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ABSTRACT

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MEASUREMENT ERROR AND THE ANALYSIS OF PANEL DATA

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STUDIES OF EDUCATIVE PROCESSES

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Abstract

Early procedures for the analysis of multivariate panel data do not rest on well-specified statistical models. Recent approaches based on path analysis suffer from the defects of variable standardization and lack of attention to measurement error. The paper formulates a measurement model for quantitatively scaled multivariate panel data. The model is applied to a data set indexing two constructs measured at three time points. Multiple measurement of each construct in conjunction with the measurement model allows estimation of a true variance-covariance matrix. Analysis of this matrix produces substantially different interpretations of variable influence than similar analyses of the original data.

Table of Contents

List of Tables

1. Problems in the Interpretation of Panel Data

Sociological and social-psychological research workers have long been concerned with the attribution of causality to variables representing basic sociological or psychological concepts. Given the difficulty of variable manipulation in many real social settings, scientists have turned to statistical methodology rather than to experimental techniques of investigation for help.

From Lazarsfeld's (1948) early discussion and exposition of the sixteen-fold table technique for qualitative data to Campbell's (1963) and Pelz and Andrews' (1964) simultaneous conception of cross-lagged panel correlation procedures for quantitative data, practicable data-analytic techniques have been available for causal attribution. These techniques, however, have been heavily criticized by statistical methodologists (e.g. Goldberger, 1971) and are consequently clouded with ambiguities of interpretation. One reason for the persistence of ambiguity is the lack of well-defined mathematical models to serve as foci for discussion and bases for critique. Our intention in this paper is to specify such models as well as to exposite technique.

In this paper we are focussing solely on quantitative variables. Consequently, the sixteen-fold table technique is not directly relevant.[1] Comparison of cross-lagged panel correlations was the first widely advocated non-experimental technique for the attribution of causality to quantitatively scaled variables in social-psychological research. In addition to the lack of a clearly stated statistical model, one universally recognized weakness of this technique is its serious distortion by commonly occurring measurement errors. One especially distorting event is systematic change in the reliability of variables over time.[2] Such events occur, for example, when a true variable's variance changes, although the quality of the measurement remains the same. This is nearly always the case when any change in a true variable takes place (Wiley and Wiley, 1970). Therefore, it is difficult to justify the use of technique in typical social research settings.

Path analysis is growingly accepted as a powerful framework for disecting social data. It originated in genetic studies early in this century and has been systematically gaining favor among social scientists as a useful devise for stripping data to bare their (in)consistencies with complex theoretical assumptions and hypotheses. Path analysis has the advantage of being completely specified in mathematical form. It is, therefore, easy to criticize on both substantive and methodological grounds. Path analysis has the advantage of being multivariate and capable of variable augmentation while maintaining the basic asymmetries which are required in the simulation of causal as opposed to simply relational networks.

The application of path analysis to quantitatively scaled panel data has been systematically developed by Duncan (1969).

However, the path-analytic framework, as it is usually implemented and specified in applied research work, suffers from at least two defects, especially, when oriented towards longitudinal data: standardization and measurement error. Standardization means rescaling of variables so that their standard deviations in the sample are in every case equal to one. This practice has been generally criticized as wasting valuable information even in cross-sectional data (Tukey, 1954; Blalock, 1967a and b). In the context of longitudinal data, these difficulties become even greater, since most such analyses are oriented toward the assessment of and the distinction between stability and change. Such standardization complicates the detection of change or stability in the structure of the effects of causal factors, because invariance in the structure of the causal network, which appears in the form of constant values for causal coefficients when the variables are not standardized, will result in non-constant values for those coefficients when they are standardized.

This problem becomes even more difficult when errors contaminate the measurement of variables because there is no longer a simply specifiable relation between the two kinds of coefficients.[3)] These measurement errors also have serious consequences for the estimation of coefficients. Wiley and Wiley (1970) and Wiley (1973) have discussed some of these distorting influences within the context of univariate longitudinal and multivariate data, respectively. A general discussion of the complex consequences in a generalized regression framework has been given by Cochran (1968; 1972). Ignoring measurement error in regression analysis complicates the regression coefficient "attenuation" problem, commonly encountered in a simpler bi-variate context, when measurement errors contaminate variable assessment. This complexification implies increases in the sizes of some coefficients as well as decreases (attenuations) in those of others. The complexity of distortions in these key-indices, used to make causal attributions, effectively distroys any hope of a simple rank-order relation between observed and real coefficients. This implies that it is necessary to explicitly incorporate measurement errors in the formulation and specification of the model that serves as a basis for data-analytic procedures. When a single psychological variable is errorfully measured this has been known as the problem of "the measurement of change" in the psychometric literature (see Cronbach and Furby, 1970). This paper is one in a series which has the intent of expositing models for the general analysis of quantitatively scaled data with measurement errors.

A basic statement of the fundamentals of such a model was given by Wiley and Wiley (1970) in a critique of a standardized path analysis specification for univariate longitudinal data (Heise, 1969). Wiley (1973) subsequently formulated a class of such models for an econometrically sophisticated audience. Keesling (1972) applied this basic model to multivariate cross-sectional data and incorporated the notion of correlations among errors. This was the first conjunction of Cronbach's multivariate generalizability theory (Cronbach et al., 1972) with complex multi-relational models, common in econometrics.

This paper presents procedures and techniques based upon statistical models which incorporate more than one theoretical concept measured at more than one point in time. The model includes measurement errors in the observed variables which may be correlated over time. The exposition and discussion is oriented toward empirical research workers in social and psychological science. We hope to make the models and procedures both comprehensible and practicable to research workers who have not received extensive advanced training in statistics.

2. A Basic Measurement Model for Quantitative Variables

In a data set representing two constructs, each measured at two points in time, there are several relatively simple pieces of information which might be desired in attributing causal impacts. In the absence of measurement error, there are three quantities which describe simple distributional characteristics of the constructs at the first time point: the two variances and the covariance. There are four more characteristics which represent the productive relations between the constructs at the first time point and at the second: the two regression coefficients relating Construct 1 at time two to the two Constructs at time one and the two regression coefficients relating Construct 2 at time two to those at time one. Finally, there are three quantities remaining: the two variances and the covariance of the residual parts of Constructs 1 and 2 at time two which are not accounted for by their time one versions. These ten quantities are equivalent to, i.e. contain the same information as and only constitute a reorganization of, the ten original quantities: four variances and six covariances. They constitute the summary of the distributional characteristics of the four measurements: two variables at each of two time points. Thus, in the absence of measurement error, we seek ten new quantities from ten old quantities, a possible task. When we add measurement error, we must add new quantities to account for the characteristics of those errors. Clearly, when we do this, we increase the number of desired quantities beyond ten and thus exceed the number of available pieces of information.

Researchers have made a variety of assumptions so that the number of values they wish to estimate is no larger than the number they possess. The simplest such assumption is that

there is no error of measurement. If all the error variances are zero, then we only need estimate and rearrange the original values: the four variances and the six covariances. There is a direct correspondence between the original values calculated and the values desired. Unfortunately, such an assumption is rarely legitimate in social science and analyses based on it are often misleading.

