, 9-7 and 61-3 successfully but wrote 51-8 when asked (orally) to subtract 15-8. When this was corrected, he still could not get answer.

After some difficulty in understanding the first story he got successive correct answers for stories related to 4+2, 4x2, 4x6 (via 6+6+6+6→12+12→24), 8÷4, gave an immediate answer of 15 for the computation 3x5 (explained to me by 3 rows of 5 circles). For the story problem of 20÷4 he made rapid guesses. When asked to try "4" to see if it worked, he knew how to go about it, but could not get problem. He also correctly shaded $\frac{1}{3}$ of a blank rectangle.

Student 2

He had considerable difficulty with drawing and labelling a diagram with +3 arrows, particularly when he had to label a dot pointing (with +3) to a known dot (eg. 8) or when he had to determine from which of several dots he could draw a +3 arrow to 5. Had no idea what the new arrow I drew (return arrow) could be for.

In summary: the student still has occasional problems with place value and larger numbers; he has good understanding and some limited skills with the processes of addition, subtraction, multiplication, and division and he can work with moderate success on arrow diagrams and with "forward moving" computations (but not with ideas of return arrows or "backward moving" computations). In short, he has shown a remarkable improvement in the basic arithmetic processes.

Uncertainties: Can the student use paper and pencil in any way to verify that 15-8=7, for example?

Follow-Up: Try the first and third story situations with this student and more arrow diagrams with return arrows, compositions, etc., but with small numbers.

The interview was carried out in the nurse's office without interruption.

Note: This student transferred to a different school after Interview 3 and hence was not available for Interview 4.

Student 3

A. Teacher's Views

Not obtained.

B. Mean percentile rank of tests through November = 21.5.

Student 3, Interview Summary #1

Behavior and Appearance

Student worked willingly and tried nearly all problems even though he was unsure of how to proceed. He seemed to be used to being told whether something he did was right or wrong, and was unsure when no specific value response was given for his actions.

Summary of Responses

Student read two of the three numbers put on the MC correctly. He hesitated when reading them and in some instances added the checkers on a board aloud to arrive at the total for that board. He had more trouble putting the numbers on the boards himself. Initially (with 3-digit numbers) he put all three numbers on the hundreds board or, in the case of 901, began to put them all on the one's board. After questioning by I. student moved the checkers to the appropriate boards, but not always in the proper configuration.

These basic weaknesses in using the MC were the bases for his being unsuccessful in adding or subtracting with the MC. He showed evidence of having some concept of how to add and subtract using the MC but his ineptness in using it caused numerous errors.

When I asked student to identify moves of the checkers as making the number larger, smaller, or the same, he successfully recognized those which reduced or increased the number when a checker was moved from 1 board to another. Forward and backward plays on a board, however, were both identified as making the number larger.

When student was asked to make forward plays to find out what number was on the MC he made all of the plays within the range of each board correctly. To get to the next board, though he said 8+8=9 (and 80+80=90), he put a checker on the 1's (10's) and the 8's (80's) squares. Such moves continued until he read the answer as 6 hundred, 13 tens, and 4.

Student did not recognize the term "backward play" but after I did 200=100+100, he continued correctly with 400=200+200, etc. while on the hundred's board. He made a seemingly random move to the 10's board and when asked what the number was at that point, he said that he wouldn't know how to figure it out.

tudent 3

Instead of asking student what $\frac{1}{2}$ of 32 was, I asked $\frac{1}{2}$ of 6. He initially said he didn't know it nor how to figure out the answer but after I told a story about two boys and six pieces of gum, he said each boy would get three pieces. When asked why, he said it was because 3+3=6. I. then asked student what $\frac{1}{2}$ of 10 was. He said he didn't know, but after I. asked him how he could figure it out, he decided it could be done with dots. He drew two circles and began making dots one at a time, alternating circles. When he reached 11 dots, I. stopped him and asked how many we had all together to divide between the two. After realizing he had one dot too many, he erased one, leaving the configuration of 6 and 4. After considerable questioning on the part of I., student arrived at five in each circle and decided that five was $\frac{1}{2}$ of 10.

Uncertainties:

Follow-Up: It would be worthwhile to see if student gains more proficiency in using the MC since as yet it offers no help in solving any problems asked. If he gains skill in putting numbers on the MC and making basic moves he will probably be able to solve addition and subtraction problems with it.

It would also be beneficial to extend fractional problems to get a better understanding of his limits in the area.

udent 3, Interview Summary #2

Behavior and Appearance

Student seemed excited about working on special problems again. He was attentive most of the time even though there was considerable noise coming from the hall and he was unsure of how to answer most of the interview questions.

Summary of Responses

Student seemed to associate multiplication with addition since, when using the MC and crossing points for multiplying he actually added the two numbers to get the answer. When I gave him the answer of 36 for 4x9 and then asked for an addition fact that showed the same thing he wrote 1+35=36. There seemed to be no realization of a discrepancy between these and the previous answer he got of 4x9=13.

In working on the arrow diagrams, student labelled the dots correctly in the diagram, but he could not work backwards from 9 to label the dots on the second diagram. He could build the road from 5 to 10, and, with a little help - after realizing another +3 arrow would put him at 13 - reached 12 successfully.

Finding the arrows that would give him the answers to the equations was a lost cause since he couldn't figure out the answers - even with the MC - and the numbers and arrows were overwhelming. With much specific questionning, student could interpret the candy arrow diagram to find the child who gets the most, and gives out the most, but even with more questions about who was giving whom candy, he couldn't determine who got the fewest.

Student did much better interpreting the string picture. He answered rather confidently and correctly - even though it took a few extra questions to figure ut the total number of boys represented.

Student 3

When asked about the largest and smallest numbers in the list, $\hat{3}3$ was chosen as largest and 0 as smallest. He knew that the ^ represented "negative" but it didn't seem to have any effect on the value of the number. (At this point we were interrupted by a teacher with a small group needing to use the room, so I asked only about the =8 equation.)

Student chose (4,4) as the response set for the equation and said it was the only one that would fit.

Uncertainties: It would probably be beneficial to give student the problem dealing with addition of positive and negative integers, since I. did not have an opportunity to look at that aspect. I am inclined toward the assumption that, since he was incorrect in his assessment of the relative size of the integers given, he will not operate on them correctly either.

Student 3, Interview Summary #3 and #4

Behavior and Appearance

Student was going on vacation the next day - it would be his last day of school. He was obviously excited about that.

Summary of Responses

For even the simple addition facts (7+2,4+3) he used his fingers to count up the first number and then count the second (adding it to the first). When the first number was >5, he didn't even work on the basis of having five fingers on one hand: he counted 1,2,3,4,5 on that hand (moving each finger every time) before moving to the next hand.

He didn't want to use the MC nor pictures to figure out the answers to the simple subtraction problems. He used only his (and I's) fingers. I. made him use the MC for 61-3 but he had to be questioned, directed, and prodded through every step from putting the digits on the correct boards, to using negative checkers, to making backward plays, etc.

The sheets containing arrow diagrams (with +9 and +1 arrows, with +3, and -2 arrows) he completed, but it required persistent prodding, questioning, repeating of numbers, etc. before he figured out an answer and wrote it in its appropriate place. (The digits 5 and 9 were frequently, but not consistently, written backwards (ƨ and ƿ).

Attitudinal Information

Student thinks he does fine in school, a little better in math than in other subjects. Math is easier because he can use the MC.

He likes to do dot-to-dot pages, matching symbols, shading fractional parts, string pictures (sets), building roads. (I. had him do one building of a road from 9 to 19 with +6 and -4 arrows. He had to count on his fingers each time he wanted to add 6. Even then he counted from 15 to 20 (instead of to 21). I. worked him through it - with frequent errors along the way. He seemed to have trouble counting backwards in the teens.)

Student 3

Student did not like the pages with "magic" numbers, addition and subtraction problems, nor those in which < > = signs had to be supplied.

He prefers class work to doing workbooks. He doesn't like to do all the work. He agreed that he would use the MC for figuring out hard problems.

Student 4

A. Teacher's Views

Student is rather emotional, with family problems at home. He has problems getting along with the other children in his class. He does, however, have a good attendance record.

Student doesn't seem to like any subject in school. He is slow in all of them. He usually will not give answers in class unless brought to the board to work. (He is being considered for testing for special education classes.)

B. Mean percentile rank of tests through November = 27.

Student 4, Interview Summary #1

Behavior and Appearance

Student is a fairly tall boy. He was friendly throughout the interview - even laughing on occasions (perhaps to cover up some rather poor responses when they were pointed out to him though he also laughed when I did something silly). Tried his best and seemed to enjoy interview. There was a tendency to give quick, unreflective answers; particularly when using the Minicomputer, student seemed to be trying to call upon some rhythmic mechanical patterns.

Summary of Responses

Student was successful on only two aspects of the interview. He was able to read and put numbers on the MC. with relative ease and he recognized quickly when the number showing on the MC was being increased or decreased by the movement of checkers. He was very unsuccessful at making any kind of plays. Occasionally, when I would direct his attention away from the MC., with fingers for example, he would see why his response (such as 2+2→1 and 80+20→10) was wrong and sometimes give the correct response. Student did seem to improve as I worked with him, his correct percentage of moves increasing to about 40%, but there seemed little hope of any significant independent work. Many items requiring plays were skipped. The student clearly did not know the addition algorithm nor understood that the number showing on the Minicomputer was unchanged by making a forward or backward play.

His understanding of $\frac{1}{2}$ was limited to a small number of concrete objects. With string pictures for $\frac{1}{2}$x32, he put in the dots but then had no idea what was represented. When "thinking about" $\frac{1}{2}$x32, he was confused with 32-2.

Uncertainties: 1. With few checkers in easy locations, can he make plays?
2. Not sure of his understanding of $\frac{1}{2}$ of 12 checkers (or coins).

Follow-Up: Basic moves on the MC and numerical skills away from MC.

The interview took place at the side of a semi-divided room. Moderate noise from class doing seat work - no apparent distractions for student.

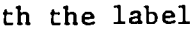
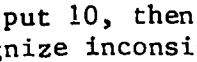
Behavior and Appearance

Student was excited to see I. and wanted to start immediately. At one point started to move checkers on MC. and asked me whether the number was larger or smaller! In a very good mood, smiling and laughing. As the interview progressed he became even more jovial and started clowning ("Come on brain," "Oh I stayed up till 5 o'clock," smacked the table when pointing out a dot. Towards the end he had had enough and was glad when it was over.

Summary

Throughout the interview the student made errors (sometimes in giving quick answers, sometimes after much reflection) and very often when the interviewer asked a related question or pointed out the consequence of his response he was able to successfully correct his response. This was particularly true with arrow diagrams and string pictures.

The student understood multiplication of x times y as taking x y's (putting 4 9's on the MC and drawing 3 sets of 3 dots for 3x3 though for 4x9 he first wanted 9 sets of 9.) He seemed unfamiliar with crossing points.

On the +1,+2,+3 arrow diagram he made frequent errors in calculation though certainly knew how to proceed with the labelling. For  put 10, then corrected. For  tried 7, then corrected to 5 and was able to recognize inconsistency of diagram. Was only able to draw 2 more arrows, from 7 to 8 and from 9 to 10. Thus though he twice mixed up his colors and was not careful of directions, the ideas were manageable. He was successful in making the road. In selecting the arrow diagram for 53-17 he was successful but subsequent questions led me to believe he was rather hazy about the differences between diagrams. For the diagram with children giving candy he made errors on each one, though with only a little prodding he was able to correct two of them. In my opinion he could successfully interpret the diagram if he took his time and was careful.

Generally good with the string pictures, though he made the classic error of putting 2 dots for Mary (instead of one in the intersection). He enjoyed it when I pointed out the two Mary's and corrected immediately. He also said the bottom dot was a girl-otherwise O.K.

His understanding of integers seemed limited to an understanding of the symbols for winning and losing games. There was no understanding of order, of operations with integers, or of integers on the number line.

For $\Delta + \bigcirc = 8$ he gave (1,7), said there was nothing else, then with prodding came up with 4 others. He was quite slow with any required computation and made errors on two of them. He said that (6,2) and (2,6) were the "same answer," that nothing could go with 8 (and he didn't mean zero!) and treated $\square + \square = 10$ as similar to the previous open sentence - the only difference being between the 8 and the 10.

For several reasons the interviewer found this the most interesting of the four interviews to date.

Uncertainties

He shows considerable understanding of arrow diagrams and string diagrams and of the concept of multiplication - though he makes frequent errors and occasionally goes off in the wrong direction. His number skills appear not to be well developed. In short he seemed to have some knowledge in all areas (except integers) but real facility in none.

Follow-Up

1. The concept of order among integers?
2. Confidence and skill in doing multiplication?
3. Development of skills in addition and subtraction of small numbers (eg. < 10)?

The interview was conducted without interruption in one-half of a semi-divided room.

Student 4, Interview Summary #3

Behavior and Appearance

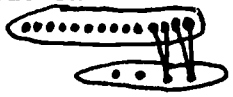
Student was cooperative and tried hard for about 20 minutes, then gradually lost concentration and made jokes. He had considerable difficulty in explaining how he did computations and seemed to think that once he had given me an answer it was pointless for him to also have to explain it.

Summary of Responses

The student again showed great inconsistencies. He was able to label dots and draw arrows on the two arrow diagrams (though unable to figure out the +9, +1 composition), but had little skill in making plays or doing computations on the Minicomputer.

The rest of the interview was devoted to probing his understanding and skill with basic numerical ideas. He read 740 as 704, then ordered it incorrectly, When counting forwards or backwards by 1's or 2's on 4 out or 5 occasions he omitted the 10 multiple eg. 77,79,82, and 62, 61, 59, 58. He could add single digit numbers successfully and 2 digit numbers with carrying when the numbers were written in vertical format. He was poorer in subtraction - making errors (9-7=1, 7=4+4) and not understanding how to carry.

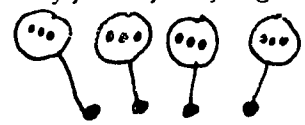
He gave some amazing (considering his previous efforts) responses to items dealing with multiplication and division. For the story problem of 4x2 he drew



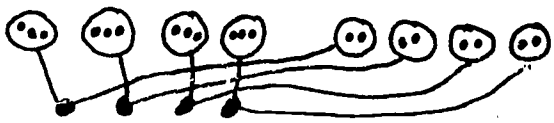
ie. many more than 8 dots. After pairing up dots for 2 of the bottom dots - he stopped - and said 8. He confirmed that he had in fact drawn more than 8 dots (evidently to have available if needed).

For the story problem 4x6, however, he used a different tactic, ie. the more common one of drawing 4 loops of 6 and counting. When shown "5x3" he said it was "5 times 3" but got into all kinds of problems drawing loops with varying numbers of dots in them.

For the story problem of 8÷4 he responded rather quickly, "2", saying that 3 would be too large. For 20÷4 story problem, he drew



counted to 12 then inserted first one then a second dot in each of the new loops shown below. He then wrote 3+2=4!



Uncertainties: No major uncertainties except why the student is so anxious to work on the MC, when he has no skill to use it.

Follow-Up:
1. It would be interesting to try to teach a mini-lesson to this student. He makes many errors and shows little understanding in some areas but in others shows a very surprising and sophisticated way of dealing with problems.
2. Subtraction with borrowing.

The interview took place at the side of a moderately noisy classroom but without interference or distraction.

Student 4, Interview Summary #4

This student left school two weeks before the end of school. He is living with his father; teacher not sure where, if anyplace, student is attending school for the rest of this year. Student will be in the Special School District next year. The teacher feels his problems are emotional more than academic. Though he is not a good student under any circumstances, she feels his emotional problems are a real barrier to achievement.

Student 5

A. Teacher's Views

This is a very good student, but she is achieving lower than expected this year. Her reading achievement is higher than expected. Reading is her best subject.

She was ready for second grade, but she does depend on others to help her with her work. She likes math in general, especially computation.

Student is very eager about school. This is about average for the class. She pays attention in math class. She tries to pay attention all of the time.

B. Mean percentile rank for tests through November = 34.5

Student 5, Interview Summary 1

Behavior and Appearance

Student is quiet, a little shy and attentive most of the time. She did not mind making estimates. She didn't appear to be nervous, worked quickly, said the moves on the MC as she made them but moves and plays did not always coincide.

Summary of Responses

She did not have any problem reading or writing numbers on the MC. For the first addition problem 37+16 (estimate 46), she put both numbers on MC correctly and proceeded to make a series of forward and backward moves all on the one's board. She made a regular (for her) error of taking one checker from 4's square and putting 2 checkers on 8's square, saying "4+4=8". She ended up with both boards filled with checkers, so I stopped her and gave the second problem. 124+398 (estimate 519). She started making backwards moves and got hung up till I asked what is 4+4. She said 8 and then immediately made the 8+2 move and continued. She made an error by taking one 200 checker and putting it on the 400 square for an answer of 722. I gave her a problem 46+38 to do on MC and she again started making backwards plays till finally told to make forward plays which she did correctly and got an answer of 84. When adding 63+28 (estimate 64) on paper she said "3 plus 8 is 14" carrying 1 for an answer of 94. Given 58+23 to add on paper she added correctly.

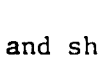
She could not do either subtraction problem. Although she did put both numbers for 63-18 (estimate 40) correctly on MC using plain and [illegible] checkers, she could not continue from there. She did not get an answer for 86-39 (estimate 63) since she said "Can't take six from nine." (I think she meant the opposite.)

She got only one move right on the conservation problem and that was the backward play 4=2+2. She said the number was the same when a checker was moved from one board to another (same relative square). She said the number was smaller when a forward play (20+20=40) was made and also when the checker was moved forward one square (100-200). When the checker was moved back one square (80-40), she said the number was larger.

Student 5

When asked to find the number on MC she made a lot of forward plays but needed a hint to make 80+20 move. She made an error by taking a checker on the 10's square and moving it on the 20's square. She did a similar move on the 100's board and ended up with 920. When I put 800 on the MC she made 4 backwards moves quickly and correctly but said the number was 710 (trying to read the number from MC).

She said she could not solve $\frac{1}{2}$x32 in her head but suggested using the MC and wanted to use the [K] checkers. When she tried to solve on MC she didn't put 32 on MC and said she couldn't do it. She did not know of another way to do it.

I gave her a string diagram  and she wrote the number sentences 12+12=24 and $\frac{1}{2}$x24=12. She did not know of any other sentence.

