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ABSTRACT

This paper reports on a study which was performed to replicate and extend the findings of Hendrix with regard to verbalization and discovery learning. College algebra classes were randomly assigned to three verbalization conditions: (1) no student verbalization of generalizations required, (2) students make written verbalization of generalization, and (3) students make written verbalization followed by teacher verbalization. In each treatment group, two algebraic and two geometric topics were covered. Learning data were gathered by posttest. The data were submitted to several analyses of variance in conjunction with pairs of other variables: mathematics aptitude, mathematics achievement, English aptitude, English achievement, sex, correct or incorrect verbalization, difficulty of generalization. In addition, analyses of variance were performed on data generated by combining groups. In all these analyses, only two significant ($p < .05$) differences were observed: high English-aptitude subjects performed more poorly in treatment group 3 when the generalization was difficult, and low English-aptitude subjects performed better overall in treatment group 3 than in the other methods combined. (DT)

THE EFFECT OF VERBALIZATION UPON CERTAIN MATHEMATICAL GENERALIZATIONS

by

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CERTAIN MATHEMATICAL GENERALIZATIONS

The purpose of this study was to replicate and extend the work of Hendrix. In her experiments reported in 1947, the subjects were taught only one generalization. In this study, the subjects were taught four generalizations from two branches of mathematics, some of which were more difficult to discover and verbalize than others. This alteration of Hendrix's design provided the opportunity to observe the effects of content and verbal complexity upon the results. Also, the sample size used in this study was much larger than the one used by Hendrix and most others who have reported studies similar to this.

Specifically, this research was designed to determine:

1. If there are significant differences among three types of student responses that could follow learning of mathematical generalizations taught by discovery: nonverbal, student verbalization only, and student verbalization with teacher assistance.

2. If there is any significant interaction between methods and generalizations chosen from algebra and geometry.

Summary of Procedure

The sample for the study consisted of approximately 160 students enrolled in eight sections of college algebra at Texas A & M University. The sections were assigned to four teachers, trained in inductive discovery teaching. Each of the classes was taught inductively four mathematical generalizations, two of which were algebraic in nature /G1 and G2/ and two of which were geometric /G3 and G4/. The generalizations were chosen so that one of the algebraic generalizations /G1/ and one of the geometric generalizations /G4/ were more difficult to both discover and verbalize than the others. Two generalizations,

one algebraic and one geometric, were taught on one day, and a posttest was administered two days later. Then approximately one week later, the remaining two generalizations were taught, and again a posttest was administered two days later. The teachers were randomly assigned to two of the three methods employed in this study. Teachers using Method 1 did not allow student verbalization of the generalizations. Teachers using Method 2 requested a written verbalization of the generalizations. Teachers using Method 3 also requested a written verbalization of the generalization, but they also presented a precise verbalization of the generalization to the class by writing it on the board. The data collected partitioned in several ways and analyzed by analysis of variance.

Conclusions

The hypotheses of this study stated in the null form are as follows:

1. There is no significant difference at the .05 level of significance in the problem solving ability of students who are required to write a verbalization of a generalization after discovering it but are not shown a correct verbalization of the generalization and students who do not verbalize a generalization upon discovery.

2. There is no significant difference at the .05 level of significance in the problem solving ability of students who are required to write a verbalization of a generalization after discovering it and are then shown a correct verbalization of that generalization and students who do not verbalize a generalization upon discovery.

3. There is no significant difference at the .05 level of significance in the problem solving ability of students who are required to write a verbalization of a generalization after discovering it and are then shown a correct verbalization of that generalization and students who are required to write a verbalization of the generalization after discovering it but

are not shown a correct verbalization of the generalization.

4. There is no significant interaction at the .05 level of significance between treatment and the generalization being taught.

Overall Data

While the data collected does reveal differences among method means and indicate there was interaction between methods and generalizations, the analysis of variance of this data show these differences to be significant at only the .286 level which due to the large sample size should be a fairly accurate probability. Therefore the null hypothesis can not be rejected at the .05 level.

Analysis of data on preciseness of verbalization

In an effort to gather further information on verbalization effects, the data was separated as follows:

1. Students who verbalized correctly initially and received no subsequent correct teacher verbalization.

2. Students who verbalized incorrectly initially and received no subsequent correct teacher verbalization.

3. Students who verbalized correctly initially and also received a subsequent correct teacher verbalization.

4. Students who verbalized incorrectly initially but received a subsequent correct teacher verbalization.

5. Students who did not verbalize or receive a subsequent correct teacher verbalization.

The analysis of variance of the data on these groups did not yield a probability lower than .2265 and consequently none of the differences obtained is significant at the .05 level of significance.

