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ABSTRACT

This booklet is designed to aid teachers in the use of the mini-calculator in the classroom. Included in this booklet are activities and suggestions for the use of the calculator from the primary grades through the secondary mathematics courses. Each topic in the booklet includes background information for the teacher, suggested activities, games, and sample problems. Included in the publication are the following topics: (1) Selecting a Mini-Calculator for Classroom Use; (2) Preparing to Use the Mini-Calculator; (3) Classroom Uses of Mini-Calculators; (4) The Keyboard; Concepts and Basic Operations; (5) Talking Mini-Calculators; and (6) a selective bibliography. (RH)

CURRICULUM OFFICE
INSTRUCTIONAL SERVICES
SCHOOL DISTRICT OF PHILADELPHIA

USING THE MINI-CALCULATOR TO TEACH MATHEMATICS

Curriculum Office
INSTRUCTIONAL SERVICES
THE SCHOOL DISTRICT OF PHILADELPHIA
1977

TO THE TEACHER:

The School District of Philadelphia, through its Division of Mathematics Education, has produced this booklet to aid teachers in the use of the mini-calculator in the classroom. Included in this booklet are activities and suggestions for the use of the calculator from the primary grades through the more sophisticated secondary mathematics courses.

THE MINI-CALCULATOR IS NOT TO BE USED AS A SUBSTITUTE FOR LEARNING BASIC NUMBER FACTS AND ALGORITHMS. It can be used to reinforce basic mathematical skills. In certain situations, it can be used to eliminate tedious time-consuming computations, permitting students to concentrate on the concepts involved. Also, there are areas in the mathematics curriculum where the mini-calculator can be used to introduce basic concepts through discovery. Finally, there is an inherent motivational value in using mini-calculators in the mathematics classroom.

Each topic in this booklet includes background information for the teacher, suggested activities, games, and sample problems. This booklet is not intended to be all inclusive. It should serve as an impetus to teachers to create their own activities involving the use of the mini-calculator. Classroom teachers are best qualified to modify and/or extend ideas within these pages to meet their individual classroom needs. The section at the end of the booklet contains additional sources relating to the use of the mini-calculator in the classroom.

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6

ACKNOWLEDGEMENTS

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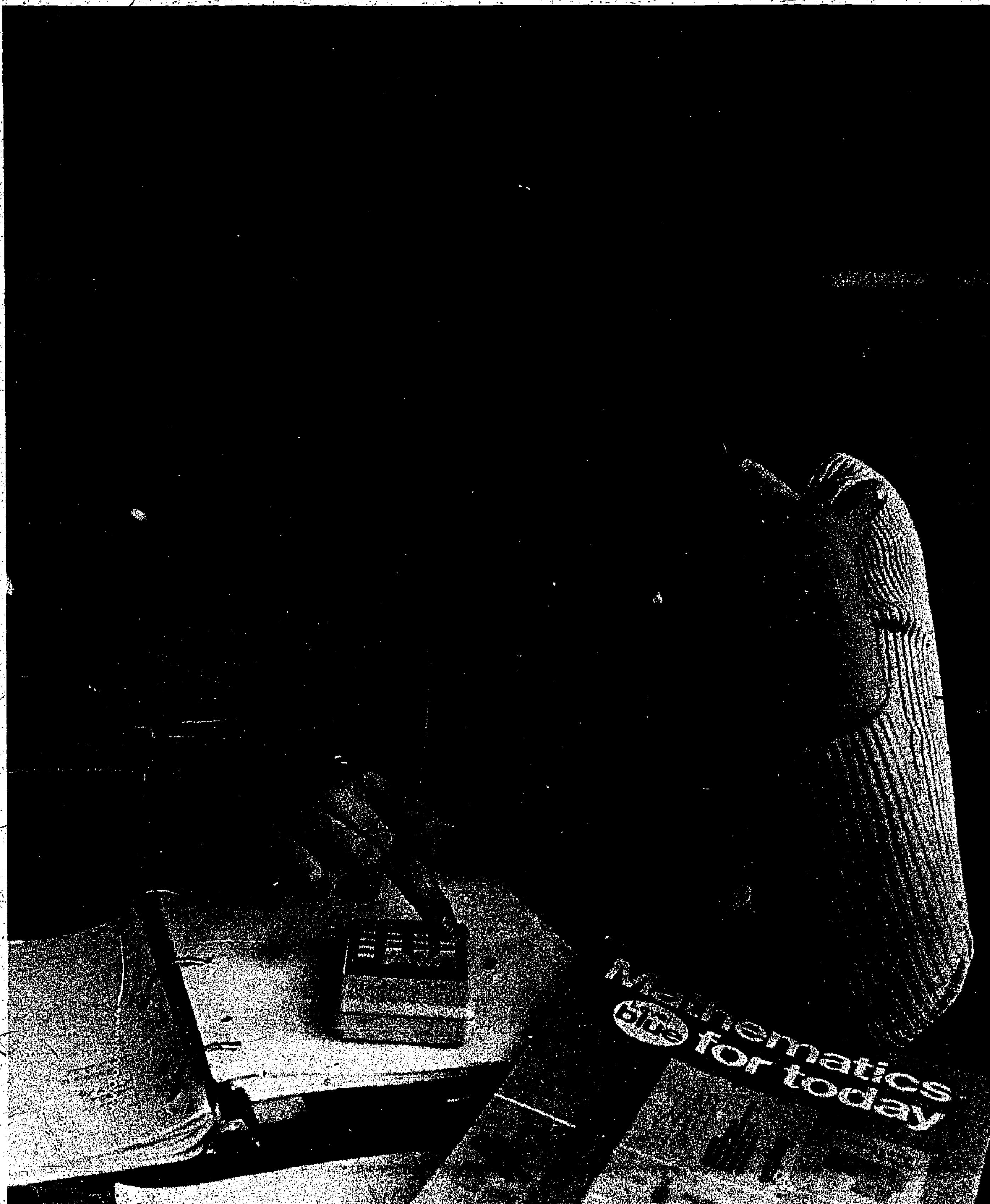
TABLE OF CONTENTS


Mathematics
plus for today

GUIDELINES REGARDING THE USE OF THE MINI-CALCULATOR IN THE CLASSROOM

The Division of Mathematics Education of the School District of Philadelphia agrees with the following statement of the National Council of Teachers of Mathematics with respect to the role of the mini-calculator in education:

> "With the decrease in cost of the mini-calculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the mini-calculator should be used in imaginative ways to reinforce learning and to motivate the learner as he/she becomes proficient in mathematics."

Additional guidelines are suggested:

1. The basic facts are to be taught prior to the use of the mini-calculator.
2. UNDER NO CIRCUMSTANCES SHOULD THE MINI-CALCULATOR BE USED IN THE TAKING OF STANDARDIZED TESTS.
3. Unless the mini-calculator is accessible to all students, it should not be used in the taking of tests.
4. Recognizing that home use of the mini-calculator will vary, it would be beneficial if schools could provide mini-calculators for student use in the school building.
5. The selection of the mini-calculator to be purchased should be guided by the level of mathematics instruction.
6. Curriculum materials and activities for the use of the mini-calculator as a motivational and reinforcing aid will be developed by the Division of Mathematics Education.

SELECTING A MINI-CALCULATOR FOR CLASSROOM USE

The purchase of mini-calculators for classroom use should be guided by the content of the course in which they will be used and the instructional level of the students using them. While a basic four-function mini-calculator would be adequate for most elementary school uses, a mini-calculator with logarithmic and trigonometric function keys would be more desirable for some advanced secondary courses.

Basic features should include:

. . . floating decimal point

. . . at least an 8 digit display

. . . single function keys

. . . algebraic rather than arithmetic logic

. . . keys that offer resistance when pressed

. . . 9 volt transistor battery power source

A rechargeable battery operated mini-calculator may be more desirable when one is purchasing a calculator for individual use. But when purchasing a set of calculators for calssroom use, the advantages of transistor battery operated units outweigh the cost of purchasing rechargeable batteries.

In addition to the above basic features, many makes and models of mini-calculators on the market today include other features teachers may wish to have on a unit for classroom use. As additional keys are added to a mini-calculator, its cost increases. This cost factor must be balanced against the increased capabilities afforded to the user.

Other features may include:

. . . a memory unit

. . . a constant key or built-in constant function

. . . automatic round-off or round-off key

. . . overflow or battery weakness indicator

Mini-calculators to be used in advanced mathematics courses should include:

. . . trigonometric functions

. . . logarithmic functions

. . . reciprocal key

. . . sign change key

. . . root and power keys

In summary, teachers should carefully consider what a unit can and cannot do prior to purchase. ALWAYS CONSULT THE USER'S MANUAL WHEN CONSIDERING A MINI-CALCULATOR FOR PURCHASE.

PREPARING TO USE THE MINI-CALCULATOR

- Check that all calculators are in working condition.
- Calculators may be powered by various sources; a. c. adaptors. rechargeable batteries or 9 volt transistor batteries. If a. c. adaptors are to be used, locate all live receptacles. The use of extension cords and multiple outlets may be necessary. Consider traffic patterns to eliminate tripping over wires.
- Stress that machines should be cleared after being turned on.
- Stress that calculators should be turned off at the end of every activity. Battery operated machines will lose their charge if not turned off. "TURN OFF YOUR MINI-CALCULATOR WHEN NOT IN USE" should appear on all activity sheets.
- Machines may misfunction if plugged in when in "ON" position.
- If calculators are to be shared, groups of not more than three students are preferred.
- SUGGESTED SECURITY MEASURES:
  - Number calculators and label with school name using an etching machine.
  - Mount machines on large size plywood boards.
  - Attach machines to a rolling work table.
  - Use teacher aides and student monitors to distribute, collect and inventory equipment.

CLASSROOM USES OF MINI CALCULATORS

USE CALCULATORS AS INSTRUCTIONAL AIDS TO:

. Emphasize the importance of place value, ch[illegible]t operation and order of operations.

. Motivate drill activities.

. Assist students in discovering number patterns.

. Reinforce decimal notation of fractions, ratios and percent.

. Focus on procedures, rather than computation, in problem solving.

. Enable students to check computations.

. Provide an effective instrument for peer tutoring.

Although this booklet has been planned for use in classrooms where mini-calculators are readily available, the use of even a single calculator can be a valuable instructional aid.

USE SINGLE CALCULATORS FOR:

. Individual checking of work.

. Demonstrations by teacher or student as class follows along with activity sheets.

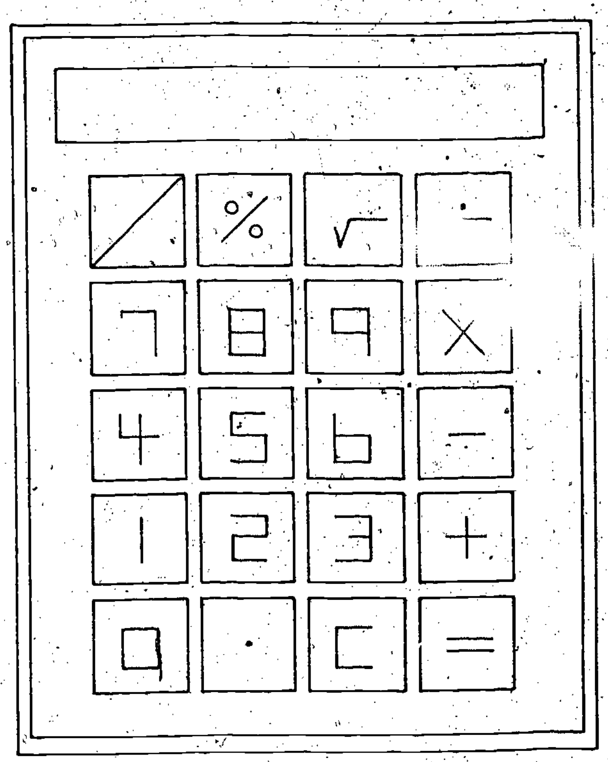
. Use in a math lab with task cards.

. Individualized, remedial, or enrichment assignments.

. Small group instruction.

11

THE KEYBOARD



The illustration above shows the basic keys and numerical arrangement, although many variations exist depending upon the make and model of the calculator. It is the order in which these keys are pressed that becomes vitally important. In addition to external variations of keys, the internal logic of various calculators may differ. It is incumbent upon the teacher to become thoroughly familiar with the calculators that are to be used in the classroom. Explicit instructions will be found in the manufacturer's instruction booklet.

Subtraction has to be performed slightly differently on machines that operate on arithmetic logic than those operating on algebraic logic. The former generally has a [+/=] and a [-/=] key, whereas the latter has separate [+], [-], and [=] keys. In a problem such as 8 - 5, keys have to be pressed in the following order:

ARITHMETIC LOGIC	ALGEBRAIC LOGIC
[8] [+/=] [5] [-/=]	[8] [-] [5] [=]
"3" will appear in the display	"3" will appear in the display

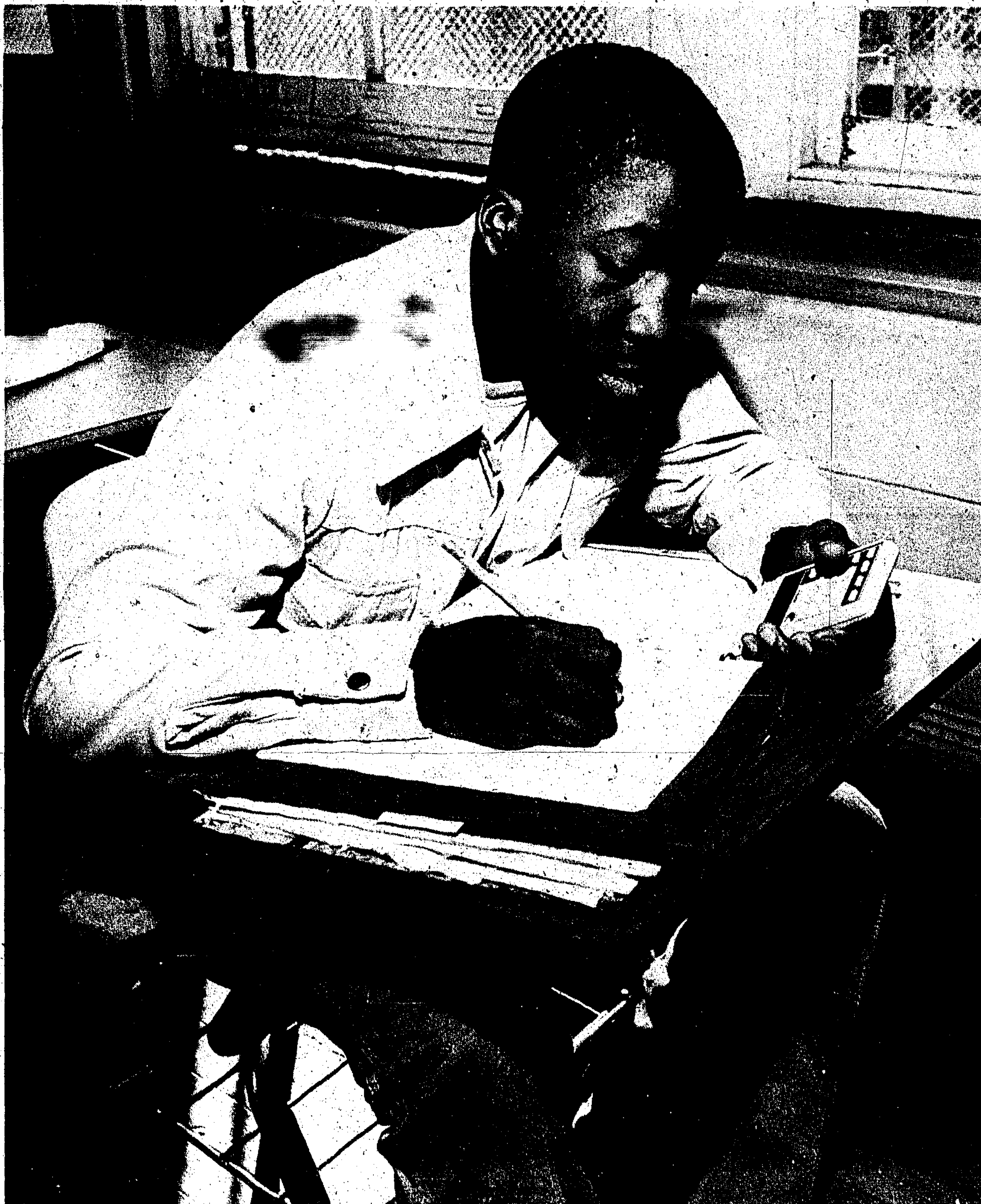
15

The internal variations of different calculators become more pronounced when chain operations are to be performed. For example, a problem such as 3 x 4 + 5 x 6 will be interpreted differently by different calculators. Some will consider the hierarchy of operations and interpret the above example as (3 x 4) + (5 x 6). Thus, the answer on the display will be 42. Others will interpret a problem in order from left to right. The above example will then become ((3 x 4) + 5) x 6 which will produce an answer of 102. Still other calculators will simply clear the storage of an accumulated product when the + key is pressed. In the above example, such calculators will clear the product of 3 x 4 when the + key is pressed, thus yielding an answer of 30.

THE TEACHER WILL HAVE TO EXPERIMENT TO DETERMINE THE WAY CHAIN OPERATIONS ARE TO BE ENTERED TO OBTAIN DESIRED RESULTS. The use of parentheses should help to eliminate this problem.

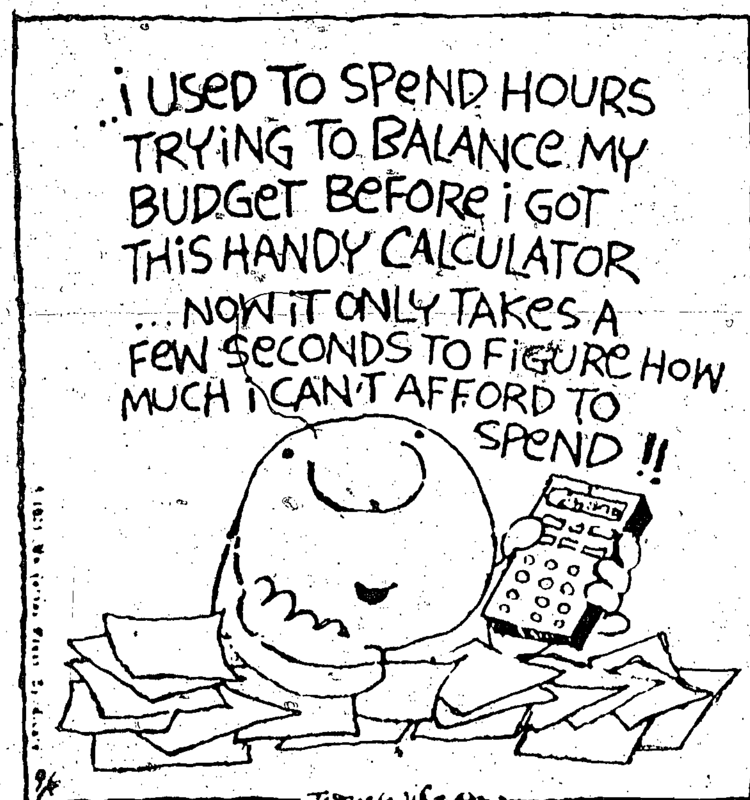
The use of special function keys such as [illegible], $\boxed{\frac{1}{x}}$, $\boxed{\sqrt{x}}$, etc. is best determined by reading the accompanying instruction booklet. Whenever possible, calculators used in the classroom should have only those functions with which the student is familiar or will become familiar during the year. The use of special function keys without an understanding of the underlying concepts should not be encouraged.