Uncertainties: Subtraction problems without borrowing; $\frac{1}{2}$x[ ] using smaller numbers than 32, more questions about her understanding of $\frac{1}{2}$, $\frac{1}{3}$ concept.

Follow-Up: Plays on MC, conservation questions, addition and subtraction problems and 1/nth concept.

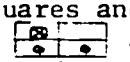
Interview took place in the cloak room in back of the classroom. It was a small area, with a lot of noise coming from classroom. Noise did not seem to bother student.

tudent 5, Interview Summary #2

Behavior and Appearance

She was a little more responsive this time, not quite as shy as during the first interview. She was very conscientious about putting the tops on the colored pens each time she used them.

Summary of Responses

She knew 2x4, 3x3, did not know 3x4 nor could she mentally find 4x9. She said she could use dots to find 4x9 and started to draw 4 strings and place 9 dots in each string. She also said she could use the MC. When she tried, she placed [K] checkers on the 8 and 1's squares and a regular checker on the 4's square and made backwards moves to arrive at  and could not give an answer. When asked about the crossing points, she started to draw 4 lines down and 9 across. She used the dots in the strings to find the answer 36. I asked for number sentences for the dot picture and she wrote 9+9+9+9=36, $\frac{1}{4}$x36=9 and 2x9=36. I think she meant to write 4x9=36. She first wrote $\frac{1}{2}$x36=9 and I said do you mean $\frac{1}{2}$ and she quickly wrote $\frac{1}{4}$. She could not find 2x27.

I gave her some problems orally, $\frac{1}{2}$x12="6", $\frac{1}{2}$x18="9", $\frac{1}{2}$x24="I don't know" and $\frac{1}{3}$x12="3".

She could correctly label the dots given in the diagram and she drew 2 more arrows, +3, +2. She used the +2, +3,+3 sequence to build the road. She was a little upset at first since she put the dot for the 12 far to the right and had to move it closer to where she had put the dot for the 9. She could not use the arrows to solve either equation. She picked 70-17=53 for the first problem but she would not give an answer. She picked 54-2=52 for the second problem and said the answer was 52. She correctly wrote the equation 53+17=70. She did not get any questions right on the candy problem, picking "b" for getting the most candy - 5 pieces, "c" getting the fewest - 2 pieces, and "a" giving the most - 6 pieces.

Student 5

She correctly identified the dots as being a boy, not tall and not wearing glasses, and she placed Mary in the diagram correctly. She described the next dot as a person wearing glasses, and I had to question her to find out it was not a boy, and not tall. Did not ask how many boys. She correctly put a dot for herself and placed the interviewer in the tall group only.

She said "negative 33" was the largest, zero was the smallest. She misinterpreted the next question and wanted to put 5 more (⌒)'s in the diagram. She matched positive and negative checkers and got the correct answer of "negative 3". She said Bill won by 4.

When given the equation, she very quickly put 5 in each box and started on the next problem. I asked why that sentence was true and she said because 5+5=10. To "Anything else?" she responded yes, 9+1, 8+2. For the second problem she put 4+4=8 and when asked said also 6+2, 5+3 and after a pause said she couldn't think of any others. She said that the answer 6+2 could not be switched around - the 6 went into the △ and the 2 into the ( ) .

Uncertainties: Not sure if she meant 2x9=36 or 4x9=36 when she wrote the number sentence for the dot picture and did not ask for the number of boys in the string diagram.

Follow-Up: Multiplication of small one digit numbers and multiplication on MC; arrow diagrams - non-numerical and equation types; concept and order of integers; concept of □+△ in open sentences.

Interview was conducted in a small cloak room in the back of a classroom where a lesson was being taught. The noise did not seem to bother student.

Student 5, Interview Summary #3

Behavior and Appearance

Student seemed very at ease with interviewer and seemed to enjoy answering questions. She did wonder when we were going to question other students.

Summary of Responses

When asked "What number is one more than 57", she said "58", "Next one", "59", "next one" "30","One more than 59", "Oh, 60", "Next", "61". "What number is 1 less than 62?" "61", "One less than 61", "50", "61", "60", "One less than 60", "59". Both times she reached a ten's number (50 and 60) she made mistakes which could be considered careless but I am not sure if just careless or lack of real understanding. She quickly identified the +9 and +1 arrows and was able to label all the dots correctly, although she had to add 30+9 and 40+9 by counting and not immediate recognition. The composition of arrows was not apparent to her. At first she said the blue arrow could be for +5 but after she labeled the other dots she said it was a +10 arrow.

She knows her basic addition facts and with any numbers which involve carrying (9+6 and 8+9) she has to count by ones for her answer. She did both 2 digit addition problems correctly.

She knew 8-3 was 5 but she said 9-7 was 3 and 15-8 was 6. She could not find any way to check her work or show subtraction.

Student 5

She correctly answered the 4+2 story problem and at first said the 4x2 story was 6 but after I repeated the question she said 8. Would not explain how she got her answer and could not find 6x4. She correctly read 5x3 and her story for 3x4=12 was 3 girls and 4 pieces of bubble gum.

She correctly answered 78-43 but did not even attempt 56-39. She wrote 15-(4+2) on the paper but could not do any more than that.

She knew $\frac{1}{2}$ of 12 was 6 and drew 3 circles and distributed dots for $\frac{1}{3}$ of 18. She said she would draw 2 circles and then dots for $\frac{1}{2}$ of 62 but she wouldn't want to.

She could not come up with one combination for 10¢ even after I showed her 2 suckers and 1 chocolate. I then drew an arrow diagram to show 2 suckers and 1 chocolate. She said she could do one and drew a diagram of +2,+4,+2,+2,+2, her mistake being in adding 2+4=4.

She could read the arrow diagram directly without any problems but does not understand return arrows. After I drew the +18 arrow she correctly identified it and answered 14+18=132, but did not transfer for $\frac{1}{2}$x114. She correctly identified a $\frac{1}{2}$x arrow drawn between 7 and 14 but could not draw any other $\frac{1}{2}$x arrows.

Uncertainties: Counting by 1's and 2's (forward and backward) to determine if first answers were careless or not.

Follow-Up: Concept of multiplication and subtraction should be checked at a later date. Composition and inverse function and problem solving should also be checked for any improvement.

Interview took place in a small cloak room where noise from the classroom could be heard but without any noticeable effect on student.

tudent 5, Interview Summary #4

She can subtract when no regrouping is necessary but (53-17="44") not when it is necessary. She can find $\frac{1}{2}$ of smaller numbers - $\frac{1}{2}$x16=8. She has the concept of multiplication although she couldn't do 2x18 mentally.

She can read an arrow diagram correctly 3x15=45 and 45-17=28 but can't use the return arrows. When I asked her what was the largest number she said "negative 2" (wrong) and the smallest was "negative 4".

She tried to do 73+18 on the MC. It was set up correctly but she did not make any forward moves correctly and would take off 2 checkers whenever 2 checkers were on a square.

She says she does pretty well in school and likes math best. Some of the math is easy and some is hard. "When you have plus and negative numbers that's hard." The math is different but doesn't know why. It's better than the others. She would rather work on workbooks by herself because it's fun. She said she would rather do the addition problems on the MC when they get hard, otherwise she does them on paper. It's not too hard to explain to her parents the arrow diagrams or any of the other problems in the workbooks.

Student 5

6AIV	pg. 3	order - likes best - easy
	pg. 7	cross out figures that do not belong - hard
	pg. 12	return arrows - doesn't like - hard
	pg. 15	secret message - like - easy
4RII	pg. 3	multiplication on MC - hard
	pg. 7	subtraction on MC - hard
4RIII	pg. 1	Venn Diagrams - easy - likes
	pg. 3	roads - easy - likes
	pg. 5	match - easy
	pg. 7	multiplication facts - easy
	pg. 11	cross out figure - doesn't like - hard
	pg. 15	open sentences
4A5	pg. 1	cut and shade - easy
	pg. 3	label dots - hard
	pg. 10	$\pm$ - hard
	pg. 12	combinatorics - easy - like

Doesn't like hard "times" problems like on 4AV pg. 7

Worksheets:

Spinner game was fun.
B156 was hard
likes reading the thermometer B
she asked to take the worksheets with the thermometer home
likes to color the flags.

Student 6

A. Teacher's Views

Student doesn't seem to try very hard in school. His teacher thinks he could do much better in his work. She thinks this apparent lack of interest is due to physical disabilities.

He is not very interested in school but he seems to like math better than any of the other subjects. He is absent frequently, mostly because of illness.

Socially, he is quite aggressive. He doesn't get along very well with other children. He seems to have only one friend in school.

B. Mean percentile rank of tests through November = 35.

Student 6, Interview Summary #1

Behavior and Appearance

Student seemed rather ill at ease when I asked him to come with me to talk about math. He seemed to have a cold and very frequently during the interview he rubbed his right eye, held it shut, or squinted. It was blood-shot and red due to all the rubbing, and probably had some irritant in it as well. Student's hands were very dirty so that using them to rub his eye could not be helping the situation. These conditions caused him to be frequently pre-occupied during the interview.

Summary of Responses

He read and showed numbers well on the MC. He could add numbers on the MC, but made all the moves sequentially on the boards and was left with an 8, 4, 1 on the ones' board when adding 37+16 and couldn't read the number. After probing on my part, he made the 4=2+2 move and then was able to convert the answer to a form he could read. He was very reticent to give estimates and when he did they were a long time in coming.

He used the ⊡ checkers appropriately to subtract, but he ran into a problem when he said 10=8+2. He put two checkers on the 8's square, causing the final answer to be wrong, but he had used the technique correctly and rather quickly. He had no idea of what to estimate for the answer to 61-18 nor for 86-39.

On the topic of conservation it seemed that whenever a checker was moved, regardless of how, student saw it as a change in the number. And all but once the change was seen as making the number "higher."

He accomplished forward and backward plays with the 14 checkers equaling 800 rather well (with this example though, there was always a 2 left with an 8 when a move to the next board was called for). His problem with reading numbers when there was more than 9 on a board and his tendency to consider any move a change in the number resulted in his reading 800 after backward plays (⊞ ⊞ ⊞) as 710, and later (⊞ ⊞ ⊞) as 796.

Student 6

He did have some understanding of fractions, but didn't use the MC successfully with $\frac{1}{2}$x32. (He seemed to have the idea of needing pairs of checkers-this he accomplished by putting 32 on twice-but that is as far as he could get.) He successfully used allocating to sets to solve the problem and even began using the same technique for $\frac{1}{3}$x32 (surprisingly he did know that $\frac{1}{3}$ of 32 would be less than $\frac{1}{2}$ of 32.)

He did not have fluency with expressing a problem in a number of different equations. $\frac{1}{2}$x32=16 belonged with the string diagram and 2x16 belonged with crossing points.

Uncertainties: I am not sure under what conditions he can and can't make the appropriate plays on the MC to result in the number being most simply expressed. Sometimes he was left with more than 10 on the board and couldn't read the number nor suggest a way to change it, but other times he successfully made the 8+2=10 plays before he had made all of the 2+2 plays.

Follow-Up:
1. The belief that any moving of a checker (or making a play) makes the number larger.
2. Making estimates of answers.
3. The concept of there being more than one way to express the same problem or a solution to it.

Student 6, Interview Summary #2

Behavior and Appearance

Student seemed more interested than the day before in working with me. His cold seemed to be lessened and he rubbed his eye only a few times during the interview. There was a woman typing in the room in which we worked, but this didn't seem to distract him.

Summary of Responses

Student could do the multiplication using the MC and crossing points. He was less clear on how the string pictures applied and couldn't offer any related addition problems. He couldn't think of an alternate way to handle the problem of 2x27 either.

In labelling dots, he was consistent with the previous interview: the first dot he labelled correctly, the second incorrectly, and then the rest were correctly done (based, however, on the erroneously labelled one). He seemed to assume the intent of a +3 arrow from the end of the one set of arrows to the beginning of the others, thus labelling 4 as a continuation of the first set. He couldn't use an arrow diagram to give the answer to a problem stated in reverse form. He can interpret relation arrow diagrams if he's led through them step by step, otherwise he gives answers apparently unrelated to anything shown in the diagram.

With string pictures he seems to be somewhat competent. He gives his answers hastily, though, overlooking some information.

When dealing with integers, he was aware of the relative size of negative and positive numbers, but he didn't handle the negative and positive scores correctly when asked how many more or less one was than another. To solve the 4+7 problem he spontaneously drew the picture to solve it.

tudent 6

Equations requiring two numbers to complete them caused no problem for David, but he didn't make a distinction between same and different shapes.

Uncertainties: It's not clear whether he doesn't understand the magnitude of difference between negative numbers and between negative and positive or whether he is just tied to a visual representation of the number using ⌒'s and ◯ to find the answers.

Follow-Up:
1. Understanding that there are many approaches to solving problems and that they can be used interchangeably.
2. Operations using negative numbers.
3. Understanding reverse(return) arrows and their use in solving problems.
4. The concept that the same shape in equations require the same number and that different shapes may have different numbers.

udent 6, Interview Summary #3 and #4

Summary of Responses

Student responded very slowly to most of the problems given him. The initial questions about labelling dots and figuring out a +10 arrow he handled very quickly. He did the simple 2-digit addition problem quickly mentally and the subtraction problem without borrowing he did mentally, but very slowly. The rest of the problems involved either slow methodical drawing of strings and dots or long pensive pauses before answers were given. For this reason, not many topics were covered.

Four review questions: 73+28, 66-49, 114+18, and 4x7 were asked at the beginning of Interview #4 followed by the balance of questions not reached in Interview #3.

Interview #3: Student figured out the answers to both addition problems (23+45, 73+49) and the first subtraction problem (78-43) correctly. He did not know how to borrow, however, so he made the expected mistake on the problem 56-39 (6-9=3).

He knew to do the portion of the equations within parentheses first, but it took a lot of prodding before he actually gave answers to the problems. In the problems using fractions (except for $\frac{1}{2}$x12 which he first said equalled 24), he used string pictures - even when it meant drawing 62 dots.

The questions regarding the amounts of candies that could be bought for 10¢ required an extreme amount of coaxing, prodding, etc. to get him to figure out even two responses. The first two questions referring to the complex arrow diagram were rather quickly answered - but only after I. pointed to the dots related to each problem.

Interview #4: Preliminary problems were given to check on previously shown weak areas. Student did the addition with carrying (73+28) correctly on paper but the subtraction with borrowing he did on paper with the usual error (66-49=23). I asked him the additional two questions (114+18 and 4x7) with a new copy of the complex arrow diagram in front of him. He responded more quickly and correctly to 114+18, but needed to use the MC to figure out the answer to 4x7 (even though it could have been determined by using the diagram).

He somewhat successfully answered the rest of the problems based on the arrow diagram. At times it meant I. had to point to the dots labelled according to the problem and ask what the return arrow would be, but he seemed to be able to use clues to the answers to make correct choices. When he was asked to draw any $\frac{1}{2}$ x arrows, student drew 2; 1 correctly, 1 incorrectly.

The straightforward problems dealing with negative numbers seemed to cause no trouble for student, but I. did have to prod him to combine three scores to arrive at the team scores.

Generally, giving the answers took an unbelievably long time. Prodding, reminding, etc. on the part of I. didn't really seem to speed things up, but only served to keep his attention on the problem immediately at hand.

Student 7

A. Teacher's Views

Student is like a clinging vine--she needs a lot of attention. Her teacher thinks she has home problems. She has a good attendance record, and likes school--but more for the student contact and the attention she receives than for the school work.

She is generally not a good student--and she seems to be getting worse. (She has such a short attention span.) She is not doing as well in math as in the other subjects.

B. Mean percentile rank of tests through November = 41.

Student 7, Interview Summary #1

Behavior and Appearance

This was a pleasant girl of above average height and weight. She appeared a bit shy and perhaps a little bit nervous. Throughout she paid very close attention, did not get flustered at her many incorrect answers and showed no great stress when she just couldn't proceed with some of the questions. She did not volunteer much nor did she chat much. At the end she said she enjoyed it which I believe she did.

Summary of Responses

1) Went slowly and made errors in reading and "writing" Minicomputer
2) When adding and subtracting on Minicomputer, after placing the two numbers on Minicomputer (sometimes with error) did not know what to do. Made what appeared to be random plays.
3) Did not seem to understand idea that Minicomputer always has a certain number on it and to find out what it is one makes plays forward and towards higher boards to get a simpler configuration. (This from the above and from "finding the number" on the conservation questions.) She did know enough to try to add mentally the amounts shown by all the various checkers but it was of course too much for her.
4) Knew when numbers were made smaller or larger (though not by how much) except when a forward play (20+20=40) was made, when she on two separate occasions said the number was larger.
5) $\frac{1}{2}$ x32 appeared to be a mystery to her. Offered no way of going about it. When asked to use the Minicomputer did not know what to do, and when asked "about what will the answer be, said "33".

Uncertainties: Not sure of her skill in simply reading and writing on Minicomputer. Not sure of her concept of $\frac{1}{2}$, let alone how to calculate.

Follow-Up: Virtually everything particularly ability to "simplify" a number shown on Minicomputer by many checkers.

Interview took place in a classroom which was vacant except for another interviewer-student pair, who faced in opposite directions and did not disturb her.

Student 7, Interview Summary #2

Behavior and Appearance

Student was more talkative than previously and when asked to explain her answers, did so more fully. Appeared to get a bit frustrated with multiplication problems, but enjoyed the interview after that until she got a bit tired of it at about the 35 minute mark during follow-up questions. Asked whether Interviewer was going to talk to any other kids and when he would be back.

Summary of Responses

Student did not seem to understand concept of multiplication; at times she started operating as if with addition (2x10→12, 2x5=7) and at other times gave other unusual answers (3x3=12, "because 6+6=12"). Correctly drew crossing points for 4x9 but could not place checkers on MC for 2x9 (although she could do 9+9).

She labelled dots correctly on arrow diagram but also labelled bottom trio of dots. Was not able to draw further arrows. Easily drew a road from 5 to 12. For selecting appropriate arrow diagram for 53-17 she at one point selected correct one but rejected it because it was "backwards" then selected diagram yielding 70 and affirmed that "70" was correct answer to 53-17. In the distribution of candy diagram she twice mixed up the arrow directions and several times changed her mind. Though given lots of time and encouraged to "look carefully," she gave no correct answers for any of the questions.

In the string picture she did quite well, making only two slips which she corrected after the interviewer paused. In one case she first placed Mary in the string for Boys and in the other case she first counted as boys only those who were not tall and those without glasses. She was able to order integers but could not "operate" with them (4+7 and 3 wins by how much over 4).

