APPENDIX C

RAW SCORE DATA

OBSERVATION 01

Group ML

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
T.E.	6	6	6	9	4	4	0	0	16	19
R.R.	6	8	7	10	0	0	0	1	13	19
D.O.	9	9	8	8	6	9	8	9	31	35
S.M.	5	7	7	9	5	8	4	4	21	28
D.L.	5	9	1	2	0	1	1	1	7	13
C.K.	5	9	4	10	2	9	6	10	17	38
K.S.	5	8	4	9	1	8	2	2	12	27
Green Form										
T.K.	4	9	5	10	5	9	3	9	17	37
D.S.	6	10	6	8	4	6	1	3	17	27
C.S.	7	9	5	9	3	8	5	6	20	32
B.S.*	8	10	4	6	4	8	2	2	18	26
C.S.	7	7	8	9	6	6	2	2	23	24
D.A.	7	8	8	9	8	10	4	4	27	31
B.H.	10	10	9	9	2	3	1	1	22	23

* Student not present at 02.

OBSERVATION 01

Group E

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
K.K.	5	10	5	8	1	7	3	3	14	28
C.M.	5	10	3	8	3	9	5	9	16	36
T.B.	8	10	9	9	6	9	6	10	29	38
A.H.	6	8	6	7	4	6	1	2	17	23
E.S.	8	9	8	9	6	7	6	6	28	31
R.B.	0	10	5	8	7	9	5	9	17	36
Green Form										
M,T.	7	9	7	10	5	6	1	1	20	26
R.H.	9	10	10	10	9	9	6	6	34	35
T.R.	6	10	4	8	5	10	2	4	17	32
K.G.	6	9	5	9	10	10	4	8	25	36
J.B.	8	10	5	7	10	10	5	9	28	36
E.J.*	10	10	5	8	10	10	10	10	35	38

* Student not present at 02.

OBSERVATION 01

Group MH

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
L.M.	8	9	4	9	4	5	1	1	17	24
C.U.	8	10	9	10	7	9	4	5	28	34
J.M.	8	10	5	8	8	10	6	7	27	35
K.R.	8	9	10	10	9	9	6	6	33	34
E.B.	7	10	8	10	5	10	6	10	26	40
C.S.	8	10	10	10	10	10	7	9	35	39
Green Form										
J.S.	7	7	9	9	7	7	5	5	28	28
P.P.	9	10	10	10	10	10	2	7	31	37
L.W.	6	10	4	7	9	9	5	6	24	32
J.F.	7	10	8	10	5	6	5	6	25	32
K.D.	8	10	9	10	8	8	6	9	31	37

OBSERVATION 02

Group ML

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
M.H.	8	10	5	10	1	6	2	2	16	28
C.S.*	9	9	9	9	4	5	1	4	23	27
C.S.*	7	9	7	9	2	6	3	3	19	27
B.H.*	9	9	8	8	3	3	3	3	23	23
T.K.*	6	8	3	9	4	8	2	8	15	33
J.T.	5	9	1	7	0	2	0	0	6	18
K.M.	5	10	2	2	1	2	0	1	8	15
D.S.*	6	10	10	10	2	5	1	6	19	31
L.K.	9	9	5	5	3	7	1	2	18	23
D.A.*	7	8	9	10	6	8	4	4	26	30
J.R.	6	9	3	9	1	1	3	3	13	22
T.O.	5	8	6	10	4	7	1	8	16	33
M.L.	5	9	2	5	1	3	0	1	8	18
Green Form										
M.A.	10	10	9	9	10	10	3	7	32	36
S.M.*	8	10	6	10	5	9	4	5	23	34
T.T.	5	10	1	5	3	4	2	2	11	21
R.R.*	7	8	4	8	3	5	3	4	17	25
K.S.	4	9	2	6	6	9	4	7	16	31
D.G.	8	9	6	9	4	5	0	0	18	23
D.O.*	10	10	8	8	7	7	2	2	27	27
J.L.	10	10	7	7	3	3	8	8	28	28
T.E.*	9	9	7	7	1	3	1	1	18	20
B.B.	10	10	7	7	5	5	2	2	24	24
K.U.	4	6	4	6	5	7	3	3	16	22
F.G.	9	10	7	10	9	9	2	5	27	34
C.K.*	5	9	4	8	4	8	2	6	15	31
D.L.*	3	10	1	5	0	0	1	1	5	16
K.S.*	2	7	4	8	3	8	0	2	9	25

* Students tested at 01.

OBSERVATION 02

Group E

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
M.T.*	8	9	3	8	1	4	1	5	13	26
J.B.*	8	9	3	8	10	10	2	6	23	33
K.G.*	10	10	7	9	7	7	4	8	28	34
R.H.*	6	10	9	9	7	10	5	5	27	34
T.R.*	6	10	5	8	4	10	3	6	18	34
B.J.*	Absent									
R.B.	8	10	2	8	5	7	4	4	19	29
L.A.	6	10	7	10	7	10	4	8	24	38
K.U.	5	10	10	10	7	9	5	8	27	37
R.K.	3	10	3	9	0	0	0	0	6	19
T.P.	6	10	7	7	5	7	3	7	21	31
M.H.	4	9	5	9	6	10	5	5	20	33
M.H.	7	9	8	10	4	9	2	4	21	32
M.E.	9	9	4	5	9	9	7	7	29	30
L.J.	9	10	9	10	9	10	6	8	33	38
P.W.	9	10	8	10	6	8	7	9	30	37
Green Form										
C.M.*	4	10	2	9	6	9	3	10	15	38
K.K.*	8	10	6	10	4	6	1	1	19	27
A.H.*	7	8	7	8	6	7	1	6	21	29
E.S.*	10	10	6	7	8	8	4	4	28	29
T.B.*	9	9	4	10	9	9	5	8	27	36
R.B.*	8	10	7	7	6	8	4	6	25	31
D.H.	6	10	1	4	1	2	1	1	9	17
D.S.	3	8	2	7	4	9	3	9	12	33
R.B.	6	9	5	6	0	2	0	0	11	17
J.M.	8	9	10	10	10	10	7	9	35	38
R.Z.	6	8	1	1	1	3	0	0	8	12
M.R.	4	9	7	10	7	8	6	8	24	35
T.H.	5	10	8	8	5	5	0	0	18	23

* Students tested at 01.

OBSERVATION 02

Group MH

Blue Form	Addition Speed	Addition Power	Subtraction Speed	Subtraction Power	Multiplication Speed	Multiplication Power	Division Speed	Division Power	Total Speed	Total Power
L.W.*	10	10	5	7	10	10	7	7	32	34
K.D.*	10	10	10	10	5	5	6	9	31	34
J.S.*	10	10	8	8	3	4	3	5	24	27
P.P.*	9	9	10	10	8	8	6	6	33	33
M.F.*	10	10	8	9	5	8	2	7	25	34
J.S.	6	10	7	9	6	8	3	5	22	32
T.G.	9	10	9	10	9	9	6	10	33	39
L.F.	10	10	9	10	8	9	3	7	30	36
D.U.	8	8	5	6	7	8	0	8	20	30
D.M.	10	10	8	10	6	9	2	5	26	34
Green Form										
J.M.*	8	10	9	10	9	9	5	9	31	38
K.R.*	10	10	10	10	9	9	2	5	31	34
C.U.*	9	9	8	10	7	8	5	9	29	36
C.S.*	10	10	10	10	9	9	7	9	36	38
E.B.	4	9	8	9	10	10	0	10	22	38
L.M.	9	9	10	10	8	9	2	2	29	30
S.O.	9	10	10	10	9	10	8	10	36	40
L.S.	5	10	5	10	4	6	1	6	15	32
D.Z.	8	10	8	10	7	10	0	7	23	37
D.H.	6	10	6	9	4	5	5	5	21	29
J.B.	4	9	3	8	1	5	4	4	12	26
S.R.	1	10	9	9	6	9	4	6	20	34
R.H.	7	9	8	8	6	9	3	7	24	33

* Students tested at 01.

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ABSTRACT

This study was an investigation of two questions.

1. How did the computational proficiency of a group of sixth grade children who had one year's experience with Developing Mathematical Processes (DMP) materials compare with that of an equivalent group of students who continued with the usual textbook program?

2. What occurs when sixth grade children study algorithms as sequences of rule statements?

The first question was approached in the following way. A random one third of a group of 90 sixth graders experienced one year of DMP developmental materials. Toward the end of the school year, the whole number computational speed and power of all of the children was measured. Though the magnitudes of the differences favored the DMP group for 6 of the 10 measures, multivariate and univariate analyses failed to show significant differences at the .05 level. In brief, the computational proficiency scores of the children using the DMP materials were as high if not higher than those of their peers who had used the more usual textbook program.

The second question, loosely related to the first, was investigated in the following way. The DMP group and a selected subgroup of the remaining population spent the last two full weeks of the school year working with a special unit. This unit required children to verbally analyze algorithms they knew how to do. While it was anticipated that there may be differences between the DMP and non-DMP groups in their ability to complete these tasks, the data were inadequate to resolve this question.

Students in general were willing to attempt the tasks and they experienced some successes in doing them. While no conclusions regarding the appropriateness of the verbal analysis of algorithms were presented, the general course of the unit was encouraging for further investigation. Perhaps the most interesting outcomes were the questions which emerged. These involve the identification and characterization of irreducible components of children's algorithmic thinking as well as the language they use to connect such components.

I

INTRODUCTION

This study was undertaken for the following two purposes: (1) to assess the computational proficiency of sixth grade children who were using developmental materials of Developing Mathematical Processes (DMP); and (2) to conduct a formative evaluation of a selected two-week unit which had computational algorithms as the primary subject matter. The rationale relating to these two purposes is presented in the following two separate sections.