Students must be given instruction on how to use the particular make and model of mini-calculator to be used in the classroom. Both its capabilities and limitations need to be stressed. This will promote optimal use of the mini-calculator in the teaching-learning process and help to avoid student frustration. The teacher should recognize that experimentation is an effective learning experience and allow time for it. The activities and games in this booklet encourage the creative use of the mini-calculator as an instructional tool.



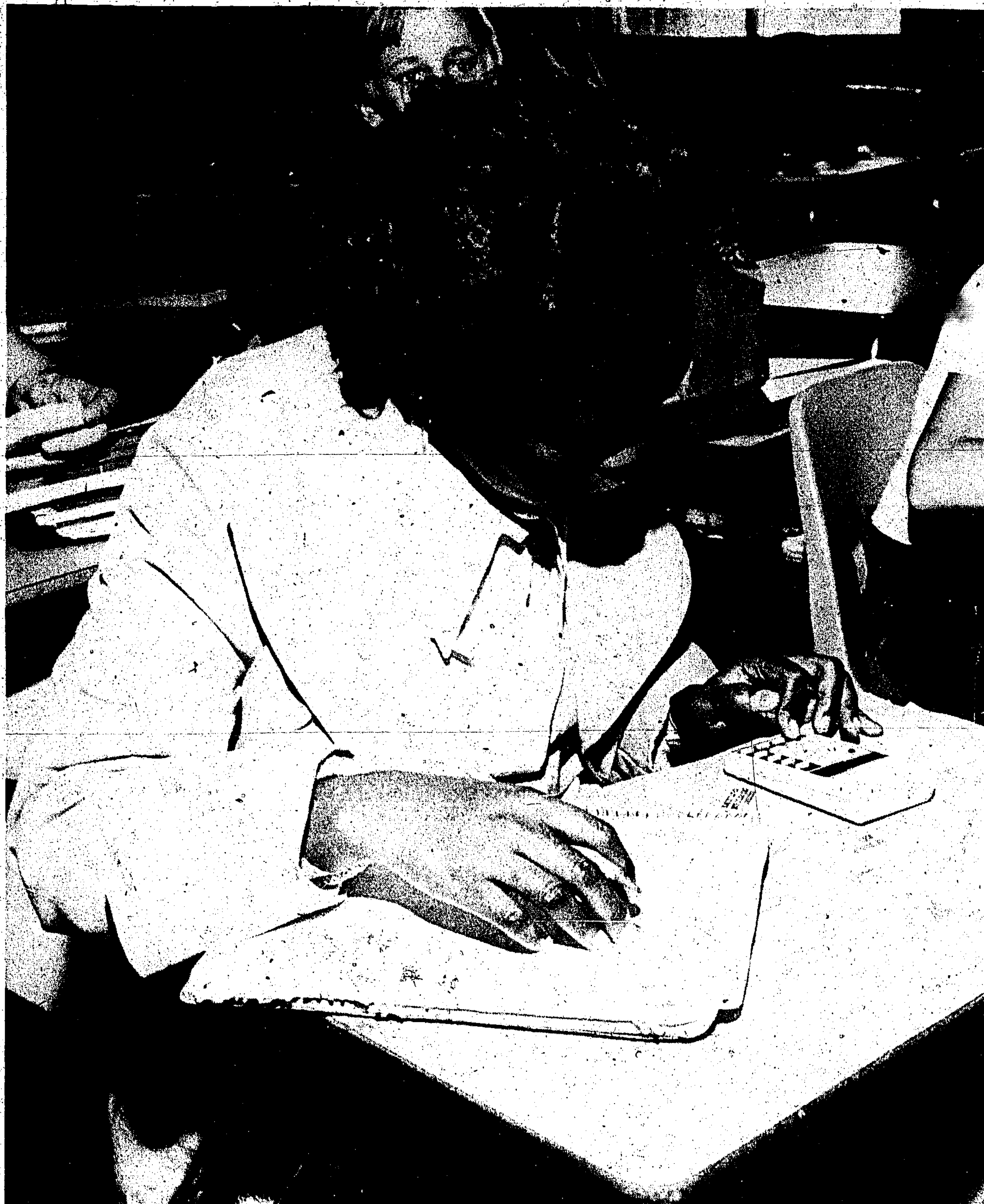
CONCEPTS AND BASIC OPERATIONS

ZIGGY



PLACE VALUE


WEE PALS
I CAN'T GO TO SLEEP, MOM!
1-28
TRY COUNTING SHEEP, MIKKI
MORRIE
OKAY!
I CAN'T! I DON'T HAVE A POCKET CALCULATOR!



PLACE VALUE

The length of display of the mini-calculator you use in the classroom will determine the size of the numbers you will be able to include in place value activities.

Students should be aware of:

a. The length of display
b. How an overflow is indicated
c. The placement of the decimal point at the end of a whole number display on most calculators
d. The omission of periodic commas in the display
Th[illegible]ing the digits of a number

Example:

DIGIT ENTERED	DISPLAY							
7								7.
2							7	2.
9						7	2	9.
8					7	2	9	8.

On your calculator, enter each of the following: Be sure to clear your calculator after each entry.

1. 13
2. 503
3. 6001
4. 520
5. 4791
6. 52163
7. 100000
8. 634521
9. 1189753
10. 84317652

Ask students to enter numbers (designated by you) on their calculators. Write the number names on the chalkboard or hold up flash cards, e. g. forty-seven, twenty-three, one thousand sixteen, etc.

Enter the following on your calculator. Remember to clear after each entry.

1. A two-place number with a "4" in the one's place.
2. A three-place number with a "6" in the ten's place.
3. A three-place number with a "0" in the ten's place.
4. A four-place number with a "0" in the ten's and one's place.

The reading of numbers and the writing of their number names are important skills to be developed in the curriculum. The calculator is a useful tool in teaching these skills.

Enter these numbers and copy your displ

1. twenty-six
2. seven hundred sixty
3. one hundred two
4. two thousand, nine hundred seven
5. fifty-one thousand, nine hundred sixty-one

Enter each of the following numbers and find their sums:

1. seventy-six
   four
   twenty
   sixty-three
   zero

   Your answer should be 163

2. seven hundred six
   five thousand, twenty-four
   one thousand, one hundred eleven
   sixty-five
   two hundred twenty

   Your answer should be 7126

WITHOUT LOOKING, PRESS ANY TWO NUMBER KEYS. COPY EACH DISPLAY AND WRITE ITS NUMBER NAME. e. g. A "1" and then a "7" are pressed.

Copy display | 1 | 7 |

Write the number seventeen

1. | | | ____________________
2. | | | ____________________
3. | | | ____________________

This type of activity should be repeated by pressing three keys, four keys, etc.

22

Circle the greatest number in each [illegible] below. Find the sum of [illegible] circled numbers using your calculator. The sum should [illegible] with total (A).

17	7	42	60
26	5	51	6
91	3	15	15
84	0	49	27

(A)	2	0	9
(B)		3	8

Now place a check next to the least number in each group. Add. Compare with (B)

Fill in the columns so that they agree with the given greatest and least totals.

1.

(A)	1	6	4	9
(B)		7	2	0

2.

(A)	7	5	2	6
(B)	1	0	0	1

The calculator may be used when teaching regrouping and expanded notation. Activities such as the following reinforce these necessary skills.

Regroup each of the following to name a decimal numeral:

	1000's	100's	10's	1's		
1.		2	3	12	→	________
2.		3	17	5	→	________
3.		10	14	7	→	________
4.		4	18	13	→	________
5.	3	17	15	10	→	________
6.	9	9	9	10	→	________
7.	4	13	5	19	→	________
8.	2	2	11	4	→	________

Use your calculator and find the sum of your answers. Did you get 25000? If not, check your work.

Rewrite each of the following as a decimal number.

1. 3000 + 400 + 30 + 7 ________
2. 500 + 70 + 2 ________
3. 20000 + 1000 + 600 + 60 + 9 ________
4. 4000 + 500 + 40 ________
5. 3000 + 900 ________
6. 1000 + 200 + 60 + 2 ________

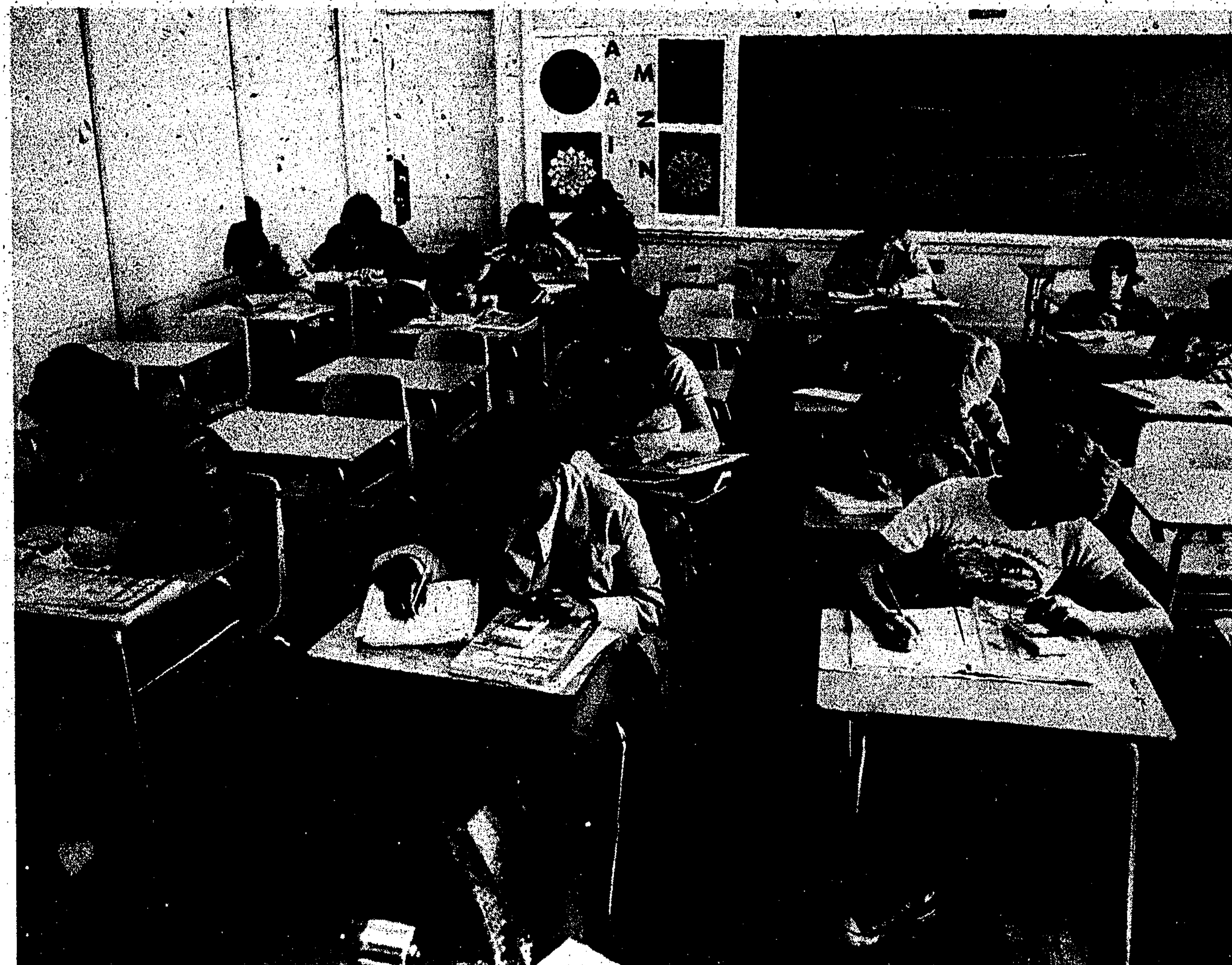
Use the calculator and find the sum of your six answers. Turn your unit upside down and read the display. If your display is not a synonym for "fat", check your work.

2

ROUNDING OFF NUMBERS

AND

ESTIMATING ANSWERS



ROUNDING OFF NUMBERS and ESTIMATING ANSWERS

The increased use of the mini-calculator will place greater emphasis on the teaching of rounding off numbers and estimating answers. An incorrect answer may appear on the display due to a student entering a wrong digit, calculator misfunction or display limitations. A student must be able to determine whether or not an answer is reasonable. The ability to estimate is a check, therefore, not only on student accuracy, but on the calculator as well.

SUGGESTIONS:

- Teach students how to round off to 10's, 100's, 1000's, etc.
- Teach students when to round off to 10's, 100's, 1000's, etc.
- Have students round off numbers to be computed, perform the indicated operation, and obtain an estimate prior to entering the problem on the calculator.
- Have students then do the problem using calculators and compare their answers to the estimate.
- Follow a similiar procedure when teaching decimal fractions.

Activities similiar to the following sharpen students' skills in rounding off numbers. The use of a "sum of rounded numbers" box provides immediate reinforcement.

Round off to nearest 10. Enter each rounded off number and find the sum. Compare with the box number.

88	18	29
71	92	38
66	48	47
37	61	66
53	27	98
320	250	290

Round off to nearest 1000. Enter each rounded number and find the sum. Compare with the box number.

2194	1770	4300
6547	2004	9076
1714	6995	6572
4981	7099	2194
6053	5473	9984
22000	23000	32000

USE YOUR CALCULATOR TO:

Square the numbers below. Round off the answers to the nearest whole number. What important events occurred on these famous dates?

NUMBER	dates	Events that occurred on the famous date.
38.626^2	______	______________________
40.249^2	______	______________________
42.142^2	______	______________________
42.567^2	______	______________________
43.185^2	______	______________________
43.794^2	______	______________________
43.920^2	______	______________________
44.056^2	______	______________________
44.283^2	______	______________________
44.373^2	______	______________________

28

A. Round off and estimate. The symbol ≈ means "is approximately equal to."

1. 795 + 602 ≈ ______	6. 8917 − 875 ≈ ______
2. 972 − 129 ≈ ______	7. 19 X 9 x 12 ≈ ______
3. 49 x 62 ≈ ______	8. 659 ÷ 11 ≈ ______
4. 99 ÷ 19 ≈ ______	9. (99−31) x 49 ≈ ______
5. 5607 + 359 ≈ ______	10. 6942 x 9 ÷ 36 ≈ ______

B. Do each problem above, using your calculator. Compare each answer with your estimate.

Estimate the answer to each problem, then do each problem using your calculator. Were your estimates reasonable? The first two have been done for you.

	ESTIMATE	CALCULATION	REASONABLE ESTIMATE?
1. 427 + 660	1100	1087	YES
2. 29 x 31	9000	899	NO
3. 1000 − 89			
4. 2136 ÷ 3			
5. 158 x 2			
6. 9857 + 4999			
7. \$5.00 − \$1.98			
8. 2673 ÷ 99			

29

At the appropriate level of instruction of your students, provide extensive multiple choice activities similar to the following.

Estimate the answers: Circle your estimate for each problem.

1. John walks at the rate of 5.2 kilometers per hour. In 4 hours he walks

   ____?____ kilometers.

   100   20   12

2. One mile is equal to 5280 feet.

   5 miles = ____?____ feet.

   10000   500   25000

3. A car costs $3875. The down payment is $967. What is the balance?

   $3000 $2500 $1800

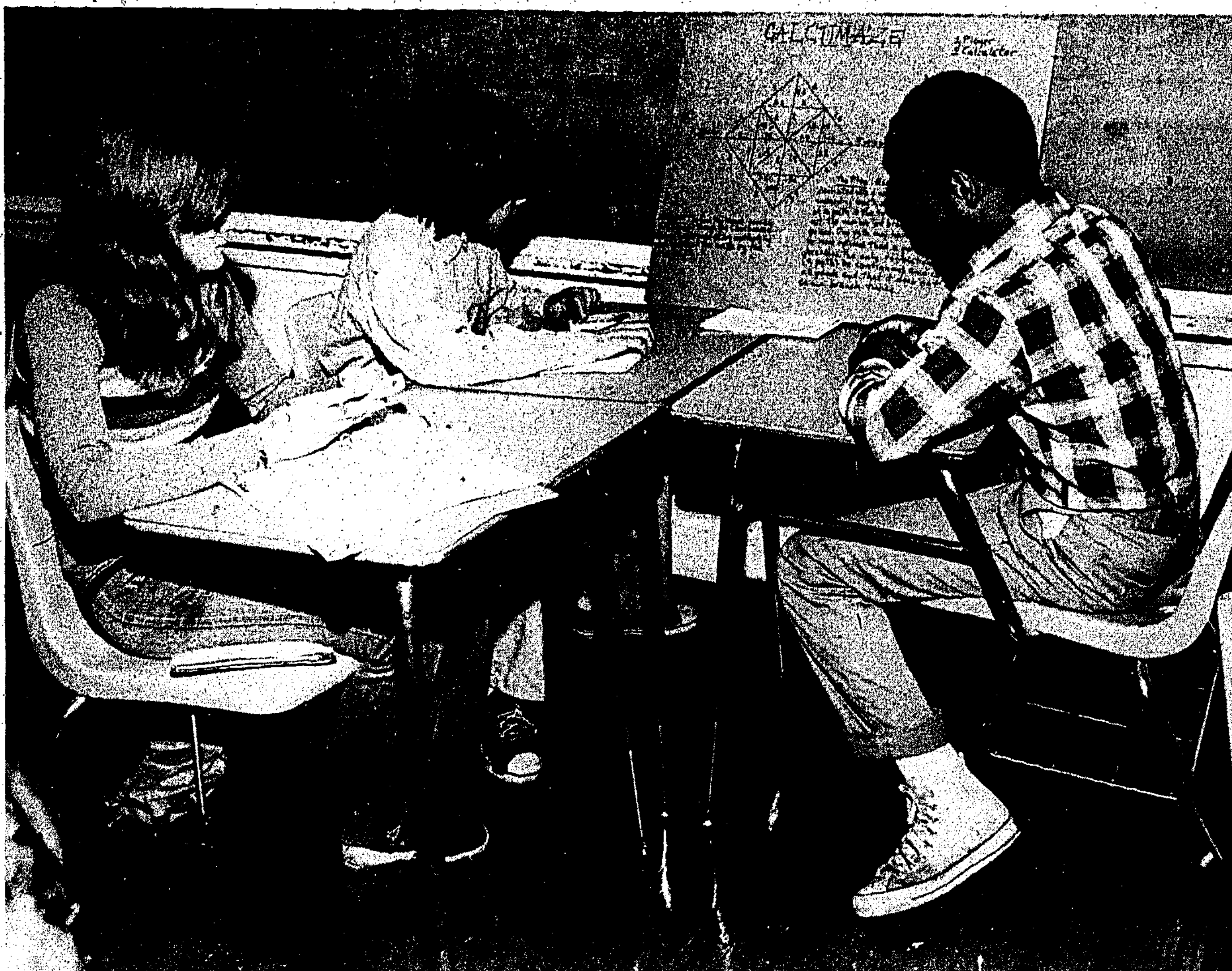
<u>Calculate the answers:</u> Use your calculator and place the answers below.

1. ________________
2. ________________
3. ________________

30

WHOLE NUMBERS

31



WHOLE NUMBERS

Basic skills are continually reinforced throughout all levels of the curriculum. When providing drill work in basic skills, use the mini-calculator for immediate reinforcement of students' responses.

