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AUTHOR Anderson, A. L.
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ABSTRACT

This booklet is the final report on a project designed to determine the extent to which students in grades 2, 3, and 4 who were having particular difficulty in learning number concepts were not yet conserving number. A test of number conservation was given to all children assigned to the remedial group; 10 of the 23 were found to be nonconservers. The nonconservers were given special instruction using manipulative materials; five of these students were classified as conservers on the posttest. A similar instructional treatment for grades 5 and 6 was developed, used, and found to be successful. Appendices to the report include a copy of the conservation test, a list of materials purchased, and copies of lesson plans and locally made activity sheets. (SD)

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Remedial Mathematics Project
Harry Gray Elementary School
Valleyview, Alberta
December, 1976

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Report Compiled by
A.L. Anderson

JE 022704

Of all the subjects taught at the elementary school level, mathematics is perhaps the most abstract of all. Many mathematics programs, particularly at the primary level, complicate the situation by presenting too formal a program too early for the majority of children. This was of particular concern to the staff at Harry Gray Elementary in Valleyview, Alberta. A number of their students at the grades II, III and IV were not responding to regular instructional programs aimed at developing number skills at their particular grade level. The mathematics program for these students was essentially that followed in the recommended textbook.

It was agreed that a formal testing program would be administered to grade II, III and IV students who were having particular difficulty coping with arithmetic concept development. If testing would indicate that students were not conserving numbers (Piaget testing) then a completely new approach to instruction would be required.

Reimer's (1968) conservation battery was selected to determine which students were in special need (Appendix J). Basically, the test measures the student's ability to conserve numbers (i.e., cardinal and ordinal). Those students who could not score 6 of the 7 subtests would be classed as non-conservers and the likelihood of success in formal mathematics is non-existent. These students will require many experiences with manipulative mathematics at their individual level so as to gradually build upon their present state to a level at which they can better cope with higher level concepts. Test results identified non-conservers from grades two, three and four (Figure 1).

Grade 2		Grade 3		Grade 4	
Conservers	Non-conservers	Conservers	Non-conservers	Conservers	Non-conservers
3	2	4	6	6	2
Total remedial group		23			
Total conservers		13			
Total non-conservers		10			

Figure 1

A program was derived whereby students would be directed to process activities that would relate directly to the development of the concept of numbers, if that were possible. Figure 2 shows that according to Piagetian classification, these students would normally be grouped in the Concrete Operational Stage and thus be conservers.

Student explorations were to fall in the area of classifying, sorting, ordering, matching, comparing, identifying, viewing and organizing. The main instructional mode aims at having students explore, manipulate, design and create within each of the outlined process areas. To facilitate this mode of operation, the school purchased a collection of manipulative material (Appendix B). Each child was to attend the remedial sessions for three half-hour periods per week. Students attending these classes would return to their normal class periods for the remainder of the normal mathematics instruction. Teachers were encouraged to accommodate these students by providing success centered materials and by being less demanding of these individuals. One weakness of the program may have been in not providing more direction for home-room situations.

	Transductive (2 - 4)	Pre-Operational	Intuitive (4 - 7)	Concrete (7 - 11)	Operational	Formal (11 - over)
- Motor stage						
er to construct						
ts						
	The object with development of language is crudely symbolized by a thought process that can be kept in the mind. - no distinction between general and particular		- thought processes governed by global perception		- reversibility of thought - conservation principle - capable of a limited form of reasoning if tied to concrete experience.	

Experiments to establish the child's concept of number

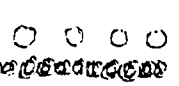
variation of quantity

(a) Continuous (liquid)

in middle container
into long, thin and
contains
amount lost or gained.



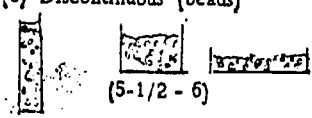
one-to-one correspondence (cardinal and ordinal)



- (1)
- (2)

Same number in (1) and (2) to this child

(b) Discontinuous (beads)



Pre-Operational
(4 - 5)

- (1) to children in this stage the quality of liquid would vary with shape of container
- (2) the number of beads will depend on shape of container

(5-1/2 - 6)

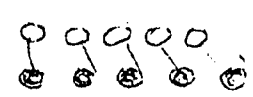
- (1) period of transition and elaboration
- (2) begins to conserve but a conflict between child's knowledge and his perceptions with latter dominant.

(6-1/2)

- (1) conservation established
- (2) number of beads same under any manipulation



Capable of placing in one-to-one correspondence but becomes confused when chips are experimentally manipulated



The one-to-one correspondence is preserved under any manipulation

number



Ten dolls and ten walking sticks seriated from small to large in even steps. The child is asked to place the dolls and walking sticks so that the doll can easily find the stick that belongs to her.

- This child finds it impossible to construct a series.
- An inability to grasp the reversibility inherent in serial elements
- Capable of setting up the one-to-one correspondence or ordering the dolls and sticks in sequence but lost when the one-to-one correspondence is manipulated
- This child is capable of ordering under any correspondence

Figure 2

Close liaison between the school and the home was maintained so that parents were aware of the objectives of the program. No students would be included whose parents were not fully supportive of the philosophy and aims of the program. Various difficulties were encountered during the operation of the project. Materials were slow in arriving to the point where planning was made difficult. In addition, the program was offered by the Administrator of two schools. Many remedial sessions were cancelled or interrupted through no fault of the operator. Some students were absent for the June testing session and because of the late June testing period, were not re-tested.

RESULTS

In June, all students who were classed as non-conservers were re-tested with Reimer's (1968) conservation battery. The re-administered test was considered valid because of the controlled interview situation. In the first testing, students who made non-conserver responses were not corrected. A "thank you" closed each question segment regardless of the student's solution response, thus non-conserver responses were not corrected.

Conservation scores in Figure 3 show individual student results in January and June, and the net improvement over the time period. Of the nine students who were re-tested, five were reclassified as conservers of numbers (6 correct responses to 7 subtests). Although this may not indicate great success, some aspects of the study were interesting. All but one student showed improvement, for example. The grade two's showed the greatest gains in that all became conservers of numbers. However, time itself may have been the significant factor. This aspect was not controlled.