GROUPING COMPUTATIONAL PROFICIENCY

The DMP curriculum is centered about processes of mathematics. As such it emphasizes a measurement approach to the learning of mathematics (Romberg, in press). Computational procedures, therefore, are presented as ways of describing or representing other less abstract situations with which the children are familiar. For example, the multiplication algorithm for whole numbers may be introduced as an efficient way to count the area of a rectangle in square units while the division algorithm can be presented as an efficient way to represent the partitioning or sharing of something. While verbal-symbolic explanations are also given to provide additional insight into computational procedures, the usual approach is to first present familiar situations and then to develop an algorithm as an efficient way of describing and solving the situation. The outline presented in Table 1 summarizes the instructional sequence employed in this study.

A contrasting approach is outlined in Table 2. The major focus of this contrasting approach is the symbolic manipulations of the algorithm to be learned. While the algorithm may be motivated by referring to proposed applications, these applications are rarely involved in either explanations of how the algorithm is worked or of why it is done that way. Furthermore, the situations of the proposed applications may be as unfamiliar to the learner as are the symbolic operations of the algorithm to be learned. This contrasting approach would provide problem situations for which the algorithm is appropriate, but only after instruction in how to do the algorithm is completed and after considerable drill and practice has been provided.

This contrasting approach, briefly identified as symbols first, then applications, can be carried out much more quickly than can the

TABLE 1

THE DMP INSTRUCTIONAL SEQUENCE FOR ALGORITHMS

	Steps	Examples
1.	Begin with a familiar situation structured in such a way that it can be represented by an algorithm. It usually involves a physical attribute.	A rectangular region subdivided into squares, e.g., graph paper, window panes, or brick wall. Attribute: area
2.	A problem related to the situation is presented that can be solved by means already known to the learner.	How many squares, panes, bricks, etc. are there? The learner could count them.
3.	Represent the situation in other ways including a symbolic representation.	There are 7 rows of 13 each. A 7 by 13 array. 7 x 13 is the total number.
4.	Solve the situation using both the attribute and the representations. Represent the solving with symbols.	Count, 7 rows of 10 is 70 and 7 rows of 3 is 21. The total is 91.
5.	Repeat 1-4 using different examples and attributes.	Regions of different dimensions, 7 by 14, 21 by 5, etc. Use arrays alone, repeated lengths or repeated weights, etc.
6.	Introduce standard algorithms to represent the actions.	$\begin{array}{r} 10 + 3 \\ \times \quad\ 7 \\ \hline 70 + 21 \end{array} \rightarrow \begin{array}{r} 13 \\ \times 7 \\ \hline 21 \\ 70 \\ \hline \end{array} \rightarrow \begin{array}{r} {}^{2}13 \\ \times 7 \\ \hline 91 \end{array} \rightarrow \begin{array}{r} 13 \\ \times 7 \\ \hline 91 \end{array}$
7.	Other applications and practice. This normally includes reversing the process as well; one may start with symbols and model them using an appropriate attribute.	Find the area of your desk top in square centimeters. Use a ruler to construct a 12" by 13" piece of paper. What could the symbols $\begin{array}{r} 24 \\ \times 6 \\ \hline \end{array}$ represent?

TABLE 2

A CONTRASTING INSTRUCTIONAL SEQUENCE FOR ALGORITHMS

Steps	Examples
1. Begin with the algorithm to be learned, motivated by proposed applications.	William Jones wanted to build a brick border along his sidewalk. It was to be 7 bricks high and 13 bricks long. He found how many bricks would be needed by multiplying 7 rows times 13 bricks per row.
2. After a few (≤, 3 ?) examples, present 1 or 2 algorithms ending with the standard algorithm.	$\begin{array}{r} 10 + 3 \\ \times \quad\quad 7 \\ \hline 70 + 21 \end{array}$ or $\begin{array}{r} 13 \\ \times 7 \\ \hline 21 \\ 70 \end{array}$ or $\begin{array}{r} 2 \\ 13 \\ \times 7 \\ \hline 91 \end{array}$ and finally $\begin{array}{r} 13 \\ \times 7 \\ \hline 91 \end{array}$
3. Practice the algorithm with diagnostic evaluations of student work.	$\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$, $\begin{array}{r} 21 \\ \times 8 \\ \hline \end{array}$, $\begin{array}{r} 45 \\ \times 9 \\ \hline \end{array}$, etc.
4. Provide applications through verbal problems.	Usual sets of word problems.

derivation of symbols from physical situations approach of Table 1. The total teaching time spent with an algorithm may be almost as much as that required for DMP, but if so, a much larger proportion of that time is spent simply practicing the algorithm.

The contrast being presented here is a contrast of approach and emphasis. It is not a question of explaining or not explaining the algorithm, nor is it a question of practicing or not practicing it. Both approaches require explanation and practice. The DMP approach places primary importance and hence the bulk of instructional time upon familiar physical or pictorial situations which are represented by the algorithms being learned. Within that framework practice is provided. The contrasting approach places primary emphasis upon the symbolic operations of the algorithm itself, and verbal explanations and practice consume the bulk of the instructional time. The first purpose of this study relates directly to this difference in approach and emphasis.

FIRST PURPOSE

A practical difference of the approach and emphasis in the DMP program, compared to common practice, is that the bulk of instructional time in DMP is spent in presenting and solving problem situations which the algorithm represents rather than in simply adding, subtracting, multiplying, or dividing numbers. Since the algorithm is developed out of a familiar context that it describes, this approach ensures that the algorithm is meaningful. However, the relatively small amount of time spent in practicing the procedure itself may have adverse effects in terms of computational proficiency when compared to more usual instructional practice. The first purpose of this study, therefore, was to compare the computational proficiency of a randomly selected group of sixth grade children who used DMP materials for one year with the computational proficiency of a group of their peers who had continued in a usual textbook oriented program.

From the point of view of the first purpose, this study was a simple comparison of the whole number computational proficiency of equivalent groups who were given different instructional treatments. However, the study had another purpose which also related to the nature of algorithms. This second purpose is presented following a discussion of algorithms themselves.

CONCERNING ALGORITHMIC LEARNING

An algorithm may be defined as a sequence of rule statements that enable anyone who knows them to complete any of a large set of similar tasks (see Suppes, 1969). For example, if one knows an appropriate algorithm, one can, theoretically at least, find the product of any two whole numbers. When a student can produce correct answers a large proportion of time (e.g., 90%) we say that he has learned an algorithm. The adequacy of this definition is demonstrated by the actual use of sets of rule statements to program machines to perform computations with complete accuracy.

There is a difficulty with this definition, however, particularly for mathematics education. That is, we cannot tell by simply watching a student work if he is conscious of a particular set of rule statements or not. We could, of course, ask the student to tell us what he is doing, transcribe his verbal responses and then see if the explanation will work as well as the student does. But, experience indicates that students exhibit a great deal of proficiency with many algorithms while evidencing little concern for or ability with precise statements of underlying rule sequences. Consequently, the defining of an algorithm as a sequence of rule statements does not seem to be useful for initial teaching and learning. An alternative is to define algorithms as sequences of behaviors that may or may not be verbalized.

Defining algorithms in this way assumes that a person may be able to compute in a meaningful way, without being able to make explicit any adequate set of underlying rule statements. If this assumption is valid, the second definition is adequate to define algorithms for initial learning and instruction. However, this second definition neglects an important content concern.

It is certainly one goal of mathematics education to produce users and inventors of algorithms in the sense of the definition first offered. At some point, therefore, the mathematics content to be studied must be algorithms as sequences of rule statements. At such a time the focus of instruction would not be upon learning to do a particular algorithm such as addition or substraction of whole numbers. Instead, ordered sets of rule statements would provide the focus of instruction and particular algorithms (in terms of either definition) would be examples to be analyzed by the student.

Objectives for such a unit of study might include the following.

1. The student is able to state (or choose) sequences of rule statements that describe a given algorithm.
2. The student is able to specify (or choose) sets of problems that a given algorithm (here, a series of rule statements) will correctly process.
3. The student is able to state (or identify) alternate sequences of rule statements that will correctly solve a set of problems that he already knows how to solve.
4. The student is able to generate an algorithm (a series of rule statements) that will process a given set of problems for which he has no formal algorithm.

Objective 4 may seem particularly difficult, but it is similar to the objective traditionally asked of second grade students who know how to add whole numbers by counting but are not able to add $\begin{array}{r}36\\ \underline{+27}\end{array}$.

The difference is that the second grade child is not usually asked to verbalize a series of rule statements that describe the behaviors he exhibits when he does addition of two-digit whole numbers. Objective 4 may be further illustrated by asking a student to describe how two common fractions can be added using an electronic calculator that can only add, subtract, multiply, and divide decimal numbers, without the neccesity of remembering or writing the results of intermediate steps.

The preceding discussion has served to provide two ways of defining algorithms and has presented some possible objectives for the study of algorithms in terms of the second definition. It is now possible to indicate the second purpose of this study.

SECOND PURPOSE

The second purpose of this study was to investigate, in an exploratory way, whether sixth graders could profitably study algorithms as sequences of verbal statements, and to further see what effect, if any, such a study may have upon their computational proficiency with the particular algorithms involved. Sixth graders were chosen because they normally know many algorithms in terms of sequence of behaviors and they should be at or near the cognitive development levels thought to be necessary for formal abstract thinking.

II

DESIGN AND PROCEDURES

The design employed in this study in response to the two purposes identified earlier is summarized in Figure 1. The following paragraphs describe this design.

SUBJECTS AND INSTRUCTION

The subjects for this study were the students of the entire sixth grade unit of a rural suburban school district in central Wisconsin. While no apparent minority groups were present in this population, the children varied widely in terms of home background, occupation of parents, academic ability, and achievement.

The children were separated into three groups for their mathematics instruction at the beginning of the year. One-third of the children were selected as the experimental group (E). This group used DMP developmental units during the entire school year. The children for this group were selected by ordering the entire population by IQ scores and choosing every third child. This procedure ensured representativeness and will be considered equivalent to random assignment. The remaining children were separated into two groups: one of middle to low ability (ML), and the other of middle to high ability (MH). The children in these two groups simply continued in the program of the preceding year that was based upon a popular textbook series. While subjects from groups ML and MH changed groups from time to time during the year, no subjects were brought into group E during the year. Furthermore, students left group E only if they were leaving the school system.

