

The survey data indicated that all the junior or community colleges studied, offer and desire to continue the offering of technical mathematics courses. The most significant finding of the survey was the indication that junior and community colleges prefer technical or applied mathematics for the vocational-technical students and that technical mathematics curricula research should be conducted toward developing a mathematics curriculum relevant to the needs of the community college student.

The following conclusions were made from the survey of ten participating junior or community colleges.

1. A study should be conducted to identify mathematical concepts requisite for job-entry competencies for vocational-technical occupations.
2. The junior or community colleges contacted were willing to participate in such a study.
3. Applied or technical mathematics programs are more valuable to the vocational-technical students than academic or traditional mathematics programs.
4. Curriculum materials for applied or technical mathematics programs need to be developed in relation to specific vocational-technical program areas.

Chapter II - Methods

Goals & Objectives

This research project was designed to develop a base for an applied or technical mathematics curriculum which would meet the needs of vocational-technical students. The major objectives accomplished during the term of project include:

1. Identifying mathematical concepts requisite for entry-level competencies in nine selected vocational-technical occupations. (Welding, Radio & T.V. Repair, Machine Shop, Diesel Mechanics, Printing, Drafting and Design, Machine Shop, and Electronics.)
2. Developing the identified mathematical concepts into an organized hierarchy of units of competencies for vocational-technical students; and
3. Developing curriculum materials, self-instructional learning packets, and pre- and post-tests for selected sequential units.

Design and Procedures

Vocational-technical programs to be used in this research project were identified in the Texas Education Agency Table, "A challenge and a Choice". Realizing that consideration of all 117 occupational programs was beyond the scope of limited research, the project included only those Industrial Occupation programs and Technical Occupation programs offered by a minimum of 12 institutions listed in the "A Challenge and a Choice" table.

Those programs selected for use in the research project and the number of institutions offering such programs are given in TABLE 1.

TABLE 1
NUMBER OF INSTITUTIONS OFFERING SELECTED VOCATIONAL PROGRAMS

Occupational Program	Number of Institutions Offering Program
Air Conditioning and Refrigeration	26
Auto Mechanics	33
Diesel Mechanics	11
Printing	12
Radio and T.V. Repair	18
Electronics Technology	29
Drafting and Design Technology	37
Machine Shop	18
Welding	28

The research project was divided into three distinct phases:

1. Identifying mathematical concepts requisite for job-entry competencies in each of nine selected occupational areas;
2. Arranging these mathematical concepts into structures of sequential units; and
3. Developing curriculum materials representing these prescribed units.

Texas Community Colleges offering the various technologies were contacted and presented a brief outline of the project. The thirty-five texts used in teaching technical mathematics courses in Texas Community Colleges were identified (See Appendix B). The project staff used these texts as guides to construct a list of mathematical competencies. Each competency statement was examined to eliminate ambiguity and duplication. Based upon the competency listings, the staff constructed a research instrument in Likert (Guilford, 1964) scale form. The Likert instrument listed the

mathematical competencies according to their skill levels including basic arithmetic, basic algebra, advanced algebra, trigonometry, and introductory calculus.

Within each level competencies were arranged in sequential order from simplest to most difficult. This arrangement provided a structured, hierarchical taxonomy of technical mathematics units that were of potential need to the technical-vocational student in the various occupational areas. (See Appendix C).

Staff members visited 10 community college in August, 1974, and carefully explained the purpose of the project. The 10 community colleges included: Del Mar Junior College, Bee County College, San Jacinto College, Alvin Junior College, St. Phillip's College, San Antonio Junior College, El Paso Community College, Eastfield Community College, Lamar University (College of Technical Arts), and Brazosport College. Each community college agreed to participate in the project and offered their administrators, teachers and advisory board members as a jury to validate the competencies identified in the instrument. The institutions were selected to represent a uniform distribution of participants from all areas of Texas. The institutions were sincerely enthusiastic about the project, as was Joe Godsey, Director of Post-Secondary Occupational Education at the Texas Education Agency, the state funding agency for Texas Vocational Programs.

The staff consulted Dr. Beryl McKinnerney, Associate Dean of Occupational Education at El Paso Community College, to further refine the competency instrument. Staff members then visited El Paso Community College and conducted the first jury validation of competencies in September, 1974. The jury commented favorably on the completeness of the instrument and

decided, under Dr. McKinnerney's supervision, to make helpful suggestions to further refine the instrument.

Staff members visited the College of Technical Arts at Lamar University and conducted the second jury validation of competencies. After further refinement, visitations were made to the remaining participant institutions, where the instrument was explained to the jury members and left with them to be completed at their convenience and later mailed to the project director.

Of the 105 jury members contacted, a total of 95 returns were received representing 87 useable responses. A profile of the jury members returning useable responses was made for each of the vocational areas. The profiles are summarized in TABLE 2, which indicates the mean number of years of teaching experience and/or industrial experience for jurors in each vocational area.

TABLE 2
PROFILE OF JURY MEMBERS

Vocational Area	Average Number of Years Teaching Experience	Average Number of Years Industrial Experience
Air Conditioning	6.9	18.4
Auto Mechanics	10	18.3
Diesel Mechanics	7.1	17.3
Drafting	9	7
Electronics	5.4	7.1
Machine Shop	10.3	13.1
Printing	7.5	8.7
Radio-T.V. Repair	5.2	14.8
Welding	5.6	12.6

All jury responses were tabulated onto a master sheet and a mean was calculated for each competency. The cut-off point for significance on the five-point Likert scale was chosen as a mean of four, reflecting agreement of jury members. A master taxonomy of the significant competencies was then composed reflecting the jury ratings. Additional taxonomies were composed for each occupational area. These were submitted to a jury for review and recommended changes and revisions were made.

Five related competencies were selected which were rated as necessary in a majority of the occupational areas. The five concepts dealing with fractions included: least common denominator, addition, subtraction, multiplication, and division. The researchers then developed self-instructional mini-module packets for each of these competencies (See Appendix D). Each packet included pre- and post-tests, and a set of audio tapes with problem workbooks and a programmed instruction unit with a set of exercises modeled after Flexer and Flexer (1971) and O'Malley and Polya (1963).

Following the initial development of the self-instructional packets, they were pilot tested at El Paso Community College, using Dr. Beryl McKinnerney as consultant, plus consultants Dr. Walter Stenning and Ms. Peggy Cooper-Harding of Texas A&M University. The modules were also pilot tested at Lamar University, College of Technical Arts, using Dr. Ken Shipper as consultant. At both schools, students were asked to complete a questionnaire which would give the researchers the reactions of the user concerning the usefulness of and the attitudes toward the individualized units.

Upon completion of the pilot tests, the materials were revised and a set of self-instructional packets was completed. These were pilot tested another time at El Paso Community College to determine the feasibility of further revision.

The last stage of the research project included the development of curriculum guides for the competencies rated as necessary for the various occupational areas. The curriculum guides were written following the sequencing indicated by the master taxonomy list and reflect observable student behaviors. Following the advice of curriculum experts, the curriculum guides do not reference any specific texts. The concensus of opinion was that this would lend greater flexibility to the implementation and continued use of guides (See Appendix E).

Chapter 3 - Results

The results of the project include:

1. A composite list of mathematical concepts requisite for entry-level competencies in the selected vocational areas;
2. A sequentially ordered taxonomy of mathematics concepts required to meet the needs of vocational-technical students in each of the ten selected vocational-technical areas with relative ratings by jurors for each occupational area;
3. Self-instructional learning packets for five related technical mathematics concepts;
4. Performance-based pre- and post-tests for the learning packets;
5. Curriculum guides for technical mathematics units in Basic Arithmetic, Basic Algebra, Advanced Algebra, and Trigonometry.

The composite and individual taxonomies were revised several times after the initial draft had been compiled. The revisions allowed the staff to update figures due to late arrival of some jurors' rating instruments. The revisions also led to a more readable format as suggested by consultants.

In order to check the effects of the self-instructional packets, each student participating in the pilot test was given a pre- and post-test over materials covered in the module he had worked. A t-test (Steel and Torrie, 1960) was used to test the null hypothesis $H_0: \mu_1 = \mu_2$ where μ_1 = the mean of pre-test scores and μ_2 = the mean of post-test scores. The test statistic used was $t = \frac{\bar{d}}{s_{\bar{d}}} = \frac{(\bar{x}_1 - \bar{x}_2)}{s_{\bar{x}_1 - \bar{x}_2}}$ since $\bar{d} = \bar{x}_1 - \bar{x}_2$. The t value corresponding to Student's t, for 61 degrees of freedom showed a highly significant difference in pre- and

post-test scores. With $\bar{x}_1=4.4$ and $\bar{x}_2=6.2$, results revealed that the two means differed at a probability level of $p<.001$.

The information on the student opinion questionnaire revealed that 79 percent of the students would use the modules, 14 percent might use them, and only 7 percent would not use the modules if they were available.

Students who found mathematics texts difficult to understand were interested in the possibility of widespread availability of cassette tapes. They were excited about being able to repeat parts of the tape if the need was felt. At El Paso Community College, the Mexican Americans were very enthusiastic over the tape aspects of the modules and suggested that translation into regional Spanish might be beneficial for those minority groups.

The technical mathematics units and curriculum materials will be made available to the ten participating junior or community colleges plus other institutions both on a state and national basis upon request for implementation into their vocational-technical programs. The structured technical mathematics program, if implemented by community colleges offering vocational-technical programs, can serve as a firm basis for granting transferability of technical mathematics into similar four-year college or university programs. The findings of the project also have implications for curriculum development in other occupational areas.

A technical mathematics curriculum utilizing self-instructional models provides vocational students the opportunity to proceed at their own rate. The students can improve a weak mathematical background as well as develop mathematical competence to meet the pre-requisites of occupational programs.

Chapter 4 - Conclusions

This research project focused on identifying technical mathematics curriculum and guides specifically relevant to vocational-technical programs in Texas community colleges. The research will help solve current articulation problems, as well as improve existing vocational-technical programs.

When implemented, results of the research should benefit vocational-technical students, teachers, and employees by lowering the attrition rate of vocational-technical students in Texas community colleges. The taxonomies, curriculum guides and mini-modules can serve as a guide for developing full-size curriculum modules in selected vocational areas.

The structured, sequential technical mathematics curriculum will help in solving the articulation problems the vocational-technical student currently encounters in transferring between programs. The structured technical mathematics program, if implemented by all community colleges offering vocational-technical programs, can serve as a firm basis for granting transferability of technical mathematics into similar four-year college or university programs.

A technical mathematics program which utilizes self-instructional modules permits the vocational students to proceed at their own rate. These modules allow students to improve a weak background in mathematics as well as develop mathematical competencies to meet the pre-requisites of their occupational programs.

The mathematical concept taxonomies and all materials that were pilot tested in this research project are now available to Texas community colleges. The taxonomy information can be used as a model to

develop and test further taxonomies of mathematical competencies for any other occupational areas not covered in this research.

The results of the pilot tests showed that the self-instructional approach can be successful for the vocational-technical student. The success of the pilot tests and the favorable attitudes of both instructors and students toward the self-instructional modules suggest that further research into the mathematical competencies for vocational-technical programs in Texas community colleges should be undertaken in the form of more comprehensive curriculum modules and guides not only for the nine occupational areas used for this project but for other occupational areas as well.

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APPENDIX A

Participants in the Community College Technical Mathematics Project

Investigators and Consultants

Institutions

INDIVIDUALS PARTICIPATING IN THE
COMMUNITY COLLEGE TECHNICAL MATHEMATICS PROJECT

Project Investigators

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OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
4. Calculations of a triangle and auxillary lines:					X	X	X	X	
5. Calculations of a circle:		X			X	X	X	X	
6. Finding the diameter of a circle when the area is given:		X			X	X		X	
7. The perimeter of the ellipse:					X			X	
8. The perimeter of polygons:								X	
<u>Area</u>									
1. As a measure of surface space:				X	X			X	
2. Calculations of four-sided figures (Parallelogram, trapezoid, square, and rectangle):				X	X			X	
3. Area calculations for a circle:		X		X	X	X		X	
4. Relationship of radius, diameter, circumference arcs, sectors, chords, and tangents:					X	X		X	
5. The properties of π					X	X		X	X
6. Area of a regular hexagon:					X				
7. Regular polygon:								X	



OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
<u>Volume</u>									
1. Measures of volume:		X		X	X		X	X	
2. Volume determined by area:				X	X			X	
3. Volume of rectangular solids (cubes):					X			X	
4. Volume of a sphere:								X	
5. Segments and zones of spheres:								X	
6. Volume of a cylinder:		X						X	
7. Surface volume of solid and hollow cylinders:								X	
8. Heights of cylinders:								X	
9. Lateral surface of cylinders:								X	
10. Volume of compound figures:								X	
11. Volume of composite figures:								X	
<u>Weight</u>									
1. Measures of mass:		X			X	X	X		
2. Specific gravity:		X		X		X			
3. Density:		X		X		X			
4. Weight standards:		X		X	X	X	X		

OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
5. Weight of materials:		X		X	X		X	X	
6. Weight of castings from patterns:		X							
<u>Time</u>									
1. Units of time:	X		X			X	X	X	X
2. Distance in relations to time:	X		X			X		X	X
3. Frequency:	X		X	X		X	X		X
4. Period:	X		X			X	X		X
<u>Money Calculations</u>									
1. Price computation:		X	X	X	X		X	X	
2. Income computation:		X	X	X	X		X	X	
3. Interest computation:		X	X		X		X	X	
<u>Scale Drawings</u>									
1. Finding the drawing measure:					X	X		X	
2. Finding the actual dimensions of an object from the measured dimension of a diagram:					X	X		X	
3. Finding the area of a figure drawn to a certain scale:					X	X		X	

OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
<u>Slide Rule</u>									
1. Reading the C and D scales:			X					X	
2. Slide rule operations (basic):			X				X		
3. Laws of exponents:			X						
4. Reciprocal, squares and square roots:			X						
5. Cubes and cube roots:			X						
6. Locating the decimal point:			X					X	
7. Combination problems:								X	
8. Extracting roots of numbers and raising powers:			X						
9. Cube roots:			X						
10. Multiplying and dividing numbers by π :			X						
11. The reciprocal scales:			X						
12. The tangent scale:			X						
13. The ARC tangent of a number:			X						
14. The sine scale:			X						
15. The ARC sine of a number:			X						
16. The ST scale:			X						
17. The cosine:			X						

OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
18. The cotangent:			X						
19. Proportion scales:			X						
20. Finding the circumference and area of a circle:								X	
<u>Geometry</u>									
1. Straight line & circle:						X		X	X
2. Polar coordinates:									X
<u>Geometrical Construction</u>									
1. Bisect or find mid-point of a line:								X	
2. Bisect an angle:								X	
3. Bisect an arc:								X	
4. Erect a perpendicular to a line at a given point on the line:								X	
5. Erect a perpendicular to a line at the end of the line:								X	
6. Erect a perpendicular through a point not on the line:								X	
7. Draw a line through given points parallel to a given line:								X	
8. Divide a given line into a number of equal parts:								X	

OCCUPATIONAL AREA

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
9. Draw an angle equal to a given angle:								X	
10. Draw an equilateral triangle of given size:								X	
11. Draw a circle through three given points:								X	
12. Find the center of a circle or circle arc:								X	
13. Inscribe a square in a circle:								X	
14. Construct a square of given size:								X	
15. Inscribe a hexagon in a circle:								X	
16. Inscribe an equilateral triangle in a circle:								X	
17. Construct a hexagon with one of the sides on a given line:								X	
18. Draw a tangent to a circle at a point on the circumference:								X	
19. Draw a tangent to a circle through a point outside the circle:								X	
20. Draw a tangent to two circles of equal size:								X	

