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AUTHOR Murray, Frank B.; Armstrong, Sharon L.
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ABSTRACT

A conservation problem of numerical equivalence which 80% of adults reliably fail and 40% of third graders pass was developed, and responses of 188 subjects (Grades 2, 3, 5, 7, 9, 11 and college) to it and related number conservation and probability problems indicated that the differences in nonconservation were rooted in subjects' different evaluations of the relevancy of the conservation question and not as much in qualitative differences in inferential competence. Still, there was evidence that adults may make exactly the same kind of nonconservation errors as have been attributed to preoperational and concrete operational children. (Author)

Adult Nonconservation of Numerical Equivalence

Frank B. Murray and Sharon Lee Armstrong

University of Delaware

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Abstract

A conservation problem of numerical equivalence which 80% of adults reliably fail and 40% of third graders pass was developed and responses of 188 subjects (grades 2, 3, 5, 7, 9, 11 and college) to it and related number conservation and probability problems indicated that the differences in nonconservation were rooted in subjects' different evaluations of the relevancy of the conservation question and not as much in qualitative differences in inferential competence. Still, there was evidence that adults may make exactly the same kind of nonconservation errors as have been attributed to preoperational and concrete operational children.

Adult Nonconservation of Numerical Equivalence[1]

Older children's superior performance on conservation problems has traditionally been taken to indicate that the older child reasons at a qualitatively higher stage than the younger child. However, when older children perform less well on a reasoning or conceptual task (e.g., Weir & Stevenson, 1959; Zeiler, 1964; Kendler & Kendler, 1959; Osler & Trautman, 1961; Osler & Weiss, 1962) it is usually taken to indicate that the older child simply has adopted a more complex hypothesis about the task than is necessary to solve the problem and not that the younger child is reasoning in a qualitatively superior manner. Whether the nonconservation response of the younger child should be attributed to some hypothetical logical deficiencies of an immature stage of reasoning or simply to a knowledge deficiency which leads to a misjudgment of the nature and effects of the conservation transformations (cf., Smedslund, 1961) or to some interaction of logic and knowledge which presumably accounts for the nonconservation décalages in the operational period is an important question in developmental psychology. A conservation problem which is very difficult for adults and relatively easy for second and third graders, for example, is uniquely related to this issue because it would be difficult to attribute adults' nonconservation and young childrens' conservation to differences in operational competence. Such a conservation problem, called the Mix 2-jar problem in this study, was developmentally examined with the intention of replicating its developmental status (Murray, Note 1; Odom, Astor & Cunningham, Note 2) and of determining the source of the adults' nonconservation error.

The equivalence conservation paradigm requires the subject initially to acknowledge an equivalence between a property of two objects. After one or both objects is transformed in some irrelevant fashion, the subject is questioned about whether the initial equivalence was preserved despite the transformation. In the main problem of this study, subjects were told that there were the same number of red beads in one jar as blue beads in another. The transformation was simply to remove six red beads, mix them in with the blue beads, and return any six beads from the mixture to the jar with the red beads. The question was whether there were as many red beads in one jar as blue beads in the other jar; that is whether the numerical equivalence was preserved despite the transformation of moving beads from one place to another. The problem differs from the usual conservation problem in two ways: (a) while the numerical equivalence is preserved, the absolute number of red beads, for example, in one jar does change after the transformation as does the number of blue beads in the other jar, and (b) unlike many other conservation problems in which conservation conclusion follows from a principle in an empirical discipline, the conclusion in this problem follows from a set of logical-mathematical relationships.[2] With respect to (a), there are conservation procedures which are conserved by young children, in which the absolute magnitude of the property of the object changes equally in the objects so that the initial equivalence of the property is preserved after the transformation (e.g., Winer, 1968, in which the same number of objects are added to each row before one row is transformed).

Murray (Note 1) found that college students generally fail to conserve numerical equivalence in the Mix 2-jar problem, which is an error since the number of red beads (or blue) in one jar always equals the number of blue beads (or red) in the other jar (see proof in footnote 2). Odom, et al. (Note 2) confirmed the finding, but found surprisingly that third graders performed at a much higher level on the problem than college students, who seemed not to understand the problem even in the few instances in which they were correct. They attributed this remarkable result to the presumed differential salience between third graders and college students of the irrelevant information in the problem, namely the mixing of the beads.