Conservation Scores			
	Testing I (January 75)	Testing II (June)	Improvement
<u>Grade 2</u>			
Darcy Barry	4/7	7/7	3/7 (conserves)
Andrew Spencer	4/7	5/7	1/7 (conserves)
Leonard Reichert	4/7	7/7	3/7 (conserves)
<u>Grade 3</u>			
Rudy Minder	4/7	1/7	3/7 (conserves)
Muriel Badjer	0/7	4/7	4/7
Rodney Ratzlaff	1/7	1/7	0/7
Brian Mcknight	1/7	2/7	1/7
Barry Schultz	1/7	6/7	5/7 (conserves)
<u>Grade 4</u>			
Wayne Fjeld	1/7	4/7	3/7
4 students were absent for re-testing			

Figure 3

Aside from the testing results, some encouraging comments from teachers were noted. Students showed real enthusiasm for the activity approach to the program. During visits to the program, students were enjoying their planned activities and were eager to linger after regular lessons.

As a result of this study, a teacher at the grade V and VI level decided to carry on a modified approach to remediation for students having difficulties in mathematics.

In September of 1975 two remedial groups were set up, one each at Grade V and VI. The primary objectives of these remedial groups were different from the previous study. Specifically, the intent at the upper level was:

- 1) to create a positive attitude toward mathematics.
- 2) to provide successful and highly motivating experiences in mathematics, and
- 3) to provide wherever possible a manipulative, explorative, discovery approach to the learning of math concepts.

The two classes met three times weekly, for a total of 1 1/2 hours of remedial instruction per week. The classes concentrated on the numbers and operations and properties strands of the Tentative Outline of the Elementary Mathematics program. Instructional emphasis was placed on developing basic number understanding and facility with the four operations (+, -, x, ÷). Appendix D indicates the basic plan and lesson format used with the remedial classes. Liaison between the remedial and regular teachers was maintained throughout the study. To determine who would participate in the grade V and VI remedial classes, the 1974 edition of Mathematics Topic Test (Form 1, University of Toronto) Number Theory was administered to the total population grade V and VI of students.

Grade 5

No. of Students	% In Each Range	Range
9	13%	1 - 6 Low
11	16%	7 - 8 Below average
26	38%	9-12 Average
15	11%	13-15 Above average
8	12%	16-25 High
69	100%	1-25

Grade 6

No. of Students	% In Each Range	Range
8	9.6%	4 - 7 Low
7	8.4%	8 -10 Below average
50	60.3%	11-19 Average
11	13.3%	20-23 Above average
7	8.4%	24-34 High
83	100%	4-34

Figure 4

Low and below average students were included in the remedial treatment class. One grade V class, less members in the remedial class, were used as a control group for the study. They would receive regular home room instruction. Students chosen from the grades V and VI remediation plus the grade V control group, were given test no.2, Addition and Subtraction of Whole Numbers and test no. 3, Multiplication and Division of Whole Numbers.

Instructional program in the remedial classes was aimed at improving basic number understanding. Specifically, emphasis was given to place value, properties of numbers, number facts, basic operations, and problem solving. Wherever possible, games, novel number activities, models and laboratory sessions were used as instructional mode. The remedial program operated for 3 1/2 months and at the end of that time, an extensive re-testing program was carried out. Form 2 of the Mathematics Topic Test (number theory, addition and subtraction, multiplication and division) were used for this re-testing of remedial and control group of students. Although the test results were not subjected to statistical analysis, some test gains are rather interesting.

Results

The motivational effect of the remedial program again was very positive. In fact, the objectives of having students developing a positive attitude from being successful in a manipulative, explorative mathematics environment were realized according to the teachers involved in the program.

Students in the remedial groups made greater percentage gains than did those students in the control group who received regular classroom instruction. (Figure 5.)

The value of a remedial program is perhaps successful to the degree that it supplies individual students with treatment for their particular difficulty within the subject. Testing results indicate that some students made substantial gains over the six months the classes operated. Two students for example made raw score gains of 11 points, which in percentage terms ranges around 300 per cent. Slightly less than half of the remedial students made point score gains of 100 percent or over.

Having a special program operating in the school seemed to have the effect of increasing teacher interest and enthusiasm. The school staff placed value on the project and as a result had a positive attitude toward research in the classroom.

REMEDIAL-GR. FIVE	NUMBER THEORY			ADD & SUBTRACT			MULT. & DIVIDE		
	TEST #1			TEST #2			TEST #3		
	SEPT.	JAN.	GAIN/LOSS	SEPT.	JAN.	GAIN/LOSS	SEPT.	JAN.	GAIN/LOSS
Beaver, Dean	6	15	+9	12	17	+5	13	11	-2
Bliss, Blaine	7	7	+5	7	4	+2	5	10	+5
Cornelson, Greg	6	17	+11	18	18	0	12	12	0
Hansen, Carey	1	9	+8	x	x	x	3	5	+2
Kaese, Donald	2	7	+5	7	11	+4	7	7	0
McCandless, Cameron	8	5	-3	x	13	x	8	7	-1
O'anski, Joanne	7	8	+1	x	15	x	7	11	+4
Pachnoski, Jamie	7	16	+9	10	15	+5	6	11	+5
Purdy, Russel	4	15	+11	18	17	-1	10	15	+5
Soester, Pat	5	6	+1	20	20	0	8	10	+2
Wallgren, Shauna	7	8	+1	6	7	+1	6	7	+1
Wood, Annelida	14	20	+6	x	x	x	x	x	x
Fields, Wayne	6	9	+3	6	4	-2	7	5	-2
Total Possible Score	32	32		32	32		32	32	
GROUP AVERAGES	5	10.2	+5.2	11	12.8	+1.8	7.6	9.3	+1.7
CONTRAST AVERAGES	11.5	16	+4.5	19.7	18.9	-0.8	11.7	12.3	+0.6