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
21. Draw an internal tangent to two circles of equal size:								X	
22. Draw an external tangent to two circles of unequal size:								X	
23. Draw an internal tangent to two unequal circles:								X	
BASIC ALGEBRA									
<u>General Numbers</u>									
1. Properties of real numbers:					X				X
2. Substitution of variables into algebraic expressions:						X		X	X
3. Translation of word expressions into algebraic expressions:					X	X		X	X
4. Order of operations:			X			X		X	X
5. Evaluating expressions containing variables:			X		X	X		X	X
6. Basic operations involving variables:			X			X		X	X
7. Constants and variables:						X		X	X
8. Positive and negative numbers:			X			X		X	X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
9. Powers and exponents:			X		X	X		X	X
10. Absolute value:								X	
11. Formulas:			X			X		X	X
12. Ratio and proportion:					X	X		X	X
<u>Arithmetic Operations with Algebraic Expressions</u>									
1. Basic operation with algebraic numbers:			X		X	X		X	X
2. Operations with signs of grouping:			X			X		X	X
3. The distributive law:			X			X			X
4. Equations involving parentheses:						X		X	X
5. Equations as symbolic sentences:					X	X		X	X
6. Removing parentheses used to indicate multiplication					X	X		X	X
7. First degree equations with signed numbers:					X	X		X	X
8. Adding and subtracting like terms:					X	X		X	X
9. Adding and subtracting polynomials			X		X	X		X	X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
10. Multiplication and division of monomials and polynomials:					X	X			X
11. Inserting signs of grouping:						X		X	X
12. Rules of exponents:			X		X	X		X	X
13. The square of a binomial sum or difference:			X		X	X			X
14. The product of a binomial sum or difference:			X		X	X			X
15. Factoring the difference of two squares:			X						X
16. Factoring out of common factors:							X		X
17. Factoring polynomials:									X
18. Multiplication by a common factor:					X	X		X	X
19. Special products or expansions:					X				
20. Extracting square roots:			X		X	X			X
21. Factoring monomials from polynomials:									X
22. Symbols of grouping:					X	X		X	X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
23. Squaring binomials:									X
<u>Algebraic Fractions</u>									
1. Properties of algebraic fractions:						X			X
2. Equations involving fractions:						X		X	X
3. Basic operations with algebraic fractions:						X			X
4. The fractional exponents:									X
5. Finding the least common denominator:						X		X	
6. Reducing to lower terms:						X			
<u>Inequalities</u>									
1. Properties of inequalities:						X			
<u>Equations</u>									
1. First degree equations in one and two variables:						X		X	
2. Literal equation formulas:						X		X	
3. Equivalent, identical, and conditional equations:								X	

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
4. Simplification of equations:						X		X	
5. Solution of equations in variables:						X		X	
6. Analysis of a formula:								X	
7. Transposition of terms:								X	
8. Transposition of factors:								X	
9. Translating problems into equations						X		X	
10. Operations with dimensional units:						X		X	
11. Solving systems of equations of two and three unknowns, algebraically:								X	
12. Dimensional units and conversion:						X		X	
13. Equations containing fractions and decimals:						X		X	
14. Degree of an Equation:								X	
15. Solving linear equations:						X		X	
16. Algebraic solutions to variables:								X	
17. Equations with unknowns in the denominators:								X	

OCCUPATIONAL AREA									
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
18. Proportions:						X		X	
19. Proportionality:						X		X	
<u>Linear Graphs</u>									
1. Coordinate notation:						X		X	X
2. Plotting points on graphs:						X		X	X
3. Graphs of ordered pairs:						X		X	X
4. Displaying data with graphs:						X		X	X
5. Solving problems by means of graphs:						X		X	X
6. Graphs of linear equations:						X		X	X
7. Determining a linear function from two points:								X	
8. Graphing parallel lines:								X	
9. Graphing perpendicular lines:								X	
10. Graphing lines parallel to the axis:								X	
11. Graphing solutions of simultaneous equations:								X	
12. The slope of a line and Y intercept:								X	

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
13. The distance formula:								X	
14. Graphic solution of system of linear equations:								X	
15. Solution by addition and subtraction:								X	
16. Applications of simultaneous equations:								X	X
<u>Quadratic Equations</u>									
1. Solution of $ax^2 + bx + c = 0$:								X	X
2. Solution by the quadratic formula:								X	
<u>Exponential Notation</u>									
1. Laws of basic operations with exponents:			X					X	X
2. The law of zero exponents:								X	X
3. The law of exponents in raising to powers:			X					X	X
4. The law of exponents in extracting roots:			X					X	X
5. The law of exponents based on 10:			X					X	X
6. Power variations:			X						X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
7. j-operator:			X						X
8. The one operator:			X						X
9. Complex numbers:			X						X
10. Complex equations:			X						X
11. Squares of numbers:			X						X
<u>Radicals</u>									
1. Irrational numbers:									X
2. The square root of positive numbers:							X		X
<u>Logarithmic Notation</u>									
1. Definition of a Logarithm:							X		
2. Properties of logarithms:									X
3. Characteristics:									X
4. Mantissa:									X
5. Determining logs from a table:							X		X
6. Finding powers using logarithms:									X
7. Finding roots using logarithms:									X
8. Interpolation using logarithms:									X
9. The logarithm of unity:							X		
10. Given log n, find n							X		X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
6. Amplitude:									X
7. Frequency:									X
8. Addition and sinusoidal waves:									X
<u>THE D.C. CIRCUIT</u>									
1. Difference of potential and electric current:		X	X	X					X
2. E.M.F. and energy considerations:		X	X	X					X
3. Ohm's law:		X	X	X					X
4. Voltage drops:		X	X	X					X
5. Energy and power dissipated in a circuit:		X	X	X					X
6. Series and parallel circuits; equivalent resistance:		X	X	X					X
7. Direction of current:		X	X						X
8. Kirchhoff's rules:		X	X	X					X
9. Battery E.M.F. and terminal voltage -- internal resistance:		X	X						X
10. The Wheatstone bridge:		X	X	X					X
<u>FUNDAMENTAL ELEMENTS IN AN A.C. CIRCUIT</u>									
1. Capacitance:			X	X					X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
2. Capacitors in series:			X	X					X
3. Capacitors in parallel:			X	X					X
4. Energy requires to charge a capacitor:			X	X					X
5. Time for charging and discharging a capacitor; the time constant:			X						X
6. Inductance:			X	X					X
7. Time constant of an L-R circuit:			X						X
8. Phase relation in A.C. circuits:			X	X					X
9. The series A.C. circuits phase diagram:			X	X					X
10. Reactance and impedance:			X	X					X
11. Resonance:			X						X
12. Power and power factor:			X	X					X

Diesel Mechanics

	<u>μ V</u>	<u>μ M</u>	<u>μ A</u>	<u>μ tot</u>	<u>%4-5</u>
BASIC ARITHMETIC					
<u>Real Numbers</u>					
1. Basic Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
2. Combined Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
3. Definition of Signed Numbers:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>67%</u>
4. Basic Operations with Signed Numbers:	<u>3</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>67%</u>
5. Rounding Off:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
6. Prime Numbers:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>67%</u>
7. Composite Numbers:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>83%</u>
8. Greatest Common Diviser:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>67%</u>
9. Least Common Multiple:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>83%</u>
10. Absolute Value:	<u>3</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>67%</u>
<u>Common Fractions</u>					
1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Reading a Ruler:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Basic Operations with Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Equivalent Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Converting Mixed Numbers to Improper Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Basic Operations with Mixed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
7. Combined Operations with Mixed Numbers:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
8. Proper and Improper Fractions:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
9. Combining Like Terms:	<u>3</u>	<u>5</u>	<u>5</u>	<u>4.3</u>	<u>83%</u>
10. Reducing to Lowest Terms:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
11. Finding the Lowest Common Denominator:	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>4.8</u>	<u>100%</u>
12. Basic Operations with Common Fractions and Mixed Fractions:	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>4.8</u>	<u>100%</u>

Decimal Fractions

1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Rounding Off:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Basic Operations:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Table of Decimal Equivalents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Fraction to Decimal Conversion:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Decimal to Fraction Conversion:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Percentage

1. Definition:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Converting Fractions to Percent:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Converting Percents to Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Converting Decimals to Percents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Converting Percents to Decimals:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Finding the Percentage Given the Base and Rate:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
7. Finding the Rate Given the Percentage and Base:	<u>4.8</u>	<u>5</u>	<u>4.5</u>	<u>4.8</u>	<u>100%</u>
8. Finding the Base Given the Rate and Percentage:	<u>4.8</u>	<u>5</u>	<u>4.5</u>	<u>4.8</u>	<u>100%</u>

	μV	μM	μA	μ_{tot}	%4-5
9. Solving Problems:	5	5	4	4.7	100%
10. Price List and Discount:	4.5	5	5	4.8	83%
11. Percentage Error and Averaging:	4.3	5	5	4.8	83%
12. Markup:	4.5	5	3	4.2	67%

Ratio and Proportion

1. Definition of Ratio:	4.5	5	4	4.5	83%
2. Reduction of Ratio to Lowest Terms:	5	5	3	4.3	83%
3. Definition of Proportion:	5	5	3	4.3	83%
4. Finding the Unknown Term in a Proportion:	5	5	3	4.3	83%
5. Application of Proportion:	5	5	3	4.3	83%
6. Principle of Levers:	4.8	5	3	4.3	83%
7. Principle of Pulleys:	4.8	5	5	4.9	100%
8. Principle of Gears:	5	5	5	5	100%
9. Principle of Hydraulic Press:	5	5	5	5	100%
10. The English System of Weights and Measures:	5	5	5	5	100%
11. The Metric of Weights and Measures:	4.5	5	5	4.8	83%
12. English-Metric Conversion:	4	5	5	4.7	67%
13. Multiplying and Dividing Measurements:	4.3	5	4	4.4	83%
14. Tables of Equivalent Measurements:	4	5	5	4.7	67%

Instruments of Measuring

1. Linear Scales:	4.3	5	5	4.8	83%
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	μV	μM	μA	μtot	%4-5
2. Vernier Calipers:	<u>4.3</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
3. Micrometers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. The Protractor:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>71%</u>
5. The Tachometer:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Length

1. Basic Operations Using Length:	<u>3</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>83%</u>
2. Length as a Measure of Distance:	<u>3</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>83%</u>

Weight

1. Measures of Mass:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>72%</u>
2. Specific Gravity:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>86%</u>
3. Weight Standards:	<u>4</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>72%</u>
4. Weight of Materials:	<u>5</u>	<u>5</u>	<u>2.5</u>	<u>4.2</u>	<u>72%</u>

Time

1. Units of Time:	<u>4.5</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>83%</u>
2. Distance in Relation to Time:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>67%</u>
3. Frequency:	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>67%</u>

Work and Power

1. Work and Power:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
2. Horsepower of a Steam Engine:	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>72%</u>
3. Horsepower of a Gas Engine:	<u>5</u>	<u>4</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
4. Brake Horsepower:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Electrical Power -- Watt:	<u>4.8</u>	<u>4</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
6. Mechanical Efficiency of Machines:					<u>100%</u>

Auto Mechanics

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	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
BASIC ARITHMETIC					
<u>Real Numbers</u>					
1. Basic Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Combined Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Definition of Signed Numbers:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
4. Graphing Signed Numbers:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>88%</u>
5. Basic Operations with Signed Numbers:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
6. Rounding Off:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
7. Prime Numbers:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>88%</u>
8. Composite Numbers:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>75%</u>
9. Greatest Common Divisor:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
10. Least Common Multiple:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
11. Absolute Value:	<u>4.3</u>	<u>4.5</u>	<u>5</u>	<u>4.6</u>	<u>88%</u>

Common Fractions

1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Reading a Ruler:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Basic Operation with Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Equivalent Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Converting Mixed Numbers to Improper Fractions:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
6. Basic Operations with Mixed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
7. Combined Operations with Mixed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
8. Proper and Improper Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
9. Complex Numbers:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>88%</u>
10. Combining Like Terms:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
11. Reducing to Lowest Terms:	<u>4.8</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
12. Finding the Lowest Common Denominator:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
13. Basic Operations with Common Fractions and Mixed Fractions:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>

Decimal Fractions

1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>88%</u>
2. Rounding Off:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>88%</u>
3. Basic Operations:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>88%</u>
4. Fraction to Decimal Conversion:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>88%</u>
5. Decimal to Fraction Conversion:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>88%</u>

Percentage

1. Definition:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
2. Converting Fractions to Percent:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
3. Converting Percents to Fractions:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
4. Converting Decimals to Percent:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
5. Converting Percents to Decimals:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
6. Finding the Percentage Given the Base and Rate:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
7. Finding the Rate Given the Percentage and Base:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
8. Finding the Base Given the Rate and Percentage:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
9. Solving Problems:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
10. Price Lists and Discount:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
11. Percentage Error and Averaging:	<u>4.3</u>	<u>5</u>	<u>3</u>	<u>4.1</u>	<u>63%</u>
12. Markup:	<u>4.3</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>75%</u>

Ratio and Proportion

1. Definition of Ratio:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
2. Reduction of Ratio to Lowest Terms:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
3. Definition of Proportion:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
4. Finding the Unknown Term in a Proportion:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
5. Application of Proportion:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>75%</u>
6. The Principle of Levers:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
7. Principle of Pulleys:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
8. Principle of Gears:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
9. Principle of Hydraulic Press:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
10. The English System of Weights and Measures:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>88%</u>
11. The Metric of Weights and Measures:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>88%</u>
12. English-Metric Conversion:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>88%</u>
13. Multiplying and Dividing Measurements:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>100%</u>
14. Tables of Equivalent Measurements:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
<u>Instruments of Measuring</u>					
1. Linear Scales:	<u>4</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>75%</u>
2. Vernier Calipers:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>75%</u>
3. Micrometers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. The Protractor:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
5. The Gauge Number of Wire:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. The Framing Square:	<u>3.5</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>50%</u>
7. The Tachometer:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Length

1. Basic Operations Using Length:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>88%</u>
2. Length as a Measure of Distance:	<u>3</u>	<u>5</u>	<u>5</u>	<u>4.3</u>	<u>88%</u>
3. Calculations of a Rectangle (Perimeter, Width, Length):	<u>3.9</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>63%</u>
4. Calculations of a Circle:	<u>4.3</u>	<u>5</u>	<u>3</u>	<u>4.1</u>	<u>75%</u>
5. Finding the Diameter of a Circle when the Area is Given:	<u>4.5</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>75%</u>

Area

1. As a Measure of Surface Space:	<u>4</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>63%</u>
2. Calculations of Four Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	<u>4</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>63%</u>
3. Area Calculations for a Circle:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>

Volume

1. Measures of Volume:	<u>4.8</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>88%</u>
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	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
2. Volume of a Cylinder:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>

Weight

1. Measures of Mass:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>88%</u>
2. Specific Gravity:	<u>4.8</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
3. Density:	<u>4.8</u>	<u>5</u>	<u>2.5</u>	<u>4.1</u>	<u>88%</u>
4. Weight Standards:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>75%</u>
5. Weights of Materials:	<u>4.8</u>	<u>5</u>	<u>2.5</u>	<u>4.1</u>	<u>88%</u>

Time

1. Units of Time:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>88%</u>
2. Distance in Relation to Time:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>75%</u>

Money Calculations

1. Price Computations:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>75%</u>
2. Income Computations:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>75%</u>
3. Interest Computation:	<u>4.5</u>	<u>5</u>	<u>2.5</u>	<u>4</u>	<u>75%</u>

BASIC ALGEBRA

Work and Power

1. Work and Power:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>88%</u>
2. Horsepower of a Steam Engine:	<u>4</u>	<u>4</u>	<u>5</u>	<u>4.3</u>	<u>75%</u>
3. Horsepower of a Gas Engine:	<u>5</u>	<u>4</u>	<u>5</u>	<u>4.7</u>	<u>88%</u>
4. Brake Horsepower:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Electrical Power-Watt:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
6. Mechanical Efficiency of Machines:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtor</u>	<u>%4-5</u>
<u>Speed Ratio and Pulley Gears</u>					
1. Gear Trains:	<u>4.8</u>	<u>3</u>	<u>5</u>	<u>4.3</u>	<u>75%</u>
2. Idlers:	<u>4.8</u>	<u>3</u>	<u>5</u>	<u>4.3</u>	<u>75%</u>
3. Finding the Number of Teeth for a Given Speed Ratio:	<u>5</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>75%</u>
4. Compound Gearing:	<u>5</u>	<u>3.5</u>	<u>3.5</u>	<u>4</u>	<u>75%</u>
5. Worm and Gear:	<u>5</u>	<u>3</u>	<u>4.5</u>	<u>4.2</u>	<u>88%</u>
6. Trains and Spur, Bevel and Worm Gearing:	<u>5</u>	<u>3</u>	<u>4.5</u>	<u>4.2</u>	<u>88%</u>
7. Pulley Trains:	<u>5</u>	<u>3</u>	<u>4.5</u>	<u>4.2</u>	<u>88%</u>
<u>THE D.C. CIRCUIT</u>					
1. Difference of Potential and Electric Current:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
2. E.M.F. and Energy Considerations:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
3. Ohm's Law:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
4. Voltage Drops:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
5. Energy and Power Dissi- pated in a Circuit:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
6. Series and Parallel Cir- cuits; Equivalent Resis- tance:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
7. Direction of Current:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
8. Kirchhoff's Rules:	<u>3.5</u>	<u>4.5</u>	<u>5</u>	<u>4.3</u>	<u>83%</u>
9. Battery E.M.F. and Termi- nal Voltage -- Internal Resistance:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
10. The Wheatstone Bridge:	<u>3</u>	<u>4.5</u>	<u>5</u>	<u>4.2</u>	<u>66%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
<u>Squares and Square Roots</u>					
1. Finding the Square of a Number:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>88%</u>
2. Finding the Square Root of a Number:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>88%</u>
3. Finding the Square Root by the Long Division Method:	<u>3.6</u>	<u>3.5</u>	<u>5</u>	<u>4</u>	<u>64%</u>
4. Basic Operations with Radicals:	<u>3.6</u>	<u>3.5</u>	<u>5</u>	<u>4</u>	<u>64%</u>
5. The Pythagorean Theorem:	<u>3.6</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>78%</u>
6. Rationalization:	<u>3.6</u>	<u>5</u>	<u>4.5</u>	<u>4.4</u>	<u>79%</u>

