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ABSTRACT

Preservice elementary teachers' use of examples in recognizing and disproving invalid deductive arguments related to topics in elementary school mathematics was examined. Two forms of the Classroom Logic Test were developed, one multiple choice and one free response. Subjects were asked to judge the correctness of arguments and, if incorrect, provide feedback. Tests were administered to 50 students enrolled in a mathematics methods course. The major finding indicated that subjects provided the list of options erred by rejecting valid arguments, while those not given the list accepted invalid arguments. Findings are related to issues in mathematics teacher education. (Author)

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Judgements of Mathematical Arguments by

Future Elementary Teachers

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Several recent studies (Eisenberg and McGinty 1975, Jansson 1975, Juraschek 1976, Damarin 1977) have addressed the issue of logical abilities among preservice elementary teachers. Most of these studies have been concerned with the ability to make appropriate inferences. In general they have shown that the preservice teachers tested perform rather poorly; that is, they draw inappropriate conclusions from pairs of statements given to them in paper and pencil tests.

While these studies are interesting, and even a bit disturbing, it is difficult to interpret their implications for teacher education. The studies have been made and reported by mathematics educators. Yet the item content used has not been mathematical. Several questions arise concerning the relationship of these studies to the education of teachers:

- Why do we care that teachers can make logical inferences?
- How is performance on traditional tests of logical inference, tests in which inferences are made about brown dogs and shiny cars, related to the ability to make logical inferences in mathematical contexts?
- What are the mental processes involved in the drawing of correct inferences from statements involving logical connectives?

Analysis of these questions points to new lines of research in the areas of logical abilities and logical training for pre-service elementary teachers.

The ability of teachers to use and interpret the language of class and conditional logic, and to draw appropriate inferences from statements in these forms, would seem to be of *a priori* importance. Moreover, there is some empirical support for the importance of this constellation of abilities. Gregory and Osborne (1975) have shown that Junior High School students of teachers who regularly use the language of conditional logic score better on tests of critical thinking than students of teachers who do not use this language.

In his comprehensive review of the literature on discovery learning, Strike (1976) argues that the "logic of verification" lies at the core of this instructional approach. Discovery learning would seem to be an appropriate instructional strategy only in situations in which that which is to be learned is a principle, rule, strategy, or another sort of generalization or generalizable phenomenon.

Structural or generalizable principles have not been the content of items in traditional tests of the ability to make logical inferences; item content has been related to hypothetical relationships among familiar objects (e.g., "If the car is shiny, then it is fast," and "The dog is brown or the ball is blue."). In vernacular speech about such familiar objects the words which serve as logical connectives are interpreted differently from the interpretations applied to them in more structured contexts demanding rigorous logical thought. It is possible, therefore, that performance on these tests would be unrelated to the ability to reason logically in structured contexts such as mathematics.

Characteristic of such situations is the need to use properties to classify objects or concepts, and to use the logical interrelationships among properties or classes to determine or interpret the truth of statements or validity of arguments. Previous research (Damarin 1976, to appear) indicates that the ability of preservice elementary teachers to use statements which convey amthematical information through logical connectives is closely related to the ability to sort the set of possible combinations of properties. This research has been "abstract," that is, it has involved symbolic tasks related to the concept of odd and even integers. Performance on these tasks differed from performance by similar subjects on traditional inference tests. The question of how performance on this type of task is related to the judgement of argument validity in a classroom setting is one of interest and importance.

<u>The Study</u>

The study was designed with two questions in mind: (1) how accurately do preservice teachers judge (children's) arguments about mathematics, and (2) to what extent do they use counter-examples when criticizing invalid arguments? Two conditions were compared; in the first subjects were given arguments and a list of combinations or properties of the phenomena to which the arguments applied; in the second condition subjects were given only the arguments.

<u>Procedures</u>. Two forms of the Classroom Logic Test (CLT) were developed. Each item contained an argument concerning a point in elementary school mathematics. Elementary education majors enrolled in a course on methods of teaching mathematics

were asked to imagine themselves as teachers and the arguments as given by their students. Under these assumptions they were to judge the arguments as correct or incorrect, and, if incorrect, provide feedback for the student making the argument. Items in CLT-A included lists of four examples (or example types) from which feedback was to be selected. CLT-B items required subjects to invent and write out their own feedback. The same arguments were presented in both forms; 5 were correct and 13 incorrect. Directions and sample items from CLT-A are presented in table 1; CLT-B directions and items differed from these only in the instructions for providing feedback.

Insert Table 1 about here

Test forms were randomly assigned to 50 students and completed during a regular session of a course on methods of teaching mathematics. Items were scored for judgements of arguments and for type of feedback provided. Responses to the two forms were compared on the basis of response frequencies and order of item difficulty.