In the present study, performance on the Mix 2-jar problem was sampled across more age groups (viz., 2, 3, 5, 7, 9, 11 graders and college students) to determine whether the younger child's performance was atypical or part of a systematic trend. Also, in an attempt to isolate the aspects of the problem which inhibited or facilitated performance, a number of control conservation and probability problems were presented and a follow up question was introduced to determine whether the subject understood that the outcome was, or was not, necessary. Specifically, subjects were asked whether the outcome was indeterminate (i.e., you can't tell whether it will be equal or not) and whether, if the transformation were repeated a number of times, the outcome would always, sometimes, or never be equal. It was speculated that poor and differential performance on the problem could be due to a number of age related factors such as (a) inferential deficiency of preoperational thought, (b) misjudgment of the transformation as a probabilistic one which precluded consideration of the outcome as determined and necessary, (c) focus of attention, or centration, on only the number of beads leaving the second jar and not as well on the number left behind in that jar and (d) simple information processing and retention overload. Various problems were constructed and presented to treat each of these alternatives. A classic conservation of number task and its counterpart in the two jar case (viz, the No mix 6 problem) were addressed to alternative (a). To speak to the second alternative, two probability tasks (Probability and Mix 3-jar problems) were presented along with a question format on all problems in which subjects had to respond whether the result on repeated transformations would always, sometimes, or never be equal. The centration factor, and also the salience of the mixing factor, was expected to be isolated in the No mix (4/2) problem in which nearly all aspects of the problem except the mixing were preserved. The final alternative was addressed by the Mix 3-jar problem which presumably made heavier processing demands upon the subjects than the other problems.

Method

Subjects

The 188 subjects for this study came from public schools in Delaware (20 second and 24 third graders from the Pleasantville Elementary School, 29 fifth graders and 30 seventh graders from the Gunning Bedford Middle School, 27 ninth graders from Stanton Junior High School, 18 eleventh graders from the John Dickinson High School, and 40 college students enrolled in EDF 410, educational psychology, at the University of Delaware. The mean age in years of subjects in each grade were 7.61 (SD = .48), 8.08 (SD = .58), 10.38 (SD = .56), 12.50 (SD = .73), 14.04 (SD = .19), 16.22 (SD = .43) and 20.53 (SD = 1.85) respectively.

Materials

The materials for the conservation of number task consisted of a 20" x 30" x ½" plywood board painted white with two rows of finishing nails one above the other 6 inches apart. The bottom row had two extra nails farther out to each side than the first row. Two rows of five differently colored plastic donuts were initially hung in one-to-one correspondence on the board in a random arrangement of colors. The extra nails on the bottom row allowed the experimenter to extend the second row of donuts at both ends during the presentation.

For the other tasks, the following materials were employed: three 7" x 4½" diameter, clear glass Libbey storage containers; 50 green, 50 blue and 50 red, 1" diameter styrofoam balls. Each jar initially contained 50 balls of one color and was identified as the red, blue, or green bead jar by the attachment of a piece of ribbon around the neck of the jar to match the color of the beads inside.

An answer booklet was provided for each student with the answers for each task on a separate page. Each task had the same set of possible answers consisting of two columns of large printed symbols. In the first column were the symbols "=", "≠", and "?" and in the second column were "A", "S", and "N", representing respectively, "the same number", "not the same number", "don't know whether they are the same or not", "always", "sometimes", and "never".

Procedure

Subjects responded in fixed order to a simple probability reasoning task and five conservation tasks. One of these was a traditional conservation of number task while the other four were conservation of equivalence tasks which differed in the types of transformation performed on the arrangement of beads in jars which initially contained equal numbers of beads (jar 1, red beads; jar 2, blue beads; jar 3, green beads). To determine subjects' judgments of the effects of the transformation and also to determine whether subjects appreciated whether the result of the transformation was necessary or merely probable they were asked two questions: Question 1, whether there were the same number of red beads in the red bead jar as blue beads in the blue bead jar (and or green beads in the green bead jar), or a different number, or whether they didn't know one way or the other; and Question 2, if, after the transformation was performed many times the number of red beads in the red bead jar would always, sometimes, or never equal the number of blue beads in the blue bead jar (etc.). Subjects were instructed to circle their responses in the answer booklet provided. A description of the transformations and correct answers in each problem follow in the order they were presented to the subjects:

1. Conservation of number. After the subjects were shown two rows of plastic donuts hung in one-to-one correspondence, one above the other, the experimenter spread the bottom row of donuts apart using the extra nails provided. (Answer: number of donuts in the bottom row is always equal to the number of donuts in the top row.)