Percentage

1. Definition:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
2. Converting Fractions to Percent:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
3. Converting Percents to Fractions:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
4. Converting Decimals to Percents:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
5. Converting Percents to Decimals:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
6. Solving Problems:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
7. Price Lists and Discount:	<u>3.4</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>50%</u>
8. Percentage Error and Averaging:	<u>3.4</u>	<u>4</u>	<u>3.5</u>	<u>4</u>	<u>50%</u>
9. Markup:	<u>3.2</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>50%</u>

Ratio and Proportion

1. Definition of Ratio:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Reduction of Ratio to Lowest Terms:	<u>4.9</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%-5</u>
3. Definition of Proportion:	<u>4.9</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Finding the Unknown Term in a Proportion:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>92%</u>
5. Application of Proportion:	<u>4.9</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. The Principle of Levers:	<u>4.6</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>88%</u>
7. The Principle of Pulleys:	<u>4.6</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>88%</u>
8. The Principle of Gears:	<u>4.6</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>88%</u>
9. Principle of the Hydraulic Press:	<u>4.6</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>88%</u>
10. The English System of Weights & Measures:	<u>4.3</u>	<u>5</u>	<u>4.5</u>	<u>4.6</u>	<u>100%</u>
11. The Metric System of Weights & Measures:	<u>4.9</u>	<u>5</u>	<u>4.5</u>	<u>4.8</u>	<u>100%</u>
12. English-Metric Conversion:	<u>4.9</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>88%</u>
13. Multiplication & Division Measurements:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
14. Tables of Equivalent Measurements:	<u>4.6</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>100%</u>

Instruments of Measuring

1. Linear Scales:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
2. Vernier Calipers:	<u>4.6</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>94%</u>
3. Micrometers:	<u>4.9</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
4. The Protractor:	<u>5</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>94%</u>

Length

1. Calculations of a Triangle and Auxiliary Lines:	<u>5</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>82%</u>
2. Calculations of a Circle:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>94%</u>
3. Finding the Diameter of a Circle When the Area is Given:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>94%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
4. The Perimeter of the Ellipse:	<u>3.4</u>	<u>5</u>	<u>4.5</u>	<u>4.3</u>	<u>67%</u>
5. The Perimeter of Polygons:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>67%</u>

Area

1. As a Measure of Surface Space:	<u>3.8</u>	<u>5</u>	<u>3.5</u>	<u>4.1</u>	<u>67%</u>
2. Calculations of Four-Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	<u>3.6</u>	<u>5</u>	<u>3.5</u>	<u>5</u>	<u>56%</u>
3. Area Calculations for a Circle:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>89%</u>
4. Relationship of Radius, Diameter, Circumference, Arcs, Sectors, Chords, and Tangents:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>89%</u>
5. The Properties of π :	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Surface Area of Solids:	<u>5</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>67%</u>
7. Area Approximation:	<u>5</u>	<u>4</u>	<u>3</u>	<u>4</u>	<u>67%</u>

Volume

1. Measures of Volume:	<u>5</u>	<u>4</u>	<u>3</u>	<u>4</u>	<u>67%</u>
2. Volume Determined by Area:	<u>5</u>	<u>4.2</u>	<u>3</u>	<u>4.1</u>	<u>78%</u>
3. Volume of Rectangular Solids (Cubes):	<u>5</u>	<u>4</u>	<u>3</u>	<u>4</u>	<u>67%</u>
4. Volume of a Sphere:	<u>5</u>	<u>4</u>	<u>3</u>	<u>4</u>	<u>67%</u>
5. Volume of a Cylinder:	<u>5</u>	<u>3.8</u>	<u>3.3</u>	<u>4</u>	<u>70%</u>
6. Heights of Cylinders:	<u>5</u>	<u>4</u>	<u>3</u>	<u>4</u>	<u>78%</u>

Weight

1. Measures of Mass:	<u>3.6</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>67%</u>
2. Specific Gravity:	<u>4.1</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>72%</u>

	<u>μ V</u>	<u>μ M</u>	<u>μ A</u>	<u>μ tot</u>	<u>%4-5</u>
3. Density:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>71%</u>
4. Weight Standards:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>71%</u>
5. Weights of Materials:	<u>4.2</u>	<u>5</u>	<u>3</u>	<u>4.1</u>	<u>78%</u>
6. Weight of Castings from Patterns:	<u>4.2</u>	<u>5</u>	<u>3</u>	<u>4.1</u>	<u>78%</u>

Time

1. Units of Time:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>92%</u>
2. Distance in Relation to Time:	<u>3.6</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>94%</u>
3. Period:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>84%</u>
4. Frequency:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>84%</u>

Money Calculations

1. Income Computation:	<u>5</u>	<u>3.6</u>	<u>3.5</u>	<u>4</u>	<u>67%</u>
2. Interest Computation:	<u>5</u>	<u>3.6</u>	<u>3.5</u>	<u>4</u>	<u>67%</u>

Scale Drawings

1. Finding the Drawing Measure:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>84%</u>
2. Finding the Actual Dimensions of an Object from the Measured Dimension of a Diagram:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>84%</u>
3. Finding the Area of a Figure Drawn to a Certain Scale:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>84%</u>

Geometry

1. Straight Line & Circle:	<u>4.6</u>	<u>3</u>	<u>5</u>	<u>4.2</u>	<u>92%</u>
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	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
BASIC ALGEBRA					
<u>General Numbers</u>					
1. Properties of Real Numbers:	<u>3.8</u>	<u>5</u>	<u>3.5</u>	<u>4.1</u>	<u>78%</u>
2. Substitution of Variables into Algebraic Expressions:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>72%</u>
3. Translation of Word Expressions into Algebraic Expressions:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>73%</u>
4. Order of Operations:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>74%</u>
5. Evaluating Expressions Containing Variables:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>78%</u>
6. Basic Operations Involving Variables:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>79%</u>
7. Axioms:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>92%</u>
8. Constants and Variables:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>92%</u>
9. Positive and Negative Numbers:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
10. Powers and Exponents:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
11. Formulas:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
12. Ratio and Proportion:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>

Arithmetic Operations with Algebraic Expressions

1. Basic Operations with Algebraic Numbers:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>94%</u>
2. Operations with Signs of Grouping:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>95%</u>
3. The Distributive Law:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>93%</u>
4. Equations Involving Parentheses:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>94%</u>
5. Equations as Symbolic Sentences:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>86%</u>

	<u>μ V</u>	<u>μ M</u>	<u>μ A</u>	<u>μ tot</u>	<u>%4-5</u>
6. Equations Involving Parentheses:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>94%</u>
7. Equations as Symbolic Sentences:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>86%</u>
8. Removing Parentheses Used to Indicate Multiplication:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>82%</u>
9. First Degree Equations with Signed Numbers:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>84%</u>
10. Adding and Subtracting Like Terms:	<u>4.6</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
11. Adding and Subtracting Polynomials:	<u>4.4</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
12. Multiplication and Division of Monomials and Polynomials:	<u>4.2</u>	<u>3</u>	<u>5</u>	<u>4.1</u>	<u>78%</u>
13. Inserting Signs of Grouping:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>94%</u>
14. Rules of Exponents:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>94%</u>
15. The Square of a Binomial Sum or Difference:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>88%</u>
16. The Product of a Binomial Sum or Difference:	<u>3.6</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>94%</u>
17. Multiplication by a Common Factor:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>94%</u>
18. Extracting Square Roots:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>94%</u>
19. Symbols of Grouping:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>90%</u>

Algebraic Fractions

1. Properties of Algebraic Fractions:	<u>4.2</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>96%</u>
2. Equations Involving Fractions:	<u>4.8</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
3. Basic Operations with Algebraic Fractions:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
5. Finding the Least Common Denominator:	<u>4.4</u>	<u>3</u>	<u>5</u>	<u>4.1</u>	<u>88%</u>
6. Reducing to Lower Terms:	<u>4.6</u>	<u>3</u>	<u>5</u>	<u>4.2</u>	<u>94%</u>

Inequalities

1. Properties of Inequalities:	<u>3.1</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>82%</u>
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Equations

1. First Degree Equations in One and Two Variables:	<u>3.8</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
2. Literal Equation Formulas:	<u>3.5</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>82%</u>
3. Equivalent, Identical, and Conditional Equations:	<u>4</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>63%</u>
4. Simplification of Equations:	<u>3.3</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>80%</u>
5. Solutions of Equations in Variables:	<u>3.3</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>82%</u>
6. Transposition of Terms and Factors:	<u>3.3</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>82%</u>
7. Translating Problems into Equations:	<u>3.5</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>84%</u>
8. Operations with Dimensional Units:	<u>3.5</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>84%</u>
9. Dimensional Units and Conversion:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>87%</u>
10. Equations Containing Fractions and Decimals:	<u>4</u>	<u>4.5</u>	<u>5</u>	<u>4.5</u>	<u>82%</u>
11. Solving Linear Equations:	<u>3.6</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>82%</u>
12. Proportions:	<u>3.2</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>72%</u>
13. Proportionality:	<u>3.2</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>72%</u>

Linear Graphs

1. Coordinate Notation:	<u>3</u>	<u>5</u>	<u>5</u>	<u>4.3</u>	<u>68%</u>
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	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
2. Plotting Points on Graphs:	<u>3.5</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>72%</u>
3. Graphs of Ordered Pairs:	<u>2.9</u>	<u>5</u>	<u>5</u>	<u>4.3</u>	<u>66%</u>
4. Displaying Data with Graphs:	<u>3.5</u>	<u>5</u>	<u>5</u>	<u>4.5</u>	<u>76%</u>
5. Solving Problems by Means of Graphs:	<u>3.2</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>70%</u>
6. Graphs of Linear Equations:	<u>3.2</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>71%</u>

Printing

117

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	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
BASIC ARITHMETIC					
<u>Real Numbers</u>					
1. Basic Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Combined Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Definition of Signed Numbers:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
4. Graphing Signed Numbers:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
5. Basic Operations with Signed Numbers:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
6. One-to-One Correspondence:	<u>4.3</u>	<u>3</u>	<u>5</u>	<u>4.1</u>	<u>66%</u>
7. Rounding Off:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
8. Prime Numbers:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
9. Composite Numbers:	<u>4.3</u>	<u>5</u>	<u>3</u>	<u>4.1</u>	<u>66%</u>
10. Greatest Common Divisor:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>83%</u>
11. Least Common Multiple:	<u>4</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>83%</u>
<u>Common Fractions</u>					
1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Reading a Ruler:	<u>5</u>	<u>3.5</u>	<u>5</u>	<u>4.5</u>	<u>83%</u>
3. Basic Operations with Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Equivalent Fractions:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
5. Converting Mixed Numbers to Improper Fractions:	<u>4.7</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
6. Basic Operations with Mixed Numbers:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
7. Combined Operations with Mixed Fractions:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
8. Proper and Improper Fractions:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>

	μV	μM	μA	μtot	%4-5
9. Combining Like Terms:	4.7	5	4	4.6	100%
10. Reducing to Lowest Terms:	5	5	3	4.3	83%
11. Finding the Lowest Common Denominator:	4.7	5	4	4.6	100%
12. Basic Operations with Common Fractions and Mixed Fractions:	5	5	3	4.3	83%

Decimal Fractions

1. Reading and Writing	5	5	4	4.7	100%
2. Rounding Off:	5	5	4	4.7	100%
3. Basic Operations:	5	5	4	4.7	100%
4. Table of Decimal Equivalents:	4.7	5	4	4.6	100%
5. Fraction to Decimal Conversion:	5	5	3	4.3	83%
6. Decimal to Fraction Conversion:	5	5	3	4.3	83%

Scientific Notation

1. The Concept of Powers of 10:	5	5	2	4	83%
2. Significant Figures:	5	5	2	4	83%
3. Rounding Off:	5	5	2	4	83%
4. Basic Operations Using Powers of 10:	5	5	2	4	83%

Percentage

1. Definition:	5	5	4	4.7	100%
2. Converting Fractions to Percents:	5	5	4	4.7	100%
3. Converting Percents to Fractions:	5	5	3	4.3	83%

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μct</u>	<u>%4-5</u>
4. Converting Decimals to Percents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>4.3</u>	<u>83%</u>
5. Converting Percents to Decimals:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
6. Solving Problems:	<u>5.7</u>	<u>5</u>	<u>3</u>	<u>4.6</u>	<u>83%</u>
7. Price Lists and Discount:	<u>4.3</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>85%</u>
8. Percentage Error and Averaging:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>83%</u>
9. Markup:	<u>4.3</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>

Ratio and Proportion

1. Definition of Ratio:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
2. Reduction of Ratio to Lowest Terms:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
3. Definition of Proportion:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
4. Finding the Unknown Term in a Proportion:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
5. Application of Proportion:	<u>5</u>	<u>4.5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
6. The English System of Weights and Measures:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
7. The Metric of Weights and Measures:	<u>4.7</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
8. English-Metric Conversion:	<u>4.7</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
9. Multiplying and Dividing Measurements:	<u>4.7</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
10. Tables of Equivalent Measurements:	<u>4.7</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>

Instruments of Measuring

1. Linear Scales:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
2. Vernier Calipers:	<u>4.3</u>	<u>4.5</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
3. The Protractor:	<u>4.3</u>	<u>4.5</u>	<u>5</u>	<u>4.3</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
<u>Length</u>					
1. Basic Operations Using Length:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Calculations of a Triangle and Auxiliary Lines:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>83%</u>
3. Calculations of a Circle:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>83%</u>
<u>Area</u>					
1. As a Measure of Surface Space:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>83%</u>
2. Calculations of Four-Sided Figures (Squares, Rectangles, etc...)	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>83%</u>
<u>Volume</u>					
1. Measures of Volume:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
<u>Weight</u>					
1. Measures of Mass:	<u>4.7</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>83%</u>
2. Weight Standards:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
3. Weights of Materials:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
<u>Time</u>					
1. Units of Time:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Frequency:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>75%</u>
3. Period:	<u>5</u>	<u>5</u>	<u>2</u>	<u>4</u>	<u>75%</u>
<u>Money Calculations</u>					
1. Price Computations:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
2. Income Computations:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
3. Interest Computations:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>

Slide Rule

1. Slide Rule Operations
(Basic):

4.3

5

3

4.1

67%

2. Locating the Decimal
Point:

4.7

5

5

4.9

100%

	μV	μM	μA	μ_{tot}	%4-5
5. Writing Linear Equations in Slope-Intercept Form:	<u>3.7</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>80%</u>
6. Slope and Parallel Lines:	<u>3.7</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>80%</u>
7. The Equations of Horizontal and Vertical Lines:	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>

TRIGONOMETRY

Right Triangles and Trigonometric Ratios

1. Right Triangles and Angle-Sum Principle:	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>83%</u>
2. Right Triangles and the Pythagorean Theorem:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
3. The Definition of the Tangent Ratio:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>80%</u>
4. The Tangent Ratio of Three Specific Angles:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>80%</u>
5. A Partial Table of Tangent Ratios:	<u>3.8</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>67%</u>
6. The Complete Table of Trig Ratios:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
7. Finding the Unknown Sides in Right Triangles:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
8. Finding the Unknown Angles in Right Triangles:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Vectors & Phasors

1. Definition of Vectors and Phasors:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>83%</u>
2. Vectors on the Rectangular Coordinate System:	<u>4.5</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>83%</u>
3. The Trig Ratios of Reference Angles:	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
4. Finding the Length and Director of a Slanted Vector:	<u>4</u>	<u>5</u>	<u>4</u>	<u>4.3</u>	<u>80%</u>

