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MONTE CARLO RESULTS FROM A
COMPUTER PROGRAM FOR TAILORED TESTING

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Abstract

INTERTAIL, the computer program which implements an approach to tailored testing outlined by Cliff (1975), was examined with errorless data in several Monte Carlo studies. Three replications of each cell of a 3 x 3 table with 10, 20 and 40 items and persons were analyzed. Mean rank correlation coefficients between the true order, specified by pre-assigned random numbers, and the computed order produced by the program ranged from .93 to .99. Other efficiency measures are reported which also support the theory as a general measuring and ordering technique. Based on these results, program modifications are proposed as well as a data scheme which is to be used in further system testing.

This report describes a computer program designed to implement the computer-interactive testing procedure proposed by Cliff (1975). The theory starts from the observation that the ordinary item score matrix in which a correct response is recorded as a 1 and an incorrect one as a 0 can be regarded as an adjacency matrix indicating the relations between a set of items and a set of persons. From that point of view the matrix should be extended so that it is items-plus-persons-by-items-plus-persons. Then there are four sections. These consist of one which is the ordinary item-by-person rights matrix, a corresponding person-by-item section which is the wrongs matrix, an item-by-item section which is all zero, and a person-by-person section which is similarly all zero. Now the interpretation is that a 1 indicates that the row element dominates the column element, regardless of which is item and which is person. The person-person and item-item sections are all zero because these relations are not observed directly.

He goes on to show that, if the data corresponds to the requirements for a Guttman scale, the supermatrix is equivalent to a type of incomplete adjacency matrix which records the relations among the members of a semiorder. Moreover, the employment of a kind of Boolean matrix algebra can be used to complete the matrix of relations. These "missing" relations are those in the item-item and person-person sections of the matrix. That is, the person-person and item-item order relations can be determined as implications of the person-item responses if the items form a Guttman scale.

This process is relevant to tailored testing because it applies

to incomplete score matrices as well as complete ones. In fact, all that is necessary to deduce the complete score matrix and the complete joint order of persons and items is the response of each person to the easiest item he would fail and the hardest one he would pass. If these are known, repeated application of the Boolean matrix algebra will succeed in completing the matrix of responses.

The workings of the matrix process goes as follows: The matrix S is the item-person matrix, $s_{ij} = 1$ if person i passes item j and zero otherwise, i.e., a rights matrix. The matrix \tilde{S} is the wrong matrix with $\tilde{s}_{ij} = 1$ if i fails j and zero otherwise. For complete data, S and \tilde{S} are complementary, but for tailored tests some elements can be zero in both. Now compute $N = \tilde{S}'S$; n_{jk} will equal the number of persons who failed item j and passed k . Similarly, we compute $X = \tilde{S}\tilde{S}'$; x_{ih} will equal the number of items that person i passed but h failed. If the items are a Guttman scale, then either n_{jk} or n_{kj} will be zero, and similarly for x_{ih} and x_{hi} . That is a consequence of the Guttman form of the score matrix; in a Guttman scale where there is an item that i passes and h fails, it is never the case that there is a different item which h passes but i fails. In a Guttman scale, then all that need be recorded is that i dominates or defeats h (1) or not (0). It is this simple sense in which we speak of the matrix algebra being Boolean; only the 1 or 0 is recorded, not the actual numbers.

This process is illustrated in Figures 1a and 1b. From some points of view, it is simpler to treat S and \tilde{S} as elements of a supermatrix A . Then N and X are sections of the supermatrix A^2 , as shown in the figures.

$$A = \begin{bmatrix} \overline{T} & \overline{S} \\ S & P \end{bmatrix} : A =$$

		Items				Persons				
		w	x	y	z	i	j	k	l	m
Items	w	0	0	0	0	0	1	1	1	1
	x	0	0	0	0	0	0	1	1	1
	y	0	0	0	0	0	0	0	1	1
	z	0	0	0	0	0	0	0	0	1
Persons	i	1	1	1	1	0	0	0	0	0
	j	0	1	1	1	0	0	0	0	0
	k	0	0	1	1	0	0	0	0	0
	l	0	0	0	1	0	0	0	0	0
	m	0	0	0	0	0	0	0	0	0

Figure 1a. Full adjacency matrix for two-set dominance data.

$$A^2 = \begin{bmatrix} \overline{N} & \overline{O} \\ O & X \end{bmatrix} : A^2 =$$

		Items				Persons				
		w	x	y	z	i	j	k	l	m
Items	w	0	1	2	3	0	0	0	0	0
	x	0	0	1	2	0	0	0	0	0
	y	0	0	0	1	0	0	0	0	0
	z	0	0	0	0	0	0	0	0	0
Persons	i	0	0	0	0	0	1	2	3	4
	j	0	0	0	0	0	0	1	2	3
	k	0	0	0	0	0	0	0	1	2
	l	0	0	0	0	0	0	0	0	1
	m	0	0	0	0	0	0	0	0	0

Figure 1b. Matrix A^2 : Ordering of items and persons.

In the tailored case, the logic works as follows: Suppose there is an item which person i passes but h fails. Then i dominates (is smarter or more knowledgeable than) h. Suppose there is another item which i himself fails. Then we need not present it to h because under the Guttman assumption the latter must fail it too. Similarly, if there is an item which h passes, we need not present it to i because he must pass it. In fact, such chains of inference can be extended over an interlocked series of persons and items to lead finally to a conclusion

that i will answer j correctly (or incorrectly). This implicational process can be symbolized quite simply with the Boolean matrix algebra. All that is necessary is to have the S and \tilde{S} matrices incomplete in the sense that $s_{ij} = \tilde{s}_{ij} = 0$ for some of the elements, and to have the A supermatrix raised to powers higher than 2. Suppose we have the results for all of the persons on some of the items. Then A^2 will contain person-person and item-item dominances that are implied by those responses. If A^3 is then computed, what it contains is the person-item responses that are implied by the relations in A^2 . If A^4 is computed, it contains the dominances that are implied at one remove; similarly, A^5 will contain the responses that are implied, but implied indirectly. This process can continue to as high a power as seems useful. It is illustrated in Figure 2.

$$\begin{array}{cc}
 A & A(A + I)^{(2)} \\
 \left[\begin{array}{cccccc}
 & & & & & \\
 & & & & & \\
 0 & & & & & \\
 1 & 1 & 0^* & 0^* & & \\
 0 & 1 & 1 & 0^* & & 0 \\
 0^* & 0 & 1 & 1 & & \\
 0^* & 0^* & 0 & 1 & & \\
 0^* & 0^* & 0^* & 0 & &
 \end{array} \right] & \left[\begin{array}{cccccc}
 & & & & & \\
 & & & & & \\
 0 & & & & & \\
 1 & 1 & 1 & 0^* & & \\
 0 & 1 & 1 & 1^* & & 0 \\
 0^* & 0 & 1 & 1 & & \\
 0^* & 0^* & 0 & 1 & & \\
 0^* & 0^* & 0^* & 0 & &
 \end{array} \right] \\
 A(A + I)^{(3)} & A(A + I)^{(4)} \\
 \left[\begin{array}{cccccc}
 0 & 1 & 1 & 1 & & \\
 0 & 0 & 1 & 1 & & 0 \\
 0 & 0 & 0 & 1 & & \\
 0 & 0 & 0 & 0 & & \\
 & & & & & \\
 & 0 & 1 & 1 & 1 & 0 \\
 & 0 & 0 & 1 & 1 & 1 \\
 & 0 & 0 & 0 & 1 & 1 \\
 & 0 & 0 & 0 & 0 & 1 \\
 0 & & & & & \\
 & 0 & 0 & 0 & 0 & 0
 \end{array} \right] & \left[\begin{array}{cccccc}
 & & & & & \\
 & & & & & \\
 0 & & & & & \\
 1 & 1 & 1^* & 1^* & & \\
 0 & 1 & 1 & 1^* & & \\
 0^* & 0 & 1 & 1 & & 0 \\
 0^* & 0^* & 0 & 1 & & \\
 0^* & 0^* & 0^* & 0 & &
 \end{array} \right]
 \end{array}$$

Figure 2. Incomplete adjacency matrix A and its powers. (Entries with asterisks represent item-person pairs which are not observed directly.)