2. Probability. An equal number of red and blue beads were put into one jar. Subjects were then directed to imagine that any six beads were taken out and placed on the table. Subjects were asked (Question 1) whether the number of red beads taken out would equal the number of blue beads taken out, etc., and (Question 2) whether on repeated trials they would be always, sometimes, or never equal. (Answer: number of red beads would sometimes be equal to the number of blue beads taken out.)

3. No mix (6). After six red beads were taken from jar 1 and placed in jar 2, subjects were directed to imagine their return to jar 1. (Answer: number of red beads in jar 1 is always equal to the number of blue beads in jar 2).

4. No mix (4/2). After six red beads were taken from jar 1 and placed in jar 2, subjects were directed to imagine that two red beads and four blue beads were returned to jar 1. (Answer: number of red beads in jar 1 is always equal to the number of blue beads in jar 2.)

5. Mix 2-jar. After six red beads were taken from jar 1 and mixed with the blue beads in jar 2, subjects were directed to imagine that the experimenter without looking took six beads from jar 2 and put them in jar 1. (Answer: number of red beads in jar 1 is always equal to the number of blue beads in jar 2.)

6. Mix 3-jar. Six red beads were taken from jar 1 and mixed in jar 2 and six green beads were taken from jar 3 and also mixed in jar 2. Subjects were then directed to imagine that without looking the experimenter took six beads from jar 2 and put them in jar 1 and took another six beads from jar 2 and put them in jar 3. (Answer: number of red beads in jar 1, number of blue beads in jar 2, and number of green beads in jar 3 are sometimes equal.)

In addition to these six problems, the second and third graders were given a warm-up problem to instruct them in the use of the answer booklets. An assistant helped the experimenter to ensure that subjects were marking the correct column of their answer booklets. The assistance of the regular classroom teacher was also employed for the second and third grades. All subjects were told by the experimenter that we were trying to see how people of different ages think about the same problems. They were assured we were not administering "an intelligence test" (for younger children--"a test to see how smart they were") and that their performance on the problems would not affect their grades.

Results

The Spearman rank order correlations between grade and the number correct on both questions were significant and positive for all but the Mix 2-jar problem, which correlated negatively with grade. The *rho* correlations were for each problem: conservation of number, .91 ($p < .01$); No mix (6), .82 ($p < .05$); No mix (4/2), .75 ($p < .05$); Probability, .71 ($p < .05$); Mix 3 jar, .71 ($p < .05$); and Mix 2 jar, -.71 ($p < .05$). The differences in the proprotions of conservers and nonconservers on the Mix 2 jar problem between grades 2, 3, 5 and grades 9, 11 and college indicated that significantly more nonconservers and fewer conservers were older subjects (Question 1, χ^2 (1) = 10.03, $p < .001$; Question 2, χ^2 (1) = 24.99, $p < .001$).

The proportions of subjects at various grade levels who were correct on both questions on each problem are presented in Table 1. Only the responses on question 2 from the older subjects (grades 9, 11 and college)

Insert Table 1 About Here

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qualified as a Guttman scale (coefficient of reproducibility = .96, scalability .65) with the following order from easiest to most difficult: Number (100%)

No mix 6 (100%), Probability (99%), Mix 3-jar (81%), No mix 4/2 (67%), Mix 2-jar (14%). The general orders of difficulty for the other grade level groups and criteria did not form acceptable scales.

A factor analysis of all subjects' responses to the six tasks revealed the same two main factors whether responses to Question 1 or Question 2 were analyzed. No factor solution was possible, however, for the performance of the second and third graders alone (sphericity test, χ^2 (15) = 21.71, $p > .05$).

The principal factors with roots greater than 1.0 were extracted via a common factors analysis of the reduced product-moment correlation matrix where the diagonal contains communality estimates which are the squared multiple correlation coefficients. The rotated factor structure obtained by a varimax procedure is given in Table 2. A minimum value of .30 was used to select salient factor loadings for interpretation of the factors as suggested by Gorsuch (1974). All of the salient factor loadings were high: all but one was above .50.