	<u>μ V</u>	<u>μ M</u>	<u>μ A</u>	<u>μ tot</u>	<u>%4-5</u>
<u>Trigonometric Ratios of Standard-Position Angles</u>					
1. Standard-Position Angles Between 0° and 360° :	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
2. Reference Angles of Standard-Position Angles Between 0° and 360° :	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
3. Trigonometric Ratios of Angles Between 90° and 360° :	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
4. Finding the Function of any Angle:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>80%</u>
5. Finding Standard-Position Angles Corresponding to Known Ratios:	<u>3.3</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>60%</u>
6. Finding the Angle Given the Function:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>80%</u>
7. Trigonometric Ratios of 0° , 90° , 180° , 270° , and 360° :	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
8. Functions of Angles in Quadrants I, II, III, and IV:	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
9. Finding Log of Trig Functions:	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>4.1</u>	<u>60%</u>
10. Definition of a "Degree" in Angle Measurements:	<u>3</u>	<u>5</u>	<u>4</u>	<u>4</u>	<u>60%</u>
11. Operations Using Minutes, Degrees & Seconds:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
12. Definition of Radian in Angle Measurement:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
13. The Relationship Between Degrees and Radians:	<u>4.7</u>	<u>5</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
14. Radians Expressed in Terms of " π ":	<u>4.7</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>80%</u>
15. Trig Ratios of Angles Expressed in Radians:	<u>4.7</u>	<u>5</u>	<u>3</u>	<u>4.2</u>	<u>80%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
<u>Oblique Triangles</u>					
1. The Definition of an Oblique Triangle:	<u>4.3</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
2. Solving Oblique Triangles By Right-Triangle Methods:	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>60%</u>
3. The Law of Sines:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>100%</u>
4. Using the Law of Sines to Find Unknown Angles Oblique Triangles:	<u>3.7</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>80%</u>
5. Limitations of Using the Law of Sines:	<u>3.7</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
6. The Law of Cosines:	<u>3.7</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
7. Using the Law of Cosines to Find Unknown Sides and Angles of Oblique Triangles:	<u>3.7</u>	<u>5</u>	<u>4</u>	<u>4.2</u>	<u>80%</u>
8. Right Triangles-Law of Sines and Law of Cosines:	<u>3.3</u>	<u>5</u>	<u>5</u>	<u>4.4</u>	<u>60%</u>
9. Strategies for Solving Triangles:	<u>3.7</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
10. The Law of Sines and Obtuse Angles:	<u>3.7</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
11. The Law of Cosines and Obtuse Angles:	<u>3.7</u>	<u>5</u>	<u>5</u>	<u>4.6</u>	<u>80%</u>
<u>Applied Geometric Problems</u>					
1. Central Angles and Arcs of Circles:	<u>4.3</u>	<u>5</u>	<u>4</u>	<u>4.4</u>	<u>100%</u>
2. Chords of Circles:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>83%</u>
3. Basic Problems Involving Chords and Circles:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
4. Applied Problems Involving Chords of Circles:	<u>4.7</u>	<u>5</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
5. Sectors and Segments of Circles:	<u>4</u>	<u>5</u>	<u>3</u>	<u>4</u>	<u>50%</u>

	μV	μM	μA	μ_{tot}	%4-5
6. Revolutions and Rotational Speed:	4.3	5	3	4.1	67%
7. Circumferential Speed:	4.7	5	3	4.2	80%
8. Angular Measures of Rotational Speed:	4	5	3	4	67%
9. Circumferential Speed and Angular Velocity (In Radians):	4	5	3	4	60%
10. Areas of Triangles:	4.8	5	5	4.9	100%

Identities and Interpolation

1. Definitions of the Cosecant, Secant, and Cotangents of an Angle:	3	5	5	4.3	40%
2. Finding the Cosecant, Secant, and Cotangent of an Angle:	3	5	5	4.3	40%
3. Ratio Identities	3	5	4	4	40%
4. Pythagorean Identities:	3.7	5	4	4.2	60%
5. Simple Identities:	3.3	5	4	4.1	60%
6. Reciprocal Identities:	3.3	5	4	4.1	60%
7. Verifying Identities:	3.3	5	5	4.4	60%
8. Similar Right Triangles:	3	5	4	4	60%

Numerical Trigonometry (Applications)

1. Measurement of Angles:	5	5	5	5	100%
2. Mapping:	3.3	5	5	4.4	60%
3. Latitudes and Departures:	3.3	5	5	4.4	67%
4. Calculations of Areas:	5	5	5	5	100%
5. Measurement of Distances in a Vertical Plane:	5	5	5	5	100%

Electronics

141

153

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
BASIC ARITHMETIC					
<u>Real Numbers</u>					
1. Basic Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Combined Operations with Whole Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Definition of Signed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Graphing Signed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Basic Operations with Signed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Rounding Off:	<u>4.8</u>	<u>4.7</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
7. Absolute Value:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
8. Greatest Common Diviser:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
9. Least Common Multiple:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
<u>Common Fractions</u>					
1. Reading and Writing:	<u>3.5</u>	<u>4</u>	<u>5</u>	<u>4.2</u>	<u>67%</u>
2. Reading a Ruler:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Basic Operations with Fractions:	<u>4.5</u>	<u>4.7</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
4. Equivalent Fractions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Basic Operations with Mixed Numbers:	<u>3.8</u>	<u>3.7</u>	<u>5</u>	<u>4.2</u>	<u>67%</u>
6. Combined Operations with Mixed Numbers:	<u>3.8</u>	<u>3.7</u>	<u>5</u>	<u>4.2</u>	<u>67%</u>
7. Proper and Improper Fractions:	<u>3.5</u>	<u>3.7</u>	<u>5</u>	<u>4.1</u>	<u>67%</u>
8. Complex Numbers:	<u>3</u>	<u>4</u>	<u>5</u>	<u>4</u>	<u>67%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
9. Combining Like Terms:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
10. Reducing to Lowest Terms:	<u>4.5</u>	<u>4.7</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
11. Finding the Lowest Common Denominator:	<u>4</u>	<u>4.3</u>	<u>5</u>	<u>4.4</u>	<u>67%</u>
12. Basic Operations with Common Fractions and Mixed Fractions:	<u>4</u>	<u>4.3</u>	<u>5</u>	<u>4.4</u>	<u>67%</u>

Decimal Fractions

1. Reading and Writing:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Rounding Off:	<u>5</u>	<u>4.7</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
3. Basic Operations:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Table of Decimal Equivalents:	<u>4</u>	<u>3.7</u>	<u>5</u>	<u>4.2</u>	<u>50%</u>
5. Fraction to Decimal Conversion:	<u>4.5</u>	<u>4.7</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
6. Decimal to Fraction Conversion:	<u>4.5</u>	<u>4.3</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>

Scientific Notation

1. The Concept of Powers of 10:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Significant Figures:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Rounding Off:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Basic Operations Using Powers of 10:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Raising to Powers Using Powers of 10:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Extracting Square Roots Using Powers of 10:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Squares and Square Roots

1. Using the Table of Square Roots:	<u>4</u>	<u>4</u>	<u>5</u>	<u>4.3</u>	<u>60%</u>
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	μV	μM	μA	μ_{tot}	%4-5
2. Finding the Square of a Number:	5	4.7	5	4.9	100%
3. Finding the Square Root of a Number:	5	4.7	5	4.9	100%
4. Basic Operations of Radicals:	4.5	4	5	4.5	83%
5. The Radical Sign and Index of Roots:	5	4.3	5	4.8	83%
6. The Pythagorean Theorem:	5	4.3	5	4.8	83%

Percentage

1. Definition:	5	5	5	5	100%
2. Converting Fractions to Percent:	5	5	5	5	100%
3. Converting Decimals to Percent:	5	4.3	5	4.8	83%
4. Converting Percents to Decimals:	5	4.3	5	4.8	83%
5. Finding the Percentage Given the Base and Rate:	5	4	5	4.7	83%
6. Finding the Rate Given the Percentage and Base:	5	4	5	4.7	83%
7. Finding the Base Given the Rate and Percentage:	5	4	5	4.7	83%
8. Solving Problems:	5	4	5	4.7	83%

Ratio and Proportion

1. Definition of Ratio:	5	5	5	5	100%
2. Reduction of Ratio to Lowest Terms:	4.5	4.7	5	4.7	100%
3. Definition of Proportion:	5	5	5	5	100%
4. Finding the Unknown Term in a Proportion:	5	5	5	5	100%

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
5. Application of Proportion:	<u>4</u>	<u>4.3</u>	<u>5</u>	<u>4.4</u>	<u>67%</u>
6. The English System of Weights and Measures:	<u>4</u>	<u>3</u>	<u>5</u>	<u>4</u>	<u>50%</u>
7. The Metric of Weights and Measures:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
8. English-Metric Conversion:	<u>5</u>	<u>3</u>	<u>5</u>	<u>4.3</u>	<u>80%</u>
9. The Binary Number System:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
10. The Computer Number System:	<u>4</u>	<u>5</u>	<u>5</u>	<u>4.7</u>	<u>83%</u>
11. Computing Like a Computer:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
12. Binary to Decimal Conversion:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
13. Decimal to Binary Conversion:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
14. Basic Operations of Binary Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
15. Multiplying and Dividing Measurements:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Instruments of Measuring

1. Linear Scales:	<u>4.5</u>	<u>5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>
2. The Gauge Number of Wire:	<u>5</u>	<u>4.7</u>	<u>4</u>	<u>4.6</u>	<u>100%</u>
3. Logarithmic Scales:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
4. Angular Scales:	<u>3.6</u>	<u>3.7</u>	<u>5</u>	<u>4.1</u>	<u>67%</u>
5. Uniform Scales:	<u>3.4</u>	<u>3.7</u>	<u>5</u>	<u>4</u>	<u>67%</u>

Area

1. Area as a Measure of Surface Space:	<u>4.2</u>	<u>4</u>	<u>5</u>	<u>4.4</u>	<u>78%</u>
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	μV	μM	μA	μ_{tot}	%4-5
2. Area Calculations of a Circle (Diameter, Circumference, Arcs, Sectors Chords and Tangents):	4.2	4	5	4.4	78%
3. The Properties of π :	4.2	4	5	4.4	78%

Time

1. Units of Time:	5	5	5	5	100%
2. Distance in Relation to Time:	5	4.3	5	4.8	83%
3. Frequency:	5	5	5	5	100%
4. Period:	5	5	5	5	100%

Geometry

1. Straight Line & Circle:	3.6	4	5	4.2	67%
2. Translation of Axis:	3.6	4	5	4.2	67%
3. Polar Coordinates:	4.2	4	5	4.4	50%

BASIC ALGEBRA

General Numbers

1. Properties of Real Numbers:	4	4	5	4.3	67%
2. Substitution of Variables into Algebraic Expressions:	5	5	5	5	100%
3. Translation of Word Expressions into Algebraic Expressions:	5	5	5	5	100%
4. Order of Operations:	5	5	5	5	100%
5. Evaluating Expressions Containing Variables:	5	5	5	5	100%
6. Basic Operations Involving Variables:	5	5	5	5	100%

	μV	μM	μA	μtot	%4-5
7. Axioms:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
8. Constants and Variables:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
9. Positive and Negative Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
10. Powers and Exponents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
11. Formulas:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
12. Ratio and Proportion:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

Arithmetic Operations with Algebraic Expressions

1. Basic Operations with Algebraic Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Operations with Signs of Grouping:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. The Distributive Law:	<u>4</u>	<u>4.7</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
4. Equations Involving Parentheses:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Equations as Symbolic Sentences:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
6. Removing Parentheses Used to Indicate Multiplication:	<u>4.5</u>	<u>4.7</u>	<u>5</u>	<u>4.7</u>	<u>100%</u>
7. First Degree Equations with Signed Numbers:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
8. Adding and Subtracting Like Terms:	<u>5</u>	<u>5.0</u>	<u>5</u>	<u>5</u>	<u>100%</u>
9. Adding and Subtracting Polynomials:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
10. Multiplication and Division of Monomials and Polynomials:	<u>5</u>	<u>4.3</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
11. Inserting Signs of Grouping:	<u>5</u>	<u>4.3</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
12. Rules of Exponents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μ_{tot}</u>	<u>%4-5</u>
13. The Square of a Binomial Sum or Difference:	<u>3.5</u>	<u>4.3</u>	<u>5</u>	<u>4.3</u>	<u>67%</u>
14. The Product of a Binomial Sum or Difference:	<u>3.5</u>	<u>4.3</u>	<u>5</u>	<u>4.3</u>	<u>67%</u>
15. Factoring the Difference of Two Squares:	<u>4</u>	<u>4</u>	<u>5</u>	<u>4.3</u>	<u>67%</u>
16. Factoring Out the Common Factors:	<u>5</u>	<u>4.7</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
17. Factoring Polynomials:	<u>4.5</u>	<u>4.3</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
18. Multiplication by a Common Factor:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
19. Special Products or Expansions:	<u>4</u>	<u>3.3</u>	<u>5</u>	<u>4.3</u>	<u>56%</u>
20. Extracting Square Roots:	<u>5</u>	<u>4.7</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
21. Factoring Monomials from Polynomials:	<u>4.5</u>	<u>4</u>	<u>5</u>	<u>4.5</u>	<u>83%</u>
22. Symbols of Grouping:	<u>5</u>	<u>4.3</u>	<u>5</u>	<u>4.8</u>	<u>83%</u>
23. Squaring Binomials:	<u>5</u>	<u>4.5</u>	<u>5</u>	<u>4.8</u>	<u>100%</u>

Algebraic Fractions

1. Properties of Algebraic Fractions:	<u>4.5</u>	<u>4.3</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
2. Graphic Representation of Algebraic Fractions:	<u>4.2</u>	<u>3.3</u>	<u>5</u>	<u>4.2</u>	<u>78%</u>
3. Equations Involving Fractions:	<u>5</u>	<u>4.7</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
4. Basic Operations with Algebraic Fractions:	<u>5</u>	<u>4.7</u>	<u>5</u>	<u>4.9</u>	<u>100%</u>
5. Special Products:	<u>3.8</u>	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>78%</u>
6. The Fractional Exponents:	<u>5</u>	<u>3.3</u>	<u>5</u>	<u>4.4</u>	<u>83%</u>
7. Factoring Fractions:	<u>4</u>	<u>3.3</u>	<u>5</u>	<u>4.1</u>	<u>78%</u>
8. Finding the Least Common Denominator:	<u>4</u>	<u>3.3</u>	<u>5</u>	<u>4.1</u>	<u>78%</u>

	μV	μM	μA	μ_{tot}	%4-5
9. Reducing to Lower Terms:	<u>4</u>	<u>3.3</u>	<u>5</u>	<u>4.1</u>	<u>78%</u>
10. Complex Fractions:	<u>4</u>	<u>3.3</u>	<u>5</u>	<u>4.1</u>	<u>78%</u>
11. Improper Fractions and Mixed Expressions:	<u>3.8</u>	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>67%</u>
12. Compound Fractions:	<u>3.6</u>	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>67%</u>
13. Quotients of Polynomials:	<u>3.6</u>	<u>3.3</u>	<u>5</u>	<u>4</u>	<u>67%</u>

Equations

1. First Degree Equations in One and Two Variables:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
2. Literal Equation Formulas:	<u>4.5</u>	<u>4.7</u>	<u>4</u>	<u>4.4</u>	<u>100%</u>
3. Simplification of Equations:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
4. Solution of Equations in Variables:	<u>4.5</u>	<u>5</u>	<u>4</u>	<u>4.5</u>	<u>100%</u>
5. Analysis of a Formula:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
6. Transposition of Terms:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
7. Transposition of Factors:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
8. Translating Problems into Equations:	<u>5</u>	<u>5</u>	<u>4</u>	<u>4.7</u>	<u>100%</u>
9. Operations with Dimensional Units:	<u>4.2</u>	<u>3.7</u>	<u>5</u>	<u>4.3</u>	<u>78%</u>
10. Solving Systems of Equations of Two and Three Unknowns Algebraically:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
11. Dimensional Units and Conversion:	<u>4.2</u>	<u>3.7</u>	<u>5</u>	<u>4.3</u>	<u>78%</u>
12. Equations Containing Fractions and Decimals:	<u>4.2</u>	<u>3.7</u>	<u>5</u>	<u>4.3</u>	<u>78%</u>
13. Degree of an Equation:	<u>4.2</u>	<u>3.7</u>	<u>4</u>	<u>4</u>	<u>78%</u>
14. Solving Linear Equations:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>

	<u>μV</u>	<u>μM</u>	<u>μA</u>	<u>μtot</u>	<u>%4-5</u>
15. Algebraic Solutions to Variables:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
16. Equations with Unknowns in the Denominators:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
17. Radical Equations:	<u>4.2</u>	<u>3.7</u>	<u>5</u>	<u>4.3</u>	<u>78%</u>
18. Proportions:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
19. Proportionality:	<u>4.2</u>	<u>3.7</u>	<u>5</u>	<u>4.3</u>	<u>78%</u>

Linear Graphs

1. Coordinate Notation:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. Plotting Points on Graphs:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
3. Graphs of Ordered Pairs:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
4. Displaying Data with Graphs:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
5. Solving Problems by Means of Graphs:	<u>5</u>	<u>5</u>	<u>3</u>	<u>4.3</u>	<u>83%</u>
6. Graphs of Linear Equations:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
7. Solve by Comparison of Equalities:	<u>4.5</u>	<u>4.7</u>	<u>3</u>	<u>4.1</u>	<u>83%</u>
8. Solve by Addition & Subtraction:	<u>4.5</u>	<u>4.7</u>	<u>3</u>	<u>4.1</u>	<u>83%</u>
9. Applications of Simultaneous Linear Equations:	<u>4.5</u>	<u>4.7</u>	<u>3</u>	<u>4.1</u>	<u>83%</u>

Quadratic Equations

1. Solution of $ax^2+bx+c=0$:	<u>4.5</u>	<u>4.3</u>	<u>5</u>	<u>4.6</u>	<u>100%</u>
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Exponential Notation

1. Laws of Basic Operations with Exponents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>
2. The Law of Zero Exponents:	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>100%</u>



least common denominator for a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to find the least common denominator of a set of fractions.