The following activity may be teacher-directed with entire class responding, or used with pairs of students working together. Problems may appear on the chalkboard, on an activity sheet or on flash cards.

Enter each problem on your calculator, respond verbally with answer <u>then</u> press $\boxed{=}$.

2 x 2	5 + 7	32 ÷ 8	11 - 3
3 x 7	4 + 3	54 ÷ 6	6 - 2
2 x 9	7 + 2	12 ÷ 3	4 - 1
8 x 3	6 + 1	56 ÷ 8	7 - 0
3 x 8	11 + 8	14 ÷ 2	10 - 9
6 x 9	9 + 7	21 ÷ 3	18 - 18
4 x 3	12 + 3	63 ÷ 9	12 - 4
7 x 1	8 + 0	36 ÷ 6	19 - 1

Use your calculator to determine which of the following products are correct. Circle only correct answers.

$$1.\ \begin{array}{r} 51 \\ \times\ 16 \\ \hline 826 \end{array} \qquad 2.\ \begin{array}{r} 37 \\ \times\ 42 \\ \hline 1544 \end{array} \qquad 3.\ \begin{array}{r} 876 \\ \times\ 456 \\ \hline 399456 \end{array} \qquad 4.\ \begin{array}{r} 2468 \\ \times\ 1357 \\ \hline 3349076 \end{array}$$

5. 24 x 68 = 1732

6. 176 x 268 = 47158

Use your calculator to determine which of the following quotients are correct. Circle only correct answers.

1. 21033 ÷ 57 = 269
2. 12177 ÷ 99 = 133
3. 1292 ÷ 17 = 76
4. 1288 ÷ 14 = 92
5. 3425904 ÷ 1929 = 1766
6. 39933 ÷ 87 = 459

It is extremely important for students to realize that the human mind can think while a mini-calculator cannot "think". The calculator has been programmed by humans to do calculations very rapidly and accurately. However, some calculations can be handled more efficiently by the human mind than by the calculator (i.e. it is faster to do some problems "in one's head" than to take the time to enter the problem on a calculator). Certainly, most of us would use a calculator to compute "26.76 x 32.056" but would do "25 x 10" mentally.

Consider multiplying whole numbers by 10, 100, 1000, etc. or multiples of 10, 100, 1000, etc. A calculator may be used to discover that multiplication by 10's, 100's, 1000's, etc. is the same as annexing a zero to the number being multiplied, 2 zeroes, 3 zeroes, etc. But, once this is learned, the use of the calculator is unnecessary. To emphasize this idea, try the following activity:

Now that you have learned some rules for multiplying by 10's, 100's and 1000's and multiples of 10's, 100's, and 1000's, the following examples may be easier to compute in your head rather than using the calculator. Pit yourself against a student with a calculator. Both of you should start at the same time, you computing mentally while your friend uses the calculator. Both of you should write down each product. See who finishes first. GET READY . . . GET SET . . . GO!

1. 34 x 10 =	________	11. 32 x 30 =	________
2. 97 x 100 =	________	12. 25 x 40 =	________
3. 170 x 1000 =	________	13. 330 x 100 =	________
4. 999 x 100 =	________	14. 37 x 1000 =	________
5. 45 x 20 =	________	15. 1000 x 875 =	________
6. 10 x 100 =	________	16. 100 x 46 =	________
7. 167 x 10 =	________	17. 10 x 340 =	________
8. 50 x 200 =	________	18. 400 x 23 =	________
9. 19 x 100 =	________	19. 50 x 30 =	________
10. 4567 x 10 =	________	20. 20 x 150 =	________

When providing practice activities for operation with whole numbers, the teacher can take advantage of some of the motivational aspects of calculator usage. When the display is turned upside down, several of the digits can be interpreted as letters. Use this not merely as an interesting diversion, but as an integral part of your activity.

Do these problems on your calculator. Place your answers below. Do the indicated operations to obtain a final result.

1. 544 + 168   2. 1300 ÷ 2   3. 158 - 85   4. 203 x 7

______ + ______ + ______ − ______ ______

1.   2.   3.   4.   RESULT

WITH YOUR ANSWER STILL ON THE DISPLAY, TURN YOUR CALCULATOR UPSIDE DOWN. YOU NOW HAVE A TALKING MACHINE.

Make up a set of problems including some with incorrect solutions. Give them to a classmate with a calculator and have him/her circle only the correct answers. Your partner scores a point for each correctly circled answer after which you and your partner reverse positions.

Do these problems on your calculator

1. 68 [illegible] 33   2. 1[illegible] x 6   3. 391 - 104   4. 75 ÷ 5

Enter [illegible] answers below [illegible]nd add.

______ + ______ ______ + ______ = ______

2.   3.   4   TOTAL

[illegible] your [illegible] the International [illegible] Signal, congratulations. If not, check your work.

35

GOAL — a calculator game for 2 players

Procedure:

1. Enter start number in the machine.
2. Use only the designated operation key.
3. Alternate play be entering a one digit number and pressing the operation key.
4. The first person to display the GOAL wins.
5. If the goal is "surpassed", game is a draw.

GAME 1 START #51 OPERATION - GOAL 0	GAME 2 START #600 OPERATION - GOAL 555	GAME 3 START #63 OPERATION - GOAL 51
GAME 4 START #73 OPERATION + GOAL 100	GAME 5 START #0 OPERATION + GOAL 18	GAME 6 START #621 OPERATION + GOAL 657
GAME 7 START #16 OPERATION x GOAL 160	GAME 8 START #1 OPERATION x GOAL 15	GAME 9 START #6 OPERATION x GOAL 1224
GAME 10 START #400 OPERATION ÷ GOAL 10	GAME 11 START #650 OPERATION ÷ GOAL 5	GOAL 12 START #48 OPERATION ÷ GOAL 1

Magic Square Repairs!

Use your calculator to find an incorrect entry in each of the magic squares below. For a square array to be a magic square, each row, column, and diagonal must have the same sum.

Example:

8	3	4
10	5	9
6	7	2

Location of incorrect entry?

Row 2 Column 1

Correct entry for location? 1

1)

7	26	22
34	18	2
14	10	30

Location of incorrect entry?
Row ______ Column ______
Correct entry for location? ______

2)

2475	1980	2079
1782	2178	2574
2277	2376	1991

Location of incorrect entry?
Row ______ Column ______
Correct entry for location? ______

3)

14	49	59	84
79	65	34	29
44	19	89	54
69	74	24	39

Location of incorrect entry?
Row ______ Column ______
Correct entry for location? ______

4)

63	21	28	42
42	28	21	63
14	56	27	35
35	49	56	14

Location of incorrect entry?
Row ______ Column ______
Correct entry for location? ______

Forbidden Keys!

In the following problems, use only the designated number keys. You may use any of the function keys on your calculator. e. g. make your display read [1] [3] by using only the [4]

[4] [÷] [4] [+] [4] [+] [4] [+] [4] [=] [1] [3]

1. Use only the [1] key 5 times to display [1] [0] [0].
2. Use only the [8] key 8 times to display [1] [0] [0] [0].
3. Display [1] [1] [1] by using only the [2] key.
4. Use only the [4] key 3 times to display [2] [0]
5. Use only the [3] key 4 times to display [2] [4]
6. Use each of the [1], [2], [3] and [4] keys but once in any order to display [2] [5]
7. Use each of the [1], [2], [3] and [4] keys but once in any order to display [2] [1]

A mini-calculator equipped with a constant key has many advantages for the classroom teacher. Check the instructional manual that accompanies your mini-calculator to determine if your machine has this feature. Depending on the model and make of the mini-calculator, the constant key will work differently.

Suggested Activities:
(An Alcor Grand Prix mini-calculator was used in the following examples.)

1. Basic counting practice

press [+] [1] [=] [=] [=] [=] [=], etc.
display will read 1, 2, 3, 4, 5 etc.

press [+] [2] [=] [=] [=] [=] [=], etc.
display will read 2, 4, 6, 8, 10 etc.

press [+] [100] [=] [=] [=] [=] [=] [=], etc.
display will read 100, 200, 300, 400, 500, etc.

38

2. Counting backwards

press $\boxed{100}$ $\boxed{-}$ $\boxed{1}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
display will read 99, 98, 97, 96, 95, etc.

press $\boxed{50}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
display will read 45, 40, 35, 30, 25, etc.

3. Drill of multiplication facts

press $\boxed{5}$ $\boxed{x}$ $\boxed{1}$ $\boxed{=}$ $\boxed{2}$ $\boxed{=}$ $\boxed{3}$ $\boxed{=}$ $\boxed{4}$ $\boxed{=}$ $\boxed{5}$ $\boxed{=}$
display will read 5, 10, 15, 20, 25 etc.

press $\boxed{8}$ $\boxed{x}$ $\boxed{1}$ $\boxed{=}$ $\boxed{2}$ $\boxed{=}$ $\boxed{3}$ $\boxed{=}$ $\boxed{4}$ $\boxed{=}$ $\boxed{5}$ $\boxed{=}$
display will read 8, 16, 24, 32, 40, etc.

4. Drill of division facts

press $\boxed{60}$ $\boxed{\div}$ $\boxed{6}$ $\boxed{=}$ $\boxed{54}$ $\boxed{=}$ $\boxed{48}$ $\boxed{=}$ $\boxed{42}$ $\boxed{=}$ $\boxed{36}$ $\boxed{=}$
display will read 10, 9, 8, 7, 6, etc

5. Multiplication as successive additions

press $\boxed{+}$ $\boxed{2}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
Display will read 2, 4, 6, 8, 10
$10 = 2 + 2 + 2 + 2 + 2 = 2 \times 5$

6 Division as successive subtractions

press $\boxed{56}$ $\boxed{-}$ $\boxed{7}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
display will read 49, 42, 35, 28, 21, 14, 7, 0.
thus, $56 \div 7 = 8$

7. Squaring numbers

press $\boxed{7}$ $\boxed{x}$ $\boxed{=}$

display will read 49

$7^2 = 49$

39

The importance of teaching the use of parentheses, order of operations and properties of whole numbers becomes apparent when using calculators in the classroom. Most calculators perform operations as they are entered. Thus, if "3 x 5 + 4 = " is entered [3] [x] [5] [+] [4] [=], the display reads "19", the correct answer. But, if "4 + 3 x 5" is entered [4] [+] [3] [x] [5] [=], an incorrect answer "35" is displayed. There are several mini-calculators on the market that have "algebraic hierarchy." These machines are programmed to receive the entire expression, then calculate it according to the process hierachy (parentheses, exponentiation, multiplication, division, addition, subtraction) automatically.

Prior to teaching order of operations, the use of parentheses around the product "3 x 5" in the above example should clear up any ambiguities. This raises another important point, though. If the calculator being used has no memory unit, a student is forced to write an intermediary answer on paper prior to arriving at a final result when working with "4 + (3 x 5)". A knowledge of the properties of whole numbers enables the student to enter many computations directly without resorting to a pencil and paper "memory". To illustrate,

4 + (3 x 5)

ENTER	DISPLAY
3	3
X	X
5	5
+	15
4	4
=	19

Provide opportunities for students to apply their knowledge of using parentheses, order of operation and properties of whole numbers. Since all three skills are not usually taught at the same time, be prepared to answer questions concerning "wrong" answers when students enter certain types of calculations.

The following activity may be used at the junior high level after all three skills have been taught in depth. Parentheses have been used only where necessary.

40

Do the following using your calculator.

1. 290 + 5 x 190 = ________

2. 627 x 41 + 75 = ________

3. (29 + 75) x 6 + 56 = ________

4. (11 x 11 + 29) x 40 = ________

5. 4282 x 5 + 11 x 6 = ________

Find the sum of your 5 results. Turn your unit upside down. Does your display reflect what life is like in our classroom? If not, check your work.

Use the calculator to discover rules for divisibility. The following activities are a suggested sequence for exploratory work. Each can be modified to meet individual classroom needs.

How may a calculator be used to determine that one number is divisible by another? Test each first number for divisibility by the second.

1. 565 by 5. 2. 343 by 3 3. 14028 by 7 4. 297636 by 4

A number is divisible by another number if there is no ________________.

What are odd and even numbers?

Check some odd numbers to see if they are divisible by 2.
Check some even numbers to see if they are divisible by 2.
Can you find an even number that is not divisible by 2?
Can you find an odd number that is divisible by 2?
Then what is the rule to tell if a number is divisible by 2?
A number is divisible by 2 if and only if

____

____

What is a short cut way of multiplying a number by 10?

If you don't remember, try these on the calculator to find out:

1. 10 x 17 2. 10 x 9 3. 10 x 230 4. 10 x 1098

Do you see a pattern?
Then every number with ________ in the one's place is divisible by 10.

Find these products using your calculator:

1. 5 x 674 2. 5 x 703 3. 5 x 27 4. 5 x 3040 5. 5 x 11311

Multiply 5 by four other numbers.
Do you see a pattern?
A number is divisible by 5 if and only if ______________________________
____.

Use your calculator to test for divisibility by 3.

1. 360 2. 5172 3. 50412 4. 622 5. 2721 6. 121212
7. 981 8. 1010101 9. 11476 10. 672000

What is the sum of the digits in each of the above?

1. ______ 2. ______ 3. ______ 4. ______ 5. ______ 6. ______ 7. ______

8. ______ 9. ______ 10. ______

Do you see a pattern? State this pattern in the form of a divisibility rule.

Further explorations:

. . . divisibility by 9
. . . divisibility by 6
. . . divisibility by 4
. . . use the calculator when determining the prime factorization of a number. In order, divide numbers by 2, 3, 5, 7, 9, 11, etc.
. . . use the calculator to show the uniqueness of a prime factorization, irrespective of order, i.e. 60 = 30 x 2 = 20 x 3 = 15 x 4 = 12 x 5 = 10 x 6 = 2 x 2 x 3 x 5.

Here is a check for divisibility by 11:
A number is divisible by 11 if the difference of the sum of alternate digits is divisible by 11. (start from the right.)

Does 11 divide 3014682?

3014682 (2 + 6 + 1 + 3) - (8 + 4 + 0)

12 - 12 = 0

∴ 11 divides 3014682

42

Test these numbers for divisibility by 11:

6456	4715
41195	713823
15248	749749

Find the missing digit that will make each resulting number divisible by 11.

5 _____ 60 2 _____ 74

638 _____ 38 40 _____ 54

Enter a 3 digit number in your dispaly. Repeat the digits to obtain a six digit number.

e.g. If "123" is your three digit number, then "123123" is your six digit number.

1. divide the display number by 7.
2. divide this quotient by 11.
3. divide this quotient by 13.

Compare your answer with your original number.

Do this several times using different three digit numbers.

Try dividing first by 13, then 11 and 7.

Try a different order of divisors.

Does the order of the divisors change the answer?

Why does this work?

4.5

DECIMAL FRACTIONS

HAZEL

"MY calculator disagrees."


G

DECIMAL FRACTIONS

As most calculators display all fractions in decimal form, greater emphasis will be placed on the teaching of decimal fractions. There is a very strong possibility, therefore, that the curriculum will change so that the teaching of decimal fractions will precede that of common fractions. In order to emphasize the location of the decimal point, many mini-calculators automatically enter any decimal fraction less than one with a zero preceding the decimal point by displaying 0 . when the machine is turned on. This terminal decimal point is usually retained in the display of every whole number. Therefore, a "floating" decimal point is a decided advantageous feature of the mini-calculator. It is an aid to complete understanding of decimal notation.

TURN ON ENTER	DISPLAY	TURN ON ENTER	DISPLAY
	0.		0.
.	0.	1	1.
3	0.3	3	13.
4	0.34	4	134.

Problems such as the following will add interest to, and give instant verification of, correct entry of decimal fractions and operations.

> Do the following problems with your calculator. Be careful of the decimal point.
>
> (1.5 x 2) ÷ (2 x 1.1133)
>
> Turn your calculator upside down to be welcomed by the calculator to decimal fractions. If you have not been greeted, check your work.
>
> Make up a problem using decimal fractions so you can answer.

The upside down calculator vocabulary may be used with or without a decimal point. The decimal point, however, can be used to separate two words, (he.is). Refer to the mini-calculator vocabulary list on page 115.

Activities and worksheets involving money may serve as an introduction to decimal fractions. More advanced topics involve rounding off, decimal and common equivalents, combinations such as 0.02 1/2, powers and roots, repeating decimals, etc. Rounding off decimal fractions to obtain an estimated answer is an important component of self-checking and verifying the accuracy of the display. Some mini-calculators automatically round off decimal fractions, including repeating decimals, that exceed the display limitations; others will truncate (cut off) at the end of the display.

Discovering the equivalence of numbers such as 0.8, .8, 0.80, etc. will reinforce the significance of the decimal point and zero in place value notation. Non-significant zeros do not appear in the displays as they are automatically dropped.

Compare the answers for each section. What do these answers indicate?

1.	26 + .9 =	2.	34 x .6 =
	026 + .90 =		034 x .60 =
	26 x 0.900 =		34 x 0.600 =
	0026 x 0.9 =		0034 x 0.6 =
3.	20 - .3 =	4.	15 ÷ .5 =
	020 - .30 =		015 ÷ .50 =
	20 - 0.300 =		15 ÷ 0.500 =
	0020 - 0.30 =		0015 ÷ 0.5 =

The placement of the decimal point in each component of a problem as well as in the answer can be reinforced by the following suggested activities and others created by the teacher. Before the calculator is used, students should be encouraged to estimate the answer, do the computation using the indicated algorithm and use the calculator as a self-check or as directed.

The mini-calculator can aid in developing the concepts needed to order decimal fractions.

Enter the decimal point. Without looking press four number keys. Record the result. Repeat this three times. Label the three resulting decimal fractions "high", "middle" and "low".

Sample:

0.	4	6	8	2	H
0.	0	1	6	5	L
0.	2	4	0	6	M

Choose a team and play the game of "Hi-Lo". Each player records and labels the three "blind" numbers as above. The numbers labeled "high" and "low" by each player are then compared. The player with the highest number scores 2 points; the lowest number scores 1 point. The winner is the first player to reach 25 points. The game can be varied by entering the decimal point after the first or second digit of each number entry.

In each of the following pairs of expressions choose the one you think has the largest answer and do the work indicated.

1.	0.03 + 0.7	0.3 + 0.07
2.	1.5 + 6	1.05 + 0.6
3.	0.03 - 0.001	0.3 - 0.001
4.	0.79 - 0.7	7.9 - 7
5.	0.007 x 0.3	0.7 x 0.03
6.	1.50 x 0.5	15 x 0.005
7.	0.16 ÷ 0.04	1.6 ÷ 0.004
8.	0.28 ÷ 0.2	2.8 ÷ 0.02

Add the eight answers. If their sum is 550.2 your estimates were correct. If not, use your mini-calculator to check your work.

Be careful when creating probl[illegible] s in[illegible]g multiplic[illegible]ion of very s[illegible]all decimal [illegible]ctions (0.0000[illegible]5 x [illegible]1, 0.000023 [illegible] .00[illegible]). Some ca[illegible]lators will d[illegible]splay $\boxed{\ .\ }$ a[illegible] the answer, as the disp[illegible] cap[illegible]city has been exceed[illegible].