Insert Table 2 About Here

The conservation of number task and the two No mix tasks loaded on the first factor while the probability problem, the Mix 2-jar and the Mix 3-jar tasks loaded on the second factor. All the problems loading on the first factor have necessary outcomes, and, accordingly, the factor was labeled Necessity. The second factor appears to be a Probability factor in the sense that two out of the three tasks have outcomes which are in fact probable and the third task, the Mix 2-jar, may have been treated as a probability problem by many of the subjects, particularly the older ones.

When a criterion of both questions correct was used for each task (0,1), a factor analysis by the procedure above yielded three factors: number, No mix 6, No mix 4/2 loaded on factor 1; probability and the Mix 3 jar loaded on factor 2; and Mix 2 jar and No mix 4/2 loaded on Factor 3. It is important to note that the second factor analysis is based upon the variable of correct and incorrect performance whereas the first analysis was based directly upon subjects' performances regardless of correctness.

Discussion

The general finding of Odom et al. (Note 2) was replicated and extended to indicate a curvilinear developmental trend for the Mix 2-jar problem. However, performance was never sufficiently high to indicate that the problem was fully understood by the subjects although the highest performance, as Odom et al. also found, was by the third graders (46% correct on questions 1 and 2). Nevertheless the relatively superior performance of the young subjects may be best explained by considering the factors which depressed the performance of the older subjects.

It is clear that the older subject's poor performance cannot be attributed to nonoperativity in its usual form, since their performance on the No Mix (6) and number conservation problems was perfect both in their judgment and their appreciation of the necessity of their judgment. This was not the case, incidentally, for second and third graders who scored significantly poorer on the second question than on the first on the No mix (6) problem (.77 vs. .50 correct) and conservation of number (.98 vs. .64 correct).

There is considerable support for the view that the poor performance of the older subjects should be attributed to their misjudgment of the Mix 2-jar transformation as a probability manipulation. This view is based partly upon their superior performance on the probability problem, although the fifth and seventh graders who have the lowest probability score (.08) also have low Mix 2-jar performance (.19). The factor analysis of the responses on question 2 provides better evidence of this view since performance on the Mix 2-jar problem loaded on a general probability factor. Also, when only the mixing feature was removed from the problem, as it was in the No mix (4/2) problem, performance by older subjects at all ages is at a reasonably high level. That the older subjects were treating the Mix 2 jar problem as a probability problem is further indicated by the high number of "sometimes" responses they gave when they were wrong on question 2 (e.g., college students gave 28 "sometimes" responses out of 32 errors).

However, the level of performance on the No mix (4/2) problem was not so high as to rule out a general centration factor as a factor which contributes to the poor performance of older subjects (cf., Case, 1975). Subjects, even without misjudging the problem as a probability problem, seem to fail to attend to what is left behind in jar 2, although they are presumably capable of simultaneously attending to what leaves and is left behind in jar 2. If they were not, the failure would indicate nonoperativity.

In a similar sense, a sign of preoperativity is the failure of the young child to recognize that if he and a friend have equal amounts of candy and his friend gives him two pieces, the child will not have just two more pieces than his friend, but four more pieces than his friend (Piaget, Note 3). While adults may, and do, make the same error, the point is that they presumably can be easily shown the error, and this ease of instruction provides evidence for the presumed operative competence (cf., Hornblum & Overton, in press). That the competence represented by success on the No mix 4/2 problem in this account is necessary for the correct performance of older subjects on the Mix 2-jar problem is supported by the Guttman scale and loadings on the third factor of the second factor analysis, which was an analysis essentially of the performance of those who were correct on the problems. Of course, why the college subjects made errors on the No mix (4/2) problem is a problem in its own right since there does not appear to be any salient irrelevant information in the problem to override or depress competence.

In sum, the older subjects have the minimal conservation competence (Number and No mix 6) to solve the Mix 2-jar problem, and it appears as well that they have the additional competence manifested in the No mix 4/2 problem, which if the Mix 2-jar transformation were not misjudged, would be adequate to insure a high level of correct performance on the Mix 2-jar problem. Even if a subject's performance on the No mix (4/2) problem were perfect, one could expect somewhat lower performance on the Mix 2-jar problem because the subject would need to generalize the 4/2 response to the other possibilities which might exist in the Mix 2-jar problem (viz., 5/1, 3/3, 2/4, 1/5, 0/6, etc.).