PROGRAMMED INSTRUCTION
for
LEAST COMMON DENOMINATORS

Before beginning this module, it is necessary to take the pre-test on Least Common Denominators. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

$$3 \times 3 = \underline{\quad\quad\quad} /$$

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

LEAST COMMON DENOMINATORS

1. In the expression $(\frac{2}{3} + \frac{4}{5} + \frac{9}{10})$, the numbers 3, 5, 10 are the _____ of the terms in which they appear?

denominators

2. The smallest number which each denominator in $(\frac{2}{3} + \frac{4}{5} + \frac{9}{10})$, divides is _____?

30 (not 50, 150, ...)

3. 30 is the smallest number that is divisible by 3, 5 and 10. Thus, the Least Common Multiple (LCM) of 3, 5 and 10 is _____?

30

4. 32 is the _____ of 8, 12 and 32?

Least Common Multiple
(or LCM)

5. What is the LCM of 3, 4, and 5? _____

60

6. Some whole numbers may be "factored", or written as the product of smaller whole numbers; e.g., $14 = 2 \times 7$. $21 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}?$

$$3 \times 7$$

7. 24 may be factored as: $24 = 12 \times \underline{\hspace{2cm}}$
 $= 4 \times \underline{\hspace{2cm}}$
 $= 3 \times \underline{\hspace{2cm}}$
 $= 3 \times 2 \times \underline{\hspace{2cm}}$

$$\begin{aligned} 24 &= 12 \times 2 \\ &= 4 \times 6 \\ &= 3 \times 8 \\ &= 3 \times 2 \times 4 \end{aligned}$$

8. A number is factored completely when it is written as the product of prime numbers. A prime number is any whole number (other than 1) that is divisible only by itself and by 1. Is 23 a prime number?

Yes

9. 24 is completely factored as $2 \times 2 \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

2, 3

10. $24 = 2 \times 2 \times 2 \times 3$

To factor a number completely, start with the smallest prime number, 2, and ask if it is a factor of the number.

To factor 36 completely, such as: $36 = 2 \times$ _____

18

11. 2 is not a factor of 9.

$$36 = 2 \times 18$$

Try the next larger prime number and

$$36 = 2 \times 2 \times 9$$

ask if 3 is a factor of 9. _____

$$36 = 2 \times 2 \times 3 \times$$

Yes, 3 is a factor of 9

$$36 = 2 \times 2 \times 3 \times 3$$

12. 36 has now been factored to its prime factors. $36 = 2 \times 2 \times 3 \times 3$.

Can any of these factors be factored? _____

No, by the definition of prime numbers.

13. To factor 294 completely, ask if 2 is a factor, then 3, then 5, etc...

Always progress in this order; never just try factors at random.

The smallest prime number that is a factor of 294 is _____?

2

14. $294 = 2 \times$ _____

147

15. Examine the final factor, 147 and decide whether it contains 3 as a factor.

$$294 = 2 \times 147$$

$$294 = 2 \times 3 \times$$

49

16. Again examine the final factor, 49. 3 is obviously not a factor of 49.

$$294 = 2 \times 147$$

$$294 = 2 \times 3 \times 49$$

$$294 = 2 \times 3 \times$$
 _____ \times _____

What factor should be tried next? _____

The next factor is 7.

7, 7

17. $294 = 2 \times 3 \times 7 \times 7$

294 is completely factored as shown here. Only the systematic approach just reviewed will ensure finding every prime factor.

Factor completely: $56 =$ _____

$$56 = 2 \times 2 \times 2 \times 7$$

18. Factor completely: $45 = \underline{\hspace{2cm}}$
 $36 = \underline{\hspace{2cm}}$

$$45 = 3 \times 3 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3$$

19. Factor completely: $60 = \underline{\hspace{2cm}}$
 $63 = \underline{\hspace{2cm}}$

$$60 = 2 \times 2 \times 3 \times 5$$
$$63 = 3 \times 3 \times 7$$

20. Complete factoring may be used to find the LCM of any series of numbers;
e.g., 10 and 15.

$$10 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$
$$15 = \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$$

$$10 = 2 \times 5$$
$$15 = 3 \times 5$$

21. $10 = 2 \times 5$

$15 = 3 \times 5$

The LCM must contain all the prime factors of 10 and 15 but without duplication of common factors.

The LCM of 10 and 15 must contain the factors , , and .

22. 2, 3, 5 (Notice that 5 appears as a factor only once.)
-

23. $10 = 2 \times 5$

$15 = 3 \times 5$

The factors in the LCM of 10 and 15 are 2, 3 and 5.

LCM of 10 and 15 = _____ x _____ x _____ = _____?

$$2 \times 3 \times 5 = 30$$

24. Factor completely: $16 =$ _____

$40 =$ _____

$$16 = 2 \times 2 \times 2 \times 2$$

$$40 = 2 \times 2 \times 2 \times 5$$

25. $16 = 2 \times 2 \times 2 \times 2$

$40 = 2 \times 2 \times 2 \times 5$

The LCM must contain each factor the maximum number of times it is used in either 16 or 40. The factors of 16 include _____ 2's?
(How many?)

The LCM must therefore contain _____ 2's?
(How many?)

four

four

26. $16 = 2 \times 2 \times 2 \times 2$

$40 = 2 \times 2 \times 2 \times 5$

We know that the LCM must contain four 2's since there are four 2's in 16.

It need contain only one _____, since this factor appears only once in 40?

5

27. $16 = 2 \times 2 \times 2 \times 2$
 $40 = 2 \times 2 \times 2 \times 5$

Thus, the LCM must contain four 2's and one 5. Are there any other factors of either 16 or 40 that must be included in the LCM? _____

no

28. $16 = 2 \times 2 \times 2 \times 2$
 $40 = 2 \times 2 \times 2 \times 5$

LCM = _____ x _____ x _____ x _____ x _____ = _____

LCM = $2 \times 2 \times 2 \times 2 \times 5 = 80$

29. Factor completely: $66 =$ _____
 $55 =$ _____
 LCM of 66 and 55 = _____

$66 = 2 \times 3 \times 11$
 $55 = 5 \times 11$
 LCM = $2 \times 3 \times 5 \times 11$

30. $66 = 2 \times 3 \times 11$
 $66 = 5 \times 11$
 LCM = $2 \times 3 \times 5 \times 11$

The only factor common to 66 and 55 is _____

11

31. Factor the following and find the L.C.M.

$$66 = 2 \times 3 \times 11$$

$$28 = \underline{\hspace{2cm}}$$

$$55 = \underline{\hspace{2cm}} 5 \times 11$$

$$35 = \underline{\hspace{2cm}}$$

$$\text{Thus, LCM} = 2 \times 3 \times 5 \times 11 \quad \text{LCM of 28 and 35} = \underline{\hspace{2cm}}$$

$$28 = 2 \times 2 \times 7$$

$$35 = \underline{\hspace{2cm}} 5 \times 7$$

$$\text{LCM} = 2 \times 2 \times 5 \times 7$$

32. Factor the following and find the L.C.M.

$$62 = \underline{\hspace{2cm}}$$

$$93 = \underline{\hspace{2cm}}$$

$$\text{LCM of 62 and 93} = \underline{\hspace{2cm}}$$

$$62 = 2 \times 31$$

$$93 = \underline{\hspace{2cm}} 3 \times 31$$

$$\text{LCM} = 2 \times 3 \times 31$$

33. The first step in adding and subtracting fractions is to determine the Least Common Denominator for the fractions involved. We will first consider several numerical fractions and then consider the general case of algebraic fractions.

34. The Least Common Denominator of the series of fractions is simply they LCM of the _____ of the fractions.

denominators

35. To find the Least Common Denominator (LCD) of $\frac{4}{27}$ and $\frac{6}{45}$, first determine the _____ of 27 and 45.

LCM

36. LCM of 27 and 45 = $\frac{\text{_____}}{\text{(Factored form)}}$ = _____
LCD of $\frac{4}{27}$ and $\frac{6}{45}$ = _____

$$3 \times 3 \times 3 \times 5 = 135$$

135

37. Find the Least Common Denominator (LCD) of $\frac{1}{6}$, $\frac{3}{8}$, and $\frac{5}{12}$.
LCD = _____?

$$2 \times 2 \times 2 \times 3 \text{ or } 24$$

38. The only factor common to the denominator of $\frac{87}{52}$, $\frac{19}{39}$, and $\frac{25}{78}$ is _____.

13

39. Find the LCD of $87/52$, $19/39$ and $25/78$.

LCD = _____?

$$2 \times 2 \times 3 \times 13 \text{ or } 156$$

40. Since the denominators of $1/3$, $1/5$ and $1/7$ cannot be factored, each must be a factor in the LCD. The factors in the LCD are _____, _____, and _____.

$$3, 5, 7$$

41. To find the LCD of $3/8$, $5/9$ and $7/12$, first find the factors of each denominator.

$$8 = 2 \times 2 \times 2$$

$$14 = \underline{\hspace{2cm}}$$

$$28 = \underline{\hspace{2cm}}$$

$$9 = 3 \times 3$$

$$12 = 2 \times 2 \times 3$$

42. $3/8$, $5/9$, $7/12$ $8 = 2 \times 2 \times 2$ $9 = 3 \times 3$ $12 = 2 \times 2 \times 3$

The maximum number of times 2 is a factor in any of the above denominators

The maximum number of times 3 is a factor in any of the above denominators

3

2

43. $8 = 2 \times 2 \times 2$
 $9 = 3 \times 3$
 $12 = 2 \times 2 \times 3$

The LCM of the denominator 8, 9, 12
is _____.

$$2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$$

44. The LCD for $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{9}$ is _____.

18

45. The LCD for $\frac{3}{14}$, $\frac{5}{28}$ and $\frac{3}{35}$ is _____.

140

46. The LCD for $\frac{3}{17}$, $\frac{2}{34}$ and $\frac{1}{68}$ is _____.

68

47. The LCD for $\frac{1}{10}$, $\frac{4}{15}$ and $\frac{7}{25}$ is _____.

150

177

48. $5/36, 29/42, 1/63$

Factor 36 = _____
(prime factors) 42 = _____
63 = _____

$$36 = 2 \times 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3 \times 3 \times 7$$

49. $5/36, 29/42, 1/63$

$$36 = 2 \times 2 \times 3 \times 3$$

$$42 = 2 \times 3 \times 7$$

$$63 = 3 \times 3 \times 7$$

LCD = _____

$$\text{LCD} = 2 \times 2 \times 3 \times 3 \times 7 = 252$$

50. You have now reviewed the most efficient way to determine the Least Common Denominator for any series of numerical fractions.

You have also calculated a sufficient number and variety of LCD's to become quite proficient in the application of the technique reviewed.

S T O P

9.

a) $16 = 8 \times 2$

b) $16 = 4 \times 2 \times 2$

c) $16 = 2 \times 2 \times 2 \times 2$

10.

a) $24 = 12 \times$ _____

b) $24 = 4 \times$ _____

c) $24 = 3 \times$ _____

d) $24 = 3 \times 2 \times$ _____

11. a) $3 = 3 \times 1$

b) $7 = 7 \times 1$

c) $8 = 2 \times 2 \times 2 \times 1$

d) $5 = 5 \times 1$

e) $12 = 3 \times 2 \times 2 \times 1$

12.

a) 11 _____

b) 13 _____

c) 29 _____

d) 12 _____

13.

$24 = 3 \times 2 \times 2 \times$ _____

14.

$18 = 3 \times$ _____ \times _____

15.

$36 =$ _____ \times _____ \times _____ \times _____

16. $72 = \underline{\hspace{2cm}}$

- 17.
- a) $35 = \underline{\hspace{2cm}}$
 - b) $16 = \underline{\hspace{2cm}}$
 - c) $60 = \underline{\hspace{2cm}}$
 - d) $63 = \underline{\hspace{2cm}}$
-

18.

$10 = 2 \times 5$
 $15 = 3 \times 5$

19.

$26 = 2 \times 13$ Answer $\underline{\hspace{2cm}}$
 $39 = 3 \times 13$

20.

$18 = \underline{\hspace{2cm}}$
 $30 = \underline{\hspace{2cm}}$

21.

18 includes $\underline{\hspace{2cm}}$ 3's
LCM includes $\underline{\hspace{2cm}}$ 3's

22.

$18 = 2 \times 3 \times 3$ Answer $\underline{\hspace{2cm}}$
 $30 = 2 \times 3 \times 5$

23.

$18 = 2 \times 3 \times 3$
 $30 = 2 \times 3 \times 5$
LCM = $\underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

24.

$$66 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$55 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

25.

$$62 = \underline{\hspace{2cm}}$$

$$93 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

26.

$$28 = \underline{\hspace{2cm}}$$

$$21 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

27.

$$27 = \underline{\hspace{2cm}}$$

$$45 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

28.

$$\text{LCD of 27 and 45} = \underline{\hspace{2cm}}$$

29.

$$35 = \underline{\hspace{2cm}}$$

$$55 = \underline{\hspace{2cm}}$$

$$77 = \underline{\hspace{2cm}}$$

$$\text{LCD} = \underline{\hspace{2cm}}$$

30.

$$\text{Find LCD for } \frac{87}{52} + \frac{19}{39} + \frac{25}{78} \quad \text{LCD} = \underline{\hspace{2cm}}$$

31. $\frac{5}{24} + \frac{3}{16} + \frac{8}{12}$ LCD = _____

32. $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{4}{5}$

33. $\frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{1+2+3}{7} = \frac{6}{7}$

34. $\frac{3}{13} + \frac{2}{39}$
 $13 = 13 \times 1$
 $39 = 13 \times 3 \times 1$
 LCD = $13 \times 3 \times 1 = 39$

35. a) $\frac{1}{2}$ is equivalent to $\frac{3}{6}$
 b) $\frac{7}{3}$ is equivalent to $\frac{21}{9}$
 c) $\frac{3}{13}$ is equivalent to $\frac{9}{39}$
-

36. a) $\frac{1}{2} \times 1 = \frac{1}{2}$ b) $\frac{7}{3} \times 1 = \frac{7}{3}$ c) $\frac{3}{13} \times 1 = \frac{3}{13}$
 $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$ $\frac{7}{3} \times \frac{3}{3} = \frac{21}{3}$ $\frac{3}{13} \times \frac{3}{3} = \frac{9}{39}$
 $\frac{1}{2} = \frac{3}{6}$ $\frac{7}{3} = \frac{21}{3}$ $\frac{3}{13} = \frac{9}{39}$

37. $\frac{2}{7} + \frac{1}{7} + \frac{3}{14}$ LCD = _____

38.

$$\frac{4}{14} + \frac{2}{14} + \frac{3}{14} = \frac{\quad}{14}$$

39.

$$\frac{1}{3} + \frac{1}{13} + \frac{1}{39} = \frac{\quad}{39}$$

40.

$$\frac{3}{7} + \frac{1}{14} + \frac{3}{28} = \frac{\quad}{\quad}$$

Subtraction of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Subtraction of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Subtraction of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Least Common Multiple
2. Factoring a Number into Prime Factors
3. Least Common Denominator

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will add and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to subtract a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to subtract a set of fractions.

PROGRAMMED INSTRUCTION
for
SUBTRACTION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Subtraction of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

$$3 \times 3 = \underline{\hspace{2cm}}$$

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

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SUBTRACTION OF FRACTIONS

1. Programmed texts are used, as workbooks. Each frame contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

$$2 \times 2 = \underline{\hspace{2cm}}$$

4

2. As soon as you have written your answer, but not before, look below and find the correct answer.

$$2 + 5 - 1 = \underline{\hspace{2cm}}$$

6

3. Compare your answer with the correct answer. If your answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you must understand each frame before you proceed to the next.

$$4 \times 2 \times 1 = \underline{\hspace{2cm}}$$

8

4. In order for you to derive the maximum benefit from this program, it is essential that you write your answers and check them immediately.

$$\frac{6 + 4}{2} = \underline{\hspace{2cm}}$$

5

5. Read all instructions carefully and work as rapidly and as accurately as you can. Remember to write your answers and to check them immediately.

$$(7 + 3)(2) = (\underline{\hspace{2cm}})(2) = \underline{\hspace{2cm}}$$

$$(10)(2) = 20$$

6. The programmed review of subtraction of fractions begins with the next frame. The first Segment of this program consists of 16 frames.

