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ABSTRACT

This study examines the extent to which modeling facilitates transformational problem solving in mathematics for children in kindergarten and first grade. Subjects were 20 children selected from each level. They responded to two problems for each of four problem types (counting on, story problems, quantitative comparisons, and ordination) under each of three modeling conditions (overt, implied and implicit). Results indicate that: (1) modeling has an effect upon the subject's ability to solve the two more difficult type of transformation problems (quantitative comparisons and ordination), and that (2) overt and implied modeling significantly affects the subject's ability to solve counting on and story problems. (Author/MS)

Language and Observation of Movement as Problem Solving Transformation Facilitators Among Kindergarten and First Grade Children

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Abstract

The concept of transformation is an important construct in Piagetian research and it has been defined and studied operationally in several studies. The present study sought to clarify these definitions and to systematically vary their usages among kindergarten and frist grade subjects. It was felt that a 4 x 3 x 2 design of problem type and transformation behaviors would provide data for obtaining new insights.

Piaget (1952) referred to transformations as a mental processing of thought related to reversibility. Steffe (1967) and LeBlanc (1968) defined transformations in terms of verbally described movement. Underhill and Shores (1975) defined transformations in terms of verbally described and physical movement.

Three modeling levels were created. Overt: verbal, visual and movement cues (initial and final stages); Implied: verbal and visual cues (initial and final stages). Implicit: visual cues only (final stages).

To examine the extent to which these modeling levels facilitate problem solving in mathematics for children in kindergarten and first grade, 20 subjects were selected from each grade. They responded to two problems of each of four problem types (counting on, story problems, quantitative comparisons, and ordination) under each of three modeling conditions.

The results indicate that modeling has an effect upon the subject's ability to solve the two more difficult types of transformation problems (quantitative comparisons and ordination). Further, overt and implied modeling significantly affected the subject's ability to solve counting on and story problems.

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Objective:

To ascertain whether the use of overt modeling and/or verbal modeling assists young children to solve four types of mathematical problems.

Theoretical background:

A child's ability accurately to solve basic mathematical problems is known to be effected by both his level of cognitive development and the effects of his initial school experiences (Underhill and Shores, 1975). In kindergarten and first grade childred aged 5 to 7 years, the ability to conserve numerousness is evolving as the children are being exposed to basic mathematical concepts of varying conceptual complexity (Piaget & Inhelder, 1965).

The conservation of numerousness construct was introduced to the mathematics education community from the translated writings of Jean Piaget (1952), the text by Flavell

(1963), and the research of Elkind (1961), Dodwell (1960), and Wohlwill (1962). Studies by Van Engen and Steffe (1966), LeBlanc (1968), Steffe (1970), and Johnson (1971) have established significant differences between conservers' and nonconservers' problem solving achievement. In addition, it was found that problems which involved transformation were significantly more difficult than those which did not involve a transformation. Transformational tasks are those which imply movement or action in the context of stated problems (Underhill and Shores, 1975).

The concept of transformation is an important construct in Piagetian research. A transformation is an act or process of alterating, or the changing of one thing into another. A transformation can exist at several different levels.

Transformations not only refer to alterations in the physical world, but also to the compensations made by the individual in his mental structures. If an object or state is known by an individual, then a transformation in the physical state is accompanied by a transformation in the cognitive structure.

In another sense, a transformation occurs when a learner states that 3 + 4 = 7. Piaget and Inhelder (1969) describe operations as reversible transformations, and they use addition of two numbers as a specific example.

The population of interest in the present study was kindergarten and first grade children, so the concept of transformation was defined within the context of pre-formal operational thought. Piaget (1972) and Sinclair (1971) characterize concrete operational thought as being limited to thinking about object experience through object-invoking mental processing. While concrete operational thought is not limited to thinking-while-manipulating, such thought is characterized by thinking about real present objects and actions.