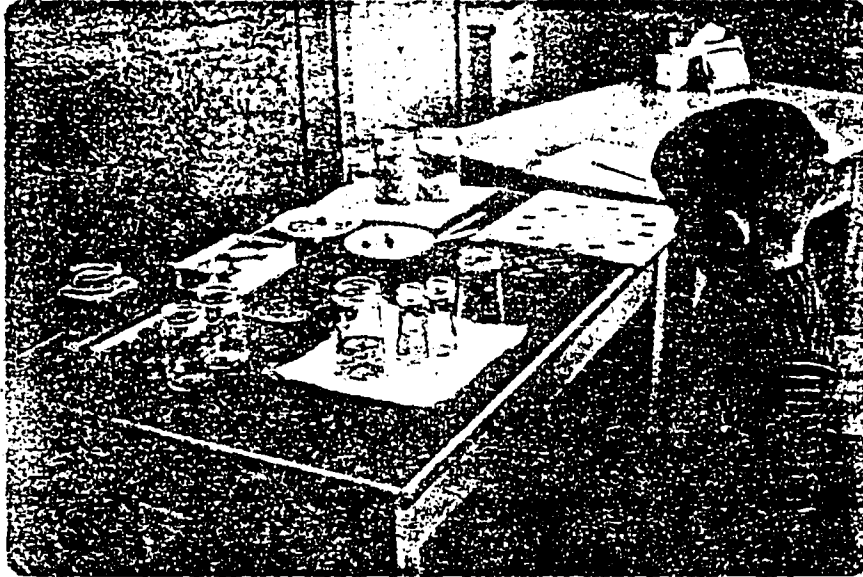
REMEDIAL-GR. SIX									
Bartlett, Terry	4	13	+9	10	12	+2	6	10	+4
Bartram, Keith	7	14	+7	22	18	-4	8	12	+4
Downar, Tim	8	11	+3	x	11	x	x	x	x
Efird, Brenda	9	17	+8	19	x	x	7	9	+2
Efird, Linda	6	13	+7	x	20	x	11	10	-1
Herbert, Lila	7	12	+5	9	18	+9	4	15	+11
Hess, Brian	9	9	0	9	13	+4	7	14	+7
Hoedl, Jodi	6	8	+2	20	13	-7	9	12	+3
Holland, Reg	11	18	+7	24	24	0	13	14	+1
Kerr, John	14	12	-2	16	23	+7	6	6	0
Knight, Darren	11	15	+4	x	24	x	x	13	x
Lee, Brenda	6	11	+5	14	14	0	4	10	+6
Tremayne, Sherry	9	13	+4	17	20	+3	9	11	+2
Wallgren, Patricia	6	16	+10	19	13	-6	7	9	+2
Total Possible Score	32	32		32	32		32	32	
GROUP AVERAGES	8	13	+5	16.3	17.2	+1.1	7.6	11.2	+3.6

x results not available

Figure 5

REIMER TEST (1968)

APPENDIX A



A SUBJECT STANDING IN FRONT OF THE TESTING TABLE
(Posed after he had been tested.)

At the beginning of each session E verified some of the personal data that had been collected for each S. This was followed by an introductory item which was designed to introduce S to the general format of the test items. S was asked to place the same number of red blocks in a row parallel to the seven blue blocks which E had placed on the table. As E pointed to the appropriate rows he asked, "Are there more blue blocks, more red blocks, or the same number of blocks in each row?" After S had agreed that there were the same number in each row, E removed two red blocks and asked, "What about now?".

The two red blocks were then replaced and again E asked, "What about now?" E then proceeded with the test proper.

The entire session lasted about fifteen minutes.

Scoring. As the test proceeded, E circled S's response on an individual score sheet. (See Appendix B.) In 1a, for example, L was circled if S said the longer (red) row had more blocks, Sh if S said the shorter (blue) row had more, and S if S said they were both the same. Unusual reactions and interesting comments were noted on the score sheet as well. A correct answer was scored as one while an incorrect answer was scored zero. It was assumed that "the present state of knowledge about conservation does not permit a much finer scaling of responses than the categorical procedure of assigning ones and zeros" (Sawada, 1966, p. 61).

For those parts of the study in which the results of a single subtest are used, a one or a zero was given to the entire subtest. If S did both items of a subtest correctly he scored a one. If one or both of the answers were incorrect a score of zero was given. It was assumed that this method of scoring individual subtests would ensure that only those subjects who were definitely conservers would be labelled as such.

APPENDIX A

CONSERVATION TEST

Introductory Item

Apparatus: 7 blue and 7 red wooden blocks each
3/4 inch cube.

E places 7 blue blocks in a straight row with approximately one-half inch spacing between blocks.

"Now, you put just as many red blocks here (E indicates a line parallel to blue blocks) as there are blue ones."

After S has completed the task E asks, "Are there more blue blocks, more red blocks, or the same number of blocks in each row?" (Before proceeding, S must agree they are equal.)

E removes the fourth and fifth red blocks.

"What about now?""*

E replaces the two red blocks.

"What about now?"

* If S fails to understand the problem whenever this question is asked, the complete question with the three alternatives is repeated.

Subtest 1: Conservation of Number Less Than Ten

Item 1a. Apparatus: Same as for the introductory item.

The final arrangement of blocks for the introductory item is used as a starting point for Subtest 1.

E spreads the red blocks to approximately one-inch spacings.

"What about now?"

"Why?"*

Item 1b. Apparatus: Same as for 1a.

E moves the red blocks back to one-half inch spacings.

"What about now?" (S must agree to equivalence before proceeding.)

E moves red blocks into a close bunch.

"What about now?"

"Why?"

Subtest 2: Conservation of Number Greater Than Ten

Item 2a. Apparatus: 30 large wooden beads, 2 identical small jars, 1 larger jar.

"Take a bead in each hand and drop them into these jars like this until all the beads are gone." (E demonstrates how the beads are dropped into the 2 smaller jars.)

* For some items this question may be changed to "How do you know?"

"Did you drop more beads into this jar, more beads into this jar, or are there the same number of beads in each?" (S must agree to equivalence before proceeding.)

E pours the beads from one jar into the larger jar.

"What about now?"

"Why?"

Item 2b. Apparatus: 2 12-inch square sheets of gray paper on which 12 yellow one-inch square pieces of paper and 12 blue one-inch square pieces of paper are arranged in two concentric circles having diameters of 4 inches and 7 1/2 inches respectively.

"What would you call these things?" (E points to several of the small "squares".)

"Are there more yellow (E uses S's own term), more blue _____, or the same number of _____ each?"

"Why?"

Distractor

Apparatus: one saucer with 3 candies and one saucer with 4 candies.

"Are there more candies in one saucer than in the other?"

"Would you like to take one of these (E points to saucer with 3 candies) and eat it?"

"What about now? Are there more candies in here, more candies in here, or the same number in each?"

"Why?"

Subtest 3: Conservation of Number in an Additive
Rearrangement

Item 3a. Apparatus: 2 sheets of blue paper having dimensions of 9" x 12", 16 Ritz crackers.

"Sometimes when you are home you like to have a lunch between meals. Suppose your mother says you may have 4 of these crackers for a morning lunch and 4 of them for an afternoon lunch. (E places 8 crackers on one sheet of paper in 2 groups of 4.) But the next day you want some lunches, too, and your mother says you may have the same thing: 4 crackers in the morning and 4 crackers in the afternoon." (E places 8 crackers on the second sheet of paper in 2 groups of 4.)