She immediately got $\square \div \square = 10$ ("same boxes") and proceeded through the number pairs for $\triangle + \bigcirc = 8$, omitting zeros and not being able to figure out 2+? and 3+? for 8. She said (1,7) and (7,1) were the same answers except they were backwards.

Some supplementary questions on the MC were asked to clarify some doubts in the Interviewer's mind from the first interview. Though her performance in reading and writing and adding on the MC was somewhat better, she displayed many of the same inconsistencies such as a 2+2→1 play, a 4→8+8 play, being stumped when reaching an 8+4, not knowing the MC's number was unchanged by making a single backward play. She had no idea how to subtract on the MC except when the minuend could be simply removed without plays (eg. 67-24). She did, however, seem to know that when a mass of checkers appears, she must make forward plays including 8+2, 80+20 in order to get to a simplified configuration.

Uncertainties: She seems rather slow with basic subtraction and addition facts (like 2+?=8).

Follow-Up:
1. Addition and subtraction with integers.
2. Progress in developing facility with M.C.
3. Interpretation of numeric and non-numeric arrow diagrams.

During the interview the rest of the class was somewhat noisily working on workbooks. Because student and I were behind shelf at back of class, they were out of sight and did not really disturb interview at all.

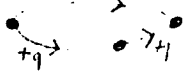
:udent 7, Interview Summary #3

Behavior and Appearance

The student was waiting at the school entrance (it was lunchtime) for me and was quite looking forward to the interview. The student was attentive throughout the very long interview and would undoubtedly have continued even longer had I requested it. She answered questions more slowly than before, but again on many occasions made errors which she probably needn't have made and which she could correct after a further query. At the end of the interview she asked if I could return next Friday for another interview.

Summary of Responses

The student was asked several questions from the elementary section of the interview. She was able to count forwards and backwards by one's and two's (rather slowly with one temporary error) and was only moderately successful with basic facts. For example; 4+3=6 corrected to 7, 8+9=16, 9-7=1 (done with fingers), 15-8=5 (when asked to check, she drew 15 dots and crossed out 8, correctly getting an answer of 7) and rewrote 36+57 as 36 she did it and 22 correctly.
+52 +19

She successfully did story problems involving 4+2, 4x2 and 4x6 (solving the latter by drawing 4 unarrayed groups of 6 dots) but while reading 5x3 correctly, claimed she couldn't tell me how to do it because she "can't do times!" She labelled and drew arrows in elementary diagrams reasonably well and correctly though when asked what the arrow from 50 to 60 in  could be for, she said, "probably minus something".

For two problems with parenthesis she calculated the result of the parenthesis eg. 4+2 in 15-(4+2)=__ and then placed 6 in the blank. When corrected she subtracted 15-6 but with error. This was repeated with 6+(2x8) with an additional problem with 8x2. She did it by crossing points, but had to recount the 8 lines when counting dots. For 56-39 she put 23 and for 78-43 with Minicomputer she did it correctly but was very slow in reading 35. I put a large number of checkers on the Minicomputer and asked her what the MC showed. She made the correct reduction (an improvement from the last interview) but read 430 as 413. When the 400 checker was removed she read 30 as 23.

We then went through about half the basketball "story", ie. with negative integers. She had to be reminded that in calculating 4+1 we joined the blue with one of the 4 red dots leaving +3 as the score. This procedure had to be repeated with 2+8 but student correctly did the remaining 3 mentally. She then correctly listed the scores from highest to lowest and with some assistance determined that Cathy would need 5 "hits" to equal Susan's score. She gave 12 as the difference between 6 and 5 - this was probably a computational error.

In summary the student has made some progress. She seemed to know what a MC was all about (improved computation and the ability to and knowledge of when to make plays), though she still was very slow and not skilled in reading numbers from a MC. She is still very poor in basic addition and subtraction facts, though again improved from the last interview. She seems more comfortable with arrow diagrams, but only the simplest variety, and with labelling dots (ie. finding numerical answers).

Student 7

Uncertainties: 1. Understanding of arrow diagrams with return arrows, compositions.
2. Her understanding of multiplication-she seems to understand and compute well in "story situations" but blocks when confronted with multiplication sign.

Follow-Up: 1 Skill in basic addition and subtraction facts.
2. Computation with MC.
The interview consumed about 40 minutes and took place in a room which was empty except for another interviewer and student.

Student 7, Interview Summary #4

The student was asked what she wanted to do. "Add" Make up a problem for me. Wrote "1001+70=" I gave her 101+101 - she responded "201" and corrected after some discussion. She said she was good at reading and writing but gave no response when asked about math. Is math easy or hard? "Easy-some of the time." She said that the minicomputer and sometimes the workbooks were easy, and subtracting "when you had to take, like two away from one" was hard. She saw her math as different from the other second graders because "they read theirs in their book - that's how they have math." Is it as much fun as yours? "No, because they have it in the afternoon." (Later in the interview, while perusing the workbooks, she mentioned that the other class had easier problems than these - pointing to an arrow diagram page.) She prefers working on the workbooks to teacher-lesson.

She has taken the MC home to show her parents but "they don't know how it works." She says she likes to do it because "you can use it to do easy problems." Show me a calculation you can use it for. She placed number (182) on the MC That's just a number. Can you show me a calculation with that number? "I don't know how to do a calculation with it. The teacher never showed us." The teacher never showed you? "Well she didn't show me." Why don't you add something to 182? She placed additional checkers on but could not read what the added number was. Let me put some checkers on and you tell me what the calculation is. "Can I make the plays?" I placed 2x86 on MC. - one in blue and one in yellow. Student couldn't tell me what the calculation was and made 2 errors in attempting to "make the plays." Would you rather use the MC. or paper and pencil to do a calculation? "Paper and pencil." Do you find that easier? "Yes."

We then leafed through the A workbooks from series 5:

- Label 2x dots - easy and could do except for one
- +4 spiral starting from 60 - liked it and could do it
- Solution set - knew how to do it.
- 3x23 with MC. - hard and didn't like
- Cutting cakes in fractions - easy
- Venn diagram problem - hard
- ½x18, 1/3x66 on MC. - hard
- +1 spiral - easy and could do it
- 4x21 with MC. - hard
- Subtraction facts - easy but not particularly good at
- Road from 0 to 13 with +3, -1 arrows - easy
- ½x problems -"hard - Oh I can do that (½x0) -"one" - corrected with help
- 22¢ with 10¢,5¢,1¢ - wanted to show me - showed 17¢ + 10¢ + 5¢
- +4 spiral from 5 - could do it
- Ways to show 6 on a MC - showed 4+2,4+1+1. Any others? "8 take away 1" corrected

Student 7

- 3x7 - didn't know. How do you figure it out when it says times? Showed 4x3 crossing points. How about this (I wrote 3x5)? "11." How did you get that? Appeared to be 3+3+5. Use your imagination - pretend you can see one, two, three lines here and one, two, three, four, five lines here. What do you think it would be? "19." Can you make up a story to go with 3x5? "No."
- $\frac{1}{2}$x[ ]=5 - Read as "$\frac{1}{2}$ of 5 is?" No, $\frac{1}{2}$ times something is 5. "10."

I wrote	She made it
62	62
-37	37
	35

Student 8

A. Teacher's Views

Student seems to enjoy school. She likes all of the subjects, but she seems to want to do best in math. She has had good attendance.

Her achievement in school subjects started out low at the beginning of the year but she was doing average work by the end of the year in all subjects and in reading she was above average.

She is a quiet girl with a small group of friends with whom she shares school work and playtime.

B. Mean percentile rank of tests through November = 43.

Student 8, Interview Summary #1

Behavior and Appearance

The student was pleasant but quiet and very careful in the work she did. She expressed out loud most of the moves she made on the MC and several times asked if it was all right if she made certain plays.

Summary of Responses

The student read numbers on the MC correctly but with some slight hesitations (involving the 10's board). She showed numbers rapidly without any noticeable problems. Pam was reticent to estimate answers and if she did guess, it was only after a long pause. She could add numbers rather quickly on the MC and could do the problem on paper (writing the numbers across, not down) handling the carrying of 1 ten successfully. With subtraction she was less successful. Since she didn't put the subtrahend on the ☒ checkers, she couldn't "match up" those to be removed. She wrote the subtraction problem on paper in the horizontal format (as she had done with addition) and made the usual error of 9-6 (for the ones). It might have been more likely for her to have seen the error if she had written the problem vertically.

Student had quite a problem with conservation of numbers on the MC. Having her start with a configuration of checkers she couldn't interpret may have precipitated the errors. But after making plays and arriving at 800 and then making backwards plays, she tried to re-calculate the number each time she was questioned with no reference to the 800 written down before her.

When student calculated $\frac{1}{2}x32$ she seemed to undo everything she had previously done correctly in making plays on the MC. She put 32 on correctly and proceeded to make "forward(?)" instead of backward plays: 10=20+20, 20=40+40, etc. - and she placed the two checkers on the next higher square each time she removed one lower one. When she had made "plays" to two checkers per square she then made the necessary plays (correctly) to put the number in its simplest form (=64). We didn't dwell on the answer, so I'm not sure if she noticed the discrepancy between it and her estimate.

Student 8

She correctly solved $\frac{1}{2}$x32 using a string diagram and counted one-half of the dots to equal 16. After a long wait she aptly started writing addition story 1+1+1+1... to also describe the string picture. She understood she would have to put in 32 ones. Student did not feel she was very good at "times" when asked if she could think of a "2x" story.

Uncertainties: I'm not sure whether she doesn't have any idea of how to do subtraction problems on the MC or whether the presence of so many checkers (blue, ⊠ , and ⊗) could have confused her when beginning the problem.

Follow-Up:
1. Subtraction with and without the MC
2. Conservation when simple plays are made
3. Computing $\frac{1}{2}$ on the MC
4. The concept that different number stories can describe the same problem.

Student 8, Interview Summary #2

Behavior and Appearance

Student worked very slowly and thoughtfully and she still expressed out loud any moves she made on the MC.

Summary of Responses

She answered the simple multiplication facts correctly and rather quickly. 4x9 was more of a problem. When she put it on the MC she put on a 9 and a 4. With a string picture she got no farther than one large string. Student was more successful using crossing points - she drew it correctly - but she counted only the enclosed squares, so she got the answer of 24. For the addition story to go with the problem she wrote 4+9=__.

I digressed to put 3x6 on the MC and ask her what multiplication fact it showed. After some questioning she arrived at the correct problem and made plays to find the correct answer. When I put 5x6 on the MC her responses were correct and they came more quickly.

When working with arrow diagrams, student labelled the first dot incorrectly but did the remaining ones correctly (based on the erroneous one). She couldn't supply any additional arrows, but easily labelled the dots preceding 9 (and explained clearly how she arrived at those answers). After some thought student successfully drew the roads to 12. The problem using arrows to find the answers to the two equations was a total loss. She was much more successful with the relations diagram, but it took some probing to draw out the correct answers.

The string picture caused no problems. She answered each question quickly and correctly. With the series of integers she showed less understanding. She knew that the -'s meant "minus", but zero was evidently the smallest number there.

I would have to call the WIN-LOSS problem a total loss since she seemed to see it as incomplete. When asked how many Mary would have to win to reach Bill's score, she said 6 (because Mary had to win 6 to get a score of 6). Neither Mary nor Tom had a higher score -- neither had a (positive) score. She seemed to see "taking away losses" as different from winning--as indicated when she said that Tom would have to "take away 3 (losses)" to get to Mary's score.

Student chose to draw a picture to figure out the answer to $4+\hat{7}=\hat{3}$. (At that point she remembered the ^ numbers were called negative.) She wanted to figure out the answer to the diagram with $\hat{2}$ and 4, even though I had skipped to the next problem.

Student gave two pairs of numbers for the solution to the first equation. She corrected herself when she wrote a 4 as part of a possible solution. When I asked her why she couldn't use 4, she explained that the other number would be 4, too, and that wouldn't be right since the shapes weren't the same. In providing a response set for $\square+\square=10$, she stuck with (5,5) as being the only one possible (for the same reason: the shapes were the same).

Uncertainties: I am not sure if student does or doesn't understand that there is more than one equation that can be used to express the same problem. She couldn't give the addition fact that went with 4x9, but I'm sure she wouldn't have been able to give the multiplication fact that went with 9+9+9+9.

Follow-Up:
1. Facility in using multiple ways of finding a solution to multiplication problem
2. Drawing function arrows that include several smaller functions or reverse functions.
3. Using given reverse arrow diagrams to solve difficult problems.
4. The relative size of negative numbers.
5. Different problems using wins and losses.

Student 8, Interview Summary #3 and #4
(one session)

Summary of Responses

Since it was so near the end of the school year when interview #3 was held, only the content covered in it was used.

Student preferred to use paper and pencil (either with calculations or with string pictures) for figuring out the answers to most problems. She used the MC only for finding $\frac{1}{2}$ of 74 and 70. She seems to have satisfactory skill in handling most calculation problems: addition with and without carrying; subtraction with and without borrowing; equations with parentheses; multiplication with rationals; those involving negative numbers. Student's facility with a complex arrow diagram was a little slow, but she did correctly interpret its parts to arrive at answers to all of the questions asked her.

Student had an erroneous concept of the relative size of negative numbers. Ranking scores in order of size she chose: 5, 3, 0, $\hat{6}$, $\hat{3}$, $\hat{2}$. She is also a little unsure (or careless) in her calculations involving negative numbers when they are not written in standard format. She figured the difference between Susan's and Mary's scores ($\hat{3},6$) as 8 and the total score for the girls' team ($\hat{2},3,6$) as 6 when she did them mentally. She drew a diagram to figure out the boys' team score, though, and answered it correctly.

I don't think student really understood the question about the number of shots Bill had to get, because in the diagram she drew she actually put 12 shots in total instead of 10.

Student 8

Attitudinal Information

Student said that she likes school. She does best in "homeroom" where they work on pages.

She things she is learning a lot in math. It is easy, but it is one of the harder classes. She thinks it is different from what other children do in math. She is the teacher when she and her friends play school and she gives them hard math problems to do.

In math student likes arrow diagrams and prefers to do problems with paper and pencil better than on the MC. She would rather work in workbooks than in class activities.

Student said she doesn't take her MC home but she does take some workbooks. Her brother, who is quite a bit older, gave her problems to do that he doesn't think she can do, but with her home-made MC she has been able to surprise him and do them.

She said she uses the MC for $\frac{1}{2}x$ and $\frac{1}{3}x$ with larger numbers, but she said they use it seldom for practice.

Student pointed out that she likes to do solution sets using dots, coloring fractional parts, secret messages, machines, building roads (with about two different kinds of arrows), money problems. She isn't too anxious to do pages full of calculation problems and she doesn't like those with lots of "times on them, because I'm not too good at times."

Student 9

A. Teacher's Views

This is an average student. She is very much oriented toward progressing in math, a highly motivated student. She has good attendance.

She is well-behaved but is withdrawn. She is even somewhat of an isolate. Her teacher feels that she doesn't have many friends.

Her father is very ill and she seems to have emotional problems related to her mother. She is very interested in her parents being proud of her.

B. Mean percentile rank of tests through November = 48.

Student 9, Interview Summary #1

Behavior and Appearance

Student appeared to be a quiet girl, who took her time before she answered any questions. She did not appear to be nervous.

Summary of Responses

She could not correctly read any numbers on the MC that had any checkers on the 10's board. 705 was the only number she could correctly read. I gave her 224 and 62 to read, without success. She put 854 on the MC instead of 864 but correctly showed 901 and 214.

Most of her estimates were off target. She correctly put the numbers for the addition problems on the MC but was not able to make all the correct plays. She started off making a 1+1=4 play and when I asked her to say the plays as she was making them, she started to make some correct plays. She misread 55 as 38. On the second addition problem, she made correct forward plays on 100's board but could not make carry plays and wanted to make backwards plays. I asked what is 20 and 80 and she responded 100. She asked if she could make an 8+8=16 play and she correctly did it. She wrote the addition problem horizontally and said "carry my 1, 74." I don't know how she arrived at that.

She first put 61-18 on MC with some colored checkers and started to make backwards plays. I asked her to show me just 61 on MC. then asked her how she was going to take away 18, and she said by making backward plays. "How do you know when you are going to be able to take away 18?" "When you make backwards plays." She made incorrect backward plays and got 41 for an answer. She wrote the subtraction problem horizontally and got 53.

She correctly identified 3 moves I made on the MC (2→20; 100→200; 10→1) and after I made a forward move 20+20=40, she said larger and when I was putting the checkers back in their original position she quickly said the same. I neglected to actually ask her about a correct backward play. All her estimates of "by how much" were answered by what square the checker was moved to and contained the word times, i.e., 2→20 larger by 20 times. 10→1 smaller by 1 times. She could not find the number 800 and did not read the number on the MC correctly. I put 800 on the MC and asked for some backward plays. She took the 800 checker and put 2 checkers on 400 square, then

Student 9

took both checkers from the 400 square and put them on the 200 square; then again both checkers were placed on the 100 square and finally put one checker on the 80 and one on the 20 square. Read her answer as 20.

She incorrectly read $\frac{1}{2}$x32 as one and a half times 32. "How would you find the answer?" "Use the MC or strings." She said she could not use the MC to find the answer and was told to use the strings she alternated putting 1 dot in each string and correctly got 16 for an answer. When asked for another number story she wrote $32x\frac{1}{2}=16$. "Another one." 16x32=16. "Use a plus sign." "I could but I just haven't thought of one."

Uncertainties: Inconsistency with plays on MC; addition if problem was given vertically.

Follow-Up: Reading numbers on MC, understanding of 10's board; conservation on MC; addition and subtraction.

Interview took place in the cafeteria, many interruptions by students looking at what we were doing and by students using drinking fountain.

Student 9, Interview Summary #2

Behavior and Appearance

Very friendly, a little excited about doing interview.

Summary of Responses

She is just starting to get the concept of multiplication. For 2x4 she said "4+4=8, 8+10=10"; 3x3, "3+3=6+3=9"; 2x5, "9"; and 4x3 = "4+4=8+3=11", 4x9, "17". She said she could show me how to do 4x9 with paper but all she did was write 4x9 and tried to work it out in her head - "4+4=8+9=18." I asked her to show me on the MC and she put checkers on the 8, 4, and 1's square but could not continue from there. I asked for 4x8 using string pictures and she drew 8 dots in each string, answer was 16. I then asked for 3x7 using crossing points, and she drew 6 horizontal lines and 7 vertical lines and got an answer of 47. Did not bother to ask 2x27.