There is a major difficulty with the foregoing. This is that test items do not form Guttman scales, but are at best quasi-scales. Consequently, the direct application of the foregoing procedure can result in erroneous or contradictory implications. Some method of making the process relatively insensitive to the probabilistic nature of test response is needed. Therefore, while the computer program essentially follows the algebraic procedure described above, an additional feature is added. This is that while all the matrices are stored in the dichotomous form, the calculations are carried out numerically and certain quantitative tests are performed before an entry is recorded in a product matrix as a 1 or a 0. This approach is taken in order to reduce the effect of non-transitivity in the data.

For example, in computing n_{jk}^* from the binary response matrices, the first operation is

$$n_{jk}^* = \sum_i s_{ij} s_{ik}.$$

Then the symmetric element n_{kj}^* is also computed in the same way.

$$n_{kj}^* = \sum_i s_{ik} s_{ij}.$$

Then the following ratio is computed

$$z_{jk} = \frac{n_{jk}^* - n_{kj}^*}{(n_{jk}^* + n_{kj}^*)^{1/2}}.$$

This may be recognized as the ratio for correlated proportions (Guilford and Fruchter, 1974). Then there is a specified criterion value for z_{jk} , and n_{jk} is recorded as 1 if the obtained ratio exceeds that criterion, n_{kj} is recorded as 1 if $-z_{jk}$ exceeds it, and both are zero if neither is the case. Thus j dominates k only if answered wrongly by "significantly" more persons. Some other function of n_{jk}^* and n_{kj}^* could obviously be used for this purpose, but this is the approach used here. The same procedure is used for person dominance.

A second major problem is that of matching the individual with an appropriate item when there is only partial information about both. Our approach to it follows from the conceptualization of the relevant score matrix as items-plus-persons by items-plus-persons. The traditional total score for a person is his sum across a row of the score matrix; the traditional item difficulty (actually, it is an easiness) is the sum down a column of it, divided by n . The present formulation extends these in several ways. First, we consider both the rights score and the wrongs score since the matrix is incomplete and there is information in both. Second, implied as well as directly observed relations are included in the scores, including the item-item and person-person relations. Finally, both persons and items are treated in exactly the same way; for both, a count is made of the number of items and persons it dominates and is dominated by, both directly and by implication. Then items and persons are matched on the basis of their net dominance scores.

This is rather easy to formulate symbolically using the present notation. Given the matrix G which is the Boolean sum of successive powers of the data matrix at a given point, the "wins" are the number of 1s in a given row and the "losses" are the number in the same column.

s_{ij} : 0000000000000000010001001000100 00001000000100000000000000000000

"and"

\tilde{s}_{hj} : 00000000000100001000000000000100 00001000000000000000000000000000

gives

00000000000000000000000000000100 00001000000000000000000000000000

Thus is this case $x_{ih} = 2$ because there are shown to be two items passed by i and failed by h . In order to find this number, the words must be unpacked to find the number of non-zero elements, which means that some of the time gained by carrying out the arithmetic 32 steps at a time is lost, but the gain is substantial, particularly since many times x_{ih} will be zero. The use of this binary storage in computation necessitates routines to perform the required packing and unpacking and utilization of special logical functions. Our program uses the logical functions of a local system (IBM 370/158). Such functions are either available for most machines or readily written by the system programmers.

An additional principle used is that of an expanding item pool. The idea is that the program initially works with only a subset of the available items. Periodically, the consistency of the items used so far is examined and those that appear to be less consistent than some input value are replaced by others from the pool.

The program described below is built around these principles and techniques. It is programmed as a rather simple main program with a number of subroutines. In addition to handling the computational and decisional aspects, these allow for different modes of operation (Monte Carlo, simulation on stored data, and true interactive) and provide for various choices concerning operating parameters, output formats, and the like.

OVERVIEW OF TAILORED TESTING PROGRAM - INTERTAIL

The current version of INTERTAIL requires that the user specify 10 initial parameters which pertain to the size of the study, and various critical values, and the mode of operation. Then, according to whether the program is in interactive or Monte Carlo mode, person-item pairs are established. In the interactive case, which is a tryout mode, the pairs are supplied by the user, while in a Monte Carlo run they are produced by the computer and they are paired at random. After this beginning set-up phase, interactive pairing and matrix powering sequence begins. The major steps involved are set up as subroutines. They are described below and flow charts for the main program and subroutines are given in the appendix.

1 - Subroutine INTERACT

In the interactive mode person-item pairs are presented and the user specifies the dominance between them. For a Monte Carlo, determining dominances is done automatically by comparing the previously assigned random numbers of person and item, and assigning a "win" to the larger. The record of wins and losses for each item and person is then logged in one of several matrices. Thus for the case of an item win over a person, the item win matrix records a 1 for this item over this person. Similarly, in the person losses matrix a 1 is recorded for the loss to this item for this person.

2 - Subroutine SQUARE

The items for which new relations have been established

are tested against all other items in order to determine the item-item dominances. Essentially this involves comparing the wins and losses between two items and testing the ratio of one item's wins to the other's wins. If the ratio is greater than some critical value then the first item dominates the second: if the ratio is less than -1.0 times the critical ratio then the second item dominates the first.

3 - Subroutine IMPLY

The process of powering the item-by-item wins and losses matrices is accomplished as many times as the user specifies. Each item of the group for which new relations have been collected, IR, is compared against all other items in the item pool, J. The number of dominances of item IR over J and also of J over IR is computed. If neither item IR nor item J currently have enough wins over the other, then any previous order between the two is removed. If however one of the two items has beaten the other frequently enough as specified by the user, then the count of item-item wins and losses are incremented and entries are added to the appropriate locations of the Item-Item dominance matrices.

4 - Subroutine REPLCE

If all the available items are in use, this section is skipped. However, if that is not the case each item is examined for its current consistency. All persons are checked against an item and the number of inconsistencies are counted (an incon-

sistency is defined as $I_x > P_y$ and $P_y > I_x$). If the ratio of the number of inconsistencies for an item divided by the number of persons is less than some user-specified value, the item is replaced by another from the item pool.

5 - Subroutine MULT

Summary implications are then computed for all relations between the items and persons. Each person is tested against each item by comparing the number of wins of the person over the item, $N1$, with the number of wins of the item over the person, $N2$. The ratio of $\frac{N1 - N2}{\sqrt{N1 + N2}}$ is tested against a user-specified critical value. If the

ratio is greater, a dominance is recorded for the person over the item; if the ratio is less than the negative user-supplied value, a dominance is recorded for the item over the person.