Thus, a meaningful comprehension of 3 + 4 = 7 suggests that learners conjure mental images of sets of real objects with number properties of "threeness" and "fourness," then if learners comprehend the operation of addition, they conceptualize a transformation in which the two sets with number properties of "threeness" and "fourness" are joined in set union to form a new superordinate set with a number property of "sevenness."

Steffe (1967, 1968) and LeBlanc (1968) defined transformational and non-transformational addition and subtraction story problems as those which do or do not provide movement cues which indicate joining or separating of sets and subsets. Here are addition examples:

Transformation: Two dogs are in the kennel. Three more dogs are placed in the kennel. Now how many dogs are in the kennel altogether?

No Transformation: Bill has three frogs. John has four frogs. How many frogs do Bill and John have altogether?

Within the context of earlier statements made by Piaget, one could say that the operation of addition is involved in both types of problems so both involve transformations. Thus, if the learner is asked to solve the problems, he is asked to complete a transformation. The Steffe and LeBlanc tasks might be more appropriately labeled as facilitative and non-facilitative addition and subtraction types relative to the addition transformation task to be completed.

It was hypothesized that the modeling of the transformation would assist children who are beginning to conserve numerousness in solving the mathematical problems. The degree of facilitation should fluxuate according to the degree to which the modeling itself varies from fully demonstrated and explained, to fully explained, to simply the implicit movement within the problem statement itself. It was anticipated that a child who observes a transformation will be able to use spatial referents as cues to assist him in recalling the untransformed set. Thus, his difficulty in solving problems will be less pronounced than that for a child who does not receive a similar modeling experience.

Among the transformational mathematical operations to which young children are introduced are the following, of varying conceptual complexity: counting on, story problems, quantitative comparisons, and ordination. Counting on requires the formation of one set and the serial addition of elements to it. It is the continuation of a simple counting sequence. Addition story problems constitute a slightly more complex task, namely the establishment of two sets of similar elements and the union of them. Quantitative comparison involves the formation of two sets, the establishment of correspondences between the sets' elements, and a judgment based on equivalence. Ordination, the most complex of the tasks in this study, posits the existence of two sets and relationships of two abstract constructs to them. Both a cardinal and ordinal (spatial position) relationship must be established with the two sets, and the positional relationship must be maintained after spatial rotation of 135^o.

The Experimental Tasks

Counting On Tasks. E placed a strip of cardboard containing a row of at least seven chips. The first n chips were covered with another piece of cardboard. S was told how many chips were covered and was requested to tell how many chips were on the cardboard in all.

Addition Tasks. E placed a cardboard piece with pictures of children and two appropriate sets of chips. E told an addition story problem. S was requested to give the answer.

Quantitative Comparisons Tasks. E placed 2 rows of chips before S. S was asked if there were the same number of chips in each row.

Order Tasks. E placed a strip of cardboard containing Four chips of four different colors and a second piece of cardboard containing two chips of two different colors at 135^{o} rotation from the first piece. E gave S a third and fourth chip to place on the second piece of cardboard.

Procedure

The subjects were presented with two items of each problem type: counting on, story problems, quantitative comparison, and ordination. Each set of 8 problems was presented, by three researchers, under three modeling conditions in the following order: (1) implicit modeling, in which the subject was presented with a transformed model and simply asked to solve the problem; (2) implied modeling, in which the subject was presented with a transformed model, and the procedures for the restoring transformation was verbally described to him; and (3) overt modeling, in which the experimenter transformed the model as the question was asked and verbally commented on the transformation as he carried it out. A random order of items within each set of eight tasks was used for each subject.

The following grid summarizes the three modeling conditions with the four problem types.

	Overt (Auditory & visual movement cues)	Implied Auditory movement cues only)	Implicit (no auditory or visual cues)
Counting On Tasks	The cardboard strip was placed before S with all of the chips showing. A second piece of cardboard was used to cover the first n chips, while E explained what he was doing.	The cardboard was placed before S with the first n chips covered. E explained that the first n chips had been covered up.	The cardboard strip was placed before S with the first n chips covered. No explanation of the covering was given beyond the statement of the problem.

