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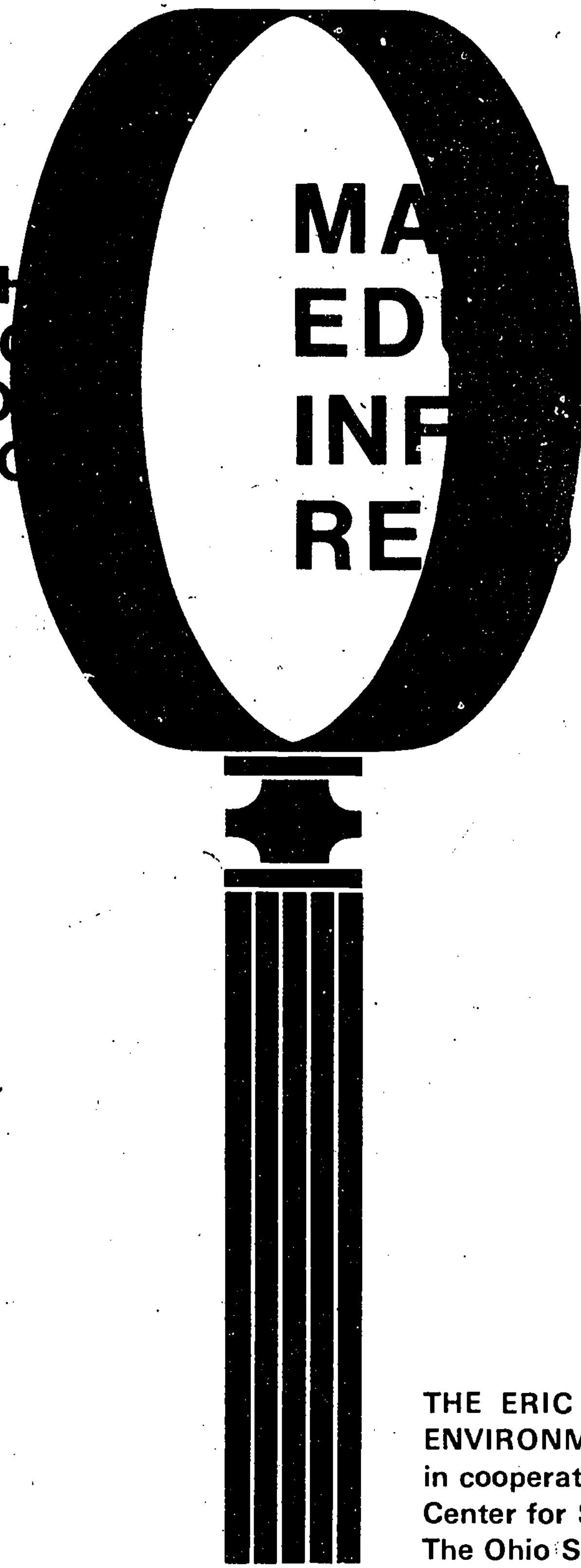
ABSTRACT

Collected in this document are papers presented at a conference on designing a mathematics methods course for secondary school teachers. Papers are organized under the following categories: (1) Realistic goals for a methods course, (2) Minimal content for a methods course, and (3) Teaching strategies used in methods courses. The seven short papers included under teaching strategies cover mathematical ideas which determine strategies, micro-teaching, modular organization, laboratory approach, student-centered strategy, discovery, and tutoring skills. The document concludes with a paper discussing creativity in teaching. Methods course syllabi are collected in Appendix I, while Appendix II gives a list of conference participants. (DT)



MATHEMATICS EDUCATION INFORMATION REPORT

THE ERIC SCIENCE, MATHEMATICS AND
ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

MATHEMATICS EDUCATION REPORTS

Papers from a Symposium
arranged by the
ILLINOIS COUNCIL ON MATHEMATICS EDUCATION

DESIGNING METHODS COURSES FOR SECONDARY SCHOOL MATHEMATICS TEACHERS

edited by
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January, 1977

Mathematics Education Reports

Mathematics Education Reports are being developed to disseminate information concerning mathematics education documents analyzed at the ERIC Information Analysis Center for Science, Mathematics, and Environmental Education. These reports fall into three broad categories. Research reviews summarize and analyze recent research in specific areas of mathematics education. Resource guides identify and analyze materials and references for use by mathematics teachers at all levels. Special bibliographies announce the availability of documents and review the literature in selected interest areas of mathematics education. Reports in each of these categories may also be targeted for specific sub-populations of the mathematics education community. Priorities for the development of future Mathematics Education Reports are established by the advisory board of the Center, in cooperation with the National Council of Teachers of Mathematics, the Special Interest Group for Research in Mathematics Education, the Conference Board of the Mathematical Sciences, and other professional groups in mathematics education. Individual comments on past Reports and suggestions for future Reports are always welcomed by the associate director.

Methods courses and student teaching experiences have formed the twin cornerstones of pre-service teacher preparation in mathematics as well as other subject areas. It is only natural then that most attempts to improve pre-service teacher education focus on these two areas. Because they are more easily controlled and manipulated, methods courses are a particularly appropriate area for experimentation and innovation. In recent years many different innovations have been tried in methods courses, none of which have yielded a consensus of agreement among the profession.

The Illinois Council on Mathematics Education, in sponsoring a special conference on designing a method's course for secondary school teachers, felt that it was particularly appropriate at this time to review different practices in methods courses and to look at further design possibilities. Although the conference was planned especially for teacher-training institutions in Illinois, it was open to the general mathematics education profession, and attended by representatives from surrounding states as well.

We believe that the problems addressed by this conference are of wide concern to mathematics teacher educators, and that the responses presented have national implications. The ERIC Center for Science, Mathematics, and Environmental Education is happy to make the proceedings of the conference available as a ~~Mathematics~~ Education Report.

Jon L. Higgins
Associate Director for
Mathematics Education

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Foreword

The Illinois Council on Mathematics Education was formed for the purpose of identifying inter-institutional problems affecting mathematics education in the State of Illinois, disseminating information about those problems, and recommending ways in which they may be solved. The Council concerns itself with issues relating to the training of teachers of mathematics; the development of new programs in mathematics education; the strengthening of existing programs for prospective teachers; and the establishment of effective avenues of communication among mathematics educators in the State, and between them and the general public and appropriate State officials.

Professor Clarence E. Hardgrove of Northern Illinois University, in introducing the opening session of the Conference, stated that the purpose of the Conference was not to debate whether or not there should be a methods course. It was assumed that there is a methods course and that it can be improved. It was also decided not to debate in the Conference whether or not such a course should be "performance based," although some of those present have strong convictions regarding this issue.

The editors wish to thank Professor Chuck Dietz of Concordia Teachers College, later of Northern Illinois University, for making available tapes of all the proceedings. We also wish to acknowledge the cooperation of Dr. Jon L. Higgins and the ERIC Center for Science, Mathematics, and Environmental Education.

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I. Realistic Goals for a Methods Course

Paper: Frank Allen

Reactors: Lewis H. Coon
J. Richard Dennis

REALISTIC GOALS FOR A METHODS COURSE
IN THE TEACHING OF SECONDARY SCHOOL MATHEMATICS

Frank B. Allen
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The goals that I propose for a methods course in the teaching of secondary school mathematics fall naturally into two categories. Those in category A pertain to the matter of providing prospective teachers with the necessary orientation. Those in category B pertain to the teaching process as it applies to mathematics. The two key words are therefore orientation and exposition.

Under A (orientation) I suggest that prospective teachers need orientation with respect to:

1. Current attitudes toward science and mathematics. (12) (13) (18)

2. Controversies about
   (a) The nature of mathematics. (7) (8) (9) (10) (13) (19) (50)
   (b) How mathematics should be taught. (4) (5) (25) (31) (32) (33) (37) (38) (46) (49)

3. The objective of mathematical instruction. (41)

4. Learning theory as it applies to mathematics. (6) (34) (35) (36) (37) (39) (40) (43)

5. The history of mathematics and its connotations for the classroom. (17) (30) (42)

6. Teaching conditions in the schools as they pertain to mathematics. (Direct observation)

7. Opportunities for professional growth for the career teacher that are provided by:
   a. continued study
   b. professional organizations.

Under B (the teaching process) I suggest that the prospective teacher should understand his role in helping pupils to learn mathematics:

1. As the pupils' counselor, friend and guide in carefully planned learning situations.

2. As one who motivates the pupil to study mathematics.

3. As a practitioner of pedagogical exposition whose purpose is to lead the pupil to an understanding of mathematics.

4. As an evaluator of the pupil's progress toward the attainment of stated objectives. (44) (45)

This two part list of goals will serve as a rough outline for my discussion. I shall spend most of my time on those points which seem to me to require explanation, amplification and, perhaps, justification. Others will be described briefly and for some, including some very important ones, I must be content with merely citing a few references. In accordance with this plan I will spend most of the time allotted to orientation on the first two points.

The assertion that the orientation of prospective teachers should begin with a study of current attitudes toward mathematics is based on the thesis that they must know something about where negative attitudes come from if they are to be successful in developing positive attitudes.

When we consider the attitudes toward mathematics which are manifested today by some young people, by elements of the academic community, by a large section of the public and even by Congress, we find ample cause for concern. Prospective mathematics teachers should know this and they should take some time to think about it.

Some current negative attitudes toward the mathematical sciences may indicate a wide-spread tendency to reject the rational process.

We recall from Philosophy 101 that there was once an Age of Reason dominated by such giants of rationality as Galileo, Bacon and Descartes. I am fearful that we may be entering an "Age of Unreason" -- an age where the rational process is consciously rejected. I am concerned about the fate of school mathematics in such an era -- in such a world.

How do we know that the rational process for making decisions is going to prevail in our world? Some very intelligent people have grave doubts. Recently addressing a group of undergraduates at Elmhurst College, Dr. Theodore Marchese of Barat College spoke as follows:

> "A second major development in Western society is far more elusive, recent and unpredictable in its effect. I refer to the clear challenge to reason as the prominent Western value. The groves of academe were planted in the French Enlightenment, with the great universities of Paris and Berlin, then the Universities of Virginia, Cornell, Hopkins, etc. The faith of the liberal man was in reason -- normative reason, logic applied to social reality -- logic applied to dominating natural reality -- were the keystones of the new university education. Mind, rationality, reason, dispassionate analysis, these would tame nature and lay smooth the way for man on earth. In the past handful of years, the rule of reason has plunged deep into crisis. As Faith made way for Reason, so Reason is now challenged by Feeling. To the young especially, the limitations of reason are manifest, its domination intolerable."

Some youth say in effect, "If the use of science and logic has gotten us into this mess then we will have nothing more to do with science and logic." This if-then statement is a mind-boggling example of using logic to reject logic.

"The clear challenge to reason," is this not a terrifying phrase? Yet there is mounting evidence that this challenge exists.

Daniel Yankelovich in describing "The New Naturalism" in the April 1, 1972 Saturday Review, speaks of "the disdain for rational procedure and disciplined thought" which characterizes this movement. (27)

He adds:

> "Perhaps no aspect of the students' cultural revolution is as poorly understood -- and as widely misinterpreted -- as student mistrust of rational, conceptual, calculative and abstract modes of thought. Faculty scholars are appalled at the seeming anti-intellectualism of the counter-culture with its stress in sense and experience and what often seems to be an ideological commitment to inarticulateness. --- The student movement reserves its most brutal shock, however, for those logical minded managers, technologists, engineers and professors -- for whom rational, orderly and logical methods are the royal road to truth."

What has all this to do with school mathematics? Well, some of us are disturbed by the fact that secondary school mathematics is the very essence of the "rational, conceptual, calculated and abstract modes of thought" which this philosophy rejects. We are discouraged by Dr. Lionel Trilling's observation that "the case against mind is being openly litigated in our culture." (28)

The effect of all this on the mathematical sciences is well established. In 1970 Alvin Weinberg (12) said in an article in Science magazine entitled "In Defense of Science":

> It is incredible, but true, that science and its technologies are today on the defensive. The attack, which is most noticeable in the United States, has been launched on four fronts. First, there are the scientific muckrakers, mostly journalists, who picture the scientific enterprise as being corrupted by political maneuvering among competing claimants for the scientific dollar. Second, there are thoughtful legislators and administrators who see a waning in the relevance of science to the public interest, especially as we address ourselves to grave social questions that are hardly illuminated by science. To deny connection between science and public affairs weakens one of the main arguments for public support of basic science: that out of basic science comes technology, which in turn improves our human condition. Third, there are the many

technological critics who urge a slowdown, or at any rate a redirection, of technology because of its detrimental side effects. And finally, there are the scientific abolitionists: the very noisy, usually young, critics who consider the whole scientific-technological, if not rationalistic mode of the past 100 years a catastrophe. To them technology is the opiate of the intellectuals; some of the more extreme would demolish human reason as the ultimate tool for achieving human well-being. The consequence, or perhaps, a further symptom, of all this harassment is a reduction in society's support for science.

This situation is brought home to the mathematical community by the recent report of the ad hoc Committee on New Priorities for Undergraduate Education in the Mathematical Sciences which was appointed in 1971 by the Board of Governors of the MAA. (18) The committee cites impressive evidence to indicate that "many students regard the physical science and mathematics to be either irrelevant or dangerous to society." The committee deplores the fact that the public is inclined to view the mathematician "as an inhabitant of the ivory tower." The Committee then proceeds to the expected recommendation that applications and modeling be given more emphasis in undergraduate mathematics. Finally this committee "joins the Committee in the Undergraduate Program in urging that teacher-training programs include significant applications of mathematics "at a level appropriate to what they will be teaching."

Thus we see that a study of current attitudes toward mathematics leads us almost immediately to the consideration of one of the major issues confronting mathematics teachers today, the importance of applications in motivating pupils to study mathematics.

We all know that there have been a variety of activities on the applications front during the last few years. (16) (26) Our students should know about these. They should know too that the tendency to prescribe more applications for whatever ails mathematics is not new.

Professor Keller (20) cites the following quotation: "---nearly all teachers of Mathematics try to find remedies for the present unsatisfactory conditions, and the cure recommended by most of them is the introduction and study of applications---" and notes that this was written by A. Schultz in 1912.

Some teachers are quite concerned about the resurgence of the idea that mathematics should be applications-oriented. They fear that other values of mathematical study will be neglected. Asked to specify which values, they cite the need of the college preparatory student for some understanding of proof, and they assert that mathematics is the place for him to obtain this. They get some tongue-in-cheek support for this position from P. J. Davis. In an article entitled "Fidelity in Mathematical Discourse: Is One and One Really Two?" in the March 1972 American Mathematical Monthly, (19) he says: "To prove is to establish beyond the question of doubt, and mathematics has been thought capable of just such a thing --Mathematics alone proves, and its proofs are held to be of universal and absolute validity, independent of position, temperature or pressure. You may be a Communist or a Whig or a lapsed Muggletonian, but if you are also a mathematician, you will recognize a correct proof when you see one." Alas, he states this thesis only for the purpose of blasting it in the sequel where he shows that, on the frontiers of mathematics, proof is not absolute but only probabilistic. He even destroys our faith in the accuracy of mathematical tables. After noting that even authoritative mathematical tables are loaded with errors he observes, "In the old days, when table making was a handicraft, some table makers felt that every entry in a table was a theorem (and so it is) and must be correct. Others took a relaxed quality control attitude. One famous table maker used to put in errors deliberately so that he would be able to spot his work when others reproduced it without permission." He also describes the sorry state of proof in college textbooks "Splicing two theorems is standard practice." In the course of a proof, one cites Euler's Theorem, say, by way of authority. The onus is now on the reader to supply the particular theorem of Euler that the author is talking about and to verify that all the conditions (in their most modern formulation) which are necessary for the applicability of the theorem are, in fact, present.

If splicing is common to lend authority, then skipping is even more common. By skipping, I mean the failure to supply an important argument. Skipping occurs because it is necessary to keep down the length of a proof, because of boredom (you cannot really expect me to go through every single step, can you?), superiority (the fellows in my club all can follow me) or out of inadvertence. Thus, far from being an exercise in reason, a convincing certification of truth, or a device for enhancing the understanding, a proof in a textbook on advanced topics is often a stylized minuet which the author dances with his readers to achieve certain social ends. What begins as reason soon becomes aesthetics and winds up as anaesthetics."

Those of us who believe in the value of proof in school mathematics can, of course, assure ourselves that the deficiencies cited by Davis need not apply to the arguments we (10) present. Nevertheless, it is discouraging to be told that proofs have little expository value at higher levels.

I have always taken pride in the idea that teaching mathematics gives me an opportunity to show young people that some results can be established by applying the rules of logic and without appeals to feeling, authority or faith. Many times over the years I have told my classes that, in mathematics, we do not prove theorems by appeals to authority--and I have urged my methods students to do the same.

Some recent developments have caused me to wonder about the validity of this position. Only recently a brilliant young mathematician told me that he and five other post-doctoral students had worked for a week in an effort to verify the proof of a certain theorem without success. They ended up accepting it on the authority of the writer who is an eminent mathematician. This enabled them to get on with the business of generating new results based on this theorem. Thus faith in authority was ultimately substituted for painstaking verification.

In a recent article entitled "Collaboration In The Mathematical Community," Bagnato (15) assures us that such reliance on faith is common practice among professional mathematicians and suggests that it has implications for the teaching of school mathematics. The story is also told of the holder of a Ph.D. in mathematics whose thesis proof was demolished by one of the first students to do doctoral work under him. For this the student received a Ph.D.!

I find all this rather scary. That men working on the frontiers of mathematics under the pressure of the "publish or perish" policy should find it necessary to rely on faith in authority instead of proof is perhaps understandable. But surely such appeals to authority have no place in the mathematics classrooms of our secondary schools where we are trying to explain why the statements we make are true. Yet I know that there are many well-informed people who feel that why was overemphasized by the "new math" programs along with structure and proof. They might be pleased to see more appeals made to intuition, authority and faith.

The thrust of my remarks is that we are here confronted with a fundamental question: What is the place of proof in the teaching of school mathematics?

Thus by considering the attitudes of various groups toward mathematics, we have already encountered three major controversies: 1) the role of applications; 2) the place of proof; and 3) the alleged deficiencies of the "new math." It is reasonable to inquire how our viewpoints on these issues are affected by our understanding of the nature of mathematics itself.

I suggest that people who are to interpret mathematics to young students should, at some time in their training period, give long and careful thought to the nature of their subject. They must acquire some degree of philosophical orientation toward mathematics. I believe that the methods course is the place for them to do this. This implies that the methods courses should be taught by mathematics professors who are, by reason of their training and interests, qualified to lead their students in an intensive study of the question "What is contemporary mathematics?"