Instruction for all groups during the school year continued as indicated until the last three weeks. During the last two full weeks of the school year, groups E and ML worked activities concerned with verbal analyses of whole number algorithms. Group MH meanwhile reviewed computational procedures and worked on practice pages from the textbook. These last two weeks contained nine arithmetic periods of 50 minutes each, with parts of the second and ninth periods involving testing. The last week of the school year was a three-day week containing no arithmetic periods.

THE ALGORITHM UNIT

The analysis of algorithms unit that groups E and ML attempted required the following three tasks:

1. Write ordered sets of rule statements that correctly describe an algorithm for computing a given problem.

Assignment of Subjects	Groups	Instruction (Purpose 1)	First Observation	Instruction (Purpose 2)	Last Observation
R	E	DMP	R 01 no test	AU	02
Ability Grouped	ML	Textbook	R 01 no test	AU	02
	MH	Textbook	R 01 no test	D & P	02

Key:
- E — Experimental Group, Random 1/3 of population
- ML — Middle to Low Ability Group
- MH — Middle to High Ability Group

- DMP — Developing Mathematical Processes developmental materials
- Textbook — A popular textbook program, 1968 copyright
- AU — Algorithm Unit
- D & P — Drill and Practice, review pages from textbook program

- R — Random selection

- 01 — Speed and Power Test of Whole Number Computations
- 02 — Speed and Power Test of Whole Number Computations

Figure 1. Design of the study.

2. Order a given set of rule statements so as to describe an algorithm for computing a given set of problems.
3. Describe the error (a systematic computational error in a given set of completed problems) that resulted in some wrong answers.

The children worked together with considerable teacher guidance to complete these tasks for the division and addition algorithm. They were then asked to complete similar but less demanding tasks for the subtraction and multiplication algorithms independently. The materials given to the teachers of groups E and ML are presented in Appendix A along with the daily plan for use of these materials.

It should be noted at this time that the tasks for each algorithm were sequenced in the order indicated. While it seems fairly obvious that the first task is much more difficult than either the second or the third, this sequence was employed to see just what kind of responses the children could make to the first rather open ended task. This is consistent with the exploratory nature of this part of the study.

TESTS USED

The tests used at observations 01 and 02 were two forms of a speed and power test for whole number computations. Each form contained four sets of ten items, one set for each of the four operations. The sets were constructed by choosing ten pairs of items that covered a broad range of computational complexities beyond the basic facts. The items of each pair were matched in every computational aspect except for the basic number combinations; more simply, they were alike except for the digits. A random sequence for each of the ten pairs was determined. One item of each pair was then randomly selected for one form, while the remaining items comprised the second form. The two forms of the test were then completed by ordering the item sets addition first, followed by subtraction, multiplication, then division. One form was color coded blue and the other green. The forms were assumed to be equivalent. These forms were used in both timed and power settings as described below. Copies of the tests along with the instructions for administering them are found in Appendix B.

The completed test forms were administered to a comparable group of subjects prior to using them in this study to determine reasonable time allotments for each set of 10 items. Minimum times required to complete each set were noted. These times were adjusted to the next larger even one-half minute to become the times allowed for working under timed conditions. The times determined were one and a half minutes for each of addition and subtraction, three minutes for multiplication, and six for division.

Each test form was used in both a timed and power setting; the timed part of each test was, of course, administered first. During the timed part, each subject was required to work with a ball-point pen. At the conclusion of the time interval for the last set of 10 problems, the pens were collected and the children were then instructed to complete or correct all items of their tests using a pencil. They were given as much time as they desired to complete this task.

Scoring of the tests then involved 10 different scores. These were the number correct under timed conditions for addition, subtraction, multiplication, division, and total, and the number correct under power conditions for the same five factors. The first set are referred to as speed scores, and the last five as the power scores.

OBSERVATION 01

It was anticipated that either the algorithm unit presented to groups E and ML or the review planned for group MH could effect computational speed or power. Therefore, the tests described in the preceding section were administered to a random one-half of each group on the second day after beginning these experiences. This was observation 01. The children being tested were given either the blue or green form of the test and were seated so that no student could see the responses of another for the same form.

These data were used to compare the computational proficiency of group E with that of the combined groups, and to provide baseline data to assess the effects of the analysis unit on speed or accuracy.

OBSERVATION 02

Observation 02 was a re-administration of the same test. It was given the last day of the two-week period used for the algorithm analysis unit. The test administrator was the same for both 01 and 02, and the arrangements, including location and time of day, were duplicated.

All students were tested at 02, however, and the random half that had been tested at 01 were given the form they had not worked previously. Students and forms were again arranged so that copying was impossible.

The data from 02 were first used with those from 01 to assess effects of the analysis unit. These data were also used to compare the computational proficiency of group E with that of the other groups.

III

DATA AND ANALYSES

DATA

Data were derived from three sources: observation 01, observation 02, and informal observations of the work of groups E and ML during the course of the algorithm unit. The raw score data from 01 and 02 are found in Appendix C.

The group means at 01 and 02 for the three groups and for the combined group ML + MH are presented in Table 3. Table 3 also contains the change scores for the random halves of groups E and ML and MH that were tested at both 01 and 02. Data from two students present at 01 but not 02 are not included in any of these means.

01 Data

The 01 data are measures of the whole number computational proficiency of a random half of all three groups. These measures follow one year's experience with DMP in the case of group E, and follow six years' experience with usual large group, textbook oriented programs for groups ML and MH. It should be remembered, however, that group E was equivalent to the combined groups ML + MH prior to beginning the years' experience with DMP.

The order of the 10 group means at observation 01 was usually MH first, followed by E and then ML. This order was followed in all but three cases: for addition power the groups MH and E had identical means, while the order was MH followed by ML and then E for addition speed and subtraction power. The differences between groups were most noticeable for speed and power measures of both multiplication and division. The means for groups MH and E for these variables were close and nearly one standard deviation above those for group ML. The means for total speed and power also followed this latter pattern.

The combined group, MH + ML, had means markedly similar to the means of group E. Apparent differences between the combined groups and group E, however, favored group E for six of the ten variables. These were: multiplication and division speed, and addition, multiplication, division, and total power. None of these differences approach one standard deviation, however.

TABLE 3

GROUP MEANS AND STANDARD DEVIATION

Groups (N)	Addition (10)	Subtraction (10)	Multiplication (10)	Division (10)	Total (40)
	Observation 01--Means for Timed Variables				
E (11)	6.18/2.44	6.09/2.17	6.00/2.86	4.00/1.95	22.27/6.72
ML (13)	6.31	6.00	3.54	2.85	18.69
MH (11)	7.64	7.82	7.45	4.82	27.73
(ML + MH) (25)	6.91/1.50	6.83/2.41	5.33/3.00	3.75/2.36	22.83/7.27
	Observation 01--Means for Power Variables				
E (11)	9.55/.68	8.45/1.04	8.36/1.57	6.09/3.18	32.45/4.91
ML (13)	8.38	8.54	6.23	4.00	27.15
MH (11)	9.55	9.36	8.45	6.45	33.82
(ML + MH) (25)	8.92/1.21	8.92/1.69	7.25/2.86	5.13/3.22	30.21/7.11
		Timed Variables for Retest Groups: 02-01 Change Scores			
E (11)	1.03	-.51	.13	-.71	-.06
ML (13)	.33	.11	-.11	-.54	-.22
MH (11)	1.36	.91	.10	-.73	1.63
		Power Variables for Retest Groups: 02-01 Change Scores			
E (11)	.00	.00	-.26	-.13	-.39
ML (13)	.38	-.11	-.33	-.16	-.22
MH (11)	.09	.00	-.36	.64	.36

TABLE 3 (continued)

Groups (N)	Addition (10)	Subtraction (10)	Multiplication (10)	Division (10)	Total (40)
	Observation 02--Timed Variables Total Groups				
E (28)	6.71/2.03	5.57/2.71	5.50/2.90	3.32/2.26	21.11/7.64
ML (28)	6.82	5.25	3.57	2.07	17.71
MH (23)	7.91	7.96	6.78	3.65	26.30
(ML + MH) (51)	7.31/2.43	6.47/2.74	5.02/2.90	2.78/2.16	21.59/7.97
	Observation 02--Power Variables Total Groups				
E (28)	9.46/.69	8.11/2.13	7.36/2.83	5.43/3.16	30.36/7.08
ML (28)	9.07	7.71	5.54	3.57	25.89
MH (23)	9.65	9.22	8.09	6.87	33.83
(ML + MH) (51)	9.33/.89	8.39/1.83	6.69/2.64	5.06/2.85	29.47/6.33

Note: The scale for each variable is indicated by parentheses. E.g., Addition (10) indicates the scale contained ten items. The standard deviations are presented following the means. E.g., 6.18/2.44 indicates a mean score of 6.18 with standard deviation of 2.44.

02 Data

The data for observation 02 were quite similar to those for 01. Here, however, the order of the group means was always MH, E, ML. The combined means of groups MH and ML were again quite similar to those for Group E. The apparent differences favored group E for the same 6 of 10 variables as for observation 01, but again did not approach one standard deviation in magnitude.

02-01 Data

The constructed measures for 02-01 were measures of changes in computational proficiency attributable to the algorithm unit in the cases of groups E and ML, and to about 8 days of review and practice (practice problem sets) in the case of group MH.

The salient features of these data were the very small magnitudes of the changes and the fact that many were negative changes. The greatest changes, however, were for the review and practice group, MH. Their means increased for all scores but division speed and multiplication power. The greatest change was only 1.63 for total speed, however. And this was a change on a scale of 40 possible points where the initial mean was only 22.83. The greatest changes in the means of group MH for individual scores were an increase of 1.36 for addition speed, followed by an increase of .91 for subtraction and a decrease of .73 for division speed.

The changes for groups E and ML were about evenly split between increases and decreases. The tendency of both groups, however, was toward general decreases as reflected in the total change scores for both speed and power variables. These ranged from -.39 to -.06.

Two curious patterns are present in these data. All three groups registered decreases for division speed and multiplication power, and all three groups showed increases for both addition speed and power. These may, of course, be just random patterns but they do not reflect the activities of the 8 instructional days in any clear way.

