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ABSTRACT

Students who behave as though an equation (function) is unchanged by a transformation of variable are said to conserve equation (function). Tasks devised to test for conservation of equation and function were administered, along with typical Piagetian tasks, to 72 students, aged 12, 14, and 17. Results suggest that conservation of equation, significantly more often than not, precedes formal thought, whereas conservation of function may or may not precede formal thought. Techniques used in this study can be extended to investigate conservation of abstract mathematical structure and its relationship to Piaget's present model of development. (Author)

Conservation of Equation and Function and Its Relationship to Formal Operational Thought

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Conservation of Equation and Function and Its Relationship to Formal Operational Thought

Objective

The primary purpose of this study was to investigate the relationship between the ability to conserve equation and function under transformations of literal variables and the ability to use formal operations in solving typical Piagetian tasks.

Theoretical Framework

Piaget (Beth & Piaget, 1966) has suggested that stages of development beyond his fourth stage of formal operations may be reached by the process of "reflective abstraction" (p. 235). He illustrates this process with the example of a student who, as a formal operational thinker, learns plane geometry as an axiomatic mathematical system, but who later, in a presumably more advanced stage of development, acquires an entirely restructured perspective of plane geometry as a result of studying more general abstract mathematical systems.

One interpretation of the process of reflective abstraction is in terms of the difference between first acquiring a concept and then, later,

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understanding that concept as an exemplar of a more general concept. In the case of the mathematics student, plane geometry is first learned as a self-contained axiomatic system. Later, when the concept of abstract mathematical structure has been acquired, plane geometry is viewed by the student as just one exemplar of the abstract Euclidean space structure.

The qualities of thought which characterize Piaget's stage of formal operations are apparently sufficient to enable a person to comprehend a particular abstract mathematical structure, such as Euclidean geometry or the real number system. However, it is not known whether formal operations are also sufficient to enable a person to comprehend the concept of abstract structure (Lovell, 1971 : Piaget 1968/1970). In order to determine whether the concept of abstract structure can be acquired at the stage of formal operations, or only at a stage beyond formal operations, it is necessary to identify tasks which test for the understanding of the concept of structure.

Many of Piaget's standard tasks embody abstract mathematical structures in their solutions (Inhelder & Piaget, 1955/1958; Piaget, Grize, Szeminska, & Bang, 1968). However, students who complete these tasks are not necessarily aware of the underlying structures (Sinclair, 1971). Therefore, Piaget's standard tasks do not seem to be sufficient for testing the understanding of the concept of structure per se. Other kinds of tasks, such as those used by Dienes and Jeeves (1965) or those used by Collis (1973), require the student to understand particular exemplars of abstract structure but still do not test the understanding of the concept of structure.

To understand a concept is to know which attributes define the concept and which attributes are irrelevant. To measure the understanding of a concept, Piaget (1941/1965) has introduced the method of observing a person's behavior before and after a transformation of irrelevant attributes. Persons who behave as though they understand that a particular concept is invariant under a transformation of irrelevant attributes are said to conserve that concept. Therefore, one method of measuring a person's understanding of the concept of abstract structure is to test the person's ability to conserve structure under a transformation of some irrelevant attribute.

In order to develop a general method for devising tasks to test for conservation of structure, it is necessary to determine which attributes are essential to a structure and which are irrelevant. The essential attribute which all abstract structures share is a distinguishable relationship, or <u>pattern</u>, among the elements of any exemplar of the structure. In fact, a structure <u>is</u> a pattern of elements. The actual elements themselves, or the operations used to combine them, are irrelevant to the structure pattern. Varying the elements or operations produces different exemplars of the structure, but the pattern itself remains invariant.

Because different elements produce different exemplars of a pattern, to <u>define</u> a pattern, whether it be an abstract mathematical structure or a simpler pattern of some sort, it is necessary to describe the relationship(s) in the pattern simultaneously for all of the elements which may comprise any exemplar of the pattern. The symbols used to represent different elements simultaneously in the definition of a pattern are called <u>variables</u>.

Because they must represent the entire domain of possible elements appearing in any exemplar of a pattern, the variables used to define a pattern must be completely arbitrary and nonconnotative, like the literal symbols commonly used as variables in mathematics. Because such mathematical variables are arbitrary and nonconnotative, they are also interchangeable. So, any pattern, regardless of its simplicity or complexity, is invariant under transformations of the variables used to define it. Therefore, one universal method for testing the understanding of a pattern, whether it be an abstract mathematical structure or a simple pattern like an equation or function, is to test for the ability to conserve the pattern under a transformation of variable.