This question seems to demand a definition--and we all know that widely acceptable definitions of mathematics are hard to come by. Sophisticated definitions such as "Mathematics is what mathematicians do" or Russell's "Mathematics is the subject where we do not know what we are talking about or whether what we are saying is true" are not much help to the beginner. To me, however, they suggest opposing viewpoints in a controversy about the nature of mathematics that has smoldered in the mathematical world for centuries. If we think of mathematicians as problem solvers who have, over the years, successfully applied mathematics to the solution of problems in the natural and social sciences, the first definition suggests that mathematics is deeply rooted in reality; that it grows and develops in response to the need for solving the problems which man encounters in the real world. Russell's definition, on the other hand, is as Henkin observes, really an assertion of the shocking thesis that all mathematics is nothing but logic (8). If we accept this along with the idea that logic is purely tautological (a companion thesis of Russell's) we might conclude that mathematics has nothing to do with reality!

Clearly we have here two conflicting viewpoints about the nature of mathematics.

The long-standing nature of this controversy was noted by Mina Rees in a memorable speech delivered at an NCTM annual meeting in 1962 (9). She said, "On one thing, however, mathematicians would probably agree: that there have been, at least since the time of Euclid, two antithetical forces at work in mathematics. These may be viewed in the great periods of mathematical development, one of them moving in the direction of constructive invention, of directing and motivating intuition, the other adhering to the ideal of precision and rigorous proof that made its appearance in Greek mathematics and has been extensively developed during the 19th and 20th centuries."

It seems reasonable to identify the first of these forces with applied or externally oriented mathematics and the second with pure or internally oriented mathematics.

I suggest that secondary school mathematics has been caught in the crossfire of this ancient controversy between the "pure" and the "applied."

I believe, for example, that the writers of the new mathematics programs of the early sixties incurred the wrath of the applied mathematicians quite unintentionally. In those days the second objective stated by the Commission on Mathematics of the College Entrance Examination Board which called for the development of "an understanding of the deductive method as a way of thinking, and a reasonable skill in applying this method to...all mathematical subjects" (14) gained wide acceptance. This was the new emphasis on <u>why</u> that was to improve school mathematics.

Now in order to stress <u>why</u>, we must give some attention to the reasoning process, and we must have some kind of structure to serve as a basis for this reasoning. In ninth-grade algebra, for example, the properties of an ordered field provided a basis for this structure. A few simple proofs were included in the SMSG ninth-grade algebra. These were well received by many teachers but they seemed pretty artificial to some applied mathematicians. Peter Lax of the Courant Institute, characterized them derisively as "logical gems" and criticized the text for lacking motivation. In his view, the way to obtain motivation "...is to present challenging problems and then show students how the theory developed helps to solve these problems" (29).

These views were also expressed by Kline who characterized these proofs as misguided efforts to achieve rigor (25) (32). In developing the theme that mathematics must be derived from and motivated by problems encountered in other fields, Kline finally thunders "To teach mathematics as a separate discipline is a perversion, a corruption and a distortion of knowledge" (25). In Kline's view, utility is the dominant value of mathematics. Yet Marshall Stone says "I hold that utility alone is not a proper measure of value, and would even go so far as to say that it is, when strictly and shortsightedly applied, a dangerously false measure of value" (7). He concurs with those who characterize our present controversy as one between liberal education and utilitarian education and adds "The opposition is between the point of view which regards as good whatever develops the intellectual and spiritual powers of the individual and the point of view that regards as good whatever works or accomplishes useful results."

I want my methods students to read the conflicting views of distinguished mathematicians. I want them to test the hypothesis that many of our current controversies about the teaching of mathematics have their roots in perennial controversies about the nature of mathematics itself.

This concludes my discussion of my first two points under orientation. In my viewpoint (3) objectives, (4) learning theory, and (5) history, are adequately covered in the references cited. Point (7) I think is self-explanatory. I conclude my discussion of orientation with some comments about teaching conditions in the schools (point 6).

In this area most of our information is derived from carefully planned visits to nearby secondary schools. We seek information about:

a. Materials of instruction including texts, syllabi, programmed materials and related items.

b. The use which the school makes of multi-sensory aids, library facilities, mathematics laboratories and resource centers.

c. The availability of computer facilities and the use made of hand-held calculators.

d. Provisions for individual differences with particular reference to individualized learning vs. homogeneous sectioning (4).

e. The departmental testing program.

Each student is expected to become thoroughly familiar with at least one text used in grades 7-9 and at least one used in grades 10-12. This involves the preparation of one unit in each chosen text complete with lesson plans, quizzes and a final examination with grading scale and key. It is here that we try to make effective application of what we have learned in sections 3, 4 and 5 (47) (48).

Now we come to a consideration of the teaching process as it pertains to mathematics. The teacher's role as a counselor, friend and guide and as a motivator of learning is, of course, tremendously important. The various competencies and personal qualities that a teacher must have in order to be successful in the classroom--any classroom--are considered at great length in the concomitant courses in Education and Psychology which our students are required to take. In this area it is our function to reinforce, in a mathematical setting, the very sound and useful ideas studied in those courses which, of course, apply to the teaching of all subjects.

However, when we come to the matter of developing effective expositions of mathematics we bear the principal responsibility. For this reason I will concentrate on the third point under section B: the teacher as a practitioner of pedagogical exposition whose purpose is to lead the pupil to an understanding of mathematics. Now clearly goal B-3 stated thus is too vague to be evaluated. What do we mean when we say that a pupil understands a mathematical concept? What do we mean by "pedagogical exposition?" Your evaluation of this goal will depend, in part, on whether or not I can answer these questions satisfactorily. Also in order to show that this goal is realistic I must try to show how it applies to the subject matter of secondary level mathematics in classroom situations.

For the purposes of this discussion we will say that a pupil understands a new idea when he perceives how it is related to knowledge he already possesses. Exposition is the process whereby this relationship is delineated. The teacher must, among other things, be an effective expositor.

In mathematics the teacher must, therefore, undertake the construction of a sequence of justifiable steps from the hypothesis of each new proposition to its conclusion, using only previously accepted propositions in the justification process.

This requires the teacher to:

1. Regard school mathematics as a hierarchy of propositions in which each proposition is implied by certain previously accepted propositions or assumptions.

2. Emphasize the structure of mathematics with due attention given to basic assumptions, definitions, undefined terms and the precise use of language, including set language.

3. Employ the rudiments of logic and set theory. (These are provided by many college texts including (21) and by some high school texts including (22) and (23).)

I will use classroom illustrations in an attempt to clarify these points and to show how they apply to pedagogical exposition.

The first three examples are designed to show what I mean by "a series of justifiable steps from the hypothesis of a proposition to its conclusion." In these I introduce the flow diagram format. This format, which has much in common with flow charts used in computer programming, is not essential to my presentation--just very convenient.

Example 1.

Solve for x: $\frac{3x + b}{2} = c$

Solution: $\underline{\frac{3x + b}{2} = c} \xrightarrow{(1)} 3x + b = 2c \xrightarrow{(2)} 3x = 2c - b \xrightarrow{(3)} x = \frac{2c - b}{3}$.

This series of three steps is called a deductive sequence. It is a proof of the implication $\frac{3x + b}{2} = c \rightarrow x = \frac{2c - b}{2}$. The pupil must supply the reasons for the numbered steps.

Example 2.

Solve for x: $5 + \sqrt{x + 7} = x$.

Here is part of the solution:

$\underline{5 + \sqrt{x + 7} = x} \xrightarrow{(1)} \sqrt{x + 7} = x - 5 \xrightarrow{(2)} x + 7 = x^2 - 10x + 25$

$\xrightarrow{(3)} x^2 - 11x + 18 = 0 \xrightarrow{(4)} (x - 9)(x - 2) = 0 \xrightarrow{(5)} (x = 9 \vee x = 2)$.

When numbered reasons are supplied we have a proof of $5 + \sqrt{x + 7} = x \xrightarrow{(6)} (x = 9 \vee x = 2)$ or equivalently that $\{x | 5 + \sqrt{x + 7} = x\} \subseteq \{9,2\}$. We omit the rest of the solution, observing that the converse of (6) turns out to be false (the solution set is $\{9\}$).

Example 3.

Solve the system: $\begin{cases} 3x + 7y = -26 \\ 4x - 3y = 27 \end{cases}$

Again we show only part of the solution:

$$\left.\begin{array}{l} \underline{3x + 7y = -26} \rightarrow 12x + 28y = -104 \\ \qquad\qquad (1) \\ \underline{4x - 3y = 27} \rightarrow 12x - 9y = 81 \end{array}\right\} \xrightarrow{(2)} 37y = -185 \xrightarrow{(3)} \left.\begin{array}{r} y = -5 \\ 3x + 7y = -26 \end{array}\right\} \xrightarrow{(4)} 3x - 35 = -26$$

$$\xrightarrow{(5)} 3x = 9 \xrightarrow{(6)} x = 3$$

So far we have proved that $\{(x,y) | 3x + 7y = -26 \wedge 4x - 3y = 27\} \subseteq \{(3, -5)\}$.

The deductive sequences in Examples 1 and 2 are linear in the sense that each implication has a simple statement (i.e., one devoid of "ands" or "ors") in its hypothesis. In example 3, I introduce the bracket form. Here we are to consider the conjunction of the bracketed statements as the hypothesis.

In examples 4 and 5, I consider the pedagogy of proof presentation. If our only interest is clarity of exposition, we need not be concerned about the method of presentation so long as the argument presented is logically correct. If we are interested in pedagogical exposition, logical correctness is, of course, required, but we are equally concerned about the method of presentation. The teacher must help pupils get the main ideas of the proof by focusing on its principal components in any order that seems appropriate. These are then arranged in the proper sequence before the details are supplied.

Example 4.

Let it be understood that we are talking about real numbers. We wish to prove the important theorem: If the product of two numbers is zero then at least one of them is zero. We must prove:

I $\quad ab = 0 \rightarrow (a = 0 \vee b = 0)$

This theorem has the form $x \rightarrow (y \vee z)$. It is important for us to know that:

II $\quad [x \rightarrow (y \vee z)] \leftrightarrow [(x \wedge \sim y) \rightarrow z]$. Using this proof pattern we see that we can establish I by proving:

III $\quad \left.\begin{matrix} ab = 0 \\ a \neq 0 \end{matrix}\right\} \rightarrow b = 0.$

Proof of III

$$\underline{a \neq 0} \xrightarrow{(1)} a \text{ has a multiplicative inverse } \frac{1}{a} \xrightarrow{(2)} \left.\begin{matrix} \frac{1}{a} \cdot 0 = 0 \\ \underline{ab = 0} \end{matrix}\right\} \xrightarrow{(3)} \left(\frac{1}{a}\right)(ab) = 0$$

$$\xrightarrow{(4)} \left(\frac{1}{a} \cdot a\right) b = 0 \xrightarrow{(5)} 1 \cdot b = 0 \xrightarrow{(6)} b = 0.$$

Now it would be a pedagogical mistake for the teacher to suddenly confront a class with the "flow diagram" proof shown in Example 4.

There are three reasons for this:

1. Details are given too much prominence and therefore the proof seems to reflect a distressing preoccupation with trivia.

2. It creates the false impression that the teacher is some kind of a super being in whose mind such proofs occur in a flash of inspiration. (This would not help us in our effort to humanize the teaching of mathematics.)

3. Pupils tend to react negatively to a formal proof in whose construction they had no part.

The construction of such a proof is an indispensable part of the teacher's preparation. By constructing a series of justifiable steps from the hypothesis of the proposition to its conclusion, he has reminded himself of the pitfalls and difficulties that beset those who are passing this way for the first time. He is aware, for example, that proof pattern II, or some comparable tautology, lurks in this argument. (The use to be made of this fact will be determined by the background of the class.) Thus prepared, the teacher can elicit the main idea of the proof by a Socratic approach involving questions such as "What if we assume that one of these, say a, is not zero. That would have to be true about b?" After some discussion, the main idea, namely $\frac{1}{a}(ab) = 0$ if $ab = 0$ will emerge and the last four steps of the proof can be quickly written down in either ledger or flow-diagram form. The argument is completed by noting that $\frac{1}{a}$ exists and that multiplication by zero is involved. When the class has justified these steps by supplying the numbered reasons, it should be clear that I is not just a new rule to be memorized but a principle that follows logically from facts already known. Moreover, the completed proof exposes some of the trivial steps where misunderstandings often lurk.

If the teacher deems it appropriate to focus attention on II this can be done by noting that we have really proved III and asking if this establishes I. Thus II will emerge as a proof pattern which will be found to be widely applicable. In fact II is involved in any situation in which we are asked to prove a disjunctive statement. (The current practice in textbooks of ignoring proof patterns of this kind <u>does not</u> make the arguments easier to follow.)

Consider the following proposition from tenth grade geometry. Here again the proof should not be put on the board in the order indicated by the numbered steps. That is the logical order not the pedagogical order. The teacher who accepts our definition of exposition is more concerned with the <u>process</u> of proof than with the facts stated in the theorem. Having stated the theorem in the bracket form (ii) he asks how can we construct a series of justifiable steps from our hypothesis to our conclusion?

Example 5.

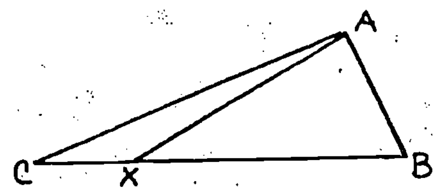
Prove: The length of a line segment joining a point in the interior of one side of a triangle to the opposite vertex is less than the length of at least one of the other two sides.

We must prove:

(i) $\left.\begin{array}{r} ABC \text{ is a triangle} \\ X \in \overset{\circ\!-\!\circ}{BC} \end{array}\right\} \rightarrow (|AX| < |AC| \vee |AX| < |AB|)$

or equivalently,

(ii) $\left.\begin{array}{r} ABC \text{ is a triangle} \\ X \in \overset{\circ\!-\!\circ}{BC} \\ |AX| \geq |AB| \end{array}\right\} \rightarrow |AX| < |AC|.$



Proof of (ii)

$\left.\begin{array}{l} \underline{ABC \text{ is a triangle}} \xrightarrow{(1)} A \notin \overleftrightarrow{BC} \\ \underline{X \in \overset{\circ\!-\!\circ}{BC}} \xrightarrow{(2)} B\text{-}X\text{-}C \xrightarrow{(3)} \overleftrightarrow{BC} = \overleftrightarrow{BX} = \overleftrightarrow{XC} \\ \qquad\quad \text{(collinear)} \qquad\quad \text{(rays)} \end{array}\right\} \xrightarrow{(4)} \left\{\begin{array}{l} \left.\begin{array}{l} \underline{B\text{-}X\text{-}C} \\ A \notin \overleftrightarrow{BX} \end{array}\right\} \\ \left.\begin{array}{l} A \notin \overleftrightarrow{XC} \\ \underline{B\text{-}X\text{-}C} \end{array}\right\} \end{array}\right.$

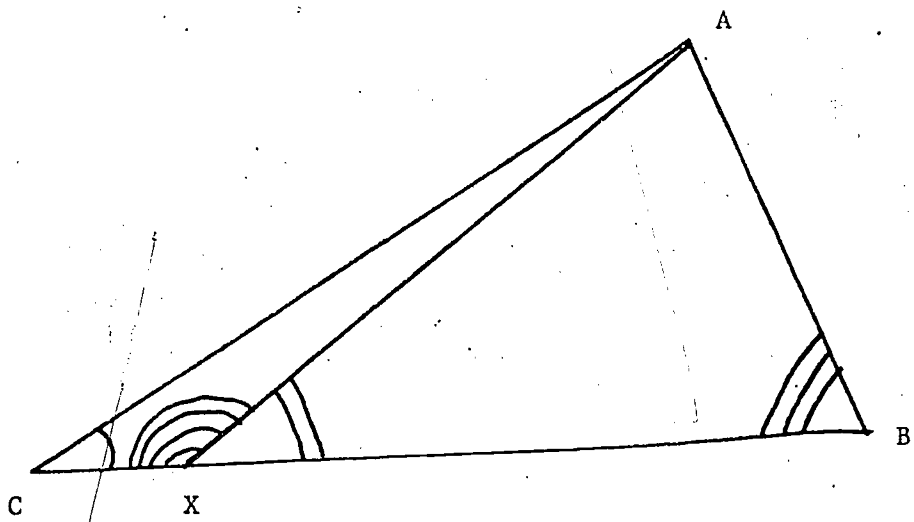
$\xrightarrow{(5)} \angle \underline{AXC}$ is an exterior angle for triangle ABX $\xrightarrow{(6)}$ $m(\angle AXC) > m(\angle ABX)$

$\xrightarrow{(5)}$ $\underline{AXB}$ is an exterior angle for triangle AXC $\xrightarrow{(6)}$ $m(\angle AXB) > m(\angle ACX)$

$\left.\begin{array}{r} \underline{ABX \text{ is a triangle}} \\ \underline{|AX| > |AB|} \end{array}\right\} \xrightarrow{(7)} m(\angle ABX) \geq m(\angle AXB)$

$\left.\begin{array}{l} \xrightarrow{(8)} m(\angle ABX) > m(\angle ACX) \\ \underline{m(\angle AXC) > m(\angle ABX)} \end{array}\right\} \xrightarrow{(9)} \left.\begin{array}{l} m(\angle AXC) > m(\angle ACX) \\ \underline{AXC \text{ is a triangle}} \end{array}\right\} \xrightarrow{(10)} |AC| > |AX|.$

In dealing with the question it is often helpful to try to "back up" from the conclusion. So by questioning the class, the teacher develops the idea that the conclusion would follow if we knew that $m(\angle AXC) > m(\angle ACX)$. Thus we have started a sequence of steps extending back from the conclusion. We can also start a sequence forward from the hypothesis by listing relations between angles that are implied by our premises. It is our hope that these two sequences can be linked together to form a series of justifiable steps from our hypothesis to our conclusion. The information about the relative size of angle measures developed by class discussion can be roughly summarized by placing arcs in angles in the manner shown in the figure below.



$m(<AXC) > m(<ACX)$

Now that the main ideas supporting our conclusion are reasonably clear, the proof can be written down in either ledger or flow-diagram form and the necessary reasons can be supplied.

Since this proof, like all proofs in geometry, involves incidence relations it serves to focus attention on an important pedagogical question: How much is the pupil allowed to infer from the drawing? This proof is taken from a college text (21) which considers plane geometry from an advanced standpoint in such a way that the drawing can be omitted. While the construction of a drawing free proof provides a worthwhile exercise for prospective teachers, it is neither reasonable or desirable at the secondary level. In a high school class, where pupils are expected to read certain incidence relations from the drawing, the proof would probably begin with the two statements following (5). Nevertheless, we can get into trouble if we allow pupils to infer too many facts from the drawing. Therefore, there is need for some ground rules about which incidence relations may be inferred from a drawing and which may not. These are supplied in some of the better high school texts such as (24).

The assertion that arguments such as these (Examples 1-5) have pedagogical value will, I believe, find some support in learning theory. Wittrock (6) says "Succinctly but abstractly stated, my hypothesis is that human learning with understanding is a generative process involving the construction of (a) organizational structures for storing and retrieving information, and (b) processes for relating new information to the stored information." For us the "stored information" consists of the postulates, definitions and theorems we have already studied and the "process for relating new information to the stored information" consists of the series of justifiable steps that constitute our deductive sequence.