A third source of data was the informal observations of the classes as they worked through the algorithm unit. Before these informal data are presented and discussed, the hypotheses and statistical tests relating to the formal data are presented.

HYPOTHESES AND TESTS

The hypotheses relating to the 01, 02 and 02-01 data were tested at the .05 level of significance. Significance levels ranging from .06 to .15 were not taken as indicative of a possible difference, but were not considered as sufficient evidence for rejecting a null hypothesis. Hypotheses relating to the 02-01 data are considered first followed by those relating to 01 and 02 data.

02-01 Hypothesis

The 02-01 data relate to the following question: Are there changes in computational proficiency that can be attributed to the analysis of algorithms unit? The null hypothesis is H_1. H_1: The means of groups ML and E for 02-01 variables are zero.

This hypothesis was tested by considering four multivariate tests. H_1 may, therefore, be regarded as four hypotheses in the obvious way. Accordingly, the data for groups E and ML were considered separately and the scores for the speed variables were considered apart from those for the power variables. The multivariate analysis for each set of scores was followed by univariate analyses to further explore the data. The statistics relating to the four tests of H_1 are recorded in Table 4.

None of the multivariate F values were significant. The only indication of possible change attributable to the algorithm unit was a possible change found in the test for the speed scores of group E. The null hypothesis could not be rejected by these data, but if differences are assumed to exist the raw scores and univariate analyses indicate a primary contrast between a decrease for division speed and an increase for addition speed. There were no other indications of changes in the performance of either group E or ML.

No further analyses of the 02-01 data were done but the data for group MH do require comment. Group MH did not experience the algorithm unit and so their 02-01 data do not relate to hypothesis H_1. However, the surprisingly small differences for this group do have a bearing on the meaning of the comparisons of 01 and 02. Briefly, even if the 02-01 scores of group MH were found to be significant, these magnitudes would hardly be taken as important.

Summary of 02-01 Data

To summarize at this point then, the 02-01 data provided no evidence that the experiences of the last two weeks of school had any marked effect on the computational skills of any of the groups. This allows 01 and 02 data to be thought of as similar tests of the computational proficiency of group E and the combined group ML + MH separated by a period of two weeks.

01 and 02 Hypotheses

The differential effects of the DMP materials upon computational skills compared to the effects of a more traditional program were assessed by comparing the means of group E with those for the combined group ML + MH. The null hypothesis is H_2 as follows:

> H_2: The means of group E are the same as those for group ML + MH at both 01 and 02.

This hypothesis was also tested by considering four multivariate tests. And so hypothesis H_2 was regarded as four hypotheses, one each for the speed and power scores, first for observation 01, and then for observation 02. Each multivariate test of a hypothesis was followed by univariate analyses of all the variables involved in the multivariate tests, and also for the total speed or total power scores. The statistics relating to these tests are presented in Table 5.

The multivariate tests failed to reveal any significant differences at either 01 or 02, and hypothesis H_2 cannot be rejected. The only multivariate test that indicated even a possible difference was that for the four 02 timed variables. As the univariate tests for these variables failed to indicate significant or possible differences for any one of the

TABLE 4

MANOVA OF GROUP MEANS FOR 02-01

Test: Means = 0

Variable	df	MS	F	p <
Group E Speed				
Multivariate	4/7	--	2.50	.14
Addition	1/10	11.64	2.81	.13
Subtraction	1/10	2.91	1.04	.33
Multiplication	1/10	.18	<1	--
Division	1/10	5.50	7.86	.02
Total Speed	1/10	.05	<1	--
Group E Power				
Multivariate	4/7	--	<1	--
Addition	1/10	.00	<1	--
Subtraction	1/10	.00	<1	--
Multiplication	1/10	.73	<1	--
Division	1/10	.18	<1	--
Total Power	1/10	1.64	<1	--
Group ML Speed				
Multivariate	4/9	--	<	--
Addition	1/12	1.38	<	--
Subtraction	1/12	.15	<	--
Multiplication	1/12	.15	<	--
Division	1/12	3.85	1.35	.27
Total Speed	1/12	.62	<	--
Group ML Power				
Multivariate	4/9	--	<	--
Addition	1/12	1.88	1.95	.19
Subtraction	1/12	.15	<	--
Multiplication	1/12	1.38	<	--
Division	1/12	.35	<	--
Total Speed	1/12	.62	<	--

TABLE 5

MANOVA OF GROUP MEANS FOR 01 AND 02:

E vs (ML + MH)

Variables	df	MS	F	p <
01 Speed				
Multivariate	1/30	--	< 1	--
Addition	1/33	4.07	1.21	.29
Subtraction	1/33	4.16	< 1	--
Multiplication	1/33	3.35	< 1	--
Division	1/33	.47	< 1	--
Total Speed	1/33	2.37	< 1	--
01 Power				
Multivariate	4/30	--	1.08	.38
Addition	1/33	2.98	2.55	.12
Subtraction	1/33	1.61	< 1	--
Multiplication	1/33	9.35	1.44	.24
Division	1/33	7.03	< 1	--
Total Power	1/33	38.06	< 1	--
02 Speed				
Multivariate	4/74	--	1.84	.13
Addition	1/77	6.50	1.22	.28
Subtraction	1/77	14.61	1.95	.17
Multiplication	1/77	4.71	< 1	--
Division	1/77	5.21	1.07	.31
Total Speed	1/77	4.18	< 1	--
02 Power				
Multivariate	4/74	--	< 1	--
Addition	1/77	.31	< 1	--
Subtraction	1/77	1.47	< 1	--
Multiplication	1/77	8.14	1.10	.30
Division	1/77	2.47	< 1	--
Total Power	1/77	14.21	< 1	--

variables, an explanation of the possible differences for the multivariate test must involve some combination of the four pairs of means. An examination of the means themselves suggest that this would be a contrast involving lower scores for group E for addition and subtraction speed, and higher scores for group E for multiplication and division speed.

The other univariate tests of differences for 01 and 02 variables were also not significant; only the test for addition power at 01 reached even the .06 to .15 levels.

<u>Summary of the 01 and 02 Data</u>

In brief then, the 01 and 02 data failed to provide any reason to reject the null hypothesis. Group E may not be considered different from the combined group ML + MH in terms of computational proficiency.

OTHER OBSERVATIONS

The second purpose of this study was to explore what happens when learners are asked to focus on algorithms as sequences of rule statements rather than as just sequences of behaviors. The 02-01 data reported and discussed previously had bearing upon this purpose. Other data relating to this purpose came from informal observations of students' work as they spent about two weeks analyzing whole number algorithms.

The rather short period of time involved and the exploratory nature of this part of the study render these data very subjective. The data to be presented therefore are descriptive only; no formal analyses of them were undertaken.

The algorithm unit required verbal analysis of the usual four whole number algorithms. The analysis required for division and addition was done with a great deal of teacher input, while that required for subtraction and multiplication was attempted by students working independent of the teacher and of each other.

The three tasks identified earlier may be more finely described as five tasks that children were asked to complete. All five tasks were attempted during that part of the unit when students were working with the teacher, but only the last three were attempted while they were working independently. The five tasks were:

A. Describe (write) exactly what must be done to compute a given set of problems.

B. Order members of a given set of rule statements to tell how to compute a given set of problems.

C. Study a given set of problems, three of which have been "worked" using a wrong pattern (algorithm), and work the remaining problems following the wrong pattern.

e.g.,

$$\begin{array}{r} 35 \\ \underline{+18} \\ 13 \end{array} \quad \begin{array}{r} 45 \\ \underline{+22} \\ 67 \end{array} \quad \begin{array}{r} 59 \\ \underline{+\ 8} \\ 17 \end{array} \quad \begin{array}{r} 98 \\ \underline{+24} \\ \end{array} \quad \begin{array}{r} 73 \\ \underline{+29} \\ \end{array}$$

D. Tell (write) what is wrong with the pattern.

E. Write the problems that have wrong answers and work them correctly.

The responses to the first task were sketchy and incomplete. That is, they often took the form--"First you do this," "then this," etc. where the "this" was accompanied by an action or several actions. The actions were often named, or names were supplied when asked for, but it is doubtful if the labels alone would be sufficient to establish that a student could complete the algorithm. Task B was usually completed after considerable time was spent identifying the rule statements with individual behaviors. This was an analysis task for most children that required a great deal of teacher help. Rule statements that were branching were most frequently omitted. For example, the statement "Regroup one as tens and ones" would be used without the preceding decision rule that indicated the need to regroup. It may be that the decision to regroup and the regrouping itself were connected too closely to be separated by the students.

Reflection at this point suggests what should perhaps have been obvious from the outset. Rule statements are statements about something. That is, an algorithm as a sequence of rule statements must be a sequence of statements ultimately about behaviors that have verbal labels. The significant questions for instruction and learning probably involve what particular behaviors (or chains of behaviors) have verbal labels, and what those labels are. This topic is returned to shortly.

Tasks C, D, and E stated earlier were carried out independently for the operations of subtraction and multiplication. Students were often successful with tasks C and E while performances for task D were less interpretable.

A student often identified the "step" that was incorrectly carried out without saying what was wrong with it. While they were not specifically asked to, no student volunteered a statement concerning what was right about the pattern. That is, answers to the question, "Why did it work sometimes?" were never volunteered.

The following examples taken from papers illustrate student responses to tasks C, D, and E for multiplication. The problem set with the following blanks, "answers", and appropriate directions was presented:

$$\begin{array}{r} 34 \\ \times 60 \\ \hline 204 \end{array} \quad \begin{array}{r} 52 \\ \times\ 3 \\ \hline 156 \end{array} \quad \begin{array}{r} 47 \\ \times 20 \\ \hline 94 \end{array} \quad \begin{array}{r} 27 \\ \times 40 \\ \hline \end{array} \quad \begin{array}{r} 53 \\ \times\ 6 \\ \hline \end{array}$$

Students filled in the blanks and explained the difficulty in the following ways.