She did not have any trouble labeling the top row of dots. She was not able to draw any other arrows at this point, even after I gave her an example of 7+2=9. She was using return arrows mentally to label 8 and 5. After she labelled 8, she drew 5+3=8, and 7+1=8. She could not find anything wrong with the diagram.

After thinking about the road for a few minutes she correctly drew +2,+3 and +2 arrows. When asked which arrow diagram would help her to find 53-17=☐ she picked 53+17=70. "What would the answer be?" She thought for a long time and eventually quit looking at the diagrams and tried to work problem out in her head. Even after she got an answer of 35 she did not pick another arrow. For 2x54=☐ she picked 54+2=56.

She had no problem with the arrow diagram about candy, although she did pick the person who received only 1 candy. I felt that was the way she interpreted the question.

She correctly identified all the dots in the diagram and placed a dot for Ms. _____ who is not tall, and not wearing glasses.

Student 9

She correctly picked 27 as the largest integer, but picked zero as the smallest and I could not get "negative" out of her (33 magic peanuts). She correctly picked Bill as the winner, but said Mary needed to win 6 more games to have the same score (she quietly said, "She hasn't won any games so she has to win 6."). She said she could figure out 4+7=__ but wanted to draw a diagram which she correctly did.

She gave "(5,3), same one but backwards, (3,5), (7,1), (6,2), 4,4)" and stopped there because "(2,6) was the same as (6,2)." "If 8 went here (△) what would go in here ◇ ?" "8 magic peanuts." For □+▭ =10 she responded 5+5, "Anything else?" "9 and 1", "8 and 2". I stopped her there. "Is there any reason why we would use 2 squares here and different shapes up here?" "There is a reason, because 3 is smaller than 5, and because 5+5 are the same size."

Uncertainties: Concept of multiplication - writing number sentences and showing work for simple multiplication problems.

Follow-Up: Multiplication, equation solving using arrow diagrams, integers.

Interview was done in the cafeteria, with only one minor interruption.

Student 9, Interview Summary #3

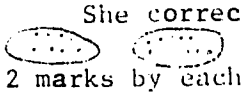
Behavior and Appearance

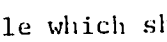
Student was friendly and seems as if she enjoys the interviews. She did not want to go out for recess but wanted to continue with the interview.

Summary of Responses

She did not have any difficulty adding one digit numbers mentally although 0+6 and 8+9 took a little longer than 7+2 and 4+3. She can correctly add 2 digit numbers using the addition algorithm.

She was a little slower in answering the subtraction problems, resorting to drawing 15 lines and crossing out 8 for 15-8.

She correctly solved the story problem although for 4x2 she originally drew  (2 circles with 10 dots in one) and then drew 4 lines and put 2 marks by each (| | | |). She started to do 4x6 the same way. At first she read 5x3 as 5-3 but said that's not right and said she forgets what that sign means. I said times and she read it and solved it mentally. Her story for 3x4=12 was "There were 3 boys and 4 girls and they had 12 pieces of gum."

By mistake I asked her to shade $\frac{1}{2}$ of the rectangle which she did () by sighting the middle of the vertical lines.

She knows she must do parenthesis first but had difficulty (?) in determining what 2x8 meant. She finally decided on 8 twos. She found $\frac{1}{2}$x12 by thinking 8+8 is 18, 7+7 is 14, 6+6=12. She found $\frac{1}{3}$x18 by using strings and was going to use strings for $\frac{1}{2}$x62 which I feel she would have done correctly but I stopped and asked $\frac{1}{2}$ of 6, $\frac{1}{2}$ of 60, $\frac{1}{2}$ of 2 and then $\frac{1}{2}$ of 62, all of which she answered correctly. I then asked $\frac{1}{2}$ of 48 and she said 24. She could not find $\frac{1}{2}$ of 74 because "you can't add the same number and get 7."

Student 9

After a long struggle she finally found that 5 pieces of chocolate would be 10 cents, and I did not ask for another way. She had great difficulty in finding out what 5 pieces of chocolate would cost.

When given the detective diagram, she said she couldn't do 2x66 because there wasn't a dot labelled 2. "Can you find 66?" "Yes, oh, 132." She found 114+18=132 but I'm not sure she really understood since when asked what could the return arrow for -85 be, she said +57 (the arrow was leaving 57).

She said a $\frac{1}{2}x$ arrow was -132 since it left the dot 132.

As a review I asked her to read some numbers on the MC. She still can't read numbers if there is more than 1 checker on the 10's boards, ie, 358 was 3 hundred fifteen eight. Working with smaller numbers, she could read the MC only if there was one checker on the 10's board. Given an addition problem without carrying she got the correct answer but could not read it correctly. She can't make any carrying moves and her backward moves are haphazard, ie. 

Uncertainties: Would have liked to see if she could have built a road with +2,+3,+4 arrows from 0 to 10.

Follow-Up: Next interview should follow up on arrow diagrams, subtraction and reading MC.

Interview took place in cafeteria, fairly noisy. Appeared to disturb student only once.

Student 9, Interview Summary #4

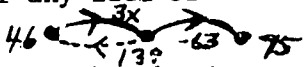
A. Review of previous weaknesses
1) Reading numbers on MC 821 ✓
435 ✓
2) Writing numbers on MC 342 ✓
3) Addition on MC 45+67 ✓

Made the 8+2=10 move but needs a hint to make 80+20=100 move. Instead of writing 7 as 4+2+1 she put 7 checkers on the 1's square and proceeded to make all the correct moves.

4) Addition algorithm
$$\begin{array}{r} 73 \\ +48 \end{array}$$ ✓

did without MC and then checked work on MC. With MC again she put 7 checkers on the 10's square and got the correct answer.

5) Subtraction algorithm
$$\begin{array}{r} 81 \\ -63 \\ \hline 22 \end{array}$$

6) Return arrows - still very weak. Does not have any idea of what the arrow could be for nor any idea of how to use an arrow diagram to solve a problem, ie. 

75+63=_____ tried to do in her head.
I then asked what could the dotted arrow be for and she said -63.

7) Given 15, 2, 0, 4 - Name the largest number, "15," the next largest, "0," the next largest "4 magic peanuts," and the smallest, "2 magic peanuts."

Student 9

8) Roads (+2,+3,+4 from 0-10) √4+3+3.
9) Open sentences □+□ =50 "40+1."
10) Answer questions from Venn diagrams 4AIV pg. 5 √
She says she does pretty well in school. She's better in reading than math. Does best on math problems - plus and take away.

Is this math different than Mrs.(teacher) math class? "They have races with numbers and we don't" and..... She likes CSMP better - "because flash cards and I can be score keeper and have math problems. It's more fun." She does not work on math outside of math class.
What do you like about math class? "Score keeper and...." Do you like workbooks? "I love workbooks." If she had to make a choice between workbooks and class activities she would choose the workbooks because "If people help me I wouldn't learn anything by myself."
Do you like working with the MC? "Yes, because I can use the checkers especially if Ms.(teacher) needs some more (magnetic) checkers."
Do you take your MC home? "I think so." What do your parents think about your math materials? "They like it when I have pluses on it but do not like it when I have -1 or -2." Do you explain how your math works? " Yes." Is it hard to explain? "Yes - the MC because they did not have anything like it when they were in school."

Math Materials:

Worksheets:	142(1A)	addition problems - some easy and some hard.
	143(1A)	open sentences - easy
	152(1B)	label dots (+13) - hard
	156(1B)	shading 1/2 and 1/3 of figure was fun.
	176(1A)	draw arrows was fun
	178(1A)	matching "they were fun some were easy"
	188(1B)	subtraction - they were fun too.
	194(#2)	fun and easy

Workbooks:	4R2 pg. 3	2x35 on MC was hard
	4R2 pg. 6	1/2x12 with string was easy
	4R3 pg. 1	(matching dots with Venn Diagram was easy
	4A6 pg. 3	secret message was fun and easy
		coloring flags fun and easy

What do you like to do best?
4A5 pg. [illegible] addition problems

What are the hardest problems?
4A5 pg. 11 return arrows

Student 10

A. Teacher's Views

This student's achievement is generally average. He is average in math but above average in reading.

He seems to like school and math specifically; the teacher hasn't heard any complaints from him. He's missed only one or two days of school this year.

He seems to have the same friends during play as during class and he especially likes playing and going to baseball games.

She is not aware of any particular family information relevant to his school behavior. He is one of five or six children.

B. Mean percentile rank of tests through November = 49.

Student 10, Interview Summary #1

Behavior and Appearance

Student was usually attentive, although when a new person arrived or noise arose in the cafeteria, he was istracted.

Summary of Responses

Student could read and show numbers on the MC and could usually make forward moves correctly on a board. He showed a definite weakness in moving forward from one board to the next, usually trying a combination of 8+8=16 and not knowing where to put it. Backward plays were truly backward since after taking a checker from, for example, 800, he would say 800=400+400 and then replace the 800 checker with a single 400 checker in the square. He did this consistently regardless of the square from which he started. The concept of conservation seemed to be supported only on a chance basis. Moving a checker from 80 to 40 did, after probing produce the answer "smaller...by 40," but a checker moved from 10 to 1 left the number the same, and a move from 4 to 2+2 produced the response "smaller...by a little...about 10." There seemed to be more likelihood of his answering nearly correctly if the move was made on the same board than if it involved 2 boards.

Solving for $\frac{1}{2}$ of 32 using the MC was described correctly, but due to his tendency to have serious problems with backward plays, I chose not to have him demonstrate it at this time. He seems to have a notion of how to approach the solution by distribution (take $\frac{1}{2}$ of 2 and $\frac{1}{2}$ of 30), but that and the process of allocating to sets are far from understood.

Uncertainties: The most intriguing point I want to pursue is how he arrived at the correct answer to 61-18 by using a number of checkers and moving them mysteriously on the table, all accompanied by considerable finger counting. It seemed to be a more effective method than using the MC.

Student 10

Follow-Up: At this time there is ample ground for follow-up work with him. All of the operations using the MC need further study. Problems using fractions and what number stories are related to them seem to be far less well understood.

Student 10, Interview Summary #2

Behavior and Appearance

Student seemed more at ease than during the previous interview. The conditions were better since we had an office to use.

Summary of Responses

Student seems to know simple multiplication facts. He chose drawing four strings and counting out nine dots for each one as the way to find the answer to 4x9. When the MC was suggested he said that you couldn't use it to find the answer. When crossing points were suggested, he drew the diagram and-even though told he did not have to count them all-he counted 37 crossing points.

When I wrote the equation 9+9+9+9=36, and asked if it was at all like 4x9=36 he did agree it was and explained generally how you had 4 - 9's in both.

After the first dot was labelled he had a rather good facility with simple arrow diagrams (counting with his fingers as he worked). He consistently miscounted for the first dot.

Using arrow diagrams to infer answers to equations was less than successful. He wrote the equation 53+17=70 correctly when a specific diagram was picked, but he did not see any way to solve the dilemma of having two equations nearly alike 70-17=53 and 53+17=70. He did not offer a suggestion for changing one or the other.

The arrow diagram referring to children giving candy was incomprehensible to him, even though he gave a deliberated answer to each question. His response that (a) gives away 4 pieces made it clear to me that the diagram wasn't clear at all.

String pictures were better understood. Initial responses were often wrong or partially correct, but more specific questions usually drew out the correct responses.

He was aware that a negative number is smaller than a positive but $\hat{6}$ was chosen as the smallest even though $\hat{3}3$ was recognized as present. He could handle the simple subtraction with a negative number, but the question regarding Bill and Mary was unclear to him.

Equations with figures needing numbers to be supplied were understood, but the necessity to put the same number in shapes that were the same was not.

Uncertainties: It is not clear how large are the number facts that he knows or does in his head, and whether he can still do them if the answers exceed 10 (or 20).

Student 10

Follow-Up: There is ample material for follow-up and nearly every topic touched up on in this interview has potential for improvement that can be surveyed as the year progresses.

Student 10, Interview Summary #3

Summary of Responses

Student did the 2-digit addition problems with and without carrying and the subtraction without borrowing correctly. He made the usual mistake in 56-39 of subtracting 6 from 9. Student knew to do the part of an equation inside the parentheses first, but his uncertain skill in mental calculation resulted in an error when adding 6+16=(24). He used a variety of techniques to determine the answers to problems involving fractions, but he was not adept enough to find $\frac{1}{2}$ x74 by either method tried: the MC or distributivity.

He hasn't enough flexibility in calculations to arrive at many solutions to the question about amount and kinds of candy purchased for 10¢. He made two correct responses and one incorrect.

He seemed to be able to interpret the complex arrow diagram rather well, giving correct responses to all of those questions where he had to imagine a return arrow to give the correct answer.

However, for those questions that required consideration of more than one point to point relationship ($\frac{1}{2}$x arrows, 4x arrows, $\frac{1}{4}$x28), he succeeded in only one instance ($\frac{1}{2}$x arrow from 14 to 7). He gave other answers, but these were incorrect.

His skill in working with negative numbers is questionable. He calculated three of the five scores correctly (all either negative or zero) and the 2 he missed should have had positive integers for their answers. He seemed to have no concept of the relative size of numbers when positive and negative were to be put in order.

Since he had had such difficulty adding two integers, calculation of team scores (involving 3 integers) was not asked, but I. did ask who he thought would have won, boys or girls. After long pondering, he chose the girls'. According to his calculated scores for the children, he would have been correct (7 vs. 8). But it was inconsistent with his choice previously of ordering numbers according to their absolute value.

Student 10, Interview Summary #4

Summary of Responses

Student does not correctly do subtraction involving borrowing with paper and pencil. He can do it using the MC.

He knows to do problems with parentheses by beginning with the part inside the ( ), but he was slow in arriving at the answer to a problem such as 5+18 when he tried to do it in his head.

Student 10

He used a string picture to figure out $\frac{1}{3}$ of 21 and can use the MC successfully to find $\frac{1}{2}$x92 - although he made several mistakes in making all of the necessary backward plays in that he began to make forward plays along with backward ones.

He was not consistent in calculations that involved negative numbers. He handled $2+\hat{8}$ and $3+\hat{6}$ correctly, but $9+\hat{4}$ he said equalled $\hat{5}$. I questioned him about whether the answers were always negative but he said "no" and showed me that if $3+\hat{6}$ were turned around to $\hat{3}+6$, it would equal a positive 3. Perhaps his mistake with $9+\hat{4}$ was just an oversight or perhaps most of his practice has been with answers being negative.

Attitudinal Information

Student thought that he was doing "okay" in school, that he did best in language and math. He said math was easy for him - he liked to do workbooks and pages of problems.

He doesn't think the math he is doing is any different because he only knows second graders in his own class.

He showed his brothers and sisters and mother and father how to use the MC. He doesn't think it's hard to teach them how to use it, but he said his father said he doesn't understand it. He likes taking extra math worksheets home to do - those left over from when the class has done them. He prefers to work in his workbooks to class activities - since he likes doing pages of problems.

He said he likes to do the pages of adding and subtracting, the arrow diagrams, and building roads.

He said he doesn't use the MC much at all unless it is to do thirds or fourths of a large number.

Student 11

A. Teacher's Views

Student is very careful with his work, to the extent of getting upset if he is not right. He does well in all subjects, but waits to begin work until he's sure of what to do.

He has good attendance, likes school in general, and likes math a little better than the other subjects. He gets along very well with his classmates in school work and during play.

B. Mean percentile rank of tests through November = 53.

Student 11, Interview Summary #1

Behavior and Appearance

Student was very cooperative but slightly ill at ease - usually when several children were standing nearby and watching what he was doing. He was usually confident that what he had done was correct, but asked for verification either from the interviewer or by working out the problem in a way he knew better in those instances when he was not so sure.

Summary of Responses

Student was very adept at reading and showing numbers on the MC and at adding, subtracting, and taking $\frac{1}{2}$ using it. It was inconsistent with this obvious skill to see him make as many errors as he did in judging the relative size of the number on the MC when I made plays or moved checkers. He also seemed to lack any notion of conservation of a number after it had undergone backward plays.

He had an unusual purpose for the special checkers: ⓐ's were "$\frac{1}{2}$ checkers," and ⊠'s were "take away" (He used them to make backward plays on 800 - which, I guess, was an indication that he saw the number as getting smaller.)

He seemed to have some concept of the distributive process -- "If it was 48 I could do $\frac{1}{2}$ of 4 is 2 and $\frac{1}{2}$ of 8 is 4. So it would be 24, because 24+24=48." But he felt this process didn't work unless each digit could be divided evenly. He did seem to have an understanding that there were several ways of expressing a problem.

Uncertainties: I am not sure whether he thinks that backward plays in and of themselves make a number change (smaller) or whether using ⊠ markers to represent the number resulted in this misconception.

Follow-Up:
1. Subtraction with borrowing without the use of the MC.
2. Conservation of a number when plays are made.
3. His peculiar use of ⓐ and ⊠ checkers.

Student 11, Interview Summary #2

Behavior and Appearance

Student seemed less responsive, less alert today. The children who would stop and watch what he was doing were more distracting to him and the interviewer as well.

Summary of Responses

Simple multiplication facts were answered very quickly and he immediately chose the MC to solve the larger problem 4x9. He added 4 - 9's using his fingers as a second way to solve the problem. (He didn't seem to notice that he'd gotten a different answer though.) The terms "crossing points" and "string pictures" seemed foreign to him although when I started showing him what they were he easily finished them correctly. (Perhaps his teacher uses different terms.)

He seemed to have no idea of how to work backwards on a number line when given the number at the end, nor any basis for drawing arrows between dots in two lines to join points on them. Using arrow diagrams with functions the reverse of what he needed were unfamiliar. The diagrams were more puzzles than helps to solve the problems. In interpreting the candy picture, he was partly successful. When questioned more specifically he could interpret the number of pieces being given and received, but for general questions, he seemed to interpret any arrow regardless of direction that touched a dot as relevant to the question asked.

He answered all of the questions about this string picture correctly and, although I forgot to ask how many boys were represented, I am confident he would have answered correctly.

He seems to be confused about the relative sizes of negative numbers. He identified them as "negative" but thought that zero was the smallest of the numbers presented. It followed reasonably that when he was asked about the relative sizes of the scores that he never passed the zero point in moving from positive to "hat" scores. $\hat{2}$ was 3 smaller than $\hat{5}$; and Mary would have to win 4 to go from $\hat{2}$ to 6.

In dealing with equations, he could give many pairs of numbers to complete them. He saw reverse pairs as different, but did not distinguish between same or different shapes when suggesting numbers for them.