There are, of course, learning theorists who disagree with Wittrock. Also, there are many well-informed mathematics educators who sincerely object to such proofs as being not only artificially rigorous but also internally oriented to the extent that they are neither practical or motivational. To this the proponents of structure would reply that they know of no other way to impart understanding, that they are concerned with clarity of exposition not with rigor, that the construction of proofs such as these can be a lot of fun and finally that this experience with proof will pay off in ways that are unforeseen and unforeseeable. There is also plenty of support for this view to be found in the literature. And so the controversy continues unabated and unresolved. The degree of acceptance that you accord to goal B-3, in the rather specific form in which I have interpreted it, depends very largely, I suspect, on where you stand in regard to this controversy.

I finish my presentation with a few remarks about evaluation, followed by a brief summary.

To gain an understanding of his role as an evaluator, the prospective teacher must, first of all, realize that evaluation of the pupil's progress toward stated objectives is an integral part of the teaching process. Therefore, he must learn the techniques and hazards of test construction and grading as they apply to mathematics (11). He must seek to develop testing procedures which are:

--particularly appropriate for school mathematics;
--effective in reducing tension while increasing motivation;
--well adjusted to the individual pupil; and
--conducive to self-evaluation.

One such procedure might be the use of open-book tests. There are situations where such tests are held to be appropriate on the grounds that, in mathematics, we are concerned with how much the pupil understands, not how much he remembers (45). Other effective procedures can be developed by the class.

Summary: The well-oriented teacher has a basis for developing a philosophy of his own which can serve to sustain his sense of mission in the years ahead. The knowledge he gains in seeking orientation can give depth and perspective to his classroom presentations. From his study of pedagogical exposition we hope that he will acquire the ability to lead his students in one of life's most satisfying experiences--understanding for oneself <u>why</u> something is true without having to accept it on faith or on the authority of "recognized experts." From his study of conflicting views about mathematics and the teaching of mathematics we can hope for the emergence of a balanced view which will prompt him to motivate some pupils by building mathematical models of the real world and motivate others by showing them that mathematics itself is a model of the rational processes to which, I hope, my opening remarks notwithstanding, our society is still committed.

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A Reaction to:
REALISTIC GOALS FOR A METHODS COURSE IN THE
TEACHING OF SECONDARY SCHOOL MATHEMATICS by Frank B. Allen

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Non-verbal Introduction

As biased as I am by more than two months of reading and rereading the responses to a statewide survey on the Preparation of Secondary Teachers of Mathematics conducted by the Illinois Council on Mathematics Education, I am going to try to avoid any reference whatever to things I have learned from that survey. I am also biased by an awareness of the great diversity of how a methods course must be taught at each unique institution in Illinois.

I will avoid any direct reference to eight basic assumptions which will be all too prevalent here today.

1. A mathematics methods course will be required and taught.

2. This is the only mathematics course in which history, proof, logic, and any exposition of teaching will be evident.

3. This is the first introduction of a methods student to what really goes on in a mathematics class other than rigor and force feeding.

4. Every student has $x + 2x + \infty$ as the amount of time and effort he can devote to a 2 (or is it almost 5) semester hour course.

5. No part of teacher preparation can wait for graduate course work although everything that is taught now will be of greater benefit after a year or two of experience.

6. There are enough students every term to offer a worthwhile methods course.

7. It is impractical to attempt or to design a competency based methods program.

8. Everyone in this audience agrees on what mathematics education is and ought to be.

My major qualifications for speaking as an expert consist of actively teaching several methods courses, attending the post NCTM meeting on Guidelines held in Chicago in April 1972, attending the reporting session of the Commission at Houston, and in attending the AOTE (Associated Organizations of Teacher Education) meeting held in St. Louis in May 1973. I'm also extremely reactionary.

According to my copy of the Random House Dictionary of the English Language, a reactor may appear as - an action in response to an effect, a response in an opposite or negative direction, a container ... for imparting heat ...., a reaction which imparts action not under control of the original effect.

Dr. Allen has been kind enough to leave a wide latitude among the subjects and goals he stated but did not cover, thus providing an opportunity to add to his presentation as this conference progresses. (The coming article, in preparation, on the results of the ICME survey of methods course practices in Illinois will expand on several of these themes.) In order not to detract from the second reactor or the next speaker, and restricted by time allotted me, I am going to restrict my remarks. This will give those following us an opportunity to fill in the gaps we leave. But I will attempt to pad and re-direct a few of Dr. Allen's comments.

I find that the use of orientation to teaching and a study of exposition afford a useful dichotomy for segmenting a methods course. However, my experience over the years has changed my outlook as I became aware that my students today do not know what the old or traditional mathematics was or is. They are products of the new math era. Thus the two or three days I can afford to give to Allen's major thesis of orientation, attitude toward mathematics, etc., must be done in a manner that allows students to become acquainted with methods and the mathematics of old so that they can compare it with what should be going on today. I find that this area makes an excellent vehicle for my students to give their first oral report on. This is an ungraded oral report in which the major objective is to assure the instructor that the student can get on his feet, speak, plan a presentation, and come across as a good communicator. The major part of my emphasis to this area must be integrated throughout the entire course due to the more important goals that must be completed in order for a methods student to become proficient enough during the methods course to display the competencies needed to survive as a partially supervised student teacher in a real classroom.

The lack of properly qualified mathematics students has been disappearing over the years, but so has their preparation in engineering and the physical sciences and their motivation to study applications only in these narrow areas. Kline and others still refuse to acknowledge that more and more of our students are not physical science oriented. The recent political appointees on the national scene in HUD, and on the state scene to the ecological groups indicate that administrative experience and possible political pull are much more important than scientific qualifications or practical experience. As in our calculus courses, this is giving us a group of students who want to study and work in areas where the professor has no expertise. Most of our calculus students today have no feeling for, nor inclination to study acceleration, surveying, vectors, or physical science problems. Their orientation is toward the business, economic, life, or social science field of applications.

Dr. Max Bell gave a presentation at the January, 1975 MAA meeting on applications at the elementary and secondary level. I hope that he and others will publish some worthwhile applications at this level. Thrall (1) presented over 200 applications of mathematics in the biological sciences. But all of these were at the post-differential equations level and utterly useless for students majoring in the life science fields at most institutions. Similar results regarding applications in mathematics have come from the efforts of others. People who know the applications either do not know what level we need the applications for or are reluctant to share them because they are at a low level.

Another of Dr. Allen's goals, the exposition of proof--by the methods course instructor--leads me to a full acknowledgment that this isn't the only course in which the methods student views proof. Unfortunately, it may be the only one in which it's done pedagogically or correctly and <u>then</u> rearranged logically. It takes a lot of time to do that. I would like to offer two other, possibly better, opportunities for a student to see proof and to view the teaching strategies in use today. At Principia and at Eastern Illinois University, methods course students visit and observe pre-calculus level courses in which they are already masters of the subject matter. Thus, for the first time they can sit in a classroom, not worry about getting their homework done, following a rigorous presentation, etc. They can concentrate on the teaching strategies in use, on the ventures, and on the chain of reasoning being presented. They can anticipate how they could present a similar lesson. Here they have an excellent chance to observe expository teaching for the first time in a real life (although non-secondary) situation. If they discuss their visits and observations with their methods course teacher and/or fellow students they will benefit greatly. A second and possibly a better approach to exposition, and to the study of how the content of advanced undergraduate courses support secondary teaching, is by the use of "shadow" courses for college geometry, for modern algebra, and analysis. Such courses were advocated by Peter Braunfeld at a recent ICME meeting and are already in use at Southern Illinois University in Carbondale. A qualified mathematics education professor meets with and teaches a separate course which meets concurrently with the base course. In the "shadow" course they study the strategies, content, and interconnection of problems and concepts presented in the base course and how they affect high school teaching and course content. For those of us without a secondary school next door, these present good alternatives to a real secondary classroom.

I find today that the students coming into our methods courses are much better prepared than they were five or ten years ago. Maybe it is the mathematics we now require them to complete before methods. Maybe it is an improved performance by those in the College of Education who really control the university curriculum for teacher preparation.

I still find that many of our students are poorly prepared in logic while others, who elected a separate logic course, are well prepared. We use the area of logic as a peer teaching vehicle in our methods class. It is a good one. We also use low level proofs and structure such as that advocated by Brother James F. Gray, and by Tomber in his Algebra for Elementary Teachers. The examples shown by Dr. Allen are both in those listings.

Among our students we find it rare that a student will have seen an exposition or even an example of discovery type learning. It just isn't done by university professors often enough to be of use for exposition; and in most university courses students are too strapped with keeping up to be aware of an exposition. We find that our methods students must visit an elementary statistics course or a course in elementary geometry or algebra to witness discovery or laboratory teaching in action.

I was more than happy to hear Dr. Allen advocate that students should work with each of grades 7 through 12. Since one-half of the jobs are at the junior high school level, we must prepare all methods students for them as well as for the upper secondary level. For that is where the jobs are.

Dr. Allen advocated that methods students should prepare unit plans for grades 7-12 and include tests in each one. I wonder who taught their professors (including myself) to properly prepare tests, analyze them, etc. I find that tests used on the college or university level are horrible examples of how test construction should not be done or taught. Where do we find instructional materials for test construction? How do we re-train university teachers to prepare and to use well-constructed tests and evaluation instruments? And don't tell me NCTM's 26th yearbook--it is another horribly outdated example.

Maybe the old triple T projects for the preparation of Teachers of Teacher Trainers missed their student body. Instead of the audience they attracted, we should have assigned post-doctoral instructors of methods courses, such as many of us in this audience, to those programs.

More important to me, and of great importance to my students, and not in any methods textbook I have discovered is: How does an observer or evaluator assess or "evaluate" the teaching of a classroom teacher or of a peer-teaching methods student?

In closing, as I reflect on Dr. Allen's presentation, I believe we have established a beginning for today and we have left plenty of untrammelled room for others to present and discuss goals yet unmentioned.

The methods course is the "nuts and bolts course" of teacher preparation. It is where we must prepare a student teacher not only to survive in the classroom, but to be a better teacher than we will ever be.

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RESPONSE TO PAPER by Frank B. Allen

J. Richard Dennis
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Dr. Allen has presented a very thoughtful and articulate analysis and discussion of goals for methods courses in the teaching of secondary mathematics. It is difficult to take issue with such careful analysis, particularly in a field like mathematics teaching in which, unlike medicine for example, there is nc commonly agreed upon theoretical basis (like anatomy, physiology, etc.) on which to structure action. I would suggest here, however, that there are other ways to look at some of the issues Dr. Allen has raised. I caution you that the ways of viewing I shall present have no more theoretical basis than Dr. Allen's.

The argument is presented that studying current attitudes toward mathematics leads us directly to the importance of applications in motivating pupils to study mathematics. I would first ask from what sources the prevailing attitudes toward mathematics are derived. I don't know for sure, but I strongly suspect that a major part of the man on the street's attitude about mathematics and mathematicians comes from observing the behavior of those who work in situations labeled as mathematical. This experience starts very early in schooling. As a citizen, I observe the behavior and attitudes of mathematics teachers in elementary and secondary schools, of mathematics professors in colleges and graduate schools, etc. At the same time that we lament about a poor image for the mathematical community, I must seriously ask the question, "Who is it that decides which behaviors are to be displayed, and the method of display?"

What I am leading to is that rather than deferring to applications as a solution to an image problem, we should take a more direct approach. What behavior is characteristically mathematical? You might be tempted to answer that deductive, logical behavior is characteristically mathematical. I wish to challenge this response. To do so, and to lend more credibility to my argument, I wish to bring before you some thoughts presented by Dr. Paul Halmos, a noted mathematician, in his 1967 centennial address at the University of Illinois. (Credibility, in this sense, is that quality derived from citing an individual listed in the <u>Who's Who</u> of the field, rather than the much larger, more often quoted, volume called <u>Who's He</u>.)

Dr. Halmos offers the following:

> Mathematics ... is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. The conviction is not likely to come early--it usually comes after many attempts, many failures, many discouragements, many false starts. It often happens that months of work result in

> the proof that the method of attack they were based on cannot possibly work, and the process of guessing, visualizing, and conclusion-jumping begins again... . The deductive stage, writing the result down, and writing down its rigorous proof are relatively trivial once the real insight arrives; it is more like the draftsman's work, not the architect's.

Dr. Halmos has described to you what he sees as mathematical behavior. By the way, the title of the talk was "Mathematics as a Creative Art." I wish to suggest that this kind of behavior is seldom publically exhibited to those who are students of mathematics or by those who are professionals of mathematics. Is it any wonder that mathematics suffers from an unfortunate public image? For the most part, the public sees the trivial part of the process.

The solutions to this, it seems to me, lie in the direction of finding ways to cause, or to assist, teachers at all levels to behave more in the creative arts mode as Dr. Halmos has described it. This would mean that teachers and students alike somehow become involved in the creation of mathematics. Let me caution those of you who jump to conclusions (a typically mathematical behavior, according to Halmos) that I do not advocate frontier mathematics creation for school classrooms. To me, the key is that the mathematics created is new to the people involved. Yes, I mean that some wheels will be reinvented. I suggest that experience with the act of invention, of creation, may account for the changes in level of personal commitment, and in the intrinsic interest and excitement of the subject, to cause more positive attitudes to take shape. One must realize, however, that attitudes are formed over a long time scale; it is reasonable to expect change in attitudes to require similar time magnitudes.

Both Dr. Allen and Dr. Halmos, whom I quoted, make references to truth. Dr. Allen talks about it when he suggests that appeals to authority have no place in school mathematics "where we are trying to explain why the statements we make are true." Let us ask exactly why are they true? And let us further ask why the postulates chosen are true, and if you don't like "truth" for this context, then ask why one should accept their authority. Or better still, why should I, John Q. Student, accept the authority of your logic? After all, have you not just charged me to not appeal to authority?

The suggestions seemed to be that deductive arguments somehow create truth and that appeal to authority in some way destroys it. Both of these seem terribly over-generalized. There are two questions involved:

1. At what point in time does one come to accept that a proposition agrees with reality?

2. At what point in time does one come to accept that a proposition could have been predicted from other propositions?

I would submit, as Halmos has, that the first of these is the advent of truth, and that it occurs in the creative stage of mathematics. I would also submit that viewing the deductive stage more as answering the question "How I could have predicted," rather than as an exercise in establishing truth, brings a perspective much more characteristic of true mathematical behavior.

In summary, then, I am making a strong appeal for treating mathematics more as a creative art, and all that this implies. There <u>is</u> reason in creative activity, but there also is emotional involvement. As Halmos concluded, mathematics is a creative art because:

(1) those working in mathematics create beautiful new concepts;
(2) those working in mathematics live, act, and think like artists; and
(3) those working in mathematics regard it so.

We must reveal more of this view of mathematical behavior at all levels. Thank you for this opportunity to present my views.

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The following were among the audience comments that came after the presentations by Professor Allen and the reactors to his talk.

Professor Kenneth Retzer of Illinois State University emphasized that he sees a natural dichotomy between conveying about mathematics (metamathematics) and the development of teaching skills.

Professor Kenneth Cummins of Kent State University spoke as follows. "In our teaching we should emphasize the formulation of conjectures as much as possible. The student should be creating mathematics as much as he can that is new <u>to him</u>. Then these conjectures are tested in the crucibles of deduction; and then comes the time for proof. There are many suggestions on proof and then we try to order them to formulate a deductive proof. The proof is really the final stage that communicates what we have done. The books are written backwards. We don't see what people went through to get the proof. The proof is the final stage. We should do a lot of this in methods."

II. Minimal Content for a Methods Course

Paper: Gerald R. Rising

Reactors: Jack E. Forbes
Katherine Pedersen

MINIMUM CONTENT FOR A ONE SEMESTER MATHEMATICS METHODS COURSE

Gerald R. Rising
State University of New York at Buffalo

1. Comment

Before I respond to the assigned topic I must devote a considerable space to introductory remarks. You have my promise that these remarks are not merely a diversion but are in fact important to my theses.

2. Credentials

First I will give my reactors something to chew on: my competence. I believe that I was chosen to address this topic because I am the junior author of a methods textbook. But while the text may be very good, I am very junior. I never took a methods course in mathematics. In fact, I was one of those liberal arts students who started: (1) as an English teacher, and (2) having taken no education courses whatsoever. My "methods" course was entirely introspective: What did my own school and college teachers do that was right? And which things that I tried in the classroom worked or failed? I was on my own in another way. With only one exception, I taught for principals whose evaluations were based not on classroom observations but on the dual records of complaints and number of students sent to the office. My numbers were low: during ten of my first twelve years I was never once observed by an administrator or supervisor.

In this regard I am reminded of (I think it was) Mark Twain who said he was amazed at how much his father learned between the time Mark was twenty and thirty. In just this way I learned slowly over the years that despite some favorable student response to my teaching, I could have used a great deal of help before and during my teaching years. Today as well as yesterday.

I am also a latecomer to methods teaching. I have taught a half dozen or so methods classes -- never to my satisfaction. Despite my earlier disclaimers, I believe that I can hold my own among mathematics teachers; I have no such confidence about my methods teaching.

But I do bring to bear prior experience as a poseur. This is my third or fourth paper on teaching methods courses. In one I even went so far as to tell science teachers how to address this problem.

At any rate, my serious deficiencies have led me to draw very heavily on my colleagues at the State University of New York at Buffalo: especially Marion Walter and Stephen Brown. To these outstanding mathematics educators should go much credit for this paper. At the same time, I retain the overall responsibility for these contents.

Next I comment briefly on the ground rules under which I am operating: I have been directed not to concern myself with the issue of competency based programs and not to attempt to justify the methods course. In my mind this course needs no justification so I am comfortable with that restriction. The other poses no problem either despite the assigned title that certainly suggests a list. But again, in fairness to any of you who do not know me, I announce that I am in direct opposition to the competency movement and related aspects of behaviorism. Now you know. That statement relates to the biases you will note in the corpus of my remarks.

And now let us turn to the matter at hand.

3. <u>The Subject and the Subjects</u>

Mr. Allen has already addressed the matter of methods course goals and we must turn our attention to content. "Minimum content for a one semester mathematics methods course." That certainly sounds well defined. But any reader who has taught a methods course knows that that is not enough. Well, secondary then, say grades seven through twelve. That too falls short. What we need most is a description of the audience, the students to be taught.

Here I choose ours at Buffalo: two dozen seniors each semester, nearly all with a mathematics major already completed. Most will have had a single course, sociology of education, during their junior year; the rest will have had no prior courses in this field. Their mathematics methods course is part of their "education semester," usually their last in the university. For the first four weeks of this semester the students study in other classes psychology of education and school organization including such things as how to operate audio-visual equipment. They then student teach full time for nine weeks. During the last weeks of the semester they return to campus to participate in electives like team teaching and counseling. Our methods course, formerly carrying two semester hours credit but now four, meets once weekly for between three and four hours through the semester. Thus we meet students before, during and after their student teaching. Some modification of the remarks in this paper will be necessary to address other circumstances.