Student 1 $\begin{array}{r} {}^{2}27 \\ \times 40 \\ \hline 108 \end{array} \quad \begin{array}{r} {}^{1}53 \\ \times\ 6 \\ \hline 318 \end{array}$ "he never x the 0 in ones colom of the bottomm numbre"

Student 2 $\begin{array}{r} {}^{2}27 \\ \times 40 \\ \hline 108 \end{array} \quad \begin{array}{r} {}^{1}53 \\ \times\ 6 \\ \hline 318 \end{array}$ "he forgot his place holders."

Student 3	$\begin{array}{r} {}^{2}27 \\ \underline{\times 40} \\ 807 \end{array}$	$\begin{array}{r} {}^{1}53 \\ \underline{\times\ 6} \\ 308 \end{array}$	"he didn't need to regroup when he did."
Student 4	$\begin{array}{r} {}^{3}27 \\ \underline{\times 40} \\ 94 \end{array}$	$\begin{array}{r} {}^{1}53 \\ \underline{\times\ 6} \\ 318 \end{array}$	"multiply tens by ones."
Student 5	$\begin{array}{r} {}^{2}27 \\ \underline{\times 40} \\ 807 \end{array}$	$\begin{array}{r} {}^{1}53 \\ \underline{\times\ 6} \\ 308 \end{array}$	"didn't need to regroup when he or she did."

All five of these students were able to identify the wrong problems and correct them.

Students 3, 4, and 5 did not follow the pattern and their explanations do not clearly describe either what they did or the wrong pattern presented. It does seem possible, however, to relate in some loose way their explanations with what they did.

Students 1 and 2 followed the pattern to produce both right and "wrong" answers; furthermore their descriptions related to the error pattern in obvious ways. The curious thing is that their explanations are quite different and that neither seems sufficient to specify the error. That is, we need to see their responses to the problems before we can be reasonably sure that they understood what was wrong.

The informal data that have been presented while not exhaustive are typical and illustrative. They do, however, seem adequate to support the generalizations to follow.

1. The appropriateness of making algorithms, in the general sense the focus of instruction for these children, is still an open question. There were positive indications for this, however, in the willingness of children to attempt the tasks and in the mixed successes described earlier.
2. The formal 02-01 data provided no evidence that the study of algorithms in the general sense increased proficiency with the particular algorithms studied as measured by speed and power measures. This finding must be viewed with great caution, however, in view of the shortness of the algorithmic unit.
3. The data derived from watching children work and from examining their independent work for certain problems indicated the presence of algorithms as adequate behavioral chains. However, while the children could describe the behavioral steps, they could not in general verbally characterize them in any adequate way.

NEXT STEPS

It was noted earlier that if algorithms are indeed sequences of rule statements, they must be statements about something. These somethings would seem ultimately to be behaviors (as when the children reported they "did this and then this," etc.) or written symbolic concepts (as "the tens digit" or "times the tens digit," etc.)

To illustrate this even further, suppose we consider the problem set and the children's responses which were presented earlier and ask the following question: What would a complete verbal description of the error pattern be? One description may be the following. "If the ones digit of the bottom number is zero, the tens digit of the bottom number is treated as if it were ones. This produces products ten times too small. However, if there is no tens digit the pattern is all right."

It is quite clear that the children would not be expected to produce this relatively sophisticated response. But it is not clear by what criteria the proposed description may be called adequate. For example, the description would not be adequate if the terms that are used are not part of the repertoire of the reader.

The best analogy may be to hardware and software components of computer technology. After all, the demonstration of the adequacy of the rule statement definition of algorithms is found in this technology. Following such an analogy, the learner must have some behaviors that are irreducible hardware components with labels such as "timzing," "borrowing," "adding digits," "regrouping," etc. These components could be relatively long chains of behaviors, perhaps even up to a complete algorithm in some cases. Presumably, though, individuals would also be expected to have software statements that string the hardware components together to form more complex algorithms.

Continuing the analogy, the hardware components would be irreducible in the sense that a person possessing them could demonstrate or label them but could not, without instruction or analysis, represent them as smaller pieces of hardware strung together by software statements. It might be that further analysis of a given hardware component is possible. Or that what is a hardware component for some learner may be a chain of hardware components for another.

Without belaboring the point, several questions of interest emerge from this analogy. For example,

a. What kinds of operation are, or should be, "hardware" and "software," for a given student or group of students?

b. Are there relationships among the number or proportions of software and hardware statements and various achievement or ability measures?

c. How are these postulated components related to cognitive development?

d. How does all of this relate to such ideas as meaningful learning, discovery, and structure in mathematics?

Which, if any, of these questions may prove fruitful is a matter for conjecture. But it seems obvious that inquiry into algorithmic learning that children have already achieved must take into account the irreducible and connecting components of their algorithms.

FINAL SUMMARY

This study was an investigation of two questions. The first concerned the discernable effects of one year's experience with DMP materials upon the computational proficiency of sixth grade children. The data failed to detect any evidence that the computational proficiency of these children was in any way impaired as compared to that of an equivalent group that continued in a standard textbook oriented program. In fact, the DMP group had means as large as, if not larger than, those of their peers for most of the measures.

The second purpose was an investigation of the feasibility of teaching sixth grade children an algorithm unit that required verbal analysis of algorithms. While the unit was accepted by the students and progress was made toward mastering objectives related to it, there was no apparent effect upon computational proficiency. The interesting questions to emerge from this part of the study involve the identification of the irreducible behavioral elements in algorithms that particular learners do or should have.

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APPENDIX A

LESSON PLANS FOR THE ANALYSIS OF ALGORITHMS UNIT

Activity .4

MATERIALS

Textbook pages 8-10 (A, B)
Sheets .4a-g (B, C)

OBJECTIVES

identifies characteristics of algorithms

VOCABULARY

no new words

ORGANIZATION

(A, B) small groups
(C) individual/large groups

PREPARATION

(A) Become familiar with the game, "Follow the Rules," on textbook page 9.

(B) Prepare Sheets .4a-c; one for each student plus some extras. Become familiar with the game, "Capture the Prize," on textbook page 10.

(C) Prepare Sheets .4d-g; one for each student.

DESCRIPTION

Activity .4 has three parts and deals with division algorithms. The first part is a small group activity that involves a game. Each group is required to "make a list of rules" that tells how to solve division problems with 1 digit quotients, and those with 2 digit quotients. The game involves verbal descriptions to help the group better understand its task. A list of rules a group writes is acceptable if another group in the class can follow the list and produce correct sentences.

The second part of the activity is also a game for two or three players. The object is to arrange a given set of direction cards to tell how to work a particular problem.

LESSON PLAN FOR 01, .3,.4,.5,.6,02, and .7

Mon (5/21)	Tue (5/22)	Wed (5/23)	Thur (5/24)	
Act. 3 Present Algor for division and Check-Up	Administer 01 Begin .4 -		Introduce .5	
Obj. familiarize them with this algor				
Fri (5/25)	**Tue (5/29)**	**Wed (5/30)**	**Thur (5/31)**	**Fri**
Conclude .4 Continue .5	Conclude .5	.6 <u>to be done independently</u> .7 may be done independent of a large group presentation	Administer 02 - - - - - - - - - - .2 may be done	

		01	Treatment	02	Act .6	Work Sheets
<u>Group</u>						
E	Ernie's	yes	act. as above	yes	yes save all work sheets	
D	Don's	yes	"	yes	"	"
O	Other	yes	D & P	yes	no	no

01 - To be administered prior to beginning Act. .4
Procedure: Choose random 1/2 of E, D, and O groups and alternate forms a and b

02 - To be administered the next day following Act. .6
Procedure: All of the children are to take the test. Those who have done one form at 01 are to do the other form

The third part is a set of four activity sheets that are to be done individually.

A. Begin part A by reading textbook page 8 with the entire group. It may be necessary to give them an example of what you do to solve one of the problems without telling which problem you solved. Be sure you look at this task before you present it to the children.

B. When a group completes the work described on textbook page 8, give them Sheets .4a-c. Have them cut out the direction cards and problems and go over the rules (textbook page 10) for "Capture the Prize." They can then begin playing in groups of 2 or 3. You may have to help them capture the first "prize" before they will see what is required.

C. Part C involves four sheets that children are to work on independently. Sheets .4d and e are descriptions of what a student might say as he works a problem. The task is to identify what problem is being worked from a given set of problems.

Sheets .4f and g contain two sets of problems that are worked following a wrong rule. The task is to identify the mistake, work another problem using the wrong rule, and then describe the error.

Conduct a large group discussion of these sheets after the students have had an opportunity to complete them.

III. Now your group is to pick one of the problems and write the best set of directions about how to solve it that you can. Try to be sure of the following.

1. If someone knows which problem you picked, the directions will tell how to solve it.
2. If they don't know which problem was picked, the directions won't give it away.

Trade lists with the other groups and see if you can guess their problem and "Follow the Rules" to solve it.

IV. When you have tried several other groups' lists of rules, repeat I, II, and III using the following set of four problems.

$3\overline{)245}$ $13\overline{)245}$ $25\overline{)1246}$ $123\overline{)3565}$

Capture the Prize

Sheet a has two sets of division problems written on it. Cut Sheet a on the lines and place set A problems in one pile and set B problems in another.

Sheet b has a set of direction cards that can be arranged to tell how to work the problems. Cut out the direction cards on Sheet b but keep one copy of the sheet to refer to.

Take turns doing the following.

1. Choose a problem from set A or set B that has not been captured and write it on Sheet .4c.
2. Arrange the direction cards to tell how to work it. Write the letters of the cards in the order you placed them.
3. If the other players agree with your list of rules, you "Capture the Prize," and write the finished sentence on your sheet. If your list won't work, you must put the prize back in the pile it came from.
4. Give the direction cards to the next person.

The game continues until all of the problems have been captured. The person with the most prizes wins. You may want to make some more prizes and play the game again.