In this study, conservation of two of the first overtly mathematical patterns which students encounter, equation and function, was investigated. Consideration was restricted to literal variables, the first true variables which students typically see.[1] And, except for one part of one task, only alphabetic transformations of variable were used, not only because such transformations are common in mathematics, but also because the simultaneous, but unrelated, linear orderings of the alphabet and of the real number system present a possible point of confusion which is useful in identifying students who do not fully understand the invariance

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[1] The empty box which appears in mathematical sentences beginning in the early elementary grades is not truly a variable; the box is used as a placeholder, and the student writes the solution to the equation <u>in</u> the box, rather than as equal to the box. Unlike a true variable, the box does not represent the solution to the equation; instead, like a blank, the box merely indicates a space in which to write the solution.

of equations and functions under transformations of variable.

The methods used in this research to test the understanding of the concepts of equation and function can be extended and generalized to study the understanding of the concept of abstract structure, at least as measured by tasks which test the ability to conserve structure under transformations, not just of variables, but also of elements and operations. Testing for the ability to conserve various kinds of patterns extends a technique used by Piaget to study cognitive development and may, therefore, eventually lead to an extension of his model to account for highly abstract mathematical thinking.

Interview Tasks

Three tasks were devised to test for the ability to conserve equation and function. Three other tasks similar to those used by Piaget and his colleagues to identify formal operational thought were designed to test for the ability to use the mathematical concepts of proportion, Cartesian product (combinations), and syllogism. Scoring criteria consistent with standard Piagetian theory were used to develop response classification categories for each task.

PROPORTION. In the proportion task the student is required to measure the height of a small tree and then the height of a big tree using long paper clips. The ratio of the heights of the two trees is 1:3. The student is next asked to measure the height of the small tree with short paper clips and then to predict the height of the big tree in short paper clips. The justification given by the student for the height estimate determines the classification of the student's response. A justification in terms of ratio, using either multiplication or division, is classified as formal operational; a justification using addition or subtraction is classified as concrete operational.

COMBINATIONS. The student is required to list the eight possible meals that could be selected from a menu containing three categories of food with two choices in each category. The student's response is classified according to the completeness of the solution and the method used to obtain the solution. Solutions which are virtually complete and/or are obtained systematically are considered formal level responses, whereas solutions which are largely incomplete and are obtained haphazardly are considered concrete level responses.

SYLLOGISM. Given that "All stigs are stonks" and "All stonks are quargs," the student is asked to explain the reasoning involved in determining whether or not the statements "All stigs are quargs" and "All stonks are stigs" are true. A student who reasons logically is classified as a formal thinker, while a student who reasons literally is classified as a concrete thinker on this task.

EQUATION. As a warm-up exercise the student is asked to solve a simple equation whose solution is $N = 4$. Next, the student is shown two identical equations with "unknown" W, and noninteger solution, and is asked whether or not the two equations are the same. The unknown in one equation is then changed from W to N, and the student is asked which solution would be larger, W or N. A student who responds that both solutions would necessarily be the same is said to conserve equation, whereas a student who believes that either W or N would be larger is classified as a nonconserver of equation (see Appendix A).

FUNCTION (Free-Response). As a warm-up exercise the student is asked to fill in several missing numbers in a chart of ordered pairs of numbers in which the columns are headed by M and N and the relationship between the columns of numbers is $N = 2M$. Next, the student is shown another chart of ordered pairs of numbers, with columns headed by B and C, and with no simple relationship between the numbers. The student is asked to supply one number which is missing from the chart. The column heading C is then changed to A, and the student is again asked to supply the missing number. A student who supplies the same number twice is said to conserve function, whereas a student who supplies different values for C and A is considered a nonconserver of function (see Appendix B).

FUNCTION (Furnished-Response). In Part (A) the student is first shown a complete chart of ordered pairs of numbers with columns headed by X and Z. The heading Z is then changed to Y and at the same time, one number is deleted from the chart. The student is asked to supply the number which is now missing. A student who supplies the number originally listed in the chart is said to conserve function; a student who supplies a number different from the "furnished" response is classified as a nonconserver (see Appendix C).