A second approach has been to increase the number of items of information available beyond the number of quantities one wishes to assess. An example of this approach is Heise's expansion of two-time point models for analysis of three-time point panel data (1969). He assumes that all the relationships among the variables may be explained by postulating that a simple Markov model holds. I.e. that variables at a specific time are determined only by those at the preceding point. He also assumes stable reliabilities for the variables. Without going into detail, this expansion gives Heise enough additional values (variances and covariances) to identify a model which includes measurement error. While an improvement over one without measurement error, this model is still quite restrictive. A discussion of the model's utility can be found in Heise (1969), and criticism of some of its assumptions in Wiley and Wiley (1970).

We believe that there is a more effective way to increase the amount of information beyond the number of quantities one needs to estimate. Instead of expanding the number of time points, we suggest expanding the number of measures of each variable at each point. Two measures of each variable at each time point produce enough additional information to allow the calculation of all the quantities in a quite general two-time point model. We can even calculate all of the important characteristics of the true variables at a single point in time. This will be illustrated in the next section. In fact, we have more information (observed variances and covariances) than we need. This extra information also permits us to assess the adequacy of the model. Increasing the number of measures of each variable beyond two, permits further loosening of assumptions. For an extensive treatment of the multiple measurement of psychological constructs see Cronbach et al. (1972).

In order to specify the model, it is necessary to make some assumptions about the structure of the measured variables. Specifying these guides us in designing our measurement operations and variable definitions so that they more closely conform to these assumptions.

A traditional way of making these specifications is, that each measurement be a linear function of the underlying (true) value and an error, and the errors be uncorrelated with the true values for every measure and errors in other measures. This means that a covariance between two different measures of the same trait should result entirely from the common true variable. A covariance between a measure of one trait and a measure of another should only result from the true relation between the traits.[4)]

In describing the measurement characteristics of our model, we will begin with two constructs measured in two versions at a single point in time. Denoting this time point by t and the version by "1", the model for a datum resulting from the first operational definition of the first construct is:

(2.1) $$x1_t = \xi_t + \varepsilon 1_t \quad ,$$

where $x1_t$ represents the observed measurement, ξ_t the true value, and $\varepsilon 1_t$ the error of measurement. We assume that the error is uncorrelated with the true values of its own and other variables as well as with other errors at the same time point. Similarly, the model for such a datum of the second construct is:

(2.2) $$y1_t = \eta_t + \delta 1_t \quad ,$$

where $y1_t$ represents the observed measurement, η_t the true value, and $\delta 1_t$ the error of measurement. Examining the variational and covariational structure of the observed variables in this simple model, we obtain:

(2.3) $$\mathrm{Var}(x1_t) = \sigma^2_{\xi_t} + \sigma^2_{\varepsilon 1_t} \quad ,$$

(2.4) $$\mathrm{Var}(y1_t) = \sigma^2_{\eta_t} + \sigma^2_{\delta 1_t} \quad , \text{ and}$$

(2.5) $$\mathrm{Cov}(x1_t, y1_t) = \sigma_{\xi_t, \eta_t}$$

for the variances and covariance of the observed variables. These three observables are composed of five basic elements. The fact that three observables are composed of five elements, implies the impossibility of the evaluation of those elements without additional information.

If we, however, take an active role in designing the data collection process, we may generate additional information which will allow us to evaluate these unobservable variances and covariances. Wiley (1973) has suggested that social measurements be designed according to strategies similar to those used in psychological measurement, that is, by generating more than one operational version of each construct. In addition to (2.1) and (2.2) above, we might assume that each of the constructs, at time t, ξ_t and η_t, has a second operational definition:

(2.6) $x2_t = \beta_t \xi_t + \varepsilon 2_t$, and

(2.7) $y2_t = \alpha_t \eta_t + \delta 2_t$,

where β_t and α_t represent the possibilities of differences in the scales of measurement between the first and second versions of each construct at time t. These addi[illegible]nal observable variables generate two more observable va[illegible]nces and five more observable covariances:

(2.8) $\text{Var}(x2_t) = \beta_t^2 \sigma^2_{\xi_t} + \sigma^2_{\varepsilon 2_t}$,

(2.9) $\text{Var}(y2_t) = \alpha_t^2 \sigma^2_{\eta_t} + \sigma^2_{\delta 2_t}$,

$$(2.10) \quad Cov(x_{2_t}, x_{1_t}) = \beta_t \sigma^2_{\xi_t} \quad ,$$

$$(2.11) \quad Cov(x_{2_t}, y_{1_t}) = \beta_t \sigma_{\xi_t, \eta_t} \quad ,$$

$$(2.12) \quad Cov(y_{2_t}, x_{1_t}) = \alpha_t \sigma_{\xi_t, \eta_t} \quad ,$$

$$(2.13) \quad Cov(y_{2_t}, y_{1_t}) = \alpha_t \sigma^2_{\eta_t} \quad , \text{ and}$$

$$(2.14) \quad Cov(x_{2_t}, y_{2_t}) = \alpha_t \beta_t \sigma_{\xi_t, \eta_t} \quad .$$

Inspecting the right-hand side of these ten compositional equations, we find nine unobservable components: $\sigma^2_{\xi_t}$, $\sigma^2_{\eta_t}$, σ_{ξ_t,η_t}, $\sigma^2_{\varepsilon_{1_t}}$, $\sigma^2_{\delta_{1_t}}$, α_t, β_t, $\sigma^2_{\varepsilon_{2_t}}$, and $\sigma^2_{\delta_{2_t}}$. As long as the quantities α_t and β_t, which are coefficients characterizing the measurement scales of the variables, are not zero (a zero value implies that the new measures do not really relate to the appropriate constructs) and as long as the two constructs are related (σ_{ξ_t, η_t} is not equal to zero), this system of ten equations with nine unknowns may be solved and the unobservable components become calculatable from the observable ones.

This change in measurement design has allowed us to separate the interrelations among the observed constructs into those three aspects specifically due to the underlying constructs and those six due to the distortions of measurement errors.

2.1 Example

In this section we will present an example which will illustrate a procedure for estimating the variances and covariances of true and error variables. The data come from a study of communication processes conducted in Central America. The two constructs, we have chosen for illustration, are television watching by children and television possession by their families. Each construct was measured twice at each of three points in time. The same four questionnaire items were used at each time point.[5)] Coding of the responses to each item at each point in time resulted in twelve variables. The covariance matrix for these variables is displayed in Table 1.

Insert Table 1 about here

We will demonstrate the estimation procedure based on the model for a single time point, presented above, using the data for Time Point 1. Since the number of quantities to be estimated, nine, is fewer than the number of observed quantities, ten, there are several possible procedures. Two commonly chosen methods of "optimal" estimation are generalized least squares and maximum likelihood (see e.g. Goldberger and Jöreskog, 1972). We have chosen a simpler method, for illustrative purposes, which consists of eliminating one of the equations and solving those remaining. This method produces a relatively but not fully efficient estimate according to the above criteria.[6)]

The remaining equation can be used for the purpose of testing the adequacy of the model and we will illustrate this use also.