Order Tasks	The cardboard strips were placed initially in parallel positions. As one was rotated through 135°, E described what he was doing.	The cardboard strips were placed initially in 135° positions. E explained how they would match if one were turned.	Same as implied with no explanation.
Story Problem Tasks	A transformation problem (Steffe) was stated as chips were used to demonstrate the action.	A transformation problem was stated. Chips were statically placed in a post-transformational position.	A non-transformation problem was stated. Chips were statically placed in two disjoint sets.
Quantitative Comparisons Tasks	Two rows of chips in one-to-one correspondence were presented. One row was linearly dispersed. The action was described.	Two rows of chips presented statically, one being more linearly dispersed. The dispersion process was explained.	Same as implied with no explanation.

From a theoretical point of view, the subject must attend to a transformation in each of the twelve tasks. Clearly, the tasks involving auditory and visual cues are much more explicit in their overt manifestations of the necessary transformations. The three cases of each class of concept tasks could be said to depend on <u>attending to and comprehending auditorially and visually presented transformation cues</u>, <u>attending to and comprehending auditorially presented transformation cues</u>, and <u>spontaneous creation of transformations unaided by experimentally visual or auditory cues</u>.

Knowledge of youngsters' performances on the twelve tasks should clarify researchers' understanding of the roles of language and observed movement in transformational thinking. If these patterns are pervasive, the practitioner is provided with an empirically verified rationale for utilizing modeling procedures during instruction.

<u>Sampling</u>

To obtain a representative sample of kindergarten ($n_k = 20$) and first grade ($n_1 = 20$) children, a large suburban school system's lists of kindergarten and first grade pupils were obtained. A random sample of twenty children were drawn from each list to serve as subjects for the study. During testing one child was removed from the sample for overt cheating (lifting the cards to count chips). He was replaced by another child drawn at random from the school's roster. All subjects were tested on May 27 and May 28, 1976. Each subject responded to the 24 tasks within a time interval of approximately 30 minutes. All tasks were individually administered.

Analysis

In the 4 x 3 x 2 (Problem Type, by Modeling Type, by Grade Level) design the subjects were used as their own controls across problem type and modeling type. An initial factor analysis of the 24 items was conducted in which the items were found to load by problem type, which confirmed the existence of conceptual distinctiveness among the problem types. A subsequent MANOVA was used to determine the effects of problem type, modeling type, and grade level for each of the four problem types.

Results

Table One presents a summary of the multivariate analysis. For the counting on and story problem items there was no significant difference across the modeling types. However, for quantitative comparison and ordinal items there was a significant difference ($p < .05$) across the modeling types.

Insert Table One about here

Table One: Summary of Multivariate Analysis of the Effects of Modeling Type and Grade Level for Each Problem Type

Problem Type: Counting On

Source of Variation	ss	df	MS	F
Main Effects	12.85	3	4.28	7.90**
Treatment	.82	2	.41	.75
Grade Level	12.03	1	12.03	22.20**
Interaction	.02	2	.01	.02
Error	61.80	114	.54	

Problem Type: Ordinal

Source of Variation	ss	df	MS	F
Main Effects	6.03	3	2.01	3.61*
Treatment	3.62	2	1.81	3.25*
Grade Level	2.41	1	2.41	4.33*
Interaction	.22	2	.11	.20
Error	63.35	114	.56	

Problem Type: Quantitative Comparison

Source of Variation	ss	df	MS	F
Main Effects	10.39	3	3.46	4.27**
Treatment	6.72	2	3.36	4.14*
Grade Level	3.68	1	3.68	4.53*
Interaction	.65	2	.33	.40
Error	92.55	114	.81	

Problem Type: Story Problem

Source of Variation	ss	df	MS	F
Main Effects	7.53	3	2.57	8.00**
Treatment	.52	2	.26	.82
Grade Level	7.01	1	7.01	22.35**
Interaction	.32	2	.16	.51
Error	35.75	114	.31	

* $p \leq .05$

** $p \leq .01$

Tables Two and Three present a summary of univariate contrasts between grade levels by problem type and modeling condition. In the quantitative comparison instance, the significant differences among the kindergarten subjects' responses were found between each of the model types (Overt > Implied > Implicit). The first grade subjects had a different pattern in their responses, with overt responses being significantly greater than both the implied and implicit responses (Overt > Implied = Implicit).