Good Luck!

7. Two or more numbers multiplied together are termed factors, and the result of the multiplication is termed their product:

$$2 \times 4 \times 5 = 40 \quad \text{Product} = \underline{\hspace{2cm}} \quad \text{Factors} = \underline{\hspace{2cm}}$$

$$\text{Product} = 40$$

$$\text{Factors} = 2, 4, 5$$

8. $2 \times 4 \times 5 = 40$ Product: 40 Factors: 2, 4, 5

If the factors are divisible only by themselves and by 1, they are called prime factors.

Are the factors of 40 listed above all prime factors?

No; 4 is divisible by 2 as well as 4 and 1.

9. $3 \times 7 \times 5 = 105$

Are 3, 7, and 5 the prime factors of 105? _____

Yes

10. The prime factors of 26 are 13 and 2; i.e., $26 = 13 \times 2$.

Similarly: $12 = 2 \times 2 \times$ _____
 $16 = 2 \times 2 \times 2 \times$ _____
 $8 =$ _____ \times _____

 $12 = 2 \times 2 \times 3$
 $16 = 2 \times 2 \times 2 \times 2$
 $8 = 2 \times 2 \times 2$

11. Recall that the Least Common Multiple (LCM) of a series of numbers is the (smallest/largest) whole number that is divisible by every member of the series.

Smallest

12. $8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

$16 = 2 \times 2 \times 2 \times 2$

LCM of 8, 12, 15 = $2 \times 2 \times 2 \times 2 \times$ _____

3

13. $14 = 2 \times 7$

$21 = 3 \times 7$

$49 = 7 \times 7$

LCM = _____ x _____ x _____ x _____

(Give prime factors)

$2 \times 3 \times 7 \times 7$

14. The Least Common Denominator is simply the LCM of the denominators of a series of fractions.

The LCD for $\frac{1}{6}, \frac{3}{8}, \frac{5}{12}$ is _____

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

LCM = $2 \times 2 \times 2 \times 3$

15. Find the Least Common Denominator (LCD) for the fractions in this expression:

$$\frac{3}{10} + \frac{2}{5} + \frac{7}{9} \quad \text{LCD } \underline{\hspace{2cm}}$$

$$10 = 2 \times 5$$

$$5 = 1 \times 5$$

$$9 = 3 \times 3$$

$$\text{LCD} = 1 \times 2 \times 3 \times 3 \times 5 = 90$$

16. Find the Least Common Denominator (LCD) of $\frac{9}{24} - \frac{5}{16} - \frac{2}{21}$.

$$\text{LCD} = \underline{\hspace{2cm}}$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$21 = 3 \times 7$$

$$\text{LCD} = 2 \times 2 \times 2 \times 2 \times 3 \times 7$$

$$= 336$$

17. $\frac{5}{9} - \frac{1}{9} - \frac{3}{9} = \left(\underline{\hspace{1cm}} \right) - \left(\underline{\hspace{1cm}} \right) - \left(\underline{\hspace{1cm}} \right) = \underline{\hspace{2cm}}$

$$\frac{(5) - (1) - (3)}{9} = \frac{1}{9}$$

18. $\frac{9}{10} - \frac{5}{10} - \frac{2}{10} = \left(\underline{\hspace{1cm}} \right) - \left(\underline{\hspace{1cm}} \right) - \left(\underline{\hspace{1cm}} \right) = \underline{\hspace{2cm}}$

$$\frac{9 - 5 - 2}{10} = \frac{2}{10}$$

19. In the last two frames, you subtracted fractions that had the same denominators. The numerators of fractions may be subtracted directly only when they share the same denominators. When the denominators of a series of fractions are different, each fraction in the series must first be changed to an equivalent fraction with the same common denominator.

20. To express $\frac{3}{8}$ as a fraction whose denominator is 32:

$$\frac{3}{8} = \frac{3}{8} \times \left(\frac{\quad}{\quad} \right) = \frac{\quad}{32}$$

$$\frac{3}{8} \times \frac{4}{4} = \frac{12}{32}$$

21. $\frac{5}{14} = \frac{\quad}{42}$

$$\frac{5}{14} \times \frac{3}{3} = \frac{15}{42}$$

22. To solve $\frac{11}{35} = \frac{?}{210}$ efficiently, consider the prime factors of the denominators: $35 = 5 \times 7$. $210 = \underline{\hspace{2cm}}$

$$5 \times 7 \times 6$$

23. $35 = 5 \times 7$

$210 = 5 \times 6 \times 7$

$\frac{11}{35} = \frac{?}{210}$

_____ is the only factor 210 that is not a factor of 35. Thus, 11 and 35 must both be multiplied by _____ to obtain the equivalent fraction _____.

6

6

$\frac{66}{210}$

24. Since $14 = 2 \times 7$ and $42 = 2 \times 3 \times 7$,

$\frac{3}{14} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{42}$

$\frac{3}{14} \times \frac{3}{3} = \frac{9}{42}$

25. To solve $\frac{15}{42} = \frac{?}{168}$

42 = _____ (Give prime

168 = _____ factors.)

$42 = 2 \times 3 \times 7$

$168 = 2 \times 2 \times 2 \times 3 \times 7$

24.

$$66 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$55 = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$$

25.

$$62 = \underline{\hspace{2cm}}$$

$$93 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

26.

$$28 = \underline{\hspace{2cm}}$$

$$21 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

27.

$$27 = \underline{\hspace{2cm}}$$

$$45 = \underline{\hspace{2cm}}$$

$$\text{LCM} = \underline{\hspace{2cm}}$$

28.

$$\text{LCD of } 27 \text{ and } 45 = \underline{\hspace{2cm}}$$

29.

$$35 = \underline{\hspace{2cm}}$$

$$55 = \underline{\hspace{2cm}}$$

$$\text{LCD} = \underline{\hspace{2cm}}$$

30.

$$\text{Find LCD for } \frac{87}{52} + \frac{19}{39} + \frac{25}{78} \quad \text{LCD} = \underline{\hspace{2cm}}$$

233

31.

$$\frac{12}{13} - \frac{2}{39} - \frac{1}{26}$$

LCD _____

32.

$$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

33.

$$\frac{6}{7} - \frac{2}{7} - \frac{1}{7} = \frac{6 - 2 - 1}{7} = \frac{3}{7}$$

34.

$$\frac{3}{13} - \frac{2}{39}$$

$$13 = 13 \times 1$$

$$39 = 13 \times 3 \times 1$$

$$\text{LCD} = 13 \times 3 \times 1 = 39$$

35.

a) $\frac{1}{2}$ is equivalent to $\frac{3}{6}$

b) $\frac{7}{3}$ is equivalent to $\frac{21}{9}$

c) $\frac{3}{13}$ is equivalent to $\frac{9}{39}$

36.

a) $\frac{1}{2} \times 1 = \frac{1}{2}$

b) $\frac{7}{3} \times 1 = \frac{7}{3}$

c) $\frac{3}{13} \times 1 = \frac{3}{13}$

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

$$\frac{7}{3} \times \frac{3}{3} = \frac{21}{9}$$

$$\frac{3}{13} \times \frac{3}{3} = \frac{9}{39}$$

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{7}{3} = \frac{21}{9}$$

$$\frac{3}{13} = \frac{9}{39}$$

37.

$$\frac{5}{7} - \frac{1}{7} - \frac{3}{14}$$

LCD = _____

38. $\frac{10}{14} - \frac{2}{14} - \frac{3}{14} = \frac{\quad}{14}$

39. $\frac{1}{3} - \frac{1}{13} - \frac{1}{39} = \frac{\quad}{39}$

40. $\frac{4}{7} - \frac{1}{14} - \frac{3}{28} = \frac{\quad}{\quad}$

Multiplication of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Multiplication of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Multiplication of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Factoring a Number into Prime Factors
2. Equivalent Fractions
3. Reduction of Fractions

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will multiply and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to multiply a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to multiply a set of fractions.

PROGRAMMED INSTRUCTION
for
MULTIPLICATION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Multiplication of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

$$3 \times 3 = \underline{\hspace{2cm}}$$

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

MULTIPLICATION OF FRACTIONS

1. Throughout this program, questions you are asked to answer will appear in boxes like this one; these boxes are called frames. The next frame appears below.

The answers appear in this position on every page.

2. Programmed texts are used, not read. Each frame contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

$$2 \times 2 = \underline{\hspace{2cm}}$$

4

3. As soon as you have written your answer, but not before, look below at the answer.

$$2 + 5 - 1 = \underline{\hspace{2cm}}$$

6

4. Compare your answer with the correct answer. If your answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you must understand each frame before you proceed to the next.

$$4 \times 2 \times 1 = \underline{\hspace{2cm}}$$

8

5. In order for you to derive the maximum benefit from this program, it is essential that you write your answers and check them immediately.

$$\frac{4 + 5}{3} = \underline{\hspace{2cm}}$$

3

6. Read all instructions carefully and work as rapidly and as accurately as you can. Remember to write your answers and to check them immediately!

$$(9 - 4) (3) = (\underline{\hspace{1cm}}) (3) = \underline{\hspace{2cm}}$$

↖

$$(5)(3) = 15$$

7. The program in this Instruction reviews a number of techniques that facilitate the rapid and efficient multiplication of fractions and fractional expressions.

Good Luck!

8. To multiply two fractions $\frac{a}{b} \times \frac{c}{d}$, simply multiply the numerators together and multiply the denominators together.

$$\frac{a}{b} \times \frac{c}{d} = \frac{(\underline{\hspace{1cm}})}{(\underline{\hspace{1cm}})}$$

$$\frac{a \times c}{b \times d} \text{ or } \frac{ac}{bd}$$

9. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$$\frac{3}{7} \times \frac{5}{8} = \frac{3 \times 5}{7 \times 8} = \underline{\hspace{2cm}}$$

$$\frac{15}{56}$$

10. $\frac{6}{11} \times \frac{7}{5} = \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$

$$\frac{6 \times 7}{11 \times 5} = \frac{42}{55}$$

11. This principle may be applied to the multiplication of any series of fractions: $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{ace}{bdf}$

$$\frac{2}{5} \times \frac{3}{7} \times \frac{6}{11} = \underline{\hspace{2cm}}$$

$$\frac{2 \times 3 \times 6}{5 \times 7 \times 11} = \frac{36}{385}$$

12. The simple steps of changing each term to prime factors and then reducing will eliminate a great deal of tedious multiplication and division that often causes careless errors. You will benefit considerably, therefore, by mastering the approach developed in the following segment. This approach, foolproof when properly applied, is based on the use of prime factors.

13. A prime number is any whole number (other than 1) that is divisible only by itself and by 1.

Which of the following numbers is/are prime?

3 6 7 13 15 18 _____

Only 3, 7, and 15

14. Two or more numbers multiplied together are termed factors, and the result of the multiplication is termed their product.

$2 \times 6 \times 5 = 60$

Factors = _____

Product = _____

2, 6, 5
60

15. A number is reduced to its prime factors when it is represented as the product of a series of prime numbers; e.g.,

$26 = 2 \times 13$ $18 = 2 \times 3 \times 3$

$27 =$ _____ \times _____ \times _____

2, 3, 3

16. Rewrite as prime factors:

$16 = 2 \times 2 \times$ _____ \times _____ $42 =$ _____ \times _____ \times _____

$30 = 2 \times$ _____ \times _____ $17 =$ _____ \times _____

$$16 = 2 \times 2 \times 2 \times 2$$

$$30 = 2 \times 3 \times 5$$

$$42 = 2 \times 3 \times 7$$

7 is already prime

17. To multiply fractions, first rewrite each number as its prime factors.

For Example: $\frac{4}{6} \times \frac{9}{16} \implies \frac{2 \times 2}{3 \times 2} \times \frac{3 \times 3}{2 \times 2 \times 2 \times 2}$

Give prime factors for: $\frac{12}{30} \times \frac{10}{15} \implies \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)}$

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} \times \frac{2 \times 5}{3 \times 5}$$

18. Note that $\frac{2 \times 2 \times 3}{2 \times 3 \times 5}$ may be regrouped as $\frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$.

Since $2/2 =$ _____ and $3/3 =$ _____, the elimination of factors common to both numerator and denominator (will/will not) change the value of the fraction.

1,1

will not

19. $\frac{2 \times 2 \times 3}{2 \times 3 \times 5} \implies \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5} \implies \frac{2}{5}$

$$\frac{2 \times 5}{3 \times 5} \implies \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)} \implies \text{_____}$$

$$\frac{5}{5} \times \frac{2}{3} \implies \frac{2}{3}$$

20. The elimination of factors common to both numerator and denominator of a fraction is termed reducing.

Reducing: $\frac{3 \times 7 \times 13}{3 \times 5 \times 7 \times 11} \implies \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(5)(\quad)}$
 $\implies \underline{\hspace{2cm}}$

$$\frac{3}{3} \times \frac{7}{7} \times \frac{13}{5 \times 11}$$

$$\implies \frac{13}{5 \times 11}$$

21. It is not actually necessary to regroup the prime factors of numerator and denominator when reducing a fraction.

Reduce directly: $\frac{2 \times 2 \times 7 \times 13}{7 \times 13 \times 13} \implies$

Do NOT Multiply.
Keep prime factors.

$$\frac{2 \times 2 \times 7 \times \cancel{13}}{7 \times \cancel{13} \times 13} \implies \frac{2 \times 2}{13}$$

22. The second step in multiplying fractions is to reduce each fraction, if possible.

$$\frac{12}{30} \times \frac{10}{35} \implies \frac{2 \times 2 \times 3}{2 \times 3 \times 5} \times \frac{2 \times 5}{5 \times 7} \implies \frac{\cancel{2} \times 2 \times \cancel{5}}{\cancel{2} \times \cancel{5} \times 5} \times \frac{\cancel{2} \times \cancel{5}}{\cancel{5} \times 7}$$

$$\implies \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)}$$

$$\frac{2}{5} \times \frac{2}{7}$$

$$23. \quad \frac{12}{30} \times \frac{10}{35} \implies \frac{\cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 5} \times \frac{2 \times \cancel{5}}{\cancel{5} \times 7} \implies \frac{2}{5} \times \frac{2}{7}$$

$$\implies \frac{(\quad)(\quad)}{(\quad)(\quad)} = \underline{\hspace{2cm}}$$

$$\frac{2 \times 2}{5 \times 7} = \frac{4}{35}$$

24. Try another example ~~one~~ by step.

$$\frac{21}{28} \times \frac{12}{20} \implies \underline{\hspace{2cm}}$$

Give prime factors. Do not reduce.

$$\frac{3 \times 7}{2 \times 2 \times 7} \times \frac{2 \times 2 \times 3}{5 \times 2 \times 2}$$

$$25. \quad \frac{21}{20} \times \frac{12}{20} \implies \frac{3 \times 7}{2 \times 2 \times 7} \times \frac{\cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 5} \text{ (each fraction reduced)}$$

$$\implies \frac{(\quad)}{(\quad)}$$

(Rewrite remaining prime factors in a single fraction.)

$$\frac{3 \times 3}{2 \times 2 \times 5}$$

$$26. \quad \frac{21}{28} \times \frac{12}{20} \implies \frac{3 \times 7}{2 \times 2 \times 7} \times \frac{2 \times 2 \times 3}{2 \times 2 \times 5}$$

$$\implies \frac{3 \times 3}{2 \times 2 \times 5} \implies \frac{9}{20}$$

(New fraction reduced.)

$$\frac{9}{20}$$

$$27. \quad \frac{28}{30} \times \frac{15}{33} \implies \text{_____} \times \text{_____}$$

Give prime factors only.
Do not reduce.

$$\frac{2 \times 2 \times 7}{2 \times 3 \times 5} \times \frac{3 \times 5}{3 \times 11}$$

$$28. \quad \frac{28}{30} \times \frac{15}{33} \implies \frac{2 \times 2 \times 7}{2 \times 3 \times 5} \times \frac{3 \times 5}{3 \times 11} \implies \left(\frac{\quad}{\quad} \right) \text{ (Give prime factors.)}$$

Reduce again to obtain
(prime factors)

$$\frac{3 \times 7 \times 5}{3 \times 5 \times 11}$$

$$= \frac{2 \times 7}{3 \times 11} = \frac{14}{33}$$

$$29. \quad \frac{35}{42} \times \frac{34}{55} \implies \frac{5 \times \cancel{7}}{2 \times 3 \times \cancel{7}} \times \frac{2 \times 17}{5 \times 11} \implies \frac{\cancel{7} \times \cancel{2} \times 17}{\cancel{2} \times 3 \times \cancel{5} \times 11}$$

$$\implies \frac{17}{3 \times 11} = \frac{17}{33}$$

Give prime factors and reduce each fraction separately.

$$\frac{12}{15} \times \frac{10}{14} = \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right)$$

$$\frac{2 \times 2 \times \cancel{5}}{\cancel{5} \times 5} \times \frac{\cancel{7} \times 5}{\cancel{2} \times 7}$$

$$30. \quad \frac{24}{28} \times \frac{10}{14} \implies \frac{\cancel{2} \times \cancel{2} \times 2 \times 3}{\cancel{2} \times \cancel{2} \times 7} \times \frac{\cancel{2} \times 5}{\cancel{2} \times 7} \implies \left(\frac{\quad}{\quad} \right) \text{ (Give prime factors.)}$$

$$\text{Reduce again to obtain } \left(\frac{\quad}{\quad} \right) = \frac{\quad}{\quad}$$

(prime factors)

$$\frac{2 \times 3 \times \cancel{7}}{\cancel{7} \times 7}$$

$$\implies \frac{2 \times 3}{7} = \frac{6}{7}$$

31. Give prime factors for each fraction and reduce. Do not multiply.

$$\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} = \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right)$$

$$\left(\frac{2 \times 2 \times 2}{3 \times 3 \times 3} \right) \times \left(\frac{\cancel{7} \times 5 \times 7}{\cancel{7} \times 2 \times 2 \times 2} \right) \times \left(\frac{2 \times 3 \times 3}{5 \times 7} \right)$$

21. Divide: $\frac{3}{7} \div \frac{4}{5} =$ _____

$$\frac{3}{4} \times \frac{5}{4} = \frac{15}{16} = \frac{15}{16}$$

You have now completed the unit on division of fractions and you should take the post-test to check your competency in this area.