The con[illegible]ts n[illegible]cessary for rounding [illegible] whole numbers as an aid to estimat[illegible]on sho[illegible]ld be extende[illegible] to decim[illegible] fractions. Rounding off t[illegible] nearest whole number is of special value. The calculator is of littl[illegible] [illegible]sistance in rounding off [illegible]ecimals unless the unit has a sp[illegible]cial rounding-off function key. It is [illegible] valuable to be able to r[illegible]cognize that the answer resulti[illegible]g from pencil and paper or calculato[illegible] [illegible]mputation is within reas[illegible]n.

By preparing flash cards of decimal fractions, activities such as the one below can be presented.

Use your calculator. Enter the numbers that are the answers to these questions as addends.

1. Round off | 2. | 2 | 4 | 2 | 6 | to the nearest whole number.
2. Round off | 0. | 9 | 9 | to the nearest tenth.
3. Round off | 0. | 0 | 3 | 8 | to the nearest hundredth.
4. Round off | 0. | 0 | 0 | 5 | 1 | to the nearest thousandth.

Look at the sum of these numbers with your mini-calculator held upside down. If it tells you what to wear on your foot, you were right. If not, check your work.

The correct digits in the answers to the following problems are given. All you have to do is put the decimal point in the right place.

1. 3.46 + 15 + 1.007 + 0.93 = 20397
2. 34.6 + 1.5 + 10.07 + 9.3 = 5547
3. 19 - 0.19 = 1881
4. 4.63 - 0.012 = 461[illegible]
5. 12.8 x 0.05 = 64
6. 2.94 x 1.6 = 470[illegible]
7. 8.84 [illegible]
8. 15.24 ÷ 0.24 = [illegible]

[illegible] your [illegible] calculator to find the sum [illegible] the eight answers. If this sum rounded off to the nearest [illegible] number is 172, you did [illegible]

The answer[illegible] some of the problems [illegible] are not accurate. Cross out the wrong answers and correct [illegible]

1. [illegible]
   [illegible]
   0.[illegible]
   + 2.[illegible]8
   97.619

2. 4.603 + 17 + 0.008 = 21.[illegible]1

3. 261.5 - 14.83 = 246.72

4. 2.008 - 0.07 = 1.938

5.
     17.5
   x 1.6
      2.8

6.
     3.04
   x 0.93
   2.8272

7. 15 ÷ 0.02 = 750

8. 2.367 ÷ 0.9 = 2.63

Use your calculator to prove you found all the errors.

Do this problem on your paper.

33928.75 x 12

Enter the answer on your calculator. If the upside down display names a famous Civil War battle you were correct.

There is at least one error in each example. Find and correct each mistake.

1.
$$\begin{array}{r} 3.41 \\ \times\ 4.2 \\ \hline 792 \\ 1364 \\ \hline 1.4442 \end{array}$$

2.
$$\begin{array}{r} 3.194 \\ .75 \\ 1.005 \\ +\ \ .6 \\ \hline 4.955 \end{array}$$

3.
$$\begin{array}{r} 24.6 \\ -\ \ .14 \\ \hline 24.54 \end{array}$$

4.
$$\begin{array}{r} 47. \\ -\ 1.3 \\ \hline 3.4 \end{array}$$

5.
$$\begin{array}{r} 2.61 \\ 0.16 \overline{)27.76} \\ 32 \\ \hline 97 \\ 96 \\ \hline 16 \end{array}$$

6.
$$\begin{array}{r} .36 \\ 1.21 \overline{)4.356} \\ 363 \\ \hline 716 \\ 716 \\ \hline \end{array}$$

Check the answers with your calculator. Did you find all the errors?

Use your calculator to find the answers to the following. Record the answers.

1.	2.	3.
213 x 10 =	314 x 100 =	716 x 1000 =
4.56 x 10 =	7.39 x 100 =	8.42 x 1000 =
0.92 x 10 =	0.79 x 100 =	0.86 x 1000 =
0.03 x 10 =	0.05 x 100 =	0.09 x 1000 =
0.000 x 10 =	0.000 x 100 =	0.0004 x 1000 =

Compare the answers. Did you find a short cut that will let you beat the calculator? Try it on the following problems. Use your calculator to check:

4.7 x 100 =	16.03 x 10 =	0.01 x 1000 =
0.001 x 10 =	29.4 x 1000 =	0.340 x 100 =

The same format as above, but using division, should lead students to discover the algorithm for division of decimal fractions.

Activities dealing with repeating decimals are usually highly motivating to students. This type of decimal fraction with its endless repetitive structure arouses interest, especially in its relation to prime number denominators and patterns so formed. Unfortunately, the display limit of the calculator may make it difficult for the student to be sure that the display shows a repeating decimal and its period. This again should prove to the student that the brain is much more powerful than any calculator because it can determine an endless repetition of digits by seeing a pattern in the division problem. $\frac{1}{7}$ will result only in a display of 0.1428571 on most calculators

50

which does not prove an exact repetitive pattern, but computation will reveal both the exact digits in the period and the repetition of the period.

$\frac{1}{7}$

```
        .1428571
    7 )1.0000000
        7
        30
        28
         20
         14
          60
          56
           40
           35
            50
            49
             10
```

Here is an interesting number cryptogram to decipher. Note that the answer is a repeating decimal.

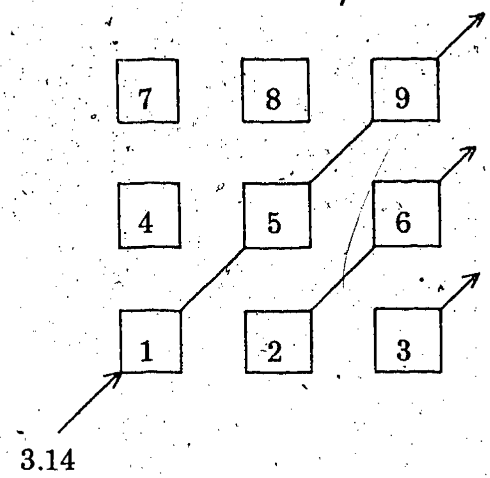
$$\frac{EVE}{DID} = .TALK = .TALK\ TALK\ TALK\ldots$$

As you use your calculator keep a record of your work.

An aid to determining the period of a repeating decimal is the value of the denominator of its common fraction equivalent. The period of repetition may be as long as one less than the value of the denominator or any factor thereof. That is, if the denominator is, say, 15, the period of repetition may extend 14 digits before repeating, or any factor of 14 including one. Thus the periodic cycle of repetition of the decimal equivalents of fifteenths may be 1, 2, 7 or 14 digits. The number of decimal places retained in a problem is, of course, determined by the required accuracy of the problem.

Note that you have to determine if the particular calculator in use truncates or rounds off at the last digit. If the calculator truncates one more than the required decimal places should be used to let the student round off the answer.

Some decimal fractions are neither terminating nor repeating. These decimals are called irrational numbers. The number π ("pi") is the most familiar irrational number, defined as the ratio of the circumference of a circle to its diameter. The computer has now carried the value of π out to a million place without any repetitive period. However, most junior high textbooks use "3.14" as an approximate value for π. An interesting activity is the arrangement of the numbers of the keyboard of the mini-calculator to extend this value:



$\therefore \pi \approx 3.14159263$

This results is a closer approximation of π - 3.1416

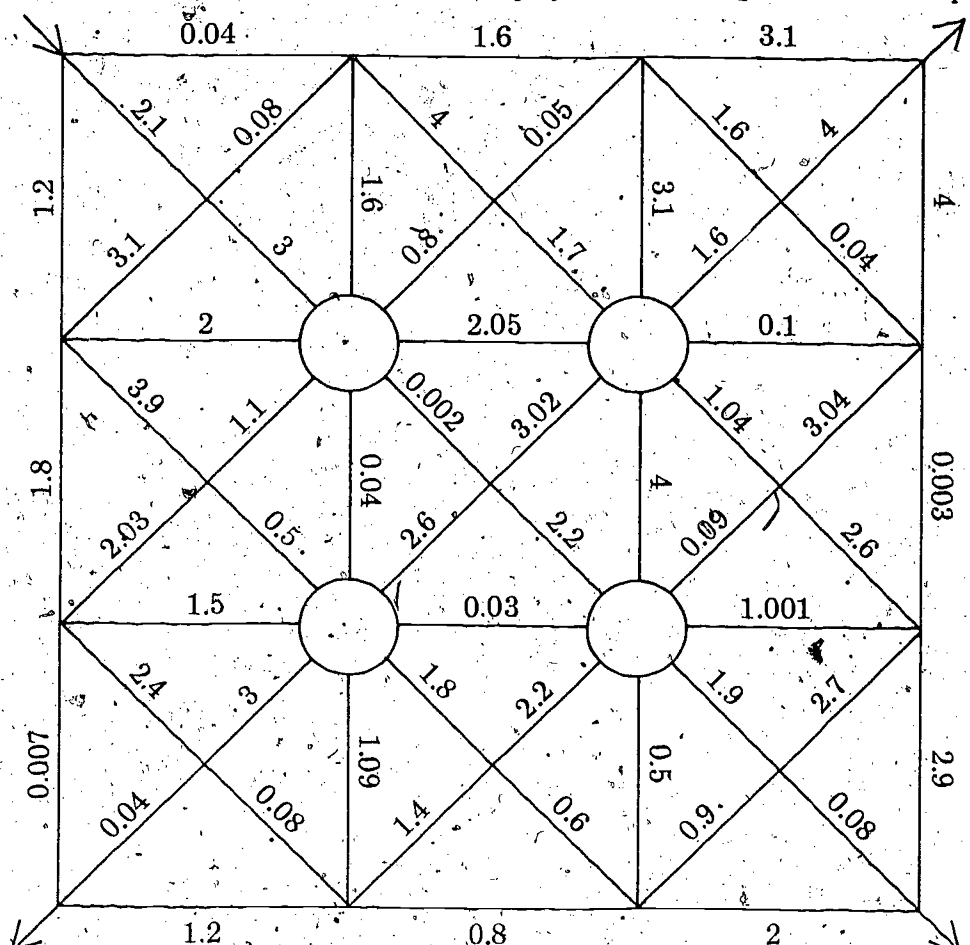
A MULTIPLICATION PATH

A game for two or three players to reinforce multiplication of decimals.

Materials: Calculators
Game board (one board per game is needed)
3 colored pencils (different colors)

Object: To achieve the highest score by multiplying the values assigned to the line segments traced while attempting to form a continuous path to an exit corner.

Play: Each player, in turn, travels one line segment. Using a colored pencil, the player traces the continuous path thus formed. The value assigned to each line segment traveled is used as a multiplier of the product already achieved by the player. Each player must enter at the corner indicated, but may exit at any other corner of the grid. Each line segment may be used only once. Paths may not cross except at the safety holes so provided. The game is over when all players who can have exited. The winner is the player with the highest cumulative product.



Variation: Have students assign values to the line segments. Set the winning score as the lowest product. Duplicate enough game sheets for the class.

ADD-O-MAZE

Enter the maze at any corner cell. Trace a path through the maze horizontally or vertically. Diagonal paths are not allowed. You may not enter the same cell twice or cross your path. Add the numbers as you go. The sum of the numbers in the cells through which you travel must total 30. You may leave the maze only through the exit cell. Don't get lost!

4	2.16	3.8	1.08
1.07	3.11	2.7	2.5
8	5.43	1.02	6
1.3	4.8	3.64	2.9

Hint: There are 8 cells in the path.

5

THE SUPER CALCULATOR

101 is a magic number in multiplication. Use your mini-calculator to record the first three examples in each group. Find the pattern -- Yes, you can! - and do the rest with your super calculator - your brain. Record your answers and use your calculator to check.

1.
   - 2.1 x 101 =
   - 3.4 x 101 =
   - 8.6 x 101 =
   - 7.5 x 101 -
   - 9.3 x 101 =
   - 6.9 x 101 =
   - 4.8 x 101 =

2.
   - 4.13 x 101 =
   - 2.65 x 101 =
   - 3.34 x 101 =
   - 1.08 x 101 =
   - 2.14 x 101 =
   - 5.23 x 101 =
   - 6.31 x 101 =

3.
   - 23.57 x 101 =
   - 31.45 x 101 =
   - 46.23 x 101 =
   - 23.45 x 101 =
   - 18.71 x 101 =

4.
   - 32.412 x 101 =
   - 41.523 x 101 =
   - 12.321 x 101 =
   - 14.543 x 101 =
   - 32.465 x 101 =

For Super calculators: Do the above examples using 1001!!

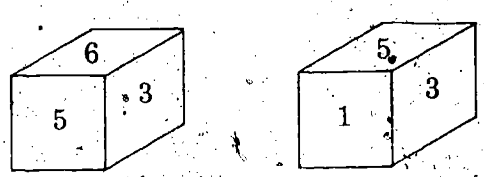
DECI-DICE

A game for 2-4 players to reinforce multiplication of decimals.

Materials: Pair of dice of different colors
Mini-calculator (s)
Score sheet and pencil

Object: To achieve the highest (lowest) score at the end of five turns by recording the cumulative product of successive throws.

Play: The players assign each color to a place value. To set the highest score as the winning score, one die is to be read as a whole number, the other as tenths. To set the score closest to zero as the winning score, one die is tenths; the other, hundredths.



High	Low
6.5	0.65

To initiate play, each player throws the dice two times, recording the throws and their product. This product serves as the initial factor. The highest product determines the starting player. Each player, in turn, builds a cumulative product using the initial factor and numbers created in each of five turns. The winner can either be the one achieving the highest cummulative product or the one whose score is closest to zero, depending on the rules established before initiating play. Once the object of the game is determined (high or low) each player tries to achieve the desired score.

SAMPLE SCORE SHEETS

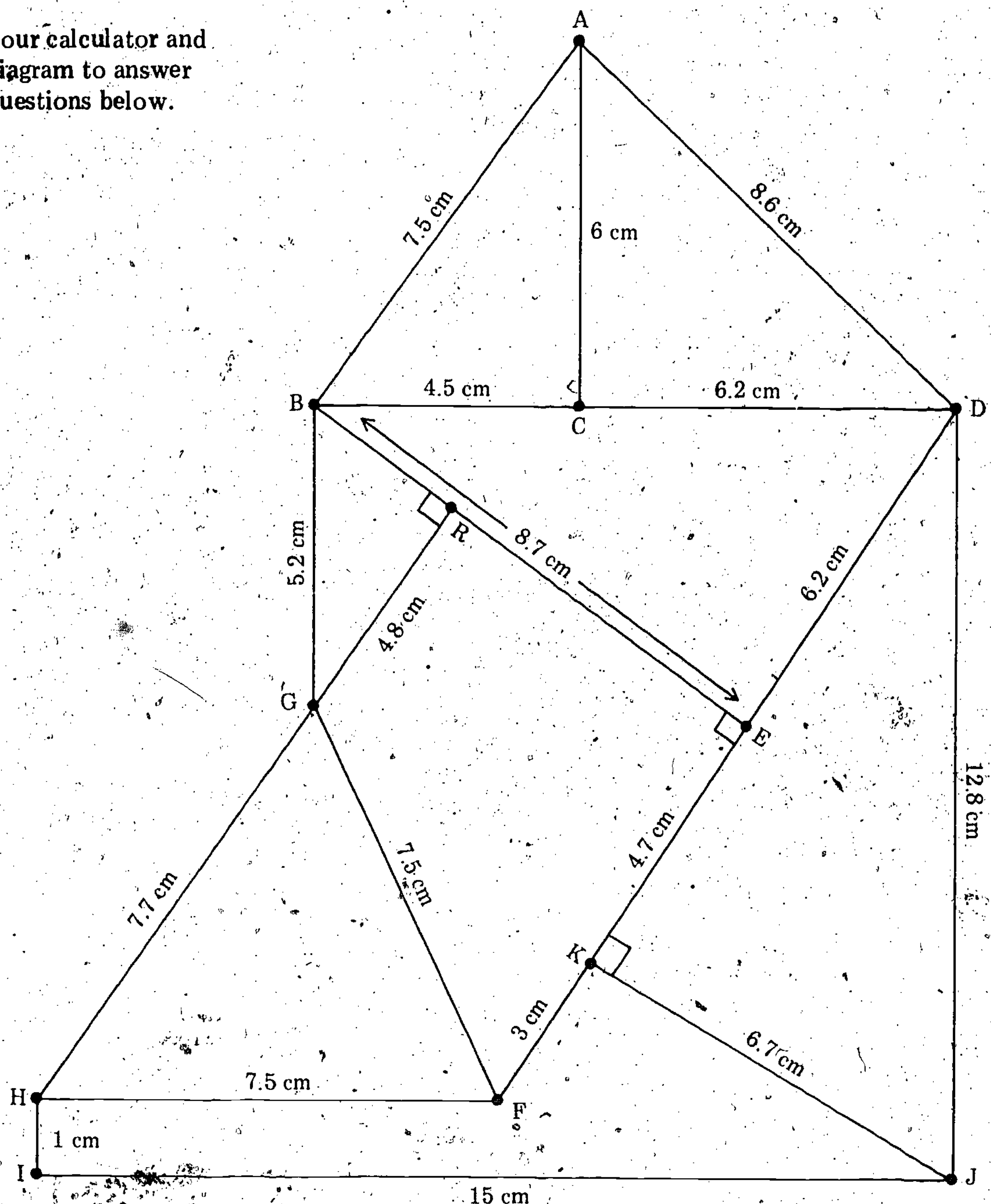
HIGH SCORE WINS		LOW SCORE WINS	
$\boxed{5}.\boxed{5} \times \boxed{3}.\boxed{4} =$	18.7	$.\boxed{5}\boxed{5} \times .\boxed{3}\boxed{4} =$	0.187
1	x 5.6	1	x 0.56
	104.72		0.10472
2	x 4.2	2	x .42
	439.824		0.0439824
3	x 6.1	3	x .61
	2682.9264		0.0268292
4	x 3.5	4	x .35
	9390.2424		0.0093902
5	x 1.3	5	x .13
	12207.315		0.0012207

Score = 12207.315

Score = 0.0012207

5ΰ

Use your calculator and the diagram to answer the questions below.



1. What is the length of the path from A to C to D? __________
2. What is the length of the shortest path
   From A to F? __________
   From B to J? __________
3. Prove m$\angle$ BCA = 90°
4. What is the area of Δ BRG? __________ Δ ACB? __________
   Δ GRFE? __________ Δ DKJ? __________
5. Which figure in the drawing has the greatest perimeter?
   Why? __________

Race to Zero - a calculator game for two players

Procedure:

1. Start with 1 2 3 4 5 6 7 8 in the display.
2. Players in turn divide by any 3 digits with decimal point in any position.
3. The first player to reach zero wins.

Question to explore:

1. Why does the machine register zero?
2. Is there a successful strategy?

The concept of infinitely small decimal fractions approaching zero as a limit can be utilized in the exercise which explores division by such numbers. As the divisor approaches zero as a limit, the quotient approaches infinity. However, this is not a proof that divisibility by zero is indeterminate. That concept depends on the application of the law $a \div b = c$ if, and only if, $c \cdot b = a$.

What happens when you divide by smaller and smaller numbers?

$5 \div 5 =$ ________
$5 \div 0.5 =$ ________
$5 \div 0.05 =$ ________
$5 \div 0.005 =$ ________
$5 \div 0.0005 =$ ________
$5 \div 0.00005 =$ ________
$5 \div 0.000005 =$ ________
$5 \div 0.0000005 =$ ________

$5 \div 0.000000000005 =$ ________

Did you need your calculator to finish the exercise?