Uncertainties: I am not sure about his ability to interpret the relations arrow diagram. He seemed to be somewhat distracted and answered haphazardly. This warrants another look.

Follow-Up:
1. Supplying arrows using a large function to portions of a diagram using several functions subsumed under it.
2. Working backwards with arrow diagrams requiring doing the reverse of the arrow indicated.
3. Relative sizes of positive and negative integers, and zero.
4. Development of the concept that different shapes used in equations require different numbers and that the same shape can use only the same number.

Student 11, Interview Summary #3

Summary of Responses

He very ably answered all of the calculation problems mentally, with the exception of the first (for which he said 77 instead of 78). He seemed to enjoy the challenge of a different type of problem when figuring out the numbers and types of candy he could buy for 10¢.

The arrow diagram he used exactly as intended. When given a problem requiring conceptualization of a return arrow, he explained how it could go there and then explained what the answer was. The only error he made was in drawing an arrow with the head pointing the wrong direction.

For the calculation of scores involving negative numbers he quickly wrote the answers. The order of the scores, however, gave support to something that had been brought out before: that the higher the number, the higher its value within the realm of positive and negative numbers as well. The order he gave was 5, 3, 0, $\hat{6}$, $\hat{3}$, $\hat{2}$.

He did not seem to grasp the concepts of "Who had the best chance to make a basket?" and how many baskets Bill would have had to have made to end up with a score of $\hat{2}$. The question about the difference between Tom and Mary's scores was also puzzling to him: "It is negative, nothing!" However, when I. rephrased the question to: "How many more shots Mary would have to make to reach Tom's score?", he responded correctly.

Student 11, Interview Summary #4

Summary of Responses

Subtraction with borrowing (76-58)
Set it up in vertical position; worked correctly.

Half of an odd number ($\frac{1}{2}$x65)
Used MC, found it was 32 with 1 left over. I. told him it could be written as $32\frac{1}{2}$.

He did mentally: $\frac{1}{3}$ of 9; $\frac{1}{3}$ of 6.

Attitudinal Information

1. "I do okay, I guess." (in school). He gets A's and B's. In math he gets A. Math is easy and fun.
2. He said he doesn't know about what other kids do in math. He doesn't talk about it when he plays with friends from other classes and schools. He does workbooks and worksheets at home but not many games.
3. He likes math. He likes number problems (pages of them).
4. He made a MC at home. He showed his parents how to use it, but they don't understand it very well.
5. He likes using the MC for some hard kinds of problems. But he uses paper and pencil mostly.

Student 11

Student likes pages of problems, arrows of +14, +13, 2x (large numbers), and he did one problem, 2x243 in his head.

He likes the workbooks with the most problems. Solution sets are okay.

Games he really doesn't like.

Student 12

A. Teacher's Views

Not obtained.

B. Mean percentile rank of tests through November = 67.

tudent 12, Interview Summary #1

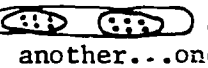
Behavior and Appearance

Student appeared to be very shy and she asked a few times during the interview f her moves on the MC were right and if she was doing it right. She did not want to guess at any of the calculations and I did not push.

Summary of Responses

Even though she misread the first two numbers given on the MC I feel she can read and write on the MC. She is not positive of all her moves and reverses moves fairly often (ie. 4+4=2). When asked does 4+4=2 she quickly corrects herself and makes correct move. Carrying move is no problem. She can add two numbers with and without MC, although she cannot do subtraction. She had no idea of subtraction on MC. She mentioned "I don't like working with these ☒'s."

She was able to successfully determine whether the number on the MC was made larger or smaller after moving various checkers. She made four backwards plays correctly and when asked what number is on the MC she said 700 and _____ and would have to make forward plays to find out.

She read "$\frac{1}{2}$ of 32" and said she could find the answer on the MC but really did not have any idea of how to go about it. She did not know any other way of finding the answer. I mentioned string pictures and drew one string and she said she remembered now. She then drew two more strings (total of 3) and started to put dots in them, one at a time and said to continue till she put the 32 dots in. I stopped her and I drew  and asked for a number sentence. She wrote 6+6=12 and when asked for another...one with x in it she wrote 12x6=__ and I said "What is 12x6?" She said "9".

Uncertainties: Reading numbers on MC with a 0 in the one's or ten's place.
Concept of $\frac{1}{2}$.

Follow-Up: Conservation of number on MC, sureness of moves on MC.

Interview took place at a table in the hall with constant interruptions (kids running and talking in the hall). Had to repeat questions to make sure she was paying attention.

Student 12, Interview Summary #2

Behavior and Appearance

Student appeared to be more at ease during this interview although she said "good" when I answered her that I would not be back tomorrow.

Summary of Responses

Basic concept of multiplication seems to be just starting to be understood. She correctly answered 2x4 and 3x3. Could not use MC to find 4x9. Had no idea - put a checker on the 8, 1, and 4'x space. When suggested, she correctly found 4x9 using crossing points. Had no idea of 2x27 but said she could use crossing points.

Could correctly label dots but could not draw any other arrows. She correctly labelled the 8 dot but could not label the 5. Two digit numbers cause problems for her since she adds as follows (10+3 - does not go 11, 12, 13 but counts on fingers to 10 and then uses 3 more fingers).

Could not correctly build a road from 5 to 12. She put 12 on right side of paper and drew a +3, +2, and +3. She was going to continue with a +2 arrow when I stopped her and pointed out that 13 was bigger than 12. She agreed but was still going to continue to draw more arrows. I put a dot and labelled it 12 near the 10 and asked her to draw an arrow from 10 to 12. She drew a +3 arrow.

When given the 6 arrow diagrams she correctly (and without hesitation) answered both questions and wrote a number sentence. But she could not correctly answer any of the questions about the children giving out candy. She paid no attention to the direction of the arrows.

She answered correctly all the questions dealing with the string diagram except the question about how many boys?

For the largest number she picked $\hat{33}$ but could not remember the word "negative" till I told her. Picked 0 for the smallest. She said Bill won and Mary needed to win 6 more games (incorrect) and that Tom had a higher score than Mary.

She said she had to use MC to find $4+\hat{7}$ but then proceeded to put 4 on MC with green checkers and then while writing $\hat{7}$ she put a $\hat{\ }$ and a green checker on the 4, 2, and 1 squares. So when she took away the matched pairs she ended up with 4. She correctly did the string picture of $4+\hat{2}$.

Quickly came up with 4+4=8 but could not come up with any others. I said <u>If I put a 5 in the △ what number would go here?</u> and she proceeded to count on her fingers 1,2,3,4,5,6,7,8 and said 3. I said 6, she said 2, then 7-1 and 1-7 which she quickly said were the same. I suggested 8; she didn't know what could go with it.

For □+□ =10 she quickly suggested (5,5), (8,2), and (9,1).

I then asked $\frac{1}{2}$x8 and she said 13. She could not explain $\frac{1}{2}$. She could not write an addition number sentence for 2x9=18.

<u>Uncertainties</u>: Not sure she understood the candy diagram.

Student 12

Follow-Up: Concept of multiplication, understanding of arrow diagrams, return arrows, and concept of integers.

Interview took place at a table in the hall again with interruptions with children running down the hall. I had to wait till I got her attention before asking a couple of questions.

Student 12, Interview Summary #3 and #4

Student transferred to another school before these interviews could be carried out.

Student 13

A. Teacher's Views

Student's general achievement in school is low. He has been recommended for special education classes, but his mother refused. He needs to be given constant attention, almost forced, to do his work. He won't do anything independently. During school student seems to be interested in anything but class activities.

Attendance has been good, but he seems to have emotional problems. He had one close friend at the beginning of the year but they had a falling out. Later the friend moved away. Student's father is dead. He has an older brother.

B. Mean percentile rank of tests through November = 71.

Student 13, Interview Summary #1

Behavior and Appearance

Student was rather quiet and shy, especially for about the first 15 minutes. Did not appear too disappointed when interview ended and went back to another table and became very active with puppets with another student (though not disruptive). Gave what appeared to be very carefully considered answers to all questions and was very hesitant to make guesses.

Summary of Responses

Student could read and write numbers on the MC fairly easily and could add successfully with the MC (though he got mixed up once with ten's and hundred's board). He had some knowledge of an addition algorithm because he twice added successfully 2 digit pairs with carrying - but always in his head.

Frequently in the interview he was reluctant to make forward moves and often stopped two or three moves short of complete simplication and mentally calculated, slowly but correctly, what the MC showed.

He was unable to make any progress with subtraction on the MC, even using the same color markers to show the two numbers. He was very good in determining changes in the number shown on MC when checkers were moved (though not by how much). Did not know (without calculating) that 800 remained on the MC after four or five backward plays.

Mentally and with MC (after some false starts and corrections by I.) got $\frac{1}{2}$x32=16 and after suggestion got it with string pictures. Was able to show the two parallel equations (though required the "two times number story" as a clue for the second one).

Student 13

All in all a good understanding of the MC and possibly a tendancy to favor mental calculations instead of the MC or written algorithms instead of the MC even when this was a slower process.

Uncertainties: 1. Written addition problems with carrying in the hundred's
2. One-half of a number on the MC
3. Subtraction (86-39) without a MC (I. forgot this).

Follow-Up: Student appeared to have surprising powers of mental calculation. This should be explored.

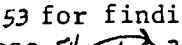
The interview took place in a cubicle at the back of the class. This was a tight squeeze. The rest of the class made moderate amounts of noise during the interview but student showed no sign of being disturbed.

Student 13, Interview Summary #2

Behavior and Appearance

Student appeared apprehensive when told by teacher he was going to do some special math problems today. He did not appear enthusiastic about interview. When told interview was over, he jumped up and went back to his room. Throughout the interview he took his time, thinking about each problem before answering. He has the basic multiplication facts and concept of repeated addition. Could not find 4x9 mentally, first said 4x9=12. Why? "9+1+3." When asked how he would find 4x9 he said "Add 4 - 9's." When asked to use string diagrams he drew 2  and then said he didn't know. He was told to use MC to find answer. He put 4 - 9's on correctly and made a few correct moves but got hung up when board was  . I suggested backwards plays and he made incorrect moves, was corrected by I. and he made a couple more moves before he made another mistake. Got answer of 35. He drew crossing point diagram and correctly explained how to find answer. I. gave him  and asked for x story. He gave an answer of 18. When asked for a number story using x or + he couldn't do it. How did you figure out 18? "3x4=12 and 3x2=6." 2x27="54" How did you figure that out? "2x20=40" and "2x7=14".

He correctly labeled the top row of dots in first arrow diagram and wanted to label the bottom row also. He labeled the first dot in bottom row 13, when asked why 13, he said 10+3=13. (Since there was a green arrow leaving the first dot in bottom row.) I. labeled a dot 9 and asked for other arrows and finally gave him an example 9+1=10. He couldn't draw any others. He labeled the second dot correctly and drew a +2 arrow from 8 to 10 - no others. He finally labeled the first dot correctly (said 7 first) and drew a +2 from 5 to 7. He might have thought he could only draw arrows from bottom row of dots to the top row. Nothing wrong with picture. Before picking up pens, he thought about drawing the road from 5 to 12, picked up pens and quickly drew road +2, +2, +3.

He picked 70  53 for finding 53-17 and when asked for an answer he said "70, no." He finally chose 54  27 and said answer was 27. He correctly identified the arrow for 2x54=104. I. felt he knew answer before picking arrow and he is weak on using diagrams.

He correctly identified who got the most candy, and who got the least. He picked (a) as giving away the most (2). He correctly told how many each person gave away but when asked again who gave the most he again picked (a).

He correctly identified in the string picture the dots for B, for G, for G, T, and B, and for G and T. But he put the dot for Mary only in G. He correctly said there were three boys.

He said 27 was the largest, 0 was the smallest and read negative 6 and negative 32. He correctly picked Bill as the winner, 8 more games for Mary to win to catch Bill, and Mary had won 3 more games than Tom. He had no problem with $4+\hat{7}=\hat{3}$.

In the first open sentence his responses were (4,4); (7,1); (6,2); (5,3); (4,4); (3,5); (2,6); (1,7) and said (7,1) and (1,7) are the same before being asked about it. (He couldn't think of any others and I said "What if I put an 8 here?" and he said "0".) In the second open sentence he wrote (5,5) and when asked for any others he started with (9,1), (7,3) and I stopped him.

I gave him the problem 86-39=__ and he mentally figured out an answer of 68. I then wrote. 47 and he thought for about 5 seconds and wrote 115 (ie. all done mentally). +68

I asked for $\frac{1}{2}$x36 on MC. He quickly made backwards plays (correctly) starting on one's board and ended up with  and said "Now take one off of all of them" and then took one checker from each pair of checkers and correctly got an answer of 18.

Uncertainties: Understanding of string diagrams - can he write multiplication number story for them; can he use them to solve multiplication problem?

Drawing of more arrows - did he misunderstand directions and think he could only draw arrows from bottom row to the top row.

Student 13, Interview Summary #3 and #4

Behavior and Appearance

Student appeared apprehensive throughout. The entire interview was characterized by very long pauses while the student considered his responses to questions. Usually after the pause he volunteered the answer, usually correct, but occasionally there was no response - he couldn't do the problem. It was impossible to determine which circumstance was prevailing because he was very loathe to explain what he was doing mentally. Only after he had given his final response was it possible (though rare) to get any explanation. The student frequently scratched - arms, face, hands in apparent nervousness.

Summary of Responses

Interviews 3 and 4 were conducted, in abbreviated form, in one sitting. Interview 3 consisted of some calculation, the "arrow" problem and the "integers" problem. The student made the usual mistake in calculating 56 getting 23 (he had previously missed the minus sign and correctly added). He was then asked 78 and responded 30. He then did 15-(4x2)= and $\frac{1}{2}$x74= mentally and correctly - very slowly for the first one and very quickly for the last one.

Student 13

He was able to use the arrow picture to get the answers to straightforward calculations but did not seem to have the idea of using a return arrow - hence he could not do the three caluclations requiring the converse relations. Similarly, though he was able to draw two $\frac{1}{2}x$ arrows by mental calculation, he did not realize the return arrow idea. He could not tell what the (7,28) arrow could be for.

The student did extremely well on the integer game problem. He got every problem except "who has the best chance," where he responded Tom, who had the highest score and how many did Bill make out of 10 to get a score of $\hat{2}$ (though he correctly responded $\hat{12}$ to Bill's score after six such games). In determining who won between the boys and the girls, he added together all the red dots and from that total subtracted the total of blue dots, thus getting correctly $9-14=\hat{5}$ for the girl's score.

Responses for the attitudinal questions were very difficult to elicit. The student likes gym and art best but said he also liked math. He thought all the students in second grade had the same math (used the MC for example). He preferred using paper and pencil to the MC for calculations and said he hadn't used the MC in quite a while. He said he had never taken the MC home with him. The thing he liked best about the math was the worksheets, but he would not say anything, even when pressed about things he didn't like in the math. (Later in the interview after going through workbook 4A5 and 4A6, he was asked to show which of the pages he liked best and which he liked least. He pointed to a code puzzle and a page with an addition and a multiplication table respectively).

We then went through 4A5 and 4A6 - for most pages he responded "yes" when asked if he liked to do that kind of thing. He never once expressed a dislike for any of the pages and seemed to particularly like the code puzzles and arrow diagrams but generally his responses were very similar. On various pages I asked him to tell me what he was supposed to do or how would he do it or what was the answer:

4AV
- Page 3 (label a return arrow on a +4 spiral with 40 in the middle) He started to calculate other dots - said he had to do that to figure out what the (unlabelled) arrow was for.
- Page 1 (string diagrams with $\chi \in A$, etc.) He couldn't remember what "c" was for and placed x in the B only string instead of A and B.
- Page 13 (draw < arrows). He correctly ordered 8+3, 8, $\hat{8+3}$ and $\hat{8+3}$.
- Page 14 (problem like 85+71=) Responded 15 instead of 14, but gave the correct explanation for the calculation.

4AVI
- Page 2 (solution set)-Said "put a 4 in there" (box in 1+[]=3+2). I said "It says 'put a dot for'." Explained in full and correctly.
- Page 5 $\frac{1}{3}x69$ (69+2) = with appropriate Minicomputer picture). I covered up all clues and asked him answer. He got 13 - subsequent discussion showed he actually worked out $\frac{1}{3}x39=$.
- Page 11 (series of calculations, paired as in 5x2 and 5x20) Asked him for only 5x20. He responded correctly.

Student 14

A. Teacher's Views

Student is a little immature. Both of her parents work and this necessitates her being left at a nursery school at 6:30 AM (before school) and returned there for one-half hour after school.

She is an average student for this class, in one of the middle reading groups. Patty is better in math than in reading but she doesn't work up to her capability and has poor study habits. She has a short attention span. She doesn't persevere in her work--she only did one and one-half workbooks from the last series.

B. Mean percentile rank of tests through November = 74.

Student 14, Interview Summary #1

Behavior and Appearance

Student is a cheerful, outgoing second grader. She did not appear to be nervous during the interview. She appeared to be confident in her work, especially on the Minicomputer but very self-conscious of all of her estimates.

Summary of Responses

She had no difficulty reading or writing numbers on the Minicomputer. Her estimates (given very reluctantly) were all off target except for the first addition problem (37+16), which she estimated the answer to be around 46. She can add two numbers with and without the Mincomputer but she can not subtract with or without the MC. Making plays on the MC did not seem to pose any problems for her, although she does make all the forward (binary) moves on one board (saying each move as she does it) without thinking of making the special play or carrying move (ie. 8+2=10) and hence has to make backwards plays to get checkers back on the 2's square.

Using the MC on the subtraction problem 61-18, she put 61 on the MC and made backwards moves saying "twenty is 10 plus a negative 10," "40 is 20 plus a negative 20." She did not seem to understand subtraction on the MC. When given the problem 56-39 and paper and pencil she arrived at the answer 53 (subtracting 6 from 9 and 3 from 8 (?).

During the "Conservation" questions I'm not sure if she was paying attention to the first move I made as she said the number was larger by 4. On all the other moves I made she gave the correct response of larger or smaller but her response to "by how much" were 2 instead of 4, 14 (18), 14 (100), 14 (9). I neglected to make a correct backwards play. She made all the forward plays and arrived at 800 for her answer and then made 4 backward plays. When I asked what number did the MC show now she said "700 and ____ - I don't know."