"Are there more crackers on this sheet, more crackers on this sheet, or the same number on both?" (Before proceeding, S must agree to equivalence.)

"But suppose that on this second day you are not very hungry in the morning so you eat only one cracker and save all the others for the afternoon." (E transfers 3 crackers from one group of 4 to the other group of 4, leaving a final arrangement of a group of 1 and a group of 7.)

"What about now? Are there more crackers on this sheet, more on this sheet, or the same number on each?"

"Why?"

Subtest 4: Conservation of Quantity

Item 4a. Apparatus: 2 identical jars with unequal amounts of colored water, 2 smaller identical jars.

E places the 2 larger jars with water in front of S.

"Is there the same amount of water in these 2 jars?"

E equalizes the contents to S's satisfaction.

"Are they the same now?" (S must agree that both jars have the same amount of water before proceeding.)

E pours the water from one of the jars into 2 smaller jars.

"Now, is there more water in this jar (E points to larger jar), more water in these 2 jars together (E points to the 2 smaller jars), or is there the same amount of water in both?"

"Why?"

Item 4b. Apparatus: 2 identical larger jars with unequal amounts of puffed wheat, a smaller jar.

E places the 2 larger jars with puffed wheat in front of S.

"Is there the same amount of puffed wheat in these 2 jars?"

E equalizes the contents to S's satisfaction.

"Are they the same now?" (S must agree that both jars have the same amount of puffed wheat before proceeding.)

E pours the puffed wheat from one of the jars into the smaller jar.

"Now, is there more puffed wheat in this jar (E points to larger jar), more in this jar (E points to smaller jar), or the same amount in both jars?"

"Why?"

Subtest 5: Conservation of Length

Item 5a. Apparatus: 2 identical strips of gray construction paper having dimensions of 6" x 1/2". One strip is cut in two with a 45° angle cut.

E places the 2 strips of paper in front of S. They are parallel, coterminous, and about 1/2 inch apart.

"Let's pretend these are 2 sidewalks. Is this sidewalk longer, is this sidewalk longer, or are they both the same length?" (Before proceeding, S must agree they are equal in length.)

"But what if I want to build this sidewalk around a corner like this? (E makes the necessary transformation with one part of the "cut" sidewalk.) Now, is this sidewalk longer, is this sidewalk longer, or are they both the same length?"

"Why?"

Item 5b. Apparatus: 2 identical sticks 6 inches in length.

E gives the 2 sticks to S and asks, "Are these 2 sticks the same length?"

E assists S in placing the sticks parallel and coterminous. (Before proceeding, S must agree they are both the same length.)

E slides one stick to the left about $3/4$ inch.

"What about now?"

"Why?"

Appendix B

SCORE SHEET

TEST	STUDENT		
<u>Introductory Item</u> (equipment)	Student must succeed on this item.		
	<u>Score</u>		<u>Attribute</u>
Subtest 1 - 7 red blocks - 7 blue blocks	(a) 0 (b) 0	1 1	Number
Subtest 2 - 30 wooden beads - blue and yellow squares	(a) 0	1	N = 10
Distractor - 2 saucers - 7 candies	(a) 0	1	Number
Subtest 3 - 2 blue paper - 16 Ritz crackers	(a) 0	1	Number
Subtest 4 - 2 large and 2 small jars of colored water - puffed wheat	(a) 0 (b) 0	1 (water) 1 (cereal)	Quantity
Subtest 5 - 6" x 1/2" paper straws	(a) 0 (b) 0	1 1	Length

Appendix C

Budget

From Algonquin Publishing

Printed

Using Cuisenaire Rods by Davidson	\$6.95
Graphs and the Child Frederique and Papy	\$9.50
Mathematics and the Child I Frederique	\$14.95
Introduction to Creative Education by Arno Stern	\$9.50
<hr/>	
Geoboard Classroom Kit by Truett	\$48.00
Cuisenaire Rods 10 colours - 200 of each 10 x \$2.00	\$20.00
Poly Math Cubes	\$12.50
Logic blocks - large size	\$9.00

From Moyer

81-3592 Wooden beads-500 assorted forms 6 colours	\$22.50
81-4528 Wooden pegs, 6 colours, 1000	\$5.65
81-4432 Peg boards (plastic) 15 x \$1.20	\$18.00
81-5816 Plain cubes (100)	\$5.35
Modern Computing Abacus 2 x \$7.25	\$15.50
81-4112 Enlarged place value sticks 3 x \$4.15	\$12.45
81-3032 Five Day Temperature Chart	\$7.50

81-1240	Capacity measures (metric)	\$5.45
661	Large Displacement can 2 x \$2.45	\$4.90
81-4824	Adjustable Counterbalance Scales	\$22.50
81-0712	Practical Scale	\$15.95
81-0716	Hook spring balance 10 x \$1.70	\$17.00
81-2030	Set of metric iron masses	\$6.95
81-7205	Metric plastic masses 5 x \$2.85	\$14.25
81-0584	Metric primary shapes	\$7.95
81-2448	Coloured gummed area paper 1 package	\$2.95
81-3528	Metric pin board 4 x \$4.25	\$17.00
81-8528	Circle pin board 20 cm. diam. 6 x \$1.65	\$9.90
81-4050	How to measure Package of 10	\$6.50
81-2232	Metric Trundle Wheel	\$6.25
81-2284	100 cm. measuring tapes 2 boxes x \$3.25	\$6.50
81-1440	Modern Math Numerals and Symbols 2 kits x \$5.10	\$10.20
81-9136	Mathematical balance Extra weights	\$5.95 \$1.25
81-3800	Mirror topic	\$5.95
81-2696	Tangram puzzles	\$4.95
81-1608	Construct-o-straws	\$2.95

From Scholar's Choice

873531	Diennes Logic Block Set Handbook	\$1.65
81528162	Orbit - Liftoff	\$5.15

APPENDIX D

1. Lesson Objective - multiplication basic facts, drill.
subpoint simple addition skills

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
paper pencils	write out times tables and by finding the sum of the digits look for patterns. eg. 9 18 = 9 27 = 9 etc.

3. Student Activity - as above. Play ping - pong. Play buzz.

4. Evaluation - Good. Great deal of enthusiasm generated. Brian Hess
Patrice show very slow work.

1. Lesson Objectives - 1) Further familiarization with Cuisnaire rods.
 2) Using rods to describe addition & subtraction operations.