In Part (B) the original, complete chart of numbers has columns headed by J and K. This time K is changed to $K + 3$ at the same time as a number in the chart is deleted. The student is again asked to supply the number which is missing. Just as for Part (A), a student who supplies the number originally listed in the chart is considered a conserver; one who does not is considered a nonconserver (see Appendix D).

Method

The six tasks were administered by the experimenter in individual interviews with 72 public-school students selected to represent both sexes and three age levels, 12-, 14-, and 17-year-olds. The students were also chosen to represent diverse levels of mathematical achievement.

The order of administration of the six tasks was varied to control for practice effects among the Piagetian tasks and learning effects among the pattern tasks. Task order was assigned to the participants using a stratified random process.

Results

Each student's response to each task was classified by the interviewer to obtain the data analyzed in this study. In order to assess the objectivity of the response classification criteria which were used, an independent rater also classified the responses using the notes recorded by the interviewer as protocol. Cramér's coefficient *V* of interrater agreement on the response classifications was acceptable ($V \geq .75$) for all tasks except SYLLOGISM ($V = .60$). The criteria used for classifying responses to SYLLOGISM apparently required too much interpretation to be very reliable.

Table 1

Joint Response Frequencies ($\underline{N}$ = 72) for Parts (A) and (B) of FUNCTION (Furnished-Response)

FUNCTION (Furnished-Response, A)	FUNCTION (Furnished-Response, B)		
	Nonconserving	Transitional	Conserving
Nonconserving	35	2	0
Transitional	8	1	0
Conserving	21	4	1

The low coefficient of association ($\underline{V}$ = .16) between scores on responses to the two parts of FUNCTION (Furnished-Response) was probably caused by the disparity between the types of variable transformation used in the two parts of the task and indicated that the score on each half of the task should be treated separately. Furthermore, because only one student interviewed in the study conserved on Part (B) of FUNCTION (Furnished-Response), the data obtained on this portion of the task were considered useless for the purposes of this study, and responses to Part (B) were eliminated entirely from the analysis of data (see Table 1).

It might be remarked that several of the very brightest students interviewed in pilot studies had conserved on Part (B) of FUNCTION (Furnished-Response), and it was conjectured that this task might yield more informative results if presented to persons older than most of the participants in this study.

Task order, as varied in this study, was found to be generally an insignificant factor affecting students' responses. However, because of the particular composition of the two function tasks used in the study, FUNCTION (Free-Response) was always presented sometime before FUNCTION (Furnished-Response), and this one constant factor in the task order may have caused a skewing of responses to FUNCTION (Furnished-Response, A) toward nonconservation. In order to determine whether or not response set did affect the results on the two function tasks, the order of these tasks should be varied in the future.

Hypotheses tested. Three hypotheses were tested using a chi-square goodness-of-fit test and a significance level of $p < .05$. It was found that:

(a) significantly more students conserved equation but not function than conserved function but not equation (see Table 2, page 11);

(b) significantly more students conserved equation but were not formal operational on PROPORTION than were formal operational on PROPORTION but did not conserve equation; significantly more students conserved equation but were not formal operational on SYLLOGISM than were formal operational on SYLLOGISM but did not conserve equation (see Table 3, page 12);

(c) significantly more students conserved on FUNCTION (Free-Response) but were not formal operational on PROPORTION than were formal operational on PROPORTION but did not conserve on FUNCTION (Free-Response) (see Table 4, page 13); significantly more students were

Table 2

Joint Response Frequencies for EQUATION and the Two Function Tasks

Type of response	EQUATION		
	Nonconserving	Transitional	Conserving
FUNCTION (Free-Response) ($\underline{N}$ = 72)			
Nonconserving	13	2	12
Transitional	2	2	4
Conserving	1	2	34
FUNCTION (Furnished-Response, A) ($\underline{N}$ = 72)			
Nonconserving	13	4	20
Transitional	2	0	7
Conserving	1	2	23

formal operational on COMBINATIONS but did not conserve on FUNCTION (Furnished-Response, A) than conserved on FUNCTION (Furnished-Response, A) but were not formal operational on COMBINATIONS (see Table 5, page 14).