Insert Tables Two & Three about here

In the ordinal instance, the significant differences among the kindergarten subjects' responses were found, with implicit responses being significantly lower than the other two modeling types (Overt = Implied > Implicit). With respect to first graders' performances on the ordinal tasks, there were no significant differences.

For the overt items there was a significant difference; children's performance with counting on and responses to story problem items were significantly higher than their performance on quantative comparison and ordinal items (CO = SP > OR = QC). For the implied items the nature and order of significance was the same as for the overt

Table Two: Summary of Univariate Contrasts Between Grade Levels by Problem Type and Modeling Condition

			MODELING CONDITION			
			Overt	Implied	Implicit	TOTALS
PROBLEM TYPE	Counting on Problems	$\bar{X}_k$=	1.01	1.05	0.90	3.05
		$\bar{X}_1$=	1.75	1.65	1.50	4.90
		F=	9.15*	7.02*	5.52*	9.65*
	Ordination Problems	$\bar{X}_k$=	0.70	0.60	0.35	1.65
		$\bar{X}_1$=	1.10	0.80	0.60	2.50
		F=	2.17	0.76	1.55	2.06
	Quantative Comparison Problems	$\bar{X}_k$=	1.10	0.70	0.35	2.15
		$\bar{X}_1$=	1.30	1.00	0.95	3.25
		F=	0.42	1.13	4.97	2.23
	Story Problems	$\bar{X}_k$=	1.45	1.25	1.50	4.20
		$\bar{X}_1$=	1.95	1.90	1.90	5.75
		F=	9.60*	13.90*	5.63*	16.03*
	TOTALS	$\bar{X}_k$=	4.35	3.60	3.10	
		$\bar{X}_1$=	6.10	5.35	4.95	
		F=	7.17*	9.92*	10.43*	

* $p \leq .05$

Table Three: Summary of Univariate Contrasts Among Modeling Conditions by Problem Type

PROBLEM TYPE			MODELING CONDITION		
			Overt	Implied	Implicit
Counting On		$\bar{X}$=	1.38	1.35	1.20
	Implied		.03		
	Implicit		.18*	.15	
Ordination		$\bar{X}$=	.90	.70	.48
	Implied		.20*		
	Implicit		.42*	.22*	
Quantative Comparisons		$\bar{X}$=	1.20	.85	.65
	Implied		.35*		
	Implicit		.55	.20*	
Story Problems		$\bar{X}$=	1.70	1.58	1.70
	Implied		.12		
	Implicit		.00	.12	

* $p \leq .05$

items. For the implicit items the counting on, quantitative comparison, and story problem responses were significantly greater than those of the Ordinal type (CO = QC = SP > OR). items the counting on, quantitative comparison, and story problem responses were significantly greater than those of the Ordinal type (CO = QC = SP > OR).

First graders were significantly better than kindergarteners in counting on and story problems responses over all types of models.

Significance of the Findings

This study indicates that modeling has an effect upon the subject's ability to solve the two more difficult types of transformation problems, quantitative comparison and ordination. Further, overt and implied modeling has significantly effected the subjects' ability to solve counting on and story problems. The effects were greater for kindergarten than for first grade children.

These findings support the hypothesis that during the child's transition from non-conserver to conserver, the use of modeling might significantly assist the teacher in facilitation of conservation-related subject matter. Researchers and practitioners need to conduct further investigation to determine the pervasiveness of the differences found in this study. If the significant differences are widespread, then considerable pedagogical change might be warranted.

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