Discover the pattern with the aid of your calculator, then apply it to the rest of the examples. Check a few to see if you were right.

1. $3 \div 9 =$
   $3 \div 99 =$
   $3 \div 999 =$
   $3 \div 9999 =$

2. $42 \div 99 =$
   $42 \div 999 =$
   $42 \div 9999 =$
   $42 \div 99999 =$

3. $341 \div 999 =$
   $341 \div 9999 =$
   $341 \div 99999 =$

4. $4781 \div 9999 =$
   $4781 \div 99999 =$
   $4781 \div 999999 =$

Can you extend this pattern to:

1. $34 \div 9 =$
   $23 \div 9 =$

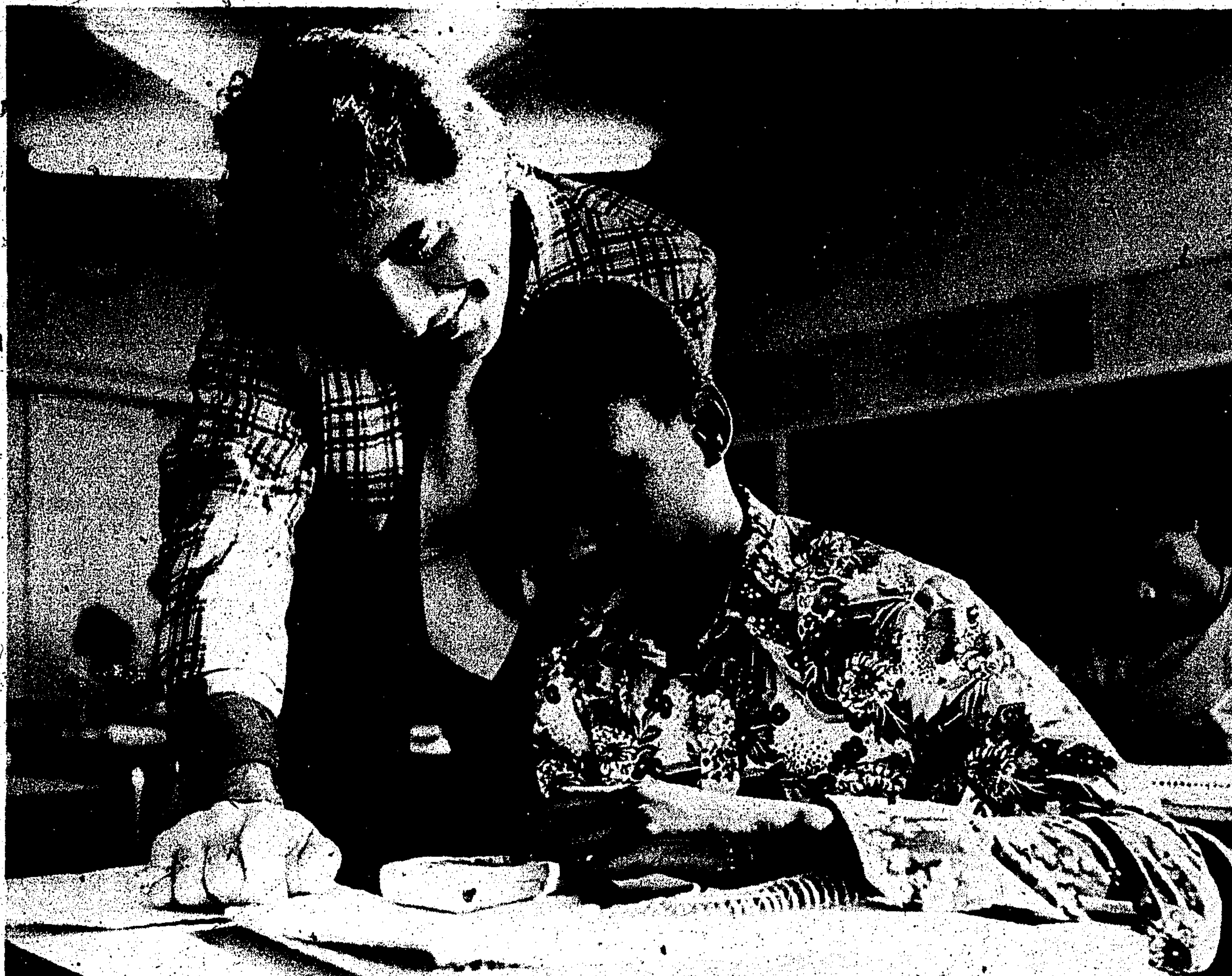
2. $403 \div 99 =$
   $324 \div 99 =$ etc?

What kind of decimal fractions did each problem generate?

59

COMMON FRACTIONS

60



COMMON FRACTIONS

The common fraction $\frac{a}{b}$ cannot be entered in that form in a calculator or be so displayed. For that reason, the students must be led to discover that the form of the common fraction implies division ($\frac{a}{b}$ means $a \div b$). As the student uses the calculator to reinforce this concept, a list of common fraction - decimal fraction equivalents should be generated. Although the increasing use of the mini-calculator and the advent of the metric system seem to make the common fraction much less important, it will still be a definite part of our industrial system. Measurements will still be given in such units as halves, quarters, fifths, etc. In addition, the common fraction will be used as an equivalent form of a repeating decimal fraction. However, computation with the common fraction may be presented as an enrichment topic.

Students should be introduced to the decimal form of the common fractions by examining the notation of dollars and cents:

$\frac{1}{4} \Longrightarrow$ "one quarter (of a dollar)" = \$0.25

$\frac{1}{2} \Longrightarrow$ "one half (of a dollar)" = \$0.50 etc.

Using the knowledge that there are 10 dimes or 20 nickels or 100 cents in a dollar can generate the decimal notation for 10ths, 20ths and 100ths.

$$7 \text{ dimes} = \frac{7}{10} = \$0.70$$

$$3 \text{ nickels} = \frac{3}{20} = \$0.15$$

$$23 \text{ cents} = \frac{23}{100} = \$0.23$$

After this method of conversion from common fraction to decimal fraction notation has been taught, the students should learn that the common fraction form ($\frac{a}{b}$), $b \neq 0$, implies division ($a \div b$). A common fraction, therefore, is entered into the calculator by performing the indicated division.

A list of common and decimal fraction equivalents should be maintained by the students. This list could be classified by "family" (denominators). In developing this list, students should be led to discover that many common fractions have the same decimal fraction notation and are, therefore, equivalent.

$$\frac{7}{8} = 0.875$$

$$\frac{14}{16} = 0.875 \qquad \therefore \frac{7}{8} = \frac{14}{16}$$

Equivalence classes can be generated and verified with the mini-calculator

$$\left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots \right\}$$

$$\left\{ \frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots \right\}$$

Write the decimal form for these common fractions. Think "money" to help you do the problems, if you can; use the calculator if you must.

$\frac{2}{5}$ =	$\frac{[illegible]}{10}$ =	$\frac{3}{20}$ =
$\frac{4}{5}$ =	$\frac{3}{6}$ =	$\frac{1}{2}$ =
$\frac{17}{34}$ =	$\frac{1}{4}$ =	$\frac{3}{4}$ =
$\frac{15}{30}$ =	$\frac{6}{8}$ =	$\frac{30}{40}$ =
$\frac{3}{5}$ =	$\frac{6}{10}$ =	$\frac{75}{100}$ =
$\frac{4}{8}$ =	$\frac{2}{4}$ =	$\frac{8}{10}$ =
$\frac{2}{8}$ =	$\frac{3}{12}$ =	$\frac{20}{50}$ =

Are any of the answers the same? Why? ______________________

Write 6 common fractions which will read as 0.25 in the display.

1. ________ 2. ________ 3. ________

4. ________ 5. ________ 6. ________

Do the indicated division with your calculator to check.

The mini-calculator can be used to determine the order of common fractions in two ways. By expressing the common fractions in their decimal form the order relationship can be easily determined. The other method uses cross multiplication to verify the relationship between two common fractions.

$\frac{2}{3}$? $\frac{3}{4}$

$\frac{2}{3} = 0.6666666$

$\frac{3}{4} = 0.75$

$8 < 9$

$\frac{2}{3} < \frac{3}{4}$

$\frac{23}{25}$? $\frac{29}{32}$

$\frac{23}{25} = 0.92$

$\frac{29}{32} = 0.90625$

$.736 > .725$

$\frac{23}{25} > \frac{29}{32}$

Use <, >, or = to show the relationship between each pair of common fractions.

1. $\frac{7}{8}$ ◯ $\frac{9}{10}$ 2. $\frac{48}{96}$ ◯ $\frac{2}{4}$ 3. $\frac{19}{11}$ ◯ $\frac{20}{13}$

4. $\frac{24}{36}$ ◯ $\frac{4}{6}$ 5. $\frac{12}{15}$ ◯ $\frac{11}{14}$ 6. $\frac{23}{46}$ ◯ $\frac{36}{72}$

7. $\frac{412}{505}$ ◯ $\frac{423}{516}$ 8. $\frac{3104}{4215}$ ◯ $\frac{2104}{3215}$ 9. $\frac{4612}{8731}$ ◯ $\frac{3712}{6521}$

The conversion of a mixed number to its decimal form relies on the commutative property for addition of rational numbers to be correctly displayed by the calculator $(4\frac{1}{5} = 4 + \frac{1}{5} = \frac{1}{5} + 4)$. Therefore, the order of entry to convert $4\frac{1}{5}$ to its decimal form is $\boxed{1}\ \boxed{\div}\ \boxed{5}\ \boxed{+}\ \boxed{4}\ \boxed{=}$

Use your calculator, if necessary, to record the decimal form equivalent of each mixed number:

1. $3\frac{1}{2} =$ 2. $2\frac{3}{8} =$ 3. $5\frac{4}{5} =$

4. $6\frac{3}{4} =$ 5. $7\frac{2}{9} =$ 6. $27\frac{3}{7} =$

7. $4\frac{4}{15} =$ 8. $17\frac{9}{7} =$ 9. $23\frac{24}{8} =$

Put a "C" on the examples for which you used your calculator.

Using the mini-calculator to compute with common fractions offers some problems. The four basic operations on common fractions can be performed by the calculator in three ways:

1. Record the decimal form of each common fraction in the problem using the calculator, if necessary, for the conversion. Perform the required operation on the resulting decimal fractions.

For example perform the four basic operations on $\frac{7}{8}$ and $\frac{1}{2}$

CALCULATOR	RECORD
$\boxed{7}$ $\boxed{\div}$ $\boxed{8}$ $\boxed{=}$ $\boxed{0.\,8\,7\,5}$	0.875
$\boxed{1}$ $\boxed{\div}$ $\boxed{2}$ $\boxed{=}$ $\boxed{0.\,5}$	0.5

$$\begin{cases} \frac{7}{8} + \frac{1}{2} = 1.375 \\ \frac{7}{8} - \frac{1}{2} = 0.375 \\ \frac{7}{8} \times \frac{1}{2} = 0.4375 \\ \frac{7}{8} \div \frac{1}{2} = 1.75 \end{cases}$$

If your mini-calculator has a memory capability, the above recording can be eliminated for addition and subtraction by using the $\boxed{m+}$ and $\boxed{m-}$ keys and getting the answer using the memory recall key. For multiplication and division it would be necessary to have as many memories as there are terms in the problem.

2. Use the algebraic algorithm for each operation. It would still be necessary to record or store the result of each step in the process.

Addition - $\left(\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}\right)$, b, d $\neq$ 0

$\frac{7}{8} + \frac{1}{2} =$

CALCULATOR	RECORD/STORE
$\boxed{7}$ $\boxed{\times}$ $\boxed{2}$ = $\boxed{1\,4.}$	14
$\boxed{1}$ $\boxed{\times}$ $\boxed{8}$ = $\boxed{8.}$	8
$\boxed{8}$ $\boxed{\times}$ $\boxed{2}$ = $\boxed{1\,6.}$	16
$\boxed{14}$ $\boxed{+}$ $\boxed{8}$ = $\boxed{2\,2.}$	22
$\boxed{22}$ $\boxed{\div}$ $\boxed{16}$ = $\boxed{1.\,3\,7\,5}$	

$$\therefore \frac{7}{8} + \frac{1}{2} = 1.375 = 1\frac{3}{8}$$

65

In similar fashion, the intermediate steps for subtraction, multiplication, and division are recorded in an exact progression according to their laws:

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \qquad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \qquad b,\ c,\ d \neq 0$$

3. A closer examination of the algebraic algorithm and some manipulation will make it possible to use the mini-calculator without recording intermediate results or using a memory. When the student learns to so manipulate algebraic equations, many formulas simplify to single chain operations done from left to right. Converting equations for calculator use is a very interesting activity.

ADD/SUBTRACT

$b, d \neq 0$

$$\frac{a}{b} \pm \frac{c}{d} = ad \div b \pm c \div d$$

$$\frac{3}{4} \pm \frac{1}{2} = 3 \times 2 \div 4 \pm 1 \div 2$$

MULTIPLY

$b, d \neq 0$

$$\frac{a}{b} \times \frac{c}{d} = ac \div b \div d$$

$$\frac{3}{4} \times \frac{1}{2} = (3 \times 1) \div 4 \div 2$$

DIVIDE

$b, c, d \neq 0$

$$\frac{a}{b} \div \frac{c}{d} = ad \div b \div c$$

$$\frac{3}{4} \div \frac{1}{2} = (3 \times 2)\ \ 4 \div 1$$

Although these chain algorithms provide the simplest way to compute with the common fraction, students should not be allowed to blindly follow the chain instruction. The students should go through the process of finding the algebraic algorithm and manipulating it for calculator use.

The direct use of the mini-calculator in division of common fractions using the transformed algebraic algorithms all but eliminates the need for reciprocals. However, it would be of interest to verify that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ with arithmetic problems and the calculator. The reciprocal key $\boxed{\frac{1}{x}}$ is really not needed as the implied division is simple to perform.

The mini-calculator can be used to self-check computations involving common fractions using the operational algorithms by converting the common fraction answer to its decimal equivalent and comparing it to the result of doing the entire problem in decimal form.

66

Do these problems on your paper:

1. $\begin{array}{r} \frac{3}{8} \\ + \frac{7}{8} \\ \hline \end{array}$	2. $\begin{array}{r} 3\frac{5}{8} \\ + 2\frac{7}{16} \\ \hline \end{array}$	3. $\begin{array}{r} 4\frac{1}{2} \\ - 1\frac{1}{4} \\ \hline \end{array}$	4. $\begin{array}{r} 3\frac{3}{8} \\ - 1\frac{3}{4} \\ \hline \end{array}$

5. $\frac{3}{4} \times \frac{2}{5}$ 6. $1\frac{3}{5} \times 3\frac{1}{2}$ 7. $2\frac{4}{5} \div 3\frac{1}{3}$ 8. $1\frac{7}{10} \div 2\frac{5}{8}$

Use your calculator to convert each answer to its decimal form and record.

1. ________ 2. ________ 3. ________ 4. ________ 5. ________

6. ________ 7. ________ 8. ________

Use your mini-calculator to do the above problems. Compare your answers with the eight decimal answers above. If they match your computations were correct. Which is the better tool for these problems - your brain or the calculator?

YOU CAN BEAT THE CALCULATOR! Do the division examples as indicated and record the answers. As soon as you see the pattern write the answers without using your machine. Check to prove that you can beat the calculator!

1. $1 \div 9 =$	2. $1 \div 11 =$
$2 \div 9 =$	$2 \div 11 =$
$3 \div 9 =$	$3 \div 11 =$
$4 \div 9 =$	$4 \div 11 =$
$5 \div 9 =$	$5 \div 11 =$
$6 \div 9 =$	$6 \div 11 =$
$7 \div 9 =$	$7 \div 11 =$
$8 \div 9 =$	$8 \div 11 =$
$9 \div 9 =$	$9 \div 11 =$

Record the decimal equivalents of the following family of common fractions:

$\frac{1}{7} =$ $\qquad\qquad$ $\frac{2}{7} =$

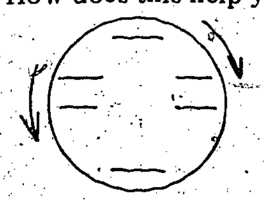
$\frac{3}{7} =$ $\qquad\qquad$ $\frac{4}{7} =$

$\frac{6}{7} =$ $\qquad\qquad$ $\frac{7}{7} =$

$\frac{9}{7}$ $\qquad\qquad$ $\frac{10}{7}$

Are these terminating or repeating decimals? What pattern do you see? (Hint: Look at the multiplication table for 7.)

What are the digits used in each decimal equivalent of the 7ths family? What is their order? Write them in the circle. How does this help you to see the pattern?



Convert the following common fractions to their decimal fraction equivalents:

1.	$\frac{3}{8} =$	2.	$\frac{7}{10} =$	3.	$\frac{3}{2} =$
4.	$\frac{1}{3} =$	5.	$\frac{5}{6} =$	6.	$\frac{3}{7} =$
7.	$\frac{9}{11} =$	8.	$\frac{6}{13} =$	9.	$\frac{5}{26} =$
10.	$\frac{17}{6} =$	11.	$\frac{15}{29} =$	12.	$\frac{8}{15} =$

Record the repeating decimals to show the period of repetition. If you cannot determine this from the display, do the indicated division and look for a repeating pattern.

An interesting topic that may be introduced at this time would be the determination whether or not a common fraction has a repeating or terminating decimal equivalent. This may be done by factoring the denominator of the common fraction to its prime factors. If, and only if, the prime factors of the denominator consist of only two's and/or five's, the decimal equivalent will terminate. If, on the other hand, any other prime number is a factor, the decimal will repeat in a cycle as indicated by the factors of the number one less than the denominator.

When common fractions whose decimal form is that of a repeating decimal are used in multiplication of whole numbers, the answer displayed by the calculator does not seem to obey the commutative property for multiplication.

$6 \times \frac{2}{3} \stackrel{?}{=} \frac{2}{3} \times 6$ $\qquad$ $6 \times \frac{3}{4} \stackrel{?}{=} \frac{3}{4} \times 6$

Using the calculator:

$(6 \times 2) \div 3 \stackrel{?}{=} (2 \div 3) \times 6$ $\qquad$ $(6 \times 3) \div 4 \stackrel{?}{=} (3 \div 4) \times 6$

$12 \div 3 \stackrel{?}{=} 0.6666666 \times 6$ $\qquad$ $18 \div 4 \stackrel{?}{=} 0.75 \times 6$

$4 \neq 3.9999996$ $\qquad$ $4.5 = 4.5$

Anticipate such confusion when preparing examples for reinforcement drill. Of course, such an interesting failure to conform to the properties of rational numbers might be a very stimulating way to analyze the role of the repeating decimal and its conversion to its common fraction equivalent.

Is this a magic square? If not, can you make it a magic square by changing the number in only one cell? Your calculator should be a big help.

$2\frac{1}{4}$	1.125	$1\frac{7}{8}$
1.5	1.75	2.125
$1\frac{5}{8}$	2.375	$1\frac{1}{4}$

What is the sum of any row, column, or diagonal of the magic square? ________

6.)