Results. Error frequencies for the judgement of arguments are presented in table 2; these frequencies were submitted to a Chi-square analysis, and a significant difference in error patterns was detected ($\chi^2 = 10.75$, $p < .01$). Subjects given form A made a greater proportion of errors by rejecting valid arguments,

Insert table 2 about here

while those given form B erred by accepting invalid arguments. When the distribution of correct and incorrect arguments was taken into account there was no difference in the overall difficulty of the two tests.

Items from each test were rank-ordered according to difficulty of argument judgements; the Spearman rank-order correlation between the two forms was found to be significant ($t = 2.75$, p .02, two-tailed test). Thus the presence or absence of examples did not have a serious differential effect on the judgement of individual arguments.

Following the analysis of argument judgements the responses to "feedback" parts of items using invalid arguments were coded to reflect use of examples, counter-examples, or (in CLT-B) other feedback, as well as omissions or wrong judgements. The coding system, and frequencies for coded responses, are presented in table 3. When distributions were compared it was found

Insert table 3 about here

that on both forms the majority of subjects who judged invalid arguments to be incorrect provided counter-examples as feedback (78.6% for CLT-A, 56.6% for CLT-B). Items were rank-ordered according to the tendency of subjects to provide counter-examples, and the Spearman rank-order correlation was computed. This coefficeint was not significant ($t = .9475$) suggesting that item content interacted with the availability of options to determine whether counterexamples would be used. Examination of individual item responses bore out this observation; several items in form A seemed to lure subjects to responses which were not produced at all by subjects using form B.

Discussion

Data analysis indicated that the presence of a list of possible examples had two effects: it enhanced the abilities of subjects to identify invalid arguments, but it also debilitated their abilities to recognize valid arguments. Subjects who were given the list of options were somewhat more likely to select counter-examples as feedback for incorrect arguments than subjects who were not given this list. Although the mathematical content of items did not seem to interact with the availability of options in determining the difficulty of judging argument validity, such an interaction was observed in the assignment of feedback.

These considerations, coupled with findings (Damarin 1976) that the ability to use the set of relevant possibilities facilitates appropriate interpretation of propositional statements, suggest that mathematical preparation of elementary teachers should focus more directly on the domains over which mathematical statements and arguments have meaning, and on the set of possibilities which must be considered before asserting the truth of a statement.

The research reported indicates that future teachers who had available the set of possibilities to be considered tended to err by rejecting valid arguments, while those not having this set available erred by accepting invalid arguments. From an instructional point of view the latter error would seem to be more serious. If a teacher disagrees with the correct conjecture of a student, the student can argue his case. When a teacher supports an incorrect conjecture or argument, on the other hand, errors of thinking are reinforced.

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Table 1

Directions and Sample Items (CLT-A)

CLASSROOM LOGIC TEST

Directions. Imagine that your class has just taken a special test for a state-wide math contest, and that you are now leading a class discussion of the problems on the test. As each student presents his interpretations and ideas concerning various problems you want to provide appropriate feedback to him or her.

For each item
Read the student's statement carefully and decide whether the argument is correct or incorrect.
Place a check (✓) which indicates your judgment concerning the student's argument.
If you check correct, go on to the next question.
If you check incorrect, read the four options, and place a check (✓) beside the one you would show the student in order to point out his error to him.

Work the Sample items below:

A. Jackie: If a number is even, then the remainder is 0 when you divide by 2. In this problem the number is even, and we're dividing by 2, so the remainder is 0.

_____ Jackie is correct

_____ Jackie is incorrect; I would show her

_____ An even number with remainder 0 when divided by 2.

_____ An even number with remainder 1 when divided by 2.

_____ An odd number with remainder 0 when divided by 2.

_____ An odd number with remainder 1 when divided by 2.

B. Tim: In this problem the number A is either 0, 1, or 2, and B is bigger than C. Therefore AB is bigger than AC.

_____ Tim is correct.

_____ Tim is incorrect. I would show him A, B, and C with

_____ B greater than C, AB greater than AC

_____ B greater than C, AB equal to AC

_____ B less than or equal to C, AB greater than AC.

_____ B less than or equal to C, AB equal to AC

Now turn to the next page and check your answers to the sample items.

Table 2

Errors on correct and incorrect arguments.

Argument is	CLT-A	CLT-B
correct	28 errors	14 errors
incorrect	101 errors	154 errors

Table 3

Coding and frequencies for feedback to arguments

Response type	CLT-A	CLT-B
Number of "incorrect" judgements	222	171
Feedback		
Counterexample(*)	179	111
A, B, C, or D (not counterexample)	38	7
E	---	18
F	---	15
O	---	12
X	5	8
N	96	135
Item omitted entirely	7	19

A
B
C
D non-conterexamples presented in CLT-A

* counterexample

E example provided or discussed; not clearly one of the above, but mathematically sound

F faulty example (e.g. arithmetic error)

O other response

X no response (although argument labelled incorrect)

N argument labelled correct

Item omitted completely