2. Lesson Design

Material (Equipment)

Teacher Clarification

Cuisnaire rods

- review meaning of equation
- review addition statement
- introduce subtraction statement with rods
- casual introduction of addition subtraction relationship & commutative principles.

3. Student Activity

- a) In 3 minutes make as many combinations (add) that yield 5 (yellow) rod.
- b) Find subtraction statements that yield answer of 5.

4. Evaluation

Addition principle grasped well
 Considerable difficulty grasping subtraction.

1. Lesson Objective - 1) Basic multiplication facts
a) knowledge of
b) introductory work with
2) Relationship between mult. & division

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
Overheads 1) these are - these aren't 2) patterns 3) find the missing # Mult. Rockets - 5 - 9	stress reciprocal relation of division and multiplication.

3. Student Activity - Do mult. rockets as quickly as possible. Find patterns on overhead 1) & 2). Find missing factor, then product on overhead 3).

4. Evaluation - Doing multiplication tables took some time but most were accurate. Follow up activity make own overheads like 1) and 2) or 3)
GRS: two students have no response to missing factors despite ease of questions.

1. Lesson Objective - Practice multiplication facts from 1×1 - 6×6

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
dice paper pencils	explain game of pig

3. Student Activity

Play Pig

4. Evaluation

Enthusiastic - Addition skills also practiced.

1. Lesson Objective

Further practice multiplication and addition skills

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
dice	introduction of class game variation for Pig.

3. Student Activity

On blackboard keep own score (this allows teacher to note who is having difficulty with the facts).

Ping Pong

4. Evaluation

Still some difficulty experienced by all but Reg Holland.
Mostly with six times table.

1. Learning Objective

Multiplication and Division facts.

2. Lesson Design

Material (Equipment)

Teacher Clarification

balance scales
(from SRA kit and
borrowed from primary)

3. Student Activity

Students place a weight on the scale. Other students attempt to balance it. Groups of 2 each with beam.

4. Evaluation

Good activity. At first great difficulty was experienced but with practice most were able to balance 1 - 9 facts by the end of the period.

1. Lesson Objective

Mult. basic facts.

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
<p>flash card drill - The Winning Touch.</p>	

3. Student Activity

Students are flashed a mult. fact - if they get fact, they keep card. Each student is given a fact in turn until all facts are given out. Winners (ie. those with most cards) play winning touch. Others redo drill with new ones. Circulate each time so all play winning touch.

4. Evaluation

Good. By reserving unknowns there is a stimulus for students to remember "mistakes" for the next time.

1. Instructional Objectives

Multiplication and Addition and Division

Use of ()

2. Lesson DesignMaterial / Equipment

Dominoes
Bingo
(Chinese Puzzle)

Teacher Clarification

On individual basis use 6
or 9 dominoes to check ability
to multiply.

3. Student Activities

Bingo, and Chinese Puzzle.

4. Evaluation

Only Russel advanced to Chinese Puzzle.

Sum of 3 products (one digit x one or two digit).

1. Lesson Objective

Using cuisnaire rods to show fractions.

2. Lesson Design

<u>Material (Equipment)</u>	<u>Teacher Clarification</u>
cuisnaire rods. student activity cards set 9 A - F	

3. Student Activity

Using the rods work through the activity cards A - F as a group, when finished F continue at own speed through G - P.

4. Evaluation

Some experienced trouble before F but others were able to work on their own. Good interest but should not be continued in next period, change of pace required.

APPENDIX D

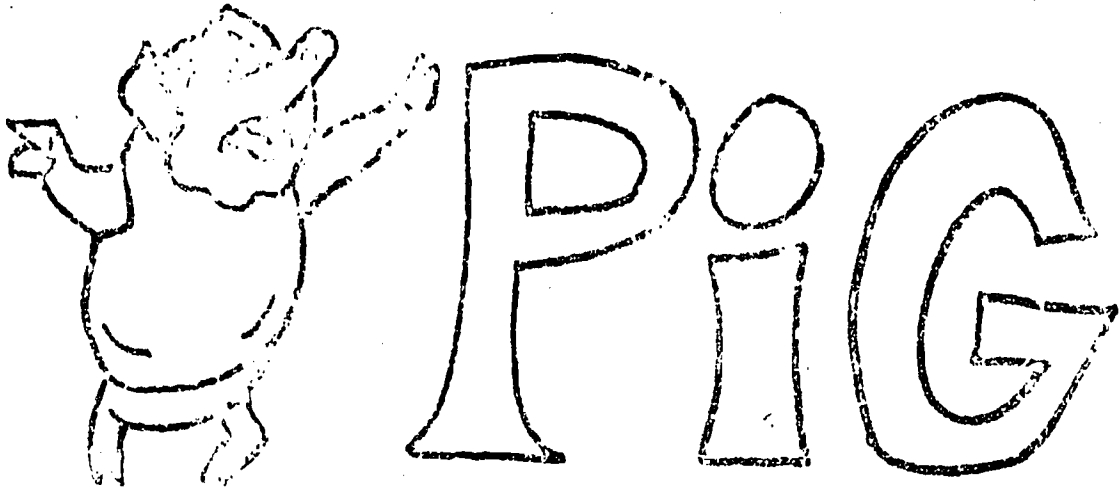
MULTIPLICATION FACTS
DIAGNOSTIC SPEED TEST
100 combinations