Student 14

When given $\frac{1}{2}$x32, student said she could solve it by using the MC, then by drawing two sets and allocating dots to each set. She did not come up with using the distributive property. She did solve the problem using the MC. When given the number sentence $\frac{1}{2}$x32=16 and asked for another number story, I could not get another one from her even though I gave her 2x16=32. She wrote 2x32=16 and when asked if that's true she said "it was a false number story."

She knew what 2x4 and 3x3 were and for 4x9 she started drawing 4 rows with 9 circles in each row, then she wrote 9+9+9+9 and said she could figure it out on the MC. I asked her to show me 4x9 using the crossing point picture which she quickly drew. For 2x27 she wrote 27+27. When asked for another way to write 27 she started writing 1+1+.... and was going to write 27-1's. I could not get her to use the distributive property.

Uncertainties: Conservation question should be repeated. I don't think her answers gave a true picture of her understanding of the MC. Multiplication using the distributive property should also be repeated in a different way. Writing number stories should also be repeated in a different question.

Follow-Up: Subtraction questions (with and without MC) should be looked at on a later date.

Student 14, Interview Summary #2

Behavior and Appearance

Student was in a cheerful mood today and had to tell me about her weekend.

Summary of Responses

I showed her the first arrow diagram and asked her to label the dots which she did without any trouble. On the second arrow diagram, she asked with what number should she start, and I replied "With any number you want to." She started with 14 and correctly labeled the other dots. I neglected to ask her to draw any more arrows. When asked to build a road from 5 to 12 she did it correctly using a +2, +3, and a +2 arrow.

Given 53-17=☐ and asked which arrow diagram would help her find the answer she first pointed to 70 ←(+17)– 53. I asked her, "What is the answer then?" and she said, "No, this one" and pointed to the correct diagram and said, "It's 36." Asked which diagram would help you solve 2x54=☐ she picked 54 ←(½x)– 27 and said the answer was 27. I neglected to ask her for an equation for 53 ←(+17)– 70.

She correctly identified the children who received the most candy, the least candy, and who gave the most candy.

When shown the string diagrams, she said, "Oh," and mentioned she liked to draw her own. She correctly identified the dots and gave the attributes without being asked. (She also labeled the dots with names of relatives who fit the description.) When asked where would I (the interviewer) belong, she pointed to wears glasses and tall (neglecting the boy part).

Student 14

She correctly identified the largest number 27, and the smallest number, negative 33. When asked what is 4+$\hat{7}$ she answered "I think negative 3." I told her to check her answer and she drew a diagram with 0's and $\hat{0}$'s. When shown Bill's and Mary's scores (4 and $\hat{3}$), she said Bill won by 5. While figuring out "by how many" she started counting on her fingers, 4,3,2,1,$\hat{1}$,$\hat{2}$,$\hat{3}$, and then looking at her hands (put together) she said, "have to take away the 4, so that's 5." I think she miscounted her fingers and would have said 6, still neglecting to count 0.

When shown the open sentence $\square+\square=10$, she said "that's easy" and wrote 5 in each square. To the question "why is that true" she said "because 5+5=10." You could draw dots to show it. When I asked if there were any other numbers that will make it true, she said, "Yes, if you make the sentence longer, and have more boxes, you can put 1+1+..." She looked at the second open sentence and put in an 8+2. I asked her to read it and she said "8+2=10. Oh!" She erased the 8+2 and put in a 7+1. When asked, she gave 6+2, 5+3, 4+4 as other answers.

As a check on some questions from the first interview, I repeated the questions involving moves on the MC. Again, when I made a legitimate forward move she said the number was larger, when a backward move was made she said the number was smaller. In each case, when asked by how much, she said 20 and 1 respectively. (as if I had just moved a checker to an adjacent square). When I took a checker from the 20's square and moved it to the 30's square, she said "larger...by 10 - no, 30." I asked her what is 3x14 and to do it in her head. She thought for a while and said I can't add the second 14. I asked her where she was now and she said 28+14. She was trying to add 14+14+14 in her head. I asked if she could try to do it differently. She said "No."

Uncertainties: Writing number stories for a given string diagram and a given arrow diagram should be checked.

Follow-Up: Weakest points shown in this interview seem to be in using arrow diagrams to solve multiplication problems, lack of use of distributive property to solve multiplication problems and size of difference between a positive and negative number.

Interview took place in an empty classroom. It was chilly in the room.

Student 14, Interview Summary #3

Behavior and Appearance

Student was in a cheerful mood again today. She had a difficult time staying with the questions and not telling stories about her relatives and how she plays. She did mention she uses magic peanuts at the grocery store and that she does draw pictures of sets at home.

Summary of Responses

She wanted to use the MC to find 56-39 and did find the correct answer. She does not seem to have any problems with the forward or backward moves on the MC. With both problems involving parentheses she did the operations inside the parentheses

tudent 14

first but in the first problem she wrote on paper 15 and got 11. She knew $\frac{1}{2}$x12 and with $\frac{1}{3}$x18 she had to distribute the dots to 3 strings to find the answers. She said $\frac{1}{2}$x62 was 13 and I asked $\frac{1}{2}$x6= "3", $\frac{1}{2}$x60= "30", $\frac{1}{2}$x62= "31". For $\frac{1}{2}$x74 she could not do it by herself and was going to distribute dots.

For the combinatorics she found two different ways of spending exactly 10¢ (0G, 2S, 1C) and (2G, 1S, 0C) but could not find any others. When asked how many "pieces" of gum she would get after the second solution, she said 2.

She found answers quickly to 2x66 and 28+38 on the arrow diagram. She could not find 114+18. I then drew a return arrow which she correctly named as +18 and she then started to name all the other return arrows in the picture. She then found $\frac{1}{2}$x114, and 57+95. She drew all the other $\frac{1}{2}$x arrows but could not name the arrow from 7 to 28.

She did not have any difficulty figuring out the score. She correctly ordered the players. For the question What is the difference between Tom's score and Mary's score she said "Mary's score is a negative number and Tom's isn't." She eventually said Mary had to make 11 more baskets to get the same score as Tom. She correctly identified who took the most and fewest shots. She correctly found the boys score by counting the red dots in all the cards and then counting the blue dots and wrote 14+$\hat{12}$=2. She was not able to find out how many shots Bob made for a score of $\hat{2}$.

Uncertainties: I neglected in the integer problem to ask who was the best shooter, and what Bob's score would be after six times.

Follow-Up: Subtraction without MC.

Interview took place in the cafeteria which was quiet for three-fourths of the interview but was very noisy for the last part.

udent 14, Interview Summary #4

Review of Weakness:

She can do subtraction problems without the MC. Multiplication still poses a slight problem as she wanted to draw a crossing point picture for 2x63. She did use the MC correctly for 2x62. Given a string diagram she wrote three correct number sentences (3x7=21). She could not use arrow diagrams for calculations which could be found by using return arrows or composite arrows.

Attitudes toward CSMP:

She likes art and gym best. After that she wants to go home because her uncle (age 13) doesn't like school so she does not either. The interviewer does not believe this really is her true feeling.

She didn't know how well in school she's doing and didn't know if she was learning a lot in math. Sometimes it's hard and sometimes it's easy. She didn't know how different this math was than math other kids do.

She likes working on workbooks better than with the rest of the class on a regular lesson because there's no one shouting. She has tried to explain the MC to her uncle (an older one) and he didn't believe it. Her mother "doesn't believe the stuff that she's done in the first and second grade" (in admiration).

Now she only likes to use the MC for hard problems.

Sample materials:

Likes		Dislikes	
5RI	pg 1 order	5RI	pg 4 label the dots
	2 + problems		5 using and
	6 x problems	5AIV	pg 3 label the dots
	7 roads		5 x problems
5AIV	pg 4 + problems		6 x table
	6 addition table		10 using and
5SVII	pg 4 + problems		11 label dots
		5SVII	pg 1 label dots and machines
			3 multiplication
			6 multiplication problems
			7 sets
			8 arrow diagrams (esp. drawing return and composition arrows)

Student 15

A. Teacher's Views

Student is highly competitive and very achievement motivated. He is extremely interested in math and particularly in competing and getting ahead of the other students in math.

Student's ability is above the material he is bei given so the teacher has trouble keeping him interested and keeping up with him. He has had perfect attendance both last year and this year.

Peer relationships are a problem for student because he tends to "lord it over them." The friends he does have are like him: bright and competitive.

Both of his parents are teachers.

B. Mean percentile rank of tests through November 88.5

Student 15, Interview Summary #1

Behavior and Appearance

Student was shorter than average with long hair almost to his eyes. He was quiet, friendly, interested and alert and was able to explain his answers very clearly. He appeared to enjoy the interview; at the end, which coincided with recess and students passing by on their way out, he said it would be fine to keep going (even though he would miss recess). He said he liked the soccer game best of the CSMP activities. He "said" each play that he made on the Mincomputer (correctly) but could probably have proceeded even faster had he chosen not to (and he was encouraged not to).

Summary

Student performed very well on virtually every task. He was able to read and write numbers on the Minicomputer, to add and subtract, to determine whether a number was increased or decreased by moving checkers on Minicomputer (and usually by how much) and understood that the number shown did not change by making a play. He added 63+28 correctly without the MC using the standard written algorithm but when subtracting without MC he got 86-39=53. His estimates for the computations were quick and usually very accurate.

He knew immediately that $\frac{1}{2}$ of 32=16, calculated mentally that $\frac{1}{2}$ of 52=26 (after first giving 25) by $\frac{1}{2}(50+2)$ and that $\frac{1}{2}$ of 64=32 by $\frac{1}{2}(60+4)$. He drew strings to show another way to find $\frac{1}{2}$ of 32. He was asked to show how to find $\frac{1}{2}$ of 78 on MC. but did not know how to proceed (he suggested using ☒ checkers). When asked to find $\frac{1}{2}$ of 32 on MC he showed two sixteens, that is, he showed the result of the manipulations but not the process. He then played forward from two sixteens to get 32 thereby demonstrating that 2x16=32.

Uncertainties: He seems to understand very well the relationship between halving and doubling and given his fine work with all other tasks, he should be asked more questions on use of MC to find $\frac{1}{2}$ or $\frac{1}{3}$ of a number.

Follow-Up: Because of his accurate estimates and his ability to use distributivity in calculating $\frac{1}{2}$ of a number, it would be interesting to probe further his rather extensive abilities in mental calculations.

The interview was conducted in one corner of the empty cafeteria. It was quiet most of the time though at the very beginning and end of the interview, there were some other students and adults in the area, but this did not disturb him in the least.

Student 15, Interview Summary #2

Behavior and Appearance

Student appeared quite comfortable in the interview situation. He was fairly outgoing in his responses, in asking questions to clarify the task, and in showing emotion (pleasure, surprise, disappointment) with some of his responses.

Summary of Responses

The student answered all multiplication questions correctly (including the mental calculations of 4x9 and 2x27). Arrow diagrams were generally well done, though he missed a few arrows that could have been drawn in the first item. He had difficulty in selecting the arrow diagram which would show the answer to 53-17 and to 2x54. This may have been due to a reluctance to select an answer that wasn't an exact equivalent (rather than implied by the potential return arrow) of the problem.

There was no difficulty with the string diagram problem or with the open sentences. Student thought of using negative integers for x+y=8. He first chose $\hat{1}$ and $\hat{7}$ then later put $\hat{1}$ and 9, then $\hat{2}$ with 10 and could have continued indefinitely. There was a problem in vocabulary with the integer problems, the student not being familiar with "negative three" for example, substituting the words "magic peanuts" on several occasions. All questions were eventually answered correctly, though student initially answered a couple of questions incorrectly, making the correction very quickly after being queried.

Using the MC to find $\frac{1}{2}$ of a given number was repeated from the first interview. Student correctly got $\frac{1}{2}$x158, commenting at the conclusion that "Now I know how to do it-when there's two on each of t'...". Asked $\frac{1}{3}$x111. Student had to be reminded of how he did the last one, then proceeded with help at one point, to successful conclusion. Tried $\frac{1}{4}$x36 - after some thinking and a review of what he had just done student said that he wanted "four parts." Started tentatively - then "Oh, yes" very excitedly and completed problem - again with some help at one particularly tricky place.

Student 15

Uncertainties: Using facts given in arrow diagram to find the answers to problems using the undrawn return arrows. Very large numbers should be used since, to this student, the problems given could be solved mentally - hence the intent of the question may not have been clear.

Follow-Up: Vocabulary development and operations with negative numbers.

The interview took place in an empty, quiet cafeteria.

Student 15, Interview Summary #3

Behavior and Appearance

Student enjoyed interview - responded quickly and explained well when asked. Not reluctant to ask for question again or for clarification when uncertain. He was still going strong and ready for more at the end of the rather long interview.

Summary of Responses

Interviewer went directly to three supplemental problems. In the combinatorics problem, student worked well and quickly, using arrow diagrams to find solutions to problems like - spend 10¢, spend 15¢ buying exactly three chocolates, spend 15¢ buying exactly two suckers, without any suckers. The arrow diagrams seemed to simplify the problem for the student and he worked very comfortably with them.

The student answered quickly and correctly each problem of the complicated arrow diagram including return arrows and compositions.

The student was able to calculate all scores in the game involving integers, and to order them and to determine who took the most and least shots and who had the "best chance" of winning. He correctly understood and answered the question "How far apart are they" for two scores but did not know how to figure out the equivalent question "How many more would Cathy (at $\hat{2}$) need to have the same score as Susan (at 3)." In determining whether the girls, as a group, or the boys had the best score, student counted only missed shots - the team with the fewest winning. He could not do the problem of 10 shots with final score $\hat{2}$ (he also persisted in using "magic two" etc. when identifying negative numbers).

Student again showed great facility in mental calculation (78-43, 15-(4+2), $\frac{1}{2}$x74, $\frac{1}{2}$x378!) but was still unable, mentally or with paper and pencil to subtract with borrowing.

Uncertainties:

Follow-Up: Subtraction algorithm with and without MC.

The interview was carried out in an empty cafeteria accompanied by music.

Student 15, Interview Summary #4

In general the student was rather quiet and non-committal towards the various attitudinal questions but quite willing to talk about the problems he had done and which ones he liked best. The partial transcript below indicates that the student was not particularly forthcoming.

I: What things do you like to do in math. No response.
I: Would you rather work on your own with a workbook or with the rest of the class on a lesson when the teacher is at the front of the class?
With the rest of the class.
I: Do you like using the minicomputer? Yes.
I: Do you ever take it home? No - once we made one in class and I took it home.
I: Did your mother ask how to do it? Yes. I showed her.
I: Does she understand it now? No
I: Was it hard to explain it to her? Yes.
I: Are you pretty good in school? I don't know.
I: What subjects are you best in? I don't know.
I: What subjects do you like best? Reading.

The student was aware that the math that the other class had was not the same as their's (both classes are in the same very large area), though he didn't think they were much different. The main differences mentioned were that they didn't have the M.C., that they had only 1 worksheet instead of 2 and that their teacher didn't show them how to do the worksheet - they just started on their own.

I asked the student to give me a problem. He wrote 153-61. When I gave him 736 he wrote 333. I explained that the last 3 seemed to be wrong. He agreed but wasn't able to correct it. He successfully did 65-39 and the problem he had given me. This was the only item repeated from the last interview.

736
-469

The following responses are from 4SVII:

He liked the following: finding the shape that is different, finding ½ of a large number, secret messages, string diagrams, multiplication problems, filling in multiplication and addition tables, number sentences (with empty boxes), a 2x spiral, what could the arrows be for. He thought the following were only "O.K." : building roads - from 7 to 7 with +8, 6 and from 2 to 24 with 4 arrows using +10 and +1, addition problems, solution sets. He didn't like: subtraction facts and drawing figures on a grid with area 6.

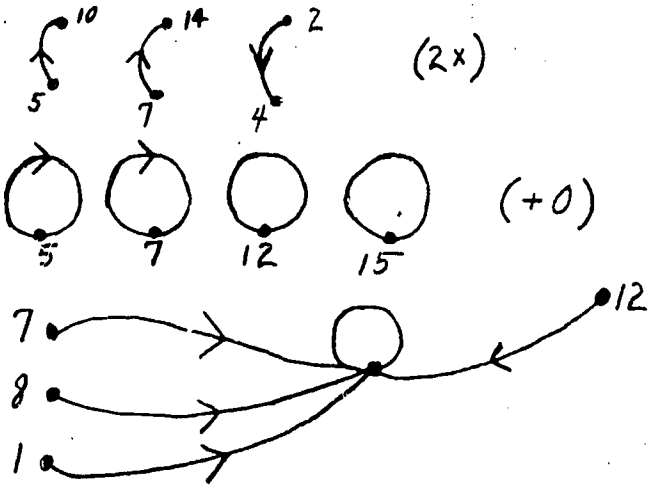
Occasionally I asked him how to do something or what the answer was.

- $\Diamond + \Diamond < 6$
  $\Diamond + \Diamond < \Diamond + 2$
  $\square = \square$

  } for each of these, 5 numbers were given - student had to decide which worked and which didn't. He got all 3 correct.

- $\frac{1}{2}x\square + \frac{1}{2}x\square = 8$: correct

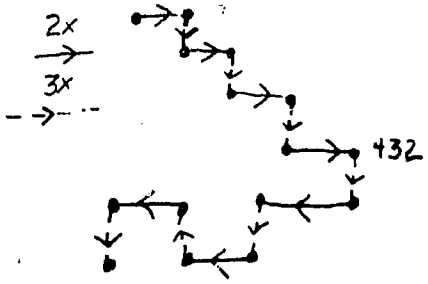
- What could the arrow be for:



(did you know I said "times something?" "Times zero.")

- 3x271=____ (713-done mentally)

- Draw an arrow for 6x



(first drew an arrow from first to about mid-point. Then drew 3 correct adjacent 6x arrows.)

- I asked if arrow diagrams sometimes made things easier. He said "Yes, sometimes." I asked (in context) Can you think of a place where you couldn't build a road? "From 0-15 with two evens." When couldn't you build a road from 0-10? "With a 100 arrow." Laughed, then said +3.

Student apparently no longer uses the minicomputer, even for hard questions, and said that he stopped using it just after Christmas.

Student 16

A. Teacher's Views

Student is a quiet girl but she likes to organize other students. She has good attendance and a student of high ability is a special friend of hers. She especially likes cats.

She likes school in general and math about the same as the other subjects. She is a very good student, doing careful and orderly work. She seems to be even better in math than most other subjects.