Sheet .4a

SET A

$3\overline{)245}$	$9\overline{)75}$	$13\overline{)75}$
$405\overline{)1285}$	$24\overline{)356}$	$124\overline{)3073}$

SET B

$5\overline{)265}$	$39\overline{)470}$	$130\overline{)4052}$
$8\overline{)2834}$	$22\overline{)4839}$	$600\overline{)123,456}$

LAST STEP: WRITE THE NUMBER SENTENCE FOR THE PROBLEM.

a WRITE THE PROBLEM IN GOOD FORM.	b FIND THE PLACE VALUE OF THE SECOND DIGIT.
c FIND THE PLACE VALUE OF THE THIRD DIGIT.	d FIND THE PLACE VALUE OF THE FIRST DIGIT.
e MAKE BEST ESTIMATE OF THE SECOND DIGIT.	f MAKE BEST ESTIMATE OF THE THIRD DIGIT.
g STOP IF DONE AND ADD ESTIMATES.	h STOP IF DONE AND ADD ESTIMATES.
i STOP IF DONE AND ADD ESTIMATES.	j MAKE BEST ESTIMATE OF THE FIRST DIGIT.
k MULTIPLY AND SUBTRACT IF POSSIBLE.	l MULTIPLY AND SUBTRACT IF POSSIBLE
m MULTIPLY AND SUBTRACT IF POSSIBLE.	n IF NECESSARY, CHANGE ESTIMATE AND REPEAT STEP ________.
o IF NECESSARY, CHANGE ESTIMTAE AND REPEAT STEP ________.	p IF NECESSARY, CHANGE ESTIMATE AND REPEAT STEP ________.

Sheet .4c

Problem	Order of Rules (Use the letters)	Captured (Write Sentence)

Janice was working some division problems. While she worked she said the following things. Circle the problem you think Janice was working and write the sentence she wrote to describe the finished problem.

"The first place is ones." "I think 6 ones."

"Six times 234 is . . ."

"Too big, . . . so I'll try 5."

"Five times 234 is . . ."

"Subtract and I get 86."

"Now, write the sentence."

$234\overline{)1256}$ $234\overline{)1156}$ $234\overline{)1458}$ $234\overline{)1200}$

Write the sentence that Janice wrote. ______________________

"The first place is tens."

"I guess 2 tens."

"Twenty times 58 is . . . ."

"Too big, . . . so only 1 ten."

"Subtract 580."

"The next place is ones."

"One more 58."

"Subtract and have a remainder of 21."

"Write the sentence and I'm done."

$58\overline{)659}$ $58\overline{)775}$ $58\overline{)639}$ $58\overline{)1239}$

Write the sentence that Janice wrote. ______________________

"This looks like a long problem."
"The first place is hundreds."
"The digit I think is 2."
"Two hundred times 25 is 5000."
"Subtract 5000 and I get . . . ."
"That's bigger than 25 so I'm not done."
"The next place is tens."
"I think 6, no 8, no 7, yes 7 is right."
"70 times 25 is . . . 1750."
"So subtract 1750 and I get . . . ."
"That's bigger than 25, so I'll do it again."
"Five 25's is 125."
"Subtract and I get 16." "Done."
"No, I must write the sentence."

$25\overline{)7000}$ $25\overline{)6000}$ $25\overline{)6891}$ $25\overline{)6881}$

What sentence should Janice have written? ______________________

____

Here are three division problems that someone worked using a wrong rule. Two of them are wrong. Look over the work carefully to see what the wrong rule is and then work the last problem following the wrong rule.

```
   12          10           3
    7           8           1
    5           2           2
6)346       12)347      123)456
 -30         -24         -246
   46         107          210
  -42         -96         -123
    4          11           87
```

$12(6)+4 = 346$ $\quad$ $10(12) + 11 = 347$ $\quad$ $3(123) + 87 = 456$

```
18)256
```

What mistake was this person making? ______________________

____

____

These three problems were worked using a wrong rule so that two of them are wrong. When you have discovered what the error is, work the next problem using the wrong rule.

$$\begin{array}{r} 40 \\ 51\overline{)235} \\ -204 \\ \hline 31 \end{array} \qquad \begin{array}{r} 30 \\ 28\overline{)853} \\ -84 \\ \hline 13 \end{array} \qquad \begin{array}{r} 60 \\ 84\overline{)539} \\ -524 \\ \hline 15 \end{array}$$

$40(51) + 31 = 235$ $\qquad$ $30(28) + 13 = 853$ $\qquad$ $60(84) + 15 = 539$

$$83\overline{)659}$$

What mistake was being made? ______________________________

____

____

Activity .5

MATERIALS

textbook pages 11 and 12 (A, B)
Sheets .5a-e (B)

VOCABULARY

algorithm

OBJECTIVES

identifies characteristics of algorithms

ORGANIZATION

(A) small groups
(B) individual
(S) small groups

PREPARATION

(A) Become well acquainted with the materials on textbook pages 11 and 12.

(B) Prepare copies of Sheets .5a-e; one for each child.

(S) Read the Description.

DESCRIPTION

During Activity .5 the children are to work in small groups to analyze addition algorithms in much the same way they did division algorithms. They are introduced to the word algorithm in this activity as "a pattern that is followed when doing a certain job." They should see that there are often several different patterns that we can follow to do the same job, and that a pattern that helps us do one job may not tell us how to do another.

A. Part A is to be done in groups of 3 or 4 where each child makes a record of the work done. Textbook page 11 describea the task they are to do.

B. Part B is to be worked by individuals using Sheets .5a, b, c, d, and e. The sheets are intended to be self-explanatory but you may want to go over the directions with the children.

S. The games introduced in Activity .4 may be modified to fit addition and played again here.

.5 (22)

You have been making lists of rules that describe what may be done to complete a division problem. These patterns that you have been describing are called <u>algorithms</u>.

We may use algorithms to help us do any of several different jobs that are very much alike in some way. For example, we usually follow an algorithm to help us add, subtract, or multiply large numbers.

For the next few days, you are going to be describing and using several different algorithms. Some of them may do the same job, and some of them may not do the job we want them to do.

There are seven sets of addition problems on page 12. These seven sets differ from one another in some way. Your group is to pick at least four sets and describe an algorithm that will solve the problems of that set. It may be that some algorithms will solve the problems of more than one set. You can play the game "Follow the Rules" to help you make your best set of rules.

You will need one sheet of paper for each list you make. On that sheet you are also to write the problems the algorithm will solve. Trade complete lists with other groups and see if you can follow what they have done.

***EXTRA ! ! for EXPERTS.

1. Describe two or more algorithms for working the same set of problems.
2. Describe an algorithm for solving the problems that <u>starts with the largest place value column</u>.
3. Describe an algorithm that will tell how to work the problems of all of the sets.

textbook page 12

SET A

35 + 24 = ______ 24 + 25 = ______

54 + 21 = ______ 17 + 60 = ______

SET B

123 + 321 = ______ 427 + 361 = ______

405 + 390 = ______ 189 + 610 = ______

SET C

37 + 25 = ______ 47 + 59 = ______

67 + 24 = ______ 56 + 27 = ______

SET D

356 + 375 = ______ 496 + 225 = ______

257 + 187 = ______ 685 + 738 = ______

SET E

27 + 31 = ______ 27 + 38 = ______

94 + 8 = ______ 7 + 62 = ______

SET F

324 + 271 = ______ 324 + 217 = ______

128 + 685 = ______ 28 + 151 = ______

SET G

27 + 16 = ______ 92 + 65 = ______

347 + 248 = ______ 408 + 731 = ______

Each of the following sets of addition problems have been worked following a pattern that sometimes gives wrong answers. See if you can work the last three examples following the same wrong rules, and then describe the mistake.

NOTE: SOME ANSWERS WILL NOT BE CORRECT.

35	45	59	98	65	73
+ 68	+ 22	+ 8	+ 24	+ 24	+ 29
13	67	17			

Tell what is wrong with the pattern. ______________________________

____

235	161	372	128	31	327
+ 178	+ 728	+ 84	+ 29	+ 984	+ 542
303	889	356			

Tell what is wrong with the pattern. ______________________________

____

345	24	345	123	347	65
+ 126	+ 63	+ 286	+ 68	+ 281	+ 28
571	97	631			

Tell what is wrong with the pattern. ______________________________

____

Use some of the direction cards of Sheet .5c to tell how to solve the problems of each set on textbook page 12. Write
44 the letters of the direction cards in the order you placed them.

SET	THE ORDER OF THE DIRECTION CARDS ON SHEET .5c
(Example) A	a, f, e, d, c, g
B	
C	
D	
E	
F	
G	

CUT OUT THESE CARDS FOR USE WITH SHEET .5b.
YOU MAY MAKE OTHER DIRECTION CARDS FROM THE BLANK CARDS.

a WRITE THE PROBLEM IN GOOD FORM.	b REGROUP IF NECESSARY AND WRITE THE UNITS OF THE SUM BELOW THE COLUMN.
c WRITE THE SUM OF THE COLUMN.	d ADD THE TENS COLUMN.
e ADD THE 100's COLUMN.	f ADD THE ONES COLUMN.
g WRITE THE SENTENCE.	h REGROUP IF NECESSARY AND WRITE THE UNITS OF THE SUM BELOW THE COLUMN.
c WRITE THE SUM OF THE COLUMN.	c WRITE THE SUM OF THE COLUMN.

46 Use the direction cards of Sheet .5e to tell how to solve the problems of each set of problems on textbook page 12. Write the letters of the direction cards in the order you placed them.

SET	THE ORDER OF THE DIRECTION CARDS ON SHEET .5e
A	
B	
C	
D	
E	
F	
G	

EXTRA ! ! FOR EXPERTS.

1. Make your own set of direction cards and challenge your friends to arrange them properly.
2. Make a flow chart that tells what to do to solve any addition problem. (See an encyclopedia to find out about flow charts.)