Since Time Point 1 was chosen for illustration, the first four by four submatrix in Table 1 is the basis for our calculations. In all that follows, x will characterize observed television possession while y will denote observed television watching. Recalling from above that the first subscript specifies the version of the construct while the second represents the time point at which it is measured, the variables, in order, are symbolized: x_{11}, x_{21}, y_{11}, and y_{21}, which represent possession version one (time one), possession version two (time one), watching version one (time one), and watching version two (time one), respectively.

According to Equation (2.5), $\hat{\sigma}_{\xi_1\eta_1} = \hat{cov}(x_{11}, y_{11})$ which from Table 1 is equal to 0.479.[7)] From Equation (2.1), $\hat{\beta}_1 = \widehat{\beta_1 \sigma}_{\xi_1,\eta_1}/\hat{\sigma}_{\xi_1,\eta_1} = \hat{cov}(x_{21}, y_{11})/\hat{\sigma}_{\xi_1,\eta_1}$ which is equal to $(0.443/0.479=)$ 0.924 which is the metric-coefficient for the second index of television possession in the scale of measurement of the first. Since both of the indices were based on dichotomous items, where the natural interpretation of the positive alternative for both items was possession, the scales of measurement should be almost the same. If this were in fact the case, β_1 would be equal to 1.000. The value which we have obtained is therefore quite reasonable. Similarly, from Equation

(2.12), $\hat{\alpha}_1 = \widehat{\alpha_1 \sigma_{\xi_1, \eta_1}} / \hat{\sigma}_{\xi_1, \eta_1} = \hat{cov}(y_{21}, x_{11}) / \hat{\sigma}_{\xi_1, \eta_1}$ which is equal to (0.312/0.479=) 0.651, the analogous coefficient for television watching. The fact that the estimate is less than one is also reasonable since the range of variation of the second watching item is less that of the first. Continuing in like fashion, from Equation (2.10), $\hat{\sigma}^2_{\xi_1} = \widehat{\beta_1 \sigma^2_{\xi_1}} / \hat{\beta}_1$ $= \hat{cov}(x_{21}, x_{11}) / \hat{\beta}_1$ equals (0.227/0.924=) 0.246, the true score variance of television possession. For television watching, from Equation (2.13), $\hat{\sigma}^2_{\eta_1} = \widehat{\alpha_1 \sigma^2_{\eta_1}} / \hat{\alpha}_1 = \hat{cov}(y_{21}, y_{11}) / \hat{\alpha}_1$ equals (1.225/0.651=) 1.882.

For television possession the variances of the errors of measurement may be estimated using Equations (2.3) and (2.8) as follows: $\hat{\sigma}^2_{\varepsilon_{11}} = (\widehat{\sigma^2_{\xi_1} + \sigma^2_{\varepsilon_{11}}}) - \hat{\sigma}^2_{\xi_1} = \hat{var}(x_{11}) - \hat{\sigma}^2_{\xi_1}$ equals (0.247 - 0.246=) 0.001, and $\hat{\sigma}^2_{\varepsilon_{21}} = (\widehat{\beta_1^2 \sigma^2_{\xi_1} + \sigma^2_{\varepsilon_{21}}}) - (\hat{\beta}_1)^2 \hat{\sigma}^2_{\xi_1}$ $= \hat{var}(x_{21}) - (\hat{\beta}_1)^2 \hat{\sigma}^2_{\xi_1}$ equals 0.248 minus (0.853)(0.246) which, in turn, equals 0.038. These small values indicate that television possession is quite reliably reported by the children. The variances of the errors of measurement for television watching may be estimated in a similar fashion using Equations (2.4) and (2.9) as follows:

$$\hat{\sigma}^2_{\delta_{11}} = (\widehat{\sigma^2_{\eta_1} + \sigma^2_{\delta_{11}}}) - \hat{\sigma}^2_{\eta_1} = \hat{var}(y_{11}) - \hat{\sigma}^2_{\eta_1}$$

equals (2.313 - 1.882=) 0.431, and

$$\hat{\sigma}^2_{\delta_{21}} = (\widehat{\alpha_1^2 \sigma^2_{\eta_1} + \sigma^2_{\varepsilon_{21}}}) - (\hat{\alpha}_1)^2 \hat{\sigma}^2_{\eta_1} = \hat{var}(y_{21}) - (\hat{\alpha}_1)^2 \hat{\sigma}_{\xi_1}$$

equals 2.264 minus (0.423)(1.882) which, in turn, equals 1.468. These values imply that the measures of television watching are considerably more errorful than those of television possession even considering the differences in measurement metric. It is also clear that the second index of watching, which is based upon long term frequencies of watching particular television programs, is much more errorful than the first, which asks about general watching in the preceding week.

Recalling that we have not used Equation (2.14), $cov(x_{21}, y_{21}) = \alpha_1 \beta_1 \sigma_{\xi_1, \eta_1}$, for estimation purposes, we may employ it to assess the adequacy of the model. This may be accomplished by comparing the tabulated value with a prediction based on our estimates of its components. The value for this covariance from Table 1 is 0.288. Predicting it from the estimates $\hat{\alpha}_1$, $\hat{\beta}_1$, $\hat{\sigma}_{\xi_1, \eta_1}$, we obtain (0.651)(0.924)(0.475) or 0.286. Since the two quantities are almost equal, we are pleased with our assessment of the adequacy of the model. Table 2 reports, for Time Point 1, all of the estimates and the ratios of the observed versus predicted covariances.

Insert Table 2 about here

We may, in the same way, estimate the corresponding quantities characterizing each of the other times of measurement. These are based on the central and final four by four subcovariance matrices of Table 1. The results for Time Point 2 and 3 are also summarized in Table 2. In each case, the covariance ratio indicates that the model is quite adequate for our data.

It should be noted that the model, as specified, assumes constancy over time in the metric of the first version of each variable. Since we do not include parameters such as α and β in these versions, we make no allowance for scale changes in them. We do, however, allow such changes in metric for the second version of each variable. Accordingly, we have subscripted the α and β values separately for each time point. This provides for a different discrepancy between version one and version two to obtain at each time point. Consequently, general metric inconsistencies for the second version of each variable are possible.

This is logically necessary for the watching variable. The first version of the watching variable has precisely the same operational definition for each time point. However, the television programs, used to specify the second version, change from time to time. Because of these changes, the metric coefficients should vary. On the other hand, the possession variable has constancy of definition for both of its versions at each point in time. As a consequence, we would not expect the true values of these metric coefficients to vary over time. If we inspect the estimates in Table 2, we, in fact, find slightly less variation in the coefficients for possession.