Here are some common fraction puzzles. Each fraction uses each digit from 1 through 9 only once, but names the unit fraction stated. The fractions are rather large. Hint: the numerators contain 4 digits; the denominators 5 digits.

$$\frac{1}{2} = \frac{6\ \_\ \_\ 9}{1\ \_\ 4\ \_\ 8} \qquad \frac{1}{3} = \frac{\_\ 5\ \_\ 2}{\_\ \_\ \_\ \_\ 9}$$

$$\frac{1}{4} = \frac{\_\ 9\ \_\ 2}{\_\ 5\ \_\ 6\ \_} \qquad \frac{1}{5} = \frac{2\ \_\ \_\ \_}{13\ \_\ \_\ \_}$$

Now you are on your own:

$$\frac{1}{6} = \qquad \frac{1}{7} =$$

$$\frac{1}{8} = \qquad \frac{1}{9} =$$

The percent key on the mini-calculator may, in the view of the mathematics teacher, be the most superfluous key on the entire keyboard. The key simply divides a number by 100, giving the percent in its decimal form. In addition, reliance on the % key may negate understanding the concepts and applications of percent.

The basic concept of the percent symbol, %, is that it is just another way of noting a denominator of 100. The student may be led to the discovery of the number 100 in the symbol "%". Converting a percent to its decimal form, therefore, implies division by 100:

$$14\% = 14 \div 100 = 0.14$$

$$\boxed{1}\ \boxed{4}\ \boxed{\div}\ \boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=} \qquad \boxed{0.}\boxed{1}\boxed{4}$$

In similar fashion, any decimal fraction can be noted as % by using the inverse operation, multiplying by 100. But the percent symbol must be affixed by the user as the calculator cannot display the % sign.

The value of the mini-calculator in percent problems, other than being used to self-check computations, is in problems involving profit and loss, interest, percent of increase and decrease, etc.

The study of ratio and proportion using cross multiplication to check equivalent ratios should precede topics in percent. Problems involving percent rely, usually, on stating the problem in formula or proportion form and performing the indicated operations.

The table of decimal and common fraction equivalents developed by the students is probably a better instructional aid than the calculator unless a series of complex percent problems are to be executed.

Random numbers can be generated with the aid of a mini-calculator with a built-in constant.

Enter $10 \div 17 =$

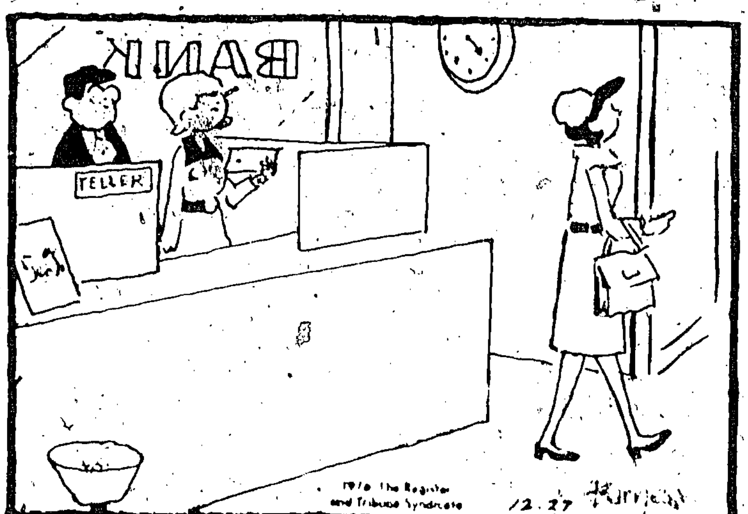
$11 \div 17 =$

$12 \div 17 =$ etc.

Record the last two, three, four, or five digits of the resulting numbers according to the desired number of digits needed in a problem situation. Other random numbers can be generated using 13 as the divisor.

This can be more easily done if the mini-calculator has a built-in constant.

NUMBER PATTERNS



"After working on her account, it's obvious Mrs. Parker didn't get a pocket calculator for Christmas."

72

NUMBER PATTERNS

The mini-calculator is an excellent aid to use in helping students discover patterns in mathematics. The recognition of number patterns in mathematical situations is an important skill to be developed. The use of the mini-calculator permits students to experiment with a suspected pattern, test hunches, arrive at a generalization or to try another approach. Furthermore, the use of the calculator affords the student the opportunity to explore number patterns involving very large and/or very small numbers.

Use your calculator to help you fill in the missing numbers in the following sequences.

1. 5, 10, 15, 20, ____, ____, ____, ...
2. 3, 6, 9, ____, ____, ____, 21, . . .
3. -4, -2, 0, ____, ____, ____, 8, ...
4. 3, 9, 27, ____, ____, ____, ...
5. 999, 899, ____, ____, 599, . . .
6. 2000, 500, ____, 31.25, ____, ____, ...

Many of the activities in this section are excellent warm-ups at the start of a lesson.

Compute

1. 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =

2. 11 + 22 + 33 + 44 + 55 + 66 + 77 + 88 + 99 =

3. 111 + 222 + 333 + 444 + 555 + 666 + 777 + 888 + 999 =

Now what do you think this answer will be?

4. 1111 + 2222 + . . . + 9999 =

Check your guess using your calculator.

5. Try 11111 + 22222 + . . . + 99999 =

Compute

1. 6 x 6 =

2. 66 x 66 =

3. 666 x 666 =

Guess the answer to the following:

4. 6666 x 6666 =

Check your guess using the calculator.

7

Multiplication by 11

1. Use your calculator to do the following multiplications:

   25 x 11 = ________ 34 x 11 = ________

   17 x 11 = ________ 24 x 11 = ________

   51 x 11 = ________ 52 x 11 = ________

   What is the pattern? ________________________________

   ____

2. What is the pattern here?

   58 x 11 = ________

   83 x 11 = ________

   96 x 11 = ________

   Is it really different from the first pattern? ________________________________

   ____

3. Does the pattern work here? ________________________________

   256 x 11 = ________

   512 x 11 = ________

Check your answer with the calculator. If you were right try this one <u>without</u> your machine:

4165275 x 11 = ________

75

Compute using your calculator.

1. $\frac{1 \times 1}{1} =$ ________

2. $\frac{22 \times 22}{1 + 2 + 1} =$ ________

3. $\frac{333 \times 333}{1 + 2 + 3 + 2 + 1} =$ ________

Guess $\frac{4444 \times 4444}{1 + 2 + 3 + 4 + 3 + 2 + 1} =$ ________

Check your guess using your calculator

Complete the following using your calculator:

1. $101 \times 1 =$ ________
2. $101 \times 101 =$ ________
3. $101 \times 101 \times 101 =$ ________

Now guess,

4. $101 \times 101 \times 101 \times 101 =$ ________

(Your calculator may be of no help with this one!)

Complete:

1. 101 x 11 = ____________

2. 101 x 111 = ____________

3. 101 x 1111 = ____________

Guess

4. 101 x 11111 = ____________

Check your guess. Try this one,

5. 101 x 111111 = ____________

Complete:

1. 101 x 22 = ____________

2. 101 x 222 = ____________

3. 101 x 2222 = ____________

Have a hunch? Guess

4. 101 x 22222 = ____________

5. 101 x 222222 = ____________

Complete:

1. 101 x 33 = ____________

2. 101 x 333 = ____________

3. 101 x 3333 = ____________

Guess! Check your guess using your calculator

4. 101 x 33333 = ____________

5. 101 x 333333 = ____________

Complete the following:

1. 101 x 44 = ____________

2. 101 x 444 = ____________

3. 101 x 4444 = ____________

Have a hunch? Guess

4. 101 x 44444 = ____________

Check your guess using your calculator

Will a pattern exist for 101 x 55, 101 x 555, etc?

If not, why not? ____

____

____

78

Fill in the missing blanks:

1. 1 x 9 + 2 = ________

2. 12 x 9 + 3 = ________

3. 123 x 9 + 4 = ________

If you see a pattern, what would the next two calculations be?:

4. ________ = ________

5. ________ = ________

Compute, using your calculator:

1. 15 x 25 = ________
2. 25 x 35 = ________
3. 35 x 45 = ________
4. 45 x 55 = ________
5. 55 x 65 = ________
6. 65 x 75 = ________
7. 75 x 85 = ________
8. 85 x 95 = ________
9. 95 x 105 = ________
10. 105 x 115 = ________

Describe the pattern above ________________________________

____

7

Compute, using your calculator:

1. 26 x 24 = ________	6. 74 x 76 = ________
2. 37 x 33 = ________	7. 53 x 57 = ________
3. 92 x 98 = ________	8. 38 x 32 = ________
4. 65 x 65 = ________	9. 15 x 15 = ________
5. 71 x 79 = ________	10. 29 x 21 = ________

What pattern do you see in the numbers above? ____________________

____

How do the answers compare to the answers of the previous exercise? ____________

____

Compute, using your calculator:

1. 2 x 142857 = ____________
2. 3 x 142857 = ____________
3. 4 x 142857 = ____________
4. 5 x 142857 = ____________
5. 6 x 142857 = ____________
6. 7 x 142857 = ____________ (This answer is interesting!)

Compare your results with "142857"

Can you describe why this works? ____

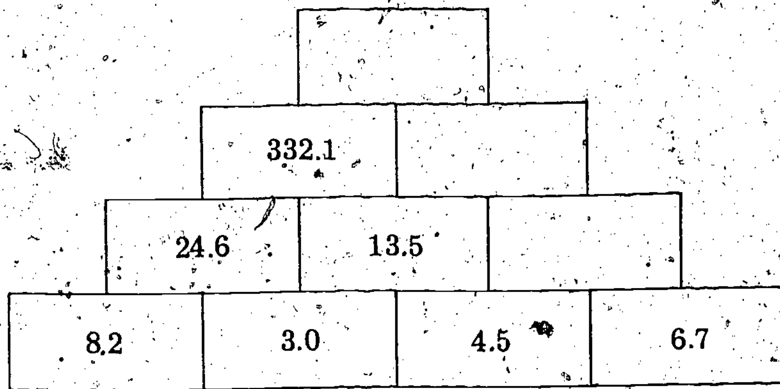
____

In each pair of computations, estimate which one is the larger product:

1.	17 x 18 x 19	16 x 17 x 18
2.	10 x 12 x 14	11 x 13 x 15
3.	9 x 12 x 14	10 x 12 x 14

Use your calculator to check your answers.

Discover the pattern and fill in the missing blanks:



If you use a calculator with an 8 digit display, is the top "box" an exact answer?

Explain. ____

____

____

Find each of the indicated products:

1. 37 x 3 =

2. 37 x 6 =

3. 37 x 9 =

4. 37 x 12 =

5. 37 x 15 =

6. 37 x 18 =

7. 37 x 21 =

8. 37 x 24 =

9. 37 x 27 =

Can you explain your results?

Find each of the indicated sums:

1. 9 x 9 + 7 =

2. 98 x 9 + 6 =

3. 987 x 9 + 5 =

4. 9876 x 9 + 4 =

Can you continue this pattern?

Find each of the indicated sums:

1. 1 x 8 + 1 =

2. 12 x 8 + 2 =

3. 123 x 8 + 3 =

4. 1234 x 8 + 4 =

Try to continue the pattern.

Continue the pattern:

1. 1 x 9 - 1 = 8

2. 21 x 9 - 1 = 188

3. 321 x 9 - 1 = 288

4. ________ = ________

5. ________ = ________

The mini-calculator is an excellent tool to use when studying palindromic numbers. Palindromic numbers read the same when the order of the digits is reversed, e.g. 44, 121, 623261, 9184819, etc. Finding palindromic "sums" is an excellent method to use for reinforcing addition of whole numbers. To find a palindromic "sum", start with a number and add to it the number obtained by reversing the digits, repeat this process until the "sum" is a palindromic number.

e. g. Start with 43

$$\begin{array}{r} 43 \\ +\ 34 \\ \hline 77 \end{array}$$

(reverse order of 43)

- one-step palindromic "sum"

Start with 68

$$\begin{array}{r} 68 \\ +\ 86 \\ \hline 154 \end{array} \longrightarrow \begin{array}{r} 154 \\ +\ 451 \\ \hline 605 \end{array} \longrightarrow \begin{array}{r} 605 \\ +\ 506 \\ \hline 1111 \end{array}$$

- three step palindromic "sum"

Use your calculator to find palindromic "sums". Keep a tally of the number of additions it takes to reach a palindromic "sum".

	Number	Palindromic "Sum"	Number of Steps
1.	95		
2.	155		
3.	273		
4.	629		
5.	198		
6.	380		

Use your calculator to find the next four terms in each of the following sequences:

1. 2, 4, 6, 10, 16, ____, ____, ____, ____,
2. 5, 10, 15, 25, 40, ____, ____, ____, ____,
3. 1, 5, 6, 11, 17, ____, ____, ____, ____,
4. 100, 100, 200, 300, 500, ____, ____, ____, ____,
5. 4, 8, 12, 20, ____, ____, ____, ____,

The pattern in each of the sequences from the previous activity was the same, i.e. each term in a sequence is the sum of the preceding two terms. This kind of pattern occurs frequently in nature and was discovered by Fibonacci and hence called The Fibonacci Sequence.

Use your calculator to find the next 10 terms in the Fibonacci Sequence:

1, 1, 2, 3, 5, 8, 13, 21, . . .

Use the Fibonacci Sequence to generate the next ten fractions in this sequence:

$\frac{1}{1}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13},$

Use the Mini-calculator to discover:

1. Patterns involving Pasacal's Triangle
2. Square Numbers
3. Triangular Numbers
4. Pentagonal Numbers
5. Rectangular Numbers
6. Patterns in a 100 Board
7. Patterns using the calendar

POWERS AND ROOTS

87

POWERS AND ROOTS

The concepts of a power as indicating the number of times the base number is used as a factor ($12^5 = 12 \times 12 \times 12 \times 12 \times 12$) and the inverse operation of finding roots are more clearly and easily demonstrated with a mini-calculator. The use of special function keys $\boxed{x^2}$, $\boxed{\sqrt{}}$, $\boxed{y^x}$, $\boxed{\sqrt[x]{y}}$, although they will all but eliminate the necessary computational skills, may be of value in advance mathematics problems. A built-in constant or a constant function key will also save time and avoid the drudgery of repetitive calculations and will reinforce the concept of a repeated factor. To find a power of a number, using a calculator with a built-in constant function, it is necessary only to enter the base number, the $\boxed{\times}$ key and then press the $\boxed{=}$ key one less time than the indicated power:

$3^4 \Rightarrow \boxed{3}\ \boxed{x}\ \boxed{=}\ \boxed{=}\ \boxed{=} \Rightarrow \boxed{8}\boxed{1.}$

$(0.1)^5 \Rightarrow \boxed{0}\boxed{.1}\ \boxed{x}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=} = \boxed{0.}\boxed{0}\boxed{0}\boxed{0}\boxed{0}\boxed{1}$

The nature of real roots can be thoroughly investigated with the aid of the mini-calculator. (If the calculator has no build-in square root key, then the procedure of nested intervals may, perhaps, be the most mathematically valid procedure.) Although other methods for determining square roots may be employed, the nested interval procedure is performed as follows:

Find $\sqrt{72}$. As $64 < 72 < 81$, $8 < \sqrt{72} < 9$. Suppose that you guess that $\sqrt{72} = 8.5$. Checking this guess, you will find that $8.5^2 = 72.25$. Therefore, $\sqrt{72} < 8.5$. Trying 8.4 as the possible answer results in 70.56. Thus $8.4 < \sqrt{72} < 8.5$. An intelligent guess might be 8.48 which when squared is 71.91 (rounded to two decimal places). This is almost 72. $8.49^2 = 72.08$. Using the nested interval again shows that $8.48 < \sqrt{72} < 8.49$. Since $8.485^2 = 71.995225$ and $8.486^2 = 72.012196$, 8.485 is a reasonable answer to the problem. If greater accuracy is desired the procedure of nested intervals can be repeated until the display limit is reached. The mini-calculator has been a valuable tool, performing the needed calculations while freeing the student to visualize the nesting or "zeroing-in" process.

The nested interval method is also useful in estimating other roots.

$4^3 < \sqrt[3]{72} < 5^3$	$64 < 72 < 125$
$4.1^3 < \sqrt[3]{72} < 4.2^3$	$68.921 < 72 < 74.088$
$4.16^3 < \sqrt[3]{72} < 4.17^3$	$71.99 < 72 < 72.51$
$4.16^3 < \sqrt[3]{72} < 4.161^3$	$71.99 < 72 < 72.04$

$$\therefore \sqrt[3]{72} \approx 4.16$$

Students should solve such problems as 3.1^4, 0.02^5, $\sqrt{46}$, $\sqrt[3]{100}$, $\sqrt[5]{40}$, etc. to determine those problems where the use of the mini-calculator is an asset, and where its display limitations will preclude solution.

The algorithms for finding square and cube roots may also be explored and the results checked with the calculator.

Another method of determining the square root of a number is Newton's Method which is mathematically more sophisticated and which requires more advanced mathematics. The calculator, however, is ideally suited to this method. Briefly, it goes as follows:

1. Guess the root. This is the first estimate.
2. Divide the radicand by the first estimate. This is the first quotient.
3. Add the first estimate and the first quotient.
4. Divide this sum by 2. This is the second estimate.
5. Divide the second estimate into the radicand. This is the second quotient.
6. Add the second estimate and the second quotient.
7. Divide this sum by 2. This is the third estimate.

Continuing this averaging process will produce a rapidly converging root. The general form is:

$$\text{approximate square root} = \frac{1}{2}\left(\frac{\text{number}}{\text{guess}} + \text{guess}\right)$$

Example: Find $\sqrt{3}$.

1. Guess, say, 1.5. This is the first estimate.
2. $3 \div 1.5 = 2$. The first quotient.
3. $2 + 1.5 = 3.5$
4. $3.5 \div 2 = 1.75$. This is the second estimate.
5. $3 \div 1.75 = 1.7142857$. This is the second quotient.
6. $1.7142857 + 1.75 = 3.4642857$
7. $3.4642857 \div 2 = 1.73214285$. This is the third estimate.
8. $3 \div 1.73214285 = 1.73195877$. This is the third quotient.
9. $1.73195877 + 1.73214285 = 3.46410162$
10. $3.46410162 \div 2 = 1.73205081$. This is the fourth estimate at which point the error is 0.00000001.

The usefulness of the calculator is obvious. The shortcoming of Newton's Method compared with nested intervals is that the former is applicable only to quadratic roots, that is, roots whose indices are powers of 2, whereas the latter method may be used to find any root.