CUT OUT THESE CARDS FOR USE WITH SHEET .5d.
YOU MAY MAKE OTHER DIRECTION CARDS FROM THE BLANK CARDS.

a WRITE THE PROBLEM IN GOOD FORM.	b WRITE THE SUM BELOW THE COLUMN.
c ADD THE COLUMN.	d START AT THE RIGHT.
e REGROUP IF NECESSARY.	f IF / THEN THIS IS THE LAST COLUMN / GO TO CARD b. THIS IS NOT THE LAST COLUMN / FIND THE NEXT COLUMN AND GO TO CARD c.
g RECORD ANY "CARRY" AND WRITE THE UNITS OF THE SUM BELOW THE COLUMN.	h WRITE THE SENTENCE.
i ADD THE COLUMN.	

Activity .6

MATERIALS

Sheets .6a-f

VOCABULARY

no new words

OBJECTIVES

identifies characteristics of algorithms

ORGANIZATION

individual

PREPARATION

Prepare copies of Sheets .6a-d for each child.

DESCRIPTION

This activity consists of six activity sheets that are to be worked individually. These sheets present the same type of tasks as Activities .4 and .5 but involve subtraction and multiplication algorithms.

You may, of course, go over the directions to be certain the children know what the tasks are, but, in general, let the children do their own best efforts first.

A large group discussion following a few days' work with this activity should summarize the characteristics of algorithm noted in the introduction to the Activities, .4, .5, and .6.

(S) Children who finish early should be asked to make algorithm flow charts for subtraction and multiplication.

The first three problems of each set have been worked following a pattern that doesn't always give the correct answer. Find the wrong pattern and follow it to get "answers" (some of them will be wrong!) for the last two problems

434	365	528	342	654
− 127	− 34	− 379	− 17	− 324
313	331	251		

Tell what is wrong with the pattern. ______________________

____

345	525	569	228	794
− 126	− 37	− 45	− 19	− 243
229	598	524		

Tell what is wrong with the pattern. ______________________

____

35	43	16	34	51
x 22	x 8	x 21	x 12	x 6
70	344	16		
70		32		
140		48		

Tell what is wrong with the pattern. ______________________

____

54	47	20	22	50
x 30	x 20	x 30	x 40	x 30
1500	800	60		

Tell what is wrong with the pattern. ______________________

____

Sheet .6b

The first three problems of each of the following sets of problems are worked following a pattern that doesn't always give the correct answer. Find the wrong pattern, and follow it to get "answers" for the last two problems.

34	52	47	27	53
x 60	x 3	x 20	x 40	x 6
204	156	94		

Tell what is wrong with the pattern. ____________________

$\overset{5}{\cancel{6}}4$	$\overset{6}{\cancel{7}}{}^{1}2$	$1\overset{2}{\cancel{3}}6$	245	379
− 23	− 29	− 24	− 28	− 145
31	43	102		

Tell what is wrong with the pattern. ____________________

Write the problems that have wrong answers above and work them correctly below.

Cut out the direction cards on Sheet .6d and place some of them in an order that will tell how to solve the problems of each set of problems below.

SET	THE LETTERS OF THE CARDS IN THE ORDER CHOSEN
35 89 36 28 -13 -45 -24 -15	a,
435 658 872 349 -224 -427 -321 -216	
82 64 86 97 -29 -26 -34 -65	
325 726 632 305 -163 - 18 -148 - 27	
34 67 60 50 x 24 x 20 x 35 x 30	A,
34 650 128 405 x 222 x 123 x 345 x 216	

(a) Write the problem in good form.	(b) Write the tens mini-product.
(c) Write the ones mini-product.	(d) Write the 100's mini-product.
(e) Multiply by the 100's.	(f) Multiply by the ones.
(g) Multiply by the tens.	(h) Add the mini-products.
(i) Write the sentence	(A) WRITE THE PROBLEM IN GOOD FORM.
(B) REGROUP IF NECESSARY.	(C) SUBTRACT AND WRITE THE ANSWER IN THE COLUMN.
(B) REGROUP IF NECESSARY	(C) SUBTRACT AND WRITE THE ANSWER IN THE COLUMN.
(B) REGROUP IF NECESSARY	(C) SUBTRACT AND WRITE THE ANSWER IN THE COLUMN.
(D) GO TO THE HUNDREDS COLUMN.	(E) START WITH THE ONES COLUMN.
(F) GO TO THE TENS COLUMN.	(G) WRITE THE SENTENCE.

Activity .7

MATERIALS

textbook pages 13, 14, 15, and 16
Sheet .7a (B)
1 centimeter graph paper (B)
several large jars or cans (B)
a container calibrated in cc (or ml) (B)
string and meter sticks (B)
pound or kilogram scale if available

VOCABULARY

no new words

OBJECTIVES

identifies situations requiring division

uses the pyramid division algorithm

ORGANIZATION

(A) large groups
(B) small groups

PREPARATION

(A) Become familiar with textbook pages 13, 14, 15, and 16.

(B) Collect a set of materials for each group of children.

1. Sheet .7a; one per person
2. 1 sheet graph paper per person
3. 1 large jar, calibrated container and dark crayon
4. ball of string and meter stick
5. place the scale where all can use it

(C) Graph paper and above apparatus.

DESCRIPTION

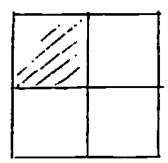
The purpose of this activity is to provide practice in division in the context of finding averages. The basic problem is to describe an average person of a group of children and of the entire class.

A. In part A is a large group discussion of what an average is, how we find one, and what it might be used for. Have the children read textbook pages 13-16.

Discuss what to do when the division is not exact (use fractional "remainders" only if your children understand them). Be sure to point out the fact that the average may not exist in the set at all. Do not helabor the point, however, and move rather quickly to part B.

B. The children are to make measures of themselves as suggested on textbook page 16 and find the average for their group and for the entire class.

Begin by assigning the children to groups of about six and discuss what they are to do. If the suggested measures cannot be made because of lack of equipment, you may substitute others. The measurement of height, waist, and little finger require only string and a meter stick (marked in mm). The measure of foot area requires graph paper with large squares. Centimeter paper is suggested though 1/2 inch or 1 inch can be used. If you use 1/2 inch graph paper, be sure to establish that the area is 1/4 of a square inch.



The measurement of the volume of the hand is a good experience for children as they can only develop understanding of capacity and volume through many such experiences. A large jar is best to use as the water level can be easily determined in this way. You can begin with an empty jar and fill it to a mark with the hand in, remove the hand and measure how many cubic centimeters (millimeters) are required to bring the level

.7 (25)

up to the mark. If the jar or can is large enough, they should see that making a fist or extending the fingers does not change the volume of the hand.

Remember the primary purpose of this activity is to practice division in the context of averaging. Consequently, you should have them show you their computations as well as the finished worksheet.

S. This activity can be extended to larger groups of children or to include many selections of six children from the entire group. Bar graphs for the entire group can be made also.

An interesting exploration for some students may be to find the average value (to the nearest whole unit) of a measurement for about 20 different groupings of five children from the class and make a graph of the data obtained. The graph will form the "bell-shaped curve" of the so called "normal distribution."

.7 (26)

Sheet .7a

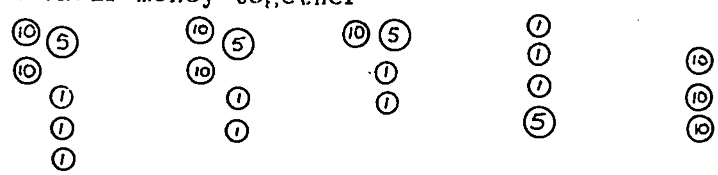
	Height in Centimeters	Area of Foot Square cm	Length of a Little Finger--mm	Volume of Hand--cc (ml)		
Person 1						56
2						
3						
4						
5						
6						
7						
Group Total						
Group Average						
Class Total						
Class Average						

What Is an Average?

We find an average when we divide a total into as many equal sized parts as we formed the total from.

If these 5 children

joined their money together



$$110¢ = 28 + 27 + 17 + 8 + 30$$

and then shared equally,

$$\begin{array}{r} 22 \\ 5\overline{)110} \\ -100 \\ \hline 10 \\ -10 \\ \hline \end{array} \qquad 110 = 5(22)$$

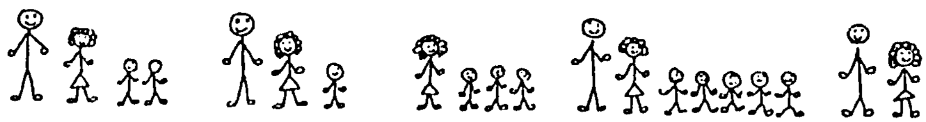
each would get an average amount.

$$110 = 22 + 22 + 22 + 22 + 22$$

The average then, describes the amount each person would have if the total were formed of equal parts.

Any time a group of things has something in common that can be measured, we can find an average.

In one apartment house, an average family had 4 persons.



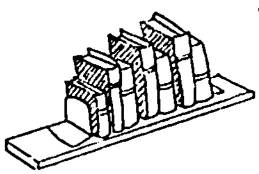
4 + 3 + 4 + 7 + 2 = 20

$$5\overline{)20}\ \ 4,\quad -20$$

5(4) = 20

On a bookshelf the average number of pages for 7 books was:

275 + 148 + 540 + 218 + 60 + 640 + 137 = 2028.



$$7\overline{)2028}\ \ 200 + 80 + 9$$

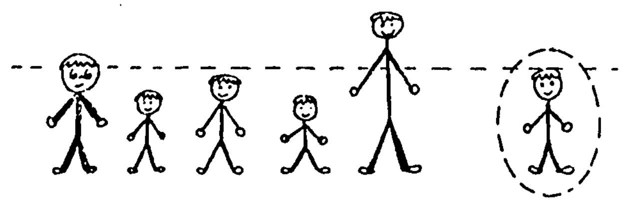
1400
628
560
68
63
5

2028 = 7(289) + 5

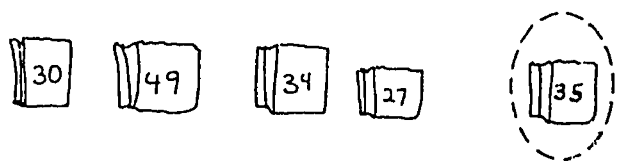
a little more than 289 pages

An average is one way to describe a typical something. But remember, the average is an idea and, very often, the "average" does not exist.