We can also assess the precisions of the measurements and the ways in which these precisions change over time. The reliability coefficient of a measure is usually defined as the proportion of variance attributable to the true variable. Table 2 conveyed the variances of the errors for each measurement operation at each time point. We may use these to assess the reliability of each measure since we also have estimates of the variances of the true variables and of the scale parameters.

Wiley and Wiley (1970) have strongly argued that this ordinary coefficient is misleading. The standard error of measurement (square root of the error variance) defines more adequately the accuracy of a measurement since it indicates, in an average sense, how far an observed measurement is likely to be from the true one. Since the standard error of measurement is not the only characteristic which influences the reliability, the traditional coefficient can vary because of changes in the true status of the variable as well as changes in the characteristics of the measurement operation. Concretely, as the variance of the true variable increases, other things being equal, the reliability increases.

Table 3 displays the reliabilities for each measurement at each point in time. The measurement properties of the variables, as indexed in this traditional way, vary widely. Since the true variances of the possession and watching constructs change over time, especially the latter, we find fluctuations in reliabilities which are not only due to changes in the average sizes of the errors but also to changes in the distributions of true watching behavior on the part of the children.

Insert Table 3 about here

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3. A Specification of the Measurement Model, Allowing Correlated Errors for Repeated Measurements

Just as we developed representations of each of the observed variances and covariances for each time point, we can construct similar representations for the cross-covariances between time points. Before, we were interested in defining, for example, the covariance of x_{12} and y_{22}, both measured at Time Point 2. Now, we are interested in defining, for example, the covariance of x_{11} and y_{22}, the first measured at Time Point 1 and the second at Time Point 2.

We specified that the errors were uncorrelated with true values and with errors in other variables. We did not specify the relations among the errors in the same variable at different points in time. Since it is reasonable to expect that the error in the measurement operation for a specific construct at one point in time may be correlated with the error in that same operation at a later point, it would seem advantageous to allow for this in the specification of the model. More concretely, it seems reasonable to assume that a child who overstates his general television watching at Time Point 1, may also overstate that at time two. It is possible to allow this additional flexibility because there are many more observed variances and covariances than values which we desire to estimate.

Of course, the allowance of correlated errors over time for specific measurement operations implies that the covariances among the resulting measurements will not directly reflect true score covariances as they do within time periods. For example:

(3.1) $\mathrm{Cov}(x_{11}, x_{12})$

$$= \mathrm{Cov}(\xi_1 + \varepsilon_{11}, \xi_2 + \varepsilon_{12})$$

$$= \mathrm{Cov}(\xi_1, \xi_2) + \mathrm{Cov}(\xi_1, \varepsilon_{12}) + \mathrm{Cov}(\varepsilon_{11}, \xi_2) + \mathrm{Cov}(\varepsilon_{11}, \varepsilon_{12})$$

$$= \sigma_{\xi_1, \xi_2} + 0 + 0 + \sigma_{\varepsilon_{11}, \varepsilon_{12}}$$

With this new specification in mind, we can turn to the general task of detailing the forty-eight symbolic compositions of the cross-covariances between time points (sixteen for each time-point pair). These are summarized together with the thirty previously specified within-time point variances and covariances (ten for each time-point) in a symbolic variance-covariance matrix (Table 4).

Insert Table 4 about here

We have forty-eight new pieces of information, but desire only twenty-four additional quantities: the twelve cross-covariances of the <u>true</u> variables (4 for each of 3 pairs of time points) and twelve error cross-covariances (also 4 for each of 3 pairs of time points).

In Table 4 the symbolic representations of the cross-covariances of the observable variables are located in the three off-diagonal blocks. The diagonal blocks symbolically represent the variances and covariances within the three time points. This sexpartite structure parallels that of the empirical values for the variances and covariances displayed in Table 1. With the three-time point data, the number of available pieces of information exceeds, by more, the number of desired quantities, than in single-time point data. This implies that there are many more ways of estimating these quantities. A statistically more elaborated solution to the problem of estimation than that given below would make more efficient use of this large amount of additional information.[8)]

3.1 Example

This subsection illustrates the computation of estimates of the cross-covariances for both the true variables and errors. We demonstrate these computations using the cross-covariances between Time Point 1 and Time Point 2. We will refer to the entries in Table 4 by the row (1 - 12) and column (I - XII) numbers corresponding to the particular cross-covariance. Since combinations of Time Points 1 and 2 are in the first off-diagonal block, the row numbers (Time Point 2) will always

be between 5 and 8, while the column numbers (Time Point 1) will always be between I and IV.

Two off-diagonal cells (6, I and 5, II) in the off-diagonal block include the cross-time covariance of true possession, σ_{ξ_1,ξ_2}. Referring to these two cells in both Table 4 and Table 1, we find that $\beta_2\sigma_{\xi_1,\xi_2}$ corresponds to 0.201 and $\beta_1\sigma_{\xi_1,\xi_2}$ corresponds to 0.196. Since we can obtain the estimates of β_2 and β_1 from Table 2 (0.998 and 0.924), we may estimate σ_{ξ_1,ξ_2} by 0.201/0.998 = 0.201, or by 0.196/0.924 = 0.212. For the final estimate, we choose the mean of these values: $\hat{\sigma}_{\xi_1,\xi_2}$ equals 0.207.

The diagonal elements of the off-diagonal block contain components representing both true and error cross-time covariances. For example, $\hat{\sigma}_{x_{11},x_{12}} = \sigma_{\xi_1,\xi_2} + \sigma_{\varepsilon_{11},\varepsilon_{12}} = 0.198$ (5, I). Subtracting the covariance of the true variables, estimated above, we obtain $\hat{\sigma}_{\varepsilon_{11},\varepsilon_{12}} = 0.198 - 0.207 = -0.009$. Similarly, $\hat{\sigma}_{x_{21},x_{22}} = \beta_1\beta_2\sigma_{\xi_1,\xi_2} + \sigma_{\varepsilon_{21},\varepsilon_{22}} = 0.205$ (6, II), therefore, $\hat{\sigma}_{\varepsilon_{21},\varepsilon_{22}} = 0.205 - (0.924)(0.998)(0.207) = 0.014$. Applying the procedure for estimating σ_{ξ_1,ξ_2} to the entries in cells 7, IV and 8, III, we obtain for watching $\hat{\sigma}_{\eta_1,\eta_2} = 1.602$. Consequently, $\hat{\sigma}_{\delta_{11},\delta_{12}} = -0.143$ and $\hat{\sigma}_{\delta_{21},\delta_{22}} = 0.617$.

If we inspect the eight remaining cells in the off-diagonal block, we find that half of them can be used, in an analogous fashion, to estimate σ_{ξ_2,η_1} and the other half to estimate σ_{ξ_1,η_2}.

The latter is a component of the following cells:

Cell 7, I : $\hat{\sigma}_{\xi_1,\eta_2} = 0.402$

Cell 7, II: $\widehat{\beta_1\sigma}_{\xi_1,\eta_2} = 0.375$

Cell 8, I : $\widehat{\alpha_2\sigma}_{\xi_1,\eta_2} = 0.212$

Cell 8, II: $\widehat{\beta_1\alpha_2\sigma}_{\xi_1,\eta_2} = 0.185$.