6 - Subroutine COUNT

Each person is compared to all other persons to determine the current order among them. This is accomplished by accumulating person X wins against person Y losses and person Y wins against person X losses. These numbers are tested in the ratio $\frac{X - Y}{\sqrt{X + Y}}$. If the ratio

is greater than a user-supplied value, the win is added to X 's total, loss to Y 's total; if the ratio is less than negative user-supplied value, a win is added to Y 's total and a loss to X 's total.

7 - Subroutine OUTPUT

The current status of the person and item orders and the contents of the super-matrix can be printed if opted. If the person-item orders are requested then they are ranked and printed according to person wins plus item wins minus the sum of person losses plus item losses ($T = PW + IW - (PL + IL)$). In the same way items are ranked by $T = PW + IW - (PL + IL)$. In addition, if they are requested, the person-item and item-item matrices are printed.

8 - Subroutine SELECT

If there are more relations to be gathered, the persons and items are matched as optimally as possible. This process first finds the person who has the fewest relations on the items, then locates the person with the next fewest, etc. An item is paired with a person by locating the item with the number of net person wins which is closest to the person's number of net item wins.

The above sequence, represented by steps 1 through 8, is repeated until the person-item super-matrix is filled (i.e., when a relation exists for each person on each item). At that point the final information can be printed by means of subroutine OUTPUT. Then, unless the user specifies that another run is to be started, the program terminates.

Monte Carlo Study

In order to examine the efficacy of INTERTAIL a series of Monte Carlo runs were designed and carried out. The scheme which was implemented assigned random numbers to the items and persons at the beginning of the session. These numbers performed as measures of item difficulty or a person's ability, and by using them dominance could be assessed directly (i.e., if the random number assigned to person P_1 was greater than that which was assigned to item I_a , then P_1 was said to have answered I_a correctly, or $P_1 > I_a$). This is essentially an errorless data technique because no allowance was made for chance factors or the interaction of ability and discrimination levels. (Subsequent studies are currently being planned which will test the system with random factors included in the data.)

Parameter Definitions

As currently written, INTERTAIL requires the user to specify 9 parameters which are used in various sections of the program. In the completely interactive approach these parameters are obtained by means of the computer prompting the user for a certain option (eg. "INPUT NUMBER OF PERSONS") and recording the subsequent response. The actual values used in the present trials are given in brackets after the individual parameter descriptions and the acronyms used in the program appear in capital letters. The parameters are (1) NPER, the number of persons in the study [10, 20, 40]; (2) NTOT, the total number of items [10, 20, 40]; (3) NITEM, the number of items, less than or equal to NTOT, which is the subset of items actually in use

at one time [10, 20, 40]; (4) NTIME, the number of cycles to be completed before an item consistency check is made in subroutine REPLAC [3]; (5) CONST, the absolute z value which must be surpassed when judging an item to be inconsistent [.99]; (6) OPTD, the optimum difference in net wins between any item and person being paired for the next cycle [1.0]; (7) RATIO, the absolute z value which must be surpassed to define an item-person or item-item dominance [1.0]; (8) NCYC, the number of cycles which must elapse before the powering process begins [3]; (9) INITCY, the number of items presented to a person before entering the major iterative process [1]. These parameters were used to test the program with nine different sized studies. Each of the nine combinations of 10, 20, or 40 persons and items was replicated three times so that solution variability could be examined.

Results

The solution INTERTAIL produces is primarily a rank ordering of the items and persons, although other indices are also given. Therefore the principle concern is the correlation of the computed rank order, given at the end of a Monte Carlo, and the true rank order of the persons and items based on the initially assigned random numbers. The mean rank order correlation coefficients (Kendall's Tau) for the nine study sizes are shown in Table 1. They are close to unity, on the average. As can be seen there is a general tendency for studies involving larger numbers of relations to produce stronger correlations, presumably because the greater the number of possible pairs, the more likely the computation of a dominance relation between

TABLE 1

Mean correlation coefficients between true order and computed order.

		Number of Items		
		10	20	40
Number of Persons	10	.94	.97	.93
	20	.95	.98	.98
	40	.95	.98	.99

any two entities becomes (that is between any item-person, item-item, or person-person).

It is noteworthy that for this errorless data there are no perfect correlations in any of the studies. This situation arises when the random assignment of numbers produces a true order such that two or more items (or two or more persons) are adjacent to each other in the matrix. In such cases, although the tied items or persons are not necessarily out of order, the routine calculation of tau in effect penalizes any inability to duplicate the true order exactly. In light of this consideration, these results were interpreted as signifying that the program did essentially recapture the true order of the original matrices; that the coefficients were less than 1.0 indicates that an order among adjacent persons is determined by chance when no item can be used to fine-tune the dominance relationships.

A second major interest in this study concerned how many responses needed to be produced relative to the total number of possible

relations. The ultimate value of a tailored approach lies in how much information a single response will generate, or how many possible relations are eliminated by the powering process. Table 2 shows the

TABLE 2

Mean percentage of possible relations
accounted for by responses.

		Number of Items		
		10	20	40
Number of Persons	10	.72	.49	.46
	20	.58	.44	.34
	40	.48	.39	.46

mean percentage of possible relations which were accounted for by responses. The values ranged from a high of 72% to a low of 34%. There is again a general trend for larger studies to yield smaller percentages, indicating the increased efficiency of the technique with bigger problems.

Figure 3 demonstrates the rates at which the various sized problems were solved (i.e., solved indicating a relationship exists between each item and person). The nine studies are separated according to number of items. Each plot reflects the program's orientation toward the person solution over the item solution. In other words for cases where the number of persons and items are not equal, a cycle is defined as pairing each person with an item and

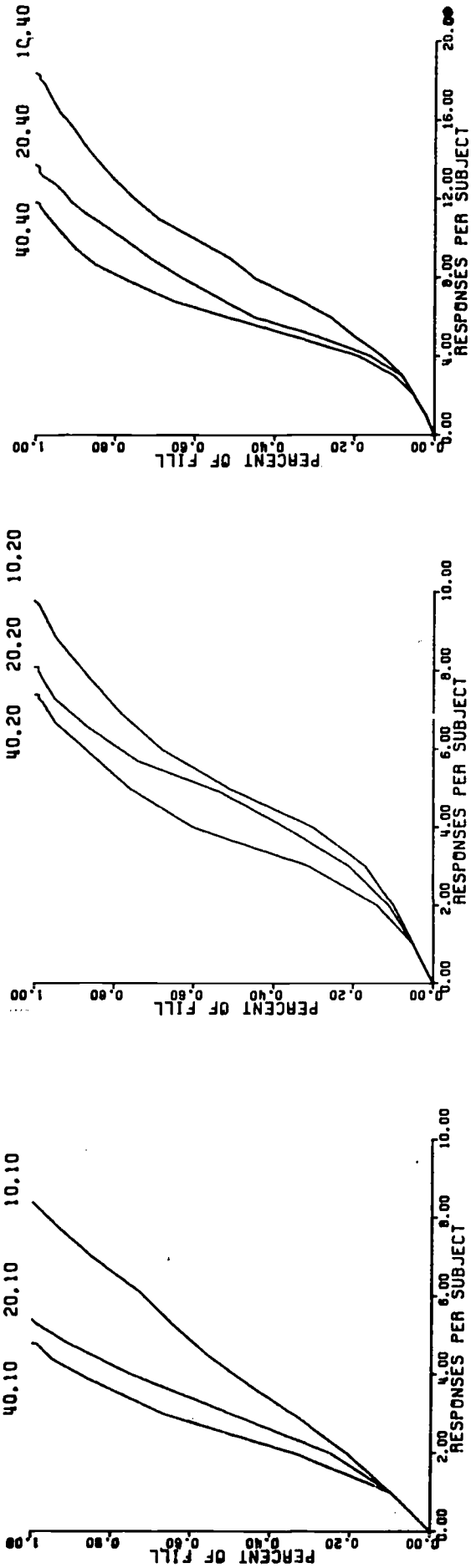


FIGURE 3.
PROGRESS CURVES FOR PERSONS AND ITEMS OF 10, 20
AND 40. PERCENT OF FILL IS GIVEN AS RATIO OF
NUMBER OF ACTUAL RESPONSES TO NUMBER OF TOTAL
POSSIBLE RELATIONS.

not the converse. This approach requires that a single item will be used more than once when the number of persons is greater than the number of items, and that some items will not be used during a cycle when the number of persons is less than the number of items. Across all studies with the same number of items there is a consistent effect for fewer responses per person as the number of total relations increases. Similarly the relative solution rates between 10, 20 and 40 persons can be seen to be approximately the same.