B. Mean percentile rank of tests through November = 88.5.

Student 16, Interview Summary #1

Behavior and Appearance:

Neatly dressed and groomed, student seemed to become nervous when she spotted the microphone. She fidgeted some, holding two small toys (a furry tiger and a rubber mouse, I think) in one hand as she worked with the other hand. She was very conscientious - responding thoughtfully to my questions - but she didn't volunteer information not requested. She was aware of the interview taking place behind us and several times recognized a question as one she had already answered or could expect.

Summary of Responses

Student has great facility with the mechanics of the MC. She read, showed added numbers on it very quickly - with no noticeable hesitation. When she "estimated" answers it was usually a case of her "figuring them out in her head." She was very careful in her work and didn't seem to like to "guess" the answer. Her handling of 124+398 needs further study. The estimate was way off but she also made an error while making the moves such that she "lost" 100.

When working with conservation, she seemed to have no problem at all following what was happening to the number. But consistently, when asked "how much larger (or smaller)" she told me what changes had occurred (ie, 8 to 4) but not how much of a change. I have little doubt she knows addition and subtraction facts related to the changes made so I assume that for the question "how much," what I wanted for an answer wasn't clear to her.

Uncertainties: Three digit number addition with estimation of the answer. Can she conceptualize what is a reasonable estimate when dealing with hundreds?

Estimation in subtraction problems (with borrowing). Can she "round off" or use distributivity to arrive at a reasonable guess?

Does she know that moving a checker on the MC from 80 to 40 is 40 less, from 2 to 20 is 18 more, from 10 to 1 is 9 less, etc.?

Follow-Up: Topics to be followed up in subsequent interviews will depend on the clarification of questions noted under uncertainties. After one interview it is unclear whether weaknesses observed were due to lack of understanding of the task or were actually skills student had not yet developed.

Interview took place in regular classroom with other interview also transpiring.

Student 16, Interview Summary #2

Behavior and Appearance

She seemed more relaxed. Very attentive. At times she anticipated a question and gave the answer before it was asked. When I asked what happened to the tiger and the mouse she had brought along last time, she said she left them at home because it was too cold for them. She related (without being asked) that today she got to play with her little sister instead because her mother brought her up to school at recess.

Summary of Responses

She seemed to understand well every topic covered. There was only one point which was not answered as hoped: the addition of a +3 arrow as the same as +1 and +2. She responded quickly, but thoughtfully, and if she thought of an answer that she thought would be better than the one given, she would say so (ie, writing the equation 70-17=53 for the arrow diagram 53 $\overset{+17}{\frown}$ 70 and then decided after looking at the other diagrams that that equation was more appropriate for the diagram above it 70 $\overset{-17}{\frown}$ 53. She then wrote a different equation (53+17=70).

Uncertainties: The only uncertainty is regarding the approaches she uses to multiplication. The question exists only because I overlooked suggesting the use of addition and crossing points. Having looked over the record of Interview #1 I found that Kim suggested 16+16=32 and 2x16=32 as other ways of writing the problem $\frac{1}{2}$x32. This indicates that the similarities or "relatedness" of the processes are understood to some degree.

Follow-Up: It seems appropriate to begin using more complex and, perhaps, multiple step problems in future interviews in order to gain insight into how she puts together the repertoire of "tools" with which she seems to have facility.

Student 16, Interview Summary #3

Behavior and Appearance

Student was glad to be able to go with me for individual work because - she later told me - the teacher doesn't call on her in math class because teacher knows she gets the answers right. Even though the interview extended beyond 45 minutes, student seemed ready to answer whatever questions I asked.

Student 16

Summary of Responses

Student did the double digit addition and subtraction without carrying or borrowing mentally, and she used paper and pencil for 73+49 correctly. She made an error in calculating 56-39 mentally as 23 and did not want to do it on the MC or on paper since she "already did it." She knew to do the operations inside the parentheses first and did the calculations mentally. She made an error in adding 16+6(=23), but seemed to recognize the correct answer when told it was 22 (because 6+6=12). She did $\frac{1}{2}$x12, $\frac{1}{3}$x18, $\frac{1}{2}$x62, $\frac{1}{2}$x74 mentally and correctly, verifying two of them on the Minicomputer.

She did both the combinatorics problems on buying candy and the detective story on using arrow diagrams correctly throughout. She also did correctly all but the last two parts of the integer problem. She thought that Bill would have to make 8 out of 10 to end up with a score of $\hat{2}$ and she thought his total score after 6 games of $\hat{2}$ would be $\hat{2}$.

Student 16, Interview Summary #4

Behavior and Appearance

Student was anxious, as usual, to work on special problems. She was confident in her ability to do anything she had been taught.

Summary of Responses

Student showed that she could ably handle the variety of calculations she was given. She seems to have acquired some flexibility in using one method or another to solve a problem, but this needs to be encouraged even more. She was frustrated by not knowing how to get the exact answer to $\frac{1}{2}$x45. (She must not be used to being asked problems she can't answer correctly.)

Other responses:

1/3 X 15	✓	Did this mentally.
1/3 X 27	✓	Used strings.
1/2 X 96	✓	Started to use strings. I. noted that it would take a lot of dots and asked if there was another way she could do it faster. Student then put 96 on the MC and made backward plays.
1/2 X 45		Said the answer was between 20 and 25. I. asked her to use the MC to see if she could find the exact answer. She made the plays, paired the checkers, but didn't know what to do with the extra one.
56 - 37	✓	Set up the problem vertically; she crossed out the 5 and changed it to 4, put the 1 ten in front of 6, then did the subtraction out loud correctly.
72 - 45	✓	Used the same process as above.
6 - 7	✓	Quickly wrote the answer correctly.

Student 16

Attitudinal Information

She thinks she is doing okay in school in general and in math specifically, but didn't know if she was better in math than other subjects (because she does "okay in all"). She thinks she is learning a lot in math and can do things that children in other classes (and in her own) cannot do - such as working with negative numbers.

She likes working with workbooks and worksheets. She likes these better than class activities because she doesn't get a chance to give answers in class very often (because her teacher knows she knows the answer).

She said that she doesn't use her MC anymore because she can do the problems in her head (this seemed to include using string pictures and paper and pencil, however). She sometimes works math problems at home. (It seems that she shows her parents and younger sister what hard problems she can do but she hasn't really shown them the method(s) used.) She doesn't think that it would be hard to teach her parents how to use the MC and other methods, though.

When I paged through the workbooks and worksheets, the ones she indicated she liked seemed to be those with more difficult problems on them. She chose those with a lot of calculation problems and I was given the impression that she thought the more simple arrow pictures, combinatorics, number sentences, etc. were a little beneath her ability and interest.

Student 17

A. Teacher's Views (1/17/75)

Student is not shy but he is quiet and firm in class. He plays with other bright students and is aggressive at play. Outstanding in all school subjects, perhaps more so in math. He has always liked math and is in the highest group for reading.

B. Mean percentile rank of tests through November = 93.

Student 17, Interview Summary #1

Behavior and Appearance

Student was a friendly and cooperative boy, of about average size. He was quite willing to chat in a very quiet, conversational tone. He appeared confident in what he was doing and in firm control of the situation. Very little hesitancy in responding and very few things like "Wait, no, that should be...etc.," ie. stuck with his answer which was usually right. Did not hesitate in asking for pen or "Can I do this?"

Summary of Responses

This student displayed a quick and sure understanding of the minicomputer and how to use it. He put numbers on the minicomputer, "read" it, and added, subtracted and took ½ of with it without error. With the exception of one (probable) slip he knew how numbers changed ordinally by moving checkers though he responded to questions like "How much smaller?" by naming the value of the square to which the checker was moved. He made rapid forward and backward plays (and said later that he liked making plays) and knew immediately that the number shown on the minicomputer had not changed during the intervening 4 backward moves.

He correctly used the traditional written algorithm for addition but for 86-39 (which he wrote in vertical format) he put 53. When asked to subtract 89-36 he answered correctly 53, was shown the discrepancy but could not figure it out. He displayed an amazing facility by mentally getting ½ of 32 (by 30+2) and then, still mentally, ½ of 76 by (50+20+6). After some prodding, he showed the corresponding number stories for ½x32=16. Without clues he showed 3 methods of halving.

Uncertainties:

Follow Up: 1. Development of written subtraction algorithm
2. "By how much" for change in number shown on minicomputer as a result of moving checkers.

Interview required 17 minutes and took place in quiet, otherwise vacant, storeroom.

Student 17, Interview Summary #2

Behavior and Appearance

Student was more talkative for this second interview. Though he had more difficulties than previously, he seemed to enjoy the interview more. At the end of the interview, he asked if the interviewer was going to return and said he would like him to return. On two or three occasions he changed his incorrect answer almost immediately to the correct one.

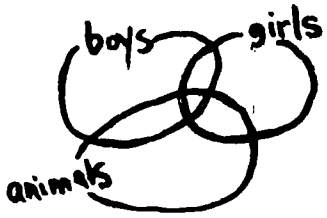
Summary of Responses

Student was very good at multiplying. He did mentally 2x4, 3x3, 4x9 (by 18+18) and 2x27. He spontaneously used a set picture and a (vertical) addition to demonstrate (using 2x18 in both cases). When asked to do it on the MC he was successful, but when asked to use crossing points he drew an 8x9 picture. When queried he corrected and was able to show that part of the picture showed 2x9.

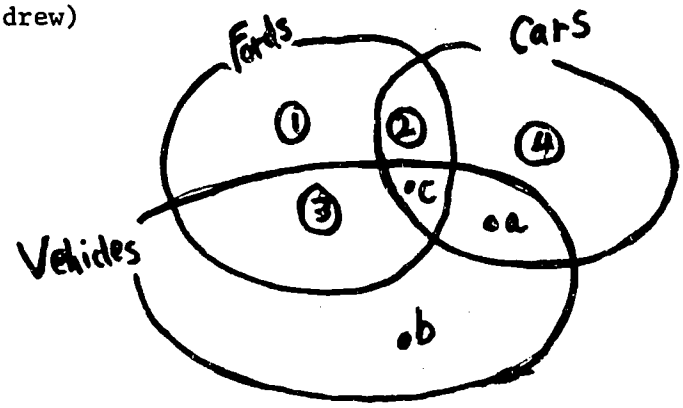
He was able to correctly label dots, to build a road and to write the equation for a given arrow. He could not find any of the +3 composite arrows. He was not able to select the arrow required to solve the two given open sentences nor could he give the equation which the return arrow of a given arrow would define (supplementary question). In the 5 dot, 8 arrow diagram for "I give you a candy," he chose the two dots with one incoming arrow instead of the dot with no incoming arrow for "gets the fewest" and repeated this error later.

On the intersecting strings he made only one slip, when placing the dot for Mary but corrected this after a query. With negative numbers he made only one error; when asked how much Bill won by he responded 1, then 4, then 4 or 5. On the open sentences his answers were all correct; when asked if "one and seven" (with pointing) was "the same answer as seven and one"(with pointing) he answered affirmatively. He did, however, proceed to give all 9 correct number pairs.

At the end of these questions I asked him what he liked to do best and he said "string pictures." When asked to make up a problem he did the following

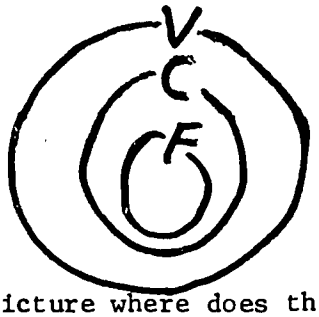


S: I am a boy. Where am I.
I: (pointed correctly). I am a tall red rooster?(correct). Short red rooster?(correct) What part can't have any dots?
S: (pointed to 3 way intersection)
I: (drew)



I: (after explaining vehicles): Red chevrolet?(correct). Blue truck?(correct) Yellow Ford?(incorrect, then correct). Any places that can't have dots?(showed (1), (2), (3), then (4) (the last two after being asked to test each remaining region). Then I drew:

Student 17



I: Now on this picture where does this go (pointed to dot "a" from previous diagram) (correct). Then dot b and dot c (both correct).

Uncertainties

1. Familiarities with crossing points (especially construction). Does he draw same picture for 3x7 and 7x3?
2. Diagrams like the candy one above. He may very well have simply responded too quickly.

Follow-Up

1. The use and interpretation of return arrows.
2. The drawing of composite arrows.
3. The difference between (or "how much bigger?" etc.) a positive and negative integ

The time required was exactly 20 minutes. Same room as previously. During the first few minutes there were children coming in and out for books with considerable talking, but the student seemed oblivious. (Interviewer was himself not aware of it until listening to the tape.)

Student 17, Interview Summary #3

Behavior and Appearance

S seemed to work at ease even with an unfamiliar interviewer. There was considerable noise in the cafeteria area where the interviews were taking place.

Summary of Responses

Student was asked all the items from Part 2 and Part 3a, 3b, and 3c. He did all parts successfully, with only occasional errors. Thus it is more convenient here to simply describe those few errors he did make.

S did all but one of the computation problems from Part 1 mentally. However, he answered $\frac{1}{2}$x18 as if it were $\frac{1}{3}$x18. When I. pointed out what the problem really was he used strings to figure out the answer.

S seemed inclined to answer questions before they were asked completely or without listening to the whole question. This, I think, was what occurred when he answered $\frac{1}{2}$x18 instead of $\frac{1}{3}$x18, and also when he was asked how many pieces of each item he could get for 10¢. His first response was to buy one of each: gum, sucker, and candy. When I emphasized that he must spend the whole 10¢ he admitted that he would have 1¢ left over. A similar example occurred when S wrote the answer to Susan's score $\hat{(4+1)}$. He first wrote $\hat{3}$ and then later, after I. questioned him about whether he was sure he changed the answer to 3.

Student 17

The only problem for which S couldn't think of an answer was dealing with the number of baskets Billy made out of his ten shots to end up with a score of 2. He didn't seem to be able to conceptualize how to figure a problem stated in such a way.

Overall his performance was exceptional and evidenced considerable insight. He seemed ready to try all but the most unusual problems.

Student 17, Interview Summary #4

Appearance and Behavior

Student appeared slightly apprehensive at the beginning of the interview but soon relaxed and responded well. He gave good answers, explained well when asked and was not bothered when he got stuck on a problem or got a wrong answer.

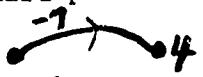
Summary of Responses

Student was asked to make up any problem he wanted for me. He wrote 52, 20 / +99 / -15 and would have continued along that line. I asked him what 100+52 was. "152." Then what is 99+52? "151." I then asked him if he could do 52 / -99 . He said he didn't know how. (after first attempting 99-52). How about this one? 1 / -3 He said "negative two" and then mentally and correctly calculated 52-99 (negative 47).

He said he liked spelling and reading in school and didn't like writing (he was just learning cursive). He said, when asked, that he liked math and especially the arrow diagrams. He doesn't use the MC any more, saying he prefers to use paper and pencil. He said the class hasn't used the MC since they moved to their new room (two months ago).

He prefers workbooks to teacher-led lessons. He has taken the MC home; his mother now knows how to do it but not his father. He has a sister in first grade studying CSMP who is "really good at it" and a brother in fifth grade who knows it because he learned it in school last year (??)

We then went through workbooks 4AVI and 4SVII. Page 1 (4AVI) was a road problem. I asked if he could make up a problem that couldn't be done-he gave "a road from 6 to 8 with +6 and +8 arrows-then almost immediately said that wouldn't work because of first a +6 arrow to get to zero, then a +8 arrow. He correctly explained how to do page 3 on solution sets. On page 4 I asked him to do 163 in his head, which he did. On page 5 I asked whether he would rather use a MC or paper and pencil to do problems like 1/3x69. +163 He said he could do it in his head and did so correctly. Also for 1/3x93. He explained how to do the flag coloring problem on page 7. At one point he said certain questions were easy - I asked whether he liked easy problems or hard problems. He said "hard ones"

On page 9 he erred in labelling the dot in  as 3. On page 10 I asked him to draw the +3 arrows where possible. He said the arrows could go both ways between 3 and 6. I convinced him otherwise but he repeated the assertion for 2 and 5 but saw it didn't work when he went through it. He explained that they had recently worked with arrows that went both ways.

Student 17

On page 12, I asked what the big arrow could be for in 
He calculated each dot then, getting 22 for last dot said +20. I then covered up the 5 in  and asked what the big arrow could be for. He said 20 (5, which he had spotted before I covered it, +15=20). He got rather mixed up on this one. We tried a third one where he could not see the dot - he couldn't do it.

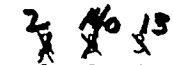
On page 14 he was able to order correctly 28+53, 27+53, 28+53 and 28+53.

We then went to Workbook 4SVII. On page 6, he correctly pointed at "impossible" regions in the boy-girl-pickle string picture. He correctly (with one or two slips which he corrected after a query) did the first and second problems of the solution sets on page 5. I asked if he liked the subtraction problems (number facts) on page 7 and he responded "yes," but unenthusiastically.

I asked if he found them boring and he agreed that he did.

We went to page 14 - to "What could the arrow be for?" He couldn't get the first arrow (2x). He got the second one (+0). When I said it could also be times something he said "times zero" then corrected to "times one." He got the third one (times zero).

We went to workbook 5SVII. On page 2 he showed me how to do the matching of congruent shapes. The last problem I gave him was a repeat from the last interview - a subtraction algorithm problem 353 - 186 which he did as follows:



```
  2  10  13
  3   5   3
- 1   8   6
-----------
  1   6   7
```

Throughout the perusal of workbooks he was asked if he liked to do the page in question. He always responded "yes." I said "I can't seem to find anything that you don't like to do" and he smiled broadly. At the end of the interview I asked if he liked math "yes." Do you like it better than reading? "Yes." This was in conflict to his earlier answer but he stuck to it when challenged.

Student 18

A. Teacher's Views

S generally does above average work in school subjects, average in math. She seems to like school in general and math specifically. She hardly ever misses school - probably not even one day this year.

It seems that student works better in a group than by herself. She is generally outgoing. She especially likes music, Brownies, outdoor activities. And she seems to get along with just about everyone in her class.

Student lives only with her mother.

B. Mean percentile rank of tests through November = 94.

udent 18, Interview Summary #1

Behavior and Appearance

Student is a cheerful girl who appeared a little shy at the beginning of the interview. She works very quickly at a given task, sometimes saying the wrong thing while making the right moves. She seems to take a clue from interviewer's delay (intentional or unintentional) to check her work. She was very attentive to everything I said. She seemed at ease about estimating her answers before she solved them. She was not overly enthusiastic about doing the math but did not mind it at all.