Dividing by the previous estimates ($\hat{\beta}_1 = 0.924$ and $\hat{\alpha}_2 = 0.571$), we obtain four distinct values for σ_{ξ_1,η_2}:

0.402 (= 0.402)

0.406 (= 0.375/0.924),

0.371 (= 0.212/0.571), and

0.351 (= 0.185/(0.924)(0.571)).

The average of these four values $0.383 = \hat{\sigma}_{\xi_1,\eta_1}$. Following the same procedure with the remaining cells we obtain 0.441 $= \hat{\sigma}_{\xi_2,\eta_1}$.

We have now estimated the four true cross-covariances relating possession and watching between Time Points 1 and 2. We have also estimated the repeated-measures error-covariances for the same two time points. The procedures we have used to produce these estimates can be equally well applied to the cross-covariances of the other two pairs of time points. The resulting error covariances are displayed in Table 5. If we combine the

---

Insert Table 5 about here

---

estimates of the true cross-covariances for each pair of time points with the estimates of the true variances and covariances within each time point, we may compose the total variance-covariance matrix of the six true measures of the two constructs at the three points in time. This estimated six by six covariance matrix is displayed in Table 6.

Insert Table 6 about here

This matrix serves as a new basic data set for all subsequent analyses of the true measures. When we compare the results of analyses based on observed data with those based on true measures, we will first analyze the observed values in Table 1 and then perform a comparative analysis of the estimated true values in Table 6.

4. Multi-relational Models for Longitudinal Data: The Effects of Measurement Error

The variances and covariances of Table 6 allow the computation of unstandardized regression analyses relating the true versions of the possession and watching variables over time. We, in fact, have all the information needed to perform multiple regressions relating any variable to any selection of other variables. The only difference between our regressions and ordinary ones which are based upon individually observed values, is that we perform our computations from summary characteristics (variances and covariances)[9] and that those summary characteristics are estimated only indirectly from the original data rather than computed directly from the observations themselves.

Regression analyses, analogous to those interrelating the <u>true</u> variables, will also be performed using the <u>fallible</u> version of those variables directly resulting from the original measurements. In computing the regression analyses involving these observed variables, we will use the variances and covariances of Table 1. These values are also summary. They were computed directly from the original individually observed values.

Another distinction between all of our regressions and those usually performed with individual values is our omission of a constant or intercept term from the specification of the regression model. This omission has no important consequences because the computation of regression coefficients from variances and covariances automatically eliminates the effects of variations in the means of the variables. The regression coefficients remain the same as they would have if the constant term had been included. The analyses will relate both television possession and television watching at adjacent time points.

We will relate television possession at Time Point 2 to possession and watching of television at the first time point. Substantively, we would expect a close relation to exist between the two measures of television possession, i.e. we do not expect families to dispose of their television sets. We might expect a small relation to exist between prior watching and later possession of television since those families who do not already own sets but have children who watch television outside of the home, may over time be influenced to acquire a set.

The additive regression model, in this case, may not be most appropriate, because we do not expect a relation between watching and subsequent possession for families who already possess television sets. I.e. our illustration of the methodology is only approximate with these data since the possession variable is a true dichotomy.

If we examine our expectations for watching, we would expect, with or without prior possession, a close relation between television watching for adjacent time points. For two children who watch television equally long at the first time point, we see several possibilities for the relation between possession and subsequent watching. It is, perhaps, most likely that any effect of television acquisition at Time Point 1 would have immediate consequences for television watching. Therefore, all effects on subsequent watching would be mediated through watching at Time Point 1 and, once we have taken this into account, we would expect no relation between television possession at that time point and later watching. However, if we had some speculation that the full effect of television acquisition on watching behavior were not felt immediately, but only later, we might expect an independent influence of possession on subsequent television watching even when we allow for prior watching.

We may test our expectations by first performing regression analyses relating the Time Point 2 values to the Time Point 1 values using the observed measurements. That is, we proceed in a fashion that is usual when such regressions are carried out without adjustment for unreliability. This implies that we fit the models:

(4.1) $x_{12} = \lambda_{1_x} x_{11} + \lambda_{1_y} y_{11} + \varepsilon_{12_x}$, and

(4.2) $y_{12} = \gamma_{1_x} x_{11} + \gamma_{1_y} y_{11} + \varepsilon_{12_y}$

in place of the models:

(4.3) $\xi_2 = \lambda_\xi \xi_1 + \lambda_\eta \eta_1 + \theta$, and

(4.4) $\eta_2 = \gamma_\xi \xi_1 + \gamma_\eta \eta_1 + \phi$

and treat our estimates of λ_{1_x}, λ_{1_y}, γ_{1_x}, and γ_{1_y} as if they were estimates of λ_ξ, λ_η, γ_ξ, and γ_η. Using the values in Table 1, we obtain the following estimates:

Possession: $\hat{\lambda}_{1_x} = 0.730$ (Possession), $\hat{\lambda}_{1_y} = 0.046$ (Watching);
Watching: $\hat{\gamma}_{1_x} = 0.665$ (Possession), $\hat{\gamma}_{1_y} = 0.494$ (Watching).

If we compare these estimates with our prior expectations under the assumption that these numbers represent the relations among the true variables, then we see that our expectations for possession are verified. There is a relatively large impact on television possession at Time Point 2 of that at Time Point 1 and there is almost no relation between watching and subsequent possession of a television set.

The results, however, for television watching at Time Point 2 were less expected. Although we do find a moderate effect of earlier watching, we discover a close relation between initial possession and subsequent watching of television. This seeming "sleeper effect" of delayed influence was only speculated about before, but seems quite apparent in the data.

Let us now compare the results of these fallible regressions with some based upon the estimated interrelations of the true variables. The estimated coefficients are:

Possession: $\hat{\lambda}_{\xi} = 0.735$ (Possession), $\hat{\lambda}_{\eta} = 0.042$ (Watching);
Watching: $\hat{\gamma}_{\xi} = -0.004$ (Possession), $\hat{\gamma}_{\eta} = 0.841$ (Watching).

Inspecting the possessions regression, we find almost no difference between our new estimates and the old ones, which indicated that watching had little or no effect on subsequent possession of a television set. However, when we look at the determinants of watching, we find a striking disparity. There is no relation at all of possession to subsequent television watching apparent in our new estimates, and the size of the coefficient relating initial and later television watching has almost doubled. The natural interpretation of these equations is, in fact, that there are no causal interconnections between television watching and possession. There is no surprising "sleeper effect".

If we examine the discrepancies between observed and true relations carefully, we may divine the cause. The accuracy of measurement of the possession variable at Time Point 1 is quite high. This may be seen from the reliabilities in Table 3 or the variances of the measurement errors in Table 2. This is not true, however, of the watching variable. The variances of the errors of measurement are substantively larger than those of the possession measurements. Even taking differences in metric into account, the reliabilities are substantially lower. The effects of these errors, on our assessment of the determinant of possession at Time Point 2, are small. The large effect is that of the most reliable variable: initial possession. The small effect is that of the unreliable variable: initial watching As a consequence, the larger errors in the initial watching measurement have little impact on the regression because the true watching variable, in fact, influences possession very little.