Finally, performance was also assessed in terms of central processing unit (CPU) time. CPU time is the actual amount of time a computer system is involved with calculation or institution of input or output. Table 3 shows the average amount of CPU time in

TABLE 3

Mean CPU Time		Number of Items		
Number of Persons		10	20	40
	10	3	10.8	64
	20	5.2	18.7	89.4
	40	11.9	36.6	132.1

seconds for the nine studies. In each case, as the maximum number of relations to be determined increases, so does the amount of CPU time.

Discussion

This preliminary examination of INTERTAIL involved errorless data and stimulus sets ranging in size from 100 to 1600 relations. The investigation gave a generally positive picture about the approach, for all indices followed rather orderly progression in solution speed and accuracy over the various study sizes. It has been suggested by Knuth (1973) that the minimum number of responses required to order N stimuli is $\log_2 N!$, where in this context, N is number of persons and items. The program produced consistently close approximations to the theoretical minimum over all study sizes.

Two major issues arose as a result of the work to this point. The first involves the problem of a stopping rule. The current version of the program halts only after a relation has been recorded for each stimulus pair. Toward the end of any problem the net information to be gained from any one response tends to decrease, that is there is usually little impact on the final order from the last few relations. Consequently a rule could be developed which would terminate the program, even with some relations outstanding. This is because the time considerations involved with collecting and processing the last responses doesn't generally alter the person order, and could be eliminated. The problem is a common one in iterative approaches, namely, how close is close enough, or what is to be gained for the final effort of gathering all relations. The issue is currently being analyzed in detail and will be specifically examined after the program has been tested with other data.

The second allied problem encountered was in the area of stimulus set size, especially in the possibility of employing only a subset of the total item pool. The idea is that if the persons can be ordered

with fewer items than the total, substantial overall efficiency could be achieved by not needlessly handling all possible pairs. This could be approached from a sort of dynamic item pool concept which would begin with a distilled sample of items and thereafter add or delete items as their relative impact to the person order was assessed. With smaller numbers of pairs this saving would be only marginally important. However as the size of the stimulus set increases it appears likely that such a trimmed item pool would yield substantial savings in computer time and the number of questions asked each subject.

The next phase will be to construct a second series of Monte Carlo studies to test the program with more realistic data. This segment will employ a model with measures corresponding to a subject's ability, and item characteristics of discrimination, difficulty and a guessing probability. Results will be used to make decisions about the stopping rule problem and the idea of a reduced item pool.

References

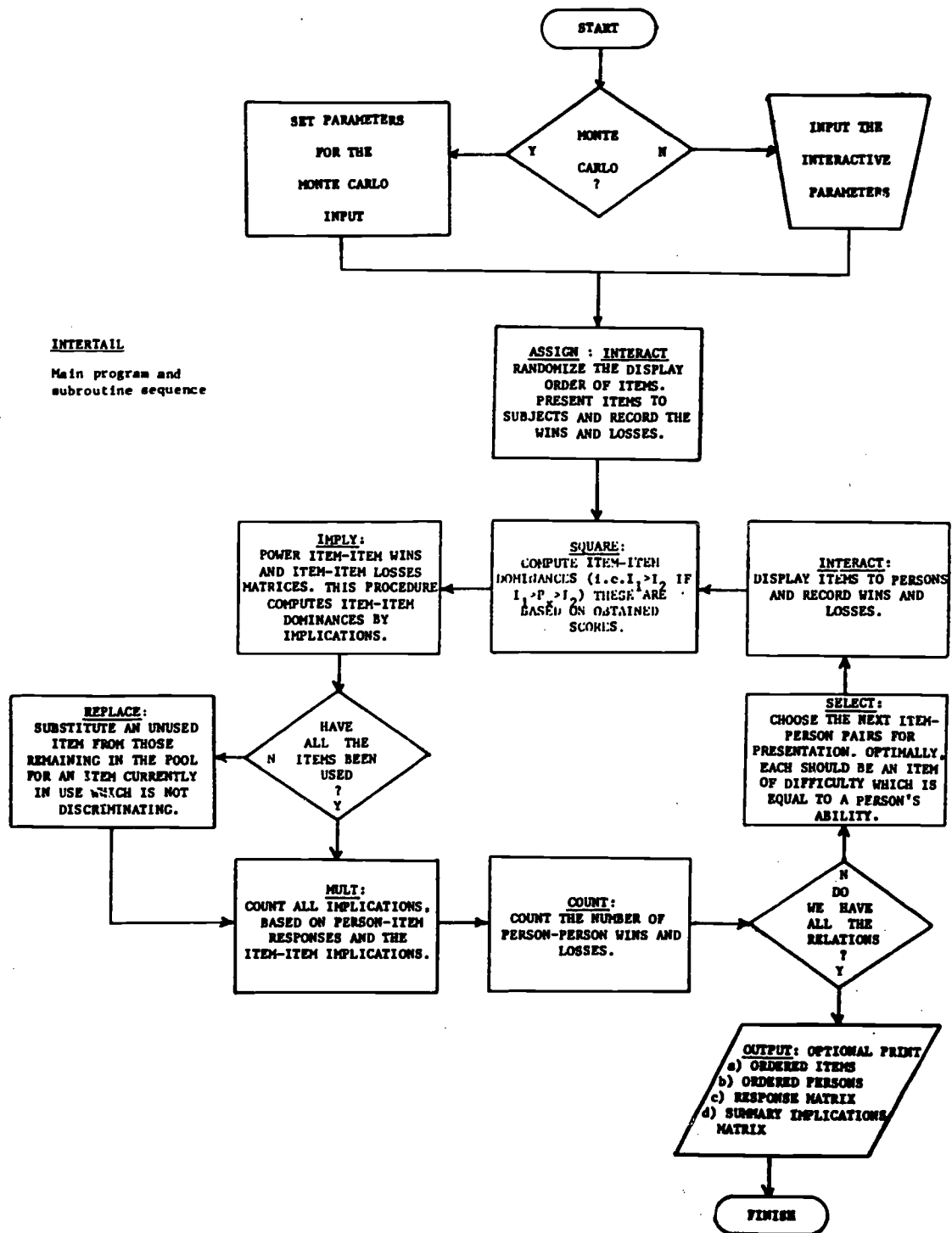
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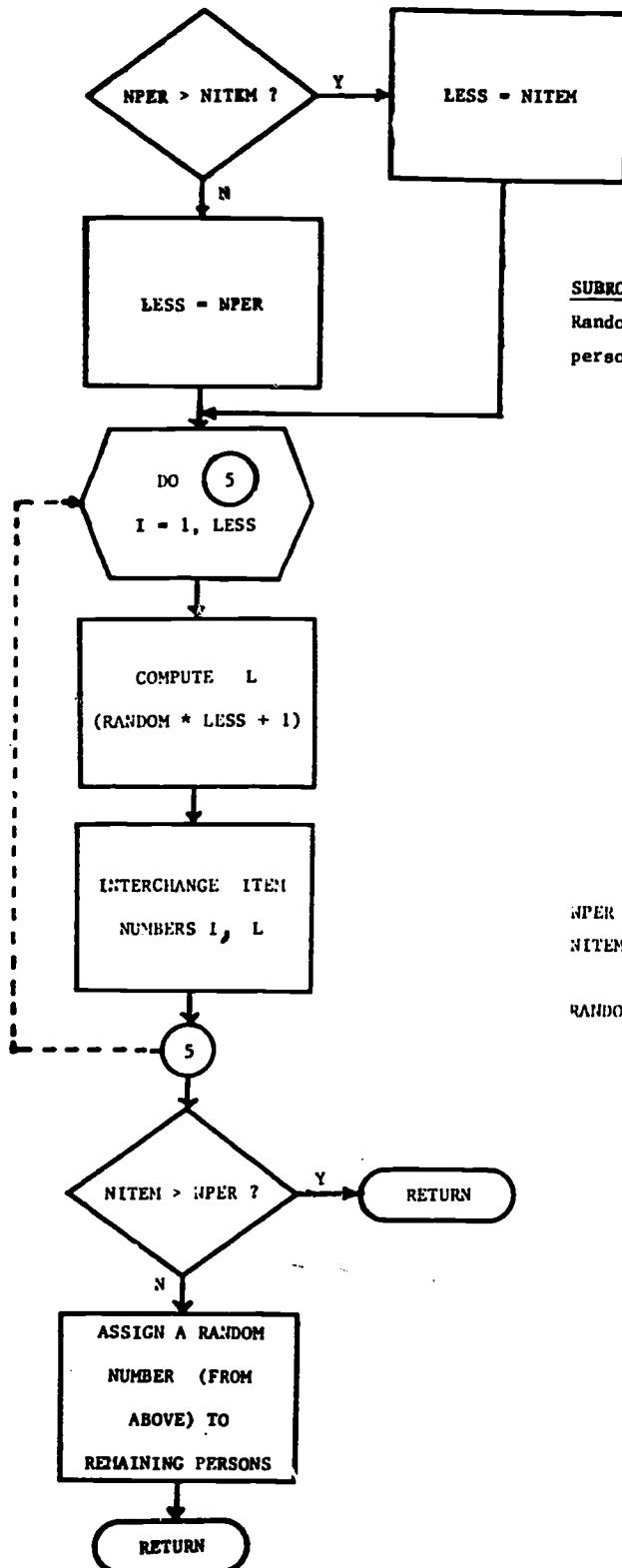
Appendix