Summary of Responses

She had no problem reading or writing numbers on the MC. She can add with the MC although I think she is careless in reading her answers. All of her estimates were fairly good ones even in subtraction. For the first addition problem which she attempted (37+16) she put 37 on the MC and then put 61 on, realized her mistake and moved the checkers around (to show 37+11) and then arrived at an answer of 48. She used the MC to add 124+398 correctly but she misread the MC and said 422, when I repeated the answer she said 422 again and then corrected herself and said 522. I gave her another addition problem 57+18 which she did correctly on the MC.

She could use the MC to do the subtraction problems but again she misread her answer and said 61-18=42 instead of 43. She could not do the subtraction problem 86-39 correctly, instead she subtracted 6 from 9 and 3 from 8 for an answer of 53.

She correctly identified all but one of the moves I made on the Minicomputer. The first time I made a backward play (4=2+2) she said the number was larger by 4. Later on she correctly identified the backward play of 40=20+20. She made the forward plays on the MC to arrive at 800. She made 8+2 (80+20) moves as soon as they were available. Her responses to how much smaller or larger the number

Student 18

was after the moves were fairly accurate estimates 80 (40), 20 (19), 100 (100) and 19 (9). After making 4 backward moves she had to figure out what number was on the MC (she did fairly quickly) and then made 3 more backwards moves and again had to figure out what number was on the MC.

She could read the card $\frac{1}{2}$x32 and when asked how she would solve it, she responded first by allocating dots to two sets, secondly by using MC. When asked to do it in her head she responded "I wouldn't know what to do." She found the answer using the MC and then started to show me how to find the answer using the string diagrams. When asked for another number sentence she first wrote down 16+16=32 and then 2x16=32. She felt the best way to show someone $\frac{1}{2}$x32 was using the string diagrams.

Uncertainties: If questioned further on how to find $\frac{1}{2}$x32 mentally she might be able to do it. This should be checked in the second interview.

Follow Up: Conservation of numbers on MC and subtraction without the MC should be checked at a later date.

Interview was conducted in the nurse's office, a small, quiet room.

Student 18, Interview Summary #2

Behavior and Appearance

Student appeared to be a little more enthusiastic when we started the interview. After both interviews she was most helpful in gathering up the materials on the table.

Summary of Responses

Student did not have any problem with 2x4 or 3x3, she did not know 4x9 and she said she could find it by "counting on my fingers" (repeated addition). When suggested she described in detail how she would use the MC and allocating dots to sets, and found the answer using crossing points. Given a string diagram she wrote 3 number sentences, starting with $\frac{1}{2}$x12=6 and when given 24+24+24=72 she wrote the other two number sentences. She solved 2x27=54 mentally by adding 27+27 and "carrying". When given 3x14 she added 14+14=28 and 28+14=32 (mentally).

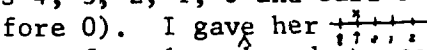
She had no problem labeling the dots and started the second picture at 2 and drew 2 more arrows +3 and +2. For her road she used +3, +2, +2.

She didn't have any problem in using the arrow diagrams to find the answers for 53-17 and 2x54 and she correctly wrote the equation 53+17=70.

She correctly identified the child who got the most candy and the one who gave out the most. For the child who received the fewest candies, she chose the child who received one and not the child who didn't receive any.

She correctly identified the dots in the string diagram but did not volunteer all the attributes for the dots selected. For interviewer (since she did not consider him to be tall) she correctly placed a dot. I neglected to ask how many boys. She drew a diagram similar to one given and asked interviewer questions and corrected interviewer for dot which belonged in the three-way intersection.

Student 18

She correctly identified and named the largest and smallest integers (27, $\hat{3}3$). Without any hesitation she correctly said $4+\hat{7}=\hat{3}$. She said Bill won the game by 7 she thought but when asked to make sure she wrote $\hat{3}\ \hat{2}\ \hat{1}\ 0\ 1\ 2\ 3\ 4$ and counted the numbers 4, 3, 2, 1, 0 and said she could only get 5 (she did not count the numbers before 0). I gave her  and asked how many steps would it take to get from 6 to $\hat{1}$ and she correctly counted 7.

Five was the correct answer for the first equation because "5 is the only number you can add to itself and get 10." There wasn't any other answer because the boxes wanted the same number.

She started the second equation with 7 and 1 and proceeded to name 6,2 - 5,3 - 4,4 - 3,5 - 2,6 and 7,1. She said there is no difference between 7,1 and 1,7 "they're just switched around."

Uncertainties: In the arrow diagram, she might have interpreted the question who got the fewest pieces of candy to mean "Of those who received candy who got the least?"

Follow-Up: Difference in scores 4 and $\hat{3}$.

Interview took 20 minutes and was given in an empty classroom - no interruptions.

Student 18, Interview Summary #3

Behavior and Appearance

Student was interested in the tasks given during this interview. She was not shy this time.

Summary of Responses

Said she forgot how to do 78-34 on paper but did get correct answer using MC. She did both problems involving parenthesis mentally. The first time she did 15-(4+2) she got 10 but when she recounted (on fingers) she got the correct answer. I neglected to ask any questions about $\frac{1}{2}x$ and $\frac{1}{3}x$.

Part 3a: She could not get one solution to the combination problem even after I gave her an example of 2X and 1C.

Part 3b: She could use the arrow diagram directly to solve the first two problems. For the first problem, 114+18, she did not come up with the return arrow by herself and could not name the return arrow when drawn. She did not get $\frac{1}{2}$ x114 until I drew return arrow and she correctly named the return arrow. Now she correctly found 57+85. She drew all the remaining $\frac{1}{2}x$ arrows but could not come up with the 4x arrow although she did name it +21.

Part 3c: She quickly understood the score cards and correctly found the rest of the scores although she did forget to put "^" on a couple of scores. She correctly ordered the scores and identified how many more baskets Cathy needed; the difference between Tom and Mary's score, the most shots and the fewest shots.

Student 18

She said (incorrectly) that Tom had the best chance of making one shot. To find the boys' and girls' scores she added the boys' final score and the girls' final score. She could not find out how many shots Bob made to have a score of $\hat{2}$. She mentally figured out $6x\hat{2}=1\hat{2}$.

Uncertainties: The combination problem, composition arrows and return arrows (involving + and -) are areas that should be explored. Rational numbers were not asked this time and should have been.

Follow-up: Subtraction without MC and problem solving.

Interview took place in nurse's office, no interruptions.

Student 18, Interview Summary #4

Review:

She can now do subtraction problems without MC. She would rather not use MC but she likes it and would use it on hard problems. She found $\frac{1}{2}x64=32$ without any hesitation but for $\frac{1}{2}x74$ she said she would need the MC. She can use an arrow diagram to solve problems without any difficulty. She found the answer directly; using a return arrow and naming the +18 arrows. When asked how she knew she said I added 12+6. She made two roads using +2,+3, and +4 arrows from 0-10 (+3,+3,+4) (+3,+3,+2,+2). From CSMP test, interviewer knew this student could use MC to do all calculations.

She didn't know how well she does in school. Does best in spelling but likes math best. Feels she's learning a lot in math. Doesn't know if it's different math. She liked most of the activities in the workbook and says most of it is "so easy." She's tried to explain the MC to parents but it's hard to explain. The hard problems for her are $\frac{1}{2}x$ an odd number. She correctly identified the odd and even numbers from 1 to 10 but could not describe it. Likes to do addition problems best and would rather work in workbooks than on a lesson.

Sample materials:

Likes

5RI	pg 4	labelling dots
	5	using and (loves doing them)
	7	roads
5RII	pg 3	labelling dots
	6	shading figures
	7	white squares on MC
5RIII	pg 1	multiplication table
	3	secret messages (loves and does first)
	4	label dots
	5	sets
	7	+ problems
	12	- problems
	8	label dots
	10	label dots
	11	roads
	14	x table

IV. Summary

The purpose of these interviews was to investigate how children thought about and understood CSMP math. At the end of this section a brief summarization of some of the content areas is given, but this barely scratches the surface of the available information. Certainly a number of issues might be raised and conjectures made, but as noted in the introduction the reader will have to do that himself, and one suspects there might be considerable disagreement among different readers.

The interviews yielded disappointingly little information about how these students felt about CSMP. While these students seemed to like their math, they did not volunteer much regarding their reasons. Many responses were rather unenthusiastic, one-word answers, although they were usually rather chatty about other things, such as what they did over the week-end, and they seemed to enjoy the interviews and showing what they could do. Perhaps this is not surprising. After all this was their regular math program - the same one they had had since beginning first grade - certainly nothing experimental or new (because every subject was to an extent new to them) or different (the students were only vaguely aware, if at all, that their math was any different than what some other second graders in their school were studying). Most of the students, when asked, said they had taken CSMP material home for their parents and almost all students preferred working on their own in their workbooks to whole group lessons.

In the Topic Area summaries which follow below, the last area, "Minicomputer" is based on Interview 1 which took place about half way through the year and does not reflect any improvements from that point. The Minicomputer tasks were more or less hierarchical in nature; others were not in that some students were good at some things and not others, and vice versa for some other students.

Summary of Performance in Topic Areas

Reading and Showing Numbers on the Minicomputer

Reading a given configuration on a Minicomputer was more difficult than putting checkers on a Minicomputer to show a given number. During the first interview it was found that all of the students could show some numbers on the Minicomputer, although two (#2,#3) could not show the number 901. One of these (#2) also seemed to be unsure about on which board unit's and ten's digits belonged, but could usually correct himself to arrive at the right configuration of checkers.

Two other students (#1,#9) could show numbers on the Minicomputer but could not read numbers that were presented to them by the interviewer. Through more specific questioning in subsequent interviews it was found that one of these (#1) could not even read correctly some numbers that were printed on paper. This same student could read numbers on the Minicomputer if they had only one checker per board, but configurations for most odd numbers were read either as one higher or lower. The other student (#9) showed during Interview 3 that she also could read only configurations that required one checker per board. But in the final interview she correctly read numbers that required two or more checkers on two or more (of the three) boards used. (By this time she could also use the Minicomputer to do addition problems involving carrying from the ones to the tens board, so there was evidence of substantial improvement.)

Addition

In Interview 1 it was found that 11 students could use the Minicomputer to do addition problems. Those who could not were the four students (#1,#2,#3,#9) who had problems reading or showing numbers on the Minicomputer, plus students #4, #5 and #7. All of these were unable to successfully make simple forward plays, certainly contributing to their lack of success with addition. In subsequent interviews only one of these students (#9) showed she had acquired skill to use the Minicomputer to solve addition problems. By the third interview she showed marked improvement, it being the fourth interview before she seemed to fully understand the process. (During this time there had been little in the way of formal class lessons on the use of the Minicomputer for addition, but the workbooks and worksheets provided opportunities for review and practice.)

While these seven students did not seem to understand the process for computing addition with the Minicomputer they all showed knowledge of the addition process by giving correct answers to the "addition facts" (7+2, 4+3, 9+6, 8+9) given in Interview 3, with two of them (#5, #7) even doing two-digit addition without carrying at that time. Three (#1, #2, #7) showed some understanding of the carrying process in addition by correctly answering all of the problems involving carrying in these interviews.

Seven students (#8,#11,#12,#14,#15,#17,#18) who had had no difficulty doing addition on the Minicomputer, could also calculate the answers to addition problems with and without carrying on paper during the first interview, and they repeated this skill (sometimes doing the calculations mentally) during subsequent interviews.

Subtraction

Six students (#8,#11,#15,#16,#17,#18) could do subtraction problems on the Minicomputer by themselves in the first interview. The twelve who could not were the seven students who could not use the Minicomputer to do addition problems (#1,#2,#3,#9,#14,#15,#17), one who had been able to use the Minicomputer for some addition and forward plays but not for backward plays (#10) and four other students for whom this was the first weakness detected (#6,#12,#13,#14). Of these 12 students, two (#10 and #14) showed improvement in using the Minicomputer for subtraction in Interviews 3 and 4.

All of these 12 students, however, did not similarly lack skill in doing subtraction on paper (without the Minicomputer). By Interview 3, students #2, #3, #4 showed knowledge of simple subtraction facts (8-3, 9-7, 15-8), students #1, #7 and #13 showed skill in doing some more difficult problems, while #6, #9, #10 and #11 did all of the more difficult problems not involving borrowing. By Interview 4, #5 finally showed she had acquired an understanding of subtraction not involving borrowing.

Subtraction with Borrowing

Understanding and correctly solving subtraction problems that included borrowing was less widespread, but by Interview 4, six of the students (#8, #11, #14, #16, #17, #18) were doing all examples successfully and three (#2, #7, #15) did one or more of the problems asked of them correctly.

Half of the students (#1,#3,#4,#5,#6,#9,#10,#12,#13) appeared not to have learned the process of subtraction with borrowing by the end of the school year although they seemed to have a basic understanding of subtraction itself since all knew basic facts and five of them could do more difficult problems not involving borrowing. In comparing interview data with test data this is the one area of substantial disagreement. Four of these students (#5,#6,#10,#13) got correct at least three of the four end-of-year items involving subtraction with borrowing.

Multiplication of Whole Numbers

In Interview 1 eleven students (#5,#6,#10,#11,#12,#13,#14,#15,#16,#17,#18) could already find the answers to both the easier and more difficult problems by one or more of the methods with which they were familiar. For the more difficult problems six students chose to sometimes use the Minicomputer to find the answer, seven used strings for at least one problem, and three used series addition. One student (#9) who could not do any multiplication in Interview 2 did show knowledge of multiplication facts during Interview 3.

Three students (#3,#4,#8) showed some mastery of easier multiplication facts during the second interview but they were not questioned with more difficult problems in subsequent interviews. The other three students (#1,#2,#7) could not figure out answers to even the easier multiplication facts (2x4, 3x3).

Multiplication of Whole Numbers by Fractions

Questions related to multiplication involving a fraction produced a similar spread in students. In Interview 1 three students (#1,#4,#12) did not seem to have any idea of how to figure out what one-half or one-third of a number was, and they didn't seem to have acquired this knowledge before the end of school. Four other students (#2,#3,#5,#7) showed they had at least a partial understanding of the process when the numbers were kept small, but they could not calculate the answer when the size of the whole number was larger than could easily be represented by strings and dots. One student (#10) showed no understanding of multiplication with fractions when larger numbers ($\frac{1}{2}$x32) were used in Interview 2, but by Interview 3 he gave evidence of understanding how to solve the easier problems and some of the more difficult ones. The other ten students (#6,#8,#9,#11,#13,#14,#15,#16,#17,#18) showed considerable understanding of the process used to solve such problems. Two used only strings and dots to arrive at a solution and the others used more than one method including mental calculations, the Minicomputer, strings, distributivity, and series addition based on estimation.

Venn Diagrams (String Pictures)

In Interview 1 all of the students seemed to be capable of interpreting a Venn Diagram, with all but one (#2) being able to correctly place or interpret dots in the intersection of two or three sets.

Arrow Diagrams

The students' understanding of concepts and operations using arrow diagrams was investigated on several dimensions. All of the students could label numerical points whose value was determined by the point of origin and the value of the arrow reaching it. Similarly all could build a road from one numbered point to another using two differently valued arrows. However, six students (#1,#2,#3,#7, #8,#9) were unable during Interview 2, to interpret a complex arrow diagram which contained all of the information needed to answer the problems given them. Student #8 did, however, demonstrate her ability to do so during Interview 3. Five of these students (#1,#2,#3,#7,#9) plus two additional (#5,#13) seemed still to be unable to conceptualize or draw in return arrows by Interview 3, so that they were unable to solve a problem that required the inverse of an operation present in the diagram. Similarly, these students plus #4, #10, #14 were unable through Interview 4 to supply composite arrows of two existing arrows in the diagram.

Integers

When considering the topic of integers with the students it was found that five (#2,#3,#4,#5,#12) had no understanding of the relative size (ordering) of a set of positive and negative numbers. It was also discovered that another five students (#1,#8,#9,#10,#11) had a consistent perception over two or more interviews that positive numbers were always larger than negative, but that for negative integers, the larger the absolute value - the larger the integer. For example, $\hat{9} > \hat{3}$. This misconception did not seem to interfere with their ability to add a mixture of positive and negative integers. However, only one of these five students (#11) could subtract a negative from a positive integer correctly.

The remaining eight students (#6,#7,#13,#14,#15,#16,#17,#18) successfully ordered and calculated with the positive and negative integers in two interviews.

Equations

Problems presented in the form of equations required the students to supply solution sets. Interview 2 included equations with two empty symbols for the addends. For "△+□=8", all students were able to supply at least some, if not all of the possible whole number pairs. For "□+□=10", all but two students (#3,#4) placed "5" in each □ as one of their answers; only eight of these (#1, #7,#8,#14,#15,#16,#17,#18) gave that as the only response they considered correct- the others listed non-identical pairs of numbers.

Combinatorics

The problem in Interview 3 dealing with combinatorics was a question given only those students who had been answering the computational problems with relative ease. Four students (#6,#8,#10,#14) gave several of the combinations of candy that could be bought for 10¢ and four (#11,#15,#16,#17) gave all possible combinations.

Minicomputer

A summary of student capabilities with the Minicomputer tasks which were posed in Interview 1 is given below. The order of students and of tasks has been rearranged somewhat to illustrate the hierarchical nature of these tasks. A check indicates that the student was able to do the task with some success - not necessarily always correctly, but sufficiently so that the interviewer could conclude that the student "knew how to do it." A question mark indicates that the interviewer was not sure and a blank indicates that the student could not do the given task.

Table 2

Summary of Minicomputer Tasks

Student Number	Putting numbers on Minicomputer	Reading numbers on Minicomputer	Make forward plays	Addition with carrying	Make backward plays	Subtraction with borrowing	Multiplication	Division	Conservation
1	✓								
9	✓								
2	?	?							
3	✓	✓							
4	✓	✓							
5	✓	✓			✓		✓		
7	✓	✓							
8	✓	✓	✓	✓	✓				
11	✓	?	✓	✓	✓				
14	✓	✓	✓	✓	✓		✓	✓	
10	✓	✓	✓	✓		✓			
6	✓	✓	✓	?	✓	?	✓		
12	✓	✓	✓	✓	✓	✓	✓	✓	
18	✓	✓	✓	?	✓	✓	✓	✓	?
13	✓	✓	✓	✓	✓		✓	✓	?
16	✓	✓	✓	?	✓	✓	✓	✓	✓
17	✓	✓	✓	✓	✓	✓	✓	✓	✓
15	✓	✓	✓	✓	✓	✓	✓	✓	✓