The most important characteristic of these students is their background in mathematics. Our mathematics department is one of those usually described by mathematicians as good -- translated to mean formal, modern, abstract and most of all tough. These youngsters have studied over forty hours of analysis, algebra and topology. The only way the program of the student preparing to teach varies from his fellows is in the substitution of geometry for another advanced course. Some, but not many, students will also have studied numerical methods or computer science and a few will have elected courses in mathematical statistics.

So they come to this single methods course - crippled. Even the best students among them -- and our students range from best to worst in the mathematics department -- have "learned" mathematics the mathematics department way. They have raced through courses barely skimming the surface. Think of all those bad learning processes of weak secondary school students and you will find them mirrored here: memorization with no understanding, no overview, all creativity suppressed by the narrow focus of the exams on the required material, passive. But in one terrible way they differ from those high school students: these are brainwashed. They think that they are mathematicians and that the content of school mathematics is trivial.

Now lest you see me as merely taking a backhand swipe at my colleagues on mathematics faculties, I submit the following: (1) I do not join with those who suggest that school teachers don't need all that high-falutin' stuff. In this regard I agree completely with Arnold Ross who has said that the beginning teacher teaches all he knows and more, the experienced teacher limits his teaching to what he knows, but the master teacher selects from what he knows what is appropriate for his students. Mathematics teachers today need a solid subject matter base. (2) I do have criticisms to direct at some college mathematics teachers and some suggestions for improvements but those are not to the point here. What does apply is the fact that these self-confident students have a weak and distorted view of both mathematics and mathematics teaching, a view that will be communicated to their students if nothing is done about it.

I realize that this overlap into the affective domain takes us close to the area of goals as well. But I feel that establishing what are the problems to be attacked is the only way to justify the contents I will -- sooner or later -- outline.

Let us approach this from another direction. What are the expectations for this course that these students bring to it? Our experience leads us to expect them to expect (expect squared) the following:

1. Students expect to be taught how to teach topic X, where the domain of X is the content of school mathematics.

2. Students want, hope for, and to some extent expect specific responses to classroom events: What do you do when a student refuses to do his homework? etc.

3. A few of the more "sophisticated" students expect to have panaceas that they have met in the popular press or in educational psychology courses explicated in greater detail. These solve-alls include mastery learning, the open classroom, behavior modification, programmed instruction or CAI, even Summerhill type deschooling. Some of these students are so tied to their solutions that they will fight every alternative.

4. Many students expect to be taught how to perform. They view teaching like acting: They want a critical run-through of their solos.

5. Many expect to waste their time. Their view of the field of education and education teachers is negative. Their attitude is that they'll learn little here.

4. A Table of Contents

In response to this, here are some methods course contents: (Finally!)

1. An overview of mathematics in the curriculum and society. Included here would be some indication of the immediate past history (new math) and contemporary directions of mathematics pedagogy.

2. Psychological theories as they relate to teaching and learning mathematics. These provide a framework in which to imbed their thinking about instructional problems.

3. An analysis of goals and objectives. By this I mean development of the hierarchy of objectives and objective construction, the idea of performance based objectives, that is translating what you want students to be like into what you want them to do, and the relation between goals and plans.

4. Planning classroom; planning for a lesson, unit and course. I only artificially separate this from ...

5. Developing different types of lessons: lessons aimed at concepts, structure and logic, lessons focused on skills, laboratory lessons, lessons devoted to problem solving, enrichment lessons with affective concerns, discovery lessons, application lessons, lessons directed at learning how to learn.

6. Classroom management. Relating these ideas to the real world of the school.

7. Gathering and using instructional techniques. This takes students to the literature of mathematics education and can lead to considering professional participation.

8. Evaluation, thus closing the circle of objectives-instruction-evaluation.

9. Activities for unsuccessful students. This would include both revising curriculum and instruction for weaker classes and providing remedial assistance.

10. Programs for the talented; a consideration of aspects of both acceleration and enrichment.

11. Utilizing facilities. This would range from textbooks to computers and would include the whole range of audio-visual aids and models.

That is, as those of you know who are familiar with the book, a thumb nail sketch of the contents of Johnson and Rising, Guidelines for Teaching Mathematics. It is rather a good book and I recommend it to all teachers.

5. A Reexamination

Now I invite you to discard, erase, trash or otherwise remove section four from your notes. That is not what I recommend for a methods course. What I described is what is usually done, what in fact I myself have done in such courses. I suggest that it is wrong, that it is in fact contributing to the continuing downhill slide of classroom teaching of mathematics. It does respond reasonably well to the student expectations listed in section three, but it makes no real inroads on their behavior. This way we merely turn out more of the same.

This is true in several ways: First a course like the one described merely organizes the student. It helps him to clean up his desk. Nothing about that will lead him to teach other than at best the way he was taught. Consider in this regard what many of you might consider the best of my section four list: discovery lessons. My experience has been that organized as in that section, the methods course trivializes discovery. Discovery turns out to be putting words into students' mouths. It is trumped up directed learning, no more creative than extending the sequence one, two, three, four, five. Now I know that I may be stepping on some toes here. Some of you may feel that you are discoverers and that you can teach discovery. I expect that perhaps one of every five of you who thinks he can, can indeed do so. But it is my claim that even that fifth teacher cannot do so in the framework I outlined.

Second, we face a much more serious problem than even I have described. I talked about student entrance problems. There are equal or greater exit problems. Much has been said in criticism of education and professional educators. You all know the chant: If you can't do, teach; and if you can't teach, teach teachers. There is much truth in what the critics have to say, but there is also a lack of understanding. I have already suggested that teaching education is much tougher than teaching mathematics. (In reconfirmation I offer my own recent delightful semester in England teaching math.) Still professional education leaves much to be desired. As we have failed, the in-school training has taken on greater importance.

Ask any beginning teacher about the best aspect of his preparation. You know the answer: student teaching. Not education courses and not content courses. Why is this? Simple. Student teaching is where the student not only meets the real world but also invests himself in that world. If we could ever get 50% of that investment in other programs we could accomplish ten times as much.

And what happens during that student teaching? In almost every case the student is stirred into the same gruel. He is further shaped (in the case of the methods course I earlier described) or substantially reshaped (even if you are able to change his behavior) into the standard mold. He becomes one more pedestrian teacher. Any of you who have taught methods classes that overlapped student teaching have witnessed this effect. Overnight students change into teachers, their ideals smashed on the reality of their first classroom discipline problem.

Granted that there are true master teachers who create in their own image (in ways that I believe mirror the sequel), there are not enough to go around and the general result is, I suggest, substantially negative. So -- returning to my point -- the methods course must not only respond to serious student problems, it must also aim at changes that will not be flattened out in either student or probationary teaching.

Third, and even more important, the kind of course that I have described in section four responds to the problem at the level of the problem rather than the solution. We cannot expect these students to develop as master teachers when our own teaching is organized at the pedestrian level.

6. Two levels of teaching

Here I present a complete oversimplification of a complex subject, for which act I apologize. It serves my purposes here and I leave it to others to consider further refinements.*

At the first level of teaching we deliver a product. Here is math. I include here much more than definitions, algorithms and skills -- or perhaps it would be more accurate to say that I extend those categories. For example, you teach solution of word problems by an algorithm. First you do this, then this, etc. You teach related rates in calculus by algorithms. The student's investment at this level is essentially passive.

The table of contents of part four is at this level. So are essentially all competency lists including that of NCTM for mathematics teachers. Even when the product is something like organizing a discovery lesson, it remains essentially in this category. This level is an engineer's conception: it is completely mechanical.

At the second higher level of teaching we are concerned with process, with what's in the student rather than what's in the mathematics. We are concerned with what is going on in the dynamic process of learning: of how personal, social, intellectual, humanistic factors bear on that

* See, for example, the work of Perry. William G. Perry, Jr. Forms of Intellectual and Ethical Development in the College Years: A Scheme. New York: Holt, 1968.

process. We are not focusing here on delivery of a specific product like the ability to factor the difference of two squares, but rather on helping the student to develop his own skills so that he can work out the next skill for himself, say in this case factoring certain quadratic trinomials.

Certainly we need both levels of activity. After all, some of our students may one day have the opportunity to appear on a TV quiz program. But there is no question in my mind that the process level is more important. If we want teachers to bring students to this second level of learning, we had better see that teachers think in these terms.

As soon as we redirect our attention to process rather than product we find ourselves sorely pressed to respond within the limitations of this one course. And we find that in order to make room for activities associated with this level we must trim those contents of section four almost out of existence.* You may see some of them in my course description, but don't count on any of your pets. In fact I hope that your reaction to omissions will force you to explore your own priorities in the light of my remarks. I see the choices as a matter of first things first. There is much to be done and we may never have another chance.

7. <u>A note on our organization</u>

We have incorporated a significant feature into the instruction of our methods classes. As an important (we feel) component of their training, our advanced doctoral students participate in methods class teaching. In this way we are developing apostles who we hope will spread the gospel. These students make excellent contributions to these classes, some of which will be noted later. The next logical step is to see that the undergraduates are placed in student teaching settings where their cooperating teacher and supervisor are also adherents. We not only have not taken that step, but we haven't been able to map out progress in that direction. This last is a function of the financial realities of the times.

8. <u>The Buffalo proposal: Introduction</u>

Under Steve Brown's direction our methods course seeks to respond to the concerns of level two. It does so by focusing on a number of themes: introspection, individual differences, surprise, mistakes, feelings, the evolution of proof, reexamining truisms, pedagogical risk, and reversing the lens from teacher to student. These themes are not addressed independently nor are they always even identified by name to

* So that you will not now reject my earlier recommendation of my methods text, I suggest that as the course activities turn to process goals the book can serve a useful reference function. I must note, however, that my co-workers, since they do not use the text, reject this notion.

the students, but the activities of the methods classroom relate to them. The classes sometimes have a free-wheeling, even temporizing quality, but a great deal of planning goes into them. Steve and the doctoral students each spend about two hours outside this class for each one with the undergraduates.

In this connection I cannot resist an aside about classroom teaching. All teachers ask for additional planning time. I doubt that they'll ever get it. That is truly unfortunate, but not necessarily wrong. It is my observation that very few use what time they now have wisely and would use additional time no better. The reason: teaching at level one doesn't require much planning -- paper correcting yes, planning no -- because all you need to do is record a few content items and activities that relate to them. When asked what do you teach, too many teachers ought to respond not algebra or calculus but Nichols or Thomas, for the book <u>is</u> their plan and their course. (This is an unfortunate American trait, only now being exported. The good teachers I met in England felt no such ties to a text.)

In detailing the activities of a single methods course at one institution, I am purposefully turning your attention from contents of <u>the</u> methods course to contents of <u>a</u> methods course. Well, perhaps there is some theoretical or abstract <u>the</u> best methods course off somewhere. In my view the one I am describing comes closest and I therefore propose as many aspects of it as are not associated with the unique personality of the instructor for export. And I even go so far as to invite any of you to join us for a semester or more to see this course in operation. If you do come, I am convinced that you too will be sold.

9. <u>The Buffalo proposal: The themes</u>

Consider now the Buffalo methods course in relation to the nine themes. I will not be able to explore them in detail, but will instead try merely to shed some light on them, often through examples.

A. <u>INTROSPECTION</u>. This is perhaps a supertheme for it pervades all the teaching in this course. The level of thinking that the students are forced to do goes far beyond the tentative and temporizing moves in this direction that I had to make as an unsupervised beginner. From the first class session when students are asked by questionnaire and follow-up interviews to assess their own attitudes and resources this is a basic element of the class. For many students this is their first experience confronting themselves. And it is often an uncomfortable experience.

The students keep logs in this course. In addition, there is a faculty log and an unsigned log. I'll cite some student comments from these logs as we progress. Everyone is required to write "something" each week <u>and</u> to read and make marginal comments on the logs of others. As one student said in her first entry: "I don't mind keeping a log and looking at other people's logs but I don't like them looking at mine." (Is that a comment on a widespread teacher characteristic: We gladly expose the hearts and souls of hundreds of young people yet we would never expose our own?)

In the methods classroom many standard teaching activities are performed by Steve and the doctoral students: these range from straightforward exposition to tricks and games. But always the questions -- Why did we do this? Could I do that? Would I? How would I do that differently? are waiting in the wings. These questions initiate discussions that help students see the elements involved in teaching. When they raise the questions -- "Why do all that? Why not just tell them?"-- and answer them for themselves in these discussions, they are moving away from delivery systems to teaching and learning.

As students are weaned into taking personal responsibilities in the methods classroom, first in small groups, then in the larger classroom setting, they are asked specific questions about themselves as well as about each other, again inviting introspection, insight and growth. Even the worst episodes -- and we all know how bad such episodes can be for beginners, to say nothing of more experienced teachers -- sometimes lead to interesting discussions. For example, one day a student presented a brief report for which it became immediately evident he was quite unprepared. After a minute or two the student stopped and said simply, "Sorry. I guess you can see that I'm unprepared." Steve's response was, "Yes, we can all see that. But here is an interesting problem that every teacher faces at one time or another. What do you do in this kind of situation? Let's spell out some of the options." And a lively discussion followed that helped students get at their own values.

B. INDIVIDUAL DIFFERENCES. In the first class session, Steve performs his famous card trick. (In planning sessions the groaning starts when he pulls out his now tattered deck for we've all seen this trick dozens of times in dozens of settings.) But it works. He performs the trick before the whole class and repeats it once or twice. Then the students break up into small groups. In one group the trick is merely repeated additional times for those students who didn't follow the steps. In another group, students who understand the mechanics of the trick but do not see the underlying structure try to determine what that structure is. (Here I had better insert that the trick is not slight-of-hand but operates on a simple mathematical structure. For those of you who are interested, a detailed description of the trick and its solution appeared in the May, 1970 Mathematics Teacher in an article by Charles Trigg.) In the third group, students who have seen through the "trick" set out to develop an explanation and to suggest modifications and generalizations.

Here, I suggest, is response to individual differences in its best form: locally. Note that the students were not grouped according to some test taken in the distant past. In this regard I recall the consternation of a sixth grade teacher for whom I was working with a group of his best students. He apologized for having to include -- for disciplinary reasons -- a stupid lad (his words) in the group. The boy did give every appearance of stupidity until we started our non-standard lesson. Then his eyes brightened up and he was the first to offer a significant student insight. His teacher was shocked.

And in this case even though graduate students monitor all three groups, the latter two are essentially independent. The teacher or a pre-trained student need only repeat the card trick over and over in the first group until everyone has mastered the mechanics and moved on. The example then is classroom manageable.

This last is important and forms a part of the course that cuts orthogonal to themes. Students do want instant applicability. They want to come away from the methods class with some things they can do themselves in the classroom. This kind of direct assistance is provided. Specific teaching techniques are demonstrated, curriculum assignments are given, student-teacher relationships are discussed, the roles of structure and discovery in recent history are reviewed but always with additional implications and values. They are generally initiated in the context of specific (planned or unplanned) class activities and are readily accepted by class members who are in the necessarily ambivalent role of student and prospective teacher.

Let us return to individual differences; small groups accentuate them: peer pressures force participation at this level allowing the differences to surface. For that reason such groups are employed in this course regularly. The focus is on variable differences rather than pre-set parametric differences. Students come to realize that the range of qualities that they bring and their charges will bring to class makes it impossible to fix them in categories.

At the same time they learn that responsibility draws out the best in students. Often when group activities are assigned, one member must represent his group in presenting results. The fact that he is the agent of his group places him under extra pressure to do well. This is part of a teacher's concern for maximizing positive pressures for learning and minimizing negative pressures. I will return to this point when I discuss feelings.

C. <u>SURPRISE</u>. Much has been made in science of the "eureka" aspect of discovery, too little of the "well I'll be damned" face of learning. The dynamics of the two responses are not dissimilar: each provides an additional charge of energy for further activity.

You know the kind of thing I am talking about in mathematical terms. Distribute a dozen decks of cards. Have each student receiving a deck shuffle his cards and then on command draw one card and hold it up. Among the cards held up, the chance of a match -- two sevens of spades for example -- is almost exactly 3:1. Surprise. This trick, which you recognize as an alternate form of the birthday problem, generates interest that extends beyond the activity. In particular here the question "Why?" leads to a significantly improved development of some aspects of compound probability.

But surprise applies beyond such mathematical oddities. For example, I recall experiencing this feeling when I first came on the wide range of applications of the distributive law. Granted that most students would not join me in being surprised at that -- or caring at all -- but the example does illustrate how surprise extends well over into pedagogical areas. In mathematics learning it usually takes the form of a surprisingly large return on a small investment. Let a course assignment tell this in a different way:

> Behind it all: The notion of surprise is going to be one of seven or so major themes in this course. We believe it is an important and neglected aspect of teaching and learning in general. We hope that we all will have the opportunity to both analyze and experience surprise many times in this course.
>
> It is perhaps not unusual to be surprised by the extraordinary -- such as travel to the moon. As an introduction to the notion of surprise, however, we wonder if it is possible to be surprised by looking at the relatively simple and uncomplicated -- but by bringing to bear fresh insights on that terrain. To a large extent we would not be surprised (ha!) if it was by such activity that great intellectual leaps were made in the evolution of ideas.
>
> So ... : Look at the following information as a start:
>
> $$\begin{aligned} 1 \times 3 &= 3 \\ 2 \times 4 &= 8 \\ 3 \times 5 &= 15 \\ 4 \times 6 &= 24 \\ 5 \times 7 &= 35 \end{aligned}$$
>
> So what? In about three pages, indicate what you see of mathematical interest in this data. How would you expand what you see? Can you describe briefly in one or two instances what might have encouraged you to make the leaps you did (or perhaps what inhibited you from doing so)?
>
> Note that we are not asking you to describe what the values of teaching this to someone might be. We are not asking you at this point to concern yourself with pedagogical questions. Instead engage in some playful mathematics on your own. Enjoy yourself and tell us what happened.

Without expanding on this topic further and taking away your own opportunity of experiencing surprise,* let me suggest that students have difficulty filling three pages before class discussion of this topic and would have difficulty pruning to a dozen pages afterward.

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* See Stephen I. Brown, "Musing on Multiplication," Mathematics Teaching, December 1974: 26-30.

D. MISTAKES. I hope that you have read in the Mathematics Teacher the fine article by Robert A. Carman on "Mathematical Mistakes" (February 1971). In that article the author wisely suggests exploration of those many avenues by which errors lead to correct answers as in canceling the sixes in 16/64 to get 1/4 -- very good but only peripherally related to this theme. Here the question is at the same time more pedagogical and more general: What do you do in the classroom when a student errs?