INTERTAIL Main Program and
Subroutine Flow Diagrams

INTERTAIL

Main program and
subroutine sequence





SUBROUTINE ASSIGN

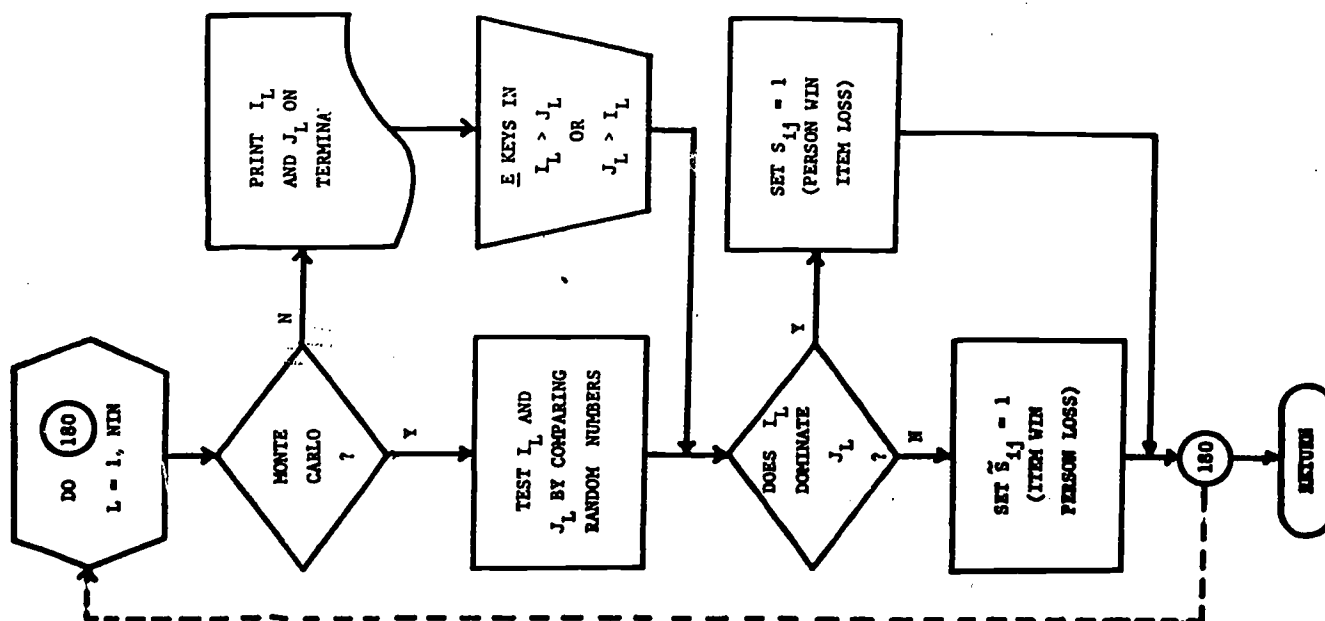
Randomizes the items and pairs each with a person