There were also several significant relationships among the task scores about which no hypotheses had been conjectured prior to the study:

(a) significantly more students conserved on FUNCTION (Free-Response) but not on FUNCTION (Furnished-Response, A) than conserved on FUNCTION (Furnished-Response, A) but not on FUNCTION (Free-Response) (see Table 6, page 15);

Table 3

Joint Response Frequencies for EQUATION and the Three Piagetian Tasks

Type of response	EQUATION		
	Nonconserving	Transitional	Conserving
PROPORTION (N = 72)			
Concrete	10	4	31
Transitional	6	1	3
Formal	0	1	16
COMBINATIONS (N = 72)			
Concrete	3	2	8
Transitional	7	2	13
Formal	6	2	29
SYLLOGISM (N = 72)			
Concrete	7	4	12
Transitional	6	2	14
Formal	3	0	24

(b) significantly more students were formal operational on COMBINATIONS but not on PROPORTION than were formal operational on PROPORTION but not on COMBINATIONS (see Table 7, page 15);

Table 4

Joint Response Frequencies for FUNCTION (Free-Response) and the Three Piagetian Tasks

Type of response	FUNCTION (Free-Response)		
	Nonconserving	Transitional	Conserving
PROPORTION ($\underline{N}$ = 72)			
Concrete	17	4	24
Transitional	7	2	1
Formal	3	2	12
COMBINATIONS ($\underline{N}$ = 72)			
Concrete	4	2	7
Transitional	10	2	10
Formal	13	4	20
SYLLOGISM ($\underline{N}$ = 72)			
Concrete	11	4	8
Transitional	9	1	12
Formal	7	3	17

(c) significantly more students were formal operational on SYLLOGISM but not on PROPORTION than were formal operational on PROPORTION but not on SYLLOGISM (see Table 7, page 15).

Relative task difficulty. The results relating relative performance

Table 5

Joint Response Frequencies for FUNCTION (Furnished-Response, A) and the Three Piagetian Tasks

Type of response	FUNCTION (Furnished-Response, A)		
	Nonconserving	Transitional	Conserving
	PROPORTION (N = 72)		
Concrete	22	5	18
Transitional	8	2	0
Formal	7	2	8
	COMBINATIONS (N = 72)		
Concrete	9	0	4
Transitional	12	2	8
Formal	16	7	14
	SYLLOGISM (N = 72)		
Concrete	12	3	8
Transitional	12	2	8
Formal	13	4	10

on the tasks can be summarized as shown in Figure 1 on page 17. The task ordering depicted in the diagram must be considered tentative, especially because of the unreliability of the response classification criteria used for SYLLOGISM. However, the diagram does give a general indication of the apparent relative difficulty of the six tasks used in this study.

Table 6

Joint Response Frequencies ($\underline{N}$ = 72) for FUNCTION (Free-Response) and FUNCTION (Furnished-Response, A)

FUNCTION (Free-Response)	FUNCTION (Furnished-Response, A)		
	Nonconserving	Transitional	Conserving
Nonconserving	22	5	0
Transitional	7	0	1
Conserving	8	4	25

Table 7

Joint Response Frequencies for PROPORTION with COMBINATIONS and SYLLOGISM

Type of response	PROPORTION		
	Concrete	Transitional	Formal
COMBINATIONS ($\underline{N}$ = 72)			
Concrete	8	1	4
Transitional	15	3	4
Formal	22	6	9
SYLLOGISM ($\underline{N}$ = 72)			
Concrete	16	3	4
Transitional	15	3	4
Formal	14	4	9

Table 8

Joint Response Frequencies ($\underline{N}$ = 72) for COMBINATIONS and SYLLOGISM

COMBINATIONS	SYLLOGISM		
	Concrete	Transitional	Formal
Concrete	3	7	3
Transitional	6	7	9
Formal	14	8	15

The easiest task seemed to be EQUATION, the hardest PROPORTION, with the others in between, shown in the figure in descending order according to increasing difficulty. Whether the differences in performance on these tasks may be due to factors other than difficulty is yet to be determined. Either longitudinal or learning transfer studies might be used to ascertain the extent to which developmental factors, for example, may affect the relative positions of the tasks in this ordering.

It had been expected that conservation of equation would be easier for more students than conservation of function. The difference in performance between the two function tasks had not been predicted and may have been caused by response set, as previously mentioned.