NAME _____
DATE _____ SCORE _____

- | | | | |
|---------|---------|---------|---------|
| 3 x 1 = | 5 x 8 = | 1 x 0 = | 4 x 3 = |
| 6 x 3 = | 9 x 3 = | 7 x 5 = | 2 x 1 = |
| 6 x 1 = | 2 x 6 = | 1 x 7 = | 6 x 6 = |
| 8 x 3 = | 2 x 0 = | 4 x 4 = | 3 x 0 = |
| 7 x 8 = | 9 x 7 = | 1 x 5 = | 4 x 8 = |
| 5 x 6 = | 0 x 2 = | 4 x 6 = | 6 x 0 = |
| 7 x 4 = | 5 x 2 = | 9 x 4 = | 8 x 8 = |
| 8 x 9 = | 0 x 1 = | 9 x 6 = | 6 x 4 = |
| 1 x 9 = | 6 x 5 = | 1 x 4 = | 9 x 2 = |
| 5 x 5 = | 0 x 6 = | 9 x 0 = | 4 x 0 = |
| 5 x 1 = | 5 x 9 = | 4 x 2 = | 3 x 7 = |
| 8 x 4 = | 2 x 8 = | 8 x 2 = | 2 x 7 = |
| 7 x 0 = | 0 x 5 = | 9 x 1 = | 2 x 3 = |
| 8 x 5 = | 1 x 3 = | 7 x 8 = | 2 x 2 = |
| 7 x 9 = | 5 x 0 = | 4 x 1 = | 0 x 4 = |
| 8 x 7 = | 5 x 7 = | 8 x 6 = | 4 x 7 = |
| 0 x 0 = | 2 x 4 = | 3 x 4 = | 2 x 5 = |
| 0 x 3 = | 3 x 6 = | 1 x 1 = | 9 x 6 = |
| 3 x 8 = | 7 x 3 = | 3 x 3 = | 4 x 5 = |
| 5 x 3 = | 7 x 1 = | 7 x 7 = | 6 x 7 = |
| 9 x 9 = | 5 x 4 = | 3 x 2 = | 0 x 8 = |
| 8 x 0 = | 1 x 6 = | 9 x 8 = | 2 x 9 = |
| 1 x 2 = | 3 x 9 = | 3 x 5 = | 6 x 2 = |
| 4 x 9 = | 1 x 8 = | 6 x 9 = | 7 x 2 = |
| 8 x 1 = | 0 x 7 = | 6 x 8 = | 0 x 9 = |

A46

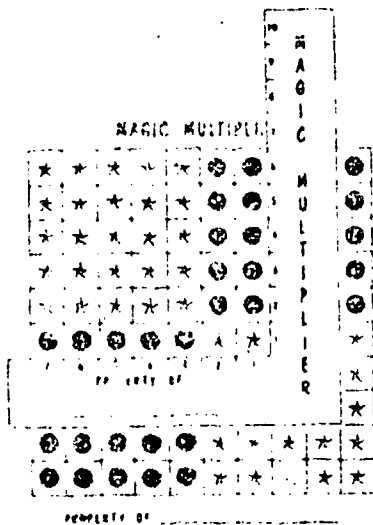
36



1st Variation - This game is played by two people with one pair of dice. The object is to be the first person to score 100 or more points by adding the totals on the dice. The players take turns rolling the dice, but may roll as many times in a row as they wish. Therefore it is possible for a player to score 100 or more points in one turn. However during any one turn, if a player rolls a one on either die, he loses all count for that turn. If he rolls a one on both dice, he loses all of his count and starts again at zero.

2nd Variation - This game is played by the whole class with the teacher rolling the dice. The object is to score the most points in ten rounds. The teacher rolls the dice until a one shows at which time a round is over. The students keep their own scores. They try to get the highest score during each round and record it before a one occurs. If a one occurs before they have recorded a score, they must record a zero for that round. Once a student writes down a score for a round, he is frozen for the remainder of that round. He begins play again during the next round. The only time a one does not stop a round is if it happens on the first roll. Play the game for ten rounds and the person with the highest score is the winner.

Before children are required to memorize multiplication facts beyond 5×5 , they should have much experience in visualizing the products as rectangular arrays. The page on the left and the L-shaped piece to be cut from the lower right of this page form a model for finding these facts. Notice that there are 25 circles and 25 stars in each region of the MAGIC MULTIPLIER.



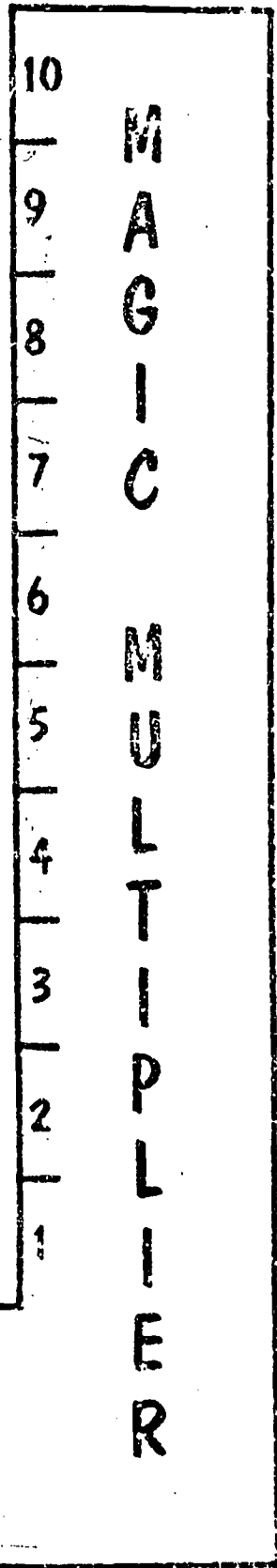
To find 6×7 , place the L-shaped piece so 6 rows and 7 columns (or 6 columns and 7 rows) appear as shown:

The 5×5 array of Stars =	25
Two columns of Circles	10
One row of Circles	5
Two more Stars	<u>2</u>
	42

Try to find 8×7 using the above method:

The 5×5 array of Stars =	25
Three rows of Circles	15
Two columns of Circles	10
Six more Stars	<u>6</u>
	56

Cut out this L-shaped piece.



M
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C

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P
L
I
E
R

7 | 6 | 5 | 4 | 3 | 2 | 1
PROPERTY OF

MAGIC MULTIPLIER

★	★	★	★	★	○	○	○	○	○
★	★	★	★	★	○	○	○	○	○
★	★	★	★	★	○	○	○	○	○
★	★	★	★	★	○	○	○	○	○
★	★	★	★	★	○	○	○	○	○
○	○	○	○	○	★	★	★	★	★
○	○	○	○	○	★	★	★	★	★
○	○	○	○	○	★	★	★	★	★
○	○	○	○	○	★	★	★	★	★
○	○	○	○	○	★	★	★	★	★

PROPERTY OF _____

FACTOR ROCKET GAMES

Rules for Game One

1. Three to six players may participate.
2. A rocket dispatcher is selected (usually the teacher or the winner of the previous game).
3. The dispatcher conceals all 25 rockets from the participants.
4. The beginning participant is selected by some random method and play continues clockwise.
5. On a player's turn he identifies a number (1-25) and the number of stages for its Factor Rocket. If the player can correctly give the number of stages and each stage (pairs of factors), he is awarded the rocket. He displays this in front of him where others can see it. If he makes a mistake, his turn is concluded and the dispatcher retains the rocket. (Players should have pencil and paper available for calculating.)
6. Play continues until all rockets are awarded. The player with the greatest number of rockets wins the game.

Rules for Game Two

Game Two is essentially the same as Game One with a different set of rockets. The rockets needed for this game are all listed below.