NPER = number of persons
NITEM = number of items

RANDOM = a random real number in the range
 $0.0 \leq X \leq 1.0$

SUBROUTINE INTERACT

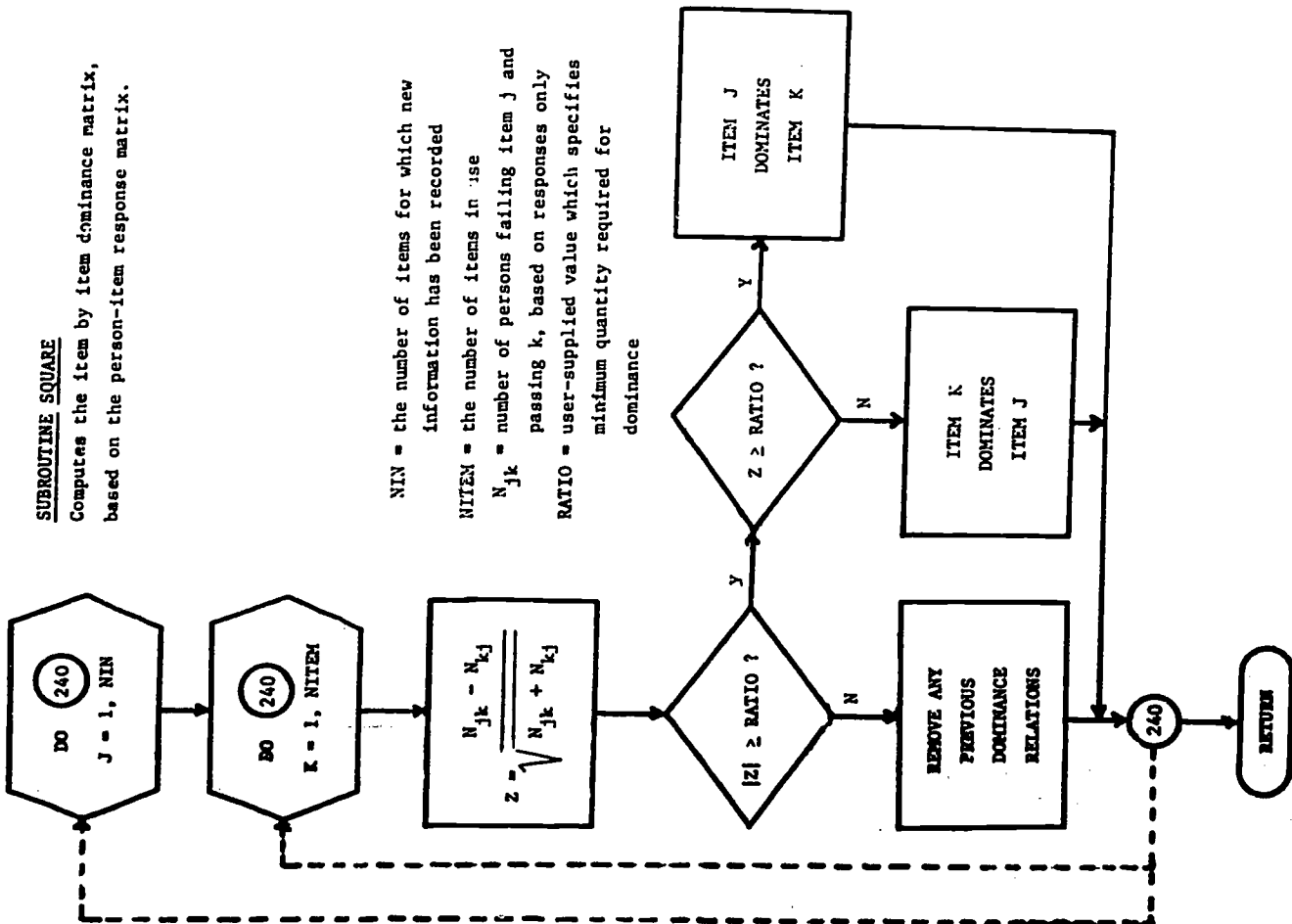
Gathers information on person-item relations by either having \bar{E} interactively assess the relation between I_L and J_L or by comparing the previously assigned random numbers of I_L to J_L in a monte carlo procedure, the larger receiving the win the lesser the loss.

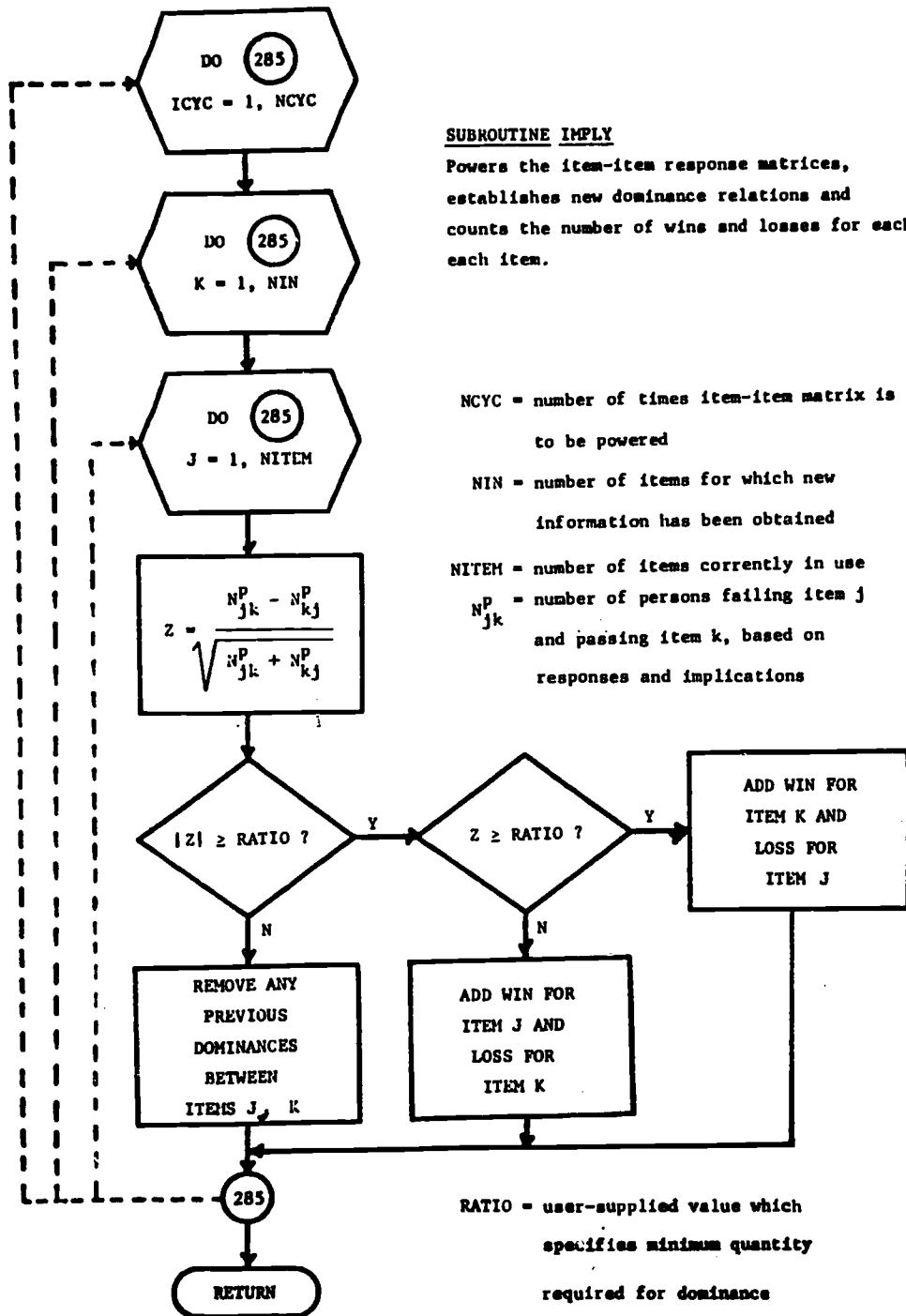


NIN = number of pairs to be presented
 J_L = the Lth item
 I_L = the Lth person
 S_{ij} = the cell of the rights submatrix corresponding to I_L, J_L
 \bar{S}_{ij} = the cell of the wrongs submatrix corresponding to I_L, J_L