STOP

Tape on
Division of Fractions

We now study the division of fractions beginning with a study of the multiplication of fractions.

To multiply two fractions, you simply multiply the numerators of the fractions together and multiply the denominators together.

In frame 1, $1/5$ is multiplied by $2/3$, and the result is $2/15$.

In frame number 2, multiply the following fractions: a) $1/7 \times 1/3 = \underline{\quad}$;
b) $3/11 \times 3/5 = \underline{\quad}$; c) $11/12 \times 5/9 = \underline{\quad}$.

Here are the answers: The answer to a) is $1/21$; The answer to b) is $9/55$;
The answer to c) is $55/108$.

In frame number 3, multiply the following fractions: a) $3/8 \times 5/7 \times 1/2$;
b) $5/12 \times 7/11 \times 1/4$; and c) $1/4 \times 3/7 \times 5/13$.

Here are the answers: The answer to a) is $15/112$; The answer to b) is $35/528$;
The answer to c) is $15/364$.

You have now seen the principle of the multiplication of fractions at work.
To multiply fractions, we simply multiply the numerators together and multiply the denominators together.

This principle may be applied to the multiplication of any series of fractions.
For example, look at frame 4: $a/b \times c/d \times e/f \times g/h = (a \times c \times e \times g)/(b \times d \times f \times h)$.

The simple steps of changing each term in the numerator and the denominator to prime factors and then reducing will eliminate a great deal of tedious multiplication that often causes errors. You will benefit considerably, therefore, by mastering the approach developed in the following frames.

A prime number is any whole number, other than one, that is divisible only by itself and one.

In frame 5, determine whether the numbers 3, 6, 7, and 12 are prime numbers or not prime numbers.

Here are the answers: a) is a prime number; b) 6 is not a prime number because you can multiply $1 \times 2 \times 3$; c) is a prime number; and 12 is not a prime number because you can multiply $1 \times 2 \times 2 \times 3$.

It's seen in that frame that the numbers 6 and 12 can be divided by 1, 2, and 3 and that consequently they are not prime numbers. Two or more numbers multiplied together are termed factors and the result of their multiplication is termed their product.

In frame number 6, we have the following multiplication: $4 \times 3 \times 6 = 72$.
The factors are 4, 3, 6 and their product is 72.

Division of Fractions

A number is reduced to its prime factors when it is represented as the product of a series of prime numbers. For example, in frame 7: $26 = 2 \times 13$; $27 = 3 \times 3 \times 3$. The product of 26 is represented by the multiplication of 2 and 13. These are both prime factors. The product of 27 is represented by $3 \times 3 \times 3$, which are all prime factors.

In frame number 8, identify which of the numbers are prime numbers.

Here are the answers: 11 is a prime number, because $1 \times 11 = 11$; 15 is not a prime number, because $1 \times 3 \times 5 = 15$; 17 is a prime number, because $1 \times 17 = 17$; 23 is also a prime number because $1 \times 23 = 23$.

In frame number 9, identify the products and factors of 27 and 105.

Here are the answers: The factors are 1, 2, 11, 3, 5, and 7. The products are 22 and 105.

In frame number 10, rewrite the numbers as prime factors.

Here are the answers: The answer to problem a) $12 = 2 \times 2 \times 3$; problem b) $30 = 2 \times 3 \times 5$; problem c) $25 = 5 \times 5$; problem d) 7 is already a prime number.

To multiply fractions, first rewrite each number as its prime factors. For example, look at frame number 11: $4/6 \times 9/16 = \{(2 \times 2)/(3 \times 2)\} \times \{(3 \times 3)/(2 \times 2 \times 2 \times 2)\}$.

In frame 12, give the factors for $12/30 \times 10/35$.

Here is the answer: $\{(2 \times 2 \times 3)/(2 \times 3 \times 5)\} \times \{(2 \times 5)/(5 \times 7)\}$.

Note that the answer given in 12 is also given in frame 13. And that $(2 \times 2 \times 3)/(2 \times 3 \times 5)$ may be regrouped as $2/2 \times 3/3 \times 2/5$. Looking at frame 13 again, since $2/2$ equals 1, and $3/3$ equals 1, what we have is $1 \times 1 \times 2/5$. Therefore, the result in frame 13 is $2/5$. This is clarified for you in frame 14.

The elimination of factors common to numerators and denominators does not change the value of a fraction. The only change is its form. For instance, $2/2$ equals 1, $3/3$ equals 1, and if you multiply any number by 1, you have a result that is equal to that number. As an example, look at frame 15. Problem a) $1 \times 1 \times 5 = 5$. In problem b) $2/2 \times 3/3 \times 5/1 = 1 \times 1 \times 5 = 5$. As it is seen, the value is not changed. The elimination of factors common to both the numerator and denominator of a fraction is termed reducing.

In frame 16, reduce the fractional multiplication.

Here is the answer: The problem again was $(2 \times 5 \times 11)/(3 \times 5 \times 7 \times 11)$; this gives us $5/5 \times 11/11 \times 2/3 \times 1/7$ which is equal to $2/(3 \times 7)$.

In frame 17, multiply and reduce the fractions.

Division of Fractions

The answer is $9/11$.

In frame 18, multiply and reduce the fractions.

Here is the way the problem would read if you have gotten a correct answer:
 $12/30 \times 10/35 = \{(1 \times 2 \times 2 \times 3)/(1 \times 2 \times 3 \times 5)\} \times \{(1 \times 2 \times 5)/(1 \times 5 \times 7)\}$.
This would equal $4/35$.

In frame 19, multiply and reduce the fractions $35/42 \times 34/55$.

The correct answer is $17/33$.

Multiply and reduce the fractions in frame 20.

The answer is $2/3$.

It was necessary to review the multiplication of fractions before we can learn how to divide fractions. In division of fractions, you use the same techniques as we learned in the multiplication of fractions, except that you encounter the concept of reciprocal.

The reciprocal of a number is defined as that number that you can multiply a number by and get an answer of 1. For example, look at frame 21: $1/2 \times 2 = 1$; $2/3 \times 3/2 = 1$; and $3/7 \times 7/3 = 1$. It is seen that $1/2$ is the reciprocal of 2, $2/3$ is the reciprocal of $3/2$, and $3/7$ is the reciprocal of $7/3$. It's rather obvious that to get the reciprocal of a number we simply turn it upside down.

In frame 22, give the reciprocals for the numbers indicated.

Here are the answers: The reciprocal of $3/5$ is $5/3$; the reciprocal of $7/11$ is $11/7$; the reciprocal of 5 is $1/5$; the reciprocal of $4/3$ is $3/4$.

In the division of fractions, you simply change the divisor to its reciprocal and multiply. For example, look at frame 23. $5 \div 1/3$ equals 5 times the reciprocal of $1/3$ or $3/1$, resulting in an answer of 15.

In frame 24, find the quotient of the following fractions. The term quotient merely means the answer to a division problem.

The answers are: Problem a) $2/15$; Problem b) $9/7$

In frame 25, divide and reduce the following fractions.

Here are the answers, as we look at the problems together: For problem a) $12/35 \div 4/7 = 3/5$; for problem b) $33/13 \div 5/39 = 99/5$.

In some cases, division of fractions is written as shown in frame 26. This type of division is better written in the form shown in frame 27.

By rewriting the fraction as shown in frame 27, it puts it in a form that will lead to fewer errors. In frame 28, do the division of the fractions.

Division of Fractions

The correct answer is $5/3$.

In frame 29, divide and reduce the fractions $\{34/39\} \div \{17/3\}$.

The correct answer is $2/13$.

Now, divide and reduce the fractions in frame 30.

The correct answer is $7/5$.

This concludes our self-study of the division of fractions. Now, please take the post-test to help determine your competency in this area.

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Name _____

Workbook for Cassette Tape
on
Division of Fractions

1.

$$\frac{1}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 3} = \frac{2}{15}$$

2.

a) $\frac{1}{7} \times \frac{1}{3} = \frac{1 \times 1}{7 \times 3} = \underline{\hspace{2cm}}$

b) $\frac{3}{11} \times \frac{3}{5} = \frac{3 \times 3}{11 \times 5} = \underline{\hspace{2cm}}$

c) $\frac{11}{12} \times \frac{5}{9} = \frac{11 \times 5}{12 \times 9} = \underline{\hspace{2cm}}$

3.

a) $\frac{3}{8} \times \frac{5}{7} \times \frac{1}{2} = \frac{3 \times 5 \times 1}{8 \times 7 \times 2} = \underline{\hspace{2cm}}$

b) $\frac{5}{12} \times \frac{7}{11} \times \frac{1}{4} = \frac{5 \times 7 \times 1}{12 \times 11 \times 4} = \underline{\hspace{2cm}}$

c) $\frac{1}{4} \times \frac{3}{7} \times \frac{5}{13} = \frac{1 \times 3 \times 5}{4 \times 7 \times 13} = \underline{\hspace{2cm}}$

4.

$$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \times \frac{g}{h} = \frac{a \times c \times e \times g}{b \times d \times f \times h}$$

5.

a) $3 = \underline{\text{Prime}} = 1 \times 3$

b) $6 = \underline{\text{Not Prime}} = 1 \times 2 \times 3$

c) $7 = \underline{\text{Prime}} = 1 \times 7$

d) $12 = \underline{\text{Not Prime}} = 1 \times 2 \times 2 \times 3$

6.

$4 \times 3 \times 6 = 72$

Factors = 4, 3, 6

Product = 72



7.
 $26 = 2 \times 13$
 $27 = 3 \times 3 \times 3$

8.
a) $11 =$ _____
b) $15 =$ _____
c) $17 =$ _____
d) $23 =$ _____

9.
 $22 = 1 \times 2 \times 11$ Factors = _____
 $105 = 1 \times 3 \times 5 \times 7$ Products = _____

10.
a) $12 = 2 \times 2 \times$ _____
b) $30 = 2 \times$ _____ \times _____
c) $25 =$ _____ \times _____
d) $7 =$ _____ \times _____

11.
 $\frac{4}{6} \times \frac{9}{16} \implies \frac{2 \times 2}{3 \times 2} \times \frac{3 \times 3}{2 \times 2 \times 2 \times 2}$

12.
 $\frac{12}{30} \times \frac{10}{35} \implies \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right)$

13.
 $\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$

14.
 $\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = 1 \times 1 \times \frac{2}{5} = \frac{2}{5}$

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15.

a) $1 \times 1 \times 5 = 5$

b) $\frac{2}{2} \times \frac{3}{3} \times \frac{5}{1} = 1 \times 1 \times 5 = 5$

16.

$$\frac{2 \times 5 \times 11}{3 \times 5 \times 7 \times 11} \implies \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right) \times \left(\frac{\quad}{\quad} \right)$$

17.

$$\frac{3}{5} \times \frac{3}{11} \times \frac{5}{11} \times \frac{11}{1} = \frac{5}{5} \times \frac{11}{11} \times \frac{3 \times 3}{11} = \underline{\hspace{2cm}}$$

18.

$$\frac{12}{30} \times \frac{10}{35} = \frac{1 \times 2 \times 2 \times 3}{1 \times 2 \times 3 \times 5} \times \frac{1 \times 2 \times 5}{1 \times 5 \times 7} = \underline{\hspace{2cm}}$$

19.

$$\frac{35}{42} \times \frac{34}{55} = \underline{\hspace{2cm}}$$

20.

$$\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} = \underline{\hspace{2cm}}$$

21.

a) $\frac{1}{2} \times 2 = 1$

c) $\frac{2}{3} \times \frac{3}{2} = 1$

d) $\frac{3}{7} \times \frac{7}{3} = 1$

22.

a) $\frac{3}{5} = \underline{\hspace{2cm}}$

c) $\frac{5}{1} = \underline{\hspace{2cm}}$

b) $\frac{7}{11} = \underline{\hspace{2cm}}$

d) $\frac{4}{3} = \underline{\hspace{2cm}}$

23. $5 \div \frac{1}{3} = 5 \times \frac{3}{1} = \frac{15}{1}$

24. a) $\frac{2}{3} \div \frac{5}{1} = \underline{\hspace{2cm}}$

b) $\frac{3}{7} \div \frac{1}{3} = \underline{\hspace{2cm}}$

25. a) $\frac{12}{35} \div \frac{4}{7} = \underline{\hspace{2cm}}$

b) $\frac{33}{13} \div \frac{5}{39} = \underline{\hspace{2cm}}$

26. $\frac{3/7}{8/17}$

27. $\frac{3/7}{8/17} = \frac{3}{7} \div \frac{8}{17}$

28. $\frac{5/13}{3/13} = \frac{5}{13} \div \frac{3}{13} = \underline{\hspace{2cm}}$

29. $\frac{34/39}{17/3} = \underline{\hspace{2cm}}$

30. $\frac{12/30}{10/35} = \underline{\hspace{2cm}}$

PRE-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(LCD, Addition and Subtraction)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of addition and subtraction of fractions. Please answer each problem carefully.

1. Find the Least Common Multiple of the following numbers: 3, 4, and 5.

ANSWER _____

2. Identify which of the following numbers are prime.

EXAMPLE: a) 3 = prime

b) 9 = _____

c) 26 = _____

d) 29 = _____

3. Completely factor the following numbers:

EXAMPLE: a) 18 = 3 x 3 x 2

b) 30 = _____

c) 63 = _____

4. Find the Least Common Denominator of the following fractions:

EXAMPLE: a) $\frac{3}{17} + \frac{7}{34} + \frac{11}{68}$, LCD = 68

b) $\frac{3}{13} + \frac{7}{26} + \frac{5}{39}$, LCD = _____

c) $\frac{2}{11} + \frac{5}{66} + \frac{2}{77}$, LCD = _____

5. Add or subtract the following fractions:

a) $\frac{3}{13} + \frac{7}{26} + \frac{5}{39} =$ _____

b) $\frac{2}{11} + \frac{5}{66} + \frac{2}{77} =$ _____

c) $\frac{1}{2} - \frac{1}{3} - \frac{1}{7} =$ _____

d) $\frac{2}{11} - \frac{5}{66} - \frac{2}{77} =$ _____

PRE-TEST: OPERATIONS ON
FRACTIONS MINI-MODULES
(Multiplication and Division)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of multiplication and division of fractions. Please answer each problem carefully.

1. Identify which of the following numbers are prime:

Example: a) $3 =$ Prime

b) $9 =$ _____

c) $26 =$ _____

d) $29 =$ _____

2. Completely factor the following numbers:

Example: a) $18 = 3 \times 3 \times 2$

b) $30 =$ _____

c) $63 =$ _____

3. Give the reciprocal of the following numbers:

Example: a) $3 =$ $1/3$

b) $7/8 =$ _____

c) $7/2 =$ _____

4. Reduce the following fractions:

Example: a) $\frac{26}{39} = \frac{2 \times 13}{3 \times 13} = \frac{2}{3} \times \frac{\cancel{13}}{\cancel{13}} = \frac{2}{3}$

b) $\frac{16}{112} =$ _____

c) $\frac{32}{56} =$ _____

5. Multiply or divide the following fractions:

a) $\frac{3}{17} \times \frac{51}{3} =$ _____

b) $\frac{7}{13} \times \frac{39}{49} =$ _____

c) $\frac{5}{26} \div \frac{25}{39} =$ _____

d) $\frac{7/22}{49/11} =$ _____

POST-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(LCD, Addition and Subtraction)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of addition and subtraction of fractions. Please answer each problem carefully.