The question is usually raised in the context of specific examples like the 16/64, and other real classroom errors like

$$(a + b)^2 = a^2 + b^2.$$

Everyone who has observed classroom teaching knows that the probabilities are high that the teacher's response is one of the following:

> Wrong. Now Bill, what do you have? ...
> How many times do I have to tell you ...
> What is the third term? ...

Or a sigh or a sneer or raised eyebrows. Not only are the pedagogical possibilities lost by such responses but the student's ego receives another bruise in the process.

There are obvious answers that you would be prepared to suggest if I opened this example to discussion. Plug in values for a and b comes immediately to the mind of each of you, I'm sure. This familiar route of counterexample, then back to scratch is still not enough. Surely the examination of this kind of situation can be and should be deeper than that. In fact two of our students have become so interested in this particular problem that they have prepared a journal article on the topic. Unfortunately I am led by past experience to expect that editors will adopt the same why-all-the-fuss attitude that leads most classroom teachers to muff these pedagogical opportunities.

A student concern that arises in this context and in several others in this course is: I like that but I doubt that I could take the time. Such questions continually draw the focus back to the product-process dichotomy. This is especially true once students are out in the schools. I believe that this is a valuable aspect of the course. Too often this kind of question is never addressed and product wins by default. Now it becomes a matter of choices. Just as with curricular choices, when you do more here you must do less somewhere else.

E. FEELINGS. I like a passage in one student's log:

> When the card trick was first done I watched with interest. When we first began to figure it out I didn't really want to. I didn't feel like thinking deeply about a trick that would easily be taught to me. When I realized that everyone

was trying to figure it out I began to find it challenging. I began to feel frustrated so I was given clues on how the cards were laid out. When I discovered how, I felt the card trick a real challenge. I wanted to discover how it was done by myself or with others who didn't know how. I felt real satisfaction when we discovered how.

I have quoted that passage, not because it is positive, but rather because it shows quite clearly the short term shifts in feelings. This student is coming to an understanding of how much feelings affect learning.

A recent episode is interesting in this connection. One of the graduate students was presenting a content lesson to the class. When the students failed to respond to a question he adopted that old teacher ploy, "I leave that question for your homework." Just as it does in any classroom this generated extreme hostility, hostility that carried over temporarily into unrelated activities. But here the hostility gives opportunities for discussion on two levels: (1) It helps students to identify this as one of so many thoughtless techniques that cry out for attention, and (2) It encourages them to think more deeply about how feelings are generated, whether they can or even should be manipulated, and how to deal with their own feelings as well as those of their students.

In the methods class the students find themselves in a sort of never-never land -- or perhaps here it is an always-always land. They are part student, part teacher, on the one hand learning some things, on the other thinking about communicating ideas. There are values in this that relate specifically to feelings. They can, as in the hostility example, see how feelings are generated because they have felt them, and yet they are then forced to step back and examine those feelings from their other role as apprentice teacher.

Our university faculty colleagues mirror the real world in having mixed feelings about feelings. Some propose a psychoanalytic plumbing of this affective area; others believe that the whole area is out of bounds. While I do not go all the way with the former group, I certainly reject the latter. I believe that we must examine the areas where feelings intersect with learning and classroom behavior in order to chip away at these barriers to teaching.

F. EVOLUTION OF PROOF. This is an area that offers exciting opportunities, but that is too little explored. Researchers please note. The basic mistake here is the widely held notion that proof is an absolute. My logician friends -- Bob Exner in particular -- tell me that proof to a mathematician is not that much unlike legal proof; it is what satisfies or convinces the recipient. No mathematician in his right mind goes to first principles in every communication. Recall in this regard that ancient proof of the Pythagorean Theorem: a diagram with the one word caption: "Look!"

Very interesting questions are raised in this course about the meaning of proof to teacher as well as student. Steve has raised this question in the interesting context of the teacher as truth withholder -- perhaps even liar. You cannot subtract four from three, divide two by three, take the square root of negative one. But later on suddenly you find that you can. Is this reasonable? Most of our students respond to this problem from a position that is essentially: no problem. That's just the way it is. Surely teachers should advance from such a posture. For entirely too long teachers have been led by the Frank Allens of the mathematics education community to focus on questions of logical presentation and format when we need to turn to questions related to our pedagogical logic.

G. REEXAMINING (PEDAGOGICAL) TRUISMS. Education is one of those delightful areas chock full with accepted ideas that are wrong. I have examined a number of the student logs for the past several years. In each case the first dozen or so pages has an average of one or two per page. These pedagogical truisms give the staff a splendid opportunity to confront the students with the same statement as a question. Two common ones are:

> Comfort promotes learning (or the obverse anxiety inhibits learning).*
>
> Order your teaching from concrete to abstract, from specific to general.

Can't you think of exceptions to those rules. I'm talking here about the sort of dictum Alexander Pope proposes: that the best way to win a friend is to let him do something for you. These aren't single exception rules of thumb; the alternate view carries as much weight as does the original statement.

H. PEDAGOGICAL RISK. This course aims at developing teachers who will have the personal strength and emotional security to be willing to generate and test pedagogical hypotheses, to experiment in their classrooms, to take risks which range over the scale in probability of success.

Here I am reminded of an episode that is taking place in hundreds of schools across this country. A classroom teacher has read about laboratory teaching. He's convinced. He develops materials. He tries them out. And the next day he apologetically returns to his original instructional format. "I'll never try that again. The kids were up the walls and I couldn't hear myself think." In reality he never gave his experiment a chance. He failed to take into account the innate conservatism of us all -- students too.

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* This paper is a direct confrontation of this truism. It may in fact lend support to the original statement.

Change itself is threatening. And if change is threatening to students it is overwhelming for most teachers. In my example the poor teacher never even got around to asking himself what he learned about his students' learning styles.

I realize that my comments here are slightly off the mark: that what Steve means (refers to) by pedagogical risks are usually smaller risks dealing with individual incidents and individual students. But I think that we are talking about the same thing. The teacher should move away from reactionary teach-as-I-was-taught ad hoc classroom decisions and should think in terms of constant attempts to improve. Introspection is obviously involved here.

I. REVERSING THE LENS FROM TEACHER TO STUDENT. Several summers ago I taught a series of demonstration lessons to a group of high school students with fifty experienced teachers observing and taking notes. This was not my first experience in this kind of setting, but the students this time were tougher to reach and I was struggling. When I examined the observing teachers' notebooks I was surprised to find no note of this. Why? Because the teachers focused on my act. They liked my turn on stage: my examples, my little twists, my change of pace. What I did -- and more -- was there. No one seemed to care that the students weren't learning anything. I was enraged. I met with the teachers to tell them that I wanted them to forget about me the next day and to see what if anything was the response of the students. After class I asked them for their observations. They were worse than the students, all examining their shoes or smiling vacantly. Finally one middle aged woman raised her hand. "I noticed that Joe is interested in Mary." I dismissed them on the spot.

If our only observations of students identify us as voyeurs, no wonder we're in trouble. No wonder too that the only attention reversal in classrooms today is in self-instructional workbook programs that close the shutter at the same time they reverse the lens. Again the methods class provides a unique opportunity for the proto-teachers to experience and think about the processes at the same time. The trick of the course is to ask over and over, "How did you react to that?" each time following up with "How would your students react and respond?"

Each of us, when we take time to address it, knows about this problem and has had personal experience with it. Somehow when we get up front we forget how it was in the back row. This course addresses that problem.

10. Comments

1. If I have no other point I have at least established that you invited the wrong guy. Steve Brown and our graduate students should be here in my place. I'm just the traveler; they're home doing the work. Earlier I suggested that many aspects of this course were exportable. Steve, I hope for our sake, is not. In this regard let me tell you a brief story that I think speaks to one facet of this fine teacher.

Some time ago I called him about something else but in our phone conversation asked him about his methods class session that day. "Wow," said Steve, "we had an unusual class. A series of events generated more hostility than I've ever experienced in a classroom before. It was a complete turnaround of the last class when everyone was excited and positive." And then after providing the details, some of which I've already noted, Steve added, "It was one of the most exciting classroom episodes I've ever experienced." I remain stunned by that remark.

2. I comment briefly on the extended role of the advanced graduate students who co-teach this course. They are not (now) student teaching supervisors but they do visit the students in their school settings. They also help with student projects and teach mini-classes on topics of special interest to them, for example, non-verbal learning. But most important each must also develop a short term research project related to the course. These tend to focus on conceptions of knowledge and linkages between school and methods class settings. Several students are expanding these projects into theses.

3. You recall that we rejected section four. As it turns out those topics are not ignored or denigrated in the course as I have seemed to do here in order to emphasize my own theses. Rather they come up in the context of our approach to students on the threshhold of teaching who have <u>within themselves</u> the beginnings of (appropriate and inappropriate) responses to those topics. For example, consider "Psychological Theories of Learning." I think that it will be easy for you to rethink my remarks in order to see how such topics as PEDAGOGICAL TRUISMS help students to develop their own theories of learning.

4. We are developing at Buffalo with Roy Callahan and Alan Riedesel as well as Steve Brown and Marion Walter whom I have already mentioned, the best mathematics education center in this country. This methods course and the associated graduate student practicum are central to that development. I meant it when I suggested earlier that you join us. I can think of few better ways to use a sabbatical. At the same time I would have to add that when I read that earlier suggestion to one of our graduate students, his response was, "Staff hell, they ought to take the course as students."

5. It would be unfair for me to omit direct credit for the success of this course to the graduate student participants some of whom helped with this report: Larry Meyerson, who will teach one section of the course himself next year, Bernie Hoerbelt, Maureen O'Grady, Carlos Vernon, Dave Alexander, Jim Comella, Jim Watson and Dorothy Buerk.

11. Conclusion

Early in the semester last year one student wrote in the anonymous log a simple descriptive note: "This course is a crock of ----!"* That sentence remained without response -- for the staff purposely did not

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* I am afraid that my failure to print this simple Anglo Saxon word dates me.

respond -- until late in the semester when another student answered. I quote from that answer:

> That isn't right. This has been a good course. I admit that there were a few times though when I would have said the same thing. For one thing I didn't like working so hard. At first I also resented many of the ideas that came up. They just seemed wrong until I thought about them and they began to make sense.
>
> The course really wasn't what any of us expected at all, but I think that most of us have gained a lot from it. I know I did. I have thought harder in this course than in any I ever took before including math. I also learned a lot about myself and how I learn. That really helped me in my teaching assignment. A lot of the things I did in student teaching came right from this class. The teacher I worked for didn't pay much attention at first, but then even he got interested.
>
> One of the to me funniest things happened when I wrote in my log that I didn't like something Bernie [a graduate student] did in class. He wrote back that he was glad that I felt that I could say that to him and he thanked me and said that he would think about what I said. I couldn't believe it as I thought it would just get him up tight. You can't help respecting a guy who would do that. I hope that I do as well with my students when I'm teaching.
>
> At the beginning of this term I was scared to death of teaching. I'm not now. My teaching assignment helped but this course did even more. Thank you everybody on the staff.

In mathematics we have the closure property. The concept has extended now beyond this subject until today we have intellectual and pedagogical closure whenever things are neatly tied up. If I had to characterize the contents of the methods course I have described and supported in this paper in a word, I would coin a new one: opensure. In this course closure is not sought, often avoided, but meanwhile the deep questions of the hidden curriculum in mathematics teaching methodology are, if not answered, at least raised.

COMMENTS ON A PAPER by Gerald Rising

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Professor Rising's description of the students who come to the methods course at Buffalo is painfully familiar. It is not unlike the description of non-teaching mathematics majors given by E. Spanier in a note in The American Mathematical Monthly of August-September, 1970. Rising comments "I do have criticisms to direct at some college mathematics teachers and some suggestions for improvements but these are not the point here." While this is not the point of this conference, perhaps it should have been! Certainly no one-term, three or four hour methods course can offset the adverse effects of some forty credit hours of inappropriate instruction in mathematics. It is in those mathematics courses that students form deep impressions of what mathematics is and of how it is taught. It is there that their attitudes and self-images are formed. If those extensive experiences are largely negative there is, I believe, little that can be done in one course to change this. This implies that we must make strenuous efforts to affect the teaching of our colleagues in mathematics. Also, we must broaden our students' pre-methods course experience by in-school observation and discussion with us of the good and bad points of what they observe. Use of our students in supervised tutorial roles in low-level courses is another way of broadening their perspectives concerning teaching and learning mathematics.

My point is simply that I doubt that we can design an effective methods course regardless of content or goals unless we can strongly affect the pre-methods course experience of our students. For the type of student Professor Rising describes, a methods course is not a rational culmination of college mathematics education. It is the start of probably life-long remediation!

The other point in Professor Rising's paper to which I wish to react is the age old dichotomous problem of "product" versus "process." No one can question that the past is full of examples of overemphasis on "product" - of training in the technology of teaching at the expense of education in the art of teaching. Rising's determination to change this by emphasizing "process" - higher order teacher behaviors - is commendable. However, I have some concerns. Certainly school mathematics has been in a process mode for the past few years in many "modern" and "math lab" programs. Many now feel that the basic technological skills have suffered. Science - and even Social Science - are literally wallowing in process, to the point where some claim that transmission of basic information in these fields is being neglected.

Professor Rising assures us that he recommends balance of product and process. Still, he says "As soon as we redirect our attention to process rather than product we find ourselves sorely pressed to respond within the limitations of this one course. And we find that in order to make room for activities associated with this level we must trim those contents of section four almost out of existence." This does not match my mold of balance! Furthermore, if I must have imbalance I would opt for overemphasis on product rather than on process. A major reason for this position is that I doubt that Professor Rising - or anyone - knows what experiences will produce a "teacher-artist." It seems likely to me that much more than appropriate sensitizing and producing reflection is involved; namely innate talent that is found in few. While I do not doubt we can produce educational experiences that will maximize the probability that innate talent will be developed I do doubt that this talent is present in the vast majority of our students. I believe that most of them - like most of us - will gradually become better and better "teacher-technicians" who (hopefully) occasionally perform an artistic act. Thus, I fear that an overemphasis on process is likely to be largely wasted!

At the same time, an underemphasis on product - the technical skills of teaching - seems detrimental to all our students. I seriously question the ability of an "artist," no matter how sensitized and reflective, to produce an object of art without the underlying technical background. For our students who will never be artists, neglecting the development of their technical skills is to leave them with little as the result of their instruction. If we attempt to make artists of all through "process education" while neglecting "product training," most will still not be artists - and will be damn poor technicians as well!

My position is not one of advocating emphasis on product while ignoring process. Rather, it is a call for balance - better balance than I see in the Buffalo program.

REACTION TO A PAPER by Gerald Rising

Katherine Pedersen
Southern Illinois University

As a reactor to Professor Rising's paper, I find myself in the difficult position of agreeing with almost all he has said -- with the exception of his disclaimer to any expertise in the area of methods of teaching mathematics. The methods course which he has described is quite removed from the usual methods course. While not presuming it to be a panacea for all frustrated methods teachers, if the course is exportable without Dr. Brown, it could provide a vehicle through which a student might develop a capacity for introspective mathematics teaching -- a capacity which will endure beyond that first classroom discipline problem.

Let me mention that by exportability I mean that: (1) all of us today could be given the materials which Dr. Brown has created as well as his instructions on how to use them; (2) we would return to our methods classes and would use these materials as he instructs; (3) we would all attain -- within a reasonable degree of variation -- the same finished product -- in this case a secondary mathematics teacher who is whatever the materials indicated he would be. Since I also have constructed a methods course at Southern Illinois University which attempts to develop secondary mathematics teachers who react on both the product level and the process level, to use Professor Rising's terms, I am inclined to ask whether or not such course materials as used by the staff at Buffalo or by myself in Carbondale are exportable. To answer my own question, "Probably not." I will return to this idea of exportability later in another context after we have looked at the product element of a methods course.

Professor Rising has succinctly described the "tell how - assign problems" syndrome which is exhibited by student teachers early in the student teaching term. In order to minimize the factors which are influencing the student in this direction, I would encourage all methods teachers to include content related to classroom management, homework, and evaluation in their course. This can quite easily be handled by a readings list with group or small-group discussion on classroom management. I might even go further and insist that the methods teacher return to a secondary classroom as a teacher for an extended length of time. Then he could experience the activities which influence a student teacher. My rationale for this is very personal -- I found that if I could not advise a future teacher on a topic as "simple" (his point of view -- not mine!) -- as simple as discipline -- he would not accept my faith in his ability to teach in a manner which varied from the accepted norm.

Continuing on the product level, I would like to propose that the topics under Professor Rising's "table of contents" such as history of teaching mathematics, psychology of learning mathematics, writing student-oriented objectives and writing lesson plans, teaching the exceptional child, and professionalism in teaching as well as the three product level topics mentioned above: classroom management, homework, and evaluation do belong in a secondary mathematics methods course -- but as topics to be covered by a readings list. No specified length of time would ever be sufficient for the process level of the methods course. So, by using a readings list, all of these become a part of the methods course while not occupying a major portion of the class time. I am not adverse to a student's clearing off his desk. I just don't want him to stop there. And, in connection with a readings list, I must state that every mathematics methods student should read the text "Guidelines for Teaching Mathematics" by Donovan Johnson and Gerald Rising. For all of you at institutions which do not offer a mathematics methods course, and some that do, why not request the methods teacher to assign this book to be read by all prospective math teachers? It is excellent.

I would like now to turn my attention to Professor Rising's statement that the usual mathematics undergraduate is ill-prepared to do mathematics much less to teach mathematics. While I do not fault this statement, I am not willing to accept it as a necessary restraint on developing a secondary mathematics methods course. Rather, I would like to open to this conference some suggestions which could initiate change in this area -- and which could possibly provide a direction in which some conference participants might want to proceed. I am speaking of the area of undergraduate mathematics teaching. Professor Rising deliberately steered clear of this topic but I would prefer not to. Now, I don't intend to expound on why I am such a fantastic teacher -- and you could use improvement. Rather, I would like to suggest that we take a good look at undergraduate mathematics textbooks. Since the textbooks determine the course content at this level as well as at the secondary level, textbooks must be written which ask where appropriate:

Why did we do this?
Could I have done such and such?
Could I do it differently?
What if ... ?

These questions sould appear on the mathematical level as well as the pedagogical level. I have been so greatly influenced by the R. L. Moore method of teaching that I cannot be too objective about this, but texts which state definitions, theorems, some proofs -- but introspective proofs, and texts which ask questions are needed. Examples of texts which come close to this are:

<u>Set Theory and Topology</u> by Strecker and Nanzetta; and
<u>Foundations of Geometry</u> by C. E. Burgess.

We could begin to influence undergraduate mathematics by writing papers which develop a single topic in a mathematically and pedagogically introspective manner. It would be hoped that these could then be accepted by most mathematics teachers for use in their own classroom. The level of mathematical integrity would have to be kept high -- as it should be. For example, the completeness of the real line could be motivated by infinite decimal representation and illustrated by reflecting on Cantor's Nested Interval Theorem. This would probably find acceptance by the usual university mathematics teacher.