SUBROUTINE SQUARE

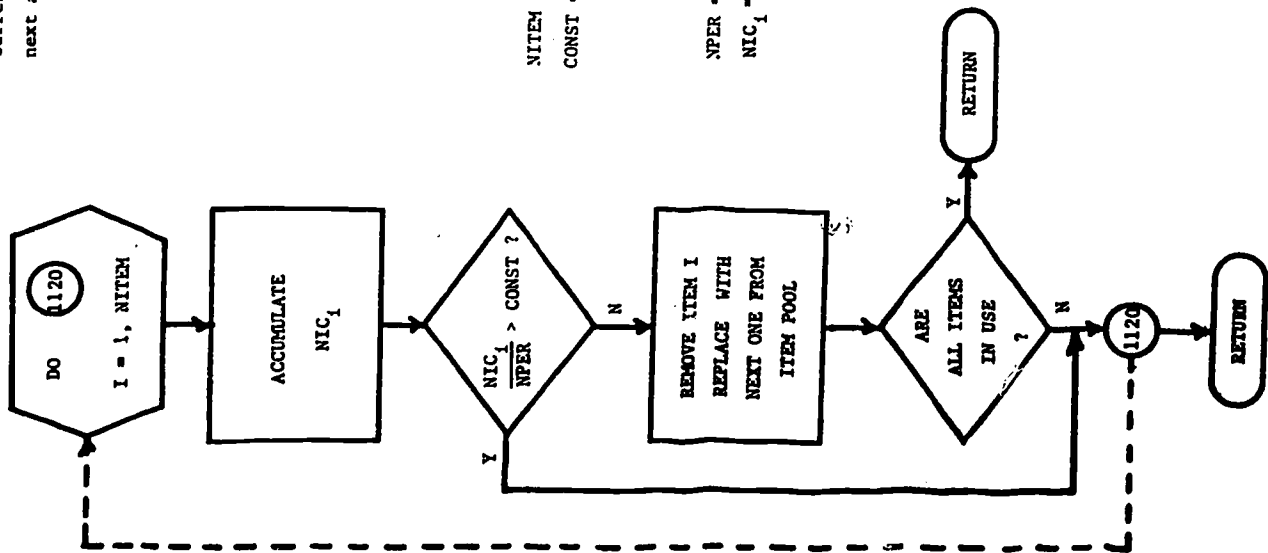
Computes the item by item dominance matrix, based on the person-item response matrix.



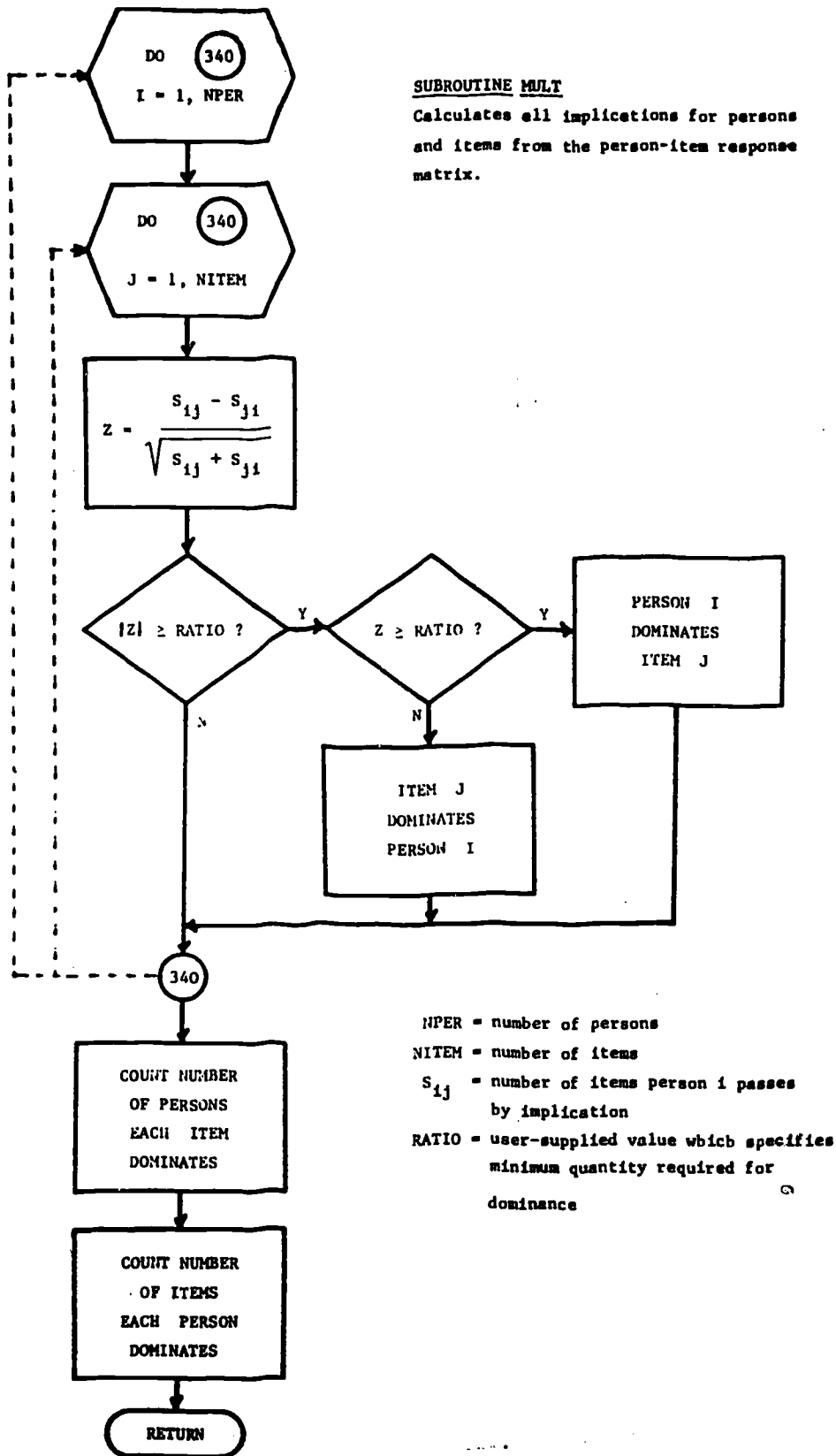


SUBROUTINE REPLAC

Removes inconsistent items from those currently in use and replaces with the next available from the item pool.