The case is considerably different, however, when we look at the regression coefficients for the determination of watching. Here, the most precisely measured determinant has no influence, while the least accurate explanatory variable has a very large impact. Since the true explanatory variables are positively related, error of measurement in the most powerful determinant (watching) not only attenuates the estimate of its effect but also weakens the "control" exerted on the relation between initial possession and subsequent watching of television. The consequence is what might be called "undercontrol". A true effect of prior watching on subsequent television watching is spuriously attributed to possession of a set, because of the high reliability of the prior possession measure, the high relation between true prior possession and true prior watching, and the lower reliability of the prior watching measure.

It should also be noted that in this case the interrelations among the errors in particular variables at the two points in time did not have a dramatic influence. These relations were small enough so that they had no material effect. If, on the other hand, they had been strong and positive, they would have partially compensated for some of the distortions in our estimated effects. However, this is only true, when there is no influence across variables. In the case of true cross-variable influence, large positive correlations among the errors in repeated measurements would result in too little attribution of influence to the other variable.

We may replicate these analyses for Time Point 2 and Time Point 3 measurements. Again, we initially examine the observed regression using the first version of each construct:

Possession: $\hat{\lambda}_{1_x} = 0.792$ (Possession), $\hat{\lambda}_{1_y} = 0.011$ (Watching);
Watching: $\hat{\gamma}_{1_x} = 0.794$ (Possession), $\hat{\gamma}_{1_y} = 0.467$ (Watching).

The pattern of results is obviously similar to that of the earlier observed regression. Turning to the true regression, we obtain:

Possession: $\hat{\lambda}_{\xi}$ = 0.917 (Possession), $\hat{\lambda}_{\eta}$ = -0.025 (Watching);
Watching: $\hat{\gamma}_{\xi}$ = 0.087 (Possession), $\hat{\gamma}_{\eta}$ = 0.857 (Watching).

The pattern of change and the resulting implications for the interpretation of influence are the same as those we found above. It is interesting to note that not only do the true effects have the same pattern for each pair of adjacent time points, but also, that the biasing effects of the measurement errors are the same. The replication did not help us see through the fog of error. Only an explicit assessment and accounting for those errors in our model and in our analyses helped us blow away the fog.

5. Summary Comments

We have reviewed some earlier attempts to deal with the analysis of panel data which have treated the problem of causality attribution to quantitatively scaled variables. Until recently, most such discussions of panel data have focussed on procedures comparing cross-lagged correlations. These practices suffer from many defects, whose severities have been difficult to assess in actual cases, because the procedures do not rest on well-specified mathematical or statistical models.

Recent concern for more mathematically explicit statement of the bases for such procedures, arising out of genetics and economics, has led to the increasing use of multi-relational models for the analysis of panel data. Also, the longstanding emphasis in individual psychology on error of measurement has gradually become more sophisticated. A combination of these concerns seems to be in order. The conjunction of measurement models, relating true variables to their measured manifestations, with multi-relational models, relating multiple measurements of many true variables, is greatly needed. Measurement errors, have large distorting effects on the assessment of variable influences when they are not explicitly taken into account within a multi-relational setting.

This paper has formulated an explicit measurement model for use with the analysis of quantitatively scaled panel data. The model incorporates more than one measurement of each construct under investigation. This multiple measurement allows the assessment of the interrelations among true variables at a particular point in time. The addition of interrelations among the errors of measurement over time allows assessment of the true cross-time relations of the underlying constructs as well as assessment of the accuracy of the measurement procedures. Once the intra- and inter-time point relations among the constructs have been assessed, multi-relational models incorporating them may be implemented. The resulting estimates serve to assess the relative degrees of inter-construct impact through time. This precipitation of true from observable, this purification from error, allows us to eliminate the distortions which these errors cause when they are not taken into account.

We illustrated these models and their implementation as practical data-analytic procedures by analyzing a three-time point data set with double measurement of each of two constructs. Our initial example allowed us to estimate the variances and covariances of the constructs separately at each time point with no contaminating effects of error. Our additional specifications of time dependencies in the errors allowed us to estimate the cross-time relations among those constructs, as well. The full set of derived interrelations among the constructs at the various time points allowed us to fit multiple regression models to assess the true impacts of the underlying constructs on one another between adjacent time points. We demonstrated that there were large effects which had severely distorting influences on substantive interpretations. These distortions were effectively removed, however, by the procedures.

Although we illustrated the models and procedures with data involving only two constructs and three time points, the general strategy is valid for any number of constructs and any number of time points. It should serve as a valuable tool for eliminating the distorting effects of measurement error in the analysis of quantitatively scaled panel data of much greater complexity.

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Footnotes

1) A discussion of the issues basic to our thesis in the context of qualitative data may be found in Murray (1971).

2) Campbell (1963, p. 240) has stated this problem clearly: "A variable which increases in reliability from Time 1 to Time 2 will, ceteris paribus, show up as an 'effect' rather than a 'cause'."

3) As stated in Wiley and Wiley (1970, p. 116): "Each standardized parameter is a function of more than one unstandardized parameter. In general, if two or more of the unstandardized parameters of a model are equal, the corresponding standardized parameters will be unequal because they are not related to the unstandardized parameters by an equivalent transformation."

4) While covariances between distinct measures are not affected by errors of measurement, under these assumptions, correlations are attenuated, since they are defined by dividing the covariance by the product of the standard deviations. These standard deviations are inflated by the measurement error, and therefore, the correlations are deflated.

5) Following are the questionnaire items, translated from Spanish with the coding of responses for each variable.

Television Watching

A. How many times did you see television in the last week?

	Coding
_____ none	0
_____ one or two times	1
_____ three or four times	2
_____ five or six times	3
_____ every day	4

B. With what frequency did you see each of the following programs?

	Every week	Once or twice per month	Rarely	Never
A. Tom Jones	_____	_____	_____	_____
B. Tarzan	_____	_____	_____	_____
C. The Office	_____	_____	_____	_____
D. Bonanza	_____	_____	_____	_____
C. Land of Giants	_____	_____	_____	_____
Coding	3	2	1	0

Codes for each program were summed and the total was recoded as follows:

Sum	Recode
0	0
1,2,3	1
4,5,6	2
7,8,9	3
10,11,12	4
13,14	5
16	6

At each time point, program names were changed to correspond to common preferences. This resulted in substantial differences in means and variances among the time points.

Television Possession

A. Of the following information media, which do you have in your home?

_____ newspapers
_____ magazines
_____ radio
_____ television
_____ books

Children who answered "television" received a score of "1", others received "0".

B. In what places do you see television programs?

_____ in your house
_____ in a friend's house
_____ in a relative's house
_____ elsewhere

Children who answered "in your house" received a score of "1", others received "0".