In the February, 1975, SIAM News, Edwin Moise has centered on the publication of "good" mathematical texts as being one of the most neglected areas of undergraduate mathematics. He further asserts that mathematical competence is not sufficient to write the books that are needed; to quote: "the trouble being that course design and text writing are arts and crafts in themselves and sometimes they have not been learned by the authors." Need I say more than to indicate that Professor Moise's talk was presented to a symposium to explore "Which mathematics pursuits should receive high priority for funding and the attention of mathematicians."

As my last point, I return to the exportability of Dr. Brown's materials, or my materials, or the materials of any other methods teacher who is trying against time to raise within a student a love and a feeling for mathematics and for doing mathematics. These materials are usually developed by a person to reflect his own teaching personality which is usually quite strong. What happens to these materials in someone else's hands, in particular, with someone who does not know the personality of the originator? I do not perceive the exportability factor to be high. To be truthful, it is difficult to indicate what a certain lesson is supposed to achieve. But this does not mean that nothing is achieved. To the contrary, Dr. Brown has indicated to us that it is possible to teach someone to teach better and has indicated a direction to pursue. The questions which further research in the process level of teaching should attempt to answer are so obvious and so numerous that I invite you to choose the ones that interest you.

Some possible questions are:

1. What is introspection?

2. How is it related to mathematical product performance?

3. Does a particular method of undergraduate teaching affect introspection?

4. Are materials exportable?

5. How do students relate to such courses?

I would like to close this reaction with a couple of remarks which might have immediate impact on mathematics methods courses. The next time any one of us is in the position to determine mathematics course content, let us refrain from packing the syllabus and let us consider the course from a point of view which includes both mathematical and pedagogical introspection as defined by Professor Rising. Let us also analyze mathematics methods courses given in our own institution with respect to mathematics' staff involvement. A methods course on the process level necessitates mathematical competence on the part of the instructor. Our future teachers of mathematics deserve this.

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II. Polya, George. "On Learning, Teaching, and Learning Teaching," Readings in Secondary School Mathematics, Douglas Aichele and Robert Reys, (Editors), Prindle, Weber, and Schmidt, Inc., 1971.

III. Moise, Edwin. "Serious Problems Need Talented Teachers," SIAM News, Volume 8, Number I, February, 1975.

Professor Rising made several comments in response to questions following the presentations by the reactors to his talk. In response to a question about performance-based objectives, he referred to his article in the November, 1973 issue of Educational Technology *-- an entire issue devoted to the teacher education aspects of such objectives. In response to a query about the logs kept by the students, he stated that these were kept atop bookcases in the seminar room and could be consulted there or taken out. The seminar room is also the focal point for discussion and exchange of ideas among students and faculty members.*

Rising stated that, despite his arguments against technologists, he sees a perfectly valid role for programmed material. "In the misuse of technology it is allowed to take over, turning teachers into clerks. Then we blame the technology. We need to imbed the technology in a reasonable program." *He stressed that the methods course should not be a remediation for those concepts that were improperly taught in the regular mathematics curriculum. He also stated that the difficulty with sending observers into the schools is that there are so few master teachers for them to emulate.*

A poll taken in response to a later question revealed that 16 of the institutions represented at the Conference required field experience prior to the student teaching term. In nine of these the field experience was an integral part of the methods course. It was also found that in 21 of the schools, microteaching or peer teaching was used.

III. Teaching Strategies Used in Methods Courses

Brief papers: A. I. Weinzweig
James Lockwood
Kenneth Retzer
Larry Sowder
Kenneth Cummins
Henry S. Kepner, Jr.
Margariete Montague

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MATHEMATICAL IDEAS DETERMINE STRATEGIES

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As trainers of teachers we must recognize that the greatest influence on the future teaching of our students is not any one course, or even sequence of courses, but rather, the myriad of courses taken before and after the official "methods" course. This is the result of the fact that most teachers tend to teach the way they were taught. If you feel that the manner in which mathematics is taught at the college level is appropriate for high school or elementary school, then a formal methods course is unnecessary. If, however, you feel that college mathematics is taught in a manner entirely unsuited for lower levels then not only does a methods course become necessary, but even more -- it must counteract these negative influences by providing a positive model of the kind of classroom behavior we want our students to emulate! If we simply lecture to them on different methods of teaching, they will dutifully memorize our brilliant discourses and repeat them on our tests -- with varying degrees of accuracy. What will they learn? You teach by lecturing -- by talking about the subject.

How then are we to convey to our students appropriate ways of teaching mathematics? Only by teaching mathematics in an appropriate way! This is my first point. You can talk about the teaching of mathematics meaningfully only in the context of teaching mathematics. That is, you operate on two levels -- on the one you teach some topic in mathematics "the way it should be done" and on another level, you can discuss what you have done and why. This has the double advantage of, first, providing a positive model of mathematics teaching, and second, providing a framework or context for discussing techniques of teaching and theories of learning.

Lest I commit the very sin against which I am inveighing -- of talking about the subject without providing a proper framework -- let me present an example which will also serve to introduce my second point.

A common definition of a Group, if one may judge by the responses of students, is the following:

A Group is a set G on which is defined a binary operation * such that:

1) G is closed under *;

2) * is associative;

3) there is an identity in G with respect to *;

4) each element of G has an inverse with respect to *.

What is a binary operation? More appropriately, what is a binary operation on the set G? The usual definition is that

$$* : G \times G \to G$$

is a function.

What does it mean that G is closed under *? Again, the usual definition is that for any pair (g,h) with $g,h \in G$, $*(g,h) = g*h \in G$.

What additional information is given in the second statement which is not already included in the assertion that * is a binary operation on G? None whatsoever! Then why is it included? Is this just a case of wearing a belt and suspenders to hold up your pants? I think not. Rather, what is operating here is something much more fundamental.

Historically (not too many years ago chronologically) when the group concept was being defined, the idea of an operation was regarded as clear and understood and was therefore never defined explicitly. In describing specific examples of groups, the operation was described explicitly, but the general idea of a binary operation was never defined. Nevertheless, they did understand what a binary operation was in the most essential way -- they could readily distinguish a binary operation from something not a binary operation. In exactly the same way, we all "know" what a chair is even though few if any of us can give a formal definition.

This situation is quite common in the development of mathematics. Someone observes that a certain phenomenon recurs sufficiently often to warrant separating it out -- or abstracting it from the context -- for further study. To facilitate discussion, we attach a name to this new concept. It is only after we have studied and understood the significance and ramifications of this new concept that we attempt a definition. Indeed, when this definition is attempted too soon, we end up with "wrong" definitions; that is, different definitions, as in the case of compactness (bicompactness, sequential compactness, etc.) until we finally settle on an appropriate one!

This point -- my second point -- is extremely important. In the normal development of mathematics, a concept is named only after sufficient experience with it to recognize it, and defined only after fuller understanding of its nature and scope. I submit that no one has ever learned a concept from a definition. It is only after much experience with the particular concept, when one has come to understand its meaning and significance, that one can appreciate that the definition does indeed describe the concept precisely and concisely.

This fact should have profound implications in the teaching of mathematics and yet it is usually overlooked for, in fact, we all too often begin with the definition!

What then are the implications of this fact in teaching -- how should one present a concept to a class? We should imitate the usual process of mathematics. Mathematical concepts arise naturally out of a particular context in response to some need. In introducing a concept we should create a context out of which will grow an awareness of the need for this concept. The student must be provided with adequate experiences so that he becomes familiar with the concept before it is even named. It is only after a concept has been well understood and the need for a definition apparent that such a definition is given.

Once again, let me illustrate my point by a concrete example. Students are quite accustomed to spot tests, so they are not too surprised when I tell them to take a sheet of paper and answer the following questions as quickly as possible. I generally restrict answers to whole numbers.

Answer each of the following as quickly as possible, restricting your answers to whole numbers, e.g., 0, 1, 2, ...

1) Give a multiple of 4.

2) Give a number between 5 and 8.

3) Add to 4, 3.

4) Square 4.

5) Subtract from 5, 2.

6) Multiply 5 by 8.

7) Give a number less than 7.

8) Give the larger of 3 and 7.

9) Give a factor of 12.

10) Divide 15 by 5.

The first thing to notice about each of these tasks is that they could have arisen quite naturally. Thus, if I have four identical packages of picture hangers and ask you to guess how many hangers I have, your guess would (or should) be a response to task (1). Similarly, my friend has 3 children, the youngest 5 and the oldest 8 years old. Guess the age of the middle child?

Second, in each case we are given one or two numbers and asked for some number related in some specified way to the given number or numbers. Thus in (1) the number given is 4 and the number we seek is related to 4 in that it is a multiple of 4. In task (3) we are given two numbers, 4 and 3, and the number we seek is their sum.

Notice that some of these tasks can be correctly answered in more than one way. Thus in (1) there are many multiples of 4 and stating any one of them successfully completes the task. On the other hand, others such as (3) have only a single correct answer. (We say that there is a unique solution to (3)!)

The task (3) is one instance of a more general task. Thus, 4 may be replaced by 10 to get another instance of the same task for which there is again only one solution, namely, 13. This new task may be stated as

3') Add to 10, 3.

Similarly we could replace 3 by 7 in (3) or (3') to get still other instances of the same task:

3'') Add to 4, 7.

3''') Add to 10, 7.

The general task of which these are instances might be described as: add to one number, another.

The tasks

Add to ___, 3.

or

Add to 4, ___.

cannot be carried out since the instructions are incomplete. In either case we have only given one number but 2 are needed!

This raises another question. How can we describe the general task -- add to one number, another. The fact that this cannot be carried out unless we specify two numbers (and in specifying these numbers we always give them in some order) is precisely what is meant by "binary" in "binary operation!"

The popularity of those little calculators leads quite naturally to the suggestion that this general task be represented by a machine where we enter the two numbers in succession (here we have the order again--even though it has yet to be made explicit!) into the input and the unique answer which completes the task is fed out of the output. Thus, the task (3), add to 4, 3, would be represented by the machine in Figure 2(a). The completion of the task is represented by Figure 2(b) where the '7' in the output is the answer or solution.

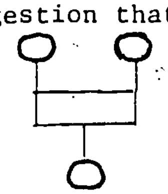
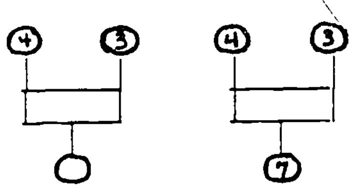


Figure 1



(a) (b)

Figure 2

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Task (8) is also a specific instance of a more general task which may also be represented by a machine. Thus, a store having a 2 for 1 sale permits you to buy any two items, for the price of the more expensive. We could then feed into our machine the prices of the two items, one in each of the entries, and the output would be the price to be paid. To distinguish this machine from the previous one, we designate the first by "Add" and the second by "Max" (Figure 3).

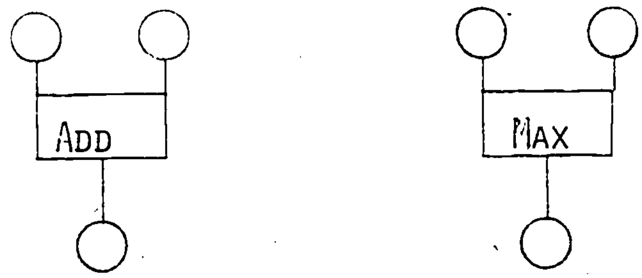


Figure 3

One word of caution is in order. We have drastically telescoped events in this presentation. Normally, the development would be spread over a much longer period of time.

At this state we would introduce many examples of machines. We do this in several ways. One is to present a context leading to a machine.

A movie theater charges $1 admission for children and $3 for adults. We have a "cost calculator" machine, where into the first entry is punched the number of children and into the second entry, the number of adults. The output is the cost (Figure 4 (a)). Thus, for 3 children and 4 adults the output as indicated in Figure 4(b) is $15.

(a) (b)

Figure 4

While our primary objective is to introduce different machines, we can maintain interest and stimulate curiosity by asking questions:

"I paid $16, what were the possible 'inputs'?"

Suppose for a special showing the prices were raised to $3 and $5 respectively. What inputs would yield an output of 13 (only 1, (1,2). Note that somewhere along the line we introduce, without fanfare, the use of ordered pairs to represent inputs.). What outputs are possible and why?

Another approach to introducing machines, which more appropriately follows after the preceding one, is to give partial information and have the students guess what the machine does. The amount of information given can be controlled by the kind of information given. Thus:

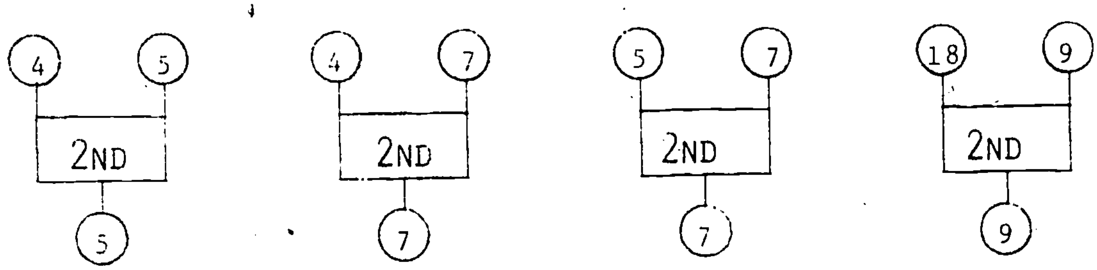


Figure 5

To encourage students to think, develop strategies, observe closely, at some point the students may specify an input and I give the output (like 20 questions).

IN	OUT
(3,4)	8
(3,5)	9
(3,6)	
(2,6)	8
(4,6)	12

Figure 6

Determining what the machine does means to be able to specify the output for a given input--not necessarily in "closed" form. Thus, in Figure 6, a student might observe that increasing the second entry by 1 increases the output by 1 whereas increasing the first entry by 1 increases the output by 2!

As a result of this guessing game, students usually notice that for some machines (such as Add or Max) reversing the entries (an input, say (3,2), and its transpose, (2,3)) yields the same output. In other machines, say 2nd, CC and that in Figure 6, reversing the entries yields a different output. This leads the students, in trying to guess what a machine does, to ask if this is the case. (To which I would answer "Yes," "No," or "You tell me"!) This leads naturally to the introduction of terminology to distinguish those machines whose outputs are unaffected by reversing the entries from those where the output is affected. It is at this point, when there is a clear need for it, that we introduce the term "commutative machine." It is meaningless to talk of commutative operations unless the student is familiar with enough operations to recognize that some operations are commutative and still others are not. Better still is to create a situation where he recognizes the need to distinguish one from the other!

Introducing the idea that a machine may "reject" certain inputs (the machine of Figure 6, where the output is 2 less than the sum of twice the first entry and the second entry, would reject the input (0,1) if we allow only whole numbers) the student is led to consider domain and codomain of machines. Similarly, the students can be led to consider hooking together two machines which, in turn, bring them to consider questions of associativity.

It is not my purpose in this presentation to give a complete exposition on the use of machines. Rather my objective was simply to illustrate, by a particular example, what I mean by "creating a context" for the introduction of a concept. While it may take some thought and a little ingenuity to develop such a context for other concepts, the payoff is well worth the investment of time and effort.

In the methods course, several examples of this kind, developed more fully, can provide an excellent framework in which to discuss the development of mathematical concepts and techniques for presenting them to a class.

MICRO-TEACHING AS A STRATEGY

James Lockwood
Northeastern Illinois University

Micro-teaching as a strategy for preparing prospective teachers has become a component of many methods courses. The purpose of this paper is to describe a micro-teaching strategy that I utilize in a mathematics methods course. It should be noted that the term "strategy" is being used in conjunction with micro-teaching in this paper in a broad sense that is in keeping with the use of the term in this conference.

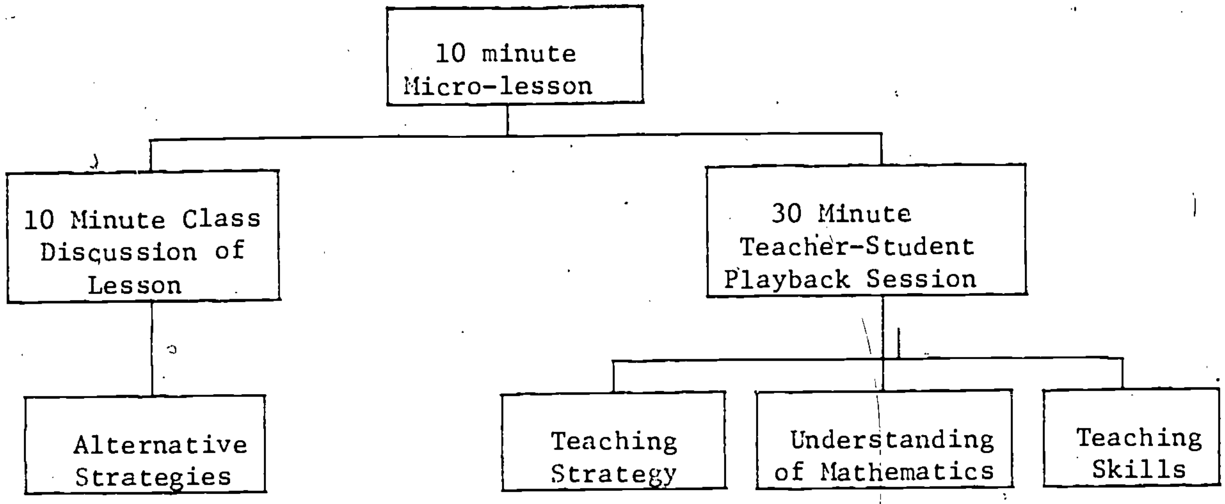
For the purposes of this paper, micro-teaching is a strategy that involves methods students in the preparation and teaching of ten minute, i.e., micro, lessons to a small group, usually six, of their peers. These lessons are audio or video taped for playback at a later time.

Many of us probably feel that the time that we have available to us in the methods class is limited in terms of the task of preparing prospective mathematics teachers for their student teaching experience and future teaching careers. The question must therefore be asked as to whether the time students spend preparing and teaching micro-lessons is time well spent. Another question that you may have would be with respect to the feasibility of using the strategy in your particular situation. If you are already involved in using micro-teaching, perhaps the ideas and suggestions to be considered will be helpful to you.

The Micro-Teaching Format

In the mathematics methods classes that I work with each student prepares and teaches two ten minute micro-lessons. The lessons are taught to a group of five or six of his peers. The student chooses a concept or principle to teach that would be taught in either a junior or senior high school mathematics class and one that they feel can be taught or introduced effectively in a ten minute lesson. The first lesson will be audio or video recorded and the second will be video recorded. The lessons are presented such that the first one is during the third or fourth week of the term and the second two or three weeks prior to the end of the term.

The following diagram indicates the strategy as it is followed after the presentation of each lesson. The strategy will now be described as it is implemented for the first lesson and then for the second lesson.