No significant differences among scores on the Piagetian tasks had been anticipated because all three tasks are typical of those used to identify formal operations. Especially surprising was the discovery

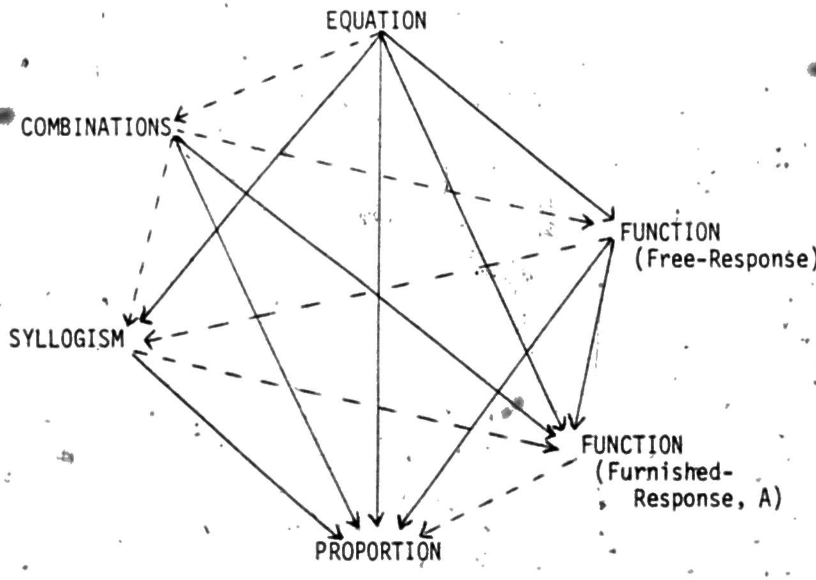

Figure 1. Tentative ordering of tasks according to difficulty, with EQUATION the easiest task and PROPORTION the most difficult. Task _X_ ----> Task _Y_ if and only if more students scored higher on Task _X_ than on Task _Y_ than conversely. Task _X_ ⟶ Task _Y_ if and only if this relationship was statistically significant ($p < .01$).

that PROPORTION, with an integer ratio of 3:1, seemed to be the most difficult of the six tasks used in the study. Judging from the results of other studies involving proportional thinking, such as that by Karplus and Peterson (1970), it had been expected that PROPORTION would be, if anything, the easiest of the Piagetian tasks.

There are two plausible explanations for the fact that more students seemed to score relatively higher on COMBINATIONS and SYLLOGISM than on PROPORTION. First, responses to COMBINATIONS and SYLLOGISM may have been scored more leniently than responses to PROPORTION. It may be, for example, that some responses to COMBINATIONS were considered systematic when, in fact, the "system" used was accidental. In the case of SYLLOGISM, the subjectivity of the response classification criteria may have resulted in more liberal interpretations of answers than was justified. A second explanation for the unexpected differences between scores on the Piagetian tasks could be that responses to the tasks depend heavily upon training, and many students may, for instance, study the Cartesian product of sets (combinations) before they study the concept of proportion.

With regard to the results relating performance on the pattern tasks with performance on the Piagetian tasks, it had been thought, from a theoretical standpoint, that more students would be formal operational and not conserve equation or function than would conserve equation or function and not be formal operational. This conjecture was generally contradicted. If it is eventually found that developmental factors play a role in the ability to conserve equation and function, it appears that the qualities of thought identified as formal operational by Piaget may be sufficient to guarantee the ability to conserve simple patterns like equation and function. If it is also confirmed that conservation of equation and conservation of function do appear substantially after concrete thought has been well established, it may possibly be concluded that the ability to conserve simple patterns constitutes an additional criterion for identifying the onset of formal thought.

<u>Other results</u>. Coefficients of association among scores on the pattern tasks were significant ($p < .05$); coefficients of association among scores on the Piagetian tasks were not significant. One reason for this result might be that the pattern tasks seem to be psychometrically more similar to each other than do the Piagetian tasks, as evidenced by the heavy concentration of entries along the main diagonals of Table 2, page 11, and Table 6, page 15. Although the diagonal entries had to be disregarded in testing the hypotheses, it is those entries which account for a large share of the coefficients of association among scores on the pattern tasks.

Still to be determined is the extent to which conservation of equation and conservation of function are distinct abilities. The results of this study seem to suggest that there may be a tendency for students to acquire conservation of equation before conservation of function more often than conversely, but many students performed similarly on both kinds of tasks, so the two abilities may be acquired almost concurrently, as the result of either development or training.

About 50% of the 12-year-olds interviewed in this study conserved equation, but only about 20% conserved function. Over 80% of the 14-year-olds conserved equation and 50% conserved function, even though these students had not, for the most part, begun a formal study of algebra.