Rockets Needed for Game Two: (For multiplication through $9 \times 9 = 81$)

In addition to the rockets needed for Game One:

Number of Stages	Number of Rockets	Nose Cone Names
Two	13	29, 31, 37, 41, 43, 47, 53, 59, 67, 69, 71, 73, 77
Three	1	49
Four	17	25, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58, 61, 65, 69, 74, 77
Five	1	81
Six	10	28, 32, 44, 45, 50, 52, 63, 68, 75, 76
Seven	1	64
Eight	8	30, 40, 42, 54, 56, 66, 70, 78
Nine	1	36
Ten	2	48, 80
Twelve	2	60, 72

Game Two is an extension of Game One with the same rules.

NOTE: Other activities of value in developing factors and primes are Factor Stacks on page 45 and Sieve of Eratosthenes on page 41.

LINE UP

DIRECTIONS: Draw a straight line connecting each problem with its correct answer. Each line will cross a letter and a number. The number tells you where to put the letter in the line of boxes at the bottom of the page.

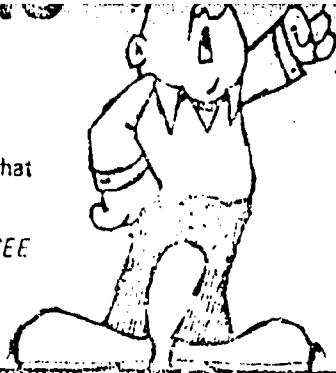
2x2		0	21
3x7		4	56
4x6	13	R	42
7x7	8	K	72
6x8		14	24
7x8	10	E	4
2x9	16	O	18
6x7		6	32
3x5	7	12	64
6x6		S	49
4x7		R	15
9x8	15	E	48
8x8		Y	28
9x9		9	9
1x9	17	I	81
4x8		3	63
3x3	11	U	36
		W	
		T	
		P	

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----

DIRECTIONS:

Shade in each box which contains a multiple of the first number in that row. Be sure to use pencil so you can erase if necessary.

YOU WILL END UP WITH A PICTURE OF SOMETHING YOU SEE EVERY DAY PLUS THE NAME OF THIS THING!



3	5	4	6	8	10	12	14	16	18	19	21	20	22	24	26	28	30	32	34	35	37
15	18	20	25	26	32	36	48	29	30	35	37	40	45	47	54	63	71	52	50	55	62
15	18	27	30	32	35	49	42	38	36	45	54	63	56	64	58	71	70	88	72	81	92
20	25	30	35	36	40	42	44	39	45	50	55	60	62	65	66	68	70	74	75	80	85
8	5	6	9	12	15	18	21	24	27	23	31	30	33	36	39	42	45	48	51	52	52
8	12	16	13	20	15	22	24	26	32	34	35	40	42	44	48	50	52	56	58	60	64
16	15	18	20	24	25	27	32	36	40	44	48	50	54	56	57	58	64	63	70	72	75
12	15	18	22	24	27	28	30	36	42	44	48	54	56	60	66	70	72	76	80	84	90
14	20	21	27	28	30	32	35	43	42	50	54	49	55	60	56	62	63	64	65	69	70
10	15	20	22	25	30	34	35	39	40	48	45	50	54	55	60	63	65	70	72	75	80

LINEUP



DIRECTIONS:

Draw a straight line connecting each problem with its correct answer. Each line will cross a letter and a number. The number tells you where to put the letter in the line of boxes at the bottom of the page.



- 9 x 10 ■
- 9 x 100 ■
- 9 x 1000 ■
- 100 x 87 ■
- 10 x 87 ■
- 1000 x 87 ■
- 540 x 100 ■
- 540 x 1000 ■
- 100 x 100 ■
- 100 x 1000 ■
- 562 x 10 ■
- 100 x 562 ■
- 562 x 1000 ■
- 80 x 100 ■
- 1000 x 80 ■
- 935 x 10 ■
- 100 x 935 ■

11 2 8 14 T E
 16 V
 12 N E O Y E
 7 C
 9
 4 5 3 O C
 17 E C 10 L
 15 R
 6 S
 13 I R

- 9000
- 870
- 10,000
- 54,000
- 5520
- 8700
- 562,000
- 90
- 93,500
- 540,000
- 900
- 6000
- 9350
- 56,200
- 100,000
- 80,000
- 87,000

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----

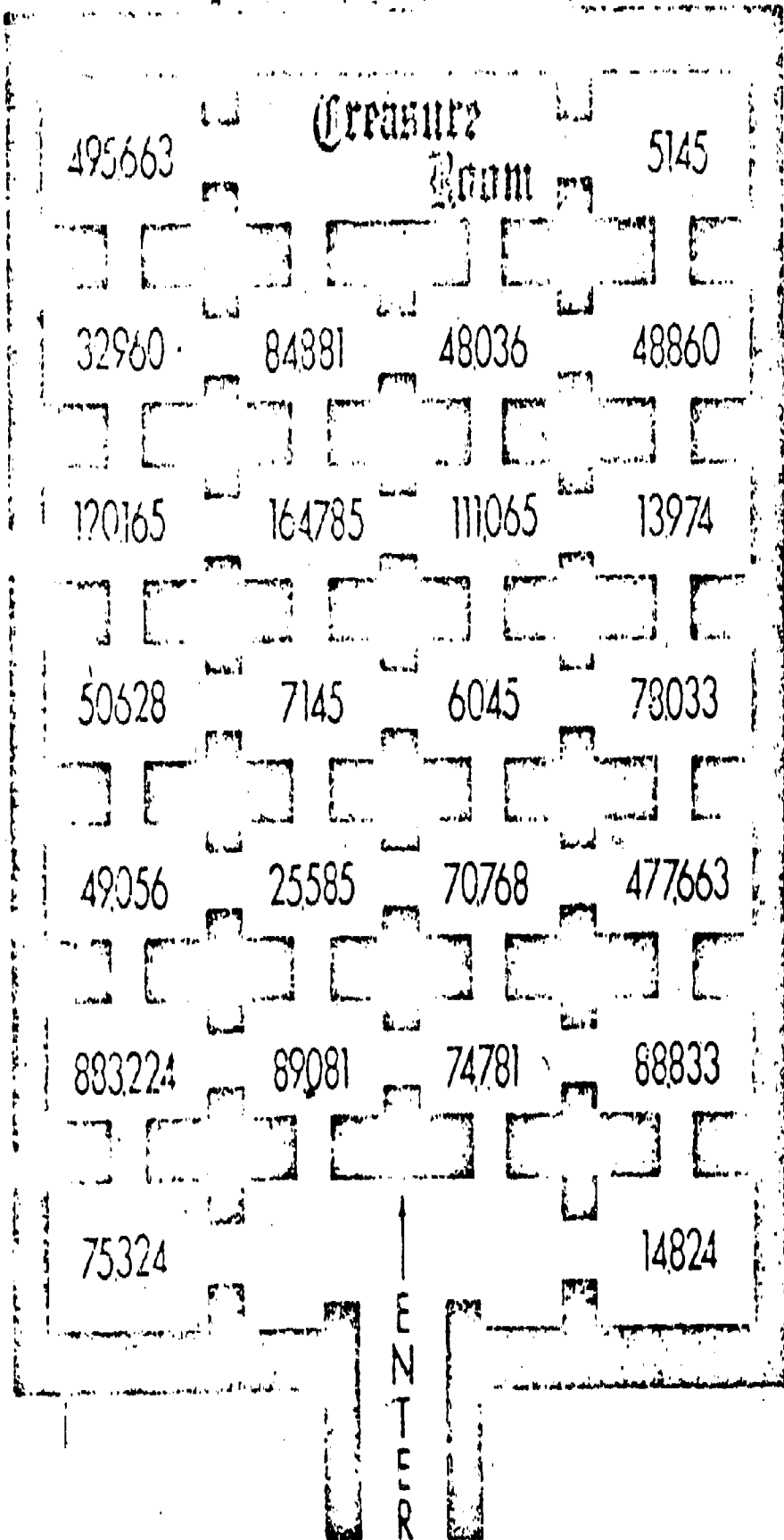
H. D. V.