The First Micro-Lesson

The student teaching the lesson indicates the grade and level of class the lesson was prepared for. The group of peers who make up the class are asked to role play in the sense that they will ask questions that are appropriate for the grade and level of student the lesson was prepared for. They are also asked not always to give the correct response to the teacher's questions. The student teaching the lesson is signaled when he has two minutes left.

The ten minute lesson is followed immediately by a ten minute discussion involving the peer group and then at a later time by a thirty minute playback session during which only the student and I are involved. The ten minute class discussion focuses on alternative strategies that could be used to teach the same concept or principle. This discussion also frequently involves a consideration of the assumptions, attitudes, and values evidenced with respect to the organization and presentation of the lesson. The discussion does not involve a valuing of the lesson itself. Any comments to the effect that it would be better to use some other strategy are avoided. This procedure is followed as a means of allaying some of the anxiety involved in the micro-teaching experience.

For some students the trauma of the micro-teaching experience can be destructive to the strategy. This trauma can be reduced in several ways. One way is to have the peer groups work together on group activities prior to the micro-teaching experience. These group experiences allow the students to get to know one another and to become used to interacting with one another, which usually has an anxiety reducing effect. The student's first lesson is viewed strictly as a trial run. Because of this it is not evaluated for grading purposes. This also helps to reduce some of the anxiety and fear involved in the first lesson.

During the week following the presentation of lessons each student is scheduled for a thirty-minute playback session. Students viewing and/or listening to the playback are apt to respond first to the way they sound and/or the way they look. While this may be of some significance, the students are asked instead to focus on specific aspects of their lesson. These specific areas are the following:

1. The manner in which the lesson was set up to gain student attention and immediately involve the class, at least at the mental level, in the discussion. (This will be discussed later in more detail with the class and will be referred to as "establishing set.") See Allen and Ryan (1969) for a discussion of set induction.

2. The clarity and appropriateness of the examples used.

3. The clarity and quality of the questions asked. (The students are aware of Bloom's Taxonomy (1956) and can therefore relate to the level of questions they are asking.)

4. The manner in which the lesson was ended in terms of tying the lesson together, i.e., relating the concept or principle taught to concepts and/or principles the students already know and to the ones the students will be studying. (This will be discussed later in more detail with the class and will be referred to as "Closure.") See Allen and Ryan (1969) for a discussion of closure.

These first four are referred to by some as "teaching skills."

5. The appropriateness of the overall strategy that was used, i.e., was the strategy appropriate in terms of the concept or principle which was being taught and the objectives for the lesson.

6. Alternative strategies that could have been used to teach the concept or principle.

The role of the instructor must be very supportive and positive for this first playback session. The tape is frequently stopped and segments replayed by either the student or the instructor. This helps to focus on certain aspects of the lesson. The students usually identify their own weaknesses and areas of concern when asked to focus on these areas. This takes the instructor out of the critic's role and instead into the role of asking questions that help the student focus on particular aspects of his lesson that may have been done very well and others that were not done so well. I have found that this role is an important one if the micro-teaching strategy is to be an effective learning experience for the prospective teacher.

The first lesson not only provides the student with an opportunity to prepare and plan a lesson but also serves as a setting for much of the discussion and some of the activities that will take place during the remainder of the course. This experience also provides the instructor

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with information about the student's understanding of mathematics. This type of feedback is helpful in determining what to emphasize in working with particular individuals in a particular methods class.

The Second Micro-Lesson

The general format indicated by the diagram is followed again when the second lesson is presented. The major changes are that this lesson is evaluated for grading purposes in terms of the six areas identified above and the understanding of the mathematics that they evidence. Also the students are now aware of terminology that makes it possible to discuss their lesson in a more precise manner. For example, the students will be able to relate to terms that are used to refer to skills such as establishing set and closure. Also they will be able to consider their overall strategy in terms of concept moves discussed by Henderson (1970) and by the concept, principle, and skill moves discussed by Cooney, Davis, and Henderson (1975). Henderson characterizes a move as "a sequence of verbal behaviors by a teacher and students toward attaining some objective." He also identifies and explicates the moves used in teaching concepts. The students' awareness of these terms and the moves in the teaching of concepts and principles is helpful in both the class discussion and the playback session. This awareness makes it possible to deal with the specifics in terms of the strategies used rather than in the generalities that are usually of little value to the student. The benefits of this background also carry over to the student teaching experience making it possible for conversations between the student and the supervisor to focus in specific aspects of the lesson. For example, if the student is asked to reflect on a particular strategy he used to teach a concept he can identify the moves that he did use and consider what moves might have strengthened the strategy for the particular concept and for the particular students he was teaching.

Reflections and Comments

Additional experience in preparing and teaching micro-lessons would be a valuable experience for many of the students that I work with in the methods class. This could be accomplished best by having a micro-teaching course that the students would take while they are in the methods class. One of the results would be that more time would be available for students to focus on each of the six areas identified in the discussion above. If such a program is possible in your institution, it should be given consideration.

The use of peer classes has advantages and disadvantages. A major advantage is the opportunity the class has of interacting with various strategies and of considering the possible alternatives to the strategies used to teach the concept or principle. However, some students are turned off by what they consider to be the artificiality of the micro-teaching setting that involves peers. For some of our general methods classes we do use a secondary school setting with secondary school students as class members. While this may make the student feel better about the experience in terms of what appears to be lower anxiety levels, it also presents problems. An example of this results from the difficulty of controlling the background level of the secondary students who make up the class and of finding secondary students who will interact in the micro-teaching setting.

Summary

In this paper a micro-teaching strategy for use in a mathematics methods course has been described. The strategy involves the students in preparing and teaching a ten minute micro-lesson and in the discussion of alternative strategies that could be used to teach the concept or principle that is the focus of a particular lesson. The instructor is involved in a thirty minute playback session in which his role is to be supportive and to encourage the student to look analytically at his own lesson. This is accomplished by asking questions, not by "telling the student what he did wrong."

The experience that students gain in the preparation and teaching of a micro-lesson is broad in the sense that they must involve themselves with curriculum materials of differing levels as they decide upon the concept or principle that they will teach. This also results in the students' reflecting on their understanding of secondary school mathematics in terms of the college mathematics they are studying. I find that the first lesson provides an effective set for the content that will be taught during the remainder of the course. The students appear to be able to better relate to and involve themselves with the content even though their teaching experience has been a very brief one. The micro-teaching experience encourages students to start to look at the teaching of mathematics in a more analytic fashion. There is a developing awareness of the many strategies that can be used to teach mathematical concepts and principles. The students' experiences with skills such as establishing set and closure and with the moves that can be used in teaching a concept or principle makes it possible to communicate precisely during the discussion of the micro-lessons and during the student teaching experience. The teaching of micro-lessons also gives the methods instructor an opportunity to evaluate the students' performance in their roles as teachers. This makes it unnecessary to rely only on the students' ability to perform on written work.

At the beginning of this paper I raised a question as to whether the time spent having students prepare and teach micro-lessons was time well spent. It should be clear that my own answer to this question is yes. While the strategy that has been considered does involve some compromises, I do feel that it represents one of the most important activities that the methods students can be involved in.

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Bloom, B. S. (Ed.) Taxonomy of Educational Objectives. The Classification of Educational Goals: Handbook I, Cognitive Domain. David McKay Co., Inc., New York, 1956.

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Henderson, Kenneth B. Concepts. The Teaching of Secondary School Mathematics, the Thirty-third Yearbook of the National Council of Teachers of Mathematics, Washington, D.C., 1970.

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MATHEMATICS EDUCATION MODULES IN
ILLINOIS STATE UNIVERSITY MATHEMATICS (ISUM) MODEL
FOR EDUCATING SECONDARY MATHEMATICS TEACHERS

Kenneth A. Retzer
Illinois State University

Context

The mathematics education component of the ISUM model is a 3 semester hour mathematics education course in the context of other stable or developing components. Other components are: mathematics content (27-29 hours), professional sequence (14 hours), student teaching (10-12 hours), experiences with out Secondary Mathematics Materials Center and communication with inservice teachers (no credit), and a preservice internship under development which would incorporate early classroom experiences and development of observational and teaching skills using strategies for teaching mathematics.

Rationale

Inadequate time exists in a 3 semester hour course for a preservice teacher to 1) acquire a historical perspective of mathematics education and of current trends; 2) become informed on research and development projects, curricular recommendations, current texts and multimedia classroom or resource materials, and benefits of professional organizations; 3) adapt what they know about general teaching methods, provision for individual differences, lesson planning, evaluation, diagnosis and remediation to the mathematics setting; 4) be equipped to handle the multiplicity of teaching trouble spots in each high school mathematics course; and 5) acquire the observation and teaching skills relevant to mathematics content through use of protocols, modeling, microteaching and feedback. Informational aspects can be modularized to permit individual selection of topics and individual pursuit of a common core of topics while preserving class time for skill acquisition and other aspects that cannot be done in modular form. Modularization can permit an inservice teacher to complete an entire set of competencies at his own initiative.

These modules don't exist in the ISUM model but are expected to be prepared as a parallel effort to preparing graduate level modules in mathematics education for our Doctor of Arts program.

Example

One such module under development is on strategies for teaching mathematics which is based on taxonomies of moves for teaching mathematical concepts, principles, and skills which are in Dynamics of Teaching Secondary School Mathematics by Cooney, Davis and Henderson and is being published by Houghton Mifflin. These moves come from systematic observation and analysis of how teachers deal with mathematical content as contrasted with teacher actions which purport to be implicit in learning theory, developmental theory, or specific topics in the mathematical curriculum. And these moves are restricted to verbal manipulation of the mathematical content rather than teaching moves related to classroom management.

Such a module will have, as ultimate objectives, demonstration of observational and teaching competencies using the moves in the taxonomies. Background reading for orientation will be outlined. Preservice teachers (PST's) will observe protocol materials on the moves; practice identifying the moves in texts, on tapes, in individualized activity packets, and in actual classroom situations; and demonstrate their ability to monitor their own manipulation of content as well as that of other teachers. In addition, PST's will use the protocols as models of the moves, plan math lessons which sequence these moves into plausible strategies, demonstrate the ability to execute such moves and to adjust lesson plans as necessitated by student response.

Joint selection by teacher and PST, of the modules most crucial to the preservice program of each PST can provide the maximal benefits of provision for individual differences.

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LABORATORY LESSONS

Larry Sowder
Northern Illinois University

Student teachers frequently seem to do most of the talking in the classroom. To combat this tendency so that there is greater opportunity for student-to-teacher or student-to-student dialogue, discovery learning is emphasized in my methods class.

One requirement along this line is that the methods student write a laboratory lesson, where "laboratory lesson" is defined to describe a lesson in which the student gathers or generates data and then manipulates these data to arrive at a conjecture (whether a proof of this conjecture is sought depends on the background of the target student or the objectives of the teacher). For example, asking students to find a shortcut for squaring a number (whose base ten numeral) ends in 5 is a laboratory lesson.

Methods students usually choose topics suggested in a methods book: properties of special sets of geometric figures, sum of the first n whole numbers, Euler's formula, Buffon's needle problem to estimate pi, the tower of Hanoi, probability experiments, laws of exponents, graphing activities, ... Quality varies from very good to unusable.

Does this activity make a difference on interaction patterns when the methods student does student teach? Data are scanty, in part because of the difficulty of requiring a laboratory lesson approach in someone else's classroom. Methods students indicate that they regard the activity as clearly worthwhile.

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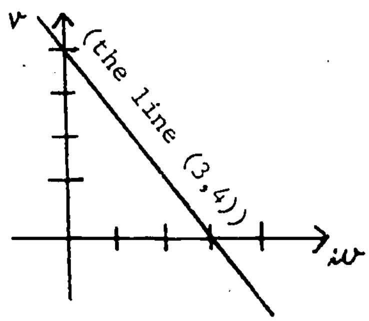
A STRATEGY IN METHODS COURSES

Kenneth Cummins
Kent State University

One facet of our course in the teaching of mathematics is to introduce and study several topics in mathematics with which the student has no prior acquaintance and to have him experience as a student the development of this topic through student-centered and laboratory approaches. It is expected that this will help encourage and prepare him to use such methodology in his own classes.

One such topic is begun by raising in a casual manner the question, "What would happen if two numbers located a line rather than a point?"



Experience in plotting lines leads naturally to "lines parallel" and "lines perpendicular" and ultimately to the "(line) equation of a point" and to the "solving of two (line) equations of points to get the (line) coordinates of their join" -- all developed with ongoing discussion and contributions by the students under teacher guidance. The student hence "lives through" what it is hoped he will encourage in his own students. Another topic often used is that of determining the equation of the "line of best fit" for given (apparently linearly related) data.

From this part of our course the students gain much practice in preparing work-study-guide-sheets, in looking at mathematics through the eyes of the student who is helping to create, and in making "mini-appearances" before the class on topics which they will teach.

FEEDBACK AS AN INTEGRAL PART
OF THE DISCOVERY STRATEGY

Henry S. Kepner, Jr.
University of Wisconsin-Milwaukee

In the secondary mathematics methods course at the University of Wisconsin-Milwaukee, the teaching strategies are chosen to demonstrate comparable classroom uses. The continuum of discovery and activity-oriented strategies are emphasized. When a teaching strategy is being used as an example of a classroom model, the instructor will stop in the session to quiz the students on the strategy being used and its appropriateness to the lesson.

A key part of each strategy is feedback. The students are involved in learning settings which provide feedback to the instructor during the learning process A frequent flaw in use of the discovery teaching strategy is the lack of involvement of many students. In the methods class, the instructor insists on every student having scratch paper ready at all times. Numerous guided discovery questions and examples to be worked are presented during the lesson for which the student is to write down his conjecture or result. The instructor moves quickly through at least part of the class to observe, provide initial assistance or clarification and to encourage involvement.

This stress on seeking student feedback has been a valuable aid in dis. very teaching strategies with all levels of students. It has typically increased student involvement and provided crucial information for the teacher. This has been true for college students entering a me ds class after three years of passive lecture courses as well as secondary students who had conditioned teachers to do their work for them.

TUTORING SKILLS AS A TEACHING STRATEGY IN THE SECONDARY SCHOOL MATHEMATICS METHODS COURSES

Margariete A. Montague
Northern Illinois University

Observation of a secondary mathematics class reveals that most teachers are frequently involved in one-to-one remedial tutoring. Factors residing with the student such as the time needed to learn the material (aptitude), time the student is willing to actively commit to learning, and ability to understand instruction, as well as factors residing with the teacher such as time allowed for learning and quality of instruction contribute to situations calling for remedial tutoring.

Tutoring encounters may be weak for several reasons: the encounter is too short (one small-scale study reported an average of 15 seconds); the encounter consists of informing the student of his errors.

Novice secondary mathematics teachers with strong qualitative preparation may not readily adapt to the role of tutor. Contributing to this deficiency are a limited knowledge of special techniques to deal with students unable to do basic arithmetic or basic algebra, an inadequate use of praise or prompting and diagnostic questions, and a restricted knowledge of strategies which help to develop students' understanding of mathematical concepts and algorithms.

To reduce the needs of the novice mathematics teacher in a one-to-one remedial teaching setting, a tutoring skills minicourse was implemented.* The information-microteaching-reteaching sequence had five major components which included the use of praise, prompting and diagnostic questions, demonstration techniques, and evaluation and practice examples. Once the students were oriented to the use and location of the videotape recorder, 16 mm projector and the printed materials, the twelve day minicourse was independent of instructor intervention.

I. Providing general and specific verbal praise; asking prompting questions.

   Day 1: Read printed materials; view instructional-model film 1.
   Day 2: Microteach/tutor one pupil for twenty minutes; critique videotapes using self-evaluation materials.

II. Asking diagnostic questions for verbal reasoning and computational problems.

* The minicourse described was influenced by the work of Borg, Kelley, Langer, and Gall as described in A Microteaching Approach to Teacher Education (1970) and Individualizing Instruction in Mathematics (1971), Macmillan.

Day 3: Read printed materials; view model film II.
Day 4: Microteach/tutor two pupils for twenty minutes each; critique videotapes.
Day 5: Microteach/tutor two other pupils for twenty minutes each; critique videotapes.

III. Using demonstration techniques such as manipulatives, diagrams, equations, and answer estimation.

Day 6: Read printed materials; view model film III.
Day 7: Microteach/tutor two pupils for twenty minutes each; critique videotapes.
Day 8: Microteach/tutor two other pupils for twenty minutes each; critique videotapes.

IV. Evaluating progress and assigning practice examples.

Day 9: Read printed materials; view model film IV.
Day 10: Microteach/tutor two pupils for twenty minutes each; critique videotapes.
Day 11: Microteach/tutor two other pupils for twenty minutes each; critique videotapes.

V. Organizing mathematics instruction for increased tutoring time.

Day 12: Read prepared materials; prepare written evaluation of minicourse; discuss evaluation with instructor.

IV. A Final Word

Reactor: Jon L. Higgins

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A FINAL WORD

Jon L. Higgins
Ohio State University

Summarizing this conference is an impossible task! But the diversity of philosophy and opinion which makes it impossible to summarize also makes the conference one of the most exciting and intellectually stimulating conferences I've attended. The invited papers by Frank Allen and Gerald Rising are as philosophically different as one can get. Allen stresses the importance of reasoning, logic and deduction in teaching. Rising, on the other hand, reminds us of the paramount importance of feeling, attitude and climate. These are polar viewpoints that cannot be satisfactorily resolved in a few minutes or in a short paper.

Let me suggest, however, that the most important goal of a methods course is to help our students develop creativity in teaching. And one mark of creative teaching is its ability to blend both reason and feeling. Perhaps an example will illustrate. Recently I watched one of our methods students tutor a seventh grader. The topic was the conversion of improper fractions to mixed numbers and vice-versa. He began by converting the fraction 13/5, pointing out in a very standard way that the fraction could be interpreted as an indicated division. The steps of the division were

$$\begin{array}{r} 2 \\ 5\,\overline{)\,13} \\ \underline{10} \\ 3 \end{array}$$

the remaining 3 must still be divided by 5, so that the answer could be written 2 3/5. But the seventh-grader seized on the word "remainder," and asked, "Do you mean that a problem like $\overset{2\text{ R }3}{5\,\overline{)\,13}}$ is the same as 2 3/5?" The tutor assured him that the answers were the same, and it was clear that the student for the first time grasped the connection between remainders and fractions. The student continued by working some practice exercises such as $\overset{2\text{ R }1}{4\,\overline{)\,9}} \longrightarrow 2\,1/4$.

Having been successful at these steps the tutor then presented the number 3 2/7 and asked how that might be converted to an improper fraction. The student's reply was, "3 2/7 is like having 3 R 2 as the answer to a division problem." After some thought he also volunteered that the divisor in the problem was the number seven. But at this point he floundered and the tutor's response was "That's not the way we work these kinds of problems," whereupon he proceeded to erase the board to begin again.