Another method of approximating square roots is to use the unit continued fraction:

$$\sqrt{2} \approx 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}}$$

$$\sqrt{3} \approx 1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \dots}}}}$$

The calculator algorithm for $\sqrt{2}$ is:

$\boxed{2}\ \boxed{\div}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{2}\ \boxed{=}$

$\boxed{\div}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{2}\ \boxed{=}$

$\boxed{\div}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{2}\ \boxed{=}$

. . . continuing to press the same keys until the number displayed does not change, then press the $\boxed{-}\ \boxed{1}\ \boxed{=}$ keys

$\sqrt{2} \approx 1.414\ 213\ 562\ 373$

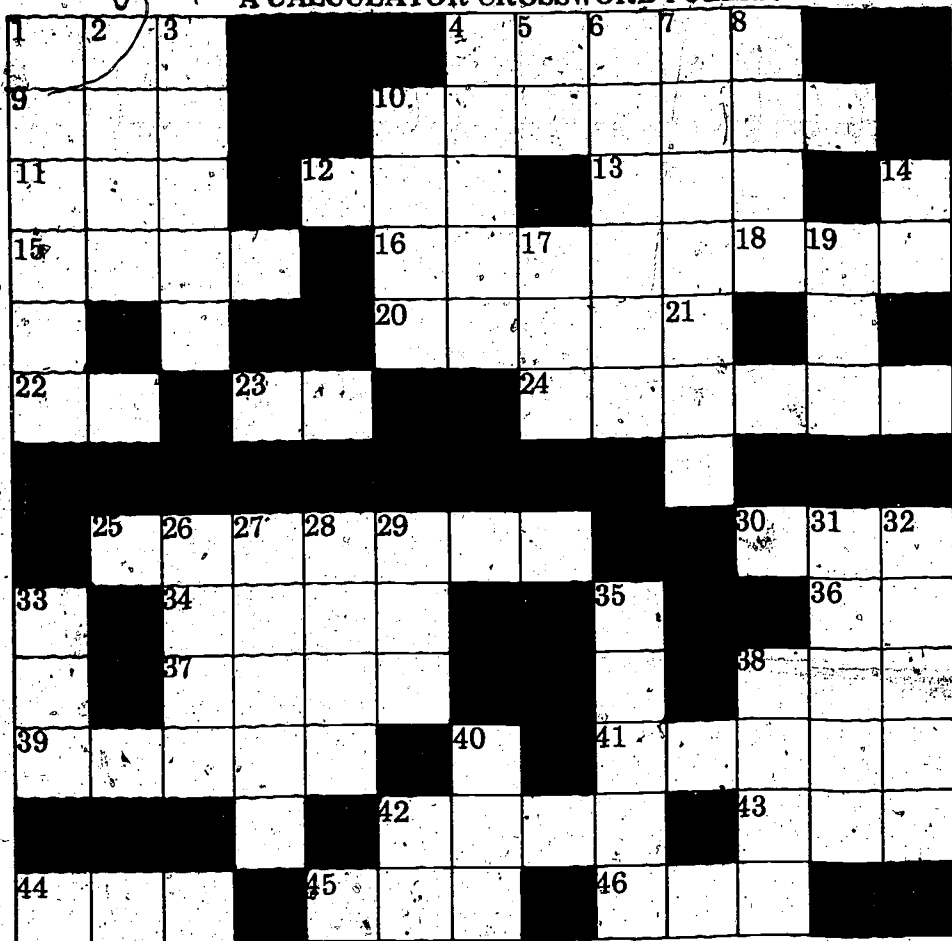
$\sqrt{3} \approx 1.732\ 050\ 8$

Use the symbols >, < or = to compare the following pairs of expressions.

1. 3^2 ◯ 2^3
2. 2^2 ◯ $2 + 2$
3. 3×5 ◯ 5^3
4. 2^3 ◯ 3^2
5. $4 + 4 + 4$ ◯ 4^3
6. $\sqrt{16}$ ◯ $\sqrt[3]{36}$
7. 4^5 ◯ 5^4
8. $3 \times 3 \times 3$ ◯ 3^3
9. $(23.5)^2$ ◯ $\sqrt[2]{532}$
10. 0.1^5 ◯ 5^1

Which problems did you do without a calculator?

A CALCULATOR CROSSWORD PUZZLE*



First find the answer to the math exercise. Then invert the calculator to spell out a word. Enter the word in the puzzle.

Across

1. $435 + 370$
4. $635^2 - 365407$
9. $5(45 + 31) - 63$
10. $(645^2 + 798^2)3 + 473051$
11. $2(85 + 30) + 25^2 - 82$
12. $(4530 \div 15) \times 2 + 101$
13. $25^2 - 7$
15. $2(36^2) + 1159$
16. $\frac{861}{3} + 22^2$
18. $3(45 + 70)$
20. $2022 \div 6$
22. $6^2 - 2$
23. $\sqrt{196}$
24. $765^2 - 47149$
25. $155^3 + 17871 + 39891$
30. $24^2 + 31$ [illegible]
34. $3327 + 180$ [illegible]
36. $456 \div 12$
37. $2(352 + 689 + 762) + 99$
38. $17^2 + 7^2$
39. $36(265 + 833) - 3094$
41. $(\frac{720}{45}) \times 4560) - 17582$
42. $3755 + 4378 - 4415$
43. $2010 \div 6$
44. $3^3 + 4^3 + 5^3 + 6^3 + 175$
45. $(311 \times 2) + 111$
46. [illegible]$^2 - 14$

Down

1. $582^2 + 345^2 + 3626$
2. $(2130 \div 3) + 5000$
3. 9623×6
4. $(3567 \times 11) - 1499$
5. $6^2 + 15$
6. $9^3 + 9^2 + 8$
7. $4^3 + 5^3 - 52$
8. $2(2341 + 227) + 527$
14. $442 \div 13$
17. $2(\frac{1386}{6}) + 175$
19. $2(103 + 49)$
21. $365 + 240$
26. $78^2 - 2511$
27. $6(3872 + 456) + 9038$
28. $3627 + \frac{1488}{12}$
29. $\frac{2366}{7}$
31. $26^3 + 27^3 - 1879$
32. $136^2 + 127^2 + 711$
33. $567 + 81 - 44$
35. $5(2356 + 8567) - 808$
38. $73^2 + 209$
40. $2130 \div 3$
42. $3(2^3 + 4) + 2$

*Rudolph and Claasen: The Calculator Book

During the month of September you start a special savings plan. On September 1st, you deposit one cent ($ .01). On September 2nd, you double the deposit of the first day and deposit two cents ($ .02). On the third day you double the deposit of the second day and deposit four cents ($ .04). If you continue to deposit money in your account following the same pattern of doubling the previous day's deposits, how much money will you deposit on the last day of the month? How much money will your deposit during the entire month of September?

Estimate your answers, then chart your deposit and totals until you see a pattern.

Day	Deposit	Total at end of day
1	$ .01	$ .01
2	$ .02	$ .03
3	$ .04	$ .07
Last	$\boxed{}$	$\boxed{}$

If your calculator has a constant key, how did you use it?

92

Use your mini-calculator to start you on this chart. As soon as you see a pattern you can complete the chart on your own. Record only the LAST DIGIT of each resulting number: $2^6 = 64$. (Look at the "4" in the circle).

POWER

Base Number	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3									
2	2	4	8	6	2	4						
3	3			1								
4								6				
5					5							5
6	6						6					
7									7			
8					8					2		
9			9			1				1		
10			0									
11						1					1	
12			8				8					
13	13				3							
14				6								
15			5						5			
16		6										6

What patterns do you see? Can you extend this chart? Do the patterns hold?

Your handwriting should be ____________

" $2(42^2 + 81^2 + 125^3) - 141263$ " will give the answer if you turn your mini-calculator upside down.

What do most people enjoy in their free time?

" $127^3 + 6317263 - 3046842$ "

Read answer on upside-down display.

Scientific notation is used to simplify complex computation with very large and/or very small numbers. This is done by converting each such number to a number between 1 and 10, times a power of ten.

$$13486500 = 1.34865 \times 10^{7}$$
$$0.00000003 = 3 \times 10^{-8}$$

The calculator can be of aid only after the problem has been stated in scientific notation. It will do the required operation on the decimal fractions, but the required number of zeros to be affixed or to precede the result of the computation must be determined by the application of the laws of exponents.

$$\frac{337560000 \times 0.008}{6738.4 \times 459.34} = \frac{3.3756 \times 10^{8} \times 8 \times 10^{-3}}{6.7384 \times 10^{3} \times 4.5934 \times 10^{2}}$$

$$= \frac{3.3756 \times 8}{6.7384 \times 4.5934} \times 10^{0} = 0.8724687 \times 10^{0} = 0.8724687$$

The students must be able to extend the calculator algorithms for common fractions to problems necessitating scientific notations. The number of factors in the numerator and denominator do not affect the basic calculator algorithms for multiplication and division of common fractions:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} = \boxed{a}\ \boxed{\times}\ \boxed{c}\ \boxed{\div}\ \boxed{b}\ \boxed{\div}\ \boxed{d}$$

$$\frac{abcd}{efg} = \boxed{a}\ \boxed{\times}\ \boxed{b}\ \boxed{\times}\ \boxed{c}\ \boxed{\times}\ \boxed{d}\ \boxed{\div}\ \boxed{e}\ \boxed{\div}\ \boxed{f}\ \boxed{\div}\ \boxed{g}$$

The use of the mini-calculator has enabled the student to concentrate on utilizing the rules of multiplication and division of powers of ten aided by the laws of exponents.

ALGEBRA

96

ALGEBRA

The mini-calculator will become a more valuable instructional tool as its use for performing arithmetic calculations frees the student to concentrate on the algebraic concepts being developed.

There are, however, many concepts whose discovery, or verification, can be strengthened by the use of the mini-calculator. The properties of the natural, whole, integer, rational and real numbers can be demonstrated without being limited to single digit numbers. Is not 179673 x 42856 = 42856 x 179673 an example of the commutative property for multiplication as well as 1 x 4 = 4 x 1?

If the mini-calculator being used can enter and display negative numbers, it becomes very useful when the concepts inherent to operations with signed numbers are studied. In preparation for solving algebraic equations, it is necessary for students to comprehend and find additive and multiplicative inverses as well as to determine the operational inverse needed for solution. When finding multiplicative inverses care must be taken, however, when dealing with rounded off numbers or repeating decimals. When such numbers are involved slight errors are produced which the student should learn to recognize. A series of zeroes before the digits on the right generally indicate this error.

Find the multiplicative inverse

1. 5
2. 0.025
3. 0.00625
4. 6.25
5. 0.625
6. $0.\overline{2}$
7. $0.\overline{285714}$
8. $0.\overline{14}$
9. $\frac{7.3}{4.6}$
10. $\frac{0.003}{0.025}$

Which of the multiplicative inverses are repeating decimal fractions?

The linear equation in slope-intercept form (a special type of function)

$$y = mx + b, \quad m \neq 0$$

is generally used to graph a line. Values of x were usually chosen from the set of integers, disregarding the concept of continuity. Of course, the choices for x were made for purposes of simplicity of computations. Using the mini-calculator, this no longer need concern us! While a rigorous treatment of continuity is beyond the scope of elementary algebra, we can show intuitively that the function y is continuous between any two values of x, i.e., y exists for all values of x. While we are limited by the capacity of a particular calculator, many students will accept two or three decimal places as sufficient proof. For example, graph:

$$y = 2x - 7$$

97

In the past, we determined the values of y for $x = -2, 0$, and perhaps 3. We can now show that y exists not only for $x = 1$ and 2, but also for $x = 1.2, 1.75$, etc.

Further, we need not restrict ourselves to such simple equations. It is not a great deal more effort to graph: $17.3y = 1.84x + 0.32$

As was indicated above, the calculator becomes a handy tool for solving equations. We no longer have to content ourselves with solving such equations as $3x - 7 = 19$. The solution set no longer needs to consist of integers with only a separate section for non-integral solutions. We can now solve equations such as $23.7x - 15.2 = 35.8$ as easily as the above example since the mini-calculator solves one as quickly as the other.

Equations of a more complex nature may need some manipulation to be transformed into an equivalent form* which can be entered on the mini-calculator unless special function keys are available. These manipulations into single-line calculator algorithms become a necessity if the calculator has no memory, or recording the results of intermediate steps become laborious. Such manipulation is, in itself, a guide to the students' ability to use algebraic laws. Rounding off the displayed answer may be necessary if any intermediate step resulted in a repeating decimal. A few such equivalents are:

$$\frac{a}{b} \pm \frac{c}{d} = ad \div b \pm c \quad d$$

$$ab \pm cd = a \times b \div d \pm c \times d$$

$$\frac{a}{b} \times \frac{c}{d} = ac \div b \div d$$

$$\frac{a}{b} \quad \frac{c}{d} = ad \div b \div c$$

$$x^2 \pm y^2 \pm z^2 = x^2 \div y \pm y \times y \div z \pm z \times z$$

This calculator algorithm limits y and z to non-zero values.

*Operations done left to right

The transformation of the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \pm\sqrt{0 - 4 \times a \times c \div b \div b + 1} \times b - b \div 2 \div a$$

needs a square root key unless the student records the resulting radicand and uses an arithmetic method to find its approximate root. However, examination of the discriminant to determine the nature of the roots can be greatly simplified with the aid of the calculator.

The students, left free to decide on the equation necessary to solve a problem, but leaving the labor to the calculator, may not cringe when word problems are discussed, especially those involving work and mixture problems. Thus the student is able to think algebraically, not arithmetically.

Realistic problems of interest to students, rather than contrived ones, can now be posed.

Solve the following:

1. Do you get good mileage on a car if it required 6.4 gallons since the last fill-up after having travelled 237.7 miles?
2. In 1973, gasoline cost about 26.9 cents per gallon. How much would the gas have cost for this trip in 1973?
3. By 1975, gasoline prices had increased to about 58.9 cents per gallon. (This may still seem cheap at the time you solve this problem). How much more would the gasoline have cost for this 237.7 mile trip in 1975 than in 1973?
4. What was the percentage increase in the cost of gasoline over the 2 year span?
5. Considering this 2 year span, what was the annual inflationary spiral for gasoline?
6. Assuming a 9% inflationary spiral from 1975 to 1976, how much will the 237.7 mile trip cost for gasoline in 1976 assuming also that the efficiency of the car remains the same?

The point of the above problems is for the students to learn what to do rather than merely to produce an answer. Of course, you must assess the students' mental processes from these answers. But just numbers are not sufficient. The numbers have to be interpreted to represent dollars, cents, miles per gallon, per cent, etc. This has to be done by the students and will indicate that the student understands the problem. Minor variations in the answers may be due to differences in calculators, i.e., truncations and rounding off. The questions should be framed so that you may ask "What would you do to determine the answer to question no. 1? "What does this answer represent?" etc.

99

The solutions of the problems and the interpretations of the answers follow:

1. $237.7 \div 6.4 =$

   The display shows 37.140625. This is to be interpreted as about 37 miles per gallon. This is generally a good mileage for cars in 1976.

2. $6.4 \times 26.9 \div 100 =$

   Display shows 1.7216. Rounded off, the gasoline cost for the trip in 1973 was $1.72.

3. $6.4 \times 58.9 \div 100 - 1.72 =$

   The display shows 2.0496. It means that the trip cost $2.05 more for gasoline in 1975 than it did in 1973.

4. $(58.9 - 26.9) \div 26.9 \times 100 =$

   The display shows 118.9591. This means that there was an approximate 119% increase in the cost of gasoline over a two-year span. The % sign must be shown, otherwise the answer is meaningless. Another way to calculate this problem is:

   $2.05 \div 1.72 \times 100 =$

   This is a bit shorter and uses the solutions of no. 2 and no. 3. The display shows 119.18604, which is approximately 119%.

5. $119 \div 2 =$

   The display will show 59.5 which is to be interpreted as the average annual percentage increase in the cost of gasoline between 1973 and 1975.

6. $(9 \div 100 \times 58.9 + 58.9) \times 6.4 \div 100 =$

   The display will show 4.108864, which is approximately $4.11, the cost for gasoline in 1976 at an assumed 9% inflationary spiral. Note that $4.11 is the full cost of gasoline, not an increase over any previous cost.

Problems similiar to these can be made up by the teacher. Note that the emphasis must be placed on the understanding of the problem.

The concept of operations with zero can be reinforced with the use of the mini-calculator. Particularly, what happens when we enter "$3 \div 0 =$"? What is shown in the display depends upon the make and type of calculator, but whatever is shown should indicate the fact that division by zero is indeterminate.

The properties of zero for addition, subtraction and multiplication are also simple to demonstrate with the use of the calculator.

Graphing polynomial functions, solving systems of linear equations, applying the Pythagorean Theorem, investigating number sequences and series and the analysis of statistical tables are but a few of the algebraic topics whose study can be aided by the use of a mini-calculator.

The calculator with a memory eliminates much of the time necessary to approach the limit of the sum of a series.

Estimate the sum of the series as the denominators become successively smaller.

$$\frac{N}{D} + \left(\frac{N}{D}\right)^2 + \left(\frac{N}{D}\right)^3 + \ldots\ldots + \left(\frac{N}{D}\right)^n$$

$$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \ldots\ldots + \left(\frac{1}{2}\right)^n$$

$$\frac{1}{3} + \ldots\ldots\ldots\ldots + \left(\frac{1}{3}\right)^n$$

$$\frac{1}{8} + \ldots\ldots\ldots\ldots + \left(\frac{1}{8}\right)^n \text{ etc.}$$

$$\text{Lim} \sum_{n=1}^{\infty} \left(\frac{N}{D}\right)^n =$$

101

GEOMETRY

GEOMETRY

Geometry is one area of the mathematics curriculum in which the calculator can be used very advantageously. It can free the teacher and students from doing time-consuming computations in order to concentrate on the geometric concepts involved.

Some specific used of the mini-calculator include:

1. evaluating mathematical formulae
2. finding measures of angles
3. finding lengths of segments
4. finding areas of polygons and circles
5. finding volumes of polyhedra and spheres
6. calculating ratios and proportions
7. determining the number of diagonals that may be constructed in a polygon of n sides, a polyhedron of m faces, etc.

If the calculator to be used has square root capability and a memory, problems can be introduced which are generally omitted in a geometry course due to the lengthy calculations involved.

Problems involving Pythagorean relationships can be explored more fully using the calculator. An approximation of π, the ratio of a circle's circumference to its diameter, is easy to handle using the calculator. A value of can be generated to a high degree of accuracy.

Given the length of a side s of a regular inscribed polygon of n sides, the length of a side t of a regular inscribed polygon of 2n sides is $t = \sqrt{2r^2 - r\sqrt{4r^2 - s^2}}$ where r is the radius of the circle.

To evaluate π, take a circle of radius 1 and inscribe a regular hexagon. The perimeter of the hexagon is 6. Double the number of sides of the hexagon, obtaining a regular inscribed dodecagon. The length of each side can be determined from the formula

$$t_{12} = \sqrt{2r - r\sqrt{4r - s_6^2}} = \sqrt{2 - \sqrt{3}} = 0.51763809$$

The perimeter of the dodecagon is $12 \times 0.51763809 = 6.2116508$. Continuing this process and dividing the perimeter by the diameter (2) we get increasingly closer approximations of π.

n	length of 1 side	perimeter	perimeter ÷ diameter
6	1	6	3
12	0.51763809	6.21165708	3.10582854
24	0.26105238	6.26525722	3.13262861
48	0.13080626	6.27870041	3.13935020
96	0.06543817	6.28206396	3.14103198

The laborious computations involved are obvious. Using a calculator with square root capability and memory greatly reduces the amount of work involved. One can now take an inscribed polygon of 192 sides, 384 sides, 768 sides, etc. Accuracy will diminish as the number of significant digits are reduced due to the display limitations of the calculator.

It is suggested that there are many problems and topics in geometry similiar to the one described where the use of the calculator is advantageous. Calculator usage is to be encouraged in these situations. Following are several such situations:

Given: an isosceles triangle ABC, AB = BC. Find the area of each of these isoceles triangles using the formula $A = \frac{1}{2}ab$

(1)	AB = 15	AC = 18
(2)	BC = 13	AC = 10
(3)	BC = 25	AC = 14
(4)	AB = 30	AC = 30
(5)	AB = 25	AC = 40
(6)	AB = 2	AC = 3

Given: right triangles with legs a and b and hypotenuse c. In each of the following examples, one of the dimensions is omitted. Fill in the blanks. (The Pythagorean relation is $a^2 + b^2 = c^2$) Carry your results to two decimal places.