The performance of the 17-year-olds in this study was not significantly different from that of the 14-year-olds, probably because a greater number of below-average students were included in the 17-year-old group than were included in the 12- or 14-year-old groups. The disparity between the population represented by the 17-year-olds interviewed compared to the other two age groups was a limitation of this study which precluded drawing valid inferences regarding the relationship between age and task performance.

A sex difference significant at the .05 level was found on both FUNCTION (Free-Response) and FUNCTION (Furnished-Response, A). In the case of FUNCTION (Free-Response) this difference was significant ($p < .05$) for the 14- and 17-year-old groups, but not for the 12-year-olds. Although such a result had not been anticipated, some other studies, such as Hilton and Berglund (1974), have found a tendency for sex differences in mathematical achievement to begin to appear around the age of 13. The fact that the conservation-of-pattern tasks seem more overtly mathematical than the Piagetian tasks may possibly account for the fact that sex differences were found on the function tasks but not on the Piagetian tasks.

One other factor which may have affected results in this study should be mentioned. In order to maximize the amount of usable information collected regarding the relationship between the ability to conserve equation and function and the ability to use formal operations, any interview in which the student seemed not to understand one or more of the tasks was discarded and replaced with an interview of a student of like sex and age. Altogether, 7 of the 72 interviews were discarded and replaced.

Discarding these interviews clearly resulted in a final sampling of students more homogeneous than the group initially selected. However, because there was no intention in this study of trying to predict performance of the total population from performance of a sample, the procedure of discarding and replacing interviews was considered admissible. Nevertheless, the reader should be aware that such a procedure was followed and thus be alerted to possible effects which this procedure may have had upon the data collected.

Implications

There are three general directions in which the research begun in this study can be continued and expanded. First, more information is needed about the nature of the ability to conserve equation and the ability to conserve function. Is one acquired before the other, or are both acquired more or less concurrently?

Secondly, is the ability to conserve equation and/or function acquired before, or concurrently with, formal operational thought? Perhaps, as the data from this study suggest, conservation of equation is accessible at the level of concrete operations, while conservation of function marks the onset of formal operations.

Finally, what is the relationship between formal thought and the ability to conserve more complex patterns? If, as seems plausible, conservation of equation and function prove to be additional criteria for identifying formal thought, perhaps it is conservation of structure which will be found to characterize a stage of thinking beyond formal operations, a stage of pattern operations. Perhaps it is conservation of structure which will eventually extend Piaget's developmental model of ordinary thinking into the realm of highly abstract thought.

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APPENDIX A

EQUATION

WHAT WOULD N HAVE TO BE FOR THIS STATEMENT TO BE TRUE?

$$2 \times N + 3 = 11$$

ARE THESE TWO STATEMENTS THE SAME?

$$7 \times W + 22 = 109$$

$$7 \times W + 22 = 109$$

IF YOU WERE TO FIGURE OUT WHAT W SHOULD BE TO MAKE THE FIRST STATEMENT TRUE AND WHAT N SHOULD BE TO MAKE THE SECOND STATEMENT TRUE, WHICH WOULD BE LARGER W OR N ?

$$7 \times W + 22 = 109$$

$$7 \times N + 22 = 109$$

APPENDIX B

FUNCTION (FREE-RESPONSE)

CAN YOU FILL IN THE MISSING NUMBERS?

M	N
2	4
3	6
4	8
5	
	14
12	

AS NEARLY AS YOU CAN TELL, WHAT WOULD C BE EQUAL TO WHEN B = 10?

B	C
1	2
2	11
4	23
5	26
9	18
10	

AS NEARLY AS YOU CAN TELL, WHAT WOULD A BE EQUAL TO WHEN B = 10?

B	A
1	2
2	11
4	23
5	26
9	18
10	

APPENDIX C

FUNCTION (FURNISHED-RESPONSE, A)

NOTICE THAT WHEN X = 12, Z = 27.

X	Z
2	7
3	9
5	13
8	19
12	27
14	31

WHAT WOULD Y BE EQUAL TO WHEN X = 12?

X	Y
2	7
3	9
5	13
8	19
12	
14	31

APPENDIX D

FUNCTION (FURNISHED-RESPONSE, B)

NOTICE THAT WHEN J = 3,
K = 8.

J	K
1	0
2	3
3	8
4	15
5	24
6	35

WHAT WOULD K + 3
BE EQUAL TO WHEN J = 3?

J	K + 3
1	0
2	3
3	
4	15
5	24
6	35