Each player chooses the numbers 1 through 9, but not in order, in the bottom row of squares and also the numbers 1 through 9, but not in order, in the leftmost column. A leader will call out a multiple such as 12 and each player will write 12 in the proper square according to the factors he has chosen.

The winner is the person whose multiples form a Horizontal, Diagonal, or Vertical line of nine numbers.

X									

MAZE



Each room in this maze contains a number. Twelve of these numbers are the correct answers to the problems below.

Work any problem and find your answer in the maze. CIRCLE the answer.

Keep working problems until you can draw a path to the treasure room that goes ONLY through rooms containing correct answers. (It might not go through all of the correct answers.)

- 1 $3 \times 2015 =$
- 2 $5 \times 5117 =$
- 3 $4 \times 8240 =$
- 4 $7 \times 7008 =$
- 5 $6 \times 8438 =$
- 6 $9 \times 8309 =$
- 7 $8 \times 8846 =$
- 8 $2 \times 6987 =$
- 9 $1 \times 78,033 =$
- 10 $3 \times 40,055 =$
- 11 $5 \times 32,957 =$
- 12 $7 \times 70,809 =$

A 35

7 by 7 GAMES

- 1- Double Dice Roll - Enter the numerals from 1 thru 6 in the 49 squares. Each student may enter as many of each numeral as he wishes. Roll two dice and call out the number on each one. Each student may then mark out one or two of his squares. The first person to mark out seven in a row horizontally, vertically, or diagonally is the winner. Time permitting you may continue playing and have more than one winner. If there is still time play blackout. The winner is the first person to have their card completely filled.

- 2- Sum Roll - Enter the numerals from 2 thru 12. Each student fills in his own card with as many of each numeral as he chooses. Roll two dice and have the children compute the sum. They then mark out any one number equal to that sum. If they cannot mark out a number they wait until the next roll. The winner is determined the same as in Game 1.

- 3- Product Dice - (1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36). Enter in the 49 squares as many of the above numbers as you wish. Roll two dice and have the students compute the product. Cross out only one product at a time. The winner is the same as in Game 1.

- 4- Fraction Time - Enter in the 49 squares any fraction with 1 thru 6 in the numerator and denominator.
e.g. $1/6$, $2/6$, $3/4$, $4/3$, $2/5$, $1/1$, ...
Roll two dice, choose one for the numerator and one for the denominator. Students mark out one square containing the number rolled.
e.g. You roll $3/6$, students may mark out either the $3/6$, the $2/4$, or the $1/2$. Or you roll $1/5$, students can mark out only the $1/5$.
Winner is same as in game 1.

FACTOR MAZE

Start

Primes

5	4	1	41	52	48	58	46
1	2	8	36	63	70	60	34
6	11	12	14	3	7	24	25
35	9	13	19	15	23	21	22
44	40	10	16	18	20	29	17
56	62	38	54	37	33	26	37
61	72	80	82	55	32	43	27
45	92	50	90	42	30	28	101

Move from "start" to finish by following a path of factors.

Finish

Start

Factors of 3

3	4	32	46	100	144	78	45
321	2	11	58	70	22	99	55
7	9	51	23	42	57	144	90
28	13	17	69	19	721	26	35
34	30	5	59	37	12	25	22
43	60	38	8	53	93	29	20
33	56	68	40	10	41	642	31
86	47	50	80	45	14	16	6

Finish

RACE TRACK

Each student in the class is given this sheet. The leader calls out an operation and a number, i.e. mult. by 9. Each student places this information on line one below. Upon doing this he immediately begins placing the answers in lane #1 as he travels around the track. He must use the numbers listed next to the inside track as part of each operation. The winner finishes first with the correct answers.

OPERATION

RACE #1 _____

2 _____

3 _____

3
5
7
2
1
1
7

3 7 10 8 1 3 6 9 7 0 4 6 8 11 3 9 7 5

COMBINATIONS

Instructions: Number combinations appear in the grid vertically, horizontally, and diagonally. If you examine the grid closely you will find many of the basic facts for addition, subtraction, multiplication, and division. See how many you can find! (Insert the correct sign of operation and the equal sign.) Do not overlap.

23	9	3	6	81	60	7	12	19	57	26
45	2	3	12	48	6 + 8 = 14			25	33	58
3	35	18	4	72	42	30	56	3	44	66
15	5	10	9	12	5	28	11	17	35	16
72	38	8	36	6	6	4	27	31	5	1
49	8	80	4	20	11	7	4	9	36	2
64	4	24	8	3 x 5 = 15				45	40	3
7	13	12	32	35	19	8	3	6	18	3
32	33	28	4	7	6	9	4	19	21	5
58	18	40	36	3	7	21	28	36	4	5
24	54	37	26	53	13	40	57	20	25	55
3	72	17	4	7	6	52	2	26	50	75