(1)	$a = 1.5$	$b = 2.0$	$c =$ ____
(2)	$a = 1.25$	$b =$ ____	$c = 3.25$
(3)	$a = 15.1$	$b = 15.1$	$c =$ ____
(4)	$a = 3.35$	$b =$ ____	$c = 6.7$
(5)	$a =$ ____	$b = \sqrt{3}$	$c = \sqrt{12}$
(6)	$a = \sqrt{6}$	$b = \sqrt{7}$	$c =$ ____

In a regular polygon of n sides, if lines are drawn from the center of the polygon to the vertices, we form n congruent triangles. Each angle in a regular polygon has a measure of $\frac{1}{2}(n-2)(\frac{360}{n})$ or $\frac{1}{2n}(n-2)(360^{\circ})$. Using the trigonometric ratios for the sine and cosine of an angle, find the length of each side in the polygon, and the area of the polygon, given the radius of the regular polygon and the number of sides. Carry results to four decimal places. (The area of a regular polygon is equal to the number of sides times the length of a side times one half the apothem (the apothem is the distance from the center to a side.)

($A = \frac{an}{2}(l)$ where a = apothem, n = number of sides l = the length of a side)

(1) $n = 4$ $r = 2$

(2) $n = 6$ $r = 3$

(3) $n = 10$ $r = 5$

(4) $n = 36$ $r = 12$

(5) $n = 120$ $r = 6$

(6) $n = 360$ $r = 10$

105

ADVANCED TOPICS

THE CALCULATOR IN TRIGONOMETRY

The emphasis on trigonometry has declined markedly in mathematics education since the mid-fifties. Once a senior course in the mathematics curriculum, trigonometry has become a part of second year algebra, or it has been integrated into a senior mathematics course usually designated as Elementary Functions.

Admittedly, the calculations that are necessary for applications of trigonometry are time consuming. Other important topics and concepts could not be explored during the given period of time. It was partially due to an effort to ease the laborious computations in trigonometry and astronomy that John Napier developed logarithms in the sixteenth century.

With the advent of the mini-calculator, trigonometry should again become a motivating and interesting topic, not only from its illustration as a set of functions based on the rotation of a vector about an origin, but from the point of view of applications.

It should be pointed out that while even the simplest calculator will be of help with the tedious multiplication and division involved in solving triangles, the calculator for problems in trigonometry should have these additional capabilities:

- At least one memory
- Square root key
- x-y exchange key
- Sine, cosine, tangent functions
- Arc-functions
- Radian degree switch

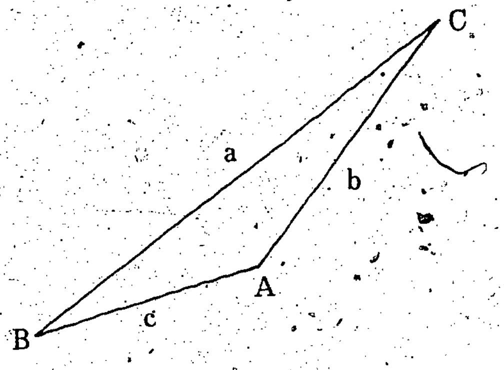
Some calculators have hard-wired routines for degree-minute-second conversions, but such routines are not essential.

In solving problems, it is possible to press the wrong keys in what might be a lengthy procedure. Hence, the following steps are essential in avoiding errors:

1. Write down the formula.
2. Substitute given values.
3. Familiarize yourself with the workings of your calculator - What your calcualtor can and can not do.
4. Write down answers to intermediate steps.
5. Check final answers.

The following exercises should illustrate the procedure:

1)



Given: Triangle ABC with
angle A = 138°45' 16"
side b = 148.15
side c = 62.340

Find: Side a
Angle B

Check answers

Using the law of cosines to find side a and angle B, we have

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$a^2 = 148.15^2 + 62.34^2 - (2)(148.15)(62.34) \cos(138^\circ 45' 16'')$$

$$a^2 = 21948.42 + 3886.28 - (-13888.43)$$

$$a^2 = 39723.13$$

$$a = 199.31$$

Here is the first answer which, with practice, may be obtained in a few minites, provided the calculator has the necessary capabilities. Certain numbers may be stored for later use for finding angle B. To find angle B, we use the law of cosines again, this time in the form

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{39723.13 + 3886.28 - 21948.42}{24849.97}$$

$$= \frac{21660.99}{24849.97} = 0.87167067$$

$$\cos^{-1}(0.87167067) \doteq 29.3466^\circ \doteq 29^\circ 20' 48''$$

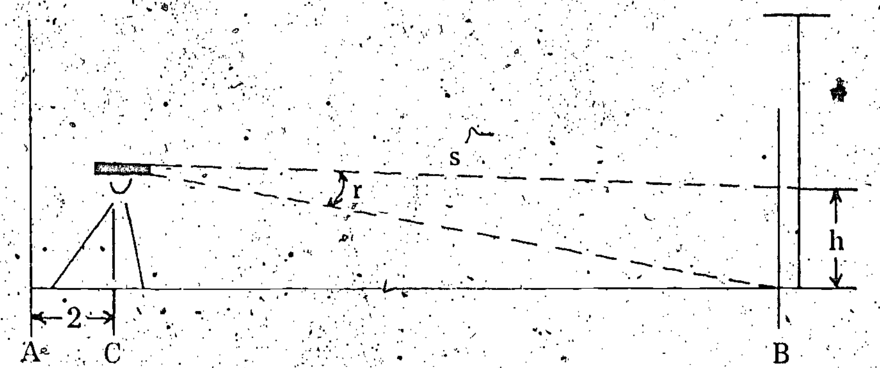
$$B = 29^\circ 20' 48''$$

To check these answers, students may recall a theorem from geometry that states a relationship between the angles of a triangle and the side opposite these angles. This is formalized by the law of sines as

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Using the calculator, we can easily find that angle C = 11.9096° or 11° 54' 25". The above stated ratios can be calculated. The degree of accuracy is dependent upon the number of digits retained.

2)



A second example might be used as a laboratory experiment. Many high schools have a rarely used transit sitting in a closet. Such a device has innumerable uses in trigonometry. In our example, let us measure the length of hallway AB without the usual tape measure.

Set the transit at point C, 2m from the wall (point A). Level transit so that sight-line s is horizontal. Have a student with target pole against the wall at end of hallway (B). Adjust target until it is in line of sight of transit. This is measure h.

Drop target to foot of pole and traverse transit to foot of pole. Take the reading of angle r.

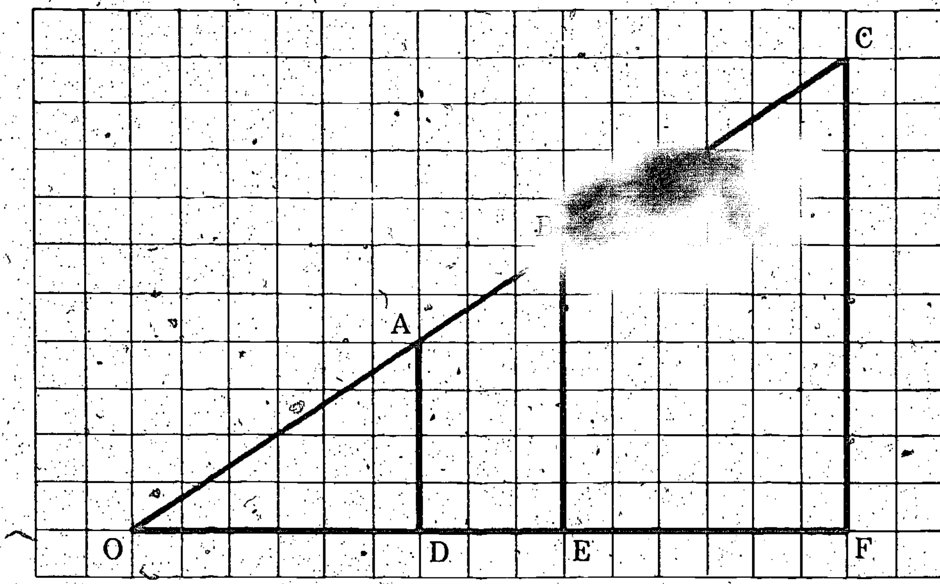
Knowing angle r, and the height h, we can calculate the length of the hallway:

$$\tan (r) = \frac{h}{s}$$

$$\text{Therefore, } s = \frac{h}{\tan r}$$

$$\text{and length of hallway } AB = s + 2m$$

The calculator is useful when teaching the concept of the ratio of the two sides of a right triangle remaining constant for each acute angle regardless of the length of the sides.

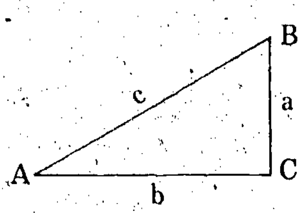


Fill in the following blanks. Use your calculator to express all ratios as decimal fractions,

AD = ______	OD = ______	AD/OD = ______
BE = ______	OE = ______	BE/OE = ______
CF = ______	OF = ______	CF/OF = ______

Compare the three ratios, what did you find? The ratio is called a tangent.

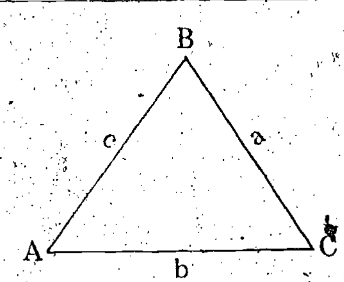
Given right triangle ABC; $\angle C = 90^\circ$



Use your calculator to determine all the other sides and angles of the triangle from the data given:

1.	c = 85	$\angle A = 35^\circ$
2.	a = .200	$\angle B = 80^\circ$
3.	a = 500	$\angle A = 55^\circ$
4.	a = 100	b = 125
5.	b = 125	c = 200

Given any triangle described by the diagram below:



Apply the Sine Law: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ and find those parts of the triangle not given.

1. $\angle A = 54^\circ$	$\angle B = 103^\circ$	$a = 36$
2. $\angle B = 38^\circ$	$\angle C = 60^\circ$	$a = 700$
3. $\angle A = 65^\circ$	$\angle B = 48^\circ$	$c = 900$
4. $a = 310$	$b = 36$	$\angle A = 22^\circ$
5. $b = 210$	$c = 18$	$\angle B = 22^\circ$
6. $\angle A = 40^\circ$	$a = 75$	$b = 90$

Using the Cosine Law; solve the following triangles for the parts not given:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

1. $a = 17$	$b = 12$	$\angle C = 60^\circ$
2. $a = 75$	$b = 40$	$\angle C = 64^\circ$
3. $b = 85$	$c = 105$	$\angle A = 51^\circ$
4. $a = 6$	$c = 3$	$\angle B = 110^\circ$

To find the radius of the circle inscribed in a triangle, use the formula:

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

where $s = \frac{a+b+c}{2}$

Find the radii of circles inscribed in triangles whose sides measure:

1. $a = 34$	$b = 26$	$c = 16$
2. $a = 15$	$b = 55$	$c = 66$
3. $a = 13$	$b = 12$	$c = 9$

Find the measure of the angles of the triangles whose side measures are given above: (use the Cosine Law and the Sine Law).

The area of a triangle is given by the formula

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

$$\text{where } s = \frac{1}{2}(a + b + c)$$

a, b and c are the measures of the sides of the triangle. Find the area of each triangle.

1. $a = 12$ $b =$ [illegible] $c = 14$
2. $a = 60$ b [illegible] $c = 100$
3. $a = 48$ b [illegible] $c = 60$

112

TALKING MINI-CALCULATORS

113

CALCULATOR VOCABULARY

Many numbers appearing in the display form words when the mini-calculator is held upside down. Encourage students to create examples and word problems which will result in a display necessary to form a desired word or phrase. Many calculator books will include many such computations and word problems.

The student must be cautioned, however, that because the mini-calculator must be turned upside down, the number in the display is in reverse order to the letters each represents when spelling a desired word. The numbers and the letters they represent are as follows:

0 → O	3 → E	7 → L
1 → I	4 → [illegible]	8 → B
2 → Z	[illegible] → S	9 → G

A partial list of calculator vocabulary is given in alphabetical order. A few proper nouns have been included. Students might make a separate list of such names.

As other words are discovered, they can be added to the mini-calculator vocabulary list:

CALCULATOR VOCABULARY - PARTIAL

Number	Word	Number	Word	Number	Word	Number	Word
38	BE	5379919	GIGGLES	0.04	hOO	3790	OGLE
338	BEE	7719	GILL	3504	hOSE	335	SEE
938	BEG	3379	GLEE	1	I	7735	SELL
7738	BELL	8179	GLIB	771	ILL	32135	SEIZE
37738	BELLE	0.79	GLO	51	IS	345	ShE
818	BIB	8079	GLOB	3751	ISLE	77345	ShELL
37818	BIBLE	38079	GLOBE	337	LEE	77145	ShILL
918	BIG	55079	GLOSS	937	LEG	3045	ShOE
39718	BILGE	0.9	GO	3781937	LEGIBLE	518045	ShOBIZ
7718	BILL	378809	GOBBLE	137	LEI	15	SI
57718	BILLS	5309	GOES	5537	LESS	39315	SIEGE
55378	BLESS	5379909	GOGGLES	73817	LIBEL	4915	SIGh
55178	BLISS	0.09	GOO	317	LIE	7715	SILL
808	BOB	4509	GOSh	0.7	LO	0715	SILO
908	BOG	34	hE	807	LOB	515	SIS
379908	BOGGLE	7334	hEEL	907	LOG	3215	SIZE
7108	BOIL	51.34	hE.IS	0.907	LOGO	372215	SIZZLE
0.08	BOO	0.7734	hELLO	7707	LOLL	491375	SLEIGH
5508	BOSS	14	hI	35007	LOOSE	0.75	SLO
883	EBB	4914	hIGh	3507	LOSE	8075	SLOB
733	EEL	7714	hILL	5507	LOSS	3075	SLOE
993	EGG	514	hIS	0	O	45075	SLOSH
0.93	EGO	5514	hISS	35380	OBESE	0.5	SO
73	EL	0.4	hO	391780	OBLIGE	805	SOB
37819173	ELIGIBLE	0.40404	hOhOhO	553580	OBSESS	7105	SOIL
773	ELL	378804	hOBBLE	40	OH	705	SOL
31573	ELSIE	0.804	hOBO	0.140	OHIO	3705	SOLE
339	GEE	304	hOE	710	OIL	0.705	SOLO
35339	GEESE	904	hOG	370	OLE	912	ZIG
739	GEL	3704	hOLE	0.370	OLEO	0.02	ZOO

Note: In order to enter words ending with the letter "o", first press the decimal point key e.g. $\boxed{.}$ $\boxed{7}$ $\boxed{7}$ $\boxed{3}$ $\boxed{4}$ "spells" hELLO.

Examples:

1. You can be stung by a "125 + 250 - 37".

2. The killing took place between Halloween and Christmas. The victim's last word was "(400^2 - 597) x 2".

3. The opposite of skinny is "20 x (2 x 30^2 - 31)".

WORD PROBLEMS TO MAKE YOUR CALCULATOR TALK!

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1. If you make $2,885.90 a month for 20 months, what have you got? (multiply)

2. There are 128 firemen at a building fire, the average temperature of which is 418 degrees. The building burns down. What did the firemen forget? (multiply)

3. If a man walks 2,946,973 inches per month, for 18 months, what does he need to replace? (multiply)

4. If the work efficiency of an elf is .001924 and Santa is down to his last 21 elves, what will he say? (multiply)

5. What European river is 720 miles long, an average of 5 fathoms deep, and an average of 273 yards wide? (720 x 5) + 273

6. What American University was founded in 1865 with 247 transfer students and 782 freshman? (1865 x 247) + 782

7. If 888 men spend 52 weeks on board ship and then in the next 2 days take over the ship, what's the Captain's name? (888 x 52 + 2)

8. What are you likely to be after 5 days of a 7,076 calorie diet? (multiply)

9. If a lion's cage has 35,007,552 bars and 18 fall out, where's the lion? (subtract)

To find the answers to the following questions, work the indicated problems on your mini-calculator, turn the calculator upside down and read your answers.

1. What is the ghost's favorite word?
   Calculate: $1.84 \div 23 =$

2. What did the society lady call the hobo?
   Calculate: $1938 \times 25 \times 4 \div 24 =$

3. What did the doctor tell Robert's mother?
   Calculate: $(555^2 + 520803) \times 93 + 70804 =$

4. What did the Casino boss think as he watched William winning at the game tables in Vegas?
   Calculate: $7 \times 5 \times 100 + 7.7718 =$

5. When the ghost frightened the little girl, what did she say?
   Calculate: $0.07 \times 0.111 \times 5 + 0.00123 =$

6. Where did you get gas today?
   Calculate: $142.15469 \times 5 =$ or $284.02212 \times 2.5 =$

7. What are the two important dates in the development of America?
   Calculate: $32159 \times 464 =$ (Read answers right side up)

A CALCULATING STORY

Directions: Solve each of the following thirty-two addition problems using your calculator. Turn the unit upside down to read each missing word from the story. Place the words in order in the blanks. Raise your hand when you can read the entire story.

1. 22864 + 14954	2. 0.084 + 0.056	3. 0.321 + 0.232	4. 3860 + 3854
5. 341 + 369	6. 4827 + 2278	7. 21013 + 16725	8. 24827 + 10180
9. 810 + 8	10. 38384 + 19354	11. 0.113 + 0.602	12. 2813303 + 2505501
13. 278 + 237	14. 1278 + 1802	15. 4271 + 2837	16. 2152 + 1553
17. 42,715 + 34,630	18. 2118 + 3220	19. 5101 + 704	20. 0.1473 + 0.6261
21. 12 + 2	22. 0.146 + 0.658	23. 4018 + 1489	24. 50137 + 7581

25. 421 + 350   26. 22222 + 35112   27. 31455 + 21590   28. 2514 + 3023

29. 21.566 + 32.138   30. 4115 + 3620   31. 3850 + 1658   32. 21215 + 34163

Deep in the _______ belt of southern _______, Farmer Jones lives with his family upon _______ _______, named for the famous _______ barons whose wealth also is derived from the _______. Farmer Jones has been on the watch, as several of his [illegible] ssing from the farm. His wife, a dainty southern _______, has ties a _____ _____ of _______ around the neck of eac[illegible] sheltered in the _____ to alert the family in case they tried to stray. "We need a watchman", they all agreed.

One sunny afternoon the Joneses were busy at their ____; ____ practicing on her ____, Mom about to _____ filet of _____ for lunch, Junior arranging his ____ collection and Farmer Jones tending his _____. Suddenly, Sis heard _____ coming from the porch. "_____!" she called. "_____!" she was answered. Before her stood an old ____, crying. "Why do you cry?" she asked. "I have suffered a tremendous _____", said the man. "My _____ were too high for me to pay and my poor _____ wife and children are hungry. I have tried to get a job but I've worn the _____ down on my _____ from walking so many miles. I'm willing to work for _____ money than others just to support my family. I will plow, dig _____, harvest the crops or tend the animals..."

"My father needs a shepherd", replied Sis, and she ran to _____ the idea to her father.

Farmer Jones agreed to hire the old man and proved to be a good _____ by inviting the entire family to join him in his _____ ings of food, shelter and comradeship. No longer did the cows stray and everyone lived happily ever after.

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ADDITIONAL SOURCE MATERIAL

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