1. Find the Least Common Multiple of the following numbers: 5, 7 and 35.

ANSWER _____

2. Identify which of the following numbers are prime.

EXAMPLE: a) 3 = prime

b) 7 = _____

c) 21 = _____

d) 31 = _____

3. Completely factor the following numbers:

EXAMPLE: a) 18 = 3 x 3 x 2

b) 40 = _____

c) 102 = _____

4. Find the Least Common Denominator of the following fractions:

EXAMPLE: a) $\frac{3}{17} + \frac{7}{34} + \frac{11}{68}$, LCD = 68

b) $\frac{1}{13} + \frac{4}{39} + \frac{2}{68}$, LCD = _____

c) $\frac{1}{5} + \frac{3}{20} + \frac{7}{100}$, LCD = _____

5. Add or subtract the following fractions:

a) $\frac{2}{11} + \frac{7}{22} + \frac{3}{88} =$ _____

b) $\frac{3}{7} + \frac{2}{17} + \frac{7}{119} =$ _____

c) $\frac{12}{13} - \frac{2}{78} - \frac{2}{117} =$ _____

d) $\frac{5}{8} - \frac{3}{64} - \frac{9}{192} =$ _____

POST-TEST: OPERATIONS ON
FRACTIONS MINI-MODULES
(Multiplication and Division)

1. Identify which of the following numbers are prime.

Example: a) $3 = \underline{\text{Prime}}$

b) $11 = \underline{\hspace{2cm}}$

c) $68 = \underline{\hspace{2cm}}$

d) $31 = \underline{\hspace{2cm}}$

2. Completely factor the following numbers:

Example: a) $18 = \underline{3 \times 3 \times 2}$

b) $68 = \underline{\hspace{2cm}}$

c) $76 = \underline{\hspace{2cm}}$

3. Give the reciprocal of the following numbers:

Example: a) $3 = \underline{1/3}$

b) $5/3 = \underline{\hspace{2cm}}$

c) $13/2 = \underline{\hspace{2cm}}$

4. Reduce the following fractions:

Example: a) $\frac{26}{39} = \frac{2 \times 13}{3 \times 13} = \frac{2}{3} \times \frac{13}{13} = \frac{2}{3}$

b) $\frac{32}{160} = \underline{\hspace{2cm}}$

c) $\frac{72}{182} = \underline{\hspace{2cm}}$

5. Multiply or divide the following fractions:

a) $\frac{22}{36} \times \frac{21}{11} = \underline{\hspace{2cm}}$

b) $\frac{17}{33} \times \frac{26}{51} = \underline{\hspace{2cm}}$

c) $\frac{7}{30} \div \frac{21}{10} = \underline{\hspace{2cm}}$

d) $\frac{17/26}{68/52} = \underline{\hspace{2cm}}$

APPENDIX E
Curriculum Guides

Arithmetic Operations with Algebraic Expressions

Concept	Behavioral Objective
20. Multiplication & division of monomials and polynomials:	From a representative group of five problems, the student will find the correct solutions to at least four by applying the rules governing multiplication and division of monomials & polynomials.
21. Equations as symbolic sentences:	The student will read an equation aloud and interpret it as a sentence.
22. Removing parentheses used to indicate multiplication:	The student will simplify equations by performing indicated multiplications in order to be able to remove parentheses. Minimum performance level will be correctly solving four problems from a set of five.
23. Adding and subtracting polynomials:	The student will demonstrate his ability to add & subtract polynomials by combining similar terms. An acceptable performance level will be 80% correct answers.
24. Inserting signs of grouping:	The student will clarify a set of problems by inserting signs of grouping.
25. Rules of Exponents:	The student will demonstrate an understanding of the rules of exponents by stating each and working an example of it. Minimum ability will be 80% accuracy.
26. The product of a binomial sum or difference:	The student will expand binomial sums or differences for ten problems. Minimum level of performance will be eight correct answers.

Arithmetic Operations with Algebraic Expressions

Concept	Behavioral Objective
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| 27. Factoring the difference of two squares: | The student will accurately factor four of five problems involving a difference of two squares and will check by multiplying the factors. |
| 28. Factoring out the common factors: | Four of five problems will be correctly solved when the student has factored out common factors. |
| 29. Factoring polynomials: | The student will completely factor eight of ten expressions involving polynomials. |
| 30. Multiplication by a common factor: | The student will demonstrate the ability to multiply by a common factor by correctly expanding four of five given expressions. |
| 31. Special products: | The student will state the expansion formulas for products of a sum and difference, the square of a sum, cube of a sum and cube of a difference. |
| 32. Extracting square roots: | The student will extract the square root of eight of ten given expressions. |
| 33. Squaring binomials: | The student will demonstrate the ability to square binomials by correctly solving at least four of five representative problems. |

Algebraic Fractions

Concept	Behavioral Objective
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| 34. Properties of algebraic fractions: | The student will state the properties of algebraic fractions and will illustrate each with an example. |
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Algebraic Fractions

Concept	Behavioral Objective
35. Basic operations with algebraic fractions:	The student will correctly solve eight of ten equations involving the four basic operations with fractions.
36. The fractional exponents:	For a given set of radical expressions, the student will simplify by expressing in terms of fractional exponents with 80% accuracy.
37. Finding the least common denominator:	The student will express each denominator as a product of prime factors and will use these prime factors to find the LCD correctly for seven out of eight problems.
38. Reducing to lower terms:	The student will factor both numerator and denominator into prime factors and will cancel common factors to reduce four of five fractions to lowest terms.

Inequalities

Concept	Behavioral Objective
39. Properties of inequalities:	The student will state the properties which are common to both equalities and inequalities, and will give examples of properties which are different for inequalities.
40. Literal equation formulas:	The student will solve eight of ten representative equations involving variables.
41. Equivalent, identical and conditional equations:	From a list of ten equations, the student will correctly identify at least eight as either equivalent, identical, or conditional.

Inequalities

Concept	Behavioral Objective
42. Simplification of equations:	Given five equations, the student will simplify at least four.
43. Solution of equations in variables:	Given ten equations, the student will solve at least eight in terms of a given variable.
44. Analysis of a formula:	Given five formulas the student will analyze at least four into component parts.
45. Transposition of terms:	The student will define the term transposition.
46. Transposition of factors:	Given five equations, the student will demonstrate the transposition of factors with 80% accuracy.
47. Translating problems into equations:	Given five word problems, the student will translate at least four into equations.
48. Operations with dimensional units:	Given ten formulas, the student will analyze eight in terms of dimensional units.
49. Solving systems of equations of two and three unknowns:	Given ten equations in terms of two and three unknowns, the student will solve at least eight algebraically.
49. Dimensional units and conversion:	Given five dimensional units, the student will convert at least four to other dimensional units.
50. Equations containing fractions and decimals:	Given ten equations containing fractions and decimals, the student will solve at least eight.

Inequalities.

Concept	Behavioral Objective
51. Degree of an Equation:	Given five equations of various degrees, the student will identify at least four.
52. Solving linear equations:	Given ten linear equations, the student will solve at least four.
53. Algebraic solutions to variables:	Given five variable equations, the student will solve at least four in terms of a specified variable.
54. Equations with unknowns in the denominators:	Given ten equations with variable fractions, the student will find the LCD for at least eight.
55. Proportions:	The student will give the definition for proportion.
56. Proportionality:	Given five proportionalities, the student will translate at least four in terms of equations.

Linear Graphs

Concept	Behavioral Objective
57. Coordinate notation:	The student will define coordinate notation in terms of a cartesian graph system.
58. Plotting points on graphs:	Given five ordered pair, the student will plot at least four.
59. Displaying data with graphs:	Given a set of data, the student will display the data by three graphs with 80% accuracy.

Linear Graphs

Concept	Behavioral Objective
60. Solving problems by means of graphs:	Given five problems, the student will solve at least four graphically.
61. Graphs of linear equations:	Given five linear equations, the student will graph at least four.
62. Determining linear function from two points:	Given ten sets of points, the student will determine the linear functions for at least eight.
63. Graphing parallel lines:	Given five equations representing parallel lines, the student will show that at least four are parallel.
64. Graphing perpendicular lines:	Given five linear equations, the student will find and graph at least four perpendicular lines.
65. Graphing lines parallel to the axis:	The student will state the linear equations parallel to the vertical and horizontal axis.
66. Graphing solutions of simultaneous equations:	Given five sets of linear equations, the student will find the solution for at least four sets.
67. The slope of a line and Y intercept:	The student will state the formula for the slope-intercept.
68. The distance formula:	Given five sets of points, the student will find the distance between at least four sets.
69. Graphic solution of system of linear equations:	The student will explain the Solution of systems of linear equations by the graphical method.

Linear Graphs

Concept	Behavioral Objective
70. Solution by addition and subtraction:	The student will explain solutions of linear equations by addition & subtraction.
71. Applications of simultaneous equations:	Given five problems of application, the student will demonstrate the solution of at least four by simultaneous equations.

Quadratic Equations

Concept	Behavioral Objective
72. Solution of $ax^2 + bx + c = 0$:	The student will solve the general quadratic equation $ax^2 + bx + c = 0$.
73. Solution by the quadratic formula:	The student will demonstrate the solution of at least four out of five quadratic equations by the use of the quadratic formula.

Exponential Notation

Concept	Behavioral Objective
74. The law of zero exponents:	The student will state the zero law for exponents.
75. The law of exponents in raising to powers:	The student will state the law for raising to a power to a power.
76. The law of exponents in extracting roots:	The student will state the law of exponents in extracting roots.

Exponential Notation

Concept

Behavioral Objective

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| 77. The law of exponents based on 10: | The student will state the law of exponents based on 10. |
| 78. Power variations: | The student will explain what is meant by power variation. |
| 79. j-operator: | The student will demonstrate the use of the j-operator. |
| 80. The one operator: | The student will demonstrate the one operator. |
| 81. Complex numbers: | The student will identify complex numbers from a list of real complex numbers. |
| 82. Complex equations: | The student will solve four out of five complex equations. |
| 83. Squares of numbers: | The student will square five numbers with 80% accuracy. |
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Radicals

Concept

Behavioral Objective

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| 84. Irrational numbers: | The student will define the term irrational number. |
| 85. The square root of positive numbers: | Given five positive numbers, the student will find the square root of at least four. |
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Logarithmic Notation

Concept

Behavioral Objective

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| 86. Definition of a logarithm: | The student will state the definition of a logarithm. |
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Logarithmic Notation

Concept	Behavioral Objective
87. Properties of logarithms:	The student will state the properties of logarithms.
88. Characteristics:	The student will define the characteristics of a number for logarithm notations.
89. Mantissa:	The student will define mantissa of a number for logarithm notation.
90. Determining logs:	Given five numbers, the student will find their logs from a table with 80% accuracy.
91. Finding powers using logarithms:	Given five numbers, the student will find the powers using logarithms of base 10.
92. Finding roots using logarithms:	Given five numbers, the student will find specified roots of at least four by use of logarithms.
93. Interpolation using logarithms:	The student will explain interpolation by use of logarithms.
94. The logarithm of unity:	The student will explain why the logarithm of unity is zero.
95. Given $\log n$, find n :	Given five problems of $\log n$, the student will find n for at least four.
96. Given n , find $\log n$:	Given five numbers the student will find $\log n$ of at least four.
97. Logarithms to the base e :	The student will define the value of e in logarithm notation.

Logarithmic Notation

Concept	Behavioral Objective
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98. Time constant curves:

Given five representative problems, the student will find at least four time constant curves.

Sets and Logic

Concept	Behavioral Objective
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99. Electric switch diagrams:

Given an electric switch diagram, the student will explain the diagram with 80% accuracy.

100. Venn diagrams:

Given three intersecting sets, the student will construct their Venn diagram with 80% accuracy.

101. Truth tables:

The student will explain the notation of truth tables with 80% accuracy.

102. Tautologies:

The student will explain tautologies with 80% accuracy.

Work and Power

Concept	Behavioral Objective
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103. Work and power:

The student will define work and power.

104. Horsepower of a steam engine:

Given five representative problems, the student will find at least four horsepowers of steam engines.

105. Horsepower of a gas engine:

Given five representative problems, the student will find at least four horsepowers of gas engines.

Work and Power

Concept	Behavioral Objective
106. Brake horsepower:	The student will define brake horsepower.
107. Electrical powerwatt:	The student will define the electrical power watt.
108. Mechanical efficiency of machines:	The student will define what the mechanical efficiency of machines mean.

Speed Ratio and Pulley Gears

Concept	Behavioral Objective
109. Gear Trains:	Given a diagram of gear trains, the student will explain the diagram with 80% accuracy.
110. Idlers:	The student will explain how an idler works with 80% accuracy.
111. Finding the number of teeth for a given speed ratio:	Given a speed ratio, the student will explain how to find the number of gear teeth with 80% accuracy.
112. Compound gearing:	The student will explain compound gearing with 80% accuracy.
113. Worm and gear:	The student will explain the advantages of a worm and gear with 80% accuracy.
114. Trains and spur, bevel, and worm gearing:	The student will explain the advantages of trains, spur, bevel and worm gearing with 80% accuracy.

Quadratic Equations and Formulas

Concept

Behavioral Objective

115. The meaning of
quadratic equations:

The student will explain the
meaning of a quadratic
equations with 80% accuracy.

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Exponents

Concept	Behavioral Objective
1. The meaning of powers whose exponents are either "1" or "0":	The student will state the meaning of exponents of 1, and 0 with 80% accuracy.
2. The meaning of powers with negative whole-number exponents:	The student will define the term negative exponents.
3. The reciprocals of powers and converting division to multiplication:	Given five division problems, the student will solve at least four by using the concept of reciprocal.
4. Converting multiplications to powers of divisions:	Given five examples, the student will convert multiplication to division by the use of powers for at least four.
5. Equivalent forms of fractions containing powers:	Given five examples of fractions containing powers, the student will convert at least four to equivalent forms.
6. Powers of fractions:	The student will work four out of five examples utilizing powers of fractions.
7. Powers of powers:	Given ten examples, the student will raise at least eight to a specific power.
8. Adding and subtracting letterterms which contain powers:	Given ten examples, the student will add and subtract letterterms which contain powers for at least eight.
9. A review of powers and the laws of exponents:	The student will give all the properties and laws for exponents with 80% accuracy.

Exponents

Concept	Behavioral Objective
10. The roots of multiplication and division:	The student will explain what is meant by the roots of multiplication and division.
11. The roots of powers:	Given five power problems, the student will find specified roots for at least four.
12. Simplifying radicals by factoring out perfect powers:	Given ten examples, the student will simplify specified radicals by factoring out perfect powers for at least eight.
13. Raising radicals to powers:	Given ten radicals, the student will raise at least eight to a specified power.
14. The meaning of powers with fraction exponents:	The student will explain the meaning of powers with fractional exponents with 80% accuracy.
15. Fractional exponents and the laws of exponents and radical powers and decimal exponents:	The student will state the laws related to fractional exponents and will correctly solve a problem involving each law.
16. The meaning of roots and radicals.	The student will state the definitions for the terms, and will find the roots to at least four out of five expressions involving radicals.

Linear Graphs and Slope

Concept	Behavioral Objective
17. Identifying linear equations in "x" and "y":	From a set of ten equations, the student will correctly identify at least eight according to whether they are linear or nonlinear.

The D. C. Circuit

Concept	Behavioral Objective
107. Kirchoff's rules:	The student will state Kirchoff's circuit rules.
108. Battery E.M.F. and terminal voltage--internal resistance:	Given a representative example, the student will calculate the internal resistance and explain how it is related to E.M.F. and terminal voltage.
109. The wheatstone bridge:	The student will demonstrate the use of the wheatstone bridge.

Fundamental Elements of an A.C. Circuit

Concept	Behavioral Objective
110. Capacitance:	The student will define capacitance.
111. Capacitors in series:	The student will wire two capacitors in series.
112. Capacitors in parallel:	The student will wire two capacitors in parallel.
113. Energy requires to charge a capacitor:	The student will calculate the energy required to charge capacitors in series and in parallel.
114. Time for charging and discharging a capacitor; the timer constant:	The student will sketch the exponential curve representing the time for charging and the time for discharging a capacitor.
115. Inductance:	The student will define inductance.
116. Time constant of an L-R circuit:	The student will explain the time constant for a L-R circuit.

Fundamental Elements in an A.C. Circuit

Concept	Behavioral Objective
117. Phase relation in A.C. circuits:	The student will explain the phase relation in an A.C. circuit.
118. The series A.C. circuits, phase diagram:	The student will draw a phase diagram for a series A.C. circuit.
119. Reactance and impedance:	The student will define reactance and impedance.
120. Resonance:	The student will define resonance.
121. Power and power factor:	The student will explain the terms of power and power factors.

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