We can find average heights of a group of people,



average size of bags of candy,

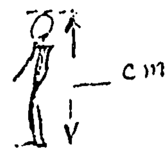


or average sunshine for a week.

DAY	HOURS WITHOUT CLOUDS
SUN	6
MON	8
TUE	12
WED	10
THUR	6
FRI	2
SAT	1
	TOTAL = 45 = 7(6) + 3
Between 6 and 7 hours of sunshine per day	

Describe an "average person" in your group and in your class. Use Sheet .7a to help you.

How tall is he?

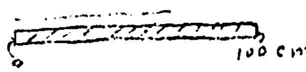


How much does he weigh? _____ lbs.

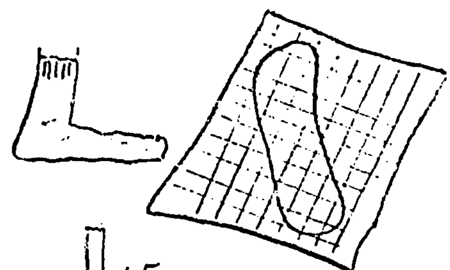
$1 \text{ Kgm} \approx 2\frac{1}{5} \text{ lbs.}$

_____ Kgms

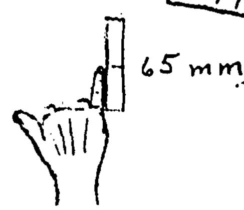
How far around the waist?



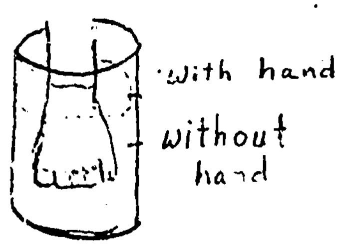
What is the area of a foot?



How long is a little finger?



What is the volume of a hand? _____ cc



APPENDIX B

TESTS AND TESTING DIRECTIONS

Directions for Administering Test Forms

Materials Required

1. One test booklet for each student.
2. A watch or clock with a second hand.
3. Each student will need a pencil and a ball-point pen (or colored pencil). You should have some extras available in case they are needed during testing.

Procedures

1. Before distributing the test booklets, tell the children the following.

TODAY I WANT YOU TO DO SOME ARITHMETIC EXERCISES. THERE ARE FOUR SHEETS WITH ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION PROBLEMS. WE WANT TO KNOW HOW WELL (SIXTH)-GRADERS CAN DO WITH PROBLEMS OF THESE KINDS. THIS WILL NOT AFFECT YOUR (GRADE, EVALUATION, CLASS STANDING, ETC. Use an appropriate term.) IN ANY WAY.

WRITE THE INFORMATION ASKED FOR ON THE OUTSIDE SHEET.

DO NOT OPEN THE BOOKLETS UNTIL I TELL YOU TO DO SO.

---

2. Distribute the test booklets and be sure that each child has a ball-point pen. The pencils should not be available at this time. Then tell the children the following.

THE EXERCISES WILL BE DONE IN THE FOLLOWING WAY.

FIRST, YOU WILL WORK ON EACH OF THE FOUR SHEETS FOR A SHORT PERIOD OF TIME. YOU ARE TO USE YOUR PEN AND WORK THE PROBLEMS IN THE ORDER THEY ARE GIVEN 1, 2, 3, etc. YOU MAY SKIP ANY PROBLEM THAT YOU CANNOT WORK.

DO AS WELL AS YOU CAN, BUT YOU WILL PROBABLY NOT HAVE TIME TO COMPLETE EACH SHEET BEFORE I SAY "STOP, PUT DOWN YOUR PENS, AND TURN TO THE NEXT SHEET."

AFTER THE LAST SHEET, THE DIVISION EXERCISES, HAS BEEN WORKED, WE WILL TAKE A BRIEF REST.

THEN YOU ARE TO GO BACK AND USE YOUR PENCIL TO CORRECT ANY ERRORS YOU MAY FIND. YOU SHOULD ALSO COMPLETE AS MANY OF THE EXERCISES AS YOU CAN. YOU WILL HAVE PLENTY OF TIME FOR THIS.

ARE THERE ANY QUESTIONS? (Review the directions as necessary.)

---

3. If there is any need for scratch paper, they may write on the back of the preceding sheet of the test booklet, or use another sheet of paper. If they do use another sheet, have them write their name on it and collect it with the booklets.

The following times and procedures apply to the various pages.

4. When they are all prepared to begin work say:

   NOW OPEN YOUR BOOKLET TO THE ADDITION PROBLEMS (pause) AND BEGIN WORK.

   Note the time here, minute and seconds. ____________________

---

5. After exactly 1 1/2 minutes say:

   STOP, AND PUT DOWN YOUR PENS.
   NOW TURN TO THE NEXT PAGE THAT HAS THE SUBTRACTION PROBLEMS.

   When they have done so say:

   BEGIN WORK.

   Note the time here, minute and seconds. ____________________

---

6. After exactly 1 1/2 minutes say:

   STOP, AND PUT DOWN YOUR PENS.
   NOW TURN TO THE NEXT PAGE THAT HAS THE MULTIPLICATION PROBLEMS.
   YOU WILL HAVE MORE TIME TO WORK ON THIS PAGE.

   When they are ready to work say:

   BEGIN WORK.

   Note the time here, minute and seconds. ____________________

---

7. After exactly 3 minutes say:

STOP, AND PUT DOWN YOUR PENS.
NOW TURN TO THE NEXT PAGE THAT HAS THE DIVISION PROBLEMS.
YOU WILL HAVE EVEN MORE TIME TO WORK ON THIS PAGE.
IF THE DIVISION IS NOT EXACT, LEAVE THE REMAINDER WHERE IT IS. YOU DO NOT NEED TO WRITE IT AT THE TOP, (OR TO WRITE THE VALIDATING SENTENCE).

When they are ready to work say:

BEGIN WORK.

Note the time here, minute and seconds. ____________________

---

8. After exactly 6 minutes say:

STOP, AND PUT DOWN YOUR PENS.
YOU MAY REST, BUT DO NOT TALK TO YOUR NEIGHBOR ABOUT THE PROBLEMS YET.

---

9. Now collect the pens (or thave them put them away) and make the pencils available.

Now say:

YOU ARE GOING TO USE YOUR PENCILS AND GO BACK AND FINISH ANY EXAMPLES THAT YOU DID NOT GET COMPLETED. YOU MAY CORRECT ANY PROBLEM YOU THINK YOU WORKED INCORRECTLY, BUT DO NOT MARK OUT THE FIRST ANSWER.

ARE THERE ANY QUESTIONS? (Answer any questions.)

WHEN YOU ARE DONE, CLOSE YOUR TEST BOOKLET AND I WILL COLLECT IT.
BEGIN.

---

10. Collect the booklets as they complete the work. They should have a relatively routine reading activity etc. to work on following completion of the test forms.

Try to arrange the children so they cannot work together, but if you observe this occurring, make a distinguishing mark on the cover of the booklet.

NAME

Teacher's name

Date Grade

LISTEN CAREFULLY AND FOLLOW ALL DIRECTIONS.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

THANK YOU ! !

(1)	(2)	(3)	(4)	(5)
		34	372	
419	2853	11	409	235
+ 268	+ 4192	+ 43	+ 67	+ 422

(6)	(7)	(8)	(9)	(10)
584	458		36	
34	322	2601	59	55
+ 278	+ 209	+ 1372	+ 87	+ 39

STOP !

DO NOT TURN THE PAG

(1)	(2)	(3)	(4)	(5)
3465	56	93	306	620
- 1297	- 26	- 47	- 168	- 69

(6)	(7)	(8)	(9)	(10)
74	846	315	204	835
- 49	- 34	- 182	- 60	- 460

STOP !

DO NOT TURN THE PAGE !

(1)	(2)	(3)	(4)	(5)
31	128	49	342	35
x 23	x 49	x 35	x 7	x 36

(6)	(7)	(8)	(9)	(10)
2364	207	804	40	8600
x 8	x 6	x 46	x 35	x 4

STOP !.

DO NOT TURN THE PAGE !

(1)	(2)	( )	(4)	(5)
$3\overline{)693}$	$30\overline{)210}$	$323\overline{)2244}$	$62\overline{)136}$	$400\overline{)1200}$

(6)	(7)	(8)	(9)	(10)
$56\overline{)6300}$	$8\overline{)969}$	$4\overline{)69}$	$323\overline{)9695}$	$53\overline{)4525}$

STOP !

DO NOT TURN THE PAGE !

NAME

Teacher's name

Date Grade

LISTEN CAREFULLY AND FOLLOW ALL DIRECTIONS.

DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

THANK YOU ! !

(1)	(2)	(3)	(4)	(5)
		52	207	
389	4355	23	646	372
+ 207	+ 1249	+ 11	+ 98	+ 626

(6)	(7)	(8)	(9)	(10)
321	238		37	
85	229	4837	78	48
+ 279	+ 410	+ 2162	+ 29	+ 37

STOP !

DO NOT TURN THE PAGE !

(1)	(2)	(3)	(4)	(5)
2433	68	82	507	350
- 1054	- 38	- 58	- 289	- 82

(6)	(7)	(8)	(9)	(10)
63	328	417	308	346
- 29	- 12	- 154	- 30	- 180

STOP !

DO NOT TURN THE PAGE !

(1)	(2)	(3)	(4)	(5)
22	242	67	422	28
x 32	x 37	x 25	x 7	x 42

(6)	(7)	(8)	(9)	(10)
6223	406	706	30	5600
x 7	x 8	x 36	x 74	x 6

STOP !

DO NOT TURN THE PAGE !

(1) $4\overline{)844}$

(2) $25\overline{)225}$

(3) $482\overline{)3621}$

(4) $54\overline{)122}$

(5) $600\overline{)1800}$

(6) $35\overline{)4200}$

(7) $7\overline{)876}$

(8) $6\overline{)72}$

(9) $223\overline{)6696}$

(10) $74\overline{)4678}$

STOP !

DO NOT TURN THE PAGE !