6) Within the context of data from a single point in time, the model may be "optimally" estimated by either generalized least squares or maximum likelihood, within the frameworks of Wiley, Schmidt, and Bramble (1973) or Jöreskog (1970). A computer program for carrying out the analysis is also available (Jöreskog, van Thillo, and Gruvaeus, 1971). The main consequenc of such "optimal" estimates is that they are generally more precise (have smaller standard errors) than the more easily computed ones proposed here. As the models become more complex, current "state-of-the-art" methods become inadequate. The extensions discussed below, for example, cannot be "optimally" estimated using existing computer programs.

7) The symbol "^" refers to the estimate of the quantity over which it appears. When one such symbol appears over more than one quantity, the result refers to a single estimate of the composite quantity. When more than one of these symbols appear in a single expression, the result refers to the composite of the estimates of the individual quantities.

8) See Footnote 6).

9) All of our regression computations were performed either by hand or using standard regression analysis computer programs. Many such programs accept summary as well as individual data as input. The summary input may take the form of variances and covariances or of correlations and standard deviations, each of which is easily convertible to the other. Programs which require means as well may be given arbitrary values in place of them and the resulting constant terms may be ignored.

Table 1. Covariance Matrix for Two Versions of Television Possession and Watching at Each of Three Time Points

	Construct	Version	Time	Symbol	(I) x_{11}	(II) x_{21}	(III) y_{11}	(IV) y_{21}	(V) x_{12}	(VI) x_{22}	(VII) y_{12}	(VIII) y_{22}	(IX) x_{13}	(X) x_{23}	(XI) y_{13}	(XII) y_{23}
(1)	1	1	1	x_{11}	.247											
(2)	1	2	1	x_{21}	.227	.248										
(3)	2	1	1	y_{11}	.479	.443	2.313									
(4)	2	2	1	y_{21}	.312	.288	1.225	2.264								
(5)	1	1	2	x_{12}	.198	.196	.432	.283	.250							
(6)	1	2	2	x_{22}	.201	.205	.437	.299	.232	.250						
(7)	2	1	2	y_{12}	.402	.375	1.459	1.053	.468	.467	2.453					
(8)	2	2	2	y_{22}	.212	.185	.906	1.212	.267	.271	1.028	1.749				
(9)	1	1	3	x_{13}	.185	.176	.375	.250	.203	.202	.397	.221	.250			
(10)	1	2	3	x_{23}	.175	.172	.351	.247	.196	.199	.381	.210	.233	.250		
(11)	2	1	3	y_{13}	.376	.330	1.308	1.020	.418	.419	1.517	.906	.502	.491	2.476	
(12)	2	2	3	y_{23}	.231	.204	.907	1.186	.274	.273	1.004	1.107	.319	.312	1.326	2.281

Table 2. Estimates of Parameters for the Three Time Points

	Time Point 1	2	3
$\hat{\beta}$	.924	.998	.978
$\hat{\alpha}$	.651	.571	.635
$\hat{\sigma}^2_{\xi}$	.246	.232	.238
$\hat{\sigma}^2_{\eta}$	1.882	1.800	2.088
$\hat{\sigma}_{\xi,\eta}$	.479	.476	.502
$\hat{\sigma}^2_{\varepsilon_1}$	.001	.018	.012
$\hat{\sigma}^2_{\varepsilon_2}$	.038	.019	.022
$\hat{\sigma}^2_{\delta_1}$	.431	.653	.387
$\hat{\sigma}^2_{\delta_2}$	1.468	1.163	1.439
$\frac{\text{observed cov}(x_2, y_2)}{\text{predicted cov}(x_2, y_2)} = \frac{\hat{\text{cov}}(x_2, y_2)}{\hat{\alpha}\hat{\beta}\hat{\sigma}_{\xi,\eta}}$	1.01	1.00	1.00

Table 3. The Reliabilities of the Variables for the Three Time Points

Construct	Version	Symbol	Time Point 1	2	3
1	1	x_1	.996	.928	.952
1	2	x_2	.847	.924	.912
2	1	y_1	.814	.734	.844
2	2	y_2	.352	.335	.369

Table 4. Symbolic Covariance Structure for Two Versions of Two Constructs at Each of Three Time Points

	Construct	Version	Time	Symbol	(I)	(II)	(III)
(1)	1	1	1	x_{11}	$\sigma^2_{\xi_1} + \sigma^2_{\varepsilon_{11}}$		
(2)	1	2	1	x_{21}	$\beta_1\sigma^2_{\xi_1}$	$\beta_1^2\sigma^2_{\xi_1} + \sigma^2_{\varepsilon_{21}}$	
(3)	2	1	1	y_{11}	σ_{ξ_1,η_1}	$\beta_1\sigma_{\xi_1,\eta_1}$	$\sigma^2_{\eta_1} + \sigma^2_{\delta_{11}}$
(4)	2	2	1	y_{21}	$\alpha_1\sigma_{\xi_1,\eta_1}$	$\alpha_1\beta_1\sigma_{\xi_1,\eta_1}$	$\alpha_1\sigma^2_{\eta_1}$
(5)	1	1	2	x_{12}	$\sigma_{\xi_1,\xi_2} + \sigma_{\varepsilon_{11},\varepsilon_{12}}$	$\beta_1\sigma_{\xi_1,\xi_2}$	σ_{η_1,ξ_2}
(6)	1	2	2	x_{22}	$\beta_2\sigma_{\xi_1,\xi_2}$	$\beta_1\beta_2\sigma_{\xi_1,\xi_2} + \sigma_{\varepsilon_{21},\varepsilon_{22}}$	$\beta_2\sigma_{\eta_1,\xi_2}$
(7)	2	1	2	y_{12}	σ_{ξ_1,η_2}	$\beta_1\sigma_{\xi_1,\eta_2}$	$\sigma_{\eta_1,\eta_2} + \sigma_{\delta_{11},\delta_{12}}$
(8)	2	2	2	y_{22}	$\alpha_2\sigma_{\xi_1,\eta_2}$	$\beta_1\alpha_2\sigma_{\xi_1,\eta_2}$	$\alpha_2\sigma_{\eta_1,\eta_2}$
(9)	1	1	3	x_{13}	$\sigma_{\xi_1,\xi_3} + \sigma_{\varepsilon_{11},\varepsilon_{13}}$	$\beta_1\sigma_{\xi_1,\xi_3}$	σ_{η_1,ξ_3}
(10)	1	2	3	x_{23}	$\beta_3\sigma_{\xi_1,\xi_3}$	$\beta_1\beta_3\sigma_{\xi_1,\xi_3} + \sigma_{\varepsilon_{21},\varepsilon_{23}}$	$\beta_3\sigma_{\eta_1,\xi_3}$
(11)	2	1	3	y_{13}	σ_{ξ_1,η_3}	$\beta_1\sigma_{\xi_1,\eta_3}$	$\sigma_{\eta_1,\eta_3} + \sigma_{\delta_{11},\delta_{13}}$
(12)	2	2	3	y_{23}	$\alpha_3\sigma_{\xi_1,\eta_3}$	$\beta_1\alpha_3\sigma_{\xi_1,\eta_3}$	$\alpha_3\sigma_{\eta_1,\eta_3}$