There are times when I find it impossible to be a non-intrusive observer, and this was certainly one of those times! I raced to the board and suggested that what the student had said so far might be written as: $3\,{}^{2}/_{7} \longrightarrow \begin{array}{r} 3\,R2 \\ 7\overline{)\,?\,} \end{array}$, and that all that remained was to figure out what number "belongs underneath the division sign." By moving first to a simpler division with no remainder we soon evolved a "new" procedure for converting mixed numbers to improper fractions. I left the tutor to provide some practice on this procedure for the short time remaining in the period.

I am sure that this illustration suggests an even more worthwhile lesson for our methods-course tutor than for the seventh-grade student. The philosophy that methods courses must teach can be summed up like this. Don't be afraid of what you don't know. Follow the leads provided by kids and hang on! You'll either know the thrill of victory or the agony of defeat!

What does it take to do this? It takes both reason as suggested by Frank Allen, and feeling as suggested by Gerry Rising. Our tutor failed on both counts. First, he failed to understand the importance of reversibility. If he knew that the equivalence relation was reversible, he certainly failed to suspect that the procedure might be reversible as well. At the same time he completely ignored the feelings of the student. Buoyed by the success of having discovered a connection between remainders and fractions, our seventh-grader was willing to think for himself and attempt another discovery. This feeling of success and confidence in mathematics is one of the most precious commodities a teacher can develop. Yet how easily, through a "logical" push for efficiency, we can destroy these attitudes by an insensitivity to feelings.

This necessary blend of reason and feeling underlies every creative teacher. But outstanding teaching requires even something more -- a sense of the unexpected. Think of good teachers you have known. Hasn't their appeal been found not in their predictability but in their sense of surprise and freshness? Good teaching cannot be codified to a mere set of rules. The best teachers know how to break rules, and more importantly, when to break them. I am reminded of Larry Ringenberg's use of for a variable. By all the "rules" that's the worst way to represent a variable. And yet, for a student, it may be the very best way, for it forces him to wonder about the crucial characteristics of the concept "variable." I'm sure you can provide many more examples of well-broken rules. A common one is the teacher who scrambles the work on the blackboard, but in the process sparks a class discussion of necessary sequence and steps. Or the teacher whose voice comes close to a whisper, but whose important words create an absolute silence in the classroom, with students on the edge of their chairs. Or the teacher whose blackboard mistakes generate a group search for problem solutions. The list could go on and on. But it suggests that a methods course should be less a course in teaching rules and more a course in creative rule-breaking.

Of course, if one is going to break rules it is nice to have some rules to break. That suggests an analytic, cognitive approach to the teaching process. But creativity also lies in the affective domain. This teaching creative rule-breaking requires both cognitive and affective techniques. One can feel without understanding, and understand without feeling. A creative teacher must both feel and understand. This is the ultimate challenge of a methods course. Because of it, it makes the teaching of methods a much more difficult undertaking than the teaching of mathematical content. If you have articulated some of these difficulties in this conference, you have also begun to suggest some of the solutions. The problem of designing an adequate methods course is not amenable to single-dimensioned solutions. It requires the very best work of all of us.

Appendix I

Methods Course Syllabi Submitted by Participants

Collected by: Eric A. Sturley
Nadine Verderber
R. N. Pendergrass

Southern Illinois University-Edwardsville

College or University: Eastern Illinois University

Course Title: Teaching Secondary Mathematics (Math 3400)

Objectives:

The general objective of the course is that the students be prepared to teach mathematics with some degree of competence as student teachers. Specifically the student who has completed the course should be able to:

1) discuss the general trends of mathematics teaching as indicated by the authors of the textbook and by writers in professional periodicals;

2) outline the major aims and accomplishments of the U.I.C.S.M., S.M.S.G., and at least one other experimental group;

3) write objectives in terms of expected student behavior (of the high school student);

4) demonstrate a number of techniques for gaining and keeping class interest;

5) write lesson plans and make presentations on some topics from arithmetic, from algebra, and from geometry (at least one laboratory lesson);

6) write test questions consistent with objectives;

7) prepare a unit;

8) evaluate a textbook using the criteria of the NCTM <u>Aids for Evaluators of Mathematics Textbooks</u>.

General Description:

Content of the course will include: history and trends, objectives and planning, motivation and materials, the teaching of arithmetic, testing and evaluation, the teaching of algebra, planning a unit, and teaching of geometry.

Textbook and References:

Johnson, Donovan A., and Rising, Gerald R. <u>Guidelines for Teaching Mathematics</u>. Wadsworth Publishing Company, Inc., 1972.

College or University: Elmhurst College

Course Title: The Teaching of Secondary School Mathematics (Math 466)

Topics Covered:

Current attitudes toward science and mathematics;
Controversies about the nature of mathematics;
Controversies about how mathematics should be taught;
The objectives of mathematical instruction;
Learning theory as it applies to mathematics;
The history of mathematics and its connotations for the classroom;
Teaching conditions in the schools as they pertain to mathematics;
Opportunities for professional growth for the career teacher that are provided by: a) continued study, and b) professional organizations;
The teaching process:

the teacher as the pupil's counselor, friend and guide in carefully planned learning situations,

the teacher as a motivator of learning,

the teacher as a practitioner of pedagogical exposition,

the teacher as an evaluator of the pupil's progress twoard the attainment of stated objectives.

Textbooks and References:

Aichele, Douglas B., and Reys, Robert E. Readings in Secondary School Mathematics. Boston: Prindle, Weber and Schmidt, Inc., 1971.

NCTM Commission on the Preservice Education of Teachers of Mathematics, Reston, Virginia. The National Council of Teachers of Mathematics, "The Guidelines for the Preparation of Teachers of Mathematics," 1973, pages 389-401.

The National Council of Teachers of Mathematics, "The Teaching of Secondary School Mathematics," Thirty-third Yearbook, 1970.

Additional selected readings

College or University: Central Michigan University

Course Title: Methods Course for Secondary Teachers of Mathematics (M 461)

General Description:

In addition to the topics listed below, the course includes a few components that deserve special mention:

1) emphasis on work with remedial students;

2) emphasis on being able to function in semi-structured and unstructured situations;

3) emphasis on building a portfolio of references and non-textual supplementary materials and learning activities.

Topics Covered:

Survey of secondary school mathematics

Computers and calculators

Professionalism

Logic

Textbook review

Remedial mathematics

Algebra

Geometry

Planning

Laboratory Activities

Micro-teaching

Classroom Management

College or University: Illinois Wesleyan University

Course Title: No separate methods course within department. Students enroll in Secondary School Practicum and participate in a Mathematics Teaching Problems Seminar during student teaching.

General Description:

Seminar met for at least one hour per week at the end of the public school day. Many of the sessions dealt with immediate problems encountered by the students. Areas of concern were discipline, motivation grading, checking homework, use of examples and models. Students were given reading assignments in the areas of evaluation, unit planning, methods and pedagogy, "New Math", philosophies and definitions of mathematics.

Textbook and References:

Bassler and Kolb. Learning to Teach Secondary School Mathematics. (supplementary text used within Teacher Education Department)

Freemont, Herbert. How to Teach Mathematics in Secondary Schools.

Adler, Irving. Readings in Mathematics.

Aichele, Douglass E. Readings in Secondary Mathematics.

Butler, Charles H. The Teaching of Secondary Mathematics.

Dubisch, Roy. The Teaching of Mathematics.

Dieudonne, J. "Response to Thom," American Scientist, Jan.-Feb., 1973.

Freemont, Herbert. How to Teach Mathematics in Secondary Schools.

Higgins, Jon L. Mathematics Teaching and Learning.

Johnson, Donovan A. and Rising, Gerald R. Guidelines for Teaching Mathematics.

Krulik, Stephen. A Handbook of Aids for Teaching Jr-Sr High School Mathematics.

Paige, Donald D. Mathematics Methods for Jr. High School Teachers.

Polya, George. "On Learning, Teaching, and Learning Teaching," American Mathematical Monthly, June-July, 1963.

Thom, Rene. "Modern Math: An Educational and Philosophical Error?" American Scientist, December, 1971.

Wilder, R. L. Foundations of Mathematics.

Selections from Mathematics Teacher.

College or University: Knox College

Course Title: Methods of Teaching Mathematics in Secondary Schools (Education 324)

General Description:

Concurrent enrollment in Education 390-1 (education block) is required. All students are required to be a teacher aide at least eight hours per week. There are informal seminar-type discussion periods one or two times per week. Opportunities are provided to work with and create laboratory-type materials. The course requires extensive and varied reading in the literature. Students are directed towards NCTM materials as well as standard mathematics methods texts. Emphasis is placed on critical analysis of the conflicting ideas in the literature. An attempt is made to help students see the effect of their discipline (math) on their developing philosophies of education. The education block requires students to prepare a unit which is also graded by the instructor of the methods course. Other assignments include the preparation of several types of examinations and an evaluation of the specific mathematical weaknesses of one or more students.

Topics Covered:

Why should mathematics be in the curriculum?

Why are you going to teach mathematics?

How can one teach problem-solving?

What mathematics courses are you going to teach in the year 2000?

Are there ways to make drill less drill-like?

Is mathematics an art form? If so, how does this affect the way one teaches it?

College or University: University of Missouri, Columbia

Course Title: Teaching Secondary School Mathematics (Education D128)

General Description:

A study of principles, techniques and materials for teaching secondary school mathematics:

Objectives:

As a result of diligent study and successful completion of the requirements for this course, the student should:

1) identify some characteristics of contemporary mathematics programs;

2) identify some current mathematics projects at the local, regional and national levels;

3) state some goals and objectives for secondary school mathematics programs;

4) become cognizant of some basic principles of learning mathematics, skills, attitudes and problem solving;

5) demonstrate different methods, techniques and/or strategies for developing a mathematical concept;

6) demonstrate effective use of audio-visual aids in teaching mathematics;

7) prepare an instructional unit for a mathematics topic;

8) demonstrate different ways to assess achievement in mathematics;

9) be familiar with related professional organizations and journals;

10) become familiar with a variety of instructional resources available to a mathematics teacher.

Textbook and References:

Required: Aichele, Douglas and Reys, Robert. Readings in Secondary School Mathematics. Boston: Prindle, Weber and Schmidt, Inc., 1971.

Johnson, Donovan and Rising, Gerald. Guidelines for Teaching Mathematics. Belmont, California: Wadsworth Publishing Company, Inc., 1972, 2nd edition.

Optional: Sobel, Max and Maletsky, Evan. Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies. Englewood Cliffs: Prentice-Hall, Inc., 1975.

College or University: Northeastern Illinois University

Course Title: Methods of Teaching Mathematics in Secondary School
(Secondary Education 72-301)

Topics Covered:

The teaching of secondary mathematics in perspective.

Planning a lesson and a unit

Developing teaching skills.

Teaching mathematical concepts and principles.

Developing mathematical skills.

Analyzing teaching.

Adapting instruction.

Objectives:

Students will be able to:

1) compare and contrast the philosophical and psychological foundations of the so-called "modern mathematics" programs with the so-called "traditional" mathematics programs;

2) discuss the pros and cons of the various models that are used in the teaching of mathematics;

3) use the knowledge of moves for teaching concepts and the moves for teaching principles to analyze various teaching strategies;

4) use the knowledge of concept moves and principle moves to develop teaching strategies;

5) discuss the similarities and differences between strategies to be used as an aid in the development of problem solving processes;

6) propose teaching strategies geared specifically to fast-learning students, slow-learning students, and inner-city students;

7) propose strategies for teaching junior high school mathematics, general mathematics, algebra, and geometry;

8) propose strategies that evidence an awareness and concern for the affective domain as well as the cognitive domain;

9) prepare lesson and unit plans in which the objectives will be stated, subject matter will be chosen, and strategies for teaching the subject matter will be identified;

10) prepare an evaluation instrument that can be used to determine student progress with respect to the stated objectives;

11) utilize at least one strategy to effectively teach a mini-lesson;

12) discuss alternative strategies that could be used to teach the subject matter presented in a mini-lesson;

13) discuss the values, assumptions, and attitudes which were apparent in the presentation of a mini-lesson;

14) discuss the presentation of a mini-lesson in terms of a teaching skill identified and discussed;

15) work with secondary students and their teachers through the field experience program;

16) relate their experiences in the field to the discussions in class;

17) apply the knowledge they have gained about themselves and teaching when tutoring students or teaching classes during their field experience;

18) re-evaluate and modify, if a need is felt, their philosophy on the basis of the new knowledge or insights gained as a result of this course.

Textbooks and References:

Johnson and Rising. Guidelines for Teaching Mathematics, 2nd ed.

Aicheles and Reys. Readings in Secondary School Mathematics (Optional).

NCTM Yearbooks and other selected readings by topic.

College or University: Northern Illinois University

Course Title: Curriculum and Instruction in Secondary School Mathematics

Topics Covered:

Why is mathematics in the secondary school curriculum?

Lesson planning, unit planning.

What is mathematics?

Behavioral objectives and taxonomies.

Multi-sensory aids.

Strategies.

Discovery learning, lab lessons.

Review of logic and proof.

Teaching selected general mathematics topics, grades 7-9.

Teaching selected algebra topics.

Teaching problem solving.

Teaching selected geometry topics - survey of geometric knowledge.

Writing tests, scoring tests - evaluation, in general.

Homework - why and how to use.

Control.

The Slow Learner.

Electronic Tools.

Micro-teaching.

Article reports.

Preparation of material for overhead projector.

Compilation of games/activities/puzzle file.

Vocabulary and spelling (sic) quizzes.

Teaching "situations" (_a la_ Bassler and Kolb).

Approaches to trigonometry.

Encouraging creativity.

Textbooks and References:

Johnson and Rising

Sobel and Maletsky (required reading)

Ideas from Bassler and Kolb.

College or University: Southern Illinois University, Carbondale

Course Title: Methods of Teaching Secondary Mathematics (Math 311)

General Description:

Students meet as a group during class sessions but the group divides in half for lab sessions. Class periods are devoted to discussion of text material and outside reading concentrating on psychology of learning mathematics and classroom management. Local school administrators and area mathematics teachers are guest speakers throughout the course. Concurrent with registration in Mathematics 311, each student registers for a general methods course offered by the Department of Secondary Education. Included in this course is a half-day per week internship in an area school. The final project for Math 311 is a detailed planning of a unit to be taught during the student teaching assignment the following semester. The lab sessions consist of student presentations.

Textbook and References:

Johnson, Donovan and Rising, Gerald. Guidelines for Teaching Mathematics. Wadsworth Publishing Company, 2nd Ed., 1972. (textbook)

Hankon, Aaron. Meaningful Mathematics Teaching.

Polya, George. How to Solve It.

Aichele and Reys. Readings in Secondary School Mathematics.

Several of the following (substitutions allowed):
Kline, Morris. Why Johnny Can't Add.
Holt, John. How Children Fail.
Biggs, Edith and Maclean, James. Freedom to Learn.
Gray, Jenny. The Teacher's Survival Guide.
Gordon, Thomas. Teacher Effectiveness Training.
Wilder, Raymond. Evolution of Mathematical Concepts.

College or University: Southern Illinois University, Edwardsville

Course Title: Teaching of Secondary Mathematics (Math 311)

Topics Covered:

History of Mathematics Education in the U.S.

Recent Developments in Mathematics Education.

Teaching Arithmetic.

Number bases, number systems.

Intuitive Geometry for the Junior High School.

Probability and Statistics in the Junior and Senior High Schools.

Logic and Sets.

Expanding the number system (irrationals, transcendental numbers, complex numbers).

High School Algebra (groups, fields, rings).

High School Geometry (and analytic).

Trigonometry, periodic functions.

Calculus.

General Teaching Methods.

Preparation of Tests.

General Description:

Students gave reports on the writings of Locke, Whitehead, Russell, Thorndike, Middlekauff, Gnedenko, Khinchin, Kline, Piaget, Fehr, Meserve, Beberman, Wertheimer, Bruner, and Gagne. Others reported on differences between old and new textbooks, visual aids, evaluation, lesson planning, programmed instruction, functions, the slow learner and non-verbal communication.

Textbook and References:

Willoughby, Stephen S. Contemporary Teaching of Secondary School Mathematics, John Wiley and Sons, 1967.

Additional readings from Mathematics Teacher, Arithmetic Teacher, NCTM Yearbooks, NEA Journal, and School Science and Mathematics.

APPENDIX II

ICME CONFERENCE ON DESIGNING A METHODS COURSE FOR SECONDARY SCHOOL MATHEMATICS TEACHERS

Conference Participants	Schools
Sister Nona Mary Allard	Rosary College
Milton Beckmann	University of Nebraska
Don Bunt	Chicago State University
Romona Choos	Chicago State University
Gerry Chrisman	Northern Illinois University
Dan Cozzo	Downers Grove S. H. S.
Don Devine	Western Illinois University
Lyman Holden	Southern Illinois University
Pat Hooper	Western Kentucky
Douglas Jones	McKendree College
Herb Kaufmann	Western Illinois University
Henry Kepner	University of Wisconsin
Herb Miller	Northern Illinois University
Douglas Nance	Central Michigan University
Mel Poage	Central Michigan University
Meredith Potter	Rockford College
William Purcell	Chicago State University
Peter Reichmann	University of Illinois, Circle
Garth Runion	Illinois State University
Larry Sowder	Northern Illinois University
Eric Sturley	Southern Illinois University
Hazel Wagner	Northwestern University
John Wheeler	University of Wisconsin
Frank Young	Knox College
Ray Zike	Downers Grove S. H. S.
Roger Chilewski	St. Xavier College
Ron Sandstrom	Illinois Wesleyan University
Lotus Hershberger	Illinois State University
Peter and April Dennis	Kirkland High School
Fred Floner	Northeastern Illinois University
Dorothy Diericky	Chicago State University
Terence Perciante	Wheaton College
Lewis H. Coon	Eastern Illinois University
Margariete Montague	Northern Illinois University
Kenneth Cummins	Kent State University
Bernadette Perham	Chicago State University
Richard Koog	Trinity Christian
Peter Pereina	DePaul University
Jeff Barnette	Northern Illinois University
Jack E. Forbes	Purdue-Calumet
Richard D. Crumley	Illinois State University
Eileen Clark	Illinois Benedictine College

I.C.M.E. Board

A. I. Weinzweig (President)	University of Illinois, Circle
Clarence Hardgrove	Northern Illinois University
John Schumaker	Rockford College
Katherine Pedersen	Southern Illinois University-Carbondal
Ross Finney	University of Illinois-Urbana
John Peterson	Eastern Illinois University
Chuck Dietz	Concordia Teachers College

Conference Committee

Chuck Dietz (Conference Chairman)	Concordia Teachers College
Clarence Hardgrove (Program)	Northern Illinois University
Jim Lockwood (Publicity)	Northeastern Illinois University
Norm Young (Arrangements)	Concordia Teachers College
Gary Knippenberg (Registration)	Concordia Teachers College
John Schumaker (Proceedings)	Rockford College
Meredith Potter (Proceedings)	Rockford College

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