While this account may provide a plausible explanation of the competence-performance factors that conspire to depress the older subjects' performance on the Mix 2-jar problem, the youngest subjects' performance is not well explained by it. One major difficulty with this account for the youngest subjects is that their Mix 2-jar performance is considerably higher than their No Mix (4/2) performance at a time when their probability performance is low. This is particularly puzzling since the No Mix (4/2) problem is, after all, a special case of the Mix 2-jar problem. If their number conservation and No Mix 6 performance were to be taken as indicators of adequate competence for the Mix 2-jar, why aren't they adequate <u>a fortiori</u> for the No Mix 4/2 problem? While this is not a problem for an account of fifth and seventh graders' performance, their very low probability task performance is a major problem, if the poor Mix 2-jar problem performance is to be attributed largely to a misjudgment of the Mix 2-jar transformation as a probability manipulation.

Perhaps the most plausible explanation of the youngest subjects' performance is that they understand very little of the tasks. For example, for the second and third graders the Spearman rank order correlation on percentages of correct responses to each problem between question 1 and question 2 was -.17 ($p > .05$). Moreover, the significant differences in the proportions of correct and incorrect responses on the number conservation problem between questions 1 and 2 (McNemar binomial, $p < .02$ grade 2, and $p < .01$ grade 3) indicates that the necessity aspect of the conservation judgment was not felt, or that they were unsure of their responses, or that they had adopted the conservative position and said "sometimes" because anything that is always true is also sometimes true. Still, more than half the subjects did appear to appreciate the necessity of their judgments. While the youngest subjects may not understand some aspects of the tasks, it was not the case that they did not understand the difference between always and sometimes. Subjects know quite well that, for example, "mommies are always and not sometimes ladies," and that "it rains sometimes and not always," etc.(Armstrong & Murray, Note 4).

In the end it is simply not clear why the youngest subjects performed so well on the Mix 2-jar problem in this sample and in the Odom et al. (Note 2) sample, but the absolute level of their performance in this case and the inconsistencies between responses to questions 1 and 2, and between the control problems do not make it likely that the performance was based upon a higher stage of reasoning than is commonly found in children of this age.

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Footnotes

[1]The assistance of David Bradley at the University of Delaware and the cooperation of the students and staff at the Pleasantville Elementary School, Gunning Bedford Middle School, Stanton Junior High School, John Dickinson High School, and College of Education, University of Delaware, are greatly appreciated.

[2]An algebraic proof for the Mix 2-jar problem in which n = number of beads in each jar initially, x and y a particular number of beads moved from one jar to another, R = red beads, and B = blue beads follows. Given: $nR = nB$; then first part of transformation yields $nR - xR = nB + xR$; then the second part yields $nR - xR + yB + (x-y)R = nB + xR = yB - (x-y)R$. The result of collapsing and simplifying terms from the second part yields $(n-y)R + yB = (n-y)B + yR$.

Table 1

Proportions of Subjects Correct on Both Questions

on Each Problem by Grade Level (N = 188)

		Problems				
Grade	Number	No Mix (6)	No Mix (4/2)	Probability	Mix 3-jar	Mix 2-jar
Second & Third	.61	.50	.18	.18	.16	.36
Fifth & Seventh	.97	.98	.51	.08	.05	.19
Ninth & Eleventh	1.00	1.00	.49	.60	.49	.04
College	1.00	1.00	.78	.7	.60	.23

Table 2

Factor Pattern Matrices of Responses to Question 2 and to Questions 1 and 2 for All Subjects for All Problems With Common Factors Analysis, Varimax Rotation (BMD03M)

	Question 2		Question 1 and 2		
	Factor 1	Factor 2	Factor 1	Factor 2	Factor 3
Problem	Necessity	Probability	Necessity	Probability	Decentration
Number	0.61*	-0.22	0.65*		
No mix (6)	0.72*	-0.00	0.66*		
No mix (4/2)	0.55*	0.11	0.37*		0.33*
Probability	-0.07	0.39*		0.63*	
Mix 3-jar	0.03	0.67*		0.65*	
Mix 2-jar	0.02	0.70*			0.46*

*Salient factor loadings >.30 n = 188.