Table 4 continued

	(IV)	(V)	(VI)	(VII)	(VIII)
(1)					
(2)					
(3)					
(4)	$\alpha_1^2\sigma^2_{\eta_1} + \sigma^2_{\delta_{21}}$				
(5)	$\alpha_1\sigma_{\eta_1,\xi_2}$	$\sigma^2_{\xi_2} + \sigma^2_{\epsilon_{12}}$			
(6)	$\alpha_1\beta_2\sigma_{\eta_1,\xi_2}$	$\beta_2\sigma^2_{\xi_2}$	$\beta_2^2\sigma^2_{\xi_2} + \sigma^2_{\epsilon_{22}}$		
(7)	$\alpha_1\sigma_{\eta_1,\eta_2}$	σ_{ξ_2,η_2}	$\beta_2\sigma_{\xi_2,\eta_2}$	$\sigma^2_{\eta_2} + \sigma^2_{\delta_{12}}$	
(8)	$\alpha_1\alpha_2\sigma_{\eta_1,\eta_2} + \sigma_{\delta_{21},\delta_{22}}$	$\alpha_2\sigma_{\xi_2,\eta_2}$	$\alpha_2\beta_2\sigma_{\xi_2,\eta_2}$	$\alpha_2\sigma^2_{\eta_2}$	$\alpha_2^2\sigma^2_{\eta_2} + \sigma^2_{\delta_{22}}$
(9)	$\alpha_1\sigma_{\eta_1,\xi_3}$	$\sigma_{\xi_2,\xi_3} + \sigma_{\epsilon_{12},\epsilon_{13}}$	$\beta_2\sigma_{\xi_2,\xi_3}$	σ_{η_2,ξ_3}	$\alpha_2\sigma_{\eta_2,\xi_3}$
(10)	$\alpha_1\beta_3\sigma_{\eta_1,\xi_3}$	$\beta_3\sigma_{\xi_2,\xi_3}$	$\beta_2\beta_3\sigma_{\xi_2,\xi_3} + \sigma_{\epsilon_{22},\epsilon_{23}}$	$\beta_3\sigma_{\eta_2,\xi_3}$	$\alpha_2\beta_3\sigma_{\eta_2,\xi_3}$
(11)	$\alpha_1\sigma_{\eta_1,\eta_2}$	σ_{ξ_2,η_3}	$\beta_2\sigma_{\xi_2,\eta_3}$	$\sigma_{\eta_2,\eta_3} + \sigma_{\delta_{12},\delta_{13}}$	$\alpha_2\sigma_{\eta_2,\eta_3}$
(12)	$\alpha_1\alpha_3\sigma_{\eta_1,\eta_3} + \sigma_{\delta_{21},\delta_{23}}$	$\alpha_3\sigma_{\xi_2,\eta_3}$	$\beta_2\alpha_3\sigma_{\xi_2,\eta_3}$	$\alpha_3\sigma_{\eta_2,\eta_3}$	$\alpha_2\alpha_3\sigma_{\eta_2,\eta_3} + \sigma_{\delta_2}$

Table 4 continued

	(IX)	(X)	(XI)	(XII)
(1)				
(2)				
(3)				
(4)				
(5)				
(6)				
(7)				
(8)				
(9)	$\sigma^2_{\xi_3} + \sigma^2_{\varepsilon_{13}}$			
(10)	$\beta_3\sigma^2_{\xi_3}$	$\beta_3^2\sigma^2_{\xi_3} + \sigma^2_{\varepsilon_{23}}$		
(11)	σ_{ξ_3, η_3}	$\beta_3\sigma_{\xi_3, \eta_3}$	$\sigma^2_{\eta_3} + \sigma^2_{\delta_{13}}$	
(12)	$\alpha_3\sigma_{\xi_3, \eta_3}$	$\alpha_3\beta_3\sigma_{\xi_3, \eta_3}$	$\alpha_3\sigma^2_{\eta_3}$	$\alpha_3^2\sigma^2_{\eta_3} + \sigma^2_{\delta_{23}}$

Table 5. Estimated Variance-covariance Matrices of the Measurement Errors

(Correlations in Parentheses)

				Time Point		
Construct	Version	Symbol	Time Point	1	2	3
1	1	x_1	1	.001		
			2	-.009 (*)	.018	
			3	.000 (.00)	.002 (.14)	.012
1	2	x_2	1	.038		
			2	.014 (.52)	.019	
			3	.005 (.24)	.003 (.15)	.022
2	1	y_1	1	.431		
			2	-.143 (-.27)	.653	
			3	-.190 (-.47)	-.068 (-.14)	.387
2	2	y_2	1	1.468		
			2	.617 (.47)	1.163	
			3	.567 (.39)	.552 (.43)	1.439

* The value for the correlation is greater than one because sampling variation has produced an abnormally small value for the Time Point 1 error variance.

Table 6. Estimated Variance-covariance Matrix of True Television Possession and Watching at Each of Three Time Points

	ξ_1	η_1	ξ_2	η_2	ξ_3	η_3
ξ_1	.246					
η_1	.479	1.882				
ξ_2	.207	.441	.232			
η_2	.383	1.602	.476	1.800		
ξ_3	.185	.379	.201	.387	.238	
η_3	.364	1.498	.425	1.585	.502	2.088

STUDIES OF EDUCATIVE PROCESSES

Report No. 1 - Annegret Harnischfeger
Personal and Institutional Characteristics Affecting Teacher Mobility. I. Problems of Teacher Mobility.
1973, iii, 59 pp., $2.00

Report No. 2 - Annegret Harnischfeger
Personal and Institutional Characteristics Affecting Teacher Mobility. II. Descriptive Characteristics of the San Jose Unified School District.
1973, vi, 48 pp., $2.00

Report No. 3 - David E. Wiley
Another Hour, Another Day: Quantity of Schooling, a Potent Path for Policy.
1973, v, 76 pp., $2.00

Report No. 4 - Annegret Harnischfeger
Personal and Institutional Characteristics Affecting Teacher Mobility. III. Teacher Characteristics and Teacher Mobility: A Confusion of Causes.
1973, viii, 74 pp., $2.00

Report No. 5 - David E. Wiley and Robert Hornik
Measurement Error and the Analysis of Panel Data
1973, iv, 45 pp., $2.00

Report No. 6 - Annegret Harnischfeger
Personal and Institutional Characteristics Affecting Teacher Mobility. IV. Schools Do Make a Difference
1973, viii, 59 pp., $2.00

These reports may be obtained by writing to David E. Wiley, Department of Education, University of Chicago, 5835 Kimbark Ave., Chicago, Ill. 60637. (Checks should be made payable to David E. Wiley, University of Chicago.)