Analysis is terms of sex, S.A.T. (Scholastic Aptitude) aptitude scores, and S.A.T. achievement scores

Additionally, further divisions into separate high (500 and above S.A.T scores) and low (below 500 S.A.T. scores) English and mathematics groups were also analyzed by analysis of variance but the probabilities obtained were not small enough to reject any of the four null hypotheses in terms of these groups. Further divisions into male and female groupings were also analyzed with only one probability significant at the .05 level found. This result was found upon analyzing female high mathematics aptitude data but is subject to suspicion due to the small sample size that was involved.

Analysis of high English aptitude data

Analysis of the overall high English aptitude data resulted in a generalization method probability that is significant at the .05 level. This finding resulted in the rejection of Hypothesis 4 for this group of students. Further analysis of the overall high English aptitude data revealed that the combination of Generalization 2 with Method 3 caused the significant result to appear. The reader should recall that Generalization 2 is the more difficult of the two algebraic generalizations and that Method 3 required student verbalization which was followed by a precise teacher verbalization.

TABLE 1

Overall High English Data
(Cell Means in percent correct)

	M1	M2	M3	G-Means
G1	87	100	100	96
G2	76	70	25	57
G3	52	42	76	56
G4	86	86	77	83
M means	75	75	69	

TABLE 2

Analysis of Variance of the Data in Table 1
Aptitude Data

Source	M.S.	D.F.	F-RATIO	P
Total	0.063	27		
Between	0.117	11		
G	0.257	3	10.0333	0.0008
M	0.009	2	0.3557	0.7104
GM	0.083	6	3.2496	0.0275
Within	0.026	16		

One possible reason for this result is that perhaps the student verbalization followed by teacher verbalization of a difficult generalization has more harmful effects than good ones. Since the high English aptitude students had approximately the same results using Methods 1 and 2 with Generalization 2, perhaps high English aptitude students verbalize covertly (internally). Therefore, it could be that Method 3 results were so poor because the teacher verbalization tended to distract and confuse students who had arrived at their own verbalization either overtly (in writing) or covertly. Perhaps this could be interpreted to mean that with high English aptitude students it is sometimes best not to contaminate a discovered concept by trying to show the students a correct verbalization.

Those who are familar with Hendrix's (1947) hypotheses realize that this result seems contrary to what she would have expected. Hendrix contended that verbalization of discovered mathematical generalizations should not be detrimental to students who have a good command of language.

Kersh, (1962) in a discussion of the results of his 1958 and 1962 studies says:

"The results of both experiments also are consistent in their failure to support the notion that attempts to provide added meaning will necessarily prolong memory for rules and procedures and will enhance their transfer. On the contrary, both experiments suggest that such attempts may well do more to interfere with learning than enhance it.

Therefore, the particular significance that was revealed in this study concerning the high English aptitude group is in agreement with Kersh's finding.

High and low English aptitude data analyzed together

An analysis that utilized high and low nglish aptitude and the three methods of this study as variables with Generalization 2 data, reveals an aptitude-method interaction probability of .0291 as seen in Table 4.

TABLE 3

Overall English Aptitude Analysis Concerning Only Generalization #2

(Cell Means in percent correct)

	M1	M2	M3	Eng. Apt. Means
H. Eng. Apt.	75	70	25	57
L. Eng. Apt.	58	28	89	58
M Means	67	49	57	

TABLE 4

Analysis of Variance for the Data in Table 3

Source	M.S.	D.F.	F-RATIO	P
Total	0.097	13		
Between	0.153	5		
Apt.	0.001	1	0.0133	0.9069
M	0.037	2	0.5988	0.5760
(Apt. M)	0.346	2	5.6632	0.0291
Within	0.061	8		

Further observation of Table 4 reveals that Generalization 2, the more difficult of the two algebraic generalizations had extremely poor posttest results with high English aptitude students who had been taught by Method 3. Also, again using Generalization 2 data, the low English aptitude students obtained noticeably lower results using Method 2. Perhaps high English aptitude students who had adequate verbalizations of their own were confused by the subsequent teacher verbalization, whereas low English aptitude students became confused as a result of writing their own verbalizations, but were aided somewhat by the subsequent teacher verbalization.

REFERENCES

Hendrix, Gertrude. "A New Clue to Transfer of Training", ELEMENTARY SCHOOL JOURNAL, 1947, 48, 197-208.

Kersh, Bert Y. "Learning by Discovery: Instructional Strategies". ARITHMETIC TEACHER, 1965, 12, 414-417.