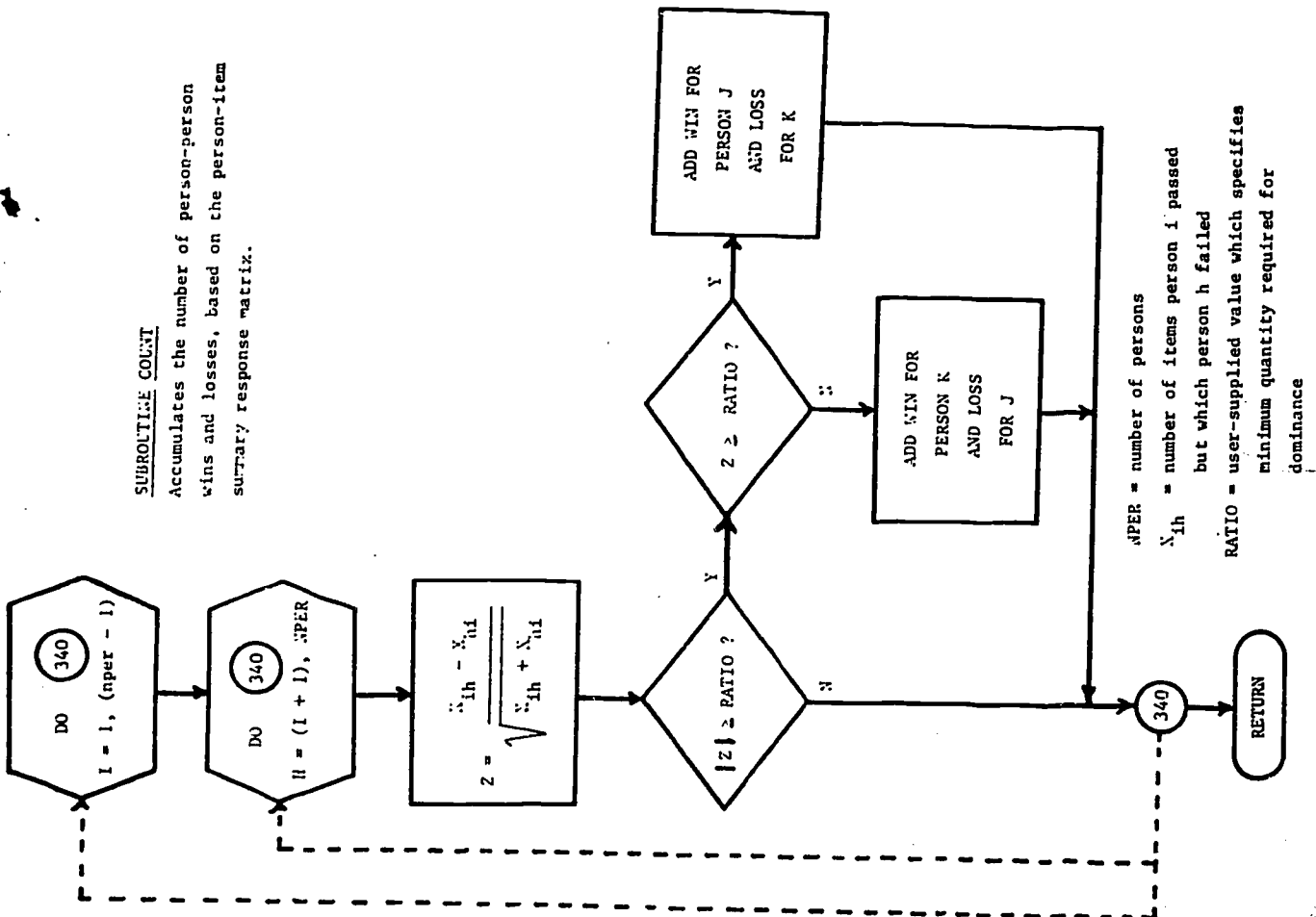


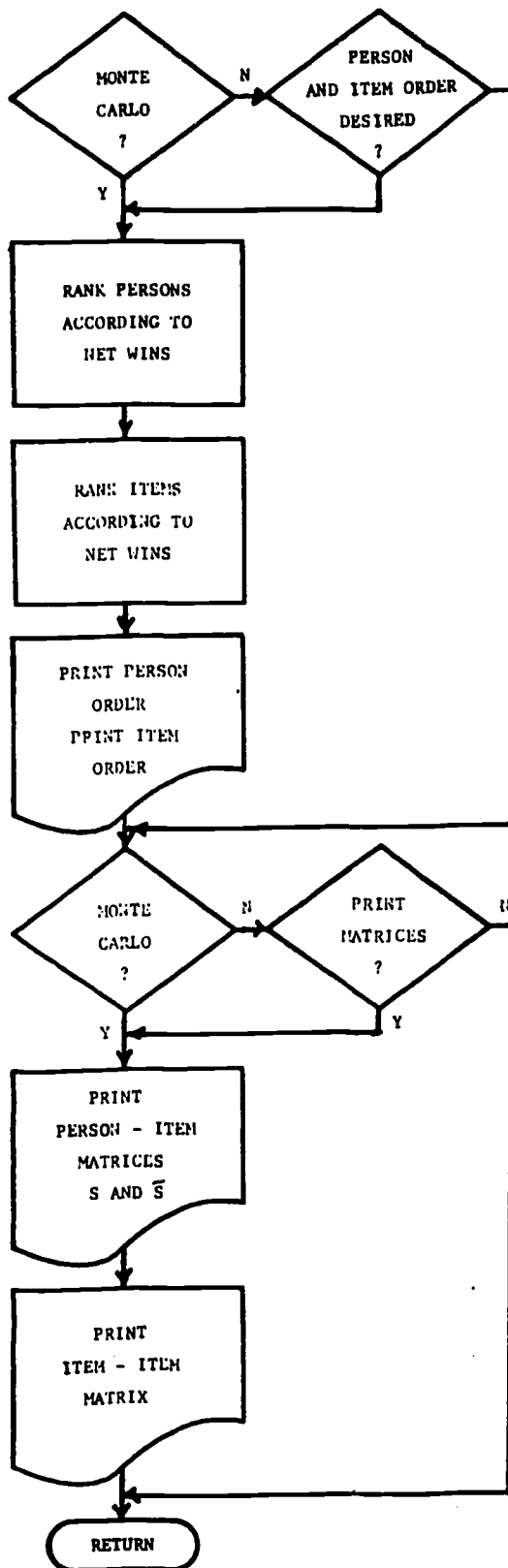
NITEM = number of items in use
 CONST = user-supplied value which indicates minimum consistency level an item must have to be retained in the active list
 NPER = number of persons
 NIC₁ = number of inconsistencies against item I, an inconsistency is defined as a person being credited as both passing and missing an item



SUBROUTINE COUNT

Accumulates the number of person-person wins and losses, based on the person-item summary response matrix.



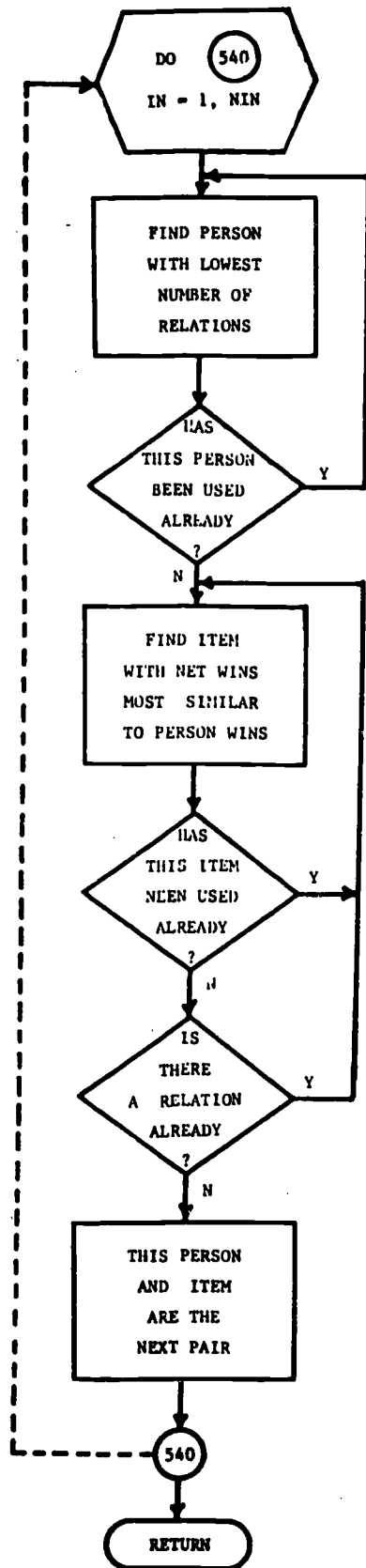


SUBROUTINE OUTPUT

Prints the person and item ranks according to net wins, the person-item matrix and the item-item matrix.

S = item-person rights matrix

\tilde{S} = item-person wrongs matrix



SUBROUTINE SELECT

Finds person-item pairs of optimally matching net wins which will be presented next.

NIN = number of pairs which will be defined if possible

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